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KIDDER-PARKER
ARCHITECTS' AND BUILDERS' HANDBOOK

KIDDER-PARKER
ARCHITECTS' AND BUILDERS' HANDBOOK

DATA FOR
ARCHITECTS, STRUCTURAL ENGINEERS,
CONTRACTORS, AND DRAUGHTSMEN

BY

THE LATE FRANK E. KIDDER, C.E., PH.D.
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COMPILED BY A STAFF OF SPECIALISTS AND
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EIGHTEENTH EDITION
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The Publishers and the Editor-in-Chief will be grateful to readers of this volume who will call attention to any errors of omission or commission therein. It is intended to make our publications standard works of study and reference, and, to that end, the greatest accuracy is sought. It rarely happens that the early editions of books are free from errors; but it is the endeavor of the Publishers to have them removed, and it is therefore desired that the Editor-in-Chief may be aided in his task of revision, from time to time, by the kindly criticism of readers.

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This Book

IS RESPECTFULLY DEDICATED TO THOSE WHOSE KINDNESS
HAS ENABLED ME TO PRODUCE IT

TO MY PARENTS

WHO GAVE ME THE EDUCATION UPON WHICH IT IS BASED

TO MY WIFE

FOR HER LOVING SYMPATHY, ENCOURAGEMENT
AND ASSISTANCE

TO ORLANDO W. NORCROSS

OF WORCESTER, MASS.

WHOSE SUPERIOR PRACTICAL KNOWLEDGE OF ALL THAT
PERTAINS TO BUILDING HAS GIVEN ME A MORE
INTELLIGENT AND PRACTICAL VIEW OF THE
SCIENCE OF CONSTRUCTION THAN I
SHOULD OTHERWISE HAVE
OBTAINED *

* Dedication to First Edition, by the late Frank E. Kidder.

EDITORIAL STAFF

EDITOR-IN-CHIEF

ASSOCIATE EDITORS

PREFACE TO EIGHTEENTH EDITION

The last complete revision of the Handbook was the sixteenth edition published in 1915. The seventeenth edition, 1921, contained certain revised chapters, but much of the former edition was retained intact.

Improved methods of construction, new materials, modification of building codes and the acceptance by architects and engineers of new unit-stresses, have necessitated another complete revision. Much material of the former edition has been omitted; many chapters have been revised or rewritten, and certain new chapters have been added. Throughout the book will be found various materials and methods which are not in general use, but investigation has shown a great diversification in the practice of construction in different localities. Consequently, it was deemed advisable to retain certain forms of construction which, although not generally used, are still standard methods in certain sections of the country.

Inasmuch as this work has been a standard for so many years, its original arrangement has been retained. Though primarily a reference book, it has also been used by colleges and technical schools as a textbook. For this reason, certain chapters have been so arranged that their use in the classroom will be facilitated. A new Index has been prepared, and it is to be hoped that its new arrangement will materially aid in the use of the Handbook in the drafting room for reference work.

Although practically all the chapters have been revised to conform with the latest research and accepted practice, the following subjects have been given particular attention and revised accordingly: Foundations; Masonry Walls, Footings, Cements and Concretes; Resistance to Tension, Properties of Iron and Steel; Resistance to Shear, Riveted Joints, Pins and Bolts; Bearing-Plates and Bases for Columns, Beams and Girders; Strength of Built-up, Flitched and Trussed Wooden Girders; Specifications for Structural Steel-Work of Buildings, Data on Structural Steel; Heating and Ventilation of Buildings; Hydraulics, Plumbing and Drainage, Gas and Gas Piping; Illumination of Buildings; and Electric Wiring.

In addition to this work of revision, the material on the following topics has been rewritten: Strength of Bricks, Stone, Mass Concrete and Masonry; Properties of Structural Shapes, Moment of Inertia, Moment of Resistance, Section-Modulus and Radius of Gyration; Strength of Columns, Posts and Struts; Strength of Steel Beams and Beam Girders; Stiffness and Deflection of Simple and Cantilever Beams; Strength and Stiffness of Restrained, Fixed and Continuous Beams; Riveted Steel Plate and Box Girders; Wood Framing; Fire-proofing of Buildings; Reinforced-Concrete Construction; Types of Roof-Trusses; Stresses in Roof-Trusses; Design and Construction of Roof-Trusses; and Acoustics of Buildings. New chapters on Elevator Service in Buildings and Architectural Shades and Shadows have been added.

The many subjects treated in Part III have been carefully revised and several new subjects added. Among these are: Copper Roofs; Estimating Costs of Buildings; Dimensions of Athletic Fields and Courts; the Roman Alphabet; Lightning Conductors; Wind Stresses in Tall Buildings; Documents of The American Institute of Architects; and Terms Defined in Building Codes.

For many years the name of Thomas Nolan has been associated with the Handbook. His decease in 1926 was a severe loss to the profession and his many friends. The writer, who for many years enjoyed his friendship and close association, is deeply indebted to him for his superior knowledge and helpfulness.

The Editor-in-Chief wishes to acknowledge with appreciation the cooperation of the Associate Editors, who have so willingly given of their time in the preparation of their contributions. In addition to them, thanks are due to the architects and manufacturers of building materials who have so graciously submitted data; to Mr. Bradley P. Kidder; and to the publishers, whose valuable suggestions and cooperation have been so helpful in the preparation of material for the new book. It is to be hoped that in its new form it will be found even more useful than before, and that it will continue to occupy its position as a standard reference work on Building Construction.

HARRY PARKER.

PHILADELPHIA, September, 1931.

PREFACE TO CORRECTED PRINTING OF THE EIGHTEENTH EDITION

SINCE the publication of the Eighteenth Edition of the Architects' and Builders' Handbook, business conditions throughout the country have retarded the building industries. The consequence of this has been a lack of development in materials and methods of construction which might normally be expected. Many materials have not been used sufficiently to prove their true value.

The second printing of the Architects' and Builders' Handbook contains minor corrections throughout the entire book. The principal changes have been made in Chapter XXX, Heating and Ventilation of Buildings. Since the first printing, the American Society of Heating and Ventilating Engineers has standardized heat-transmission tables. These new standards have necessitated a revision of obsolete tables, and other omissions and additions have been made to bring the chapter into conformity with present-day practice.

HARRY PARKER.

PHILADELPHIA, September, 1935.

PREFACE TO SIXTEENTH EDITION

The changes in the fifteenth edition, published in 1908, consisted principally of the rewriting of the two chapters on Fireproofing of Buildings and Reinforced Concrete.

In 1912 the undersigned was asked to undertake the revision of the entire book with the cooperation of a corps of Associate Editors, each highly qualified to render the necessary assistance in matters pertaining to his own work. On account of the comprehensive nature of the contents of the Pocket-Book, the many recent changes and rapid developments in different fields of architectural construction, and the consequent effect of such changes on the interrelated subjects treated, the Editor-in-Chief decided to rewrite and reset the entire book. After more than three years of arduous labor, in which the Associate Editors and many other contributors have most ably and generously assisted, the New Kidder is about to be published.

It was decided to retain Mr. Kidder's original arrangement of the subject-matter which is divided into three Parts, Part I dealing with practical applications of Arithmetic, Geometry and Trigonometry, Part II with the Materials of Construction and the Strength and Stability of Structures, and Part III with miscellaneous useful information for architects and builders. Each of the twenty-nine chapters of Part II, however, has the name of the Associate Editor who revised or rewrote it printed with the chapter-caption. Part I has been carefully checked and much of the matter rearranged. The twenty-eight chapters of Part II have been rewritten and one new chapter has been added on Reinforced-Concrete Mill and Factory-Construction. Part III has been largely rewritten and all subjects retained have been thoroughly revised. To this part, also, much new matter has been added, such as extended tables of Specific Gravities and Weights of Substances, Architectural Acoustics, Waterproofing for Foundations, the Quantity System of Estimating, the Standard Documents of the American Institute of Architects, Educational Societies of the World and extended lists of Architectural Schools, Books and Periodicals.

The Editor-in-Chief has, with very few exceptions, personally checked on every page of manuscript, galley-proof and page-proof the equations, formulas, computations and problems, and has read or examined carefully every word, figure and illustration, every detail of syntax, paragraphing, punctuation and typography, and every arrangement of tables, captions, classifications, notation, Table of Contents and Index.

He is responsible for many changes in the form of presentation of data which it is hoped will add to the Pocket-Book still more of that efficiency and practical helpfulness for which it has been so long noted. Some of these changes may be briefly mentioned. The text has been entirely reset; the type, while slightly smaller, is clearer and has the lines and paragraphs separated by wide leads; a special type is used for the tables; the paragraphing is revised throughout and every paragraph has a black-face type caption descriptive of the subject-matter; words in italics or with quotation-marks are seldom used, words in small caps taking their place; every chapter is divided into numbered chapter-subdivisions which are briefly descriptive of the classified matter; the number of cross-

references is largely increased and the page-numbers of such references are almost always added; many tables and diagrams which in the former editions read lengthwise of the page have been reset or reengraved to read across the page for greater convenience; the number of illustrations has been largely increased, many old cuts reused have been reengraved, and some diagrams printed with lines of different colors to make the demonstrations clearer; a descriptive caption has been added to every illustration; the abbreviations Chap. I, Chap. II, etc., have been printed with each page-caption of the left-hand pages, thus avoiding the necessity of referring to the Table of Contents to locate any particular chapter.

The Editor-in-Chief decided to change some of the unit stresses, especially those for the different woods, and in some cases to recommend more conservative values, and he believes that results based upon such stresses conform to the best engineering practice. This change necessitated the revision of many tables and problems throughout the book which had to be entirely recalculated. Numerous practical problems with complete solutions have been added. The derivation of many of the formulas used has been explained, either in the body of the text or in extended foot-notes, for those who wish to understand as well as to use such formulas, and numerous cross-references accompanying them enable the reader to use the Pocket-Book as a textbook for certain parts of the mechanics of materials as well as a handbook for office work. The tables of the properties of structural shapes, of safe loads for columns, beams and girders, etc., have been revised and numerous new tables added. The Editor has found that it is the consensus of opinion among architects that the insertion of these tables is a great convenience and for their ordinary office work condenses into one handy volume much of the essential data of several manufacturers' handbooks.

The difficulty of securing a unity of treatment and of avoiding repetitions and contradictions in a book of reference the data of which cover so many subjects and is written by so many contributors has been fully realized; but it is believed that in these respects the New Kidder is reasonably successful and will meet with the approval of all who use it.

Acknowledgments and thanks are due the Associate Editors for their hearty cooperation and generous contributions of the time and labor taken from their professional work. Acknowledgment is made, also, of the valuable assistance of all others who have furnished new or revised old data, and of many helpful suggestions from Mrs. F. E. Kidder and from the publishers.

The Editor-in-Chief expresses the hope that for the architects and builders of this country the new Pocket-Book will continue to be, as Mr. Kidder expressed it in his preface to the first edition in 1884, "a compendium of practical facts, rules and tables presented in a form as convenient for application as possible, and as reliable as our present knowledge will permit"; and also, in its present extension and fuller development, a work which will lead to a still clearer understanding of the essential principles of sound architectural construction.

THOMAS NOLAN.

PHILADELPHIA, September, 1915.

PREFACE TO FOURTEENTH EDITION

It is now nearly twenty years since the author, then quite a young man, completed the first edition of this work, which, although containing but 586 pages, had required about three years for its preparation. At that time the author thought he had covered all of those practical details relating to the planning and construction of buildings, with which the architect was concerned, tolerably well, and it would appear as though the purchasers of the book thought so too, but as the years have come and gone, so many and such great improvements have taken place in the building world, so many articles invented, new methods of construction developed, higher standards established, that the present edition, although containing nearly three times as many pages, is perhaps not more complete, for the times, than was the first edition.

When preparing the first edition, it was the aim of the author to give to architects and builders a handbook which should be, in its field, as useful and reliable as Trautwine's had been to civil engineers; and with that object constantly in view, the book has been revised from time to time to meet the changed conditions in building construction and equipment.

About three years ago it was thought, by the publishers and the author, that a thorough and complete revision of the book should be undertaken, and although the re-writing of a work of this character, even with the thirteenth edition to work from, involved many months of close and constant application, the utilization of those hours which one ordinarily takes for recreation, and at the best more or less interruption to his regular business, and consequent reduction in income, the writer undertook to prepare a work of a still wider scope, and which should be thoroughly up-to-date in every particular, or at least as far as is practicable, in a work requiring a period of three years in its preparation, and from that time to this he has spared no labor or expense to make the book as useful and complete as he possibly could, without making it too bulky.

In this revision the author has had in view:

1st. A reference-book which should contain some information on every subject (except design) likely to come before an architect, structural engineer, draughtsman, or master-builder, including data for estimating the approximate cost.

2d. To as thoroughly cover the subject of architectural engineering as is practicable in a handbook.

3d. To present all information in as simple and convenient a form for immediate application as is consistent with accuracy. To this end a great many new tables, arranged and computed by the author, have been inserted.

At the time the first edition was written, the term "Architectural Engineering" had not been used in its present application, and the term "Structural Engineering," when used, referred almost exclusively to bridge work.

To-day, structural and architectural engineers are concerned almost exclusively with building construction, and their work is more closely allied to that of the architect than to that of the civil engineer; hence the author has had in mind the needs of the structural engineer and draughtsman as well as those of the architect and builder, and the book should be of nearly equal value to both.

Where it was impossible, for lack of space, to go extensively into any subject, references to other books or sources of information have been given, so that in this way the book may serve as a general index to the many lines of work, materials, and manufactured products entering into the planning, construction, and equipment of buildings.

To attain the objects in view, it has been necessary to add considerably to the number of pages, but as experience has shown that the book is used principally at the desk or draughting-table, and is seldom carried in the pocket, it is believed that the convenience of having everything in one book will more than offset any disadvantage resulting from increase in bulk.

Nearly the entire book has been re-written, and great pains have been taken to furnish reliable data. A large number of experts in various lines have assisted the author, as is manifest by the foot-notes and references. To all of such, and to the many authors of technical works, and to the publishers of technical journals, who have kindly consented to the use of cuts and data, the author takes pleasure in acknowledging his indebtedness. Also to Mr. E. S. Hand, of New York, who, for many years, has rendered material assistance in collecting data along the line of manufactured products.

The names and addresses of manufacturers have been given solely for the convenience of the users of the book, and not for any pecuniary considerations; in fact, if money considerations had solely appealed to the writer, this book would never have been re-written, because a technical work of this character can never adequately compensate, in money, for the time, labor, and thought required in its preparation. The many words of appreciation which have come to the author from hundreds of those who have found the book useful have been a great stimulus to further increase its usefulness.

As in the former prefaces, the author requests that any one discovering errors in the work or who may have any suggestions looking to the further improvement of the book, will communicate the same to him, that the book may be made as complete and reliable as possible.

Finally, the author desires to acknowledge his indebtedness to the publishers, who have heartily seconded his efforts in every particular, and who have spared no pains or expense to make a perfect handbook.

F. E. KIDDER.

DENVER, COLO., July 18th, 1904.

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PART I

PRACTICAL
ARITHMETIC, GEOMETRY AND TRIGONOMETRY

RULES, TABLES AND PROBLEMS

1. PRACTICAL ARITHMETIC

Mathematical Signs and Characters *

The following signs and characters are generally used to denote and abbreviate the several mathematical operations:

The sign = means equal to, or equality;
− means minus or less, or subtraction;
+ means plus, or addition;
× means multiplied by, or multiplication;
÷ or / means divided by, or division;
^{2} ^{3} are indexes or powers, meaning that the number to which they are added is to be squared (^{2}) or cubed (^{3});
: is to, :: so is, : to — are signs of proportion;
√ is the RADICAL SIGN and means that the square root of the number before which it is placed is to be extracted;
$\sqrt[3]{}$ means that the cube root of the number before which it is placed is to be extracted;
—— the BAR indicates that all the numbers under it are to be taken together;
( ) the PARENTHESIS means that all the numbers between are to be taken as one quantity;
. means decimal parts; thus, 2.5 means $2\frac{5}{10}$, 0.46 means $\frac{46}{100}$.
° means degrees, ′ minutes and ″ seconds;
∴ means hence;
′ means feet;
″ means inches.

Involution

To Square a Number, multiply the number by itself, and the product will be the square; thus, the square of $18 = 18^2 = 18 \times 18 = 324$.

The Cube of a Number is the product obtained by multiplying the number by itself, and that product by the number again; thus, the cube of $14 = 14^3 = 14 \times 14 \times 14 = 2\,744$.

The Fourth Power of a Number is the product obtained by multiplying the number by itself four times; thus, the fourth power of $10 = 10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$.

Evolution

Square Root. Rule for extracting the square root of a number:

(1) Divide the given number into periods of two figures each, commencing at the right if it is a whole number, and at the decimal point if there are decimals; thus, 10236.8126.

(2) Find the largest square in the left-hand period, and place its root in the quotient; subtract the said square from the left-hand period, and to the remainder bring down the next period for a new dividend.

(3) Double the root already found, and annex one cipher for a trial-divisor; see how many times it will go in the dividend, and put the number in the quotient and also in place of the cipher in the divisor. Multiply this final

* See, also, Introduction to Part II.

divisor by the number in the quotient just found, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend and proceed as before. If it should be found that the trial divisor cannot be contained in the dividend, bring down the next period for a new dividend, annex another cipher to the trial divisor, put a cipher in the quotient and proceed as before.

Example. $\dot{1}0\dot{2}3\dot{6}.8\dot{1}2\dot{6}$ (101.17, the square root

	1
201)	0236
	201
2021)	3581
	2021
20227)	156026
	141589
	14437

Cube Root. To extract the cube root of a number, point off the number from right to left into periods of three figures each, and, if there is a decimal, commence at the decimal point and point off into periods, going both ways.

Ascertain the highest root of the first period, and place it to the right of the number, as in long division; cube the root thus found and subtract from the first period; to the remainder annex the next period; square the root already found, multiply by three and annex two ciphers for the trial divisor. Find how many times this trial divisor is contained in the dividend and write the result in the root.

Add together the trial divisor, three times the product of the first figure of the root by the second with one cipher annexed, and the square of the second figure in the root; multiply the sum by the last figure in the root, and subtract from the dividend; to the remainder annex the next period and proceed as before.

When the trial divisor is greater than the dividend, write a cipher in the root, annex the next period to the dividend and proceed as before.

Example. Required, the cube root of 493039 or $\sqrt[3]{493039}$

		$49\dot{3}03\dot{9}$(79, the cube root
$7 \times 7 \times 7 =$		343
$7 \times 7 \times 3 =$	14700	150039
$7 \times 9 \times 3 =$	1890	
$9 \times 9 =$	81	
	16671	150039

Example. Required, the cube root of 403583.419 or $\sqrt[3]{403583.419}$

		$40\dot{3}58\dot{3}.41\dot{9}$(73.9, the cube root
$7 \times 7 \times 7 =$		343
$7 \times 7 \times 3 =$	14700	60583
$7 \times 3 \times 3 =$	630	
$3 \times 3 =$	9	
	15339	46017
$73 \times 73 \times 3 =$	1598700	14566419
$73 \times 9 \times 3 =$	19710	
$9 \times 9 =$	81	
	1618491	14566419

Example. Required, the cube root of 158252.632929 or $\sqrt[3]{158252.632929}$

$$
\begin{array}{r|l}
 & 15\dot{8}25\dot{2}.63\dot{2}92\dot{9}(54.09\text{, the cube root} \\
5 \times 5 \times 5 = & 125 \\
\hline
5 \times 5 \times 3 = 7500 & 33252 \\
5 \times 4 \times 3 = 600 & \\
4 \times 4 = \underline{16} & \\
8116 & 32464 \\
\hline
540 \times 540 \times 3 = 87480000 & 788632929 \\
540 \times 9 \times 3 = 145800 & \\
9 \times 9 = \underline{81} & \\
87625881 & \underline{788632929}
\end{array}
$$

TABLES

OF

SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS AND RECIPROCALS

From 1 to 1054

The following table, taken from Searles' Field Engineering, will be found of great convenience in finding the square, cube, square root, cube root and reciprocal of any number from 1 to 1054. The reciprocal of a number is the quotient obtained by dividing 1 by the number. Thus, the reciprocal of 8 is $1 \div 8 = 0.125$.

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
1	1	1	1.0000000	1.0000000	1.000000000
2	4	8	1.4142136	1.2599210	.500000000
3	9	27	1.7320508	1.4422496	.333333333
4	16	64	2.0000000	1.5874011	.250000000
5	25	125	2.2360680	1.7099759	.200000000
6	36	216	2.4494897	1.8171206	.166666667
7	49	343	2.6457513	1.9129312	.142857143
8	64	512	2.8284271	2.0000000	.125000000
9	81	729	3.0000000	2.0800837	.111111111
10	100	1000	3.1622777	2.1544347	.100000000
11	121	1331	3.3166248	2.2239801	.090909091
12	144	1728	3.4641016	2.2894286	.083333333
13	169	2197	3.6055513	2.3513347	.076923077
14	196	2744	3.7416574	2.4101422	.071428571
15	225	3375	3.8729833	2.4662121	.066666667
16	256	4096	4.0000000	2.5198421	.062500000
17	289	4913	4.1231056	2.5712816	.058823529
18	324	5832	4.2426407	2.6207414	.055555556
19	361	6859	4.3588989	2.6684016	.052631579
20	400	8000	4.4721360	2.7144177	.050000000
21	441	9261	4.5825757	2.7589243	.047619048
22	484	10648	4.6904158	2.8020393	.045454545
23	529	12167	4.7958315	2.8438670	.043478261
24	576	13824	4.8989795	2.8844991	.041666667
25	625	15625	5.0000000	2.9240177	.040000000
26	676	17576	5.0990195	2.9624960	.038461538
27	729	19683	5.1961524	3.0000000	.037037037
28	784	21952	5.2915026	3.0365889	.035714286
29	841	24389	5.3851648	3.0723168	.034482759
30	900	27000	5.4772256	3.1072325	.033333333
31	961	29791	5.5677644	3.1413806	.032258065
32	1024	32768	5.6568542	3.1748021	.031250000
33	1089	35937	5.7445626	3.2075343	.030303030
34	1156	39304	5.8309519	3.2396118	.029411765
35	1225	42875	5.9160798	3.2710663	.028571429
36	1296	46656	6.0000000	3.3019272	.027777778
37	1369	50653	6.0827625	3.3322218	.027027027
38	1444	54872	6.1644140	3.3619754	.026315789
39	1521	59319	6.2449980	3.3912114	.025641026
40	1600	64000	6.3245553	3.4199519	.025000000
41	1681	68921	6.4031242	3.4482172	.024390244
42	1764	74088	6.4807407	3.4760266	.023809524
43	1849	79507	6.5574385	3.5033981	.023255814
44	1936	85184	6.6332496	3.5303483	.022727273
45	2025	91125	6.7082039	3.5568933	.022222222
46	2116	97336	6.7823300	3.5830479	.021739130
47	2209	103823	6.8556546	3.6088261	.021276600
48	2304	110592	6.9282032	3.6342411	.020833333
49	2401	117649	7.0000000	3.6593057	.020408163
50	2500	125000	7.0710678	3.6840314	.020000000
51	2601	132651	7.1414284	3.7084298	.019607843
52	2704	140608	7.2111026	3.7325111	.019230769
53	2809	148877	7.2801099	3.7562858	.018867925
54	2916	157464	7.3484692	3.7797631	.018518519
55	3025	166375	7.4161985	3.8029525	.018181818
56	3136	175616	7.4833148	3.8258624	.017857143
57	3249	185193	7.5498344	3.8485011	.017543860
58	3364	195112	7.6157731	3.8708766	.017241379
59	3481	205379	7.6811457	3.8929965	.016949153
60	3600	216000	7.7459667	3.9148676	.016666667
61	3721	226981	7.8102497	3.9364972	.016393443
62	3844	238328	7.8740079	3.9578915	.016129032

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0412401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314432	8.2462113	4.0816551	.014705882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8 4261498	4.1408178	.014084507
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013698630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444815	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2736185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4479602	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778688	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857375	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009708738
104	10816	1124864	10.1980390	4.7026694	.009615385
105	11025	1157625	10.2469508	4.7176940	.009523810
106	11236	1191016	10.2956301	4.7326235	.009433962
107	11449	1225043	10.3440804	4.7474594	.009345794
108	11664	1259712	10.3923048	4.7622032	.009259259
109	11881	1295029	10.4403065	4.7768562	.009174312
110	12100	1331000	10.4880885	4.7914199	.009090909
111	12321	1367631	10.5356538	4.8058955	.009009009
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008849558
114	12996	1481544	10.6770783	4.8488076	.008771930
115	13225	1520875	10.7238053	4.8629442	.008695652
116	13456	1560896	10.7703296	4.8769990	.008620690
117	13689	1601613	10.8166538	4.8909732	.008547009
118	13924	1643032	10.8627805	4.9048681	.008474576
119	14161	1685159	10.9087121	4.9186847	.008403361
120	14400	1728000	10.9544512	4.9324242	.008333333
121	14641	1771561	11.0000000	4.9460874	.008264463
122	14884	1815848	11.0453610	4.9596757	.008196721
123	15129	1860867	11.0905365	4.9731898	.008130081
124	15376	1906624	11.1355287	4.9866310	.008064516

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
125	15625	1953125	11.1803399	5.0000000	.008000000
126	15876	2000376	11.2249722	5.0132979	.007936508
127	16129	2048383	11.2694277	5.0265257	.007874016
128	16384	2097152	11.3137085	5.0396842	.007812500
129	16641	2146689	11.3578167	5.0527743	.007751938
130	16900	2197000	11.4017543	5.0657970	.007692308
131	17161	2248091	11.4455231	5.0787531	.007633588
132	17424	2299968	11.4891253	5.0916434	.007575758
133	17689	2352637	11.5325626	5.1044687	.007518797
134	17956	2406104	11.5758369	5.1172299	.007462687
135	18225	2460375	11.6189500	5.1299278	.007407407
136	18496	2515456	11.6619038	5.1425632	.007352941
137	18769	2571353	11.7046999	5.1551367	.007299270
138	19044	2628072	11.7473401	5.1676493	.007246377
139	19321	2685619	11.7898261	5.1801015	.007194245
140	19600	2744000	11.8321596	5.1924941	.007142857
141	19881	2803221	11.8743421	5.2048279	.007092199
142	20164	2863288	11.9163753	5.2171034	.007042254
143	20449	2924207	11.9582607	5.2293215	.006993007
144	20736	2985984	12 0000000	5.2414828	.006944444
145	21025	3048625	12 0415946	5.2535879	.006896552
146	21316	3112136	12.0830460	5.2656374	.006849315
147	21609	3176523	12.1243557	5.2776321	.006802721
148	21904	3241792	12.1655251	5.2895725	.006756757
149	22201	3307949	12.2065556	5.3014592	.006711409
150	22500	3375000	12.2474487	5.3132928	.006666667
151	22801	3442951	12.2882057	5.3250740	006622517
152	23104	3511808	12.3288280	5.3368033	.006578947
153	23409	3581577	12.3693169	5.3484812	.006535948
154	23716	3652264	12.4096736	5.3601084	.006493506
155	24025	3723875	12.4498996	5.3716854	.006451613
156	24336	3796416	12.4899960	5.3832126	.006410256
157	24649	3869893	12.5299641	5.3946907	.006369427
158	24964	3944312	12.5698051	5.4061202	.006329114
159	25281	4019679	12.6095202	5.4175015	.006289308
160	25600	4096000	12.6491106	5.4288352	.006250000
161	25921	4173281	12.6885775	5.4401218	.006211180
162	26244	4251528	12.7279221	5.4513618	.006172840
163	26569	4330747	12.7671453	5.4625556	.006134969
164	26896	4410944	12.8062485	5.4737037	.006097561
165	27225	4492125	12.8452326	5.4848066	.006060606
166	27556	4574296	12.8840987	5.4958647	.006024096
167	27889	4657463	12.9228480	5.5068784	.005988024
168	28224	4741632	12.9614814	5.5178484	.005952381
169	28561	4826809	13.0000000	5.5287748	.005917160
170	28900	4913000	13.0384048	5.5396583	.005882353
171	29241	5000211	13.0766968	5.5504991	.005847953
172	29584	5088448	13.1148770	5.5612978	.005813953
173	29929	5177717	13.1529464	5.5720546	.005780347
174	30276	5268024	13.1909060	5.5827702	.005747126
175	30625	5359375	13.2287566	5.5934447	.005714286
176	30976	5451776	13.2664992	5.6040787	.005681818
177	31329	5545233	13.3041347	5.6146724	.005649718
178	31684	5639752	13.3416641	5.6252263	.005617978
179	32041	5735339	13.3790882	5.6357408	.005586592
180	32400	5832000	13.4164079	5.6462162	.005555556
181	32761	5929741	13.4536240	5.6566528	.005524862
182	33124	6028568	13.4907376	5.6670511	.005494505
183	33489	6128487	13.5277493	5.6774114	.005464481
184	33856	6229504	13.5646600	5.6877340	.005434783
185	34225	6331625	13.6014705	5.6980192	.005405405
186	34596	6434856	13.6381817	5.7082675	.005376344

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7286543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291005
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189057	13.8924440	5.7789966	.005181347
194	37636	7301384	13.9283883	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356688	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284767	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421356	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674643	.004950495
203	41209	8365427	14.2478068	5.8771307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5 9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907264	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986486	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941048	14.8996644	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464286
225	50625	11390625	15.0000000	6.0822020	.004444444
226	51076	11543176	15.0332964	6.0911994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6 1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919334	6.2144650	.004166667
241	58081	13997521	15.5241747	6.2230843	.004149378
242	58564	14172488	15.5563492	6.2316797	.004132231
243	59049	14348907	15.5884573	6.2402515	.004115226
244	59536	14526784	15.6204994	6.2487998	.004098361
245	60025	14706125	15.6524758	6.2573248	.004081633
246	60516	14886936	15.6843871	6.2658266	.004065041
247	61009	15069223	15.7162336	6.2743054	.004048583
248	61504	15252992	15.7480157	6.2827613	.004032258

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
249	62001	15438249	15.7797338	6.2911946	.004016064
250	62500	15625000	15.8113883	6.2996053	.004000000
251	63001	15813251	15.8429795	6.3079935	.003984064
252	63504	16003008	15.8745079	6.3163596	.003968254
253	64009	16194277	15.9059737	6.3247035	.003952569
254	64516	16387064	15.9373775	6.3330256	.003937008
255	65025	16581375	15.9687194	6.3413257	.003921569
256	65536	16777216	16.0000000	6.3496042	.003906250
257	66049	16974593	16.0312195	6.3578611	.003891051
258	66564	17173512	16.0623784	6.3660968	.003875969
259	67081	17373979	16.0934769	6.3743111	.003861004
260	67600	17576000	16.1245155	6.3825043	.003846154
261	68121	17779581	16.1554944	6.3906765	.003831418
262	68644	17984728	16.1864141	6.3988279	.003816794
263	69169	18191447	16.2172747	6.4069585	.003802281
264	69696	18399744	16.2480768	6.4150687	.003787879
265	70225	18609625	16.2788206	6.4231583	.003773585
266	70756	18821096	16.3095064	6.4312276	.003759398
267	71289	19034163	16.3401346	6.4392767	.003745318
268	71824	19248832	16.3707055	6.4473057	.003731343
269	72361	19465109	16.4012195	6.4553148	.003717472
270	72900	19683000	16.4316767	6.4633041	.003703704
271	73441	19902511	16 4620776	6.4712736	.003690037
272	73984	20123648	16.4924225	6.4792236	.003676471
273	74529	20346417	16.5227116	6.4871541	.003663004
274	75076	20570824	16.5529454	6.4950653	.003649635
275	75625	20796875	16.5831240	6.5029572	.003636364
276	76176	21024576	16.6132477	6.5108300	.003623188
277	76729	21253933	16.6433170	6.5186839	.003610108
278	77284	21484952	16.6733320	6.5265189	.003597122
279	77841	21717639	16.7032931	6.5343351	.003584229
280	78400	21952000	16.7332005	6.5421326	.003571429
281	78961	22188041	16.7630546	6.5499116	.003558719
282	79524	22425768	16.7928556	6.5576722	.003546099
283	80089	22665187	16.8226038	6.5654144	.003533569
284	80656	22906304	16.8522995	6.5731385	.003521127
285	81225	23149125	16.8819430	6.5808443	.003508772
286	81796	23393656	16.9115345	6.5885323	.003496503
287	82369	23639903	16.9410743	6.5962023	.003484321
288	82944	23887872	16.9705627	6.6038545	.003472222
289	83521	24137569	17.0000000	6.6114890	.003460208
290	84100	24389000	17.0293864	6.6191060	.003448276
291	84681	24642171	17.0587221	6.6267054	.003436426
292	85264	24897088	17.0880075	6.6342874	.003424658
293	85849	25153757	17.1172428	6.6418522	.003412969
294	86436	25412184	17.1464282	6.6493998	.003401361
295	87025	25672375	17.1755640	6.6569302	.003389831
296	87616	25934336	17.2046505	6.6644437	.003378378
297	88209	26198073	17.2336879	6.6719403	.003367003
298	88804	26463592	17.2626765	6.6794200	.003355705
299	89401	26730899	17.2916165	6.6868831	.003344482
300	90000	27000000	17.3205081	6.6943295	.003333333
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543608	17.3781472	6.7091729	.003311258
303	91809	27818127	17.4068952	6.7165700	.003300330
304	92416	28094464	17.4355958	6.7239508	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278689
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
308	94864	29218112	17.5499288	6.7533134	.003246753
309	95481	29503629	17.5783958	6.7606143	.003236246
310	96100	29791000	17.6068169	6.7678995	.003225806

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7824229	.003205128
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968844	.003184713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554496	17.7763888	6.8112847	.003164557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753443	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003058104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313008	.003003003
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6 9520533	.002976190
337	113569	38272753	18.3575598	6.9589431	.002967359
338	114244	38614472	18.3847763	6 9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18 4661853	6.9863681	.002932551
342	116964	40001688	18.4932420	6 9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7.0135791	.002898551
346	119716	41421736	18 6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815417	7.0405806	.002865330
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002808989
357	127449	45499293	18.8944436	7.0939709	.002801120
358	128164	45882712	18.9208879	7.1005885	.002793216
359	128881	46268279	18.9472953	7.1071937	.002785595
360	129600	46656000	18.9736660	7.1137866	.002777778
361	130321	47045881	19.0000000	7.1203674	.002770083
632	131044	47437928	19.0262976	7.1269360	.002762431
363	131769	47832147	19.0525589	7.1334925	.002754821
364	132496	48228544	19.0787840	7.1400370	.002747253
365	133225	48627125	19.1049732	7.1465695	.002739726
366	133956	49027896	19.1311265	7.1530901	.002732240
367	134689	49430863	19.1572441	7.1595988	.002724796
368	135424	49836032	19.1833261	7.1660957	.002717391
369	136161	50243409	19.2093727	7.1725809	.002710027
370	136900	50653000	19.2353841	7.1790544	.002702703
371	137641	51064811	19.2613603	7.1855162	.002695418
372	138384	51478848	19.2873015	7.1919663	.00268817[illegible]

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
373	139129	51895117	19.3132079	7.1984050	.002680965
374	139876	52313624	19.3390796	7.2048322	.002673797
375	140625	52734375	19.3649167	7.2112479	.002666667
376	141376	53157376	19.3907194	7.2176522	.002659574
377	142129	53582633	19.4164878	7.2240450	.002652520
378	142884	54010152	19.4422221	7.2304268	.002645503
379	143641	54439939	19.4679223	7.2367972	.002638522
380	144400	54872000	19.4935887	7.2431565	.002631579
381	145161	55306341	19.5192213	7.2495045	.002624672
382	145924	55742968	19.5448203	7.2558415	.002617801
383	146689	56181887	19.5703858	7.2621675	.002610966
384	147456	56623104	19.5959179	7.2684824	.002604167
385	148225	57066625	19.6214169	7.2747864	.002597403
386	148996	57512456	19.6468827	7.2810794	.002590674
387	149769	57960603	19.6723156	7.2873617	.002583979
388	150544	58411072	19.6977156	7.2936330	.002577320
389	151321	58863869	19.7230829	7.2998936	.002570694
390	152100	59319000	19.7484177	7.3061436	.002564103
391	152881	59776471	19.7737199	7.3123828	.002557545
392	153664	60236288	19.7989899	7.3186114	.002551020
393	154449	60698457	19.8242276	7.3248295	.002544529
394	155236	61162984	19.8494332	7.3310369	.002538071
395	156025	61629875	19.8746069	7.3372339	.002531646
396	156816	62099136	19.8997487	7.3434205	.002525253
397	157609	62570773	19.9248588	7.3495966	.002518892
398	158404	63044792	19.9499373	7.3557624	.002512563
399	159201	63521199	19.9749844	7.3619178	.002506266
400	160000	64000000	20.0000000	7.3680630	.002500000
401	160801	64481201	20.0249844	7.3741979	.002493766
402	161604	64964808	20.0499377	7.3803227	.002487562
403	162409	65450827	20.0748599	7.3864373	.002481390
404	163216	65939264	20.0997512	7.3925418	.002475248
405	164025	66430125	20.1246118	7.3986363	.002469136
406	164836	66923416	20.1494417	7.4047206	.002463054
407	165649	67419143	20.1742410	7.4107950	.002457002
408	166464	67917312	20.1990099	7.4168595	.002450980
409	167281	68417929	20.2237484	7.4229142	.002444988
410	168100	68921000	20.2484567	7.4289589	.002439024
411	168921	69426531	20.2731349	7.4349938	.002433090
412	169744	69934528	20.2977831	7.4410189	.002427184
413	170569	70444997	20.3224014	7.4470342	.002421308
414	171396	70957944	20.3469899	7.4530399	.002415459
415	172225	71473375	20.3715488	7.4590359	.002409639
416	173056	71991296	20.3960781	7.4650223	.002403846
417	173889	72511713	20.4205779	7.4709991	.002398082
418	174724	73034632	20.4450483	7.4769664	.002392344
419	175561	73560059	20.4694895	7.4829242	.002386635
420	176400	74088000	20.4939015	7.4888724	.002380952
421	177241	74618461	20.5182845	7.4948113	.002375297
422	178084	75151448	20.5426386	7.5007406	.002369668
423	178929	75686967	20.5669638	7.5066607	.002364066
424	179776	76225024	20.5912603	7.5125715	.002358491
425	180625	76765625	20.6155281	7.5184730	.002352941
426	181476	77308776	20.6397674	7.5243652	.002347418
427	182329	77854483	20.6639783	7.5302482	.002341920
428	183184	78402752	20.6881609	7.5361221	.002336449
429	184041	78953589	20.7123152	7.5419867	.002331002
430	184900	79507000	20.7364414	7.5478423	.002325581
431	185761	80062991	20.7605395	7.5536888	.002320186
432	186624	80621568	20.7846097	7.5595263	.002314815
433	187489	81182737	20.8086520	7.5653548	.002309469
434	188356	81746504	20.8326667	7.5711743	.002304147

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
435	189225	82312875	20.8566536	7.5769849	.002298851
436	190096	82881856	20.8806130	7.5827865	.002293578
437	190969	83453453	20.9045450	7.5885793	.002288330
438	191844	84027672	20.9284495	7.5943633	.002283105
439	192721	84604519	20.9523268	7.6001385	.002277904
440	193600	85184000	20.9761770	7.6059049	.002272727
441	194481	85766121	21.0000000	7.6116626	.002267574
442	195364	86350888	21.0237960	7.6174116	.002262443
443	196249	86938307	21.0475652	7.6231519	.002257336
444	197136	87528384	21.0713075	7.6288837	.002252252
445	198025	88121125	21.0950231	7.6346067	.002247191
446	198916	88716536	21.1187121	7.6403213	.002242152
447	199809	89314623	21.1423745	7.6460272	.002237136
448	200704	89915392	21.1660105	7.6517247	.002232143
449	201601	90518849	21.1896201	7.6574138	.002227171
450	202500	91125000	21.2132034	7.6630943	.002222222
451	203401	91733851	21.2367606	7.6687665	.002217295
452	204304	92345408	21.2602916	7.6744303	.002212389
453	205209	92959677	21.2837967	7.6800857	.002207506
454	206116	93576664	21.3072758	7.6857328	.002202643
455	207025	94196375	21.3307290	7.6913717	.002197802
456	207936	94818816	21.3541565	7.6970023	.002192982
457	208849	95443993	21.3775583	7.7026246	.002188184
458	209764	96071912	21.4009346	7.7082388	.002183406
459	210681	96702579	21.4242853	7.7138448	.002178649
460	211600	97336000	21.4476106	7.7194426	.002173913
461	212521	97972181	21.4709106	7.7250325	.002169197
462	213444	98611128	21.4941853	7.7306141	.002164502
463	214369	99252847	21.5174348	7.7361877	.002159827
464	215296	99897344	21.5406592	7.7417532	.002155172
465	216225	100544625	21.5638587	7.7473109	.002150538
466	217156	101194696	21.5870331	7.7528606	.002145923
467	218089	101847563	21.6101828	7.7584023	.002141328
468	219024	102503232	21 6333077	7.7639361	002136752
469	219961	103161709	21.6564078	7.7694620	.002132196
470	220900	103823000	21.6794834	7.7749801	.002127660
471	221841	104487111	21.7025344	7.7804904	.002123142
472	222784	105154048	21.7255610	7.7859928	.002118644
473	223729	105823817	21.7485632	7.7914875	.002114165
474	224676	106496424	21.7715411	7.7969745	.002109705
475	225625	107171875	21.7944947	7 8024538	002105263
476	226576	107850176	21.8174242	7 8079254	.002100840
477	227529	108531333	21.8403297	7.8133892	.002096436
478	228484	109215352	21.8632111	7 8188456	.002092050
479	229441	109902239	21.8860686	7.8242942	.002087683
480	230400	110592000	21.9089023	7.8297353	.002083333
481	231361	111284641	21.9317122	7.8351688	.002079002
482	232324	111980168	21.9544984	7 8405949	.002074689
483	233289	112678587	21.9772610	7 8460134	.002070393
484	234256	113379904	22.0000000	5 8514244	.002066116
485	235225	114084125	22 0227155	7.8568281	.002061856
486	236196	114791256	22.0454077	7.8622242	.002057613
487	237169	115501303	22.0680765	7.8676130	.002053388
488	238144	116214272	22.0907220	7.8729944	.002049180
489	239121	116930169	22.1133444	7.8783684	.002044990
490	240100	117649000	22.1359436	7.8837352	.002040816
491	241081	118370771	22.1585198	7.8890946	.002036660
492	242064	119095488	22.1810730	7.8944468	.002032520
493	243049	119823157	22.2036033	7.8997917	.002028398
494	244036	120553784	22.2261108	7.9051294	.002024291
495	245025	121287375	22.2485955	7.9104599	.002020202
496	246016	122023936	22.2710575	7.9157832	.002016129

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
497	247009	122763473	22.2934968	7.9210994	.002012072
498	248004	123505992	22.3159136	7.9264085	.002008032
499	249001	124251499	22.3383079	7.9317104	.002004008
500	250000	125000000	22.3606798	7.9370053	.002000000
501	251001	125751501	22.3830293	7.9422931	.001996008
502	252004	126506008	22.4053565	7.9475739	.001992032
503	253009	127263527	22.4276615	7.9528477	.001988072
504	254016	128024064	22.4499443	7.9581144	.001984127
505	255025	128787625	22.4722051	7.9633743	.001980198
506	256036	129554216	22.4944438	7.9686271	.001976285
507	257049	130323843	22.5166605	7.9738731	.001972387
508	258064	131096512	22.5388553	7.9791122	.001968504
509	259081	131872229	22.5610283	7.9843444	.001964637
510	260100	132651000	22.5831796	7.9895697	.001960784
511	261121	133432831	22.6053091	7.9947883	.001956947
512	262144	134217728	22.6274170	8.0000000	.001953125
513	263169	135005697	22.6495033	8.0052049	.001949318
514	264196	135796744	22.6715681	8.0104032	.001945525
515	265225	136590875	22.6936114	8.0155946	.001941748
516	266256	137388096	22.7156334	8.0207794	.001937984
517	267289	138188413	22.7376340	8.0259574	.001934236
518	268324	138991832	22.7596134	8.0311287	001930502
519	269361	139798359	22.7815715	8.0362935	.001926782
520	270400	140608000	22.8035085	8.0414515	.001923077
521	271441	141420761	22.8254244	8.0466030	.001919386
522	272484	142236648	22.8473193	8.0517479	.001915709
523	273529	143055667	22.8691933	8.0568862	.001912046
524	274576	143877824	22.8910463	8.0620180	.001908397
525	275625	144703125	22.9128785	8.0671432	.001904762
526	276676	145531576	22.9346899	8.0722620	.001901141
527	277729	146363183	22.9564806	8.0773743	.001897533
528	278784	147197952	22 9782506	8.0824800	.001893939
529	279841	148035889	23.0000000	8.0875794	.001890359
530	280900	148877000	23.0217289	8.0926723	.001886792
531	281961	149721291	23.0434372	8.0977589	.001883239
532	283024	150568768	23.0651252	8.1028390	.001879699
533	284089	151419437	23.0867928	8.1079128	.001876173
534	285156	152273304	23.1084400	8.1129803	.001872659
535	286225	153130375	23.1300670	8.1180414	.001869159
536	287296	153990656	23.1516738	8.1230962	.001865672
537	288369	154854153	23.1732605	8.1281447	.001862197
538	289444	155720872	23.1948270	8.1331870	.001858736
539	290521	156590819	23.2163735	8.1382230	.001855288
540	291600	157464000	23.2379001	8.1432529	.001851852
541	292681	158340421	23.2594067	8.1482765	.001848429
542	293764	159220088	23.2808935	8.1532939	.001845018
543	294849	160103007	23.3023604	8.1583051	.001841621
544	295936	160989184	23.3238076	8.1633102	.001838235
545	297025	161878625	23.3452351	8.1683092	.001834862
546	298116	162771336	23.3666429	8.1733020	.001831502
547	299209	163667323	23.3880311	8.1782888	.001828154
548	300304	164566592	23.4093998	8.1832695	.001824818
549	301401	165469149	23.4307490	8.1882441	.001821494
550	302500	166375000	23.4520788	8.1932127	.001818182
551	303601	167284151	23.4733892	8.1981753	.001814882
552	304704	168196608	23.4946802	8.2031319	.001811594
553	305809	169112377	23.5159520	8.2080825	.001808318
554	306916	170031464	23.5372046	8.2130271	.001805054
555	308025	170953875	23.5584380	8.2179657	.001801802
556	309136	171879616	23.5796522	8.2228985	.001798561
557	310249	172808693	23.6008474	8.2278254	.001795332
558	311364	173741112	23.6220236	8.2327463	.001792115

No.	Squares	Cubes	Square roots	Cube roots	Reciprocals
559	312481	174676879	23.6431808	8.2376614	.001788909
560	313600	175616000	23.6643191	8.2425706	.001785714
561	314721	176558481	23.6854386	8.2474740	.001782531
562	315844	177504328	23.7065392	8.2523715	.001779359
563	316969	178453547	23.7276210	8.2572633	.001776199
564	318096	179406144	23.7486842	8.2621492	.001773050
565	319225	180362125	23.7697286	8.2670294	.001769912
566	320356	181321496	23.7907545	8.2719039	.001766784
567	321489	182284263	23.8117618	8.2767726	.001763668
568	322624	183250432	23.8327506	8.2816355	.001760563
569	323761	184220000	23.8537209	8.2864928	.001757469
570	324900	185193000	23.8746728	8.2913444	.001754386
571	326041	186169411	23.8956063	8.2961903	.001751313
572	327184	187149248	23.9165215	8.3010304	.001748252
573	328329	188132517	23.9374184	8.3058651	.001745201
574	329476	189119224	23.9582971	8.3106941	.001742160
575	330625	190109375	23.9791576	8.3155175	.001739130
576	331776	191102976	24.0000000	8.3203353	.001736111
577	332929	192100033	24.0208243	8.3251475	.001733102
578	334084	193100552	24.0416306	8.3299542	.001730104
579	335241	194104539	24.0624188	8.3347553	.001727116
580	336400	195112000	24.0831891	8.3395509	.001724138
581	337561	196122941	24.1039416	8.3443410	.001721170
582	338724	197137368	24.1246762	8.3491256	.001718213
583	339889	198155287	24.1453929	8.3539047	.001715266
584	341056	199176704	24.1660919	8.3586784	.001712329
585	342225	200201625	24.1867732	8.3634466	.001709402
586	343396	201230056	24.2074369	8.3682095	.001706485
587	344569	202262003	24.2280829	8.3729668	.001703578
588	345744	203297472	24.2487113	8.3777188	.001700680
589	346921	204336469	24.2693222	8.3824653	.001697793
590	348100	205379000	24.2899156	8.3872065	.001694915
591	349281	206425071	24.3104916	8.3919423	.001692047
592	350464	207474688	24.3310501	8 3966729	.001689189
593	351649	208527857	24.3515913	8.4013981	.001686341
594	352836	209584584	24.3721152	8.4061180	.001683502
595	354025	210644875	24.3926218	8.4108326	.001680672
596	355216	211708736	24 4131112	8.4155419	.001677852
597	356409	212776173	24.4335834	8 4202460	.001675042
598	357604	213847192	24.4540385	8.4249448	.001672241
599	358801	214921799	24.4744765	8.4296383	.001669449
600	360000	216000000	24.4948974	8.4343267	.001666667
601	361201	217081801	24.5153013	8.4390098	.001663894
602	362404	218167208	24.5356883	8.4436877	.001661130
603	363609	219256227	24.5560583	8.4483605	.001658375
604	364816	220348864	24.5764115	8.4530281	.001655629
605	366025	221445125	24.5967478	8.4576906	.001652893
606	367236	222545016	24.6170673	8.4623479	.001650165
607	368449	223648543	24.6373700	8.4670001	.001647446
608	369664	224755712	24.6576560	8.4716471	.001644737
609	370881	225866529	24.6779254	8.4762892	.001642036
610	372100	226981000	24.6981781	8 4809261	.001639344
611	373321	228099131	24.7184142	8.4855579	.001636661
612	374544	229220928	24.7386338	8.4901848	.001633987
613	375769	230346397	24.7588368	8.4948065	.001631321
614	376996	231475544	24.7790234	8.4994233	.001628664
615	378225	232608375	24.7991935	8.5040350	.001626016
616	379456	233744896	24.8193473	8.5086417	.001623377
617	380689	234885113	24.8394847	8.5132435	.001620746
618	381924	236029032	24.8596058	8.5178403	.001618123
619	383161	237176659	24.8797106	8.5224321	.001615509
620	384400	238328000	24.8997992	8.5270189	.001612903

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
621	385641	239483061	24.9198716	8.5316009	.001610306
622	386884	240641848	24.9399278	8.5361780	.001607717
623	388129	241804367	24.9599679	8.5407501	.001605136
624	389376	242970624	24.9799920	8.5453173	.001602564
625	390625	244140625	25.0000000	8.5498797	.001600000
626	391876	245314376	25.0199920	8.5544372	.001597444
627	393129	246491883	25.0399681	8.5589899	.001594896
628	394384	247673152	25.0599282	8.5635377	.001592357
629	395641	248858189	25.0798724	8.5680807	.001589825
630	396900	250047000	25.0998008	8.5726189	.001587302
631	398161	251239591	25.1197134	8.5771523	.001584786
632	399424	252435968	25.1396102	8.5816809	.001582278
633	400689	253636137	25.1594913	8.5862047	.001579779
634	401956	254840104	25.1793566	8.5907238	.001577287
635	403225	256047875	25.1992063	8.5952380	.001574803
636	404496	257259456	25.2190404	8.5997476	.001572327
637	405769	258474853	25.2388589	8.6042525	.001569859
638	407044	259694072	25.2586619	8.6087526	.001567398
639	408321	260917119	25.2784493	8.6132480	.001564945
640	409600	262144000	25.2982213	8.6177388	.001562500
641	410881	263374721	25.3179778	8.6222248	.001560062
642	412164	264609288	25.3377189	8.6267063	.001557632
643	413449	265847707	25.3574447	8 6311830	.001555210
644	414736	267089984	25.3771551	8.6356551	.001552795
645	416025	268336125	25.3968502	8.6401226	.001550388
646	417316	269586136	25.4165301	8 6445855	.001547988
647	418609	270840023	25.4361947	8.6490437	.001545595
648	419904	272097792	25.4558441	8.6534974	.001543210
649	421201	273359449	25.4754784	8.6579465	.001540832
650	422500	274625000	25.4950976	8.6623911	.001538462
651	423801	275894451	25.5147016	8.6668310	.001536098
652	425104	277167808	25.5342907	8.6712665	.001533742
653	426409	278445077	25.5538647	8.6756974	.001531394
654	427716	279726264	25.5734237	8.6801237	.001529052
655	429025	281011375	25.5929678	8.6845456	.001526718
656	430336	282300416	25.6124969	8.6889630	.001524390
657	431649	283593393	25.6320112	8.6933759	.001522070
658	432964	284890312	25.6515107	8.6977843	.001519757
659	434281	286191179	25.6709953	8.7021882	.001517451
660	435600	287496000	25.6904652	8 7065877	.001515152
661	436921	288804781	25.7099203	8.7109827	.001512859
662	438244	290117528	25.7293607	8.7153734	.001510574
663	439569	291434247	25.7487864	8.7197596	.001508296
664	440896	292754944	25.7681975	8.7241414	.001506024
665	442225	294079625	25.7875939	8.7285187	.001503759
666	443556	295408296	25.8069758	8.7328918	.001501502
667	444889	296740963	25.8263431	8.7372604	.001499250
668	446224	298077632	25.8456960	8.7416246	.001497006
669	447561	299418309	25.8650343	8.7459846	.001494768
670	448900	300763000	25.8843582	8.7503401	.001492537
671	450241	302111711	25.9036677	8.7546913	.001490313
672	451584	303464448	25.9229628	8.7590383	.001488095
673	452929	304821217	25.9422435	8.7633809	.001485884
674	454276	306182024	25.9615100	8.7677192	.001483680
675	455625	307546875	25.9807621	8.7720532	.001481481
676	456976	308915776	26.0000000	8.7763830	.001479290
677	458329	310288733	26.0192237	8.7807084	.001477105
678	459684	311665752	26.0384331	8.7850296	.001474926
679	461041	313046839	26.0576284	8.7893466	.004472754
680	462400	314432000	26.0768096	8.7936593	.001470588
681	463761	315821241	26.0959767	8.7979679	.001468429
682	465124	317214568	26.1151297	8.8022721	.001466276

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
683	466489	318611987	26.1342687	8.8065722	.001464129
684	467856	320013504	26.1533937	8.8108681	.001461988
685	469225	321419125	26.1725047	8.8151598	.001459854
686	470596	322828856	26.1916017	8.8194474	.001457726
687	471969	324242703	26.2106848	8.8237307	.001455604
688	473344	325660672	26.2297541	8.8280099	.001453488
689	474721	327082769	26.2488095	8.8322850	.001451379
690	476100	328509000	26.2678511	8.8365559	.001449275
691	477481	329939371	26.2868789	8.8408227	.001447178
692	478864	331373888	26.3058929	8.8450854	.001445087
693	480249	332812557	26.3248932	8.8493440	.001443001
694	481636	334255384	26.3438797	8.8535985	.001440922
695	483025	335702375	26.3628527	8.8578489	.001438849
696	484416	337153536	26.3818119	8.8620952	.001436782
697	485809	338608873	26.4007576	8.8663375	.001434720
698	487204	340068392	26.4196896	8.8705757	.001432665
699	488601	341532099	26.4386081	8.8748099	.001430615
700	490000	343000000	26.4575131	8.8790400	.001428571
701	491401	344472101	26.4764046	8.8832661	.001426534
702	492804	345948408	26.4952826	8.8874882	001424501
703	494209	347428927	26.5141472	8.8917063	.001422475
704	495616	348913664	26.5329983	8.8959204	001420455
705	497025	350402625	26.5518361	8.9001304	.001418440
706	498436	351895816	26.5706605	8.9043366	.001416431
707	499849	353393243	26.5894716	8.9085387	.001414427
708	501264	354894912	26.6082694	8.9127369	.001412429
709	502681	356400829	26.6270539	8.9169311	.001410437
710	504100	357911000	26.6458252	8.9211214	.001408451
711	505521	359425431	26.6645833	8.9253078	.001406470
712	506944	360944128	26.6833281	8.9294902	.001404494
713	508369	362467097	26.7020598	8.9336687	.001402525
714	509796	363994344	26.7207784	8.9378433	.001400560
715	511225	365525875	26.7394839	8.9420140	.001398601
716	512656	367061696	26.7581763	8.9461809	.001396648
717	514089	368601813	26.7768557	8.9503438	.001394700
718	515524	370146232	26.7955220	8.9545029	.001392758
719	516961	371694959	26.8141754	8.9586581	.001390821
720	518400	373248000	26.8328157	8.9628095	.001388889
721	519841	374805361	26 8514432	8.9669570	.001386963
722	521284	376367048	26.8700577	8.9711007	.001385042
723	522729	377933067	26.8886593	8.9752406	.001383126
724	524176	379503424	26.9072481	8.9793766	.001381215
725	525625	381078125	26.9258240	8.9835089	.001379310
726	527076	382657176	26.9443872	8.9876373	.001377410
727	528529	384240583	26.9629375	8.9917620	.001375516
728	529984	385828352	26.9814751	8.9958829	.001373626
729	531441	387420489	27.0000000	9.0000000	.001371742
730	532900	389017000	27.0185122	9.0041134	.001369863
731	534361	390617891	27.0370117	9.0082229	.001367989
732	535824	392223168	27.0554985	9.0123288	.001366120
733	537289	393832837	27.0739727	9.0164309	.001364256
734	538756	395446904	27.0924344	9.0205293	.001362398
735	540225	397065375	27.1108834	9.0246239	.001360544
736	541696	398688256	27.1293199	9.0287149	.001358696
737	543169	400315553	27.1477439	9.0328021	.001356852
738	544644	401947272	27.1661554	9.0368857	.001355014
739	546121	403583419	27.1845544	9.0409655	.001353180
740	547600	405224000	27.2029410	9.0450419	.001351351
741	549081	406869021	27.2213152	9.0491142	001349528
742	550564	408518488	27.2396769	9.0531831	001347709
743	552049	410172407	27.2580263	9.0572482	001345895
744	553536	411830784	27.2763634	9.0613098	001344086

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
745	555025	413493625	27.2946881	9.0653677	.001342282
746	556516	415160936	27.3130006	9.0694220	.001340483
747	558009	416832723	27.3313007	9.0734726	.001338688
748	559504	418508992	27.3495887	9.0775197	.001336898
749	561001	420189749	27.3678644	9.0815631	.001335113
750	562500	421875000	27.3861279	9.0856030	.001333333
751	564001	423564751	27.4043792	9.0896392	.001331558
752	565504	425259008	27.4226184	9.0936719	.001329787
753	567009	426957777	27.4408455	9.0977010	.001328021
754	568516	428661064	27.4590604	9.1017265	.001326260
755	570025	430368875	27.4772633	9.1057485	.001324503
756	571536	432081216	27.4954542	9.1097669	.001322751
757	573049	433798093	27.5136330	9 1137818	.001321004
758	574564	435519512	27.5317998	9.1177931	.001319261
759	576081	437245479	27.5499546	9.1218010	.001317523
760	577600	438976000	27.5680975	9.1258053	.001315789
761	579121	440711081	27.5862284	9.1298061	.001314060
762	580644	442450728	27.6043475	9.1338034	.001312336
763	582169	444194947	27.6224546	9.1377971	.001310616
764	583696	445943744	27.6405499	9.1417874	.001308901
765	585225	447697125	27.6586334	9.1457742	.001307190
766	586756	449455096	27.6767050	9.1497576	.001305483
767	588289	451217663	27.6947648	9.1537375	.001303781
768	589824	452984832	27.7128129	9.1577139	.001302083
769	591361	454756609	27.7308492	9.1616869	.001300390
770	592900	456533000	27.7488739	9.1656565	.001298701
771	594441	458314011	27.7668808	9.1696225	.001297017
772	595984	460099648	27.7848880	9.1735852	.001295337
773	597529	461889917	27.8028775	9.1775445	.001293661
774	599076	463684824	27.8208555	9.1815003	.001291990
775	600625	465484375	27.8388218	9.1854527	.001290323
776	602176	467288576	27.8567766	9.1894018	.001288660
777	603729	469097433	27.8747197	9.1933474	.001287001
778	605284	470910952	27.8926514	9.1972897	.001285347
779	606841	472729139	27.9105715	9.2012286	.001283697
780	608400	474552000	27.9284801	9.2051641	.001282051
781	609961	476379541	27 9463772	9.2090962	.001280410
782	611524	478211768	27.9642629	9.2130250	.001278772
783	613089	480048687	27.9821372	9.2169505	.001277139
784	614656	481890304	28.0000000	9.2208726	.001275510
785	616225	483736625	28 0178515	9.2247914	.001273885
786	617796	485587656	28.0356915	9.2287068	.001272265
787	619369	487443403	28.0535203	9.2326189	.001270648
788	620944	489303872	28.0713377	9.2365277	.001269036
789	622521	491169069	28.0891438	9.2404333	.001267427
790	624100	493039000	28.1069386	9.2443355	.001265823
791	625681	494913671	28.1247222	9.2482344	.001264223
792	627264	496793088	28.1424946	9.2521300	.001262626
793	628849	498677257	28.1602557	9.2560224	.001261034
794	630436	500566184	28.1780056	9.2599114	.001259446
795	632025	502459875	28.1957444	9.2637973	.001257862
796	633616	504358336	28.2134720	9.2676798	.001256281
797	635209	506261573	28.2311884	9.2715592	.001254705
798	636804	508169592	28.2488938	9.2754352	.001253133
799	638401	510082399	28.2665881	9.2793081	.001251564
800	640000	512000000	28.2842712	9.2831777	.001250000
801	641601	513922401	28.3019434	9.2870440	.001248439
802	643204	515849608	28.3196045	9.2909072	.001246883
803	644809	517781627	28.3372546	9.2947671	.001245330
804	646416	519718464	28.3548938	9.2986239	.001243781
805	648025	521660125	28.3725219	9.3024775	.001242236
806	649636	523606616	28.3901391	9.3063278	.001240695

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
807	651249	525557943	28.4077454	9.3101750	.001239157
808	652864	527514112	28.4253408	9.3140190	.001237624
809	654481	529475129	28.4429253	9.3178599	.001236094
810	656100	531441000	28.4604989	9.3216975	.001234568
811	657721	533411731	28.4780617	9.3255320	.001233046
812	659344	535387328	28.4956137	9.3293634	.001231527
813	660969	537367797	28.5131549	9.3331916	.001230012
814	662596	539353144	28.5306852	9.3370167	.001228501
815	664225	541343375	28.5482048	9.3408386	.001226994
816	665856	543338496	28.5657137	9.3446575	.001225490
817	667489	545338513	28.5832119	9.3484731	.001223990
818	669124	547343432	28.6006993	9.3522857	.001222494
819	670761	549353259	28.6181760	9.3560952	.001221001
820	672400	551368000	28.6356421	9.3599016	.001219512
821	674041	553387661	28.6530976	9.3637049	.001218027
822	675684	555412248	28.6705424	9.3675051	.001216545
823	677329	557441767	28.6879766	9.3713022	.001215067
824	678976	559476224	28.7054002	9.3750963	.001213592
825	680625	561515625	28.7228132	9.3788873	.001212121
826	682276	563559976	28.7402157	9.3826752	.001210654
827	683929	565609283	28.7576077	9.3864600	.001209190
828	685584	567663552	28.7749891	9.3902419	.001207729
829	687241	569722789	28.7923601	9.3940206	.001206273
830	688900	571787000	28.8097206	9.3977964	.001204819
831	690561	573856191	28.8270706	9.4015691	.001203369
832	692224	575930368	28.8444102	9.4053387	.001201923
833	693889	578009537	28.8617394	9.4091054	.001200480
834	695556	580093704	28.8790582	9.4128690	.001199041
835	697225	582182875	28.8963666	9.4166297	.001197605
836	698896	584277056	28.9136646	9.4203873	.001196172
837	700569	586376263	28.9309523	9.4241420	.001194743
838	702244	588480472	28.9482297	9.4278936	.001193317
839	703921	590589719	28.9654967	9.4316423	.001191895
840	705600	592704000	28.9827535	9.4353880	.001190476
841	707281	594823321	29.0000000	9.4391307	.001189061
842	708964	596947688	29.0172363	9.4428704	.001187648
843	710649	599077107	29.0344623	9.4466072	.001186240
844	712336	601211584	29.0516781	9.4503410	.001184834
845	714025	603351125	29.0688837	9.4540719	.001183432
846	715716	605495736	29.0860791	9.4577999	.001182033
847	717409	607645423	29.1032644	9.4615249	.001180638
848	719104	609800192	29.1204396	9.4652470	.001179245
849	720801	611960049	29.1376046	9.4689661	.001177856
850	722500	614125000	29.1547595	9.4726824	.001176471
851	724201	616295051	29.1719043	9.4763957	.001175088
852	725904	618470208	29.1890390	9.4801061	.001173709
853	727609	620650477	29.2061637	9.4838136	.001172333
854	729316	622835864	29.2232784	9.4875182	.001170960
855	731025	625026375	29.2403830	9.4912200	.001169591
856	732736	627222016	29.2574777	9.4949188	.001168224
857	734449	629422793	29.2745623	9 4986147	.001166861
858	736164	631628712	29.2916370	9.5023078	.001165501
859	737881	633839779	29.3087018	9.5059980	.001164144
860	739600	636056000	29.3257566	9.5096854	.001162791
861	741321	638277381	29.3428015	9.5133699	.001161440
862	743044	640503928	29.3598365	9.5170515	.001160093
863	744769	642735647	29.3768616	9.5207303	.001158749
864	746496	644972544	29.3938769	9.5244063	.001157407
865	748225	647214625	29.4108823	9.5280794	.001156069
866	749956	649461896	29.4278779	9.5317497	.001154734
867	751689	651714363	29.4448637	9.5354172	.001153403
868	753424	653972032	29.4618397	9.5390818	.001152074

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
869	755161	656234909	29.4788059	9.5427437	.001150748
870	756900	658503000	29.4957624	9.5464027	.001149425
871	758641	660776311	29.5127091	9.5500589	.001148106
872	760384	663054848	29.5296461	9.5537123	.001146789
873	762129	665338617	29.5465734	9.5573630	.001145475
874	763876	667627624	29.5634910	9.5610108	.001144165
875	765625	669921875	29.5803989	9.5646559	.001142857
876	767376	672221376	29.5972972	9.5682982	.001141553
877	769129	674526133	29.6141858	9.5719377	.001140251
878	770884	676836152	29.6310648	9.5755745	.001138952
879	772641	679151439	29.6479342	9.5792085	.001137656
880	774400	681472000	29.6647939	9.5828397	.001136364
881	776161	683797841	29.6816442	9.5864682	.001135074
882	777924	686128968	29.6984848	9.5900939	.001133787
883	779689	688465387	29.7153159	9.5937169	.001132503
884	781456	690807104	29.7321375	9.5973373	.001131222
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	709732288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998328	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117318
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111
901	811801	731432701	30.0166620	9.6584684	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103753
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550528	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188354	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083423
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.001076426
930	864900	804357000	30.4959014	9.7610001	.001075269

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
931	866761	806954491	30.5122926	9.7644974	.001074114
932	868624	809557568	30.5286750	9.7679922	.001072961
933	870489	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784616	.001069519
936	876096	820025856	30.5941171	9.7819466	.001068376
937	877969	822656953	30.6104557	9.7854288	.001067236
938	879844	825293672	30.6267857	9.7889087	.001066098
939	881721	827936019	30.6431069	9.7923861	.001064963
940	883600	830584000	30.6594194	9.7958611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896888	30.6920185	9.8028036	.001061571
943	889249	838561807	30.7083051	9.8062711	.001060445
944	891136	841232384	30.7245830	9.8097362	.001059322
945	893025	843908625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896809	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382879	9.8339238	.001051525
952	906304	862801408	30.8544972	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192497	9.8511280	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838668	9.8648483	.001041667
961	923521	887503681	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056347	31.0322413	9.8751135	.001038422
964	929296	895841344	31.0483494	9.8785305	.001037344
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805405	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287648	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330048	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2409987	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966168	31.3368792	9.9396363	.001018330
983	966289	949862087	31.3528308	9.9430092	.001017294
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146
989	978121	967361669	31.4483704	9.9631981	.001011122
990	980100	970299000	31.4642654	9.9665549	.001010101
991	982081	973242271	31.4801525	9.9699095	.001009082
992	984064	976191488	31.4960315	9.9732619	.001008065

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
993	986049	979146657	31.5119025	9.9766120	.001007049
994	988036	982107784	31.5277655	9.9799599	.001006036
995	990025	985074875	31.5436206	9.9833055	.001005025
996	992016	988047936	31.5594677	9.9866488	.001004016
997	994009	991026973	31.5753068	9.9899900	.001003009
998	996004	994011992	31.5911380	9.9933289	.001002004
999	998001	997002999	31.6069613	9.9966656	.001001001
1000	1000000	1000000000	31.6227766	10.0000000	.001000000
1001	1002001	1003003001	31.6385840	10.0033322	.0009990010
1002	1004004	1006012008	31.6543836	10.0066622	.0009980040
1003	1006009	1009027027	31.6701752	10.0099899	.0009970090
1004	1008016	1012048064	31.6859590	10.0133155	.0009960159
1005	1010025	1015075125	31.7017349	10.0166389	.0009950249
1006	1012036	1018108216	31.7175030	10.0199601	.0009940358
1007	1014049	1021147343	31.7332633	10.0232791	.0009930487
1008	1016064	1024192512	31.7490157	10.0265958	.0009920635
1009	1018081	1027243729	31.7647603	10.0299104	.0009910803
1010	1020100	1030301000	31.7804972	10.0332228	.0009900990
1011	1022121	1033364331	31.7962262	10.0365330	.0009891197
1012	1024144	1036433728	31.8119474	10.0398410	.0009881423
1013	1026169	1039509197	31.8276609	10.0431469	.0009871668
1014	1028196	1042590744	31.8433666	10.0464506	.0009861933
1015	1030225	1045678375	31.8590646	10.0497521	.0009852217
1016	1032256	1048772096	31.8747549	10.0530514	.0009842520
1017	1034289	1051871913	31.8904374	10.0563485	.0009832842
1018	1036324	1054977832	31.9061123	10.0596435	.0009823183
1019	1038361	1058089859	31.9217794	10.0629364	.0009813543
1020	1040400	1061208000	31.9374388	10.0662271	.0009803922
1021	1042441	1064332261	31.9530906	10.0695156	.0009794319
1022	1044484	1067462648	31.9687347	10.0728020	.0009784736
1023	1046529	1070599167	31.9843712	10.0760863	.0009775171
1024	1048576	1073741824	32.0000000	10.0793684	.0009765625
1025	1050625	1076890625	32.0156212	10.0826484	.0009756098
1026	1052676	1080045576	32 0312348	10.0859262	.0009746589
1027	1054729	1083206683	32.0468407	10.0892019	.0009737098
1028	1056784	1086373952	32.0624391	10 0024755	.0009727626
1029	1058841	1089547389	32.0780298	10.0957469	.0009718173
1030	1060900	1092727000	32.0936131	10.0990163	.0009708738
1031	1062961	1095912791	32.1091887	10.1022835	.0009699321
1032	1065024	1099104768	32.1247568	10.1055487	.0009689922
1033	1067089	1102302937	32.1403173	10.1088117	.0009680542
1034	1069156	1105507304	32.1558704	10.1120726	.0009671180
1035	1071225	1108717875	32.1714159	10.1153314	.0009661836
1036	1073296	1111934656	32.1869539	10.1185882	.0009652510
1037	1075369	1115157653	32.2024844	10.1218428	.0009643202
1038	1077444	1118386872	32.2180074	10.1250953	.0009633911
1039	1079521	1121622319	32.2335229	10.1283457	.0009624639
1040	1081600	1124864000	32.2490310	10.1315941	.0009615385
1041	1083681	1128111921	32.2645316	10 1348403	.0009606148
1042	1085764	1131366088	32.2800248	10.1380845	.0009596929
1043	1087849	1134626507	32.2955105	10.1413266	.0009587738
1044	1089936	1137893184	32.3109888	10.1445667	.0009578544
1045	1092025	1141166125	32.3264598	10.1478047	.0009569378
1046	1094116	1144445336	32.3419233	10 1510406	.0009560229
1047	1096209	1147730823	32.3573794	10.1542744	.0009551098
1048	1098304	1151022592	32.3728281	10.1575062	.0009541985
1049	1100401	1154320649	32.3882695	10.1607359	.0009532888
1050	1102500	1157625000	32.4037035	10.1639636	.0009523810
1051	1104601	1160935651	32.4191301	10.1671893	.0009514748
1052	1106704	1164252608	32.4345495	10.1704129	.0009505703
1053	1108809	1167575877	32.4499615	10.1736344	.0009496676
1054	1110916	1170905464	32.4653662	10.1768539	.0009487666

2. WEIGHTS AND MEASURES

Measures of Length

12 inches = 1 foot				
3 feet = 1 yard	= 36 inches			
$5\frac{1}{2}$ yards = 1 rod	= 198 inches	= $16\frac{1}{2}$ feet		
40 rods = 1 furlong	= 7 920 inches	= 660 feet	= 220 yards	
8 furlongs = 1 mile	= 63 360 inches	= 5 280 feet	= 1 760 yards	= 320 rods
1 yard = 0.0005682 of a mile				

Gunter's Chain

7.92 inches = 1 link
100 links = 1 chain = 4 rods = 66 feet
80 chains = 1 mile

Ropes and Cables

6 feet = 1 fathom 120 fathoms = 1 cable's length

Table Showing Inches Expressed in Decimals of a Foot

In.	0	1	2	3	4	5	6	7	8	9	10	11
0	Foot	.0833	1667	.2500	3333	4167	.5000	.5833	.6667	.7500	.8333	9167
1-32	0026	0859	.1693	.2526	3359	4193	.5026	5859	6693	7526	8359	.9193
1-16	0052	0885	1719	2552	.3385	4219	.5052	.5885	6719	.7552	.8385	.9219
3-32	.0078	.0911	.1745	.2578	.3411	4245	5078	5911	.6745	7578	.8411	.9245
1-8	.0104	.0938	.1771	.2604	3438	4271	.5104	.5938	.6771	.7604	.8438	.9271
5-32	0130	.0964	.1797	.2630	.3464	4297	.5130	5964	6797	7630	.8464	.9297
3-16	.0156	0990	.1823	2656	3490	.4323	.5156	.5990	6823	.7656	.8490	.9323
7-32	.0182	.1016	.1849	.2682	.3516	.4349	.5182	6016	.6849	.7682	.8516	.9349
1-4	.0208	1042	.1875	.2708	.3542	.4375	.5208	.6042	6875	.7708	.8542	9375
9-32	.0234	.1068	1901	.2734	.3568	.4401	.5234	.6068	.6901	.7734	.8568	9401
5-16	.0260	.1094	.1927	.2760	.3594	.4427	.5260	6094	.6927	.7760	.8594	.9427
11-32	.0286	.1120	.1953	.2786	3620	.4453	.5286	6120	.6953	.7786	.8620	.9453
3-8	.0313	.1146	.1979	.2813	3646	.4479	.5313	.6146	.6979	7813	.8646	.9479
13-32	.0339	.1172	.2005	.2839	3672	.4505	5339	.6172	.7005	.7839	.8672	.9505
7-16	.0365	.1198	.2031	.2865	3698	4531	.5365	.6198	.7031	.7865	.8698	.9531
15-32	.0391	.1224	.2057	.2891	3724	.4557	.5391	.6224	.7057	7891	.8724	.9557
1-2	.0417	.1250	.2083	.2917	3750	.4583	.5417	6250	.7083	.7917	.8750	9583
17-32	.0443	.1276	.2109	.2943	3776	4609	.5143	6276	7109	7943	.8776	.9609
9-16	0469	.1302	.2135	.2969	3802	.4635	.5469	.6302	.7135	7969	.8802	9635
19-32	.0495	.1328	.2161	.2995	.3828	.4661	.5495	.6328	.7161	.7995	.8828	.9661
5-8	0521	1354	.2188	.3021	3854	4688	5521	.6354	7188	.8021	.8854	9688
21-32	0547	.1380	.2214	.3047	3880	.4714	.5547	.6380	.7214	.8047	.8880	9714
11-16	.0573	.1406	2240	.3073	.3906	.4740	.5573	.6406	.7240	.8073	.8906	.9740
23-32	.0599	.1432	.2266	.3099	.3932	4766	.5599	.6432	.7266	.8099	.8932	9766
3-4	.0625	.1458	.2292	.3125	.3958	.4792	.5625	.6458	.7292	.8125	.8958	.9792
25-32	.0651	.1484	.2318	.3151	.3984	.4818	.5651	.6484	.7318	.8151	.8984	.9818
13-16	.0677	.1510	.2344	.3177	.4010	.4844	.5677	.6510	.7344	.8177	.9010	.9844
27-32	.0703	.1536	.2370	.3203	.4036	.4870	.5703	.6536	.7370	.8203	.9036	.9870
7-8	.0729	.1563	.2396	.3229	.4063	.4896	.5729	.6563	.7396	.8229	.9063	.9896
29-32	.0755	.1589	.2422	.3255	.4089	.4922	.5755	.6589	.7422	.8255	.9089	.9922
15-16	.0781	.1615	.2448	.3281	.4115	.4948	.5781	.6615	.7448	8281	.9115	.9948
31-32	.0807	.1641	.2474	.3307	.4141	.4974	.5807	.6641	.7474	.8307	.9141	.9974
	0	1	2	3	4	5	6	7	8	9	10	11

Decimal Equivalents for Fractions of an Inch

1/32	1/64	Decimals	Fractions	1/32	1/64	Decimals	Fractions
	1	0.015625			33	0.515625	
1	2	0.03125		17	34	0.53125	
	3	0.046875			35	0.546875	
2	4	0.0625	1/16	18	36	0.5625	9/16
	5	0.078125			37	0.578125	
3	6	0.09375		19	38	0.59375	
	7	0.109375			39	0.609375	
4	8	0.125	1/8	20	40	0.625	5/8
	9	0.140625			41	0.640625	
5	10	0.15625		21	42	0.65625	
	11	0.171875			43	0.671875	
6	12	0.1875	3/16	22	44	0.6875	11/16
	13	0.203125			45	0.703125	
7	14	0.21875		23	46	0.71875	
	15	0.234375			47	0.734375	
8	16	0.25	1/4	24	48	0.75	3/4
	17	0.265625			49	0.765625	
9	18	0.28125		25	50	0.78125	
	19	0.296875			51	0.796875	
10	20	0.3125	5/16	26	52	0.8125	13/16
	21	0.328125			53	0.828125	
11	22	0.34375		27	54	0.84375	
	23	0.359375			55	0.859375	
12	24	0.375	3/8	28	56	0.875	7/8
	25	0.390625			57	0.890625	
13	26	0.40625		29	58	0.90625	
	27	0.421875			59	0.921875	
14	28	0.4375	7/16	30	60	0.9375	15/16
	29	0.453125			61	0.953125	
15	30	0.46875		31	62	0.96875	
	31	0.484375			63	0.984375	
16	32	0.5	1/2	32	64	1.	1

Nautical Measures

A nautical or sea-mile is the length of a minute of longitude of the earth at the equator at the level of the sea. It is assumed that 6086.07 ft = 1.152664 statute or land-miles by the United States Coast Survey.

3 nautical miles = 1 league

Miscellaneous Measures

1 palm = 3 inches	1 span = 9 inches
1 hand = 4 inches	1 meter = 3.2809 feet

Measures of Surface

144 square inches	= 1 square foot	
9 square feet	= 1 square yard	= 1 296 square inches
100 square feet	= 1 square (architects' measure)	

Land Measure

$30\frac{1}{4}$ square yards	= 1 square rod		
40 square rods	= 1 square rood	= 1 210 square yards	
4 square roods }	= 1 acre	= 4 840 square yards	
10 square chains }	= 160 square rods		
{ 640 acres	= 1 square mile	= 3 097 600 square yards	= }
{ 102 400 square rods	= 2 560 square roods		}
208.71 feet square	= 1 acre	= 43 560 square feet	

A SECTION of land is a square mile, and a QUARTER-SECTION is 160 acres

Measures of Volume

1 gallon, liquid measure = 231 cubic inches, and contains 8.339 avoirdupois pounds of distilled water at 39.8° F., or 58 333 grains

1 cubic foot contains 7.48 liquid gallons, or 6.428 dry gallons

1 gallon, dry measure = 268.8 cubic inches

1 bushel (Winchester) contains 2 150.42 cubic inches, or 77.627 pounds distilled water at 39.8° F.

A heaped bushel contains 2 747.715 cubic inches

Dry Measure

2 pints	= 1 quart	=			67.2 cubic inches
4 quarts	= 1 gallon	= 8 pints	=		268.8 cubic inches
2 gallons	= 1 peck	= 16 pints	= 8 quarts	=	537.6 cubic inches
4 pecks	= 1 bushel	= 64 pints	= 32 quarts	=	8 gallons
					= 2 150.42 cubic inches

1 cord of wood = 128 cubic feet

Liquid Measure

4 gills	= 1 pint	=		16 fluid ounces
2 pints	= 1 quart	= 8 gills	=	32 fluid ounces
4 quarts	= 1 gallon	= 32 gills	= 8 pints	= 128 fluid ounces

In the United States and Great Britain 1 barrel of wine or brandy = $31\frac{1}{2}$ gallons, and contains 4 211 cubic feet.

A hogshead is 63 gallons, but this term is often applied to casks of various capacities.

Cubic Measure

1 728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

In MEASURING WOOD, a pile of wood cut 4 feet long, piled 4 feet high, and 8 feet on the ground, making 128 cubic feet, is called a CORD.

16 cubic feet make one cord-foot.

A PERCH OF STONE is nominally $16\frac{1}{2}$ feet long, 1 foot high and $1\frac{1}{2}$ feet thick, and contains $24\frac{3}{4}$ cubic feet.

A perch of stone is, however, often computed differently in different localities; thus, in most if not all of the States and Territories west of the Mississippi, stone-masons figure rubble by the perch of 16½ cubic feet. In Philadelphia, 22 cubic feet are called a perch. In Chicago, stone is measured by the cord of 100 cubic feet.

A TON of shipping is 42 cubic feet in Great Britain and 40 cubic feet in the United States.

Fluid Measure

60 minims	=	1 fluid drachm
8 fluid drachms	=	1 ounce
16 ounces	=	1 pint
8 pints	=	1 gallon

Miscellaneous Measures

Butt of Sherry = 108 gallons
Pipe of Port = 115 gallons
Butt of Malaga = 105 gallons
Puncheon of Scotch Whiskey = 110 to 130 gallons
Puncheon of Brandy = 110 to 120 gallons
Puncheon of Rum = 100 to 110 gallons
Hogshead of Brandy = 55 to 60 gallons
Hogshead of Claret = 46 gallons

Measures of Weight

The standard AVOIRDUPOIS POUND is the weight of 27.7015 cubic inches of distilled water weighed in air at 39.83° F., with the barometer at 30 inches. It contains 7 000 grains. One pound avoirdupois = 1.2153 pounds troy.

Avoirdupois, or Ordinary Commercial Weight

1 drachm	=	27.343 grains	
16 drachms	=	1 ounce	(oz)
16 ounces	=	1 pound	(lb)
100 pounds	=	1 hundredweight	(cwt)
20 hundredweight	=	1 ton	

In collecting duties upon foreign goods at the United States custom-houses, and also in freighting coal and selling it by wholesale,

28 pounds	=	1 quarter
4 quarters, or 112 pounds	=	1 hundredweight
20 hundredweight	=	1 long ton = 2 240 pounds
A stone	=	14 pounds
A quintal	=	100 pounds

The following measures are sanctioned by custom or law: 1 bushel = 1.244 cubic feet or 1¼ cubic feet, nearly.

32 pounds of oats	=	1 bushel
45 pounds of Timothy-seed	=	1 bushel
48 pounds of barley	=	1 bushel
56 pounds of rye	=	1 bushel
56 pounds of Indian corn	=	1 bushel
50 pounds of Indian meal	=	1 bushel
60 pounds of wheat	=	1 bushel
60 pounds of clover-seed	=	1 bushel
60 pounds of potatoes	=	1 bushel

56 pounds of butter	=	1 firkin
100 pounds of meal or flour	=	1 sack
100 pounds of grain or flour	=	1 cental
100 pounds of dry fish	=	1 quintal
100 pounds of nails	=	1 cask
196 pounds of flour	=	1 barrel
200 pounds of beef or pork	=	1 barrel
80 pounds of lime	=	1 bushel

Troy Weight

USED IN WEIGHING GOLD OR SILVER

24 grains	=	1 pennyweight	(pwt)
20 pennyweights	=	1 ounce	(oz)
12 ounces	=	1 pound	(lb)

A CARAT of the jewelers, for precious stones, is, in the United States, 3.2 grains, but it varies according to different authorities. In London, 3.17 grains, in Paris, 3.18 grains are divided into 4 jewelers' grains. The international carat is 3.168 grains or 200 milligrams. In troy, apothecaries' and avoirdupois weights, the grain is the same, 1 pound troy being equal to 0.82286 pound avoirdupois.

Apothecaries' Weight

USED IN COMPOUNDING MEDICINES AND IN PUTTING UP MEDICAL PRESCRIPTIONS

20 grains (gr)	=	1 scruple (℈)	8 drachms	=	1 ounce (oz)
3 scruples	=	1 drachm (ʒ)	12 ounces	=	1 pound (lb)

Measures of Value

UNITED STATES STANDARD

10 mills	=	1 cent	10 dimes	=	1 dollar
10 cents	=	1 dime	10 dollars	=	1 eagle

The STANDARD of gold and silver is 900 parts of pure metal and 100 of alloy in 1 000 parts of coin.

The FINENESS expresses the quantity of pure metal in 1 000 parts.

The REMEDY OF THE MINT is the allowance for deviation from the exact standard fineness and weight of coins.

Weights of Coins

Double eagle	=	516	troy grains
Eagle	=	258	troy grains
Dollar (gold)	=	25.8	troy grains
Dollar (silver)	=	412.5	troy grains
Half-dollar	=	192	troy grains
5-cent piece (nickel)	=	77.16	troy grains
Cent (bronze)	=	48	troy grains

Measures of Time

60 seconds = 1 minute		365 days = 1 common year	
60 minutes = 1 hour		366 days = 1 leap-year	
24 hours = 1 day			

A SOLAR DAY is measured by the rotation of the earth upon its axis, with respect to the sun.

In ASTRONOMICAL COMPUTATIONS and in NAUTICAL TIME the day commences at noon, and in the former it is counted throughout the 24 hours.

In CIVIL COMPUTATIONS the day commences at midnight, and is divided into two parts of 12 hours each.

A SOLAR YEAR is the time in which the earth makes one revolution around the sun. Its average time, called the MEAN SOLAR YEAR, is 365 days, 5 hours, 48 minutes and 49.7 seconds, or nearly 365¼ days.

A MEAN LUNAR MONTH, or LUNATION of the moon, is 29 days, 12 hours, 44 minutes, 2 seconds and 5.24 thirds. It is equal, on the average, to 29.53 days.

The Calendar, Old and New Style

The JULIAN Calendar was established by Julius Cæsar, 44 B.C., and by it one day was inserted in every fourth year. This was the same thing as assuming that the length of the solar year was 365 days and 6 hours, instead of the value given above, thus introducing an accumulative error of 11 minutes and 12 seconds every year. This calendar was adopted by the church in 325 A.D., at the Council of Nice. In the year 1582 the annual error of 11 minutes and 12 seconds had amounted to 10 days, which, by order of Pope Gregory XIII, was suppressed in the calendar, and the 5th of October reckoned as the 15th. To prevent the repetition of this error, it was decided to leave out three of the inserted days every 400 years, and to make this omission in the years which are not exactly divisible by 400. Thus, of the years 1700, 1800, 1900 and 2000, all of which are leap-years according to the Julian Calendar, only the last is a leap-year according to the REFORMED or GREGORIAN Calendar. This Reformed Calendar was not adopted by England until 1752, when 11 days were omitted from the calendar. The two calendars are now often called the OLD STYLE and the NEW STYLE. The latter style is now adopted in every Christian country except Russia.

Circular and Angular Measures

USED FOR MEASURING ANGLES AND ARCS, AND FOR DETERMINING LATITUDE AND LONGITUDE

60 seconds (″)	= 1 minute	(′)
60 minutes	= 1 degree	(°)
360 degrees	= 1 circumference	(C)

The SECOND is usually subdivided into tenths and hundredths.

A MINUTE of the circumference of the earth is a geographical mile.

The DEGREES of the earth's circumference on a meridian average 69.16 common miles.

The Metric System

The METRIC SYSTEM is a system of weights and measures based upon a unit called a METER.

The METER was intended to be one ten-millionth part of the distance from

the equator to either pole, measured on the earth's surface at the level of the sea.

The NAMES of derived metric denominations are formed by prefixing to the name of the primary unit of measure:

Milli, a thousandth	Hecto, one hundred
Centi, a hundredth	Kilo, a thousand
Deci, a tenth	Myria, ten thousand
Deca, ten	

This system, first adopted by France, has been extensively adopted by other countries, and is much used in the sciences and the arts. It was legalized in 1866 by Congress to be used in the United States, and is already employed by the Coast Survey, and, to some extent, by the Mint and the General Post-Office.

Linear Measures

The METER is the primary unit of lengths

10 millimeters (mm)	= 1 centimeter (cm)	= 0.3937 inch
10 centimeters	= 1 decimeter (dm)	= 3.937 inches
10 decimeters	= 1 METER (m)	= 39.37 inches
10 meters	= 1 decameter	= 393 7 inches
10 decameters	= 1 hectometer	= 328 feet 1 inch
10 hectometers	= 1 KILOMETER (km)	= 0.62137 mile
10 kilometers	= 1 myriameter	= 6.2137 miles

The METER is used in ordinary measurements; the CENTIMETER, or MILLIMETER, in reckoning very small distances; and the KILOMETER, for roads or great distances.

A CENTIMETER is about $\frac{3}{8}$ of an inch; a METER is about 3 feet $3\frac{3}{8}$ inches; a KILOMETER is about 200 rods, or $\frac{5}{8}$ of a mile.

Measures of Surface

The SQUARE METER is the primary unit of ordinary surfaces.

The ARE, a square, each of whose sides is ten METERS, is the unit of land measures.

100 square millimeters (mm^2) = 1 square centimeter (cm^2) = 0.155 square inch
100 square centimeters = 1 square decimeter = 15.5 square inches
100 square decimeters = 1 square METER (m^2) = 1 550 square inches, or 1.196 square yards
100 centiares, or square meters = 1 ARE (a) = 119.6 square yards
100 ares = 1 hectare (ha) = 2.471 acres

A SQUARE METER, or one CENTIARE, is about $10\frac{3}{4}$ square feet, or $1\frac{1}{5}$ square yards, and a HECTARE is about $2\frac{1}{2}$ acres.

Cubic Measure

The CUBIC METER, or STERE, is the primary unit of a volume.

1 000 cubic millimeters (mm^3) = 1 cubic centimeter (cm^3) = 0.061 cubic inch
1 000 cubic centimeters = 1 cubic decimeter (dm^3) = 61.022 cubic inches
1 000 cubic decimeters = 1 cubic METER (m^3) = 35.314 cubic feet

The STERE is the name given to the cubic meter in measuring wood and timber. A tenth of a stere is a DECISTERE, and ten steres are a DECASTERE.

A CUBIC METER, or STERE, is about $1\frac{1}{3}$ cubic yards, or about $2\frac{1}{5}$ cord feet.

Liquid and Dry Measures

The LITER is the primary unit of measures of capacity, and is a cube, each of whose edges is a tenth of a meter in length.

The HECTOLITER is the unit in measuring large quantities of grain, fruits, roots and liquids.

10 milliliters (ml)	= 1 centiliter (cl)	= 0.338 fluid ounce
10 centiliters	= 1 deciliter	= 0.845 liquid gill
10 deciliters	= 1 LITER (l)	= 1.0567 liquid quarts
10 liters	= 1 decaliter	= 2.6417 gallons
10 decaliters	= 1 HECTOLITER (hl)	= 2 bushels, 3.35 pecks
10 hectoliters	= 1 kiloliter	= 28 bushels, 1½ pecks

A CENTILITER is about ⅓ of a fluid ounce; a LITER is about 1⅛ liquid quarts, or 9/10 of a dry quart; a HECTOLITER is about 2⅚ bushels; and a KILOLITER is one cubic meter, or stere.

Weights

The GRAM is the primary unit of weights, and is the weight in a vacuum of a cubic centimeter of distilled water at the temperature of 39.2° F.

10 milligrams (mg)	= 1 centigram (cg)	= 0.1543	troy grain
10 centigrams	= 1 decigram (dg)	= 1.543	troy grains
10 decigrams	= 1 GRAM (g)	= 15.432	troy grains
10 grams	= 1 decagram	= 0.3527	avoirdupois ounce
10 decagrams	= 1 hectogram	= 3.5274	avoirdupois ounces
10 hectograms	= 1 KILOGRAM (kg)	= 2.2046	avoirdupois pounds
10 kilograms	= 1 myriagram	= 22 046	avoirdupois pounds
10 myriagrams	= 1 quintal (q)	= 220.46	avoirdupois pounds
10 quintals	= 1 TONNEAU (t)	= 2204.6	avoirdupois pounds

1 kilogram per kilometer = 0.67195 pound per 1 000 feet
1 pound per thousand feet = 1.4882 kilograms per kilometer
1 kilogram per square millimeter = 1 423 pounds per square inch
1 pound per square inch = 0.000743 kilogram per square millimeter

The GRAM is used in weighing gold, jewels, letters and small quantities of things. The KILOGRAM, or, for brevity, KILO, is used by grocers; and the TONNEAU, or METRIC TON, is used in finding the weight of very heavy articles.

A GRAM is about 15½ grains troy; the KILO about 2⅕ pounds avoirdupois; and the METRIC TON, about 2 205 pounds.

A KILO is the weight of a liter of water at its greatest density; and the METRIC TON, of a cubic meter of water.

Metric numbers are written with the decimal point (.) at the right of the figures denoting the unit; thus the expression, 15 meters 3 centimeters, is written, 15.03 m.

When metric numbers are expressed by figures, the part of the expression at the left of the decimal point is read as the number of the unit, and the part at the right, if any, as a number of the lowest denomination indicated, or as a decimal part of the unit; thus, 46.525 m is read 46 meters and 525 millimeters, or 46 and 525 thousandths meters.

In writing and reading metric numbers, according as the scale is 10, 100 or 1 000, each denomination should be allowed one, two or three orders of figures.

Metric Conversion Table

The following metric conversion table has been compiled by C. W. Hunt, and is most convenient in dealing with metric weights and measures:

Millimeters × 0.03937	= inches
Millimeters ÷ 25.4	= inches
Centimeters × 0.3937	= inches
Centimeters ÷ 2.54	= inches
Meters × 39.37	= inches. (Act of Congress)
Meters × 3.281	= feet
Meters × 1.094	= yards
Kilometers × 0.621	= miles
Kilometers ÷ 1.6093	= miles
Kilometers × 3280.7	= feet
Square millimeters × 0.0155	= square inches
Square millimeters ÷ 645.1	= square inches
Square centimeters × 0.155	= square inches
Square centimeters ÷ 6.451	= square inches
Square meters × 10.764	= square feet
Square kilometers × 247.1	= acres
Hectares × 2.471	= acres
Cubic centimeters ÷ 16.383	= cubic inches
Cubic centimeters ÷ 3.69	= fluid drachms. (U. S. Pharmacopœia)
Cubic centimeters ÷ 29.57	= fluid ounce. (U. S. Pharmacopœia)
Cubic meters × 35.315	= cubic feet
Cubic meters × 1.308	= cubic yards
Cubic meters × 264.2	= gallons (231 cubic inches)
Liters × 61.022	= cubic inches. (Act of Congress)
Liters × 33.84	= fluid ounces. (U S. Pharmacopœia)
Liters × 0.2642	= gallons (231 cubic inches)
Liters ÷ 3 78	= gallons (231 cubic inches)
Liters ÷ 28.316	= cubic feet
Hectoliters × 3.531	= cubic feet
Hectoliters × 2.84	= bushels (2 150 42 cubic inches)
Hectoliters × 0.131	= cubic yards
Hectoliters × 26.42	= gallons (231 cubic inches)
Grams × 15.432	= grains. (Act of Congress)
Grams × 981	= dynes
Grams (water) ÷ 29.57	= fluid ounces
Grams ÷ 28.35	= ounces avoirdupois
Grams per cubic centimeter ÷ 27.7	= pounds per cubic inch
Joule × 0.7373	= foot-pounds
Kilograms × 2.2046	= pounds
Kilograms × 35.3	= ounces avoirdupois
Kilograms × 0.0011023	= tons (2 000 pounds)
Kilograms per sq cm × 14.223	= pounds per square inch
Kilogrammeters × 7.233	= foot-pounds
Kilograms per meter × 0.672	= pounds per foot
Kilograms per cubic meter × 0.062	= pounds per cubic foot
Kilograms per cheval-vapeur × 2.235	= pounds per horse-power
Kilowatts × 1.34	= horse-power
Watts ÷ 746	= horse-power
Watts × 0.7373	= foot-pounds per second
Kilogram-calorie × 3.968	= British thermal units (B.T.U.)
Cheval-vapeur × 0.9863	= horse-power
(Centigrade × 1.8) + 32	= degrees Fahrenheit
Gravity, Paris	= 980.94 centimeters per second

Metric Conversion Tables. This and the following table from Molesworth's Metrical Tables will be found of great convenience in figuring plans to be executed in countries using the metric system.

Feet Converted into Meters

Feet	0	1	2	3	4
0	.... .	0 304794	0.609589	0 914383	1 21918
10	3 047945	3.35274	3.65753	3 96233	4 26712
20	6 095890	6.40068	6.70548	7 01027	7 31507
30	9.143835	9.44863	9.75342	10 0582	10.3630
40	12.19178	12.4966	12.8014	13.1062	13.4110
50	15 23972	15.5445	15 8493	16.1541	16.4589
60	18 28767	18 5925	18 8973	19 2020	19 5068
70	21 33561	21.6404	21.9452	22 2500	22.5548
80	24 38356	24.6884	24.9931	25.2979	25.6027
90	27 43150	27.7363	28 0411	28 3459	28 6507

Scripture and Ancient Measures and Weights

Scripture Long Measures

	Inches
Digit	= 0.912
Palm	= 3.648
Span	= 10.944

	Feet	Inches
Cubit	= 1	9.888
Fathom	= 7	3.552

Egyptian Long Measures

Nahud cubit = 1 foot 5.71 inches   Royal cubit = 1 foot 8.66 inches

Grecian Long Measures

	Feet	Inches
Digit	=	0.7554
Pous (foot)	= 1	0.0875
Cubit	= 1	1.5984⅜

	Feet	Inches
Stadium	= 604	4.5
Mile	= 4 835	

Jewish Long Measures

Cubit	= 1.824 feet	Mile	= 7 296 feet
Sabbath-day's journey	= 3 648 feet	Day's journey	= 33.164 miles

Roman Long Measures

	Inches
Digit	= 0.72575
Uncia (inch)	= 0.967
Pes (foot)	= 11.604

	Feet	Inches
Cubit	= 1	5.406
Passus	= 4	10.02
Mille (millarium)	= 4 842	

Roman Weight

Ancient libbra = 0.7094 pound

Miscellaneous

	Feet
Arabian foot	= 1.095
Babylonian foot	= 1.140
Egyptian finger	= 0.06145

	Feet
Hebrew foot	= 1.212
Hebrew cubit	= 1.817
Hebrew sacred cubit	= 2.002

Feet Converted into Meters (Continued)

Feet	5	6	7	8	9
0	1.52397	1.82877	2 13356	2.43836	2.74315
10	4.57192	4.87671	5.18151	5.48630	5.79110
20	7.61986	7.92466	8.22945	8.53425	8.83904
30	10.6678	10.9726	11.2774	11.5822	11.8870
40	13.7158	14.0205	14.3253	14.6301	14.9349
50	16.7637	17.0685	17.3733	17.6781	17.9829
60	19.8116	20.1164	20.4212	20.7260	21.0308
70	22.8596	23.1644	23.4692	23.7740	24.0788
80	25.9075	26.2123	26.5171	26 8219	27.1267
90	28.9555	29 2603	29.5651	29 8699	30.1747

Example. 44 ft = 13.411 meters = 134.11 decimeters = 1 341.1 centimeters = 13 411 millimeters.

The above-mentioned work contains eighty pages of conversion tables similar to the above.

Inches and Sixteenths Converted into Millimeters

Inches	0	1	2	3	4	5
. . .	. .	25.400	50 799	76 199	101 60	127 00
1/16	1 5875	26 987	52 387	77.786	103.19	128.59
1/8	3.1749	28.574	53 974	79.374	104.77	130.17
3/16	4.7624	30.162	55.561	80.961	106.36	131.76
1/4	6.3499	31.749	57.149	82.549	107.95	133.35
5/16	7.9374	33.337	58.736	84.136	109.54	134.94
3/8	9 5248	34 924	60.324	85 723	111.12	136.52
7/16	11.112	36 512	61.911	87.311	112.71	138 11
1/2	12.700	38.099	63 499	88.898	114 30	139.70
9/16	14 287	39.687	65 086	90.486	115 89	141.28
5/8	15.875	41.274	66.674	92 073	117.47	142.87
11/16	17.462	42.862	68.261	93.661	119.06	144.46
3/4	19.050	44.449	69 849	95.248	120.65	146.05
13/16	20 637	46.037	71.436	96.836	122.24	147 63
7/8	22 225	47.624	73 024	98.423	123 82	149 22
15/16	23 812	49 212	74 611	100 01	125 41	150 81

Inches	6	7	8	9	10	11
. . . .	152 40	177 80	203 20	228.60	254 00	279 39
1/16	153.98	179.38	204.78	230.18	255.58	280.98
1/8	155.57	180.97	206.37	231.77	257.17	282.57
3/16	157.16	182.56	207.96	233.36	258.76	284.16
1/4	158.75	184.15	209 55	234.95	260.35	285 74
5/16	160.33	185.73	211 13	236.53	261.93	287.33
3/8	161.92	187.32	212.72	238.12	263.52	288.92
7/16	163.51	188.91	214.31	239.71	265.11	290.51
1/2	165.10	190.50	215.90	241.30	266.70	292.09
9/16	166.68	192.08	217.48	242.88	268.28	293.68
5/8	168.27	193.67	219.07	244.47	269.87	295.27
11/16	169.86	195.26	220.66	246.06	271.46	296.86
3/4	171.45	196.85	222.25	247.65	273.05	298.44
13/16	173.03	198.43	223.83	249.23	274.63	300.03
7/8	174.62	200.02	225.42	250.82	276.22	301.62
15/16	176.21	201.61	227.01	252.41	277.81	303.21

For meters, move the decimal point THREE figures forward.

Example. 8 3/16 inches = 207.96 millimeters = 20.796 centimeters = 2.0796 decimeters = 0.20796 meter.

3. GEOMETRY AND MENSURATION

Definitions

A POINT is that which has only position.

A PLANE is a surface in which, any two points being taken, the straight line joining them will be wholly in the surface.

A CURVED LINE is a line of which no part is straight (Fig. 1).

Fig. 1. Curved Line

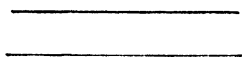

Fig. 2. Parallel Lines

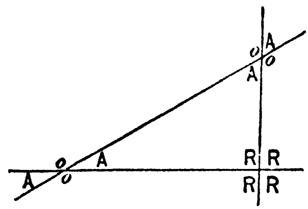

Fig. 3. Angles

PARALLEL LINES are such as are wholly in the same plane, and have the same direction (Fig. 2).

A BROKEN LINE is a line composed of a series of dashes; thus, —— — — ——.

An ANGLE is the opening between two lines meeting at a point, and is termed a RIGHT ANGLE when the two lines are perpendicular to each other, an ACUTE ANGLE when it is less or sharper than a right angle, and an OBTUSE ANGLE when it is greater than a right angle. Thus, in Fig. 3,

$A\ A\ A\ A$ are ACUTE ANGLES,

$o\ o\ o\ o$ are OBTUSE ANGLES and $R\ R\ R\ R$ are RIGHT ANGLES.

Polygons

A POLYGON is a portion of a plane bounded by straight lines.

A TRIANGLE is a polygon of three sides.

A SCALENE TRIANGLE has none of its sides equal; an ISOSCELES TRIANGLE has two of its sides equal; an EQUILATERAL TRIANGLE has all three of its sides equal.

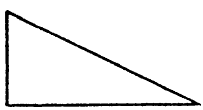

Fig. 4. Right-angled Triangle

Fig. 5. Scalene Triangle

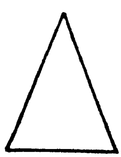

Fig. 6. Isosceles Triangle

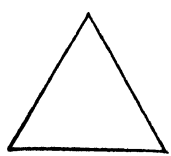

Fig. 7. Equilateral Triangle

A RIGHT-ANGLED TRIANGLE is one which has a right angle. The side opposite the right angle is called the HYPOTHENUSE; the side on which the triangle is supposed to stand is called its BASE and the other side, its ALTITUDE.

A QUADRILATERAL is a polygon of four sides.

Quadrilaterals are divided into classes, as follows: the TRAPEZIUM (Fig. 8), which has no two of its sides parallel; the TRAPEZOID (Fig. 9), which has two of its sides parallel; and the PARALLELOGRAM (Fig. 10), which is bounded by two pairs of parallel sides.

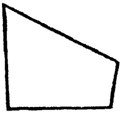

Fig. 8. Trapezium

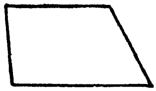

Fig. 9. Trapezoid

Fig. 10. Parallelogram

A parallelogram whose sides are not equal and whose angles are not right angles is called a RHOMBOID (Fig. 11); when the sides are all equal, but the angles are not right angles, it is called a RHOMBUS (Fig. 12), and when the angles are right angles, it is called a RECTANGLE (Fig. 13). A rectangle, all of whose sides are equal, is called a SQUARE (Fig. 14). Polygons, all of whose sides are equal, are called REGULAR POLYGONS.

Fig 11.
Rhomboid

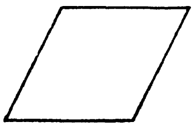

Fig. 12.
Rhombus

Fig 13
Rectangle

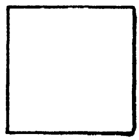

Fig. 14.
Square

Besides the square and equilateral triangles, there are: the PENTAGON (Fig. 15), which has five sides; the HEXAGON (Fig. 16), which has six sides; the HEPTAGON (Fig. 17), which has seven sides; and the OCTAGON (Fig. 18), which has eight sides.

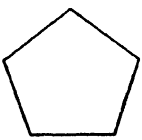

Fig. 15.
Pentagon

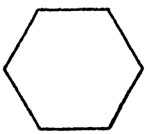

Fig. 16.
Hexagon

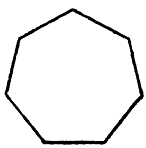

Fig. 17.
Heptagon

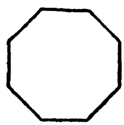

Fig. 18.
Octagon

The ENNEAGON or NONAGON has nine sides; the DECAGON has ten sides; and the DODECAGON has twelve sides.

For all polygons, the side upon which it is supposed to stand is called its BASE; the perpendicular distance from the highest side or angle to the base (prolonged, if necessary) is called the ALTITUDE; and a line joining any two angles not adjacent is called a DIAGONAL.

A PERIMETER is the bounding line of a plane figure.

A CIRCLE is a portion of a plane bounded by a curve, all the points of which are equidistant from a point within, called the CENTER (Fig. 19).

The CIRCUMFERENCE is the curve which bounds the circle.

A RADIUS is any straight line drawn from the center to the circumference.

Any straight line drawn through the center to the circumference on each side is called a DIAMETER.

An ARC of a circle is any part of its circumference.

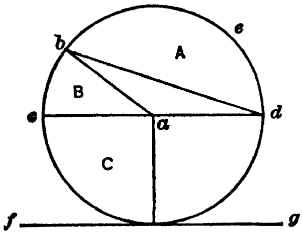

Fig. 19. Circle and Parts

A CHORD is any straight line joining two points of the circumference, as *bd*, Fig. 19.

A SEGMENT is a portion of the circle included between the arc and its chord, as *A*, Fig. 19.

A SECTOR is the space included between an arc and two radii drawn to its extremities, as *B*, Fig. 19. In the figure, *ab* is a radius, *cd* a diameter and *db* a chord SUBTENDING the arc *bed*. A TANGENT is a right line which in passing a curve touches without cutting it, as *fg*, Fig. 19.

Volumes

A PRISM is a volume whose ends are equal and parallel polygons and whose sides are parallelograms.

A prism is TRIANGULAR, RECTANGULAR, etc., according as its ends are TRIANGLES, RECTANGLES, etc.

A CUBE is a rectangular prism all of whose sides are squares.

A CYLINDER is a volume of uniform diameter, bounded by a curved surface and two equal and opposite parallel circles.

A PYRAMID is a volume whose base is a polygon and whose sides are triangles meeting in a point called the VERTEX. A pyramid is triangular, quadrangular, etc., according as its base is a triangle, quadrilateral, etc.

A CONE is a volume whose base is a circle, from which the remaining surface tapers uniformly to a point or vertex (Fig. 20).

A CONIC SECTION is the plane figure made by a plane cutting a cone.

An ELLIPSE is the section of a cone cut by a plane passing obliquely through both sides, as at *ab*, Fig. 21.

A PARABOLA is a section of a cone cut by a plane parallel to its side, as at *cd*.

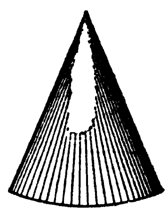

Fig. 20. Cone

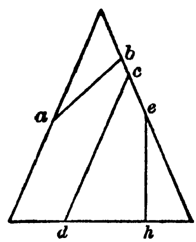

Fig. 21. Cone with Section-lines

A HYPERBOLA is a section of a cone cut by a plane making a greater angle with the base than that made by the side of the cone, as at *eh*.

In the ellipse, the TRANSVERSE AXIS, or LONG DIAMETER, is the longest line that can be drawn in it. The CONJUGATE AXIS, or SHORT DIAMETER, is a line drawn through the center at right-angles to the long diameter.

A FRUSTUM OF A PYRAMID or CONE is that which remains after cutting off the upper part of it by a plane parallel to the base.

A SPHERE is a volume bounded by a curved surface, all points of which are equidistant from a point within, called the center.

Mensuration treats of the measurement of lines, surfaces and volumes.

Rules

To compute the area of a square, a rectangle, a rhombus or a rhomboid.

Rule. Multiply the length by the breadth or height. Thus, in Figs. 22, 23 or 24, the area = $ab \times bc$.

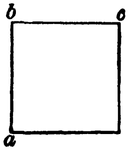

Fig. 22. Square

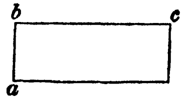

Fig. 23. Rectangle

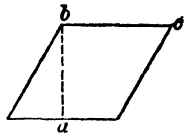

Fig. 24. Parallelogram

To compute the area of a triangle.

Rule. Multiply the base by the altitude and divide by 2. Thus, in Fig. 25, the area of $abc = \dfrac{ab \times cd}{2}$.

To find the length of the hypothenuse of a right-angled triangle when both sides are known.

Rule. Square the length of each of the sides making the right angle, add their squares together and take the square root of their sum. Thus (Fig. 26), the length of $ac = 3$, and of $bc = 4$; then

$$ab = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$$

$$\sqrt{25} = 5, \quad \text{or} \quad ab = 5$$

To find the length of the base or altitude of a right-angled triangle when the length of the hypothenuse and one side is known.

Rule. From the square of the length of the hypothenuse subtract the square of the length of the other side and take the square root of the remainder.

To find the area of a trapezium (Fig. 27).

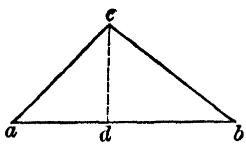

Fig. 25.
Scalene Triangle

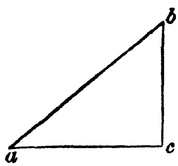

Fig. 26
Right-angled Triangle

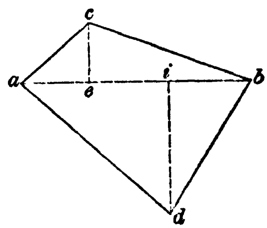

Fig. 27.
Trapezium

Rule. Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles and divide the product by 2. Thus,

$$\frac{ab \times (ce + di)}{2} = \text{area}$$

To find the area of a trapezoid (Fig. 28).

Rule. Multiply the sum of the two parallel sides by the perpendicular distance between them and divide the product by 2.

To compute the area of an irregular polygon.

Rule. Divide the polygon into triangles by means of diagonal lines and then add together the areas of all the triangles, as *A*, *B* and *C* (Fig. 29).

To find the area of a regular polygon.

Rule. Multiply the length of a side by the perpendicular distance to the center (as *ao*, Fig 30) multiply that product by the number of sides and divide the result by 2

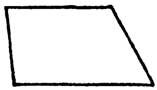

Fig 28 Trapezoid

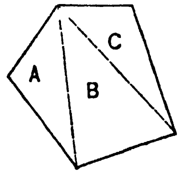

Fig 29 Irregular Polygon

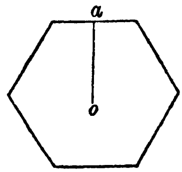

Fig 30 Regular Polygon

To compute the area of a regular polygon when the length, only, of a side is given.

Rule. Multiply the square of the side by the multiplier opposite the name of the polygon in column *A* of the following table

Table of Factors for Determining the Elements of Polygons

Name of polygon	Number of sides	A Factor for area	B Factor for radius of circumscribing circle	C Factor for length of the sides	D Factor for radius of inscribed circle
Triangle	3	0 433013	0 5773	1 732	0 2887
Tetragon	4	1	0 7071	1 4142	0 5
Pentagon	5	1 720477	0 8506	1 1756	0 6882
Hexagon	6	2 598076	1	1	0 866
Heptagon	7	3 633912	1 1524	0 8677	1 0383
Octagon	8	4 828427	1 3066	0 7653	1 2071
Nonagon	9	6 181824	1 4619	0 684	1 3737
Decagon	10	7 694209	1 618	0 618	1 5383
Undecagon	11	9 365564	1 7747	0 5634	1 7028
Dodecagon	12	11 196152	1 9319	0 5176	1 866

To compute the radius of a circle circumscribed about a regular polygon when the length, only, of a side is given.

Rule. Multiply the length of a side of the polygon by the number in column *B* of table

Example. What is the radius of a circle that will contain a hexagon, the length of one side being 5 in?

Solution. $5 \times 1 = 5$ in

To compute the length of a side of a regular polygon inscribed in a given circle, when the radius of the circle is given.

Rule. Multiply the radius of the circle by the number opposite the name of the polygon in column *C* of table

Example. What is the length of the side of a pentagon contained in a circle 8 ft in diameter?

Solution. 8 ft diameter ÷ 2 = 4 ft radius; 4 × 1 1756 = 4 7024 ft

To compute the length of a side of a regular polygon, when the radius of the inscribed circle is given.

Rule. Divide the radius of the inscribed circle by the number opposite the name of the polygon in column D of table.

To compute the radius of a circle that can be inscribed in a given regular polygon, when the length of a side is given.

Rule. Multiply the length of a side of the polygon by the number opposite the name of the polygon in column D.

Example. What is the radius of the circle that can be inscribed in an octagon, the length of one side being 6 in?

Solution. $6 \times 1.2071 = 7.2426$ in.

Circles

To compute the circumference of a circle.

Rule. Multiply the diameter by 3.1416. For many purposes, the multiplier $3\frac{1}{7}$ gives sufficiently accurate results.

Example. What is the circumference of a circle 7 in in diameter?

Solution. $7 \times 3.1416 = 21.9912$ in, or $7 \times 3\frac{1}{7} = 22$ in, the error in this last result being 0.0088 in.

To find the diameter of a circle when the circumference is given.

Rule. Divide the circumference by 3.1416, or for a very close approximate result, multiply by 7 and divide by 22.

To find the radius of an arc when the chord and rise or versed sine are given.

Rule. Square ONE-HALF the CHORD and the RISE; divide the sum of these squares by twice the rise; the result will be the radius.

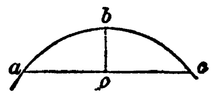

Fig. 31. Circular Arc, Chord and Rise

Example. The length of the chord ac, Fig. 31, is 48 in, and the rise, bo, is 6 in. What is the radius of the arc?

Solution. $$\text{Radius} = \frac{oc^2 + bo^2}{2\,bo} = \frac{24^2 + 6^2}{12} = 51 \text{ in}$$

To find the rise or versed sine of a circular arc, when the chord and radius are given.

Rule. Square the radius; also square one-half the chord; subtract the latter from the former and take the square root of the remainder. Subtract the result from the radius and the remainder will be the rise.

Example. A given arc has a radius of 51 in and a chord of 48 in. What is the rise?

Solution. $$\text{Rise} = \text{radius} - \sqrt{\text{radius}^2 - \tfrac{1}{2}\text{chord}^2} = 51 - \sqrt{2\,601 - 576}$$
$$= 51 - 45 = 6 \text{ in} = \text{rise}$$

To compute the area of a circle

Rule. Multiply the square of the diameter by 0.7854, or multiply the square of the radius of 3.1416.

Example. What is the area of a circle 10 in in diameter?

Solution. $10 \times 10 \times 0.7854 = 78.54$ sq in, or $5 \times 5 \times 3.1416 = 78.54$ sq in.

Tables of Areas and Circumferences of Circles

The following tables will be found very convenient for finding the circumferences and areas of circles.

Areas and Circumferences of Circles

For diameters from 1/10 to 100, advancing by tenths

Dia.	Area.	Circum.	Dia.	Area.	Circum	Dia.	Area.	Circum.
0.0			5.0	19.6350	15.7080	10.0	78.5398	31.4159
.1	0.007854	0.31416	.1	20.4282	16 0221	.1	80.1185	31.7301
.2	0.031416	0.62832	.2	21.2372	16 3363	.2	81.7128	32 0442
.3	0.070686	0.94248	.3	22.0618	16.6504	.3	83.3229	32.3584
.4	0.12566	1.2566	.4	22.9022	16.9646	.4	84.9487	32.6726
.5	0.19635	1.5708	.5	23.7583	17.2788	.5	86.5901	32.9867
.6	0.28274	1.8850	.6	24.6301	17.5929	.6	88.2473	33.3009
.7	0.38485	2.1991	.7	25.5176	17.9071	.7	89.9202	33.6150
.8	0.50266	2.5133	.8	26.4208	18.2212	.8	91.6088	33.9292
.9	0.63617	2.8274	.9	27.3397	18.5354	.9	93.3132	34.2434
1.0	0.7854	3.1416	6.0	28.2743	18.8496	11.0	95.0332	34.5575
.1	0.9503	3.4558	.1	29.2247	19.1637	.1	96.7689	34.8717
.2	1.1310	3.7699	.2	30.1907	19.4779	.2	98.5203	35.1858
.3	1.3273	4.0841	.3	31.1725	19.7920	.3	100.2875	35.5000
.4	1.5394	4.3982	.4	32.1699	20.1062	.4	102.0703	35.8142
.5	1.7671	4.7124	.5	33.1831	20.4204	.5	103.8689	36.1283
.6	2.0106	5.0265	.6	34.2119	20.7345	.6	105.6832	36.4425
.7	2.2698	5.3407	.7	35.2565	21.0487	.7	107.5132	36 7566
.8	2.5447	5.6549	.8	36.3168	21.3628	.8	109 3588	37.0708
.9	2.8353	5.9690	.9	37.3928	21.6770	.9	111.2202	37.3850
2.0	3.1416	6.2832	7.0	38.4845	21.9911	12.0	113.0973	37.6991
.1	3 4636	6.5973	.1	39 5919	22.3053	.1	114.9901	38 0133
.2	3.8013	6.9115	.2	40.7150	22.6195	.2	116.8987	38 3274
.3	4.1548	7.2257	.3	41.8539	22.9336	.3	118.8229	38.6416
.4	4.5239	7.5398	.4	43.0084	23.2478	.4	120.7628	38.9557
.5	4.9087	7.8540	.5	44.1786	23.5619	.5	122.7185	39.2699
.6	5.3093	8.1681	.6	45.3646	23.8761	.6	124.6898	39.5841
.7	5.7256	8.4823	.7	46 5663	24.1903	.7	126.6769	39.8982
.8	6.1575	8.7965	.8	47.7836	24.5044	.8	128.6796	40.2124
.9	6.6052	9.1106	.9	49.0167	24.8186	.9	130.6981	40 5265
3.0	7.0686	9.4248	8.0	50 2655	25 1327	13.0	132.7323	40.8407
.1	7.5477	9.7389	.1	51.5300	25.4469	.1	134.7822	41.1549
.2	8.0425	10.0531	.2	52.8102	25.7611	.2	136.8478	41 4690
.3	8.5530	10.3673	.3	54.1061	26.0752	.3	138.9291	41.7832
.4	9.0792	10.6814	.4	55.4177	26.3894	.4	141.0261	42.0973
.5	9.6211	10.9956	.5	56.7450	26.7035	.5	143.1388	42.4115
.6	10.1788	11.3097	.6	58 0880	27.0177	.6	145.2672	42.7257
.7	10.7521	11.6239	.7	59.4468	27.3319	.7	147.4114	43 0398
.8	11.3411	11.9381	.8	60.8212	27.6460	.8	149 5712	43.3540
.9	11.9459	12.2522	.9	62.2114	27.9602	.9	151.7468	43.6681
4.0	12.5664	12.5664	9.0	63 6173	28.2743	14.0	153.9380	43.9823
.1	13.2025	12.8805	1	65.0388	28.5885	.1	156.1450	44.2965
.2	13.8544	13.1947	.2	66.4761	28.9027	.2	158.3677	44 6106
.3	14.5220	13.5088	.3	67.9291	29.2168	.3	160.6061	44.9248
.4	15.2053	13.8230	.4	69.3978	29.5310	.4	162.8602	45.2389
.5	15.9043	14.1372	.5	70.8822	29.8451	.5	165.1300	45.5531
.6	16.6190	14.4513	.6	72.3823	30.1593	.6	167.4155	45.8673
.7	17.3494	14.7655	.7	73.8981	30.4734	.7	169.7167	46.1814
.8	18.0956	15.0796	.8	75.4296	30.7876	.8	172.0336	46.4956
.9	18.8574	15.3938	.9	76.9769	31.1018	.9	174.3662	46.8097

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area	Circum	Dia.	Area	Circum.	Dia	Area	Circum.
15.0	176.7146	47.1239	20.0	314.1593	62.8319	25.0	490.8739	78.5398
.1	179.0786	47.4380	.1	317.3087	63.1460	.1	494.8087	78.8540
.2	181.4584	47.7522	.2	320.4739	63.4602	.2	498.7592	79.1681
.3	183.8539	48.0664	.3	323.6547	63.7743	.3	502.7255	79.4823
.4	186.2650	48.3805	.4	326.8513	64.0885	.4	506.7075	79.7965
.5	188.6919	48.6947	.5	330.0636	64.4026	.5	510.7052	80.1106
.6	191.1345	49.0088	.6	333.2916	64.7168	.6	514.7185	80.4248
.7	193.5928	49.3230	.7	336.5353	65.0310	.7	518.7476	80.7389
.8	196.0668	49.6372	.8	339.7947	65.3451	.8	522.7924	81.0531
.9	198.5565	49.9513	.9	343.0698	65.6593	.9	526.8529	81.3672
16.0	201.0619	50.2655	21.0	346.3606	65.9734	26.0	530.9292	81.6814
.1	203.5831	50.5796	.1	349.6671	66.2876	.1	535.0211	81.9956
.2	206.1199	50.8938	.2	352.9894	66.6018	.2	539.1287	82.3097
.3	208.6724	51.2080	.3	356.3273	66.9159	.3	543.2521	82.6239
.4	211.2407	51.5221	.4	359.6809	67.2301	.4	547.3911	82.9380
.5	213.8246	51.8363	.5	363.0503	67.5442	.5	551.5459	83.2522
.6	216.4243	52.1504	.6	366.4354	67.8584	.6	555.7163	83.5664
.7	219.0397	52.4646	.7	369.8361	68.1726	.7	559.9025	83.8805
.8	221.6708	52.7788	.8	373.2526	68.4867	.8	564.1044	84.1947
.9	224.3176	53.0929	.9	376.6848	68.8009	.9	568.3220	84.5088
17.0	226.9801	53.4071	22.0	380.1327	69.1150	27.0	572.5553	84.8230
.1	229.6583	53.7212	.1	383.5963	69.4292	.1	576.8043	85.1372
.2	232.3522	54.0354	.2	387.0756	69.7434	.2	581.0690	85.4513
.3	235.0618	54.3496	.3	390.5707	70.0575	.3	585.3494	85.7655
.4	237.7871	54.6637	.4	394.0814	70.3717	.4	589.6455	86.0796
.5	240.5282	54.9779	.5	397.6078	70.6858	.5	593.9574	86.3938
.6	243.2849	55.2920	.6	401.1500	71.0000	.6	598.2849	86.7080
.7	246.0574	55.6062	.7	404.7078	71.3142	.7	602.6282	87.0221
.8	248.8456	55.9203	.8	408.2814	71.6283	.8	606.9871	87.3363
.9	251.6494	56.2345	.9	411.8707	71.9425	.9	611.3618	87.6504
18.0	254.4690	56.5486	23.0	415.4756	72.2566	28.0	615.7522	87.9646
.1	257.3043	56.8628	.1	419.0963	72.5708	.1	620.1582	88.2788
.2	260 1553	57.1770	.2	422.7327	72.8849	.2	624.5800	88.5929
.3	263.0220	57.4911	.3	426.3848	73.1991	.3	629.0175	88.9071
.4	265.9044	57.8053	.4	430.0526	73.5133	.4	633.4707	89.2212
.5	268.8025	58.1195	.5	433.7361	73.8274	.5	637.9397	89.5354
.6	271.7164	58.4336	.6	437.4354	74.1416	.6	642.4243	89.8495
.7	274.6459	58.7478	.7	441.1503	74.4557	.7	646.9246	90.1637
.8	277.5911	59.0619	.8	444.8809	74.7699	.8	651.4407	90.4779
.9	280.5521	59.3761	.9	448.6273	75.0841	.9	655.9724	90.7920
19.0	283.5287	59.6903	24.0	452.3893	75.3982	29.0	660.5199	91.1062
.1	286.5211	60.0044	.1	456.1671	75.7124	.1	665.0830	91.4203
.2	289.5292	60.3186	.2	459.9606	76.0265	.2	669.6619	91.7345
.3	292.5530	60.6327	.3	463.7698	76.3407	.3	674.2565	92.0487
.4	295.5925	60.9469	.4	467.5947	76.6549	.4	678.8668	92.3628
.5	298.6477	61.2611	.5	471.4352	76.9690	.5	683.4928	92.6770
.6	301.7186	61.5752	.6	475.2916	77.2832	.6	688.1345	92.9911
.7	304.8052	61.8894	.7	479.1636	77.5973	.7	692.7919	93.3053
.8	307.9075	62.2035	.8	483.0513	77.9115	.8	697.4650	93.6195
.9	311.0255	62.5177	.9	486.9547	78.2257	.9	702.1538	93.9336

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area	Circum.	Dia.	Area	Circum.	Dia.	Area	Circum.
30.0	706.8583	94.2478	35.0	962.1128	109.9557	40.0	1256.6371	125.6637
.1	711.5786	94.5619	.1	967.6184	110.2699	.1	1262.9281	125.9779
.2	716.3145	94.8761	.2	973.1397	110.5841	.2	1269.2348	126.2920
.3	721.0662	95.1903	.3	978.6768	110.8982	.3	1275.5573	126.6062
.4	725.8336	95.5044	.4	984.2296	111.2124	.4	1281.8955	126.9203
.5	730.6167	95.8186	.5	989.7980	111.5265	.5	1288.2493	127.2345
.6	735.4154	96.1327	.6	995.3822	111.8407	.6	1294.6189	127.5487
.7	740.2299	96.4469	.7	1000.9821	112.1549	.7	1301.0042	127.8628
.8	745.0601	96.7611	.8	1006.5977	112.4690	.8	1307.4052	128.1770
.9	749.9060	97.0752	.9	1012.2290	112.7832	.9	1313.8219	128.4911
31.0	754.7676	97.3894	36.0	1017.8760	113.0973	41.0	1320.2543	128.8053
.1	759.6450	97.7035	.1	1023.5387	113.4115	.1	1326.7024	129.1195
.2	764.5380	98.0177	.2	1029.2172	113.7257	.2	1333.1663	129.4336
.3	769.4467	98.3319	.3	1034.9113	114.0398	.3	1339.6458	129.7478
.4	774.3712	98.6460	.4	1040.6212	114.3540	.4	1346.1410	130.0619
.5	779.3113	98.9602	.5	1046.3467	114.6681	.5	1352.6520	130.3761
.6	784.2672	99.2743	.6	1052.0880	114.9823	.6	1359.1786	130.6903
.7	789.2388	99.5885	.7	1057.8449	115.2965	.7	1365.7210	131.0044
.8	794.2260	99.9026	.8	1063.6176	115.6106	.8	1372.2791	131.3186
.9	799.2290	100.2168	.9	1069.4060	115.9248	.9	1378.8529	131.6327
32.0	804.2477	100.5310	37.0	1075.2101	116.2389	42.0	1385.4424	131.9469
.1	809.2821	100.8451	.1	1081.0299	116.5531	.1	1392.0476	132.2611
.2	814.3322	101.1593	.2	1086.8654	116.8672	.2	1398.6685	132.5752
.3	819.3980	101.4734	.3	1092.7166	117.1814	.3	1405.3051	132.8894
.4	824.4796	101.7876	.4	1098.5835	117.4956	.4	1411.9574	133.2035
.5	829.5768	102.1018	.5	1104.4662	117.8097	.5	1418.6254	133.5177
.6	834.6898	102.4159	.6	1110.3645	118.1239	.6	1425.3092	133.8318
.7	839.8185	102.7301	.7	1116.2786	118.4380	.7	1432.0086	134.1460
.8	844.9628	103.0442	.8	1122.2083	118.7522	.8	1438.7238	134.4602
.9	850.1229	103.3584	.9	1128.1538	119.0664	.9	1445.4546	134.7743
33.0	855.2986	103.6726	38.0	1134.1149	119.3805	43.0	1452.2012	135.0885
.1	860.4902	103.9867	.1	1140.0918	119.6947	.1	1458.9635	135.4026
.2	865.6973	104.3009	.2	1146.0844	120.0088	.2	1465.7415	135.7168
.3	870.9202	104.6150	.3	1152.0927	120.3230	.3	1472.5352	136.0310
.4	876.1588	104.9292	.4	1158.1167	120.6372	.4	1479.3446	136.3451
.5	881.4131	105.2434	.5	1164.1564	120.9513	.5	1486.1697	136.6593
.6	886.6831	105.5575	.6	1170.2118	121.2655	.6	1493.0105	136.9734
.7	891.9688	105.8717	.7	1176.2830	121.5796	.7	1499.8670	137.2876
.8	897.2703	106.1858	.8	1182.3698	121.8938	.8	1506.7393	137.6018
.9	902.5874	106.5000	.9	1188.4724	122.2080	.9	1513.6272	137.9159
34.0	907.9203	106.8142	39.0	1194.5906	122.5221	44.0	1520.5308	138.2301
.1	913.2688	107.1283	.1	1200.7246	122.8363	.1	1527.4502	138.5442
.2	918.6331	107.4425	.2	1206.8742	123.1504	.2	1534.3853	138.8584
.3	924.0131	107.7566	.3	1213.0396	123.4646	.3	1541.3360	139.1726
.4	929.4088	108.0708	.4	1219.2207	123.7788	.4	1548.3025	139.4867
.5	934.8202	108.3849	.5	1225.4175	124.0929	.5	1555.2847	139.8009
.6	940.2473	108.6991	.6	1231.6300	124.4071	.6	1562.2826	140.1153
.7	945.6901	109.0133	.7	1237.8582	124.7212	.7	1569.2962	140.4292
.8	951.1486	109.3274	.8	1244.1021	125.0354	.8	1576.3255	140.7434
.9	956.6228	109.6416	.9	1250.3617	125.3495	.9	1583.3706	141.0575

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area.	Circum.	Dia.	Area.	Circum.	Dia.	Area.	Circum.
45.0	1590.4313	141.3717	50.0	1963.4954	157.0796	55.0	2375.8294	172.7876
.1	1597.5077	141.6858	.1	1971.3572	157.3938	.1	2384.4767	173.1017
.2	1604.5999	142.0000	.2	1979.2348	157.7080	.2	2393.1396	173.4159
.3	1611.7077	142.3142	.3	1987.1280	158.0221	.3	2401.8185	173.7301
.4	1618.8313	142.6283	.4	1995.0370	158.3363	.4	2410.5126	174.0442
.5	1625.9705	142.9425	.5	2002.9617	158.6504	.5	2419.2227	174.3584
.6	1633.1255	143.2566	.6	2010.9020	158.9646	.6	2427.9485	174.6726
.7	1640.2962	143.5708	.7	2018.8581	159.2787	.7	2436.6899	174.9867
.8	1647.4826	143.8849	.8	2026.8299	159.5929	.8	2445.4471	175.3009
.9	1654.6847	144.1991	.9	2034.8174	159.9071	.9	2454.2200	175.6150
46.0	1661.9025	144.5133	51.0	2042.8206	160.2212	56.0	2463.0086	175.9292
.1	1669.1360	144.8274	.1	2050.8395	160.5354	.1	2471.8130	176.2433
.2	1676.3853	145.1416	.2	2058.8742	160.8495	.2	2480.6330	176.5575
.3	1683.6502	145.4557	.3	2066.9245	161.1637	.3	2489.4687	176.8717
.4	1690.9308	145.7699	.4	2074.9905	161.4779	.4	2498.3201	177.1858
.5	1698.2272	146.0841	.5	2083.0723	161.7920	.5	2507.1873	177.5000
.6	1705.5392	146.3982	.6	2091.1697	162.1062	.6	2516.0701	177.8141
.7	1712.8670	146.7124	.7	2099.2829	162.4203	.7	2524.9687	178.1283
.8	1720 2105	147.0265	.8	2107.4118	162.7345	.8	2533.8830	178.4425
.9	1727.5697	147.3407	.9	2115.5563	163.0487	.9	2542.8129	178.7566
47.0	1734.9445	147.6550	52.0	2123.7166	163.3628	57.0	2551.7586	179 0708
.1	1742.3351	147.9690	.1	2131.8926	163.6770	.1	2560.7200	179.3849
.2	1749.7414	148.2832	.2	2140.0843	163.9911	.2	2569.6971	179.6991
.3	1757.1635	148.5973	.3	2148.2917	164.3053	.3	2578.6899	180.0133
.4	1764.6012	148.9115	.4	2156.5149	164.6195	.4	2587.6985	180.3274
5	1772.0546	149.2257	.5	2164.7537	164.9336	.5	2596 7227	180.6416
.6	1779 5237	149.5398	.6	2173.0082	165.2479	.6	2605.7626	180.9557
.7	1787.0086	149.8540	.7	2181.2785	165.5619	.7	2614.8183	181.2699
.8	1794.5091	150 1681	.8	2189.5644	165.8761	.8	2623.8896	181.5841
.9	1802.0254	150.4823	.9	2197.8661	166 1903	.9	2632.9767	181.8982
48 0	1809.5574	150.7964	53.0	2206.1834	166.5044	58.0	2642.0794	182.2124
.1	1817.1050	151.1106	.1	2214.5165	166.8186	.1	2651.1979	182.5265
.2	1824.6684	151.4248	.2	2222.8653	167.1327	.2	2660.3321	182.8407
.3	1832.2475	151.7389	.3	2231.2298	167.4469	.3	2669.4820	183.1549
.4	1839.8423	152.0531	.4	2239.6100	167.7610	.4	2678.6476	183.4690
.5	1847.4528	152.3672	.5	2248.0059	168.0752	.5	2687.8289	183.7832
.6	1855.0790	152.6814	.6	2256.4175	168.3894	.6	2697.0259	184.0973
.7	1862.7210	152.9956	.7	2264.8448	168.7035	.7	2706.2386	184.4115
.8	1870.3786	153.3097	.8	2273.2879	169.0177	.8	2715.4670	184.7256
.9	1878.0519	153.6239	.9	2281.7466	169.3318	.9	2724.7112	185.0398
49.0	1885.7409	153.9380	54.0	2290.2210	169.6460	59.0	2733.9710	185.3540
.1	1893.4457	154.2522	.1	2298.7112	169.9602	.1	2743.2466	185.6681
.2	1901.1662	154.5664	.2	2307.2171	170.2743	.2	2752.5378	185.9823
.3	1908.9024	154.8805	.3	2315.7386	170.5885	.3	2761.8448	186.2964
.4	1916.6543	155.1947	.4	2324.2759	170.9026	.4	2771.1675	186.6106
.5	1924.4218	155.5088	.5	2332.8289	171.2168	.5	2780.5058	186.9248
.6	1932.2051	155.8230	.6	2341.3976	171.5310	.6	2789.8599	187.2389
.7	1940.0042	156.1372	.7	2349.9820	171.8451	.7	2799.2297	187.5531
.8	1947.8189	156.4513	.8	2358.5821	172.1593	.8	2808.6152	187.8672
.9	1955.6493	156.7655	.9	2367.1979	172.4735	.9	2818.0165	188.1814

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area.	Circum.	Dia.	Area.	Circum.	Dia.	Area.	Circum.
60.0	2827.4334	188.4956	65.0	3318.3072	204.2035	70.0	3848.4510	219.9115
.1	2836.8660	188.8097	.1	3328.5253	204.5176	.1	3859.4544	220.2256
.2	2846.3144	189.1239	.2	3338.7590	204.8318	.2	3870.4736	220 5398
.3	2855.7784	189.4380	.3	3349.0085	205.1460	.3	3881.5084	220.8540
.4	2865.2582	189.7522	.4	3359.2736	205.4602	.4	3892.5590	221.1681
.5	2874.7536	190.0664	.5	3369.5545	205.7743	.5	3903.6252	221.4823
.6	2884.2648	190.3805	.6	3379.8510	206.0885	.6	3914.7072	221.7964
.7	2893.7917	190.6947	.7	3390.1633	206.4026	.7	3925.8049	222.1106
.8	2903.3343	191.0088	.8	3400.4913	206.7168	.8	3936.9182	222.4248
.9	2912.8926	191.3230	.9	3410.8350	207.0310	.9	3948.0473	222.7389
61.0	2922.4666	191.6372	66.0	3421.1944	207.3451	71.0	3959.1921	223.0531
.1	2932.0563	191.9513	.1	3431.5695	207.6593	.1	3970.3526	223.3672
.2	2941.6617	192.2655	.2	3441.9603	207.9734	.2	3981.5289	223.6814
.3	2951.2828	192.5796	.3	3452.3669	208.2876	.3	3992.7208	223.9956
.4	2960.9197	192.8938	.4	3462.7891	208.6017	.4	4003.9284	224.3097
.5	2970.5722	193.2079	.5	3473.2270	208.9159	.5	4015.1518	224.6239
.6	2980.2405	193.5221	.6	3483.6807	209.2301	.6	4026.3908	224.9380
.7	2989.9244	193.8363	.7	3494.1500	209.5442	.7	4037.6456	225.2522
.8	2999.6241	194.1504	.8	3504.6351	209.8584	.8	4048.9160	225.5664
.9	3009.3395	194.4646	.9	3515.1359	210.1725	.9	4060.2022	225.8810
62.0	3019.0705	194.7787	67.0	3525.6524	210.4867	72.0	4071.5041	226.1947
.1	3028.8173	195.0929	.1	3536.1845	210.8009	.1	4082.8217	226.5088
.2	3038.5798	195.4071	.2	3546.7324	211.1150	.2	4094.1550	226.8230
.3	3048.3580	195.7212	.3	3557.2960	211.4292	.3	4105.5040	227.1371
.4	3058.1520	196.0354	.4	3567.8754	211.7433	.4	4116.8687	227.4513
.5	3067.9616	196.3495	.5	3578.4704	212.0575	.5	4128.2491	227.7655
.6	3077.7869	196.6637	.6	3589.0811	212.3717	.6	4139.6452	228.0796
.7	3087.6279	196.9779	.7	3599.7075	212.6858	.7	4151.0571	228.3938
.8	3097.4847	197.2920	.8	3610.3497	213.0000	.8	4162.4846	228.7079
.9	3107.3571	197.6062	.9	3621.0075	213.3141	.9	4173.9279	229.0221
63.0	3117.2453	197.9203	68.0	3631.6811	213.6283	73.0	4185.3868	229.3363
.1	3127.1492	198.2345	.1	3642.3704	213.9425	.1	4196.8615	229.6504
.2	3137.0688	198.5487	.2	3653.0754	214.2566	.2	4208.3519	229.9646
.3	3147.0040	198.8628	.3	3663.7960	214.5708	.3	4219.8579	230.2787
.4	3156.9550	199.1770	.4	3674.5324	214.8849	.4	4231.3797	230.5929
.5	3166.9217	199.4911	.5	3685.2845	215.1991	.5	4242.9172	230.9071
.6	3176.9043	199.8053	.6	3696.0523	215.5133	.6	4254.4704	231.2212
.7	3186.9023	200.1195	.7	3706.8359	215.8274	.7	4266.0394	231.5354
.8	3196.9161	200.4336	.8	3717.6351	216.1416	.8	4277.6240	231.8495
.9	3206.9456	200.7478	.9	3728.4500	216.4556	.9	4289.2243	232.1637
64.0	3216.9909	201.0620	69.0	3739.2807	216.7699	74.0	4300.8403	232.4779
.1	3227.0518	201.3761	.1	3750.1270	217.0841	.1	4312.4721	232.7920
.2	3237.1285	201.6902	.2	3760.9891	217.3982	.2	4324.1195	233.1062
.3	3247.2209	202.0044	.3	3771.8668	217.7124	.3	4335.7827	233.4203
.4	3257.3289	202.3186	.4	3782.7603	218.0265	.4	4347.4616	233.7345
.5	3267.4527	202.6327	.5	3793.6695	218.3407	.5	4359.1562	234.0487
.6	3277.5922	202.9469	.6	3804.5944	218.6548	.6	4370.8664	234.3628
.7	3287.7474	203.2610	.7	3815.5350	218.9690	.7	4382.5924	234.6770
.8	3297.9183	203.5752	.8	3826.4913	219.2832	.8	4394.3341	234.9911
.9	3308.1049	203.8894	.9	3837.4633	219.5973	.9	4406.0916	235.3053

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area	Circum.	Dia.	Area	Circum.	Dia	Area	Circum.
75.0	4417.8647	235.6194	80.0	5026.5482	251.3274	85.0	5674.5017	267.0354
.1	4429.6535	235.9336	.1	5039.1225	251.6416	.1	5687.8614	267.3495
.2	4441.4580	236.2478	.2	5051.7124	251.9557	.2	5701.2367	267.6637
.3	4453.2783	236.5619	.3	5064.3180	252.2699	.3	5714.6277	267.9779
.4	4465.1142	236.8761	.4	5076.9394	252.5840	.4	5728.0345	268.2920
.5	4476.9659	237.1902	.5	5089.5764	252.8982	.5	5741.4569	268.6062
.6	4488.8332	237.5044	.6	5102.2292	253.2124	.6	5754.8951	268.9203
.7	4500.7163	237.8186	.7	5114.8977	253.5265	.7	5768.3490	269.2345
.8	4512.6151	238.1327	.8	5127.5819	253.8407	.8	5781.8185	269.5486
.9	4524.5296	238.4469	.9	5140.2818	254.1548	.9	5795.3038	269.8628
76.0	4536.4598	238.7610	81.0	5152.9973	254.4690	86.0	5808.8048	270.1770
.1	4548.4057	239.0752	.1	5165.7287	254.7832	.1	5822.3215	270.4911
.2	4560.3673	239.3894	.2	5178.4757	255.0973	.2	5835.8539	270.8053
.3	4572.3446	239.7035	.3	5191.2384	255.4115	.3	5849.4020	271.1194
.4	4584.3377	240.0177	.4	5204.0168	255.7256	.4	5862.9659	271.4336
.5	4596.3464	240.3318	.5	5216.8110	256.0398	.5	5876.5454	271.7478
.6	4608.3708	240.6460	.6	5229.6208	256.3540	.6	5890.1407	272.0619
.7	4620.4110	240.9602	.7	5242.4463	256.6681	.7	5903.7516	272.3761
.8	4632.4669	241.2743	.8	5255.2876	256.9823	.8	5917.3783	272.6902
.9	4644.5384	241.5885	.9	5268.1446	257.2966	.9	5931.0206	273.0044
77.0	4656.6257	241.9026	82.0	5281.0173	257.6106	87.0	5944.6787	273.3186
.1	4668.7287	242.2168	.1	5293.9056	257.9247	.1	5958.3525	273.6327
.2	4680.8474	242.5310	.2	5306.8097	258.2389	.2	5972.0420	273.9469
.3	4692.9818	242.8451	.3	5319.7295	258.5531	.3	5985.7472	274.2610
.4	4705.1319	243.1592	.4	5332.6650	258.8672	.4	5999.4681	274.5752
.5	4717.2977	243.4734	.5	5345.6162	259.1814	.5	6013.2047	274.8894
.6	4729.4792	243.7876	.6	5358.5832	259.4956	.6	6026.9570	275.2035
.7	4741.6765	244.1017	.7	5371.5658	259.8097	.7	6040.7250	275.5177
.8	4753.8894	244.4159	.8	5384.5641	260.1239	.8	6054.5088	275.8318
.9	4766.1181	244.7301	.9	5397.5782	260.4380	.9	6068.3082	276.1460
78.0	4778.3624	245.0442	83.0	5410.6079	260.7522	88.0	6082.1234	276.4602
.1	4790.6225	245.3584	.1	5423.6534	261.0663	.1	6095.9542	276.7743
.2	4802.8983	245.6725	.2	5436.7146	261.3805	.2	6109.8008	277.0885
.3	4815.1897	245.9867	.3	5449.7915	261.6947	.3	6123.6631	277.4026
.4	4827.4969	246.3009	.4	5462.8840	262.0088	.4	6137.5411	277.7168
.5	4839.8198	246.6150	.5	5475.9923	262.3230	.5	6151.4348	278.0309
.6	4852.1584	246.9292	.6	5489.1163	262.6371	.6	6165.3442	278.3451
.7	4864.5128	247.2433	.7	5502.2561	262.9513	.7	6179.2693	278.6593
.8	4876.8828	247.5575	.8	5515.4115	263.2655	.8	6193.2101	278.9740
.9	4889.2685	247.8717	.9	5528.5826	263.5796	.9	6207.1666	279.2876
79.0	4901.6699	248.1858	84.0	5541.7694	263.8938	89.0	6221.1389	279.6017
.1	4914.0871	248.5000	.1	5554.9720	264.2079	.1	6235.1268	279.9159
.2	4926.5199	248.8141	.2	5568.1902	264.5221	.2	6249.1304	280.2301
.3	4938.9685	249.1283	.3	5581.4242	264.8363	.3	6263.1498	280.5442
.4	4951.4328	249.4425	.4	5594.6739	265.1514	.4	6277.1849	280.8584
.5	4963.9127	249.7566	.5	5607.9392	265.4646	.5	6291.2356	281.1725
.6	4976.4084	250.0708	.6	5621.2203	265.7787	.6	6305.3021	281.4867
.7	4988.9198	250.3850	.7	5634.5171	266.0929	.7	6319.3843	281.8009
.8	5001.4469	250.6991	.8	5647.8296	266.4071	.8	6333.4822	282.1150
.9	5013.9897	251.0133	.9	5661.1578	266.7212	.9	6347.5958	282.4292

Areas and Circumferences of Circles (Continued)

Advancing by tenths

Dia.	Area	Circum.	Dia.	Area	Circum.	Dia.	Area	Circum.
90.0	6361.7251	282.7433	93.5	6866.1471	293.7389	97.0	7389.8113	304.7345
.1	6375.8701	283.0575	.6	6880.8419	294.0531	.1	7405.0559	305.0486
.2	6390.0309	283.3717	.7	6895.5524	294.3672	.2	7420.3162	305.3628
.3	6404.2073	283.6858	.8	6910.2786	294.6814	.3	7435.5922	305.6770
.4	6418.3995	284.0000	.9	6925.0205	294.9956	.4	7450.8839	305.9911
.5	6432.6073	284.3141	94.0	6939.7732	295.3097	.5	7466.1913	306.3053
.6	6446.8309	284.6283	.1	6954.5515	295.6239	.6	7481.5144	306.6194
.7	6461.0701	284.9425	.2	6969.3406	295.9380	.7	7496.8532	306.9336
.8	6475.3251	285.2566	.3	6984.1453	296.2522	.8	7512.2078	307.2478
.9	6489.5958	285.5708	.4	6998.9658	296.5663	.9	7527.5780	307.5619
91.0	6503.8822	285.8849	.5	7013.8019	296.8805	98.0	7542.9640	307.8761
.1	6518.1843	286.1991	.6	7028.6538	297.1947	.1	7558.3656	308.1902
.2	6532.5021	286.5133	.7	7043.5214	297.5088	.2	7573.7830	308.5044
.3	6546.8356	286.8274	.8	7058.4047	297.8230	.3	7589.2161	308.8186
.4	6561.1848	287.1416	.9	7073.3033	298.1371	.4	7604.6648	309.1327
.5	6575.5498	287.4557	95.0	7088.2184	298.4513	.5	7620.1293	309.4469
.6	6589.9304	287.7699	.1	7103.1488	298.7655	.6	7635.6095	309 7610
.7	6604.3268	288.0840	.2	7118.1950	299.0796	.7	7651.1054	310.0752
.8	6618.7388	288.3982	.3	7133.0568	299.3938	.8	7666.6170	310 3894
.9	6633.1666	288.7124	.4	7148.0343	299.7079	.9	7682.1444	310.7035
92.0	6647.6101	289.0265	.5	7163.0276	300.0221	99 0	7697.6874	311.0177
.1	6662.0692	289.3407	.6	7178.0366	300.3363	.1	7713.2461	311.3318
.2	6676.5441	289.6548	.7	7193.0612	300.6504	.2	7728 8206	311.6460
.3	6691.0347	289 9690	.8	7208.1016	300.9646	.3	7744.4107	311.9602
.4	6705.5410	290.2832	.9	7223.1577	301.2787	.4	7760.0166	312.2743
.5	6720.0630	290.5973	96.0	7238.2295	301.5929	.5	7775.6382	312.5885
.6	6734.6008	290.9115	.1	7253.3170	301.9071	.6	7791.2754	312.9026
.7	6749.1542	291.2256	.2	7268.4202	302.2212	.7	7806.9284	313.2168
.8	6763.7233	291.5398	.3	7283 5391	302.5354	.8	7822 5971	313.5309
.9	6778.3082	291.8540	.4	7298.6737	302.8405	.9	7838.2815	313.8451
93.0	6792.9087	292.1681	5	7313.8240	303.1637	100.0	7853.9816	314.1593
.1	6807.5250	292 4823	.6	7328.9901	303.4779			
.2	6822.1569	292.7964	.7	7344.1718	303 7920			
.3	6836.8046	293.1106	.8	7359.3693	304.1062			
.4	6851.4680	293.4248	.9	7374.5824	304.4203			

Areas of Circles

Advancing by eighths

AREAS

Dia.	0.0	0⅛	0.¼	0.⅜	0.½	0.⅝	0.¾	0⅞
0	0.0	0.0122	0.0490	0.1104	0.1963	0.3068	0.4417	0.6013
1	0.7854	0.9940	1.227	1.484	1.767	2.073	2.405	2.761
2	3.1416	3.546	3.976	4.430	4.908	5.411	5.939	6.491
3	7.068	7.669	8.295	8.946	9.621	10.32	11.04	11.79
4	12.56	13.30	14.18	15.03	15.90	16.80	17.72	18.66
5	19.63	20.62	21.64	22.69	23.75	24.85	25.96	27.10
6	28.27	29.46	30.67	31.91	33.18	34.47	35.78	37.12
7	38.48	39.87	41.28	42.71	44.17	45.66	47.17	48.70
8	50.26	51.84	53.45	55.08	56.74	58.42	60.13	61.86
9	63.61	65.39	67.20	69.02	70.88	72.75	74.66	76.58
10	78.54	80.51	82.51	84.54	86.59	88.66	90.76	92.88
11	95.03	97.20	99.40	101.6	103.8	106.1	108.4	110.7
12	113.0	115.4	117.8	120.2	122.7	125.1	127.6	130.1
13	132.7	135.2	137.8	140.5	143.1	145.8	148.4	151.2
14	153.9	156.6	159.4	162.2	165.1	167.9	170.8	173.7
15	176.7	179.6	182.6	185.6	188.6	191.7	194.8	197.9
16	201.0	204.2	207.3	210.5	213.8	217.0	220.3	223.6
17	226.9	230.3	233.7	237.1	240.5	243.9	247.4	250.9
18	254.4	258.0	261.5	265.1	268.8	272.4	276.1	279.8
19	283.5	287.2	291.0	294.8	298.6	302.4	306.3	310.2
20	314.1	318.1	322.0	326.0	330.0	334.1	338.1	342.2
21	316.3	350.4	354.6	358.8	363.0	367.2	371.5	375.8
22	380.1	384.4	388.8	393.2	397.6	402.0	406.4	410.9
23	415.4	420.0	424.5	429.1	433.7	438.3	443.0	447.6
24	452.3	457.1	461.8	466.6	471.4	476.2	481.1	485.9
25	490.8	495.7	500.7	505.7	510.7	515.7	520.7	525.8
26	530.9	536.0	541.1	546.3	551.5	556.7	562.0	567.2
27	572.5	577.8	583.2	588.5	593.9	599.3	604.8	610.2
28	615.7	621.2	626.7	632.3	637.9	643.5	649.1	654.8
29	660.5	666.2	671.9	677.7	683.4	689.2	695.1	700.9
30	706.8	712.7	718.6	724.6	730.6	736.6	742.6	748.6
31	754.8	760.9	767.0	773.1	779.3	785.5	791.7	798.0
32	804.3	810.6	816.9	823.2	829.6	836.0	842.4	848.8
33	855.3	861.8	868.3	874.9	881.4	888.0	894.6	901.3
34	907.9	914.7	921.3	928.1	934.8	941.6	948.4	955.3
35	962.1	969.0	975.9	982.8	989.8	996.8	1003.8	1010.8
36	1017.9	1025.0	1032.1	1039.2	1046.3	1053.5	1060.7	1068.0
37	1075.2	1082.5	1089.8	1097.1	1104.5	1111.8	1119.2	1126.7
38	1134.1	1141.6	1149.1	1156.6	1164.2	1171.7	1179.3	1186.9
39	1194.6	1202.3	1210.0	1217.7	1225.4	1233.2	1241.0	1248.8
40	1256.6	1264.5	1272.4	1280.3	1288.2	1296.2	1304.2	1312.2
41	1320.3	1328.3	1336.4	1344.5	1352.7	1360.8	1369.0	1377.2
42	1385.4	1393.7	1402.0	1410.3	1418.6	1427.0	1435.4	1443.8
43	1452.2	1460.7	1469.1	1477.6	1486.2	1494.7	1503.3	1511.9
44	1520.5	1529.2	1537.9	1546.6	1555.3	1564.0	1572.8	1581.6
45	1590.4	1599.3	1608.2	1617.0	1626.0	1634.9	1643.9	1652.9

Circumferences of Circles

Advancing by eighths

CIRCUMFERENCES

Dia.	0.0	$0.\frac{1}{8}$	$0.\frac{1}{4}$	$0.\frac{3}{8}$	$0.\frac{1}{2}$	$0.\frac{5}{8}$	$0.\frac{3}{4}$	$0.\frac{7}{8}$
0	0.0	0.3927	0.7854	1.178	1.570	1.963	2.356	2.748
1	3.141	3.534	3.927	4.319	4.712	5.105	5.497	5.890
2	6.283	6.675	7.068	7.461	7.854	8.246	8.639	9.032
3	9.424	9.817	10.21	10.60	10.99	11.38	11.78	12.17
4	12.56	12.95	13.35	13.74	14.13	14.52	14.92	15.31
5	15.70	16.10	16.49	16.88	17.27	17.67	18.06	18.45
6	18.84	19.24	19.63	20.02	20.42	20.81	21.20	21.59
7	21.99	22.38	22.77	23.16	23.56	23.95	24.34	24.74
8	25.13	25.52	25.91	26.31	26.70	27.09	27.48	27.88
9	28.27	28.66	29.05	29.45	29.84	30.23	30.63	31.02
10	31.41	31.80	32.20	32.59	32.98	33.37	33.77	34.16
11	34.55	34.95	35.34	35.73	36.12	36.52	36.91	37.30
12	37.69	38.09	38.48	38.87	39.27	39.66	40.05	40.44
13	40.84	41.23	41.62	42.01	42.41	42.80	43.19	43.58
14	43.98	44.37	44.76	45.16	45.55	45.94	46.33	46.73
15	47.12	47.51	47.90	48.30	48.69	49.08	49.48	49.87
16	50.26	50.65	51.05	51.44	51.83	52.22	52.62	53.01
17	53.40	53.79	54.19	54.58	54.97	55.37	55.76	56.15
18	56.54	56.94	57.33	57.72	58.11	58.51	58.90	59.29
19	59.69	60.08	60.47	60.86	61.26	61.65	62.04	62.43
20	62.83	63.22	63.61	64.01	64.40	64.79	65.18	65.58
21	65.97	66.36	66.75	67.15	67.54	67.93	68.32	68.72
22	69.11	69.50	69.90	70.29	70.68	71.07	71.47	71.86
23	72.25	72.64	73.04	73.43	73.82	74.22	74.61	75.00
24	75.39	75.79	76.18	76.57	76.96	77.36	77.75	78.14
25	78.54	78.93	79.32	79.71	80.10	80.50	80.89	81.28
26	81.68	82.07	82.46	82.85	83.25	83.64	84.03	84.43
27	84.82	85.21	85.60	86.00	86.39	86.78	87.17	87.57
28	87.96	88.35	88.75	89.14	89.53	89.92	90.32	90.71
29	91.10	91.49	91.89	92.28	92.67	93.06	93.46	93.85
30	94.24	94.64	95.03	95.42	95.81	96.21	96.60	96.99
31	97.39	97.78	98.17	98.57	98.96	99.35	99.75	100.14
32	100.53	100.92	101.32	101.71	102.10	102.49	102.89	103.29
33	103.67	104.07	104.46	104.85	105.24	105.64	106.03	106.42
34	106.81	107.21	107.60	107.99	108.39	108.78	109.17	109.56
35	109.96	110.35	110.74	111.13	111.53	111.92	112.31	112.71
36	113.10	113.49	113.88	114.28	114.67	115.06	115.45	115.85
37	116.24	116.63	117.02	117.42	117.81	118.20	118.60	118.99
38	119.38	119.77	120.17	120.56	120.95	121.34	121.74	122.13
39	122.52	122.92	123.31	123.70	124.09	124.49	124.88	125.27
40	125.66	126.06	126.45	126.84	127.24	127.63	128.02	128.41
41	128.81	129.20	129.59	129.98	130.38	130.77	131.16	131.55
42	131.95	132.34	132.73	133.13	133.52	133.91	134.30	134.70
43	135.09	135.48	135.87	136.27	136.66	137.05	137.45	137.84
44	138.23	138.62	139.02	139.41	139.80	140.19	140.59	140.98
45	141.37	141.76	142.16	142.55	142.94	143.34	143.73	144.12

Areas and Circumferences of Circles

FROM 1 TO 50 FEET

Advancing by one inch

Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in
1 0	0 7854	3 1⅝	5 0	19 635	15 8½	9 0	63 6174	28 3¼
1	0.9217	3 4⅝	1	20 2947	15 11⅝	1	64 8006	28 6⅜
2	1 069	3 8	2	20 9656	16 2¾	2	65 9951	28 9½
3	1.2271	3 11	3	21 6475	16 5¾	3	67 2007	29 ⅝
4	1.3962	4 2⅛	4	22 34	16 9	4	68 4166	29 3¾
5	1 5761	4 5⅜	5	23 0437	17 ⅛	5	69 644	29 7
6	1 7671	4 8½	6	23 7583	17 3¼	6	70.8823	29 10⅛
7	1 9689	4 11⅝	7	24 4835	17 6⅜	7	72.1309	30 1¼
8	2 1816	5 2¾	8	25 2199	17 9⅝	8	73 391	30 4⅜
9	2.4052	5 5⅞	9	25.9672	18 ¾	9	74.662	30 7½
10	2 6398	5 9	10	26 7251	18 3⅞	10	75 9433	30 11⅝
11	2 8852	6 ¼	11	27.4943	18 7⅛	11	77.2362	31 1¾
2 0	3 1416	6 3⅜	6 0	28 2744	18 10⅛	10 0	78.54	31 5
1	3 4087	6 6½	1	29 0649	19 1¼	1	79 854	31 8⅛
2	3 6869	6 9⅝	2	29 8668	19 4⅜	2	81.1795	31 11¼
3	3 976	7 ¾	3	30 6796	19 7½	3	82 516	32 2⅜
4	4 276	7 3⅞	4	31 5029	19 10⅝	4	83 8627	32 5½
5	4 5869	7 7	5	32 3376	20 1⅞	5	85.2211	32 8⅝
6	4 9087	7 10¼	6	33 1831	20 4⅞	6	86 5903	32 11¾
7	5 2413	8 1⅜	7	34 0391	20 8⅛	7	87 9697	33 2⅞
8	5 585	8 4½	8	34 9065	20 11½	8	89.3608	33 6⅛
9	5 9395	8 7⅝	9	35 7847	21 2⅜	9	90 7627	33 9¼
10	6 3049	8 10¾	10	36 6735	21 5½	10	92 1749	34 ⅜
11	6.6813	9 1⅞	11	37.5736	21 8¾	11	93.5986	34 3½
3 0	7.0686	9 5	7 0	38 4846	21 11⅞	11 0	95 0334	34 6⅝
1	7.4666	9 8¼	1	39 406	22 3	1	96.4783	34 9¾
2	7.8757	9 11⅜	2	40 3388	22 6⅛	2	97.9347	35 ⅞
3	8.2957	10 2½	3	41 2825	22 9¼	3	99.4021	35 4⅛
4	8.7265	10 5⅝	4	42 2367	23 ⅜	4	100.8797	35 7¼
5	9 1683	10 8¾	5	43.2022	23 3⅝	5	102 3689	35 10⅝
6	9.6211	10 11⅞	6	44 1787	23 6¾	6	103 8691	36 1½
7	10 0846	11 3	7	45 1656	23 9⅞	7	105.3794	36 4½
8	10.5591	11 6⅛	8	46 1638	24 1⅛	8	106 9013	36 7¾
9	11 0446	11 9⅜	9	47.173	24 4⅛	9	108 4342	36 10⅞
10	11.5409	12 ½	10	48 1962	24 7¼	10	109.9772	37 2¾
11	12.0481	12 3⅝	11	49.2236	24 10⅜	11	111.5319	37 5¼
4 0	12.5664	12 6¾	8 0	50 2656	25 1½	12 0	113 0976	37 8⅜
1	13.0952	12 9⅞	1	51 3178	25 4⅝	1	114 6732	37 11½
2	13.6353	13 1	2	52 3816	25 7⅞	2	116.2607	38 2⅝
3	14 1862	13 4⅛	3	53 4562	25 11	3	117 859	38 5¾
4	14 7479	13 7¼	4	54 5412	26 2⅛	4	119 4674	38 8⅞
5	15 3206	13 10½	5	55 6377	26 5¼	5	121 0876	39 0
6	15 9043	14 1⅝	6	56 7451	26 8⅜	6	122 7187	39 3¼
7	16 4986	14 4⅝	7	57 8628	26 11½	7	124 3598	39 6⅜
8	17 1041	14 7⅞	8	58.992	27 2¾	8	126.0127	39 9½
9	17.7205	14 11	9	60 1321	27 5¾	9	127.6765	40 ⅝
10	18 3476	15 2⅛	10	61 2826	27 9	10	129.3504	40 3¾
11	18.9858	15 5¼	11	62.4445	28 ⅛	11	131.036	40 6⅞

Areas and Circumferences of Circles (Continued)

Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in
13 0	132 7326	40 10	18 0	254 4696	56 6½	23 0	415 4766	72 3
1	134.4391	41 1⅛	1	256 8303	56 9⅝	1	418 4915	72 6⅛
2	136 1574	41 4⅜	2	259 2033	57 ⅞	2	421 5192	72 9⅜
3	137.8867	41 7½	3	261 5872	57 4	3	424 5577	73 ½
4	139 626	41 10⅝	4	263 9807	57 7⅛	4	427 6055	73 3⅝
5	141.3771	42 1⅝	5	266 3864	57 10¼	5	430.6658	73 6¾
6	143.1391	42 4⅞	6	268 8031	58 1⅜	6	433 7371	73 9⅞
7	144 9111	42 8	7	271 2293	58 4½	7	436 8175	74 1
8	146 6949	42 11⅛	8	273 6678	58 7⅝	8	439 9106	74 4⅛
9	148 4896	43 2¼	9	276.1171	58 10¾	9	443 0146	74 7¼
10	150.2943	43 5½	10	278 5761	59 2	10	446 1278	74 10⅝
11	152.1109	43 8⅝	11	281 0472	59 5⅛	11	449.2536	75 1⅜
14 0	153 9384	43 11¾	19 0	283 5294	59 8¼	24 0	452 3904	75 4¾
1	155 7758	44 2⅞	1	286 021	59 11½	1	455.5362	75 7⅞
2	157.625	44 6	2	288 5249	60 2½	2	458.6948	75 11
3	159 4852	44 9⅛	3	291 0397	60 5⅝	3	461 8642	76 2⅛
4	161.3553	45 ¼	4	293 5641	60 8¾	4	465 0428	76 5¼
5	163 2373	45 3½	5	296 1107	60 11⅞	5	468 2341	76 8½
6	165 1303	45 6⅝	6	298 6483	61 3⅛	6	471.4363	76 11⅝
7	167 0331	45 9¾	7	301 2054	61 6¼	7	474 6476	77 2¾
8	168 9479	46 ⅞	8	303 7747	61 9½	8	477.8716	77 5⅞
9	170.8735	46 4	9	306 355	62 ½	9	481.1065	77 9
10	172 8091	46 7⅛	10	308 9448	62 3⅝	10	484 3506	78 ⅛
11	174.7565	46 11¼	11	311.5469	62 6¾	11	487.6073	78 3¼
15 0	176.715	47 1½	20 0	314 16	62 9⅞	25 0	490 875	78 6½
1	178 6832	47 4⅝	1	316 7824	63 1⅛	1	494 1516	78 9½
2	180 6634	47 7¾	2	319.4173	63 4¼	2	497.4411	79 ¾
3	182 6545	47 10⅞	3	322 063	63 7⅜	3	500 7415	79 3⅞
4	184.6555	48 2½	4	324.7182	63 11½	4	504 051	79 7⅛
5	186 6684	48 5⅛	5	327 3858	64 1⅝	5	507 3732	79 11⅛
6	188.6923	48 8¼	6	330.0643	64 4¾	6	510 7063	80 1¼
7	190.726	48 11⅜	7	332 7522	64 7⅞	7	514 0484	80 4⅜
8	192.7716	49 2⅝	8	335.4525	64 11	8	517 4034	80 7⅝
9	194 8282	49 5¾	9	338.1637	65 2¼	9	520 7692	80 10¾
10	196.8946	49 8⅞	10	340 8844	65 5⅜	10	524 1441	81 1⅞
11	198.973	50 0	11	343.6174	65 8¼	11	527 5318	81 5
16 0	201.0624	50 3⅛	21 0	346 3614	65 11⅜	26 0	530 9304	81 8⅛
1	203 1615	50 6¼	1	349.1147	66 2¾	1	534 3379	81 11¼
2	205.2726	50 9⅝	2	351 8804	66 5⅞	2	537 7583	82 2⅜
3	207.3946	51 ½	3	354.6571	66 9	3	541 1896	82 5¼
4	209.5264	51 3¾	4	357.4432	67 ⅛	4	544 6299	82 8⅝
5	211.6703	51 6½	5	360 2417	67 3⅜	5	548 083	82 11⅞
6	213.8251	51 10	6	363 0511	67 6½	6	551.5471	83 3
7	215.9896	52 1⅛	7	365.8698	67 9⅝	7	555.0201	83 6⅛
8	218 1662	52 4¼	8	368.7011	68 ¾	8	558 5059	83 9¼
9	220 3537	52 7⅜	9	371.5432	68 3⅞	9	562 0027	84 ⅜
10	222.551	52 10½	10	374 3947	68 7	10	565 5084	84 3½
11	224.7608	53 1⅝	11	377.2587	68 10¼	11	569.027	84 6⅝
17 0	226.9806	53 4⅞	22 0	380.1336	69 1⅜	27 0	572 5566	84 9⅞
1	229 2105	53 8	1	383 0177	69 4½	1	576 0949	85 1
2	231.4525	53 11⅛	2	385 9144	69 7⅝	2	579 6463	85 4¼
3	233.7055	54 2⅜	3	388.822	69 10¾	3	583 2085	85 8⅛
4	235.9682	54 5⅜	4	391.7389	70 1⅞	4	586.7796	85 11⅜
5	238.243	54 8½	5	394 6683	70 5	5	590 3637	86 1½
6	240.5287	54 11⅝	6	397.6087	70 8¼	6	593 9587	86 4⅝
7	242.8241	55 2⅞	7	400.5583	70 11⅜	7	597.5625	86 7⅞
8	245.1316	55 6	8	403 5204	71 2½	8	601 1703	86 11
9	247.45	55 9⅛	9	406 4935	71 5⅝	9	604 807	87 2⅛
10	249.7781	56 ¼	10	409 4759	71 8¾	10	608 4436	87 5¼
11	252.1184	56 3½	11	412 4707	71 11⅞	11	612.0931	87 8¾

Areas and Circumferences of Circles (Continued)

Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in
28 0	615.7536	87 11½	33 0	855.301	103 8	38 0	1134.118	119 4½
1	619.4228	88 2⅝	1	859.624	103 11⅛	1	1139.095	119 7⅝
2	623.105	88 5¾	2	863.961	104 2¼	2	1144.087	119 10¾
3	626.7982	88 9	3	868.309	104 5⅜	3	1149.089	120 2
4	630.5002	89 ⅛	4	872.665	104 8⅝	4	1154.110	120 5⅛
5	634.2152	89 3¼	5	877.035	104 11¾	5	1159.124	120 8⅜
6	637.9411	89 6⅜	6	881.415	105 2⅞	6	1164.159	120 11⅜
7	641.6758	89 9½	7	885.804	105 6	7	1169.202	121 2½
8	645.4235	90 ⅝	8	890.206	105 9⅛	8	1174.259	121 5⅝
9	649.1821	90 3¾	9	894.619	106 ¼	9	1179.327	121 8¾
10	652.9495	90 6⅞	10	899.041	106 3⅜	10	1184.403	121 11⅞
11	656.73	90 11⅛	11	903.476	106 6⅝	11	1189.493	122 3⅛
29 0	660.5214	91 1¼	34 0	907.922	106 9¾	39 0	1194.593	122 6¼
1	664.3214	91 4⅜	1	912.377	107 ⅞	1	1199.719	122 9½
2	668.1346	91 7½	2	916.844	107 4	2	1204.824	123 ½
3	671.9587	91 10⅝	3	921.323	107 7⅛	3	1209.958	123 3⅝
4	675.7915	92 1¾	4	925.810	107 10¼	4	1215.099	123 6¾
5	679.6375	92 4⅞	5	930.311	108 1⅜	5	1220.254	123 9⅞
6	683.4943	92 8⅛	6	934.822	108 4⅝	6	1225.420	124 1⅛
7	687.3598	92 11¼	7	939.342	108 7¾	7	1230.594	124 4¼
8	691.2385	93 2⅜	8	943.875	108 10⅞	8	1235.782	124 7⅜
9	695.1028	93 5½	9	948.419	109 2	9	1240.981	124 10½
10	699.0263	93 8⅝	10	952.972	109 5⅛	10	1246.188	125 1⅝
11	702.9377	93 11⅞	11	957.538	109 8¼	11	1251.408	125 4¾
30 0	706.86	94 2⅞	35 0	962.115	109 11⅜	40 0	1256.64	125 7⅞
1	710.791	94 6	1	966.770	110 2⅝	1	1261.879	125 11
2	714.735	94 9¼	2	971.299	110 5¾	2	1267.133	126 2¼
3	718.69	95 ⅜	3	975.908	110 8⅞	3	1272.397	126 5⅜
4	722.654	95 3½	4	980.526	111 0	4	1277.669	126 8½
5	726.631	95 6⅝	5	985.158	111 3⅛	5	1282.955	126 11⅝
6	730.618	95 9¾	6	989.803	111 6¼	6	1288.252	127 2¾
7	734.615	96 ⅞	7	994.451	111 9⅜	7	1293.557	127 5⅞
8	738.624	96 4	8	999.115	112 ½	8	1298.876	127 9
9	742.645	96 7¼	9	1003.79	112 3¾	9	1304.206	128 ¼
10	746.674	96 10⅜	10	1008.473	112 6⅞	10	1309.543	128 3⅜
11	750.716	97 1½	11	1013.170	112 10	11	1314.895	128 6½
31 0	754.769	97 4⅝	36 0	1017.878	113 1⅛	41 0	1320.257	128 9⅝
1	758.831	97 7¾	1	1022.594	113 4¼	1	1325.628	129 ¾
2	762.906	97 10⅞	2	1027.324	113 7⅜	2	1331.012	129 3⅞
3	766.992	98 2	3	1032.064	113 10⅝	3	1336.407	129 7
4	771.086	98 5⅛	4	1036.813	114 1¾	4	1341.810	129 10⅛
5	775.191	98 8⅜	5	1041.576	114 4⅞	5	1347.227	130 1⅜
6	779.313	98 11½	6	1046.349	114 8	6	1352.655	130 4½
7	783.440	99 2⅝	7	1051.130	114 11⅛	7	1358.091	130 7⅝
8	787.581	99 5¾	8	1055.926	115 2¼	8	1363.541	130 10¾
9	791.732	99 8⅞	9	1060.731	115 5⅜	9	1369.001	131 1⅞
10	795.892	100 0	10	1065.546	115 9¼	10	1374.47	131 5
11	800.065	100 3⅛	11	1070.374	115 11⅝	11	1379.952	131 8⅛
32 0	804.25	100 6⅜	37 0	1075.2126	116 2⅞	42 0	1385.446	131 11⅜
1	808.442	100 9½	1	1080.059	116 6	1	1390.247	132 2½
2	812.648	101 ⅝	2	1084.920	116 9⅛	2	1396.462	132 5⅝
3	816.865	101 3¾	3	1089.791	117 ¼	3	1401.988	132 8¾
4	821.090	101 6⅞	4	1094.671	117 3½	4	1407.522	132 11⅞
5	825.320	101 10	5	1099.564	117 6½	5	1413.07	133 3
6	829.579	102 1⅛	6	1104.469	117 9⅝	6	1418.629	133 6⅛
7	833.837	102 4⅜	7	1109.381	118 ¾	7	1424.195	133 9¼
8	838.108	102 7½	8	1114.307	118 4	8	1429.776	134 ½
9	842.391	102 10⅝	9	1119.244	118 7⅛	9	1435.367	134 3⅝
10	846.681	103 1¾	10	1124.180	118 10¼	10	1440.967	134 6¾
11	850.985	103 4⅞	11	1129.148	119 1⅜	11	1446.580	134 9⅞

Areas and Circumferences of Circles (Continued)

Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in	Diam., ft in	Area, sq ft	Circum., ft in
43 0	1452.205	135 1	46 0	1661 906	144 6⅛	49 0	1885 745	153 11¼
1	1457 836	135 4⅛	1	1667 931	144 9¼	1	1892 172	154 2⅜
2	1463 483	135 7¼	2	1673 97	145 ⅜	2	1898 504	154 5½
3	1469.14	135 10½	3	1680 02	145 3½	3	1905 037	154 8⅝
4	1474 804	136 1⅝	4	1686 077	145 6⅝	4	1911 497	154 11⅞
5	1480 483	136 4¾	5	1692 148	145 9⅞	5	1917 961	155 2⅞
6	1486 173	136 7⅞	6	1698 231	146 1⅛	6	1924.426	155 6
7	1491 870	136 11	7	1704 321	146 4⅛	7	1930 919	155 9¼
8	1497 582	137 2⅛	8	1710 425	146 7¼	8	1937 316	156 ⅛
9	1503 305	137 5¼	9	1716 541	146 10⅜	9	1943 914	156 3½
10	1509 035	137 8⅜	10	1722 663	147 1½	10	1950 439	156 6⅝
11	1514 779	137 11⅝	11	1728 801	147 4⅝	11	1956 969	156 9¾
44 0	1520 534	138 2¾	47 0	1734 947	147 7¾	50 0	1963 5	157 ⅞
1	1526 297	138 5⅞	1	1741 104	147 11	....	....	....
2	1532 074	138 9	2	1747 274	148 2⅛		....	....
3	1537 862	139 ⅛	3	1753 455	148 5¼	.	....	....
4	1543 658	139 3¼	4	1759 643	148 8⅜		....	....
5	1549.478	139 6⅜	5	1765 845	148 11½	.	.	....
6	1555 288	139 9⅝	6	1772 059	149 2⅝	..	...	....
7	1561.116	140 ¾	7	1778 28	149 5⅞	...	....	....
8	1566 959	140 3⅞	8	1784 515	149 8⅞	....	..	....
9	1572 812	140 7½	9	1790 761	150 ⅛	...	...	....
10	1578 673	140 10⅛	10	1797 015	150 3¼	....	...	....
11	1584.549	141 1¼	11	1803 283	150 6⅜	....	...	....
45 0	1590.435	141 4⅜	48 0	1809 562	150 9½	....	....	....
1	1596.329	141 7½	1	1815 848	151 ⅝	....	....	....
2	1602.237	141 10⅔	2	1822 149	151 3¾	....	....	....
3	1608 155	142 1⅞	3	1828 460	151 6⅞	....	....	....
4	1614 082	142 5	4	1834 779	151 10⅛	....	....	....
5	1620 023	142 8⅛	5	1841 173	152 1¼	....	....	....
6	1625.974	142 11¼	6	1847 457	152 4⅜	...	....	....
7	1631 933	143 2⅛	7	1853 809	152 7½	....	....	....
8	1637.907	143 5½	8	1860 175	152 10⅝	...	....	....
9	1643 891	143 8¾	9	1866 552	153 1¾	..	...	....
10	1649.883	143 11⅞	10	1872 937	153 4⅞	..	...	....
11	1655 889	144 3	11	1879 335	153 8⅛	....	....	....

Circular Arcs

To find, by the following table, the length of a circular arc when its chord and height, or versed sine is given.

Rule. Divide the height by the chord; find in the column of heights the number equal to this quotient; take out the corresponding number from the column of lengths; and multiply this number by the given chord.

Example. The chord of an arc is 80 and its versed sine is 30. What is the length of the arc?

Solution. $30 \div 80 = 0.375$. The length of an arc for a height of 0.375 is from table, 1.34063. $80 \times 1.34063 = 107.2504$ = length of arc.

Table of Circular Arcs

Hts	Lengths	Hts	Lengths	Hts	Lengths	Hts	Lengths	Hts	Lengths
.001	1.00001	.062	1.01021	.123	1.03987	.184	1.08797	.245	1.15308
.002	1.00001	.063	1.01054	.124	1.04051	.185	1.08890	.246	1.15428
.003	1.00002	.064	1.01088	.125	1.04110	.186	1.08984	.247	1.15549
.004	1.00004	.065	1.01123	.126	1.04181	.187	1.09079	.248	1.15670
.005	1.00007	.066	1.01158	.127	1.04247	.188	1.09174	.249	1.15791
.006	1.00010	.067	1.01193	.128	1.04313	.189	1.09269	.250	1.15912
.007	1.00013	.068	1.01228	.129	1.04380	.190	1.09365	.251	1.16034
.008	1.00017	.069	1.01264	.130	1.04447	.191	1.09461	.252	1.16156
.009	1.00022	.070	1.01301	.131	1.04515	.192	1.09557	.253	1.16279
.010	1.00027	.071	1.01338	.132	1.04584	.193	1.09654	.254	1.16402
.011	1.00032	.072	1.01376	.133	1.04652	.194	1.09752	.255	1.16526
.012	1.00038	.073	1.01414	.134	1.04722	.195	1.09850	.256	1.16650
.013	1.00045	.074	1.01453	.135	1.04792	.196	1.09949	.257	1.16774
.014	1.00053	.075	1.01493	.136	1.04862	.197	1.10048	.258	1.16899
.015	1.00061	.076	1.01533	.137	1.04932	.198	1.10147	.259	1.17024
.016	1.00069	.077	1.01573	.138	1.05003	.199	1.10247	.260	1.17150
.017	1.00078	.078	1.01614	.139	1.05075	.200	1.10347	.261	1.17276
.018	1.00087	.079	1.01656	.140	1.05147	.201	1.10447	.262	1.17403
.019	1.00097	.080	1.01698	.141	1.05220	.202	1.10548	.263	1.17530
.020	1.00107	.081	1.01741	.142	1.05293	.203	1.10650	.264	1.17657
.021	1.00117	.082	1.01784	.143	1.05367	.204	1.10752	.265	1.17784
.022	1.00128	.083	1.01828	.144	1.05441	.205	1.10855	.266	1.17912
.023	1.00140	.084	1.01872	.145	1.05516	.206	1.10958	.267	1.18040
.024	1.00153	.085	1.01916	.146	1.05591	.207	1.11062	.268	1.18169
.025	1.00167	.086	1.01961	.147	1.05667	.208	1.11165	.269	1.18299
.026	1.00182	.087	1.02006	.148	1.05743	.209	1.11269	.270	1.18429
.027	1.00196	.088	1.02052	.149	1.05819	.210	1.11374	.271	1.18559
.028	1.00210	.089	1.02098	.150	1.05896	.211	1.11479	.272	1.18689
.029	1.00225	.090	1.02145	.151	1.05973	.212	1.11584	.273	1.18820
.030	1.00240	.091	1.02192	.152	1.06051	.213	1.11690	.274	1.18951
.031	1.00256	.092	1.02240	.153	1.06130	.214	1.11796	.275	1.19082
.032	1.00272	.093	1.02289	.154	1.06209	.215	1.11904	.276	1.19214
.033	1.00289	.094	1.02339	.155	1.06288	.216	1.12011	.277	1.19346
.034	1.00307	.095	1.02389	.156	1.06368	.217	1.12118	.278	1.19479
.035	1.00327	.096	1.02440	.157	1.06449	.218	1.12226	.279	1.19612
.036	1.00345	.097	1.02491	.158	1.06530	.219	1.12334	.280	1.19746
.037	1.00364	.098	1.02542	.159	1.06611	.220	1.12444	.281	1.19880
.038	1.00384	.099	1.02593	.160	1.06693	.221	1.12554	.282	1.20014
.039	1.00405	.100	1.02645	.161	1.06775	.222	1.12664	.283	1.20149
.040	1.00426	.101	1.02698	.162	1.06858	.223	1.12774	.284	1.20284
.041	1.00447	.102	1.02752	.163	1.06941	.224	1.12885	.285	1.20419
.042	1.00469	.103	1.02806	.164	1.07025	.225	1.12997	.286	1.20555
.043	1.00492	.104	1.02860	.165	1.07109	.226	1.13108	.287	1.20691
.044	1.00515	.105	1.02914	.166	1.07194	.227	1.13219	.288	1.20827
.045	1.00539	.106	1.02970	.167	1.07279	.228	1.13331	.289	1.20964
.046	1.00563	.107	1.03026	.168	1.07365	.229	1.13444	.290	1.21102
.047	1.00587	.108	1.03082	.169	1.07451	.230	1.13557	.291	1.21239
.048	1.00612	.109	1.03139	.170	1.07537	.231	1.13671	.292	1.21377
.049	1.00638	.110	1.03196	.171	1.07624	.232	1.13785	.293	1.21515
.050	1.00665	.111	1.03254	.172	1.07711	.233	1.13900	.294	1.21654
.051	1.00692	.112	1.03312	.173	1.07799	.234	1.14015	.295	1.21794
.052	1.00720	.113	1.03371	.174	1.07888	.235	1.14131	.296	1.21933
.053	1.00748	.114	1.03430	.175	1.07977	.236	1.14247	.297	1.22073
.054	1.00776	.115	1.03490	.176	1.08066	.237	1.14363	.298	1.22213
.055	1.00805	.116	1.03551	.177	1.08156	.238	1.14480	.299	1.22354
.056	1.00834	.117	1.03611	.178	1.08246	.239	1.14597	.300	1.22495
.057	1.00864	.118	1.03672	.179	1.08337	.240	1.14714	.301	1.22636
.058	1.00895	.119	1.03734	.180	1.08428	.241	1.14832	.302	1.22778
.059	1.00926	.120	1.03797	.181	1.08519	.242	1.14951	.303	1.22920
.060	1.00957	.121	1.03860	.182	1.08611	.243	1.15070	.304	1.23063
.061	1.00989	.122	1.03923	.183	1.08704	.244	1.15189	.305	1.23206

Table of Circular Arcs (Continued)

Hts	Lengths	Hts	Lengths	Hts	Lengths	Hts	Lengths	Hts	Lengths
.306	1.23349	.345	1.29209	.384	1.35575	.423	1.42402	.462	1.49651
.307	1.23492	.346	1.29366	.385	1.35744	.424	1.42583	.463	1.49842
.308	1.23636	.347	1.29523	.386	1.35914	.425	1.42764	.464	1.50033
.309	1.23781	.348	1.29681	.387	1.36084	.426	1.42945	.465	1.50224
.310	1.23926	.349	1.29839	.388	1.36254	.427	1.43127	.466	1.50416
.311	1.24070	.350	1.29997	.389	1.36425	.428	1.43309	.467	1.50608
.312	1.24216	.351	1.30156	.390	1.36596	.429	1.43491	.468	1.50800
.313	1.24361	.352	1.30315	.391	1.36767	.430	1.43673	.469	1.50992
.314	1.24507	.353	1.30474	.392	1.36939	.431	1.43856	.470	1.51185
.315	1.24654	.354	1.30634	.393	1.37111	.432	1.44039	.471	1.51378
.316	1.24801	.355	1.30794	.394	1.37283	.433	1.44222	.472	1.51571
.317	1.24948	.356	1.30954	.395	1.37455	.434	1.44405	.473	1.51764
.318	1.25095	.357	1.31115	.396	1.37628	.435	1.44589	.474	1.51958
.319	1.25243	.358	1.31276	.397	1.37801	.436	1.44773	.475	1.52152
.320	1.25391	.359	1.31437	.398	1.37974	.437	1.44957	.476	1.52346
.321	1.25540	.360	1.31599	.399	1.38148	.438	1.45142	.477	1.52541
.322	1.25689	.361	1.31761	.400	1.38322	.439	1.45327	.478	1.52736
.323	1.25838	.362	1.31923	.401	1.38496	.440	1.45512	.479	1.52931
.324	1.25988	.363	1.32086	.402	1.38671	.441	1.45697	.480	1.53126
.325	1.26138	.364	1.32249	.403	1.38846	.442	1.45883	.481	1.53322
.326	1.26288	.365	1.32413	.404	1.39021	.443	1.46069	.482	1.53518
.327	1.26437	.366	1.32577	.405	1.39196	.444	1.46255	.483	1.53714
.328	1.26588	.367	1.32741	.406	1.39372	.445	1.46441	.484	1.53910
.329	1.26740	.368	1.32905	.407	1.39548	.446	1.46628	.485	1.54106
.330	1.26892	.369	1.33069	.408	1.39724	.447	1.46815	.486	1.54302
.331	1.27044	.370	1.33234	.409	1.39900	.448	1.47002	.487	1.54499
.332	1.27196	.371	1.33399	.410	1.40077	.449	1.47189	.488	1.54696
.333	1.27349	.372	1.33564	.411	1.40254	.450	1.47377	.489	1.54893
.334	1.27502	.373	1.33730	.412	1.40432	.451	1.47565	.490	1.55091
.335	1.27656	.374	1.33896	.413	1.40610	.452	1.47753	.491	1.55289
.336	1.27810	.375	1.34063	.414	1.40788	.453	1.47942	.492	1.55487
.337	1.27964	.376	1.34229	.415	1.40966	.454	1.48131	.493	1.55685
.338	1.28118	.377	1.34396	.416	1.41145	.455	1.48320	.494	1.55884
.339	1.28273	.378	1.34563	.417	1.41324	.456	1.48509	.495	1.56083
.340	1.28428	.379	1.34731	.418	1.41503	.457	1.48699	.496	1.56282
.341	1.28583	.380	1.34899	.419	1.41682	.458	1.48889	.497	1.56481
.342	1.28739	.381	1.35068	.420	1.41861	.459	1.49079	.498	1.56681
.343	1.28895	.382	1.35237	.421	1.42041	.460	1.49269	.499	1.56881
.344	1.29052	.383	1.35406	.422	1.42221	.461	1.49460	.500	1.57080

Table of Lengths of Circular Arcs whose Radius is 1

Rule. Knowing the measure of the circle and the measure of the arc in degrees, minutes and seconds; take from the table the lengths opposite the number of degrees, minutes and seconds in the arc, and multiply their sum by the radius of the circle.

Example. What is the length of an arc subtending an angle of 13° 27′ 8″ with a radius of 8 ft?

Solution. Length for 13° = 0.2268928

27′ = 0.0078540

8″ = 0.0000388

13° 27′ 8″ = 0.2347856

8

Length of arc = 1.8782848 ft

Lengths of Circular Arcs. Radius = 1

Sec	Length	Min	Length.	Deg	Length	Deg	Length
1	0.0000048	1	0.0002909	1	0.0174533	61	1.0646508
2	0.0000097	2	0.0005818	2	0.0349066	62	1.0821041
3	0.0000145	3	0.0008727	3	0.0523599	63	1.0995574
4	0.0000194	4	0.0011636	4	0.0698132	64	1.1170107
5	0.0000242	5	0.0014544	5	0.0872665	65	1.1344640
6	0.0000291	6	0.0017453	6	0.1047198	66	1.1519173
7	0.0000339	7	0.0020362	7	0.1221730	67	1.1693706
8	0.0000388	8	0.0023271	8	0.1396263	68	1.1868239
9	0.0000436	9	0.0026180	9	0.1570796	69	1.2042772
10	0.0000485	10	0.0029089	10	0.1745329	70	1.2217305
11	0.0000533	11	0.0031998	11	0.1919862	71	1.2391838
12	0.0000582	12	0.0034907	12	0.2094395	72	1.2566371
13	0.0000630	13	0.0037815	13	0.2268928	73	1.2740904
14	0.0000679	14	0.0040724	14	0.2443461	74	1.2915436
15	0.0000727	15	0.0043633	15	0.2617994	75	1.3089969
16	0.0000776	16	0.0046542	16	0.2792527	76	1.3264502
17	0.0000824	17	0.0049451	17	0.2967060	77	1.3439035
18	0.0000873	18	0.0052360	18	0.3141593	78	1.3613568
19	0.0000921	19	0.0055269	19	0.3316126	79	1.3788101
20	0.0000970	20	0.0058178	20	0.3490659	80	1.3962634
21	0.0001018	21	0.0061087	21	0.3665191	81	1.4137167
22	0.0001067	22	0.0063995	22	0.3839724	82	1.4311700
23	0.0001115	23	0.0066904	23	0.4014257	83	1.4486233
24	0.0001164	24	0.0069813	24	0.4188790	84	1.4660766
25	0.0001212	25	0.0072722	25	0.4363323	85	1.4835299
26	0.0001261	26	0.0075631	26	0.4537856	86	1.5009832
27	0.0001309	27	0.0078540	27	0.4712389	87	1.5184364
28	0.0001357	28	0.0081449	28	0.4886922	88	1.5358897
29	0.0001406	29	0.0084358	29	0.5061455	89	1.5533430
30	0.0001454	30	0.0087266	30	0.5235988	90	1.5707963
31	0.0001503	31	0.0090175	31	0.5410521	91	1.5882496
32	0.0001551	32	0.0093084	32	0.5585054	92	1.6057029
33	0.0001600	33	0.0095993	33	0.5759587	93	1.6231562
34	0.0001648	34	0.0098902	34	0.5934119	94	1.6406095
35	0.0001697	35	0.0101811	35	0.6108652	95	1.6580628
36	0.0001745	36	0.0104720	36	0.6283185	96	1.6755161
37	0.0001794	37	0.0107629	37	0.6457718	97	1.6929694
38	0.0001842	38	0.0110538	38	0.6632251	98	1.7104227
39	0.0001891	39	0.0113446	39	0.6806784	99	1.7278760
40	0.0001939	40	0.0116355	40	0.6981317	100	1.7453293
41	0.0001988	41	0.0119264	41	0.7155850	101	1.7627825
42	0.0002036	42	0.0122173	42	0.7330383	102	1.7802358
43	0.0002085	43	0.0125082	43	0.7504916	103	1.7976891
44	0.0002133	44	0.0127991	44	0.7679449	104	1.8151424
45	0.0002182	45	0.0130900	45	0.7853982	105	1.8325957
46	0.0002230	46	0.0133809	46	0.8028515	106	1.8500490
47	0.0002279	47	0.0136717	47	0.8203047	107	1.8675023
48	0.0002327	48	0.0139626	48	0.8377580	108	1.8849556
49	0.0002376	49	0.0142535	49	0.8552113	109	1.9024089
50	0.0002424	50	0.0145444	50	0.8726646	110	1.9198622
51	0.0002473	51	0.0148353	51	0.8901179	111	1.9373155
52	0.0002521	52	0.0151262	52	0.9075712	112	1.9547688
53	0.0002570	53	0.0154171	53	0.9250245	113	1.9722221
54	0.0002618	54	0.0157080	54	0.9424778	114	1.9896753
55	0.0002666	55	0.0159989	55	0.9599311	115	2.0071286
56	0.0002715	56	0.0162897	56	0.9773844	116	2.0245819
57	0.0002763	57	0.0165806	57	0.9948377	117	2.0420352
58	0.0002812	58	0.0168715	58	1.0122910	118	2.0594885
59	0.0002860	59	0.0171624	59	1.0297443	119	2.0769418
60	0.0002909	60	0.0174533	60	1.0471976	120	2.0943951

To compute the cord of an arc when the chord of half the arc and the versed sine are given.

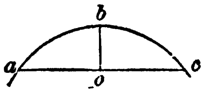

Fig. 32. Circular Arc, Chord and Rise

(The versed sine is the perpendicular *bo*, Fig. 32.)

Rule. From the square of the chord of half the arc subtract the square of the versed sine, and take twice the square root of the remainder.

Example. The chord of half the arc is 60, and the versed sine 36. What is the length of the chord of the arc?

Solution. $60^2 - 36^2 = 2\,304$; $\sqrt{2\,304} = 48$; and $48 \times 2 = 96$, the chord.

To compute the chord of an arc when the diameter and versed sine are given.

Multiply the versed sine by 2 and subtract the product from the diameter; then subtract the square of the remainder from the square of the diameter and take the square root of that remainder.

Example. The diameter of a circle is 100 and the versed sine of an arc 36. What is the chord of the arc?

Solution. $36 \times 2 = 72$; $100 - 72 = 28$; $100^2 - 28^2 = 9\,216$; $\sqrt{9\,216} = 96$, the chord of the arc.

To compute the chord of half an arc when the chord of the arc and the versed sine are given.

Rule. Take the square root of the sum of the squares of the versed sine and of half the chord of the arc.

Example. The chord of an arc is 96 and the versed sine 36. What is the chord of half the arc?

Solution. $\sqrt{36^2 + 48^2} = 60$.

To compute the chord of half an arc when the diameter and versed sine are given.

Rule. Multiply the diameter by the versed sine and take the square root of their product.

To compute a diameter.

Rule 1. Divide the square of the chord of half the arc by the versed sine.

Rule 2. Add the square of half the chord of the arc to the square of the versed sine and divide this sum by the versed sine.

Example. What is the radius of an arc whose chord is 96 and whose versed sine is 36?

Solution. $48^2 + 36^2 = 3\,600$; $3\,600 \div 36 = 100$, the diameter; and the radius $= 50$.

To compute the versed sine.

Rule. Divide the square of the chord of half the arc by the diameter.

To compute the versed sine when the chord of the arc and the diameter are given.

Rule. From the square of the diameter subtract the square of the chord and extract the square root of the remainder; subtract this root from the diameter and halve the remainder.

To compute the length of an arc of a circle when the number of degrees and the radius are given.

Rule 1. Multiply the number of degrees in the arc by 3.1416 multiplied by the radius and divide by 180. The result will be the length of the arc in the same unit as the radius.

Rule 2. Multiply the radius of the circle by 0.01745 and the product by the degrees in the arc.

Example. The number of degrees in an arc is 60 and the radius is 10 in. What is the length of the arc in inches?

Solution. $10 \times 3.1416 \times 60 = 1884.96$; and $1884.96 \div 180 = 10.47$ in. Or, $10 \times 0.01745 \times 60 = 10.47$ in.

To compute the length of the arc of a circle when the length is given in degress, minutes and seconds.

Rule. (1) Multiply the number of degrees by 0.01745329 and the product by the radius. (2) Multiply the number of minutes by 0.00029 and that product by the radius. (3) Multiply the number of seconds by 0.0000048 times the radius. (4) Add together these three results for the length of the arc. (See also, table, page 57.)

Example. What is the length of an arc of 60° 10′ 5″, the radius being 4 ft?

Solution.
(1) $60° \times 0.01745329 \times 4 = 4.188789$ ft
(2) $10' \times 0.00029 \times 4 = 0.0116$ ft
(3) $5'' \times 0.0000048 \times 4 = 0.000096$ ft
(4) The length of the arc $= 4.200485$ ft

To compute the area of a sector of a circle when the degrees of the arc and the radius are given (Fig. 33).

(The degrees of the arc are the same as the angle *aob*.)

Rule. Multiply the number of degrees in the arc by the area of the whole circle and divide by 360.

Example. What is the area of a sector of a circle whose radius is 5 and length of arc 60°?

Solution. Area of circle $= 10 \times 10 \times 0.7854 = 78.54$

Hence, area of sector $= \dfrac{78.5 \times 60}{360} = 13.09$

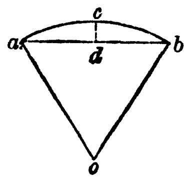

Fig. 33. Sector of Circle

Note. If the length of the arc is given in degrees and minutes, reduce it to minutes, multiply by the area of the whole circle and divide by 21 600.

To compute the area of a sector of a circle when the length of the arc and radius are given.

Rule. Multiply the length of the arc by half the length of the radius. The product is the area.

To compute the area of a segment of a circle when the chord and versed sine of the arc and the radius or diameter of the circle are given.

(The versed sine is the distance *cd*, Fig. 33.)

Rule 1. When the segment is less than a semicircle. (1) Find the area of the sector having the same arc as the segment. (2) Find the area of a triangle formed by the chord of the segment and the radii of the sector. (3) Take the difference of these areas.

Rule 2. When the segment is greater than a semicircle. Find, by the preceding rule, the area of the lesser portion of the circle and subtract it from the area of the whole circle. The remainder will be the area.

To compute the area of the surface of a sphere.

Rule. Multiply the diameter by the circumference. The product will be the area of the surface.

Example. What is the area of the surface of a sphere 10 in in diameter?

Solution. Circumference of sphere = $10 \times 3.1416 = 31.416$ in; $10 \times 31.416 = 314.16$ sq in, the area of surface of sphere.

To compute the total area of the surface of a segment of a sphere.

Rule. Multiply the height (*bc*, Fig. 34) by the circumference of the sphere and add the product to the area of the base.

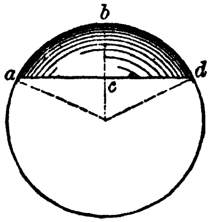

Fig. 34.
Segment of Sphere.

To find the area of the base, having the diameter of the sphere and the length of the versed sine of the arc *abd*, find the length of the chord *ad* by the rule on page 58. Having, then, the length of the chord *ad* for the diameter of the base, find the area of the base.

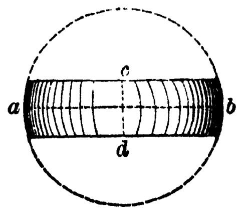

Fig. 35
Zone of Sphere

Example. The height, *bc*, of a segment *abd*, is 36 in, and the diameter of the sphere is 100 in (Fig. 34). What is the area of the convex surface and the area of the whole surface?

Solution. $100 \times 3.1416 = 314.16$ in, the circumference of sphere
$36 \times 314.16 = 11309.76$ sq in, the area of the convex surface
$100 - (36 \times 2) = 28$
$\sqrt{100^2 - 28^2} = 96$, the chord *ad*
$96^2 \times 0\ 7854 = 7238\ 2464$ sq in, the area of the base
$11309.76 + 7238\ 2464 = 18548.0064$ sq in, the total area

To compute the total area of the surface of a spherical zone.

Rule. Multiply the height, *cd* (Fig. 35), by the circumference of the sphere for the convex surface and add to it the area of the two ends for the total area.

Spheroids, or Ellipsoids of Revolution

Definition. Spheroids, or ellipsoids, are figures generated by the revolution of a semiellipse about one of its diameters.

When the revolution is about the long diameter, they are PROLATE; and when it is about the short diameter, they are OBLATE.

A PROLATE SPHEROID is approximately cigar-shaped and an OBLATE SPHEROID is, in form, somewhat like a watch.

To compute the area of the surface of a spheroid.

Let $a = \frac{1}{2}$ the long axis; let $b = \frac{1}{2}$ the short axis;

let
$$\frac{a^2 - b^2}{a^2} = e^2 \quad \text{or} \quad e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Then, the area of the SURFACE OF THE OBLATE SPHEROID

$$= 2\,\pi a^2 + \frac{\pi b^2}{e} \log\left(\frac{1 + e}{1 - e}\right)$$

and the area of the SURFACE OF THE PROLATE SPHEROID

$$= 2\,\pi b^2 + 2\,\pi ab\,\frac{\sin^{-1} e}{e}$$

In the first formula, NATURAL LOGARITHMS must be used. The natural logarithm may be obtained by multiplying the common logarithm by 2.302. The value of the expression $\sin^{-1} e$ may be determined by finding the angle whose natural sine is equal to e and dividing this angle by 57.3.

Note. Although the above formulas are complicated, no simpler rules that give correct results can be given.

To compute the area of the surface of a cylinder.

Rule. Multiply the length of the cylinder by the circumference of one of the ends and add to the product the areas of the two ends.

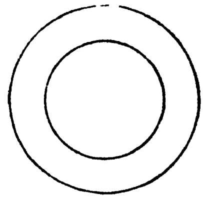

Fig. 36. Circular Ring

To compute the area of a circular ring (Fig. 36).

Rule. Find the area of both circles and subtract the area of the smaller from the area of the larger; the remainder will be the area of the ring.

To compute the area of the surface of a cone.

Rule. Multiply the circumference of the base by one-half the slant-height or side of the cone, for the convex area. Add to this the area of the base, for the whole area.

Example. The diameter of the base of a cone is 3 in and the slant-height 15 in. What is the area of the surface of the cone?

Solution.	3×3.1416	= 9 4248	= circumference of base
	$9\ 4248 \times 7\frac{1}{2}$	= 70 686 sq in	= area of convex surface
	$3 \times 3 \times 0.7854$	= 7 068 sq in	= area of base
Area of entire surface of cone		= 77.754 sq in	

To compute the area of the surface of the frustum of a cone (Fig. 37).

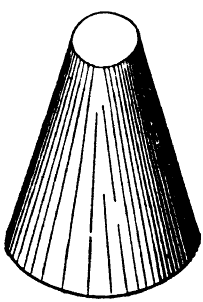

Fig. 37. Frustum of Cone

Rule. Multiply the sum of the circumferences of the two ends by the slant-height of the frustum and divide by 2, for the area of the convex surface. Add the areas of the two ends.

To compute the area of the surface of a pyramid.

Rule. Multiply the perimeter of the base by one-half the slant-height and add to the product the area of the base.

To compute the area of the surface of the frustum of a pyramid.

Rule. Multiply the sum of the perimeters of the two ends by the slant-height of the frustum, halve the product, and add to the result the areas of the two ends.

Mensuration of Solids

To compute the volume of a prism. (For definition of a prism see under Volumes, Article 3, Part I.)

Rule. Multiply the area of the base or end by the altitude or perpendicular height.

This rule applies to prisms with bases or ends of any shape, as long as these bases or ends are parallel.

To compute the volume of a prismoid.

Definition. A prismoid is a solid with parallel but unequal ends or bases and with quadrilateral sides.

Rule. To the sum of the areas of the two ends or bases add four times the area of the middle section parallel to them, and multiply this sum by one-sixth of the altitude or perpendicular height.

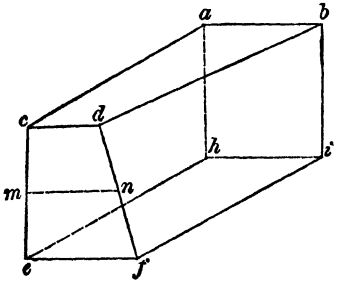

Fig 38. Quadrangular Prismoid

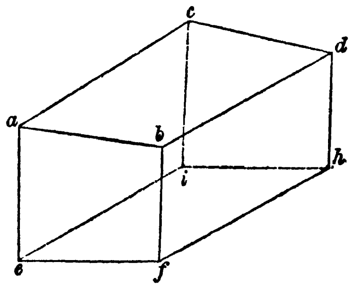

Fig. 39. Prism Truncated Obliquely

Example. What is the volume of a quadrangular prismoid, as Fig. 38, in which $ab = 6$ in, $cd = 4$ in, $ac = he = 10$ in, $ce = 8$ in, $ef = 8$ in and $ih = 6$ in?

Solution. Area of top $= \dfrac{6+4}{2} \times 10 = 50$ sq in

Area of bottom $= \dfrac{8+6}{2} \times 10 = 70$ sq in

Area of middle section $= \dfrac{6+6}{2} \times 10 = 60$ sq in

$$[50 + 70 + (4 \times 60)] \times 8/6 = 480 \text{ cu in}$$

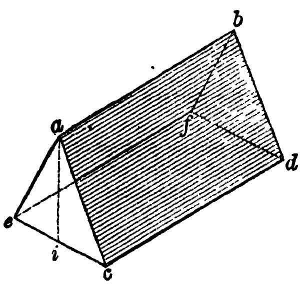

Fig. 40. Wedge or Right Triangular Prism

Note. The length of the end of the middle section (as at mn, in Fig. 38) $= \dfrac{cd + ef}{2}$

To find the volume of a prism truncated obliquely.

Rule. Multiply the area of the base by the average height of the edges.

Example. What is the volume of a truncated prism (Fig. 39) in which $ef = 6$ in, $fh = 10$ in, $ea = 10$ in, $ci = 12$ in, $dh = 10$ in and $fb = 8$ in?

Solution. Area of base $= 6 \times 10 = 60$ sq in

$$\text{Average height of edges} = \frac{10 + 12 + 8 + 10}{4} = 10 \text{ in}$$

$$60 \times 10 = 600 \text{ cu in}$$

To compute the volume of a wedge or right triangular prism when the ends are parallel and equal.

Rule. Multiply the area of one end by the length of the wedge.

To compute the volume of a wedge when the ends are not parallel.

Rule. Add together the lengths of the three edges, *ab*, *cd* and *ef* (Fig. 40); multiply their sum by the altitude or perpendicular height of the wedge, and then by the breadth of the back, and divide the product by 6.

Regular Polyhedrons

Definition. A regular polyhedron is a solid contained within a certain number of similar and equal plane faces, all of which are equal regular polygons. The following is a list of all the regular polyhedrons:

(1) The TETRAHEDRON, or pyramid
(2) The HEXAHEDRON, or cube, which has six square faces.
(3) The OCTAHEDRON, which has eight triangular faces.
(4) The DODECAHEDRON, which has twelve pentagonal faces.
(5) The ICOSAHEDRON, which has twenty triangular faces.

To compute the volume of a regular polyhedron.

Rule 1. When the radius of the circumscribing sphere is given. Multiply the cube of the radius of the sphere by the multiplier opposite to the polyhedron in column 2 of the following table.

Rule 2. When the radius of the inscribed sphere is given. Multiply the cube of the radius of the inscribed sphere by the multiplier opposite to the polyhedron in column 3 of the table.

Rule 3. When the area of the surface of the polyhedron is given. Cube the surface given, extract the square root, and multiply the root by the multiplier opposite to the polyhedron in column 4 of the table.

Table of Factors for Determining the Volumes of Regular Polyhedrons

Figure	1 Number of sides	2 Factor for volume by radius of circumscribing sphere	3 Factor for volume by radius of inscribed sphere	4 Factor for volume by surface
Tetrahedron	4	0.5132	13.85641	0.0517
Hexahedron . . .	6	1.5396	8.0000	0.06804
Octahedron . . .	8	1.33333	6.9282	0.07311
Dodecahedron	12	2.78517	5.55029	0.08169
Icosahedron .	20	2.53615	5.05406	0.0856

To compute the volume of a cylinder.

Rule. Multiply the area of the base by the altitude or length.

To compute the volume of a cone.

Rule. Multiply the area of the base by one-third the altitude.

To compute the volume of the frustum of a cone (Fig. 41).

Rule. Add together the squares of the diameters of the two ends or bases and the product of the two diameters; multiply this sum by 0.7854, and this product by the altitude, and then divide this last product by 3.

Example. What is the volume of a frustum of a cone 9 in in height, 5 in in diameter at the base and 3 in in diameter at the top?

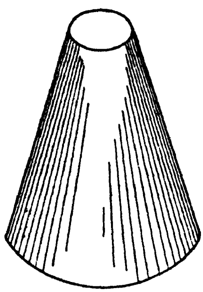

Fig. 41. Frustum of Cone

Solution. $5^2 + 3^2 = 34$. $3 \times 5 = 15$. $15 + 34 = 49$, the sum of the squares of the two diameters added to the product of the diameters of the ends. $49 \times 0.7854 = 38.4846$.

$$\frac{38.4846 \times 9}{3} = 115.4538 \text{ cu in}$$

To compute the volume of a pyramid.

Rule. Multiply the area of the base by the altitude or perpendicular height, and take one-third of the product.

To compute the volume of the frustum of a pyramid.

Rule. Find the height that the pyramid would be if the top were put on, and then compute the volume of the completed pyramid and the volume of the part added; subtract the latter from the former, and the remainder will be the volume of the frustum.

To compute the volume of a sphere.

Rule. Multiply the cube of the diameter by 0 5236.

To compute the volume of a segment of a sphere.

Rule 1. To three times the square of the radius of its base add the square of its height; multiply this sum by the height and the product by 0.5236.

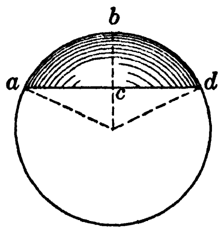

Fig. 42. Segment of Sphere

Rule 2. From three times the diameter of the sphere subtract twice the height of the segment; multiply this remainder by the square of the height and the product by 0.5236.

Example. The segment of a sphere has a radius, *ac* (Fig. 42), of 7 in for its base, and a height, *cb*, of 4 in; what is its volume?

Solution. (By Rule 1.) $3 \times 7^2 = 147$, and $147 + 4^2 = 163$, or three times the square of the radius of the base plus the square of the height. $163 \times 4 \times 0.5236 = 341.3872$ cu in = the volume of the segment.

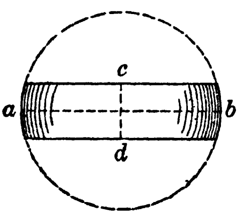

Fig. 43. Zone of Sphere

Second Solution. By the rule for finding the diameter of a circle when a chord and its versed sine are given, we find that the diameter of the sphere in this case is 16.25 in; then, by Rule 2, $(3 \times 16.25) - (2 \times 4) = 40.75$; and $40.75 \times 4^2 \times 0.5236 = 341.3872$ cu in, the volume of the segment.

To compute the volume of a spherical zone.

Definition. The part of a sphere included between two parallel planes (Fig. 43).

Rule. To the sum of the squares of the radii of the two ends add one-third of the square of the height of the zone; multiply this sum by the height and that product by 1.5708.

To compute the volume of a prolate spheroid.

Rule. Multiply the square of the short axis by the long axis and this product by 0.5236.

To compute the volume of an oblate spheroid.

Rule. Multiply the square of the long axis by the short axis and this product by 0.5236.

To compute the volume of a paraboloid of revolution (Fig. 44).

Rule. Multiply the area of the base by half the altitude.

To compute the volume of a hyperboloid of revolution (Fig. 45).

Rule. To the square of the radius of the base add the square of the middle diameter; multiply this sum by the height and the product by 0.5236.

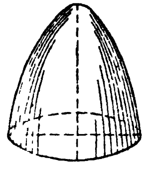

Fig 44. Paraboloid of Revolution

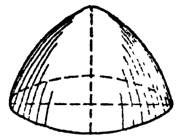

Fig. 45 Hyperboloid of Revolution

To compute the volume of any figure of revolution.

Rule. Multiply the area of the generating surface by the circumference described by its center of gravity.

To compute the volume of an excavation, where the ground is irregular and the bottom of the excavation is level (Fig. 46).

Rule. Divide the surface of the ground to be excavated into equal squares of about 10 ft on a side, and ascertain by means of a level the height of each corner, *a*, *a*, *a*, *b*, *b*, *b*, etc., above the level to which the ground is to be excavated. Then add together the heights of all the corners that come in one square only. Next take twice the sum of the heights of all the corners that come in two squares, as *b*, *b*, *b*; next three times the sum of the heights of all the corners that come in three squares, as *c*, *c*, *c*; and then four times the sum of the heights of all the corners that belong to four squares, as *d*, *d*, *d*, etc. Add together all these quantities, and multiply their sum by one-fourth the area of one of the squares. The result will be the volume of the excavation.

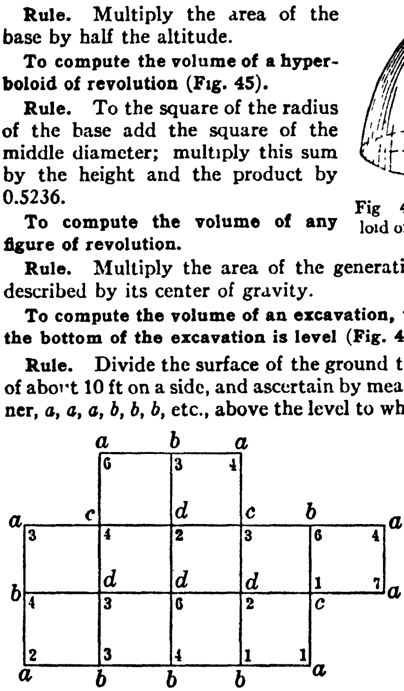

Fig. 46. Plan of Excavation

Example. Let the plan of an excavation for a cellar be as shown in Fig. 46, and the heights of each corner above the proposed bottom of the cellar be as given by the numbers in the figure. Then the volume of the cellar will be as follows, the area of each square being $10 \times 10 = 100$ sq ft:

Volume $= \frac{1}{4}$ of 100 (a's + 2 b's + 3 c's + 4 d's)

The a's in this case	$= 4 + 6 + 3 + 2 + 1 + 7 + 4$	$= 27$
$2 \times$ the sum of the b's	$= 2 \times (3 + 6 + 1 + 4 + 3 + 4)$	$= 42$
$3 \times$ the sum of the c's	$= 3 \times (1 + 3 + 4)$	$= 24$
$4 \times$ the sum of the d's	$= 4 \times (2 + 3 + 6 + 2)$	$= 52$
		145

Volume $= 25 \times 145 = 3\,625$ cu ft, the quantity of earth to be excavated.

4. GEOMETRICAL PROBLEMS

Problem 1. To bisect, or divide into equal parts, a given line, ab (Fig. 47).

From *a* and *b*, with any radius greater than half of *ab*, describe arcs intersecting in *c* and *d*. The line *cd*, connecting these intersections, will bisect *ab* and be perpendicular to it.

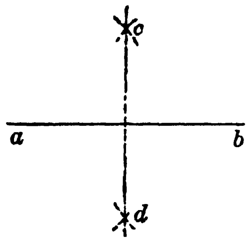

Fig. 47. Line Bisected

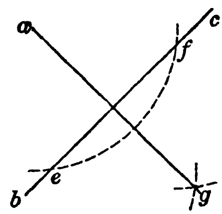

Fig. 48. Perpendicular from Point to Given Line

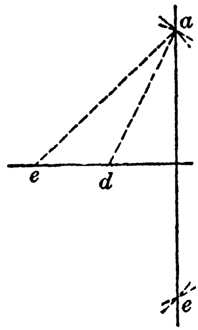

Fig. 49. Perpendicular from Point to Given Line

Problem 2. To draw a perpendicular to a given straight line from a point without it.

First Method (Fig. 48). From the point *a* describe an arc cutting the line *bc* in two places, as *e* and *f*. From *e* and *f* describe two arcs, with the same radius, intersecting in *g*; then a line drawn from *a* to *g* is perpendicular to the line *bc*.

Second Method (Fig. 49). From any two points, *d* and *c*, at some distance apart in the given line, and with radii *da* and *ca* respectively, describe arc

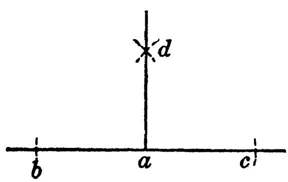

Fig. 50. Perpendicular from Point in Given Line

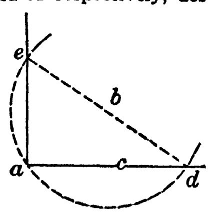

Fig. 51. Perpendicular from Extremity of Given Line

cutting at *a* and *e*. Draw *ae*, which is the perpendicular required. This method is useful where the given point is opposite the end of the line, or nearly so.

Problem 3. To draw a perpendicular to a straight line from a given point, a, in that line.

First Method (Fig. 50). With any radius, from the given point *a* in the line, describe arcs cutting the line in the points *b* and *c*. Then with *b* and *c* as centers, and with any radius greater than *ab* or *ac*, describe arcs cutting each other at *d*. The line *da* is the perpendicular required.

Second Method (Fig. 51), when the given point is at the end of the line. From any point, b, outside of the line, and with a radius ba, describe a semicircle passing through a and cutting the given line at d. Through b and d draw a straight line intersecting the semicircle at e. The line ea will then be perpendicular to the line ac at the point a.

Third Method (Fig. 52), or the 3, 4 and 5 Method. From the point a on the given line measure off 4 in, or 4 ft, or 4 of any other unit and with the same unit of measure describe an arc, with a as a center and 3 units as a radius. Then from b describe an arc with a radius of 5 units, cutting the first arc in c. Then ca is the perpendicular required. This method is particularly useful in laying out a right angle on the ground, or framing a house where the foot is used as the unit and the lines are laid off by the straight-edge.

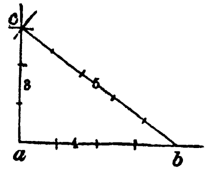

Fig 52. Perpendicular from Extremity of Given Line

In laying out a right angle on the ground, the proportions of the triangle may be 30, 40 and 50, or any other multiple of 3, 4 and 5; and it can best be laid out with the tape. Thus, first measure off, say 40 feet from a (Fig. 52) on the given line; then let one person hold the end of the tape at b, another hold the tape at the 80-ft mark at a, and a third person take hold of the tape at the 50-ft mark, with his thumb and finger, and pull the tape taut. The 50-ft mark will then be at the point c in the line of the perpendicular.

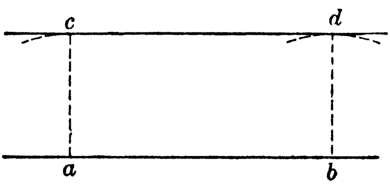

Fig. 53. Straight Line Parallel to Given Line

Problem 4. To draw a straight line parallel to a given line at a given distance away (Fig. 53).

From any two points near the ends of the given line describe two arcs about opposite the given line. Draw the line cd tangent to these arcs and it will be parallel to ab.

Problem 5. To construct an angle equal to a given angle (Fig. 54).

With the point A, at the apex of the given angle, as a center, and any radius, describe the arc BC. With the point a, at the vertex of the new angle, as a center, and with the same radius as before, describe an arc, as BC. With BC as a radius and b as a center, describe an arc cutting the other arc at c. Then will cab be equal to the given angle CAB.

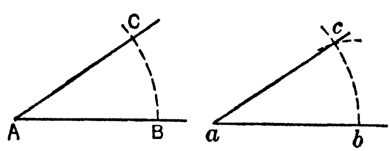

Fig. 54. Angle Equal to Given Angle

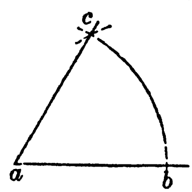

Fig. 55. Angle of 60°

Problem 6. From a point on a given line to draw a line making an angle of 60° with the given line (Fig. 55).

Take any distance, as ab, as a radius, and with a as a center, describe the arc bc. With b as a center and with the same radius, describe an arc cutting the

first one at c. Draw from a a line through c, and it will make with ab an angle of 60°.

Problem 7. From a given point, A, on a given line, AE, to draw a line making an angle 45° with the given line (Fig. 56).

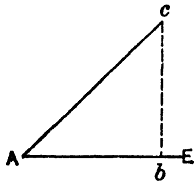

Fig. 56. Angle of 45°

Measure off from A, on AE, any distance, Ab, and at b draw a line perpendicular to AE. Measure off on this perpendicular bc equal to Ab and draw a line from A through c. This line Ac will make an angle of 45° with AE.

Problem 8. From any point, A, on a given line, to draw a line which will make any desired angle with the given line (Fig. 57).

To solve this problem the tables of chords on pages 81 to 89 are used. Find in the table the length of chord to a radius 1, for the given angle. Then take any radius, as large as convenient and describe an arc of a circle bc, with A as a center. Multiply the chord of the angle, found in the table, by the length of the radius Ab, and with the product as a new radius and with b as a center, describe a short arc cutting bc in d. Draw a line from A through d and it will make the required angle with DE.

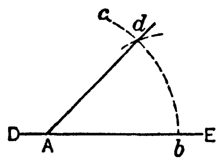

Fig. 57 Line Making Any Angle with Given Line

Example. Draw a line from A on DE, making an angle of 44° 40′ with DE (Fig. 57).

Solution. The largest convenient radius for the arc is 8 in. With A as a center and 8 in as a radius, describe the arc bc. In the table of chords, the chord for an angle or arc of 44° 40′ to a radius 1 is 0.76. Multiplying this by 8 in, the length of the new radius is 6.08 in; and with this as radius and with b as a center, describe an arc cutting bc in d. Ad will be the line required.

Problem 8a. To lay off a given angle approximately, by means of an ordinary two-foot rule.

Tables of Angles Corresponding to Openings of a Two-Foot Rule *

In.	Deg.	Min.	In.	Deg.	Min.	In.	Deg.	Min.	In.	Deg.	Min.	In.	Deg.	Min.
¼	1	12		11	22	4½	21	37	.	32	3	8¾	42	46
..	1	48	2½	11	58		22	13	6¾	32	40		43	24
½	2	24	.	12	34	¾	22	50	.	33	17	9	44	3
.	3	00	¾	13	10		23	27	7	33	54	.	44	42
¾	3	36	..	13	46	5	24	3		34	33	¼	45	21
	4	11	3	14	22	.	24	39	¼	35	10	.	45	59
1	4	47	.	14	58	¼	25	16	..	35	47	½	46	38
	5	23	¼	15	34	.	25	53	½	36	25		47	17
¼	5	58		16	10	½	26	30	.	37	3	¾	47	56
	6	34	½	16	46	.	27	7	¾	37	41		48	35
½	7	10		17	22	¾	27	44	.	38	19	10	49	15
.	7	46	¾	17	59		28	21	8	38	57		49	54
¾	8	22		18	35	6	28	58		39	35	¼	50	34
..	8	58	4	19	12	.	29	35	¼	40	13		51	13
2	9	34	.	19	48	¼	30	11	.	40	51	½	51	53
...	10	10	¼	20	24	.	30	49	½	41	29		52	33
¼	10	46		21	00	½	31	26	.	42	7			.

* Trautwine.

Lay one leg of the rule on the paper or board with its inner edge coinciding with the given line. Open the rule until the distance between the inner edges at the ends correspond with that given for the angle in the following table; then draw a line by marking along the inner edge of the other leg, and it will give the desired angle within a very close approximation.

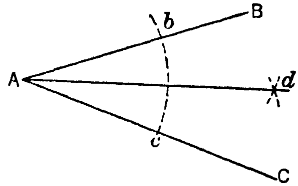

Fig. 58. Angle Bisected

Problem 9. To bisect a given angle, as BAC (Fig. 58).

With A as a center and any radius, describe an arc, as cb. With c and b as centers, and any radius greater than one-half of cb, describe two arcs, intersecting in d. Draw from A a line through d and it will bisect the angle BAC.

Problem 10. To bisect the angle included between two lines, as AB and CD, when the vertex of the angle is not on the drawing (Fig. 59).

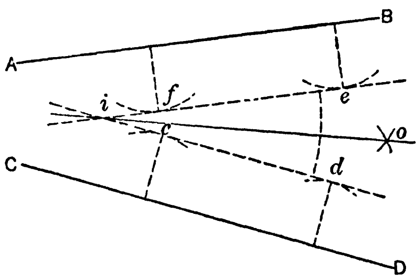

Fig 59. Angle Bisected. Angle not on Drawing

Draw fe parallel to AB and cd parallel to CD, so that they intersect, as at i. Make radii at f, e, c, and d equal. Bisect angle eid, as in Problem 9; draw a line through i and o. This will bisect the angle between the given lines.

Problem 11. Through two given points, B and C, to describe an arc of a circle with a given radius (Fig. 60).

With B and C as centers and with a radius equal to the given radius, describe two arcs intersecting at A. With A as a center and the same radius, describe the arc bc, which will pass through the given points, B and C.

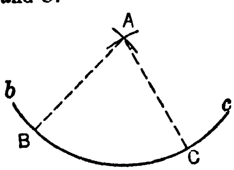

Fig 60. Circular Arc through Two Given Points

Problem 12. To find the center of a given circle (Fig. 61).

Draw any chord in the circle, as ab, and bisect this chord by the perpendicular cd. This line will pass through the center of the circle and ef will be a diameter of the circle. Bisect ef, and the center o will be the center of the circle.

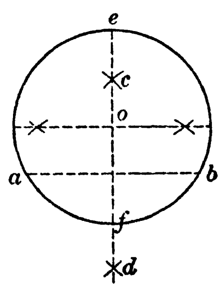

Fig. 61. Center of Given Circle

Problem 13. To draw a circular arc through three given points, as A, B and C (Fig. 62).

Draw lines from A to B and from B to C. Bisect AB and BC by the lines aa and cc and prolong these lines until they intersect at o, which will be the center for the arc sought. With o as a center and Ao as a radius, describe the arc ABC.

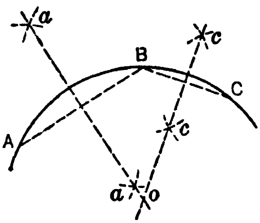

Fig. 62. Circular Arc through Three Given Points

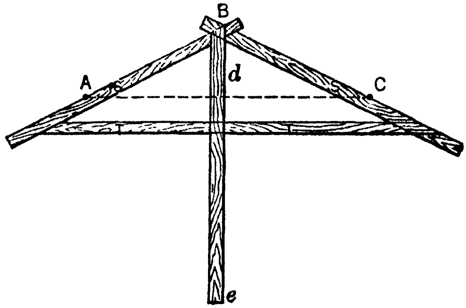

Fig. 63. Frame for Drawing Circular Arc

Problem 14. To describe a circular arc passing through three given points when the center is not available, by means of a triangle (Fig. 63).

Let *A*, *B* and *C* be the given points. Insert two stiff pins or nails at *A* and *C*. Place two strips of wood, *SS*, as shown in the figure, one against *A*, the other against *C*, and inclined so that their intersection shall come at the third point, *B*. Fasten the strips together at their intersection and nail a third strip, *T*, to their other ends, so as to make a firm triangle. Place the pencil-point at *B*, and, keeping the edges of the triangle against *A* and *C*, move the triangle to the left and right. The pencil-point will describe the required arc.

When the points *A* and *C* are at the same distance from *B*, if a strip of wood is nailed to the triangle, so that its edge *de* is at right-angles to a line joining *A* and *C*, as the triangle is moved one way or the other, the edge *de* will always point to the center of the circle. This principle is used in linear perspective.

Problem 15. To describe a circular arc which will be tangent at a given point, A, to a straight line, and pass through a given point, C, outside the line (Fig. 64).

Draw from *A* a line perpendicular to the given line. Connect *A* and *C* by a straight line and bisect this line by the perpendicular *ac*. The point where these two perpendiculars intersect is the center of the circle.

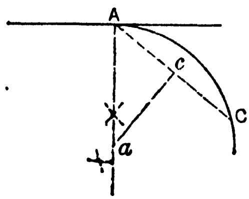

Fig. 64. Circular Arc Tangent to Line at Given Point

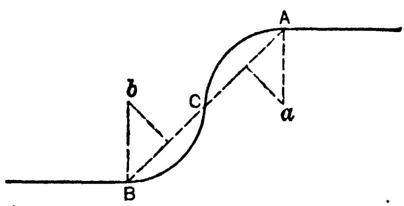

Fig. 65. Reversed Curve between Parallel Lines

Problem 16. To connect two parallel lines by a reversed curve composed of two circular arcs of equal radius, and tangent to the lines at given points, as A and B (Fig. 65).

Join *A* and *B* and divide the line into two equal parts at *C*. Bisect *CA* and *CB* by perpendiculars. At *A* and *B* erect perpendiculars to the given lines, and the intersections *a* and *b* will be the centers of the arcs composing the required curve.

Problem 17. On a given line, as AB (Fig. 66), to construct a compound curve of three arcs of circles, the radii of the two side arcs being equal and of a given length, and their centers being in the given line. The central arc is to pass through a given point, C, on the perpendicular bisecting the given line, and is to be tangent to the other two arcs.

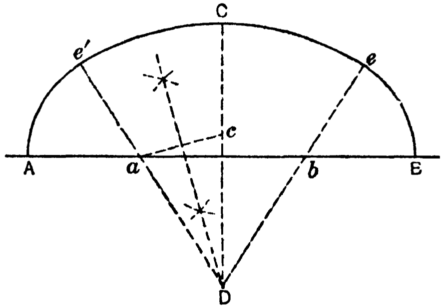

Fig. 66. Curve of Three Circular Arcs

Draw the perpendicular *CD*. Lay off *Aa*, *Bb* and *Cc*, each equal to the given radius of the side arcs, draw *ac*; bisect *ac* by a perpendicular The intersection of this line with the perpendicular *CD* is the required center of the central arc. Through *a* and *b* draw the lines *Dc* and *De′*; from *a* and *b*, with the given radius, equal to *Aa*, *Bb*, describe the arcs *Ae′* and *Be*, from *D* as a center, and with *CD* as a radius, describe the arc *eCe′* which completes the curve required.

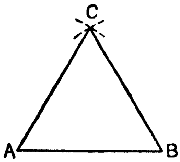

Fig 67. Equilateral Triangle on Given Base

Problem 18. To construct a triangle upon a given straight line or base, the length of the two sides being given (Figs. 67 and 68).

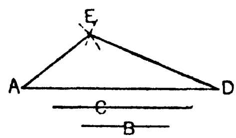

Fig. 68. Scalene Triangle on Given Base

First. An equilateral triangle (Fig. 67). With the extremities *A* and *B* of the given line as centers and with *AB* as a radius describe arcs cutting each other at *C*. Join *AC* and *BC*.

Second. A scalene triangle (Fig. 68). Let *AD* be the given base and the other two sides be equal to *C* and *B*. With *D* as a center, and with a radius equal to *C*, describe at *E* an arc of indefinite length. With *A* as a center and with *B* as a radius, describe an arc cutting the first at *E*. Join *E* with *A* and *D*. *ADE* is the required triangle.

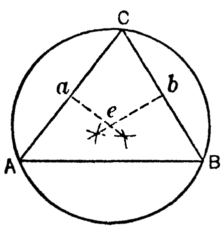

Fig. 69. Triangle and Circumscribed Circle

Problem 19. To describe a circle about a triangle (Fig. 69).

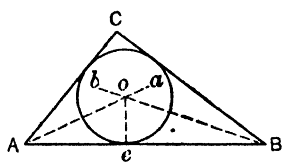

Fig. 70. Triangle and Inscribed Circle

Bisect two of the sides, as *AC* and *CB*, of the triangle, and at their centers, erect perpendicular lines, as *ae* and *be*, intersecting at *e*. With *e* as a center, and *eC* as a radius, describe a circle. It will pass through *A* and *B*.

Problem 20. To inscribe a circle in a triangle (Fig. 70).

Bisect two of the angles, *A* and *B*, of the triangle by lines cutting each other at *o*. With *o* as a center, and with *oe* as a radius, describe a circle. It will be tangent to the other two sides.

Problem 21. To inscribe a square in a circle and to describe a circle about a square (Fig. 71).

To inscribe the square. Draw two diameters, AB and CD, at right-angles to each other. Join the points A, D, B and C. $ADBC$ is the inscribed square.

To describe the circle. Draw the diagonals as before, intersecting at E, and with E as a center and AE as a radius, describe the circle.

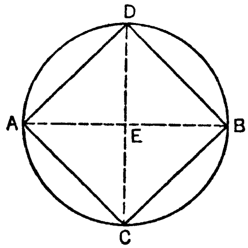

Fig. 71. Inscribed Square and Circumscribed Circle

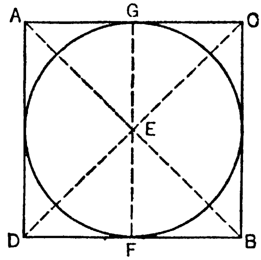

Fig 72. Inscribed Circle and Circumscribed Square

Problem 22. To inscribe a circle in a square and to describe a square about a circle (Fig. 72).

To inscribe the circle. Draw the diagonals AB and CD, intersecting at E. Draw the perpendicular EG to one of the sides. Then with E as a center, and EG as a radius, describe a circle. It will be tangent to all four sides of the square.

To describe the square. Draw two diameters, AB and CD, at right-angles to each other, and prolonged beyond the circumference. Draw the diameter GF, bisecting the angle CEA or BED. Draw lines through G and F perpendicular to GF, and terminating in the diagonals. Draw AD and CB to complete the square.

Problem 23. To inscribe a pentagon in a circle (Fig. 73).

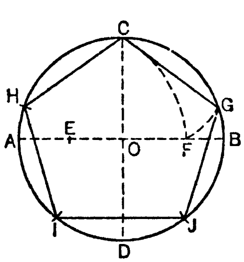

Fig. 73. Circle and Inscribed Pentagon

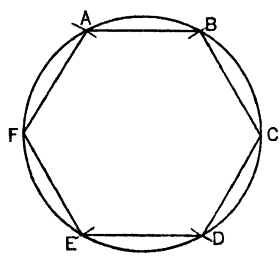

Fig. 74. Circle and Inscribed Hexagon

Draw two diameters, AB and CD, at right-angles to each other. Bisect AO at E. With E as a center and EC as a radius, cut OB at F. With C as a center and CF as a radius, cut the circle at G and H. With these points as centers and the same radius, cut the circle at I and J. Join I, J, G, C and H. $IJGCHI$ is the inscribed regular pentagon.

Problem 24. To inscribe a regular hexagon in a circle (Fig. 74).

Lay off on the circumference the radius of the circle six times, and connect the points.

Problem 25. To construct a regular hexagon upon a given straight line, AB (Fig. 75).

From A and B, with a radius equal to AB, describe arcs intersecting at O. With O as a center and a radius equal to AB, describe a circle, and from A or B lay off the lengths BC, CD, DE, EF and FA on the circumference of the circle. $ABCDEFA$ is the required regular hexagon.

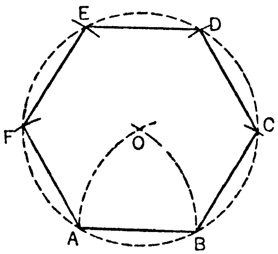

Fig. 75. Regular Hexagon on Given Line

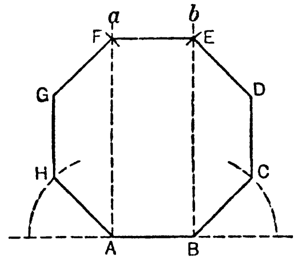

Fig. 76. Regular Octagon on Given Line

Problem 26. To construct a regular octagon upon a given straight line, AB (Fig. 76).

Produce the line AB both ways and draw the perpendiculars Aa and Bb, of indefinite length. Bisect the external angles at A and B and make the length of the bisecting lines equal to AB. From H and C draw lines parallel to Aa or Bb and equal in length to AB. From G and D as centers describe arcs, with a radius AB, cutting the perpendiculars Aa and Bb in F and E. Draw GF, FE and ED. $ABCDEFGHA$ is the required octagon.

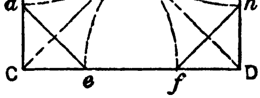

Fig. 77. Square and Inscribed Regular Octagon

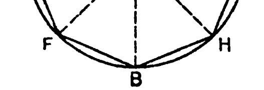

Fig. 78. Circle and Inscribed Regular Octagon

Problem 27. To construct a regular octagon in a square (Fig. 77).

Draw the diagonals AD and BC and from A, B, C and D, with a radius equal to AO, describe arcs cutting the sides of the square in a, b, c, d, e, f, h and i. Draw ai, hf, ed and cb. $aihfedcba$ is the required octagon.

Problem 28. To inscribe a regular octagon in a circle (Fig. 78).

Draw two diameters, AB and CD, at right-angles to each other. Bisect the angles AOD and AOC by the diameters EF and GH. $AEDHBFCGA$ is the required octagon.

Problem 29. To inscribe a circle within a regular polygon.

First. When the polygon has an even number of sides, as in Fig. 79. Bisect two opposite sides at A and B, draw AB and bisect it at C by a diagonal, DE, connecting two opposite angles, as D and E. The circle drawn with a radius CA and with C as a center is the inscribed circle required.

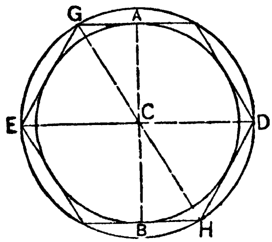

Fig. 79. Regular Polygon, Even Number of Sides, with Inscribed and Circumscribed Circles

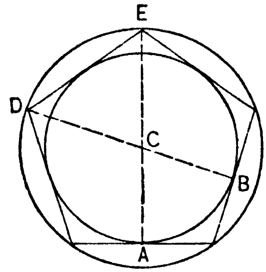

Fig. 80. Regular Polygon, Odd Number of Sides, with Inscribed and Circumscribed Circles

Second. When the number of sides is odd, as in Fig. 80. Bisect two of the adjacent sides as at A and B, and draw lines AE and BD, to the opposite angles, and intersecting at C. The circle drawn with C as a center and CA as a radius is the inscribed circle required.

Problem 30. To draw a circumscribing circle around a regular polygon.

First. When the number of sides is even, as in Fig. 79. Draw two diagonals from opposite angles, as ED and GH, intersecting at C. The circle drawn with C as a center and with CD as a radius is the circumscribing circle required.

Second. When the number of sid s is odd, as in Fig. 80. Determine the center, C, as in the last problem. The circle drawn with C as a center and CD as a radius, is the circumscribing circle required.

Problems on the Ellipse, the Parabola, the Hyperbola and the Cycloid

The Ellipse

Problem 31. To describe an ellipse, the length and breadth, or the two axes, being given.

First Method (Fig. 81), the two axes, AB and CD, being given. On AB and CD as diameters and from the same center, O, describe the circles $AGBH$ and $CLDK$. Take any convenient number of points on the circumference of the outer circle, as b, b', b'', etc., and from them draw lines to the center, O, cutting the inner circle at the points a, a', a'', etc., respectively. From the points b, b', etc., draw lines parallel to the shorter axis CD; and from the points a,

a', etc., draw lines parallel to the longer axis AB, and intersecting the first set of lines at c, c', c'', etc. These last points will be points in the ellipse, and by determining a sufficient number of them, the ellipse can be drawn.

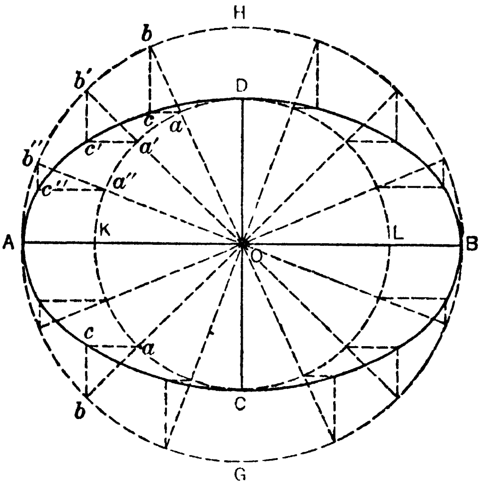

Fig. 81. Ellipse Described on Given Axes

Second Method (Fig. 82). Take the straight-edge, made of a stiff piece of paper, cardboard or wood, and from some point as a, mark off ab equal to half the shorter diameter CD, and ac equal to half the longer diameter AB. Place the straight-edge so that the point b is on the longer and the point c on the shorter diameter. Then will the point a be over a point in the ellipse. Make on the paper a dot at a and move the straight-edge around, always keeping the points b and c over the major and minor axes respectively. In this way any number of points in the ellipse may be determined and the ellipse drawn.

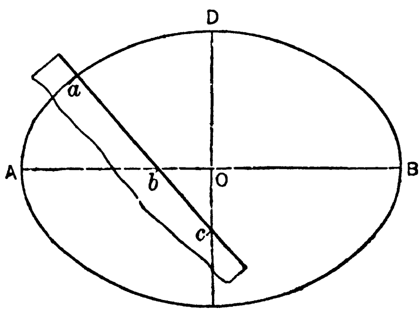

Fig. 82. Ellipse Described with Straight-Edge

Third Method (Fig. 83). Given, the two axes, AB and CD. From the point D as a center, and a radius AO, equal to one-half of AB, describe an arc cutting AB at F and F'. These two points are called the foci of the ellipse.

Note. One property of the ellipse is, that the sums of the distances of any two points on the circumference from the foci are the same. Thus $F'D + DF = F'E + EF$ or $F'G + GF$.

Fix two pins in the axis AB at F and F' and loop upon them a thread, or cord equal in length, when fastened to the pins, to AB, so as, when stretched as per dotted line FDF', it will just reach to the extremity D of the short axis. Place a pencil-point inside the chord, as at E, and move the pencil along, keeping the cord stretched tight. The pencil-point will trace the ellipse required.

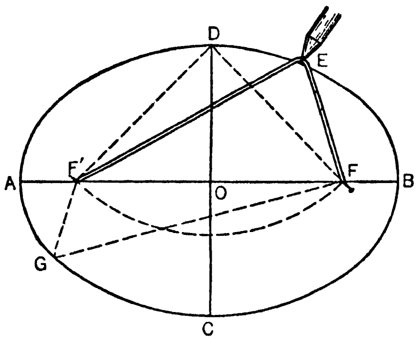

Fig. 83. Ellipse Described with String and Pencil

Problem 32. To draw a tangent to an ellipse at a given point on the curve (Fig. 84).

Let it be required to draw a tangent at the point E on the ellipse shown. First determine the foci F and F' as in the third method for describing an ellipse, and from E draw lines EF and EF'. Prolong EF' to a, so that Ea equals EF. Bisect the angle aEF by describing arcs from a and F as centers, as shown at b, and through b draw a line through E. This line is the tangent required. If it is required to draw a line normal to the curve at E, as, for instance, the joint of

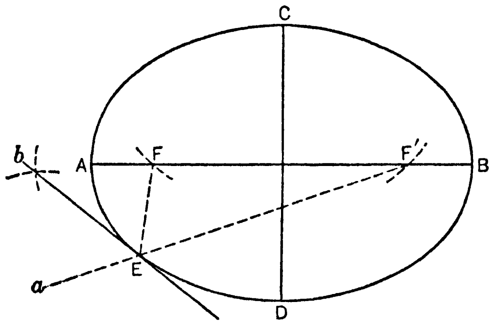

Fig. 84. Tangent Drawn to Point on Ellipse

an elliptical arch, bisect the angle FEF', and draw the bisecting line through E, and it will be the normal to the curve and the proper line at that point for the joint of an elliptical arch.

Problem 33. To draw a tangent to an ellipse from a given point outside of the curve (Fig. 85).

From the given point T as a center, and with a radius equal to the distance to the nearer focus F, describe an arc of a circle. From F' as a center, and with a radius equal to the length of the longer axis of the ellipse, describe arcs cutting the circle just described at a and b. Draw lines from F' to a and b, cutting the ellipse at E and G. Draw lines from T through E and G and they will be the tangents required.

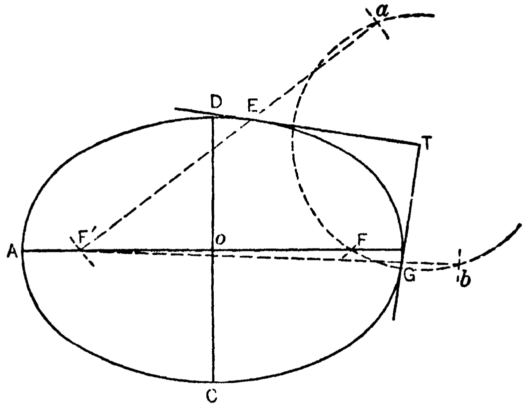

Fig. 85. Tangent Drawn to Ellipse from Point Outside

Problem 34. To describe an ellipse approximately, by means of circular arcs.

First. With arcs of two radii (Fig. 86). Take half the difference of the two axes AB and CD, and set it off from the center O to a and c on OA and OC; draw ac and on AB set off half ac from a to d, draw di parallel to ac; set off Oe equal to Od; join ei and draw em and dm parallel respectively to id and ie.

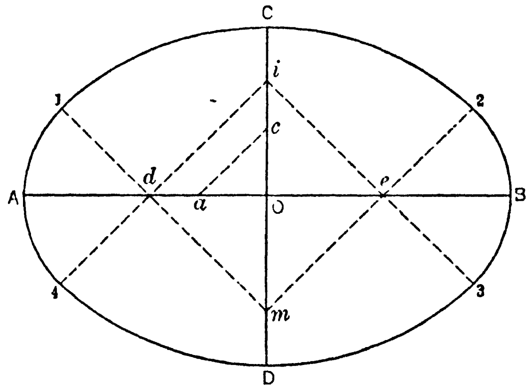

Fig. 86. Ellipse Described with Circular Arcs of Two Radii

With m as a center and with a radius mC, describe an arc through C, terminating in the points 1 and 2 on md and me produced. With i as a center, and with iD as a radius, describe an arc through D, terminating in points 3 and 4 on ie and id produced. With d and e as centers, describe arcs through A and B, connecting the points 1 and 4 and 2 and 3. The four arcs thus described form approximately an ellipse. This method is not satisfactory when the conjugate or minor axis is less than two-thirds the transverse or major axis.

Another method of approximating an ellipse by means of arcs of two radii, is shown in Fig. 87, the axis major AB and the semiminor axis OC being given. Draw the rectangle $AabBA$, and the diagonal CB. Lay off Cc equal to the difference between OB and OC. Bisect cB at M and erect the perpendicular YD, intersecting CO produced at Y and OB, at x. Make $Ox' = Ox$. Then will x, x', and Y be the three centers required, the curves becoming tangent at D and at the corresponding point on the left-hand side of the ellipse. This method results in a curve which is slightly fuller at the haunches than the curve drawn by the preceding method.

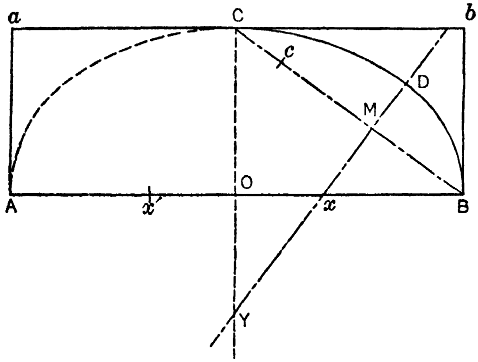

Fig. 87. Ellipse Described with Circular Arcs of Two Radii

Second. With arcs of three radii (Fig. 88). On the transverse or major axis

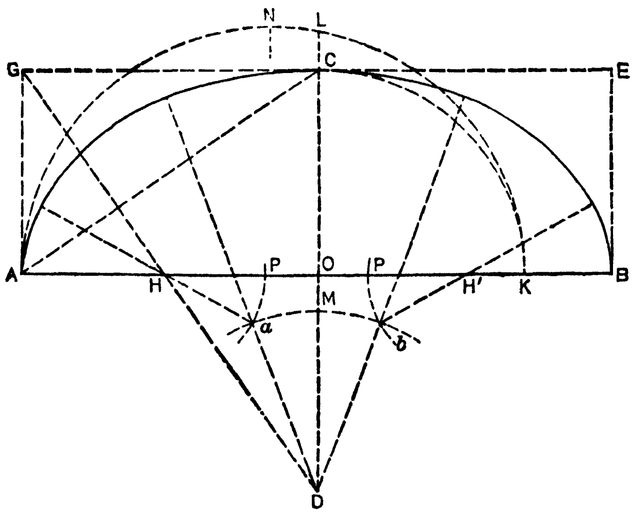

Fig. 88. Ellipse Described with Circular Arcs of Three Radii

AB draw the rectangle $AGEBA$, equal in height to OC, half the conjugate or minor axis. Draw AC and draw GD perpendicular to AC. Set off OK equal to OC, and on AK as a diameter describe the semicircle ANK. Extend OC to L and to D. Set off OM equal to CL, and with D as a center and with a radius

DM, describe an arc. With *A* and *B* as centers and with a radius *OL*, cut *AB* at *P* and *P*. From *H* as a center, and with a radius *HP*, cut the arc *ab* at *a*. *H'* and *b* are determined in like manner. The points *H*, *a*, *D*, *b* and *H'*, are the centers of the arcs required.

Produce the lines *aH*, *Da*, *Db*, and *bH'*, and thus determine the lengths of the arcs. This method is practicable for all ellipses. It is often employed for vaults, stone arches and bridges.

The Parabola

Problem 35. To describe a parabola when the vertex A, the axis AB and a point, M, of the curve are given (Fig. 89).

Construct the rectangle *ABMCA*. Divide *MC* into any number of equal parts, four for instance. Divide *AC* in like manner. Connect *A*1, *A*2 and *A*3. Through 1', 2', 3', draw parallels to the axis *AB*. The intersections I, II and III, of these lines, are points in the required curve.

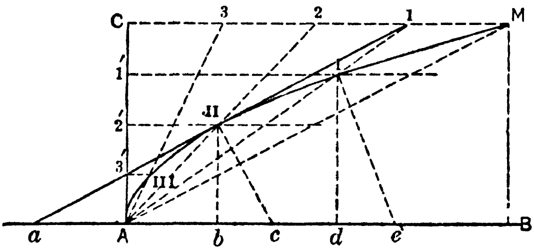

Fig. 89. Parabola and Tangent to Point on Parabola

Problem 36. To draw a tangent to a given point, II, of the parabola (Fig. 89).

From the given point II let fall a perpendicular on the axis *AB* at *b*. Produce the axis to the left of *A*. Make *Aa* equal to *Ab*. A line drawn through *a* and II is the tangent required. The lines perpendicular to the tangent are called NORMALS.

To draw a normal to any point, as I, the tangent to any other point, II being given.

Draw the normal II*c*. From I, let fall a perpendicular I*d*, on the axis *AB*. Lay off *de* equal to *bc*. The line I*e* is the normal required. The tangent may be drawn at I by laying off a perpendicular to the normal I*e* at I.

The Hyperbola

If from any point, *P*, of an hyperbola, two straight lines are drawn to two fixed points, as *F* and *F'*, the foci of the hyperbola, their DIFFERENCE is always the same.

Problem 37. To describe an hyperbola when a vertex, a, the given difference ab and one of the foci, F are given (Fig. 90).

Draw the axis *AB* of the hyperbola, with the given distance *ab* and the focus

F marked on it. From *b* lay off bF_1 equal to aF to determine the other focus F_1. Take any point, as 1 on AB, and with $a1$ as a radius and F as a center, describe two short arcs above and below the axis. With $b1$ as a radius, and F' as a center, describe arcs cutting those just described, at P and P'. Take several points, as 2, 3 and 4, and determine the corresponding points P_2, P_3 and P_4 in the same way. The curve passing through these points is an hyperbola.

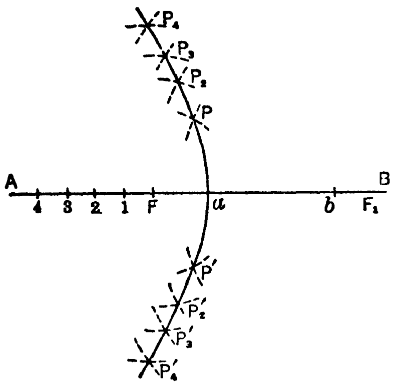

Fig. 90. Hyperbola Described

To draw a tangent to any point of an hyperbola, draw lines from the given point to each of the foci and bisect the angle thus formed. The bisecting line is the tangent required.

The Cycloid

The CYCLOID is the curve described by a point on the circumference of a circle rolling in a straight line.

Problem 38. To describe a cycloid (Fig. 91).

Draw the straight line AB. Describe the generating circle tangent to this line at its middle point D, and through the center C, of the circle, draw the line

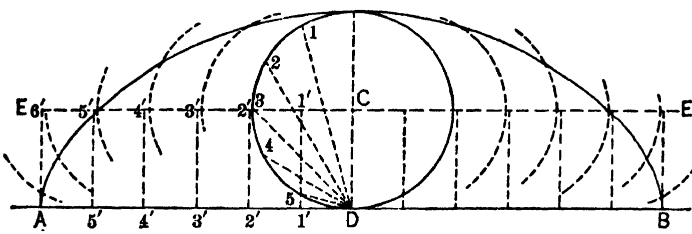

Fig. 91. Cycloid Described

EE parallel to AB. Let fall a perpendicular from C upon AB. Divide the semi-circumference into any number of equal parts, for example, six. Lay off on AB and CE distances $C1'$, $1'2'$, etc., equal to the divisions of the circumference. Draw the chords $D1$, $D2$, etc. From the points $1'$, $2'$, $3'$, etc., on the line CE, with radii equal to the generating circle, describe arcs as shown. From the points $1'$, $2'$, $3'$, $4'$, $5'$, etc., on the line BA, and with radii equal respectively to the chords $D1$, $D2$, $D3$, $D4$, $D5$, describe arcs cutting the preceding arcs. The intersections are points of the required cycloid.

Table of Chords. Radius = 1.000

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0'	0.0000	0.0175	0.0349	0.0524	0.0698	0.0872	0.1047	0.1221	0.1395	0.1569	0.1743	0'
1	0.0003	0.0177	0.0352	0.0526	0.0701	0.0875	0.1050	0.1224	0.1398	0.1572	0.1746	1
2	0.0006	0.0180	0.0355	0.0529	0.0704	0.0878	0.1053	0.1227	0.1401	0.1575	0.1749	2
3	0.0009	0.0183	0.0358	0.0532	0.0707	0.0881	0.1055	0.1230	0.1404	0.1578	0.1752	3
4	0.0012	0.0186	0.0361	0.0535	0.0710	0.0884	0.1058	0.1233	0.1407	0.1581	0.1755	4
5	0.0015	0.0189	0.0364	0.0538	0.0713	0.0887	0.1061	0.1235	0.1410	0.1584	0.1758	5
6	0.0017	0.0192	0.0366	0.0541	0.0715	0.0890	0.1064	0.1238	0.1413	0.1587	0.1761	6
7	0.0020	0.0195	0.0369	0.0544	0.0718	0.0893	0.1067	0.1241	0.1415	0.1589	0.1763	7
8	0.0023	0.0198	0.0372	0.0547	0.0721	0.0896	0.1070	0.1244	0.1418	0.1592	0.1766	8
9	0.0026	0.0201	0.0375	0.0550	0.0724	0.0899	0.1073	0.1247	0.1421	0.1595	0.1769	9
10	0.0029	0.0204	0.0378	0.0553	0.0727	0.0901	0.1076	0.1250	0.1424	0.1598	0.1772	10
11	0.0032	0.0207	0.0381	0.0556	0.0730	0.0904	0.1079	0.1253	0.1427	0.1601	0.1775	11
12	0.0035	0.0209	0.0384	0.0558	0.0733	0.0907	0.1082	0.1256	0.1430	0.1604	0.1778	12
13	0.0038	0.0212	0.0387	0.0561	0.0736	0.0910	0.1084	0.1259	0.1433	0.1607	0.1781	13
14	0.0041	0.0215	0.0390	0.0564	0.0739	0.0913	0.1087	0.1262	0.1436	0.1610	0.1784	14
15	0.0044	0.0218	0.0393	0.0567	0.0742	0.0916	0.1090	0.1265	0.1439	0.1613	0.1787	15
16	0.0047	0.0221	0.0396	0.0570	0.0745	0.0919	0.1093	0.1267	0.1442	0.1616	0.1789	16
17	0.0049	0.0224	0.0398	0.0573	0.0747	0.0922	0.1096	0.1270	0.1444	0.1618	0.1792	17
18	0.0052	0.0227	0.0401	0.0576	0.0750	0.0925	0.1099	0.1273	0.1447	0.1621	0.1795	18
19	0.0055	0.0230	0.0404	0.0579	0.0753	0.0928	0.1102	0.1276	0.1450	0.1624	0.1798	19
20	0.0058	0.0233	0.0407	0.0582	0.0756	0.0931	0.1105	0.1279	0.1453	0.1627	0.1801	20
21	0.0061	0.0236	0.0410	0.0585	0.0759	0.0933	0.1108	0.1282	0.1456	0.1630	0.1804	21
22	0.0064	0.0239	0.0413	0.0588	0.0762	0.0936	0.1111	0.1285	0.1459	0.1633	0.1807	22
23	0.0067	0.0241	0.0416	0.0590	0.0765	0.0939	0.1114	0.1288	0.1462	0.1636	0.1810	23
24	0.0070	0.0244	0.0419	0.0593	0.0768	0.0942	0.1116	0.1291	0.1465	0.1639	0.1813	24
25	0.0073	0.0247	0.0422	0.0596	0.0771	0.0945	0.1119	0.1294	0.1468	0.1642	0.1816	25
26	0.0076	0.0250	0.0425	0.0599	0.0774	0.0948	0.1122	0.1296	0.1471	0.1645	0.1818	26
27	0.0079	0.0253	0.0428	0.0602	0.0776	0.0951	0.1125	0.1299	0.1473	0.1647	0.1821	27
28	0.0081	0.0256	0.0430	0.0605	0.0779	0.0954	0.1128	0.1302	0.1476	0.1650	0.1824	28
29	0.0084	0.0259	0.0433	0.0608	0.0782	0.0957	0.1131	0.1305	0.1479	0.1653	0.1827	29
30	0.0087	0.0262	0.0436	0.0611	0.0785	0.0960	0.1134	0.1308	0.1482	0.1656	0.1830	30
31	0.0090	0.0265	0.0439	0.0614	0.0788	0.0962	0.1137	0.1311	0.1485	0.1659	0.1833	31
32	0.0093	0.0268	0.0442	0.0617	0.0791	0.0965	0.1140	0.1314	0.1488	0.1662	0.1836	32
33	0.0096	0.0271	0.0445	0.0619	0.0794	0.0968	0.1143	0.1317	0.1491	0.1665	0.1839	33
34	0.0099	0.0273	0.0448	0.0622	0.0797	0.0971	0.1145	0.1320	0.1494	0.1668	0.1842	34
35	0.0102	0.0276	0.0451	0.0625	0.0800	0.0974	0.1148	0.1323	0.1497	0.1671	0.1845	35
36	0.0105	0.0279	0.0454	0.0628	0.0803	0.0977	0.1151	0.1325	0.1500	0.1674	0.1847	36
37	0.0108	0.0282	0.0457	0.0631	0.0806	0.0980	0.1154	0.1328	0.1502	0.1676	0.1850	37
38	0.0111	0.0285	0.0460	0.0634	0.0808	0.0983	0.1157	0.1331	0.1505	0.1679	0.1853	38
39	0.0113	0.0288	0.0462	0.0637	0.0811	0.0986	0.1160	0.1334	0.1508	0.1682	0.1856	39
40	0.0116	0.0291	0.0465	0.0640	0.0814	0.0989	0.1163	0.1337	0.1511	0.1685	0.1859	40
41	0.0119	0.0294	0.0468	0.0643	0.0817	0.0992	0.1166	0.1340	0.1514	0.1688	0.1862	41
42	0.0122	0.0297	0.0471	0.0646	0.0820	0.0994	0.1169	0.1343	0.1517	0.1691	0.1865	42
43	0.0125	0.0300	0.0474	0.0649	0.0823	0.0997	0.1172	0.1346	0.1520	0.1694	0.1868	43
44	0.0128	0.0303	0.0477	0.0651	0.0826	0.1000	0.1175	0.1349	0.1523	0.1697	0.1871	44
45	0.0131	0.0305	0.0480	0.0654	0.0829	0.1003	0.1177	0.1352	0.1526	0.1700	0.1873	45
46	0.0134	0.0308	0.0483	0.0657	0.0832	0.1006	0.1180	0.1355	0.1529	0.1703	0.1876	46
47	0.0137	0.0311	0.0486	0.0660	0.0835	0.1009	0.1183	0.1357	0.1531	0.1705	0.1879	47
48	0.0140	0.0314	0.0489	0.0663	0.0838	0.1012	0.1186	0.1360	0.1534	0.1708	0.1882	48
49	0.0143	0.0317	0.0492	0.0666	0.0840	0.1015	0.1189	0.1363	0.1537	0.1711	0.1885	49
50	0.0145	0.0320	0.0494	0.0669	0.0843	0.1018	0.1192	0.1366	0.1540	0.1714	0.1888	50
51	0.0148	0.0323	0.0497	0.0672	0.0846	0.1021	0.1195	0.1369	0.1543	0.1717	0.1891	51
52	0.0151	0.0326	0.0500	0.0675	0.0849	0.1023	0.1198	0.1372	0.1546	0.1720	0.1894	52
53	0.0154	0.0329	0.0503	0.0678	0.0852	0.1026	0.1201	0.1375	0.1549	0.1723	0.1897	53
54	0.0157	0.0332	0.0506	0.0681	0.0855	0.1029	0.1204	0.1378	0.1552	0.1726	0.1900	54
55	0.0160	0.0335	0.0509	0.0683	0.0858	0.1032	0.1206	0.1381	0.1555	0.1729	0.1902	55
56	0.0163	0.0337	0.0512	0.0686	0.0861	0.1035	0.1209	0.1384	0.1558	0.1732	0.1905	56
57	0.0166	0.0340	0.0515	0.0689	0.0864	0.1038	0.1212	0.1386	0.1560	0.1734	0.1908	57
58	0.0169	0.0343	0.0518	0.0692	0.0867	0.1041	0.1215	0.1389	0.1563	0.1737	0.1911	58
59	0.0172	0.0346	0.0521	0.0695	0.0869	0.1044	0.1218	0.1392	0.1566	0.1740	0.1914	59
60	0.0175	0.0349	0.0524	0.0698	0.0872	0.1047	0.1221	0.1395	0.1569	0.1743	0.1917	60

Table of Chords (Continued). Radius = 1.0000

M.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	M.
0′	0.1917	0.2091	0.2264	0.2437	0.2611	0.2783	0.2956	0.3129	0.3301	0.3473	0.3645	0′
1	0.1920	0.2093	0.2267	0.2440	0.2613	0.2786	0.2959	0.3132	0.3304	0.3476	0.3648	1
2	0.1923	0.2096	0.2270	0.2443	0.2616	0.2789	0.2962	0.3134	0.3307	0.3479	0.3650	2
3	0.1926	0.2099	0.2273	0.2446	0.2619	0.2792	0.2965	0.3137	0.3310	0.3482	0.3653	3
4	0.1928	0.2102	0.2276	0.2449	0.2622	0.2795	0.2968	0.3140	0.3312	0.3484	0.3656	4
5	0.1931	0.2105	0.2279	0.2452	0.2625	0.2798	0.2971	0.3143	0.3315	0.3487	0.3659	5
6	0.1934	0.2108	0.2281	0.2455	0.2628	0.2801	0.2973	0.3146	0.3318	0.3490	0.3662	6
7	0.1937	0.2111	0.2284	0.2458	0.2631	0.2804	0.2976	0.3149	0.3321	0.3493	0.3665	7
8	0.1940	0.2114	0.2287	0.2460	0.2634	0.2807	0.2979	0.3152	0.3324	0.3496	0.3668	8
9	0.1943	0.2117	0.2290	0.2463	0.2636	0.2809	0.2982	0.3155	0.3327	0.3499	0.3670	9
10	0.1946	0.2119	0.2293	0.2466	0.2639	0.2812	0.2985	0.3157	0.3330	0.3502	0.3673	10
11	0.1949	0.2122	0.2296	0.2469	0.2642	0.2815	0.2988	0.3160	0.3333	0.3504	0.3676	11
12	0.1952	0.2125	0.2299	0.2472	0.2645	0.2818	0.2991	0.3163	0.3335	0.3507	0.3679	12
13	0.1955	0.2128	0.2302	0.2475	0.2648	0.2821	0.2994	0.3166	0.3338	0.3510	0.3682	13
14	0.1957	0.2131	0.2305	0.2478	0.2651	0.2824	0.2996	0.3169	0.3341	0.3513	0.3685	14
15	0.1960	0.2134	0.2307	0.2481	0.2654	0.2827	0.2999	0.3172	0.3344	0.3516	0.3688	15
16	0.1963	0.2137	0.2310	0.2484	0.2657	0.2830	0.3002	0.3175	0.3347	0.3519	0.3690	16
17	0.1966	0.2140	0.2313	0.2486	0.2660	0.2832	0.3005	0.3178	0.3350	0.3522	0.3693	17
18	0.1969	0.2143	0.2316	0.2489	0.2662	0.2835	0.3008	0.3180	0.3353	0.3525	0.3696	18
19	0.1972	0.2146	0.2319	0.2492	0.2665	0.2838	0.3011	0.3183	0.3355	0.3527	0.3699	19
20	0.1975	0.2148	0.2322	0.2495	0.2668	0.2841	0.3014	0.3186	0.3358	0.3530	0.3702	20
21	0.1978	0.2151	0.2325	0.2498	0.2671	0.2844	0.3017	0.3189	0.3361	0.3533	0.3705	21
22	0.1981	0.2154	0.2328	0.2501	0.2674	0.2847	0.3019	0.3192	0.3364	0.3536	0.3708	22
23	0.1983	0.2157	0.2331	0.2504	0.2677	0.2850	0.3022	0.3195	0.3367	0.3539	0.3710	23
24	0.1986	0.2160	0.2333	0.2507	0.2680	0.2853	0.3025	0.3198	0.3370	0.3542	0.3713	24
25	0.1989	0.2163	0.2336	0.2510	0.2683	0.2855	0.3028	0.3200	0.3373	0.3545	0.3716	25
26	0.1992	0.2166	0.2339	0.2512	0.2685	0.2858	0.3031	0.3203	0.3376	0.3547	0.3719	26
27	0.1995	0.2169	0.2342	0.2515	0.2688	0.2861	0.3034	0.3206	0.3378	0.3550	0.3722	27
28	0.1998	0.2172	0.2345	0.2518	0.2691	0.2864	0.3037	0.3209	0.3381	0.3553	0.3725	28
29	0.2001	0.2174	0.2348	0.2521	0.2694	0.2867	0.3040	0.3212	0.3384	0.3556	0.3728	29
30	0.2004	0.2177	0.2351	0.2524	0.2697	0.2870	0.3042	0.3215	0.3387	0.3559	0.3730	30
31	0.2007	0.2180	0.2354	0.2527	0.2700	0.2873	0.3045	0.3218	0.3390	0.3562	0.3733	31
32	0.2010	0.2183	0.2357	0.2530	0.2703	0.2876	0.3048	0.3221	0.3393	0.3565	0.3736	32
33	0.2012	0.2186	0.2359	0.2533	0.2706	0.2878	0.3051	0.3223	0.3396	0.3567	0.3739	33
34	0.2015	0.2189	0.2362	0.2536	0.2709	0.2881	0.3054	0.3226	0.3398	0.3570	0.3742	34
35	0.2018	0.2192	0.2365	0.2538	0.2711	0.2884	0.3057	0.3229	0.3401	0.3573	0.3745	35
36	0.2021	0.2195	0.2368	0.2541	0.2714	0.2887	0.3060	0.3232	0.3404	0.3576	0.3748	36
37	0.2024	0.2198	0.2371	0.2544	0.2717	0.2890	0.3063	0.3235	0.3407	0.3579	0.3750	37
38	0.2027	0.2200	0.2374	0.2547	0.2720	0.2893	0.3065	0.3238	0.3410	0.3582	0.3753	38
39	0.2030	0.2203	0.2377	0.2550	0.2723	0.2896	0.3068	0.3241	0.3413	0.3585	0.3756	39
40	0.2033	0.2206	0.2380	0.2553	0.2726	0.2899	0.3071	0.3244	0.3416	0.3587	0.3759	40
41	0.2036	0.2209	0.2383	0.2556	0.2729	0.2902	0.3074	0.3246	0.3419	0.3590	0.3762	41
42	0.2038	0.2212	0.2385	0.2559	0.2732	0.2904	0.3077	0.3249	0.3421	0.3593	0.3765	42
43	0.2041	0.2215	0.2388	0.2561	0.2734	0.2907	0.3080	0.3252	0.3424	0.3596	0.3768	43
44	0.2044	0.2218	0.2391	0.2564	0.2737	0.2910	0.3083	0.3255	0.3427	0.3599	0.3770	44
45	0.2047	0.2221	0.2394	0.2567	0.2740	0.2913	0.3086	0.3258	0.3430	0.3602	0.3773	45
46	0.2050	0.2224	0.2397	0.2570	0.2743	0.2916	0.3088	0.3261	0.3433	0.3605	0.3776	46
47	0.2053	0.2226	0.2400	0.2573	0.2746	0.2919	0.3091	0.3264	0.3436	0.3608	0.3779	47
48	0.2056	0.2229	0.2403	0.2576	0.2749	0.2922	0.3094	0.3267	0.3439	0.3610	0.3782	48
49	0.2059	0.2232	0.2406	0.2579	0.2752	0.2925	0.3097	0.3269	0.3441	0.3613	0.3785	49
50	0.2062	0.2235	0.2409	0.2582	0.2755	0.2927	0.3100	0.3272	0.3444	0.3616	0.3788	50
51	0.2065	0.2238	0.2411	0.2585	0.2758	0.2930	0.3103	0.3275	0.3447	0.3619	0.3790	51
52	0.2067	0.2241	0.2414	0.2587	0.2760	0.2933	0.3106	0.3278	0.3450	0.3522	0.3793	52
53	0.2070	0.2244	0.2417	0.2590	0.2763	0.2936	0.3109	0.3281	0.3453	0.3625	0.3796	53
54	0.2073	0.2247	0.2420	0.2593	0.2766	0.2939	0.3111	0.3284	0.3456	0.3628	0.3799	54
55	0.2076	0.2250	0.2423	0.2596	0.2769	0.2942	0.3114	0.3287	0.3459	0.3630	0.3802	55
56	0.2079	0.2253	0.2426	0.2599	0.2772	0.2045	0.3117	0.3289	0.3462	0.3633	0.3805	56
57	0.2082	0.2255	0.2429	0.2602	0.2775	0.2948	0.3120	0.3292	0.3464	0.3636	0.3808	57
58	0.2085	0.2258	0.2432	0.2605	0.2778	0.2950	0.3123	0.3295	0.3467	0.3639	0.3810	58
59	0.2088	0.2261	0.2434	0.2608	0.2781	0.2953	0.3126	0.3298	0.3470	0.3642	0.3813	59
60	0.2091	0.2264	0.2437	0.2611	0.2783	0.2956	0.3129	0.3301	0.3473	0.3645	0.3816	60

Table of Chords (Continued). Radius = 1.0000

M.	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	M.
0′	0.3816	0.3987	0.4158	0.4329	0.4499	0.4669	0.4838	0.5008	0.5176	0.5345	0.5513	0′
1	0.3819	0.3990	0.4161	0.4332	0.4502	0.4672	0.4841	0.5010	0.5179	0.5348	0.5516	1
2	0.3822	0.3993	0.4164	0.4334	0.4505	0.4675	0.4844	0.5013	0.5182	0.5350	0.5518	2
3	0.3825	0.3996	0.4167	0.4337	0.4508	0.4677	0.4847	0.5016	0.5185	0.5353	0.5521	3
4	0.3828	0.3999	0.4170	0.4340	0.4510	0.4680	0.4850	0.5019	0.5188	0.5356	0.5524	4
5	0.3830	0.4002	0.4172	0.4343	0.4513	0.4683	0.4853	0.5022	0.5190	0.5359	0.5527	5
6	0.3833	0.4004	0.4175	0.4346	0.4516	0.4686	0.4855	0.5024	0.5193	0.5362	0.5530	6
7	0.3836	0.4007	0.4178	0.4349	0.4519	0.4689	0.4858	0.5027	0.5196	0.5364	0.5532	7
8	0.3839	0.4010	0.4181	0.4352	0.4522	0.4692	0.4861	0.5030	0.5199	0.5367	0.5535	8
9	0.3842	0.4013	0.4184	0.4354	0.4525	0.4694	0.4864	0.5033	0.5202	0.5370	0.5538	9
10	0.3845	0.4016	0.4187	0.4357	0.4527	0.4697	0.4867	0.5036	0.5204	0.5373	0.5541	10
11	0.3848	0.4019	0.4190	0.4360	0.4530	0.4700	0.4869	0.5039	0.5207	0.5376	0.5543	11
12	0.3850	0.4022	0.4192	0.4363	0.4533	0.4703	0.4872	0.5041	0.5210	0.5378	0.5546	12
13	0.3853	0.4024	0.4195	0.4366	0.4536	0.4706	0.4875	0.5044	0.5213	0.5381	0.5549	13
14	0.3856	0.4027	0.4198	0.4369	0.4539	0.4708	0.4878	0.5047	0.5216	0.5384	0.5552	14
15	0.3859	0.4030	0.4201	0.4371	0.4542	0.4711	0.4881	0.5050	0.5219	0.5387	0.5555	15
16	0.3862	0.4033	0.4204	0.4374	0.4544	0.4714	0.4884	0.5053	0.5221	0.5390	0.5557	16
17	0.3865	0.4036	0.4207	0.4377	0.4547	0.4717	0.4886	0.5055	0.5224	0.5392	0.5560	17
18	0.3868	0.4039	0.4209	0.4380	0.4550	0.4720	0.4889	0.5058	0.5227	0.5395	0.5563	18
19	0.3870	0.4042	0.4212	0.4383	0.4553	0.4723	0.4892	0.5061	0.5230	0.5398	0.5566	19
20	0.3873	0.4044	0.4215	0.4386	0.4556	0.4725	0.4895	0.5064	0.5233	0.5401	0.5569	20
21	0.3876	0.4047	0.4218	0.4388	0.4559	0.4728	0.4898	0.5067	0.5235	0.5404	0.5571	21
22	0.3879	0.4050	0.4221	0.4391	0.4561	0.4731	0.4901	0.5070	0.5238	0.5406	0.5574	22
23	0.3882	0.4053	0.4224	0.4394	0.4564	0.4734	0.4903	0.5072	0.5241	0.5409	0.5577	23
24	0.3885	0.4056	0.4226	0.4397	0.4567	0.4737	0.4906	0.5075	0.5244	0.5412	0.5580	24
25	0.3888	0.4059	0.4229	0.4400	0.4570	0.4740	0.4909	0.5078	0.5217	0.5415	0.5583	25
26	0.3890	0.4061	0.4232	0.4403	0.4573	0.4742	0.4912	0.5081	0.5249	0.5418	0.5585	26
27	0.3893	0.4064	0.4235	0.4405	0.4576	0.4745	0.4915	0.5084	0.5252	0.5420	0.5588	27
28	0.3896	0.4067	0.4238	0.4408	0.4578	0.4748	0.4917	0.5086	0.5255	0.5423	0.5591	28
29	0.3899	0.4070	0.4241	0.4411	0.4581	0.4751	0.4920	0.5089	0.5258	0.5426	0.5594	29
30	0.3902	0.4073	0.4244	0.4414	0.4584	0.4754	0.4923	0.5092	0.5261	0.5429	0.5597	30
31	0.3905	0.4076	0.4246	0.4417	0.4587	0.4757	0.4926	0.5095	0.5263	0.5432	0.5599	31
32	0.3908	0.4079	0.4249	0.4420	0.4590	0.4759	0.4929	0.5098	0.5266	0.5434	0.5602	32
33	0.3910	0.4081	0.4252	0.4422	0.4593	0.4762	0.4932	0.5100	0.5269	0.5437	0.5605	33
34	0.3913	0.4084	0.4255	0.4425	0.4595	0.4765	0.4934	0.5103	0.5272	0.5440	0.5608	34
35	0.3916	0.4087	0.4258	0.4428	0.4598	0.4768	0.4937	0.5106	0.5275	0.5443	0.5611	35
36	0.3919	0.4090	0.4261	0.4431	0.4601	0.4771	0.4940	0.5109	0.5277	0.5446	0.5613	36
37	0.3922	0.4093	0.4263	0.4434	0.4604	0.4773	0.4943	0.5112	0.5280	0.5448	0.5616	37
38	0.3925	0.4096	0.4266	0.4437	0.4607	0.4776	0.4946	0.5115	0.5283	0.5451	0.5619	38
39	0.3927	0.4098	0.4269	0.4439	0.4609	0.4779	0.4948	0.5117	0.5286	0.5454	0.5622	39
40	0.3930	0.4101	0.4272	0.4442	0.4612	0.4782	0.4951	0.5120	0.5289	0.5457	0.5625	40
41	0.3933	0.4104	0.4275	0.4445	0.4615	0.4785	0.4954	0.5123	0.5291	0.5460	0.5627	41
42	0.3936	0.4107	0.4278	0.4448	0.4618	0.4788	0.4957	0.5126	0.5294	0.5462	0.5630	42
43	0.3939	0.4110	0.4280	0.4451	0.4621	0.4790	0.4960	0.5129	0.5297	0.5465	0.5633	43
44	0.3942	0.4113	0.4283	0.4454	0.4624	0.4793	0.4963	0.5131	0.5300	0.5468	0.5636	44
45	0.3945	0.4116	0.4286	0.4456	0.4626	0.4796	0.4965	0.5134	0.5303	0.5471	0.5638	45
46	0.3947	0.4118	0.4289	0.4459	0.4629	0.4799	0.4968	0.5137	0.5306	0.5474	0.5641	46
47	0.3950	0.4121	0.4292	0.4462	0.4632	0.4802	0.4971	0.5140	0.5308	0.4576	0.5644	47
48	0.3953	0.4124	0.4295	0.4465	0.4635	0.4805	0.4974	0.5143	0.5311	0.5479	0.5647	48
49	0.3956	0.4127	0.4298	0.4468	0.4638	0.4807	0.4977	0.5145	0.5314	0.5482	0.5650	49
50	0.3959	0.4130	0.4300	0.4471	0.4641	0.4810	0.4979	0.5148	0.5317	0.5485	0.5652	50
51	0.3962	0.4133	0.4303	0.4474	0.4643	0.4813	0.4982	0.5151	0.5320	0.5488	0.5655	51
52	0.3965	0.4135	0.4306	0.4476	0.4646	0.4816	0.4985	0.5154	0.5322	0.5490	0.5658	52
53	0.3967	0.4138	0.4309	0.4479	0.4649	0.4819	0.4988	0.5157	0.5325	0.5493	0.5661	53
54	0.3970	0.4141	0.4312	0.4482	0.4652	0.4822	0.4991	0.5160	0.5328	0.5496	0.5664	54
55	0.3973	0.4144	0.4315	0.4485	0.4655	0.4824	0.4994	0.5162	0.5331	0.5499	0.5666	55
56	0.3976	0.4147	0.4317	0.4488	0.4658	0.4827	0.4996	0.5165	0.5334	0.5502	0.5669	56
57	0.3979	0.4150	0.4320	0.4491	0.4660	0.4830	0.4999	0.5168	0.5336	0.5504	0.5672	57
58	0.3982	0.4153	0.4323	0.4493	0.4663	0.4833	0.5002	0.5171	0.5339	0.5507	0.5675	58
59	0.3985	0.4155	0.4326	0.4496	0.4666	0.4836	0.5005	0.5174	0.5342	0.5510	0.5678	59
60	0.3987	0.4158	0.4329	0.4499	0.4669	0.4838	0.5008	0.5176	0.5345	0.5513	0.5680	60

Table of Chords (Continued). Radius = 1.0000

M.	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	M.
0′	0.5680	0.5847	0.6014	0.6180	0.6346	0.6511	0.6676	0.6840	0.7004	0.7167	0.7330	0′
1	0.5683	0.5850	0.6017	0.6183	0.6349	0.6514	0.6679	0.6843	0.7007	0.7170	0.7333	1
2	0.5686	0.5853	0.6020	0.6186	0.6352	0.6517	0.6682	0.6846	0.7010	0.7173	0.7335	2
3	0.5689	0.5856	0.6022	0.6189	0.6354	0.6520	0.6684	0.6849	0.7012	0.7176	0.7338	3
4	0.5691	0.5859	0.6025	0.6191	0.6357	0.6522	0.6687	0.6851	0.7015	0.7178	0.7341	4
5	0.5694	0.5861	0.6028	0.6194	0.6360	0.6525	0.6690	0.6854	0.7018	0.7181	0.7344	5
6	0.5697	0.5864	0.6031	0.6197	0.6363	0.6528	0.6693	0.6857	0.7020	0.7184	0.7346	6
7	0.5700	0.5867	0.6034	0.6200	0.6365	0.6531	0.6695	0.6860	0.7023	0.7186	0.7349	7
8	0.5703	0.5870	0.6036	0.6202	0.6368	0.6533	0.6698	0.6862	0.7026	0.7189	0.7352	8
9	0.5705	0.5872	0.6039	0.6205	0.6371	0.6536	0.6701	0.6865	0.7029	0.7192	0.7354	9
10	0.5708	0.5875	0.6042	0.6208	0.6374	0.6539	0.6704	0.6868	0.7031	0.7195	0.7357	10
11	0.5711	0.5878	0.6045	0.6211	0.6376	0.6542	0.6706	0.6870	0.7034	0.7197	0.7360	11
12	0.5714	0.5881	0.6047	0.6214	0.6379	0.6544	0.6709	0.6873	0.7037	0.7200	0.7362	12
13	0.5717	0.5884	0.6050	0.6216	0.6382	0.6547	0.6712	0.6876	0.7040	0.7203	0.7365	13
14	0.5719	0.5886	0.6053	0.6219	0.6385	0.6550	0.6715	0.6879	0.7042	0.7205	0.7368	14
15	0.5722	0.5889	0.6056	0.6222	0.6387	0.6553	0.6717	0.6881	0.7045	0.7208	0.7371	15
16	0.5725	0.5892	0.6058	0.6225	0.6390	0.6555	0.6720	0.6884	0.7048	0.7211	0.7373	16
17	0.5728	0.5895	0.6061	0.6227	0.6393	0.6558	0.6723	0.6887	0.7050	0.7214	0.7376	17
18	0.5730	0.5897	0.6064	0.6230	0.6396	0.6561	0.6725	0.6890	0.7053	0.7216	0.7379	18
19	0.5733	0.5900	0.6067	0.6233	0.6398	0.6561	0.6728	0.6892	0.7056	0.7219	0.7381	19
20	0.5736	0.5903	0.6070	0.6236	0.6401	0.6566	0.6731	0.6895	0.7059	0.7222	0.7384	20
21	0.5739	0.5906	0.6072	0.6238	0.6404	0.6569	0.6734	0.6898	0.7061	0.7224	0.7387	21
22	0.5742	0.5909	0.6075	0.6241	0.6407	0.6572	0.6736	0.6901	0.7064	0.7227	0.7390	22
23	0.5744	0.5911	0.6078	0.6244	0.6410	0.6575	0.6739	0.6903	0.7067	0.7230	0.7392	23
24	0.5747	0.5014	0.6081	0.6247	0.6412	0.6577	0.6742	0.6906	0.7069	0.7232	0.7395	24
25	0.5750	0.5917	0.6083	0.6249	0.6415	0.6580	0.6745	0.6909	0.7072	0.7235	0.7398	25
26	0.5753	0.5920	0.6086	0.6252	0.6418	0.6583	0.6747	0.6911	0.7075	0.7238	0.7400	26
27	0.5756	0.5922	0.6089	0.6255	0.6421	0.6586	0.6750	0.6914	0.7078	0.7241	0.7403	27
28	0.5758	0.5925	0.6092	0.6258	0.6423	0.6588	0.6753	0.6917	0.7080	0.7243	0.7406	28
29	0.5761	0.5928	0.6095	0.6260	0.6426	0.6591	0.6756	0.6920	0.7083	0.7246	0.7408	29
30	0.5764	0.5931	0.6097	0.6263	0.6429	0.6594	0.6758	0.6922	0.7086	0.7249	0.7411	30
31	0.5767	0.5934	0.6100	0.6266	0.6432	0.6597	0.6761	0.6925	0.7089	0.7251	0.7414	31
32	0.5769	0.5936	0.6103	0.6269	0.6434	0.6599	0.6764	0.6928	0.7091	0.7254	0.7417	32
33	0.5772	0.5939	0.6106	0.6272	0.6437	0.6602	0.6767	0.6931	0.7094	0.7257	0.7419	33
34	0.5775	0.5942	0.6108	0.6274	0.6440	0.6605	0.6769	0.6933	0.7097	0.7260	0.7422	34
35	0.5778	0.5945	0.6111	0.6277	0.6443	0.6608	0.6772	0.6936	0.7009	0.7262	0.7425	35
36	0.5781	0.5947	0.6114	0.6280	0.6445	0.6610	0.6775	0.6939	0.7102	0.7265	0.7427	36
37	0.5783	0.5950	0.6117	0.6283	0.6448	0.6613	0.6777	0.6941	0.7105	0.7268	0.7430	37
38	0.5786	0.5953	0.6119	0.6285	0.6451	0.6616	0.6780	0.6944	0.7108	0.7270	0.7433	38
39	0.5789	0.5956	0.6122	0.6238	0.6454	0.6619	0.6783	0.6947	0.7110	0.7273	0.7435	39
40	0.5792	0.5959	0.6125	0.6291	0.6456	0.6621	0.6786	0.6950	0.7113	0.7276	0.7438	40
41	0.5795	0.5961	0.6128	0.6294	0.6459	0.6624	0.6788	0.6952	0.7116	0.7279	0.7441	41
42	0.5797	0.5964	0.6130	0.6296	0.6462	0.6627	0.6791	0.6955	0.7118	0.7281	0.7443	42
43	0.5800	0.5967	0.6133	0.6299	0.6465	0.6630	0.6794	0.6958	0.7121	0.7284	0.7446	43
44	0.5803	0.5970	0.6136	0.6302	0.6467	0.6632	0.6797	0.6961	0.7124	0.7287	0.7449	44
45	0.5806	0.5972	0.6139	0.6305	0.6470	0.6635	0.6799	0.6963	0.7127	0.7289	0.7452	45
46	0.5808	0.5975	0.6142	0.6307	0.6473	0.6638	0.6802	0.6966	0.7129	0.7292	0.7454	46
47	0.5811	0.5978	0.6144	0.6310	0.6476	0.6640	0.6805	0.6969	0.7132	0.7295	0.7457	47
48	0.5814	0.5981	0.6147	0.6313	0.6478	0.6643	0.6808	0.6971	0.7135	0.7298	0.7460	48
49	0.5817	0.5984	0.6150	0.6316	0.6481	0.6646	0.6810	0.6974	0.7137	0.7300	0.7462	49
50	0.5820	0.5986	0.6153	0.6318	0.6484	0.6649	0.6813	0.6977	0.7140	0.7303	0.7465	50
51	0.5822	0.5989	0.6155	0.6321	0.6487	0.6651	0.6816	0.6980	0.7143	0.7306	0.7468	51
52	0.5825	0.5992	0.6158	0.6324	0.6489	0.6654	0.6819	0.6982	0.7146	0.7308	0.7471	52
53	0.5828	0.5995	0.6161	0.6327	0.6492	0.6657	0.6821	0.6985	0.7148	0.7311	0.7473	53
54	0.5831	0.5997	0.6164	0.6330	0.6495	0.6660	0.6824	0.6988	0.7151	0.7314	0.7476	54
55	0.5834	0.6000	0.6166	0.6332	0.6498	0.6662	0.6827	0.6991	0.7154	0.7316	0.7479	55
56	0.5836	0.6003	0.6169	0.6335	0.6500	0.6665	0.6829	0.6993	0.7156	0.7319	0.7481	56
57	0.5839	0.6006	0.6172	0.6338	0.6503	0.6668	0.6832	0.6996	0.7159	0.7322	0.7484	57
58	0.5842	0.6009	0.6175	0.6341	0.6506	0.6671	0.6835	0.6999	0.7162	0.7325	0.7487	58
59	0.5845	0.6011	0.6178	0.6343	0.6509	0.6673	0.6838	0.7001	0.7165	0.7327	0.7489	59
60	0.5847	0.6014	0.6180	0.6346	0.6511	0.6676	0.6840	0.7004	0.7167	0.7330	0.7492	60

Table of Chords (Continued). Radius = 1.0000

M.	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	M.
0′	0.7492	0.7654	0.7815	0.7975	0.8135	0.8294	0.8452	0.8610	0.8767	0.8924	0.9080	0′
1	0.7495	0.7656	0.7817	0.7978	0.8137	0.8297	0.8455	0.8613	0.8770	0.8927	0.9082	1
2	0.7498	0.7659	0.7820	0.7980	0.8140	0.8299	0.8458	0.8615	0.8773	0.8929	0.9085	2
3	0.7500	0.7662	0.7823	0.7983	0.8143	0.8302	0.8460	0.8618	0.8775	0.8932	0.9088	3
4	0.7503	0.7664	0.7825	0.7986	0.8145	0.8304	0.8463	0.8621	0.8778	0.8934	0.9090	4
5	0.7506	0.7667	0.7828	0.7988	0.8148	0.8307	0.8466	0.8623	0.8780	0.8937	0.9093	5
6	0.7508	0.7670	0.7831	0.7991	0.8151	0.8310	0.8468	0.8626	0.8783	0.8940	0.9095	6
7	0.7511	0.7672	0.7833	0.7994	0.8153	0.8312	0.8471	0.8629	0.8786	0.8942	0.9098	7
8	0.7514	0.7675	0.7836	0.7996	0.8156	0.8315	0.8473	0.8631	0.8788	0.8945	0.9101	8
9	0.7516	0.7678	0.7839	0.7999	0.8159	0.8318	0.8476	0.8634	0.8791	0.8947	0.9103	9
10	0.7519	0.7681	0.7841	0.8002	0.8161	0.8320	0.8479	0.8636	0.8794	0.8950	0.9106	10
11	0.7522	0.7683	0.7844	0.8004	0.8164	0.8323	0.8481	0.8639	0.8796	0.8953	0.9108	11
12	0.7524	0.7686	0.7847	0.8007	0.8167	0.8326	0.8484	0.8642	0.8799	0.8955	0.9111	12
13	0.7527	0.7689	0.7849	0.8010	0.8169	0.8328	0.8487	0.8644	0.8801	0.8958	0.9113	13
14	0.7530	0.7691	0.7852	0.8012	0.8172	0.8331	0.8489	0.8647	0.8804	0.8960	0.9116	14
15	0.7533	0.7694	0.7855	0.8015	0.8175	0.8334	0.8492	0.8650	0.8807	0.8963	0.9119	15
16	0.7535	0.7697	0.7857	0.8018	0.8177	0.8336	0.8495	0.8652	0.8809	0.8966	0.9121	16
17	0.7538	0.7699	0.7860	0.8020	0.8180	0.8339	0.8497	0.8655	0.8812	0.8968	0.9124	17
18	0.7541	0.7702	0.7863	0.8023	0.8183	0.8341	0.8500	0.8657	0.8814	0.8971	0.9126	18
19	0.7513	0.7705	0.7865	0.8026	0.8185	0.8344	0.8502	0.8660	0.8817	0.8973	0.9129	19
20	0.7546	0.7707	0.7868	0.8028	0.8188	0.8347	0.8505	0.8663	0.8820	0.8976	0.9132	20
21	0.7549	0.7710	0.7871	0.8031	0.8190	0.8349	0.8508	0.8665	0.8822	0.8979	0.9134	21
22	0.7551	0.7713	0.7873	0.8034	0.8193	0.8352	0.8510	0.8668	0.8825	0.8981	0.9137	22
23	0.7554	0.7715	0.7876	0.8036	0.8196	0.8355	0.8513	0.8671	0.8828	0.8984	0.9139	23
24	0.7557	0.7718	0.7879	0.8039	0.8198	0.8357	0.8516	0.8673	0.8830	0.8986	0.9142	24
25	0.7560	0.7721	0.7882	0.8042	0.8201	0.8360	0.8518	0.8676	0.8833	0.8989	0.9145	25
26	0.7562	0.7723	0.7884	0.8044	0.8204	0.8363	0.8521	0.8678	0.8835	0.8992	0.9147	26
27	0.7565	0.7726	0.7887	0.8047	0.8206	0.8365	0.8523	0.8681	0.8838	0.8994	0.9150	27
28	0.7568	0.7729	0.7890	0.8050	0.8209	0.8368	0.8526	0.8684	0.8841	0.8997	0.9152	28
29	0.7570	0.7731	0.7892	0.8052	0.8212	0.8371	0.8529	0.8686	0.8843	0.8999	0.9155	29
30	0.7573	0.7734	0.7895	0.8055	0.8214	0.8373	0.8531	0.8689	0.8816	0.9002	0.9157	30
31	0.7576	0.7737	0.7898	0.8058	0.8217	0.8376	0.8534	0.8692	0.8848	0.9005	0.9160	31
32	0.7578	0.7740	0.7900	0.8060	0.8220	0.8378	0.8537	0.8694	0.8851	0.9007	0.9163	32
33	0.7581	0.7742	0.7903	0.8063	0.8222	0.8381	0.8539	0.8697	0.8854	0.9010	0.9165	33
34	0.7584	0.7745	0.7906	0.8066	0.8225	0.8384	0.8542	0.8699	0.8856	0.9012	0.9168	34
35	0.7586	0.7748	0.7908	0.8068	0.8228	0.8386	0.8545	0.8702	0.8859	0.9015	0.9170	35
36	0.7589	0.7750	0.7911	0.8071	0.8230	0.8389	0.8547	0.8705	0.8861	0.9018	0.9173	36
37	0.7592	0.7753	0.7914	0.8074	0.8233	0.8392	0.8550	0.8707	0.8864	0.9020	0.9176	37
38	0.7595	0.7756	0.7916	0.8076	0.8236	0.8394	0.8552	0.8710	0.8867	0.9023	0.9178	38
39	0.7597	0.7758	0.7919	0.8079	0.8238	0.8397	0.8555	0.8712	0.8869	0.9025	0.9181	39
40	0.7600	0.7761	0.7922	0.8082	0.8241	0.8400	0.8558	0.8715	0.8872	0.9028	0.9183	40
41	0.7603	0.7764	0.7924	0.8084	0.8244	0.8402	0.8560	0.8718	0.8874	0.9031	0.9186	41
42	0.7605	0.7766	0.7927	0.8087	0.8246	0.8405	0.8563	0.8720	0.8877	0.9033	0.9188	42
43	0.7608	0.7769	0.7930	0.8090	0.8249	0.8408	0.8566	0.8723	0.8880	0.9036	0.9191	43
44	0.7611	0.7772	0.7932	0.8092	0.8251	0.8410	0.8568	0.8726	0.8882	0.9038	0.9194	44
45	0.7613	0.7774	0.7935	0.8095	0.8254	0.8413	0.8571	0.8728	0.8885	0.9041	0.9196	45
46	0.7616	0.7777	0.7938	0.8098	0.8257	0.8415	0.8573	0.8731	0.8887	0.9044	0.9199	46
47	0.7619	0.7780	0.7940	0.8100	0.8259	0.8418	0.8576	0.8734	0.8890	0.9046	0.9201	47
48	0.7621	0.7782	0.7943	0.8103	0.8262	0.8421	0.8579	0.8736	0.8893	0.9049	0.9204	48
49	0.7624	0.7785	0.7946	0.8105	0.8265	0.8423	0.8581	0.8739	0.8895	0.9051	0.9207	49
50	0.7627	0.7788	0.7948	0.8108	0.8267	0.8426	0.8584	0.8741	0.8898	0.9054	0.9209	50
51	0.7629	0.7791	0.7951	0.8111	0.8270	0.8429	0.8587	0.8744	0.8900	0.9056	0.9212	51
52	0.7632	0.7793	0.7954	0.8113	0.8273	0.8431	0.8589	0.8747	0.8903	0.9059	0.9214	52
53	0.7635	0.7796	0.7956	0.8116	0.8275	0.8434	0.8592	0.8749	0.8906	0.9062	0.9217	53
54	0.7638	0.7799	0.7959	0.8119	0.8278	0.8437	0.8594	0.8752	0.8908	0.9064	0.9219	54
55	0.7640	0.7801	0.7962	0.8121	0.8281	0.8439	0.8597	0.8754	0.8911	0.9067	0.9222	55
56	0.7643	0.7804	0.7964	0.8124	0.8283	0.8442	0.8600	0.8757	0.8914	0.9069	0.9225	56
57	0.7646	0.7807	0.7967	0.8127	0.8286	0.8444	0.8602	0.8760	0.8916	0.9072	0.9227	57
58	0.7648	0.7809	0.7970	0.8129	0.8289	0.8447	0.8605	0.8762	0.8919	0.9075	0.9230	58
59	0.7651	0.7812	0.7972	0.8132	0.8291	0.8450	0.8608	0.8765	0.8921	0.9077	0.9232	59
60	0.7654	0.7815	0.7975	0.8135	0.8294	0.8452	0.8610	0.8767	0.8924	0.9080	0.9235	60

Table of Chords (Continued). Radius = 1.0000

M.	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	M.
0′	0.9235	0.9389	0.9543	0.9696	0.9848	1.0000	1.0151	1.0301	1.0450	1.0598	0′
1	0.9238	0.9392	0.9546	0.9699	0.9851	1.0003	1.0153	1.0303	1.0452	1.0601	1
2	0.9240	0.9395	0.9548	0.9701	0.9854	1.0005	1.0156	1.0306	1.0455	1.0603	2
3	0.9243	0.9397	0.9551	0.9704	0.9856	1.0008	1.0158	1.0308	1.0457	1.0606	3
4	0.9245	0.9400	0.9553	0.9706	0.9859	1.0010	1.0161	1.0311	1.0460	1.0608	4
5	0.9248	0.9402	0.9556	0.9709	0.9861	1.0013	1.0163	1.0313	1.0462	1.0611	5
6	0.9250	0.9405	0.9559	0.9711	0.9864	1.0015	1.0166	1.0316	1.0465	1.0613	6
7	0.9253	0.9407	0.9561	0.9714	0.9866	1.0018	1.0168	1.0318	1.0467	1.0616	7
8	0.9256	0.9410	0.9564	0.9717	0.9869	1.0020	1.0171	1.0321	1.0470	1.0618	8
9	0.9258	0.9413	0.9566	0.9719	0.9871	1.0023	1.0173	1.0323	1.0472	1.0621	9
10	0.9261	0.9415	0.9569	0.9722	0.9874	1.0025	1.0176	1.0326	1.0475	1.0623	10
11	0.9263	0.9418	0.9571	0.9724	0.9876	1.0028	1.0178	1.0328	1.0477	1.0626	11
12	0.9266	0.9420	0.9574	0.9727	0.9879	1.0030	1.0181	1.0331	1.0480	1.0628	12
13	0.9268	0.9423	0.9576	0.9729	0.9881	1.0033	1.0183	1.0333	1.0482	1.0630	13
14	0.9271	0.9425	0.9579	0.9732	0.9884	1.0035	1.0186	1.0336	1.0485	1.0633	14
15	0.9274	0.9428	0.9581	0.9734	0.9886	1.0038	1.0188	1.0338	1.0487	1.0635	15
16	0.9276	0.9430	0.9584	0.9737	0.9889	1.0040	1.0191	1.0341	1.0490	1.0638	16
17	0.9279	0.9433	0.9587	0.9739	0.9891	1.0043	1.0193	1.0343	1.0492	1.0640	17
18	0.9281	0.9436	0.9589	0.9742	0.9894	1.0045	1.0196	1.0346	1.0495	1.0643	18
19	0.9284	0.9438	0.9592	0.9744	0.9897	1.0048	1.0198	1.0348	1.0497	1.0645	19
20	0.9287	0.9441	0.9594	0.9747	0.9899	1.0050	1.0201	1.0351	1.0500	1.0648	20
21	0.9289	0.9443	0.9597	0.9750	0.9902	1.0053	1.0203	1.0353	1.0502	1.0650	21
22	0.9292	0.9446	0.9599	0.9752	0.9904	1.0055	1.0206	1.0356	1.0504	1.0653	22
23	0.9294	0.9448	0.9602	0.9755	0.9907	1.0058	1.0208	1.0358	1.0507	1.0655	23
24	0.9297	0.9451	0.9604	0.9757	0.9909	1.0060	1.0211	1.0361	1.0509	1.0658	24
25	0.9299	0.9454	0.9607	0.9760	0.9912	1.0063	1.0213	1.0363	1.0512	1.0660	25
26	0.9302	0.9456	0.9610	0.9762	0.9914	1.0065	1.0216	1.0366	1.0514	1.0662	26
27	0.9305	0.9459	0.9612	0.9765	0.9917	1.0068	1.0218	1.0368	1.0517	1.0665	27
28	0.9307	0.9461	0.9615	0.9767	0.9919	1.0070	1.0221	1.0370	1.0519	1.0667	28
29	0.9310	0.9464	0.9617	0.9770	0.9922	1.0073	1.0223	1.0373	1.0522	1.0670	29
30	0.9312	0.9466	0.9620	0.9772	0.9924	1.0075	1.0226	1.0375	1.0524	1.0672	30
31	0.9315	0.9469	0.9622	0.9775	0.9927	1.0078	1.0228	1.0378	1.0527	1.0675	31
32	0.9317	0.9472	0.9625	0.9778	0.9029	1.0080	1.0231	1.0380	1.0529	1.0677	32
33	0.9320	0.9474	0.9627	0.9780	0.9932	1.0083	1.0233	1.0383	1.0532	1.0680	33
34	0.9323	0.9477	0.9630	9.9783	0.9934	1.0086	1.0236	1.0385	1.0534	1.0682	34
35	0.9325	0.9479	0.9633	0.9785	0.9937	1.0088	1.0238	1.0388	1.0537	1.0685	35
36	0.9328	0.9482	0.9635	0.9788	0.9939	1.0091	1.0241	1.0390	1.0539	1.0687	36
37	0.9330	0.9484	0.9638	0.9790	0.0942	1.0093	1.0243	1.0393	1.0542	1.0690	37
38	0.9333	0.9487	0.9640	0.9793	0.9945	1.0096	1.0246	1.0395	1.0544	1.0692	38
39	0.9335	0.9489	0.9643	0.9795	0.9947	1.0098	1.0248	1.0398	1.0547	1.0694	39
40	0.9338	0.9492	0.9645	0.9798	0.9050	1.0101	1.0251	1.0400	1.0549	1.0697	40
41	0.9341	0.9495	0.9648	0.9800	0.9952	1.0103	1.0253	1.0403	1.0551	1.0699	41
42	0.9343	0.9497	0.9650	0.9803	0.9955	1.0106	1.0256	1.0405	1.0554	1.0702	42
43	0.9346	0.9500	0.9653	0.9805	0.9957	1.0108	1.0258	1.0408	1.0556	1.0704	43
44	0.9348	0.9502	0.9655	0.9808	0.9960	1.0111	1.0261	1.0410	1.0559	1.0707	44
45	0.9351	0.9505	0.9658	0.9810	0.9962	1.0113	1.0263	1.0413	1.0561	1.0709	45
46	0.9353	0.9507	0.9661	0.9813	0.9965	1.0116	1.0266	1.0415	1.0564	1.0712	46
47	0.9356	0.9510	0.9663	0.9816	0.9967	1.0118	1.0268	1.0418	1.0566	1.0714	47
48	0.9359	0.9512	0.9666	0.9818	0.9970	1.0121	1.0271	1.0420	1.0569	1.0717	48
49	0.9361	0.9515	0.9668	0.9821	0.9972	1.0123	1.0273	1.0423	1.0571	1.0719	49
50	0.9364	0.9518	0.9671	0.9823	0.9975	1.0126	1.0276	1.0425	1.0574	1.0721	50
51	0.9366	0.9520	0.9673	0.9826	0.9977	1.0128	1.0278	1.0428	1.0576	1.0724	51
52	0.9369	0.9523	0.9676	0.9828	0.9980	1.0131	1.0281	1.0430	1.0579	1.0726	52
53	0.9371	0.9525	0.9678	0.9831	0.9982	1.0133	1.0283	1.0433	1.0581	1.0729	53
54	0.9374	0.9528	0.9681	0.9833	0.9985	1.0136	1.0286	1.0435	1.0584	1.0731	54
55	0.9377	0.9530	0.9683	0.9836	0.9987	1.0138	1.0288	1.0438	1.0586	1.0734	55
56	0.9379	0.9533	0.9686	0.9838	0.9990	1.0141	1.0291	1.0440	1.0589	1.0736	56
57	0.9382	0.9536	0.9689	0.9841	0.9992	1.0143	1.0293	1.0443	1.0591	1.0739	57
58	0.9384	0.9538	0.9691	0.9843	0.9995	1.0146	1.0296	1.0445	1.0593	1.0741	58
59	0.9387	0.9541	0.9694	0.9846	0.9998	1.0148	1.0298	1.0447	1.0596	1.0744	59
60	0.9389	0.9543	0.9696	0.9848	1.0000	1.0151	1.0301	1.0450	1.0598	1.0746	60

Table of Chords (Continued). Radius = 1.0000

M.	65°	66°	67°	68°	69°	70°	71°	72°	73°	M.
0′	1 0746	1 0893	1 1039	1 1184	1 1328	1 1472	1 1614	1 1756	1.1896	0′
1	1 0748	1 0895	1 1041	1 1186	1 1331	1 1474	1 1616	1 1758	1.1899	1
2	1 0751	1.0898	1 1044	1 1189	1 1333	1 1476	1.1619	1 1760	1.1901	2
3	1 0753	1 0900	1 1046	1 1191	1 1335	1 1479	1.1621	1.1763	1.1903	3
4	1 0756	1 0903	1 1048	1 1194	1 1338	1 1481	1 1624	1 1765	1.1906	4
5	1 0758	1 0905	1 1051	1 1196	1 1340	1 1483	1.1626	1 1767	1.1908	5
6	1.0761	1 0907	1 1053	1 1198	1 1342	1 1486	1 1628	1 1770	1 1910	6
7	1 0763	1 0910	1 1056	1 1201	1 1345	1.1488	1 1631	1 1772	1 1913	7
8	1.0766	1 0912	1 1058	1 1203	1 1347	1.1491	1.1633	1 1775	1.1915	8
9	1 0768	1 0915	1 1016	1 1206	1 1350	1 1493	1.1635	1.1777	1.1917	9
10	1.0771	1 0917	1.1063	1 1208	1 1352	1.1495	1.1638	1 1779	1.1920	10
11	1.0773	1 0920	1 1065	1 1210	1 1354	1.1498	1.1640	1 1782	1.1922	11
12	1 0775	1 0922	1 1068	1 1213	1 1357	1 1500	1.1642	1 1784	1 1924	12
13	1 0778	1 0924	1.1070	1 1215	1 1359	1 1502	1.1645	1.1786	1 1927	13
14	1.0780	1 0927	1.1073	1 1218	1.1362	1 1505	1.1647	1 1789	1.1929	14
15	1 0783	1 0929	1 1075	1 1220	1 1364	1 1507	1 1650	1 1791	1.1931	15
16	1.0785	1 0932	1.1078	1 1222	1 1366	1.1510	1.1652	1 1793	1 1934	16
17	1.0788	1 0934	1.1080	1 1225	1 1369	1.1512	1 1654	1 1796	1 1936	17
18	1 0790	1 0937	1.1082	1 1227	1 1371	1 1514	1.1657	1 1798	1 1938	18
19	1.0793	1 0939	1.1085	1 1230	1 1374	1 1517	1.1659	1 1800	1 1941	19
20	1.0795	1 0942	1.1087	1 1232	1 1376	1.1519	1.1661	1.1803	1.1943	20
21	1 0797	1 0944	1.1090	1 1234	1 1378	1.1522	1.1664	1 1805	1.1946	21
22	1 0800	1 0946	1.1092	1 1237	1 1381	1.1524	1 1666	1 1807	1.1948	22
23	1 0802	1 0949	1.1094	1 1239	1 1383	1.1526	1.1668	1 1810	1 1950	23
24	1.0805	1 0951	1.1097	1 1242	1 1386	1 1529	1 1671	1 1812	1 1952	24
25	1 0807	1 0954	1.1099	1 1244	1 1388	1 1531	1.1673	1 1814	1 1955	25
26	1 0810	1 0956	1.1102	1 1246	1 1390	1 1533	1 1676	1.1817	1 1957	26
27	1 0812	1 0959	1.1104	1 1249	1 1393	1 1536	1 1678	1 1819	1.1959	27
28	1 0815	1 0961	1.1107	1 1251	1 1395	1 1538	1 1680	1 1821	1 1962	28
29	1 0817	1 0963	1.1109	1 1254	1 1398	1 1541	1.1683	1 1824	1.1964	29
30	1 0820	1 0966	1 1111	1 1256	1 1400	1 1543	1.1685	1 1826	1 1966	30
31	1 0822	1 0968	1.1114	1 1258	1 1402	1 1545	1 1687	1 1829	1 1969	31
32	1 0824	1 0971	1.1116	1 1261	1 1405	1 1548	1.1690	1 1831	1 1971	32
33	1 0827	1 0973	1 1119	1 1263	1 1407	1 1550	1.1692	1 1833	1 1973	33
34	1 0829	1 0976	1.1121	1 1266	1 1409	1 1552	1.1694	1 1836	1 1976	34
35	1 0832	1 0978	1.1123	1 1268	1 1412	1 1555	1.1697	1 1838	1 1978	35
36	1 0834	1 0980	1.1126	1 1271	1 1414	1 1557	1.1699	1 1840	1 1080	36
37	1 0837	1 0983	1 1128	1 1273	1 1417	1 1560	1.1702	1 1843	1.1983	37
38	1 0839	1 0985	1.1131	1 1275	1.1419	1 1562	1.1704	1 1845	1 1985	38
39	1 0841	1 0988	1.1133	1 1278	1 1421	1 1564	1.1706	1 1847	1 1987	39
40	1.0844	1 0990	1.1136	1 1280	1 1424	1.1567	1.1709	1 1850	1.1990	40
41	1.0846	1 0993	1 1138	1 1283	1 1426	1 1569	1.1711	1 1852	1 1992	41
42	1 0849	1 0995	1 1140	1 1285	1 1429	1 1571	1 1713	1 1854	1 1994	42
43	1.0851	1 0997	1 1143	1 1287	1 1431	1 1574	1.1716	1 1857	1 1997	43
44	1.0854	1 1000	1.1145	1 1290	1 1433	1 1576	1.1718	1 1859	1.1999	44
45	1 0856	1 1002	1 1148	1 1292	1 1436	1 1579	1 1720	1 1861	1 2001	45
46	1 0859	1 1005	1.1150	1 1295	1 1438	1 1581	1.1723	1 1864	1.2004	46
47	1 0861	1 1007	1 1152	1 1297	1 1441	1 1583	1.1725	1 1866	1 2006	47
48	1.0863	1 1010	1.1155	1 1299	1 1443	1 1586	1.1727	1 1868	1 2008	48
49	1 0866	1 1012	1.1157	1 1302	1 1445	1 1588	1.1730	1 1871	1.2011	49
50	1 0868	1 1014	1.1160	1 1304	1.1448	1 1590	1.1732	1.1873	1.2013	50
51	1 0871	1 1017	1.1162	1 1307	1.1450	1 1593	1.1735	1 1875	1.2015	51
52	1 0873	1 1019	1.1165	1 1309	1.1452	1 1595	1.1737	1.1878	1 2018	52
53	1 0876	1 1022	1.1167	1 1311	1 1455	1 1598	1.1739	1 1880	1 2020	53
54	1 0878	1.1024	1 1169	1 1314	1 1457	1 1600	1.1742	1 1882	1 2022	54
55	1 0881	1 1027	1.1172	1.1316	1 1460	1 1602	1.1744	1 1885	1.2025	55
56	1 0883	1 1029	1.1174	1 1319	1 1462	1.1605	1.1746	1 1887	1.2027	56
57	1 0885	1 1031	1.1177	1 1321	1.1464	1 1607	1.1749	1 1889	1.2029	57
58	1 0888	1.1034	1 1179	1 1323	1.1467	1 1609	1.1751	1 1892	1.2032	58
59	1.0890	1.1036	1.1181	1 1326	1.1469	1 1612	1.1753	1.1894	1.2034	59
60	1.0893	1 1039	1.1184	1.1328	1.1472	1.1614	1.1756	1.1896	1.2036	60

Table of Chords (Continued). Radius = 1.0000

M.	74°	75°	76°	77°	78°	79°	80°	81°	82°	M.
0′	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	1.3121	0′
1	1.2039	1.2178	1.2316	1.2453	1.2589	1.2724	1.2858	1.2991	1.3123	1
2	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860	1.2993	1.3126	2
3	1.2043	1.2182	1.2320	1.2457	1.2593	1.2728	1.2862	1.2996	1.3128	3
4	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865	1.2998	1.3130	4
5	1.2048	1.2187	1.2325	1.2462	1.2598	1.2733	1.2867	1.3000	1.3132	5
6	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869	1.3002	1.3134	6
7	1.2053	1.2191	1.2329	1.2466	1.2602	1.2737	1.2871	1.3004	1.3137	7
8	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2374	1.3007	1.3139	8
9	1.2057	1.2196	1.2334	1.2471	1.2607	1.2742	1.2876	1.3009	1.3141	9
10	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878	1.3011	1.3143	10
11	1.2062	1.2201	1.2338	1.2475	1.2611	1.2746	1.2880	1.3013	1.3145	11
12	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882	1.3015	1.3147	12
13	1.2066	1.2205	1.2343	1.2480	1.2616	1.2751	1.2885	1.3018	1.3150	13
14	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887	1.3020	1.3152	14
15	1.2071	1.2210	1.2348	1.2484	1.2620	1.2755	1.2889	1.3022	1.3154	15
16	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891	1.3024	1.3156	16
17	1.2076	1.2214	1.2352	1.2489	1.2625	1.2760	1.2894	1.3027	1.3158	17
18	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896	1.3029	1.3161	18
19	1.2080	1.2219	1.2357	1.2493	1.2629	1.2764	1.2898	1.3031	1.3163	19
20	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	1.3033	1.3165	20
21	1.2085	1.2224	1.2361	1.2498	1.2634	1.2769	1.2903	1.3035	1.3167	21
22	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905	1.3038	1.3169	22
23	1.2090	1.2228	1.2366	1.2503	1.2638	1.2773	1.2907	1.3040	1.3172	23
24	1.2092	1.2231	1.2368	1.2505	1.2641	1.2775	1.2909	1.3042	1.3174	24
25	1.2094	1.2233	1.2370	1.2507	1.2643	1.2778	1.2911	1.3044	1.3176	25
26	1.2097	1.2235	1.2373	1.2509	1.2645	1.2780	1.2914	1.3046	1.3178	26
27	1.2099	1.2237	1.2375	1.2512	1.2648	1.2782	1.2916	1.3049	1.3180	27
28	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918	1.3051	1.3183	28
29	1.2104	1.2242	1.2380	1.2516	1.2652	1.2787	1.2920	1.3053	1.3185	29
30	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	1.3055	1.3187	30
31	1.2108	1.2247	1.2384	1.2521	1.2656	1.2791	1.2925	1.3057	1.3189	31
32	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927	1.3060	1.3191	32
33	1.2113	1.2251	1.2389	1.2525	1.2661	1.2795	1.2929	1.3062	1.3193	33
34	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931	1.3064	1.3196	34
35	1.2117	1.2256	1.2393	1.2530	1.2665	1.2800	1.2934	1.3066	1.3198	35
36	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936	1.3068	1.3200	36
37	1.2122	1.2260	1.2398	1.2534	1.2670	1.2804	1.2938	1.3071	1.3202	37
38	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940	1.3073	1.3204	38
39	1.2127	1.2265	1.2402	1.2539	1.2674	1.2809	1.2942	1.3075	1.3207	39
40	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	1.3077	1.3209	40
41	1.2131	1.2270	1.2407	1.2543	1.2679	1.2813	1.2947	1.3079	1.3211	41
42	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949	1.3082	1.3213	42
43	1.2136	1.2274	1.2412	1.2548	1.2683	1.2818	1.2951	1.3084	1.3215	43
44	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954	1.3086	1.3218	44
45	1.2141	1.2279	1.2416	1.2552	1.2688	1.2822	1.2956	1.3088	1.3220	45
46	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958	1.3090	1.3222	46
47	1.2145	1.2283	1.2421	1.2557	1.2692	1.2827	1.2960	1.3093	1.3224	47
48	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962	1.3095	1.3226	48
49	1.2150	1.2288	1.2425	1.2562	1.2697	1.2831	1.2965	1.3097	1.3228	49
50	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	1.3099	1.3231	50
51	1.2154	1.2293	1.2430	1.2566	1.2701	1.2836	1.2969	1.3101	1.3233	51
52	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971	1.3104	1.3235	52
53	1.2159	1.2297	1.2434	1.2571	1.2706	1.2840	1.2973	1.3106	1.3237	53
54	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976	1.3108	1.3239	54
55	1.2164	1.2302	1.2439	1.2575	1.2710	1.2845	1.2978	1.3110	1.3242	55
56	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1.2980	1.3112	1.3244	56
57	1.2168	1.2306	1.2443	1.2580	1.2715	1.2849	1.2982	1.3115	1.3246	57
58	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985	1.3117	1.3248	58
59	1.2173	1.2311	1.2448	1.2584	1.2719	1.2854	1.2987	1.3119	1.3250	59
60	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	1.3121	1.3252	60

Table of Chords (Concluded). Radius = 1.0000

M.	83°	84°	85°	86°	87°	88°	89°	M.
0′	1.3252	1 3383	1 3512	1.3640	1 3767	1 3893	1 4018	0′
1	1 3255	1 3385	1 3514	1 3642	1 3769	1 3895	1 4020	1
2	1 3257	1.3387	1 3516	1 3644	1 3771	1 3897	1 4022	2
3	1 3259	1 3389	1 3518	1 3646	1 3773	1 3899	1 4024	3
4	1 3261	1 3391	1 3520	1 3648	1 3776	1 3902	1 4026	4
5	1 3263	1 3393	1 3523	1 3651	1 3778	1 3904	1 4029	5
6	1.3265	1 3396	1 3525	1 3653	1 3780	1 3906	1 4031	6
7	1 3268	1 3398	1 3527	1 3655	1 3782	1 3908	1 4033	7
8	1 3270	1 3400	1 3529	1 3657	1 3784	1 3910	1 4035	8
9	1 3272	1 3402	1 3531	1 3659	1 3786	1 3912	1 4037	9
10	1.3274	1 3404	1 3533	1 3661	1 3788	1 3914	1.4039	10
11	1 3276	1 3406	1 3535	1 3663	1 3790	1 3916	1 4041	11
12	1 3279	1 3409	1 3538	1 3665	1 3792	1 3918	1 4043	12
13	1 3281	1 3411	1 3540	1 3668	1 3794	1 3920	1 4045	13
14	1 3283	1 3413	1 3542	1 3670	1 3797	1 3922	1 4047	14
15	1 3285	1 3415	1 3544	1 3672	1 3799	1 3925	1 4049	15
16	1 3287	1 3417	1 3546	1 3674	1 3801	1 3927	1 4051	16
17	1.3289	1 3419	1 3548	1 3676	1 3803	1 3929	1 4053	17
18	1 3292	1 3421	1 3550	1 3678	1 3805	1 3931	1 4055	18
19	1 3294	1 3424	1 3552	1 3680	1 3807	1 3933	1 4058	19
20	1 3296	1 3426	1 3555	1.3682	1 3809	1 3935	1 4060	20
21	1 3298	1 3428	1 3557	1 3685	1 3811	1 3937	1 4062	21
22	1 3300	1 3430	1 3559	1 3687	1 3813	1 3939	1 4064	22
23	1 3302	1 3432	1 3561	1 3689	1 3816	1 3941	1 4066	23
24	1 3305	1 3434	1 3563	1 3691	1 3818	1 3943	1 4068	24
25	1 3307	1 3437	1 3565	1 3693	1 3820	1 3945	1 4070	25
26	1 3309	1 3439	1 3567	1 3695	1 3822	1 3947	1 4072	26
27	1 3311	1 3441	1 3570	1 3697	1 3824	1 3950	1 4074	27
28	1 3313	1 3443	1 3572	1 3699	1 3826	1 3952	1 4076	28
29	1 3315	1 3445	1 3574	1 3702	1 3828	1 3954	1 4078	29
30	1 3318	1 3447	1 3576	1 3704	1 3830	1 3956	1 4080	30
31	1 3320	1 3449	1 3578	1 3706	1 3832	1 3958	1 4082	31
32	1 3322	1 3452	1.3580	1 3708	1 3834	1 3960	1 4084	32
33	1 3324	1 3454	1 3582	1 3710	1 3837	1 3962	1 4086	33
34	1 3326	1 3456	1 3585	1 3712	1 3839	1 3964	1 4089	34
35	1 3328	1 3458	1 3587	1 3714	1 3841	1 3966	1 4091	35
36	1 3331	1 3460	1 3589	1 3716	1 3843	1 3968	1 4093	36
37	1 3333	1 3462	1 3591	1 3718	1 3845	1 3970	1 4095	37
38	1 3335	1 3465	1 3593	1 3721	1 3847	1 3972	1 4097	38
39	1 3337	1 3467	1 3595	1 3723	1 3849	1 3975	1.4099	39
40	1.3339	1 3469	1 3597	1 3725	1 3851	1 3977	1 4101	40
41	1 3341	1 3471	1 3599	1 3727	1 3853	1.3979	1.4103	41
42	1 3344	1.3473	1 3602	1 3729	1 3855	1 3981	1 4105	42
43	1 3346	1 3475	1 3604	1 3731	1 3858	1 3983	1 4107	43
44	1.3348	1 3477	1 3606	1 3733	1 3860	1 3985	1.4109	44
45	1.3350	1 3480	1 3608	1 3735	1 3862	1 3987	1.4111	45
46	1 3352	1 3482	1 3610	1 3738	1 3864	1 3989	1.4113	46
47	1 3354	1 3484	1 3612	1 3740	1 3866	1 3991	1.4115	47
48	1 3357	1 3486	1 3614	1 3742	1 3868	1 3993	1 4117	48
49	1 3359	1 3488	1 3617	1 3744	1 3870	1 3995	1 4119	49
50	1 3361	1 3490	1.3619	1 3746	1.3872	1.3997	1.4122	50
51	1 3363	1 3492	1.3621	1.3748	1 3874	1 3999	1 4124	51
52	1 3365	1 3495	1.3623	1 3750	1 3876	1 4002	1.4126	52
53	1 3367	1 3497	1 3625	1 3752	1 3879	1 4004	1 4128	53
54	1.3370	1.3499	1 3627	1.3754	1 3881	1 4006	1 4130	54
55	1.3372	1.3501	1.3629	1.3757	1 3883	1 4008	1.4132	55
56	1 3374	1 3503	1 3631	1 3759	1 3885	1.4010	1 4134	56
57	1 3376	1 3505	1 3634	1.3761	1.3887	1 4012	1 4136	57
58	1 3378	1 3508	1 3636	1.3763	1.3889	1.4014	1.4138	58
59	1 3380	1 3510	1 3638	1.3765	1.3891	1 4016	1.4140	59
60	1.3383	1 3512	1.3640	1.3767	1.3893	1.4018	1.4142	60

Lengths and Bevels of Hip-Rafters and Jack-Rafters

Method of Determining the Lengths and Bevels. The lines *ab* and *bc* (Fig. 92) represent the outside of the walls at the angle of a building; *be* is the seat of the hip-rafter and *gf* of a jack-rafter. Draw *eh* at right-angles to *be* and make it equal to the rise of the roof; join *b* and *h* and *hb* will be the length of the hip-rafter. Through *e* draw *di* at right-angles to *bc*. With *b* as a center

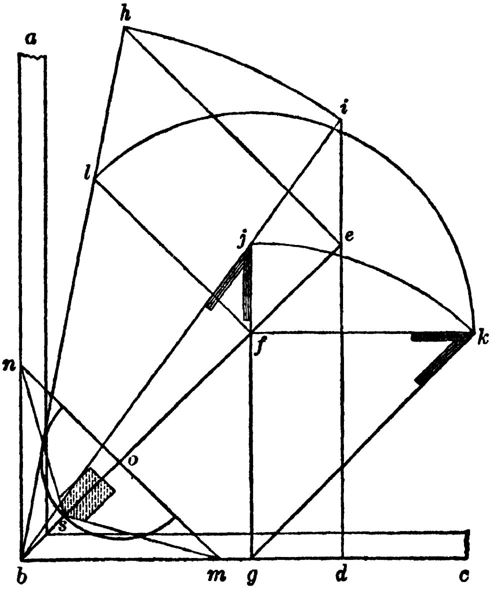

Fig. 92. Lengths and Bevels of Hip-rafters and Jack-rafters

and with the radius *bh*, describe the arc *hi*, cutting *di* in *i*. Join *b* and *i* and extend *gf* to meet *bi* in *j*; then *gj* is the length of the jack-rafter. The length of each jack-rafter is found in the same manner, by extending its seat to cut the line *bi*. From *f* draw *fk* at right-angles to *fg*; also *fl* at right-angles to *be*. Make *fk* equal to *fl* by the arc *lk*, or make *gk* equal to *gj* by the arc *jk*; then the angle at *j* is the TOP BEVEL of the jack-rafters, and the angle at *k* the DOWN BEVEL.

Backing of the Hip-Rafter. At any convenient point in *be* (Fig. 92), as *o*, draw *mn* at right-angles to *be*. From *o* describe a circle, tangent to *bh*, cutting *be* in *s*. Join *m* and *s* and *n* and *s*. The lines *ms* and *ns* form at *s* the proper angle for beveling the top of the hip-rafter.

5. TRIGONOMETRY

It is not the purpose of the author to teach the principles or uses of trigonometry; but for the benefit of those readers who have already acquired a knowledge of this science, the following convenient formulas and tables of natural sines, cosines, tangents and cotangents have been inserted. To those who know how to apply these trigonometric functions, they will often be found of great convenience and utility. These tables are taken, by permission, from Searles' Field Engineering, John Wiley & Sons, Inc., publishers.

Trigonometric Functions

Let A (Fig. 93) = angle BAC = arc BF and let the radius $AF = AB = AH = 1$. Then

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = BG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2\,BC$

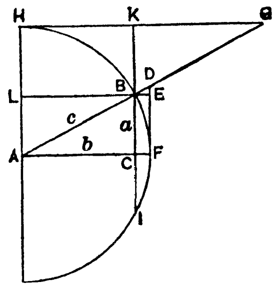

Fig 93. Functions of Right-angled Triangle

In the right-angled triangle ABC (Fig. 93) let $AB = c$, $AC = b$ and $BC = a$. Then

(1) $\sin A = \frac{a}{c} = \cos B$

(2) $\cos A = \frac{b}{c} = \sin B$

(3) $\tan A = \frac{a}{b} = \cot B$

(4) $\cot A = \frac{b}{a} = \tan B$

(5) $\sec A = \frac{c}{b} = \operatorname{cosec} B$

(6) $\operatorname{cosec} A = \frac{c}{a} = \sec B$

(7) $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$

(8) $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$

(9) $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$

(10) $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$

(11) $a = c \sin A = b \tan A$

(12) $b = c \cos A = a \cot A$

(13) $c = \frac{a}{\sin A} = \frac{b}{\cos A}$

(14) $a = c \cos B = b \cot B$

(15) $b = c \sin B = a \tan B$

(16) $c = \frac{a}{\cos B} = \frac{b}{\sin B}$

(17) $a = \sqrt{(c+b)(c-b)}$

(18) $b = \sqrt{(c+a)(c-a)}$

(19) $c = \sqrt{a^2 + b^2}$

(20) $C = 90° = A + B$

(21) area $= \frac{ab}{2}$

Solution of Oblique Triangles

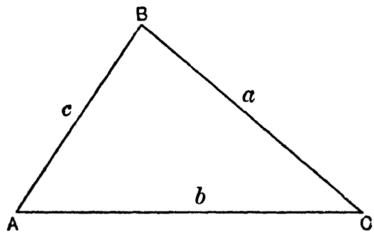

Fig 94. Oblique-angled Triangle

	Given	Required	Formulas
(22)	A, B, a	C, b, c	$C = 180° - (A + B)$ $\quad b = \dfrac{a}{\sin A} \cdot \sin B$ $c = \dfrac{a}{\sin A} \sin (A + B)$
(23)	A, a, b	B, C, c	$\sin B = \dfrac{\sin A}{a} \cdot b$ $\quad C = 180° - (A + B)$ $c = \dfrac{a}{\sin A} \cdot \sin C$
(24)	C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$
(25)	....	$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \dfrac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
(26)	.. ...	A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$ $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$
(27)	. ...	c	$c = (a + b) \dfrac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \dfrac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$
(28)	. ....	Area	$K = \frac{1}{2} ab \sin C$
(29)	a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$; $\quad \sin \frac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{bc}}$
(30)	....	....	$\cos \frac{1}{2} A = \sqrt{\dfrac{s(s - a)}{bc}}$; $\tan \frac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{s(s - a)}}$
(31)	....	. ... ..	$\sin A = \dfrac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc}$ $\operatorname{vers} A = \dfrac{2(s - b)(s - c)}{bc}$
(32)	....	Area	$K = \sqrt{s(s - a)(s - b)(s - c)}$
(33)	A, B, C, a	Area	$K = \dfrac{a^2 \sin B \sin C}{2 \sin A}$

Oblique Triangles. General Formulas

(34) $\sin A = \dfrac{1}{\operatorname{cosec} A} = \sqrt{1 - \cos^2 A} = \tan A \cos A$

(35) $\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \operatorname{vers} A \cot \tfrac{1}{2} A$

(36) $\sin A = \sqrt{\tfrac{1}{2} \operatorname{vers} 2A} = \sqrt{\tfrac{1}{2}(1 - \cos 2A)}$

(37) $\cos A = \dfrac{1}{\sec A} = \sqrt{1 - \sin^2 A} = \cot A \sin A$

(38) $\cos A = 1 - \operatorname{vers} A = 2 \cos^2 \tfrac{1}{2} A - 1 = 1 - 2 \sin^2 \tfrac{1}{2} A$

(39) $\cos A = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} \cos 2A}$

(40) $\tan A = \dfrac{1}{\cot A} = \dfrac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$

(41) $\tan A = \sqrt{\dfrac{1}{\cos^2 A} - 1} = \dfrac{\sqrt{1 - \cos^2 A}}{\cos A} = \dfrac{\sin 2A}{1 + \cos 2A}$

(42) $\tan A = \dfrac{1 - \cos 2A}{\sin 2A} = \dfrac{\operatorname{vers} 2A}{\sin 2A} = \operatorname{exsec} A \cot \tfrac{1}{2} A$

(43) $\cot A = \dfrac{1}{\tan A} = \dfrac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}$

(44) $\cot A = \dfrac{\sin 2A}{1 - \cos 2A} = \dfrac{\sin 2A}{\operatorname{vers} 2A} = \dfrac{1 + \cos 2A}{\sin 2A}$

(45) $\cot A = \dfrac{\tan \tfrac{1}{2} A}{\operatorname{exsec} A}$

(46) $\operatorname{vers} A = 1 - \cos A = \sin A \tan \tfrac{1}{2} A = 2 \sin^2 \tfrac{1}{2} A$

(47) $\operatorname{vers} A = \operatorname{exsec} A \cos A$

(48) $\operatorname{exsec} A = \sec A - 1 = \tan A \tan \tfrac{1}{2} A = \dfrac{\operatorname{vers} A}{\cos A}$

(49) $\sin \tfrac{1}{2} A = \sqrt{\dfrac{1 - \cos A}{2}} = \sqrt{\dfrac{\operatorname{vers} A}{2}}$

(50) $\sin 2A = 2 \sin A \cos A$

(51) $\cos \tfrac{1}{2} A = \sqrt{\dfrac{1 + \cos A}{2}}$

(52) $\cos 2A = 2 \cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$

(53) $\tan \tfrac{1}{2} A = \dfrac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \dfrac{1 - \cos A}{\sin A} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

(54) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

(55) $\cot \tfrac{1}{2} A = \dfrac{\sin A}{\operatorname{vers} A} = \dfrac{1 + \cos A}{\sin A} = \dfrac{1}{\operatorname{cosec} A - \cot A}$

(56) $\cot 2A = \dfrac{\cot^2 A - 1}{2 \cot A}$

Oblique Triangles. General Formulas (Continued)

$$(57)\ \text{vers}\ \tfrac{1}{2} A = \frac{\tfrac{1}{2}\ \text{vers}\ A}{1 + \sqrt{1 - \tfrac{1}{2}\ \text{vers}\ A}} = \frac{1 - \cos A}{2 + \sqrt{2\,(1 + \cos A)}}$$

$$(58)\ \text{vers}\ 2A = 2 \sin^2 A$$

$$(59)\ \text{exsec}\ \tfrac{1}{2} A = \frac{1 - \cos A}{(1 + \cos A) + \sqrt{2\,(1 + \cos A)}}$$

$$(60)\ \text{exsec}\ 2A \quad \frac{2 \tan^2 A}{1 - \tan^2 A}$$

$$(61)\ \sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$(62)\ \cos (A \pm B) = \cos A \cos B \pm \sin A \sin B$$

$$(63)\ \sin A + \sin B = 2 \sin \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B)$$

$$(64)\ \sin A - \sin B = 2 \cos \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B)$$

$$(65)\ \cos A + \cos B = 2 \cos \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B)$$

$$(66)\ \cos B - \cos \mathrm{A} = 2 \sin \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B)$$

$$(67)\ \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B)$$

$$(68)\ \cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$$

$$(69)\ \tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B}$$

$$(70)\ \tan A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}$$

′	0° Sine	0° Cosin	1° Sine	1° Cosin	2° Sine	2° Cosin	3° Sine	3° Cosin	4° Sine	4° Cosin	′
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10° Sine	10° Cosin	11° Sine	11° Cosin	12° Sine	12° Cosin	13° Sine	13° Cosin	14° Sine	14° Cosin	′
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21935	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97509	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

'	25° Sine	25° Cosin	26° Sine	26° Cosin	27° Sine	27° Cosin	28° Sine	28° Cosin	29° Sine	29° Cosin	'
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87026	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
'	Cosin 64°	Sine 64°	Cosin 63°	Sine 63°	Cosin 62°	Sine 62°	Cosin 61°	Sine 61°	Cosin 60°	Sine 60°	'

′	30° Sine	30° Cosin	31° Sine	31° Cosin	32° Sine	32° Cosin	33° Sine	33° Cosin	34° Sine	34° Cosin	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	59° Cosin	59° Sine	58° Cosin	58° Sine	57° Cosin	57° Sine	56° Cosin	56° Sine	55° Cosin	55° Sine	′

'	35° Sine	35° Cosin	36° Sine	36° Cosin	37° Sine	37° Cosin	38° Sine	38° Cosin	39° Sine	39° Cosin	'
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	57928	.81513	59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
'	54° Cosin	54° Sine	53° Cosin	53° Sine	52° Cosin	52° Sine	51° Cosin	51° Sine	50° Cosin	50° Sine	'

′	40° Sine	40° Cosin	41° Sine	41° Cosin	42° Sine	42° Cosin	43° Sine	43° Cosin	44° Sine	44° Cosin	′
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

′	0° Tang	0° Cotang	1° Tang	1° Cotang	2° Tang	2° Cotang	3° Tang	3° Cotang	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	89° Cotang	89° Tang	88° Cotang	88° Tang	87° Cotang	87° Tang	86° Cotang	86° Tang	′

′	4° Tang	4° Cotang	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.23190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	85° Cotang	85° Tang	84° Cotang	84° Tang	83° Cotang	83° Tang	82° Cotang	82° Tang	′

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97433	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18055	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20801	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20021	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22536	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

′	12° Tang	12° Cotang	13° Tang	13° Cotang	14° Tang	14° Cotang	15° Tang	15° Cotang	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4 29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3 90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01570	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	26
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	1
59	.30541	3.27426	.22460	3.08073	.34400	2.90696	.36364	2.74997	0
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	2
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

′	20° Tang	20° Cotang	21° Tang	21° Cotang	22° Tang	22° Cotang	23° Tang	23° Cotang	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	69° Cotang	69° Tang	68° Cotang	68° Tang	67° Cotang	67° Tang	66° Cotang	66° Tang	′

′	24° Tang	24° Cotang	25° Tang	25° Cotang	26° Tang	26° Cotang	27° Tang	27° Cotang	′
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91552	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91414	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91288	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91142	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91017	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

′	28° Tang	28° Cotang	29° Tang	29° Cotang	30° Tang	30° Cotang	31° Tang	31° Cotang	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	61° Cotang	61° Tang	60° Cotang	60° Tang	59° Cotang	59° Tang	58° Cotang	58° Tang	′

′	32°		33°		34°		35°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70170	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.63314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

′	36° Tang	36° Cotang	37° Tang	37° Cotang	38° Tang	38° Cotang	39° Tang	39° Cotang	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

′	40° Tang	40° Cotang	41° Tang	41° Cotang	42° Tang	42° Cotang	43° Tang	43° Cotang	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
	Cotang	Tang			Cotang	Tang			Cotang	Tang	
′	45°		′	′	45°		′	′	45°		′

Natural Secants and Cosecants

Degrees	Secants 0′	10′	20′	30′	40′	50′	60′	
0	1 00000	1 00001	1 00002	1.00004	1.00007	1 00011	1 00015	89
1	1 00015	1 00021	1 00027	1 00034	1 00042	1 00051	1 00061	88
2	1 00061	1 00072	1 00083	1 00095	1 00108	1 00122	1 00137	87
3	1 00137	1 00153	1 00169	1 00187	1 00205	1 00224	1 00244	86
4	1 00244	1 00265	1 00287	1 00309	1 00333	1 00357	1 00382	85
5	1 00382	1 00408	1 00435	1 00463	1 00491	1 00521	1 00551	84
6	1 00551	1 00582	1 00614	1 00647	1 00681	1 00715	1 00751	83
7	1 00751	1 00787	1 00825	1 00863	1 00902	1 00942	1 00983	82
8	1 00983	1 01024	1 01067	1 01111	1 01155	1 01200	1 01247	81
9	1 01247	1 01294	1 01342	1 01391	1 01440	1 01491	1 01543	80
10	1 01543	1 01595	1 01649	1 01703	1 01758	1 01815	1 01872	79
11	1 01872	1 01930	1 01989	1 02049	1 02110	1 02171	1 02234	78
12	1 02234	1 02298	1 02362	1 02428	1 02494	1 02562	1 02630	77
13	1 02630	1 02700	1 02770	1 02842	1 02914	1 02987	1 03061	76
14	1 03061	1 03137	1 03213	1 03290	1 03368	1 03447	1 03528	75
15	1 03528	1 03609	1 03691	1 03774	1 03858	1 03944	1 04030	74
16	1.04030	1 04117	1 04206	1 04295	1 04385	1 04477	1 04569	73
17	1 04569	1 04663	1 04757	1 04853	1 04950	1 05047	1 05146	72
18	1 05146	1 05246	1 05347	1 05449	1 05552	1 05657	1 05762	71
19	1 05762	1 05869	1 05976	1 06085	1 06195	1 06306	1 06418	70
20	1 06418	1 06531	1 06645	1 06761	1 06878	1 06995	1 07115	69
21	1 07115	1 07235	1 07356	1 07479	1 07602	1 07727	1 07853	68
22	1 07853	1 07981	1 08109	1 08239	1 08370	1 08503	1 08636	67
23	1 08636	1 08771	1 08907	1 09044	1 09183	1 09323	1 09464	66
24	1 09464	1 09606	1 09750	1 09895	1 10041	1 10189	1 10338	65
25	1 10338	1 10488	1 10640	1 10793	1 10947	1 11103	1 11260	64
26	1 11260	1 11419	1 11579	1 11740	1 11903	1 12067	1 12233	63
27	1 12233	1 12400	1 12568	1 12738	1 12910	1 13083	1 13257	62
28	1 13257	1.13433	1 13610	1 13789	1 13970	1 14152	1 14335	61
29	1 14335	1 14521	1 14707	1 14896	1 15085	1 15277	1 15470	60
30	1 15470	1 15665	1 15861	1 16059	1 16259	1 16460	1 16663	59
31	1 16663	1 16868	1 17075	1 17283	1 17493	1 17704	1 17918	58
32	1 17918	1 18133	1 18350	1 18569	1 18790	1 19012	1 19236	57
33	1 19236	1 19463	1 19691	1 19920	1 20152	1 20386	1 20622	56
34	1 20622	1 20859	1 21099	1 21341	1 21584	1 21830	1 22077	55
35	1 22077	1 22327	1 22579	1 22833	1 23089	1 23347	1 23607	54
36	1 23607	1 23869	1 24134	1 24400	1 24669	1 24940	1 25214	53
37	1 25214	1 25489	1 25767	1 26047	1 26330	1 26615	1.26902	52
38	1 26902	1 27191	1 27483	1 27778	1 28075	1 28374	1 28676	51
39	1 28676	1 28980	1 29287	1 29597	1 29909	1 30223	1 30541	50
40	1 30541	1 30861	1 31183	1 31509	1 31837	1 32168	1 32501	49
41	1 32501	1 32838	1 33177	1 33519	1 33864	1 34212	1 34563	48
42	1 34563	1 34917	1 35274	1 35634	1 35997	1 36363	1 36733	47
43	1 36733	1 37105	1 37481	1 37860	1 38242	1 38628	1 39016	46
44	1.39016	1 39409	1 39804	1 40203	1 40606	1 41012	1.41421	45
	60′	50′	40′	30′	20′	10′	0′	Degrees
	Cosecants							

Natural Secants and Cosecants (Continued)

Degrees	Cosecants 0′	10′	20′	30′	40′	50′	60′	
0	∞	343 77516	171 88831	114 59301	85 94561	68 75736	57.29869	89
1	57 29869	49 11406	42 97571	38 20155	34 38232	31 25758	28 65371	88
2	28 65371	26 45051	24 56212	22 92559	21 49368	20 23028	19 10732	87
3	19 10732	18 10262	17 19843	16 38041	15 63679	14 95788	14 33559	86
4	14 33559	13 76312	13.23472	12 74550	12 29125	11 86837	11 47371	85
5	11 47371	11.10455	10 75849	10 43343	10 12752	9 83912	9 56677	84
6	9 56677	9 30917	9 06515	8 83367	8 61379	8 40466	8 20551	83
7	8 20551	8 01565	7.83443	7 66130	7 49571	7 33719	7 18530	82
8	7 18530	7 03962	6 89979	6 76547	6 63633	6 51208	6 39245	81
9	6 39245	6 27719	6 16607	6 05886	5 95536	5 85539	5 75877	80
10	5 75877	5 66533	5 57493	5 48740	5 40263	5 32049	5 24084	79
11	5 24084	5 16359	5 08863	5 01585	4 94517	4 87649	4 80973	78
12	4 80973	4.74482	4 68167	4 62023	4 56041	4 50216	4 44541	77
13	4 44541	4 39012	4 33622	4 28366	4 23239	4 18238	4 13357	76
14	4 13357	4 08591	4 03938	3 99393	3 94952	3 90613	3 86370	75
15	3 86370	3 82223	3 78166	3 74198	3 70315	3 66515	3 62796	74
16	3 62796	3 59154	3 55587	3 52094	3 48671	3 45317	3 42030	73
17	3 42030	3 38808	3 35649	3 32551	3 29512	3 26531	3 23607	72
18	3 23607	3 20737	3 17920	3 15155	3 12440	3 09774	3 07155	71
19	3 07155	3 04584	3 02057	2 99574	2.97135	2 94737	2 92380	70
20	2 92380	2 90063	2 87785	2 85545	2 83342	2 81175	2 79043	69
21	2 79043	2 76945	2.74881	2 72850	2 70851	2 68884	2 66947	68
22	2 66947	2 65040	2 63162	2 61313	2 59491	2 57698	2 55930	67
23	2 55930	2 54190	2 52474	2 50784	2 49119	2 47477	2 45859	66
24	2 45859	2 44264	2 42692	2 41142	2 39614	2 38107	2 36620	65
25	2 36620	2 35154	2 33708	2 32282	2 30875	2 29487	2 28117	64
26	2 28117	2 26766	2 25432	2 24116	2 22817	2 21535	2 20269	63
27	2 20269	2 19019	2 17786	2 16568	2 15366	2 14178	2 13005	62
28	2 13005	2 11847	2 10704	2 09574	2 08458	2 07356	2 06267	61
29	2 06267	2 05191	2 04128	2 03077	2 02039	2.01014	2 00000	60
30	2 00000	1 98998	1 98008	1 97029	1 96062	1 95106	1 94160	59
31	1 94160	1 93226	1 92302	1 91388	1 90485	1 89591	1 88708	58
32	1 88708	1 87834	1 86990	1 86116	1 85271	1 84435	1 83608	57
33	1 83608	1 82790	1 81981	1 81180	1.80388	1 79604	1 78829	56
34	1 78829	1 78062	1 77303	1 76552	1 75808	1 75073	1 74345	55
35	1 74345	1 73624	1 72911	1 72205	1 71506	1 70815	1 70130	54
36	1 70130	1 69452	1 68782	1 68117	1 67460	1 66809	1 66164	53
37	1 66164	1 65526	1 64894	1 64268	1 63648	1 63035	1 62427	52
38	1 62427	1 61825	1 61229	1 60639	1 60054	1 59475	1 58902	51
39	1 58902	1.58333	1.57771	1 57213	1 56661	1 56114	1 55572	50
40	1 55572	1 55036	1 54504	1 53977	1 53455	1 52938	1 52425	49
41	1 52425	1 51918	1 51415	1 50916	1 50422	1 49933	1 49448	48
42	1 49448	1 48967	1 48491	1 48019	1 47551	1 47087	1 46628	47
43	1.46628	1 46173	1 45721	1 45274	1.44831	1 44391	1 43956	46
44	1 43956	1 43524	1 43096	1 42672	1 42251	1 41835	1 41421	45
	60′	50′	40′	30′	20′	10′	0′	Degrees
	Secants							

PART II

STRENGTH OF MATERIALS AND STABILITY OF STRUCTURES

INTRODUCTION

EXPLANATION OF SUBJECT-MATTER AND NOTATION

1. Introduction to Part II

Subject-Matter of Part II. In the thirty-six chapters of Part II are given the necessary rules, formulas and data for computing the strength and stability of all ordinary forms of building-construction, whether of wood, steel, concrete or masonry, and in fact of all but the more intricate problems of steel construction, with which few architects care to cope, and which, indeed, are more especially within the province of the engineer.

The Rules and Formulas have been reduced to their simplest forms, and, in general, require only an elementary knowledge of mathematics to understand them. The application of the formulas is explained and in most cases their derivation, and it is believed that the formulas, constants and working stresses are representative of conservative and approved contemporary practice.

Constants and Working Stresses. In the use of constants for the strength of materials, the authors and editors have been guided by the practice of leading structural engineers, by the available records of tests and by their own experience of many years as practicing and consulting architects and engineers The varying conditions of building-construction have been taken into account and an attempt made to adapt the values to the practical conditions usually governing such construction. Every possible precaution has been taken to prevent the misapplication of rules and formulas and to insure absolute safety without undue waste of materials.

Tables. Much thought and labor have been expended on the preparation of the numerous tables, to insure their accuracy and to arrange them in the most convenient form for use by architects and builders. Many of these tables were computed by the authors and editors, all have been carefully verified, and it is believed that they may be used with perfect confidence. In all cases, unless otherwise noted, they give the same values that would be obtained by using the formulas specially referred to, while they afford a great saving of time and labor and reduce to a minimum the danger of errors in making the necessary computations.

Treatment of the Subject. Owing to the nature of the subjects treated and the large number of pages required to include them all in one book of reference, some forms of construction are treated rather briefly. The intention is to give the data needed for immediate use rather than a complete discussion of all the principles involved. Those who wish more complete discussions of masons' work, carpenters' work, steelwork, etc., are referred to treatises on these branches of building-construction. References are made in the different chapters to various other books and periodicals containing more complete information on some of the subjects. Part II deals principally with foundations, walls and piers, arches, columns, beams and girders, floors, mill-construction, fireproofing, reinforced-concrete construction, roof-trusses, wind-bracing, and domes and vaults.

2. Explanation of the Notation or Symbols Used in Part II

Besides the usual mathematical signs and characters in general use, the following abbreviations and symbols are frequently used:

A area of cross-section; also, a constant used in Chapter XX;
$a, b, c, \ldots m$, etc., known or given distances;
b breadth, as of beams;
C coefficient of strength;
c normal distance from neutral axis of cross-section of beam to most distant fiber in same;
d diameter, as of rivets; exterior diameter; depth, as of beams;
d_1 interior diameter;
E modulus of elasticity;
E_s, E_c modulus of elasticity for steel and concrete respectively (as in reinforced concrete);
e total deformation or change in length, as in a bar;
F shearing-modulus of elasticity,
f maximum deflection for a beam; unit stress;
h distance between parallel axes for moments of inertia;
I moment of inertia about a line;
I/c section-modulus or section-factor;
J polar moment of inertia;
J' polar moment of inertia of bolts about shaft-axis;
K total elastic resistance of a bar; resilience, work; also, a factor or constant used in formulas for reinforced concrete;
l length; span of a beam;
M bending moment;
M_{max} maximum bending moment;
M_1, M_2, etc., bending moments at supports of beams;
M_r or SI/c moment of resistance;
n number of loads, spans, etc.;
P external force, concentrated load;
P_1, P_2, P_3, etc., concentrated loads on beams;
p pitch of rivets; eccentricity of load on column; ratio of cross-section of steel to cross-section of beam (reinforced concrete);
r radius of curvature; radius; radius of gyration; ratio of Es for steel to Ec for concrete (reinforced concrete);
R_1, R_2, R_3, etc., reactions at the supports of a beam;
S unit stress, with subscripts t, c and s for unit stress in tension, compression and shear, respectively; section modulus,
S_b buckling resistance in webs of steel beams;
S_h horizontal unit shearing-stress in beams;
S_e elastic limit;
S_f modulus of rupture, or computed flexural strength;
t_1, t_2, etc , thicknesses;
V vertical shear;
W weight of a bar or beam; total uniform load on beam (may include weight of beam);
wl total uniform load on a beam (may include weight of beam);
w weight of a cubic unit of material; uniform load on beam, per linear unit of length;
x, y, z, variable distances;
α, β, etc., material constants;

ϕ constant depending upon material;
θ an angle.

Greek letters are used generally for signs of operation, for abstract numbers and for angles. Σ is employed as a symbol of summation.

The following are the Greek letters most in use:

α Alpha,	β Beta,	ϵ Epsilon,	η Eta,
θ Theta,	κ Kappa,	λ Lambda,	μ Mu,
ν Nu,	π Pi,	ρ Rho,	σ Sigma,
τ Tau,	ϕ Phi,	ψ Psi,	ω Omega.

Note. In a few places in the book it has been considered necessary or advisable by some of the associate editors to give a different meaning to one or more of the above symbols or to introduce different symbols for the meanings given in the list, but in all such cases the new symbols or meanings have been very clearly indicated.

The term BREADTH is used to denote the horizontal thickness of a beam or the smaller dimension of the cross-section of a rectangular column, post or strut, and is always measured in inches unless expressly stated otherwise.

The term DEPTH denotes the vertical height of a beam or girder, and is always measured in inches unless expressly stated otherwise.

The term LENGTH denotes the distance between supports and is always measured in feet unless expressly stated otherwise.

Abbreviations. In order to shorten the formulas, the tabulations of computations, etc , and throughout the text generally, to economize space, the units of measurement are generally abbreviated. For example, foot and feet are abbreviated, ft; inch and inches, in; pound and pounds, lb; square, sq; cubic, cu; linear, lin; inch-pound or inch-pounds, in-lb; foot-pound or foot-pounds, ft-lb; ounces, oz; horse-power, h p ; gallons, gal; etc.; and no periods are placed after these abbreviations, except at the ends of sentences. Where the word TON is used in this volume, it always means the net ton of 2 000 lb.

CHAPTER I

EXPLANATION OF TERMS USED IN ARCHITECTURAL ENGINEERING

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1. Definitions of Some of the Terms Used in the Mechanics of Materials *

Terms Used in Architectural Engineering. The following terms frequently occur in discussions of the principles of architectural engineering and an understanding of their meaning is essential.

Mechanics is the branch of physics that treats of the phenomena caused by the action of forces on material bodies.

Applied Mechanics treats of the laws of mechanics as applied to construction in the useful arts, as in beams, trusses, arches, etc.

Mechanics of Materials treats of the effects of forces in causing changes in the size and shape of bodies.

Rest is the relation that exists between two points when the straight line joining them does not change in length or direction. A body is at rest relatively to a point when any point in the body is at rest relatively to the first-mentioned point.

Motion is the relation that exists between two points when the straight line joining them changes in length or direction, or in both. A body moves relatively to a point when any point in the body moves relatively to the first-mentioned point.

Force is that which changes, or tends to change, the state of rest or motion of the body acted upon. It is a cause regarding the essential nature of which we are ignorant. In the mechanics of materials we do not deal with the nature of forces, but only with the laws of their action

Equilibrium is that condition of a body in which the forces acting upon it balance or neutralize each other; or, it is that condition of a force-system in which the resultant of the force-system is zero.

Statics is the branch of Mechanics that treats of the conditions of equilibrium. It is divided into:

(1) Statics of rigid bodies.

(2) Statics of practically incompressible fluids.

In building-construction we have to deal only with the former.

Structures are artificial constructions in which all the parts are intended to be in equilibrium and at rest relatively to each other, as in the case of a bridge-truss or roof-truss. They consist of two or more solid bodies, generally called PIECES or MEMBERS, which are connected at different parts of their surfaces called JOINTS.

* In addition to the terms defined here, many others are defined in the chapters of Part II, and especially in Chapters VI, IX, X, XIV, XV, XIX and XXIII.

In general there are three conditions of equilibrium in a structure.

(1) The external forces acting upon the whole structure must balance each other. These forces are·

(*a*) The weight of the structure;

(*b*) The loads it carries;

(*c*) The upward supporting forces, reactions or resistance under or around the foundations.

(2) The forces acting upon each piece of the structure must balance each other. These forces are, for each piece:

(*a*) The weight of the piece;

(*b*) The loads it carries;

(*c*) The resistances or reactions at its joints.

(3) The forces acting upon each of the parts into which any piece may be supposed to be divided must balance each other.

The Stability of a Structure requires the fulfillment of conditions (1) and (2), that is, the ability of the structure to resist the DISPLACEMENT of any of its parts.

The Strength of a Piece or Member consists in the fulfillment of condition (3), that is, the ability of a piece to resist BREAKING.

The Stiffness of a Piece or Member consists in the ability of a piece to resist BENDING

The Theory of Structures is divided into two parts:

(1) That which treats of strength and stiffness, dealing only with single pieces and generally known as the STRENGTH OF MATERIALS or the MECHANICS OF MATERIALS, before defined.

(2) That which treats of stability, dealing with the structures themselves.

Stress is an internal force that resists a change in shape or size caused by external forces. When the applied external forces reach certain intensities the internal stresses hold them in equilibrium.

The Intensity of a Stress is measured by the UNIT STRESS. (See Unit Stress.) The INTENSITY OF THE STRESS per square inch on any normal surface of a solid is the total stress divided by the area of the section in square inches. Thus, if a bar 10 ft long and 2 in square has a load of 8 000 lb pulling in the direction of its length, the stress on any normal section of the bar is 8 000 lb; and the intensity of the stress per square inch is 8 000/4 sq in = 2 000 lb per sq in.

Deformation.* When a solid body is acted upon by an external force an alteration takes place in the volume and shape of the body, and this alteration is called the DEFORMATION. In the case of the bar given above, the deformation is the amount that the bar stretches under its load.

The Ultimate Strength is the highest unit stress a piece of material can sustain and it is the unit stress at or just before rupture.

The Working Unit Stress is the ultimate stress divided by the factor of safety.

The Safe Load is the load that a piece can support without exceeding the working unit stresses.

* In mechanics the term STRAIN is now synonymous with the term DEFORMATION. On account of the tendency to confuse the terms STRAIN and STRESS the term DEFORMATION is used to denote change in shape and the term STRAIN is omitted in all discussions in the Handbook.

The Factor of Safety * of a piece of material under stress is the ratio of the ultimate strength of the material to the actual unit stress on the section-area; or it is the number by which the ultimate unit stress must be divided to give the working unit stress. In designing a piece of material to sustain a certain load, it is required that it shall be perfectly safe under all circumstances; and hence it is necessary to make an allowance for any defects in the material, workmanship, etc. It is obvious, that, for materials of different composition, different factors of safety are required. Thus, steel being more homogeneous than wood and less liable to defects, does not require as high a factor of safety. Again, different kinds of stresses require different factors of safety. Thus, a long wooden column or strut requires a higher factor of safety than a wooden beam. As the factors of safety thus vary for different kinds of stresses and materials, the proper factors for the different kinds of stresses and conditions are given in considering the resistance of the various materials to those stresses under varying conditions.

The Unit Stress is the stress on a unit of section-area, and is usually expressed in pounds per square inch. (See Intensity of Stress)

Dead Loads and Live Loads. The term DEAD LOAD means a load that is applied and increased gradually and that finally remains constant, such as the weight of a structure itself.

The term LIVE LOAD means a load that is applied suddenly and causes vibrations, such as a train traveling over a railway bridge. It has been found by experience that the effect of a live load on a beam or other piece of material has twice the destructive tendency of a dead load of the same magnitude or intensity. Hence a piece of material designed to carry a live load should have a factor of safety twice as large as one designed to carry a dead load. The load due to a crowd of people walking on a floor is usually considered to produce an effect which is a mean between that of a dead load and a live load, and a suitable factor of safety is adopted accordingly. In municipal ordinances and laws relating to the allowable loads for floors, the loads to be supported by the floors, exclusive of their inherent construction and stationary fixtures, are generally referred to as the LIVE LOADS no matter of what they may consist; but the term does not have the exact significance given to it by many engineers and as explained in the paragraph above.

The Modulus of Rupture or Computed Flexural Strength is the value of the UNIT FIBER-STRESS S, computed from the flexure-formula $M = SI/c$, when a beam is ruptured under a transverse load. Its value is intermediate between the ultimate tensile and compressive strengths of a material.

The Elastic Limit is that unit stress at which the deformation of a piece of material begins to increase in a faster ratio than the applied loads. It is sometimes called the ELASTIC STRENGTH.

The Modulus of Elasticity or Coefficient of Elasticity. In treatises on physics this is often called YOUNG'S MODULUS. It we take a bar of any elastic material, say one inch square, of any length, and secured at one end, and to the other apply a force, say a certain number of pounds P, pulling in the

* The ELASTIC LIMITS of materials must be considered in deciding upon working unit stresses and in forming a judgment of the security of materials under stress. When the elastic limit is considered the actual allowable unit stress is made a certain percentage of it, as 35 or 50%, according to varying conditions. Both ULTIMATE STRENGTHS and ELASTIC LIMITS must be taken into account in practice. But the use of the FACTOR OF SAFETY, as determined by the old method, is still a great help in the study and application of the principles of the mechanics of materials, and is used frequently in the Handbook.

direction of its length, we shall find by careful measurement that the bar has been stretched or elongated by the action of the force. If we divide the TOTAL ELONGATION e, in inches, by the original length l of the bar, in inches, we shall have e/l, the UNIT ELONGATION ϵ, or the elongation of the bar per unit of length; and if we divide the unit stress S, developed (that is, in this case, the external force P, divided by the area of the cross-section A, or P/A) by this ratio we shall have what is known as the MODULUS OF ELASTICITY, E. Expressed in symbols and by equations, $E = S/\epsilon = \frac{P/A}{e/l}$. Hence, we may define the MODULUS OF ELASTICITY as the ratio of the unit stress to the unit deformation. Another definition is, the force which would elongate a bar of 1 sq in in cross-section to double its original length, if that could be done without exceeding the ELASTIC LIMIT of the material This is evident from the above equation; for if $A = 1$ and $e = l$, E will equal P. These formulas apply only when the unit stress S or P/A is less than the ELASTIC LIMIT of the material. ϵ is an ABSTRACT NUMBER, because e and l are both linear quantities, and hence E is expressed in the same unit as S, that is, in POUNDS PER SQUARE INCH.

As an example of one method of determining the modulus of elasticity of any material the following illustration is given·

Suppose we have a bar of wrought iron, 2 in square and 10 ft long, securely fastened at one end, and to the other end we apply a tensile force of 40 000 lb. This force causes the bar to stretch, and by careful measurement we find the elongation to be 0.0414 in. As the bar is 10 ft, or 120 in long, if we divide 0.0414 by 120, we shall have the elongation of the bar per unit of length. Performing this operation, we have as the result, 0 00034 in. As the bar is 2 in square, the area of cross-section is 4 sq in, and hence the stress per square inch is 10 000 lb. Dividing 10 000 by 0 00034, we have, as the MODULUS OF ELASTICITY of the bar, 29 400 000 lb per sq in. This is the method generally employed to determine the MODULUS OF ELASTICITY of iron ties; but E can also be determined from the DEFLECTION of beams, and it is in that way that its values for most woods have been found. The modulus of elasticity is used in the determination of the STIFFNESS of beams.

The Moment of a Force with respect to an axis is the product obtained by multiplying the magnitude of the force by the shortest distance from the axis to its line of action The shortest distance is called the LEVER-ARM of the force. The moment of the force is the measure of the tendency of the force to cause ROTATION about the axis. (See Chapter VI and IX.)

The Center of Gravity of a body is the point in the body through which the RESULTANT of the forces exerted by gravity upon all the particles of the body passes. A body may be balanced upon a point placed above or below the center of gravity, because the RESULTANT of any number of forces may be held in equilibrium by an equal and opposite force. Another definition of the CENTER OF GRAVITY of a body or bodies is: a point such that there is NO TENDENCY TO ROTATION about any axis drawn through it. (For center of gravity of surfaces, lines and solids, see Chapter VI.)

2. Classification of the Principal Stresses Caused in Bodies by External Forces

Tension is the stress that resists the tendency of two forces acting away from each other to PULL APART two adjoining planes of a body.

Compression is the stress that resists the tendency of two forces acting toward each other to PUSH TOGETHER two adjoining planes of a body.

Shear is the stress that resists the tendency of two equal parallel forces acting in opposite directions to cause two adjoining planes of a body to SLIDE one on the other.

Torsion is the stress that resists the tendency of forces to TWIST a body.

Combined Stresses. Parts of structures are often acted upon by several external forces which develop stresses of different character, such as combined flexure and compression, flexure and tension, flexure and torsion, shear and axial compression or tension, torsion and compression, etc.

CHAPTER II

FOUNDATIONS

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1. Definition of the Word and Terms Used

Definitions. The word FOUNDATION is derived from the Latin verb FUNDARE, meaning to establish or lay the base, bottom, keel or foundation of anything. The English word is used in the broadest possible way to describe the base, physical or otherwise, on which anything is supported, and in technical language it may be used to describe any part of a structure on which a subsequent operation or construction is superimposed. Thus a plaster wall may be called the FOUNDATION for a fabric to be stretched thereon and the fabric in turn becomes the FOUNDATION for various coats of paint or other decorations. More specifically and in relation to a building or other complete structure the word FOUNDATION is unfortunately applied indiscriminately (1) to construction below grade, such as footing courses, cellar walls, etc., forming the lower section of the structure; (2) to the natural material, the particular part of the earth's surface on which the construction rests; and (3) to special construction such as piling or piers used to transmit the loads of the building to firm substrata. In view of the indefinite meaning of the word it is advisable to use it either to distinguish work below grade, or below the tier of beams nearest to grade, from work above grade. In even a still more restricted sense, it might include only the work below the cellar or basement-floor to rock or other solid foundation-bed.

The Foundation-Bed. The natural material on which the construction rests is called the FOUNDATION-BED. Walls, piers and columns below grade are called, in general, FOUNDATION WALLS, PIERS AND COLUMNS to distinguish them from similar construction above grade and occasionally those only below the basement-floor are so called; the lower portions of walls, piers or columns which are spread to provide a safe base will be called FOOTING COURSES.

2. General Requirements

The Object of Foundations. The object to be borne in mind in designing any foundation is to provide a safe and permanent base for the superstructure such that the movement of the base and of the superimposed structure shall be the least possible and shall result in the least possible damage to the structure. To fully meet the above requirements the design and construction should fulfill the following conditions:

(1) **The Materials of Construction** should be proof against all deteriorating influences, or, if any of the materials are liable to deterioration, they should be permanently protected.

(2) **Stresses and Future Changes.** No part of the foundation-structure should, under any combination of loadings, be stressed beyond safe limits, and the possibility of future additions or changes in the superstructure, or of a change in the use of the building, should be kept in mind.

(3) **The Load on the Natural Bed** should be kept within the safe limit for such material, under the worst conditions to which it may be exposed. In fixing this limit the amount of settlement allowable will in many cases determine the limit rather than the safe ultimate bearing capacity.

(4) **Adjoining Excavations.** The possible danger to the structure or to the stability of the foundation-bed from adjoining excavations or other disturbing causes should be guarded against as far as possible.

Physical Conditions of the Site. In order to meet the above requirements, the design should be suited to the physical conditions existing at the location. The architect or engineer should personally examine the site. He should secure all available information relative thereto and, if necessary, should make borings and tests so as to secure reliable information on which to base his design for the foundation. The first step is, therefore, a detailed and exhaustive study of the site to determine the characteristics of the foundation-bed on which the structure is to rest.

3. Geological Considerations

Character of the Foundation-Bed. A knowledge of geology is of material assistance in many cases in making a proper estimate of the character of the foundation-bed. While it is not proposed in the limits of this chapter to go into any general geological discussion the following notes may be of value in assisting the architect to determine whether any given deposits can be relied upon as affording a stable foundation-bed. Broadly speaking, as the location of the building may be in any part of the world, so the materials encountered may belong to any one of the many geological formations forming the surface of the earth. For practical purposes, however, the materials met with are roughly divided into ROCK, or materials other than rock, roughly defined as EARTH.

4. Composition and Classification of Rocks

Composition of Rocks. Rocks, and the earthy deposits derived from rocks, are composed of various minerals of which many hundred kinds are known, each varying from the others in some particular of chemical composition form of crystallization or other characteristic. A rock or an earthy deposit may consist almost entirely of a single mineral, but it is usually composed of several distinct minerals or of mixtures of minerals. The principal classes of rock-forming minerals are:

(1) **The Silica Minerals** composed of silica (SiO_2) in different forms;

(2) **Silicates** or combinations of silica, with various metallic bases;

(3) **Calcareous Minerals** composed of calcite or carbonate of lime ($CaCO_3$) and its combinations.

(1) **Silica Minerals** are different forms of the oxide of silicon, known as SILICA. In the crystalline state silica is known as :

Quartz. This is the most abundant of all minerals. Owing to its hardness and insolubility it resists decomposition and abrasion better than the minerals with which it is associated and grains of it form the principal constituent of sandy deposits. In finely comminuted particles it forms a part of most of the clays.

Flint, Chert, Agate, etc., are non-crystalline varieties of silica. SILICA also forms the cementing material in many sandstones and other rocks.

(2) **Silicates** or combinations of silica with various bases are second in importance only to quartz.

Feldspar, an important constituent of granite and other igneous rocks, is a silicate of alumina with potash, soda or lime. When exposed to the action of water it slowly decomposes, forming silicate of alumina, the base of clay. The decomposition of granite results in the formation of clay and crystals of quartz and mica. The mica is very slowly affected and the quartz is practically unchanged.

Mica. The various mica minerals are silicates of alumina, with potash and and other constituents. All varieties are soft and split into thin elastic plates. Small particles of mica are frequently found in sand.

Hornblende and Augite are silicates of lime, magnesia, iron and alumina and are of frequent occurrence.

Chlorite, Talc and Soapstone Travertine are hydrated silicates formed from other silicates by a chemical change in which a certain amount of water is absorbed These minerals are soft and have a SOAPY FEEL. Special care should be taken in building foundations on rock of this character to guard against any sliding on the foundation-bed or between parts of the foundation-bed.

(3) **Calcareous Minerals.** The following are the principal calcareous minerals:

Calcite ($CaCO_3$), carbonate of lime, when pure and crystallized, is known as ICELAND SPAR. It is soluble in water containing CO_2. Calcite in varying degrees of purity forms limestone and marbles. As a result of its solubility caverns and voids are frequently found in limestone

Dolomite is a carbonate of lime and magnesia. It forms the so-called DOLOMITIC LIMESTONES, which are less soluble than the calcite limestone.

Selenite, Lypsum, Alabaster, Anhydrite, Aragonite and Apatite are other and less important lime-minerals.

Classification of Rocks. Rocks are classified not only according to the minerals of which they are composed, but also according to the way in which they have been formed, as:

(1) **Igneous Rocks,** which have solidified from a molten condition;

(2) **Sedimentary Rocks,** which have been formed under water by mechanical pressure or by cementation due to chemical or organic processes;

(3) **Metamorphic or Plutonic Rocks,** which have changed from their original character as igneous or sedimentary rocks

(1) **Igneous or Plutonic Rocks** are not truly stratified. They may be granular, crystalline or glassy in texture. GRANITE, SYENITE, BASALT, TRAP, etc., are examples. LAVA, PUMICE and OBSIDIAN are volcanic products, as are also certain deposits of mud and ash. With the exception of volcanic ash and mud, the igneous rocks are enduring and are not liable to present any unforeseen weakness as foundation-beds.

(2) **Sedimentary Rocks** are composed of sand, clay and other materials resulting from the breaking down of the original igneous rocks. These materials were deposited in horizontal beds generally by settling from water, and the consolidation into rock was generally affected under water by chemical, mechanical or organic action. The resultant rock-masses are stratified as a result of their constituent materials having been deposited in layers. As sand and clay are the most abundant products of rock-decomposition, so the sedimentary rocks are most frequently SILICEOUS (sandy) or ARGILLACEOUS (clayey).

Sandstone is composed of grains of sand cemented together by silica, oxides of iron, or carbonate of lime. The durability of sandstone depends on the solubility of the cementing material. Carbonate of lime being soluble, sandstones containing it as cementing material yield to the weather and are not as reliable as sandstones having silica or iron oxide as cementing material.

Argillaceous Rocks contain clay with fine sand, mud, etc, and while SHALE and some other varieties are compact and hard when first uncovered, they are liable to deterioration when exposed to frost, water and other disintegrating agencies.

Limestone is composed more or less of carbonate of lime derived from the calcareous skeletons of marine animal and vegetable organisms. The character of limestone varies greatly. In so-called FOSSILIFEROUS LIMESTONES, fossils of shells or corals indicate clearly its origin, but in other limestones there are no fossils or other indications of the organic origin of the calcareous material. Admixtures of sand, clay, or other impurities may make it difficult to distinguish certain limestones from sandstones or shales.

Dolomite is a limestone containing a high percentage of magnesia.

Chalk is a soft limestone composed of the fine shells of minute marine organisms In general, the purer the limestone the more soluble it is and the greater the danger from fissures or caverns due to the action of water.

(3) **Metamorphic or Plutonic Rocks** are rocks which have been formed from sedimentary or igneous rocks by heat, compression, or moisture, acting alone or in combination. Thus by heat from a nearby intrusion of molten rock, limestone is changed into a crystalline marble. The general effect of METAMORPHISM is to produce a hard or durable rock.

Quartzite, a metamorphosed sandstone, is a crystalline rock of great hardness and durability.

Slate is a hard dense rock, sometimes with a well-defined tendency to split into thin plates. It has been formed by metamorphic action from clayey shales and is generally durable, but liable to slide along planes which are sometimes parallel to the cleavage, or along seams which are not parallel to the cleavage.

Gneiss is a "laminated metamorphic rock that usually corresponds mineralogically to some one of the plutonic types" * There are many varieties, best classified in accordance with the igneous rocks to which they most nearly correspond in composition. Some varieties resemble granite, but the laminated or striped aspect is generally characteristic. They are generally compact and durable.

Schists are similar to gneiss but are more finely foliated or striped. In MICA-SCHIST there are layers or foliations composed of fine grains or plates of mica. Mica-schists are liable to decomposition and it frequently happens that excavations have to be carried to great depths through decomposed rock of this character before solid rock is encountered. The material resulting from the decomposition of this rock contains fine grains of mica and other fine material and, when wet, acts as QUICKSAND.

Rock as a Foundation. All rock, if sound and not liable to slippage, is proverbially a solid foundation and capable of supporting any weight which a building is likely to impose on it. Care should be taken that rock liable to disintegration is protected from the weather, water-action, or other disintegrating influences.

* Kemp.

5. Geology of Earthy Material

Earth and Soil. Materials other than rock, resulting from the disintegration of rock-masses, are broadly classed as EARTH. The word SOIL, when used to designate any earthy material not rock, is a misnomer, in that the idea of FERTILITY, or the lack of it, is conveyed when the word SOIL is used.

The agencies producing the disintegration of the rock masses which form or underlie the entire surface of the earth, are various, but for the purpose of this chapter they may be defined as (1) CHEMICAL and (2) MECHANICAL.

(1) Chemical Agencies. By CHEMICAL ACTION OR DECOMPOSITION, a rock-mass of great strength and hardness and of complicated mineralogical structure may disintegrate into a noncoherent mass of elementary minerals. Thus a feldspathic granite under the combined action of water and varying temperature disintegrates, the crystals of feldspar changing chemically and forming the hydrated silicate of aluminum known as CLAY, while the crystals of quartz, mica or hornblende, being more resistant to chemical action, retain their chemical identity but become detached particles of SAND.

(2) Mechanical Agencies. By the MECHANICAL AGENCIES, such as the action of frost, moving water or ice, fragments of rock are detached from the ledge of which they originally formed part and are subsequently transported, by the action of glaciers or streams, or by the wave-action in bodies of water. The attrition between the materials thus roughly thrown about breaks up the rock-masses into smaller and smaller pieces without altering the composition of the rock-material.

Flowing Water. As flowing water more readily transports small particles than large ones, the larger pieces of rock move intermittently during periods of storm or flood and are deposited as soon as the velocity of the water falls; while the smaller particles are held in suspension longer and, as the velocity of the stream falls, are deposited in the order of their size, the largest first. The rapid upper courses of streams and rivers in mountainous regions constantly roll and grind together the materials in their rocky beds, the heavy masses being moved slowly. The attrition between the fragments forms GRAVEL and SAND which are washed down stream to be deposited, as the current slackens, first as BEDS OF GRAVEL, then as SAND-BARS, and finally, in the slow-moving lower levels, as BEDS OF SILT and ALLUVIUM.

Glaciers and Glacial Deposits. The action of glaciers is similar to the action of streams. Glacial deposits, the so-called GLACIAL DRIFTS, are composed of sand, clay, gravel and boulders but, in general, there is a noticeable difference between glacial deposits and deposits made by rivers or streams. In glacial deposits the boulders frequently exhibit groovings or scratches on their faces and the edges and surfaces of the boulders are generally sharp, so that a boulder may appear as if it had been recently fractured. They rarely exhibit the smooth, water-worn and rounded surfaces found on boulders formed by water-action. Moreover, the glacial boulders may be found singly, or unassociated with other boulders in a deposit of sand or gravel. The deposit differs from a river-deposit in that there is no classification as to size; the boulders may occur on the surface or may be disseminated as if by accident through the sand and gravel forming the body of the deposit. Such glacial deposits partake of the character of a rough artificial fill without the stratification or classification as to size which is characteristic of river-deposits. In glacial moraines or dumping grounds it not infrequently happens that the surface-water finds underground passages forming so-called SINK-HOLES. A line of glacial deposits extends across the continent of North America from

Long Island westward. The southern limits can be determined by reference to geological maps.

Glacial and River-Deposits Distinguished. It is important to distinguish between GLACIAL and RIVER-DEPOSITS, because, while the occurrence of glacial boulders gives, in general, little or no information as to the character and value of the surrounding deposits, the occurrence of boulders, on the other hand, in river-deposits is generally an indication that the bed of which they form a part has been thoroughly consolidated as a result of the river-action which formed it; and, also, because such deposits generally extend down to rock or to some compact material which at the time the deposit was made was capable of resisting the action of rapidly flowing water.

Wave-Action on Lakes and Along Coast-Lines is constantly working on the materials composing the beach. Rock-masses are broken away from cliffs and ground together, producing boulders, gravel and sand. The sand, being carried more readily by the tidal currents, is deposited in the more sheltered locations and forms BEACHES, while the larger rock-masses remain near the point of origin in BARS and REEFS.

Beds of Sand, Gravel and Boulders deposited by the action of waves on the SHORES OF SEAS OR LAKES are not necessarily constant in character and tests should be made to determine the character of the material underlying such BEACH-FORMATIONS. In large river-valleys where the general formation is composed of silt or other fine material little reliance should be placed on the occurrence of BEDS OF GRAVEL, even if such beds extend over large areas. Tests should be made to determine that such beds are not underlain by less trustworthy materials. Where tributary streams discharge into large valleys they may deposit BARS OF SAND, GRAVEL and BOULDERS on top of the silt, peat, or other materials formerly deposited by the main river. The general topographical conditions should serve as an indication of danger in such cases.

Results of Chemical and Mechanical Action. As a result of the foregoing brief description of the agencies at work it may be seen that ICE, WAVE and STREAM-ACTION alike tend to disrupt rock-masses and to produce boulders, gravel, sand and finer materials. The ultimate result of the combination of CHEMICAL ACTION and MECHANICAL ACTION is to reduce the hardest rocks to the finest sand, the most impalpable clays, silts and muds; and the ACTION OF WIND, WAVE and MOVING WATER is to classify such materials in deposits of grains of uniform size.

6. Materials Composing Foundation-Beds

Classification and Definitions. The following list includes the materials which are most frequently encountered, with their definitions:

Rock (solid rock, bed-rock, or ledge). Undisturbed rock-masses forming an undisturbed part of the original rock-formation.

Decayed Rock (rotten rock). Sand, clays and other materials resulting from the disintegration of rock-masses, lacking the coherent qualities but occupying the space formerly occupied by the original rock.

Loose Rock. Rock-masses detached from the ledge of which they originally formed a part.

Boulders. Detached rock-masses larger than gravel, generally rounded and worn as a result of having been transported by water or ice a considerable distance from the ledges of which they originally formed a part.

Gravel. Detached rock-particles, generally water-worn, rounded and intermediate in size between sand-particles and boulders.

Sand. Noncoherent rock-particles smaller than ¼ in in maximum dimension.

Clay. The material resulting from the decomposition and hydration of feldspathic rocks, being hydrated silicate of alumina, generally mixed with powdered feldspar, quartz and other materials.

Hard-Pan. Any strongly coherent mixture of clay or other cementing material with sand, gravel, or boulders.

Silt. A finely divided earthy material deposited from running water.

Mud. Finely divided earthy material generally containing vegetable matter and deposited from still or slowly moving water.

Dirt. Loosely used to describe any earthy material.

Soil. Earthy material capable of supporting vegetable life and generally limited to material containing decayed vegetable or animal matter.

Mould. Earthy material containing a large proportion of humus or vegetable matter.

Loam. Earthy material containing a proportion of vegetable matter.

Peat. Compressed and partially carbonized vegetable matter.

7. Characteristics of the Materials of Foundation-Beds

Solid Rock, or, as it is locally known, BED-ROCK, or LEDGE, is proverbially a solid foundation. The harder rocks, such as granite, trap, slate, sandstone, limestone, etc., are all capable of carrying the load of any ordinary structure. The softer rocks, among which may be classed the shales, shaley slates and certain marley limestones and clay stones, should not be loaded with more than 15 tons per sq ft unless they are tested for greater loads. In all cases where foundations are to be placed on what is supposed to be solid rock, care should be taken to determine whether or not the supposed solid consists of a detached portion and, also, in case the bedding-planes of the rock are inclined, if there is danger from a slip of the layer forming the foundation-bed.

Decayed Rock. Certain igneous or metamorphic rocks such as granites, gneisses, etc., frequently disintegrate, forming so-called ROTTEN ROCK or DECAYED ROCK. The decayed rock is generally found in conformity with the ledge of which it originally formed a part. It may retain the stratification, color and markings of the solid rock, but as a result of the disintegrating effect of water or other agents, it has lost the solid character of the original rock. When struck with a hammer it does not give the characteristic ringing sound of solid rock. It may be fairly compact and hard, or so soft as to be readily excavated with pick and shovel. The amount of such disintegrated rock overlying the solid rock varies greatly; in some cases the removal of a few inches will disclose the solid rock, in other cases the layer of decayed rock may be many feet in thickness. Test-borings in rotten rock give samples similar to the samples from solid rock; so that it frequently happens that while the foundations are planned for solid rock the excavations disclose a thick layer of rotten rock. In such cases, if it is impracticable to carry the footings down to solid rock, it may be necessary to increase the size of the footings or to adopt some other expedient.

Loose Rock. Where a rock-mass detached from the ledge of which it originally formed a part is encountered it must not be loaded in excess of the safe capacity of the material by which it is surrounded. If the voids between adjoining pieces of loose rock are completely filled in with hard-pan, compact

gravel, sand, or clay, the loading may be the same as for the filling-in material, but care should be taken to determine that no voids exist. In natural rock-fills, as in artificial rock-fills, it may happen that large voids exist between the rock-masses, forming passageways for streams of water, in which case there is extreme danger of settlements.

Boulders, Gravel and Sand. Boulders are rock-masses which have been transported by water or ice-action. Boulders are sometimes found disseminated through sand and clay and in such cases the load should be limited by the safe load of the material in which they are found. At other times boulders are found in beds, packed closely together, with the interstices filled in with gravel, sand, or clay. In such cases it is usually safe to assume that no further consolidation of the mass is likely to take place. If the bed of boulders extends to rock, they will safely sustain any load which will not crush them.

Gravel. The name GRAVEL is given to rock particles larger than sand and smaller than the rock-masses known as BOULDERS. If compact, and if no underlying bed of poorer material exists, gravel forms a most desirable foundation-bed, equal to sand or boulders in supporting power and not as liable to be disturbed by adjoining excavations or pumping operations. If cemented it may partake of the quality of hard-pan or rock. Care, however, should be taken to determine whether or not the bed of gravel has been deposited over a layer of silt or quicksand. It is possible for this dangerous condition to exist.

Sand. Sand is composed of comminuted rock-material. As quartz is the most abundant rock-mineral and as its hardness and insolubility make it highly resistant to disintegrating action, it will be found to be the principal constituent of most deposits of sand or sandy material. Grains of mica, feldspar, garnet and other minerals are frequently found. Sand is described as being FINE, MEDIUM, or COARSE, according to the size of the grains of which it is composed.

Coarse Sand may contain particles of gravel, but after eliminating all particles which will not pass a screen with 4 meshes to the inch it will be found that a large proportion of the remaining material is too coarse to pass a 40-mesh sieve.

Fine Sand, on the other hand, may contain no particles which will not pass a 20-mesh sieve, and a considerable proportion which will pass a 100-mesh sieve.

Very Fine Sand is frequently mistaken for clay and, indeed, generally does contain some clay, as clay generally contains fine sand.

Uniform Sand is sand in which there is relatively a small variation in the size of the particles.

Balanced Sand is sand in which the size of the particles varies from large to small and in which there is no great difference in the numbers of particles of each size.

Clean Sand contains no clay or loam, but a pure sand containing a large percentage of fine particles is often considered to be NOT CLEAN.

Sharp Sand is clean sand containing coarse, angular grains. When firmly grasped in the hand it gives a NOTE, due to the particles slipping over each other. Sharp sand is generally esteemed for use in mortar, although it requires more cement to fill the voids and, in the writer's opinion, is not as desirable as a clean, rounded sand.

Rounded or Buckshot Sand is composed of rounded grains not cemented together.

Quicksand. This term is popularly used to describe any fine sand, or mixture of fine sand and clay, which, when wet, forms a soft, unstable material. In the popular mind quicksand is supposed to have some mysterious and peculiar qualities which result in a tendency to FLOW LIKE WATER and to SUCK IN animate and inanimate objects. These manifestations are connected with various theories as to the composition of quicksand, some persons insisting that quicksand must contain flakes of mica or some slippery mineral, others that the particles must be extremely fine or spherical in shape, while others contend that there must be a certain proportion of fine clay with the sand The fact is that any uncemented sand, when subjected to the action of moving water, will move and that any sand moving as the result of the action of water becomes a quicksand The finer the sand the more readily it is affected by a current of water, so that fine sands are more troublesome than coarse sands. A coarse sand, having large voids, permits the flow of a certain amount of water through them; if this flow has not sufficient velocity to disturb the particles of the sand, the sand can be drained without moving it. In a fine sand, having very small voids, a similar flow of water will cause the whole mass to move and there is great difficulty in draining it without producing a current sufficient to cause it to move or flow.

Excavations in Quicksand are made difficult by the tendency of the sand forming the sides of the excavation to flow into the excavation; and even if the sides of the excavation are protected, it not infrequently happens that the bottom of the excavation will LIFT, that is, there will be a movement of material from points outside of the line into the excavation, the movement in general following a curved line, and carrying the sand under the protected side walls of the excavation. In such cases some advantage may be gained by surrounding the excavation with driven wells or well-points and draining the soil by continued pumping through sand, in other cases, wooden or steel sheeting may be driven to a point below the depth to which the excavation is to be carried, or to some underlying layer of impervious material, in which case the sheeting will act as a coffer-dam to cut off the flow of material. Such sheeting, however, must be practically watertight, as extremely fine sand, when in the condition of quicksand, will flow through very small apertures.

Quicksand as a Foundation-Bed is objectionable on account of the danger of its moving or flowing, in case it finds any outlet such as would be afforded by an adjoining excavation. Cases are known where excavations have permitted the escape of quicksand and resulted in the settlement of buildings at a very considerable distance. Such settlements have occurred not only when the footings themselves rested on quicksand, but also when they were on a stratum of coarse sand, gravel or clay of good quality which rested on an underlying stratum of quicksand.

Pockets of Quicksand. It frequently happens that pockets of fine sand are found in deposits of mixed character. Where such pockets are small in extent the fine sand may be removed and the spaces filled with concrete. Where the pockets are larger it may be necessary to carry piers through them to a better foundation-bed, to drive piles, or to resort to other expedients.

Fine Dry Sand is readily converted into quicksand by the addition of water, which fact should be carefully borne in mind in considering the load on fine sand, as a material which in dry weather is apparently safe, may be, in wet weather, an extremely dangerous one. It is frequently stated that confined quicksand is a perfectly reliable material on which to found a building. While this, as a theory, cannot be controverted, it is a dangerous assumption

to act on because of the impossibility of providing that the fine sand shall be always confined.

Variation in the Size of Grains of Sand. The accompanying diagram (Fig. 1) shows graphically the results of sieve-tests on characteristic sands. The dash-line curve (1) is an average, giving the results of sieve-tests on several so-called quicksands; the full-line curve (2) gives the result of sieve-tests

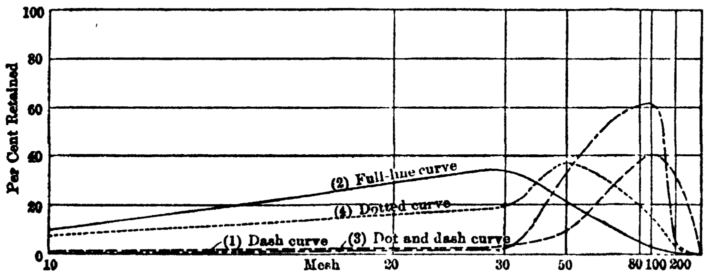


Fig. 1. Graphical Illustration of Results of Sieve-tests on Sands

on a natural sand which would be classed as a good building sand; the dot and-dash curve (3) gives the result of sieve-tests on a fine beach sand remarkable for the uniformity of the size of its grains For purposes of comparison and in order to show the variation in sands which appear to be substantially the same, the dotted curve (4) has been added. This shows the result of tests on a bank sand apparently as coarse as sand (2), but containing a much larger percentage of fine particles between 0 015 and 0.0055 in in diameter. Fine sand frequently contains a considerable proportion of clay. A chemical analysis of a so-called QUICKSAND from the down-town section of New York City, reported on to the writer by Dr. C. F. McKenna, is as follows:

Silica	73.76%
Alumina and oxide of iron	18 52%
Lime	1 60%
Magnesia	1.48%
Loss on ignition	2.26%

A rational analysis shows the following composition:

Quartz, as given	39 38%
Clay and mica, as given	23.94%
Feldspathic detritus	36.68%

On the other hand, a sample of extremely fine sand from Michigan, of which 75% passed a 200-mesh sieve, appears to be absolutely pure quartz.

Clay. When pure, clay consists of hydrated silica of alumina, the product of decomposition of feldspar. Ordinarily, various impurities are mixed with the clay, so that, in general, clay may be considered a mixture of hydrated silica of alumina with other finely divided minerals. Mixtures of clay and sand are found, varying from beds of nearly pure clay to beds of nearly pure sand, and no definite classification can be made.

The Effect of Moisture on Clay. Clay as generally found in excavations is in a plastic condition due to the presence of moisture, the amount of water

present varying greatly. On drying, the clay shrinks in volume and loses its plasticity, becoming a firm and coherent mass resembling in consistency a sun-dried brick. Large masses of clay are liable to crack into a number of small fragments during the process of drying, as the result of the shrinkage in volume. When these lumps are crushed or ground the clay becomes an extremely fine or impalpable powder. The loss in volume due to the change in the condition of the clay from a moist, plastic state to a thoroughly air-dried condition may amount to from 10% to 20% of the original volume. Compact, moist clay is impervious to water in the sense that water cannot pass through it as it would through porous sand; but when clay is exposed to water the clay gradually absorbs the water, so that eventually the entire mass becomes saturated and softened by the water.

Clay as a Foundation-Bed. Clay is not a reliable material on which to found a building; first, because of the PLASTICITY of the clay when wet, and secondly, because of its TENDENCY TO SHRINK on losing its contained moisture. The plasticity of clay increases with the percentage of contained water, so that a firm, hard clay may be converted into a liquid puddle by being agitated in the presence of a sufficient amount of water. The plasticity is also increased by pressure, as is shown by the action of clay in a brick-machine. Clay, in a foundation-bed under moderate pressure imposed on it by the footing of a structure, frequently develops this QUALITY OF PLASTICITY, the clay moving out from beneath the footing and causing serious settlements and displacements of the footings. This movement of the clay may be a local movement, as referred to each footing, in which case the clay flows from beneath the footing laterally toward the side and then upward, causing the surface of the adjacent material to rise and to form so-called BULGES or WAVES. If this motion is uniform from the center toward the sides, the footing may settle vertically, but more frequently the movement will not be symmetrical and the footing will settle more on one side than on the other. Such movements of the clay may be reduced or prevented in some cases by the simple device of loading the surrounding soil, as, for example, by a concrete floor.

Movements of Clay Foundation-Beds. The movement of the clay may be on a larger scale, amounting to a general flow of the clay underlying the entire building toward some point where the pressure on the clay is less than the pressure resulting from the load of the building. Such general movements are more likely to happen if the building is located on the side of a hill, so that the clay finds some outlet at a point below the level of the footings. It frequently happens that adjoining excavations cause settlements to buildings at a considerable distance, by affording an outlet to a bed of clay. As noted elsewhere, beds of clay resting on inclined strata of rock or other material are liable to move downward, sometimes with a slow, almost imperceptible movement, and at other times forming landslides of greater or less magnitude.

Protection of Clay Foundation-Beds. Where the foundation-bed is clay, or sand with a considerable amount of clay, it is advisable to protect it from water-action, so far as is possible, by a system of drains surrounding the site of the building and by diverting the surface-water from the building. Care should be taken in back-filling around exterior walls to prevent any accumulation of water which might affect the material under the footing. The neglect of such precaution has frequently results in serious settlements during, or immediately after, construction.

Mud, Silt, Peat and Other Unstable Materials. When the site of a structure is in a marsh or on materials which are not capable of affording a safe foundation, the only alternative is to resort to the use of piles, piers, or coffer-

dams sunk to an underlying and firmer strata. Such constructions will be described in subsequent divisions of this chapter.

Filled Ground. All artificial fills and some natural fills are liable to a more or less uniform but continuous settlement or shrinkage due to the gradual consolidation of the material of which the fill is composed. Where the fill is of solid rock this consolidation may amount to little, but where the fill is of earth, and especially where it is of mixed materials, the shrinkage will not only be large in amount but will continue for a very long period. For example, where dirt has been thrown on top of a rock-fill each rain-storm will wash some of the dirt into the voids in the rock-fill, and this action will be continuous until all of the voids are filled in Any vegetable matter, or other matter liable to decay and shrinkage in volume, will increase the total shrinkage of the mass. Certain natural deposits, such as beds of peat or soils containing vegetable matter, are apt to shrink in volume from the same causes. When it is necessary to found a building on such material it is inevitable that the footings will settle with the mass, notwithstanding that the unit load on the foundation-bed is so small as to be negligible. In such cases the settlements may be vertical and uniform; but if the depth of the fill under one part of the building is greater than the depth under another part, the settlements will not be uniform, as the shrinkage in the fill will, in general, be in proportion to the depth of the fill. No important building should be founded on such material and, wherever possible, the footings should be carried down through the filled-in material to some more reliable underlying stratum.

8. Allowable Loads on Materials of Foundation-Beds

General Considerations. Owing to the infinite number of variations in the materials encountered and the conditions affecting the reliability of such materials, no general or definite rule can be given, and every case should be carefully investigated before determining the allowable unit load on the foundation-bed. If the material and conditions are uniform over the entire site of the building a uniform unit load may be used, but in practice it is frequently found that entirely different conditions exist under different portions of the same building and in such cases great care must be exercised in determining the unit loads. For instance, one section of a building may rest on rock and another section on a light compressible soil or on a clay of doubtful stability. In such cases the unit load on the compressible soil or on the clay must be reduced as much as possible so as to reduce the differences in settlements between the two sections of the building to a minimum. If the entire building were on a compressible soil a very considerable settlement might be allowable, provided it was uniform; but in this particular case it is known beforehand that the part of the building on rock will not settle at all and that any settlements of other parts of the building must be considered as unequal settlements, and, as such, liable to produce cracks and distortions in the building. It is also important to remember that a certain unit load on compressible soil may be safe, in that the soil will ultimately safely support that load; but the use of that load would nevertheless be inadvisable on account of the excessive settlements. In this connection it may be said that a considerable settlement, if uniform, in a detached building may be a matter of no importance; but that where a building is to be constructed in contact with adjoining buildings or where additions are to be made to an existing building, the total amount of settlement becomes a matter of prime importance. These and other considerations, such as the character of the proposed building and of the material composing it, should be borne in mind in selecting

the unit load for any given foundation-bed, irrespective of the allowed pressure as given by building codes or by examples quoted in this chapter.

Safe Loads on Rock. The safe unit load on rock may often amount to more than the crushing strength of brickwork or stone masonry, and in nearly all cases any material worthy of the name of rock is capable of supporting from 15 to 40 tons per sq ft.

Safe Loads on Sand, Gravel and Boulders. When compact and confined laterally these materials are capable of supporting 10 tons per sq ft without appreciable settlement. It rarely happens, however, that it is advisable to load such materials with more than 5 tons per square foot.

Safe Loads on Loose Sand. By loose sand is meant sand which has not been thoroughly compacted and which may settle by its own weight independently of a superimposed load. All such materials should be tested and the unit load reduced in accordance with the result of such tests.

Loads on Fine Sand or Quicksand. It is probable that fine sand, if absolutely confined, will sustain as heavy a load as coarse sand, but in view of the fact that if afforded the slightest opportunity it is liable to lateral displacement, it is inadvisable to found any structure on such material When it is imperative to place the footings on such material the unit load should be reduced as much as possible and preferably to less than 2 tons per sq ft, and great care should be taken to connect all footings with a continuous layer of concrete so as to prevent any flow of material into the cellar-excavation. Care should be taken, also, that any sumps, pump-pits, drainage-arrangements and sewer-connections for the building do not permit the escape of any quicksand.

Safe Loads on Hard-pan and certain cemented sands partaking of the nature of hard-pan may approximate rock in hardness and reliability. Such materials, however, are liable to soften if exposed to water. If these materials, when uncovered, are dry, experiments should be made to determine how they behave when wet, and if the level of the water in the ground is liable to change so as to reach the layer of hard-pan, the load should be correspondingly reduced. Cemented hard-pan containing gravel has been frequently loaded with more than 10 tons per sq ft. Care should be taken, however, to determine that the layer of hard-pan is continuous to a solid substratum, as it frequently happens that layers of hard-pan and fine sand or clay are deposited alternately.

Safe Loads on Clay. Ordinary clay should not be loaded with more than 2 tons per sq ft. If soft and plastic, a load of 2 tons per sq ft may produce inadmissible settlements. Clay containing so large a percentage of sand as to lose its plasticity has been loaded with from 4 to 6 tons per sq ft without undue settlements, and sand or gravel containing sufficient clay to act as a cementing material will partake of the qualities of hard-pan. In general, however, clay is the most dangerous of all the materials on which structures are founded and the unit load should be reduced to a minimum and every precaution taken to prevent the flow of material. Undue reliance should not be placed upon loading-tests of clayey soils. It is probable that a loading on a large area which will produce a movement of the clay will on a small area have no effect, so that it is unsafe to rely upon the results of a test-load applied to an area smaller than the actual supporting area to be used From the experience gained in the construction of large buildings in Albany and Chicago which were FLOATED on clay, the allowable unit load has been generally reduced to 2 tons per sq ft and, in the writer's experience, a load of less than 2 tons per sq ft on clay has produced settlements varying from nothing to 12 in.

9. Unit Loads on Foundation-Beds Allowed by Building Codes

Variations in Building Codes. Table I gives an outline of the requirements of different cities as to the allowable unit loads on different materials, as contained in their respective BUILDING CODES or REGULATIONS. While the allowed loads given may in some cases be based upon actual experience in the respective localities, it is more likely that they are based upon the individual experience of the authors of the codes, or are copied from other codes. The architect should, therefore, not place too much reliance on the unit loads allowed by the codes, but should investigate each case and determine for himself the proper allowance to be made.

Special Requirements of Some Building Codes. The New Orleans code limits the maximum load to 1 400 lb per sq ft, the entire city being on an alluvial-delta formation.

The Buffalo code limits the load on SOIL to 3½ tons per sq ft; if the SOIL is other than hard clay or gravel the supporting areas "shall be extended as directed."

The Cincinnati code limits the load on soils INFERIOR to those listed, to 1 ton per sq ft.

10. Investigation of the Site

General Considerations. To determine the character of the materials which will be encountered at the level of a foundation-bed, the architect should first get as definite information as possible from others as to their experience in making excavations and erecting buildings in that vicinity. In some localities the subsoil conditions are uniform over large areas, while in other localities important variations may occur within the limits of a city lot. Abrupt changes in surface-topography, changes in the character of the surface-soil or in the native vegetation, proximity to old or existing water-courses are suggestive of subsurface irregularities. In such cases, and in all cases where there is any doubt as to subsurface conditions, a sufficient number of exploratory borings or testpits should be made to determine the facts. This exploratory work should go below the level of the proposed footings, should determine the ground-water level and insure that no unsuspected layer of quicksand or other unsuitable material underlies the foundation-bed. The methods in use for such explorations are as follows:

Testing in an Open Pit. For shallow work an open pit is the most satisfactory method as it allows actual inspection of the undisturbed material over a considerable area. If the excavation is in firm material, no sheet-piling or other protection may be required; but if in flowing material, or if carried deeper than adjoining footings, timber sheeting or steel sheeting should be employed. If the excavation is carried no deeper than the proposed footing-level, the underlying material should be tested by one of the methods hereinafter described.

Testing with Steel Bars. A steel bar with a pointed end or a steel pipe provided with a steel point is driven to the required depth by a maul or by a falling weight. While no samples can be obtained by this crude method, it may determine the ground-water level, and a little practice will enable one to distinguish sandy from clayey soils by the sound given out when the bar is twisted. The difficulty of driving is a rough index of the degree of the compressibility of the soil. It should be remembered, however, that any dry material will afford considerable resistance to the bar and that a small boulder

Table I. Loads in Tons per Square Foot on Foundation-Beds Allowed by Building Codes

Character of foundation-bed Loads in tons per sq ft	Akron, 1920	Atlanta, 1911	Boston, 1926	Chicago, 1924	Cleveland, 1927	Denver, 1927	Louisville, 1923	Minneapolis, 1911	Newark, 1924	New York, 1922	Pacific Coast, 1928	Philadelphia, 1929	Portland, Ore., 1924	Richmond, 1908–12	St. Louis, 1917	St. Paul, 1910	Jacksonville, 1922	Prop. N. Y. Code, 1929
Quicksand or alluvial soil	½				½													
Adobe											½							
Soft or wet clay, at least 15′ thick	1	1		1¼	2			1	1	1		1	1	1			1	1
Soft clay, sand loam or silt											1							
Soft clay and wet sand	1½				1½											1		
Sand and clay mixed or in layers		2		1½				2	2	2	2		2	2		2	2	2
Firm clay									2	2								2
Wet sand									2	2								
Dry clay and sand	2				2											3		
Fine wet sand	2				2							2						2
Soft clay held against displacement			2															
Clay in thick beds, mod dry						2-4							3					
Dry, solid clay				2¼											2½		3	
Loam clay or fine sand, firm and dry							2½	3										
Clean sand				2½														
Firm dry loam	2½	2-3				1-2												
Firm dry sand	3	2-3				2-4			3	3							3	3
Quicksand when drained					3													
Hard clay		3-4			3		4	4								4		
Fine grained wet sand			3											4				
Moderately dry sand											3							
Moderately dry sand and clay											3		3					
Clay or fine sand, firm and dry														3				
Very firm, coarse sand		3-4				4-6	4	4	4		4	4	4	4		4	4	4
Gravel		3-4					4	4	6	6				4			4	[illegible]
Dry hard clay										4	4	4	4					
Clay in thick beds always dry	4					4-6												
Stratified stone and clay																		
Fine dry clay		2-3							4									
Plastic clay, mixed or unmixed with sand			4															
Fine grained dry sand			4		4							4						
Compact coarse sand and gravel																	4	
Gravel and coarse sand in thick beds	5				8							6						
Wet or dry, med or coarse sand			5							4								
Hard blue clay—sand mix			5															
Firm coarse sand and gravel																6		
Gravel, compact sand and hard yellow clay			6			8-10						5						
Gravel or coarse sand well cemented											6		8					
Hard-pan									10	10								10
Hard shale, unexposed	6				6													
Shale and hard-pan			10									8						
Disintegrated ledge rock			5															
Soft rock									8	8								8
Medium rock									15	15		24						25
Rock	10	15	100		10	10-200			40	40	Rock unit 20% of crushing strength	48	16					40

will stop it; so that not much reliance can be placed on a report that the BAR DROVE HARD or that it REACHED ROCK.

Testing with Post-Hole Diggers. For shallow explorations in easily excavated material, the ordinary post-hole digger used for fence-posts, or the longer and larger ones used for telegraph-poles, can be used to depths of from 6 to 8 ft.

Testing with Augers. In clay or similar material a single or double-twist, carpenter's auger welded to a long rod, or the so-called POD-AUGER may give satisfactory samples. In gravel or loose and sandy material, the sides of the hole fall in, clogging the operation and destroying the samples.

Testing by Dry-Pipe Borings. A POD-AUGER or the above-described CARPENTER'S AUGER can be used inside a casing-pipe. The pipe should be driven so as to keep close to the bottom of the hole made by the auger. The pipe prevents the material falling from the sides of the hole and the auger excavates and loosens the material ahead of the pipe and facilitates driving. The above methods are not generally successful for deep holes or where gravel, boulders or compact material interferes with driving the pipe.

Testing with Wash-Pipes. For test-borings over 10 ft in depth the method in most frequent use is the WASH-PIPE METHOD. In this method a wrought-iron or steel pipe known as the CASING-PIPE or DRIVE-PIPE is driven into the earth in much the same way as in the DRY-PIPE METHOD, but the driving of the pipe is facilitated by the use of a JET OF WATER. The lower end of the casing-pipe is provided with a hollow SHOE or reinforcement, slightly larger in outside diameter than the casing. This serves to protect the pipe from injury in driving through gravel or hard-pan, and forms a hole slightly larger than the diameter of the casing. The upper end of the drive-pipe is protected from injury by an annular drive-head which has a threaded part fitting the thread on the casing-pipe and a central hole to admit the jet-pipe. The jet-pipe is small enough to permit it to freely enter the casing-pipe. The lower end is contracted so as to produce a jet-action. The upper end is connected with a water-supply which must be under considerable pressure. The driving-mechanism consists of a cast-iron weight with a central vertical hole large enough to admit the wash-pipe, and stationary verticals supporting a BLOCK-AND-FALL and an arrangement which releases the weight when it has reached a predetermined height. With this arrangement, water is continuously pumped through the jet-pipe, the length of which is regulated so that the jet-action loosens the material immediately below or AHEAD of the casing. Some of the jetting water returns to the surface outside of the casing and thus lubricates the surface in contact with the outside material. Another part of the water returns to the surface in the annular space between the wash-pipe and the casing, carrying with it particles of the material loosened by the jet. As the jet loosens and washes away the material immediately below the casing, the latter is driven deeper by repeated blows of the ram, the driving and washing being carried on at the same time. The operation is thus continuous until the top of the casing comes close to the surface of the ground, when the hammer drive-head and hose-connection are removed to permit additional lengths of pipe to be added to the casing and wash-pipes, after which the hose-connection, drive-head and hammer are replaced and the operation is resumed.

Borings can be made by this method to great depths in sand, clay or other suitable material. Samples of the material encountered are obtained by settlement from the water returning between the jet-pipe and wash-pipe. These

samples are not accurate samples as the water separates the materials. The finer particles do not settle readily and the large and heavy particles may not be brought up at all. It is evident that such samples do not give any index as to the solidity of the original deposits. If large gravel, hard-pan or boulders are encountered there will be great difficulty in forcing the casing past such obstructions. In such cases a DRILL-ROD is sometimes substituted for the JET and the obstruction broken up into small pieces or pushed to one side; but in either case it is difficult to get any sample or real indication of the character of the obstruction. If solid rock or large boulders are encountered, no further progress can be made with the casing and no sample can be obtained by this method. Resort must then be had to one of the CORE-BORING METHODS described hereafter, to determine the character of the obstruction encountered.

Testing by Core-Borings. These borings can be made through rock or boulders and accurate samples obtained In all core-boring methods the hole is made by rotating a pipe-like tool which makes an annular cut in the rock and leaves a cylindrical core which is afterwards detached and brought to the surface by a gripping-tool called the CORE-LIFTER. The cutting is done in different ways

Diamond Bits are annular rings fitted on the lower end of the hollow pipe used as the rotating drill-rod and furnished with a number of small diamonds arranged so as to form cutting-edges, which, when rotated in contact with the rock, gradually wear away the required annular space. The diamonds employed are known as BORT, BLACK DIAMONDS, or CARBONS, and their only resemblance to the stones used by jewelers is the necessary hardness The carbons are skillfully secured in a soft metal bed. in sockets drilled in the bit, and they project below the bit and also sufficiently inside and outside to insure the cutting of a groove large enough to provide clearance for the bit and the attached drill-rod or pipe.

Shot-Drills. The same result is arrived at by the SHOT-DRILL METHOD, by which particles of chilled cast iron called SHOT are used as the abrasive or cutting-agent. The shot is poured loose into the hole and forced against the rock by the rotating bit.

Efficiency of Drill-Methods. Both of the drill-methods mentioned are expensive, but as they are the only methods which will give an accurate sample in rock, one or the other must be employed where the accurate determination of rock is necessary. If the core corresponds to the known underlying rock-formation and the rock is continuous for a length of from 8 to 20 ft, it is safe to assume that solid rock has been reached. If, however, the core is of different rock from the known underlying formation, the probability is that a boulder has been encountered. If the core is not continuous it may indicate that there are seams in the rock or that there are detached rock-masses. The above-described methods are used after the overlying earth has been penetrated by one of the PIPE-SINKING METHODS previously described.

The Results of Pipe-Borings are frequently misleading and misinterpreted, and great care should be taken to compare the samples with samples obtained from other borings where the exact character of the materials tested is known.

11. Loading-Tests

General Considerations. Loading-tests of the materials forming the foundation-bed are made to assist in determining its safe bearing capacity. It is not known to what extent the supporting power of a given soil varies with the

area subjected to the unit load, and tests on small areas are not a safe guide for the safe load on large areas. Moreover, the underlying material is not tested, except by a very small increment of load. On account of the expense involved, tests on large areas are rarely made, the usual test being on an area of about 1 sq ft. The test should be made on an undisturbed portion of the foundation-bed, leveled to receive the test-load, and for a space around the area tested, so that the adjoining material is not reinforced or SURCHARGED by a bank of unexcavated material. The load should be applied with the least possible jar or movement of the surface in contact with the material of the foundation-bed.

Explanation of Methods. A convenient arrangement for this purpose consists of a vertical timber or post carrying a platform to receive the test-load, and having four horizontal guys at the top to keep the post in a vertical position. The bottom of the post, forming the loading-area, should be approximately 12 by 12 in and its exact area should be known. The platform, sufficiently strong to support the load to be applied, should be concentric with the post and as close to the bottom of the post as practicable. The load may be pig iron, cement or sand in bags, or any other convenient material. The guys should be not less than four in number, should be attached to the top of the post and should lead horizontally so as not to pull up or down on it. Levels should be read to a point on the post above or below the load, as may be most convenient. The load should be applied gradually and with the least possible jar, care being taken, also, to keep the loading uniform on opposite sides of the post, which should be always vertical. Levels should be taken at frequent intervals during the application of the load. The level observed when the platform is first in position may be taken as zero and successive settlements referred to it. When the proposed unit load has been reached, no additional load should be added until no further settlement is observed. After this, first 50 and later 100% overload may be added and the total and periodic settlements observed. If the settlement under a test-load of twice the proposed load is not excessive, the test is considered satisfactory.

12. Topographical and Special Conditions

Excavations over Inclined Strata. In case the site of a proposed building is on a slope, and especially if the slope is steep, there may be danger from a slip of the material forming the foundation-bed. (See Solid Rock, Sec. 7) This may occur if there is an inclined plane of separation between layers of the underlying rock, or between the rock-surface and the material overlying the rock, or if inclined strata or beds of clay occur below the foundation-bed. Slips in such locations are the more likely to occur if water is present, as the water increases the weight of the soil and also reduced the COEFFICIENT OF FRICTION against sliding. Such conditions are frequently indicated by the appearance of springs or springy ground below the site. Where the base of the slope reaches a stream or river there may be danger from the washing away of banks which have been supporting the side slopes of the valley. In the case of deep valleys with steep clay banks, or in any location where landslides have been known to occur, great care should be taken to extend the footings to a bed that will not be affected by any landslide. It sometimes happens that there is a slow, continuous and general movement of the material forming the side slope of a valley toward the center of the valley; but such conditions are rare, fortunately, as, in general, no adequate protection is possible. In certain limestone formations there is danger from natural caves formed in the limestone by the action of water.

Excavations Near Navigable Waters. When buildings are located near navigable waters, it not infrequently happens that dredging-operations at a considerable distance induce a flow of fine sand or clay from strata underlying the adjoining banks. This has occurred where the existence of such strata was not suspected. This danger is especially to be guarded against in marshy localities adjoining waters which are, or may be, used as navigable streams, or in locations near the water-front where it is likely that docks will be constructed.

Damage from Adjoining Excavations. Common and statute laws make general provision for the protection of property-owners against damage resulting from the acts of others in making such excavations; but an owner has usually no control over such operations, whether on adjoining properties or streets, and in general will prefer the assurance of safety to the possibility of damage to his building and the expense and uncertainty of a lawsuit. While it is not always possible to guard fully against the effects of adjoining excavations, and while the expense of so doing is not always justifiable, due consideration should be given to the matter. The following suggestions, therefore, may be of value.

Depth of Adjoining Excavations. Footings adjacent to property-lines or situated where there is a probability of future additions to a building, or footings of a building which adjoins property liable to become the site of building-operations, should go down at least as deep as the maximum probable depth of the adjacent work. In estimating these probabilities, the character of the location should be taken into account. In medium-priced residential sections footings are rarely carried much deeper than 10 ft, a sufficient depth for a cellar of medium height below grade. In high-priced residential sections it is not unusual to have both a basement and a cellar, in which case a depth of cellar below grade up to 20 ft may be expected. Cellars for residences are rarely carried below 10 ft, if in reaching that depth the excavation goes below the water-level. In fact, a high water-level discourages deep excavation, not only on account of the increased difficulty and expense of excavation but also on account of the expense of waterproofing. In business sections, especially in locations where high ground-rental prevails, there is not only an increasing tendency but a demand for deep basements. Deep basements allow the placing of boiler-rooms and mechanical equipment at the lowest level, permitting the renting of one or even two or three basements Basement space is especially valuable to banks, not only for the storage of valuable papers, securities, etc., in their three- or four-storied security vaults, but also as banking and coupon rooms. To create such basements, it has been necessary to make excavations 80 to 100 ft below grade, even though the water-level was within a few feet of the street level. The Barclay-Vesey Building of the New York Telephone Company in New York has five basements aggregating a volume of 3 629 800 cu ft below grade. The Federal Reserve Bank of New York with its five basements has a basement volume of 3 230 350 cu ft, approximately one-fifth of which is used for a specie and security vault. Other banks, as the Chase National Bank and Irving Trust Company, have three-story vaults and public banking-rooms in the basements.

Sewers and Trenches as Affecting Foundations. In cities and towns consideration should be given to the possibility of the construction of trenches in the streets. For the majority of localities it will be sufficient to consider the probable depth of a sewer of the proper depth to serve the street. In other localities it will be necessary to consider the broader question as to the

probability of deeper excavations for trunk sewers, subways, etc. As such constructions are controlled by broad topographical considerations, no general rules can be given and the local city engineer should be consulted.

Foundations Near Mines, Shafts, Wells, etc. In mining-districts local authorities should be consulted as to danger from the caving of OLD MINE-WORKINGS. No adequate provision can be made in the foundation against such widespread caving or subsidence as may result from mining-operations. In some cases, successive falls of rock-fragments from the roof may gradually fill the voids left by the mining-operations, as the loosely piled fragments of the roof will occupy more space as FILL than they did as part of the solid roof-mass. It sometimes happens that where the original working is deep, progressive falling of the roof fills all voids, and no surface-settlements result. In other cases the overburden may settle as a solid mass, causing a settlement at the surface equal to the thickness of the old working. Precautionary measures may involve the filling in of the workings, a subject outside the limits of this chapter. In the case of an important building a local mining engineer should be consulted or, if possible, the location of the building changed to a safer site MINING-SHAFTS, DEEP WELLS, SHAFTS FOR TUNNELS, etc., may cause disturbances of the soil, but in such cases the settlement is generally concentrated around the shaft or well, and buildings at a reasonable distance are slightly affected, if at all.

Foundations Near Tunnels and Trenches for Railroads and Subways. In large cities the necessities of transportation are increasingly calling for construction of underground RAILROADS, TUNNELS and SUBWAYS. Such constructions are generally planned to follow streets. Railroad tunnels for trunk lines can be expected to follow direct lines to centrally located stations or terminals along routes which avoid, as far as possible, difficulties of construction, condemnation of real estate and damage to high-priced properties. The depth of excavation will generally be as shallow as practicable Where the tunnel has to dip to pass underneath some obstruction, the approach-grades will probably be at the maximum or limiting grade of the particular section.

Relation of Subways to Foundations of the Most Important Buildings. In SUBWAY-CONSTRUCTION for rapid-transit passenger service, the lines can be operated on sharper curves and with steeper grades than would be used in the case of a trunk-line railroad. This permits the lines to follow closely the lines of the city streets. For traffic-considerations the locations will, in general, follow the principal arteries of surface traffic, and stations will, in general, be located at intersections of important streets, where there is the greatest congestion of population. As such conditions are caused by the existence of trade-centers, and call for the construction of high buildings, it may readily be seen that the heaviest and most important buildings are most likely to have their foundations affected by the construction of a subway in their immediate vicinity. Where there is reason to apprehend the construction of such SUBWAYS or TUNNELS, information should be sought as to the probable depth of the excavation, the depth at which water is encountered, the character of the material, the probable width of the construction as affecting the use of sidewalk vaults, and the method to be employed in making excavations Where the excavations for such tunnels and subways have been carried below the levels of the footings of adjoining buildings, as in Baltimore, Boston, Brooklyn, Chicago, New York City and Philadelphia, buildings along the routes have been seriously affected. Such results have not been limited to any particular methods used in the construction of the tunnels, as even where the excavations were wholly, or partly, in rock, serious damage has been done.

13. Loads Coming on the Footings

The Loads to be Considered in the design of the footings of a structure are:

(1) **The Dead Loads,** or the loads due to the actual weight of the completed structure, ready for occupancy.

(2) **The Live Loads,** or the loads due to the occupancy of the building and also to the weight of snow on the roof

(3) **The Wind-Loads,** or the vertical components of stresses in the structure, produced by wind-pressure

(1) **The Dead Load.** The dead load of any structure can be accurately calculated If the structure is properly designed the part of the dead load supported by each element of the foundation can be definitely stated. The total dead load becomes effective as soon as the building is completed, and remains constant thereafter unless additions or alterations are made to or in the structure.

(2) **The Live Load.** The live load of any structure is the sum of the roof-loads and floor-loads In designing the roof and floors the calculations for strength are based on an assumed unit load which should be the maximum load, consistent with the probable use of the structure, to which any portion of the roof or floor may ever be subjected. The assumed live load is, therefore, probably greater than the average load for the entire area of a floor or the entire area of the roof. Moreover, as it is improbable that conditions of maximum loading will ever occur simultaneously on the roof and on all of the several floors, it is probable that the maximum load on the footings will be less than the sum of maximum loads on the roof and on the several floors.

The Minimum Live Load for an unloaded building is zero.

The Actual Live Load will vary from zero to a maximum, which maximum will generally be less than the total assumed live load.

The Ratio of the Probable Maximum Live Load to the Assumed Live Load varies in different buildings, so that no table or general rule can be given.

The Probable Maximum Live Load. As it is important to know, approximately at least, the maximum live loads to which the footings will be subjected, and as this maximum may be only a fraction of the assumed live loads, the architect should make a careful study of the conditions of loading to which the building will probably be subjected and estimate the probable maximum live load for the entire building.

Data for Estimating Live Loads. In estimating the probable maximum live loads for different uses, the following notes may be of value. In certain buildings the assumed unit loading on the roof and on parts of each floor may be reached at various times, but it is unlikely that the maximum loading of all parts of the building will occur at the same time. In buildings of many stories the probability of having maximum loads on all of the floors at the same time decreases with the number of stories.

Ordinary Household and Office-Furniture weighs from 5 to 10 lb per sq ft of space occupied. While safes, bookcases or filing-cases may produce local loadings of from 10 to 100 lb sq ft, the average load on office-floors rarely reaches 10 lb per sq ft.

Residences, Apartments and Parts of Hotels not used for public assemblies are rarely loaded with more than 5 lb per sq ft of floor-area.

Retail and Wholesale Stores require a large percentage of the floor-area for the use of salespeople and customers, and not over 50% of the floor-area is

used for the storage of stock. In estimating the weight of miscellaneous stocks, an average between the lightest and heaviest classes should be taken for the weight per cubic foot, and also, in figuring the total space occupied by stock, an average should be taken between the maximum and minimum amount of stock carried. In RETAIL DRY-GOODS STORES the floor-load for the entire building may amount to not more than 25 lb per sq ft, but in WHOLESALE STORES, and especially in grocery and hardware stroes, the average load may greatly exceed this figure.

In Workshops, Loft-Buildings and Buildings for Manufacturing, the actual live loads will, of course, vary with the class of material handled and the weight of the machinery used, and no general estimate can be made. Where the character of the occupancy to be expected is known it is possible to make a close approximation of the weights of machinery, fixtures and average stock on each floor.

Storehouses. In buildings used, in whole or in part, for STORAGE PURPOSES a floor may be used for light, bulky materials which, when stowed so as to leave gangways and working-spaces, will give a resultant load much below the assumed load. On the other hand, the heaviest materials may be compactly piled from floor to ceiling in defiance of building regulations, posted notices and common sense. Raw materials or crated or baled materials can be packed closer than miscellaneous articles, and are therefore liable to increase the loads. Security vaults in addition usually provide for a live load of approximately 600 lb per sq ft, and in the case of bullion storage the live load may be as high as 1 800 lb per sq ft or more.

The Ratio of the Total Probable Maximum Live Load to the Total Assumed Live Load having been determined for the entire building, the probable maximum live load for any element of the footing may be readily obtained by multiplying the assumed or calculated live load for that element by this ratio.

(3) **The Wind-Load** is generally calculated on the assumption that the wind may exert a uniform pressure, frequently taken at 30 lb per sq ft, on the entire external area of any side of the building. This assumption makes no deduction for the protecting influences of adjoining buildings. In a building of any size it is improbable that the maximum pressure will be reached over the entire exposed area at the same instant of time, and consequently, if the assumed pressure represents the maximum pressure, the average, at any time, will be less than the calculated total.

General Effect of Wind-Pressure. The horizontal pressure of the wind tends to increase the load on footings on the leeward side of the building and to decrease the load on footings on the windward side. In many buildings diagonal bracing, called wind-bracing, or other special construction, is used to prevent the building from being deformed by the wind-pressure and to convert the horizontal stresses due to the wind-pressure into vertical components, acting along defined lines of support, that is, into either uplifts or loads on certain walls, piers or columns. Where the uplift on any element of the structure is less than the dead load on the same element, the uplift is ignored. Where the vertical component increases the compression in any element it is called the WIND-LOAD IN THAT ELEMENT of construction and on the corresponding footing. The design is generally based upon concentrating all of the wind-load on certain external footings. If, on account of the general rigidity of the building, or on account of any other reason, the wind-stresses reach footings not designed to receive wind-loads, the amounts figured on the external footings will be reduced correspondingly. It is

probable that the maximum effect of the wind results from a series of impulses of short duration and that the effect of such pulsations may be partially overcome by the inertia and elasticity of the buildings; so that the resultant load reaching the footing may be only a part of the theoretical load for the instant during which the maximum pressure is exerted. (See, also, Chapter on Wind-Bracing of Tall Buildings.)

The Probable Maximum Wind-Load acting on the footing is, therefore, less than the theoretical load due to the maximum wind-pressure. If the assumed wind-load represents approximately the maximum wind-pressure, as recorded by a wind-gauge, it would appear safe to assume that only 50% of the assumed wind-load would act to produce a settlement in the footings of a building. Some authors recommend that in proportioning footings all wind-loads be ignored; but this, especially in the case of high and narrow buildings, is manifestly improper. The minimum wind-load is negative, being actually an uplift from which the load may vary to the maximum, but the maximum will be reached only at rare intervals and will endure for a short period only.

The Combined Wind-Load and Live Load. It is improbable that the maximum wind-load and the maximum live load will occur at the same time, which consideration should be borne in mind when the estimate is being made as to the effective wind-load.

14. Assumed Loads Specified by Building Codes

Table II. Requirements of Building Codes for Assumed Loads for Office-Buildings

City	Requirements
Akron, Ohio..	Live load, 80 lb per sq ft Footings, piers and walls designed for dead load and the following live load: Roof, full live load; top floor, 90%; for each lower floor, a reduction of 5% until a reduction of 50% of full live load is reached, when such reduced load shall be used for remaining floors. Wind load, 20 lb per sq ft
Atlanta, Ga....	Live load, 75 lb per sq ft above 1st floor; 150 lb per sq ft on 1st floor. Wind-load, 30 lb per sq ft Footings designed for dead load and 60% of live load and wind-load
Boston, Mass.....	Live load, 60 lb per sq ft above 1st floor; 125 lb per sq ft on 1st floor Wind-load: for 40 ft in height 10 lb per sq ft. Portion from 40 to 80 ft above ground, 15 lb per sq ft. Portion more than 80 ft above ground, 20 lb per sq ft Foundation designed for dead load and 75% of live load
Buffalo, N. Y.....	Live load, 50 lb per sq ft above 1st floor. 120 lb per sq ft on 1st floor if used as stores Walls, piers and footings designed for dead load and the following live load: Roof, full live load; top floor, 85%. For each succeeding lower floor a reduction of 5% until 50% of full live load is reached, when such reduced load shall be used for remaining floors Wind-load, 30 lb per sq ft

Table II (Continued). Requirements of Building Codes for Assumed Loads for Office-Buildings

City	Requirements
Chicago, Ill	Live load, 50 lb per sq ft; 50% of the live load used for piles Piers and walls designed for 85% of live load on top floor and reduced by 5% for each lower floor until 50% is reached, when such reduced loads shall be used for the remaining floors Wind-load, 20 lb per sq ft
Cincinnati, Ohio	Live load, 50 lb per sq ft above 1st floor; 100 lb for 1st floor. Live load reduced by 5% for each floor below the top until 20% is reached, when such reduced loads shall be used for remaining floors. Wind-load, 20 lb per sq ft above surrounding buildings
Cleveland, Ohio .	Live load, 70 lb per sq ft Piers, footings and wall designed for dead load and the following live load: Roof, 100%, top floor, 90%; for each lower floor, a reduction of 5% until a reduction of 50% of full live load is reached, when such reduced load shall be used for the remaining floors Wind-load, 20 lb per sq ft
Jacksonville, Fla...	Live load, 90 lb per sq ft in entrances and public rooms, 60 lb per sq ft in other rooms Footings designed for dead load and 50% of live load Wind-load, 30 lb per sq ft
Louisville, Ky.....	Live load, 75 lb per sq ft above 1st floor; 150 lb per sq ft on 1st floor Footings designed for dead load and 40% of live load Wind-load, 30 lb per sq ft
Minneapolis, Minn	Live load, 75 lb per sq ft above 1st floor, 100 lb per sq ft for 1st floor. Wind-load, 30 lb per sq ft Roof and top floor, full live load For each succeeding lower floor, a reduction of 5% until 50% is reached, such reduction being used for the remaining floors
Newark, N. J. .	Live load, 60 lb per sq ft above 1st floor, 100 lb per sq ft on 1st floor Footings designed for dead load and the following live load: Roof and top floor, full live load; for each succeeding lower floor, a reduction of 5% until 50% of full live load is reached, when such reduced load shall be used for remaining floors Wind-load, 30 lb per sq ft
New Orleans, La..	Live load, 75 lb per sq ft Foundation and piers designed for dead load and 20% of live load
New York, N. Y .	Dead load shall include weight of partitions or an allowance of 20 lb per sq ft for same Live loads: Roof, 40 lb per sq ft when pitch is 20° or less and 30 lb per horizontal square foot, with pitch more than 20°. Roof, full load: top floor 85%; for each lower floor, a reduction of 5% until 50% of full live load is reached when such reduced load shall be used for remaining floors

Table II (Continued). Requirements of Building Codes for Assumed Loads for Office-Buildings

City	Requirements
New York, N. Y.	Wind load: Buildings over 100 ft high, 20 lb per sq ft above 100 ft level. Buildings less than 100 ft high, or when height is less than two and one-half times least width, wind may be neglected; otherwise 20 lb per sq ft on upper 50% of height The overturning moment is not to exceed 80% of the moment of stability of the dead loads, unless anchored to foundation
Pacific Coast	Live load, 50 lb per sq ft Piers, walls and foundations designed for dead and the following live loads: 1 story 100%, 2 stories 90%, 3 stories 80%, 4 stories 70%, 5 stories 60%, 6 stories 55%, and 7 stories or more 50% of full live load Wind-load, 10 lb per sq ft for buildings less than 40 ft, 20 lb per sq ft for buildings more than 40 ft above ground
Philadelphia, Pa	Live load, 60 lb per sq ft Columns, piers, walls and foundations designed for dead load, the following reductions of live load per increment are permissible. Carrying one floor, no per cent, two floors, 10%; three floors, 20%, four floors, 30%; five floors, 40%, six floors (or more), 50% Wind-load, not less than 20 lb per sq ft over entire surface except isolated buildings where it shall not be less than 25 lb per sq ft
Portland, Ore	Live load, 60 lb per sq ft above ground floor, 125 lb per sq ft on ground floor Piers and walls designed for dead load and the following live load. Roof, full live load, top floor 95% and for each succeeding lower floor a reduction of 5% until 50% of full live load is reached, when such reduced load shall be used for remaining floors Wind-load, 20 lb per sq ft
Richmond, Va.	Foundations designed for dead load and 60% of the live load
St Louis, Mo.	Live load, 60 lb. per sq ft; 1st floor, 100 lb. Loads carried by the soil, total dead load and 10 lb per sq ft of all the floor area Wind-load, 30 lb per sq ft
St Paul, Minn	Live load, 60 lb per sq ft above the 1st floor. First floor, 125 lb Roof and top floor, full load; for each lower floor, a reduction of 5% until 50% of the full live load is reached, when such reduced load shall be used for the remaining floors. Footings designed for dead load and live load Wind-load, 30 lb

Minimum live loads allowable for use in design of buildings as recommended by the Building Code Committee of the Bureau of Standards, U. S. Department of Commerce, are:

Residences, hospitals or hotel guest rooms and tenement houses	40 lb per sq ft
Office-buildings, churches, schools, theaters, etc.	50 lb per sq ft

In this case provision must be made in the floor design to satisfy the presence of a concentrated load of 2 000 lb on an area of 2 ft 6 in square.

Corridors, lobbies, public spaces, assembly halls, stairways	100 lb per sq ft
General storage purpose floors	250 lb per sq ft
Special storage purpose floors, printing plants, wholesale stores	100 lb per sq ft
Light manufacturing, retail salesrooms, stables	75 lb per sq ft
Garages, all type of vehicles	100 lb per sq ft
Garages, passenger cars only	80 lb per sq ft

Sidewalks, 250 lb per sq ft or 8 000 lb concentrated, whichever gives the largest moment or shear.

Roof loads, 30 lbs per sq ft as live load, or 20 lb per sq ft normal to the roof surface if roof slopes 45° or more.

Reductions in live loads, except in buildings for storage purposes, the following reductions in assumed total floor live loads are permissible in designing all columns, piers, walls, foundations, trusses, and girders:

Carrying 1 floor	0%
Carrying 2 floors	10%
Carrying 3 floors	20%
Carrying 4 floors	30%
Carrying 5 floors	40%
Carrying 6 floors	45%
Carrying 7 or more floors	50%

The dead load in a building includes the weight of all permanent construction and stationary construction entering into the building.

Reduction in Assumed Loads. The building codes of various cities contain rules governing the assumptions to be made as to live loads and wind-loads, and these rules generally provide for some REDUCTION IN THE ASSUMED LOADS. Generally, it will be found possible to meet these requirements and at the same time arrange for the proper proportioning of the supporting areas. Table II gives briefly the requirements of the building codes of several cities, as to assumed loads for office-buildings.

15. Proportioning the Supporting Areas for Equal Settlement

The Minimum Areas of Support. The actual dead loads and the assumed live loads and wind-loads for each lineal foot of wall and for each column, pier, or other supporting element of the building down to the level of the footings having been calculated, a foundation-plan should be prepared giving the amount and center of action of all loads. For safety under the worst possible combination of loads, each footing should be ample to support the total of the dead loads, live loads and wind-loads coming on it. The MINIMUM AREAS OF SUPPORT for any footing are obtained by dividing the total of the dead loads,

live loads and wind-loads by the safe supporting power of the foundation-bed. If the foundation-bed is rock, or can be considered as incompressible under the unit load, the minimum areas so obtained may be used for the footings. On compressible materials, or generally on all materials other than rock, the use of these minimum areas will not result in uniform settlements owing to the fact that the actual live loads and wind-loads are not consistent with the assumed live loads and wind-loads.

The Actual Loads on the Footings. In accordance with what has been previously said, let us assume that the dead load is constant, and that for a building under consideration the probable maximum live load is 50% of the assumed live load, that the probable maximum wind-load is 40% of the assumed wind-load, and that on the completion of the building, for a short period, the live loads and wind-loads reduce to zero. The ACTUAL LOADS ON THE FOOTINGS would then be:

(1) Upon completion of the building, the dead load only;
(2) Under the maximum load due to occupancy and to snow on the roof, the dead load plus 50% of the assumed live load;
(3) When loaded as in (2) and subject, in addition, to the maximum probable wind-action,
  (*a*) The footings on the leeward side of the building will sustain the total dead load, plus 50% of the assumed live load, plus 40% of the assumed wind-load;
  (*b*) The footings on the windward side of a building will sustain the total dead load, plus 50% of the assumed live load, minus 40% of the assumed uplift;
  (*c*) Other footings will support the total dead load, plus 50% of the assumed live load, plus zero wind-load;
(4) Intermediate conditions as to live loads and wind-loads will produce loadings intermediate between (1) and (3).

Variations in Unit Loads on Foundation-Beds. With such known variations it is, therefore, impossible to proportion the supporting areas so that the unit load on the foundation-bed shall be uniform at all times. If the supporting areas are proportioned in the ratio of the dead load only, the building, on completion, and before occupancy, will uniformly load the supporting areas, and at that time all of the footings should show equal settlements; but subsequently, when the supporting areas have been subjected to the full effects of the live loads and wind-loads, certain supporting areas, having a high percentage of live loads, or of live loads and wind-loads, will be subject to a higher unit load, and the corresponding footings will consequently settle more than other footings supporting a low percentage of live loads, or live loads and wind-loads.

Non-Uniformity in Footing-Settlements. If, on the other hand, the supporting areas are proportioned on the basis of the dead loads, plus the maximum live loads, plus the maximum wind-loads, even if the MAXIMUM LOADS are the PROBABLE ACTUAL MAXIMUM LOADS, and not the FICTITIOUS ASSUMED LOADS, it is inevitable that upon the completion of the building and before occupancy, the supporting areas having a lower percentage of live loads and wind-loads will have a higher unit load, and the corresponding footings will have settled more than other footings supporting a high percentage of live loads and wind-loads. On this basis, the footings will not come to a uniform settlement until they have been subjected to the maximum live loads and wind-loads.

Arbitrary Rules for Proportioning Supporting Areas. Various ARBITRARY RULES have been recommended for the proportioning of the supporting areas to secure equal settlements. These rules generally provide for a reduction in the assumed live loads and wind-loads, but do not take into consideration the fact that a large proportion of the total settlement of certain footings may take place subsequently to the completion of the building and after other footings may have reached practically their full settlement.

Rational Rule for Proportioning Supporting Areas. The rule hereinafter recommended provides not only for a reduction of the assumed loads on a more rational basis, but also for the proportioning of the footings for the mean load, instead of for the ultimate load, and it is believed that the resulting settlements will be as nearly uniform as possible. The rule is based on the proportioning of the footings in accordance with the loads which will act on the footings at the time when all of the dead loads and one-half of the probable maximum live loads and wind-loads exist. The reason for taking one-half of the probable maximum wind-loads and live loads is that these loads vary from zero to a maximum, the average being one-half of the maximum.

Provision for Variations in Loads. On the completion of the building and before the live loads or wind-loads have gone on the footings, the settlements will not be uniform, because areas designed for a high percentage of live loads and wind-loads will have much less than their average load and will therefore have settled less than footings having a low percentage of live loads and wind-loads. When these same footings have been subjected to the maximum probable live loads and wind-loads, the settlements will again be unequal, because the areas have been proportioned for only one-half of the probable maximum live loads and wind-loads; but the footings which originally were the highest will now be the lowest. The inevitable movement due to the variation in the live loads and wind-loads will be equally divided, one-half of the settlement being required to bring the footing to the level of a footing having the dead loads only, and the other half of the settlement carrying it an equal distance below the same footing In other words, the method provides for the least possible variation between footings having different proportions of live loads and wind-loads

The Mean Load. For lack of a better name, the loads taken for the proportioning of the footings, consisting of the total dead loads, one-half of the probable maximum live loads and one-half of the probable wind-loads coming on each footing, will be called the MEAN LOAD.

The Mean Unit Load. The areas will be made such that the load on the foundation-bed due to the mean loads will be uniform, and this uniform load which, in general, will be considerably less than the allowable unit load on the foundation-bed will be called the MEAN UNIT LOAD.

The Minimum Unit Load. The necessity for providing for the worst possible condition of loading is satisfied if the supporting area for all footings is sufficiently large to support the total of the dead loads and the assumed live loads and wind-loads at the allowable unit pressure. The resulting areas of support are the MINIMUM AREAS, and any change in these areas necessary to make them proportionate to the mean loads must be effected by increasing some areas rather than by diminishing any. Any mean unit load which would give, when divided into the mean loads, areas, all of which would be larger than the minimum areas, would serve as the mean unit load, but it is more economical to determine the LOWEST POSSIBLE MEAN UNIT LOAD which, when applied to the mean loads, will give the least possible increase of the

areas. This can be done by determining which one of the minimum areas carries the LEAST MEAN LOAD PER SQUARE FOOT. This area may be selected by calculating the mean load on each of the minimum areas, or more simply, by comparing the table of assumed loads and a table giving the mean loads, and noting which footing has the LARGEST PERCENTAGE OF REDUCTION between the assumed load and the mean load. The resulting mean load on this footing will be the MINIMUM UNIT LOAD which can be used as a MEAN UNIT LOAD.

The Method Reduced to Rule. The method can be reduced to rule as follows:

(1) Prepare a table giving in vertical columns or table-divisions for each footing, the dead loads, the assumed live loads, the assumed wind-loads and the total of these three loads. This table is called the TABLE OF ASSUMED LOADS.

(2) Prepare a similar table giving the dead loads, one-half of the maximum probable live loads, one-half the maximum probable wind-loads and the total of these three loads This table will be called the TABLE OF MEAN LOADS.

(3) By a comparison of the two tables, find the supporting area which has suffered the greatest percentage of reduction between the total assumed loads and the total mean loads and find the unit load resulting from the mean load on this area This unit load will be called the MEAN UNIT LOAD.

(4) Divide the total mean load as given in the table of mean loads for each footing by the mean unit load. The result will be the required AREA OF SUPPORT.

Short Method for Determining the Mean Unit Load. From the foregoing it follows that the MEAN UNIT LOAD can be obtained more directly by the following rule. Find the supporting area which has suffered the largest percentage of reduction between the total assumed load and the total mean load and multiply the allowable unit load on the foundation-bed by the ratio obtained by dividing the total mean load by the total assumed load.

Illustrative Example. The following example is figured out more fully than is necessary in practice in order to fully explain the method and also to compare the method with other methods frequently used and recommended. Ordinarily the wind-loads on a building of the size and type assumed in the example would be ignored, but they have been considered here to make the example complete.

A factory-building (Fig. 2) is to have four floors above the basement, each capable of supporting an assumed unit load of 200 lb per sq ft. The load on the flat roof is assumed at 50 lb per sq ft. The horizontal wind-pressure is assumed as a uniform pressure of 40 lb per sq ft, on the sides AB and CD only. The vertical component of the wind-pressure is to be taken care of by the footings of the side walls. There is also an interior self-supporting chimney and ventilating shaft which is protected from the wind and which carries no floor-loads

The foundation-bed is a uniform, sandy material which is expected to compress uniformly and at the rate of ½ in per ton of load per sq ft of supporting area. The MAXIMUM UNIT LOAD on the foundation-bed is taken at 4 tons, corresponding to a settlement of 2 in for the assumed load. The calculated dead loads of the building, including all construction down to the level of the footings, the summation of the assumed live loads and the vertical components of the assumed wind-loads are given in Table III.

A careful study of the probable loading of the building shows that the maximum live loads at any one time will not exceed 60% of the total assumed live loads, and that the maximum wind-loads will be less than 50% of the

assumed wind-loads, for the reason that the assumed wind-pressure is based upon the highest recorded pressure on a limited area in an exposed situation, whereas the proposed building will be in a sheltered situation. Having, therefore, determined the probable maximum live loads and wind-loads at 60% and 50% respectively of the assumed loads, the so-called mean loads,

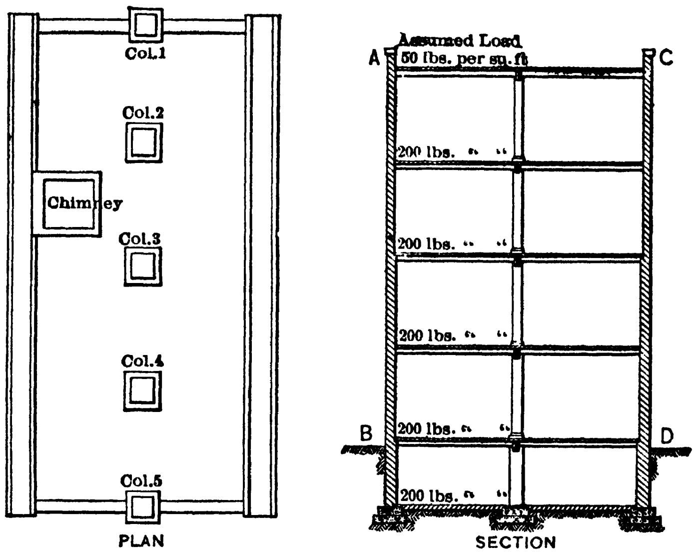


Fig. 2. Foundation-plan and Section of Factory-building

corresponding to loads half-way between the minimum and maximum loads, will be one-half of the probable maximum loads, or 60% × ½ = 30% of the assumed live loads and 50% × ½ = 25% of the assumed wind-loads. Table IV gives the dead loads and the mean live loads and wind-loads separately, and the total of the dead loads and mean loads, which total is to be used in proportioning the areas for least variation in settlement. This is known as the total mean load.

Table III. Dead Loads and Assumed Live and Wind-Loads

Element of footing	Division 1, dead loads only, lb	Division 2, assumed live loads, lb	Division 3, assumed wind-loads, lb	Division 4, total dead, live and wind-loads, lb
Side walls per lin ft	14 000	8 400	2 000	24 400
Columns 1 and 5...	137 500	160 000	. . . . .	297 500
Columns 2, 3 and 4	90 000	340 000	....	430 000
Chimney.........	320 000	....	....	320 000

Table-columns are called divisions to avoid confusion with building-columns.

Table IV. Dead Loads, Mean Live and Wind-Loads and Total Dead and Mean Loads

Element of footing	Division 5, dead loads, unchanged, lb	Division 6, one-half of 60% of assumed live loads, lb	Division 7, one-half of 50% of assumed wind-loads, lb	Division 8, total mean loads, lb
Side walls per lin ft	14 000	2 520	500	17 020
Columns 1 and 5	137 500	48 000	....	185 500
Columns 2, 3 and 4	90 000	102 000	....	192 000
Chimney..... ...	320 000	....	... .	320 000

Table-columns are called divisions to avoid confusion with building-columns.

Comparing the two tables it will be seen that the interior columns of the building, columns 2, 3 and 4, had originally the largest percentage of live loads plus wind-loads, and have consequently suffered the greatest reduction in the amount of total load. The minimum areas of support for columns 2, 3 and 4, and also for the other elements of the footings, are obtained by dividing the total assumed loads given in division 4, Table III, by 8 000, the allowable unit load in pounds on the foundation-bed. The resulting areas are given in division 9, Table V. No reduction can be made in these areas without exceeding the limitation that the most disadvantageous combinations of loading, however improbable, shall not exceed the safe unit load. The adjustment of the areas to the probable mean loading, as given in Table IV, the table of mean loads, must be accomplished solely by increasing the sizes of certain footings.

If we divide the total mean loads in division 8, Table IV, by the minimum areas given in division 9, Table V, we will get the mean load per square foot on the minimum areas for each element of the footing. The results given in division 10, Table V, show that the mean load for columns 2, 3 and 4 is only 3 568 lb per sq ft, while under the chimney the load is 8 000 lb per sq ft. As no reduction in area is permissible it is necessary to increase the footings under the chimney, side walls and columns 1 and 5 until the mean unit load corresponds to the mean unit load for columns 2, 3 and 5. This is done by dividing the mean loads given in division 8, Table IV, by 3 568, the mean unit load as determined for columns 2, 3 and 4. The resulting areas are given in division 11, Table V, and are the areas which should be used.

Table V. Mean Loads on Minimum Areas and Areas for Mean Loads

Element of footing	Division 9, minimum areas, sq ft	Division 10, mean loads on minimum areas, lb per sq ft	Division 11, areas for mean loads, sq ft
Side walls per lin ft......	3 05	5 580	4 7
Columns 1 and 5 .. .	37 2	4 986	51.9
Columns 2, 3 and 4.......	53 8	3 568	53.8
Chimney................	40 0	8 000	89.7

Table-columns are called divisions to avoid confusion with building-columns.

The method of calculation can be shortened and reduced to a rule as follows: Compare Table IV, the table of mean loads, with Table III, the table of assumed loads, and find the element of support which has suffered the highest percentage of reduction between the total assumed load and the total mean load, and note the corresponding minimum area of support of the allowable unit load on the foundation-bed. Divide the mean load for the same element of support by the number of square feet in the minimum area of support. The result will be the unit load for mean settlement. Then divide the mean loads for each element of support by the mean unit load. The results will be the required areas as given in Table V.

Or the mean unit load may be determined by multiplying the allowable unit load by the ratio obtained by dividing the mean load for the element of support having suffered the highest percentage of reduction by the assumed load for the same element.

Resulting Settlements. The following Tables VI, VII and VIII show the comparative settlements which may be expected if the supporting areas are proportioned in accordance with different assumptions as to load. In all the tables it is assumed that the foundation-bed will settle ½ in per ton of load, and that the total assumed load will never load the foundation-bed in excess of 4 tons per sq ft.

In Table VI the footings are proportioned in the ratio of the DEAD LOADS only.

In Table VII the footings are proportioned in the ratio of the TOTAL ASSUMED LOADS.

In Table VIII the footings are proportioned in the ratio of the MEAN LOADS.

In each table, division 1 gives the dead load coming on the footings on the completion of the building. Division 2 gives the load coming on the footings when the building is subjected to the maximum probable live loads and wind-loads. Division 3 gives the supporting areas in accordance with the assumed loading. Division 4 gives the settlements for the unloaded building. Division 5 gives the settlement after the addition of the maximum probable live loads and wind-loads.

Explanation of Table VI. The method of proportioning the areas in the ratio of dead loads only may, in the form of a rule, be stated as follows:

Compare the table-division of dead loads, Table VI, with the division of assumed live loads, find the element of support which has the highest percentage of live loads to dead loads, and note the corresponding minimum area of support at the allowable unit load on the foundation-bed. Divide the dead load for the same element of support by the number of square feet in this minimum area of support, and the result will be the unit load due to the dead load only. Then divide the dead loads for all other elements of support by this unit load, and the results will be the areas required. Thus, in Table VI, it is seen by referring to Table III that columns 2, 3 and 4 have the greatest percentage of live load to dead load, and their minimum area of support, as in Table V, is 53.8 sq ft. Then, 90 000 ÷ 53 8 = 1 675 lb, the unit load due to the dead load only. The area for columns 1 and 5 is 137 500 ÷ 1 675 = 82.1 sq ft. The process is similar for the other elements.

The calculations for settlements are readily made, when the amount of compressibility of the foundation-bed is known, by multiplying the unit load on the foundation-bed of each element of support by the amount of compressibility of the foundation-bed per unit of load. Thus, in the above example the amount of compressibility is given as ½ in per ton. In Table VI the unit loads, due to dead loads for each element of support, are the same, or 1 675l lb

= 0.838 ton per sq ft, which, multiplied by ½ = 0.42 in. Similarly, the unit loads due to maximum probable loads for each element of support are determined, and these loads, in tons, multiplied by one-half, give the settlements in inches as given in division 5 of Table VI.

Table VI. Footings Proportioned in the Ratio of the Dead Loads Only

Probable settlement where supporting areas are proportioned in the ratio of dead loads only					
	Division 1	Division 2	Division 3	Division 4	Division 5
Element of footing	Dead loads only, lb	Maximum probable loads, lb	Areas, sq ft	Settlements	
				Empty, in	Loaded, in
Side walls per lin ft	14 000	20 040	8 3	0 42	0 60
Columns 1 and 5..	137 500	233 500	82 1	0 42	0 71
Columns 2, 3 and 4	90 000	294 000	53 8	0 42	1 36
Chimney.....	320 000	320 000	191 0	0 42	0 42
Maximum variation, empty				0 00	.... .
Maximum variation, loaded.. .					0 94

Table-columns are called divisions to avoid confusion with building-columns.

Explanation of Table VII. The areas given in Table VII are obtained by dividing the total maximum dead loads, live loads and wind-loads by the allowed unit, 8 000 lb per sq ft, and are the minimum areas given in Table V. The settlements for the loaded building are based on the maximum probable loads as given in division 2 of Table VII.

Table VII. Footings Proportioned in the Ratio of the Total Assumed Loads

Probable settlement where supporting areas are proportioned in the ratio of total assumed loads					
	Division 1	Division 2	Division 3	Division 4	Division 5
Element of footing	Dead loads only, lb	Maximum probable loads, lb	Areas, sq ft	Settlements	
				Empty, in	Loaded, in
Side walls per lin ft	14 000	20 040	3 1	1 13	1.61
Columns 1 and 5...	137 500	233 500	37 2	0.92	1 57
Columns 2, 3 and 4	90 000	294 000	53 8	0.42	1 36
Chimney..........	320 000	320 000	40 0	2.00	2 00
Maximum variation, empty.			......	1.58	....
Maximum variation, loaded.			.. ....	. ...	0 64

Table-columns are called divisions to avoid confusion with building-columns.

Explanation of Table VIII. The areas in Table VIII are obtained as already explained and as given in division 11, Table V, and the methods used in determining the settlements are similar to those used for the preceding tables. In Table VIII it will be noted that columns 2, 3 and 4 have a settlement of 1.36 − 0.42 = 0.94 in, as a result of the addition of the live loads and wind-loads. Half of this settlement is required to bring these footings down to the level of the chimney-footing, and the other half of the settlement brings them below the chimney-footing. There is no way to prevent this settlement of 0.94 in, but its effect on the building is reduced to a minimum by having the settlement of the footings of columns 2, 3 and 4 start above the chimney-footing and finish below it. The chimney-footing does not change its elevation after the completion of the building, and compared with it, the variation in level of the other footings is the minimum. In their mean position, halfway in their movement, these other footings will be at the same level as the chimney-footing.

Table VIII. Footings Proportioned in the Ratio of the Mean Loads

Probable settlement where supporting areas are proportioned in the ratio of total mean loads					
	Division 1	Division 2	Division 3	Division 4	Division 5
Element of footing	Dead loads only, lb	Maximum probable loads, lb	Areas, sq ft	Settlements	
				Empty, in	Loaded, in
Side walls per lin ft	14 000	20 040	4.7	0 74	1.06
Columns 1 and 5 ..	137 500	233 500	51 9	0.66	1 12
Columns 2, 3 and 4.	90 000	294 000	53.8	0 42	1 36
Chimney.... ...	320 000	320 000	89.7	0 89	0 89
Maximum variation, empty.......... . .			. ..	0.47	.. ...
Maximum variation, loaded..... .			. ..	....	0 47

Table-columns are called divisions to avoid confusion with building-columns.

16. Determining the Supporting Areas

General Requirements. In laying out the AREAS OF SUPPORT for any structure it should be borne in mind, as previously explained, that (1) the total of the dead loads, assumed live loads and assumed wind-loads should not load the foundation-bed in excess of the allowable load on it; (2) when the foundation-bed is compressible the areas of support should be calculated by the method of mean loads; and (3) the center of gravity of the supporting area should coincide with the center of action of the load to be supported. To these may be added a further condition that (4) economy will be furthered by keeping the supporting areas simple in outline and by arranging each area as compactly as possible around the center of the load to be supported.

(1) The first condition is necessary in order to provide that no possible condition of loading will exceed the allowable pressure on the foundation-bed.

(2) The second condition provides for making the settlements of different footings as nearly equal as possible.

(3) The third condition provides that the settlements of each footing shall be uniform, that is, that the footing shall not settle out of level.

(4) The fourth condition provides for economy in design in the footing itself and for economy in making the excavation for the footing, especially in the case of deep excavations requiring sheathing for the protection of their sides.

In the case of a free-standing structure, the total load of which is not in excess of the supporting capacity of the entire area of the building at the safe unit load on the foundation-bed, it will generally be possible to arrange simple supporting areas whose centers will correspond with the centers of the loads. The disposition of such areas is considered in succeeding paragraphs in the discussions of CONCENTRIC LOADING. In buildings having restricted sites, where walls or columns are placed close to adjoining property-lines, it will frequently be impossible to arrange for simple concentric loadings and necessary to use offset footings, cantilevers or other devices to transfer the loads to supporting areas located on the property. Such supporting areas are discussed in succeeding paragraphs relating to ECCENTRIC FOOTINGS.

Footings with a Concentric Load. In order to have the load on the foundation-bed uniform under a footing it is necessary that the center of gravity of the supporting area should coincide with the center of gravity of the load, otherwise the area is said to be ECCENTRICALLY LOADED and the resulting load on the foundation-bed will not be uniform. Any variation in the loading of a compressible foundation-bed under a footing will result in an unequal settlement of the footing and this in turn will result in unequal stresses in the wall, pier, or column supported by the area.

Wall-Footings with Concentric Load. In the case of a WALL, the footing should project an equal distance on each side so that the center of gravity of the supporting area will coincide with the center of gravity of the wall and of the loads transmitted by the wall. The width of the supporting area should vary with the load on the wall, irrespectively of any change in the thickness of the wall.

Footing for a Concentric Isolated Load. In the case of a SIMPLE CONCENTRATED LOAD, as, for example, a load from a COLUMN or PIER, the footing may be CIRCULAR, SQUARE, RECTANGULAR, or IRREGULAR in outline, but the center of gravity of the area must coincide with the center of gravity of the load. Theoretically the CIRCULAR SHAPE gives the most economical footing, as the supporting areas extend radially the least possible distance from the center or axis of the load. Where deep excavation is necessary the circular form may lend itself to an economical method of excavation, as, for example, when cylindrical piers are sunk by the pneumatic method or by dredging. In general, however, for ordinary footings the RECTANGULAR FORM is preferable, in that it lends itself to an economical arrangement of grillage-beams. The SQUARE is the most economical rectangle as the sum of the bending movements in the grillage and bolsters is reduced to a minimum.

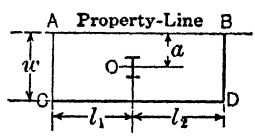


Fig. 3 Elongated Supporting Area. Concentric Load

Elongated Supporting Areas. When the supporting area for an isolated load cannot be made a circle or a square, for example, when the square or the circle would overlap an adjoining property-line or interfere with an adjoining supporting area, the necessary area may frequently be made RECTANGULAR in form, as $ABDC$ (Fig. 3), having a width w, twice the distance a between the center of the load O and the limiting line AB.

The required length l equals the required area divided by w and the area should be centered on O, that is, l_1 must equal l_2.

Combinations of Simple Areas. Two Adjacent Isolated Areas. When adjoining supporting areas overlap or when, for other reasons, it is desirable to COMBINE ADJACENT FOOTINGS, the best arrangement may be obtained as follows: Knowing the supporting area required for each of two adjacent concentrated loads and the distance between the centers of the loads, the sum of the two areas should be divided by twice the distance between the load-centers. The quotient will be the width or the dimension of the required rectangle of support taken at right-angles to the line connecting the load-centers; and the other dimension of the rectangle will be twice the distance between the load-centers. The center of the area should be placed so as to coincide with the center of gravity of the two loads, when it will be found that each load will be concentric with its own area of support. Where a row of columns requires areas which nearly overlap, the COMBINATION OF THE AREAS frequently results in economy in excavation and form-work. Footing widths or projections beyond the column center are very often controlled by the billet or grillage of the column. These can, of course, be modified to a certain extent so as to satisfy the footing dimensions and designs. In this case the economics of the footing and grillage design should be carefully investigated and compared.

Supporting Area for a Concentrated Load in the Line of a Wall. If one or more concentrated loads are carried in the line of a wall the ADDITIONAL SUPPORTING AREAS required for such concentrated load may be provided in either of two ways.

(1) If the concentrated loads rest on the wall, as, for example, when the wall supports the ends of girders and when the conditions are such that the concentrated loads are distributed along given lengths of it, then, all that is necessary is to increase the width of the footing for the given lengths sufficiently to provide for the total of the uniformly distributed and concentrated loads.

(2) If a concentrated load is on the center line of the wall but cannot be distributed by the wall, as when a considerable load is carried by a pier or column to the level of the footings, then one-half the additional area for the concentrated load should be placed on either side of the wall-footing, so that a line connecting the centers of the two areas will pass through the center of the load. In general, it is desirable that the additional areas, together with the area for the wall lying between them, should APPROXIMATE A SQUARE. Knowing the width of the footing required to support the wall and the additional area required to support the concentrated load, the length of the side of the required square can be determined by the following formula (Fig. 4):

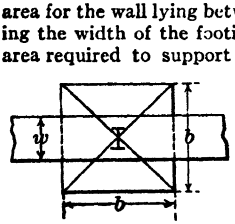


Fig. 4. Square Supporting Area. Wall and Concentric Isolated Load

Let w = the width of the footing;
A = the area required to support the concentrated load;
b = the side of the square which will support a length of wall equal to b, and also provide an additional area equal to A. Then

$$b = w/2 + \sqrt{A + \left(\frac{w}{2}\right)^2}$$

Supporting Area for Concentrated Load not in the Center Line of a Wall. The same additional supporting area is required for this as for a concentrated

load on the center line of a wall, but the total area must be divided unequally between the two sides of the wall-footing, the larger portion being placed on the side of the eccentric load. The simplest way to determine the location of the supporting areas for this combination is to determine the size of the required square as if the concentrated load were concentric with the center line of the wall. The next step is to calculate the load due to the wall for the length of this square and determine the location of the center of gravity of the combined loads, that is, the center of gravity of this wall-load and the concentrated load. The center of the supporting area is then placed concentrically with the center of gravity of the combined loads. In Fig. 5 let

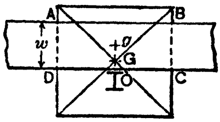


Fig. 5. Square Supporting Area. Wall and Eccentric Isolated Load

w = the required width of the wall-footing;
O = the concentrated load;
A = the area required for the support of the concentrated load. Then, as before, the length of the side of the required square will be

$$b = AB = \tfrac{1}{2} w + \sqrt{A + \left(\frac{w}{2}\right)^2}$$

The center of gravity of the wall-load contained between the lines AD and BC is at g, and the amount of the load is evidently the load per foot multiplied by the distance $AB = b$. Knowing the position and amount of the loads at O and g, the center of gravity of the combined loads is determined, say at G. This fixes the center for the square.

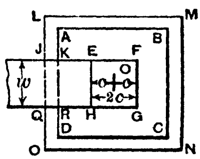


Fig 6 Square Supporting Area Isolated Load on End of Wall

Supporting Area for a Concentrated Load on the End of a Wall. A somewhat different treatment is required for this, but the supporting area may be best determined as follows (Fig. 6): Knowing the width w of the footing required for the support of the wall, the additional area required for the concentrated load O and the distance c from the center of the concentrated load from the end of the wall, proceed in this way. Determine the square whose area corresponds to the sum of the areas required for the support of the concentrated load and for a length of wall equal to twice the projection of the wall beyond the center of the concentrated load. Plot this square $ABCD$ on the foundation-plan and also the total area required for the support of the wall. The square $ABCD$ includes an area sufficient for the support of the concentrated load and for a section of the wall $EFGH$ corresponding to a length of wall equal to twice the projection c multiplied by the width of the footing. It is evident that the area $KEHR$ is loaded both by the wall and the concentrated load; in other words, that the square $ABCD$ is too small by the amount of the rectangle $KEHR$. The required square $LMNO$ will be approximately the square which will contain the original area $ABCD$ plus the area $KEHR$, plus twice the area $JKRQ$. The length of the side $LM = MN$ will be approximately the length of the original square plus one-half of the area $KEHR$ divided by the length of the original square. The resulting square should be moved from the position shown on the drawing so that its center coincides with the center of gravity of the combined concentrated load and the wall-load back as far as the square goes on the wall. A

further approximation may be necessary where accuracy is required. The final result should be that the area of the square *LMNO* should be sufficient to support the concentrated load *O* and that portion of the wall-load *JFGQ* resting on the square, and that the center of gravity of the square should coincide with the center of gravity of the combined loads.

17. Offset Footings

Supporting Areas for Non-Concentric Loads. When walls, columns, or piers are placed close to property-lines the required supporting areas cannot be placed concentrically with the loads without overlapping the property-lines. In such cases recourse must be had to some method which will transfer the loads to supporting areas not concentric with the loads. An attempt to accomplish this result, the method known as OFFSETTING THE FOOTING has been largely used, especially for side walls adjoining property-lines. While theoretically faulty, if not useless, it is indisputable that OFFSET FOOTINGS have generally served the purpose for which they were designed. In the typical construction a cellar wall rests on a course of concrete or of flat stones forming a footing course considerably wider than the wall, the projection being entirely on one side of the wall. The load acting on one side of the center of the footing loads the supporting area unequally. The VARYING LOAD on the supporting area can be calculated as follows: In Fig. 7, let

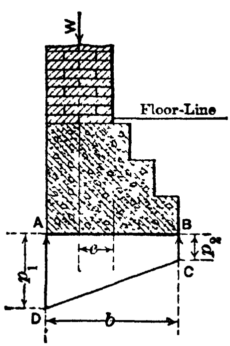


Fig. 7. Offset Footing. Varying Pressure on Foundation-bed

W = the total load per unit of length coming on the supporting area;
e = the eccentricity of load, that is, the distance between the center of the load and the center of the supporting area;
b = the width of the footing = the width of the supporting area = AB;
p_1 = the unit load, or pressure on the foundation-bed at A, the edge of the footing nearest the load;
p_2 = the unit load, or pressure on the foundation-bed at B, the edge of the footing farthest from the load;
y = any ordinate, from A to B.

Then the AVERAGE PRESSURE on the foundation-bed will evidently be W/b. The pressure at A, the edge nearest to the point of application of the load, will be $p_1 = W/b(1 + 6\,/eb)$, or the MAXIMUM LOAD will equal the average load plus six times the average load multiplied by the ratio of the ECCENTRICITY divided by the width of the footing.

Similarly, the pressure at B, the edge farthest from the point of application of the load, will be $p_2 = W/b(1 - 6\,e/b)$, or the MINIMUM LOAD equals the average load minus six times the average load multiplied by the ratio of the eccentricity divided by the width of the footing.

When the ECCENTRICITY equals $\frac{1}{6}$ the width, the pressure at B becomes zero. If the eccentricity exceeds $\frac{1}{6}$ the width there will be an uplift at B, or the footing will have a tendency to overturn. This relation is generally expressed by saying that to avoid an upward reaction the center line of the load must fall within the MIDDLE THIRD of the base.

Load-Diagrams for Offset Footings. If in the diagram (Fig. 8) the figure *ADEC* represents the load-diagram on the foundation-bed for a width of footing *AD* and the load *AC* is the maximum permissible load, then the area *ADEC* represents the maximum support afforded by the footing *AD*. If the width is increased until the load falls on the limit of the middle third or to the width *AB*, then the load at *B* is zero and the support is represented by the triangle *ABC*, the area of which is less than the area *ADEC*. Moreover, if the width of the footing is reduced until its center is concentric with the load-center, then the load-diagram becomes *AFGC*, the area of which is greater than either *ABC* or *ADEC*. From the foregoing it is evident that any advantage gained by offsetting the footing must be obtained at the cost of concentrating the support given to the wall away from the center line of the wall.

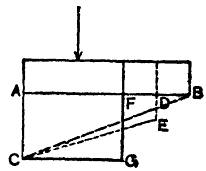


Fig 8 Pressure-diagrams for Footings

Eccentric Loading Due to Offset Footings. In Fig. 9, representing a simple case of ECCENTRIC LOADING due to OFFSET FOOTINGS, the load on the foundation-bed at *E* is perhaps twice the average load and at *F* about zero. Under these conditions the projecting portion of the footing may shear, as indicated, along the line *DG*. If it does not shear and if there is any settlement due to the load, the settlement will be unequal and the footing course will tend

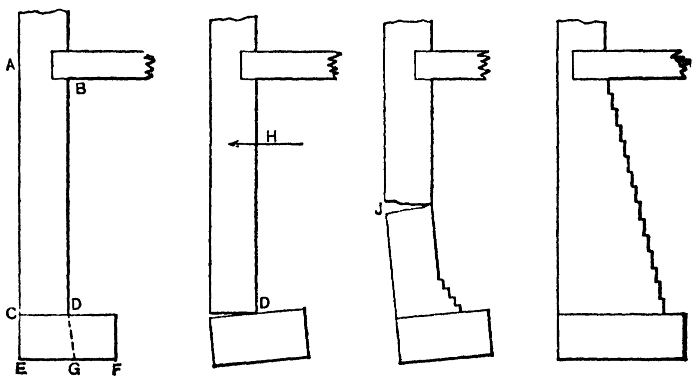


Fig. 9 Fig. 10 Fig. 11 Fig. 12

Figs. 9, 10 and 11. Eccentric Loading and Tendencies to Failure Due to Offset Footings; Fig. 12. Improved Type of Construction

to rotate into the position shown in Fig. 10. The entire load will then be transmitted through the inner lower corner *D* of the cellar wall, rendering the wall unstable and developing a tendency to move in the direction *H*.

The cellar wall may successfully resist this tendency by its own rigidity assisted by the first-floor beams acting as ties or by the external resistance afforded by an abutting wall or bank of earth, or it may partially or completely fail, developing a horizontal crack as indicated in Fig. 11 at *J*.

In this figure it will be noted that the base of the wall itself is offset. This is done to prevent the separate rotation of the footing course; but this con-

struction does not diminish the TENDENCY TO ROTATION of the entire base of the wall and to the formation of a crack at J.

An improved type of construction is illustrated in Fig. 12, in which the floor-beams are anchored into the wall and the cellar wall has a continuous stepped batter from the level of the footing up to the level of the beams. The beams should evidently be arranged as tension-members, should run across the building and should be anchored in the opposite wall. While this method may have some effect it is of doubtful efficacy and should never be used for piers.

18. The Use of Cantilevers in Foundations

Application of the Principle of the Lever. The use of the CANTILEVER, in transferring a load to a supporting area not concentric with the load, is based upon the PRINCIPLE OF THE LEVER and involves a girder or cantilever connecting the two loads, and a supporting area or areas the CENTER OF ACTION of which lies between the two loads Part or all of the load on one side counterbalances the load on the other side of the center of the supporting area.

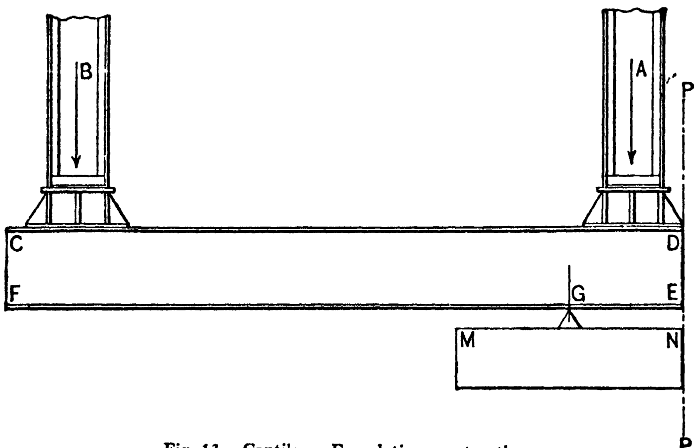


Fig. 13. Cantilever Foundation-construction

Illustrative Example. If an exterior column A (Fig. 13) carrying a load of 400 tons and requiring 100 sq ft of supporting area, at 4 tons per sq ft, the column-center being 18 in from a property-line PP which forms the limit of the building plot, it is evidently impractical to employ a concentric footing 3 by $33\frac{1}{3}$ ft for its support. If, however, a sufficient counterweight can be found in the shape of an adjacent interior column-load, as at B, the exterior load can be transferred by a girder or cantilever construction $CDEF$ to a supporting area MN lying between the two loads, and entirely within the limits of the property.

In Fig. 13 let PP represent the property-line, A the center of the load on column A, and B the center of the load on column B. Let the load on A be 400 tons, on B, 200 tons, and the distance AB between centers, 20 ft. Assume that a rigid girder $CDEF$ supports and connects the two columns. If now a FULCRUM or point of support G is provided for the girder at some point between

A and B, the load on that point can be readily determined from the PRINCIPLE OF THE LEVER by multiplying the load on A, 400 tons, by the distance AB, 20 ft, and dividing the product by the distance BG, 19 ft; or, the load on G = 400 × 20/19 = 421 tons+. The area required for the support of this load, at 4 tons per sq ft, is 421/4 = 105¼ sq ft. The uplift at B, or the part of the load B required to counterbalance the overhanging load A is, from the principle of the lever, the product of the load A by the lever-arm AG divided by the lever-arm BG. The load on the footing for B is the difference between the original load and the uplift; but in view of the possibility of a reduction in the load A, which would decrease the uplift at B, it is well to provide for a possible increase in the load B.

Determination of the Area of Support. In determining the AREA OF SUPPORT for A, having assumed one dimension of the supporting area to be twice the distance GP, or say 5 ft, the other dimension will be 105¼ sq ft/5 = 21 ft ½ in. If the length 21 ft ½ in, as determined, is found to be excessive, then the point G must be moved to the left and the corresponding length of the supporting area must be determined as before When the length of the supporting area for the fulcrum of the cantilever is limited, so that the length parallel to the property-line is fixed, the width of the area can be determined experimentally or by the use of the formula

$$b = (L + a) - \sqrt{(L + a)^2 - 2\,WL/lp}$$

in which L = the distance between centers of the two loads:
W = the load nearest to the property-line;
l = the length of the supporting area;
p = the unit load on the supporting area; and
a = the distance between the center of action of the load to be cantilevered and the edge of the supporting area nearest to the property-line.

If the position of the center of gravity of the load A combined with that part of the load on B which is borne by the cantilever is determined, it will be found to coincide with the fulcrum or point of support G of the cantilever, thus demonstrating that the use of the cantilever provides a means of combining two loads so that their center of gravity falls on the center of a supporting area not concentric with either load.

The Grillage Fulcrum. Of course in practice the KNIFE-EDGE FULCRUM shown in the diagram is not used. The bottom flange of the girder forming the cantilever rests on the DISTRIBUTING GRILLAGE directly, as is shown in Fig. 14, which may be considered a typical arrangement.

The Girdering-Method for Two Equal Loads. When it is desirable to support two or more adjacent concentrated loads on a single supporting area the method called GIRDERING is employed. In the case of two concentrated loads, let A and B (Fig. 15) represent two columns. Let W_1 represent the load on A and W_2 represent the load on B. Let L represent the distance between the centers of the two loads. Let G represent the center of gravity of the combined loads. Let p represent the allowable unit load on the foundation-bed. The required area of support will be $(W_1 + W_2)/p$. This area may be of any desired shape, provided that its center of gravity coincides with the center of gravity of the combined loads at G. In general, however, the most economical arrangement will result when each load is as nearly as possible over the center of gravity of its own required area. If, however, this is impracticable, as for

example, when either column is near a property-line or an adjoining footing, it will be necessary to distribute the loads of both columns over the area lying between the two columns. In the case of two columns equally loaded, as in Fig. 15, the distance a, from the center of column A to the property-line PP, determines the maximum allowable extension beyond column A. The dimensions of the area are obtained by making the length l of the footing equal to the distance L between the columns plus twice the extension a. Knowing the length of the required area the width b is determined by simple division.

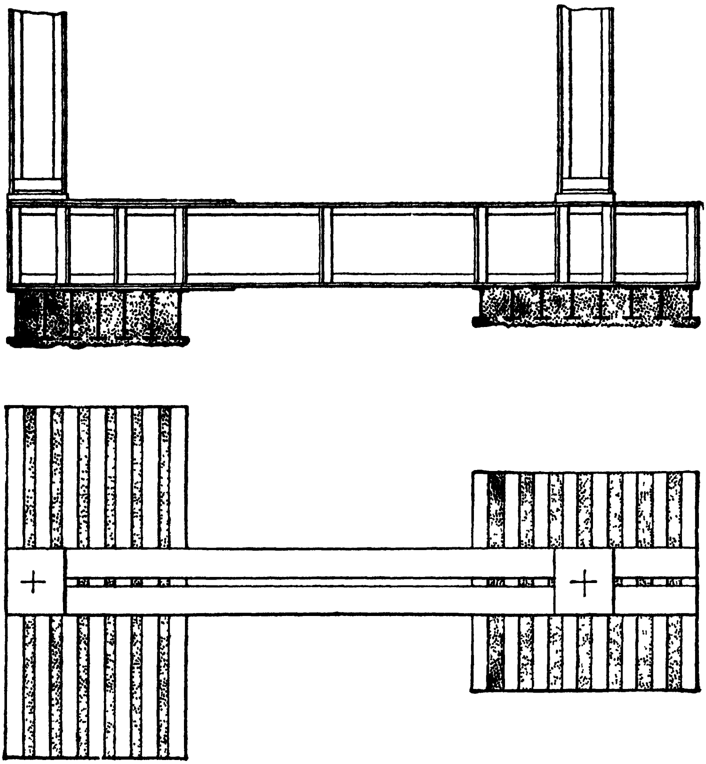


Fig. 14. Cantilever Foundation. Grillage Fulcrum

The Girdering-Method for Two Unequal Loads. In the case of columns not equally loaded, the SUPPORTING AREA may be a TRAPEZOID, as in Fig. 16, the center of gravity of which must coincide with the center of gravity of the two loads. Knowing the sum of the two loads and the required area for their support, and fixing the total length l of the footing in accordance with the requirements that the footing shall not project beyond the line PP, the widths of the footing at the small and large end b_1 and b_2, respectively, can be determined as follows: Let B represent the distance from the small end of the

trapezoid to the center of gravity of the two loads and let A represent the area of the trapezoid. Then

$$b_2 = 2\,A/l(3\,B/l - 1)$$

and

$$b_1 = 2\,A/l(2 - 3\,B/l)$$

also

$$A = (b_1 + b_2)l/_2 \quad \text{and} \quad b_1 + b_2 = 2\,A/l$$

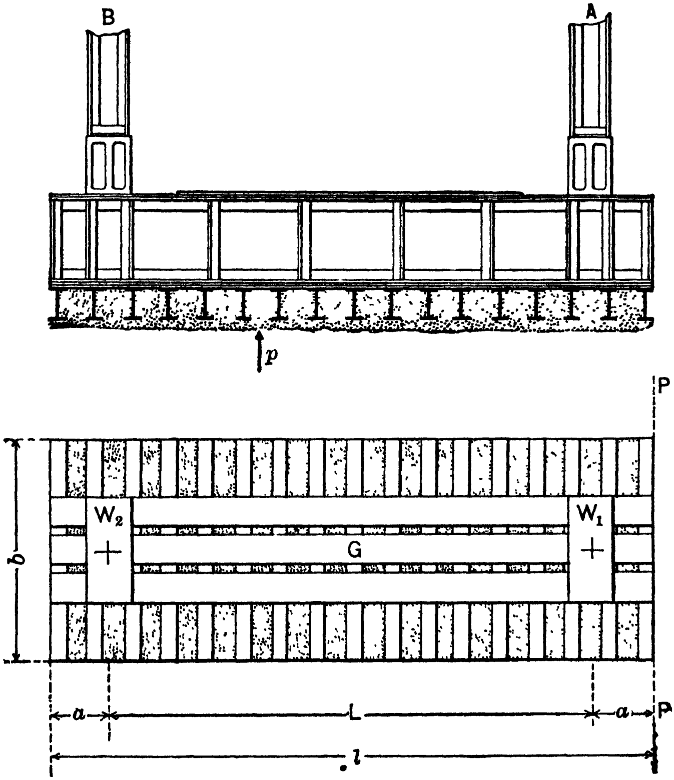


Fig. 15. Girdering-method of Foundations. Two Equal Loads

Cantilevering an Exterior Wall. In the case of a wall the same principles apply, but the cantilevering effect must be distributed along the length of the wall. This can be accomplished by placing a girder under the wall, the girder in turn resting on the cantilever, or by using a number of cantilevers arranged in fan shape and radiating from the interior load-center. In narrow buildings the cantilevers may run from wall to wall.

Double Cantilevering. The considerations controlling the design of the supporting areas required are the same as outlined in the preceding paragraphs.

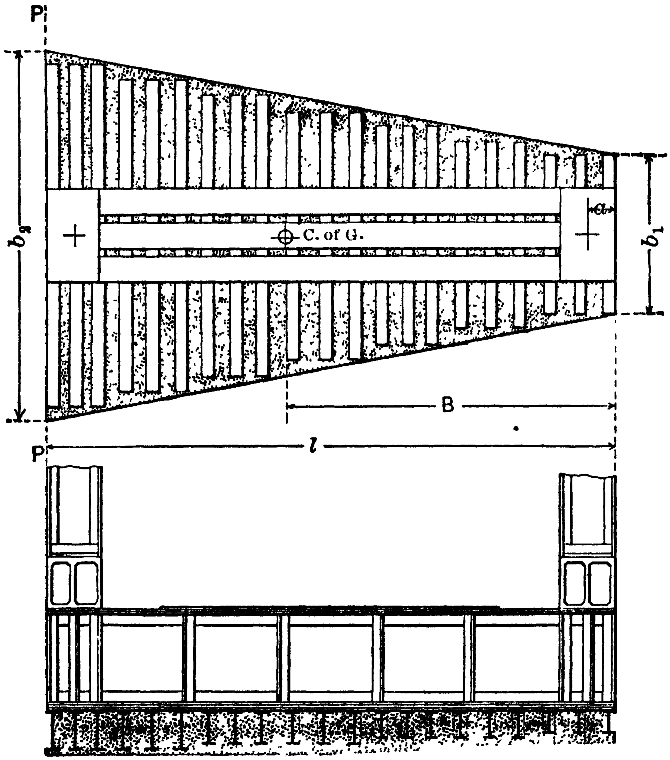


Fig. 16. Girdering-method of Foundations. Two Unequal Loads

19. Stresses on Footing Courses

Size and Form of Footing Courses. The footing courses of all walls and piers should be larger than the superimposed construction in order to secure STABILITY AGAINST OVERTURNING and to reduce the UNIT LOAD on the foundation-bed. When the change in size is accomplished abruptly as when a wall rests on a grillage or a slab of plain or reinforced concrete the footing is called a SPREAD FOOTING. When the base of the wall is thickened by means of offset courses so that its bottom course is substantially as large as the footing course the construction is known as a STEPPED FOOTING. It is evident that no hard and fast line can be drawn between the two classes. Whatever the form of the footing is it must be strong enough to distribute the more or less concentrated load coming on it, into a uniform pressure or load on the foundation-bed.

The Unit Loads of Footing Courses. If the load on the upper surface of a footing course is uniformly distributed the intensity of the load, or in other words the UNIT LOAD ON THE FOOTING, is obtained by dividing the total load by the area of the base of the wall, pier, or other construction at that level. The load on the foundation-bed should be UNIFORMLY DISTRIBUTED and in fact, if the foundation-bed is compressible and the load concentric with the supporting area, it may safety be assumed as uniform, since a compressible material will adjust itself until the loading at different points is substantially uniform. The unit load on the foundation-bed is evidently the total load divided by the supporting area. If the area of the footing course varies between the top and bottom of the footing the INTENSITY of the load will vary, and if uniformly distributed, the unit load at any level is obtained by dividing the total load by the area of the footing at that level.

The Weight of the Footing Itself. This is generally so small when compared with the superimposed loads that it may be ignored without serious error.

The Transmitting of Loads by Footings. If we neglect the weight of the footing we can consider the footing course as transmitting the imposed load to the foundation-bed or as being subject to two equal loads; one, the SUPERIMPOSED LOAD, more or less concentrated on the center line of the footing and acting downward; the other, the REACTION due to the loading of the foundation-bed assumed uniformly, distributed over the supporting area and acting upward. These loads or forces being equal and opposite in direction the stresses developed in the footings are due to the differences in the distribution of these loads, and the footing courses simply act to convert concentrated into distributed loads.

Manner of Failure of Footings. A footing may fail in several ways: (1) by SHEARING; (2) by direct CRUSHING; (3) by SPREADING; and (4) by BENDING or RUPTURE.

(1) Failure of Footings by Shearing. This is illustrated in Fig. 17, showing a wall the weight of which has caused it to SHEAR along the lines *EG* and *FH*.

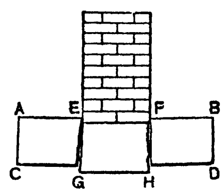


Fig. 17. Failure of Footing by Shearing

The force tending to cause SHEAR is the weight due to the wall less the reaction of the foundation-bed acting on the under side of the section *EFGH*. Since the load is supposed to be uniformly distributed this is equivalent to the product of the area corresponding to the width *CD* minus the width *GH* times the length of the wall considered, by the unit loading on the foundation-bed.

For a 1-ft length of wall the force causing shear, V, is

$$V = W\,(l - w)/l$$

in which W = the load due to wall per foot of length in pounds;
l = the width of footing;
w = the width of base of wall.

Or, since $W/l = p$ = the unit load on the foundation-bed in pounds per square foot,

$$V = p\,(l - w)$$

Since p is in terms of feet, l and w also must be in feet. The resistance to

shear, R, under the conditions illustrated in Fig. 17, taken for a 1-ft length b of the wall, is determined by equation

$$R = 2 \times d \times b \times v$$

in which v = the safe resistance of the material to shear, in pounds per square inch;
d = the depth of the footing in inches; and
b = the length of wall considered = 12 in.

Placing $V = R$, we have

$$2\,dbv = p\,(l - w)$$

Or, since $(l - w)/2$ = the projection of the footing

$$cp = 12\,dv$$

The depth of the footing, therefore, must not be less than

$$d = cp/12\,v$$

in which c is in feet.

Shear in Footings of Piers and Columns. FAILURE BY SHEAR is most likely to occur in footings for piers and columns. The FORCE TENDING TO CAUSE SHEAR is the total load on the column or pier less the reaction of the foundation-bed on the area immediately under the column-base. The resistance offered is determined by multiplying the perimeter of the column-base by the depth of the footing and by the allowable unit shear. When the area of the column-base is small, the entire load may be taken as producing shear. When reinforced concrete is used for the footing, there must be a sufficient number of stirrups to take care of the shear. Where steel beams are employed the cross-section of the beams must be sufficient to take care of the shear, otherwise additional web-plates should be added.

(2) Failure of Footings by Direct Crushing. The failure of footings by DIRECT CRUSHING of the materials composing the footings rarely, if ever, occurs. Where, however, the concentrated load, due to a pier or column, is distributed by beams or girders which have thin webs, the webs may fail by BUCKLING. Such beams or girders should have their webs reinforced by vertical STIFFENERS or by additional WEB-PLATES, and the spaces between the beams or girders should be filled with concrete or grout. Where the load transmitted by the column-base exceeds the safe unit load on the material of the footing the area of the column-base may be increased, or a block of granite may be interposed between the concrete or masonry footing and the base of the columns. In this case, however, such granite blocks should be considered as a footing course and designed to resist bending, by formulas hereinafter given.

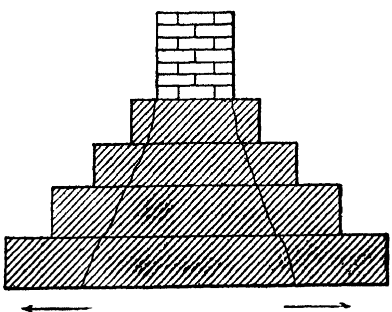


Fig. 18. Failure of Footing by Spreading

(3) Failure of Footings by Speading. Failure of the footings by SPREADING may occur under walls or piers, as shown in Fig. 18, especially when the foundation-bed is of clay or other yielding material, which has, under the load

of the footing, a tendency to FLOW along the lines indicated by arrows in the figure. This tendency should be provided against by making the bottom layer continuous and adequate to resist the tension. Vertical joints, such as are made in footings composed of masonry, are sources of weakness, and should be avoided. The TENDENCY TO SPREAD is greatest in footings having a spread which is wide compared with the width of the superimposed wall or other construction. The writer knows of at least one important footing which has failed in this way, the cracks in general following the joints of the masonry substantially as shown in Fig. 18.

(4) **Failure of Footings by Bending or Rupture.** A footing may fail by BENDING OR RUPTURE as a beam or girder. In the case of a wall, if the footing bends, as shown in Fig. 19, the concentration of the load on the lower edges

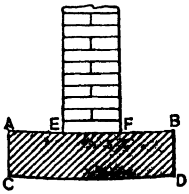


Fig. 19

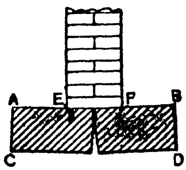


Fig. 20

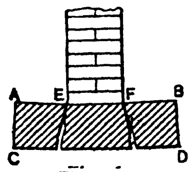


Fig. 21

Figs. 19, 20 and 21. Failures of Footings by Bending

of the wall, as at E and F, may cause the base of the wall to fail. This possibility should be borne in mind in designing footings where the load on the wall approaches the allowable unit load for the material composing it, and especially where the width of the footing is much greater than its own width. If the footing fails by RUPTURE the rupture may occur either under the center line of the wall, as in Fig. 20, or at points close to the outer edge of the wall, as in Fig. 21. Fig. 20 illustrates the objection to using a footing course composed of masonry or stones which do not extend the full width of the footing. The joints in such construction prevent the footing course from acting in TENSION and the footing as a whole from acting as a BEAM.

20. Methods of Calculating Bending-Stresses in Wall-Footings

Assumptions Made in Determining Bending-Stresses in Footings. Two methods for the calculation of the BENDING-STRESSES IN FOOTING COURSES are in general use. Both are based upon the assumption that the REACTION of the foundation-bed is UNIFORM; but the methods differ in the assumption made as to how the footing course and the base of the superstructure act. Neither assumption can be held to be wholly correct.

The First Method of Determining Bending-Stresses in Footings. This method is based upon the assumption that the pressure of the wall on the footing is uniform over the area and remains so at all times.

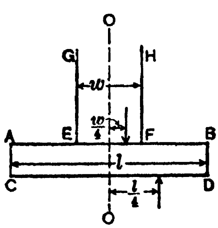


Fig. 22. Bending-stresses in Footings. First Method

If, in Fig. 22, $ABCD$ represents a footing course supporting a centrally located wall $EFGH$, and if

W = the load of the wall in pounds per linear foot;
w = the width of the wall in feet;
and l = the width of the footing in feet;
then $\frac{1}{2}(l - w)$ = the projection AE or FB,
and $W/l = p$ = the unit load per square foot on the foundation-bed.

Considering the forces acting on the right of the center line of the wall for a 1-ft length of wall, it is evident that the uplift on the half-footing OD will equal $\frac{1}{2} W$ and that its CENTER OF ACTION will lie half-way between O and D, or at a distance $\frac{1}{4} l$ from the center line OO; and, similarly, that the load due to one-half the wall will be $\frac{1}{2} W$ and that its CENTER OF ACTION will be at a distance $\frac{1}{4} w$ from the center line OO. The resulting moments will be

$$M_1 = \tfrac{1}{2} W \times \tfrac{1}{4} l = \tfrac{1}{8} Wl$$

and

$$M_2 = \tfrac{1}{2} W \times \tfrac{1}{4} w = \tfrac{1}{8} Ww$$

and as these two moments act in opposite directions, the resultant moment tending to produce bending in the footing will be the difference between the two, or the bending moment at the center line OO is

$$M_0 = M_1 - M_2$$

or

$$M_0 = \tfrac{1}{8} W (l - w)$$

Or, since

$W/l = p$ and $\frac{1}{2}(l - w) = c$, the projection,

Equation (1) may be written in either of the forms

$$\left.\begin{aligned} M_0 &= \tfrac{1}{8} p (l - w) l \\ \text{or} \quad M_0 &= \tfrac{1}{4} Wc \end{aligned}\right\} \qquad (1)$$

The Error Involved in this first method is due to the assumption that the pressure on the upper surface of the footing remains UNIFORMLY DISTRIBUTED, as if the base of the wall acted as a FLUID, in which case the distribution of the load would remain constant and the formula would be correct. But the base of the wall is not a FLUID, but a SOLID which will resist DEFORMATION. If, as in Fig. 19, the footing course $ABCD$ deflects and the base of the wall is assumed to be incompressible, the entire load of the wall will be communicated to the footing through the edges E and F. While such a concentration is, of course, impossible (as the edges E and F will crush or compress until a considerable area of the base of the wall is in contact with the footing) the result is that the weight of the wall is concentrated near the outer edges of its base. Equation (1) gives results which are too large; but as it errs on the side of safety, it is recommended for general use.

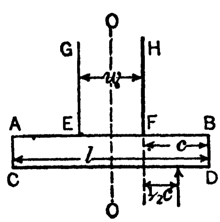


Fig. 23. Bending-stresses in Footings. Second Method

The Second Method of Determining Bending-Stresses in Footings, also in common use, takes into consideration only the projecting portion of the footing as follows:

If in Fig. 23 $ACBD$ represents a footing course supporting a centrally located wall $EFGH$, and if we use the notation of the preceding method, then, if we assume that the footing acts as a FIXED BEAM and the projections AE and FB as CANTILEVERS

rigidly supported by the wall, and denote the projection of the footing on either side of the wall by c the reaction of the foundation-bed on this projecting portion c, per unit length of wall, will be pc. The CENTER OF ACTION of this force will be at a distance $\frac{1}{2} c$ from E or F and its moment at E or F will be

$$M = cp \times \tfrac{1}{2} c = \tfrac{1}{2} pc^2$$

or, since

$$c = \tfrac{1}{2} (l - w)$$

the value of M may be given in the form

$$M = \tfrac{1}{8} p (l - w)^2 \qquad (2)$$

The Error Involved in this second method is due to the assumption that the uplift on the projection P can be resisted by the extreme outer edge of the base of the wall. If the uplift on the projecting part is concentrated on the edge, then the edge must either compress or fail by crushing, which, in either case, would throw the center of support for the cantilever back from the edge of the wall; and this is contrary to the assumption used in calculating the moment. This method takes into consideration only the intensity of the reaction or uplift and the length of the projection, and is known as the PROJECTION-METHOD.

Comparison of Results. Comparing the results of the two methods, it will be seen that the load cannot act at the two edges E and F as assumed in Equation (2), nor ordinarily can it be uniformly distributed as assumed in Equation (1), but that the INTENSITY OF THE LOAD PER UNIT OF AREA will vary, being a MINIMUM at the center and a MAXIMUM near the edges of the base of the wall. The exact positions of the CENTERS OF ACTION are affected by various considerations which cannot be fully discussed in this chapter.

New Formula for Determining Bending Moments in Footings. The writer has devised a formula which gives values for the bending moment M half-way between the values given by Equations (1) and (2), and which closely corresponds to the assumption that, considering the forces on either side of the center of the wall, the CENTER OF ACTION of the half-load of the wall is at the center of the half-wall, when the projection equals zero, and, as the projection increases, moves toward a position which is two-thirds of the distance from the center of the wall to its edge. This formula may be expressed as follows:

$$M = \tfrac{1}{8} p (l - w) (l - \tfrac{1}{2} w) \qquad (3)$$

Or, substituting the value of p in terms of W,

$$M = \tfrac{1}{8} W (l - w) (1 - w/2\, l)$$

Weight and Pressure-Units. In practice W, the weight due to the wall, is generally given in pounds per linear foot of wall, and the allowable pressure on the foundation-bed, while frequently given in tons per square foot, should be reduced to pounds per square foot.

The Required Width of the Footing in feet is obtained by dividing the weight of the wall in pounds per linear foot of wall by the allowable unit load on the foundation-bed expressed in pounds per square foot.

Moment-Units. The moment tending to produce rupture may be calculated in foot-pounds or inch-pounds. If in Equations (1), (2) and (3) the dimensions l, w and c are in feet and p is in pounds per square foot, the resulting bending moment will be in foot-pounds per linear foot of wall. As the MOMENT OF RESISTANCE is generally stated in inch-pounds it is more con-

venient to have the MAXIMUM BENDING MOMENT OF MOMENT OF RUPTURE * in inch-pounds. Thus, for Equation (1)

$$M \text{ (in inch-pounds per foot of wall)} = 12\,M \text{ in foot-pounds,}$$

or
$$M \text{ (in inch-pounds)} = \tfrac{3}{2}\, p\,(l - w)\,l \qquad (1)'$$

Equation (2) in the same way becomes

$$M \text{ (in inch-pounds)} = \tfrac{3}{2}\, p\,(l - w)^2 \qquad (2)'$$

Or, using the more convenient form,

$$M = \tfrac{1}{2}\, pc^2$$

if we express the projection c in inches, instead of in feet, we will have

$$M \text{ (in inch-pounds per foot of wall)} = \tfrac{1}{24}\, pc^2$$

Similarly, Equation (3) becomes

$$M \text{ (in inch-pounds per foot of wall)} = \tfrac{3}{2}\, p\,(l - w)\,(l - \tfrac{1}{2}w). \qquad (3)'$$

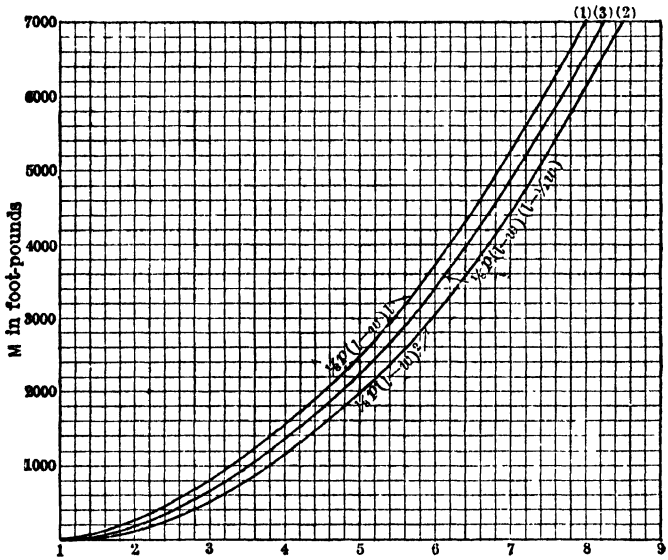


Fig. 24. Graphical Comparison of Bending Moments in Footings

Until Equations (3) or (3)′ are more generally accepted, an engineer or designer will avoid criticism and be perfectly safe in using Equation (1), and in the following pages the writer will use Equations (1) or (1)′ unless the contrary is stated.

Example. The following is an example illustrating the application of the foregoing formulas:

* In the flexure-formula the moment of resistance is made equal to the bending moment at any cross-section of the footing, and the maximum bending moment is sometimes called the moment of rupture.

A 24-in wall transmits to the footing 42 000 lb per linear foot of wall. The allowable unit load on the foundation-bed is 3 600 lb per sq ft. What is the width and required MOMENT OF RESISTANCE * of the footing?

$$42\,000/3\,600 = 11\tfrac{2}{3} \text{ ft}$$

Then, by Equation (1), we have

$$M = \tfrac{1}{8} \times 3\,600\,(11\tfrac{2}{3} - 2)\,11\tfrac{2}{3} = 50\,750 \text{ ft-lb}$$

If Equation (2) is used, we have

$$M = \tfrac{1}{8} \times 3\,600\,(11\tfrac{2}{3} - 2)^2 = 42\,050 \text{ ft-lb}$$

and by Equation (3),

$$M = \tfrac{1}{8} \times 3\,600\,(11\tfrac{2}{3} - 2)\,(11\tfrac{2}{3} - 1) = 46\,400 \text{ ft-lb}$$

Comparing the results we see that the moment by Equation (3) is the average of the moments by Equations (1) and (2).

Graphical Comparison of Bending Moments in Footings. Fig. 24 is a graphical comparison of the moments for varying ratios of l to w calculated by Equations (1), (2) and (3) on the assumption that

w = the width of wall = 1 ft;
p = the unit load on the foundation-bed = 1 000 lb per sq ft; and
$r = l/w$.

The load on the wall, in pounds, for any value of l, is 1 000 l.

Comparing the curves of Equations (1) and (2) it will be seen that the results are widely apart, the percentage of variation being highest in the case of small projections. When l is less than twice w, or in other words, when the projection is less than one-half the width of the wall, Equation (2) gives moments less than half the moments given by Equation (1). Equation (2) may be used for small projections. Equation (1) gives results which are too large, especially where the projections are small. Equation (3), giving results half-way between those of Equations (1) and (2) and in accordance with a reasonable hypothesis, would appear to be preferable, but is not in accordance with present practice.

21. Bending Moments in Footings of Columns and Piers

General Statement of the Problem. Fig. 25 represents in plan a pier or column resting on a footing which projects on four sides. The base of the column or pier is represented by $ABCD$, and the footing and its area of support by $EFGH$. That part of the footing included in the areas $MNOP$ and $QRST$ can be considered as acting in the same way as projecting footings under a wall, but the uplift on the four corners $EQMA$, etc., on which no superimposed wall-load is imposed, also causes bending moments.

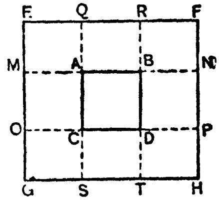


Fig. 25. Plan of Column-footing with Four Equal Projections

Different Theories. There are several theories, more or less complicated and unsatisfactory, as to how the UPLIFT ON THE FOUR CORNER-AREAS should be determined. The discussion of these theories would be out of place in this chapter. In a square footing, if the projection is not over one-half the width of the superimposed

* In the flexure-formula the moment of resistance is made equal to the bending moment at any cross-section of the footing, and the maximum bending moment is sometimes called the moment of rupture.

base, the four corner-areas will not aggregate over 25% of the total area of the footing, and it may then be assumed that the bending moment is the same as if the base of the column or pier extended like a wall across the entire footing, as is shown in Fig. 26. To insure these conditions, when the projection of the footing exceeds $\frac{1}{2}w$, and in all cases when the footing is not homogeneous, as when a grillage of steel is used, the load of the column must be distributed over the width of the footing by a GIRDER or BOLSTER or by an extension of the column-base. In case the footing is in several layers, each layer must extend the full width of the underlying layer. With such construction it is evident that the bending moment will be the same as if the GIRDER or BOLSTER were a wall and Equation (1) will be applicable.

Bending Moments in Column-Footings. For column-footings Equation (1) can be used, taking the total load in place of the load per foot, and the result will then be the total bending moment.

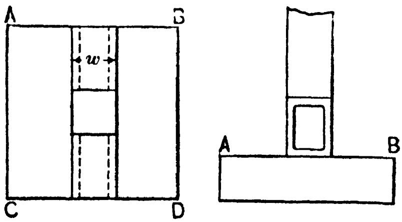


Fig. 26. Column-footing Treated Like Wall-footing

Example. A column carrying 96 tons is to be supported on a square concrete slab. The cast-iron column-base is 2 ft square. The allowable pressure on the foundation-bed is 6 tons per sq. ft. What is the MAXIMUM BENDING MOMENT in the slab?

The area of support = 96/6 = 16 sq ft = 4 by 4 ft. The projection is $\frac{1}{2}(4 - 2) = 1$ ft, or one-half the width of the base, and by the foregoing rule we can calculate the bending moment as if the base of the column extended in one direction across the footing. Applying a convenient form of Equation (1)

$$M = \frac{1}{8} \times 192\,000 \text{ lb } (4 - 2) = 48\,000 \text{ ft-lb, or } 576\,000 \text{ in-lb}$$

The footing must therefore be of sufficient depth to resist this bending moment.

If in this example the allowable unit pressure on the foundation-bed is 2 tons instead of 6 tons per sq ft, the supporting area and the area of the bottom concrete footing course will be 96/2 = 48 sq ft. If the footing course can be a square its dimensions will be, with sufficient exactness, 7 by 7 ft. By the rule given, since the projection exceeds one-half the width of the base, there should be a BOLSTER extending across the footing. The bolster will be, therefore, 7 ft long and may properly be composed of two or more steel beams. The cast-iron base may be dispensed with, in which case the base of the column will be provided with a steel base or with flange-angles. Let us assume that the column-base is 1 ft 6 in square and the width of the bolster 2 ft.

The bending moment in the bolster is determined, then, by Equation (1), using $1\frac{1}{2}$ ft, the width of the column-base, for w, and 7 ft, the length of the bolster, for l.

$$M = \frac{1}{8} \times 192\,000\,(7 - 1\tfrac{1}{2}) = 132\,000 \text{ ft-lb} = 1\,584\,000 \text{ in-lb}$$

The bending moment in the slab is determined in the same way by Equation (1), using 2 ft, the width of the bolster, for w, and 7 ft, the length of the slab, for l.

$$M = \frac{1}{8} \times 192\,000\,(7 - 2) = 120\,000 \text{ ft-lb} = 1\,440\,000 \text{ in-lb.}$$

Footings Other than Square in Plan. In case it is necessary to use some other shape than a square for the supporting area the resulting moments in

the slab and bolster will vary from those calculated above. If in the foregoing example the supporting area, for any reason, is necessarily made 6 by 8 ft, giving 48 sq ft as the required area, and if the bolster is parallel with the 6-ft side, the moment in the bolster will be

$$M = \tfrac{1}{8} \times 192\,000\,(6 - 1\tfrac{1}{2}) = 108\,000 \text{ ft-lb} = 1\,296\,000 \text{ in-lb}$$

and the moment in the slab will be

$$M = \tfrac{1}{8} \times 192\,000\,(8 - 2) = 144\,000 \text{ ft-lb} = 1\,728\,000 \text{ in-lb}$$

or, the moment in the bolster is less and the moment in the slab is greater than in the case of the 7 by 7-ft supporting area. If the bolster runs parallel with the long side, the moments will be, for the bolster,

$$M = \tfrac{1}{8} \times 192\,000\,(8 - 1\tfrac{1}{2}) = 156\,000 \text{ ft-lb}$$

and for the slab,

$$M = \tfrac{1}{8} \times 192\,000\,(6 - 2) = 96{,}000 \text{ ft-lb}$$

In footings having more than two layers, each layer must be investigated separately, using l for the length of the layer which is being determined and w for the width of the superimposed layer.

Compound Footings. In COMPOUND FOOTINGS where, for example, a wall and a column or two or more columns are supported by a single footing, or where loads are cantilevered, the loads will in general be distributed to the supporting area by GIRDERS or CANTILEVERS. The shears and bending moments of such girders or cantilevers must be determined for each case by the methods used in the calculations of beams and girders.

22. Design of the Footings

Materials used for Footings. To possess the required strength the SAFE MOMENT OF RESISTANCE of the footing must be at least equal to the MOMENT OF RUPTURE, calculated as explained in the preceding paragraphs. Masonry, whether of brickwork or stone, is not generally suitable for any but the lightest buildings, as its tensional strength is low. Concrete, plain or reinforced, or grillages of steel embedded in concrete, are generally employed.

Footings of Homogeneous Slabs. If the footing is composed of a SLAB OF HOMOGENEOUS MATERIAL, as a block of granite or other reliable building stone, or a single layer of concrete, the MOMENT OF RESISTANCE is, by the well-known flexure-formula for rectangular cross-sections, $M_r = \tfrac{1}{6} bd^2 f$, in which

d = the depth or thickness of the footing, in inches;
b = the breadth of the footing, in inches;
f = the allowable unit tensile stress of the material, in pounds per square inch;
M_r = the moment of resistance

Placing M, the moment of the forces tending to cause rupture, equal to M_r, for a length of wall equal to 1 foot we have

$$b = 12 \text{ in}$$

and

$$d^2 = \tfrac{1}{2} M/f \tag{4}$$

Substituting in Equation (4) the value for M in inch-pounds as determined by formulas (1), (2) and (3) and a value for f as given in the following paragraph, the required depth d can be determined.

Safe Tensional Strength for Materials in Footings. The values of f, the ALLOWABLE UNIT TENSILE STRESS, for concrete or stone must include a high FACTOR OF SAFETY, as experiments show wide variations in the tensional strength and in the MODULUS OF RUPTURE OR FLEXURAL STRENGTH of such materials. The following values for f in pounds per square inch include a factor of safety of from 8 to 10 and should not be exceeded.

	f in lbs per sq in
For brickwork or masonry in lime mortar	from 0 to 10
For brickwork or masonry in cement mortar.....	from 10 to 40
For concrete, 1 : 3 : 6.....	from 15 to 25
For concrete, 1 : 2½ : 5......................	from 20 to 40
For concrete, 1 : 2 : 4...	from 30 to 50
For sandstone or limestone in monolithic blocks...	from 75 to 150
For granite in monolithic blocks	from 100 to 250

Example of Concrete-Footing Design. Concrete Cast as a Unit. A concrete footing course 4 ft wide supports a wall 2 ft thick. The load on the foundation-bed is 28 000 lb per lin ft of wall, or 7 000 lb per sq ft. Assuming a value for f of 35 lb per sq in, what is the required depth for the concrete footing course?

The moment of rupture from one form of Equation (1) is

$$M = \tfrac{3}{2} W (l - w), \quad \text{or} \quad \tfrac{3}{2} \times 28\,000\,(4 - 2) = 84\,000 \text{ in-lb}$$

Substituting in Equation (4)

$$d^2 = \tfrac{1}{2} \times 84\,000/35 = 1\,200, \quad \text{or} \quad d = 35 \text{ in}$$

By Equation (2)′ the moment of rupture is

$$M = \tfrac{1}{24}\,pc^2 = \tfrac{1}{24} \times 7\,000 \times 12 \times 12 = 42\,000 \text{ in-lb}$$

and

$$d^2 = \tfrac{1}{2} \times 42\,000/35 = 600, \quad \text{or} \quad d = 24 \text{ in} +$$

The depth determined by Equations (1) or (1)′, as previously noted, errs on the side of safety. The result by Equations (2) or (2)′ conforms more nearly with usual practice, and as the projection is small compared with the width of the wall, it may be used, or an intermediate value, as determined by Equations (3) or (3)′, may be considered amply safe.

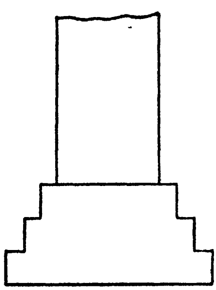


Fig. 27. Concrete Stepped Wall-footing

Stepped Footings. If the concrete footing is cast in one uninterrupted operation so as to act as a SINGLE GIRDER for its entire depth, a considerable saving of material may be effected by forming steps, as shown in Fig. 27. If the steps are of equal height the total projection should be equally divided between the steps. If the footing is cast in several layers, or if a granite slab is superimposed on a bed of concrete, then each layer must be figured separately and the width of the superimposed layer used in place of w, the width of the wall.

Caution in Design of Footings of Several Layers. Equation (2) should not be used where the footing consists of several layers, as the error due to the erroneous assumption is cumulative and would result in a serious concentration on the outer edges of the upper layers.

Example of Footings of Several Layers. In the case of footings cast in separate layers the calculations should be made as follows: Let l_1 = the

length of the footing having a moment, M. From Equation (1), reduced to inch-pounds,

$$l_1 = \frac{2\,M}{3\,W} + w$$

Having decided on the depth of each layer, say 15 in, and a value of f, say 35 lb, for concrete, then, from the flexure-formula, $M = M_r = \frac{1}{6} \times 12 \times 15^2 \times 35 = 15\ 750$ in-lb, which, substituted in the above equation, will give the value of l_1, or the length of the top course. Having determined l_1, the length of the second course, l_2, is found in the same way, using l_1 for w, and so on until the required width of the footing is reached. The dimensions l and w are to be taken in feet.

Comparison of Unit and Separate-Layer Footings. Footings made in separate layers are very uneconomical in the amount of material required, when compared with those cast in one operation. If the footing in the previous example is designed on the separate-layer basis and the courses assumed to be 15 in thick, their lengths are as follows:

$$l_1 = \frac{2\,M}{3\,W} + w = [(2 \times 15\ 750)/(3 \times 28\ 000)] + 2 = 2\ 375\text{ ft}$$

Also

$$l_2 = 2.75\text{ ft},\quad l_3 = 3.125\text{ ft},\quad l_4 = 3.50\text{ ft}\quad\text{and}\quad l_5 = 3.875\text{ ft}$$

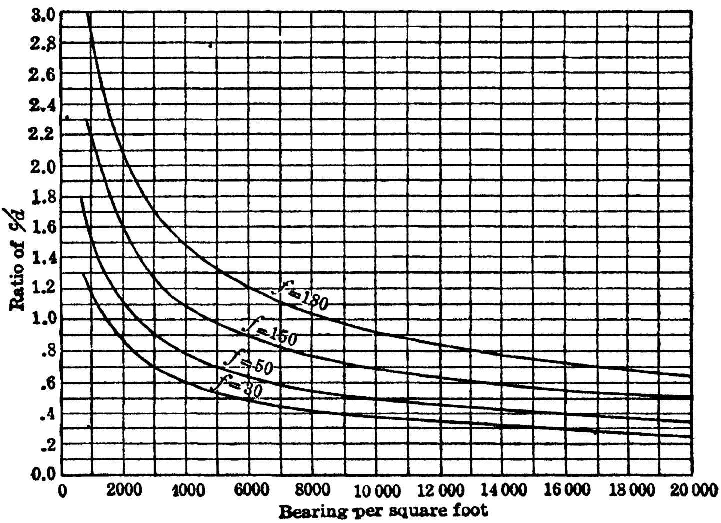


Fig. 28. Diagram Showing Ratio of Projection to Depth of Footings

As l_5 is nearly 4 ft, the required length, it may be made so by increasing the thickness of the bottom course to 16 in. The total thickness of the footing is therefore (4 × 15 in) + 16 in = 76 in instead of 35 in, as previously determined by Equation (1) for the footing cast as a unit.

Rule-of-Thumb Methods for Projections and Steps in Footings. Various ARBITRARY RULES are in use which purport to give for different materials of

construction so-called SAFE PROJECTIONS for given depths of footing or to give the SAFE RATIO between the projection and the depth of a footing. These rules ignore the fact that the uplift varies and they are entirely unreliable, although such RULES-OF-THUMB are often incorporated in the building codes of cities.

Example. The safe projection for offsets in brickwork is frequently given in building codes and in text-books as 3 in for a double course of bricks or for a depth of about 5 in, the corresponding ratio being 0.6. If we assume the value of f for brickwork at 20 lb per sq in, this offset will be safe when the uplift is less than 2 666 lb per sq ft, but not safe when the uplift is over 2 666 lb per sq ft.

Ratio of Projection to Depth of Footing. For footings of homogeneous material, however, having a small projection and where Formula (2) can be used safely, it is possible to calculate a so-called SAFE RATIO OF PROJECTION for a given unit load. From Equation (2)′ and Equation (4), derived from the formula for the MOMENT OF RESISTANCE for beams of homogeneous material and rectangular cross-section, the following formula may be derived:

$$c/d = \sqrt{48\,f/p} \tag{5}$$

in which all dimensions are in inches, f in pounds per square inch, and p in pounds per square foot. The quantity c/d is the ratio of the projection to the depth of the beam or footing. For a given value of f the ratio will vary inversely as the square root of p.

The diagram (Fig. 28) shows curves for different values of f and p from which the ratio of projection to depth of footing may be taken. Thus, for a concrete footing for which the allowable unit stress, f, in tension is, say 30 lb per sq in, if the load, p, on the foundation-bed is 3 000 lb per sq ft, the allowable projection will be 0 69 times the depth of the footing course. If the concrete is 12 in thick, the allowable offset will be 8.3 in. Conversely, for a given offset, say 12 in, when the unit load is 3 000 lb and $f = 30$ lb as before, the required depth will be 1.45 times the offset.

23. Steel Grillages in Foundations

Advantages in the Use of Steel-Beam Grillages. When it is desirable to avoid the deep excavation required for concrete or masonry footings, and when the load of a wall has to be distributed over a wide area of support, STEEL RAILS or STEEL BEAMS are frequently advantageously used to give the required moment of resistance with a minimum of depth. Steel beams are generally cheaper and preferable to rails, although second-hand rails have frequently been used as an expedient.

Preparing the Bed and Setting the Beams. The foundation-bed should be first covered with a layer of concrete not less than 6 in in thickness and so mixed and compacted as to be as nearly impervious to moisture as possible. The beams should be placed on this layer, the upper surfaces brought to a line and the lower flanges carefully grouted so as to secure an even bearing. Subsequently, concrete should be placed between and around the beams so as to permanently protect them.

Requirements for Steel Grillages. In determining the number and size of the beams for any given footing the following points should be considered:

(1) The beams must resist the MAXIMUM BENDING MOMENT, and this without undue DEFLECTION.

(2) The beams must resist the SHEARING-STRESSES, the meeting of which requirement ordinarily provides against CRUSHING or BUCKLING.

(3) The beams must not be spaced so far apart that there is danger of the concrete filling between the beams failing to DISTRIBUTE THE LOAD.

(4) The beams must not be spaced so near together as to prevent the placing of concrete between them. The clear space between the flanges of the top layer should preferably be not less than 2 in and should be somewhat more for the lower layers.

(5) Where the BENDING MOMENT is the governing feature, of two beams of equal weight, the deeper beam should be used. Thus, if the required section-modulus is 146, a 20 in 81.4-lb beam with a section-modulus of 146.63 might be used; but a 24 in 79.9-lb beam is stiffer and stronger, having a section-modulus of 173.93.

(6) Where the SHEAR is the governing feature, of two beams of equal weights, the smaller beam is the stronger. Thus, the SHEARING VALUE of a 20-in 81.4-lb beam is greater than that of a 24-in 79.9-lb beam and is nearly equivalent to that of a 24-in 90-lb beam. However, on account of the greater STIFFNESS of the deeper beam it is sometimes advisable to use it even though the cost is increased.

(7) Recently the various steel companies have been manufacturing H-beams, with heavy flanges and light webs, which gives an effective and economical section for beams and columns, but these should not be used in grillages. The thin web has insufficient strength against buckling unless reinforced with web plates.

Spacing of Beams in Grillage. Table IX gives the LIMITING SPACING for steel beams, based upon the safe capacity of the concrete filling acting as a beam, for loads of from 1 to 6 tons per sq ft. Since, however, in such small spans there is considerable ARCHING EFFECT, the concrete will safely distribute the load on larger spans than those given in the table, provided a sufficient number of tie-rods of proper size are used to take up the THRUST of the arches.

Table IX. The Limiting Spacing for Steel Beams Used with Concrete Filling

Depths of beams	Spacing of beams for the following pressures per square foot											
	1 ton		2 tons		3 tons		4 tons		5 tons		6 tons	
in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
6	1	3	0	11	0	10	0	9	0	8	0	7
7	1	6	1	1	0	11	0	10	0	9	0	8
8	1	8	1	3	1	1	0	11	0	10	0	9
9	1	11	1	5	1	2	1	0	0	11	0	10
10	2	1	1	6	1	4	1	2	1	1	1	0
12	2	5	1	10	1	6	1	4	1	3	1	2
15	3	0	2	3	1	10	1	8	1	6	1	5
18	3	8	2	8	2	3	1	11	1	9	1	8
20	4	0	2	11	2	5	2	2	1	11	1	10
24	4	9	3	6	2	11	2	7	2	4	2	2

The Design of a Wall-Footing of steel beams is illustrated by the following example: A 24-in wall carries 42 000 lb per lin ft. What should be the size

and spacing of steel beams to distribute the load over the foundation-bed at 3 600 lb per sq ft? The required width of the footing is 42 000/3 600 = 11 ft 8 in and the bending moment by Equation (3) is 556 800 in-lb per lin ft of wall. The amount of shear, by the formula given, is $V = W(l - w)/l$, or 34 800 lb. As the beams are in double shear, the single shear per linear foot of wall is 17 400 lb. The required section-modulus per linear foot of wall is obtained by dividing the bending moment by the allowable fiber-stress in the steel, or 556 800/16 000 (assumed fiber-stress) = 34.8. By referring to tables giving the section-moduli of steel beams we find that a 12-in 31.8-lb beam has a section-modulus of 35.97. To satisfy the condition of bending, the beams must not be spaced more than 35.97/34.8 = 1.03 ft center to center. To satisfy the condition of web-crippling due to direct compression, the unit compressive stress must not exceed the value of S_b (allowable buckling resistance), which for a 12-in 31 8-lb beam is 13 060 lb per sq in. The area of the beam resisting compression is the length over which the load is distributed, times the web-thickness. Some authorities consider that the load is distributed over a length equal to the loaded portion of the beam plus one-half the depth of the beam, but in this example the length of only the loaded portion is taken. In this case the area is therefore 24 × 0.35 = 8.4 sq in. If the beams are spaced 1.03 ft on centers the unit direct compression is 42 000 × 1.03/8.4 = 5 150 lb, which is well within the allowed stress, 13 060 lb. To satisfy the condition of web-crippling due to shear, the shearing-stress must not exceed the value as derived from the formula for allowable shear. The approximate, allowed, unit shearing value may be obtained by dividing the value of S_b, by the factor F, the values of which are given in Table IXa, following. For example, for a 12-in 31.8-lb beam this shearing value = 13 060/1.65 = 7 915 lb per sq in. The shearing capacity of the beam is obtained by multiplying this unit stress by the depth of beam times the web-thickness, or 7 915 × 12 × 0.35 = 33 240 lb, or much more than required. Only one of the conditions of web-crippling need be considered by applying the following rule: If the shear divided by the depth of the beam is greater than the total load divided by the product of the distance (over which the load is distributed) by the factor F, investigate for shear; if otherwise, investigate for direct compression. This rule may also be expressed as follows: According as $(l - w)/l$ is greater or less than $2\,D/w'F$, investigate for shear or for compression. Here l = length of beam, w = loaded portion of beam, D = depth of beam, w' = length of beam over which the load is assumed to be distributed (often taken = $w + \frac{1}{2}D$) and F = the factor for the given beam obtained from Table IXa. All dimensions must be taken in the same unit. If, instead of the 12-in beam, 15-in 42.9-lb beams, having a section-modulus of 58.91 are used, the spacing will be 58.91/34 8 = 1.7 ft, or say 1 ft 8 in. By referring to Table IX, it is seen that the spacing of the beams is well within the safe limit of the concrete and no tie-rods are necessary.

The Design of Column-Footings of steel beams and slabs for individual columns has, in recent years, been generally modified. Formerly, designers used the two or three tiers of steel beams with a slab as a column base. The present tendency is to use a single heavy slab, or one tier of beams with a slab, or one tier of beams with a slab stiffened by wing-plates to the column.

In designing grillages for the heavy loads being carried by columns of the modern office-building, the designer should determine not only the economical limit of the load which can be carried on a single slab and at which point it is cheaper to use beams and slab, but he must also consider several other

factors. The deeper the grillage, the greater the excavation and concrete filling in the pit for the placement of the grillage. When using wing-plates to stiffen the slab of the grillage, will it be possible to have them extend above the finished floor level of the lowest basement and thus minimize the excavation? Thirdly, can the mill roll the billets of the dimensions required? Many engineers prefer the single billet regardless of the load and location of the column, whether an interior column supported in a pit or on a pier, or an exterior column supported on a coffer-dam wall, the controlling factor being the size and thickness which the mills can produce. The Chase National Bank in New York rests on billets, as examples of which the following are given: Single billet, 80 in × 10½ in × 6 ft 8 in with a load of 3 168 000 lb; in some cases two billets were placed alongside of one another and the load distributed from the column through the addition of wing-plates or cantilevers as required. As an example, in one case where 2–58½ in × 15 in × 9 ft 3 in billets, with a load of 6 460 500 lb, were used, the wing-plates were 3⅞ in thick.

Table IXa. Values of Factor * for Shearing Values for Various Beams

Beams	For standard-weight beams	For heavy-weight beams
12-in beam	1 65	1 52
15-in beam	1 71	1 50
18-in beam	1.76	1.58
20-in beam	1 77	1 62
24-in beam	1.91	1 67

* The factors, F, which have been deduced to be used in connection with S_b allowable buckling resistance, to give the safe unit shearing value based on web-crippling, will help greatly in investigations of shears in case tables of safe shears are not obtainable. It is to be noted, however, that the values derived from the use of F are approximate only, as this factor is a little different for every beam; and to give its value for every beam would require as much space as complete tables of safe shears The values of F are not given for the new sections of light beams as they are not usually good sections for grillages. It may be mentioned that the standard weight for each size of beam for which F is given is always the next weight higher than the minimum weight given in tables, except for the 20-in beams, for which the minimum weight, 65 4 lb, is also standard weight. The rule given above for determining whether web-crippling based on shear or on direct compression is the determining condition eliminates one of the calculations to be made in investigating grillages.

The Barclay-Vesey Telephone Building, the Irving Trust Company Building and many others are supported on single tier of grillage-beams with a slab stiffened by wing-plates. The maximum loads on such grillages in the above cases are 2 823 000 lb and 5 207 000 lb, respectively. The make-up of these grillages are: 9–24 in I 115 lb × 8 ft 6 in, 1–40 in × 4 in × 6 ft 10 in with wing-plates, and 9–24 in I 120 lb × 8 ft 4 in, 1–100 in × 5½ in × 3 ft 6 in with wing-plates, respectively.

The writer believes it unnecessary to repeat here the method of designing grillages and billets, as many textbooks and handbooks such as those published by the American Institute of Steel Construction, Inc., and the leading steel companies give examples illustrating each type.

24. Reinforced-Concrete Footings

Advantages and Disadvantages. Reinforced concrete has in recent years been largely used for footings. The arguments in favor of its use are:

(1) Low cost of the footing-construction;

(2) Reduction in the amount of excavation required;

(3) Convenience, as compared with the use of steel-beam grillages, in that the reinforcing-steel is readily obtainable, can be cut to length on the work and handled without derricks.

The objections urged are:

(1) Danger of defective workmanship, as the strength of the footing depends upon the proper mixing and placing of the concrete, the proper placing of the reinforcement and the complete union of the concrete with the reinforcement. The danger of defective workmanship is increased by reason of the usual difficulties of foundation-work, in that water and mud are generally present and the difficulty of careful work and inspection is greater.

(2) Danger of the deterioration of the steel reinforcement either by rusting or by electrolysis. This danger is increased by the presence of moisture and by the relatively small cross-section of the reinforcing-bars. In this connection it is well to remember that in reinforced-concrete girders as usually designed the concrete on the tension side is stressed beyond its elastic limit, as a result of which numerous fine cracks are developed under the figured load.

Use of Reinforced Concrete for Foundations. From the foregoing it is apparent that great care should be used in connection with reinforced concrete in foundations, especially as any defect is difficult to detect or repair. Reinforced concrete is used not only for so-called MATS or SLABS but is frequently used for DISTRIBUTING-GIRDERS, BOLSTERS and even for CANTILEVERS.

The Methods Used in Calculating the Strength of Reinforced-Concrete Slabs, Girders, etc., are explained in other chapters. The stresses coming on the reinforced-concrete construction are to be determined in the same way as explained for footings of other materials.

25. Timber Footings for Temporary Buildings

Timber Footings. For buildings of moderate height timber may be used to give the necessary spread to the footings, provided water is always present. The footings should be built by covering the bottom of the trenches, which should be perfectly level, with 2-in planks laid close together and longitudinally with the wall. Across these planks heavy timbers should be laid, spaced about 12 in on centers, the size of the timbers being proportioned to the transverse stress. On top of these timbers again should be spiked a floor of 3-in planks, of the same width as the masonry footings which are laid upon it. A section of such a footing is shown in Fig. 29. All of the timber-work must be kept below low-water mark, and the space between the transverse timbers should be filled with sand, broken stone, or concrete. The best woods for such foundations are oak, long-leaf yellow pine and Norway pine. Many of the old buildings in Chicago rest on timber footings.

Calculations for the Sizes of the Cross-Timbers. The sizes of the transverse timbers should be computed by the following formula:

$$\text{Breadth in inches} = \frac{2 \times p \times c^2 \times s}{d^2 \times A},$$

p representing the bearing resistance of the foundation-bed in pounds per square foot, c the projection of the transverse timbers beyond the 3-in planks, in feet, s the distance on centers of the timbers in feet, and d the assumed depth of the beam in inches. A is the constant for strength. The values recommended for it are 88.9 for long-leaf yellow pine, 66.7 for white oak, and 61.1 for common white pine or spruce.

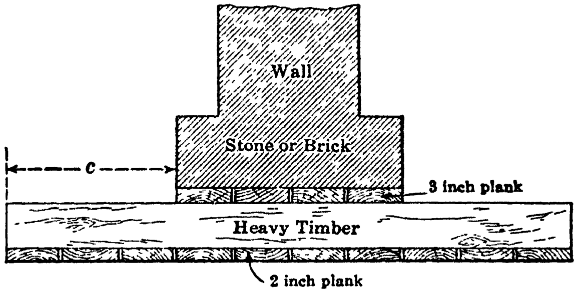


Fig. 29. Spread Footing of Timber

Example. The side walls of a given building impose on the foundation a pressure of 20 000 lb per lin ft; the soil will only support, without excessive settlement, 2 000 lb per sq ft. It is decided for economy to build the footings as shown in Fig. 29, using long-leaf yellow-pine timber. What should be the size of the transverse timbers?

Solution. Dividing the total pressure per linear foot by 2 000 lb, we have 10 ft for the width of the footings. The masonry footing we will make of granite or other hard stone, 4 ft wide, and solidly bedded on the planks in Portland cement mortar. The projection c of the transverse beams will then be 3 ft. We will space the beams 12 in on centers, so that $s = 1$, and will assume 10 in for the depth of the beams. Then, by the formula,

$$\text{the breadth in inches} = \frac{2 \times 2\,000 \times 9 \times 1}{100 \times 88.9} = 4.04 \text{ in}$$

and we should use 6 by 10-in timbers, spaced 12 in on centers. If spruce timber were used we should substitute 61.1 for 88.9, and the result would be 5.9 in.

Foundations for Temporary Buildings. When temporary buildings are to be built on a compressible soil, the foundations may, in some parts of the country, be constructed more cheaply of timber than of any other material, and in such cases the durability of the timber need not be considered, as when it is sound it will last two or three years in almost any place, if thorough ventilation is provided. The World's Fair buildings at Chicago (1893) were, as a rule, supported on timber platforms, proportioned so that the maximum load on the soil would not exceed 1¼ tons per sq ft. Only in a few places over MUD-HOLES were pile foundations used.

26. General Conditions Affecting Foundations and Footings

Type of Foundations. The following are the principal types of foundations in general use:

(1) Spread footings
  (*a*) Individual
  (*b*) combined
(2) Mats
  (*a*) Plain
  (*b*) Two-way continuous perforated
  (*c*) With stiffening frame
  (*d*) With stiffening walls
(3) Wood piles (single, spliced, lagged, etc.)
(4) Precast concrete piles—various types and shapes
(5) Uncased cast-in-place concrete piles
  (*a*) Straight shaft
  (*b*) Enlarged base
(6) Concrete piles cast in place within casing
  (*a*) Tapered piles
  (*b*) Cylindrical piles
  (*c*) Enlarged base
(7) Steel pipe piles
  (*a*) Open end
  (*b*) Closed end
(8) Composite piles—various combinations
(9) Caisson piles
(10) Open caissons
  (*a*) Timber sheeted; sheeting driven progressively with excavation
  (*b*) Steel sheeted; sheeting driven in advance of excavation
  (*c*) Vertical lagging placed against side of excavation (Chicago Method)
  (*d*) Horizontal lagging (New York Method)
(11) Compressed-air caissons—various types
(12) Coffer-dam walls used to permit excavation
  (*a*) Single-line wood sheeting
  (*b*) Single-line steel sheeting
  (*c*) Cribs, earth or rock filled
  (*d*) Double row sheeting
(13) Walls used as coffer-dam and as permanent construction
  (*a*) Built as filling to double row sheeting
  (*b*) Built as open or pneumatic caissons

A detailed description of all the types enumerated above would be too lengthy for the discussion of this chapter. The writer will therefore limit the following description to the fundamentals of design and general notes.

Each structure presents its own particular requirements, and each location presents its own natural peculiarities which sometimes facilitate economical design and construction and at other times demand extreme care and skill to insure safety during and after construction. The design and the methods of construction used should be the best for the structure in combination with the site.

Present-day requirements for tall buildings now approaching a height of 100 stories have greatly increased the size of footings, the depths of the excavation, the lateral thrusts on exterior walls and in general the complications,

difficulties and magnitude of the problems involved. The demand for speed in execution of the work has greatly increased.

Bearing in mind that the foundations of a building come first in the building program, that the design should anticipate all of the demands of the structure and the peculiarities of the site, and that the safety and value of the entire project depend on a successful foundation, sometimes also on the economy of the design and frequently on its rapid construction, it may not be out of place to recommend that architects and owners contemplating construction of important or difficult work should consult engineers who are experienced and competent in this particular field. The writer believes that the security, economy and speed so obtained, in general, greatly overbalance proper charges for such services.

General Considerations. Where the footings of a building rest on wet sand, or on clay, it is important that any movement of the material forming the foundation-bed be prevented if possible. In many cases it is advisable to connect all footings with a concrete floor to prevent any UPLIFT of the foundation-bed between the footings. Where unequal settlement is apprehended it is inadvisable to have long columns firmly attached to the footings, as any unequal settlement of the footings develops a BENDING-STRESS in the column; such bending-stresses, in the case of long columns, may become extremely serious, resulting possibly in the rupture or distortion of the columns. In such cases it has even been proposed to design the bases of the columns with BALL-AND-SOCKET joints which would allow unequal settlement of the footings without distortion or bending of the columns. Such connections, however, could not be generally used because of the necessity of bracing the structure against the horizontal pressure of the wind, but they would be entirely practicable in the case of long interior columns.

The Minimum Depth of Footings is limited by the depth of the cellar, by the requirements of the cellar as to whether part of the footings can project above the cellar-floor level, by the depth of the footing itself and by the ground water-level. The minimum depth will be advantageously exceeded if, by a slight increase in depth, a material capable of sustaining a higher unit load is found on which to rest the footings; or if, as explained in previous articles of this chapter, greater security is afforded by locating the footing at a greater depth. These considerations will influence the design of a footing and in all cases should be taken into consideration. In some cases it may be cheaper to abandon the use of a SPREAD FOOTING of any type and resort to PILES or MASONRY CONSTRUCTION going to ROCK or to some other solid substratum. Where there is any question on this point, careful comparison should be made of the advantages and costs of the two methods. In general, however, it will be cheaper to spread footings immediately below the cellar-excavation level than to employ any of the various deep-foundation methods.

Deep Foundations are necessary when the material at the level at which SPREAD FOOTINGS would ordinarily be constructed is not suitable, or in case it is desirable for any reason to carry the foundations of the building down to an underlying stratum of greater supporting power. Recourse must then be had to one of the above enumerated types of foundations, other than spread footings or mats.

27. Wooden-Pile Foundations

The Use of Wooden Piles. When it is required to build upon a compressible soil that is constantly saturated with water and of considerable depth, the most practicable method of obtaining a solid and enduring foundation for

buildings of moderate height is by driving wooden piles. Many buildings in the city of Boston, Mass., and several tall office-buildings of New York City and Chicago, rest on wooden piles, and they are extensively used for supporting buildings, grain-elevators, etc., erected along the water-front of coast and lake cities. The durability of wooden piles in ground constantly saturated with water is beyond question, as they have been found in a perfectly sound condition after the lapse of from six to seventeen centuries.

Municipal Requirements. The laws of Boston require that wooden piles shall be capped with block-granite levelers or with Portland-cement concrete, and that the spacing shall not exceed 3 ft between centers. The laws of Chicago require that wooden piles shall be driven to rock or hard-pan and capped with grillage of timber, concrete, or steel, or a combination of these. The laws of New York specify a minimum diameter of 5 inches and a maximum spacing of 3 ft between centers.

The Maximum Loads Allowed on Wooden Piles in various cities are as follows: Atlanta, 20 tons; Buffalo, 25 tons; Minneapolis, 20 tons; Richmond, 25 tons; St. Louis, as many tons as the piles will safely support; Chicago, 25 tons; Louisville, 20 tons; St. Paul, 25 tons; New York, 20 tons; Portland, Ore., 25 tons; Cleveland, 25 tons. Most of the above cities also limit the allowed load by Wellington's formula which is hereinafter given under the heading, Bearing-Power of Piles.

Kinds of Wood Used for Piles. Wooden piles are made from the trunks of trees and should be as straight as possible and not less than 5 in in diameter at the small end for light buildings, or 8 in for heavy buildings. The woods generally used for piles are spruce, hemlock, white pine, Norway pine, long-leaf and short-leaf yellow pine, pitch-pine, cypress, Douglas fir, and occasionally oak, hickory, elm, black gum and basswood. There does not appear to be much difference in the woods as to durability under water, but the tougher and stronger woods are to be preferred, especially where the piles are to be driven to hard-pan and heavily loaded.

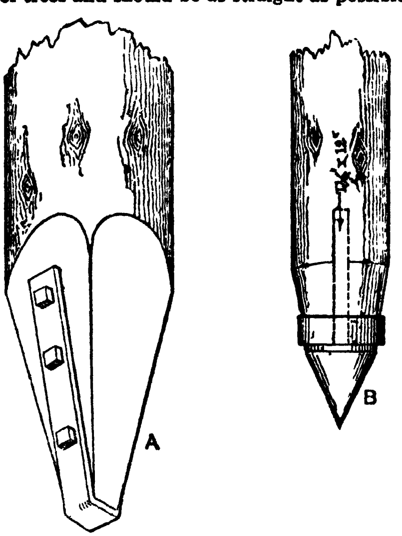


Fig. 30. Points of Wooden Piles Prepared for Driving

Preparing Wooden Piles for Driving. The piles should be PREPARED FOR DRIVING by cutting off all limbs close to the trunk and sawing the ends square. It is probably better to remove the bark, although piles are more often driven with the bark on, and it is doubtful if the bark makes much difference one way or the other. For driving in soft and silty soils, experience has shown that the piles drive better with a square point. When the penetration

is less than 6 in at each blow the top of the pile should be protected from BROOMING by putting on an IRON RING, about 1 in less in diameter than the head of the pile, and from 2½ to 3 in wide by ⅝ in thick. The head should be chamfered to fit the ring. When driven into compact soil, such as sand, gravel, or stiff clay, the point of the pile should be SHOD with iron or steel. The method shown at *A*, Fig. 30, answers very well for all but very hard soils, and for these a CAST CONICAL POINT about 5 in in diameter, secured by a long DOWEL, with a RING around the end of the pile, as shown at *B*, makes the best shoe. Piles that are to be driven in or exposed to salt water should be thoroughly impregnated with creosote, dead oil, or coal-tar, or some mineral poison to protect them from the TEREDO or SHIPWORM, which will completely honeycomb an ordinary pile in three or four years.

Driving Wooden Piles with the Drop-Hammer. The piles should always be driven to an even bearing, which is determined by the PENETRATION under the last four or five blows of the hammer. The usual method of driving piles for the support of buildings is by a succession of blows given with a block of cast iron or steel, called the HAMMER, which slides up and down between the uprights of a machine called a PILE-DRIVER. The machine is placed over the pile, so that the hammer descends fairly on its head, the piles always being driven with the small end down. The hammer is generally raised by steam-power and is dropped either automatically or by hand. The usual weight of the hammers used for driving piles for building foundations is from 1 500 to 2 500 lb, and the fall varies from 5 to 20 ft, the last blows being given with a short fall. Occasionally, hammers weighing up to 4 000 pounds and over are used.

Driving Wooden Piles with the Steam-Hammer. Steam-hammers are to a considerable extent taking the place of the ordinary drop-hammers in large cities, as they will drive many more piles in a day, and with less damage to the piles. The steam-hammer delivers short, quick blows, from sixty to seventy to the minute, and seems to jar the piles down, the short interval between the blows not giving time for the soil to settle around them.* In driving piles care should be taken to keep them plumb, and when the penetration becomes small, the fall should be reduced to about 5 ft, the blows being given in rapid succession. Whenever a pile refuses to sink under several blows, before reaching the average depth, it should be cut off and another pile driven beside it. When several piles have been driven to a depth of 20 ft or more and refuse to sink more than ½ in under five blows of a 1 200-pound hammer falling 15 ft, it is useless to try them further, as the additional blows only result in brooming and crushing the heads and points of the piles, and splitting and crushing the intermediate portions to an unknown extent.

Spacing Wooden Piles. Piles should be spaced not less than 2 ft on centers. When long piles are driven closer than 2 ft on centers there is danger that they may force each other up from their solid bed on the bearing stratum. Driving the piles close together also breaks up the ground and diminishes the bearing power. When three rows of piles are used the most satisfactory spacing is 2 ft 6 in on centers across the trench and 3 ft on centers longitudinally, provided this number of piles will carry the weight of the building. If they will not, then the piles must be spaced closer together longitudinally, or another row of piles driven; but in no case should the piles be less than 2 ft apart on centers, unless driven by means of a water-jet. The number of piles under

* The 5 000 piles, averaging 48 ft in net length, under the Chicago Post Office were driven with a steam-hammer weighing 4 400 lb and delivering 60 blows per minute.

the different portions of the building should be proportioned to the weight which they are to support, so that each pile will receive very nearly the same load.

Capping Wooden Piles. The tops of the piles should invariably be cut off at or a little below low water-mark, otherwise they will soon commence to decay. They should then be capped, either with large stone blocks, or concrete, or with timber or steel grillage.

Concrete Capping. The general practice in use to-day is the reinforced concrete cap, the method being to excavate 6 in to 12 in below the tops of the piles and one foot outside of the piles. Concrete is then placed around and above the piles. Approximately 3 in above the tops of the piles a layer of reinforcement running in both directions is placed, the concreting is continued until the required thickness of the cap has been completed. Caps are usually 18 in or more in thickness, depending upon the number and spacing of piles in the group and the design requirements for bending and shear. On this foundation the grillages of the column, or the wall to be supported, is placed. This construction, in the opinion of the writer, makes the best type of capping, as the reinforced concrete cap ties all the piles of the group together.

Timber-Grillage Capping. Most of the pile foundations of Chicago have heavy timber grillages bolted to the tops of the piles and stone or concrete footings laid on top of the grillages. The timbers for the grillages should be at least 10 by 10 in in cross-section, and should have sufficient transverse strength to sustain the load from center to center of piles, using a low fiber-stress. They should be laid longitudinally on top of the piles and fastened to them by means of DRIFT-BOLTS, which are plain bars of iron, either round or square in section, and driven into holes about 20% smaller in section than the bolts themselves. Round or square bars 1 in in section are generally used, the holes being bored by a ¾-in auger for the round bolts and by a ⅞-in auger for the square bolts. The bolts should enter the piles at least 1 ft. If heavy stone or concrete footings are used and the space between the piles and timbers is filled with concrete level with the tops of the timbers, no more timbering is required; but if the footings are made of small stones and no concrete is used, a solid floor of cross-timbers, at least 6 in thick for heavy buildings, should be laid on top of the longitudinal capping and drift-bolted to them. Where timber grillage is used it should, of course, be kept entirely below the lowest recorded water-line, as otherwise it will rot and allow the building to settle. It has been proved conclusively, however, that any kind of sound timber will last practically forever if completely immersed in water.

The Advantages of Timber Grillage are that it is easily laid and effectually holds the tops of the piles in place. It also tends to distribute the pressure evenly over the piles, as the transverse strength of the timber will help to carry the load over a single pile, which for some reason may not have the same bearing capacity as the others. Steel beams, embedded in concrete, are sometimes used to distribute the weight over piles, but some other form of construction can generally be employed at less expense and with equally good results.*

Specifications for Wooden-Pile Foundations. The contractor is to furnish and drive the piles indicated on sheet No. ___.

The piles are to be of sound spruce (hemlock, long-leaf yellow pine), per-

* For a description of the pile foundations and capping of the Chicago Post Office, see Freitag's Architectural Engineering, pages 350 to 352.

fectly straight from end to end, trimmed close, and cut off square to the axis at both ends.

They are to be not less than 6 in in diameter at the small end, 10 in at the large end, when cut off, and of sufficient length to reach solid bottom, the nec-

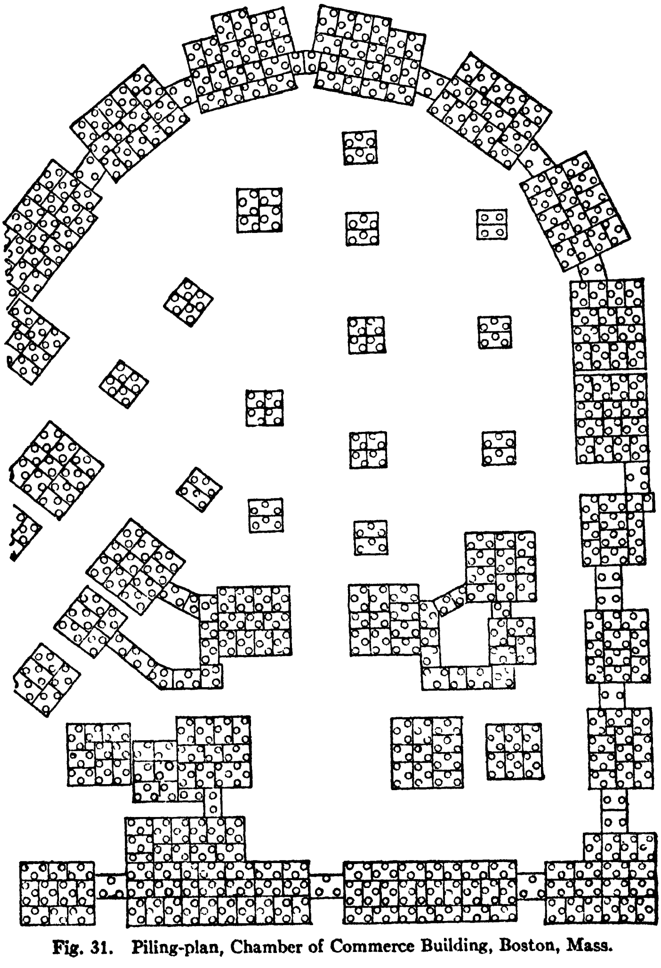


Fig. 31. Piling-plan, Chamber of Commerce Building, Boston, Mass.

essary length of piles to be determined by driving test-piles in different parts of the foundation.

All piles are to be driven vertically, in the exact positions shown by the plan, until they do not move more than 5 in under the last five blows of a hammer weighing 2 000 lb and falling 20 ft. All split or shattered piles are

to be removed if possible and a good one driven in place of each imperfect one. In cases where such piles cannot be removed an additional pile is to be driven for each imperfect one. If the piles show a tendency to BROOM, they are to be bound with wrought-iron rings, $2\frac{1}{2}$ in wide and $\frac{5}{8}$ in thick.

All piles, when driven to the required depth, are to be sawed off square for a horizontal bearing at the grade indicated on the drawings.

The Bearing Power of Piles. In regard to their use for supporting buildings, piles may be divided into two classes: (1) Those which are driven to ROCK or HARD-PAN, that is, firm GRAVEL or CLAY and (2) those which do not reach HARD-PAN.

(1) A pile belonging to this class when driven through a soil that is sufficiently firm to brace the pile at every point, may be computed to sustain a load equal to the safe resistance to crushing on the least cross-section. If the surrounding soil is plastic the bearing power of the pile will be its safe load computed as a column, having a length equal to the length of the pile when capped. Test-piles driven on the site of the Chicago Public Library Building, through 27 ft of soft, plastic clay, 23 ft of tough, compact clay and 2 ft into hard-pan, sustained a load of 50.7 tons per pile for two weeks without apparent settlement. There are many instances where piles driven to the depth of 20 ft in hard clay sustain from 20 to 40 tons, and a few instances where they sustain up to 80 tons per pile.

(2) A pile belonging to this class depends for its bearing power upon the FRICTION, COHESION and BUOYANCY of the soil into which it is driven. The safe load for such piles is usually determined by the average penetration of the pile under the last four or five blows of the hammer. Several engineers have formulated rules for determining the safe loads for piles of this class, but there are so many conditions that modify the amount of the penetration, and its exact determination, and so many varying conditions of driving and of soil, that it is considered impossible to formulate any rule that can be considered entirely satisfactory for all the conditions under which such piles are driven.

The Engineering News Formula. The formula generally used by engineers was derived by M. A. Wellington, and is often referred to as the ENGINEERING NEWS FORMULA:

$$\text{The safe load in tons} = 2\,wh/\,(S + 1) \text{ for drop-hammer}$$

$$\text{The safe load in tons} = 2\,wh/\,(S + 0.1) \text{ for steam hammer}$$

in which w = the weight of the hammer in tons;
h = the height of fall of the hammer in feet;
S = the penetration in inches under the last blow or the average under the last five blows.

When loads are based on this formula the piles should be driven until the penetration does not exceed the limit assumed, or if this is found to be impracticable, new calculations must be made based on the smallest average penetration that can be obtained, and a greater number of piles used. In localities where piling is commonly used for foundations, the least penetration that can be obtained within practical limits of length of pile can generally be ascertained by observation, or by consulting somebody who is experienced in driving piles. The longer the pile the less, as a rule, will be the final SET or penetration. Where there is no experience to guide one it will be necessary to drive a few piles to determine the length of pile required, or the least SET for a given length of pile. Some piles will have to be driven further than

others to bring them to bearings of equal resistance. When the piles are to be loaded to more than 50% of the assumed safe load, the final set of each pile should be carefully measured by an inspector, the BROOM and SPLINTERS being removed from the head of the pile for the last blow.

Safe Loads for Piles. In the past the drop-hammer has been in more general use than is the case to-day. Table X, computed by the above formula for drop-hammers, gives the safe loads for different penetrations under different falls of the hammer weighing one ton. For a hammer of different weight, multiply the safe load in the table by the actual weight of the hammer in tons. Thus, for a hammer weighing 1 000 lb, the values in the table should be multiplied by ½, and for a 1 500-lb hammer, by ¾.

For steam hammers it would be useless to set up a table of penetration versus safe carrying capacity, as the weights of striking parts and the length of strokes are continually being modified by the manufacturers. It is therefore advisable to apply the above formula only after determining the type and specifications of the hammer to be used. When a double-acting hammer is used with steam or compressed air to increase the blow of the driving ram, the stroke times the weight of the ram plus the area of the piston multiplied by the mean effective pressure of the steam or compressed air on the piston during the stroke gives the energy of the ram in foot tons per blow and should be substituted for the term wh in the above formula.

Table X. Safe Loads in Tons for Piles

For hammer weighing 1 ton

Penetration of pile in inches	Height of the fall of the hammer in feet												
	3	4	5	6	8	10	12	14	16	18	20	25	30
0 25	4 8	6 4	8 1	9 7	12 9	16 1	19 4	22 5	25 8	29 1	32 3		.
0 50	4 0	5 3	6 7	8 0	10 7	13 3	16 1	18 7	21.3	24 0	26.6	33 3	
0 75	3 4	4 6	5 7	6 9	9 2	11 5	13 8	16 1	18 4	20 7	23.0	28 8	34.5
1.00	3.0	4 0	5 0	6 0	8 0	10 0	12 0	14 0	16 0	18.0	20 0	25 0	30 0
1 25	..	3 6	4 5	5 4	7 1	8 9	10 7	12 5	14 3	16 1	17 9	22 3	26 7
1 50	..	3 2	4 0	4 8	6 4	8 0	9 6	11 2	12 8	14 4	16.0	20 0	24 0
1.75		..	3 6	4 4	5 8	7.3	8 8	10.2	11 7	13 1	14 6	18 2	21 9
2 00	.	..	3 3	4 0	5 3	6.7	8 0	9 3	10.7	12.0	13.3	16 7	20.0
2.50			.	3 4	4 6	5 7	6 9	8 0	9 1	10 3	11.4	14 3	17.1
3 00				3 0	4 0	5 0	6 0	7 0	8 0	9 0	10.0	12 5	15.0
3 50	.	.	.	.	3 6	4 4	5 3	6 2	7 1	8 0	8 9	11 1	13 3
4.00	...	..	.	..	3 2	4.0	4 8	5 6	6.4	7.2	8 0	10 0	12 0
5.00	.	..	..		..	3.3	4 0	4 7	5.3	6.0	6.7	8.3	10.0
6 00	..	...	..	.	....	....	3.4	4.0	4.6	5.1	5.7	7.1	8.6

Example of Computations for Pile Foundations. Suppose that from the observations of the pile-driving for an adjacent building it is found that piles driven from 20 to 30 ft take a set of 1 in under a 1 200 hammer falling 20 ft, and that additional blows result in about the same set.

From Table X we find that the safe load for a fall of 20 ft and a penetration of 1 in is 20 tons. Multiplying by the weight of the hammer in tons, 0.6, we have 12 tons as the safe load per pile. Suppose that the total load on 1 lin ft of footing is 13 tons. As we must have at least two rows of piles, and each two piles will support 24 tons, it follows that the spacing of the piles

longitudinally should be 24/13 = 1 ft 10 in. As this is too close, we should use three rows of piles, spaced 2 ft apart laterally, and the longitudinal spacing would then be 36/13 = 2 ft 9 in. The width of the capping would be about 5 ft. If the load on the piles under the interior columns, for example, is 105.8 tons, this, divided by 12, the safe load for one pile, gives nine piles, or three rows of three piles each, which should be spaced 2 ft 6 in apart, each way.

Some Actual Loads on Wooden Piles. The following examples of the actual loads supported by piles, under well-known buildings, and of loads which piles have borne for a short time without settlement, should be of value when designing pile foundations.

BOSTON. At the South Station three piles were loaded with about 60 tons of pig iron, 20 tons per pile, without settlement. The allowed load was 10 tons per pile.

Piles 12 in in diameter at the butt and 6 in at the point driven 31 ft into hard, blue clay near Haymarket Square, failed to show movement under 30 tons, the ultimate load being probably 60 tons. Other piles driven 17.9 ft sustained a load of 31 tons each. The average penetration under the last ten blows of a 1 710-lb hammer falling from 9 to 12 ft varied from 0.4 to 0.95 in per blow for fifteen piles.

Piles 25 ft long under the Chamber of Commerce Building penetrated about 3 in under the last blow of a 2 000-lb hammer falling about 15 ft.

CAMBRIDGE. The buildings of the Massachusetts Institute of Technology built in 1915–16 are supported on spruce and oak piles with a design load of 10 and 14 tons, respectively. The pile tips penetrated to the stiff clay strata and were so spaced as to load the clay at a unit of ¾ of a ton per sq ft plus the overburden. Notwithstanding the apparently low unit load, the buildings have settled for a long period of time.*

CHICAGO. In the Public Library Building the piles were proportioned to 30 tons each and were tested to 50.7 tons without settlement.

In the Schiller Building the estimated load was 55 tons per pile; the building settled from 1½ to 2¼ in.

In the Passenger Station of the Northern Pacific Railroad, at Harrison Street, piles 50 ft long were designed to carry 25 tons each and did so without perceptible settlement.

The Art Institute Building, parts of the Stock Exchange Building and also a large number of warehouses and other buildings on the banks of the river rest on piles.

NEW YORK CITY. The Ivins (Park Row) Building is supported by about 3 500 14-in spruce piles, arranged in clusters of fifty or sixty, for single columns, and a corresponding number under piers supporting two or more columns. The piles were driven to refusal of 1 in under a 20-ft fall of a 2 000-lb hammer. The material is fine, dense sand to a depth of over 90 ft. But few piles could be driven more than 15 or 20 ft. The average maximum load per pile is 9 tons.

The American Tract Society's Building is supported on piles.

BROOKLYN. Piles under the Government Graving Dock, driven 32 ft, on the average, into fine sand mixed with fine mica and a little vegetable loam, are supposed to sustain from 10 to 15 tons each.

NEW ORLEANS. Piles driven from 25 to 40 ft into a soft alluvial soil carry safely from 15 to 25 tons, with a factor of safety of from 6 to 8.

* Boston Society of Civil Engineers, Proceedings of January, 1918.

28. Concrete-Pile Foundations

Durability of Wooden and Concrete Piles. Concrete piles, either plain or reinforced, possess many advantages over wooden piles and, in general, can be used in all places where wooden piles can be driven. Concrete piles, compared with wooden piles, have primarily the advantage of greater PERMANENCE. Timber piles, kept constantly wet and protected from the action of the TEREDO or other destructive influences, may be practically everlasting, but cannot be counted upon above water level; whereas concrete piles should be proof against all deteriorating actions, whether wet or dry, except the action of freezing on wet concrete.

Strength of Wooden and Concrete Piles. Concrete piles without reinforcement, if made of good concrete, should have nearly the same CRUSHING STRENGTH per square inch as ordinary yellow-pine piles, and with properly placed reinforcement concrete piles should have a much higher crushing strength per square inch than timber piles. Moreover, timber piles do not have UNIFORM CROSS-SECTIONS. For instance, a slender timber pile 40 ft in length and 12 in in diameter at the butt, is probably not over 6 in in diameter at the point. In direct compression the load on a point-bearing pile of the above dimensions is limited to the safe load on the point of the pile, where it is 6 in in diameter; and a cylindrical concrete pile, 12 in in diameter and under similar conditions, will have a cross-section of 113 sq in at all points, compared with the cross-section of 28 sq in at the point of the timber pile. Moreover, if we consider both piles as LONG COLUMNS, it must be borne in mind that a timber pile may not be straight and that it may, therefore, be subject to STRESSES and DEFORMATIONS due to ECCENTRIC LOADING, which are avoided in a straight concrete pile.

Reinforced-Concrete Piles. In practice concrete piles are generally reinforced, and if a pile is to be considered as a long column the reinforcement may be increased at the center, so as to provide for stresses due to handling and to its acting as a long column. The concrete piles may be formed complete, above ground, in which case they may be straight or tapered, with square, circular or other cross-sections. The reinforcement may consist of a number of vertical rods generally disposed symmetrically around the axis of the pile. The vertical rods should be connected by horizontal wiring or by spiral reinforcement. As before stated, the reinforcement may be increased at the central section so as to provide against stresses due to the use of the pile as a LONG COLUMN, in which case the additional reinforcement should be placed near the periphery of the cross-section.

Types of Concrete-Pile Reinforcement. There are many TYPES OF REINFORCEMENT, one method even employing a woven-wire fabric which is laid out flat on a table and covered with a thin layer of concrete, the entire mat comprising the wire fabric and the concrete being then rolled into a solid cylindrical form which, when set, forms the finished pile. The concrete piles may be FORMED IN PLACE by any one of several different methods.

The Raymond Method. In the so-called RAYMOND PILE METHOD a steel MANDREL of tapered form is driven into the ground, and when the required penetration has been obtained this mandrel is collapsed and withdrawn, leaving a hole corresponding to the size of the extended mandrel in the ground; this hole is then filled with concrete. The reinforcement may be placed in the hole prior to the placement of the concrete. This method, as described, is applicable only to such material as will stand while the mandrel

is being withdrawn and the hole is being filled with concrete. In most cases the method used is as here described except that a thin shell of steel is placed on the mandrel before driving. When the mandrel is collapsed the shell is left in the ground, thus forming a lining for the hole which is subsequently filled with concrete or with reinforcing-rods and concrete, as before described. Raymond piles have been extensively used, especially for FRICTION-PILES or SKIN-BEARING PILES in soft and artificially filled-in ground. An improved form of lining-shell employed in the Raymond method is combined with a spiral reinforcement inside of the shell, which materially assists in preventing the collapse of the shell.

The Simplex Method of Forming Concrete Piles in Place. THE SIMPLEX METHOD differs from the Raymond method and may be briefly described as follows: A steel pipe, generally cylindrical in form, of the required size and length and fitted with a detachable cast-iron conical DRIVING-POINT, is driven into the ground to the required depth; the pipe is then partially filled with concrete. A piston-like PLUNGER, smaller in diameter than the inside diameter of the pipe, is then placed on the concrete and the pipe is partially withdrawn, leaving the driving-point and part of the superimposed concrete in the ground. This operation is repeated until the pile is built up to the required height. In certain materials, instead of using a detachable driving-point, the driving-point consists of two jaws hinged to the lower end of the pipe, so arranged that while during the driving they form a driving-point, when the pipe is withdrawn they open and form an extension of the cylindrical pipe. In other words, the jaws are formed of steel plates previously bent to the same radius as the radius of the pipe and so hinged that when they are in their open-position the plates forming the jaws constitute an extension of the cylindrical surface of the pipe. It is evident that plain reinforcing-bars can be placed in position before concrete is put into the pipe.

Caution for Concrete Piles Built in Place. Care should be taken in designing and placing the reinforcing for all concrete piles BUILT IN PLACE, that the subsequent placing of the concrete does not throw the reinforcement out of position and that all voids between the reinforcement and the shell are completely filled.

The Pedestal Pile is designed to give an ENLARGED CROSS-SECTION at the base of the pile. The method is similar to that of the RAYMOND METHOD, the increase in diameter being obtained as follows: After the pipe has been driven, the driving-core is withdrawn and the pipe partially filled with concrete. Then the concrete in the pipe is rammed, forcing the concrete out of the pipe and compressing the material below the pipe, so that the concrete is forced into the soil. A repetition of this operation results in forming a BASE or MUSHROOM below the pipe larger in diameter than the diameter of the pipe. Finally the pipe is withdrawn, the filling and ramming-operations continuing meanwhile, until the pile is carried up to the required height.

Composite Piles. Protected piles, for use in localities where the TEREDO affects the life of timber piles under water, are composed of timber piles with concrete coatings held in position by steel reinforcements in the shape of expanded metal or wire netting. Such piles are to be considered as timber piles rather than as concrete piles.

Timber Piles with Concrete Caps. In some localities where the permanent water-level is considerably below the level of the required excavation, timber piles have been driven with a FOLLOWER, the follower consisting of a steel pipe

or cylindrical shell. When the head of the pile is driven to a safe distance below low water the PIPE-FOLLOWER is filled with concrete and withdrawn, leaving the concrete pier resting on a timber pile. This composite pile would appear to possess the advantage of combining the cheapness of a timber pile below the water-level with the permanency of a concrete pile above the water-level. Great care, however, should be used in adopting this method on account of the difficulty of securing proper connection between the concrete and the wooden pile.

The Methods used in Driving Built-up Piles are practically the same as are used in driving wooden piles, except that a CUSHION of wood, rope, or other material is placed on the head of the pile to be driven to cushion the blow of the hammer. Steam-driven or air-driven RECIPROCATING HAMMERS are preferable to the ordinary DROP-HAMMERS. In stiff materials the use of a WATER-JET is advisable and, in fact, in many cases indispensable. In lifting concrete piles use is made of a special SLING which is attached to a pile at two points, each point one-quarter of the length of the pile from the end. The sling should have a SPREADER so that the stress due to the oblique pull of the CHAIN-SLING is taken up by the spreader rather than by the pile.

The Casting of Concrete Piles. Concrete piles should be CAST IN ONE PIECE by a continuous operation so that there will be no PLANE OF WEAKNESS formed between partially set concrete and fresh concrete. They may be cast either in a vertical position, in forms, or in a horizontal position. Square-section concrete piles have been cast in a horizontal position and side-forms, only, used, the previously cast concrete pile, protected by paper, forming the bottom form. In some cases, where it is intended to use a WATER-JET in sinking a pile, the latter is cast around an iron pipe which is afterwards used for the water-jet. In general, however, this is dispensed with and an external detachable pipe used for the water-jet.

Incidental Advantages of Concrete Piles. In many cases, where concrete piles are more expensive than timber piles, the saving in excavation and footings more than offsets the increased cost. For example, if the excavation for the cellar of a building does not go down to water-level, the use of timber piles will necessitate excavating down to a point below water-level in order that the piles may be cut off low enough to keep their heads always wet. Concrete piles, however, can be driven from the level of the bottom of the cellar-excavation, and this additional excavation and the necessary construction between the excavation-level and the level of the cut-off for the timber piles thus avoided Moreover, as one concrete pile may have a SUPPORTING POWER equal to the supporting power of four wooden piles, the size of the footings will be much smaller with concrete piles than with wooden piles.

Comparison of Wooden and Concrete Piles under Piers. The footings for a column or pier 24 in sq in section, requiring for its support, say, sixteen wooden piles, spaced 2 ft 6 in from center to center, will be, allowing for slight inequalities in driving, approximately 10 ft square, the projections being 4 ft beyond the size of the base. Such a footing will ordinarily require a steel grillage or reinforced-concrete base, or, if made of ordinary concrete, will be of very considerable depth; whereas, if four concrete piles, placed 3 ft from center to center, are used, instead of wooden piles, the area of the base will be a little over 4 ft square and the projection will be only 1 ft. A suitable footing would consist of a reinforced-concrete cap not over 2 ft in thickness. The saving in cost of excavation, concrete and steel in the footing is all in favor of the use of concrete piles.

Concrete Piles under Walls. In the case of a continuous wall, where the load per linear foot of wall is not great, a single row of concrete piles is often sufficient to support the weight of the wall. In such cases, the piles should not be placed in straight lines but should be STAGGERED, and a sufficient footing should be constructed connecting the heads of the piles, so as to afford stability to the wall.

The Method Employed in Calculating Reinforcement for Concrete Piles is the same as that employed in calculating ordinary reinforced-concrete columns, the only difference being that where a pile is not point-bearing, but is dependent on the surrounding material for its support, it need not be considered as a LONG COLUMN. POINT-BEARING PILES deriving their support from some solid material on which their lower extremity rests, must be considered as LONG COLUMNS, on the assumption that the material surrounding the piles may fail to support them. In the case of FRICTION-PILES, depending for their support upon the surrounding material, this assumption cannot be made, as any failure of the material will involve a settlement of the pile. It should be borne in mind that any structure supported on piles supported by SKIN-FRICTION is dependent for its stability upon the continued supporting power of the material surrounding the piles. In many cases buildings resting on piles driven into soft ground have settled as the result of the consolidation and settlement of the material surrounding the piles, notwithstanding the fact that the piles when driven were amply able to support the loads for which they were designed.

Loads Allowed on Concrete Piles. The building laws of most cities allow on concrete piles from 350 to 500 lb per sq in on the concrete plus from 6 000 to 7 500 lb per sq in on the vertical reinforcement. On this statement it would appear possible to design a short concrete pile 12 in square, on which the allowed load would be 100 tons, and it is possible that such a pile, tested as a SHORT COLUMN, would develop in a testing-machine a strength justifying the use of such construction; but, bearing in mind that the character of the support for the base of such a column is underground and cannot be inspected, and bearing in mind also the uncertainties attending the manufacture of the pile, it is evident that it would be improper to load a pile to this extent in practice. It would, however, be considered good practice to load concrete piles up to one-third of a test-load applied to not less than 3% of the piles used. In ordinary practice, reinforced-concrete piles are loaded up to 500 lb per sq in of cross-section.

Pipe-Pile Foundations. In recent years many buildings have been supported on concrete-filled pipe-piles bearing on rock or hard-pan. This type can be advantageously used when cellar space below ground water-level is not desired, when encumbrances, such a boulders, timber cribbing, etc., are not to be encountered between the cellar subgrade and the bearing strata and where this distance is not excessive. The New York City code requires that the length of pipe-piles should not be more than 40 times their inside diameter. The pipes are usually 10 to 18 in in diameter, although cylinders up to 52 in in diameter have been used, having a thickness of 3/8 to 5/8 in. The pipe is driven in sections with a steam hammer, and, as additional sections are required, these are attached to the driven section by means of a cast-iron or steel internal sleeve and redriven. When the pile has reached its bearing level it is cleaned out by blowing or dug out by means of augers, pipe "orange peels" or other similar tools. The pile is then pumped out and concreted. The New York City code allows that these piles, if filled with 1 : 2 : 4 concrete, may be loaded to 500 lb per sq in on the concrete and 7 500 lb per

sq in on the steel. The effective steel area is to be taken as the circumference multiplied by the shell thickness minus $\frac{1}{16}$ in. The writer believes the load on the pile should be reduced 5% for each splice in excess of one in its length. Piles should be spaced at least the pipe's diameter plus 10 in, and not less than the diameter of a circular area which, when divided into the load on the pile, will give a unit load on the bearing strata not greater than that allowed by the code.

29. Foundation Piers and Foundation Walls

Foundation Piers and Walls as distinguished from ordinary CELLAR PIERS and WALLS, extend from the level of the underside of the cellar-floor to rock or other solid foundation-bed. In general, such piers and walls are composed of concrete and are of such dimensions that the safe unit loads on the concrete forming them are not exceeded. If the foundation-bed is rock, compact hardpan, or gravel, there need be little or no enlargement of the base of the pier or wall, as the safe unit loads on such natural foundation-beds are generally equal to the safe unit loads on the concrete forming the body of the pier or wall. The design of such piers and walls is therefore an entirely simple matter governed by the principles already outlined, and by certain considerations mentioned hereafter.

The Methods used in the Construction of Foundation Piers and Walls are, however, necessarily varied to suit different materials and to meet different conditions encountered, and the design of a pier necessarily differs with different methods of construction. For example, if the construction is to be executed by means of the ordinary SHEET-PILING METHOD, piers and walls will have in general rectangular outlines. But if the CHICAGO METHOD or the PNEUMATIC CAISSON is employed, it will generally be cheaper to use piers having a circular cross-section and the support for walls may be a succession of cylinders rather than continuous walls. The detailing of the concrete structure constituting the piers or walls is simple after a determination is made of the methods by which the construction is to be put in place. This subject is discussed in the following chapter-subdivision, Methods of Excavating for Foundations.

30. Methods of Excavating for Foundations

Simple and Complex Excavations. Excavations in earth for footings of walls and piers may vary from simple trenches and pits of the required sizes and depths to accommodate the footings, up to deep subaqueous excavations requiring all the resources of engineering skill.

The Sides of Excavations. If the earth is firm and the depth not excessive the sides of the excavation may be self-supporting, in which case the excavation may be made the neat size of the footing and the sides of the excavation may take the place of forms for the concrete deposited to form the footing. Where the excavation is deep, and especially where the earth is not firm, the sides of the excavation must be sloped or, if made vertical, must be supported by bracing or by some form of sheet-piling. Where the excavation is over 8 ft in depth it will generally be cheaper to support the sides of the excavation than to slope them. Where the excavation adjoins a property-line it will generally be inadvisable to slope the excavation on account of damage to the adjoining property, and in such cases it will be necessary to use sheeting, even if sloping the earth would be cheaper.

Bracing in many cases will serve to support the sides of the excavation without the necessity of close SHEETING. The BRACING may consist simply of short pieces of PLANK placed against opposite sides of the excavation and held in position by horizontal timber STRUTS secured by WEDGES; or, especially in narrow trenches, some form of an EXTENSIBLE SEWER-BRACE may be used. Fig. 32 represents a usual form of EXTENSIBLE BRACE. Generally, however, the sides of an excavation will not stand with a vertical face, even if braced in this manner, for any length of time, and if the material is loose sand or soft clay, such bracing is entirely inadequate. In such cases, and in fact generally, some form of CONTINUOUS SHEET-PILING must be employed.

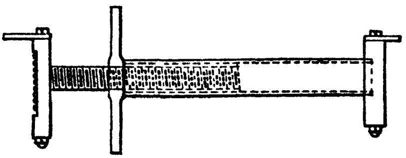


Fig. 32. Extensible Brace for Narrow Excavations

Ordinary Wooden Sheet-Piling consists of a continuous line of vertical planks held against the sides of the excavation by horizontal timbers known as WALES, WALING or BREAST-TIMBERS, these wales, or breast-timbers being in turn supported either by CROSS-BRACES extending across the excavation to an opposite wall or side of the excavation, or by inclined struts known as SHORES or PUSHERS, extending to the bottom of the excavation where HEELS or inclined platforms are sunk in the undisturbed material to afford points of support.

Earth-Pressure on Sheet-Piling. The load on the sheeting due to the EARTH-PRESSURE may be calculated on the assumptions made for the design of RETAINING-WALLS, but the thickness of the sheeting planks, the sizes and spacing of the breast-pieces and braces, if figured on this basis, will in general exceed the sizes constantly used with success and safety in such work. The probable reason for this is that an earth bank, when steadied and in part supported by the sheeting, does not, for a considerable time, lose the COHESION between its particles natural to most earth banks in their original and undisturbed state. Or, in other words, under these conditions no real ANGLE OF FRICTION is developed in the earth-mass. Local experience and practice should be consulted and will generally serve as a guide. Earth banks apparently similar will, however, act very differently and no general rule can be given. It should be borne in mind that the earth composing a bank should be, as far as possible, protected from jar, from the action of water and from alternating freezing and thawing; and that permanent work should be completed as rapidly as possible so as to avoid the deteriorating effects of time and exposure on the structure of the bank.

The Thickness of the Sheeting Planks required may be calculated on the assumption that the earth bank is composed of loose material having a definite ANGLE OF SLOPE and COEFFICIENT OF FRICTION; but practically, under favorable conditions, 2-in planks may be used for a depth of drive of 16 ft, 3 in planks up to 24 ft and 4-in planks up to 32 ft; and timbers, 8 by 11 in, have been driven in favorable material to a depth of over 40 ft.

Depths and Numbers of Drives. Ordinarily the depth to which a plank can be driven is limited by its ability to resist the shock due to driving, and in unfavorable material a plank may become shattered before it is driven to the above-quoted depths. If the required depth cannot be reached by the first planks or DRIVE, a second, and sometimes a third and fourth set of planks are

employed. As the BREAST-PIECES supporting the first line of planks must remain in place, the planks in the second set or DRIVE have to be placed inside of the breast-pieces, thus reducing the size of the excavation by the amount of the necessary offset. Where more than one drive is required the first drive should be started at a sufficient distance outside to allow the planks forming the second or the second and third drives to be placed outside of the required area for the bottom of the excavation.

Cutting and Fitting Sheeting Planks. The sheeting planks may be SQUARE-EDGED where there is no water or fine loose sand, but where water or running sand is to be excluded the planks should be TONGUED AND GROOVED, or SPLINED. The use of tongued and grooved planks has the additional advantage that the planks are more readily kept in line. It is usual to cut the

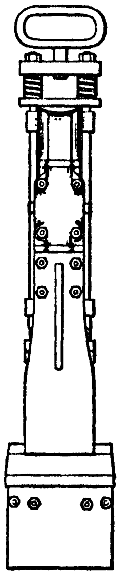


Fig. 33 Small Power-hammer for Driving Sheeting Planks

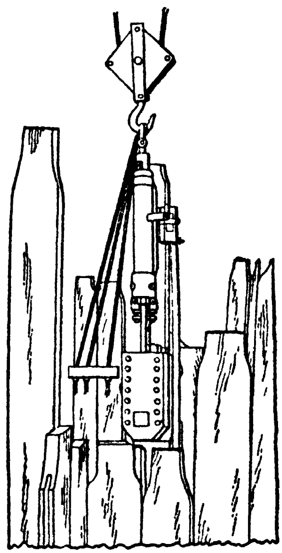


Fig. 34. Large-size Power-hammer and Sheeting Planks

bottom edge of each plank on a slight angle, so that in driving it is WEDGED against the preceding plank. The top of each plank may be fitted to receive an iron DRIVING-CAP; or, if this is not used, the upper corners of the plank should be cut off so that the effect of the blows will be concentrated along its vertical axis, and the tendency of the plank to SPLIT, due to a blow on one corner, thus diminished.

The Means Employed for Driving the Sheeting vary with the depth and the size of the sheeting. For small jobs and for moderate depths of drive, the primitive method of DRIVING BY HAND with ringed wooden MAULS still prevails. For work involving a considerable amount of driving, and in all cases for long drives, POWER-HAMMERS driven either by steam or compressed air are preferably employed. A small-sized power-hammer (Fig. 33) resem-

bles a STEAM-DRILL and may be handled by two or three men without any special lifting-appliances. The larger sizes of power-hammers (Figs. 34 and 35) are practically small, power, pile-driving hammers arranged with a special DRIVING-HEAD TO fit the sheeting employed. Such hammers are handled by DERRICKS or are carried in a frame similar to a pile-driver frame. Ordinary DROP-HAMMERS are sometimes used, but are not as advantageous as the RECIPROCATING POWER-HAMMERS, as the blow struck by the drop-hammer shatters the plank, while the frequent light blows of the power-hammer tend to keep the planks and the adjacent material in motion and accomplish the required work with less damage to the sheet-piling. The weights and dimensions of several types of pile-driving hammers are given in Table XI.

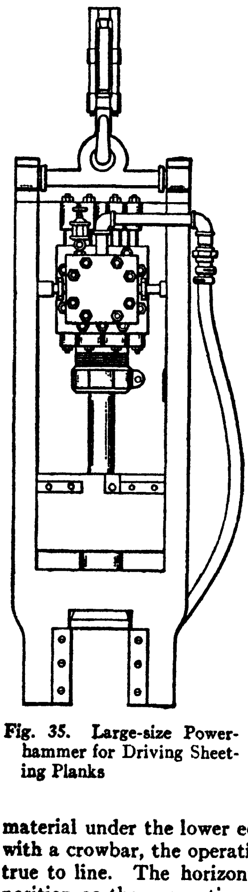


Fig. 35. Large-size Power-hammer for Driving Sheeting Planks

The SUBMARINE PILE-HAMMER, or a hammer which can drive piles under water, has recently come into use. The exhaust of the hammer is brought to the surface through a hose. The motive power may be either steam or compressed air. Considerable economy and advantage can be exercised through the use of the submarine hammer, a few of these advantages are:

(1) Shorter piles can be used.
(2) The wastage due to cut-off is reduced.
(3) The use of followers is avoided.
(4) Driving is not affected by rise and fa' o tide.
(5) The work of underwater cutting to bring pile heads to proper level is reduced.

Manner of Driving Sheeting Piles. In practice, a shallow excavation is first made to the proper line for the outside of the sheeting planks. The top BREAST-TIMBER is temporarily secured in place and the lower end of the planks placed between this timber and the bank. If the planks are long, temporary TOP GUIDES or STAY-BRACES are arranged so as to keep the planks vertical until they have been driven well into the ground and guided by the permanent BREAST-PIECES. The planks are then driven as the excavation progresses, each plank being driven a few inches in turn. As the driving goes on the material under the lower edge of the planks is loosened with a shovel or with a crowbar, the operation being so conducted that the planks are held true to line. The horizontal breast-timbers and their braces are placed in position as the excavation progresses. If inclined braces are to be used the excavation in the center is taken out first, leaving a sloping bank against the sides of the excavation. This permits of the placing of the inclined braces and of the heels for their points of support before there is any danger to the bank. After the first breast-piece and its inclined brace are set in place, the second and subsequent breast-pieces and braces are put in as the excavation proceeds.

Sheet-Piling for Excavations Below Water-Level. These excavations may be made by the SHEET-PILING METHOD if there is not too much water and if water can be drained out of the material without inducing a flow of sand or clay below the bottom of the sheet-piling. In some cases, where unfavorable conditions exist, but where there is an underlying stratum of impervious material, it is possible to drive the sheeting in advance of the excavation, so that the bottom of the sheeting makes a tight joint with the impervious stratum, cutting off the flow of water and material. Where a considerable amount of water finds its way into the excavation, the water must be led to a SUMP or depression from which it is ejected by means of a PUMP or a STEAM-SYPHON. Where the foundation-bed is below water-level and the material is sand, clay, or other material which would be softened by the action of the water, it should be protected by having the sump at a considerable distance from the area to be used for the support of the footing. This may be accomplished by making the area to be sheeted and excavated large enough to accommodate the sump outside of the supporting area, or by sinking a separate excavation to be used exclusively as a sump; or the same result may in some cases be accomplished by the use of WELL-POINTS, driven to a point below the level of the footing in which continued pumping may reduce the level of the water to a point below the footing. Care also should be taken, when the level for the footing is reached, to prevent the foundation-bed from being disturbed and softened by unnecessary tramping of workmen over the surface of the excavation.

The foundation-bed should be left as nearly as possible in its original or natural condition.

Well-Point System of Drainage. The principle of draining water from the ground by the use of a pipe with an opening near the lower end, protected by a screen, was used in the early days of the development of the Mid-West as a means of water-supply. Recently this principle has been used in lowering the level of the ground water to facilitate construction. A pipe is fitted with a jetting point and a perforated section at one end. The jetting point permits driving the pipe into the ground to the desired elevation. The perforated section generally has in addition a fine mesh covering which acts as a filter to draw the water and not fine materials. A series of these pipes are sunk around the periphery of the area in which the water-level is to be lowered. These are then connected to a header pipe and a pump. The success of this method is dependent on the stratification and the percolability of the soil as these control the hydraulic gradient that can be artificially produced. It has been found that a properly designed well-point system can economically lower the water-level 15 to 20 ft in favorable material by means of a single stage or set of pipes. For greater depths, multi-stages or additional sets of well-points must be installed. The effect on the water-level by these additional stages is, however, considerably less than that of the initial stage. The well-point system was successfully used during the recent installation of the foundations and basement construction of the Western Union Building in New York City, the water-level being reduced 30 ft. This was largely due to the character of sand encountered, it being rather fine, and therefore the water could be maintained at a steep gradient.

Steel Sheeting has been largely employed recently in place of wooden sheeting. It has the advantage that it can be driven in advance of the excavation, thereby reducing the likelihood of any flow of material under the sheeting. It also has the advantages of affording greater strength for a given thickness of sheeting, of being driven to a greater depth, and in many cases

Table XI. Weights and Dimensions of Pile-Driving Hammers

Size no.	Average weight	Weight of striking part	Dimensions over all			Cylinder			Steam-boiler horse-power required	Compressed air, free air per minute	Size of hose	Duty, size of piles hammer will drive
			Height	Width	Depth	Diameter	Stroke	Strokes per minute				
	lb	lb	in	in	in	in	in			cu ft	in	
Union Pile-Hammers Manufactured by Union Iron Works, Hoboken, N. J.												
00	21 000	5 500	156	36	25	14	36	90	100‡	.....	3	....
0	13 575	2 650	121	30½	23½	10½	24	110	50‡	.....	2	....
1	8 950	1 540	102	28	20¼	9½	21	130	35‡	600§	1½	....
2	5 875	950	86¾	25	16½	7¼	16	145	25‡	300§	1¼	....
3	4 500	680	76¾	23½	14¾	6¼	14	170	20‡	250§	1¼	....
4	2 575	360	63¼	20	12½	5¼	12	200	12‡	150§	1	....
5	1 500	210	50½	17¼	10¼	4¼	9	240	10‡	100§	1	....
6	850	100	42½	14	8¼	3¼	7	340	8‡	75§	¾	....
8	220	40	35	8	5	2⅜	6½	450	....	60§	¾	3″¶
9	97	24½	29½	6¾	4¼	2	4	550	....	45§	½	2″ and 3″¶

McKiernan-Terry Pile-Hammers
Manufactured by McKiernan-Terry Drill Co, New York, N Y

11–B–2	13 185	3 625	119	30	26	12⅜	20	120	60	600	2	21"¶	Largest‖	
10–B–2	10 000	2 500	110	28	24	10	20	115	50	500	2	20"	Largest	
9–B–2	6 760	1 500	92	24	20	8½	16	140	40	400	1½	17"	Largest	
7	5 000	800	73	27	21	12½	9½	225	35	350	1½	10"×14"	Largest	
6	2 900	400	63	24	19	9¾	8¾	275	25	275	1¼	6"×12"	12"	
5	1 500	200	57	19	14	7	7	300	20	200	1¼	4"×12"	12"	
3	675	68	62	14	13	3¼	5¾		15	90	1	3"×12"	12"	
2	343	48	37	11	10	4¹⁄₁₆	5¼	.	10	85	¾	3"×8"	9"	
1	145	21	47	9	11	2¼	3¾	.	10	75	¾	2"×10"	9"	
0	95	5½	24	8	12	2¼	4	.	5	45	¾	2"×10"	9"	

Warrington-Vulcan Steam Pile-Hammer
California Pile-Hammer (Double Acting, Compound)
Manufactured by Vulcan Iron Works, Chicago, Ill

0*	16 250	7 500	180	....	.	16½	48	50	60	1 450	2½	24"¶		22"**
1†	10 000	5 000	159	.	.	13½	36	60	40	975	2	18"	Heaviest‖	18"
1*	9 600	5 000	156	....	..	13½	36	60	40	975	2	18"	Heaviest	18"
2†	6 600	3 000	144	...	.	10½	36	70	25	580	1½	14"	14"	14"
2*	6 300	3 000	138	...	..	10½	36	70	25	580	1½	14"	14"	14"
3*	3 700	1 800	114	....	...	8	30	80	18	380	1¼	10"	12"	..
4*	1 400	550	84	....	...	4	24	80	8	62	1	8"	8"	..
E	3 800	950	93	...	..	10½–7⅝	16	150	25	460	2	12" or 8"×12"		14"
F	1 800	330	71	...	...	6¹⁵⁄₁₆–5⁵⁄₁₆	12	190	15	167	1½	8" or 6"×10"		12"
G	750	100	47	....	....	4¹¹⁄₁₆–3¾	8	270	7	65	1	4"× 8"		12"

* Standard base. † McDermid base. ‡ 100 lb pressure. § 90–100 lb pressure. For Warrington-Vulcan steam hammer blows per min. is minimum, and steam or air pressure at hammer is 80 lb. For California S. H (E, F, G) steam or air pressure at hammer is 85 lb. Union and McKiernan-Terry pile-hammers are double-acting. ¶ Size wood sheeting, square or round piling, in. ‖ Size steel sheet piling, in. ** Largest diameter of concrete, pile, in.

Table XII. Steel Sheet Piling

Section no.	Type of section	Size, in	Thickness of web, in	Width of wall, in	Weight in lb per lin ft of bar	Weight in lb per sq ft of wall	Area of section in sq in	Section mod. of bar, in³	Section mod. per lin ft of wall, in³
Lackawanna Steel Sheet Piling Manufactured by Bethlehem Steel Co.									
S.P. 8	Straight	8 1/2	13/64	2 1/2	14 7	20 8	4 32	1 10	1 55
S.P. 8a	Straight	8 1/2	3/8	2 1/2	17.8	25 1	5 23	1.10	1 55
S.P. 12	Straight	12 3/4	3/8	3 45/64	37 2	35.0	10.94	4 00	3 77
S.P. 12b	Straight	12 3/4	1/2	3 45/64	40 9	38 5	12.03	4 02	3 78
S.P. 15	Straight	15	3/8	3 11/16	38.4	30.7	11.30	3.97	3 17
A.P. 14	Arch	14	3/8	3 11/16	40.8	35.0	12.01	7 61	6.52
A P. 15	Arch	15	9/16	4 1/8	58.1	46.5	17.09	11.86	9.49
A.P. 16	Arch	16	3/8	3 11/16	29.3	22.0	8.63	4.87	3.65
D.P. 165	Deep arch	16	3/8	10	33.3	25 0	9.80	13.42	10 07
D.P. 166	Deep arch	16	1/2	12	42 6	32 0	12 54	20 28	15.21
Carnegie Steel Sheet Piling Manufactured by Carnegie Steel Co									
M 106	Arch	14	3/8	6 1/4	36 9	31 6	10 85	10 34	8 86
M 107	Straight	15	3/8	3 1/4	38.4	30.7	11.29	4.10	3 28
M 108	Straight	15	1/2	3 1/4	42.8	34 2	12.59	4 10	3.28
M 110	Deep arch	16	31/64	12	42 6	32.0	12.53	20 34	15 26
M 111	Arch	16	3/8	6 3/4	29 3	22 0	8 62	7 74	5 80
M 112	Straight	16	3/8	2 11/16	30 6	23.0	9.00	2 50	1.88
M 113	Straight	16	1/2	2 11/16	36 2	27.2	10 64	3.28	2 46
M 114	Deep arch	16	3/8	10	33 3	25 0	9 80	14 60	10 95
J & L Steel Piling Manufactured by Jones & Laughlin Steel Corp									
E.C. 22 0	Arch	17 3/4	3/8	4 3/4	32 50	22.00	9.56	5.18	3 50
E.C. 23.75	Arch	16 1/2	3/8	6 5/64	32 50	23.75	9.56	6.53	4 75
C. 27	Deep arch	14 1/2	3/8	8	32 50	27 00	9.56	8.19	6 77
D.C. 25	Deep arch	16	3/8	11 3/16	3 33	25 00	9 80	13 80	10 35
Larssen Sheet Piling Manufactured by Vereinigte Stahlwerke A. G. Dortmunder Union, Dortmund, Germ									
S. W.	Straight	11	35/64	1 15/16	29 10	31.75	8.56	1.83	2.00
Ib	Arch	14 3/16	1/4	3 15/16	18 56	15 7	5 45	2 75	2.33
Ia	Arch	15 3/4	9/32	5 1/8	22.31	17 0	6 56	3 99	3.04
I	Arch	15 3/4	5/16	5 7/8	26 25	20 0	7.73	5 46	4.16
II	Arch	15 3/4	13/32	7 7/8	32 81	25 0	9.66	9.43	7.17
IIa	Deep arch	15 3/4	5/16	10 5/8	30 84	23 5	9 08	13.38	10 19
III	Deep arch	15 3/4	9/16	9 3/4	42.00	32.0	12.34	12 41	9 48
IIIa	Deep arch	15 3/4	7/16	11 3/8	39.05	29 75	11.47	13 95	10 62
IV	Deep arch	15 3/4	39/64	12 1/4	50.53	38 5	14 88	19 25	14 66
V	Deep arch	16 9/16	7/8	13 1/2	67.63	49 0	19.91	25 81	18 63
VI†	Deep arch	16 9/16	7/8	17 3/8	82.81	60 0	24 35		
I	Box	18 1/8	13/32	10 5/8	76 51	59.6	22.50	73.26	48 50*
Ic	Box	18 1/8	17/32	10 7/8	91.46	69 5	26 86	89 27	59.10*
II	Box	18 1/8	37/64	14 17/32	104 60	78 2	30 75	134 88	89 30*

* Interlocked. † Not available

Wemlinger corrugated steel sheet piling

Thickness		Depth of Corrugations, in	Length in feet	Thickness of clips, in	Approx. weight per lin ft or sq ft, lb	Section mod. per single section, in³
Gauge	Inches					
10	0.134	1 7/8	8–16	3/16	9.0	1.06
11	0.125	1 7/8	8–16	3/16	8 75	0.97
12	0.109	1 7/8	8–10–12	1/8	7 50	0.87

of being withdrawn and used over again. As generally manufactured, it has the further advantage of being INTERLOCKING, so that there is less danger of its getting out of line and leaving openings between adjacent pieces.

All of these advantages have been considered by engineers in using steel instead of wooden sheeting.

The Use of Steel Sheeting. The fundamental idea of steel sheeting is not new, as CAST-IRON SHEET-PILING was used in England as far back as 1822 and various combinations of steel plates have been used in coffer-dams. The general use of steel sheeting started in this country in 1899 when Luther P. Friestedt drove experimental INTERLOCKING CHANNEL-BAR SECTIONS. Since that time it has come into general use, and with its aid many excavations have been made with STEEL SHEET-PILING which would have been impracticable with timber sheeting.

Earth-Pressure on Steel Sheeting. In using STEEL SHEETING, it should be borne in mind that the EARTH-PRESSURE coming on the steel sheeting is the same as the earth-pressure coming on timber sheeting, and the breast-pieces and braces should be as strong as in the case of timber sheeting. Certain forms or sections of steel sheeting offer considerable resistance to bending due to the lateral earth-pressure. With such forms the horizontal breast-pieces may be spaced farther apart than with ordinary timber sheeting or steel sheeting not having this property; but the strength of the breast-pieces and of their braces must be sufficient to take up the entire load coming on the sheeting, irrespective of the spacing between such breast-pieces, for in case there is a failure in these the entire sheeting will fail.

Different Forms of Steel Sheeting. Various TYPES OF STEEL SHEETING are on the market. In making a selection between different forms of sheeting,

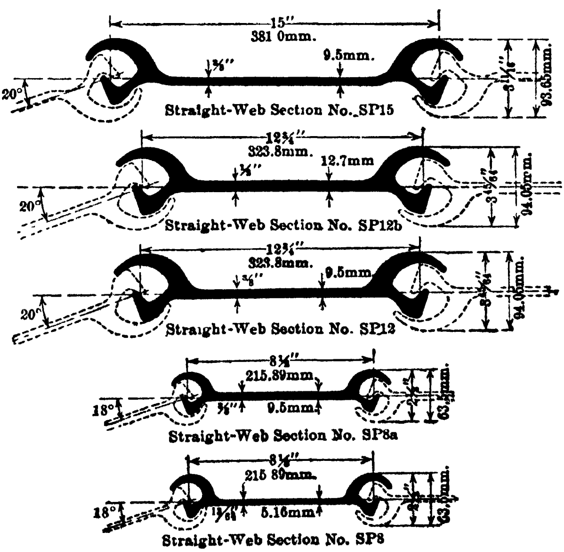
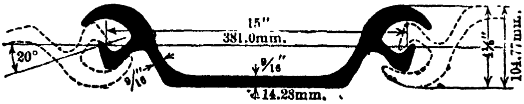
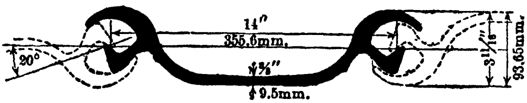


Fig. 36. Lackawanna Steel Sheet Piling

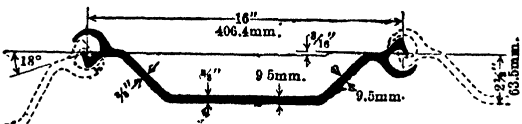
the character of the material to be encountered should be borne in mind, as the simpler, straight, heavy web sheeting will penetrate hard and gravelly soils with less danger of deformation than the arch or deep arch lighter web sheeting. The various manufacturers of sheet-piling publish catalogues



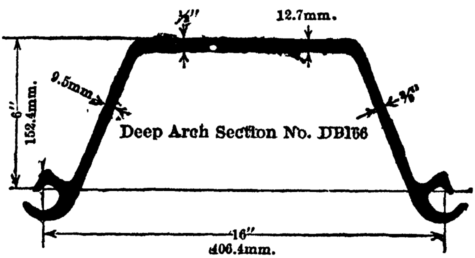
Arched-Web Section No. AP15



Arched-Web Section No. AP14



Arched-Web Section No. AP16



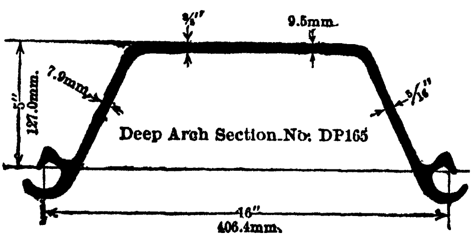


Fig. 36 (Continued). Lackawanna Steel Sheet Piling

containing complete data as to weights and dimensions of the different sections. These catalogues may be obtained from the manufacturer, but for convenience illustrations and some data of the principal sections are given in the following pages.

The Poling-Board or Chicago Method is a special method of excavation in general use in Chicago and in occasional use elsewhere for excavations which go to a great depth in clay or in other suitable material. It has the

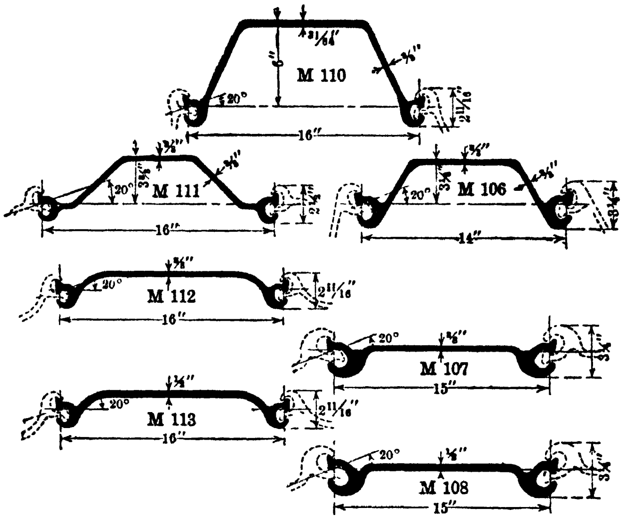


Fig 37. Carnegie Steel Sheet Piling

advantage over the ordinary sheet-piling method that the lining of the excavation is not driven. The method is not generally used for trenches or for square excavations, as a circular excavation is more readily handled. The success of the method depends entirely upon the character of the material

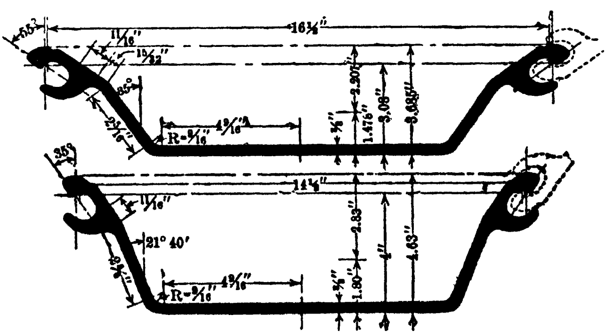
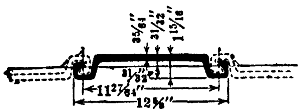


Fig. 38. Jones & Laughlin Steel Piling Sections

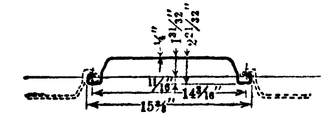
to be encountered, as the excavation is first made and the sides of the excavation afterwards supported. The method in detail for a circular excavation for a pier-foundation may be described as follows:

(1) A circular excavation slightly in excess of the size required for the pier

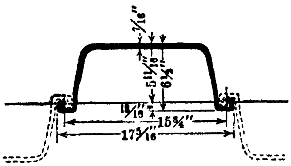
is carried down to a depth of 5 ft, great care being taken to have the sides of the excavation vertical and true to the circle.



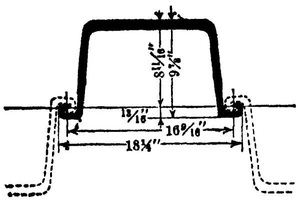
LARSSEN Straight Web Section



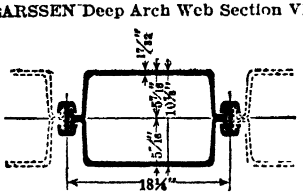
LARSSEN Deep Arch Web Section I B



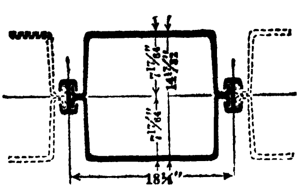
LARSSEN Deep Arch Web Section III A



LARSSEN Deep Arch Web Section VI



Box Section I



Box Section II

Fig. 39. Larssen Sheet Piling

(2) Vertical planks called LAGGING-PIECES, 5 ft in length and slightly beveled on their edges so that each piece may be considered as a stave with radial joints corresponding to the size of the required circle, are set in place against the walls of the excavation. These planks are held in place by two or more steel rings, generally made in quadrants, so that they may be conveniently handled and bolted together. The planks are wedged firmly against the walls of the excavation by means of wooden wedges driven between the planks and the iron rings

(3) As soon as the first set of lagging is complete, the excavation is carried down for another section, 5 ft in depth, and another section of lagging is put in place and secured in the same manner.

Depth and Character of Excavations in Poling-Board Method. In the manner described above the excavation may be carried down indefinitely, a depth of 100 ft having frequently been attained. In the case of the Cleveland Union Passenger Terminal, piers were carried by this method to rock at a depth of 243 ft below street-level, or 205 ft below sub-grade for the station tracks In many cases the bottom of the excavation is BELLED OUT to a larger diameter than the excavation for the main shaft of the pier with the object of reducing the load on the foundation-bed to a unit load less than the safe unit load on the main shaft of the pier. This method is not adapted for running sand nor for clay that is not solid enough to stand with vertical sides during the necessary interval between making the excavation and placing the lagging. In some cases where a stratum of quicksand has been encountered, the excavation has been carried past it by the use of a cylin-

drical shell of steel, forced by jacks through it to an underlying impervious layer of clay; but in general this method is dependent for its success upon a continuous body of impervious material.

The Open-Caisson Method or Well-Curb Method is used for piers to be carried to a considerable depth, and has advantages over the sheet-piling method in certain materials. It is a development of the old method used in sinking masonry wells and, in its modern form, consists of a structure which eventually forms part of the pier itself and which is arranged with an open chamber at its base in which men may excavate the material under the structure and allow it to settle as the excavation proceeds. It is evident that a central opening or shaft must be left in the structure to permit of the passage of men and material.

Details of the Open-Caisson Method. In detail, the method may be described as follows: First a CURB or CUTTING-EDGE of timber or steel, follow-

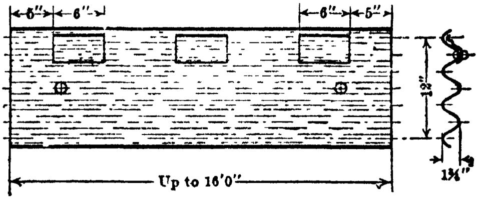


Fig. 40. Wemlinger Corrugated Steel Sheet Piling

ing the outline of the pier, is constructed on the surface of the ground. The outer face of this curb is generally vertical and is protected with a steel plate which extends below the main section of the curb, so as to form a cutting-edge or sharp downward projection serving to penetrate the soil slightly in advance of the excavation. On this curb a wall of timber, concrete, or masonry is constructed, inside of which the so-called WORKING-CHAMBER affords room for the workmen to be employed in excavating. Above the working-chamber the walls may continue to a height corresponding to the required height of the pier, leaving the central space to be filled in after the required depth is reached; or a roof may be built over the working-chamber and the entire cross-section of the pier filled with concrete or masonry excepting only a small central opening large enough to accommodate a HOISTING-TUB or BUCKET and to permit of the ingress and egress of the men employed in sinking the construction. In practice, the excavation is started before the pier-structure is carried up to its final height, after which the excavation and the building up of the pier progresses simultaneously, the constantly increasing weight of the structure aiding the sinking of the pier. When the excavation has reached rock or a firm substratum, further excavation is stopped and the working-chamber and the central opening are packed full of concrete, leaving finally a complete pier-structure extending from the rock to the proper level to receive the steel grillage or other construction coming on the pier.

Advantages of the Open-Caisson Method. This method of construction has the advantages that the workmen at all times are protected, that obstructions, such as boulders or logs, may be removed from under the cutting-edge, and that when rock is encountered, ample opportunity is afforded for the proper preparation of the rock-surface to receive the final concrete filling.

If a moderate amount of water is encountered, not accompanied by a flow of material, it can generally be taken care of by means of pumps.

Dredged Wells are similar to the open caissons described in the previous paragraphs and are used where large quantities of water are encountered. The construction of the piers is similar to that of the piers used in the open-caisson method; but the central shaft and working-chamber are designed to permit of the use of a CLAM-SHELL DREDGE or other form of dredge, and the water is allowed to rise to its natural level in the working-chamber and shaft. This method can be used to advantage when a considerable amount of water-bearing sand or other material is found overlying level rock or other firm foundation-bed. When the dredging and the sinking of the pier-structure have been carried down to the hard underlying strata it is sometimes possible to pump out the water. If this is not practicable the bottom may be prepared by divers for the reception of the concrete filling, and the concrete may be deposited through water, care being taken to use some special arrangement to protect the concrete from being injured by loss of its cement-content, in the process of deposition.

The Well-Digger's Method is also occasionally used in making PIT-EXCAVATIONS under walls or in cramped locations. By this method the sides of the excavation are supported by planks placed horizontally. The method of placing the planks is as follows: A shallow excavation, the depth of a plank, is made by ordinary methods, and a SET, consisting of four planks fitting the four sides of the excavation, is secured in place. Before proceeding with the general excavation of the pit a trench is dug directly alongside and underneath one of the side planks of the FIRST SET. As soon as this trench is deep enough to accommodate the planks for the SECOND SET, the side of the trench under the plank already in place is cut to a vertical face, the plank placed in position and the loose earth temporarily back-filled against it. As soon as the four planks forming the SECOND SET have been put in place by this method, the two side planks are wedged against the bank, the end-planks being used as struts The end-planks are wedged into position and nailed or cleated to the side planks, forming a PRESSURE-RESISTING FRAME supporting the side of the excavation. A continuation of this method enables the excavation to be carried on indefinitely, provided there is no flow of water or run of material causing an inflow of material into the excavation.

The Pneumatic-Caisson Method. Where piers or foundation walls have to be carried to a considerable depth through water-bearing materials, and especially where large bodies of quicksand are encountered, the PNEUMATIC-CAISSON METHOD must be resorted to. This method is based upon the PRINCIPLE OF A DIVING-BELL and may be briefly described as follows: The construction of the pier is similar to the piers previously described as used in the open-caisson and dredged-well construction, except that the working-chamber and shaft are made air-tight and connected with a device called an AIR-LOCK, so that compressed air may be introduced into the working-chamber. The object of the compressed air is to prevent water entering into the working-chamber. This is accomplished in accordance with the well-known PRINCIPLE OF THE DIVING-BELL by having the compressed air constantly kept at a pressure which will counterbalance the water-pressure at the level of the cutting-edge of the working-chamber. The pressure of the air evidently must vary with the depth of the cutting-edge below water-level. A column of water 1 in square in cross-section weighs .43⅓ lb per vertical ft, and it will therefore be counterbalanced by an air-pressure of .43⅓ lb per sq in over the normal air-pressure. If the column of water is 30 ft in height, it will

weigh thirty times .43⅓ lb, or will be counterbalanced by an air-pressure of 13 lb per sq in above the atmospheric pressure.

The Maximum Air-Pressure in the Pneumatic Caisson in which men can work for short periods is about 50 lb per sq in above atmospheric pressure, corresponding to a depth below water-level of about 115 ft. At this depth the work is carried on in shifts of one-half hour duration. Great care must be exercised in decompressing from the high air-pressures to atmospheric, as the physiological effects of compressed air are often very serious. BENDS, or CAISSON-DISEASE, causes severe pains in the joints, may injure the ear-drums and result in deafness, so that working under high pressure is extremely hazardous.

Table XIII gives working and decompression periods as required by the New York State Labor Laws as of August 1, 1929. The working period or shift is the time under pressure and does not include the time in entering or leaving the working chambers. The maximum pressure reached at any time during the shift is the governing factor in the number of working hours. The "sand-hogs" must be examined by physicians at regular intervals.

Table XIII. Working and Decompression Periods Required by New York State Labor Laws

Shifts and intervals of work for each 24-hour period

Air pressure in pounds	Working periods per day	Minimum rest intervals in open air, hours	Payment rate per day, Oct., 1929
0–18	2–4-hr shifts	½	12.00
18–26	2–3-hr shifts	1	12.50
26–33	2–2-hr shifts	2	13 00
33–38	2–1½-hr shifts	3	13 50
38–43	2–1-hr shifts	4	14.00
43–48	2–¾-hr shifts	5	14 50
48–50	2–½-hr shifts	6	15.00
Decompression periods			
Air pressure in pounds		**Minimum number of minutes**	
0–10	. . . . ..	1	....
10–15	.... . .	2	....
15–20	....	5	....
20–25	....	10	....
25–30	....	12	....
30–36	. . . .	15	....
36–40	. . ..	20	....
40–50	. . . .	25	....

In sinking a bridge pier at Vicksburg, Miss., the pressure reached 54 pounds. The men worked two 25-minute shifts and the decompression period was 1 minute per lb of pressure. No cases of bends were reported.

The Air-Lock Used in Connection with the Pneumatic Caisson is a device for the purpose of retaining the air in the caisson and at the same time per-

mitting the passage of men and material in and out. It consists essentially of a metallic AIR-TIGHT CHAMBER or SHELL connected to the working-chamber either directly or to an air-tight lining or extension of the central shaft-opening. This air-chamber has two doors, one at the bottom, opening downward into the shaft, and the other in the upper head of the air-lock chamber, also opening downward and affording a direct connection to the open air. In the operation of an air-lock one of these two doors must at all times be closed so as to prevent the free escape of air through the air-lock. If the bottom door is closed, it will be held firmly to its seat by the uplift of the compressed air in the shaft, which is at all times in direct communication with the working-chamber. If, under these conditions, the upper door is open, the interior of the air-lock will be in direct communication with the open air and the air contained in the lock will evidently be at atmospheric pressure. Workmen and materials may then enter the air-lock. In order to pass into the shaft and working-chamber, it is necessary, first, to close the upper door, and secondly, to shift the so-called EQUALIZING VALVE and admit compressed air into the space between the two doors, until the air-pressure is brought up to the air-pressure in the working-chamber and shaft. Pressure on the upper side of the lower door will then equal the pressure on the lower side and the lower door may be opened, the upper door being firmly held against its seat by the compressed air in the air-lock. As soon as the lower door opens, the men and material may be passed into the shaft and working-chamber. In coming out the operations are reversed; men and material enter the air-lock through the open lower door, the lower door is closed and held tightly against its seat, and the equalizing valve is shifted, affording a connection between the interior of the air-lock and the external air. The compressed air escapes through the equalizing valve, reducing the pressure in the air-lock to atmospheric pressure, and the upper door has atmospheric pressure on both sides of it. It may then be opened, giving free connection with the outside air.

The Design of Pneumatic Caissons. The first consideration is, of course, to have the final structure a permanent and sufficient pier to carry the load to be imposed upon it. To this end the cross-section of the pier at all points from top to bottom should be capable of carrying safely the maximum load. As the cross-section of the pier is generally, in the finished pier, composed of solid concrete, the cross-section will be determined by the allowable load on the concrete. For piers the cross-section will generally be square or circular; for walls the caisson will generally be not less than 6 ft in width, as it is difficult to sink caissons having a width less than 6 ft. If the caisson is to be carried to solid rock, the bearing on the rock need be no larger than the cross-section of the concrete pier; but if the excavation does not go to rock, it is frequently desirable to BELL OUT the base of the pier so as to reduce the loading on the foundation-bed to a unit load less than that allowable on concrete. The operation of BELLING OUT is difficult in some materials; in a compact material it can be generally accomplished without serious difficulty.

Piers Sunk by the Pneumatic-Caisson Method may be constructed of various combinations of materials. The side walls and roof of the working-chamber were formerly frequently constructed of timber. In many cases they are now formed of steel; but in recent designs the working-chamber is generally formed of reinforced concrete, the only structural steel used being an angle or a plate and angle composing the cutting-edge. The outside of the caisson is preferably made vertical. The superimposed pier is generally of the same size as the working-chamber, at least it is generally so in piers sunk for buildings.

A Typical Design for a Caisson Built of Reinforced Concrete is given in Fig. 41, in which *AB* is the angle-iron and plate forming the so-called CUTTING-EDGE and *C* is the WORKING-CHAMBER formed by the side walls *DE* and *DE* and by the roof *EE*. The concrete side walls are reinforced with steel rods attached to the cutting-edge, and extending upward into the body of the pier, and the roof and body of the pier are reinforced to take care of stresses due to construction and sinking. In building up the working-chamber, the INTERIOR FORMS are arranged so as to support the concrete which makes the roof. These are subsequently removed. The exterior forms may constitute a permanent part of the structure, in which case they are called a COFFERDAM, or they may be removed as soon as the concrete has sufficiently set. At the center of the pier an opening is left to serve as the SHAFT or opening connecting the working-chamber with the AIR-LOCK. The sides of this opening or of the upper part of it, only, are lined with an AIR-TIGHT STEEL SHELL. To the upper end of the steel shell the air-lock is connected. If the height of the pier does not exceed 40 ft the construction of it may be completed before the excavation is commenced. Generally, however, the construction of the pier is stopped as soon as the working-chamber and from 5 to 10 ft of the superimposed pier has been constructed; then sufficient excavation is done, without the use of compressed air, to carry the cutting-edge down to water-level. This is called DITCHING THE CAISSON and is done so that the caisson will have some slight lateral support from the soil before the construction is carried up high enough to make it top-heavy. When the entire pier or the first section is finished, excavation is resumed and the whole structure is sunk as the excavation progresses, care being taken to remove any obstruction from beneath the cutting-edge. During the progress of sinking compressed air is conducted to the working-chamber through the SUPPLY-PIPE *G*, the excavated material being hoisted through the SHAFT *F*. The shaft *F* is fitted with a LADDER for the use of the workmen.

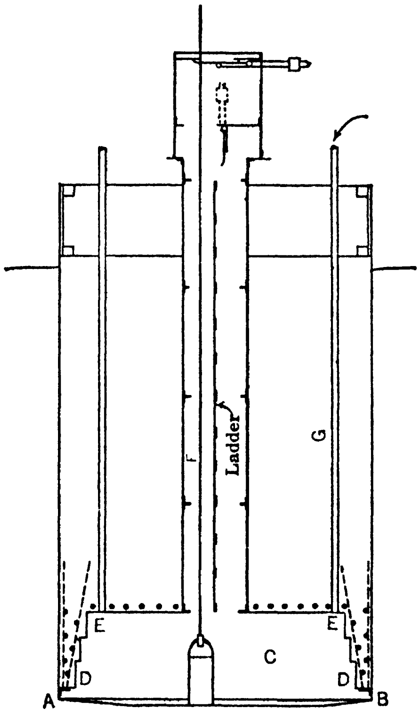


Fig. 41. Typical Form of Reinforced-concrete Caisson

Details of Caisson-Sinking and Filling. In sinking the caisson and superimposed pier, care must be taken to maintain it in a vertical position. This

end may be accomplished in large caissons by means of the excavation itself. In case one side of the caisson is high the excavation on that side will be carried somewhat in advance of the excavation on the low side, and the material under the cutting-edge of the high side will be removed while a bank of material is kept under the cutting-edge of the low side. These methods, however, are of little avail when the caisson is narrow. In such cases that part of the caisson which is above ground is held in position by GUIDES or other devices; but it frequently happens that the caisson in its final condition is considerably out of its correct location and considerably out of plumb. In general, therefore, the size of the caisson should be made larger than the minimum size necessary, in order to allow for errors in its final location. When the caisson has reached the required depth the foundation-bed is prepared for the reception of the concrete filling and the working-chamber filled with it, care being taken that it completely fills all voids and is in perfect contact with the roof. Finally, the air-lock and the steel lining of the shaft are removed and the shaft-opening filled with concrete to the proper level to receive the GRILLAGE or other construction forming the base of the column which is to rest on the caisson.

The Height of Caisson-Piers. The height of a pier cannot be exactly fixed until it is known to what depth the caisson must sink in order to reach the foundation-bed. If the rock is found at a greater depth than anticipated, additional height is added to the top of the pier after the caisson is in its final position; but if, on the other hand, the rock is found unexpectedly high, the top of the pier will have to be cut off. If the finished elevation of the pier is to be below the level of the general excavation, it is usual to extend the exterior surface of the pier to the required height by means of a temporary chamber-structure called a COFFER-DAM, the height of which corresponds to the depth of the finished surface below the level of the general excavation. Inside of this COFFER-DAM some STEEL GRILLAGES may conveniently be set.

Coffer-Dam Wall Construction by Pneumatic Caissons. A COFFER-DAM WALL around the perimeter of the lot is constructed by the sinking and sealing to rock or other bearing strata of a series of pneumatically constructed caissons. Adjacent caissons are not sunk simultaneously as this procedure involves the danger of a "blow-out" and loss of air. One caisson being lower than the other and therefore using a higher pressure, the air from the lower caisson may flow through the sand and gravel to the working-chamber of the other. The danger is not only the sudden loss of air, but a loss of material which may cause a bad lean to the caisson itself or the heaving of materials in the working-chambers, thereby making it doubly hazardous for the laborers. Caissons which are to form a coffer-dam are usually spaced approximately 1 ft 6 in to 2 ft apart. The ends of the caissons have keys formed as shown in Fig. 42. Assuming that two adjacent caissons are sunk in place, the first step in the joint construction is the driving of steel sheeting on each side of the caissons, 10 or 12 ft below the top of the same. This space is then excavated to, or somewhat below, water-level. The external forms of the caissons are stripped, as far as possible. A PONY SHAFT is placed between the caissons, and the balance of the space between the two caissons is filled with concrete. An air-lock is attached to the pony shaft and compressed air is applied. One or two men only can work in these joints. They proceed by excavating 3 or 4 ft and building a form between the caissons. After excavating, they place a second form and construct a small concrete wall on each side; and thus line the shaft as the excavation proceeds. Finally rock is reached and cleaned, after which the shaft is sealed and concreted to its

proper elevation. When all the joints between the caissons have been completed, the coffer-dam wall is complete and the excavation within the coffer-dam may proceed to its desired elevation.

Coffer-Dam Wall of the Federal Reserve Bank of New York. Fig. 43 illustrates a cross-section through the coffer-dam wall of the Federal Reserve Bank of New York. In most cases the rock-level was very close to the anticipated elevation. In this particular case the caisson straddled a ROCK POCKET, that is to say, at each end of the caisson, the rock was encountered

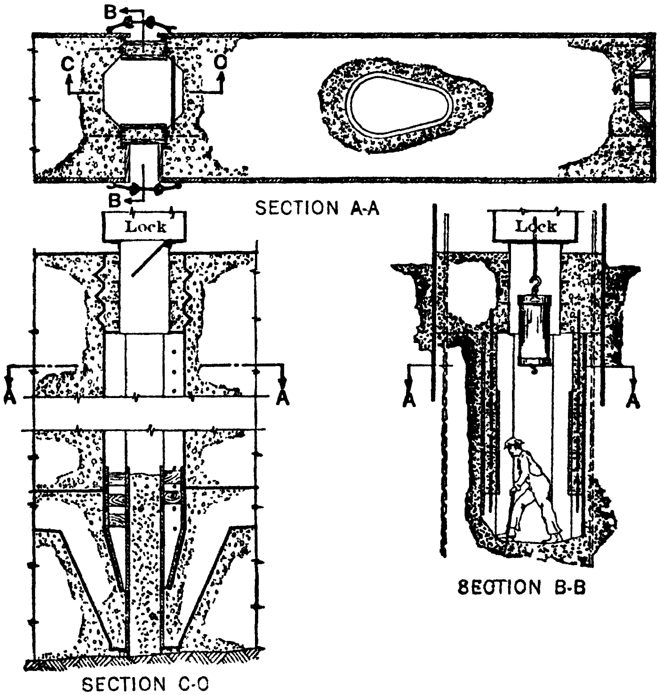


Fig. 42. Coffer-Dam Wall Construction by Pneumatic Caissons

very near the anticipated elevation, from which points the rock surface pitched downward at a very steep angle toward the center. It was therefore necessary, in order to reach hard rock, to underpin the cutting edges in stages as shown. Upon the completion of this underpinning, which was hazardous on account of the high air-pressure, the thinness of the underpinning walls and the lack of lateral stiffness, the deepened working-chamber was sealed and concreted under air. The shaft was then concreted, completing the caisson.

Coffer-Dam Wall Construction by Open-Trench Methods. Under certain conditions coffer-dams to rock below water-level can be constructed by the OPEN-TRENCH METHOD, dependent, however, upon the character of the mate-

rial overlying the rock. A double line of steel sheeting is driven, creating walls for a trench. Tee connections are placed at regular intervals so as to subdivide the trench into rectangular boxes. Excavation, caulking and bracing of the sheeting proceeds in alternate boxes until rock is reached, after which either the whole trench is filled with concrete and necessary steel or, for a thinner wall, a form is placed for the inside face of the wall and the space between it and the outer sheeting is then concreted. The success of this method is dependent on many factors, but primarily on being able to drive the sheeting, without breaking the connections between sheets at any point, to a water-tight seal. There are many hazards in the use of this method, for example, the unexpected presence of boulders preventing the driving of the sheeting, or of a water-bearing strata below the strata on which the sheeting has landed, as well as any failure in the water-tightness and the bracing of the sheeting. It is generally possible to convert an open-trench box into a PNEUMATIC CHAMBER by placing a deck, fitted with shafts and air-locks. The details, methods and dangers of this are too numerous to be discussed in this chapter. This open-trench method has been highly developed and has proved very successful. It has recently been used where 10, 15 or 25 years ago pneumatic caissons would have been deemed necessary.

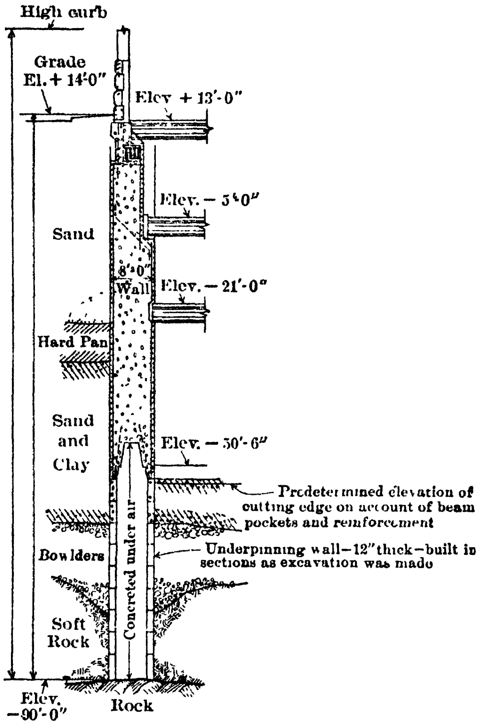


Fig. 43. Coffer-Dam Wall of the Federal Reserve Bank of New York

Temporary and Permanent Cross-Lot Bracing. Permanent coffer-dam walls are usually not designed to act as gravity sections or as cantilevers to resist overturning. It is therefore necessary to support these walls temporarily by timber or steel struts, installed progressively as the excavation is made, and to maintain them until the permanent structural floors of the building are in place which can then support these walls.

If the excavation within the coffer-dam is relatively shallow, SPUR-BRACING, or diagonal timbers bearing on DEADMEN or cribs at the subgrade level, may be very effectively used.

When the excavation is to be carried to a considerable depth within the coffer-dam, spur-bracing usually cannot be used. Spurs placed at the proper angle would become of such a length that their effectiveness would be greatly reduced and difficulty of their maintenance very much increased, and they would become a hindrance to the installation of the permanent construction. To overcome these difficulties, CROSS-LOT BRACING, that is, continuous horizontal struts from one side to the other of the coffer-dam, is used. See Fig. 44. These struts may be of timber or of steel sections or a combination of both may be used. The struts are designed as horizontal columns of sufficient strength to support the walls and should be so arranged

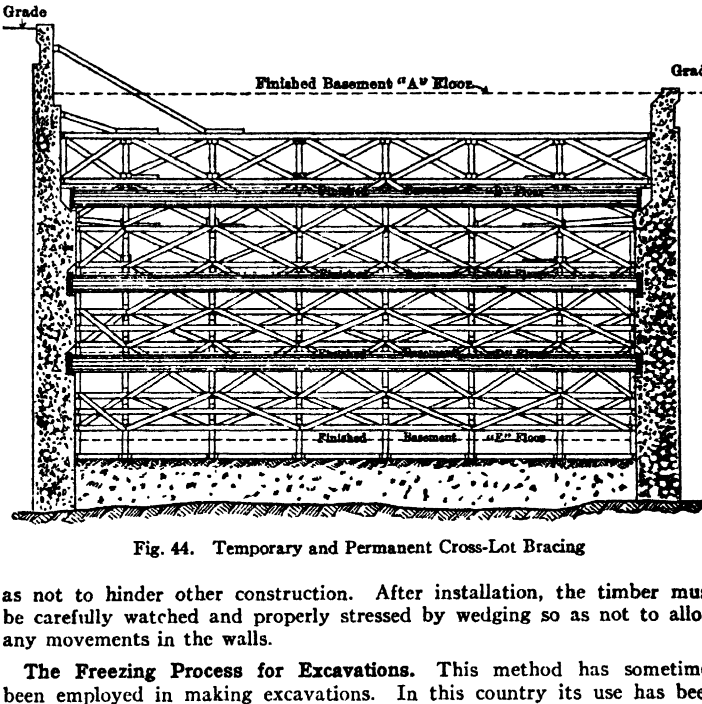


Fig. 44. Temporary and Permanent Cross-Lot Bracing

as not to hinder other construction. After installation, the timber must be carefully watched and properly stressed by wedging so as not to allow any movements in the walls.

The Freezing Process for Excavations. This method has sometimes been employed in making excavations. In this country its use has been limited to one or two mining-shafts, but in Germany it has been resorted to in making excavations for building-foundations. The method consists in driving steel pipes into the ground. These pipes are closed at the bottom and at the top are connected to smaller pipes through which brine, at an extremely low temperature, is made to circulate. The refrigerating effect results in freezing the water contained in the soil, converting quicksand to a frozen mass resembling soft sandstone. When the freezing has progressed sufficiently to form a solid wall or coffer-dam around the excavation, the material inside the frozen wall may be excavated. The method has the advantage, theoretically, of being applicable to excavations of any depth. There are many precautions necessary, and for the present, at any rate, it should only be considered as a possibility.

Compressol System. In sand piles, the sand is placed in an open-ended pipe or similar arrangement which can be forced down into a compressible material. The sand is then rammed or forced by means of jacks out of the pipe into the surrounding materials for the purpose of compressing and stiffening the same. Under the name COMPRESSOL, the French have employed, in certain silts and similar soils, a pier of sand, gravel, or lean concrete which is poured into a pit formed in the material by repeated blows from a heavy steel weight. The mass of metal used is shaped like an ordinary top or plumb-bob. Under favorable conditions this is a logical method, but so far as the writer knows, it has not been used in the United States.

31. Protection of Adjoining Structures

General Considerations. The COMMON LAW provides that any person making an excavation is responsible for resulting damage to adjoining property. STATUTE LAWS as embodied in the building codes of different cities may modify or limit this responsibility, but, in general, excavations should be made in such a manner as to cause the least possible damage to surrounding property. Where there are no adjoining structures it is generally sufficient to slope the sides of the excavation so as to prevent the sliding of material into the excavation, or, at least, to sheet-pile and brace the sides of the excavation; but where the excavation is to be made alongside of an existing structure, and carried below the footings of such structure, it is necessary to take special measures for its protection. Such work is described as SHORING, UNDERPINNING and PROTECTING ADJOINING STRUCTURES, and may involve the carrying of the weight of part or all of the buildings on temporary supports, the removal of the old footings and the construction of new footings at lower elevations.

Shoring. When the excavation for the new building does not go much below the adjoining footings and when the material is fairly solid, it may suffice to transfer a portion of the load of the wall to temporary footings. This may be accomplished by means of heavy inclined posts called SHORES, arranged to act as INCLINED COLUMNS or STRUTS. Each SHORE consists of a post, the lower end of which rests on a PLATFORM, generally consisting of planks and timbers arranged so as to form a temporary spread footing. This platform should be placed at a depth which will insure that subsequent operations will not undermine it. The upper end of the post fits into a hole or niche cut into the wall to be supported. The post itself may be a timber with a square cross-section, usually 12 by 12 in, and of the required length. Provision is made, between the platform and the lower end of the post, for WEDGES or JACKS, so that when operated their lifting effect transfers part of the weight of the wall from its footing to the temporary foundation or platform. During this operation all parts of the temporary structure are in compression and brought into bearing, and the material under the platform is compressed and solidified as much as possible.

Kinds of Shores. If the SHORE is to act preferably for LIFTING only, it is kept as nearly vertical as possible and is known as a LIFTING SHORE. If it is to act preferably to combine a horizontal PUSHING action with the lifting action, it is placed at a considerable angle from the vertical and is then known as a PUSHING SHORE or STEADYING SHORE. In arranging such shores care should be taken to have the niche cut close to a floor-level of the building to be shored, as otherwise the horizontal component of the thrust of the shores might buckle the wall.

Numbers and Sizes of Shores. Where a wall is light, a number of smaller shores should be used in preference to a few large ones. Where a wall is high, two or more shores of varying lengths may be used, and these may conveniently be placed in the same vertical plane and rest on the same platform.

Wedges and Screw-Jacks. In transferring the load of a wall from its own footing to the temporary platform, use is made of wooden or steel WEDGES, SCREW-JACKS, or HYDRAULIC JACKS; or wedges and jacks may be used in combination. Wooden wedges should be made of hard wood and are generally arranged in pairs, both wedges being driven at the same time. The lifting effect of such wooden wedges is powerful, but where a considerable settlement of the temporary foundation is anticipated, it is more convenient to use screw-jacks, as they can take up a considerable settlement.

Materials and Types of Screw-Jacks. The SCREW-JACKS usually manufactured for this purpose are made of cast iron and have rough threads, with too coarse a pitch to have much lifting effect. Screw-jacks of a better kind are made of steel and have a machine-thread of small pitch. Such jacks can be obtained capable of lifting weights up to 100 tons. Figs 45 and 46 represent standard forms of screw-jacks When a single screw-jack is used in connection with a post or shore, a hole to receive the threaded portion of the jack is bored in the end of the timber used for the shore, the end being squared to receive the nut. Such an arrangement is called a PUMP and is illustrated in Fig. 47. When a lifting effect greater than that exerted by a single jack is required, the jacks are arranged in pairs in connection with a short timber or CROSS-HEAD. Such an arrangement is illustrated in Fig. 48. It has the advantage that after operating the jacks, blocking and wedges can be placed between the platform-timbers and the cross-head so that the post resting on the cross-head has a direct and solid bearing on the platform. By this method the load of the wall can be thrown on the platform by the jacks and after the blocking and wedging is in position the jacks can be removed.

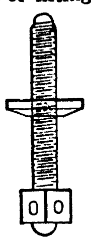


Fig. 45. Standard Type of Steel Screw-jack

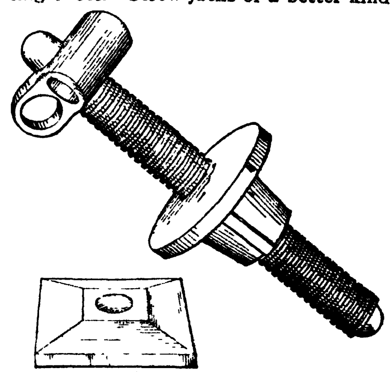


Fig. 46. Standard Type of Steel Screw-jack

Hydraulic Jacks. Where excessively heavy loads are to be lifted, HYDRAULIC JACKS may be used in place of screw-jacks, but an objection to them is that they are liable to SLACK BACK under the load. While the load, therefore, should not be permanently supported on hydraulic jacks, they may be used to take the load temporarily, while the blocking and wedging are being placed between the cross-head and the temporary footing. In this way an indefinite number of shores may be set and taken care of with a single pair of hydraulic jacks.

Example of Shoring. Fig. 49 shows the method used in SHORING the ornamental front wall of a heavy building, advantage having been taken of the

numerous deep margin-drafts shown in the section. In order to avoid the necessity of cutting niches for the tops of the shores, nine hardwood blocks, *a, a*, etc., were fitted to the margin-draft grooves in the masonry. Nine similar blocks, *b, b*, etc., were gained into and bolted to the vertical timber *VV*, space being left between the *a* blocks and the *b* blocks for adjusting wedges *w, w*, etc. Three headpieces, T_1, T_2 and T_3 were keyed and bolted to *VV* and transmitted to it the uplift of the three shores, S_1, S_2 and S_3. Each shore

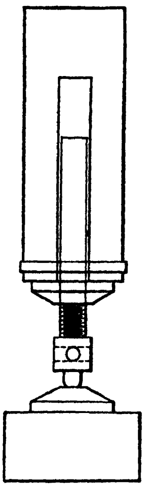


Fig. 47. Pump, or Screw-jack let into End of Shore

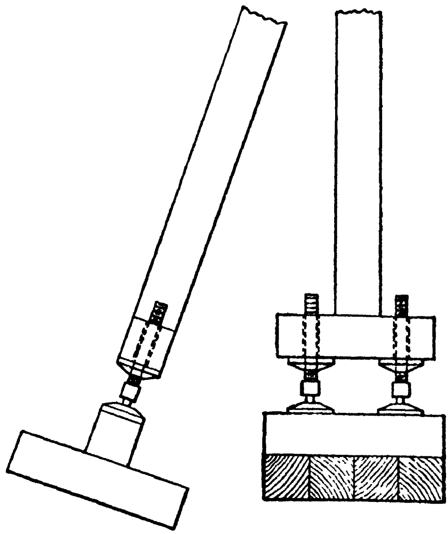


Fig. 48. Shore, Screw-jacks and Timber Cross-head

had a 60-ton screw-jack at its base. Each shore is shown fitted with a pump or detached extension-piece arranged for the screw-jack.

Needling. NEEDLES OR GIRDERS are employed when part or all of the weight of the wall has to be carried, as when the old footing is to be removed and the wall UNDERPINNED or carried down to a new footing at a greater depth.

Example of Needling and Underpinning. Fig. 50 represents a typical case of UNDERPINNING, the several operations being as follows:

(1) **The General Excavation** is carried down to within a few inches of the bottom of the footing *BB* under the wall *W*.

(2) **The Pit** *DDDD*, properly braced and protected by sheet piling, is sunk to approximately the level of the proposed excavation, this PIT being placed at a safe distance from the existing wall. In good material it may be safe to have this pit approach to within a few feet of the footing course of the wall, but in material which is liable to run it should not approach the wall closer than its depth. No hard and fast rule can be given, and in every case great care should be taken to prevent any movement of the material from under the adjoining footing.

(3) **The Platforms.** On the bottom of this pit-excavation, a PLATFORM *FF* is placed, generally composed of heavy timbers resting on a base of heavy

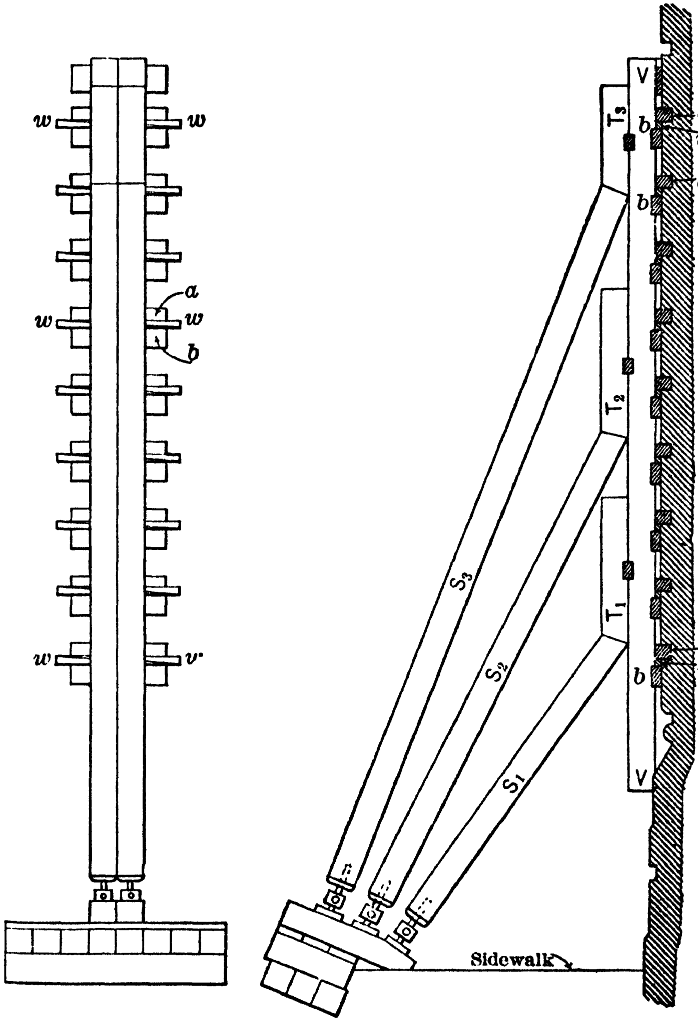


Fig. 49. Shoring an Ornamented Wall

planks, and acting as a support for the outer end of the needle. During the construction of this pit a similar pit may be dug on the inside of the wall to provide for the support of the inside end of the needle; but as this involves the

destruction of the cellar-floor the method of procedure inside the building is generally different from this. If the material is solid it is sometimes sufficient to place the platform for the support of the inside end of the needle directly on the cellar-floor and at such a distance from the wall that the necessary excavation for the new footing will not disturb it; or the platform may be placed on the cellar-floor and a line of sheeting *LL*, properly braced, and so placed that the excavation can be made for the new footing. This is generally sufficient to prevent any serious settlement of the temporary platform for the inside end of the needle.

(4) The Insertion of the Needles. Having provided a support for each end of the NEEDLE it only remains to cut a hole through the wall, as at *A*, insert the needle *GG*, put the post and blocking *MN* under the outside end of the needle, and the blocking and jacks under the inside end. The post *MN*

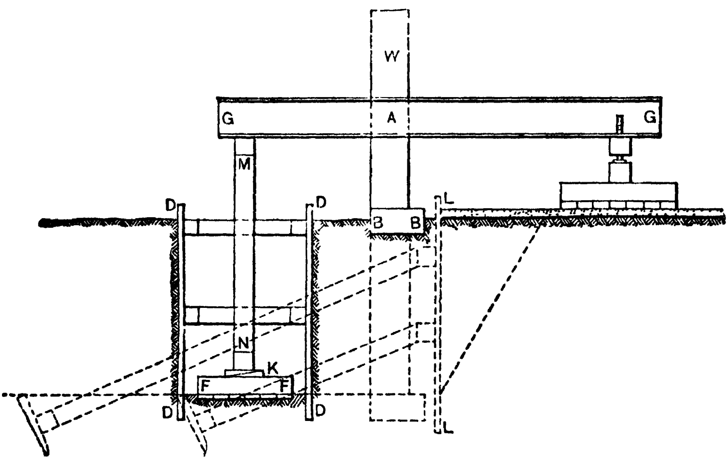


Fig 50. Wall-needling and Underpinning

may be fitted with wedges as shown at *K*, or with one or more screw-jacks. The needle *GG* may consist of one or more heavy timbers or one or more steel I beams. In any case, the load to come on this needle should be figured and its strength made ample to safely support such load. As soon as the weight of the wall *W* is transferred to the needles and to the temporary platforms prepared to receive the load, that part of the wall which is below the needles and all of the footing may be removed and all of the excavation for the new footing made.

Needling a Brick Wall. Fig. 51 shows the elevation of a brick wall supported by NEEDLES. If the needles are carrying the entire weight of the wall, it is evident that at the level of their upper surfaces the entire weight will be transferred through those parts of the wall which are immediately above them, and that above these points the material composing the wall will CORBEL OUT in both directions as indicated in Fig. 51 by the heavy zigzag lines *AAAAA*. All of the wall below this line will be supported simply by COHESION to the part of the wall above it. An experienced man can determine the

location of this line by the sound given by the wall on being struck by a hammer. All of the wall below this line is HANGING and liable to fall as soon as the support given by the footing is removed. The hanging parts of the wall may be removed or suspended by rods and chains to the needles. If they are not so suspended a crack will form along the line *AAAAA*.

Transferring the Load to the New Underpinning. As soon as the new footing has been put in place and the new wall carried up ready to receive the old wall, provision must be made for REVERSING THE OPERATION, that is, for

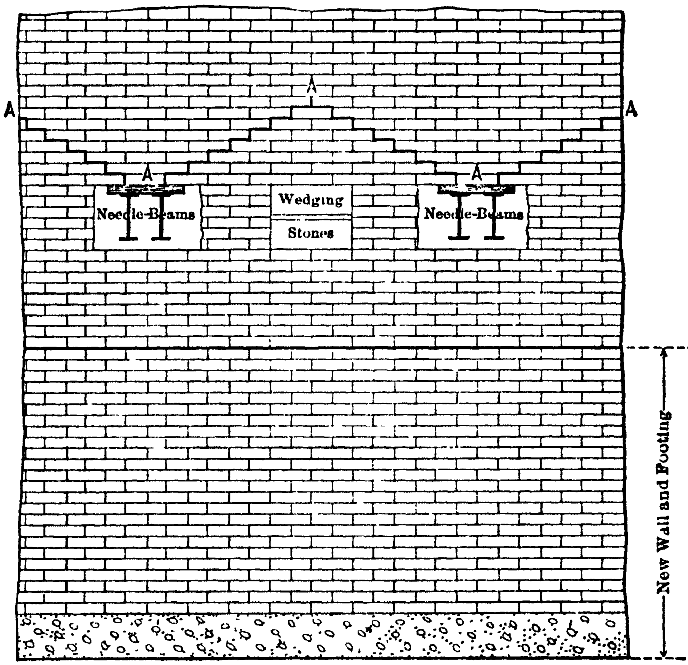


Fig. 51. Needling a Brick Wall

transferring the load onto the new underpinning wall and footing. This is generally done by means of a number of GRANITE BLOCKS or PLATES grouted for bearing, set in pairs between the needles and fitted with STEEL WEDGES. After setting these blocks, the space between the base of the old wall and the top of the upper wedging block is filled in with brickwork, the mortar in the last joint being compacted by means of PIECES OF SLATE driven in so as to wedge the mortar between the bricks. This brickwork should be laid up in Portland-cement mortar so as to reduce the time of setting. As soon as it is sufficiently set, the wedges are driven home so as to throw at least a portion of the weight of the wall on the new footing. As a result of this it frequently happens that this footing settles, the load being restored to the needles. This necessitates continued driving on the wedges until it has reached its final settlement, which will be evidenced by a lifting of the wall sufficient to par-

tially relieve the stress in the needles and by the fact that the wedges remain tight.

Removal of the Needles, etc. As soon as the entire weight of the wall has been transferred to the footing and the footing has demonstrated that it is capable of supporting the weight of the wall without further settlement, all of the temporary work, including the needles, can be removed, the needle-holes bricked up and the repairs made to the cellar of the adjoining building.

The Figure-Four Method of Needling. In certain cases it is impractical to employ a NEEDLE-BEAM projecting on both sides of the wall, as for example when the occupancy of the adjoining building is such as to make it impractical to have a needle-beam projecting into the cellar space. In such cases the so-called FIGURE-FOUR NEEDLE has been employed (Fig. 52). In this case the needle *AB* acts as a CANTILEVER. Part of the load of the wall is carried by the inclined shore *C* and another equal or nearly equal part is carried by the needle at *B*, the needle-beam *AB* being really balanced on the block *dd*.

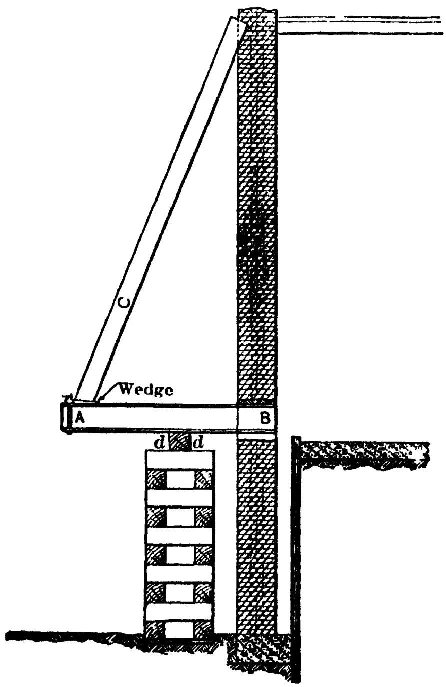


Fig. 52. The Figure-four Method of Needling

Spring-Needles. Fig. 53 shows a method frequently employed, known as the SPRING-NEEDLE METHOD. In this case the needle engages with the wall to be supported and also with an adjoining wall. A temporary platform is placed as close to the wall to be supported, *W*, as is practicable. The uplift of the jack tending to lift the needle-beam acts on both walls, but on account of its being located nearer to the wall to be lifted, a large proportion of its effect is exerted thereon.

Pipes or Cylinders for Underpinning are frequently used for the support of a wall and have many advantages, as they not only afford a support for the footing through the operations affecting the stability of the wall, but also form a permanent support. The operation in brief is as follows: A hole or niche is cut in the wall and footing to be supported, of sufficient size to permit the introduction of a section of STEEL PIPE, in such manner that the center of the pipe will come below the center of the wall to be supported, the height being sufficient to accommodate a section of pipe and also the means employed to drive it. The pipe may be driven (1) by HYDRAULIC JACKS or by SCREW-

JACKS, placed between the top of the pipe and the wall itself. The patented BREUCHAUD METHOD and the PRETEST METHOD use this general method in combination with certain patented improvements; (2) it may be driven by means of a POWER-HAMMER driven either by steam or compressed air; or (3) in some cases, where the material is fine sand or clay, the pipe may be JETTED or the JET-METHOD may be used in combination with either jacks or power-hammers. In any case the first section of pipe is driven into the ground and additional sections are added until the lower end of the pipe encounters rock or some material possessing sufficient stability to insure the required support. The material entering the pipe is removed by a WATER-JET or by other means and the space filled with concrete. As soon as the concrete has set sufficiently, the pipe is capped with a ¾-in or 1-in steel plate on

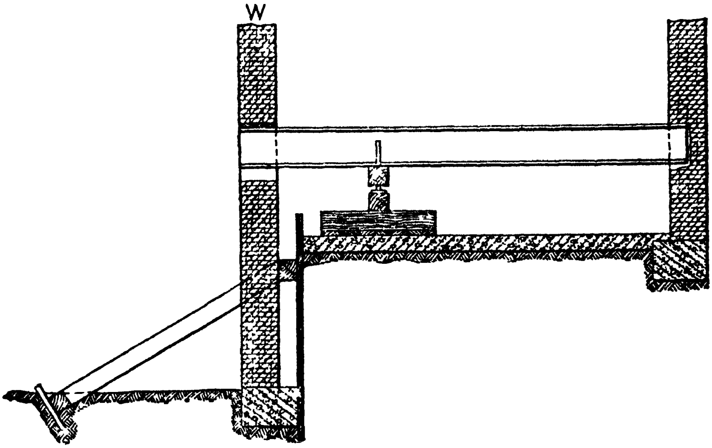


Fig. 53. The Spring-needle Method of Underpinning

which short steel I-beams are arranged to distribute the support of the pipe over a considerable part of the base of the wall to be supported. These I-beams correspond to the wedging blocks used in the ordinary methods before described. Provision is frequently made for STEEL WEDGES between the cap and the base of the steel beam, but it is generally found sufficient to thoroughly grout in the space between the base of the wall and the steel beams after the niche itself has been bricked up.

Cylinders for Underpinning Very Heavy Walls. The description in the preceding paragraph is intended to cover the use of pipes varying in size from 6 to 20 in in diameter, according to the loads to be carried. In the case of excessively heavy walls, CAST-IRON or STEEL CYLINDERS are used in place of steel pipes. These cylinders are arranged in sections, each section making a water-tight joint with the preceding section. They are generally used where water is encountered and where it is necessary to carry down the underpinning to rock at great depths. Under such conditions, these cylinders are usually sunk by the PNEUMATIC-CAISSON METHOD, although there have been cases where the OPEN METHOD has been successfully used. Such cylinders have been sunk to a depth of 70 ft below water-level; 4 ft 4 in diameters have been used and single cylinders have been designed to carry as much as 950 tons.

CHAPTER III

MASONRY WALLS. FOOTINGS FOR LIGHT BUILDINGS.* CEMENTS AND CONCRETES †

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1. Footings for Light Buildings

Footing Courses in General. Every foundation or bearing wall overlying anything except solid rock should rest on a footing or base projecting beyond the wall on each side. On wet or very compressible soils these footings may be built of steel beams or of reinforced concrete, as described in Chapter II, but on reasonably firm soils and for buildings of moderate size and weight the footings are generally of concrete, stone, or brick. Footings answer two important purposes:

(1) By distributing the weight of a structure over a larger area of bearing surface, the pressure per square foot on the foundation-bed is diminished and the tendency to vertical settlement correspondingly lessened.

(2) By increasing the area of the base of a wall, footings add to its stability and form a protection against the danger of the work being thrown out of plumb by any forces that may act on it. Nearly every building law requires that every foundation wall or pier and every cellar or basement wall or pier shall have a footing at least 12 in wider, that is, 6 in on each side, than the thickness of the wall or pier, and this may be considered as the minimum projection, except in rare instances where there may be a special reason for making it less. On firm soils and for comparatively light buildings a projection of 6 in on each side of a wall will generally reduce the unit pressure, that is, the pressure per square foot, to the safe resistance of the soil, but it is always wise to proportion the footings to a uniform unit pressure, as explained in Chapter II. To have any useful effect, footings must be well bedded and have sufficient transverse strength to resist the upward reactions on the projections.

Stone Footings for Walls with Light Loads. Except in cases where light loads are involved, basement walls are now seldom built of stone and consequently stone footings have also largely gone out of use, giving way to concrete. Most Building Laws restrict stone footings to dwellings and other buildings of the lightest type in which the cellar walls might be of stone. In some cities the Building Laws require that all footings should have at least 6 in projection on each side of the cellar wall and should be at least 12 in

* For a complete discussion of foundations in general and the mechanical principles involved in their strength and stability, for walls, piers, etc., below the basement or cellar floor, see Chapter II.

† This chapter has been revised and rewritten from the original chapter by the late Thomas Nolan.

deep, irrespective of the safe bearing pressure upon the soil. In the case of lightly loaded dwellings and small buildings, the loads per unit of foundation-bed are often much less than the allowable pressure on the soil, and in these cases it is still considered good practice to start the stone cellar wall directly upon the foundation-bed without a projecting footing. It must be understood, however, that the stone cellar wall should be at least 24 in thick and that the bottom course should be of heavy flat stones each one extending through the wall and well bedded in mortar.

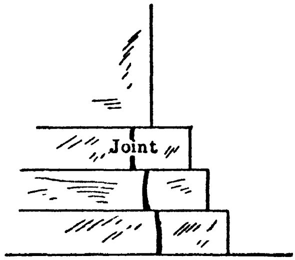

Fig. 1. Stone Footing. Openings at Joints

Wherever the Building Code requires projecting footings or the questionable bearing qualities of the soil render them necessary, stone footings should project in one or more steps, the amount of each projection and its ratio to the thickness of the step receiving careful consideration. If practicable, the footings should consist of stones having a width equal to that of the footing. If impracticable to obtain stones of sufficient size, then two stones should be used, meeting under the middle line of the wall. In any event each footing course should extend inside the course above for a distance equal to at least one and one-half times the projection, otherwise the stones will not transmit properly the loads and reactions, and the joints will tend to open, as in Fig. 1. Stone footings should be of hard, strong and durable stones always laid on their natural bed and solidly bedded in mortar The most common difficulty in using footing stones of large size is to obtain a proper bedding, as large stones are more troublesome to bed than small ones. They should be laid in a thick bed of mortar and worked sidewise with a bar until firmly settled in place.

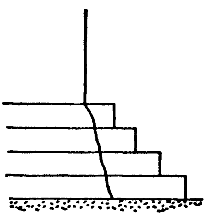
Fig. 2. Crack in Footing from Excessive Offsets

Offsets in Stone Footings. As stated above, stone footings can be used only for the lightest buildings. In such cases and where the loads per square foot of foundation-bed are much less than the allowable soil pressure, the thickness of each stone course or step may be made equal to one and one-half times its projection beyond the course above. But wherever the foundation-bed is of poor quality or wherever the loads begin to be moderately heavy, concrete should be used for the footings, and the flexure formulas should be employed to determine the thickness and projection of the steps. If the projection of the footing or offset be too great for the strength of the stone or concrete, the footing will crack as shown in Fig. 2.

Concrete Footings. For buildings of great weight, except the very heaviest, and especially for those built on a clay soil, concrete generally makes the best footings. When the concrete is properly made and used, it attains a strength equal to that of most stones, and under walls, being devoid of joints, it is like a CONTINUOUS BEAM, having sufficient strength to span any soft spots that happen to be in the foundation-bed. When deposited in thin layers and well rammed, concrete becomes firmly bedded on the bottom of the trenches, so that there is no possible chance for settlement except that due to the compression of the soil.

Preparing the Trenches. For footings, concrete made with Portland cement only should be used, and it should have a thickness of at least 8 in, even under light buildings; and for buildings of more than two stories, a thickness of at least 12 in. On firm soils, such as hard clay, the trenches should be accurately dug and trimmed to the exact width of the footings, so that the concrete will fill them. When the foundation-bed is of loose gravel or sand it is generally necessary to set up planks to confine the concrete and form the sides of the footings. These planks may be held in place by stakes; they should be left in place until the concrete has become hard, which generally requires from two to four days, after which they may be pulled up and dirt filled in against the concrete. The proportions and manner of mixing concrete are described in the latter part of this chapter.

Depositing the Concrete. Concrete should be used as soon as mixed and should always be deposited in layers, which as a rule should not exceed 6 in in thickness, especially for the first layer. On small jobs where the work is done by hand the concrete is usually carried to the trenches in wheel-barrows and dumped into the trenches The height from which the concrete is dumped, however, should not exceed 4 ft above the bottom of the trench, because when it falls from a greater height the heavy particles are apt to separate from the lighter ones. As soon as the concrete has been deposited in the trenches, it should be leveled off and then tamped with a wooden rammer weighing about 20 lb, until the water in the concrete is brought to the surface. Concrete should not be permitted to dry too quickly, and if twenty-four hours elapse between the deposits of the successive layers, the top of each layer should be sprinkled with water and grouted with neat cement before the next is put in place.

Brick Footings. When the soil is dry and the cellar walls are of brick, the footings may be of brick although at the present time concrete is more gen-

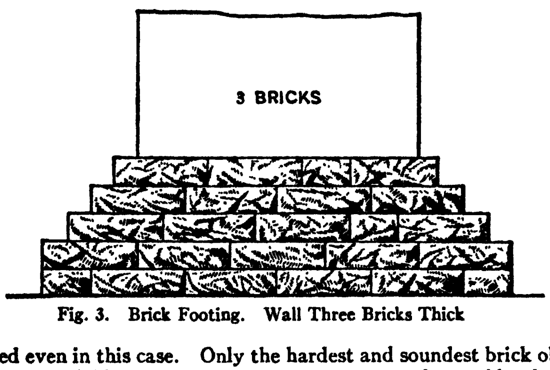

Fig. 3. Brick Footing. Wall Three Bricks Thick

erally used even in this case. Only the hardest and soundest brick obtainable should be used, laid up in Portland-cement mortar thoroughly slushed and grouted so that all joints are filled. The footings should start with a double course on the foundation-bed. The outside of the work should be laid all headers, and no course should project more than one-fourth the length of a brick beyond the one above it. See Fig. 3.

2. Cellar Walls and Basement Walls

Definitions. These terms are generally applied to walls which are below the surface of the ground or below the water-table of first-floor beams, which

support the superstructure and which go down to the FOUNDATION WALLS, properly so called. (See Chapter II.) Walls whose chief office is to withhold a bank of earth, such as the walls around areas, are called RETAINING-WALLS. (For retaining-walls, see Chapter IV.)

Materials for Cellar and Basement Walls. These walls may be built of brick, stone or concrete. BRICK is suitable only in very dry soils or for a party wall with a cellar or basement on each side of it. PORTLAND-CEMENT CONCRETE is an excellent material for foundation walls, and is being more extensively used in their construction every year. The concrete may be filled in between wooden forms, which hold it in place until it has set, or concrete blocks molded so as to form a solid wall may be used. If POURED CONCRETE be used the forms should be removed as soon as the concrete has set and the walls should be sprinkled once or twice a day, if the weather be dry, so that the concrete will not dry too quickly. Good hard LEDGE-STONE, especially if it come from the quarry with flat beds, makes not only a strong wall but, if well built, one that will stand the effects of moisture and the pressure of the earth much better than a brick wall Between a good stone wall and a wall of Portland-cement concrete, there is probably not much choice, except perhaps in the matter of expense, the relative cost of stonework and concrete varying in different localities. A wall built of soft stones, or stones that are very irregular in shape, with no flat surfaces, is greatly inferior to a concrete wall, or even to a wall of good hard bricks, and should be used only for dwellings or light buildings. Stone walls should never be less than 18 in thick, and should be well bonded, with full and three-quarter headers, and all spaces between the stones should be filled solid with mortar and broken stones or spalls. The MORTAR for stonework should be made of cement and rather coarse sand. The outside walls of cellars and basements should be PLASTERED smooth on the outside with 1 : 2, or 1 : 1½ cement mortar, from ½ to ¾ in thick. In heavy-clay soils it is a good idea to BATTER the walls on the outside, making them from 6 in to 1 ft thicker at the bottom than at the top.

Thickness of Cellar and Basement Walls. This is usually governed by that of the walls, above, and also by the depth of the wall. Nearly all building regulations require that the thickness of the cellar and basement wall, to the depth of 12 ft below the grade-line, shall be 4 in greater than the thickness of the wall above for brick, and 8 in greater for stone, and that for every additional 10 ft or part thereof in depth, the thickness shall be increased 4 in. In all large cities the thickness of the walls of buildings is controlled by law. For buildings in which the thickness of the walls is not so governed, the following table will serve as a guide:

Table I. Thickness of Cellar and Basement Walls

Height of building	Dwellings, hotels, etc.		Warehouses	
	Brick, in	Stone, in	Brick, in	Stone, in
Two stories	12 or 16	20	16	20
Three stories	16	20	20	24
Four stories	20	24	24	28
Five stories	24	28	24	28
Six stories	28	32	28	32

3. Walls of the Superstructure

Brick and Stone Walls. Very little is known regarding the stability of walls of buildings beyond what has been gained by practical experience. The only stresses in any horizontal sections of such walls, which can be determined with any accuracy, are the direct weight of the walls above and the pressure due to the floors and roof. In most walls, however, there is a tendency to BUCKLE, to overcome which it is necessary to make them thicker than would be required to resist the DIRECT CRUSHING STRESS. The resistance to fire should also be taken into account in deciding upon the thickness of any given wall. The strength of a wall depends also very much upon the quality of the materials used and upon the way in which the wall is built. A wall bonded every 12 in in height, and with every joint slushed full with good rich mortar, is as strong as a poorly built wall 4 in thicker. Walls laid with cement mortar are also much stronger than those laid with lime mortar, and a brick wall built with bricks that have been well wet just before laying is very much stronger than one built with dry bricks.

Thickness of External Walls. In nearly all the larger cities of the country the minimum thickness of the walls is prescribed by law or ordinance, and as these requirements are generally ample they are commonly adhered to by architects when designing brick buildings. Table II gives the thickness of brick walls required for MERCANTILE BUILDINGS in representative cities of different sections of the United States, and affords about as good a guide as one can have, because the values given, as a rule, represent the judgment of well-qualified and experienced persons. Walls for DWELLINGS are generally permitted to be 4 in less in thickness than for warehouses, although in some cities little or no distinction is made between business blocks and dwellings.

In compiling Table II the top of the second floor was taken at 19 ft above the sidewalk, and the height of the other stories at 13 ft 4 in, including the thickness of the floor, as the New York and Boston laws and the laws of some other cities give the height of the walls in feet instead of in stories. When the height of stories exceeds these measurements the thickness of the walls in some cases will have to be increased. The Chicago ordinance (1928) specifies that "where 12-in walls are used, the story-heights shall not exceed 18 ft, where 16-in walls are used, the story-heights shall not exceed 24 ft, and where 20-in walls are used, the story-heights shall not exceed 30 ft."

General Rule for Thickness of Walls. Although there are some differences in the thickness given in Table II, more indeed than there should be, a general rule might be formulated from it, for MERCANTILE BUILDINGS over four stories in height, which would be somewhat as follows:

For bricks equal to those used in Boston or Chicago, make the thickness of the three upper stories 16 in, of the next three below 20 in, the next three 24 in, and the next three 28 in. For a poorer quality of material make only the two upper stories 16 in thick, the next three 20 in, and so on down. In buildings less than five stories in height the top story may be 12 in thick.

In determining the thickness of walls the following general principles should be recognized:

(1) That walls of warehouses and mercantile buildings should be heavier than those used for living or office purposes.

(2) The high stories and clear spans exceeding 25 ft require thicker walls.

(3) That the length of a wall is a source of weakness, and that the thickness should be increased 4 in for every 25 ft over 100 or 125 ft in length. In New York the thicknesses given in the table must be increased for buildings exceed-

Table II. Thickness of Brick Walls—Building Codes

Height and location of building		Stories 1st	2d	3d	4th	5th	6th	7th	8th
Two stories	Boston	12	12						
	New York	12	12						
	Chicago	12	12						
	Philadelphia	13	13						
	Denver	12	12						
	San Francisco	17	13						
Three stories	Boston	12	12	12					
	New York	16	16	12					
	Chicago	16	12	12					
	Philadelphia	18	13	13					
	Denver	16	12	12					
	San Francisco	17	17	13					
Four stories	Boston	16	12	12	12				
	New York	15	15	15	12				
	Chicago	20	16	16	12				
	Philadelphia	18	18	13	13				
	Denver	16	16	12	12				
	San Francisco	17	17	17	13				
Five stories	Boston	16	16	12	12	12			
	New York	20	16	16	16	16			
	Chicago	20	20	16	16	16			
	Philadelphia	22	18	18	13	13			
	Denver	20	20	16	16	12			
	San Francisco	21	17	17	17	13			
Six stories	Boston	16	16	16	12	12	12		
	New York	24	20	20	16	16	16		
	Chicago	20	20	20	16	16	16		
	Philadelphia	22	22	18	18	13	13		
	Denver	20	20	20	16	16	12		
	San Francisco	21	21	17	17	17	13		
Seven stories	Boston	20	16	16	16	12	12	12	
	New York	28	24	24	20	20	16	16	
	Chicago	20	20	20	20	16	16	16	
	Philadelphia	26	22	22	18	18	13	13	
	Denver	24	20	20	20	16	16	12	
	San Francisco	.	.	..	.			..	...
Eight stories	Boston	20	20	16	16	16	12	12	12
	New York	32	28	24	24	20	20	16	16
	Chicago	24	24	20	20	20	16	16	16
	Philadelphia	26	26	22	22	18	18	13	13
	Denver	24	24	20	20	20	16	16	12
	San Francisco	.	....	...	....	....	....	....	....

Table II (Continued). Thickness of Brick Walls—Building Codes

Height and location of building		Stories											
		1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th
Nine stories	Boston			..			..			..	..	..	..
	New York	32	32	28	24	24	20	20	16	16	..	..	
	Chicago	24	24	24	20	20	20	16	16	16	..	..	
	Philadelphia	30	26	26	22	22	18	18	13	13	..	..	..
	Denver	..	..	..	..	..	..	..	..	..	..	..	..
	San Francisco	..	..	..	..	..	..	..	..	..	..	..	..
Ten stories	Boston	..			..					..	..	..	..
	New York	36	32	32	28	24	24	20	20	16	16	..	
	Chicago	28	28	24	24	24	20	20	20	16	16	..	
	Philadelphia	30	30	26	26	22	22	18	18	13	13	..	..
	Denver	..			..	..				..	..	..	
	San Francisco										..		
Eleven stories	Boston			..	..				..	..	..		
	New York	36	36	32	28	28	24	24	20	20	16	16	..
	Chicago	28	28	24	24	24	20	20	20	16	16	16	
	Philadelphia	34	30	30	26	26	22	22	18	18	13	13	..
	Denver	..			..		..				..	..	
	San Francisco	..						..	..	..	..	..	..
Twelve stories	Boston									..	..		
	New York	40	36	36	32	32	28	24	24	20	20	16	16
	Chicago	28	28	28	24	24	24	20	20	20	16	16	16
	Philadelphia	34	34	30	30	26	26	22	22	18	18	13	13
	Denver	..	..	..	..	..	..	..	..	..	..	..	..
	San Francisco	..	..	..	..	..	..	..	..	..	..	..	..

Boston (1926) and Denver (1927) make no provision for brick-bearing walls over 8 stories in height. San Francisco (1928) does not permit brick-bearing walls over 86 ft in height.

ing 105 ft in depth on the lot. In Western cities the tables are compiled for warehouses 125 ft in depth, as that is the usual depth of lots in those cities.

(4) That walls with over 33% of openings should be increased in thickness.

(5) That partition walls may be 4 in less in thickness than the outside walls if not over 60 ft long, but that no partition should be less than 8 in thick.

(6) That bearing walls shall be supported at not less than 18 times their thickness in a vertical direction, nor less than 20 times their thickness in a horizontal direction, cross walls and floors being considered as constituting supports.

Walls Faced with Ashlar. "In reckoning the thickness of walls, ashlar shall not be considered unless the walls are at least 16 in thick and the ashlar is at least 8 in thick, or unless alternate courses are at least 4 and 8 in to allow bonding with the backing. Ashlar shall be held by metal clamps to the backing or be properly bonded to the same.*

Stone Walls should generally be 4 in thicker than required for brick walls.

* Boston Building Law, in force in 1926.

Hollow Walls. Hollow walls of brick or stone are undoubtedly desirable for dwellings, and might well be used for other buildings not more than four or five stories in height, on account of the security afforded from the weather. Owing to the fact that they are usually more expensive than solid walls and occupy more space, they are not very extensively used in this country, their place being taken by hollow terra-cotta block and hollow concrete walls.

The Boston building law requires that vaulted walls shall contain, exclusive of withes, the same amount of material as is required for solid walls, and the masonry on the inside of the air-space in walls over two stories in height shall be not less than 8 in thick, and the parts on either side shall be securely tied together with ties not more than 2 ft apart in each direction.

Walls of Concrete Blocks. Blocks made of Portland-cement concrete, and formed in molds, are frequently used for building walls and partitions that are comparatively thin and bear light loads Patents have been taken out on different forms of blocks and on machines or processes for making the same and many buildings have been erected with walls built of these blocks. Most of the blocks are molded so as to form hollow walls. Block construction of this kind has an advantage over poured walls, in that the blocks are thoroughly seasoned before they are set and hence no provision is required for expansion or contraction. For the thin, light walls above mentioned the concrete-block construction is better adapted than solid concrete. The expense of forms is avoided and also the tendency to crack and to leave an unsatisfactory surface-finish. Concrete blocks may be substituted for any ordinary stone or brick masonry Building laws usually require the thickness of walls of hollow concrete blocks to be not less than that required for brick walls. They should not be used in party walls nor for exterior walls over four stories high.

Walls of Hollow Terra-Cotta Blocks. Exterior bearing walls of hollow terra-cotta block are now permitted by the majority of Building Codes, but they are generally limited to four stories or about 40 ft in height. The walls should be from 8 in to 12 in thick, depending upon their height and loads, laid in cement mortar, with each tile extending through the wall. The New York Building Law of 1926 specifies hollow block walls of residences outside the fire limits to have a thickness of not less than 8 in for the uppermost 20 ft, 10 in for the next lower 10 ft, and 12 in for the next lower 10 ft. The tile should be hard burned and dense so as to be impervious to moisture, or if of semi-porous quality should be veneered on the outside with brick or stone or covered with at least 3/4 in of cement stucco. The building codes do not generally permit party walls to be of hollow clay tile or blocks.

The use of both hollow terra-cotta and hollow concrete blocks has greatly increased of late years, especially for residences, garages and light manufacturing and retail store buildings. Many special shapes have been patented and put on the market which have various advantages as to bonding, insulation, strength, or ease of fitting and laying. Setting the blocks with the hollow spaces or cells horizontal is called SIDE CONSTRUCTION, and with the cells vertical, END CONSTRUCTION. End construction is considered as capable of bearing greater loads, but this advantage is offset in side construction by the better bed presented for the horizontal mortar-joints and greater ease in laying. Special tiles are required for sills and window-jambs and, in side construction, for corners also, so that the ends of the cells will not be exposed.

Party Walls. There is much diversity in building regulations regarding the thickness of party walls, although they all agree in that such walls should never be less than 12 in thick. About one-half of the laws require that party

walls shall be of the same thickness as external walls; the remainder are about equally divided between making the party walls 4 in thicker or thinner than for independent side walls. When the walls are proportioned by the rule previously given, the thickness of the party walls should be increased 4 in in each story. The floor-load on party walls is obviously twice that on side walls, and the necessity for thorough fire-protection is greater in the case of party walls than in other walls.

Enclosing Walls for Steel Skeleton Construction. In buildings of the skeleton type the outer masonry walls are supported in every story by the steel framework and carry nothing but their own weight. Such walls may, therefore, be considered as only one story high and are usually made only 12 in thick for the whole height of a very tall building. These ENCLOSING WALLS may be of brick or stone masonry throughout, furred on the inside with terra-cotta or metal furring, or they may be of hollow terra-cotta blocks faced on the outside with brick or stone.

Reinforced-concrete buildings may be made of the SKELETON TYPE, and in this case the enclosing walls at each story are sometimes built of hollow terra-cotta blocks with an outer coating of cement stucco.

It may be interesting to note at this point the great saving in space attained through the development of the SKELETON FRAME METHOD of construction over the former SELF-SUSTAINING WALLS. A few of the earlier tall buildings were erected with self-sustaining walls starting from the foundations, while columns were introduced merely to support the floors and to give additional stiffness. "The World Building, New York City, erected in 1890, is an extreme example of high building construction with self-sustaining walls. The main roof is 191 ft above the street level, making 13 main stories, above which is a dome containing 6 stories, in all, a height of 275 ft above the street. The self-sustaining walls are built of sandstone, brick and terra-cotta, the thickness increasing from 2 ft at the top to as much as 11 ft 4 in near the bottom, where the walls are offset to a concrete footing 15 ft wide. The walls are vertical on the outside faces, the thickness being varied by inside offsets, so that the columns are recessed into the walls at the bottom, but emerge and are some distance clear of the walls at the top."*

4. Natural Cements and Mortars †

Properties and Uses of Natural Cements. The first hydraulic cements used in this country were NATURAL CEMENTS, manufactured by the calcination of argillaceous limestone containing sufficient silica, alumina and iron oxide to confer hydraulic properties when the burned rock was pulverized and gauged with water. These natural cements were very widely manufactured and used until recent years, when they have been practically completely replaced by Portland cement. Natural cements vary in color from light yellow to dark brown according to the content of oxide of iron, and in distinction to Portland cements they are not uniform in their composition or behavior. The chemical composition and physical characteristics of various natural cements vary within wide limits, not only between cements manufactured in different mills, but between the products of the same mill at different times. Natural cements set more rapidly than Portland cements and are slower in developing strength. The production of natural cement in the United States for 1913 was 800 000

* From Architectural Engineering, by J. F. Freitag.

† Practical data relating to Cements, Limes and Plasters were furnished the Editor by the Charles Warner Company of Wilmington, Del.

barrels, while during the same year the production of Portland cement was 82 000 000 barrels and in 1927, 150 000 000 bushels; from which it is seen that the natural-cement industry is relatively almost extinct. Natural cement may be used in massive masonry where weight rather than strength is the essential feature. It is used, also, for certain special purposes, such as in the manufacture of safes and in certain industries where a quick-setting cement is necessary. Where economy is the governing factor, a comparison may be made between the use of natural cement and a leaner mixture of Portland cement that will develop the same strength.

Weight. The specifications of the American Society for Testing Materials require that a bag of natural cement shall contain 94 lb, net, of cement, and that each barrel of natural cement shall contain three bags of this NET WEIGHT.

Strength. A natural-cement mortar, in order to comply with the requirements of the standard specifications of the American Society for Testing Materials, must show a TENSILE STRENGTH for the neat cement, of at least 150 lb per sq in, when one week old, and 250 lb at the end of 28 days; or, when mixed with three parts of standard Ottawa sand, 50 lb at the end of one week, and 125 lb at the end of 28 days. The strength of 1 : 2 natural-cement mortar is about equal to that of 1 : 4 Portland-cement mortar.

Proportions of Natural Cement and Sand for Mortar and Concrete. For mortar for rubble-stone masonwork and ordinary brickwork, one part of natural cement may be mixed with three parts of sand, by measure.

Hydraulic Lime. A product closely related to natural cement is HYDRAULIC LIME. This is manufactured in the same way as natural cement, but the rock used contains sufficient lime to permit it to slake like quicklime. When the resulting product is pulverized, it sets and hardens as an hydraulic cement. Hydraulic limes are largely manufactured in Europe, and especially in France and Belgium, but in the United States they have been manufactured only in a few localities. This is due to the fact that while rock of suitable composition is widely found, the impurities are not uniformly distributed through it, but are found in layers or seams which prevent the material from being uniformly burned. The portion of the rock immediately adjacent to and including the seam of impurities overburns, frequently melting like a slag, while the purer portions consist simply of quicklime; and while the resulting mass slakes partly, the product when pulverized is unreliable as a cement.

Grappier Cement is a BY-PRODUCT produced during the calcination of HYDRAULIC LIME.

La Farge Cement is an imported NON-STAINING GRAPPIER CEMENT. It develops nearly the same strength as the Portland cements.

5. Artificial Cements and Mortars

The Artificial Cements used in the United States include Portland cement and Puzzolan or slag cement.

Portland Cement. The principal artificial cement in this country to-day is PORTLAND CEMENT. It is manufactured from two raw materials which are ground to extreme fineness to secure an intimate mix before burning, and it is from this fact that it derives its name, ARTIFICIAL CEMENT. These materials must be so proportioned that in the finished cement, silica, alumina, iron oxide and lime will be present in a certain ratio which must be maintained within close limits. In the Lehigh Valley region of Pennsylvania, in which are located some of the leading Portland-cement mills of the United States,

the raw materials used are limestone and cement-rock. The cement-rock is an impure limestone carrying argillaceous or clay-matter. In order to bring the lime-content up to the required percentage, it is usually found necessary in this region to add limestone. In other districts the raw materials used are limestone and clay, limestone and shale, marl and clay and also blast-furnace slag and limestone. The product from the last-mentioned mixture should not be confused with the common slag cement or Puzzolan cement, as the slag is simply used as a raw material supplying silica, alumina, iron oxide and lime; and with the exception of the use of slag to furnish these ingredients, the process of manufacture and the properties are substantially the same as for the other Portland cements. The raw mix in a Portland cement mill is analyzed at most mills several times each hour to keep the composition of the cement within the proper limits. The raw material, which is pulverized as fine as the finished cement, is burned in rotary kilns, the fuel used in most instances being powdered coal. From the kiln it issues in the form of CLINKER, the name given to the semivitrified product. After cooling, calcium sulphate in the form of gypsum is added to control the set and the product is pulverized and packed for shipment. The manufacture and properties of Portland cement have been made the subject of careful study by the American Society for Testing Materials and by the American Society of Civil Engineers. The result of this study is embodied in the standard specifications of the American Society for Testing Materials, extracts from which are given in the paragraphs following. These specifications furnish a reliable guide for the acceptance or rejection of any shipment of cement and have been very widely adopted by the leading architects and engineers of this country. These specifications do not stipulate that Portland cement shall consist of any one particular composition, but in this respect confine themselves to the limitation of the magnesia (MgO) and anhydrous sulphuric acid (SO_3) content. The reason for this is that with different raw materials it is found necessary to vary the composition of the cement to obtain the correct physical properties in the finished material. Different cements which satisfy the requirements of these standard specifications are generally considered satisfactory cements for use, although the composition of one may vary in some particulars from that of another. The CHEMICAL COMPOSITION of a good brand of Portland cement is about as follows: Lime, 62; silica, 23; alumina, 8; and impurities, such as iron oxide, magnesia, and sulphuric acid, 7.

STANDARD SPECIFICATIONS FOR PORTLAND CEMENT.* The following extracts give the most important requirements for Portland cement:

(1) DEFINITION. Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum. (2) CHEMICAL LIMITS. The following limits shall not be exceeded:

Loss on ignition, per cent	4 00
Insoluble residue, per cent	0 85
Sulphuric anhydride (SO_3), per cent	2.00
Magnesia (MgO), per cent	5.00

(3) FINENESS. The residue on a standard No. 200 sieve shall not exceed 22 per cent by weight. (4) SOUNDNESS. A pat of neat cement shall remain

* From the Standard Specifications and Tests for Portland Cement revised, 1933, by the American Society for Testing Materials.

firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness. (5) TIME OF SETTING. The cement shall not develop initial set in less than 45 minutes when the Vicat needle is used, or 60 minutes when the Gilmore needle is used. Final set shall be attained within 10 hours. (6) TENSILE STRENGTH. The average tensile strength in pounds per square inch of not less than three standard mortar briquettes composed of 1 part cement and 3 parts standard sand, by weight, shall be equal to or higher than the following:

Age at test, days	Storage of briquettes	Tensile strength, lb per sq in
7	1 day in moist air, 6 days in water . .	275
28	1 day in moist air, 27 days in water . .	350

(7) The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days. (8) PACKAGES AND MARKING. The cement shall be delivered in packages as specified, with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk When shipped in bulk, this information shall be contained in the shipping advices accompanying the shipment. A bag shall contain 94 lb net. A barrel shall contain 376 lb net. (9) STORAGE. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness. (10) INSPECTION. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 12 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 33 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test need not be made if waived by the purchaser. (11) REJECTION. The cement may be rejected if it fails to meet any of the requirements of these specifications.

Puzzolan or Slag Cements are not used extensively and never in important work. Their manufacture and properties may be briefly described as follows: Blast-furnace basic slag is granulated by running it in a molten condition into water. This accomplishes two objects. The slag is broken up into fine particles and the sudden chilling enhances its hydraulic properties. These particles are dried and ground with hydrated lime, in the proportion of from 15 to 25% of hydrated lime and from 75 to 85% of granulated slag. Such cement, known as SLAG CEMENT, is slow-setting and slow-hardening, and does not develop as much strength as natural or Portland cement. Slag cements are characterized by their light lilac color, their extreme fineness and their low specific gravity. They are considered unreliable for use except for foundation-work under ground where they are not exposed to air or running water.

Stainless Cements. Any ordinary Portland or natural cement will stain limestones, some porous marbles, some granites and some other light-colored stones. The best non-staining material is lime, that is, lime free from excess of iron oxide. White Portland cements, called NON-STAINING CEMENTS, have been developed and where care is used in their manufacture and they are free or comparatively free from iron oxide, they are satisfactory. Among the non-staining cements which have been extensively used for masonry on

which staining would be objectionable, is La Farge Cement, before mentioned. It is made at Teil, France, is light-colored and contains a small percentage of iron and soluble salts. There are on the market other non-staining cements made in the United States whose compressive strength is equal to that of standard Portland cement. For setting stones, and in order to RETARD THE SETTING of the cement until the stones are well bedded, 1 part by volume of lime-paste is usually mixed with 4 parts of the cement.

Quick-Setting Cements. QUICK-SETTING CEMENTS, also called ALUMINA CEMENTS because of high alumina content from bauxite, an aluminum ore, were developed several years ago in Europe and have recently come into use in this country. Their advantages over common Portland cement lie in the fact that after setting for 24 hours the alumina cements attain a compressive strength equal to the strength developed in common cement only at an age of 28 days. Initial set, however, does not take place any earlier, so that the mixing, conveying and depositing of the quick-setting cements can be accomplished by the same methods as are used for the standard cements. Quick-setting cement has been used in many structures and for concrete roads with perfectly satisfactory results. The quick hardening produces considerable heat which is advantageous in cold-weather construction.

Storage of Cement. Cement should be properly protected after delivery at the building site from injury through contact with dampness. Piles should be limited to twelve sacks in height to prevent caking in the bottom sacks. Cement should be used as soon as possible after delivery, for under average conditions it is subject to deterioration when stored for any length of time.

Cost of Portland Cement. Portland cement can now (1931) be purchased in this country at prices averaging $1.75 per barrel, free on board cars at the mills. The cost of the sacks and the freight are extra. The retail price for single barrels varies from about $2.10 to $2.70 per barrel, with an average of $2.45. An extra charge of 10 cts per bbl for bags is made when the cement is delivered in paper bags. The extra charge is 40 cts, if delivered in cloth, but the mills refund this 40 cts when the bags are returned in good condition. It is generally cheaper in the end to buy the cement in cloth bags and return the empty bags. Testing costs from 3 to 5 cts per bbl, or from $5 to $6 per carload. Unloading and storing cost about 15 cts per bbl, and about 5 cts per bbl are usually added to the costs to allow for handling and returning empty sacks, and freight-charges for and damage to same. Teaming costs about 10 cts per bbl per mile. Although the cost of cement is usually quoted in barrels it is now generally shipped only in paper or cloth bags or in bulk in box-cars.

Water Required in Mixing Cement Mortar. Good Portland cement requires relatively little water to make a good mortar. Neat cement will take from 20 to 22% (by weight) of water to produce the normal consistency, a quick-setting cement requiring more water than one that is slow-setting. If a greater quantity of water is required, it indicates the presence of an excess of free lime. When sand is mixed with cement, in the proportion of 3 to 1, not more than from 9 to 12½% (by weight) of water will be required. Natural cements and slag cements require more water than do Portland cements. Too much water drowns the cement, retards the setting and weakens the mortar. Cements can also be weakened or even spoiled by a deficiency of water.

Portland-Cement Mortar. For first-class mortar not more than 3 bbl of sand should be added to 1 bbl of cement. Ten or 15% of the cement by volume may be replaced by an equal amount of hydrated lime to give greater

workability to the mortar. The strength of the mortar appears to be somewhat increased by the addition of not more than 15% of hydrated lime. For rubble stonework under ordinary conditions a mortar composed of 4 parts of sand to 1 of cement will answer every purpose, and be much stronger than lime mortar. For the top surface of floors and walks, from 1 to 1½ parts of sand may be mixed with 1 part of cement. 1 to 3 Portland-cement mortar has about the same strength at the end of one year as 1 to 1 natural-cement mortar. Mortar made with fine sand requires a much larger quantity of cement to obtain a given strength than mortar made with coarse sand.

Effects of Low Temperatures and Freezing on Cement Mortars. The rate of setting and hardening of cement mortar is greatly affected by the temperature, and the exposure and loading of new work often depends upon the prevailing temperature. The freezing of natural-cement mortars should be entirely avoided as it seriously injures them. Although freezing greatly retards the hardening of Portland-cement mortars and concretes, it does not appear to injure them. Thin coats of mortar, such as plaster, and troweled surfaces or those on which free moisture is formed should not be applied in freezing weather as they are apt to scale. In general, it is undesirable to work with mortar or concrete in freezing weather, unless they are suitably heated and protected, as the difficulties of properly mixing and placing the materials are then increased. It must be admitted, however, that successful work with Portland-cement mortar and concrete has been done in temperatures considerably below freezing.

Quantity of Mortar Required for Masonry and Plastering.* "One bbl of Portland cement and 3 bbl of sand, thoroughly and properly mixed, will make 3⅓ bbl, or 12 cu ft of good strong mortar. This will be sufficient to lay up 1½ cu yd of rough stone, or about 750 bricks, with from ¼ to ⅜-in joints, or cover 125 sq ft of surface, 1 in thick, or 250 sq ft, ½ in thick."

"One bbl of natural cement and 2 bbl of lime, mixed with about ½ bbl of water, will make 8 cu ft of mortar, sufficient to lay 522 common bricks, with from ¼ to ⅜-in joints, or about 1 cu yd of rough rubble."

For the top coat of walks or floors, 1 bbl of Portland cement and 1 of sand will cover from 75 to 80 sq ft, ½ in thick, or from 50 to 56 sq ft, ¾ in thick.

One bbl of Portland cement and 1½ bbl of sand will cover from 110 to 120 sq ft of floor, ½ in thick, or from 75 to 80 sq ft, ¾ in thick.

The Mixing of Mortar. Mortar may be mixed by hand or by mechanical mixers, the latter being preferable for the mixing of large quantities. When the mixing is by hand, it should be done on platforms made water-tight to prevent the loss of cement. The cement and sand should be mixed dry in small batches and in the proportions required, the platform being clean. Water is added and the whole mass remixed until it is homogeneous and leaves the mixing hoe clean when drawn out. Mortar should never be retempered after it has begun to set.

Adhesive Strength of Portland Cement, Sulphur and Lead for Anchoring Bolts. "Fourteen holes were drilled in a ledge of solid limestone, seven of them being 1⅜ and seven 1⅝ in in diameter, and all being 3½ ft deep. Seven ¾ and seven 1-in bolts were prepared with thread and nut on one end and with the other end plain but ragged for a length of 3½ ft.

"Four were anchored with sulphur, four with lead and six with cement, mixed neat. Half the number of the ¾-in and 1-in bolts being thus anchored

* These figures can be considered as approximately only, as the amount of mortar will vary on different jobs.

with each of the three materials, all stood until the cement was two weeks old. Then a lever was rigged and the bolts pulled, with the following results.

"Sulphur: Three bolts out of four developed their full strengths, 16 000 and 31 000 lb. One 1-in bolt failed by drawing out, under 12 000 lb. Lead: Three bolts out of four developed their full strength, as above. One 1-in bolt pulled out, under 13 000 lb. Cement: Five of the bolts out of six broke without pulling out. One 1-in bolt began to yield in the cement at 26 000 lb, but sustained the load a few seconds before it broke.

"While this experiment demonstrated the superiority of cement, both as to strength and ease of application, it did not give the strength per square inch of area. To determine this, four specimens of limestone were prepared, each 10 in wide, 18 in long and 12 in thick, two of them having 1¾-in holes, and two of them 2¾-in holes drilled in them. Into the small holes 1-in bolts were cemented, one of them being perfectly plain round iron, and the other having a thread cut on the portion which was embedded in the cement. Into the 2¾-in holes were cemented 2-in bolts similarly treated, and the four specimens were allowed to stand 13 days before completing the experiment. At the end of this time they were put into a standard testing-machine and pulled. The plain 1-in bolt began to yield at 20 000 lb and the threaded one at 21 000 lb. The 2-in plain bolt began to yield at 34 000 lb and the threaded one at 32 000 lb, the force in all cases being very slowly applied. The pump was then run at a greater speed, and the stones holding the 2-in bolts split at 67 000 lb in the case of the smooth one and at 50 000 lb in the case of the threaded one.

"It is thus seen that for anchoring bolts in stone, cement is more reliable, stronger and easier of application than either lead or sulphur, and that its resistance is from 400 to 500 lb per sq in of surface exposed. It is also a well-ascertained fact that it preserves iron rather than corrodes it. The cement, however, takes much longer to attain its strength than do sulphur or lead. The cement used throughout the experiment was an English Portland cement."

6. Concrete

Properties and Uses of Concrete.* There is probably no material that is so enduring or better adapted for foundations, walks and basement floors, etc., than cement concrete, and for certain classes of buildings it is used with advantage for the walls, floors and interior supports. There are now thousands of buildings in this and other countries in which all of the structural portions are formed of reinforced concrete, and the use of Portland-cement concrete for a great variety of purposes is rapidly extending, due to the reduced price of Portland cement, and to a better appreciation and understanding of its properties and merits. Concrete may be defined as an artificial stone, made by uniting cement, water and what is called an aggregate, consisting of small and large particles of sand or screenings and gravel or broken stone; and when made with good Portland cement, in proper proportions, it becomes so hard and strong that when pieces of it are broken, the line of fracture often passes through the particles of stone, showing that the adhesion of the cement to the stone is greater than the cohesive strength of the stone itself.

The Aggregates.† "Extreme care should be exercised in selecting the aggregates for mortar and concrete, and careful tests made of the materials for the

* For further information about concrete, see Chapter on Reinforced concrete.

† Most of the matter of this paragraph, and of following paragraphs relating to concrete, consists of data and conclusions formulated by the joint committees of the Am. Soc. C. E., Am. Soc. for Test. Mats., Am. Ry. Eng. and Maint. of Way Asso., and Portland Cement Association.

purpose of determining their qualities and the grading necessary to secure maximum density. A convenient coefficient of density is the ratio of the sum of the volumes of materials contained in a unit volume to the total unit volume.

"(1) **Fine Aggregates** should consist of sand, crushed stone, or gravel screenings, graded from fine to coarse and passing when dry a screen having ¼-in diam holes; it preferably should be of siliceous material, and should be clean, coarse, free from dust, silt, soft particles, vegetable loam, organic impurities, or other deleterious matter, and not more than 30% should pass a sieve having 50 meshes per lin in. Between these limits fine aggregate should be well graded from fine to coarse. Fine aggregates should always be tested. In large work the quality of the sand may be tested for grading by sieving, for silt by decantation, and for organic matter by the calorimetric test. Fine aggregates should be of such quality that mortar composed of one part Portland cement and three parts fine aggregate by weight when made into briquettes will show a tensile or compressive strength at least equal to the strength of 1 : 3 mortar of the same consistency made with the same cement and standard Ottawa sand. This is a natural sand obtained at Ottawa, Ill, passing a screen having 20 meshes and retained on a screen having 30 meshes per lin in. It is prepared and furnished by the Ottawa Silica Company, at Ottawa, Ill., under the direction of the Special Committee on Uniform Tests of Cement of the American Society of Civil Engineers. If the aggregate be of poorer quality the proportion of cement should be increased in the mortar to secure the desired strength. If the strength developed by the aggregate in the 1 : 3 mortar be less than 70% of the strength of the Ottawa-sand mortar, the material should be rejected. To avoid the removal of any coating on the grains, which may affect the strength, bank sands should not be dried before being made into mortar, but should contain natural moisture. The percentage of moisture may be determined upon a separate sample for correcting weight. From 10 to 40% more water may be required in mixing bank or artificial sands than for standard Ottawa sand to produce the same consistency.

"(2) **Coarse Aggregates** should consist of crushed stone or gravel which is retained on a screen having ¼-in diam holes and graded from the smallest to the largest particles; they should be clean, hard, insoluble, durable and free from all deleterious matter. Aggregates containing dust and soft, flat or elongated particles should be excluded from important structures."

Any kind of stone is suitable for the coarse aggregate which has such strength that the strength of the concrete is not limited by the strength of the stone. Great strength is of little advantage beyond this minimum. The stones generally employed are granites, traps and limestones. Shales, slates, and sandstones of deficient strength should be tested before use. Pit-run gravel generally makes a good aggregate, but the proportion of small to large particles is apt to be changeable. It should, therefore, be screened and remixed in the required proportions. "The maximum size of the coarse aggregate is governed by the character of the construction. For reinforced concrete and for small masses of unreinforced concrete, the aggregate must be small enough to produce with the mortar a homogeneous concrete of viscous consistency which will pass readily between and easily surround the reinforcement and fill all parts of the forms. For concrete in large masses the size of the coarse aggregate may be increased, although it should be noted that the danger of separation from the mortar becomes greater as the size of the coarse aggregate increases."

The use to be made of the concrete determines the maximum size of the coarse aggregate. When used in mass-concrete construction, such as heavy walls, the maximum size may run up to 2½ and 3 in with good results. For reinforced work and thin walls, however, it is necessary to reduce the maximum size to 1 in or less. It has been found that the following are the maximum sizes for the coarse aggregate of plain or mass-concrete in the best practice: for foundations, 2½ in; for abutments, 2 in; for arch-rings, 1¼ in; and for copings, thin walls, etc., 1 in.

Size of Aggregates. The joint code of the American Concrete Institute adopted in 1928 recommends that the size of aggregate shall be not larger than ⅕ the narrowest dimensions between forms of the member for which the concrete is to be used nor larger than ¾ of the minimum clear spacing between reinforcing bars. Most Building Codes limit coarse aggregate for reinforced concrete to the 1¼-in size and for mass concrete not reinforced to 2 in. Some codes, however, permit much larger stones in rubble concrete but specify that there shall be at least 6 in of mortar between any two stones or between any stone and the form-work. Rubble concrete is permitted only in mass concrete without reinforcement. It should not be used for a projecting footing.

Cinders. CINDER CONCRETE is much used in some localities for reinforced floor and roof slabs of short span and for fireproofing. It should not be used for walls, columns, beams or other structural purposes. The cinders should be hard, well-burned, vitreous clinker, reasonably free from sulphides, fine ashes, unburned coal or coke and foreign matter. Sulphur in any form is liable to corrode and destroy the metal reinforcement. Cinders from anthracite coal should be used in preference to those from soft coal, since the latter are apt to contain these harmful sulphides.

Admixtures. Substances have frequently been mixed with concrete to accelerate its set, improve its workability, increase its waterproof qualities, harden its surface or lend to it other advantages. Some of these substances are proprietary and their composition is secret. Such patented admixtures should be avoided and standard chemical compounds only should be employed whose effect upon the concrete has been well proved. CALCIUM CHLORIDE, HYDRATED LIME and KAOLIN are the chemicals most commonly used. Calcium chloride accelerates the setting of the concrete and acts as a surface hardener, while hydrated lime and kaolin render the concrete more workable and thereby reduce somewhat the required quality of mixing water. Not more than 3% of commercial calcium chloride, 8% of hydrated lime or 8% of kaolin, each by weight of cement, should be used since larger proportions often reduce the strength of the cement. Integral waterproofing compounds are put on the market by many manufacturers and are fairly effective for waterproofed cement mortar coats as used in the surface coating method of waterproofing. In large masses of concrete, however, they are powerless to prevent the passage of water through settlement cracks, fill-lines, joints or pockets. In general, it is better for reinforced concrete in building construction to increase the cement rather than to depend upon chemicals for hardness or workability, and to give strict attention to the proportioning, mixing and placing of the concrete rather than to rely upon added compounds to render it waterproof. *

Water for Mixing Concrete. The water used in mixing concrete should be free from oil, acid, alkalies, organic matter, or salt.

* From Bulletins of the Lewis Institute.

Preparing and Placing Concrete. Proportions.* The materials to be used in concrete should be carefully selected, of uniform quality, and proportioned with a view to securing a workable and economical mixture with as nearly as possible a maximum density.

Unit of Measure. The unit of measure should be the cubic foot. A bag of cement, containing 94 lb, net, should be considered the equivalent of 1 cu ft The measurement of the fine and coarse aggregates should be by loose volume. Water is measured by the gallon.

Ratios of Cement, Sand and Aggregate. Many methods have been devised in the past for the proportioning of concrete, and the question is still the subject of continual study and research. The method most used until recently has been called the method of ARBITRARY PROPORTIONS, by which the cement, sand and coarse aggregates are proportioned by fixed volumes of each without reference to the characteristics of the aggregates nor to the amount of water to be used in the mix or already contained upon the aggregates. Thus a proportion of 1 : 2 : 4, or one part cement to two parts sand and four parts coarse aggregate, all by volume, was considered a sufficient specification to produce a concrete with an ultimate strength of 2 000 lb in 28 days. While it is true that much satisfactory concrete has been designed and mixed according to this formula, the adequate strength derived has in most cases resulted from the large factor of safety and not from scientific proportioning. Workability and flow were obtained by adding water according to the judgment of the superintendent or the builder without regard to its influence upon the strength of the concrete. Although it is evident that this method is neither exact nor economical it is still widely used because of its simplicity.

Water-Cement Ratio. It is well known that the cement and the water are the two chemically active elements in the concrete. By their combination they form a paste or glue which coats and surrounds the particles of the inert aggregates and upon hardening binds the entire mass together. The strength of the mixture then depends directly upon the strength of the paste. If there be an excess of water the paste becomes thin and watery and its holding strength is reduced. The actual amount of water required to hydrate thoroughly the cement is very small in comparison with that required for the workable plastic consistency of the concrete. These considerations have given rise to the recent WATER-CEMENT RATIO theory, which has been adopted, after many tests, by the Concrete Institute and by the revised Building Codes of several cities. This theory is based upon the principle that for a given set of materials and conditions the strength of concrete is determined entirely by the amount of mixing water in proportion to the amount of cement, provided that the mixture be of workable plasticity. The method of ARBITRARY PROPORTIONS considers concrete as a mass of aggregate, its interstices filled with a mortar consisting of sand and cement, the interstices of the sand in turn being filled with the cement. The WATER-CEMENT RATIO theory, on the other hand, is based upon the conception that concrete is a mass of cement-and-water paste in fixed proportions and that the aggregates are embedded in the paste. If a small amount of aggregate be mixed with the paste the concrete will be fluid; when more aggregate is added the concrete will become plastic and finally upon the addition of still more aggregate the concrete becomes stiff. The strength of the concrete remains the same for a given water-cement ratio of paste irrespective of the amount of aggregate embedded in the paste, as has been proved by many series of tests. The quantity of aggregate varies according to the consistency or degree of work-

* See, also, chapter on Reinforced Concrete.

ability desired in the concrete. In large masses with little reinforcement the concrete can be drier and stiffer than when intended for thin walls or beams with a complex system of reinforcement, the water-cement ratio and therefore the strength of the concrete being alike in both cases.

The quantity of aggregate to be mixed with a fixed amount of cement paste with a chosen water-cement ratio depends also upon the grading of sand and the coarse aggregate and upon their proportions to each other. Since cement is more costly than the aggregates, the most economical concrete is the one which contains the largest proportion of aggregate to cement paste and still preserves its workability. To attain this end both the sand and the coarse aggregate should be well graded from fine to coarse and the relative proportion of sand to coarse aggregate should be carefully studied. Excess sand is expensive and causes shrinking, while excess coarse aggregate produces a harsh, honeycombed and unworkable concrete. Tests have shown that, in general, satisfactory proportions of sand to total aggregate range from one-third to one-half by volume. For good economy, as little sand should be used as will give a workable mixture, and the coarse aggregate should be as large in size as the character of the work and the spacing of the reinforcement will permit. With these points in mind trial batches of concrete can easily be made, using a fixed water-cement ratio and different proportions of any satisfactory coarse and fine aggregates locally and cheaply obtainable, varying the mixtures until the proper consistency is secured to give workability and minimum segregation of aggregate during and after placing, together with economy in cost. In making the trial batches it is often found helpful to begin with a maximum amount of coarse aggregate and to add the sand until the mixture is of workable and homogeneous consistency. If the batch be too dry, water must not be added but the combined aggregate must be reduced In fixing the water-cement ratio the water contained on the aggregate or absorbed by it before mixing should be considered, the quantity of such water sometimes being very considerable. The Portland Cement Association publishes the following tables of absorbed and free water carried by average aggregates.

Table III. Absorption of Aggregates

Material	Per cent by weight
Average sand . .	1.00
Pebbles and crushed stone	1.00
Trap-rock and granite	0.50
Porous sandstone ..	7 00

Table IV. Free Water Carried by Aggregates

Material	Gal per cu ft
Very wet sand	0 75 to 1.00
Moderately wet sand	0 50
Moist sand ..	0 25
Moist gravel and crushed rock . .	0.25

The amount of absorbed or free water carried by the aggregates should be deducted from the total amount of mixing water fixed by the chosen water-cement ratio.

The following figure and tables are based upon a wide range of tests and experience in connection with the water-cement ratio method of proportioning concrete.

Effect of Quantity of Mixing Water on Strength of Concrete. Fig. 4 * shows graphically the relation between the amount of mixing water and the

* Taken from Portland Cement Association data.

compressive strength of the concrete at 28 days for many different mixtures and consistencies and expresses the very evident decrease in the strength of the concrete as the water is increased. Curve A is formed from mixes made in the laboratory, and Curve B, about 500 lb less, from strengths as they may be expected under field conditions.

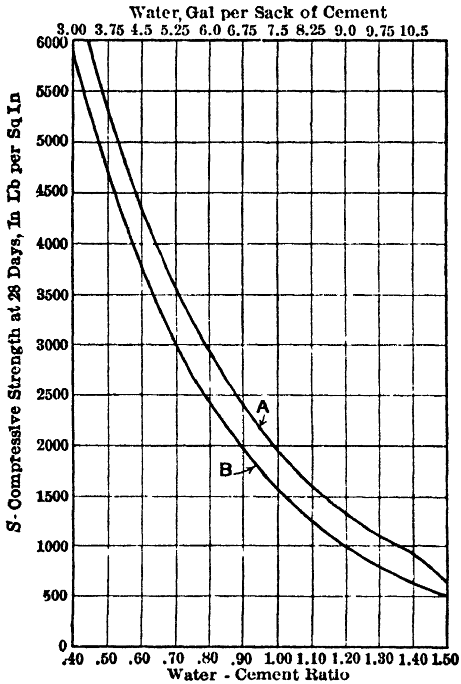

FIG. 4. Effect of Quantity of Mixing Water on Strength of Concrete

The American Concrete Institute published in 1925 the following table of water-cement ratios for ultimate strengths of concrete and made the following recommendations in their Standard Building Regulations.

Table V. Proportion of Mixing Water to Cement

Ultimate strength used in design, lb per sq in	Water-cement ratio U. S. gal of water per sack of concrete
1 500	8¼
2 000	7¼
2 500	6½
3 000	5¼

"Water and moisture contained in the aggregates must be included in determining the ratio of water to cement."

"All structural drawings and plans submitted for approval shall show the strength of concrete to be used and the water-cement ratio necessary to produce the strength as per this table. Such note indicating the water required shall clearly state that this quantity of water includes that contained in the aggregates."

Concrete Proportions and Consistency. The proportions of aggregates to cement for concrete of any water-cement ratio shall be such as to produce concrete that will work readily into the corners and angles of the form and around the reinforcement without excessive puddling or spading and without permitting free water to collect on the surface. The combined aggregate shall be of such composition of sizes that when separated by the No. 4 standard sieve, the weight retained on the sieve shall not be less than one-half or more than two-thirds of the total, nor shall the amount of coarse material be such as to produce harshness in placing nor honeycombing in the structure. When forms are removed, the faces and corners of the members shall show smooth and sound throughout.

Compressive Strength. At the present day the Building Laws of most cities limit the allowable working stress for compression in concrete to 500 lb per sq in for direct compression and 650 lb for compression in flexure. It is therefore wasteful of cement and aggregates to design a concrete with an ultimate strength of more than 2 000 lb per sq in, which gives a sufficient factor of safety for all requirements.

Mixing Concrete. Even on small jobs concrete is most satisfactorily mixed in MECHANICAL MIXERS, consisting of rotating drums furnished with blades or paddles in their interiors to stir up and thoroughly mingle together the cement, water and aggregate. The drums are revolved by power at a speed of about 200 peripheral feet per minute, greater speed producing a less thorough mix. If necessary to mix the concrete by hand it is best first to mix the dry cement and sand together with a shovel or hoe in a tight wood or metal box until the mixture assumes a uniform color. The coarse aggregate and the water are then added and the whole mass shoveled over until it becomes homogeneous and of uniform color. Mechanical mixers are manufactured in a variety of sizes and of both batch and continuous type. The BATCH MIXER, in which one batch of concrete is mixed and discharged at a time, is considered more thorough than the CONTINUOUS MIXER, which is continuously charged with the materials and continuously discharges the finished concrete. One minute is usually allowed for mixing each batch, but this time should be considered as an absolute minimum, since strength, impermeability and hardness are all increased by mixing for two minutes or more. The cement, water and aggregates should all be carefully measured before mixing. The measurements are usually by volume, one 94-lb bag of cement being considered as a cu ft and one cu ft of water as containing 7.5 gal and weighing 62.5 lb. The effort should be to maintain the same proportioning of ingredients in all the successive batches with equal amounts of water so that no variation in the strength or workability of the concrete will result.

Transporting Concrete. Concrete should be transported from the mixer to its destination in the forms with the least loss of time and with the minimum separation or segregation of ingredients. In small jobs the concrete is conveyed in wheelbarrows, but in constructions of any size it is more economical to use towers and hoists to elevate the concrete in buckets to the proper level for distribution. From the proper level the concrete is then conveyed to the

forms by steel troughs or chutes or by buggies which are steel two-wheeled carts. If chutes be used, they are suspended at the right slope from cables running from the hoist tower to one or more secondary towers, or they may be swung in the desired direction by booms supported on the tower. If buggies be used they are either filled at the mixer and carried up on the hoist or they may be filled from the buckets at the top of the hoist. In both cases they are wheeled by hand on runways from the hoist to the form. The hoist tower is generally placed close to the mixer. The number and arrangement of mixers, towers, etc., depend upon the conditions at each site. Carry-alls are considered less costly for jobs up to 2 000 cu yd, and chutes, also called the gravity method, more economical for larger constructions.

Placing Concrete, Methods. The forms should be cleaned of all chips and shavings and are often wet or oiled just before they are filled. Concrete should not be dropped from any distance so as to cause separation or segregation of the ingredients. In large horizontal surfaces such as floor slabs. the concrete should be placed in horizontal layers of equal thickness all over the area. Beams should be poured in horizontal layers and columns filled on one operation to the bottoms of the beams or of the drops of the floor slabs. The reinforcement should be completely embedded in the concrete, the forms should be filled in every corner and no air-pockets or honeycombing should occur. Thorough spading or TAMPING is therefore necessary throughout the pouring. All LAITANCE * should be removed. The day's work should be stopped at predetermined points so that the construction joints will form definite horizontal or vertical planes in favorable positions. In walls these joints should be horizontal and level, in beams they should be vertical and occur at the centers of the beams and slabs where the shearing-stress is least. Before recommencing work the old surfaces should be roughened and cleaned of laitance, thoroughly soaked with water and coated with neat cement.

"Mixing and Depositing Concrete in Freezing Weather. Concrete should not be mixed or deposited at a freezing temperature, unless special precautions are taken to avoid the use of materials covered with ice-crystals or containing frost, and to provide means to prevent the concrete from freezing after being placed in position and until it has thoroughly hardened. As the coarse aggregate forms the greater portion of the concrete, it is particularly important that this material be heated to well above the freezing-point."

The Joint Committee on Concrete report that the chemical reactions involved in the setting of concrete are retarded or stopped in cold weather and that the temperature of concrete should be maintained at a minimum of 50° F. for not less than 72 hours after placing. During the winter season it is consequently necessary to heat the water and aggregates before mixing and to maintain the required temperature in the concrete after it is placed. Steam furnishes the best means of heating, for the pipes can be led to the water barrels or tanks and through the piles of aggregates. Wood fires may also be built through which the water-pipes are passed and the aggregate may be piled over sewer-pipes or sheet metal, under which fires are built. Protection of the newly laid concrete is furnished by curtains of duck hung from the exterior girders and completely surrounding the portions of the building recently poured, and the heat is provided by salamanders distributed over the floor areas and near the exterior columns. Green concrete, not protected

* Laitance is a whitish, gelatinous substance of about the same composition as cement but with little tendency to harden. It accompanies a disintegration of some of the cement from the surface of concrete which is exposed to the action of water in which it is deposited. The concrete is thus weakened and the laitance, also, weakens the bond between old and new material and should be removed before fresh concrete is placed.

by earth banks or form-work, should be covered with canvas, tar paper or hay at the end of each day's work.

"Rubble Concrete. Where the concrete is to be deposited in massive work, its value may be improved and its cost materially reduced by the use of clean stones thoroughly embedded in the concrete and as near together as is possible while still entirely surrounded by concrete.

"Depositing Concrete under Water. In placing concrete under water it is essential to maintain still water at the place of deposit. The use of TREMIES,* properly designed and operated, is a satisfactory method of placing concrete through water. The coarse aggregate should be smaller than ordinarily used, and never more than 1 in in diameter. The use of gravel facilitates mixing and assists the flow of concrete through the tremies. The mouth of the tremie should be buried in the concrete so that it is at all times entirely sealed and the surrounding water prevented from forcing itself into the tremie; the concrete will then discharge without coming in contact with the water. The tremie should be suspended so that it can be lowered quickly when it is necessary either to choke off or prevent a too rapid flow; the lateral flow should preferably be not over 15 ft. The flow should be continuous in order to produce a monolithic mass and to prevent the formation of laitance in the interior. In large structures it may be necessary to divide the mass of concrete into several small compartments or units, filling one at a time. With proper care it is possible in this manner to obtain as good results under water as in the air."

Curing. In the hardening processes of concrete some of the chemical changes take place very slowly. Water is required for these reactions, and if the water evaporates in the early days of setting a weaker concrete will result than when the full amount of water is present. It is necessary, then, to keep the concrete properly moist for at least 10 days after it is laid. Floors are covered with burlap, sand or earth to prevent evaporation, and beams, columns and walls should be sprinkled or sprayed as soon as the forms are removed. This treatment is particularly necessary in building-construction where the members are relatively slender and exposed to air circulation on all sides and floor and wall areas are extensive and of no great thickness. The moisture in heavy construction of massive character such as dams, piers and reservoirs evaporates much more slowly.

Shrinkage of Concrete and Temperature-Changes. "Shrinkage of concrete, due to hardening and contraction from temperature-changes, causes cracks, the size of which depends on the extent of the mass. The resulting stresses are important in monolithic construction and should be considered carefully by the designer; they cannot be counteracted successfully, but the effects can be minimized. Large cracks produced by quick hardening or wide ranges of temperature can be broken up to some extent into small cracks by placing reinforcement in the concrete; in long continuous lengths of concrete, it is better to provide shrinkage-joints at points in the structure where they will do little or no harm. Reinforcement is of assistance and permits longer distances between shrinkage-joints than when no reinforcement is used. Small masses or thin bodies of concrete should not be joined to larger or thicker masses without providing for shrinkage at such points. Fillets similar to those used in metal castings, but of larger dimensions, for gradually

* A tremie is a round or square box or tube of wood or plate iron open at the top and bottom. The diameter varies from 12 to 24 in. The tremie rests in the deposited concrete, extends above the water-level and is kept full of concrete, which escapes at the bottom as the tube is shifted over the surface.

reducing from the thicker to the thinner body, are of advantage. Shrinkage-cracks are likely to occur at points where fresh concrete is joined to that which is set, and hence in placing the concrete, construction-joints should be made on horizontal and vertical lines, and, if possible, at points where joints would naturally occur in dimension-stone masonry." Roof slabs, parapet walls and exterior walls, especially, should be reinforced against temperature changes.

Effect of Heat on Concrete Fireproofing. "The actual fire-tests of concrete and reinforced concrete have been limited, but experience, together with the results of tests thus far made, indicates that concrete, on account of its low rate of heat-conductivity and the fact that it is incombustible, may be used safely for fireproofing purposes. The dehydration of concrete probably begins at about 500° F. and is completed at about 900° F.; but experience indicates that the volatilization of the water absorbs heat from the surrounding mass, which, together with the resistance of the air-cells, tends to increase the heat-resistance of the concrete, so that the process of dehydration is very much retarded. The concrete that is actually affected by fire remains in position and affords protection to the concrete beneath it. Tests show that temperatures of 1 600° F. on the face of concrete are reduced to 500° F. 2 in below the surface in 2 to 4 hours. Limestone aggregate stands excessive heat better than granite, trap-rock, sandstone or quartz. The thickness of the protective coating required depends on the probable duration of a fire which is likely to occur in the structure and should be based on the rate of heat-conductivity. The question of the conductivity of concrete is one which requires further study and investigation before a definite rate for different classes of concrete can be fully establisned. However, for ordinary conditions it is recommended that the metal in girders and columns be protected by a minimum of 2 in of concrete; that the metal in beams be protected by a minimum of 1½ in of concrete, and the metal in floor-slabs be protected by a minimum of 1 in of concrete. It is recommended that in monolithic concrete columns, the concrete to a depth of 1½ in be considered as protective covering and not included in the effective section. It is recommended that the corners of columns, girders and beams be beveled or rounded, as a sharp corner is more seriously affected by fire than a round one."

Waterproofing Concrete. "Many expedients have been used to render concrete impervious to water under normal conditions, and also under pressure-conditions that exist in reservoirs, dams and conduits of various kinds. Experience shows, however, that where concrete is proportioned to obtain the greatest practicable density with low water ratio to cement and good curing, the resulting concrete is impervious under moderate pressure. A concrete of harsh consistency is more or less pervious to water, and compounds of various kinds have been mixed with the concrete, or applied as a wash to the surface for the purpose of making it water-tight. Many of these compounds are of but temporary value, and in time lose their power of imparting impermeability to the concrete. In the case of subways, long retaining-walls and reservoirs, provided the concrete itself is impervious, cracks may be so reduced by horizontal and vertical reinforcement properly proportioned and located, that they are too minute to permit leakage or are soon closed by infiltration of silt. Coal-tar preparations applied either as a mastic or as a coating on felt or cloth-fabric are used for waterproofing, and should be proof against injury by liquids or gases. For retaining-walls and similar walls in direct contact with the earth, the application of one or two coatings of hot coal-tar pitch to the thoroughly dried surface of concrete is an efficient method of preventing the penetration of moisture from the earth."

Surface-Finish of Concrete. "Concrete is a material of an individual type and should not be used in imitation of other structural materials. One of the important problems connected with its use is the character of the finish of exposed surfaces. The finish of the surface should be determined before the concrete is placed, and the work conducted so as to make possible the finish desired. For many forms of construction the natural surface of the concrete is unobjectionable; but frequently the marks of the boards and the flat, dead surface are displeasing, thus making some special treatment desirable. A treatment of the surface either by scrubbing it while green, by rubbing with carborundum, or by tooling it after it is hard, which removes the film of mortar and brings the aggregates of the concrete into relief, is frequently used to remove the form-markings, break the monotonous appearance of the surface, and make it more pleasing. The plastering of surfaces should be avoided, for even if carefully done, the plaster is likely to peel off under the action of frost or temperature-changes."

Quantities of Materials Required per Cubic Yard of Cement Mortar. A standard barrel of cement weighs 376 lb and contains about 3.8 cu ft. It has been found by experience that 109 lb of cement produces 1 cu ft of cement paste of normal working consistency. Nine cubic feet or 981 lb of paste and 27 cu ft of ordinary building sand will make one cu yd of 1 : 3 straight cement mortar. Therefore 2.6 bbl of cement will be required to make 1 cu yd of 1 : 3 mortar. It has been found when cement is mixed with sand in 1 : 3 ratio that the bulk of the mixture is not increased beyond the original bulk of the sand.

When 10% of the cement by volume is replaced with hydrated lime, as is often done to improve the workable condition of the mortar, the quantities required for a cubic yard of 1 : 3 mortar would be as follows, one sack of lime weighing 50 lb:

Cement	Lime	Sand	Lime		Cement	Sand
%	%	Ratio	lb	Sacks	lb	cu yd
90	10	1 : 3	40	0 8	882	1 00

In laying common brick the quantity of mortar required to lay 1 000 brick varies with the thickness of the wall and the width of the mortar joints. The following quantities are based on the standard size brick 8 in × 2¼ in × 3¾ in, with a small allowance for waste:

Table VI. Mortar Required to Lay 1000 Bricks

	Width of mortar joint					
	⅛ in	¼ in	⅜ in	½ in	⅝ in	¾ in
Quantity of mortar....	4½ cu ft	9 cu ft	13½ cu ft	18 cu ft	22½ cu ft	27 cu ft

Quantities of Material Required per Cubic Yard of Concrete. Although the water-cement ratio method of proportioning concrete is rapidly taking the place of proportioning by arbitrary quantities, the latter method is still retained in many building codes and in many architects' offices. Consequently the processes of calculating the required ingredients will be given herein for both methods of proportioning.

(1) **Arbitrary Quantities.** The usual proportions specified, depending upon the use to which the concrete is to be put, are 1 : 1½ : 3; 1 : 2 : 4; 1 : 2½ : 5; and 1 : 3 : 6, the first figure signifying the parts by volume of cement, the second figure the parts of sand or fine aggregate, and the third, parts of crushed stone or coarse aggregate.

The amount of cement in barrels is first determined by the formula

$$\text{Cement} = \frac{10}{c + s + g},$$

where c = number of parts of cement, s = number of parts of fine aggregate, g = number of parts of coarse aggregate, and 10 is a number arrived at after many experimental trials. The amount of cement being thus determined the amounts of fine and coarse aggregate can easily be calculated, since there are 3.8 cu ft in a bbl and 27 cu ft in a cu yd. The above-mentioned ratios of ingredients would give the following amounts of cement in barrels and of aggregates in cubic yards:

1 : 2 : 4 Concrete

$$\text{Cement} = \frac{10}{1 + 2 + 4} = 1.43 \text{ bbl}$$

$$\text{Fine aggregate} = \frac{2 \times 1.43 \times 3.8}{27} = 0.4 \text{ cu yd}$$

$$\text{Coarse aggregate} = \frac{4 \times 1.43 \times 3.8}{27} = 0.8 \text{ cu yd}$$

1 : 3 : 6 Concrete

$$\text{Cement} = \frac{10}{1 + 3 + 6} = 1.00 \text{ bbl}$$

$$\text{Fine aggregate} = \frac{3 \times 1 \times 3.8}{27} = 0.42 \text{ cu yd}$$

$$\text{Coarse aggregate} = \frac{6 \times 1 \times 3.8}{27} = 0.84 \text{ cu yd}$$

1 : 2½ : 5 Concrete

$$\text{Cement} = \frac{10}{1 + 2\tfrac{1}{2} + 5} = 1.23 \text{ bbl}$$

$$\text{Fine aggregate} = \frac{2.5 \times 1.23 \times 3.8}{27} = 0.43 \text{ cu yd}$$

$$\text{Coarse aggregate} = \frac{5 \times 1.23 \times 3.8}{27} = 0.86 \text{ cu yd}$$

1 : 1½ : 3 Concrete

$$\text{Cement} = \frac{10}{1 + 1\tfrac{1}{2} + 3} = 1.81 \text{ bbl}$$

$$\text{Fine aggregate} = \frac{1.5 \times 1.81 \times 3.8}{27} = 0.38 \text{ cu yd}$$

$$\text{Coarse aggregate} = \frac{3 \times 1.81 \times 3.8}{27} = 0.76 \text{ cu yd}$$

These calculations are based upon laboratory conditions and are accurate. In the field, however, there is bound to be a certain waste of material, and allowances are usually made by builders in calculating the ingredients required to produce a full cubic yard of concrete in the required ratio. For this purpose 1 cu yd of coarse aggregate and ½ cu yd of fine aggregate are often provided for all mixtures, the amount of cement only being varied for the different ratios. Mr. Allen of the Aberthaw Construction Co. of Boston states that "when estimating quantities, it is not safe to allow less than the following amounts of cement for different proportions of mix."

1 : 1½ : 3 mix..................	2.00 bbl cement per cu yd
1 : 2 : 4 mix....................	1.66 bbl cement per cu yd
1 : 2½ : 5 mix..................	1 40 bbl cement per cu yd
1 : 3 : 6 mix..................	1.20 bbl cement per cu yd

(2) Water-Cement Ratio. In this method the amount of each material in the mixture including the water is found by testing experimental samples or is taken from tables published by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete. The volume of the concrete is equal to the sum of the absolute volumes of the cement, the aggregates and the water, so long as the concrete is plastic. The weights and specific gravities of the materials are used in calculating the volumes as follows:

$$\text{Absolute volume} = \frac{\text{unit weight}}{\text{specific gravity} \times \text{unit weight of water}}$$

Suppose that a mixture has been decided upon consisting of one bag of cement, 2 cu ft of fine aggregate and 3.7 cu ft of coarse aggregate mixed with 6.7 gal of water to a bag of cement. The weights may be taken as 94 lb per bag or 1 cu ft for the cement, 110 lb per cu ft for the fine aggregate, 100 lb per cu ft for the coarse aggregate and 62.5 lb per cu ft for water. The specific gravity of cement is 3.1 and of the usual aggregates about 2.65. One cubic foot of water contains 7.5 gal.*

The volume of concrete is computed as follows:

$$\text{Cement} = \frac{94 \times 1}{3.1 \times 62\ 5} = 0.49 \text{ cu ft}$$

$$\text{Fine aggregate} = \frac{110 \times 2}{2.65 \times 62.5} = 1.3 \text{ cu ft}$$

$$\text{Coarse aggregate} = \frac{100 \times 3.7}{2.65 \times 62.5} = 2.2 \text{ cu ft}$$

$$\text{Volume of water} = \frac{62.5 \times 6.7}{1 \times 7.5 \times 62.5} = 0.89 \text{ cu ft}$$

Total volume of concrete 4.88 cu ft

The materials required for 1 cu yd of concrete would then be:

$$\text{Cement} = \frac{27}{4.88} = 5.53 \text{ bags or } 1.38 \text{ bbl}$$

* From publications of the Portland Cement Association.

Fine aggregate $= \dfrac{5.53 \times 2}{27} = 0.41$ cu yd

Coarse aggregate $= \dfrac{5.53 \times 3.7}{27} = 0.76$ cu yd

Water $= 6.7 \times 5.53 = 37.25$ gal

Costs of Concrete. The cost of concrete will naturally vary in different parts of the country, depending upon the cost of the cement, the aggregates and the labor. To the cost of the cement, sand and crushed stone delivered at the building site must be added the costs of unloading and storing the materials and of power and water. Credits may be deducted from the cement cost for returned empty bags and for cash discounts. The labor costs depend upon the size of the building and the methods of mixing and placing the concrete. Mechanical mixers are now used in all but the smallest jobs, the size of the mixer depending upon the amount of concrete to be placed and the speed of placing. The concrete may be wheeled to the forms by hand or may be hoisted up a tower and flowed down spouts and chutes to the forms. The cost of a plant with hoists, towers and chutes is high but the labor cost of placing the concrete is reduced.

As an example of the expense of a cubic yard of 1 : 2 : 4 concrete in place the following calculation is shown:

Cement, 1.43 bbl at $2.85	$4.07
Sand, 0.4 cu yd at $2.10	.84
Stone, 0.8 cu yd at $3.00	2.40
Labor, power and water	2.30
Machinery	1.50
	$11.11

Common labor at 75 cts per hour will vary from $1.50 to $3.00 per cu yd, and cost of plant from $1.00 to $3.00.

If the work be carried on in freezing weather the cost of warming the water and aggregates and of protecting the green concrete must be added to the total expense per cubic yard.

The Weight of Concrete varies from 110 to 155 lb per cu ft, according to the material used. Concrete of the usual proportions weighs from 140 to 150 lb per cu ft. Trap-rock concrete weighs from 148 to 155; limestone or gravel concrete, from 142 to 148; and cinder concrete from 80 to 115 lb per cu ft.

Commonly used averages are 150 lb per cu ft for stone concrete and 108 lb per cu ft for cinder concrete.

Weights of 110 lb per cu ft for sand and 100 lb per cu ft for crushed stone are generally employed in calculating aggregates.

Earlier Examples of Portland-Cement Concrete.* From the foregoing it is seen that for foundation-work reinforced concrete and mass-concrete varies in proportions from a 1 : 1½ : 3 to a 1 : 3 : 6 mix. Some of the earlier examples are added for comparison.

Foundations of the United States Naval Observatory, Georgetown, D. C.: 1 part cement, 2½ sand, 3 gravel, 5 broken stone. (1 bbl of cement, 380 lb, made 1.18 yd of concrete.)

* See chapter on Reinforced Concrete.

Foundations of the Cathedral of St. John the Divine, New York: 1 part Portland cement, 2 parts sand, 3 parts quartz gravel of pieces from 1½ to 2 in in diameter. (17 000 bbl of cement made 11 000 yd of concrete.)

Manhattan Life Insurance Building, New York, filling of caissons: 1 part Alsen Portland cement, 2 parts sand, 4 parts broken stone.

Johnston Building (15 stories), New York, filling of caissons: 1 part Portland cement, 3 parts sand, 7 parts stone, finished on top for brickwork with 1 part cement and 3 parts gravel.

Professor Baker states that the concrete foundations under the Washington Monument were made of 1 part Portland cement, 2 parts sand, 3 parts gravel and 4 parts broken stone, and that this mixture stood, when six months old, a load of 2 000 lb per sq in, or 144 tons per sq ft.

CHAPTER IV

RETAINING-WALLS, BREAST-WALLS AND VAULT-WALLS

By

GRENVILLE TEMPLE SNELLING

MEMBER OF AMERICAN INSTITUTE OF ARCHITECTS

1. Mechanical Principles Involved

General Principles. Before discussing more in detail the problems relating to masonry structures, in which, if improperly constructed, a tendency to slide or overturn on their bases may be developed, a familiarity with what are known as the THEOREM OF FRICTION and the THEOREM OF THE MIDDLE THIRD will be of assistance in comprehending the methods indicated for rendering such structures stable.

Theorem of Friction. If a body rests on an inclined plane it will remain stationary until the angle ϕ, that the plane makes with the horizontal,

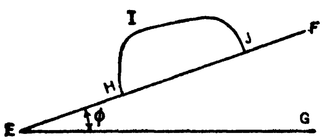

Fig. 1

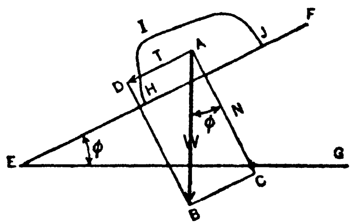

Fig 2

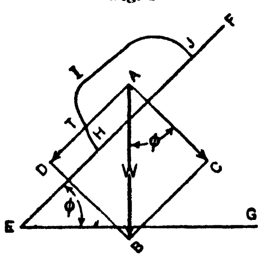

Fig. 3

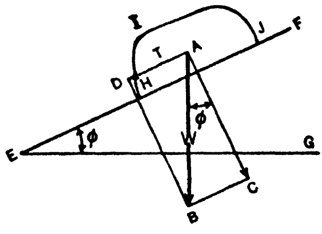

Fig. 4

Figs. 1, 2, 3 and 4. Body on Inclined Plane. Graphical Representation of Forces

becomes so great that the FRICTION developed between the surfaces of the body and the plane is no longer sufficient to prevent the body from sliding down the plane (Fig. 1).

Assume the body *HIJ* resting on the plane *EF*. The weight, *W*, of this

body is shown graphically by the line AB, applied at its center of gravity A (Fig. 2). This weight can be resolved into two component forces, one, AC, normal to the inclined plane and the other, AD, parallel to it. It is the parallel or tangential force which tends to pull the body down the plane and which is resisted by the friction developed between the two surfaces. The friction developed between any two surfaces in contact depends upon the nature of the materials of which they are composed and the intensity of the forces pressing them together; and it resists the tendency to slide only up to a certain point. As the angle ϕ, which the inclined plane makes with the horizontal, increases, the tangential component T, of the weight W, increases, until it becomes greater than the frictional resistance, and the body moves down the plane (Fig. 3). From trigonometry,

$$T = W \sin \phi$$

$$N = W \cos \phi, \quad \text{or,} \quad T = N \tan \phi$$

There is evidently a position of the plane, intermediate between the positions shown in Figs. 1 and 3, in which the component force T is just balanced by the friction and in which the body remains at rest although just on the point of sliding (Fig. 4). If the angle which the inclined plane makes with the horizontal, at the moment when the body is just about to slide, be designated by ϕ, the friction developed between the two surfaces will be equal to $N \tan \phi$, since, when the angle of inclination of the plane to the horizontal is ϕ, the tangential component of the weight just balances the friction. From the equation $T = N \tan \phi$ it is evident that the friction is directly proportional to N and to $\tan \phi$ Tan ϕ is then known as the COEFFICIENT OF FRICTION and ϕ as the ANGLE OF REPOSE, or, in the case of stone surfaces, it is often known as the ANGLE OF FRICTION.

The following Table I gives the average values of these constants as determined by experiment.

Table I. Coefficients and Angles of Friction

Kind of Surface	Coefficient of friction, tan ϕ	Angle of friction, ϕ
Granite, limestone and marble:		
Soft dressed upon soft dressed	0.70	35° 00′
Hard dressed upon hard dressed	0 55	28 50
Hard dressed upon soft dressed	0 65	33 00
Stone, brick or concrete:		
Masonry upon masonry	0 65	33 00
Masonry upon wood (with the grain)	0 60	31 00
Masonry upon wood (across the grain)	0 50	26 40
Masonry upon dry clay	0 50	26 40
Masonry upon wet or moist clay	0 33	18 20
Masonry upon sand	0 40	21 50
Masonry upon gravel	0 60	31 00
Soft stone upon steel or iron	0 40	21 50
Hard stone upon steel or iron	0 30	16 40

In this discussion only the weight AB (Figs. 2, 3 and 4), of the body has been considered; but the body might be subjected to the action of other forces besides the force of gravity, in which case these other forces would be combined with the weight in order to find the resultant, this resultant being again resolved into a tangential and a normal component. Since the angle BAC

is equal to the angle FEG (Figs. 2, 3 and 4), given a certain normal pressure exerted by the body on the plane, the amount of the tangential pressure T depends upon the angle FEG. The problem in actual practice reduces itself to so arranging the conditions that no matter what the position of the plane may be, the angle ϕ, which the resultant W, makes with the normal N, to the plane, will not be greater than the ANGLE OF FRICTION OR REPOSE.

Theorem of the Middle Third. When any surface is subjected to pressure from the action of any force or forces, this TOTAL PRESSURE may be considered as a SYSTEM OF AN INFINITE NUMBER OF PARALLEL FORCES, equal or unequal in intensity. These forces will have a RESULTANT, whose MAGNITUDE, DIRECTION and POINT OF APPLICATION can be determined, either graphically, or by moments, as explained in Chapter VI. The determination of these three elements of this resultant force may at times become of the utmost importance to the engineer.

Pressure of this nature is technically known as the STRESS to which the surface in question is subjected. (See Chapter I.) When the INTENSITY OF A STRESS is not the same at different points of a surface, it is called a VARYING STRESS, while if, on the contrary, its intensity remains the same at every point of the surface, it is called a UNIFORM STRESS.

When a stress varies it may do so in one or two ways. It may vary UNIFORMLY, that is to say, in a uniform manner, following some definite law of variation, so that, knowing this law, its intensity may be determined for any given point of the surface; or NON-UNIFORMLY, following no law. When a stress varies in the former manner it is called a UNIFORMLY VARYING STRESS. This is the case most frequently met with in engineering problems.

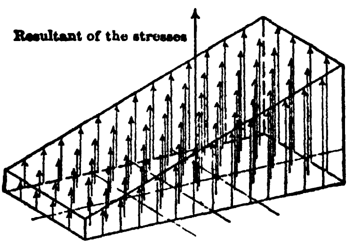

Fig. 5. Resultant within Middle Third

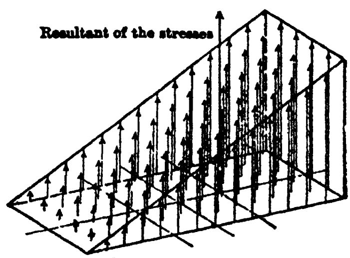

Fig. 6. Resultant at Middle Third

In dealing with ISOLATED FORCES, such as concentrated loads on a beam, we are usually interested in determining the MAGNITUDE and POINT OF APPLICATION of the RESULTANT of these forces. When, however, the question is one of STRESS, or of an unlimited number of forces, the problem that usually presents itself is one in which the resultant is known, in magnitude, direction and point of application, and in which it is required to determine the DISTRIBUTION OF THE STRESS to which the surface is subjected. Or, in actual practice, it is required to so arrange the parts of the structure that this resultant shall have such a magnitude, direction and point of application that the stress to which the surface under consideration is subjected shall not exceed certain LIMITS OF SAFETY, determined beforehand by experience. For example, when the resultant of a known amount of pressure or stress acts at the CENTER OF GRAVITY of the surface subjected to the stress, this stress in UNIFORMLY DISTRIBUTED over the surface.

When the resultant acts at a distance of two-thirds the total width of the surface from one edge or boundary line of the surface, and at one-third the width from the other edge, the stress is UNIFORMLY VARYING; and its INTENSITY at the edge farthest from the point of application of the resultant is ZERO and at the other edge a MAXIMUM or twice the average stress. When, however, the total amount of the stress remaining the same, the point of application of the resultant is at a greater distance from one edge than two-thirds the total width of the surface, a certain part of the surface adjacent to the edge furthest from the resultant is subjected to a STRESS OF A CONTRARY NATURE to that distributed over the rest of the area; that is to say, if the stress to which the major part of the surface is subject is a COMPRESSIVE stress, the stress acting on the remainder of the surface is a TENSILE stress. The stresses in a surface resulting from three different positions of the resultant force may be illustrated graphically, as shown in Figs. 5, 6 and 7.

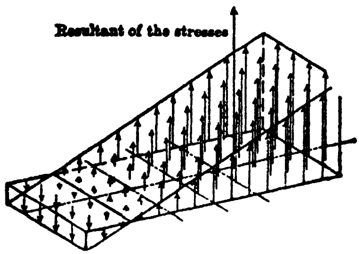

Fig. 7. Resultant beyond Middle Third

2. Retaining-Walls

Definitions. A RETAINING-WALL is a wall built to resist the pressure of earth, sand, or other filling or backing deposited behind it after it is built, as distinguished from a BREAST-WALL or FACE-WALL, which is a similar structure built to prevent the fall of earth which is in its undisturbed, natural position, but from which part has been excavated, leaving a vertical or inclined face. Fig. 8 is an illustration of the two kinds of wall.

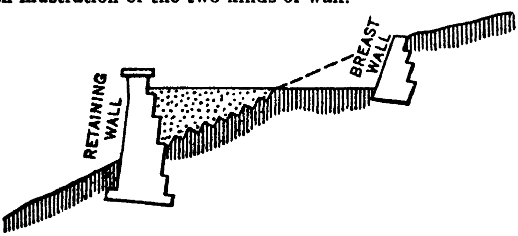

Fig. 8. Retaining-wall and Breast-wall

Theories of Retaining-Walls. A great deal has been written on the THEORY OF RETAINING-WALLS, and many theories, involving elaborate calculations for determining the CONJUGATE PRESSURES in the earth-backing behind the wall, have been developed for computing the THRUST which a bank of earth exerts against such a wall, and for determining the FORM of wall which offers the greatest resistance with the least amount of material. There are so many conditions, however, upon which the thrust exerted by the backing depends, such as the cohesion of the earth, the dryness of the material, the mode of backing up the wall, etc., that in practice it is impossible to determine the exact thrust which will be exerted against a wall of a given height. It is necessary, therefore, in designing retaining-walls, to be guided by experience rather than by theory. As the theories of retaining-walls are so vague

and unsatisfactory, we shall not include any in this work, but offer, rather, such suggestions, rules and cautions as have been established by practice and experience. A construction suggested from empirical data, which has been found to work well in practice, for determining the THRUST OF THE EARTH-BACKING and the DIMENSIONS OF THE WALL to properly resist this thrust, is given on the following page.

In designing a retaining-wall the backing as well as the wall itself must be carefully considered. THE TENDENCY OF THE BACKING TO SLIP is very much less when the material is in a dry state than when it is saturated with water, and hence every precaution should be taken to secure good drainage. Besides surface-drainage, there should be openings left in the wall for the water which may accumulate behind it to escape.

The manner in which the material is filled against the wall, also, affects the stability of the backing. If the ground is made irregular, with steppings, as shown in Fig 8, and the earth well rammed in layers inclined DOWN FROM the wall, the pressure will be very trifling, provided that attention is paid to drainage. If, on the other hand, the earth is tipped in the usual manner, in layers sloping DOWN TOWARDS the wall, almost the full pressure of the earth will be exerted against it, and it must be made strong enough to withstand such pressure.

Slopes of Repose and Angles of Repose. Cases may occur in practice in which the conditions are not such as are shown in Fig. 8, which shows only a limited amount of fill or new material put in behind the wall on top of the original slope of the grade; cases in which, on the contrary, the wall has been built on the natural surface of the ground with a view to creating an entirely new terrace or embankment and where all the material back of the wall is new.

All of this material does not beat upon the wall and tend to overturn it, for sand or loose earth taken from an excavation and deposited on the surface of the ground does not spread itself out like a liquid but piles up in a mound. This PILING UP is due to the FRICTION developed between the separate particles as they slide one over the other while being dumped. This phenomenon is observed in the action of any solid material broken up into separate particles; and although the SLOPE OF THE SIDES of such a mound varies with different materials, it is, in general, the same for the same material. The angle of this slope is known as the ANGLE OF NATURAL SLOPE of the material. This angle for the materials generally used for fill is given in the following Table II.

Table II. Slopes of Repose, Angles of Repose and Weights of Loose Materials

Kind of earth	Slope of repose*	Angle of repose	Weight in lb per cu ft
Sand, clean	1 5 to 1	33° 41′	90
Sand and clay	1 33 to 1	36 53	100
Clay, dry	1 33 to 1	36 53	100
Clay, damp, plastic	2 to 1	26 34	100
Gravel, clean	1.33 to 1	36 53	100
Gravel and clay	1 33 to 1	36 53	100
Gravel, sand and clay	1.33 to 1	36 53	100
Soil	1.33 to 1	36 53	100
Soft rotten rock	1.33 to 1	36 53	110
Hard rotten rock	1 to 1	45 00	100
Bituminous cinders	1 to 1	45 00	65
Anthracite ashes	1 to 1	45 00	30

* The slope is that of horizontal to vertical projection.

Pressures on Retaining-Walls. Even under the conditions shown in Fig. 8, only a part of the filled-in material will exert a pressure on the wall. It would be natural to suppose that the part of the fill exerting pressure on the wall would be determined by the ANGLE OF NATURAL SLOPE, all material from a natural horizontal grade up to this angle being able to take care of itself, and all the material above the angle needing the wall to hold it in place. Experiment shows that this is not strictly true, for as the earth settles into place certain forces of INTERNAL ELASTICITY and tendencies toward a state of EQUILIBRIUM come into play creating INTERNAL STRESSES which produce the CONJUGATE PRESSURES already referred to. The exact determination of these INTERNAL STRESSES demands relatively complicated calculations which would be out of place in a book of this character. The construction given in the following paragraphs for determining the SLOPE OF THE CLEAVAGE-PLANE, between that part of the backing which sustains itself and the triangular fill which actually bears on the wall, is sufficiently accurate, however, for all practical purposes.

The Slope of the Cleavage-Plane. The following construction (Figs. 9 and 10), based upon empirical data, for determining first, the PRISM OF EARTH

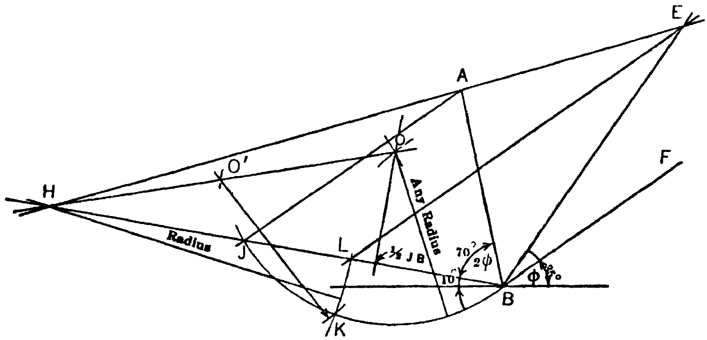

Fig. 9. Method of Determining the Prism of Earth

which exerts pressure on the back of the wall and secondly, the proper DIMENSIONS for the wall, has been found to work well in practice, when certain necessary precautions are taken. These include proper DRAINAGE behind the wall, proper RAMMING of the fill and efficient BRACING of the wall during its construction.

In the calculations to determine the pressure of the earth and the weight of the wall, a slice 1 ft thick is first considered. Then the area of the triangle ABE is proportional to the volume and weight of the slice of earth causing pressure on the wall, and as the area of the cross-section of the wall is proportional to the volume and weight of the slice of the wall itself.

To determine the PRISM OF EARTH which exerts pressure against the back of the wall, decide first upon the BATTER to be given to the back of the wall. In this case it made 80° with the horizontal, an angle slightly greater than that advised by Trautwine. Draw BH (Fig. 9), making an angle ABH, equal to 2ϕ, with the back of the wall; continue this line until it meets at H the slope of the surface of the earth back of the wall, prolonged. From A, the top of the wall, draw AJ parallel to BF the natural slope of the fill. This has been taken at 35°, as a fair average value. Erect a perpendicular from the middle

of JB, and with any point, O, as a center, on this perpendicular, describe an arc passing through J and B. Draw HO and bisect it, and with O' as a center and OO' as a radius, describe the arc cutting the arc JKB at K. Again, with a radius HK and with H as center, describe the arc KL, and finally, from L, draw LE parallel to JA. The intersection of this line with the surface of the ground locates the point E. The line EB is the line of the CLEAVAGE-PLANE which separates the part of the backing which bears against the wall from the part which exerts no lateral pressure.

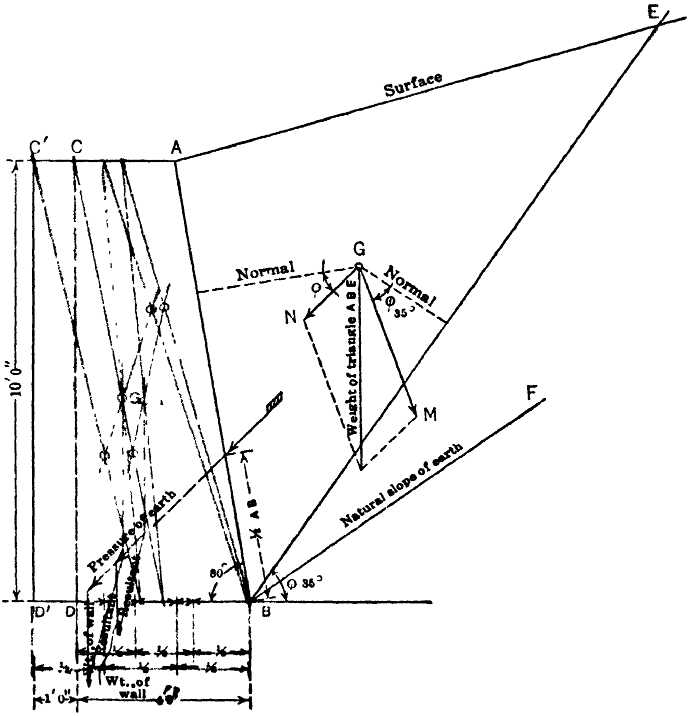

Fig. 10. Method of Determining Dimensions of Retaining-wall

Having found the DIMENSIONS OF THE VOLUME OF EARTH, the thrust of which must be resisted by the wall, the next step is to determine what the DIMENSIONS OF THE WALL should be to properly resist this thrust. Usually one or two trials are necessary before the proper solution of the problem is found. In the example given, a preliminary trial was made with a thickness at the base of 4 ft. This construction is shown with the green lines (Fig. 10).

After drawing the triangle representing the base of the PRISM OF EARTH, find its center of gravity, G (Chap. VI). From this point draw two normals, one to the back of the wall and the other to the line of the CLEAVAGE-PLANE. Draw the two lines, GM and GN, making angles ϕ with these normals. Lay off vertically from the center of gravity, at any convenient scale of so many square

inches to the linear inch, the area of the triangle of the base of the prism, the area, as already explained, being proportional to the volume of the prism and its weight. Resolve this weight-line along the two lines *GM* and *GN* (Chap. VI). This will give the MAGNITUDE and DIRECTION of the THRUST or pressure of the earth against the wall. Apply this pressure at a point on the back of the wall one-third of the distance from the bottom, as shown by the arrow. This is the force which may tend to OVERTURN the wall and which tends to make it SLIDE along the base. (See Fig. 6.)

To resist these OVERTURNING and SLIDING-TENDENCIES, the weight of the wall combined with the pressure of the earth behind it should produce a resultant which satisfies the following conditions. First, its MAGNITUDE should not be great enough to cause a unit pressure on the foundation-bed greater than it can safely bear; secondly, it should pass within the MIDDLE THIRD of the base so that the stress over the entire area of the base will be a COMPRESSIVE stress; and thirdly, it should make an angle with a normal to the plane of the foundation-bed not greater than the ANGLE OF FRICTION between the stone, brickwork, concrete, or other masonry of the footings and the sand, clay, or rock of the foundation-bed.

In order to determine these conditions, the CENTER OF GRAVITY of the cross-section of the wall must be determined and a vertical line drawn through this point until it intersects the line of the EARTH-THRUST produced. It is at this intersection of the LINES OF ACTION of the two forces that their RESULTANT acts. To find the CENTER OF GRAVITY of the cross-section of the wall, the method of dividing the trapezoid into two triangles has been followed, the center of gravity of each triangle being found and these two points being joined by a line. The intersection of this line with the median line drawn between the base and the top of the wall is the center of gravity of the trapezoid. In this example, for convenience, the scale used for the composition of the forces of the pressure of the earth and the weight of the wall is one-half the scale used for the resolution of the forces representing the weight of the earth-prism.

In the first trial, shown by the green lines, the first and third conditions necessary to insure stability are fulfilled; but the second is not, the resultant passing outside the MIDDLE THIRD of the base. This indicates, theoretically, a slight TENSILE stress or a tendency for the joints at the back of the wall to open. Another trial, therefore, is shown with the red lines, the thickness of the wall being increased as shown by the rectangle *CC'D'D*. In this second trial the WEIGHT OF THE WALL is necessarily increased while the EARTH-THRUST remains the same. As in this case the resultant passes within the middle third, it is concluded that a wall of these dimensions, 5 ft base by 10 ft height and with an 80° batter, will be safe and will properly resist the thrust of the earth-backing.

Details of Construction. Retaining-walls are generally built with a BATTERING, that is, a SLOPING face, as walls of this form are the strongest for a given amount of material; and if the courses are INCLINED DOWN TOWARDS THE BACK, the tendency to slide on each other will be resisted, and it will not be necessary to depend upon the adhesion of the mortar. The importance of making the resistance independent of the adhesion of the mortar is obviously very reat, as it would otherwise be necessary to delay the backing up of the wall until the mortar had thoroughly set, which might require several months.

In brickwork it is advisable to let every third or fourth course below the frostline project an inch or two. This increases the friction of the earth against the back and causes the resultant of the forces acting behind the wall to become more nearly vertical, and to fall farther within the base, increasing

the stability. It also conduces to strength to make the courses of varying heights throughout the thickness of the wall, and to have some of the stones, especially those near the back, sufficiently high to extend through two or three courses. By this means the whole masonry becomes more effectually interlocked or bonded together as one mass and is less liable to bulge. The courses of masonry are often laid with their beds SLOPING IN, as in Fig. 15, to overcome the tendency of the courses to slide on each other.

Where the ground freezes to a great depth, the back of the wall should be SLOPED FORWARD for three or four feet below its top surface, as at OC (Fig. 11),

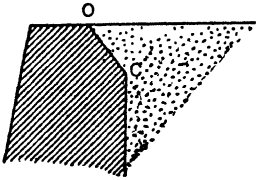

Fig. 11. Retaining-wall for Deep-freezing Earth

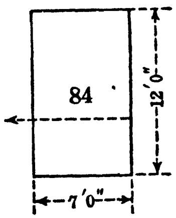

Fig. 12. Retaining-wall with Rectangular Cross-section

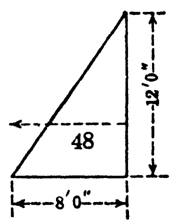

Fig. 13. Retaining-wall with Triangular Cross-section

and this slope should be quite smooth, so as to lessen the hold of the frost and prevent displacement.

Figs. 12, 13, 14 and 15 show the approximate RELATIVE VERTICAL SECTIONAL AREAS of walls of different shapes that would be required to resist the pressure of a bank of earth 12 ft high. The first three examples are calculated to resist the maximum thrust of wet earth, while the last shows the modified form usually adopted in practice.

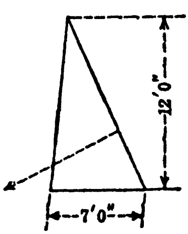

Fig. 14. Retaining-wall with Triangular Cross-section. Area, 42 sq in

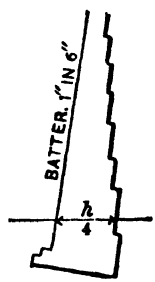

Fig. 15. Retaining-wall with Stepped Back

Notes on the Thickness of Retaining-Walls. As has been stated, about the only practical rules for retaining-walls are the empirical rules based upon experience and tests. Trautwine* gives the following Table III for the thickness at the base of vertical retaining-walls with a sand backing deposited in the usual manner. The first column contains the vertical height CD (Fig. 16) of the earth as compared with the vertical height of the wall, AB. The latter is assumed to be 1, so that the table begins with a backing of the same height as the wall. These vertical walls may be battered to any extent not exceeding 1½ in to 1 ft, or 1 in 8, without affecting their stability and without increasing the base.

If the wall is built as in Fig. 17, with the ground practically level with the top, the top of the wall should be not less than 18 in thick, and the thicknesses at a, a, etc., just above each step, should be from one-third to two-fifths of the

* The Civil Engineer's Pocket-Book, John C. Trautwine.

Table III. Proportions of Retaining-Walls

(Thickness of wall at the base in parts of the height, AB, Fig 16)

Total height of the earth compared with the height of the wall above ground	Wall of cut stone in mortar	Wall of rubble or brick, good mortar	Wall of good, dry rubble
1	0 35	0 40	0.50
1.1	0 42	0 47	0 57
1 2	0 46	0 51	0 61
1.3	0 49	0 54	0 64
1.4	0.51	0 56	0 66
1.5	0 52	0.57	0 67
1.6	0 54	0 59	0 69
1.7	0.55	0 60	0 70
1.8	0 56	0 61	0.71
2	0 58	0 63	0 73
2.5	0 60	0 65	0 75
3	0 62	0 67	0 77
4	0 63	0 68	0 78
6	0.64	0 69	0 79
14	0 65	0 70	0.80
25	0 66	0 71	0 81
or more	0 68	0 73	0 83

height from the top of the wall to each of these levels. If the earth is banked above the top of the wall, the thicknesses should be increased as indicated by the table given above. If built upon ground that is affected by frost or surface-water, the footings should be carried sufficiently below the surface of the ground at the base to insure against heaving or settling.

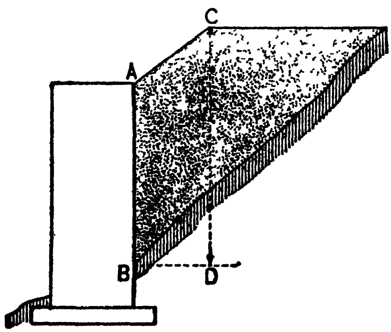

Fig. 16. Retaining-wall with Raised Sand Backing

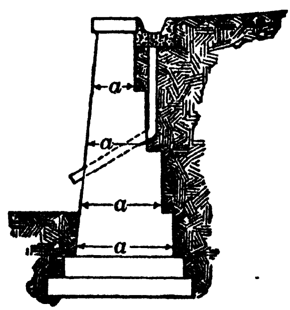

Fig. 17. Retaining-wall with Stepped Back

Reinforced-Concrete Retaining-Walls. With the constantly increasing use of REINFORCED CONCRETE for various purposes, there has come, also, the construction of retaining-walls in this material. Figs. 18,* 19 * and 20 * show three designs by A. L. Johnson for retaining-walls to satisfy the require-

* Plain and Reinforced Concrete, Taylor and Thompson.

ments of banks 5, 10 and 20 ft high. The wall shown in Fig. 20 is reinforced at intervals with COUNTERFORTS. The walls themselves in Figs. 18 and 19 act as CANTILEVER BEAMS. The FOOTINGS, in all three cases, are subjected to two principal external forces, the resultant of the resisting upward pressure of the foundation-bed and the resultant of the downward pressures of the fill. In Fig. 20 the COPING acts as a BEAM FIXED AT BOTH ENDS, with a span equal to the distance between the counterforts, and loaded with the

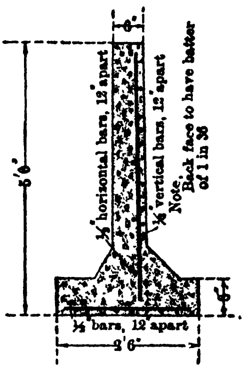

Fig. 18. Reinforced-concrete Retaining-wall, 5 ft High

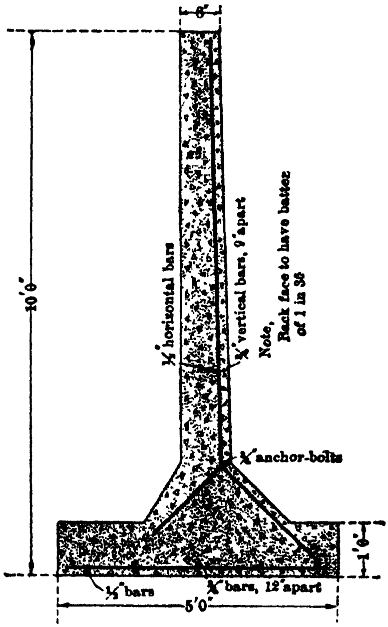

Fig. 19. Reinforced-concrete Retaining-wall, 10 ft High

proper proportion of the load due to the pressure of the fill behind the wall and transmitted to the coping by the wall. The wall itself in this case acts as a FLOOR-SLAB supported on all four sides and subjected to an approximately evenly distributed load. The counterforts are in tension. The MAXIMUM BENDING MOMENTS for these various cases can be determined (Chapter IX) and the necessary DIMENSIONS and REINFORCEMENT to be provided decided by the rules given in Chapter XXIII.

3. Breast-Walls

Breast-Walls. Where the ground to be supported is firm, and the strata are horizontal, the office of a BREAST-WALL (Fig. 8) is more to protect than to sustain the earth. It should be borne in mind that a trifling force skillfully applied to unbroken ground will keep in its place a mass of material, which, if once allowed to move, would crush a heavy wall. Great care, therefore,

should be taken not to expose the newly opened ground to the influence of air and water longer than is requisite for sound work, and to avoid leaving the smallest space for motion between the back of the wall and the ground. The strength of a breast-wall must be proportionately increased when the

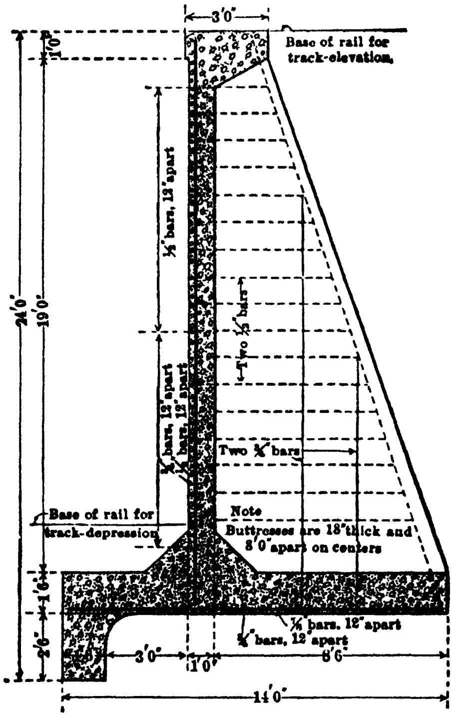

Fig. 20. Reinforced-concrete Retaining-wall with Counterforts and Apron

strata to be supported incline down towards the wall; where they incline down from it, the wall need be little more than a THIN FACING to protect the ground from disintegration. The preservation of the NATURAL DRAINAGE is one of the most important points to be attended to in the erection of breast-walls, as upon this their stability in a great measure depends. No rule can be given for the best way to do this; it is a matter for attentive consideration in each particular case.

4. Vault-Walls

Vault-Walls. In large cities it is customary to utilize the space under the sidewalk for storage or other purposes. This necessitates a wall at the curb-line to hold back the earth and the street-pressures and also the weight of the sidewalk. Where practicable the space should be divided by partition-walls about every 10 ft, and when this is done the outer wall may be advanta-

geously built of hard bricks in the form of arches, as shown in Fig. 21. The THICKNESS of the arch should be at least 16 in for a depth of 9 ft and the RISE of the arch from one-eighth to one-sixth of the span. If partitions are not practicable, each sidewalk-beam may be supported by a heavy I-beam column, with either flat or segmental arches between, of either brick or concrete. Fig. 22 shows a detail of the outer walls of the vault under the sidewalk around the Singer Building, New York City. These walls consist of a core formed by two-ring brick arches with vertical axes, built between the flanges of 8-in vertical steel I-beams spaced about 5 ft apart and bedded at the bottom in a concrete footing. Their tops are joined by 6-in horizontal I-beams and braced laterally by the sidewalk-beams, 5 ft apart. The arches themselves are segmental, with a

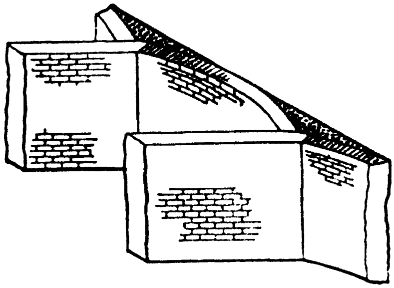

Fig 21. Vault-wall with Partitions

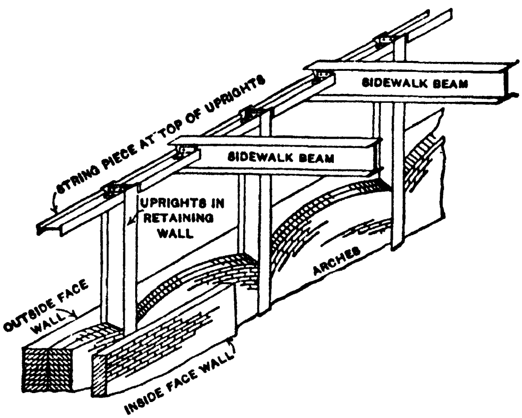

Fig. 22. Vault-walls of Singer Building, New York City

rise of about 6 in, and are built up solid against an 8-in outside face-wall. A 4-in plain curtain wall is built inside against the flanges of the vertical beams, inclosing segmental air-chambers in front of each arch.

CHAPTER V

STRENGTH OF BRICKS, STONE, MASS-CONCRETE AND MASONRY *

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1. Crushing Strength of Stonework, Brickwork, Bricks, etc.

Stresses in Masonry. By the term strength of masonry is generally meant its resistance to a direct COMPRESSIVE force or load, and this is the only direct stress to which masonry should be subjected. Stone lintels and footings may be subjected to a TRANSVERSE or BENDING STRESS, but they can hardly be included in the term masonry, as they consist of single pieces. There are also tendencies to bend and to split apart in brick walls and piers, as they are usually high in proportion to their lateral dimensions, but the stresses thus developed cannot be accurately determined and should be avoided as much as possible. It is impossible to fix values for the strength of brickwork or stonework with anything like the exactness possible for wooden or steel members, for the reason that there is not only a great variation in the strength of different kinds of brick and stone, even when taken from the same kiln or quarry, but the strength of walls and piers is also greatly affected by the kind and quality of the mortar used, the way in which the work is built and bonded, and the amount of moisture in the materials when they are laid. All that can be done, therefore, is to give values which will be safe for the different kinds of masonry built in the usual manner.

Working Compressive Strength of Masonry. The building laws of most of the larger cities of this country specify the maximum loads per square foot allowed to be placed upon different kinds of masonry, and these laws must govern the architects in such cities. When there is no restriction of this kind, Table I gives an idea of the maximum loads which it is safe to put upon the different kinds of work mentioned. Table II gives the maximum safe loads specified in the building laws of several cities, and the remaining tables of the chapter give records of numerous tests made to determine the ultimate compressive strengths of various kinds of brick, terra-cotta block, building stone, mortar and concrete, and are of value in determining the safe loads for special cases. In determining the safe compressive resistance of masonry from tests on the ultimate compressive strength of work of the same kind, a factor of safety of at least 10 should be allowed for piers and 20 for arches.

* This chapter has been revised and rewritten from the original chapter by the late Thomas Nolan.

Table I. Safe Working Loads for Masonry—Compression

In pounds per square inch

NATURAL STONE

Sandstone	400	Limestone......	700	Slate...........	1 000
Marble..........	600	Granite	1 000		

BRICKWORK

Grade of brick	Allowable working stresses			
Av compressive strength	Portland cement mortar	Natural cement mortar	Cement lime mortar	Lime mortar
8 000 plus	400	300	300	100
4 500 to 8 000	250	200	200	100
2 500 to 4 500	175	140	140	75
1 500 to 2 500	125	100	100	50

MASONRY

	Portland cement mortar	Natural cement mortar	Cement lime mortar	Lime mortar
Dressed ashlar granite	800	640	640	400
Dressed ashlar limestone	500	400	400	250
Dressed ashlar marble	500	400	400	250
Dressed ashlar sandstone. . . .	400	320	320	160
Rubble stone...	140	100	100	....
Hollow clay or cinder block .	80	70	70	... .
Solid clay or cinder block .. .	125	100	100	. ...
Neat cement grout, under bases	1 000			

STONE CONCRETE

1 : 2 : 4 mixture; 7½ gal of water to 1 bag cement

In direct compression	500	Shear	40
Bending in extreme fiber......	650		

CINDER CONCRETE

1 : 2 : 5 mixture

Compression	300

MORTAR

	Pounds per square inch
Portland-cement mortar, 1 cement, 1/10 lime, 3 sand	325
Cement-lime mortar, 1 cement, 1 lime, 6 sand	125
Cement-lime mortar, 1 cement, 1 lime, 4 sand	200
Lime mortar, 1 lime, 3 sand	40

The following SAFE WORKING STRESSES for brick and stone masonry and concrete are given in the Chicago Building Code:

	Lb per sq in	Tons per sq ft
Common bricks, crushing strength 1 800 lb per sq in:		
In lime mortar	100	7 1/5
In lime-and-cement mortar	125	9
In natural-cement mortar	150	10 4/5
In Portland-cement mortar	175	12 3/5
Select, hard, common bricks, crushing strength equal to 2 500 lb per sq in:		
In 1 part Portland cement, 1 lime-paste and 3 sand	175	12 3/5
In 1 : 3 Portland-cement mortar	200	14 2/5
Pressed and sewer-bricks, crushing strength equal to 5 000 lb per sq in: 1 : 3 Portland-cement mortar	250	18
Paving bricks, in 1 : 3 Portand-cement mortar	350	25 1/5
Concrete, natural cement, 1 : 2 : 5	150	10 4/5
Concrete, Portland cement, 1 : 3 : 6, machine-mixed	300	21 3/5
Concrete, Portland cement, 1 : 3 : 6, hand-mixed	250	18
Concrete, Portland cement, 1 : 2 : 4, machine-mixed	400	28 4/5
Concrete, Portland cement, 1 : 2 : 4, hand-mixed	350	25 1/5
Rubble, uncoursed, in lime mortar	60	4 1/5
Rubble, uncoursed, in Portland-cement mortar	100	7 1/5
Rubble, coursed, in lime mortar	120	8 3/5
Rubble, coursed, in Portland-cement mortar	200	14 2/5
Ashlar, limestone, in Portland-cement mortar	400	28 4/5
Ashlar, granite, in Portland-cement mortar	600	43 1/5
Committee on Code Requirements for Indiana limestone recommends:		
Limestone-ashlar masonry, in lime mortar, equivalent to 1 : 3 cement-and-lime mortar	200 to 250	14 2/5 to 18
In natural-cement-and-lime mortar, 1 : 3	400 to 500	28 4/5 to 36

Brick Piers. As a rule brickwork is subject to its full safe resistance only when used in piers, and in small sections of walls, under bearing-plates. In the latter case but a few courses receive the full load, and hence a greater unit stress may be allowed than for piers. Aside from the quality of the work and materials the two elements which most influence the strength of brick piers are the ratio of height to least lateral dimension and the method of bonding. When the height of a brick pier exceeds six times its least lateral dimension the load per square foot should be reduced from the values given in Table I.

Formulas for the Safe Strength of Brick Piers exceeding six diameters in height. From the records of numerous tests on the strength of brick piers, from some formulas published by Ira O. Baker, and also from personal observation, Mr. Kidder deduced the following formulas for the maximum working

Table II. Comparison of Building Laws

Materials	Boston, 1926	New York, 1927	Philadelphia, 1929	Chicago, 1928	Denver, 1927	San Francisco, 1928
	Allowable pressure, lb per sq in					
Granite	830	1 000		600	800	400
Marble	550	600			500	. .
Limestone	550	700		400	500	. .
Sandstone	415	400		400	400	. . . .
Rubble, coursed	.	140	.	200	140	. . . .
Rubble, ordinary	.	110	70–140	100	. .	
Hard-burned brick. Lime mortar	110	110	95	100	90	100
Hard-burned brick. Lime and cement mortar	200	160	200	175	130	140
Hard-burned brick. Portland-cement mortar	275	250	250	200	170	200
Stone concrete, 1 : 2 : 4	450	500	400*	400	400	750
Stone concrete, 1 : $2\frac{1}{2}$: 5	360	400	. . .	350	. . .	. . .
Stone concrete, 1 : 3 : 6	290	.		300	.	. .
Hollow T-C block	150 gross	100 gross	80 gross	350 net	80 gross	. . . .
Hollow concrete block	100 gross	75 gross	80 gross	. . .	80 gross	. . .
Grout-Neat Portland cement	830	1 000		. . .	. . . .	. . .

* For 2 000-lb concrete. For stronger concrete 20% of ultimate strength may be used for allowable compressive stress.

loads for first-class brickwork in piers whose height exceeds six times the least lateral dimension.

For piers laid with rich lime mortar:

$$\text{Safe load per square inch} = 110 - 5\, H/D \tag{1}$$

For piers laid with 1 : 2 natural-cement mortar:

$$\text{Safe load per square inch} = 140 - 5\tfrac{1}{2}\, H/D \tag{2}$$

For piers laid with 1 : 3 Portland-cement mortar:

$$\text{Safe load per square inch} = 200 - 6\, H/D \tag{3}$$

H representing the height in feet, and D the least lateral dimension in feet.*

For a pier 20 ft high and 2 ft square these formulas will reduce the safe load to 4.3 tons per sq ft for lime mortar, 6.1 tons for natural-cement mortar and 10 tons for Portland-cement mortar. No pier over 8 ft high should be less

* For piers faced with pressed bricks, laid with joints ¼ in or less in thickness, and backed with common bricks in lime mortar, only the dimensions of the backing should be considered in figuring their strength If the backing is laid in cement mortar and the face-bricks are well tied to the backing, the full section of the pier may be considered. For piers veneered with stone or terra-cotta, 4 in thick, only the strength of the backing should be considered.

than 12 by 12 in in cross-section and when from 6 to 8 ft high piers should be at least 8 by 12 in in cross-section.

The following is the Chicago law (1928): "Isolated piers of concrete, brick or masonry shall not be higher than six times their smallest dimensions unless the above unit stresses are reduced according to the following formula:

$$P = C\,(1.25 - H/20\,D) \qquad (4)$$

in which P is the reduced allowed unit load, C the unit stress above referred to, H the height of the pier in feet and D the least dimension of the pier in feet. No pier shall exceed in height twelve times the least dimension. The weight of the pier shall be added to other loads in computing the load on the pier."

Brick piers intended to carry more than 50% of the safe loads given above should not be built in freezing weather nor with dry bricks. Lime mortar should never be used for building piers.

Effect of Bond on the Strength of Brickwork. Brick piers, loaded to the point of destruction, fail by the splitting and bulging out of the piers themselves, following bending and shearing stresses in the bricks and not by direct crushing of the bricks, showing that piers are weakest in their bond and mortar beds and in the tensile or transverse strengths of the bricks. It is very important, therefore, to have the brickwork well bonded, and all joints smoothly filled with mortar. The strength of a brick pier intended to carry an extreme load would probably be increased by bonding frequently with hoop-iron in addition to the regular brick-bond.

Bond-Stones in Brick Piers. Many competent architects and builders consider that the strength of a brick pier is increased by inserting bond-stones, from 5 to 8 in in thickness and the full size of the pier in cross-section, every 3 or 4 ft in height.

For example, the Building Laws for the City of New York (1927) require bond-stones every 30 in in height, and at least 4 in in thickness, to be built into brick piers which contain less than 9 superficial feet of section, and which support any beam, girder, arch, or column on which a wall rests, or lintel spanning an opening over 10 ft and supporting a wall. The New York laws allow perforated steel or cast-iron plates of the full cross-section of the pier to be used instead of the bond-stones. On the other hand, there are many architects who consider that bond-stones in a brick pier do more harm than good, and the author is of the opinion that this is generally the case. The Boston Building Laws do not require intermediate bond-stones. If bond-stones are used, they should be bedded so as to bear rather more heavily on the inner portion of the pier than on the outer 4 in, for unless this is done the outer shell will take most of the load, and will be likely to bulge away from the core. A pier which supports a girder or column should have a cap-stone or iron plate of sufficient strength to distribute the pressure over the cross-section of the pier.

Walls Faced with Stone, Terra-Cotta, or Cement Blocks. Brick walls faced with blocks or ashlar of any material should always have the backing laid in cement mortar. The aggregate thickness of the mortar joints in the backing is so much greater than in the facing, that any shrinkage or compression of the mortar tends to throw undue weight on the facing and to separate it from the backing. Veneering generally should be tied to the backing at least every 18 in in height. Stone courses, up to about 36 in in height, usually have anchors in the bed-joints only. Anchors are placed in the side joints also, when the height of the courses is more than 36 in. The Building

Ordinances of several large cities require that all bearing walls faced with bricks laid in running bond, and all walls faced with stone ashlar less than 8 in thick, shall be of such thickness as to make the wall independent of the facing conform to that required for unfaced walls. Ashlar 8 in thick and bonded into the backing may be counted as part of the thickness of the wall, the allowable stress of the backing then being used in calculating the wall.

Crushing Height of Bricks and Stone. If we assume that the weight of brickwork is 120 lb per cu ft, and that it would commence to crush under 700 lb per sq in, then a wall of uniform thickness would have to be 840 ft high before the bottom courses would commence to crush from the weight of the brickwork above. Average sandstones, at 145 lb per cu ft, would require a column 5 950 ft high to crush the bottom stones, and an average granite, at 165 lb per cu ft, would require a column 10 470 ft high. The Merchants' shot-tower at Baltimore is 246 ft high, and its base sustains a pressure of 6½ tons per sq ft, the tons being long tons of 2 240 lb. The base of the granite pier of Saltash Bridge (by Brunel), of solid masonry to the height of 96 ft, and supporting the ends of two iron spans of 455 ft each, sustains 9½ tons per sq ft.

Stone Piers. Piers of good strong building stone laid in courses the full cross-sections of the piers, with the top and bottom courses bedded true and even, may be built to support very heavy loads. The height of such piers, however, should not exceed ten times the least lateral dimension, and when it exceeds eight times the thickness, the load should be reduced. The joints should not exceed ⅜ in in thickness and should be spread with 1 : 3 Portland-cement mortar, kept back 1 in from the face of the pier to prevent spalling of the edges. A test of the strength of a limestone pier 12 in square is described under Marbles and Limestones, in this chapter. Rubble-work should not be used for piers whose height exceeds five times the least dimension, or in which the latter is less than 20 in.

Compressive Tests on Brick Piers and Walls. Two very instructive series of tests have recently been made by the Bureau of Standards at Washington, one on large brick piers by Mr. J. G. Bragg and the other by Messrs. Stang, Parsons and McBurney on large sections of brick walls. It has been shown by experience in testing a wide range of materials that there is a distinct divergence between the strength as shown by tests on small specimens and the actual strength obtained in practice with full-sized material. The importance of these recent tests upon brick piers and walls lies in the fact that the specimens approached much more closely to the true conditions in building, the piers being uniformly 10 ft 0 in high with areas of 79 to 1 024 sq in, and the walls being all 6 ft 0 in long and 9 ft 0 in high.

Table III gives the results of Mr. Bragg's tests, as published in Technologic Paper No. 111. The bricks were of three varieties: (1) best hard-burned, (2) medium-burned, (3) soft-burned, inferior qualities, and were selected from four districts, Pittsburgh, New York, Chicago, and New Orleans.

The conclusions deduced from the tests may be briefly summarized as follows:

(1) The primary failure of brick piers is caused by transverse failure of the individual bricks rather than by crushing of the bricks. Therefore the component parts of the pier should be made as deep as possible, by laying the bricks on edge and by breaking joints every few courses instead of every course. Likewise the mortar joints should be as thin as possible and of uniform thickness, and for this reason regularity in the shape of the bricks is important.

Table III. Compressive Strength of Large Brick Piers

Pittsburgh district—cement mortar

	Piers											Bricks—test data			
	Construction data							Test data							
No. of test	Mark on pier	District	Grade of bricks	Bond header and stretcher	Courses	Mortar	Height in ft	Age in days	Area in sq in	Max. load in lb	Max. load in lb per sq in	Mod. of elasticity	Av. compressive strength, flat	Av. compressive strength on edge	Av. transverse strength	Per cent of absorption
1	B 13	B	1	1 : 1	46	1 cement, 3 sand	10	30	930	2 520 000	2 710	2 500 000	11 990	8 900	1 945	4.08
2	14	B	1	1 : 3	46	do.	10	30	930	2 550 000	2 740	3 200 000	11 990	8 900	1 945	4.08
3	15	B	1	1 : 6	46	do.	10	30	930	2 697 000	2 900	3 200 000	11 990	8 900	1 945	4.08
4	17	B	2	1 : 1	45	do.	10	32	856	1 714 000	2 000	2 250 000	7 880	6 450	1 370	7 46
5	18	B	2	1 : 3	45	do.	10	32	885	1 834 000	2 070	2 000 000	7 880	6 450	1 370	7 46
6	27	B	2	1 : 6	44	do.	10	33	946	824 000	870	833 000	2 450	2 040	675	15.16
7	B 1	B	3	1 : 1	41	do.	10	29	1 024	524 000	510	700 000	1 659	1 350	345	16.28
8	2	B	3	1 : 3	41	do.	10	29	1 043	580 000	560	733 000	1 659	1 350	345	16.28
9	3	B	3	1 : 6	41	do.	10	32	1 024	660 000	650	533 000	1 659	1 350	345	16.28
Pittsburgh district—cement and lime mortar																
10	B 31	B	1	1 : 1	45	1 (15% lime, 85% cement), 3 sand	10	90	841	3 211 000	3 800	3 500 000	11 965	10 050	2 775	1.28
11	29	B	1	1 : 3	45	do.	10	90	841	2 714 000	3 220	3 050 000	11 965	10 050	2 775	1.28
12	30	B	1	1 : 6	45	do.	10	90	841	2 787 000	3 300	3 500 000	11 965	10 050	2 775	1.28
13	20	B	2	1 : 1	45	do.	10	33	908	1 594 000	1 760	1 550 000	7 880	6 450	1 370	7.46
14	28	B	2	1 : 3	44	do.	10	32	961	838 000	870	750 000	2 450	2 040	675	15.16
15	22	B	2	.1 : 6	45	do.	10	30	878	1 545 800	1 760	1 625 000	7 880	6 450	1 370	7.46

Pittsburgh district—lime mortar															
16	B 7	B	1	1 : 1	45	1 lime, 6 sand	10	120	940	1 360 000	1 450	725 000	11 990	8 900	1 945	4 08
17	8	B	1	1 : 3	45	do.	10	120	940	1 197 500	1 270	416 700	11 990	8 900	1 945	4.08
18	9	B	1	1 : 6	45	do.	10	120	940	1 280 000	1 360	750 000	11 990	8 900	1 945	4 08
19	10	B	2	1 : 1	44	do.	10	120	906	764 000	840	620 000	7 880	6 450	1 370	7 46
20	11	B	2	1 : 3	44	do.	10	120	906	804 000	890	687 000	7 880	6 450	1 370	7 46
21	12	B	2	1 : 6	44	do.	10	120	900	892 000	990	620 000	7 880	6 450	1 370	7 46
22	B 4	B	3	1 : 1	41	1 lime, 3 sand	10	120	1 024	215 500	210	300 000	1 659	1 350	345	16 28
23	5	B	3	1 : 3	41	do.	10	120	1 024	182 000	178	300 000	1 659	1 350	345	16 28
24	6	B	3	1 : 6	41	do	10	120	1 024	129 000	126	270 000	1 659	1 350	345	16.28
New Orleans district—cement and lime mortar																
25	B 47	D	1	1 : 1	41	1 (15% lime, 85% cement), 3 sand	10	29	841	1 223 000	1 450	1 533 000	7 340	4 910	733	16.80
26	48	D	1	1 : 3	41	do.	10	29	841	1 475 000	1 760	1 533 000	7 340	4 910	733	16 80
27	52	D	2	1 : 1	41	do.	10	31	841	1 370 000	1 630	1 563 000	6 880	5 490	893	16 40
28	53	D	2	1 : 3	41	do.	10	30	841	1 506 000	1 790	1 750 000	6 880	5 490	893	16 40
29	49	D	3	1 : 1	41	do	10	30	841	1 580 000	1 880	1 833 000	6 510	5 700	1 090	17.10
30	50	D	3	1 : 3	41	do	10	29	841	1 422 000	1 690	1 583 000	6 510	5 700	1 090	17 10
31	51	D	3	1 : 6	41	do	10	31	841	1 397 000	1 660	1 583 000	6 510	5 700	1 090	17 10
New York district—cement and lime mortar																
32	B 44	A	1	1 : 1	45	1 (15% lime, 85% cement), 3 sand	10	32	791	921 500	1 170	875 000	5 630	6 440	601	16 40
33	45	A	1	1 : 3	45	do	10	32	791	1 033 500	1 300	875 000	5 630	6 440	601	16 40
34	46	A	1	1 : 6	45	do	10	31	791	998 000	1 260	875 000	5 630	6 440	601	16 40
35	38	A	2	1 : 1	45	do.	10	30	784	1 001 000	1 280	875 000	4 430	5 449	616	18.60
36	39	A	2	1 : 3	45	do	10	31	791	1 011 000	1 280	1 125 000	4 430	5 449	616	18.60
37	40	A	2	1 : 6	45	do	10	31	791	967 500	1 220	817 000	4 430	5 449	616	18.60
38	41	A	3	1 : 1	45	do	10	31	791	840 500	1 070	484 000	2 710	2 970	497	19 30
39	42	A	3	1 : 3	45	do	10	31	791	840 500	1 060	667 000	2 710	2 970	497	19.30
40	43	A	3	1 : 6	45	do	10	30	791	808 000	1 020	600 000	2 710	2 970	497	19 30
Chicago district—cement and lime mortar																
41	B 35	C	1	1 : 1	46	1 (15% lime, 85% cement), 3 sand	10	32	841	706 700	840	833 000	3 200	3 010	1 180	16 20
42	36	C	1	1 : 3	46	do	10	29	812	641 500	790	750 000	3 200	3 010	1 180	16.20
43	37	C	1	1 : 6	46	do.	10	30	812	660 500	810	750 000	3 200	3 010	1 180	16 20
44	32	C	2	1 : 1	46	do	10	30	812	606 500	750	833 000	3 150	2 710	1 140	16 20
45	33	C	2	1 : 3	46	do.	10	30	812	572 000	700	650 000	3 150	2 710	1 140	16 20
46	34	C	2	1 : 6	46	do	10	30	812	578 500	710	580 000	3 150	2 710	1 140	16 20

Table IV

(A) Walls of series 1

Type of wall	Mortar	Kind of brick	Workmanship (type of bed joints)	Curing conditions	Wall Nos.			Average secant modulus of elasticity	Average stress at first crack	Wall strength			
					A	B	C			A	B	C	Average
								lb per sq in	lb per sq in	lb per sq in	lb per sq in	lb per sq in	lb per sq in
8-in solid........	Lime	Chicago	Furrowed	Ordinary	1	2	3	54 000	200	255	250	315	275
	Cement-lime	do.	do.	do.	7	8	9	459 000	520	575	620	590	595
	Cement	do	do.	do.	13	14	15	554 000	640	660	750	580	665
		Mississippi	do.	do.	155	..	.	493 000	870	870	..	.	870
		do	Spread	do.	159	..	...	974 000	1 480	1 480	....	.	1 480
		New England	Furrowed	do.	154	..	...	2 040 000	1 500	2 030	.		2 030
12-in solid.......	Lime	Chicago	do.	do.	4	5	6	69 600	210	295	350	250	300
	Cement-lime	do	do	do.	10	11	12	556 000	510	580	575	580	580
	Cement	do	do.	do.	16	17	.	686 000	600	655	655	.	655
8-in all-rowlock...	do	Mississippi	do	do.	156	...		379 000	335	440	...		440
8-in all-rowlock (Flemish bond)	do	do	do	do.	157	..		509 000	270	535		. .	535
8-in rowlock-back	do	do	do.	do	158			453 000	485	745			745

(3) Walls of series 2

Type of wall	Mortar	Kind of brick	Workmanship (type of bed joints)	Curing conditions	Wall Nos. A	Wall Nos. B	Wall Nos. C	Average secant modulus of elasticity	Average stress at first crack	Wall strength A	Wall strength B	Wall strength C	Wall strength Average
8-in solid... ...	Cement-lime	Detroit	Spread	Ordinary	19	20	21	638 000	515	1 060	905	760	910
		Mississippi	do.	do	34	35	36	1 031 000	1 125	965	1 175	1 340	1 160
		New England	do.	do.	94	95	96	2 049 000	1 165	1 785	1 875	1 710	1 790
	Cement	Chicago	do.	do.	160	161	162	1 041 000	865	800	870	965	880
		do	do.	do.	163	164	165	1 014 000	830	885	925	755	855
		Detroit	do.	do.	22	23	24	727 000	790	1 305	895	1 050	1 080
		do	do	Wetted	33	.	...	767 000	610	1 055	....	....	1 055
		Mississippi	do.	Ordinary	37	38	39	1 300 000	1 270	1 335	1 405	1 405	1 380
		New England	do	do.	97	98	99	2 360 000	1 750	2 040	2 950	2 915	2 635

Table IV (Continued)

(B) Walls of series 2 (continued)

Type of wall	Mortar	Kind of brick	Workmanship (type of bed joints)	Curing conditions	Wall Nos. A	Wall Nos. B	Wall Nos. C	Average secant modulus of elasticity	Average stress at first crack	Wall strength A	Wall strength B	Wall strength C	Wall strength Average
								lb per sq in	lb per sq in	lb per sq in	lb per sq in	lb per sq in	lb per sq in
12-in solid	Cement-lime	Detroit	Spread	Ordinary	25	26	27	727 000	590	1 050	940	965	985
		Mississippi	do.	do.	40	41	42	1 288 000	1 155	1 375	1 305	1 225	1 300
		New England	do.	do.	100	101	102	2 461 000	1 100	1 815	1 990	1 855	1 890
	Cement	Chicago	Shoved	do.	18		..	1 350 000	1 165	1 165	....	.	1 165
		do.	Spread	do.	166	167	.	1 060 000	875	890	1 080	..	985
		do.	do.	169 days old	168	.	...	......	1 065	1 200	...	..	1 200
		Detroit	do.	Ordinary	28	29	30	935 000	800	1 165	1 345	1 115	1 210
		do.	do.	Wetted	31	32	..	809 000	665	1 040	1 165	.	1 105
		Mississippi	do.	Ordinary	43	44	45	1 538 000	1 390	1 710	1 855	1 350	1 640
		do.	do.	Wetted	46	47	...	1 945 000	1 380	1 205	1 875	...	1 590
		do.	Shoved	Ordinary	48	..	..	1 082 000	1 200	1 465	....	..	1 465
12-in solid	Cement	New England	Spread	do.	103	104	105	2 652 000	2 301	2 440	3 230	2 695	2 790
		do.	do.	Wetted	106	107	...	2 455 000	1 750	1 750	3 270	....	2 510
		do.	Shoved	Ordinary	108	.	.	2 024 000	2 530	2 720	....	....	2 720
8-in all-rowlock	Cement-lime	Mississippi	Spread	do.	49	50	51	709 000	600	745	665	890	765
		New England	do.	do.	109	110	111	1 103 000	415	1 055	780	795	875
	Cement	Mississippi	do.	do.	52	53	54	883 000	785	850	735	1 175	920
		New England	do.	do.	112	113	114	1 209 000	430	1 005	1 065	795	955
12-in all-rowlock	Cement-lime	Mississippi	do.	do.	55	56	57	687 000	565	645	725	750	705
		New England	do.	do.	115	116	117	1 105 000	450	805	785	640	740
	Cement	Mississippi	do.	do.	58	59	60	681 000	665	795	745	870	800
		New England	do	do.	118	119	120	1 270 000	665	775	935	895	870

Table IV (Continued)

(B) Walls of series 2 (continued)

Type of wall	Mortar	Kind of brick	Workmanship (type of bed joints)	Curing conditions	Wall Nos. A	Wall Nos. B	Wall Nos. C	Average secant modulus of elasticity	Average stress at first crack	Wall strength A	Wall strength B	Wall strength C	Wall strength Average
								lb per sq in	lb per sq in	lb per sq in	lb pcr sq in	lb per sq in	lb per sq in
8-in all-rowlock (Flemish bond)	Cement-lime	Mississippi	do.	do.	61	62	63	743 000	545	800	750	695	750
		New England	do.	do.	121	122	123	1 199 000	600	555	745	610	640
	Cement	Mississippi	do.	do.	64	65	66	738 000	530	825	720	855	800
		New England	do.	do.	124	125	126	1 712 000	630	680	925	720	775
12-in all-rowlock (Flemish bond)	Cement-lime	Mississippi	do.	do.	67	68	69	739 000	485	570	640	810	675
		New England	do.	do.	127	128	129	1 108 000	575	890	900	700	830
	Cement	Mississippi	do.	do.	70	71	72	804 000	500	550	660	920	710
		New England	do.	do.	130	131	132	1 422 000	605	940	1 005	880	940
8-in rowlock-back	Cement-lime	Mississippi	do.	do.	73	74	75	857 000	810	950	905	965	940
		New England	do.	do.	133	134	135	1 151 000	530	815	1 155	680	880
	Cement	Mississippi	do	do.	76	77	78	859 000	905	925	890	950	920
		New England	do.	do.	136	137	138	2 074 000	910	1 270	1 245	1 105	1 205
12-in rowlock-back (heavy duty)	Cement-lime	Mississippi	do	do.	79	80	81	833 000	515	775	830	950	850
		New England	do	do.	139	140	141	1 316 000	600	1 150	830	910	965
	Cement	Mississippi	do	do.	82	83	84	782 000	755	1 035	835	955	940
		do	do	Wetted	85	86		866 000	650	1 025	905	. .	965
		New England	do.	Ordinary	142	143	144	1 694 000	775	1 635	1 525	1 610	1 590
		do.	do.	Wetted	145	146	. .	1 803 000	740	1 400	1 250	. . . .	1 325
12-in rowlock . . (standard)	Cement	Mississippi	do.	Ordinary	87	.	. . .	749 000	985	1 010	. . .	. . .	1 010
		New England	do.	do.	147	.	. .	1 633 000	450	1 160	. . . .	. . . .	1 160
4-in economy	Cement-lime	Mississippi	do	do	88	89	90	1 155 000	1 375	1 370	1 365	1 565	1 435
		New England	do.	do.	148	149	150	1 826 000	1 235	1 960	1 975	1 880	1 940
	Cement	Mississippi	do	do.	91	92	93	1 257 000	1 575	1 650	1 350	1 875	1 625
		New England	do.	do.	151	152	153	2 460 000	2 275	2 755	3 520	3 160	3 145

(2) The kind of mortar used is important. Pure lime mortar gave the weakest results, but it was found that in a mortar of 1 part Portland cement to 3 parts sand, 25% by volume of the cement could be replaced by hydrated lime without affecting the strength of the piers. The workability of the mortar was likewise improved and smoother, more even beds and fuller joints resulted. Equal parts by volume of cement and lime, however, caused a lessening in the strength of the piers.

(3) Varying the number of header courses does not appreciably affect the ultimate strength of the pier, but its strength is slightly increased by the introduction of wire mesh in all horizontal joints. This increase did not take place, however, when the mesh was introduced in every fourth joint only

The wall tests of Messrs. Stang, Parsons and McBurney, published in Research Paper No. 108, are as follows:

The foregoing investigation deals with the compressive tests under central loading of 168 brick walls each 6 ft long and about 9 ft high and of 129 wallettes or small walls each about 18 in long and 34 in high, each wallette corresponding in kind of brick, method of laying, mortar mixture, and workmanship with one of the large walls. It was expected that tests upon sample wallettes exactly similar in every way except in size would give a more consistent measure of the strength of a proposed construction than would tests upon individual bricks, since the laying, mortar, and workmanship are such important factors in the strength of any brick masonry. These tests verify this supposition, the solid walls averaging in strength about 86% of the strength of the wallettes, and the hollow walls about 77%. The strengths of the solid walls were more closely related to the shearing strength of the brick than to the compressive or tensile strength.

The workmanship was divided into two types. In the first, the walls were built by contract on a lump sum basis without supervision, the longitudinal vertical joints being very short of mortar and the horizontal beds deeply furrowed with the trowel. In the second type the walls were laid up by day's work with careful supervision. The vertical joints were well filled and the horizontal mortar beds smoothly spread.

The first type were built of Chicago brick and the average strengths of the walls with various mortars were as follows, the strength of a half-brick flatwise being 3 280 lb per sq in.

Lime mortar walls	287 lb per sq in
Cement lime mortar walls	587 lb per sq in
Cement mortar walls	661 lb per sq in

For the second type the results were as follows:

Kind of brick	Compressive strength of half-brick flatwise	Average compressive strength of solid walls	
		Cement-lime mortar	Cement mortar
	lb per in	lb per sq in	lb per sq in
Chicago.	3 280	. . . .	895
Detroit	3 580	945	1 145
Mississippi.	3 410	1 300	1 550
New England.	8 600	1 875	2 855

It is seen that both the kind of mortar and the workmanship very materially affect the strength of the walls. Those in which the beds were smooth and the vertical joints well filled were stronger than walls in which the mortar beds were furrowed by from 24 to 109%.

The tests also show that the strengths of hollow walls varied about as the net area in compression.

2. Strength of Terra-Cotta and Terra-Cotta Piers

General Properties of Terra-Cotta. The lightness of terra-cotta, combined with its great compressive strength, together with its relatively high resistance to the effects of heat and fire, renders it an especially valuable building material. Terra-cotta for building purposes, whether plain or ornamental, is generally made of hollow blocks formed with webs to give extra strength and keep the work true while drying. This is necessary because good, well-burned terra-cotta cannot safely be made more than about 1½ in thick, whereas, when required to bond with brickwork, it must be at least 4 in thick. When the terra-cotta work does not project beyond the face of the wall these hollow spaces are generally filled with concrete or brickwork.

Although individual hollow tile show greater strength when tested on end than on the sides, this advantage does not in general hold true when the tile are laid in end construction in a wall, unless the mortar is very rich and the central webs are in bearing on the webs of the tile below. This is seldom the case, owing to the difficulty of spreading mortar upon the central webs without excessive loss of material and time.

Tests on Terra-Cotta Tile Walls. In 1926 the results of a series of tests made by Messrs. Stang, Parsons and Foster were published by the United States Bureau of Standards. The walls, 70 in number, were 6 ft long, 9 ft high and either 8 or 12 in thick, and were built with ordinary workmen under average indoor conditions. The mortar mixes represent four commonly used volume proportions as follows:

Mortar No. 1. Lime mortar (1¼ L : 3 S) by volume 1¼ parts lime to 3 parts sand

Mortar No. 2. Cement-lime mortar (1 C : 1¼ L : 6 S) by volume 1 part cement to 1¼ parts lime to 6 parts sand

Mortar No. 3. Cement-lime mortar (1 C : 1¼ L : 4 S) by volume 1 part cement to 1¼ parts lime to 4 parts sand

Mortar No. 4. Cement mortar (1 C : 3 S) by volume 1 part cement to 3 parts sand

The average results of the tests of the mortar specimens are given in Table V.

Table V. Average Strength of Mortar

Mortar Nos.	Proportions by volume	Specimens tested	Average compressive strength	Average tensile strength
			lb per sq in	lb per sq in
1	1¼ L : 3 S	12	85	14
2	1 C : 1¼ L : 6 S	81	760	80
3	1 C : 1¼ L : 4 S	105	1 190	135
4	1 C : 3 S	12	1 990	155

The physical properties of the tile are shown in Table VI, and the results of the compressive tests in Table VII.

Table VI. Physical Properties of Hollow Tile and Brick

Tile Lot Number	Kind of clay	Source	Design of tiles	Nominal size in inches	Position in which tested	Average area under compression		Average compressive strength		Average absorption	Average weight
						Gross	Net	Gross area	Net area		
						sq in	sq in	lb per sq in	lb per sq in	per cent	lb
1	Fire clay, dense burning	Ohio	6-cell	8 ×12 ×12	End	98.3	40.1	1 830	4 470	11.5	35.2
1	do.	do	do	do	Side	97.4	22.1	500	2 210	13.2	....
2	do.	do.	do.	12 ×12 ×12	End	152.3	57.4	1 860	4 940	9.8	50.0
2	do	do	do	do	Side	146.0	32.6	580	2 620	9.5	..
3	do.	do.	3-cell	3¾ ×12 ×12	End	48.9	22.9	1 740	3 720	11.2	19.4
4	Fire clay, open burning	New Jersey	6-cell	8 ×12 ×12	do	99.5	46.3	1 270	2 740	16.9	36.2
4	do.	do	do	do	Side	95.5	24.7	330	1 290	17.0	...
5	Shale	Kentucky	do	do	End	94.1	41.4	2 840	6 470	6.7	38.6
5	do.	do.	do	do	Side	91.8	22.6	1 740	7 080	6.4	...
6	Surface clay	New Jersey	XXX	do	End	90.6	44.6	2 100	4 260	11.5	35.8
6	do	do	do	do	Side	91.2	26.3	430	1 500	9.9	....
7	Fire clay, dense burning	Ohio	do	do	End	91.7	35.5	2 030	5 280	8.5	30.5
8	do.	do.	H-shaped	8 ×10¼ ×12	Side	99.5	30.8	1 040	3 350	12.6	34.9
9	do.	do	Double shell	8 ×12 ×5	End	94.5	45.4	3 320	6 900	6.2	17.1
10	do.	do	2-cell	8 ×5 ×12	Side	92.9	20.2	820	3 740	6.8	14.4
11	do.	do	1-cell	3¾ ×5 ×12	do	46.3	14.8	1 190	3 720	8.1	9.0
12	Hydraulic pressed brick	Virginia	. . .	2¼ ×3¾ ×8	Flat	30.5	30.5	5 690	5 690	13.6	...
13	Fire clay, dense burning	Ohio	3-cell	8 ×5 ×12	Side	94.4	24.0	900	3 540	7.4	14.4
14	Surface clay	Georgia	T-shaped	8 ×6¼ ×12	do	93.3	28.0	340	1 150	14.4	15.8
15	do.	New Jersey	do.	do	do.	96.3	32.2	630	1 870	8.8	18.0

Table VII. Results of Compressive Tests of Hollow-Tile Walls

Walls 6 ft 0 in long, 9 ft 0 in high

A. Centrally Loaded

Wall designation	Wall thickness	Tiles		Mortar volume ratio	Modulus of elasticity gross area	Compressive strength of walls		
	Inches	Kind of clay	Description of tiles and sizes in inches		Lb/sq in	Specimen A Lb/sq in	Specimen B Lb/sq in	Average Lb/sq in
1–E–1	8	Ohio fire clay	6-cell, 8 × 12 × 12	1¼L : 3S	150,000	90	90	90
1–S–1	8	do	do.	do.	278 000	170	160	165
1–E–2	8	do.	do.	1C : 1¼L : 6S	660,000	270	280	275
4–E–2	8	New Jersey fire clay	do	do.	632 000	290	300	295
6–E–2	8	New Jersey surface clay	XXX, 8 × 12 × 12	do.	884 000	370	.....	370
1–S–2	8	Ohio fire clay	6-cell, 8 × 12 × 12	do.	415 000	240	330	285
8–S–2	8	do	H-shaped, 8 × 10¼ × 12	do.	803 000	480	430	455
1–M–2	8	do.	6-cell, 8 × 12 × 12	do.	370 000	200	......	200
1–E–3	8	do.	do.	1C : 1¼L : 4S	694 000	480	440	460
5–E–3	8	Kentucky shale	do.	do	1 076 000	590	510	550
6–E–3	8	New Jersey surface clay	XXX, 8 × 12 × 12	do.	. .	......	650	650
7–E–3	8	Ohio fire clay	do	do	1 072 000	530	690	610
9–E–3	8	do	Double shell, 8 × 12 × 5	do.	1 216 000	540	570	555
1–S–3	8	do	6-cell, 8 × 12 × 12	do.	562 000	410	400	405
4–S–3	8	New Jersey fire clay	do	do.	340,000	240	220	230
5–S–3	8	Kentucky shale	do.	do.	1 264 000	490	510	500
6–S–3	8	New Jersey surface clay	XXX, 8 × 12 × 12	do	526,000	230	250	240
10–S–3	8	Ohio fire clay	2-cell, 8 × 5 × 12	do	569,000	380	290	335
13–S–3	8	do	3-cell, 8 × 5 × 12	do	750,000	420	460	440
14–S–3	8	Georgia surface clay	T-shaped, 8 × 6¼ × 12	do	350,000	330	280	305
15–S–3	8	New Jersey surface clay	do	do.	536,000	450	410	430
1–E–4	8	Ohio fire clay	6-cell, 8 × 12 × 12	1C : 3S	846 000	370	340	355
1–E–4	8	do.	do	do	457 000	370	350	360

Table VII (Continued). Results of Compressive Tests of Hollow-Tile Walls

Walls 6 ft 0 in long, 9 ft 0 in high

A. Centrally Loaded—Continued

Wall designation	Wall thickness	Tiles		Mortar volume ratio	Modulus of elasticity gross area	Compressive strength of walls		
	Inches	Kind of clay	Description of tiles and sizes in inches		Lb/sq in	Specimen A Lb/sq in	Specimen B Lb/sq in	Average Lb/sq in
2–E–2	12	Ohio fire clay . . .	6-cell, 12 × 12 × 12	1C : $1\frac{1}{4}$L : 6S	850 000	430	240	335
(1 + 3)–E–2	12	do .	{6-cell, 8 × 12 × 12 / 3-cell, $3\frac{3}{4}$ × 12 × 12}	do.	870 000	280	360	320
2–S–2	12	do .	6-cell, 12 × 12 × 12	do.	682 000	370	400	385
(10 + 11 + 12)–S–3	12	do	{2-cell, 8 × 5 × 12 / 1-cell, $3\frac{3}{4}$ × 5 × 12 / faced with brick}	1C : $1\frac{1}{4}$L : 4S	792 000	560	560	560
14–S–3	12	Georgia surface clay	T-shaped, 8 × $6\frac{1}{4}$ × 12	do	411 000	360	320	340
B Eccentrically Loaded Eccentricity 2 in								
1–E–2	8	Ohio fire clay	6-cell, 8 × 12 × 12	1C : $1\frac{1}{4}$L : 6S	605 000	190	170	180
1–S–2	8	do	do.	do	513 000	100	120	110
8–S–2	8	do. . . .	H-shaped, 8 × $10\frac{1}{4}$ × 12	do.	637 000	230	170	200
1–M–2	8	do	6-cell, 8 × 12 × 12	do	. . . . . .	. . . . .	110	110
1–E–3	8	do.	do	1C : $1\frac{1}{4}$L : 4S	723 000	330	320	325
10–S–3	8	do.	2-cell, 8 × 5 × 12	do	544 000	240	240	240
13–S–3	8	do	3-cell, 8 × 5 × 12	do.	606 000	200	290	245
2–E–2	12	do.	6-cell, 12 × 12 × 12	1C : $1\frac{1}{4}$L : 6S	515 000	180	140	160
2–S–2	12	do .	do	do	342 000	120	100	110

Symbols listed in column 1 represent Tile lot number Construction—edge, side or mixed Mortar number.

Strength of Terra-Cotta Brackets or Consoles. A cornice-modillion made by the Northwestern Terra-Cotta Company, 11½ in high at the wall-line, 8 in wide on the face, and with a projection of 2 ft, was built into a wall and the upper surface loaded with 2 tons of pig iron without any effect upon the modillion. Another bracket, 5½ in high, 6 in wide and with a 14-in projection, made in the East, broke at the wall-line under 2 650 lb, while a duplicate of it sustained 2 400 lb for one month without breaking.

The Weight of Terra-Cotta. The weight of terra-cotta in solid blocks is 120 or 122 lb per cu ft. When made in hollow blocks 1½ in thick the weight varies from 65 to 85 lb per cu ft, the smaller pieces weighing the most. For pieces 12 by 18 in or larger on the face, 70 lb per cu ft will probably be a fair average. The tables in the manufacturers' catalogues give the various bearing-areas, weights per square foot, thicknesses of parts, sizes of blocks, etc., for porous and semiporous blocks for all purposes.

3. Crushing Strength of Building Stones

(1) Sandstones

Longmeadow, Mass., Stone.* Reddish-brown sandstone, two blocks about 4 by 4 in in cross-section and 8 in in height.

Block No. 1 commenced to crack at 10 333 lb per sq in, and flew from the machine in fragments at 13 596 lb per sq in.

Block No. 2 commenced to crack at 3 012 lb per sq in and failed completely at 9 121 lb per sq in.

Sandstone from Norcross Brothers' Quarries, East Longmeadow, Mass., Soft Saulsbury Stone.* Block No. 1, 4 by 4 by 8 in high, commenced to crack at 8 250 lb and failed at 8 812 lb per sq in.

Block No. 2, 4 by 4 by 8 in high, commenced to crack at 6 500 lb and failed at 8 092 lb per sq in.

Hard Saulsbury Stone.* Block No. 1, 4 by 4 by 8 in high (about), commenced to crack at 12 716 lb and failed at 13 520 lb per sq in.

Block No. 2, same size as No. 1, commenced to crack at 13 953 lb and failed at 14 650 lb per sq in.

Kibbe Stone.* Block No. 1, 6 by 6 by 6 in, commenced to crack at 12 590 lb and failed at 12 619 lb per sq in.

Block No. 2, same size as No. 1, commenced to crack at 12 185 lb and failed at 12 874 lb per sq in.

Brown Stone from the Shaler & Hall Quarry Company, Portland, Conn.† The results of the tests are as follows:

Table VIII. Crushing Strength of Brown Sandstone

Dimensions			Sectional area	First crack	Ultimate strength	Classification
Height in	Compressed, surface in		sq in	lb	lb per sq in	
2.50	2.50	2.45	6 13	84 800	13 980	1st quality
2.50	2.48	2.47	6.13	81 700	13 330	1st quality
2 98	3 00	2.95	8.85	123 200	13 920	2d quality
2.95	2.98	2.97	8.85	122 000	15 020	3d quality
2.51	2.55	2.53	6.45	63 850	9 900	Bridge
2.48	2.48	2.52	6.25	58 340	9 330	Bridge

* These tests were made with the United States testing-machines at Watertown Arsenal, Mass.

† From tests made by Colt's Patent Fire-arms Manufacturing Company.

Brown Stone from the Middlesex Quarry Company, Portland, Conn.* Four nearly cubical blocks, about 1½ in square. Pressure per square inch at time of failure: No. 1, 10 928 lb; No. 2, 10 322 lb; No. 3, 8 252 lb and No. 4, 6 322 lb.

Red Sandstone † from Greenlee & Son's Quarries at Manitou, Col. One specimen failed at 11 000 lb per sq in; weight, 140 lb per cu ft.

Light-Red Laminated Sandstone from St. Vrain Cañon, Col., a very hard stone, excellent for walks and foundations. Crushing strength on bed, 11 505 lb per sq in; weight, 150 lb per cu ft.

Gray Sandstone (free-working) from Trinidad, Col. Crushing strength, 10 000 lb per sq in; weight, 145 lb per cu ft.

Gray Sandstone from Fort Collins, Col. (laminated and similar in quality to the St. Vrain stone). Crushing strength on bed, 11 700 lb per sq in; weight, 140 lb per cu ft. One ton of this stone measures just a perch in the wall.

Gray Sandstone from McDermott, Ohio. Crushing strength, 9 000 lb per sq in.

Gray Sandstone from Amherst, Ohio. Crushing strength, 8 000 lb per sq in.

(2) Granite

Red Granite from Platte Cañon, Col. Crushing strength per square inch, 14 600 lb; weight per cubic foot, 164 lb.

Granite from Colorado Cañon. Crushing strength, 22 000 to 28 000 lb per sq in.

Granite from Quincy, Mass. Average crushing strength, 10 000 lb per sq in.

(3) Lava Stones

Lava Stone from the Kerr Quarries, near Salida, Col. Four cubical blocks. The results of the tests are as follows:

Table IX. Crushing Strength of Lava Stone

Dimensions			Sectional area	First crack	Ultimate strength	
Height in	Compressed surface in		sq in	lb	lb	lb per sq in
4.00	4 00	4 00	16 00	165 900	165 000	10 369
4 00	4.00	4.00	16 00	174 100	174 100	10 881
2.00	2 00	1.99	3 98	36 400	37 100	9 322
1.99	1.99	1 99	3.96	38 200	38 200	9 646

Lava Stone Curry's Quarry, Douglas County, Col. Crushing strength, 10 675 lb per sq in; weight, 119 lb per cu ft. Experience has shown that this

* These tests were made with the United States testing-machines at Watertown Arsenal, Mass.

† These tests were made with the United States testing-machines at Watertown Arsenal, Mass.

stone is not suitable for piers, or where any great strength is required, as it cracks very easily.

(4) Marble and Limestone

White marble quarried at Sutherland Falls, Vt. Two cubical blocks about 6 in square.*

Block No. 1 commenced to crack at 9 750 lb per sq in and failed suddenly at 11 250 lb per sq in.

Block No. 2 did not crack until it suddenly gave way at 10 243 lb per sq in.

The Bureau of Standards gives the following crushing strengths per sq in for marbles:

Gantt's Quarry, Ala., 14 000 lbs.

Dolomite, Cockeysville, Md., 22 000 lb.

Calcite, Ball Ground, Georgia, 10 000 lb.

Tests at Watertown Arsenal give the following crushing strengths per sq in for marbles:

White Dolomite, Lee, Mass., 18 047 lb.

Calcite, Rutland, Vt, 11 525 to 14 397 lb.

Calcite, So. Dorset, Vt., on bed, 11 300 lb; on edge, 9 100 lb.

Norwegian Dolomite, 24 891 lb; Carrara, 6 329 lb; Tyrolese Statuary, 16 036 lb.

Test of a Limestone Pier. A pier of Lemont limestone, 1 sq ft in cross-section and 9 ft in height, composed of seven stones with bearing surfaces planed perfectly true and parallel to the natural bed and the joints washed with a thin grout of the best English Portland cement, was tested at the Watertown Arsenal for William Sooy Smith, and only commenced to crack when the full power of the machine, 400 tons, was exerted.

Tests on Limestone. In 1927 the Bureau of Standards of the U. S. Department of Commerce published in Technologic Paper No. 349 a series of tests on limestones performed by Mr. D. W. Kessler and Mr. W. H. Sligh. These tests treat very thoroughly of the strength, absorption, specific gravity, porosity, efflorescence, weathering and other physical properties of a great variety of limestones, the samples being taken from Alabama, Illinois, Indiana, Kentucky, Minnesota, Missouri, New York, Texas and Kansas. Because of the many samples tested and the wide range of localities represented and also because limestone both in its finer and coarser grades is now so generally employed for all kinds of buildings throughout the United States, representative extracts from the published tables of the tests are given herein Only the average compressive strengths, absorptions and weights are included. The stones showing the greatest strength and weight are generally the dolomitic limestones with a high proportion of magnesium. Table X, which follows, is an abridgment of the tests published by the Bureau of Standards.

* Tested at the United States Arsenal, Watertown, Mass.

Table X. Compressive Tests on Limestones

State	Test No.	Trade Name	Location of Quarry	Manner of Testing*	Average compressive strength lb per sq in	Average absorption by volume	Weight per cu ft
Alabama	2	Alabama	Russellville	Perpend	13 900	11.0	147
				Parallel	11 700		
do.	3	Rockwood Alabama	do	Perpend	5 050	10.1	144
				Parallel	5 350		
Illinois	7	Joliet	Joliet	Perpend	22 100	6.5	163
				Parallel	24 400		
Indiana	8	Statuary Buff	Dark Hollow	Perpend	6 900	10.0	142
				Parallel	8 350		
do.	16	Select Buff	do	Perpend	6 350	10.8	144
				Parallel	5 750		
do.	42	Standard Buff	Bedford	Perpend	3 750	14.1	142
				Parallel	3 550		
do	44	do.	Dark Hollow	Perpend	8 900	9.3	148
				Parallel	7 500		
do	55	Rustic Buff	Bedford	Perpend	7 900	9.9	148
				Parallel	6 450		
do	56	do	Dark Hollow	Perpend	5 350	12.9	141
				Parallel	4 450		
do.	67	Select Gray	do.	Perpend	7 350	9.1	149
				Parallel	6 150		
do.	72	do.	Bedford	Perpend	11 000		
				Parallel	9 200		
do.	84	Standard Gray	Dark Hollow	Perpend	6 650	8.3	148
				Parallel	5 400		
do.	88	Variegated	Bedford	Perpend	8 000	12.0	144
				Parallel	7 700		

* Loads applied perpendicular or parallel to the stratification.

Table X. (Continued). Compressive Tests on Limestones

State	Test No.	Trade Name	Location of Quarry	Manner of Testing*	Average compressive strength lb per sq in	Average absorption by volume	Weight per cu ft
Indiana	89	Variegated	Dark Hollow	Perpend	9 750	9.4	146
				Parallel	8 800	......	......
Kentucky	108	Bowling Green Stone	Bowling Green	Perpend	4 100	10.6	146
				Parallel	2 900	......	......
do.	110	do.	do.	Perpend	6 900	11 3	144
				Parallel	6 300	......	......
Minnesota	112	Buff Travertine	Mantorville	Perpend	11 700	11.3	151
				Parallel	12 600	......	......
do.	114a	Golden Buff Kasota	Kasota	Perpend	15 000	9 8	157
				Parallel	10 000	......	......
do.	115	Yellow Fleuri	do.	Perpend	12 000	12 2	152
				Parallel	11 300	......	......
do.	116	Mankato Cream	Mankato	Perpend	14 500	10.7	153
				Parallel	13 000	......	......
do.	118	Kato	do.	Perpend	14 200	7.5	159
				Parallel	13 000	......	......
do.	119	Travertine	Winona	Perpend	17 000	7.2	158
				Parallel	8 900	......	......
New York	129	Onondaga Gray	Near Syracuse	Perpend	17 900	0.19	168
				Parallel	18 700	......	......
Texas	133	Leuders Stone	Lueders	Perpend	8 900	14 4	139
				Parallel	8 600	......	......
Kansas	134	Silverdale	Silverdale	Perpend	6 700	18 7	131
				Parallel	6 300	......	......

* Loads applied perpendicular or parallel to the stratification.

(5) Bricks and Various Stones

Table XI gives the crushing strength of various kinds of bricks and building stones, the pressure being normal to the plane of the bed.

Table XI. Crushing Strength of Bricks and Stone

Pressure at right angles to bed

Kind of brick or stone	Crushing strength, lb per sq in
Bricks:	
Common, Massachusetts	10 000
Common, St. Louis, Mo	6 417
Common, Washington, D. C.	7 370
Paving, Illinois	6 000 to 13 000
Granites:	
Blue, Fox Island, Me	14 875
Gray, Vinal Haven, Me.	13 000 to 18 000
Westerly, R I	15 000
Rockport and Quincy, Mass	17 750
Milford, Conn	22 600
Staten Island, N Y.	22 250
East St Cloud, Minn	28 000
Gunnison, Col	13 000
Red, Platte Cañon, Col	14 600
Limestones:	
Glens Falls, N. Y.	11 475
Joliet, Ill.	12 775
Bedford, Ind	6 000 to 10 000
Salem, Ind.	8 625
Red Wing, Minn.	23 000
Stillwater, Minn.	10 750
Sandstones:	
Dorchester, N. B. (brown)	9 150
Mary's Point, N B. (fine grain, dark brown)	7 700
Connecticut brown stone* (on bed)	7 000 to 13 000
Longmeadow, Mass. (reddish brown)	7 000 to 14 000
Longmeadow, Mass (average, for good quality)	12 000
Little Falls, N. Y.	9 850
Medina, N. Y	17 000
Potsdam, N. Y. (red)	18 000 to 42 000
Cleveland, Ohio	6 800
North Amherst, Ohio	6 212
Berea, Ohio	8 000 to 10 000
Hummelstown, Pa.	12 810
Fond du Lac, Minn.	8 750
Fond du Lac, Wis.	6 237
Manitou, Col. (light red)	6 000 to 11 000
St. Vrain, Col. (hard laminated)	11 505
Marbles:	
Lee, Mass.	22 900
Rutland, Vt.	10 746
Montgomery Co., Pa.	10 000
Colton, Cal.	17 783
Italy	12 156
Flagging:	
North River, N. Y.	13 425

* This stone should not be set on edge.

(6) Additional Data on the Strength of Building Stones

Average Data for Building Stones of Good Quality. The following average relative values * are given by R. P. Miller.† SANDSTONE: weight, 150 lb per cu ft; specific gravity, 2.40; crushing strength, 8 000 lb per sq in; shearing strength, 1 500 lb per sq in; modulus of rupture, 1 200 lb per sq in; modulus of elasticity, 3 000 000 lb per sq in. GRANITE: weight, 170; specific gravity, 2.72; crushing strength, 15 000; shearing strength, 2 000; modulus of rupture, 1 500; modulus of elasticity, 7 000 000. LIMESTOME: weight, 170; specific gravity, 2.72; crushing strength, 6 000; shearing strength, 1 000; modulus of rupture, 1 200; modulus of elasticity, 7 000 000. MARBLE: weight, 170; specific gravity, 2 72; crushing strength, 10 000; shearing strength, 1 400; modulus of rupture, 1 400; modulus of elasticity, 8 000 000. SLATE: weight, 175; specific gravity, 2.80; crushing strength, 15 000; modulus of rupture, 8 500; modulus of elasticity, 14 000 000. TRAP-ROCK: weight, 185; specific gravity, 2 96; crushing strength, 20 000.

The following average relative values are given by A. I. Frye.‡ They are the results of tests made on small cubes of the materials. SANDSTONE: crushing strength, 9 000 lb per sq in; GRANITE and GNEISS: crushing strength, 17 733 lb per sq in. LIMESTONES and MARBLES: crushing strength, 14 445 lb per sq in. SLATE: crushing strength, 10 000; ultimate tensional strength, 3 000; modulus of rupture, 5 000 lb per sq in.

When stones are not tested, Frye recommends the following average values for ultimate strengths to be used in determining the safe stresses. SANDSTONE: crushing strength, 5 000; ultimate tensional strength, 150; modulus of rupture, 1 200 lb per sq in. GRANITE and GNEISS: crushing strength, 12 000; modulus of rupture, 1 600 lb per sq in. LIMESTONES and MARBLES: crushing strength, 8 000; ultimate tensional strength, 800; modulus of rupture, 1 500 lb per sq in.

The following working unit stresses in pounds per square inch for stone slabs or single blocks of stone are recommended by W. J. Douglass.§ SANDSTONE: compression, 700; tension (direct and flexural), 75; shear, 150. GRANITE, SYENITE and GNEISS: compression for hard, 1 500; for medium, 1 200; for soft, 1 000; tension (direct and flexural), 150; shear, 200. LIMESTONE: compression for hard, 1 000; for medium, 800; for soft, 700; tension (direct and flexural), 125; shear, 150. MARBLE: compression for hard, 900; for soft, 700; tension (direct and flexural), 125; shear, 150. BLUESTONE FLAGGING: compression, 1 500; tension (direct and flexural), 200.

(7) Comparison of Tests on Building Stones

From the values derived as a result of the above tests it would appear that the average crushing strength of any class of stone might be misleading because of the wide range in the crushing strength of different varieties of the same basic stone, some marbles, for example being twice as strong in compression as some granites. If an exact working stress be desired it should be derived from tests on samples from the actual quarry furnishing the stone

* The values in all cases are as follows: weight, in lb per cu ft; strength modulus of rupture and modulus of elasticity, in lb. per sq. in.

† American Civil Engineers' Pocket Book (1912), page 357.

‡ Civil Engineers' Pocket-Book (1913), page 511.

§ American Civil Engineers' Pocket Book (1912), page 575.

which it is proposed to employ. The results of many of such tests are given in the preceding paragraphs and tables.

4. Compressive Strength of Mortars and Concretes

The Compressive Strength of Lime Mortar. The crushing strength of common lime mortar, six months old and composed of 1 part lime to 6 parts sand by measure, varies from 85 to 300 lb per sq in or from 6 to 21.6 tons per sq ft. Lime mortar alone should never be used where any but moderate loads are to bear upon the work, nor where the full loading is to be applied before the mortar has had time to harden.

The Compressive Strength of Natural-Cement Mortar. The crushing strength * of natural-cement mortar, neat, averaged, for 7 days, 2 010; for 28 days, 2 689; for 3 months, 3 646; and for 6 months, 5 052 lb per sq in. When mixed with 2 parts of standard quartz sand, the mortar averaged in crushing strength, for 7 days, 940; for 28 days, 1 390; for 3 months, 1 730; and for 6 months, 2 012 lb per sq in. For 2 years, an additional increase of 18% and 6% may be assumed for the neat and sanded mortars, respectively, of natural cement.

The Compressive Strength of Portland-Cement Mortar. The crushing strength * of Portland-cement mortar, neat, averaged, for 7 days, 5 915; for 28 days, 7 041; for 3 months, 7 347; and for 6 months, 9 760 lb per sq in. When mixed with 3 parts of standard quartz sand, the mortar averaged, in crushing strength, for 7 days, 941; for 28 days, 1 290; for 3 months, 1 490; and for 6 months, 1 529 lb per sq in. When mixed with 3 parts of Ottawa sand, the mortar averaged, in crushing strength, for 7 days, 1 199; for 28 days, 1 796; for 3 months, 1 887; and for 6 months, 2 181 lb per sq in. For 2 years, an additional increase of about 16% and 18% may be assumed for the neat and sanded mortars, respectively, of Portland cement.

Relation of Compressive to Tensile Strength of Mortars. While it may be stated as a very general guide that the compressive strength of hydraulic cement mortars is from six to ten times the tensile strength, these ratios are variable and cannot be used as a reliable basis for calculations. The tensile strength of Portland-cement mortars, under normal conditions, increases rapidly during the first few days, the rate of change gradually falling off. In 7 days the tensile strength is generally from one-half to two-thirds of the ultimate strength, which is practically reached in 2 or 3 months. The compressive strength, however, continues to increase with age and the rate of increase varies according to a somewhat different law.

Form of Specimen for Compression-Tests. For compression-tests of concrete in general, 4 to 12-in cubes of the mixture have been the standard forms of test-specimens; but since the advent of reinforced-concrete construction and the growth of the importance of determining the elastic properties of concrete, it has been found that a cylindrical test-specimen gives more definite results than a cube. A common shape of such cylinder is one in which the height is about twice the diameter, and the cylinders are not less than 6 by 12 in for aggregate less than 2 in and 8 by 16 in for aggregate larger than 2 in. It is found that the compressive strengths of these cylinders of concrete are from 10 to 15% less than those of the cubes, but for cylinders of still greater slenderness the compressive strengths remain about constant for heights up to about seven diameters.

* From compression-tests made by W. P. Taylor on cylindrical specimens 1 in in height, about 1⅛ in in diameter and 1 sq in in cross-section.

Crushing Strength of Concrete Affected by Area of Bearing Surface. Professor Hool states that if a load be applied over the central part, only, of the bearing surface of a concrete test-specimen in compression, the unit load will be greater than if it be applied over the entire surface; and this is due to the fact that the outer parts tend to assist the inner part to resist the stress. This was shown by tests made on some of the 12-in concrete cubes used in the tests made for the Boston Elevated Railway Company. Thirty-six of these concrete cubes were crushed by applying the load over the entire upper bearing-surface of 144 sq in and an equal number of similar concrete cubes were then crushed by applying the stress over a smaller area, 10 by 10 in, or 100 sq in. After this, the cubes of a third set were crushed by the application of the stress over the still smaller area, 8 by 8¼ in, or 66 sq in. The tests of the second set gave unit crushing strengths 12% higher than the first, and those of the third set unit crushing strengths 28% higher than the first.

Working Stress for Bearing on Concrete. "When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 32.5% of the compressive strength may be allowed."

The 1929 Philadelphia code allows 37½% of ultimate strength for direct compression when surface is 3 or more times the loaded area. The Boston code of 1926 allows 35% of the ultimate strength when the surface is twice the loaded area.

Effect of Consistency on the Crushing Strength of Concrete. A series of tests * made by the United States Geological Survey show very clearly the relation of the strength of concrete to the consistency or amount of water used in mixing. At the age of 1 month the mean compressive strengths in pounds per square inch were, for the wet concrete: granite, 3 155; gravel, 2 300; limestone, 4 195. For the medium concrete: granite, 4 090; gravel, 3 545; limestone, 2 975. For the damp concrete: granite, 4 520; gravel, 4 610; limestone, 4 365. At the end of 3 months the values for the granite aggregates were, for the wet concrete, 4 755; for the medium, 4 990; and for the damp, 5 445.

Comparison of Compressive Strengths of Gravel and Stone Concretes. Concretes made with broken stone have, generally, a somewhat greater compressive strength than those made with gravel. From tests made by E. Candlot, the average compressive strength at 30 and 180 days, of concrete made with 1½-in maximum-size broken stone, was 20% greater than that of concrete made of gravel of about the same size, the percentage of voids being nearly the same, 40% voids for the gravel and 47.4% voids for the broken stone. The average difference at 12 months, however, was reduced to 9%.

Compressive Strength of Concrete. As has been set forth in Chapter III the most important element in the strength of concrete of workable consistency is the ratio of the water to the cement, the strength diminishing rapidly as the water is increased. Closely related also to the water-cement ratio in determining strength are the considerations of age of concrete and curing. Tests made by the Portland Cement Association show that for the same water-cement ratio and mix, the sizes of the aggregates and their grading have much less effect upon the compressive strength of the concrete than has been supposed. The real significance of grading of the aggregate is in relation to the workability and economy of concrete mixtures rather than to the

* Made in 1908. See Bulletin No. 344, United States Geological Survey.

compressive strength. As regards the kind of aggregate, naturally, materials which are structurally weak cannot produce the same strength as sound materials, but the usual run of sand, stone and gravel as found in most localities is at least as strong as the paste and the compressive strength is little affected by the type of aggregate. Such small increases in strength of concrete as do occur for the harder materials are probably due to angularity and roughness of surface rather than to greater strength of aggregate, since the roughness increases the bond between paste and aggregate. As has been stated, the allowable working compressive stress for cinder concrete is limited to 300 lb per sq in in the majority of Building Codes, while the allowable stress for stone concrete in direct compression is about 500 lb per sq in and 650 lb in flexure but without reference to the strength of the aggregate so long as it is sound material.

Effect of Curing. Moisture is required to produce the chemical actions of hardening cement and since these actions continue for considerable periods it is necessary to protect the concrete for several days from drying conditions and to keep it moistened to avoid evaporation. Test cylinders of concrete surrounded by damp sand for 120 days have shown from 2½ to 3 times the strength of similar cylinders exposed to room atmosphere for the same period. Concrete should therefore be protected from evaporation and dampened as required for periods of from five to ten days to attain its full strength even at later periods.

Effect of Age. It is well established that concrete increases in strength with age under the right conditions. Age and curing cannot be separated, an increase in age merely providing for further chemical combinations if the conditions be favorable for continued reaction. Tests by the Portland Cement Association show, for a ratio of 7 gal of water to one sack of cement, the following compressive strengths per square inch: 3 days, 850 lb; 7 days, 1 600 lb; 28 days, 2 800 lb; 3 months, 4 200 lb; 1 year, 5 000 lb. These strengths were found with concretes of different mixes, varying from 1 : 2 to 1 : 8, but with the constant water-cement ratio of 7 gal of water to 1 sack of cement.

Allowable Working Stresses. The Building Codes of many cities still continue to consider 2 000 lb, per sq in as the ultimate compressive strength of stone concrete and, basing their regulations upon this figure, limit the allowable working stresses as follows in the majority of cases.

	Per Sq in
Direct compression	500 lb
Extreme fiber stress in compression	650 lb
Shearing stress without reinforcement	40 lb

Tests show, however, that plastic concrete of workable consistency can be designed with ultimate strengths ranging from 1 000 lb to 3 000 lb per sq in at 28 days depending upon the ratio of water to cement in the paste. Where the water-cement ratio method of mixing concrete has been adopted the allowable working stresses are expressed as percentages of the ultimate strength.

The following assumed strengths at 28 days of concrete mixed according to the water-cement ratio were adopted in their Joint Code in 1928 by the American Concrete Institute and the Reinforcing Steel Institute and have been followed with slight amendments by several municipalities in revising their Building Codes.

Table XII. Strength of Concrete According to Water Cement Ratio

Water-cement ratio U. S. gallons per 94-lb sack of cement	Approximate mix, volume of Portland cement to sum of separate volumes of fine and coarse aggregate	Assumed compressive strength at 28 days in lb per sq in
Dry Plastic Concrete		
8¼	1 : 7	1 500
7½	1 : 6	2 000
6¾	1 : 5¼	2 500
6	1 : 4½	3 000
Moderately Wet Concrete		
8¼	1 : 6½	1 500
7½	1 : 5½	2 000
6¾	1 : 4¾	2 500
6	1 : 4	3 000

The following unit stresses in lb per sq in were adopted for use in design, f'_c equaling the minimum ultimate strength at 28 days.

Description	Allowable Unit Stresses			
	For any strength of concrete fixed by test	When strength of concrete is fixed by water-cement ratio		
		$f'_c =$ 2 000 lb per sq in	$f'_c =$ 2 500 lb per sq in	$f'_c =$ 3 000 lb per sq in
Flexure: f_c Extreme fiber stress in comp. (f_c)	$0.40 f'_c$	800	1 000	1 200
Extreme fiber stress adjacent to supports of continuous beams (f_c)	$0.45 f'_c$	900	1 125	1 350
Shear (v) Beams without web-reinforcement (v_c)	$0.02 f'_c$	40	50	60
Bearing (f_c) Direct compression	$0.25 f'_c$	500	625	750

Building Laws for Working Loads on Masonry. As previously mentioned the building codes of most cities specify working loads to be used for masonry and as shown in Table II these loads vary greatly. It is important, therefore, that the architect should be acquainted with the municipal codes by which the construction of his building is governed. As building laws and regulations are constantly changing this information should be obtained from the code itself, care being taken that the latest edition and all supplements are consulted. A few requirements, peculiar to the codes in which they are found, will be cited.

The Chicago code (1928) gives for eight classes of brickwork bearing values ranging from 100 lb per sq in for common brick with good lime mortar to 350 lb per sq in for paving brick with 1 to 3 Portland-cement mortar. This code discriminates between concrete mixed by hand and by machine, values of from 250 to 350 lb per sq in being given for hand-mixed concrete and from 300 to 400 lb per sq in for the same mixture if mixed by machine. The values in the Buffalo code are exceptionally low, common brick laid in lime mortar being allowed but 3 tons and concrete in foundations but 4 tons per sq ft. The Louisville code introduces values for "Louisville-cement mortar." The practice of stating values of local material is to be commended. The Denver code gives a value of 6 tons per sq ft on common brick in lime mortar, 5½ tons for hollow tile in cement mortar and 8 tons for terra-cotta, solid, in cement. The Seattle code gives for the allowable compressive stress of mass concrete 20% of its compressive strength in twenty-eight days. The building code recommended by the National Board of Fire Underwriters is followed by a number of cities. This code includes in its list of allowable compression values, 1 000 lb per sq in for Portland-cement grout, neat, between steel members in foundations. For natural-cement concrete values are given of 150 lb per sq in for a 1 : 2 : 4 mixture and 80 lb per sq in for a 1 : 2 : 5 mixture. The average ultimate compressive strength for terra-cotta blocks designed to be laid normally with the cells vertical, and which are tested with the cells in that position, must not be less than 1 200 lb per sq in. The allowable working stress on such blocks must not exceed 180 lb per sq in.

CHAPTER VI

FORCES AND MOMENTS

By

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1. Composition and Resolution of Forces

Composition and Resolution of Forces. Imagine a round ball placed on a plane, frictionless surface at A (Fig. 1), the surface being perfectly level, so that the ball has no tendency to move until some force is applied to it. If, now, the force, P, is applied to the ball in the direction indicated by the arrow, the ball will move in that direction. If, instead of one force only, two forces, P and P_1, are applied to the ball, it will not move in the direction of either of the forces, but will move in the direction of the RESULTANT of these forces, or in the direction Ab. If the magnitudes of the forces P and P_1 are indicated by the lengths of the lines, then, if we complete the parallelogram $ABDC$, the diagonal DA represents the direction and magnitude of a single force which has the same effect on the ball as that resulting from the two forces P and P_1. If, in addition to the two forces P and P_1, the third force, P_2, is applied, the ball will move in the direction of the resultant of all three forces, and this resultant is obtained by completing the parallelogram $ADEF$, of which the resultant DA and the third force P_2 are two adjacent sides. The diagonal R of this second parallelogram is the resultant of all three forces, and the ball will move in the direction Ae. In the same way the resultant of any number of forces may be found. Again, suppose a ball, whose weight is indicated by the length of the line W (Fig. 2), is suspended by two inclined cords. What are the magnitudes of the pulls or stresses which are developed in the cords and which keep the ball suspended at the point A? This is the converse of the last case. Instead of finding the diagonal or the resultant, the diagonal, which is the line W, is given, and the sides of the parallelogram are to be found. To find these the lines representing the directions of P and P_1 are prolonged and from B lines parallel to them are drawn to complete the parallelogram. Then CA is the required magnitude of the stress in cord P, and BC of that in cord P_1. Thus one force may have the same effect as many, or many the same effect as one.

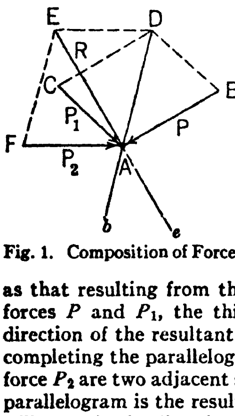

Fig. 1. Composition of Forces

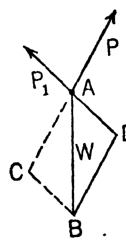

Fig. 2. Resolution of Forces

Forces Represented by Straight Lines. In considering the action of forces, it is convenient to represent them graphically by straight lines with arrowheads, as in Fig. 3. The length of the line, if drawn to a scale of pounds, represents the MAGNITUDE OF THE FORCE in pounds; the position of the line indicates

its LINE OF ACTION; the arrow-head indicates its SENSE or the direction in which it acts; and the point A its POINT OF APPLICATION. Thus the magnitude, direction and point of application are indicated and the force is completely represented.

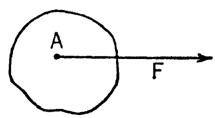

Fig 3. Force Represented by a Straight Line

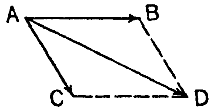

Fig. 4. Parallelogram of Forces

Parallelogram of Forces. If two forces applied at one point are represented in magnitude and direction by two straight lines inclined to each other, their resultant is the diagonal of the PARALLELOGRAM formed on those lines. Thus, if the lines AB and AC (Fig. 4) represent two forces acting at a point A, to find the force which will have the same effect as the two forces, the parallelogram $ABDC$ is completed and the diagonal AD drawn. This line represents the RESULTANT of the two forces. When the two given forces act at right-angles to each other, the magnitude of the resultant is equal to the square root of the sum of the squares of the magnitudes of the other two forces.

Triangle of Forces. If three forces acting at a point are represented in magnitude and direction by the sides of a TRIANGLE taken in order, they are in equilibrium. Let P, Q and R (Fig. 5) represent three forces acting at the point O. If a triangle can be drawn, like that shown at the right in Fig. 5, having sides respectively parallel to the directions of the forces and taken in the same order, the forces are in equilibrium. If such a triangle cannot be drawn, the forces are not in equilibrium.

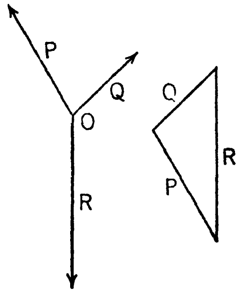

Fig 5. Triangle of Forces

The Polygon of Forces. If any number of forces acting at a point can be represented in magnitude and direction by the sides of a POLYGON taken in order, they are in equilibrium. This follows directly from the preceding theorem.

2. Moments of Forces

Moments. In considering the stability of structures and the strength of materials, we are often obliged to take into consideration the moments of the forces acting on a structure or on some part of a structure; and a knowledge of the general PRINCIPLES OF MOMENTS is essential to the proper understanding of these subjects. When we speak of the MOMENT OF A FORCE, we must have in mind some fixed point or line with respect to which the moment is taken. The moment of a force with respect to any given point, or CENTER OF MOMENTS, is the product of the magnitude of the force and the perpendicular distance from the point to the LINE OF ACTION of the force; or, in other words, the moment of a force is the product of the magnitude of the force by the ARM with which it acts. Thus if we have the force F (Fig. 6), and wish to determine its moment with respect to the point P, we determine the perpendicular distance Pa, between the point and the line of action of the force, and multiply it by the magnitude of the force. For example, if the magnitude

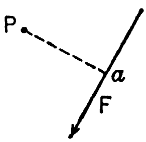

Fig. 6. Moment of a Force

of the force F is 500 lb and the distance Pa is 2 in, the moment of the force with respect to the point P is 500 lb $\times$ 2 in = 1 000 in-lb.*

Parallel Forces. If any body is in a state of rest or equilibrium under the action of parallel forces, the sum of the forces acting in one direction equals the sum of the forces acting in the opposite direction. Thus if we have the parallel forces P^1, P^2, P^3 and P^4 acting on the rod AB (Fig. 7), in a direction opposite to that of the forces P_1, P_2 and P_3, then, if the rod is in equilibrium, the sum of the forces P^1, P^2, P^3 and P^4 must equal the sum of the forces P_1, P_2 and P_3.

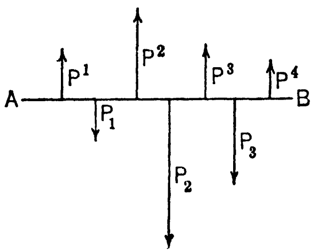

Fig. 7. Algebraic Sum of Unlike Parallel Forces

Parallel Forces Opposite in Character. If any number of parallel forces, not all acting in the same direction, act on a body, if the body is in equilibrium, the sum of the moments of the forces tending to turn the body in one direction about any given point is equal to the sum of the moments of the forces tending to turn it in the opposite direction. Let F_1, F_2 and F_3 (Fig. 8) represent three parallel forces acting on the rod AB. If the rod is in equilibrium, the sum of the forces F_2 and F_3 is equal to F_1. Also, if we take the end of the rod, A, for the center of moments, the moment of F_1 is equal to the sum of the moments of F_2 and F_3 about that point, because the moment of F_1 measures the tendency to turn the rod CLOCKWISE, and the sum of the moments of F_2 and F_3 measure the tendency to turn the rod CONTRA-CLOCKWISE, and there is no more tendency to turn the rod one way than the other. For example, let the magnitude of forces F_2, F_3 each be represented by 5 force-units, the distance Aa by 2 length-units and the distance AB by 4 length-units. The magnitude of the force F_1 must equal the sum of the magnitudes of the forces F_2 and F_3, or 10 force-units, and its moment with respect to any point in the plane of the forces must equal the sum of the moments of F_2 and F_3 with respect to the same point. If we take A as the center of moments, the moment of $F_3 = 5 \times 2 = 10$, and of $F_2 = 5 \times 4 = 20$. Their sum equals 30; hence the moment of F_1 must be 30. Dividing the moment 30 by the force $F_1 = 10$ force-units, we have for the arm, 3 length-units; or the force F_1 must act at a distance of 3 units from A to keep the rod in equilibrium. If we take b as the center of moments, the force F_1 has no moment, as the length of its lever-arm is zero; and, for equilibrium, the moment of F_2 about b must equal the moment of F_3 about the same point; or, as in this case the magnitudes of the forces F_2 and F_3 are equal, they must both be applied at the same distance from b, showing that b must be half-way between a and B, as was demonstrated before.

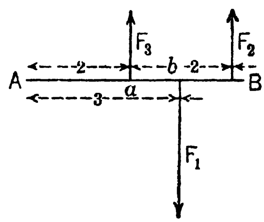

Fig 8. Algebraic Sum of Moments of Unlike Parallel Forces

Three Parallel Forces. THE PRINCIPLE OF THE LEVER. This principle is based upon the two preceding propositions and is of great importance and

* The expressions POUND-FEET and POUND-INCHES are often given to these products to distinguish them from FOOT-POUND and INCH-POUNDS, by which WORK and ENERGY are measured.

convenience. If a body is in equilibrium under the action of three parallel forces acting in the same plane, each force is proportional to the normal distance between the other two. Thus, if, as in Figs. 9, 10 and 11, three forces,

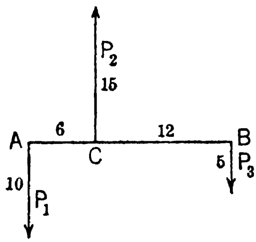

Fig 9. Principle of the Lever

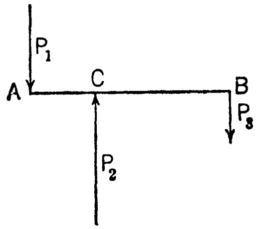

Fig. 10 Principle of the Lever

P_1, P_2 and P_3, act on the rod AB, in order that it may be in equilibrium, the following relations must obtain between the magnitudes of the forces and the distances between their points of application;

$$\frac{P_1}{CB} : \frac{P_2}{AB} : \frac{P_3}{AC}$$

or

$$P_1 : P_2 : P_3 :: CB : AB : AC$$

This is the case of the COMMON LEVER and shows the method of determining what weight a given lever will raise. The proportion is also true for any arrangement of the forces (as shown in Figs 9, 10 and 11), provided, of course, the forces are lettered in the order shown in the figures.

For example, let the distance AC be 6 in and the distance CB be 12 in. If a weight of 500 lb is applied at the point B, how much will it raise at the other end and what support will be required at C (Fig. 10)?

Applying the rule just given, we have the proportion:

$$P_3 : P_1 :: AC : CB \quad \text{or} \quad 500 : P_1 :: 6 : 12$$

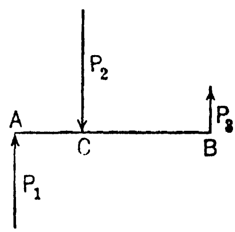

Fig. 11. Principle of the Lever

Hence $P_1 = 1\,000$ lb; or 500 lb applied at B will lift 1 000 lb resting on or suspended at A. The supporting force at C must, by the principles of PARALLEL FORCES IN EQUILIBRIUM, be equal to the sum of the forces P_1 and P_3, or 1 500 lb in this case.

3. Center of Gravity

General Principles. The LINES OF ACTION of the force of gravity converge towards the center of the earth but the distance of the center of the earth from the bodies which we have occasion to consider, compared with the size of those bodies, is so great, that we may consider the lines of action of the forces as parallel. The number of the forces of gravity acting upon a body may be considered as equal to the number of particles composing the body. The CENTER OF GRAVITY of a body may be defined as the point through which the resultant

of the parallel forces of gravity, acting upon the body, passes for every position of the body. If a body is supported at its center of gravity and turned about that point, it will remain in equilibrium in all positions. The resultant of the parallel forces of gravity acting upon a body is obviously equal to the WEIGHT OF THE BODY; and if a force, equal in magnitude to the resultant, is applied, acting in a line passing through the center of gravity of the body, and in a direction opposite to that of the resultant, the body will be in equilibrium.

Center of Gravity of a Straight Line. The word LINE here means a material line whose transverse section is very small, such as a very fine wire. The center of gravity of a straight line or rod of uniform size and material is at its middle point. This proposition is too evident to require demonstration.

The Center of Gravity of the Perimeter of a Triangle is at the center of the circle inscribed in the triangle formed by the lines joining the middle points of the sides of the given triangle. Thus, let ABC (Fig. 12) be the given triangle. To find the center of gravity of its perimeter, find the middle points, D, E and F, and connect them by straight lines. The center of the circle inscribed in the triangle formed by these lines will be the center of gravity sought.

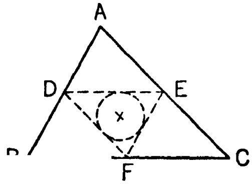

Fig. 12. Center of Gravity of Perimeter of Triangle

Center of Gravity of Symmetrical Lines. The center of gravity of a line which is symmetrical with reference to a point is at that point. Thus the center of gravity of the circumference of a circle or of an ellipse is at the geometrical center of the figures. The center of gravity of the perimeter of an equilateral triangle, or of a regular polygon, is at the center of the inscribed circle. The center of gravity of the perimeter of a square, rectangle, or parallelogram is at the intersection of the diagonals of those figures.

Center of Gravity of a Surface. A SURFACE here means a very thin plate or shell. If a surface can be divided by a line into two symmetrical halves, the center of gravity will be on that line; if it can thus be divided by two lines, the center of gravity will be at their intersection.

Center of Gravity of Regular Figures. The center of gravity of the surface of a circle or an ellipse is at the geometrical center of the figure; of an equilateral triangle or regular polygon, at the center of the inscribed circle; of a parallelogram, at the intersection of the diagonals; of the surface of a sphere, or of an ellipsoid of revolution, at the geometrical center of the body; and of the convex surface of a right cylinder, at the middle point of the axis of the cylinder.

Center of Gravity of Irregular Figures. Any figure bounded by straight lines may be divided into rectangles and triangles, and, the center of gravity of each part being found, the center of gravity of the whole figure may be determined by treating the centers of gravity of the separate parts as particles whose weights are proportional to the areas of the parts they represent. See Figs. 21 and 22.

Center of Gravity of Triangles. To find the center of gravity of a triangle, draw a line from each of two angles to the middle of the opposite side. The intersection of the two lines is the center of gravity.

Center of Gravity of Quadrilaterals. To find the center of gravity of any quadrilateral, draw the diagonals, and from that end of each diagonal which is farthest from the intersection, lay off, toward the intersection, the length of

its shorter segment. The two points thus formed, together with the point of intersection, will form a triangle whose center of gravity is that of the quadrilateral. Thus, let Fig. 13 be a quadrilateral whose center of gravity is to be found. Draw the diagonals AD and BC, and from A lay off $AF = DE$, and from B lay off $BH = CE$. From E draw a line to the middle of FH, and from F a line to the middle of EH. The point of intersection of these two lines is the center of gravity of the quadrilateral. This is a method commonly used for finding the centers of gravity of the voussoirs of an arch.

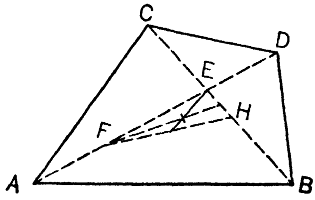

Fig. 13. Center of Gravity of Quadrilateral

Table of Centers of Gravity. Let a be a line drawn from the vertex of a figure to the middle point of the base, and D the distance from the vertex to the center of gravity of the figure. Then (Fig. 14):

In an isosceles triangle........	$D = \frac{2}{3}a$
In a segment of a circle, vertex at center of circle	$D = \frac{\text{chord}^3}{12 \times \text{area}}$
In a sector of a circle, vertex at center of circle.	$D = R \times \frac{2 \times \text{chord}}{3 \times \text{arc}}$
In a semicircle, vertex at center of circle......	$D = \frac{4R}{3\pi} = 0.4244R$
In a quadrant of a circle..............	$D = \frac{3}{5}R$
In a semiellipse, vertex at center of circle.....	$D = 0.4244a$
In a parabola, vertex at intersection of axis with curve....	$D = \frac{3}{5}a$
In a cone or pyramid.....................	$D = \frac{3}{4}a$

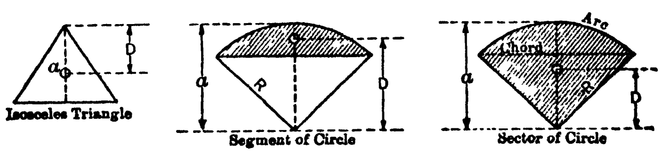

Fig. 14. Center of Gravity of Triangle, Segment and Sector

In a frustum of a cone or pyramid, let h = the height of the complete cone or pyramid, h_1 = the height of the frustum, and let the vertex be at the apex of the complete cone or pyramid; then,

$$D = \frac{3\,(h^4 - h_1^4)}{4\,(h^3 - h_1^3)}$$

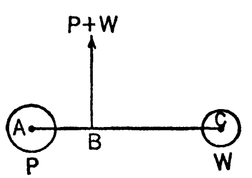

Fig. 15. Center of Gravity of Two Heavy Particles

Center of Gravity of Two Heavy Particles. Let P be the weight of a particle at A (Fig. 15), and W that of a particle at C. The center of gravity is at some point, B, on the line joining A and C. The point B must be so situated that if the two particles were held

together by a stiff wire and supported at B by a force equal in magnitude to the sum of P and W they would be in equilibrium. The problem then is solved by the PRINCIPLE OF THE LEVER, and we have the proportion (see Three Parallel Forces. The Principle of the Lever),

$$P + W : P :: AC : BC$$

or

$$BC = \frac{P \times AC}{P + W}$$

If $W = P$, then $BC = AB$, or the center of gravity will be half-way between the two particles. This problem is of great importance and has many practical applications

Center of Gravity of Several Heavy Particles. Let W_1, W_2, W_3, W_4 and W_5 (Fig. 16) be the weights of the particles. Join W_1 and W_2 by a straight line and find their center of gravity A, as in the preceding problem. Join A with W_3 and find the center of gravity B, which will be the center of gravity of the three weights W_1, W_2, W_3. Proceed in the same way with each weight. The last center of gravity found will be the center of gravity of all the particles In both of these cases the lines joining the particles are supposed to be horizontal lines, or else the horizontal projections of the straight lines which join the points.

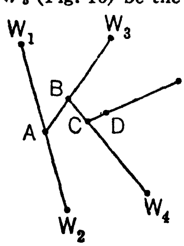

Fig. 16. Center of Gravity of Several Heavy Particles

Center of Gravity of Compound Sections Found by Moments. To determine the strength of a beam having an unsymmetrical cross-section, it is first necessary to determine the distance of the center of gravity of this section from the upper or lower surface of the beam Various other computations, also, involve finding the center of gravity of an irregular figure, so that the problem is one of practical importance. If the figure of which the center of gravity is to be found can be divided into parts which are themselves regular figures, the readiest and simplest method of finding the distance of the center of gravity from one edge of the section is by means of MOMENTS. To explain this method assume a T-shaped section of uniform thickness, hinged on a wire XX, as in Fig. 17. The T section is made up of two rectangles, one forming the flange, the other the web. The center of gravity of each rectangle is at its own center of figure and may be readily found. If the T section is placed horizontally, as in the figure, the axis XX being fixed, it will immediately, by the force of gravity, revolve about the axis until it becomes vertical, and the sum of the moments of the forces causing the revolution is $A' \times d' + A'' \times d''$, A' representing the weight of the web and A'' the weight of the flange. To hold the T section in a horizontal position, there must be a moment of some force acting in an opposite, or upward, vertical direction and just equal to the sum of the two moments causing revolution downwards. If the force A, of this moment, tending to cause revolution upward, is equal to the weight of the entire T sec-

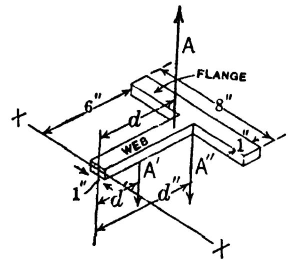

Fig. 17. Center of Gravity of Compound Sections by Moments

tion, it must be applied at the center of gravity of the entire figure to make its moment just equal to the sum of the moments of the two downward forces. But the moment of A is $A \times d$, therefore d is the distance from the end of the web, or from the axis XX, to the center of gravity of the entire figure. Therefore, since $A \times d = A' \times d' + A'' \times d''$,

$$d = \frac{A' \times d' + A'' \times d''}{A} \tag{1}$$

As the weight of any homogeneous material of uniform thickness is proportional to the area, A, A' and A'' may be used to represent areas as well as weights. Expressing formula (1) as a rule, we have:

Center of Gravity of Compound Figures. The distance of the center of gravity of a compound figure from any line of reference is equal to the sum of the products, obtained by multiplying the area of each of the simple parts into which the compound figure is divided by the distances of its center of gravity from the line of reference, divided by the area of the entire figure. This rule applies to any compound figure.

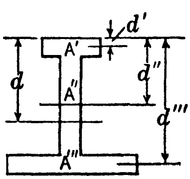

Fig. 18. Center of Gravity of Tees, Angles, Channels, etc.

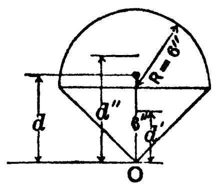

Fig. 19. Center of Gravity of Irregular I Section

Example I. Assume that the T section shown in Fig. 17 has the dimensions indicated. Then A' equals 6, A'' equals 8, and A equals 14 sq in; and d' equals 3 and d'' equals $6\frac{1}{2}$ in. The sum of the products of A' by d' and A'' by d'' is 18 + 52 or 70 sq in $\times$ in, and this divided by 14 sq in, the area of the entire figure, gives 5 in for the distance d. The distance d of the center of gravity from the top of the webs, in each of the figures shown in Fig. 20, is found by the following formula:

$$d = \frac{\text{area of the web or webs} \times d'/2 + \text{area of flange} \times d''}{\text{area of the web or webs} + \text{area of flange}} \tag{2}$$

For a section like that shown in Fig. 18, in which A', A'' and A''' represent the areas of the respective rectangles, the distance d of the center of gravity from the top may be found by the formula

$$d = \frac{A' \times d' + A'' \times d'' + A''' \times d'''}{A' + A'' + A'''} \tag{3}$$

Example II. To show the application of the rule for finding the center of gravity of compound figures, take the one shown in Fig. 19. The distance d

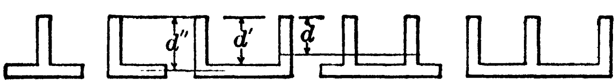

Fig. 20. Center of Gravity of Irregular Figures

of the center of gravity of the entire figure from the vertex O is found as follows: The area of the triangle is 36 sq in and of the semicircle 56.5 sq in. From the Table of Centers of Gravity the distance of the center of gravity of an isosceles triangle from the vertex is two-thirds its height, which gives 4 in as the

value for d'. The center of gravity for a semicircle is 0.4244 R from its base, so that d'' equals 8.54 in. Then,

$$d = \frac{36 \times 4 + 56.5 \times 8.54}{36 + 56.5} = 6.77 \text{ in}$$

This method of finding the center of gravity is similar to that explained in Chapter IX for finding the supporting forces or reactions. In the latter case, however, the problem is to find the balancing forces instead of the lever-arms.

Additional Methods of Determining Graphically the Center of Gravity of Irregular Plane Figures.* The center of gravity may be obtained graphically by means of the FORCE-POLYGON and the EQUILIBRIUM-POLYGON. The figure or section considered, Fig. 21, is divided into parts whose centers of gravity

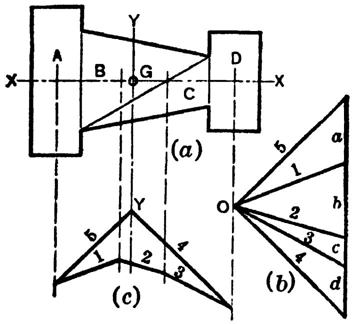

Fig. 21. Center of Gravity Determined Graphically

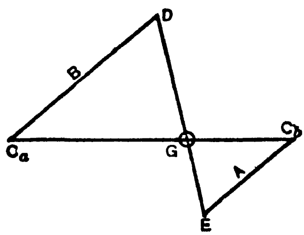

Fig. 22. Center of Gravity Determined Graphically. Second Method

can be located and areas calculated. The force-polygon (b) and equilibrium-polygon (c) are drawn. The figure (a) is divided into rectangles and triangles, A, B, C and D, and vertical lines are drawn through their centers of gravity. In (b) the vertical lines a, b, c and d are respectively proportional in length to the areas A, B, C and D. The pole, O, is located by the intersection of lines drawn at angles of 45° from the extremities of the line $abcd$. The rays 1, 2, 3, 4 and 5 are drawn from 0, as shown, and the corresponding parallel lines drawn in (c). (See Chapter X.) The vertical line through Y, produced upwards, is a GRAVITY-AXIS and its intersection G with the horizontal gravity-axis XX is the center of gravity of the figure. If the figure is not symmetrical about XX a second gravity-axis may be found by turning the figure through 90° and repeating the construction. The intersection of the two gravity-axes will be the center of gravity of the figure. ANOTHER METHOD is shown in Fig. 22. Let the centers of gravity of two areas A and and B be at the points C_a and C_b, respectively. From C_a the line C_aD is drawn in any direction, and its length represents on some given scale the area B. From C_b the line C_bE is drawn parallel to it and its length on the same scale represents the area A. The intersection of the line joining D and E with C_aC_b is at the center of gravity of the areas A and B. For three areas A, B and C, the construction can be repeated by considering A and B as a single area; and so on for any number of areas.

* Condensed from data by Robins Fleming.

CHAPTER VII

STABILITY OF PIERS AND BUTTRESSES*

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Mechanical Principles. A pier or buttress may be considered STABLE when the forces acting upon it do not cause it to ROTATE OR TIP OVER nor cause any course of masonry to SLIDE on its bed; some parts, however, of the masonry may be CRUSHED. When a pier sustains a vertical load only, it might be considered STABLE, but it might not have sufficient STRENGTH. It is only when the pier receives a THRUST, as from a rafter or an arch, that its stability must be considered. In order that there may be no rotation, the MOMENT OF THE THRUST (Chapter VI) against the pier about any point in its outside edge must not exceed the MOMENT OF THE WEIGHT of the pier about the same point.

To illustrate let us consider the pier shown in Fig. 1. Let us suppose that this pier receives the foot of a rafter which exerts a THRUST T in the direction AB. The tendency of this thrust is to cause the pier to rotate about the outer edge b_1, and the MOMENT OF THE THRUST about this point, which is the measure of this tendency to rotate, is $T \times a'b_1$, $a'b_1$ being the lever-arm of the moment. For UNSTABLE EQUILIBRIUM, only, the MOMENT OF THE WEIGHT of the pier about the same edge must just equal $T \times a'b_1$. The resultant force representing the weight of the pier acts vertically through its center of gravity which in this case is equidistant from its sides; and its lever-arm is b_1c, or one-half its thickness.

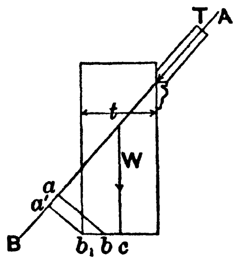

Fig. 1. Pier with Thrust

Hence, for equilibrium of moments, we must have the equation

$$T \times a'b_1 = W \times b_1c$$

But in this condition the least additional thrust, or the crushing of the outer edge, will cause the pier to rotate; hence, for safety, we must use some FACTOR OF SAFETY. This is sometimes done by making the moment of the weight equal to that of the thrust when referred to a point in the bottom of the pier, a certain distance in from the outer edge. This distance for piers or buttresses should not be less than one-fourth the thickness of the pier.

Representing this point in the figure by b, we have the necessary equation for the safe stability of the pier

$$T \times ab = W \times t/4$$

t being the width of the pier.

We cannot from this equation determine the dimensions of a pier to resist a given thrust, because we have the distance ab, t and W, all unknown quantities. Hence we must first assume a tentative size for the pier, find the length of the line ab, and see if the MOMENT OF THE WEIGHT of the pier is equal to the MOMENT OF THE THRUST. If it is not we must assume another size for the pier. In point of fact the steps of the problem usually present themselves in the

* See, also, Chapter XXIX, Section 2, Vaults.

inverse order, the pier or buttress being given and the determination of its stability being required. The size of the pier or buttress is usually first determined rather from the architectural exigencies of the design than from the engineering requirements for the stability of the structure. If upon investigation these are not in accord, it is the duty of the designers to use their ingenuity in seeing that both conditions are fulfilled.

The Stability of Piers and Buttresses. When it is desired to determine if a given pier or buttress is capable of resisting a given thrust, the problem can be solved GRAPHICALLY in the following manner. Let $ABCD$ (Fig. 2) represent a pier which sustains a given thrust T at B. To determine whether the pier will safely sustain this thrust, we proceed as follows:

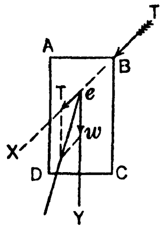

Fig. 2. Graphical Determination of Thrust on Pier

Draw the indefinite line BX in the direction of the thrust. Through the center of gravity of the pier, which in this case is midway between AD and BC, draw a vertical line intersecting the line of the thrust at e. As a force may be considered to act anywhere along its line of action, we may consider that the thrust and the weight act at the point e. The resultant of these two forces is obtained by laying off the thrust T from e on eX, and the weight of the pier W, from e on eY, both to the same scale of so many pounds to the inch, completing the parallelogram and drawing the diagonal. If this diagonal, prolonged, cuts the base of the pier at less than one-third the width of the base from the outer edge, the pier is generally considered unstable and its dimensions are changed. (See Chapter IV, Theorem of the Middle Third)

The Stability of Buttress with Offsets. THE STABILITY OF A PIER may be increased by adding to its weight by placing some heavy material on top, for example, or by increasing its width at the base by means of OFFSETS, as in Fig. 3. Figs. 3 and 4 show the method of determining the stability of a buttress with offsets. The first step is to find the vertical line passing through the center of gravity of the whole pier. This is best done by dividing the buttress into quadrilaterals, as $ABCD$, $DEFG$ and $GHIK$ (Fig. 3), finding the center of gravity of each quadrilateral by either the method of diagonals or triangles as explained in Chapter VI, and then measuring the perpendicular distances X_1, X_2, X_3 from the different centers of gravity to the line KI. (See, also, Chapter VIII.)

Multiply the area of each quadrilateral by the distance of its center of gravity from the line KI and add together the areas and the products. Divide the sum of the products by the sum of the areas and the result will be the distance of the center of gravity of the whole buttress from KI. This distance we denote by X_0. This calculation is a practical application of the theorem in mechanics that the MOMENT OF THE RESULTANT of any number of forces about a given point is equal to the SUM OF MOMENTS of the individual forces about that point.

Example 1. Let the buttress shown in Fig. 3 have the dimensions shown. Then the areas of the quadrilaterals and the distances from their centers of gravity to KI are as follows:

First area	= 35 sq ft	$X_1 = 0'.95$	First area $\times X_1 = 33.25$
Second area	= 23 sq ft	$X_2 = 2'.95$	Second area $\times X_2 = 67.85$
Third area	= 11 sq ft	$X_3 = 4'.95$	Third area $\times X_3 = 54.45$
Total area,	69 sq ft		Total moments, 155.55

The sum of the moments of the areas is 155.55, and dividing this by the total area, we have 2.25 as the distance X_0. Measuring this to the scale of the drawing from KI, we have a point through which the vertical line passing through the center of gravity must pass.

This line, passing through the center of gravity of the buttress, can be found GRAPHICALLY, also, by the method of the EQUILIBRIUM POLYGON (Fig. 3). In order to do this, lay off at any convenient scale beginning at some conven

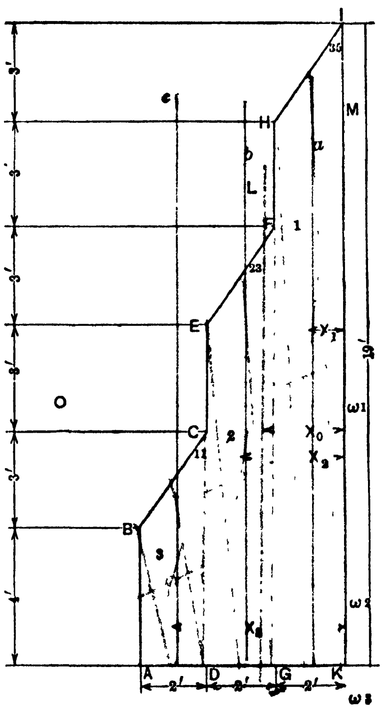

Fig. 3. Buttress with Offsets

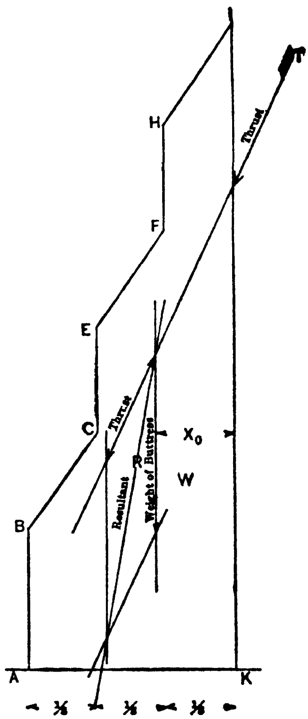

Fig. 4. Resultant Thrust on Buttress with Offsets

ient point M, Mw 1, w 1 w 2, and w 2 w 3, the areas of the various quadrilaterals composing the buttress. Through the center of gravity of each quadrilateral draw a vertical (green) line. Draw the lines MO and w 3 O, intersecting at some conveniently chosen POLE-POINT, O. Draw Ow 1 and Ow 2. Through a, where MO intersects the vertical (green) line drawn through the center of gravity of the first quadrilateral draw ab parallel to Ow 1, and through b, where ab intersects the (green) line through the center of gravity of the second quadrilateral, draw bc parallel to Ow 2. Finally draw cL parallel to Ow 3. Where this line intersects MO at L will be the point through which the (heavy red) line, passing through the center of gravity of the buttress taken as a whole, should be drawn. The distance X_0, meas-

ured from IK, should then be 2.25 ft or very nearly this, allowing for slight errors of drawing, and the same as that found by MOMENTS. Fig. 5 shows the same method of determining the position of the center of gravity of a buttress similar to the one illustrated in Fig. 9.

After this line is found, the method of determining the stability of the pier is the same as that given for the pier in Fig. 2 and Fig. 4. If the buttress is more than one foot thick, that is, at right-angles to the plane of the paper, its cubic contents must be determined in order to find its weight. It is easier, however, to divide the total thrust by the thickness of the buttress. This gives the thrust per foot of thickness of the buttress.

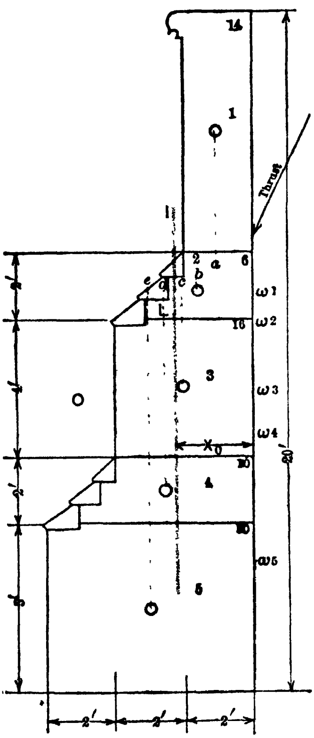

Fig. 5. Center of Gravity of Wall and Buttress

The Line of Pressure or Line of Resistance.* The LINE OF RESISTANCE or the LINE OF PRESSURE of a pier or buttress is a line drawn through the CENTER OF PRESSURE of each joint. The CENTER OF PRESSURE of any joint is the point in which the resultant of the forces acting on that portion of the pier above the joint cuts it. The line of pressure, or of resistance, when drawn in a pier, shows how near the greatest stress on any joint comes to the edges of that joint. It can be drawn by the following method.

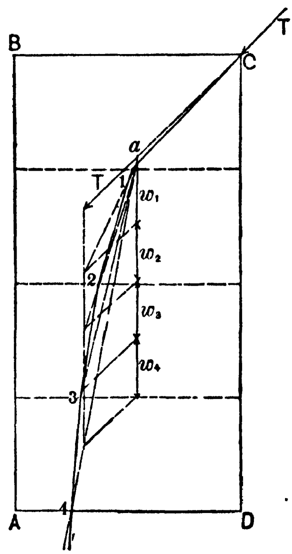

Fig. 6. Line of Pressure in Pier

Let $ABCD$ (Fig. 6) be a pier whose LINE OF PRESSURE we wish to draw. Let T be the thrust against the pier. First, divide the height of the pier into several parts, each 2 or 3 feet high, as shown by the horizontal dotted lines. It is more convenient to make the courses or

* This line is called, interchangeably, the line of pressure, the line of resistance, the resistance-line, etc.

parts equal in height. Prolong the LINE OF THE THRUST, and draw a vertical line through the center of gravity of the pier, intersecting the line of thrust at the point a. From a lay off to a scale the thrust T, and the weights of the different parts of the pier, commencing with the weight of the upper portion. Thus, w_1 represents the weight of the portion above the first joint; w_2 represents the weight of the second part; and so on. The sum of the w's will represent the weight of the whole pier.

Draw a parallelogram, with T and w_1 for its two sides. Draw the diagonal and produce it beyond the parallelogram, if necessary. Its point of intersection with the first joint will be a point in the line of pressure. Draw a second parallelogram, with T and $w_1 + w_2$ for its two sides. Draw the diagonal intersecting the second joint at the point 2. Continue in this way with the rest of the partial weights, the last diagonal intersecting the base AD, in the point 4. Join the points 1, 2, 3 and 4. The resulting broken line $C1234$ is the LINE OF PRESSURE or LINE OF RESISTANCE.

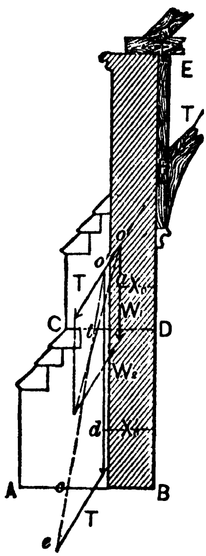

Fig. 7. Stability of Wall and Buttress

We have taken the simplest case as an example; but the same principles are true for any case. If the line of pressure of the pier at any point falls at a distance from the outside edge of the joint less than ONE-THIRD THE WIDTH OF THE JOINT, the pier is generally considered unsafe.

The Stability of a Wall and Buttress. By Moments and Graphical Method. The following example illustrates the application of these principles.

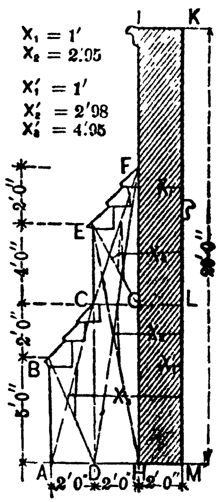

Fig. 8. Stability of Wall and Buttress

Example 2. Let Fig. 7 represent the section of a side wall of a church, with a buttress against it. Opposite the buttress, on the inside of the wall, is a hammer-beam truss, which we will suppose exerts an outward thrust on the wall of the church, amounting to about 9 600 lb. We will further consider that the resultant of the thrust acts at P, and at an angle of 60° with the horizontal. The dimensions of the wall and buttress are given in Fig. 8. The buttress is 2 ft thick, at right-angles with the plane of the paper. Has the buttress the proper SIZE and FORM to enable the wall to resist the thrust of the truss?

The first point to decide is whether or not the LINE OF PRESSURE cuts the joint CD at a safe distance in from C. To ascertain this we must determine the position of the center of gravity of the wall and buttress above the joint CD (Fig. 7). One way to determine this is by the METHOD OF MOMENTS, the MOMENTS OF THE AREAS being taken about the line KM as an axis, or line of reference (Fig. 8), as already explained. The distance X_1 is, of course, half the thickness of the wall or 1 ft. We next find the center of gravity of the part

CEFG (Fig. 8) by the method of diagonals; and scaling the distance X_2, we find it to be 2.95 ft.

The area $CEFG = A_2 = 10$ sq ft; and the area $GIKL = A_1 = 26$ sq ft. Let $A =$ the total area above CL.

Then we have

$$\begin{array}{lll} X_1 = 1 \text{ ft} & A_1 = 26 \text{ sq ft} & A_1 \times X_1 = 26 \text{ sq ft} \times \text{ft} \\ X_2 = 2.95 \text{ ft} & A_2 = 10 \text{ sq ft} & A_2 \times X_2 = 29.5 \text{ sq ft} \times \text{ft} \\ & A = 36 \text{ sq ft} & 36)55.5 \text{ sq ft} \times \text{ft} \\ & & X_0 = 1.5 \text{ ft} \end{array}$$

Expressed in EQUATIONS OF MOMENTS OF AREAS, this may be written as follows, A representing the total area above the line CL (Fig. 8):

$$A \times X_0 = (A_1 \times X_1) + (A_2 \times X_2)$$

Hence,

$$X_0 = \frac{(A_1 \times X_1) + (A_2 \times X_2)}{A}$$

The center of gravity is at a distance 1.5 ft from the line ED (Fig. 7). In Fig. 7 measure the distance $X_0 = 1.5$ ft, and through the point a draw a vertical line intersecting the line of the thrust prolonged at O. If the thrust is 9 600 lb, for example, for a buttress 2 ft thick, it will be half that, or 4 800 lb, for a buttress 1 ft thick. We will call the weight of the masonry of which the buttress and wall is built, 150 lb per cu ft. Then the thrust is equivalent to $4\,800/150 = 32$ cu ft of masonry. Laying this off to a scale from O, in the direction of the thrust, and the area of the masonry, 36 sq ft, from O on the vertical line, completing the rectangle, and drawing the diagonal, we find that the diagonal cuts the joint CD at t, within the limits of safety. We must next find where the LINE OF PRESSURE cuts the base AB.

First, determine the position of the center of gravity of the whole figure. This is determined by finding, as explained for the distances X_1 and X_2, the distances X_2', X_3', in Fig. 8, and making the following computation, letting $A' =$ the total area above AM.

$$\begin{array}{lll} X_1' = 1 \text{ ft} & A_1' = 40 \text{ sq ft} & A_1 \times X_1 = 40 \text{ sq ft} \times \text{ft} \\ X_2' = 2.98 \text{ ft} & A_2' = 24 \text{ sq ft} & A_2 \times X_2 = 71.52 \text{ sq ft} \times \text{ft} \\ X_3' = 4\ 95 \text{ ft} & A_3' = 12 \text{ sq ft} & A_3 \times X_3 = 59.40 \text{ sq ft} \times \text{ft} \\ & A' = 76 \text{ sq ft} & 76)170.92 \text{ sq ft} \times \text{ft} \\ & & X_0' = 2.25 \text{ ft} \end{array}$$

This, also, may be expressed in EQUATIONS OF MOMENTS OF AREAS, as explained for the part above the line CL.

Then from the line EB (Fig. 7) lay off the distance $X_0' = 2.25$ ft, and draw through d a vertical line intersecting the line of the thrust at O'. On this vertical line, measure down from O' the whole area 76, to scale, as explained above, and from the lower extremity of this line representing the area, lay off, at the proper angle, the thrust $T = 32$. Draw the line $O'e$, intersecting the base at c. This is the point where the LINE OF PRESSURE cuts the base; and, as it is at a safe distance in from A, the buttress has sufficient stability. If there were more offsets, we should proceed in the same way, finding where the LINE OF PRESSURE cuts the joint at the top of each offset. The reason for doing this is

because the LINE OF PRESSURE might cut the base at a safe distance from the outer edge, while higher up it might come outside of the buttress or too near the outside face, thus making the buttress unstable. The method given in these examples is applicable to piers of any shape or material. If the LINE OF PRESSURE makes an angle of less than 30° with any horizontal joint, the stones above the joint may SLIDE at this joint, or at least have a strong tendency to do so. SLIDING can be prevented either by doweling, or by inclining the joints. Such conditions, however, are rare in architectural construction.

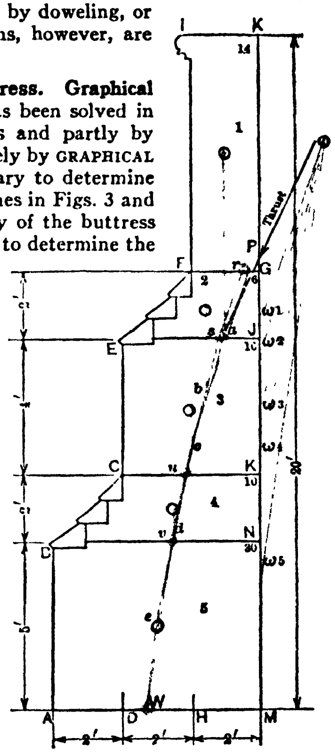

Fig. 9. Line of Pressure in Wall and Buttress

The Stability of a Wall and Buttress. Graphical Method. This same example, which has been solved in the foregoing case partly by MOMENTS and partly by GRAPHICAL METHODS, can be solved entirely by GRAPHICAL METHODS. In this case it is not necessary to determine the position of the line (the heavy red lines in Figs. 3 and 5) passing through the center of gravity of the buttress taken as a whole. It is necessary, only, to determine the (red) lines passing through the centers of gravity of the various trapezoids and rectangles into which it has been subdivided. To determine the position of the LINE OF PRESSURE and the various CENTERS OF PRESSURE on the different joints, the method shown in Fig 6 may be used. The construction shown in that figure, in which the complete parallelograms of the forces acting at each joint are drawn, may be simplified. One-half the parallelogram, only, or the TRIANGLE OF THE FORCES acting at each joint, may be drawn and the whole construction placed at one side of the figure and afterwards transferred to the figure itself by means of parallel lines.

Draw the joint-planes *FG*, *EJ*, *CK* and *BN* and calculate the areas of the various parts of the wall and buttress, such as *IKGF*, *FGJE*, *EJKC*, *CKNB* and *BNMA*, Fig. 9. These are respectively 14 sq ft, 6 sq ft, 16 sq ft, 10 sq ft and 30 sq ft. Lay off these areas to a scale of so many square units to a linear unit, at *Pw* 1, *w* 1 *w* 2, *w* 2 *w* 3, *w* 3 *w* 4 and *w* 4 *w* 5, along the line *KM*, beginning at the point of application of the THRUST. Lay off, at the same scale, the thrust *OP* for one foot of thickness of the wall, and let this thrust be 4 800 lb. Draw *Ow* 1, *Ow* 2, *Ow* 3, etc. Then *Ow* 1 will be the resultant of the thrust and the weight of the buttress above the joint *FG*, *OW* 2 will be the resultant of this last resultant and the weight of that part of the buttress between the joints *FG* and *EJ*, and so on until *Ow* 5 is reached, which is the resultant of the total weight of the buttress and the thrust as well as the resultant of the rectangle *BNMA* and the previous resultant. Prolong the thrust *OP*, until it cuts the first (red) line through the center of gravity of the first rectangle *IKGF*, at *a*. Through this point draw a (green) line parallel to *Ow* 1 and pro-

long it backward until it intersects the joint *FG* at the point within the small (red) circle. This determines the CENTER OF PRESSURE on this joint. Next, draw *ab* (green) parallel to *Ow* 2 and prolong it backward until it intersects the joint *EJ*, at the CENTER OF PRESSURE on that joint. Repeat this operation to obtain the CENTERS OF PRESSURE on each successive joint, drawing *bc*, *cd* and *de* parallel respectively to *Ow* 3, *Ow* 4 and *Ow* 5.

It must be remembered, however, that *cd* does not have to be prolonged backward, as it cuts the joint *CK* below and to the left of the line passing through the center of gravity of *EJKC*. Finally, join the various CENTERS OF PRESSURE by the (red) broken line, which is the LINE OF PRESSURE in the buttress. As this line lies within the MIDDLE THIRD of the construction, and the resultants of the pressures on the various joint-planes do not make with the normals to the joint-planes angles greater than the ANGLE OF FRICTION, the conditions for stability may be considered to be satisfied.

CHAPTER VIII

THE STABILITY OF MASONRY ARCHES *

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1. Arches

The Lintel and the Arch. When an opening is made in a masonry wall it is necessary to provide some means of spanning such opening to support the superimposed masonry. Two methods have been employed by constructors for this purpose. The first involves the use of the BEAM, GIRDER, CAP, or LINTEL, and the second the throwing of an ARCH from one side of the opening to the other. LINTELS are made of various materials, as wood, stone, reinforced concrete, cast iron and steel, and have cross-sections of different shapes. They are placed across the tops of the openings and transfer laterally the loads above, causing VERTICAL REACTIONS, only, in the side supports. An ARCH, on the contrary, is a particular arrangement of blocks of stone or other material, put together, generally along a curved line, in such a way that they resist the load by a balancing of certain THRUSTS and COUNTERTHRUSTS. An arch exerts on its supports an OUTWARD THRUST as well as a VERTICAL PRESSURE; and it is this outward thrust which requires that the arch should be used with caution when the abutments are not amply large and strong. The mechanical principles involved in the spanning of an opening by a lintel are much simpler than those of the arch and, historically, the lintel very considerably antedates the arch.

Definitions. Before taking up the principles of the arch, we will define the principal terms relating to it. The distance *ec* (Fig. 1) is called the SPAN of the arch; *ai*, the rise; *b*, the CROWN; the lower boundary line *eac*, the SOFFIT or INTRADOS; the outer boundary line, the BACK or EXTRADOS. The terms SOFFIT and BACK are also applied to the entire lower and upper curved surfaces of the whole arch. The sides of the arch which are seen are called the FACES The blocks of which the arch itself is composed are called VOUSSOIRS; the center one, *K*, is called the KEYSTONE; and the lowest ones, *SS*, the SPRINGERS. In SEGMENTAL arches, or those of which the intrados is not a complete semicircle, the springers generally rest upon two stones, as *RR*, which have their upper surfaces cut to receive them; these stones are called SKEWBACKS. The line connecting the lower edges of the springers is called the SPRINGING-LINE; the sides of the arch are called the HAUNCHES; and the loads in the triangular spaces, between the haunches and a horizontal line drawn from the crown, are called the SPANDRELS. The blocks of masonry, or other material, which support two successive arches, are called PIERS; and the extreme blocks which, in the case of stone bridges, generally support, on one side, embankments of earth, are called ABUTMENTS. A pier strong enough to resist the thrust of one of two successive arches, in case the other one falls down

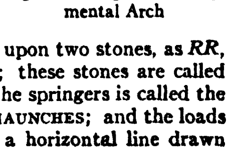

Fig. 1. Diagram of Segmental Arch

* See, also, Chapter XXIX.

is sometimes called an ABUTMENT-PIER. Besides their own weight, arches usually support permanent loads or SURCHARGES of masonry or of earth.

Forms of Arches. In using arches in architectural constructions, the FORMS of the arches are generally governed by the style of the building, or by a limited amount of space, rather than by engineering considerations. The problem, therefore, that usually presents itself to the architect is not to design the form and dimensions of an arch that will most economically and, from an engineering point of view, efficiently bear its load, but rather to determine if an arch of a certain form and of certain dimensions will be stable and safe under its load. The SEMICIRCULAR and SEGMENTAL forms of arches are the best as regards stability, and are the simplest to construct. ELLIPTICAL and THREE-CENTERED arches are not as strong as circular arches, and should only be used where they can be given all the strength desirable.

The Strength of an Arch depends very much upon the care with which it is built and upon the quality of the materials. In stone arches, special care should be taken to cut and lay the beds of stones accurately, and to make the bed-joints thin and close, in order that the arches may be stressed as little as possible in settling. To insure this, arches are sometimes built DRY, grout or liquid mortar being afterwards run into the joints; but the advantage of this method is doubtful.

Brick Arches.* (See Figs. 2, 3, 4 and 5.) These may be built either of WEDGED-SHAPED bricks, molded or rubbed so as to fit to the radius of the soffit

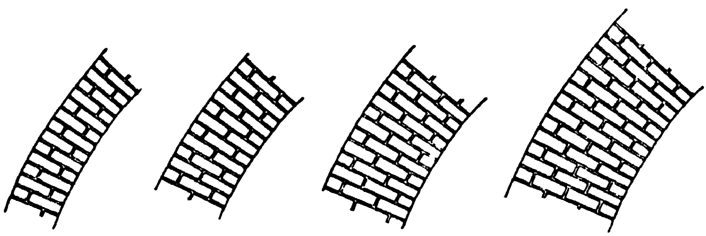

Fig. 2 Fig. 3 Fig. 4 Fig. 5

Brick Arches

or of bricks of COMMON SHAPE. The former method is undoubtedly the best, as it enables the bricks to be thoroughly bonded, as in a wall; but, as it involves considerable expense to make the bricks of the proper shape, it is very seldom employed. When bricks of the ordinary shape are used, they are accommodated to the curved figure of the arch by making the bed-joints thinner towards the intrados than they are at the extrados; or, if the curvature is sharp, by driving thin pieces of slate into the outer edges of those joints; and different methods are followed for BONDING them.

The usual method is to build the arch in concentric rings, each one-half brick thick; that is, to lay all the bricks as STRETCHERS and depend upon the tenacity of the mortar for the connection of the several rings. Brick masonry constructed in this way is deficient in strength, unless the bricks are laid in cement mortar which is at least as tenacious as themselves. Another way is to introduce courses of HEADERS at intervals, and to connect pairs of half-brick

* For illustrations of the different methods of building brick arches, see Chapter VII Building Construction and Superintendence, Part I, Masons' Work, F. E. Kidder.

rings together. This may be done either by thickening with pieces of slate the joints of the outer ring of a pair of half-brick rings, so that there will be the same number of courses of stretchers in each ring between two courses of headers; or by placing the courses of headers at such distances apart, that between each pair of them there will be one course of stretchers more in the outer than in the inner ring. The former method is best suited to arches of long raidus; the latter, to those of short radius. HOOP-IRON laid around the arch, between half-brick rings, as well as longitudinally and radially, is very useful for strengthening brick arches. The bands of hoop-iron which traverse the arch radially may also be bent, and prolonged into the bed-joints of the backing and spandrels. By the aid of HOOP-IRON BOND, Sir Marc-Isambard Brunel built a half-arch of bricks, laid in strong cement mortar, which stood, projecting from its abutment like a bracket to the distance of 60 ft, until it was destroyed by the undermining of its foundations.

Rule for the Radius of Brick Arches. A good RULE for the radius of segmental brick arches over windows, doors and other small openings is to make the RADIUS EQUAL TO THE WIDTH OF THE OPENING. This gives a good rise to the arch and a pleasing proportion. In common brickwork, when no particular architectural effect is desired, such as in the rowlock arches thrown over the openings in cellar walls, a RULE in very common use is to make the RISE of the arch at the crown AN INCH IN HEIGHT FOR EVERY FOOT OF SPAN.

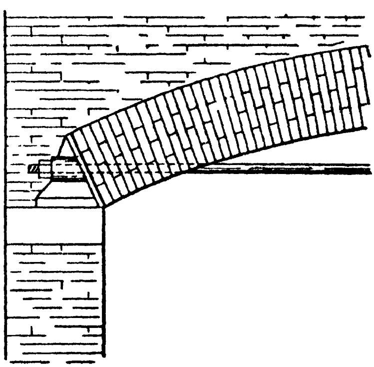

Fig. 6. Segmental Brick Arch, Cast-iron Skewback and Wrought-iron Tie-rod

Segmental Arches with Tie-Rods. It is often desirable to span openings in a wall by means of arches when there are not a sufficient number of abutments to withstand the thrusts. In cases of this kind each arch can be sprung from two cast-iron SKEWBACKS, held in place by IRON RODS as is shown in Fig. 6. When this is done, it is necessary to proportion the size of the rods to the THRUST of the arch. The HORIZONTAL THRUST of the arch may be very closely determined by the following formula:

$$\text{Horizontal thrust} = \frac{\text{load on arch} \times \text{span}}{8 \times \text{rise of arch in feet}}$$

If the load is concentrated at the center of the arch, the thrust will be twice that given by this formula.

The TENSIONAL STRESS in the rod or rods will equal the HORIZONTAL THRUST of the arch and if there are two rods, the stress in each will be one-half the thrust. If there are three rods, then each must resist one-third the thrust. Knowing the stresses in the rods, the size of each may be determined from Table II, Chapter XI.

Example 1. Let us assume that a brick arch, like the one shown in Fig. 6, has a span of 15 ft, a rise at the center of 1 ft 6 in, and that it supports a 12-in brick wall. The weight of all the brick masonry above the arch does not come upon it. Usually only an EQUILATERAL TRIANGLE of brickwork is considered,

the base of the triangle being the span. Assume, therefore, an equilateral triangle the sides of which are each 15 ft long. The altitude of this triangle is about 12.6 ft and its area will equal 15 ft × 12.6 ft × ½ = 94½ sq ft. If the wall is 12 in thick there will be 97½ cu ft of brickwork within this triangle of the wall; and since ordinary brickwork weighs about 115 lb per cu ft, its weight will be about 10 867 lb. Substituting these values in the formula,

$$\text{The horizontal thrust} = \frac{10\,867 \times 15}{8 \times 1.5} = 13\,584 \text{ lb}$$

Looking in Table II, Chapter XI, it appears that one 1½-in or two 1⅛-in plain, round wrought-iron rods, or one 1⅛-in or two ¾-in round, upset, steel rods should be used.

Centers for Arches. A CENTER is a temporary structure, generally of timber, on which the voussoirs of an arch are supported while the arch is being built. It consists of parallel frames or ribs, placed at convenient distances apart, curved on the outside to a line parallel to that of the soffit of the arch, and supporting series of transverse planks, upon which the arch-stones rest. The center commonly used is one which can be lowered, or STRUCK all in one piece, by driving out wedges from below it, so as to remove at once the support from every point of the arch. The center of an arch should not be struck until the solid part of the backing has been built and the mortar has had time to set and harden; and when an arch forms one of a series of arches with piers between them, no center should be struck so as to leave a pier with an arch abutting against one side of it only, unless the pier has sufficient stability to act as an abutment. When possible, the STRIKING of the center of large brick arches should be delayed for two or three months after the arch is built, and during the period that they are in place they should be EASED from time to time. This is done by EASING OUT the wedges under the centers a little at a time so as to let them down gradually and thus adjust any slight settling or shrinkage of the masonry as it occurs.

Mechanical Principles of the Arch. In designing an arch, the first question to be settled is the FORM of the arch; and in regard to this, as already noted, there is generally little choice. When the abutments are of ample size, the SEGMENTAL ARCH is the strongest; but when it is necessary to make the abutments of the arch as small as possible, the SEMICIRCULAR or the POINTED ARCH should be used.

Depth of Keystone. Having decided upon the form of the arch, the DEPTH OF THE ARCH-RING must next be decided. This is generally determined by computing the required DEPTH OF THE KEYSTONE and making the depth of the whole ring the same or a little larger. In considering the strength of an arch, the depth of the keystone is considered to be only the distance from the extrados to the intrados of the arch; and if the keystone projects above the arch-ring, as in Fig. 1, the projection is considered a part of the load on the arch. There are several rules for determining the depth of the keystone, but all are empirical; and they differ so greatly that it is difficult to recommend any particular one.

Rankine's Formula for Depth of Keystone. Professor Rankine's rule is often quoted, and gives results which are probably true enough for most arches. It applies to both CIRCULAR and ELLIPTICAL ARCHES and is as follows. Take a mean proportional between the inside radius at the crown, and 0 12 of a foot for a single arch, and 0.17 of a foot for an arch forming one of a series: or,

Depth in feet of keystone for single arch $= \sqrt{(0.12 \times \text{radius at crown})}$

Depth in feet of keystone for arch of a series $= \sqrt{(0.17 \times \text{radius at crown})}$

The dimensions given by this formula seem to agree very well with those generally used in practice in arches of a certain kind. The formula, however, gives the same depth of keystone for spans of any length, provided the radius is the same; and in this particular it would seem that the rule is not satisfactory.

Trautwine's Formula for Depth of Keystone. Trautwine, from calculations made for a large number of arches, deduced a formula for the depth of keystone, which seems to agree with theory more closely than Rankine's formula. His rule is, for CUT STONE,

$$\text{Depth of key in feet} = \left(\frac{\sqrt{\text{radius} + \text{half span}}}{4}\right) + 0.2 \text{ ft}$$

For SECOND-CLASS work this depth may be increased about one-eighth part, or for BRICKWORK or FAIR RUBBLE, about one-third.

Tables for Depths of Keystones. Table I gives a few examples of the DEPTHS OF THE KEYSTONES of some bridges, together with the depths which would be required by Trautwine's or Rankine's formula. From this table it is seen that the results of both formulas agree very well with dimensions used in actual practice.

Table I. Depths of Keystones of Some Arches of Circular Arc

Name or location of structure	Span ft	Rise ft	Radius ft	Actual depth of key ft	Calculated depth of key: Trautwine's Rule ft	Calculated depth of key: Rankine's Rule ft	Engineer
Cabin John, Washington aqueduct. ...	220.0	57 25	134 25	4 16	4 11	4.00	Meigs
Grosvenor bridge, Chester, England.	200.0	42 00	140.00	4 00	4.07	4.10	Hartley
Dora Riparia, Turin, Italy	148 0	18 00	160 10	4 92	4.03	4 38	Mosca
Tongueland, England	118 0	38 00	64.80	3.50	3.00	2.79	Telford
Dean bridge, Scotland, in a series...........	90.0	20 00	48.90	3.00	2 62	2.88	Telford
Falls bridge, Philadelphia & Reading Railroad	78.0	25.00	43.00	3.00	2.46	2.27	Steele
Chestnut St. bridge, Philadelphia, brick in cement..........	60.0	18.00	34 00	2.50	2.20	2.00*	Kneass
Philadelphia & Reading Railroad.........	44.0	8.00	34.30	2 50	2.08	2 02	Steele
Philadelphia & Reading Railroad	31 2	5 00	26 80	1.66	1 83	1.79	Steele

* For first-class cut-stone work.

Table II * gives the DEPTHS OF KEYSTONES for arches of first-class cut stone according to Trautwine's Formula. For second-class cut stone, add about

* Taken from The Civil Engineer's Pocket Book, John C. Trautwine.

one-eighth part and for fair rubble or for brickwork about one-third part, as stated with formula.

Table II. Depths of Keystones for Arches of First-Class Cut-Stone Masonry

Span	Rise, in parts of the span						
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
ft	ft	ft	ft	ft	ft	ft	ft
2	0.55	0.56	0.58	0.60	0.61	0.64	0.68
4	0.70	0.72	0.74	0.76	0.79	0.83	0.88
6	0.81	0.83	0.86	0.89	0.92	0.97	1.03
8	0.91	0.93	0.96	1.00	1.03	1.09	1.16
10	0.99	1.01	1.04	1.07	1.11	1.18	1.26
15	1.17	1.19	1.22	1.26	1.30	1.40	1.50
20	1.32	1.35	1.38	1.43	1.48	1.59	1.70
25	1.45	1.48	1.53	1.58	1.64	1.76	1.88
30	1.57	1.60	1.65	1.71	1.78	1.91	2.04
35	1.68	1.70	1.76	1.83	1.90	2.04	2.19
40	1.78	1.81	1.88	1.95	2.03	2.18	2.33
50	1.97	2.00	2.08	2.16	2.25	2.41	2.58
60	2.11	2.18	2.26	2.35	2.44	2.62	2.80
80	2.44	2.49	2.58	2.68	2.78	2.98	3.18
100	2.70	2.75	2.86	2.97	3.09	3.32	3.55
120	2.94	2.99	3.10	3.22	3.35	3.61	3.88
140	3.16	3.21	3.33	3.46	3.60	3.87	4.15
160	3.36	3.44	3.58	3.72	3.87	4.17	..
180	3.56	3.63	3.75	3.90	4.06	4.38	..
200	3.74	3.81	3.95	4.12	4.29	.	.
220	3.91	4.00	4.13	4.30	4.48	. .	..
240	4.07	4.15	4.30	4.48	...		.
260	4.23	4.31	4.47	4.66	. .		
280	4.38	4.46	4.63	....	. .	...	..
300	4.53	4.62	4.80	.. ..	.		

Example 2. Having decided what the thickness of the arch-ring will be it remains to determine whether such an arch would be stable if built. The following example will illustrate the method of determining this.

Consider an unloaded semicircular arch of 20-ft span.

First, to find the depth of the keystone, we will use Rankine's Formula.

$$\text{Depth of key} = \sqrt{0.12 \times 10} = \sqrt{1.2} = 1.1 \text{ ft}$$

Trautwine's Formula gives nearly the same result,

$$\text{Depth of key} = \frac{\sqrt{10 + 10}}{4} + 0.2 \text{ ft} = 1.3 \text{ ft}$$

But if we should compute the stability of a 20-ft semicircular arch with a keystone 1.3 ft deep, we should find that the arch is very unstable; hence, in this case, we cannot use the formula and must act upon our own judgment. In the opinion of the author, the arch-ring of such an arch should be at least 2½ ft deep and the stability of the arch should be tested for that thickness. In all calculations on the arch, it is customary to consider it 1 ft thick at right-angles to its face. This allows the AREAS OF THE FACES to be substituted for the ACTUAL WEIGHTS of the voussoirs and their loads. This method was

used in the discussion of Retaining-Walls, Chapter IV, and Piers and Buttresses, Chaper VII. Furthermore, it is evident that if an arch 1 ft thick is stable, any number of arches of the same dimensions built alongside of it would be stable. In determining the stability of masonry arches it is also customary to neglect any increase in the strength of the arch from the mortar in the joints, or in other words, to consider the arch as laid up dry.

Graphic Determination of the Stability of Arches. An arch has already been defined as a particular arrangement of blocks of stone or other material, these blocks being called the VOUSSOIRS. For the sake of simplicity consider an UNLOADED ARCH. In such an arch each voussoir is subjected to the action of three forces, (1) the thrust that it receives from the voussoir next above it in the arch-ring, (2) the force of gravitation, or its own weight and (3) the reaction to the resultant thrust. The first two forces combine into one and form the thrust that this voussoir exerts on the one next below it in the arch-ring (Fig 7). The points in which these various thrusts cut the joints are called the CENTERS OF PRESSURES of the joints, while the line joining these centers of pressure is called the LINE OF PRESSURE or LINE OF RESISTANCE.* In order that an arch may be absolutely stable, this line of resistance must fall within the MIDDLE THIRD of the arch-ring (See Theorem of the Middle Third, Chapter IV.) If the arch is stable the centers of pressure on the various joint-lines are within the middle third of the voussoir-depths and the angles made by the different thrusts with the normals to the joints are less than the ANGLE OF FRICTION of the material of which the arch is constructed. If these conditions

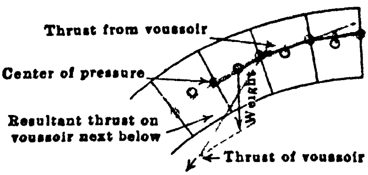

Fig. 7. Equilibrium of Forces on Voussoir

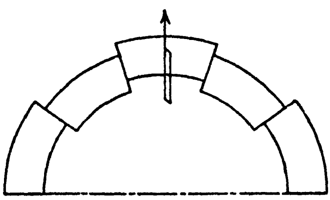

Fig. 8. Failure of Semicircular Arch. Haunches Sliding Down

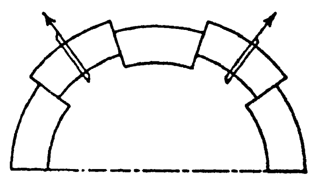

Fig. 9. Failure of Semicircular Arch. Haunches Sliding Up

are not fulfilled the CRITERIA OF SAFETY, explained in Chapter VII in the discussion of the Stability of a Buttress, will not be satisfied; and at any joint where these conditions do not obtain, the voussoir above the joint will tend to SLIDE along the joint-plane if the angle made by the thrust with a normal to the joint is greater than the angle of friction. If the center of pressure lies outside the middle third, there will be a tendency for the voussoir to OVERTURN. When these tendencies reach extreme limits actual FAILURE may occur. Figs. 8, 9, 10 and 11 illustrate some of the ways in which an arch may fail, Figs. 8 and 9 showing different parts of the masonry sliding on the joints and Figs. 10 and 11 the failures caused by the passing of the line of pressure near the intrados or extrados.

* This line is called, interchangeably, the LINE OF PRESSURE, the LINE OF RESISTANCE the RESISTANCE-LINE, etc. (See, also, Chapter XXIX.)

Before passing to the actual discussion of the GRAPHIC METHOD for determining the stability of arches, a consideration of the action of the STRESSES developed in a construction of this kind will assist in a clearer understanding of the subject.

Fig. 8 shows how, if the line of resistance along the HAUNCHES of the arch should turn sharply downward and in so doing make with a normal to one of the joints an angle greater than the angle of friction, the voussoirs at this point

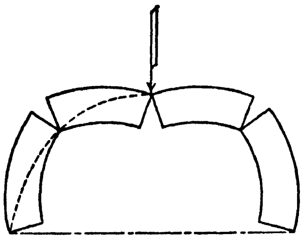

Fig. 10. Failure of Semicircular Arch. Opening of Arch-ring

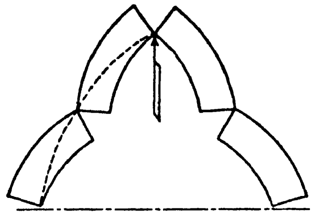

Fig. 11 Failure of Pointed Arch. Opening of Arch-ring

would tend to slide inward on their joint-planes, forcing outward the voussoirs at the spring and crown of the arch. Fig. 9 shows how failure of the arch would occur under similar conditions, but with the line of resistance turning sharply upward instead of downward. In these two cases it is conceivable that, although the RESISTANT THRUST at the joint where failure takes place makes an angle with the normal greater than the angle of friction, its point of application is still within the middle third of the joint.

Figs. 10 and 11, on the contrary, illustrate methods of failure in which, although the angle made by the thrust may be such as to cause no SLIPPING of one joint on another, its point of application is sufficiently outside the middle third of the arch-ring itself at the crown to cause OVERTURNING In Fig. 10 the line of resistance passes high up, or perhaps entirely outside of the arch-ring, in the voussoirs at the CROWN of the arch and low down along the HAUNCHES. In Fig 11 exactly contrary conditions exist

The ten ways in which a masonry arch may fail have been classified as follows: * "(1) By CRUSHING of the masonry, (2) By SLIDING of one voussoir upon another; (3) By one voussoir or section of masonry OVERTURNING about an adjacent voussoir or section; (4) By SHEARING in a horizontal or vertical plane, this applying to solid concrete arches and not to voussoirs; (5) AS A COLUMN when the ratio of the unsupported length of an arch to its least width is greater than twelve; (6) From STRIKING THE CENTERING before the mortar is hard or when the arch, although stable under the full load, is not stable under its weight alone; (7) By STRIKING THE CENTERING or loading the arch during construction unsymmetrically; (8) By SETTLEMENT of the foundations; (9) By SLIDING upon the foundations; (10) By OVERTURNING about any point in the pier or abutment. Methods (8) and (9) are the most common ways of failure. All methods of failure, however, must be guarded against in design."

While some of these ways of failure may seem other than those illustrated in the foregoing figures, they may be perhaps more properly considered CAUSES OF FAILURE than WAYS OF FAILURE; and all, with the exception of the first,

* W. J. Douglas in American Civil Engineering Pocket-Book, page 625.

bring about a position of the line of resistance in the arch-ring which causes failure in one of the ways noted.

In regard to the method of failure (1), the conditions may be such that the loading, although symmetrical, is so excessive that although the line of resistance remains within the middle third, the total pressure on a joint is sufficient to CRUSH THE MATERIAL of which the arch is constructed. Such conditions, however, are not common.

From the foregoing discussion it is evident that in order to determine whether or not a given arch is stable, it is necessary to find the TRUE LINE OF RESISTANCE corresponding to the conditions of loading, form and dimensions of that particular arch. It is always possible, in every arch-ring, to pass one MAXIMUM and one MINIMUM LINE OF RESISTANCE. The TRUE LINE OF RESISTANCE will lie somewhere between these two. The method of procedure, therefore, is to pass tentatively, a line of resistance, either a maximum or a minimum one, and see if it remains within the middle third. If it does not, as it may not be the true line of resistance, it does not mean necessarily that the arch is not stable. The next step then, is to note where it departs farthest from the middle third, and to pass a second line of resistance through the same point on the crown-joint and the point on the line of the middle third where the original line departs farthest from the middle third. If this second line of resistance remains within the middle third it is reasonable to assume that the arch is stable. In these various operations it is only necessary to consider half the arch when the loading is symmetrical, and this is usually the case in architectural problems. The NUMBER OF VOUSSOIRS, also, into which we divide the half-arch, is immaterial and the joints need not coincide with those of the actual arch.

In order to pass a line of resistance through an arch-ring, the THRUST exerted by the other half AT THE CROWN-JOINT on the half-arch is first determined. This thrust is then combined with the resultant of the weight of the first voussoir and its load to determine the thrust exerted by this voussoir on the one next below it, and this thrust, in turn, is combined in the same way with the resultant of the weight and the load of the second voussoir, and so on down to the springing-joint, for each succeeding voussoir. The points in which the various lines representing the thrusts cut the joints are known as the CENTERS OF PRESSURE, and the line joining them is the LINE OF PRESSURE or LINE OF RESISTANCE. In performing this operation, the CENTER OF GRAVITY of each voussoir as well as the line passing through the center of gravity of the whole half-arch must be located. The face of each voussoir may be considered a TRAPEZOID, and any one of the methods for finding the center of gravity of this figure may be used for finding the center of gravity of each voussoir. The method of dividing the trapezoid into TRIANGLES is here employed and is shown at the side of the arch in Fig. 12. (See, also, in Chapters VI and VII.) As the determination of the position of the line passing through the center of gravity of the half-arch is the problem of finding the RESULTANT OF A SYSTEM OF PARALLEL FORCES, the method involving the drawing of the EQUILIBRIUM-POLYGON may be used. The most convenient way to determine the stability of an arch is to use the GRAPHIC METHOD. The STEPS in this method are outlined in the preceding paragraphs. Each of the operations will now be considered in detail.

First Step. Draw one-half the arch to as large a scale as convenient, and divide it into voussoirs of equal size. In the example shown in Fig. 12, the arch-ring is divided into ten voussoirs of equal face-areas. As already pointed out, it is not necessary that these should represent the actual voussoirs of which the arch is built. Next, the face-area of each of these voussoirs is to be

found. Where the arch-ring is divided into voussoirs of equal size, this is most easily done by computing the total area of the arch-ring and dividing this total area by the number of voussoirs. The FORMULA for finding the area of one-half the arch-ring is as follows:

$$\text{Area in square feet} = 0.7854\ (r^2 - r_1^2)$$

In this formula r is the outside radius and r_1 the inside radius in feet. In this problem, for example, if the

$$\text{Area of the arch-ring} = 0.7854\ (12.5^2 - 10^2) = 44.2 \text{ sq ft}$$

as there are ten equal voussoirs, the area of each voussoir is 4.42 sq ft. Having drawn out one-half of the arch-ring, divide the crown-joint into three equal parts, and with radii of $O'E$ and $O'F$ describe the arcs dividing the arch-ring into thirds.

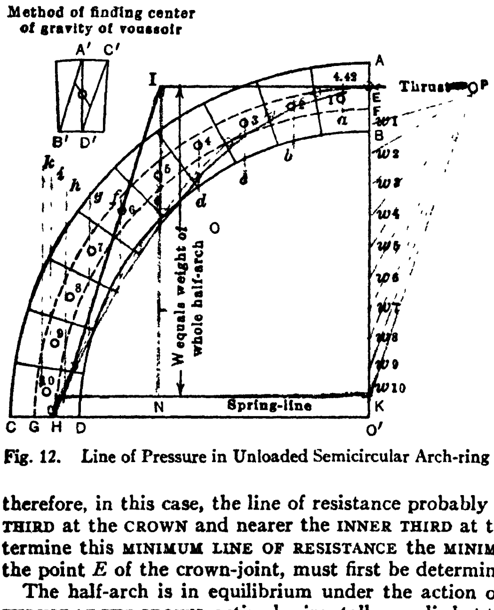

Fig. 12. Line of Pressure in Unloaded Semicircular Arch-ring

Second Step. Choose the points E and H through which to pass a MINIMUM LINE OF RESISTANCE. The points F and G, through which a MAXIMUM LINE OF RESISTANCE can be passed, could equally well have been chosen. It should be noted that an unloaded semicircular arch is more apt to fail by opening at the intrados at the crown and at the extrados at the haunch, and therefore, in this case, the line of resistance probably passes nearer the OUTER THIRD at the CROWN and nearer the INNER THIRD at the HAUNCH. To determine this MINIMUM LINE OF RESISTANCE the MINIMUM THRUST, applied at the point E of the crown-joint, must first be determined.

The half-arch is in equilibrium under the action of three forces: (1) the THRUST AT THE CROWN, acting horizontally, applied at the point E and preventing the half-arch from overturning inward; (2) the WEIGHT OF THE HALF-ARCH considered as a vertical force, acting through its center of gravity and tending to overturn it inwards about the point D; and (3) A FORCE EQUAL AND OPPOSITE TO THE RESULTANT of these two forces and passing from H to I. I is the intersection of the weight-line through the center of gravity of the half-arch, with the line of action of the thrust at the crown, prolonged. It is thus possible to construct the TRIANGLE OF THESE THREE FORCES and determine the magnitudes of the thrusts, when the position of the weight-line of the half-arch is determined. It is first necessary to draw a vertical line through the center of gravity of each voussoir. The center of gravity of one of the voussoirs may be found by the METHOD OF TRIANGLES, as shown in the supplementary figure at the side of the arch-ring.

Having determined the positions of the centers of gravity of the voussoirs,

locate them on the voussoirs as shown. From the point E (Fig. 12) lay off vertically, to a scale of so many SQUARE UNITS TO A LINEAR UNIT, the area of each voussoir, one below the other, commencing with the top voussoir. The length of the line EK will then equal the total area of the arch-ring. From E and K (Fig. 12) draw 45° lines intersecting at O. Draw Ow 1, Ow 2, Ow 3, etc. Then where OE intersects the first vertical line through the center of gravity of the first voussoir at a, draw a line parallel to Ow 1, intersecting the second vertical at b. Draw bc parallel to Ow 2, cd parallel to Ow 3 and so on to k. Draw kL parallel to Ow 10 and prolong it downward until it intersects EO prolonged, at L. A vertical line drawn through L will pass through the center of gravity of the half arch-ring. This is an application to a practical problem of the method of finding, by the EQUILIBRIUM-POLYGON, the line of action of the resultant of a SYSTEM OF PARALLEL FORCES. The weights of the individual voussoirs act along parallel vertical lines and the weight of the half-arch is their resultant in magnitude.

Third Step. To determine the THRUST AT THE CROWN and the REACTION AT THE SPRING, draw a horizontal line through E, the upper part of the middle third, and a vertical line through L, the two lines intersecting at I (Fig. 12). For the arch to be stable, it is, in general, considered necessary for the LINE OF RESISTANCE to pass within the MIDDLE THIRD. First, assume that the line of pressure or resistance starts at E and comes out at H. Draw a line IH the direction of the line of action of the resultant of the thrust at the crown and the weight of the half-arch, and draw, also, a horizontal line opposite the point w 10, between N and M. This horizontal line MN represents the magnitude of the horizontal thrust at the crown, for INM is the TRIANGLE OF THE THREE FORCES in equilibrium, the THRUST at the crown, the WEIGHT of the half-arch and the REACTION at the spring. Draw w 10 O^p parallel to HI, and the lines O^pw 1, O^pw 2, O^pw 3, etc. O^pE, equal to NM, is the thrust at the crown, and w 10 O^p, equal to MI, the reaction at the spring. INM and EKO^p are similar triangles.

Fourth Step. It is required next, to determine the LINE OF RESISTANCE through the arch-ring. The thrust at E is combined with the weight of the first voussoir; their resultant is found and in turn combined with the weight of the second voussoir; and so on for all the voussoirs. The intersections of these resultants with the joint-lines are the CENTERS OF PRESSURE; the line joining these centers of pressure is the LINE OF RESISTANCE.

These resultants could be determined by drawing a series of PARALLELOGRAMS OF FORCES over each voussoir. This would complicate the figure and involve unnecessary labor. It is found more convenient to draw the TRIANGLES OF FORCES one after the other, at the right-hand side of the figure and then transfer the results thus obtained by means of parallel lines to the figure itself, especially as the weights of the voussoirs have already been laid off along the line EK, at Ew 1, w 2, w 3, w 4, w 5, etc.

Then from the point where O^pE prolonged intersects the first vertical in voussoir number 1, draw a (green) line to the second vertical, parallel to O^pw_1; from this point, a (green) line to the third vertical, parallel to O^pw 2 and so on. The last line should pass through H. Join the various points, where these (green) lines cut the joints at the centers of pressure, by the broken (red) line. This last line drawn is the LINE OF RESISTANCE. If this line lies entirely within the MIDDLE THIRD of the arch-ring, the arch may be considered to be stable. But suppose that the line of resistance passes not only outside of the middle third but also outside of the arch-ring itself; it is still possible that the arch is not unstable. This is the case in Fig. 12 and we will

next determine if a line of resistance can be drawn which will remain within the limits of the middle third of the arch-ring.

Fifth Step. The Second Trial. Reproducing the condition of Fig. 12 in Fig. 13, without the construction lines, it is seen that the LINE OF RESISTANCE leaves the arch-ring at R and enters it again at S, while it is furthest from it at U. If, at U, a perpendicular is erected to a straight line joining the two points R and S, this perpendicular line VW, called the LINE OF FRACTURE, will be approximately the trace of the plane along which, with the line of resistance under consideration, the arch will tend to fail, presumedly by TURNING OVER to the right about the point V. This shows that the THRUST AT THE CROWN, assumed to be applied at the point E, while of sufficient intensity to maintain equilibrium about H, is not of sufficient intensity to maintain equilibrium about V. If now a SECOND THRUST, of sufficient intensity to maintain equilibrium about V, or better, about X, can be applied at E without being so great in magnitude that it will OVERTURN THE ARCH OUTWARD about G, or some other point on the outer line of the middle third, it

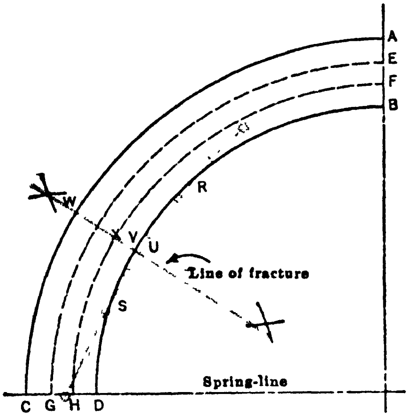

Fig. 13. Line of Fracture in Unloaded Semicircular Arch-ring

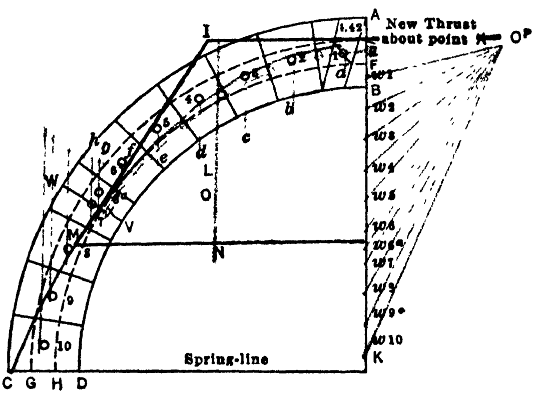

Fig. 14. Second Line of Pressure in Unloaded Semicircular Arch-ring

is reasonable to conclude that the line of resistance resulting from this thrust is very nearly the TRUE LINE OF RESISTANCE in the arch-ring and that the arch is stable.

In order to determine this NEW LINE OF RESISTANCE the NEW THRUST AT THE

CROWN must be found (Fig. 14). The preliminary steps required for this are the same as before until the seventh voussoir is reached. This is divided into two voussoirs by the line VW (Fig. 14), one being $w6\ w6^a$ and the other the remainder of this seventh voussoir, and this division must be allowed for along the load-line EK, at $w6\ w6^a$. The line $w6\ w6^a$ represents the area of voussoir 6^a, and the line $w6^a\ w7$ the area of the remainder of the seventh voussoir.

The vertical line IL, passing through the center of gravity of that part of the half-arch above the line VW, is found by prolonging backwards the line hg, parallel to $O\ w6^a$, until it intersects OE at L. To find the NEW THRUST AT THE CROWN by completing the TRIANGLE OF FORCES for this thrust and the force equal and opposite to their resultant, the inclined (blue) line must be drawn through the point X and the horizontal (blue) line through $w6^a$. The new thrust then is as before NM, equal to O^pE. This thrust is laid off at O^pE, the (green) lines O^pw 1, O^pw 2, O^pw 3, etc., being drawn as before and the new line of resistance being drawn through the points where the parallels to these

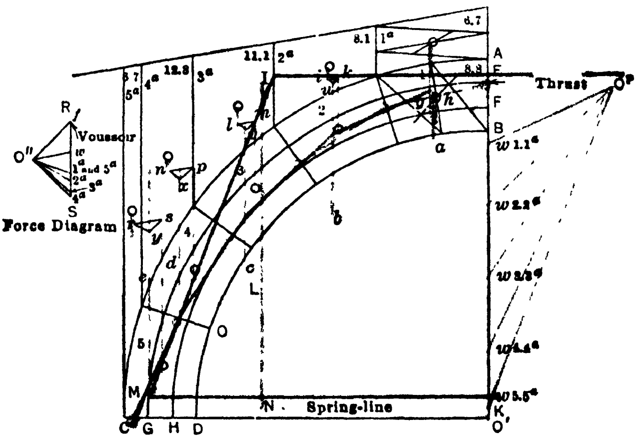

Fig. 15. Line of Pressure in Loaded Semicircular Arch-ring.

(green) lines cut the joints. This NEW LINE OF RESISTANCE, if drawn correctly, should pass through X. It lies within the middle third, except for a short distance at the springing, and hence it is justifiable to consider the arch stable. If it had passed outside the middle third to any great extent, in this second trial, this presumption would not have been justified.

This discussion explains the method of determining the stability of an UNLOADED SEMICIRCULAR ARCH. Such cases very seldom occur in practice, but they serve to illustrate the methods which apply generally to all other cases. With LOADED ARCH-RINGS there is slight difference in the method of determining the position of the center of gravity.

Example 3. A LOADED OF SURCHARGED SEMICIRCULAR ARCH (Fig. 15) will be considered next. Assume the same arch shown in Figs. 12, 13 and 14, and suppose it to be loaded with a wall of masonry of the same thickness and weight per square foot as that of the arch-ring, the upper surface of the wall being an inclined plane, 1 ft above the arch-ring at the crown, and 8 ft above it at the spring. The assumption of the particular load in this case is a purely

arbitrary one for the purpose of illustrating the method of solution. The determination of the ACTUAL LOAD that comes upon an arch in any given case is by no means easy, so numerous are the uncertain elements that affect the transmission of this load to the arch-ring.

The customary procedure is to assume that the load is itself transmitted to the arch-ring VERTICALLY DOWNWARD. Each voussoir thus receives that portion of the load which is included between two vertical lines drawn to the points of intersection of the joints on either side of that voussoir with the extrados. Having made this assumption it is necessary next to determine how much of the total superimposed masonry bears upon the arch-ring.

It is a matter of common observation that if an opening is made in a wall, especially in a wall that has stood for some time, the major portion of the masonry above this opening is self-supporting, limited portions only, bounded by a somewhat irregular line, falling down into the opening, as shown in Fig. 16. The profile of this boundary-line depends upon the nature of the material of which the wall is constructed, the size of the stones, bricks, etc. the character of the bond and the quality of the mortar. This being the case, all the masonry above an arch should not be considered as the load on it. Some authorities recommend considering as the proper load, for brickwork, a TRIANGULAR PART of the wall, the sides of which triangle have an inclination to the horizontal of 45°; others assume an inclination of 60° (Fig. 16). The exact determination of this load by mechanical laws is difficult if not impossible. It is better to consider each case separately and by a careful study of the conditions to determine as closely as possible just what portion of the weight of the superimposed masonry is transmitted to the arch. Having assumed a load for this particular arch-ring (Fig. 15), the procedure is as follows:

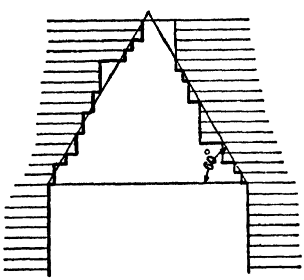

Fig. 16. Triangle of Loading over Opening

First Step of Example 3. This involves the finding of the CENTER OF GRAVITY of the ARCH-RING AND LOAD COMBINED. Divide the arch-ring into five voussoirs of equal size. In this case the area of each voussoir is equal to 44.2 sq ft ÷ 5, or 8.8 sq ft. (See under First Step, Fig. 12, preceding example.) The surcharge or load, also, is divided into five parts, not necessarily equal, by drawing vertical lines to the points of intersection of the joints and the extrados. The approximate area of each one of these surcharges is found by multiplying half the sum of the lengths of the two parallel vertical sides by the length of the horizontal distance between them.

The positions of the center of gravity of each voussoir and of the center of gravity of each voussoir-surcharge are determined as in the preceding example. The CENTERS OF GRAVITY of these SURCHARGES can be found by dividing each TRAPEZOIDAL FIGURE into TRIANGLES as shown, remembering that the MEDIAL LINE in this case joins the middle points of the two parallel faces. As the latter are vertical, the medial lines approach a horizontal direction. This construction is shown on surcharge 1^a, Fig. 15. Having drawn the lines of action of the weights of the various voussoirs and of their loads through their respective centers of gravity, the lines of action of the combined weight of each voussoir and its load must be found. The construction for this

operation is shown at the left of Fig. 15. The method used, that of the EQUILIBRIUM-POLYGON, is the same as that employed in the previous example to find the line passing through the center of gravity of the half-arch, only in this case the forces are reduced to two. Furthermore, as the areas of the various voussoirs are equal it is possible to superimpose the different FORCE-DIAGRAMS, one over the other, and so save considerable labor. Begin, therefore, by laying off along the line RS at the left of the loaded arch, and at any convenient scale, fw, the area (weight) of a voussoir; then from w, in turn, the distances $w\ 1^a$, $w\ 2^a$, $w\ 3^a$, etc., representing the areas of the successive surcharges, 1^a, 2^a, 3^a, etc., always at the same scale. The scale to be employed later for laying off the combined weights of the voussoirs and their loads along the line AK is the best one to choose, but the difference in scales is not important. In this particular instance the two points 1^a and 5^a coincide because the two areas 1^a and 5^a, although of different shapes, are each equal to 6.7 sq ft. This is a mere coincidence. Next draw fO'' and $4^a\ O''$ at 45° to RS, and in turn, $O''w$, $O''1^a$, $O''2^a$, etc. As the problem which presents itself is to combine the weight of each voussoir with its individual surcharge, and as the weights of all the voussoirs are equal, and, furthermore, as the forces which are to be combined to find their resultant are only two, the two POLE-LINES or RAYS $O''f$ and $O''w$ in the FORCE-DIAGRAM serve in each case, and the FUNICULAR POLYGON is reduced to a TRIANGLE. Draw gh, ik, lm, np and rs parallel to $O''w$, and ht, ku, mv, px and sy parallel to $O''f$; and draw gt, iu, lv, nx and ry parallel respectively to $O''1^a$, $O''2^a$, $O''3^a$, $O''4^a$ and $O''5^a$. The points t, u, v, x and y are the points through which to draw the heavy (red) lines of action of the combined weights of the voussoirs and their surcharges.

Having found and drawn these lines, the procedure for finding the line IN is the same as in the previous example, except that the distances $Ew1\ 1^a$, $w^1\ 1^a$, $w2\ 2^a$, etc., instead of being equal to the weights of the voussoirs alone, are equal to the combined weights of each voussoir and its surcharge, $Ew1\ 1^a$, being equal to $f\ 1^a$, $w1\ 1^a$ to $w2\ 2^a$ being equal to $f\ 2^a$, etc.

The line EO is drawn at 45° to AO', but as the position of the POLE-POINT, O, is entirely arbitrary, the line $Ow\ 5\ 5^a$ has been drawn in this case in such a way that O falls well over toward the left of the figure, thus avoiding a certain amount of confusion in the drawing which would have resulted if $Ow\ 5\ 5^a$ had made an angle of 45° with AO'. The lines ab, bc, cd and de are drawn respectively parallel to $w1\ 1^aO$, $w2\ 2^aO$, etc., and eL is produced backward parallel to $Ow\ 5\ 5^a$ until it intersects EO at L, which is the point through which the heavy (red) line IN, passing through the center of gravity of the whole half-arch and its surcharge, should be drawn. A vertical line drawn through L will pass through the center of gravity of the arch-ring and its load. If this were an arch designed for a building and if the only abutments possible were of such size and form that it was essential for the thrust exerted by the last or fifth voussoir on these abutments to approach more nearly the vertical, the architectural expedient of increasing slightly the weight of the surcharge, 5^a, on this voussoir by adding some piece of ornament, such as a cartouche, could be resorted to. A case of this kind in actual practice is the archway over the entrance to the service-courtyard of the Grand Opera House in Paris, where the pyramidal stone ornaments which surmount the cornice on either side of the central motive were added after the original design was made, with this end in view. In the example illustrated in Fig. 15 the areas of the faces of the surcharges are shown by the figures on these faces. For the second surcharge from the crown, for example, the area is 8.1 sq ft.

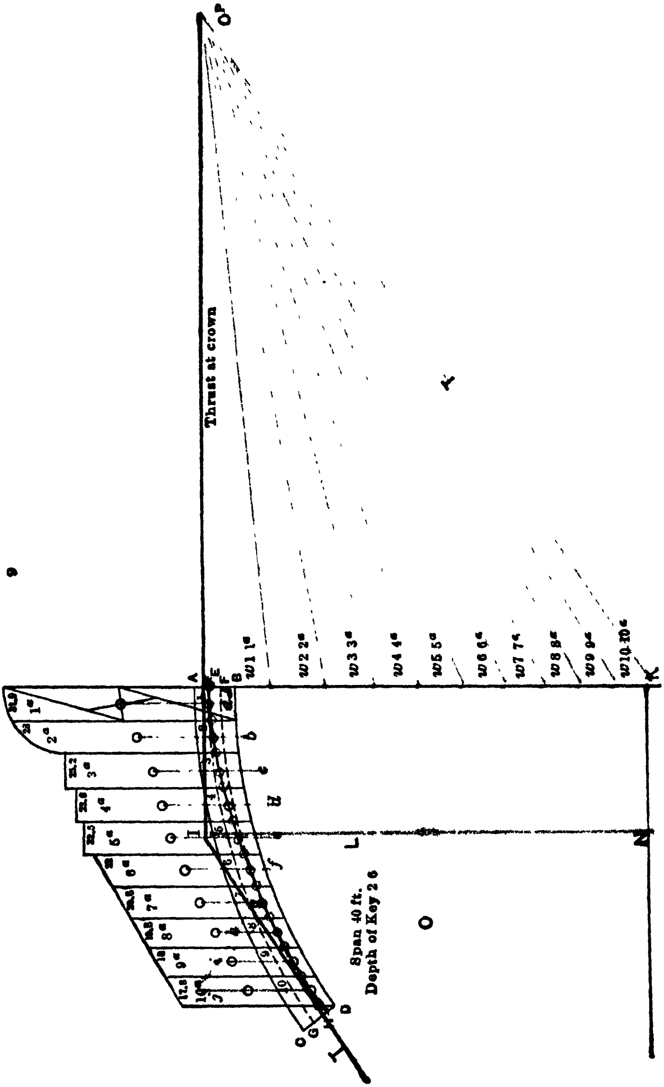

Fig. 17. Line of Pressure in Loaded Segmental Arch-ring

Second Step of Example 3. This involves the determination of the THRUST AT THE CROWN and the LINE OF RESISTANCE. The method of finding this thrust at the crown is similar to that employed in the previous example. In that example, however, it was found that this thrust, applied at E and determined by assuming H as the point of application of the reaction at the spring, produced a line of resistance which fell considerably below the middle third. But instead of performing the operations required by a second trial, as in the previous example, the expedient is tried of slightly increasing the inclination to the vertical of the (blue) line IM, and so assuming a somewhat greater THRUST AT THE CROWN. As the line of resistance, as shown in Fig. 15, passed with this thrust departs but slightly from the middle third near the springing, we are justified in assuming that this arch is stable under the given conditions. The method used for this example may be used, also, for a SEMIELLIPTICAL ARCH.

Example 4. This example (Fig. 17) illustrates the application of the preceding methods, with some variations, to the determination of the position of the center of gravity of a LOADED SEGMENTAL ARCH, the thrusts at the crown and spring and the line of pressure or resistance through the arch-ring. In this case, instead of dividing the arch-ring into a certain number of voussoirs with joints radiating from a center and considering the surcharge on each individual voussoir, the method of dividing the arch-ring and its load into VERTICAL SLICES, in this case 2 ft wide, and computing the areas of the entire slices has been adopted. Having computed the areas of the slices, including in each case the combined areas of the sliced part of the arch-ring and its surcharge, we lay them off in order from E, to a convenient scale, and then proceed as in the previous examples. The remaining steps required to determine the thrusts at the crown and at the spring and the line of resistance are also the same as explained in the foregoing paragraphs. In a FLAT SEGMENTAL ARCH there is practically no need of dividing the arch-ring into voussoirs by JOINTS RADIATING FROM A CENTER, in order to determine its stability. Of course, when built, they must be made to radiate.

Fig. 17 shows the GRAPHICAL ANALYSIS of an arch of 40-ft span and carrying a load 13½ ft high at the crown. The depth of the arch-ring is 2 ft 6 in. It is seen that the line of resistance lies entirely within the middle third, and that the arch is therefore stable. It is to be noted that the line of resistance in a SEGMENTAL ARCH should be drawn through the LOWER or INNER EDGE of the middle third at the springing. It is to be noted, also that the horizontal thrust at the crown and the thrust T against the supports are very great when compared with those in a SEMICIRCULAR ARCH; and hence, although the SEGMENTAL ARCH is the stronger of the two, it requires much heavier abutments. The foregoing examples serve to show the various methods of determining the stability and thrusts of any arch used in buildings.

CHAPTER IX

REACTIONS AND BENDING MOMENTS FOR BEAMS

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1. Reactions for Simple Beams

Definition of Reaction. One of the fundamental principles of static equilibrium is that the sum of all the forces acting upon a body in one direction must be balanced by the sum of another set of forces acting in the opposite direction. Therefore, in the case of a beam or girder, the loads acting downward must be balanced by an equal set of forces at the supports, acting upward. These upward forces are called THRUSTS, or REACTIONS, and in computing the strength of beams one of the first steps is to determine them, since the loads are usually given in intensity and position.

The Principle of Moments. The reactions may be determined by the application of another fundamental principle of static equilibrium for forces acting in the same plane. The algebraic sum of the moments of all the forces

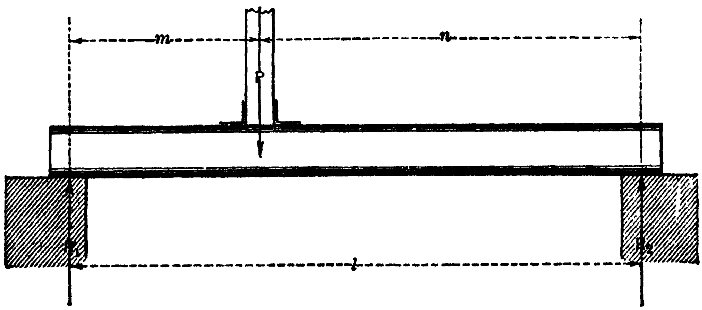

Fig. 1. Simple Beam. One Concentrated Load

taken about any point in the plane in which they act must be zero. The MOMENT OF A FORCE about a point is the product of the magnitude or intensity of the force by the perpendicular distance between the LINE OF ACTION of the force and the point. The perpendicular distance is called the LEVER-ARM, and the point the CENTER OF MOMENTS. Forces acting upward are considered POSITIVE and those acting downward are considered NEGATIVE. The center of moments may be taken at any point in the plane of action of the forces, but it is more convenient to take it at one of the reactions. For example, the beam in Fig. 1 supports a concentrated load P at the distance m from the left support. To find the left reaction take the center of moments at the right reaction. Then the EQUATION OF MOMENTS is

$$R_1 l - Pn = 0$$

from which $$R_1 = Pn/l \qquad (1)$$

In like manner, to find R_2 the center of moments is taken at R_1 and the equation of moments is

$$R_2 l - Pm = 0$$

from which
$$R_2 = Pm/l \tag{1'}$$

From the first principle of statics mentioned, $R_1 + R_2$ must equal P; hence, as a check, $Pn/l + Pm/l = P$.

Example 1. Let a beam 15 ft in span support a concentrated load of 700 lb, 6 ft from the left end; or, $P = 700$, $m = 6$ and $n = 9$. Then, from Formula (1), $R_1 = 700 \times 9/15 = 420$ lb. $R_2 = 700 \times 6/15 = 280$ lb, and $420 + 280 = 700$ lb.

For a concentrated load at the middle, or for a uniform load over a simple beam, it is evident without applying the conditions of equilibrium, that each reaction is one-half the load, for, in Formulas (1) and (1)′, m and n each equal $\frac{l}{2}$ and R_1 and $R_2 = \frac{1}{2} P$.

For any number of concentrated loads (Fig. 2) the reactions may be found by adding together the reactions found by Formula (1) due to each load separately, or they may be computed in one operation by the following formula:

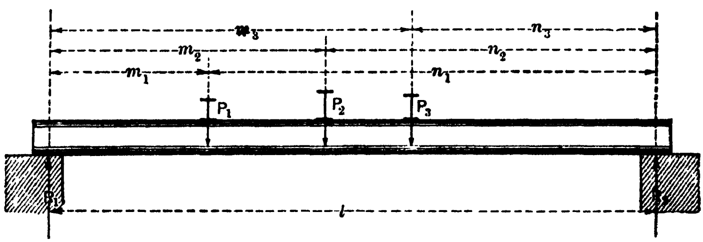

Fig. 2. Simple Beam. Three Concentrated Loads

To find the right reaction, the center of moments is taken at the left support, and the equation of moments is

$$R_2 l - P_1 m_1 - P_2 m_2 - P_3 m_3 = 0$$

hence,
$$R_2 = \frac{P_1 m_1 + P_2 m_2 + P_3 m_3}{l} \tag{2}$$

In like manner, to find R_1 the center of moments is taken at R_2 and the equation of moments is

$$R_1 l - P_1 n_1 - P_2 n_2 - P_3 n_3 = 0$$

from which
$$R_1 = \frac{P_1 n_1 + P_2 n_2 + P_3 n_3}{l} \tag{3}$$

Example 2. Suppose the beam in Fig. 2 is 20 ft in length. Let there be three concentrated loads of 500, 800 and 600 lb placed 5, 9 and 12 ft respectively from the left support. Then $l = 20$, $m_1 = 5$, $m_2 = 9$, $m_3 = 12$,

$P_1 = 500$, $P_2 = 800$ and $P_3 = 600$. Substituting in Formulas (2) and (3),

$$R_2 = \frac{500 \times 5 + 800 \times 9 + 600 \times 12}{20} = 845 \text{ lb}$$

$$R_1 = \frac{500 \times 15 + 800 \times 11 + 600 \times 8}{20} = 1\ 055 \text{ lb}$$

and

$$500 + 800 + 600 = 845 + 1\ 055 = 1\ 900 \text{ lb}$$

To find the reactions for a combination of uniformly distributed and concentrated loads, to each of the reactions obtained by Formulas (1) or (2) for the concentrated loads, add one-half the distributed load. Thus, suppose the 20-ft beam in this example weighs 40 lb per linear ft. This is considered as a uniformly distributed load and for the entire beam it is 40 lb $\times$ 20 = 800 lb. By the rule, one-half of this is added to each reaction, so that the total reactions are, $R_2 = 845 + 400 = 1\ 245$ lb and $R_1 = 1\ 055 + 400 = 1\ 455$ lb.

Example 3. For a distributed load applied over only a part of the span, as in Fig. 3, assume the load to be CONCENTRATED AT THE MIDDLE of the part over

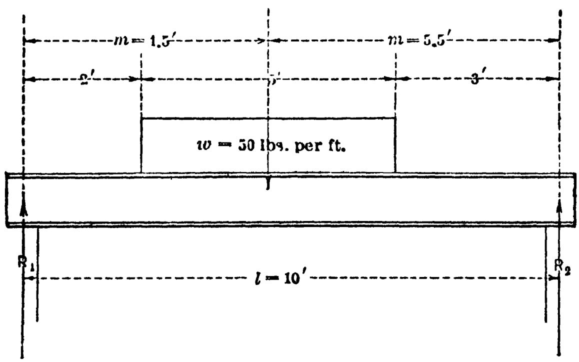

Fig. 3. Simple Beam. Distributed Load over Part of Span

which it acts and use Formulas (1) and (1)′. For example, let w (Fig. 3) equal 50 lb per linear ft, applied for a distance of 5 ft over the beam. Then W, the total load, is 50 lb $\times$ 5 = 250 lb. This may be assumed to be concentrated at its center, 4.5 ft from the left support. Then $P = 250$, $m = 4.5$ and $n = 5.5$; and from Formulas (1) and (1)′,

$$R_1 = \frac{250 \times 5.5}{10} = 137.5 \text{ lb}$$

and

$$R_2 = \frac{250 \times 4.5}{10} = 112.5 \text{ lb}$$

Therefore, for any combination of concentrated and uniform loads distributed over the entire beam, or over only part of it, find the reactions due to the concentrated loads by Formulas (1) or (2), and to them add the reactions due to the uniformly distributed loads.

2. Bending Moments in Cantilever and Simple Beams *

Definitions. The bending moment is a measure of the tendencies of forces to break a beam by BENDING or FLEXURE. Fig. 4 shows the manner in which a simple beam, supported at the ends, breaks when subjected to a load greater than it can bear. The effect of a load upon a beam is to cause it to SAG, or BEND. The bending of the beam shortens, or compresses, the upper fibers and stretches, or elongates, the lower fibers. So long as the resistance of the fibers to shortening, or compression, and to stretching, or tension, is greater than the tendency of the load to disrupt them, the beam carries the load; but, when the load causes a greater tension, or compression, on the fibers than they are capable of resisting, the beam breaks. The stretching of the fibers before breaking allows the beam to bend; hence, the name BENDING MOMENT has been given to the forces causing a beam to BEND and perhaps ultimately to BREAK.

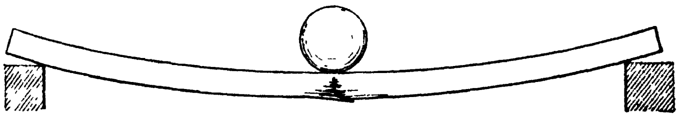

Fig 4 Manner of Rupture of Simple Beam

In order to calculate the FLEXURAL STRENGTH OF A BEAM, it is necessary to ascertain the nature and extent, first, of the EXTERNAL FORCES acting to break the beam, and secondly of the INTERNAL FORCES OR STRESSES tending to resist rupture † The external forces tending to break the beam by flexure are the DOWNWARD LOADS and the UPWARD REACTIONS. Each acts with a LEVERAGE equal to the perpendicular distance from its LINE OF ACTION to the section at which the beam tends to break. The algebraic sum of the moments of these external forces on the left, or right, of any section is called the BENDING MOMENT for that section, since it is the MOMENT OF THE RESULTANT OF THE FORCES which tends to bend the beam at that section. It is generally designated by M. Then, from the definition, the BENDING MOMENT for any section of a beam resting on two supports and in a state of flexure under a load or loads is M = the moment of either reaction minus the sum of the moments of the loads between that reaction and the section. The moment of the reaction is UPWARD, or POSITIVE, and the moment of any load DOWNWARD, or NEGATIVE, if the part of the beam on the left of the section is considered.

3. Bending Moments in Beams for Different Kinds of Loading

Case I

Beam Fixed at One End and Loaded with a Concentrated Load *P*, Near the Free End (Fig. 5).

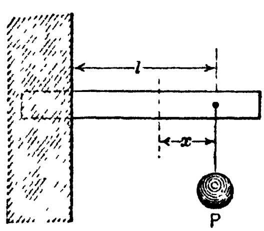

Fig. 5. Cantilever Beam. Concentrated Load near Free End

Maximum bending moment, at wall $= P \times l$
Bending moment at any other section $x = Px$

Note. If l is in feet, the bending moment will be in foot-pounds; if l is in inches, the bending moment will be in inch-pounds.

* See, also, Chapter XV.
† See Chapter X for a discussion of these internal stresses and of the resisting moment.

Case II

Beam Fixed at One End and Loaded with a Uniformly Distributed Load W (Fig. 6).

Maximum bending moment, at wall $= W \times l/2$

At any other section x, $M = wx \times x/2 = wx^2/2$

Note. $W = wl$ and $w =$ the load per unit of length.

Case III

Beam Fixed at One End and Loaded with Both a Concentrated and a Uniformly Distributed Load (Fig. 7).

Maximum bending moment, at wall $= P \times l_2 + W \times l_1/2$

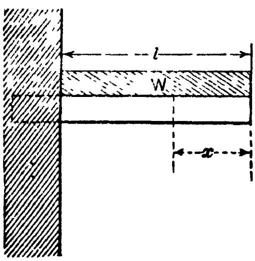

Fig. 6. Cantilever Beam. Uniformly Distributed Load

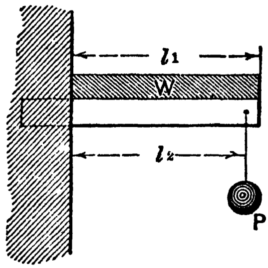

Fig. 7. Cantilever Beam. Distributed Load and Load at Free End

Case IV

Beam Supported at Both Ends and Loaded with a Concentrated Load at the Middle (Fig. 8).

Maximum bending moment, under the load $= Pl/4$

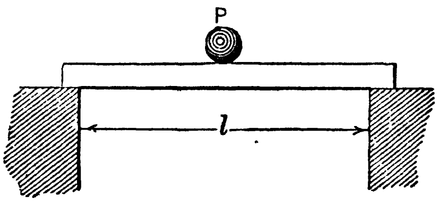

Fig. 8. Simple Beam. Concentrated Load at the Middle

Case V

Beam Supported at Both Ends and Loaded with a Uniformly Distributed Load W (Fig. 9).

Maximum bending moment, at the middle $= Wl/8$

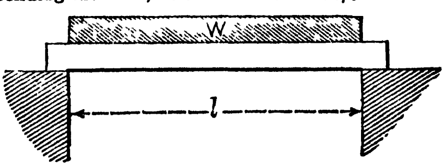

Fig. 9. Simple Beam. Uniformly Distributed Load

Case VI

Beam Supported at Both Ends and Loaded with a Concentrated Load not at the Middle (Fig. 10).

Maximum bending moment, under the load = *Pmn/l*

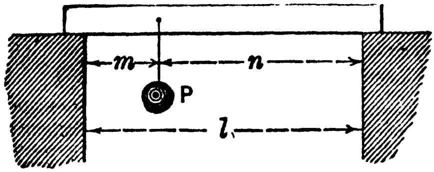

Fig. 10. Simple Beam. Concentrated Load not at the Middle

Case VII

Beam Supported at Both Ends and Loaded Symmetrically with Two Equal Concentrated Loads (Fig. 11).

Maximum bending moment = Pm and is the same for any section of the beam between the two loads.

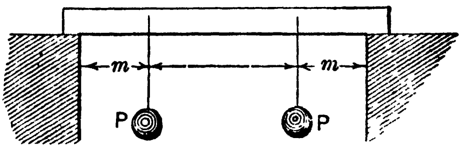

Fig. 11. Simple Beam. Two Concentrated Loads Symmetrically Placed

From these examples it will be seen that all the quantities which enter into the computation of the bending moment are the load, the span and the distance of the point of application of the load from the center of moments.

Case VIII

Beam Supported at Both Ends and Loaded with a Distributed Load Over Part of the Span (Fig. 12).

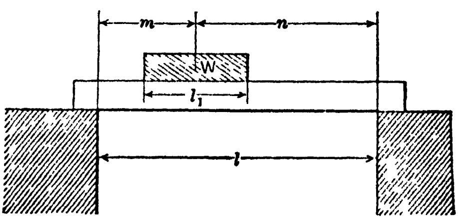

Fig. 12. Simple Beam. Distributed Load over Part of Span

If assumed under the center of the load, $M^*\text{max} = Wmn/l - Wl_1/8$
When m and n are equal the bending moment $= W \times l/4 - W \times l_1/8$

* This is only approximately correct when m and n are unequal. For the exact value, find the section of zero shear; the maximum bending moment will be at that section.

Example 4. In Fig. 12 let $W = 800$ lb, $m = 8$ ft, $n = 12$ ft, $l = 20$ ft and $l_1 = 8$ ft. Then the bending moment

$$= \frac{800 \times 8 \times 12}{20} - \frac{800 \times 8}{8} = 3\,840 - 800 = 3\,040 \text{ ft-lb,} \quad \text{or} \quad 36\,480 \text{ in-lb}$$

Example 5. In Fig. 12 let $m = n = 10$ ft, $l = 20$ ft, $l_1 = 4$ ft and $W = 600$ lb. Then the bending moment

$$= \frac{600 \times 20}{4} - \frac{600 \times 4}{8} = 3\,000 - 300 = 2\,700 \text{ ft-lb,} \quad \text{or} \quad 32\,400 \text{ in-lb}$$

The Bending Moment for any Case Other than the Above may easily be obtained by the GRAPHIC METHOD, which will now be explained.

4. Graphic Method of Determining Bending Moments in Beams

Beam with One Concentrated Load (Fig. 13).

The BENDING MOMENT of a beam supported at both ends and loaded with one concentrated load may be determined GRAPHICALLY, as follows:

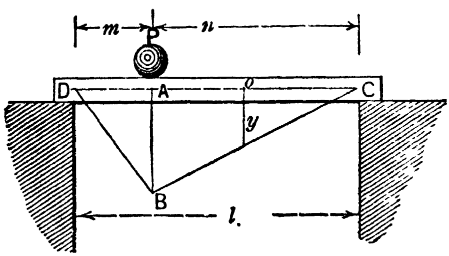

Fig. 13. Bending-moment Diagram. One Concentrated Load

Let P be the load, applied as shown. Then, by the rule under Case VI, the MAXIMUM BENDING MOMENT is under the load and $= Pmn/l$

Draw the beam, with the given span, accurately to scale, and measure down the line AB, to a scale of FOOT-POUNDS to the LINEAR INCH, a distance equal to the bending moment. Connect B with each end of the beam. To find the bending moment at any other point of the beam, as at o, draw the vertical y to BC. Its length, measured to the same scale to which AB is drawn, will give the bending moment at o. The

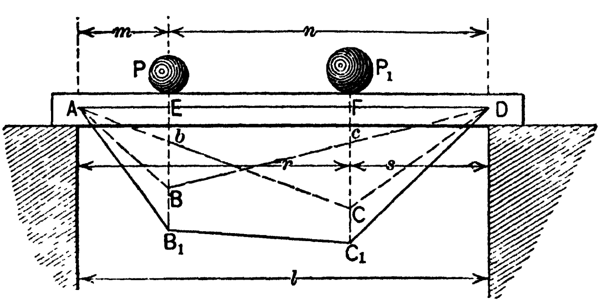

Fig. 14. Bending-moment Diagram. Two Concentrated Loads

figure $DBCAD$ is called the BENDING-MOMENT DIAGRAM and the lines BD and BC are called INFLUENCE LINES for the bending moments.

Beam with Two Concentrated Loads (Fig. 14).

To draw the bending-moment diagram for a beam with two concentrated loads, draw the dotted lines ABD and ACD, giving the BENDING-MOMENT DIAGRAMS for each load separately. EB is laid out to scale, equal to Pmn/l and FC equal to P_1rs/l.

The bending moment at the point E is equal to EB(from the load P) + Eb (from the load P_1), or $M = EB + Eb = EB_1$; and at F the bending moment is equal to $FC + Fc = FC_1$. The BENDING-MOMENT DIAGRAM for both loads is AB_1C_1D and the MAXIMUM BENDING MOMENT is, in this particular case, the line FC_1 measured to scale.

Beam with Any Number of Concentrated Loads (Fig. 15).

Proceed as in the last case, and draw the BENDING-MOMENT DIAGRAM for each load separately. Make $AD = A1 + A2 + A3$, $BE = B1 + B2 + B3$

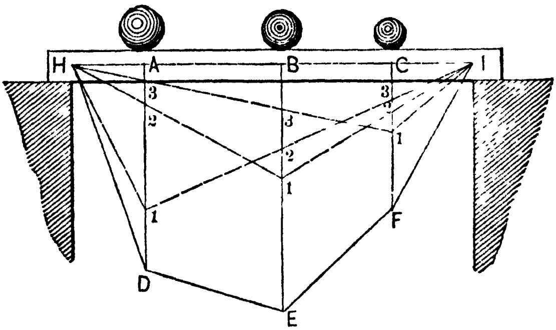

Fig. 15. Bending-moment Diagram. Three Concentrated Loads

and $CF = C1 + C2 + C3$. The figure $HDEFIH$ will then be the BENDING-MOMENT DIAGRAM corresponding to all the loads. The BENDING-MOMENT DIAGRAM for a beam with any number of concentrated loads may be drawn in the same way.

Beam with a Uniformly Distributed Load (Fig. 16).

Draw the beam with the given span, accurately to a scale as before, and at the middle of the beam draw the vertical line AB, to a scale of a certain number of FOOT-POUNDS to the LINEAR INCH, equal to $Wl/8$, from Case V, W representing the whole distributed load. Connect the points C, B, D by a PARABOLA to obtain the BENDING - MOMENT DIAGRAM. To find the bending moment at any point a, draw the vertical line ab, measure it to the same scale to which AB is drawn, and it will be the bending moment desired. Methods for drawing the PARABOLA will be found in Part I.

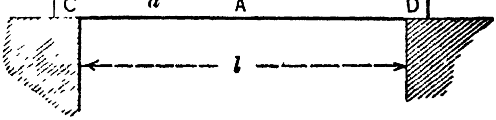

Fig. 16. Bending-moment Diagram. Distributed Load over Whole Beam

Beam Loaded with Both Distributed and Concentrated Loads (Fig. 17).

To determine the bending moments in this case, combine the BENDING-MOMENT DIAGRAMS for the concentrated loads and for the distributed load, as shown in Fig. 17. The bending moment at any section of the beam will then be limited by the line ABC on top and by the line $CDEFA$ on the bottom; and the MAXIMUM BENDING MOMENT will be the longest vertical line that can be drawn between these two bounding lines.

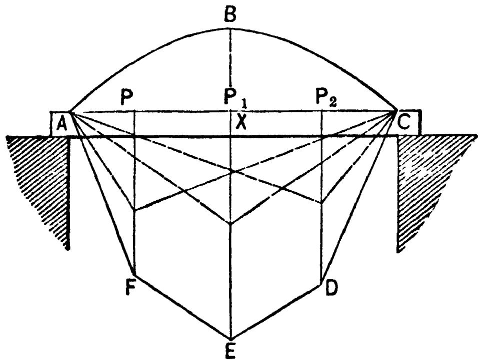

Fig. 17. Bending-moment Diagram. Distributed and Concentrated Loads

For example, the bending moment at X is BE. The point of MAXIMUM BENDING MOMENT depends upon the position of the concentrated loads and the relative magnitude of the distributed load; it may or may not occur at the middle of the beam or under one of the concentrated loads.

Example 6. What is the greatest bending moment in a beam of 20 ft span (Fig. 18), loaded with a distributed load of 800 lb, a concentrated load of 500 lb 6 ft from one end, and a concentrated load of 600 lb 7 ft from the other end?

Solution. (1) The maximum bending moment due to the distributed load, from Case V, is $Wl/8$, or $800 \times 20/8 = 2\ 000$ ft-lb. Lay off vertically over the middle of the beam, and at any convenient scale, say 4 000 ft-lb to the inch, $B1 = 2\ 000$ ft-lb, and draw a parabola through the points A, B and C.

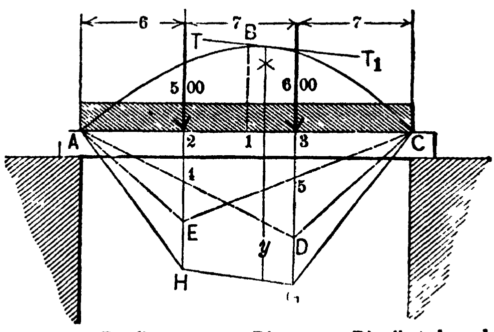

Fig. 18. Bending-moment Diagram. Distributed and Concentrated Loads

(2) The maximum bending moment for the concentrated load of 500 lb, from Case VI, is $500 \times 6 \times 14/20$, or 2 100 ft-lb. Draw $E2 = 2\ 100$ ft-lb to the same scale as $B1$, and then draw the lines AE and CE.

(3) The maximum bending moment for the concentrated load of 600 lb, in like manner, is $600 \times 7 \times 13/20$, or 2 730 ft-lb. Draw $D3 = 2\ 730$ ft-lb and connect D with A and C.

(4) Make EH equal to the distance from 2 to 4, and DG to the distance from 3 to 5, and draw $AHGC$.

The MAXIMUM BENDING MOMENT will be represented by the longest vertical line which can be drawn between the parabola ABC and the broken line $AHGC$. In this example the longest vertical line which can be drawn is Xy, and it should scale 5 645 ft-lb.

The position of the line Xy is determined by drawing the line TT_1 parallel to HG and tangent to ABC. Draw Xy vertically through point of tangency.

5. Reactions and Bending Moments for Beams with Triangular Loading and for Beams Fixed at Both Ends *

Beams with Triangular Loading have reactions and bending moments as follows:

Beam Supported at Both Ends, Fig. 19 (a)

End-reactions: $R_1 = R_2 = \frac{1}{2} W$

Bending moment at any point $= Wx(\frac{1}{2} - 2x^2/3l^2)$

Maximum bending moment, at center $= Wl/6$

Beam Supported at Both Ends, Fig. 19 (b)

End-reactions:

$R_1 = W/3 + Wb/3l$; $R_2 = \frac{2}{3} W - Wb/3l$

Bending moment at left of apex

$$= Wx\left(\frac{l^2 - b^2 - x^2}{3l(l - b)}\right)$$

Bending moment at right of apex

$$= \frac{Wx_1}{3}\left(2 - \frac{b^2 + x_1^2}{bl}\right)$$

Cantilever Beam, Fig. 19 (c)

Reaction: $R_1 = W$

Bending moment at any point $= Wx^3/3l^2$

Maximum bending moment (at R_1) $= Wl/3$

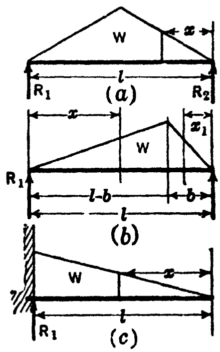

Fig. 19. Triangular Loading on Beams

Beams of Cases IV, V, and VI, with Fixed Ends, have reactions and bending moments as follows:

Case IV A. Beam Fixed at Both Ends, with a Concentrated Load P at the Middle (Fig. 8)†

End-reactions: $R_1 = R_2 = \frac{1}{2} P$

Maximum positive bending moment, under the load $= Pl/8$

Maximum negative bending moment, at ends $= Pl/8$

Case V A. Beam Fixed at Both Ends, with a Uniformly Distributed Load W (Fig. 9) †

End-reactions: $R_1 = R_2 = \frac{1}{2} W$

Maximum negative bending moment, at ends $= Wl/12$

Maximum positive bending moment, at center $= Wl/24$

Case VI A. Beam Fixed at Both Ends, with a Concentrated Load P at Distance m from Left End and Distance n from Right End (Fig. 10)†

End-reactions: $R_1 = Pn^2(3m + n)/l^3$; $R_2 = Pm^2(3n + m)/l^3$

Maximum bending moment, negative, at left end, $M_1 = Pmn^2/l^2$

at right end, $M_2 = Pm^2n/l^2$

Bending moment under load, positive $= R_1m - M_1$

* From notes by Robins Fleming.

† The figure referred to shows loading but does not show "fixed ends."

CHAPTER X

PROPERTIES OF STRUCTURAL SHAPES

MOMENT OF INERTIA, MOMENT OF RESISTANCE, SECTION MODULUS AND RADIUS OF GYRATION

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1. General Remarks

The Important Properties of Structural Sections are the area, the position of the centroid and the moment of inertia; less frequently the product of inertia, the radii of gyration, section-modulus, moment of resistance, coefficient of strength, bending factor and kern area. Of these properties the AREA of the cross-section and the POSITION OF THE CENTROID or center of gravity are familiar properties having a significance which can be seen. The other properties of a section represent entirely MATHEMATICAL ABSTRACTIONS to which it is impossible or at least useless to give any physical significance. It is important to understand this quite thoroughly. These terms originated in the theory of bending and are used in connection with this theory. Because they occur over and over again in computing stresses in beams and columns, it is convenient to give to them a definite terminology and to list them in tables to which reference can be conveniently made. Thus we avoid individual computation of each particular case.

Terminology for some of these properties is practically standard in the literature of mechanics and structural engineering.

Area is represented by A
Moment of inertia is represented by I
Section-modulus is usually represented by S
Radius of Gyration is usually represented by r or sometimes by the Greek letter ρ.

THE MOST COMMONLY USED PROPERTIES OF A SECTION, except the area and position of the centroid, are the MOMENT OF INERTIA and RADIUS OF GYRATION. The moment of inertia is used in computing stresses in beams and the radius of gyration is used in the design of columns.

2. Moment of Inertia, Product of Inertia and Radius of Gyration

The Moment of Inertia of an area about any axis is defined as the sum of the products of the differential areas, into which the section may be divided, multiplied by the squares of their distances from a given axis. Moment of inertia of a section is usually given about an axis through the centroid of the section. Evidently the section has as many moments of inertia as there are axes which can be drawn through the centroid. The moment of inertia about one of these axes is the greatest and that about some other axis the smallest.

These two axes are called the PRINCIPAL AXES of the section and are invariably at right-angles to each other. Where the section is symmetrical about any axis, this axis is one of the principal axes and the other principal axis is normal to this.

The Product of Inertia is a property less frequently used than the moment of inertia. By product of inertia about any two axes is meant the sum of the products of the differential areas, into which the section may be divided, by the products of the distances of each of the areas from each of these two axes. Products of inertia are usually referred to axes through the centroid. Evidently, there are as many products of inertia as there are pairs of axes. The product of inertia about the principal axes is always zero. Any pair of axes about which the product of inertia is zero, whether or not these axes are normal to each other, are called CONJUGATE AXES. The principal axes are that pair of conjugate axes which are normal to each other.

The relations of the moments of inertia about any two axes and of the product of inertia about these axes to the corresponding quantities for any other pair of axes may be traced out in various mathematical ways. These relations are sometimes developed by graphical constructions known as the CIRCLE OF INERTIA and the ELLIPSE OF INERTIA. The relations of certain properties of the circle and of the ellipse are used to deduce from known values certain derived values. The product of inertia ard the graphical constructions involved in the circle of inertia and in the ellipse of inertia are so infrequently needed in structural analysis that they are rarely important to the designer.

For sections most frequently used, the moments of inertia about the principal axes are given in Tables I to IX.

Parallel Axis Rule· The moment of inertia of a figure about any axis equals the moment of inertia of the figure about an axis through its centroid parallel to the given axis, plus the product of the area of the figure by the square of the distance of its centroid from the given axis. This may be written

$$I_x = I_0 + Ax^2 \tag{1}$$

in which I_x is the moment of inertia of the area about axis xx;
I_0 is the moment of inertia of the area about an axis parallel to xx through the centroid of the area;
A is the area;
x is the perpendicular distance between axis xx and the parallel axis through the centroid

$$\text{Conversely, we may write } I_0 = I_x - Ax^2 \tag{2}$$

From these two formulas we can deduce either the moment of inertia about an axis through the centroid from a given value of the moment of inertia about any other axis or we may deduce the moment of inertia about any other axis from a given moment of inertia through the centroid. These two rules enable us to deduce readily any desired moment of inertia from the tabulated values of area, position of centroid and centroidal moment of inertia of standard shapes. For application of these formulas, see Examples 6, 7 and 8 at the end of the chapter.

The Radius of Gyration is entirely a mathematical abstraction which is defined as the square root of the quotient obtained by dividing the moment of inertia by the area. The term is used and has been tabulated

because this quotient $\frac{I}{A}$ occurs so frequently in analyses involving flexure and especially in evaluating the buckling tendency of columns. Just as there may be an unlimited number of centroidal moments of inertia, so there may be an unlimited number of radii of gyration. It is therefore necessary to state the axis about which the radius of gyration is given. Radii of gyration about the principal axes of the section will be respectively the least and greatest radii of gyration. In column computations the LEAST RADIUS OF GYRATION is the one to be used in determining the allowable axial stress.* The principal axes of the section are usually obvious because of the symmetry of the section.

The Moment of Inertia of Two Areas about the Centroid of these Areas may be computed directly from the formula

$$I = \frac{A_1 A_2}{A_1 + A_2} d^2 + \Sigma I_0 \tag{3}$$

in which I is the moment of inertia of the areas about the centroid of the two areas;
A_1 and A_2 are the two areas;
d is the distance between the centroids of the areas;
ΣI_0 is the sum of the centroidal moments of inertia of the areas

For applications of this formula, see Examples 2, 3 and 4 at the end of the chapter.

3. Section-Modulus, Moment of Resistance, Coefficient of Strength

These terms are all used in the theory which deals with the design of beams. The SECTION-MODULUS is strictly a function of the dimensions of the section. The other two involve also the strength of the material of which the section is composed.

Section-Modulus is the quotient got by dividing the moment of inertia about any axis through the centroid (I) by the distance from this axis to the most remote fiber on either side of the axis (c).

$$S = \frac{I}{c} \tag{4}$$

The beam formula † may be written

$$M = Ss \tag{5}$$

in which M is the bending moment on the section in inch-pounds;
S is the section-modulus of the section;
s is the bending stress in the outermost fiber

Evidently unsymmetrical sections, in which the centroid is not equally distant from the outermost fiber above and below, have two section-moduli about any centroidal axis, each corresponding to fibers at one edge of the section.

Since the section-modulus has been defined as $\frac{I}{c}$ and also as $\frac{M}{s}$, its value

* See Chapter XIV.
† See Chapter XV.

may be deduced either by computing the moment of inertia and dividing by the distance c or in some cases conveniently by computing the moment which could be carried by the cross-section with unit stress intensity in the outer fiber.

The Moment of Resistance of a section is obtained by multiplying the allowable fiber stress by the section-modulus. That is, $M = sS$. Because the computation occurs so frequently, it is convenient to tabulate the values for certain standard allowable fiber stresses. Tables III to IX give moments of resistance for various standard sections for an allowable fiber stress in the outer fiber of 18 000 lb per sq in.

The Coefficient of Strength may be defined as the total load on a span one foot long which would produce a bending moment equal to the moment of resistance permitted for the section. The product (WL) of load by span in feet may be computed for any given span and a beam selected having this coefficient of strength. The values of the coefficient of strength are tabulated and given in some handbooks for various beam sections. They have been omitted here because of the greater convenience of determining beam sizes by other methods.

4. Kern. Bending Factor

The kern of a section is the area within which a compressive load must act in order to give only compression over the cross-section. It is a rather complicated and somewhat needless conception but is useful where the kern distances are known by inspection or where they have been computed and tabulated. The kerns of two standard types of sections should be remem-

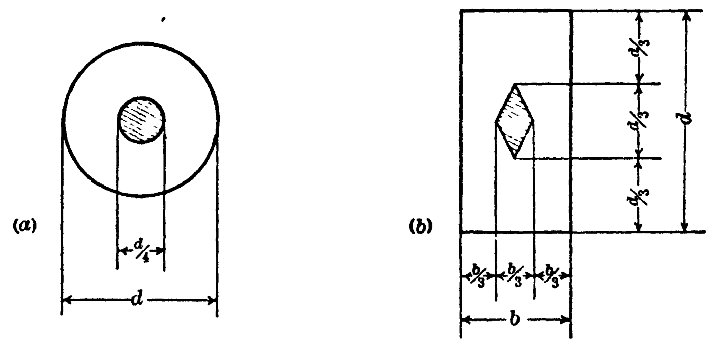


FIG 1. Kerns of Circular and Rectangular Sections

bered. For RECTANGULAR SECTIONS the kern is a lozenge-shaped figure whose apices lie at distances one-third of the width from the outside of the section, as shown in Fig. 1b. For a CIRCULAR SECTION the kern is a circle about the center of the section having a diameter equal to one-fourth the diameter of the section. (See Fig. 1a.)

In sections subject to compression where tension must be avoided, as in masonry structures, the resultant pressure on any plane section should lie within the kern of that section.

The kern is sometimes useful in computing stresses due to a combination of bending and direct stress. The stress in the outermost fiber may be

computed by a formula similar to the beam formula, sometimes known as the KERN FORMULA, which may be written as follows:

$$M_k = sS \qquad (6)$$

in which M_k is the moment of the resultant of the external forces on one side of the section about the point on the kern corresponding to the outermost fiber;

S is the section-modulus for this fiber;

s is the stress in the outermost fiber

Some engineers find these conceptions useful. Thus on a rectangular section subject to an eccentric load it is sometimes convenient to take moments about the opposite edge of the middle third of the section and divide by the section-modulus to get the fiber stress. But it is never necessary to use the kern and it is often not especially convenient.

The bending factor for a section is a constant by which moments on the section may be divided to deduce equivalent axial loads. This means that if a section is subject to bending, the bending moment may be divided by the bending factor for the section to get an equivalent axial load, and the required area of cross-section then deduced by dividing this equivalent load by the allowable fiber stress. The bending factor may be computed as $\frac{S}{A}$, the quotient obtained by dividing the section-modulus by the area.

The bending factor is useful in the design of columns.* The column is ordinarily designed primarily for an axial load. The column having been so designed is sometimes known to be subject to a certain bending moment. From this bending moment the equivalent axial load is deduced and from this is deduced the additional area needed to provide for the bending. Bending factors for various sections are given in the column tables in Chapter XIV, where their use is explained.

5. Graphical Analysis of Section

The moment of inertia of an irregular section may be obtained graphically as shown in Fig. 2. Let it be required to determine graphically the moment of inertia of the section shown about a vertical axis through its centroid.

(1) Divide the section into a number of small vertical strips of equal width. For accuracy, twice as many sections should be used as are shown in Fig. 2.

(2) Lay off vertically to some scale along line $a \ldots g$ the areas of these strips.

(3) Draw verticals $f_1\, f_2 \ldots f_7$ through the mid-points of the strips.

(4) Choose any point O called a POLE, at some convenient horizontal distance H called the POLE DISTANCE from ab.

(5) Draw any STRING hs parallel to the first RAY Oa.

(6) From the intersection of this string with f_1, draw a line parallel to Ob to intersect f_2, and continue this process until the last string intersects f_7, the last vertical. Draw through this intersection on the last string ks parallel to the last ray Og, thus completing the STRING POLYGON † ksh.

(7) THE CENTROID OF THE FIGURE is vertically above s, the intersection of the first and last strings of the polygon.

* See Chapter XIV.

† Called also equilibrium polygon or funicular polygon.

(8) THE MOMENT OF INERTIA of the figure about the vertical axis through the centroid equals the shaded area (in this figure designated as A), enclosed within the string polygon multiplied by the pole distance H.

$$I = HA$$

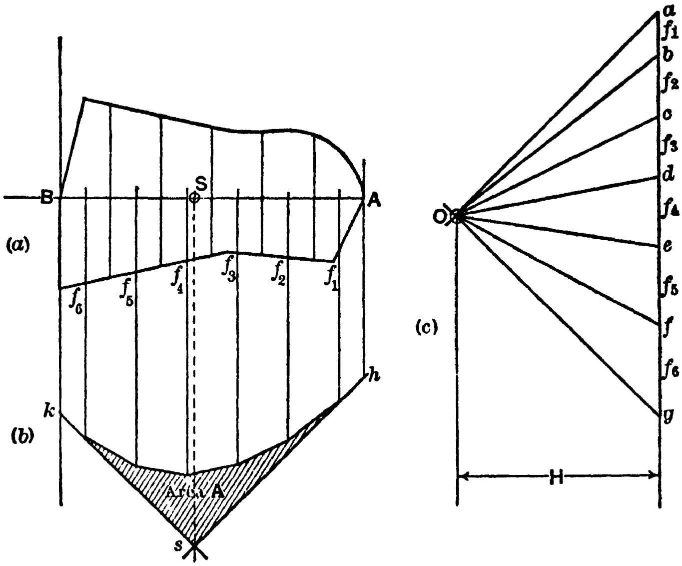


FIG. 2. Graphical Determination of the Properties of an Irregular Section

No difficulty as to scale need arise. The area of the polygon is measured to the scale to which the figure was drawn, and the pole distance is measured to the scale to which the areas were laid off on line ab.

6. Commonly Used Properties

Common Geometrical Figures. The following properties of sections should be remembered.

Area	Moment of inertia about centroidal axis parallel to base	Section-modulus
Rectangle, Height d, Width b: bd	$\frac{1}{12} Ad^2$	$\frac{1}{6} Ad$
Triangle, Base b, Height d: $\frac{bd}{2}$	$\frac{1}{18} Ad^2$ (Centroid $\frac{d}{3}$ above base)	$\frac{1}{6} Ad$ (for base); $\frac{1}{12} Ad$ (for vertex)
Circle, Diameter d: $\frac{\pi d^2}{4}$	$\frac{1}{16} Ad$ (About any diameter through centroid)	$\frac{1}{8} Ad$

From these properties, the properties of common combinations of geometrical figures may be readily computed. Illustrations are given in Examples 1, 2 and 3 at the end of the chapter.

Approximate Values of S for Steel I Beams: The section-modulus of a rolled steel beam is approximately $\frac{9}{8}$ of the weight per foot times the depth in feet. While the rule is very rough, it is sometimes useful. From the section-modulus the moment of inertia may be got by multiplying by half the depth of the beam in inches.

Approximate Values for Radii of Gyration for various types of figures are shown in Chapter XIV. These approximate values are useful in estimating the radius of gyration of columns in order to compute allowable fiber stresses prior to final design of the column section.

Section-Modulus to Be Deducted for Rivet Holes. It is customary to compute the section-modulus of the net section of girders in applying the beam formula to them. The net section is obtained by subtracting the rivet holes from the gross section. It is often convenient to compute the section-modulus of the holes directly and subtract this from the section-modulus of the gross section. If the holes are symmetrically placed on the two sides of a symmetrical section, as is usually the case, the section-modulus to be deducted for rivet holes may be computed as

$$S = A\left(\frac{d_1}{d}\right)^2 d \tag{7}$$

in which S is the section-modulus to be deducted;
A is the area of the holes in one flange;
d_1 is the distance between centers of holes in upper and lower flanges;
d is the overall depth of the section

In applying this formula it is convenient to multiply the thickness of each hole or row of holes by $\left(\frac{d_1}{d}\right)^2$, add these products and then multiply by the width of the holes times the value of d. See Example 5 at the end of the chapter, also Example 1 in Chapter XV and Examples 1 and 2 in Chapter XIX.

7. Compound Shapes

A very common problem is to determine the moment of inertia or radius of gyration or section-modulus of a section built up of several elementary shapes. A certain order is preferably followed in such computations. That recommended here is as follows: (*a*) Select a convenient trial axis. This will be taken through the centroid if the position of this is evident by inspection. (*b*) Tabulate for each member its area (A), the distance (x) of its centroid from the trial axis of the figure and its moment of inertia about its own centroid (I_0). These values can be had from data in the tables, which give areas, position of centroid and centroidal moment of inertia for standard sections. Call values of x positive when measured above or to the right of the trial axis. (*c*) Compute Ax. Compute Ax^2 and set above the values for I_0. (*d*) Add all figures in the columns for area, for Ax and for Ax^2 plus I_0. This gives the area, the statical moment about the trial axis and the moment of inertia about the trial axis of the section. (*e*) Divide the statical moment by the area to get the distance from the trial axis to the true centroidal axis (x). (*f*) Multiply the area by the square of this distance

and subtract from the moment of inertia just found. This gives the moment of inertia (I) about the centroidal axis. Equation 2. (g) Section-modulus (S) may now be deduced by dividing this moment of inertia by the distance from the centroidal axis to the outermost fiber. (h) The radius of gyration (r) may be deduced as the square root of the quotient obtained by dividing the moment of inertia I by the area A.

This procedure is indicated in tabulated form in Fig. 3, and its application is illustrated in Examples 6, 7 and 8 at the end of this chapter and in Example

	Member	Area (A)	Distance of Centroid from Trial Axis (x)	Values of Ax	Values of Ax^2 and of I_o
(d)	Totals	A		Ax	Ax^2+I_o

(e) $\bar{x}=\dfrac{\text{Total } Ax}{\text{Total } A}$ Subtract: $(\text{Total } A)\bar{x}^2$

(f) -- I

(g) $S=\dfrac{I}{c}$

(h) $r=\sqrt{\dfrac{I}{A}}$

FIG. 3. Tabular Form for Analyzing Compound Sections

2, Chapter XIX. Of course in those cases where the centroid of the section is evident by inspection, computations for its position are not required.

8. Examples

The following examples illustrate common problems in computation of the properties of sections. The solution of all such problems may be effected by use of the tables and application of the method explained for compound shapes. In many cases, however, the expressions for the moment of inertia of two areas and the expression for computing the section-modulus to be deducted for rivet holes are very convenient. The application of these expressions is shown by several examples.

Example 1. Find the moment of inertia, section-modulus, radius of gyration and bending factor of the hollow cast-iron column shown. See Fig. 4.

For the full rectangle,

$A = 6'' \times 6'' = 36\ \square''$ $I = \frac{1}{12}\ 36 \times 6^2 = 108.00$

For the hole,

$A = 5\frac{1}{2}'' \times 5\frac{1}{2}'' = 30.25\square''$ $I = \frac{1}{12}\ 30.25 \times \overline{5.5}^2 = 76.25$

Total A = 5.75 I = 31.75

Section-modulus, $S = \dfrac{I}{c} = \dfrac{31.75}{3} = 10.58$ (Formula 4)

Radius of gyration, $r = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{31\ 75}{5.75}} = 2.35$

Bending factor, $k = \dfrac{S}{A} = \dfrac{10\ 58}{5.75} = 1.84$

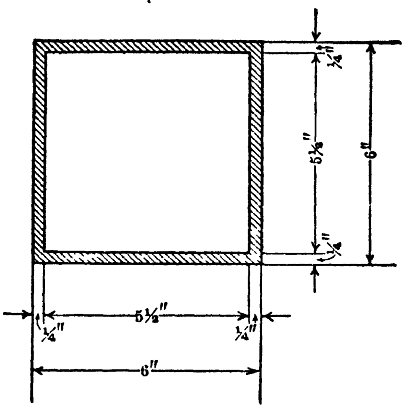


Fig. 4. Example 1

Example 2. Find the section-moduli for a trapezoid having bases of 12 in and 8 in and an altitude of 6 in See Fig 5

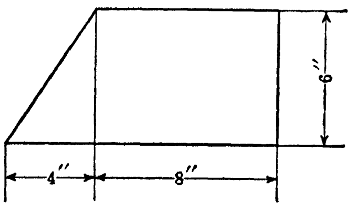


Fig. 5. Example 2

Rectangle 8 in × 6 in

$A = 48$ $\qquad I = \frac{1}{12}\, 48 \times 6^2 =$ 144

Triangle 4 in × 6 in

$A = 12$ $\qquad I = \frac{1}{18}\, 12 \times 6^2 =$ 24.

Distance between centroids $\frac{6}{2} - \frac{6}{3} = 1$ in

$$\frac{12 \times 48 \times 1^2}{12 + 48} = 9.6$$

From Equation (3), Centroidal $I =$ 177.6

From centroid to long base $= \dfrac{12 \times \frac{6}{3} + 48 \times \frac{6}{2}}{12 + 48} = 2.8$ in

$$S = \frac{177.6}{2.8} = 63.5$$

From centroid to short base $= 6 - 2.8 = 3.2$ in

$$S = \frac{177.6}{3.2} = 55.5$$

Example 3. Find the section-moduli of the section shown in Fig. 6.

Vertical area 1 in $\times$ 6 in, Area $= 6.00$

$$I_0 = \tfrac{1}{12}\, 6 \times 6^2 = \qquad 18.00$$

Horizontal area 1 in $\times$ 8 in, Area $= 8.00$

$$I_0 = \tfrac{1}{12}\, 8 \times 1^2 = \qquad 0.7$$

Distance between centroids $= 3.5$ in

$$\frac{6 \times 8}{6 + 8}\, \overline{3.5}^2 = \qquad 42.0$$

From equation (3), Total I = 60.7

Centroid, $\dfrac{8}{6 + 8} 3.5 = 2.0''$ below center of vertical area

For top fibers $c = 5.0''$ $\quad S = \dfrac{60.7}{5} = 12.1$

For bottom fibers $c = 2.0''$ $\quad S = \dfrac{60.7}{2} = 30.3$

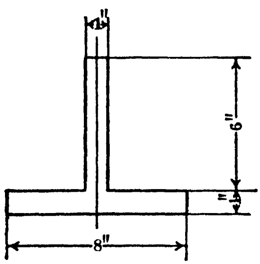


FIG 6. Example 3

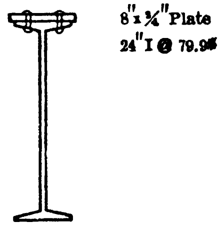


FIG. 7 Example 4

Example 4. Find the centroidal moment of inertia of the section shown in Fig 7.

For the beam $A = 23.33$ $\qquad I = 2\,087.2$ (From Table IV)
For the plate $A = 8 \times \tfrac{3}{4} = 6.00$ $\qquad I =$ ——— (negligible)
The distance between centroids is $12\tfrac{3}{8}$ in

$$\frac{23.33 \times 6.00}{23.33 + 6.00} \times \overline{12.375}^2 = \quad 730$$

From Equation (3) $\qquad$ Total $I = 2\,817$

Example 5. Find the section-modulus to be deducted for rivet holes in the plate girder shown in Fig 8, having

1 web $30'' \times \tfrac{3}{8}''$

4 angles $6'' \times 4'' \times \tfrac{3}{4}''$

2 cover plates $14'' \times \tfrac{1}{2}''$

The angles are 30.5 in back to back, the gauge is $2\frac{1}{4}$ in. Hence the inner rivets are 26.0 in apart. The out-to-out depth is 31.5 in and the outer rows

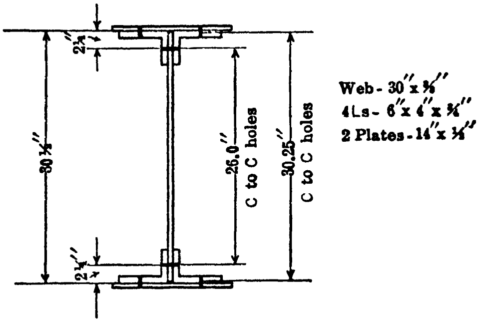


FIG 8. Example 5

of rivets are 30.25 in on centers. Assuming $\frac{3}{4}$-in diameter rivets and $\frac{7}{8}$-in diameter holes, apply Formula (7).

Multiply the thickness of the holes by $\left(\frac{d_1}{d}\right)^2$

For the inner row $1\frac{7}{8} \times \left(\frac{26.0}{31.5}\right)^2 = 1.27$

For the outer rows $2 \times 1\frac{1}{4} \times \left(\frac{30\ 25}{31\ 5}\right)^2 = 2.31$

$3\ 58$

Section-modulus to be subtracted, $3.58 \times \frac{7}{8} \times 31.5 = 99$

Example 6. Compute the moment of inertia of the section shown in Fig. 9.

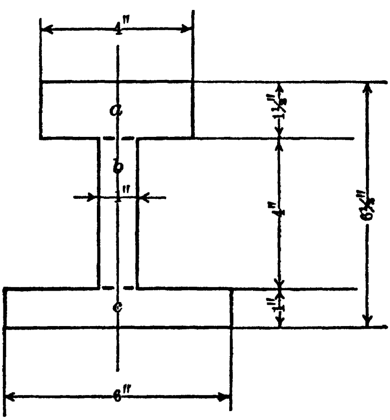


FIG. 9. Example 6

Divide the section into three areas *a, b, c,* as shown, assume a trial axis through the center of section *b*

	A, Area	x	Ax	$Ax^2 + I_0$
a	$4 \times 1\frac{1}{2} = 6$	$+2.75$	$+16.5$	$6 \times \overline{2.75}^2 = 45.5$ $\frac{1}{12}\, 6 \times (\frac{3}{2})^2 = 1.1$
b	$4 \times 1 = 4$	0	.	0 $\frac{1}{12}\, 4 \times 4^2 = 5.3$
c	$6 \times 1 = 6$	-2.50	-15.0	$6 \times \overline{2.50}^2 = 37.5$ $\frac{1}{12}\, 6 \times 1^2 = 0.5$
	$A = 16$		$+1.5$	89.9
				$A\bar{x}^2 = 0.1$
		$x = \dfrac{+1.5}{16} = +0.09$		$I = 89.8$

Example 7. Compute the section-modulus of the net section of the plate girder shown in Fig. 10.

Web 36 in × ½ in
4 angles 6 in × 6 in × ½ in
2 cover plates 14 in × ½ in

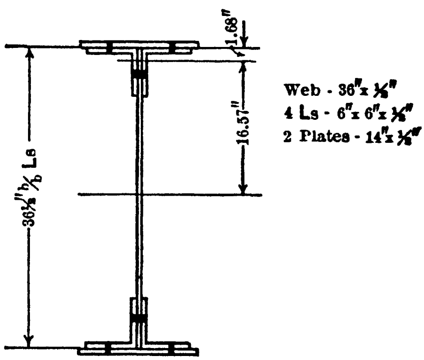


FIG. 10. Example 7

The area, the centroidal moment of inertia and the distance from the back to the centroid of a 6 in × 6 in × ½ in angle are found in Table II, as

$$A = 5.75,$$
$$I = 19.91,$$
$$x = 1.68$$

For the web in Table I we find I of plate 36 in × ½ in = 1 944.

	Area A	x	$Ax^2 + I_0$
Web 36 × ½	18.00	0	0 1 940
4 ∠s 6 × 6 × ½	23.00	16.57	6 300 80
2 Covers 14 × ½................	14.00	18.50	4 800 0
		$I =$	13 120
		$c = 18.75$	
		$S = \dfrac{13\,120}{18.75}$	$= 700.3$ gross

For rivet-holes deduct (Formula (7))

$$\text{Outer holes} \quad 2 \times 1 \times \left(\frac{36.5}{37.5}\right)^2 = 1.90$$

$$\text{Inner holes} \quad 1\tfrac{1}{2} \times \left(\frac{32.0}{37.5}\right)^2 = 1.10$$

$$3.00 \times \tfrac{7}{8} \times 37.5 = \quad 98.00$$

$$\text{Net section-modulus} = 700 - 98 = \quad 602.00$$

Example 8. Find radii of gyration about both principal axes of the heavy column section shown in Fig 11

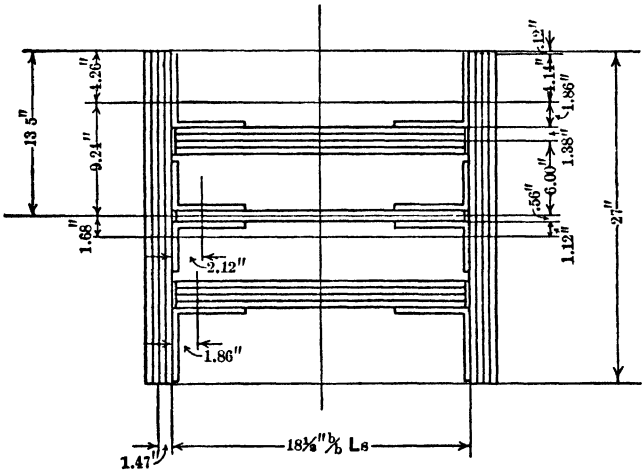


Fig. 11. Example 8

It is not necessary to treat separately each plate in a group. In the computations the layers of plates are treated as if they were one thick plate. The areas of these plates are computed. Their moments of inertia are taken from Table I. The areas, moments of inertia and distances to centroid from backs of the angles are taken from Table II

The procedure of computation follows that already explained. Two separate computations, one for properties about axis XX and the other for properties about axis YY, are here grouped in one table.

	Area	About axis XX		About axis YY	
		y	$Ay^2 + I_0$	x	$Ax^2 + I_0$
Center web 2–18 in × 9/16 in.....	20.3	0	0 0	0	0 547
Side webs 8–18 in × 11/16 in...	99.0	6 00	3 560 0	0	0 2 670
4 Center ∠s 6 in × 4 in × 7/8 in	32 0	1 68	90 40	7 13	1 630 110
4 Side ∠s 6 in × 6 in × 1 in. ...	44.0	9.24	3 760 140	7.39	2 390 140
Covers 6–27 in × 3/4 in 2–27 in × 11/16 in . .	158 6	0	0 9 640	10.72	18 400 0
	353 9		17 230		25 887

$$r = \sqrt{\frac{17\ 230}{353.9}} = 6\ 98 \text{ in} \qquad r = \sqrt{\frac{25\ 890}{353.9}} = 8.55 \text{ in}$$

9. Tables of Properties

Table I gives the moment of inertia about the centroid for plates of various widths and thicknesses. These tables are useful in computing the moments of inertia of girder and column sections. From the moments of inertia the section-moduli and radii of gyration can be computed. For thicknesses not given, add values to give total thickness.

Table II gives properties of standard angles. These are, successively, the dimensions of the section, the weight per foot, the area in square inches, the moment of inertia, the section-modulus, radius of gyration and distance to the centroid from the back of the leg for an axis through the centroid parallel to the shorter leg (axis X–X) and corresponding quantities for an axis through the centroid parallel to the longer leg (axis Y–Y) and finally the least radius of gyration (about an inclined axis Z–Z).

The properties of rolled steel sections commonly used as beams are given in

Table III, for Standard Channels
Table IV, for Standard I Beams
Table V, for Bethlehem I Beams
Table VI, for Bethlehem Girder Beams
Table VII, for Carnegie Beams
Table VIII, for Special Sections Rolled by Carnegie and by Phoenix
Table IX, for floor joists rolled by Bethlehem and by Jones and Laughlin
For a description of these sections, see Chapter XV.
For remarks on stock sizes, see Chapter XV.

The properties listed are, in order, for Standard Beams and Bethlehem Beams:

(A) Dimensions of Section

(1) The depth out to out of section in inches
(2) The weight per foot in pounds
(3) The area in square inches
(4) The flange width, b
(5) The web thickness, t
(6) The clear distance between fillets of flanges, c
(7) The standard gauge for rivet holes in the flanges, g
(8) The thickness of the flange at center of rivet hole, u

(B) Properties of Section

(9) The moment of inertia about the major axis
(10) The section-modulus about the major axis
(11) The radius of gyration about the major axis
(12) The moment of inertia about the minor axis
(13) The section-modulus about the minor axis
(14) The radius of gyration about the minor axis

(C) Strength of the Section as a Beam

(15) The bending moment in inch-pounds which the beam will carry with an extreme fiber stress of 18 000 lb per sq in
(16) The allowable shear on the section at 12 000 pounds per sq in maximum intensity
(17) The permissible reaction for web buckling if the beam has an end bearing 3½ in long
(18) The permissible load on one standard connection such as is shown in Table IV, Chapter XV.

Slight changes in the arrangement of these data occur for the other sections. It has been thought best to condense into these tables all information needed for the design of these sections. The use of the properties just listed under (*C*) and Strength of the Sections are explained in Chapter XV.

Table I.* Moments of Inertia of Rectangles

Axis through center and normal to depth

Depth in inches	Widths of rectangles in inches						
	1/4	5/16	3/8	7/16	1/2	9/16	5/8
2	0.17	0.21	0.25	0.29	0.33	0.38	0.42
3	0.56	0.70	0.84	0.98	1.13	1.27	1.41
4	1.33	1.67	2.00	2.33	2.67	3.00	3.33
5	2.60	3.26	3.91	4.56	5.21	5.86	6.51
6	4.50	5.63	6.75	7.88	9.00	10.13	11.25
7	7.15	8.93	10.72	12.51	14.29	16.08	17.86
8	10.67	13.33	16.00	18.67	21.33	24.00	26.67
9	15.19	18.98	22.78	26.58	30.38	34.17	37.97
10	20.83	26.04	31.25	36.46	41.67	46.87	52.08
11	27.73	34.66	41.59	48.53	55.46	62.39	69.32
12	36.00	45.00	54.00	63.00	72.00	81.00	90.00
13	45.77	57.21	68.66	80.10	91.54	102.98	114.43
14	57.17	71.46	85.75	100.04	114.33	128.63	142.92
15	70.31	87.89	105.47	123.05	140.63	158.20	175.78
16	85.33	106.67	128.00	149.33	170.67	192.00	213.33
17	102.35	127.94	153.53	179.12	204.71	230.30	255.89
18	121.50	151.88	182.25	212.63	243.00	273.38	303.75
19	142.90	178.62	214.34	250.07	285.79	321.52	357.24
20	166.67	208.33	250.00	291.67	333.33	375.00	416.67
21	192.94	241.17	289.41	337.64	385.88	434.11	482.34
22	221.83	277.29	332.75	388.21	443.67	499.13	554.58
23	253.48	316.85	380.22	443.59	506.96	570.33	633.70
24	288.00	360.00	432.00	504.00	576.00	648.00	720.00
25	325.52	406.90	488.28	569.66	651.04	732.42	813.80
26	366.17	457.71	549.25	640.79	732.33	823.88	915.42
27	410.06	512.58	615.09	717.61	820.13	922.64	1 025.16
28	457.33	571.67	686.00	800.33	914.67	1 029.00	1 143.33
29	508.10	635.13	762.16	889.18	1 016.21	1 143.23	1 270.26
30	562.50	703.13	843.75	984.38	1 125.00	1 265.63	1 406.25
32	682.67	853.33	1 024.00	1 194.67	1 365.33	1 536.00	1 706.67
34	818.83	1 023.54	1 228.25	1 432.96	1 637.67	1 842.38	2 047.08
36	972.00	1 215.00	1 458.00	1 701.00	1 944.00	2 187.00	2 430.00
38	1 143.17	1 428.96	1 714.75	2 000.54	2 286.33	2 572.13	2 857.92
40	1 333.33	1 666.67	2 000.00	2 333.33	2 666.67	3 000.00	3 333.33
42	1 543.50	1 929.38	2 315.25	2 701.13	3 087.00	3 472.88	3 858.75
44	1 774.67	2 218.33	2 662.00	3 105.67	3 549.33	3 993.00	4 436.67
46	2 027.83	2 534.79	3 041.75	3 548.71	4 055.67	4 562.63	5 069.58
48	2 304.00	2 880.00	3 456.00	4 032.00	4 608.00	5 184.00	5 760.00
50	2 604.17	3 255.21	3 906.25	4 557.29	5 208.33	5 859.38	6 510.42
52	2 929.33	3 661.67	4 394.00	5 126.33	5 858.67	6 591.00	7 323.33
54	3 280.50	4 100.63	4 920.75	5 740.88	6 561.00	7 381.13	8 201.25
56	3 658.67	4 573.33	5 488.00	6 402.67	7 317.33	8 232.00	9 146.67
58	4 064.83	5 081.04	6 097.25	7 113.46	8 129.67	9 145.87	10 162.08
60	4 500.00	5 625.00	6 750.00	7 875.00	9 000.00	10 125.00	11 250.00

* This table may be used in computing the moments of inertia of plate girders, columns and other compound sections in which plates are used.

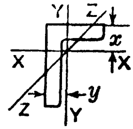
Table I * (Continued). Moments of Inertia of Rectangles

Axis through center and normal to depth

Depth in inches	Widths of rectangles in inches					
	11/16	3/4	13/16	7/8	15/16	1
2	0.46	0.50	0 54	0 58	0.63	0 67
3	1.55	1.69	1 83	1.97	2 11	2 25
4	3.67	4 00	4 33	4.67	5 00	5 33
5	7.16	7 81	8 46	9.11	9.77	10 42
6	12.38	13.50	14.63	15 75	16.88	18.00
7	19.65	21 44	23 22	25 01	26.80	28 58
8	29 33	32 00	34.67	37.33	40 00	42 67
9	41.77	45 56	49 36	53 16	56.95	60.75
10	57.29	62.50	67 71	72 92	78.13	83 33
11	76.26	83 19	90.12	97 05	103 98	110 92
12	99.00	108.00	117 00	126 00	135 00	144 00
13	125 87	137.31	148 75	160.20	171.64	183.08
14	157.21	171.50	185 79	200 08	214.38	228 67
15	193.36	210 94	228 52	246.09	263.67	281.25
16	234 67	256.00	277.33	298 67	320 00	341 33
17	281.47	307.06	332.65	358 24	383.83	409.42
18	334.13	364.50	394.88	425 25	455.63	486.00
19	392.96	428 69	464 41	500 14	535 86	571 58
20	458.33	500.00	541.67	583.33	625.00	666 67
21	530.58	578.81	627.05	675 28	723 52	771 75
22	610.04	665.50	720 96	776 42	831 87	887 33
23	697.07	760.44	823 81	887 18	950 55	1 013 92
24	792.00	864 00	936.00	1 008.00	1 080 00	1 152 00
25	895.18	976.56	1 057 94	1 139.32	1 220.70	1 302.08
26	1 006.96	1 098 50	1 190 04	1 281.58	1 373 13	1 464 67
27	1 127.67	1 230 19	1 332 70	1 435 22	1 537 73	1 640 25
28	12 57 67	1 372 00	1 486.33	1 600 67	1 715.00	1 829.33
29	1 397.29	1 524 31	1 651.34	1 778 36	1 905 39	2 032 42
30	1 546.88	1 687.50	1 828 13	1 968.75	2 109 38	2 250 00
32	1 877.33	2 048 00	2 218 67	2 389 33	2 560.00	2 730 67
34	2 251 79	2 456 50	2 661 21	2 865 92	3 070.63	3 275 33
36	2 673 00	2 916 00	3 159 00	3 402 00	3 645.00	3 888 00
38	3 143 71	3 429 50	3 715 29	4 001.08	4 286 88	4 572.67
40	3 666 67	4 000 00	4 333.33	4 666.67	5 000.00	5 333 33
42	4 244.63	4 630.50	5 016 38	5 402 25	5 788 13	6 174 00
44	4 880 33	5 324 00	5 767 67	6 211.33	6 655 00	7 098 67
46	5 576.54	6 083 50	6 590 46	7 097.42	7 604.38	8 111 33
48	6 336.00	6 912.00	7 488 00	8 064.00	8 640 00	9 216.00
50	7 161.46	7 812 50	8 463 54	9 114.58	9 765 63	10 416.67
52	8 055.67	8 788 00	9 520.33	10 252 67	10 985 00	11 717.33
54	9 021.38	9 841.50	10 661.63	11 481.75	12 301.88	13 122.00
56	10 061.33	10 976 00	11 890.67	12 805.33	13 720.00	14 634 67
58	11 178.29	12 194.50	13 210.71	14 226.92	15 243.12	16 250 33
60	12 375.00	13 500 00	14 625.00	15 750 00	16 875.00	18 000.00

* This table may be used in computing the moments of inertia of plate girders, columns and other compound sections in which plates are used.

Table II.* Standard Angles



Sizes—Weights—Areas

Technical Functions

For Allowable Loads as Beams, see Table IV, Chapter XV

Size in inches	Thickness	Weight per foot	Area in sq in	Axes X-X I	X-X S	X-X r	X-X x	Y-Y I	Y-Y S	Y-Y r	Y-Y y	Z-Z r
1¾×1¾	1/8	1.44	0.42	0.13	0.10	0.55	0.48					0.35
	3/16	2.12	0.62	0.18	0.14	0.54	0.51					0.34
	1/4	2.77	0.81	0.23	0.19	0.53	0.53					0.34
2×2	1/8	1.65	0.48	0.19	0.13	0.63	0.55					0.40
	3/16	2.44	0.72	0.27	0.19	0.62	0.57					0.39
	1/4	3.19	0.94	0.35	0.25	0.61	0.59					0.39
	5/16	3.92	1.15	0.42	0.30	0.60	0.61					0.39
2½×2	1/8	1.86	0.55	0.35	0.20	0.80	0.74	0.20	0.13	0.61	0.49	0.43
	3/16	2.75	0.81	0.51	0.29	0.79	0.76	0.29	0.20	0.60	0.51	0.43
	1/4	3.62	1.06	0.65	0.38	0.78	0.79	0.37	0.25	0.59	0.54	0.42
	5/16	4.50	1.31	0.79	0.47	0.78	0.81	0.45	0.31	0.58	0.56	0.42
2½×2½	1/8	2.08	0.61	0.38	0.20	0.79	0.67					0.50
	3/16	3.07	0.90	0.55	0.30	0.78	0.69					0.49
	1/4	4.1	1.19	0.70	0.39	0.77	0.72					0.49
	5/16	5.0	1.47	0.85	0.48	0.76	0.74					0.49
	3/8	5.9	1.73	0.98	0.57	0.75	0.76					0.48
3×2½	1/4	4.5	1.31	1.17	0.56	0.95	0.91	0.74	0.40	0.75	0.66	0.53
	5/16	5.6	1.62	1.42	0.69	0.94	0.93	0.90	0.49	0.74	0.68	0.53
	3/8	6.6	1.92	1.66	0.81	0.93	0.96	1.04	0.58	0.74	0.71	0.52
	7/16	7.6	2.22	1.88	0.93	0.92	0.98	1.18	0.66	0.73	0.73	0.52
3×3	1/4	4.9	1.44	1.24	0.58	0.93	0.84					0.59
	5/16	6.1	1.78	1.51	0.71	0.92	0.87					0.59
	3/8	7.2	2.11	1.76	0.83	0.91	0.89					0.58
	7/16	8.3	2.43	1.99	0.95	0.91	0.91					0.58
	1/2	9.4	2.75	2.22	1.07	0.90	0.93					0.58
3½×2½	1/4	4.9	1.44	1.80	0.75	1.12	1.11	0.78	0.41	0.74	0.61	0.54
	5/16	6.1	1.78	2.19	0.93	1.11	1.14	0.94	0.50	0.73	0.64	0.54
	3/8	7.2	2.11	2.56	1.09	1.10	1.16	1.09	0.59	0.72	0.66	0.54
	7/16	8.3	2.43	2.91	1.26	1.09	1.18	1.23	0.68	0.71	0.68	0.54
	1/2	9.4	2.75	3.24	1.41	1.09	1.20	1.36	0.76	0.70	0.70	0.53
3½×3½	5/16	7.2	2.09	2.45	0.98	1.08	0.99					0.69
	3/8	8.5	2.48	2.87	1.15	1.07	1.01					0.68
	7/16	9.8	2.87	3.26	1.32	1.07	1.04					0.68
	1/2	11.1	3.25	3.64	1.49	1.06	1.06					0.68
	9/16	12.4	3.62	3.99	1.65	1.05	1.08					0.68
	5/8	13.6	3.98	4.33	1.81	1.04	1.10					0.68
4×3	5/16	7.2	2.09	3.38	1.23	1.27	1.26	1.65	0.73	0.89	0.76	0.65
	3/8	8.5	2.48	3.96	1.46	1.26	1.28	1.92	0.87	0.88	0.78	0.64
	7/16	9.8	2.87	4.52	1.68	1.25	1.30	2.18	0.99	0.87	0.80	0.64
	1/2	11.1	3.25	5.05	1.89	1.25	1.33	2.42	1.12	0.86	0.83	0.64
	9/16	12.4	3.62	5.55	2.09	1.24	1.35	2.66	1.23	0.86	0.85	0.64
	5/8	13.6	3.98	6.03	2.30	1.23	1.37	2.87	1.35	0.85	0.87	0.64

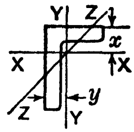
* From Steel Construction, published by American Institute of Steel Construction.

Table II * (Continued). Standard Angles

Sizes—Weights—Areas

Technical Functions

For Allowable Loads as Beams, see Table IV, Chapter XV



Size in inches	Thickness	Weight per foot	Area in sq in	Axes X-X I	X-X S	X-X r	X-X x	Y-Y I	Y-Y S	Y-Y r	Y-Y y	Z-Z r
4×3½	5/16	7.7	2.25	3.56	1.26	1.26	1.18	2.55	0.99	1.07	0.93	0.73
	3/8	9.1	2.67	4.18	1.49	1.25	1.21	2.99	1.17	1.06	0.96	0.73
	7/16	10.6	3.09	4.76	1.72	1.24	1.23	3.40	1.35	1.05	0.98	0.72
	1/2	11.9	3.50	5 32	1.94	1.23	1.25	3.79	1.52	1 04	1 00	0.72
	9/16	13 3	3.90	5.86	2.15	1.23	1.27	4.17	1.68	1.03	1 02	0.72
	5/8	14 7	4.30	6.37	2.35	1.22	1.29	4.49	1.83	1.02	1 04	0 72
	11/16	16.0	4.68	6.86	2.56	1.21	1.32	4.86	2.00	1.02	1.07	0.72
	3/4	17 3	5 06	7.32	2.75	1 20	1 34	5.18	2 15	1 01	1 09	0.72
4×4	5/16	8.2	2.40	3.71	1.29	1.24	1.12	.	..	.	..	0.79
	3/8	9.8	2.86	4.36	1.52	1.23	1.14	..	..	..	..	0 79
	7/16	11.3	3.31	4.97	1.75	1.23	1.16	.	..		..	0.78
	1/2	12 8	3.75	5.56	1.97	1.22	1.18					0 78
	9/16	14.3	4.18	6.12	2.19	1.21	1.21	..	..	.	.	0.78
	5/8	15.7	4.61	6.66	2.40	1.20	1.23	.	..	..		0.77
	11/16	17.1	5.03	7.17	2.61	1.19	1.25	..		.	.	0.77
	3/4	18 5	5.44	7 66	2.81	1.19	1.27	..		.		0 77
5×3	5/16	8.2	2.40	6.26	1.89	1.61	1.68	1.75	0.75	0.85	0.68	0 66
	3/8	9.8	2.86	7.37	2.24	1.61	1.70	2.04	0.89	0.84	0.70	0.65
	7/16	11.3	3.31	8.43	2.58	1.60	1.73	2.32	1.02	0.84	0.73	0.65
	1/2	12.8	3.75	9.45	2.91	1.59	1.75	2.58	1.15	0.83	0 75	0.65
	9/16	14.3	4.18	10.43	3.23	1.58	1.77	2.83	1.27	0.82	0.77	0.65
	5/8	15.7	4.61	11.37	3.55	1.57	1.80	3.06	1.39	0.82	0.80	0.64
	11/16	17.1	5.03	12.28	3.86	1.56	1.82	3.29	1.51	0.81	0 82	0.64
	3/4	18 5	5.44	13.15	4 16	1 55	1.84	3 51	1.62	0 80	0 84	0 64
5×3½	5/16	8.7	2.56	6.60	1.94	1.61	1.59	2.72	1.02	1.03	0 84	0.77
	3/8	10.4	3.05	7.78	2.29	1.60	1.61	3.18	1.21	1 02	0 86	0.76
	7/16	12 0	3.53	8.90	2.64	1.59	1.63	3.63	1.39	1.01	0 88	0.76
	1/2	13 6	4.00	9.99	2 99	1.58	1.66	4.05	1.56	1 01	0.91	0 75
	9/16	15.2	4.47	11.03	3.32	1.57	1 68	4.45	1.73	1.00	0.93	0.75
	5/8	16.8	4.92	12.03	3.65	1.56	1.70	4 83	1.90	0.99	0.95	0.75
	11/16	18 3	5.37	12.99	3.97	1.56	1.72	5 20	2.06	0.98	0.97	0.75
	3/4	19 8	5 81	13 92	4.28	1.55	1.75	5 55	2.22	0 98	1.00	0.75
6×3½	3/8	11.7	3.42	12.86	3.24	1.94	2.04	3.34	1.23	0.99	0 79	0.77
	7/16	13 5	3 97	14.76	3.75	1.93	2.06	3.81	1.41	0 98	0.81	0.76
	1/2	15.3	4 50	16.59	4.24	1.92	2.08	4.25	1.59	0.97	0.83	0.76
	9/16	17.1	5 03	18.37	4.72	1.91	2.11	4.67	1.77	0.96	0 86	0.75
	5/8	18.9	5.55	20.08	5.19	1.90	2.13	5.08	1.94	0.96	0.88	0.75
	11/16	20 6	6.06	21.74	5.65	1.89	3.15	5.47	2.11	0.95	0.90	0.75
	3/4	22 4	6.56	23.34	6.10	1.89	2.18	5.84	2.27	0.94	0 93	0.75
	13/16	24.0	7.06	24.89	6 55	1.88	2.20	6.20	2.43	0.94	0 95	0.75
	7/8	25 7	7.55	26 39	6.98	1.87	2 22	6.55	2.59	0.93	0 97	0 75
6×4	3/8	12.3	3.61	13.47	3.32	1.93	1.94	4.90	1.60	1.17	0 94	0.88
	7/16	14.3	4.18	15.46	3.83	1.92	1.96	5.60	1.85	1.16	0.96	0.87
	1/2	16.2	4.75	17.40	4.33	1.91	1.99	6.27	2.08	1.15	0.99	0.87
	9/16	18 1	5.31	19.26	4.83	1.90	2.01	6.91	2.31	1.14	1 01	0.87
	5/8	20.0	5.86	21.07	5.31	1.90	2.03	7.52	2.54	1.13	1.03	0.86
	11/16	21.8	6.40	22.82	5.78	1.89	2.06	8.11	2.76	1.13	1.06	0.86
	3/4	23.6	6.94	24.59	6.25	1.88	2.08	8.68	2.97	1.12	1.08	0.86
	13/16	25.4	7.47	26.15	6.70	1.87	2.10	9.23	3.18	1.11	1.10	0.86
	7/8	27.2	7.98	27.73	7.15	1.86	2.12	9.75	3.39	1.11	1.12	0.86

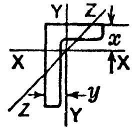
* From Steel Construction, published by American Institute of Steel Construction.

Table II * (Continued). Standard Angles

Sizes—Weights—Areas

Technical Functions

For Allowable Loads as Beams, see Table IV, Chapter XV



Size in inches	Thickness	Weight per foot	Area in sq in	X-X I	X-X S	X-X r	X-X x	Y-Y I	Y-Y S	Y-Y r	Y-Y y	Z-Z r
6×6	3/8	14.9	4.36	15.39	3.53	1.88	1.64	...	...	...	...	1.19
	7/16	17.2	5.06	17.68	4.07	1.87	1.66	...	...	...	...	1.19
	1/2	19.6	5.75	19.91	4.61	1.86	1.68	...	...	...	...	1.18
	9/16	21.9	6.43	22.07	5.14	1.85	1.71	...	...	...	...	1.18
	5/8	24.2	7.11	24.16	5.66	1.84	1.73	...	...	...	...	1.17
	11/16	26.5	7.78	26.19	6.17	1.83	1.75	...	...	...	...	1.17
	3/4	28.7	8.44	28.15	6.66	1.83	1.78	...	...	...	...	1.17
	13/16	31.0	9.09	30.06	7.15	1.82	1.80	...	...	...	...	1.17
	7/8	33.1	9.73	31.92	7.63	1.81	1.82	...	...	...	...	1.16
	15/16	35.3	10.37	33.72	8.11	1.80	1.84	...	...	...	...	1.16
	1	37.4	11.00	35.46	8.57	1.80	1.86	...	...	...	...	1.16
7×3½	3/8	13.0	3.80	19.62	4.34	2.27	2.48	3.47	1.25	0.96	0.73	0.76
	7/16	15.0	4.40	22.56	5.01	2.26	2.50	3.95	1.44	0.95	0.75	0.76
	1/2	17.0	5.00	25.41	5.68	2.25	2.53	4.41	1.62	0.94	0.78	0.75
	9/16	19.1	5.59	28.18	6.34	2.25	2.55	4.86	1.80	0.93	0.80	0.75
	5/8	21.0	6.17	30.86	6.96	2.24	2.57	5.28	1.97	0.93	0.82	0.75
	11/16	23.0	6.75	33.47	7.60	2.23	2.60	5.69	2.14	0.92	0.85	0.74
	3/4	24.9	7.31	35.99	8.22	2.22	2.62	6.08	2.31	0.91	0.87	0.74
	13/16	26.8	7.87	38.45	8.83	2.21	2.64	6.46	2.48	0.91	0.89	0.74
	7/8	28.7	8.42	40.82	9.42	2.20	2.66	6.83	2.64	0.90	0.91	0.74
	15/16	30.5	8.97	43.13	10.00	2.19	2.69	7.18	2.80	0.89	0.94	0.74
	1	32.3	9.50	45.37	10.58	2.19	2.71	7.53	2.96	0.89	0.96	0.74
8×6	7/16	20.2	5.93	39.23	7.07	2.57	2.45	19.25	4.23	1.80	1.45	1.30
	1/2	23.0	6.75	44.31	8.02	2.56	2.47	21.68	4.79	1.79	1.47	1.30
	9/16	25.7	7.56	49.26	8.95	2.55	2.50	24.04	5.34	1.78	1.50	1.30
	5/8	28.5	8.36	54.10	9.87	2.54	2.52	26.33	5.88	1.77	1.52	1.29
	11/16	31.2	9.15	58.82	10.77	2.54	2.54	28.56	6.40	1.77	1.54	1.29
	3/4	33.8	9.94	63.42	11.67	2.53	2.56	30.72	6.92	1.76	1.56	1.28
	13/16	36.5	10.72	67.92	12.55	2.52	2.59	32.82	7.44	1.75	1.59	1.28
	7/8	39.1	11.48	72.32	13.41	2.51	2.61	34.86	7.94	1.74	1.61	1.28
	15/16	41.7	12.25	76.59	14.27	2.50	2.63	36.85	8.43	1.73	1.63	1.28
	1	44.2	13.00	80.78	15.11	2.49	2.65	38.78	8.92	1.73	1.65	1.28
8×8	1/2	26.4	7.75	48.65	8.37	2.51	2.19	...	...	...	...	1.59
	9/16	29.6	8.68	54.09	9.31	2.50	2.21	...	...	...	...	1.58
	5/8	32.7	9.61	59.43	10.30	2.49	2.23	...	...	...	...	1.58
	11/16	35.8	10.53	64.64	11.25	2.48	2.25	...	...	...	...	1.58
	3/4	38.9	11.44	69.74	12.18	2.47	2.28	...	...	...	...	1.57
	13/16	42.0	12.34	74.72	13.11	2.46	2.30	...	...	...	...	1.57
	7/8	45.0	13.23	79.58	14.02	2.45	2.32	...	...	...	...	1.56
	15/16	48.1	14.12	84.34	14.91	2.44	2.34	...	...	...	...	1.56
	1	51.0	15.00	88.98	15.80	2.44	2.37	...	...	...	...	1.56
	1 1/16	54.0	15.87	93.53	16.67	2.43	2.39	...	...	...	...	1.56
	1 1/8	56.9	16.73	97.97	17.53	2.42	2.41	...	...	...	...	1.55

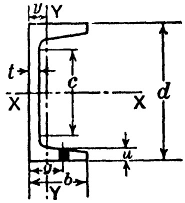
* From Steel Construction, published by American Institute of Steel Construction.

Table III.* Standard Channels

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
W is Maximum Load on one Standard Connection.
y is Distance in inches between Center of Gravity and back of Channel.

Note: The attention of the reader is called to remarks on Stock Sizes, Chapter XV.



	3 in			4 in		
Depth = *d* in	3	3	3	4	4	4
Weight per foot	4 1	5 0	6 0	5 4	6 25	7 25
Area, sq in	1 19	1 46	1 75	1 56	1 82	2 12
b in	1 41	1.50	1 60	1.58	1 65	1.72
t in	170	258	356	.180	.247	.320
c in	1 789	1 789	1.789	2 700	2 700	2 700
g in	7/8	7/8	7/8	1	1	1
u in	1/4	1/4	1/4	5/16	5/16	5/16
AXES X–X *I*	1 6	1 8	2 1	3 8	4 1	4 5
AXES X–X *S*	1 07	1 2	1.4	1 9	2 05	2 25
AXES X–X *r*	1 17	1 12	1 08	1 56	1 50	1 47
AXES Y–Y *I*	0 20	0 25	0 31	0 32	0 38	0.44
AXES Y–Y *S*	0 21	0.24	0 27	0.29	0 32	0 35
AXES Y–Y *r*	0.41	0 41	0 42	0 45	0 45	0 46
AXES Y–Y *y*	0 44	0 44	0 46	0 46	0 46	0 46
Max. Mom., in-lb.	19 200	21 600	25 200	34 200	36 900	40 500
V.	6 100	9 300	12 800	8 600	11 900	15 400
W	7 700	11 600	11 900	8 100	11 100	11 900

	5 in			6 in		
Depth = *d* in	5	5	5	6	6	6
Weight per foot	6 7	9 0	11 5	8 2	10 5	13 0
Area, sq in .	1 95	2 63	3 36	2 39	3 07	3 81
b in	1.75	1.89	2 03	1.92	2 03	2.16
t in . .	.190	.325	472	.200	.314	437
c in	3 609	3 609	3 609	4 518	4 518	4 518
g in .	1 1/8	1 1/8	1 1/8	1 1/8	1 1/8	1 3/8
u in	5/16	5/16	5/16	5/16	3/8	3/8
AXES X–X *I*	7 4	8 8	10 4	13 0	15 1	17 3
AXES X–X *S*	2 96	3 52	4 16	4 33	5 03	5 77
AXES X–X *r*.	1 95	1.83	1 76	2 34	2 22	2 13
AXES Y–Y *I*.	0 48	0 64	0 82	0.70	0 87	1.1
AXES Y–Y *S*...	0 38	0 45	0 54	0 50	0.57	0 65
AXES Y–Y *r*..	0 50	0 49	0 49	0 54	0 53	0 53
AXES Y–Y *y*	0 49	0 48	0 51	0 52	0 50	0 52
Max. Mom., in-lb.	53 300	63 400	74 900	78 000	90 200	103 800
V..........	11 400	19 500	28 500	14 400	22 600	31 500
W	8 600	11 900	11 900	9 000	11 900	11 900

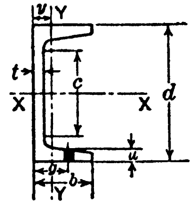
* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table III* (Continued). Standard Channels

Dimensions—Functions

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
W is Maximum Load on one Standard Connection.
y is Distance in inches between Center of Gravity and back of Channel.

Note: The attention of the reader is called to remarks on Stock Sizes, Chapter XV.



		6 in	7 in				
Depth = *d* in..		6	7	7	7	7	7
Weight per foot		15 5	9.8	12 25	14.75	17 25	19.75
Area, sq in .		4 54	2 85	3 58	4 32	5 05	5 79
b in.. .		2 28	2 09	2 19	2 30	2 40	2.51
t in ..		559	.210	.314	.419	.524	.629
c in		4 518	5 429	5 429	5 429	5 429	5.429
g in.... ...		1⅜	1¼	1¼	1¼	1½	1½
u in .		⅜	⅜	⅜	7/16	7/16	7/16
AXES X-X	*I*	19.5	21 1	24 1	27 1	30.1	33.1
	S	6 5	6 03	6 88	7 74	8 60	9 46
	r	2 07	2 72	2 59	2 51	2 44	2 39
AXES Y-Y	*I*	1 3	0 98	1 2	1 4	1 6	1 8
	S	0 73	0 63	0 71	0 79	0 86	0.96
	r .	0 53	0 59	0 58	0 57	0 56	0 56
	y	0 55	0 55	0 53	0 53	0 55	0 58
Max. Mom., in-lb.		117 000	108 500	123 900	139 400	154 800	170 200
V. . .		40 200	17 600	26 500	35 200	44 000	52 800
W		11 900	9 500	11 900	11 900	11 900	11 900

		8 in					9 in
Depth = *d* in..		8	8	8	8	8	9
Weight per foot		11 50	13 75	16.25	18 75	21.25	13 4
Area, sq in		3 36	4 02	4 76	5 49	6 23	3 89
b in ...		2.26	2.34	2 44	2 53	2 62	2 43
t in		220	.303	.395	487	579	.230
c in		6 338	6.338	6 338	6 338	6 338	7.247
g in .		1⅜	1⅜	1½	1½	1½	1⅜
u in		⅜	⅜	7/16	7/16	7/16	7/16
AXES X-X	*I*	32 3	35 8	39 8	43 7	47.6	47.3
	S .	8 08	8 95	9 95	10 92	11 9	10.51
	r	3 10	2 99	2 89	2 82	2 77	3.49
AXES Y-Y	*I*	1.3	1 5	1 8	2 0	2 2	1 8
	S..	.79	.86	94	1.0	1.1	0 97
	r ..	.63	.62	61	60	.60	67
	y.	.58	.56	56	.57	.59	.61
Max. Mom., in-lb.		145 400	161 100	179 100	196 700	214 200	189 200
V........		21 100	29 100	37 900	46 800	55 600	24 800
W.... .		19 800	23 800	23 800	23 800	23 800	20 700

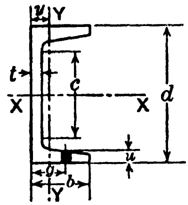
* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table III * (Continued). Standard Channels

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
W is Maximum Load on one Standard Connection.
y is Distance in inches between Center of Gravity and back of Channel.

Note: The attention of the reader is called to remarks on Stock Sizes, Chapter XV.



			9 in			10 in		
Depth = *d* in.			9	9	9	10	10	10
Weight per foot.			15 0	20.0	25.0	15.3	20.0	25 0
Area, sq in			4.39	5 86	7.33	4.47	5 86	7.33
b in.			2.49	2.65	2.81	2.60	2 74	2.89
t in.			.285	.448	.612	.240	.379	.526
c in.			7.247	7.247	7.247	8.158	8.158	8.158
g in.			1⅜	1½	1½	1½	1½	1¾
u in			7/16	½	½	7/16	7/16	½
AXES	X-X	*I*	50.7	60 6	70.5	66 9	78.5	90.7
		S	11.27	13 47	15 66	13 38	15 7	18.14
		r	3 40	3 22	3 10	3 87	3 66	3 52
	Y-Y	*I*	1.9	2.4	3 0	2 3	2.8	3 4
		S	1.0	1.2	1 4	1.2	1 3	1.5
		r	.67	65	.64	.72	.70	.68
		y	.59	59	.61	64	.61	62
Max. Mom., in-lb.			202 800	242 400	282 000	240 900	282 600	326 500
V			30 800	48 500	66 100	28 800	45 500	63 100
W			23 800	23 800	23 800	21 600	23 800	23 800

			10 in		12 in			
Depth = *d* in			10	10	12	12	12	12
Weight per foot.			30 0	35.0	20 7	25 0	30 0	35.0
Area, sq in			8.80	10.27	6 03	7.32	8.79	10 26
b in.			3.03	3.18	2.94	3.05	3.17	3.29
t in.			.673	.820	.280	.387	.510	.632
c in.			8.158	8 158	9 910	9 910	9.910	9.910
g in.			1¾	1¾	1¾	1¾	1¾	2
u in			½	½	½	½	½	⅝
AXES	X-X	*I*	103.0	115.2	128.1	143 5	161 2	178.8
		S	20 6	23.04	21.35	23 92	26.87	29.8
		r	3 42	3.34	4.61	4 43	4 28	4 18
	Y-Y	*I*	4.0	4 6	3.9	4.5	5.2	5.9
		S	1.7	1.9	1.7	1.9	2.1	2.3
		r	.67	.67	.81	.79	.77	.76
		y	65	.69	.70	.68	.68	.69
Max. Mom., in-lb			370 800	414 700	384 300	430 500	483 600	536 400
V			80 800	98 400	40 300	55 700	73 400	91 000
W			23 800	23 800	23 860	23 860	23 860	23 860

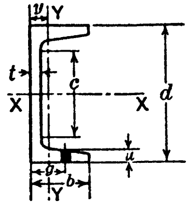
* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table III * (Continued). Standard Channels

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
W is Maximum Load on one Standard Connection.
y is Distance in inches between Center of Gravity and back of Channel.

Note: The attention of the reader is called to remarks on Stock Sizes, Chapter XV.



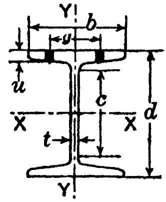
			12 in	15 in					
Depth = *d* in			12	15	15	15	15	15	15
Weight per foot.			40 0	33 9	35 0	40 0	45.0	50 0	55 0
Area, sq in . . .			11 73	9 90	10 23	11 70	13 17	14.64	16 11
b in . . .			3 42	3 40	3 42	3 52	3.62	3 72	3 81
t in . . .			.755	400	422	520	.618	716	.814
c in			9.910	12 353	12 353	12 353	12.353	12 353	12.353
g in . . .			2	2	2	2	2	2½	2½
u in			5/8	5/8	5/8	5/8	5/8	11/16	11/16
AXES	X–X	*I* . .	196 5	312 6	318 7	346 3	373 9	401 4	429.0
		S . . .	32 75	41 68	42 49	46 17	49 85	53 52	57 2
		r . . .	4 09	5 62	5 58	5 44	5 33	5 24	5.16
	Y–Y	*I* . . .	6 6	8 2	8 4	9 3	10 3	11 2	12.1
		S	2.5	3 2	3 2	3 4	3 6	3.8	4.1
		r . .	75	.91	91	.89	.88	.87	.87
		y	72	79	79	78	.79	.80	.82
Max. Mom., in-lb			589 500	750 300	764 900	831 100	897 300	963 400	1 029 600
V .			108 700	72 000	76 000	93 600	111 200	128 900	146 500
W			23 860	36 000	38 000	46 800	47 700	47 700	47 700

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table IV.* Standard Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.



For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.

			3 in			4 in		
Depth = *d* in.			3	3	3	4	4	4
Weight per foot			5.7	6 5	7 5	7 7	8 5	9 5
Area, sq in.			1.64	1.88	2.17	2 21	2 46	2 76
b in.			2.33	2 41	2.51	2 66	2.72	2 80
t in.			.170	.251	.349	.190	.253	.326
c in.			1.843	1.843	1.843	2 717	2 717	2.717
g in			1½	1½	1½	1½	1½	1½
u in			5/16	5/16	5/16	5/16	5/16	5/16
AXES	X-X	*I*	2.5	2 7	2 9	6 0	6 3	6 7
		S	1.67	1 80	1 93	3 0	3 15	3 35
		r	1.23	1 19	1 15	1 64	1 60	1 56
	Y-Y	*I*	0 46	0 51	0 59	0 77	0 83	0 91
		S	0.40	0 43	0 47	0 58	0 61	0 65
		r	0 53	0 52	0 52	0 59	0 58	0 58
Max. Mom., in-lb.			30 000	32 400	34 800	54 000	56 700	60 300
V			6 100	9 000	12 600	9 100	12 100	15 600
R			6 100	9 000	12 600	9 100	12 100	15 600
W			7 650	11 300	11 900	8 550	11 400	11 900

			4 in		5 in		6 in	
Depth = *d* in			4	5	5	5	6	6
Weight per foot.			10.5	10.0	12.25	14 75	12 5	14 75
Area, sq in.			3 05	2.87	3 56	4.29	3 61	4.29
b in.			2 87	3.00	3.14	3 28	3 33	3.44
t in.			.400	210	347	.494	230	343
c in.			2 717	3 589	3 589	3 589	4 465	4 465
g in.			1½	1¾	1¾	1¾	2	2
u in			5/16	⅜	⅜	⅜	⅜	⅜
AXES	X-X	*I*	7.1	12 1	13.5	15 0	21 8	23.8
		S	3.55	4.84	5.40	6.00	7.27	7.93
		r	1 52	2 05	1 95	1 87	2 46	2 36
	Y-Y	*I*	1 0	1 2	1.4	1.7	1 8	2.1
		S	0 70	0 82	0 91	1.0	1.1	1 2
		r	0 57	0 65	0 63	0 63	0 72	0 69
Max. Mom, in-lb.			63 900	87 120	97 200	108 000	130 800	142 800
V			19 200	12 600	20 800	29 600	16 600	24 700
R			19 200	12 600	20 800	29 600	16 600	24 700
W			11 900	9 450	11 900	11 900	10 350	11 900

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

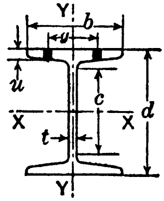
Table IV * (Continued). Standard Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



	6 in	7 in			8 in	
Depth = *d* in	6	7	7	7	8	8
Weight per foot	17 25	15.3	17 5	20 0	18 4	20.5
Area, sq in	5.02	4 43	5 09	5 83	5 34	5.97
b in	3.57	3 66	3 76	3.86	4 00	4 08
t in	.465	250	345	450	270	349
c in	4 465	5 339	5 339	5 339	6 211	6 211
g in	2	2¼	2¼	2¼	2¼	2¼
u in	3/8	3/8	3/8	3/8	5/16	5/16
Axes X-X *I*	26.0	36 2	38 9	41.9	56.9	60 2
Axes X-X *S*	8 67	10 34	11 11	11.97	14 22	15 05
Axes X-X *r*	2 28	2 86	2 77	2 68	3 26	3 18
Axes Y-Y *I*	2.3	2.7	2.9	3 1	3 8	4 0
Axes Y-Y *S*	1 3	1 5	1 6	1 6	1 9	2 0
Axes Y-Y *r*	68	0 78	0 76	0 74	0 84	0 82
Max. Mom., in-lb.	156 000	186 200	200 100	215 500	256 100	270 900
V	34 500	21 000	29 000	37 800	25 900	33 500
R	34 500	19 700	27 200	35 400	22 300	28 800
W	11 900	11 250	11 900	11 900	23 800	23 800

	8 in		9 in			
Depth = *d* in	8	8	9	9	9	9
Weight per foot.	23.0	25 5	21 8	25 0	30 0	35 0
Area, sq in	6 71	7 43	6 32	7 28	8 76	10.22
b in.	4 17	4 26	4 33	4.44	4 60	4.76
t in	441	532	290	.397	561	.724
c in	6 211	6 211	7 085	7 085	7 085	7 085
g in.	2¼	2¼	2½	2½	2½	2½
u in	5/16	1/2	1/2	1/2	1/2	1/2
Axes X-X *I*	64.2	68.1	84.9	91.4	101.4	111 3
Axes X-X *S*	16.05	17.02	18.87	20 31	22 53	24 73
Axes X-X *r*	3.09	3 03	3 67	3 54	3 40	3 30
Axes Y-Y *I*	4 4	4.7	5 2	5 6	6.4	7 3
Axes Y-Y *S*	2 1	2 2	2.4	2.5	2.8	3 0
Axes Y-Y *r*	0 81	0 80	0 90	0 88	0 85	0 84
Max. Mom., lb-in.	288 900	306 450	339 600	365 600	405 600	445 200
V	42 300	51 100	31 300	42 900	60 600	78 200
R	36 400	43 900	25 000	34 200	48 400	62 400
W	23 800	23 800	23 800	23 800	23 800	23 800

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

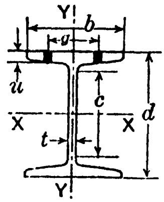
Table IV * (Continued). Standard Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



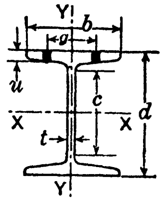
	10 in				12 in	
Depth = *d* in	10	10	10	10	12	12
Weight per foot	25.4	30.0	35.0	40.0	31.8	35.0
Area, sq in	7.38	8.75	10.22	11.69	9.26	10.20
b in	4.66	4.80	4.94	5.09	5.00	5.08
t in	.310	.447	.594	.741	.350	.428
c in	7.959	7.959	7.959	7.959	9.762	9.762
g in	2¾	2¾	2¾	2¾	3	3
u in	1½	1½	1½	1½	9/16	9/16
AXES X-X *I*	122.1	133.5	145.8	158.0	215.8	227.0
AXES X-X *S*	24.42	26.70	29.16	31.60	35.97	37.83
AXES X-X *r*	4.07	3.91	3.78	3.68	4.83	4.72
AXES Y-Y *I*	6.9	7.6	8.5	9.4	9.5	10.0
AXES Y-Y *S*	3.0	3.2	3.4	3.7	3.8	3.9
AXES Y-Y *r*	0.97	0.93	0.91	0.90	1.01	0.99
Max. Mom., in-lb	439 600	480 600	524 900	568 800	647 400	681 000
V	37 200	53 600	71 300	88 900	50 400	61 600
R	27 900	40 200	53 500	66 700	34 100	41 700
W	23 800	23 800	23 800	23 800	23 860	23 860

	12 in				15 in	
Depth = *d* in	12	12	12	12	15	15
Weight per foot	40.8	45.0	50.0	55.0	42.9	45.0
Area, sq in	11.84	13.10	14.57	16.04	12.49	13.12
b in	5.25	5.36	5.48	5.60	5.50	5.54
t in	.460	.565	.687	.810	.410	.452
c in	9.333	9.333	9.333	9.333	12.453	12.468
g in	3	3	3½	3½	3½	3½
u in	¾	¾	¾	¾	5/8	5/8
AXES X-X *I*	268.9	284.1	301.6	319.3	441.8	453.6
AXES X-X *S*	44.82	47.35	50.27	53.22	58.91	60.48
AXES X-X *r*	4.77	4.66	4.55	4.45	5.95	5.88
AXES Y-Y *I*	13.8	14.8	16.0	17.3	14.6	15.0
AXES Y-Y *S*	5.3	5.5	5.8	6.2	5.3	5.4
AXES Y-Y *r*	1.08	1.06	1.05	1.04	1.08	1.07
Max. Mom., in-lb.	806 700	852 300	904 800	957 900	1 060 300	1 088 600
V	66 200	81 400	98 900	116 600	73 800	81 400
R	44 900	55 100	67 000	79 000	43 800	49 200
W	23 860	23 860	23 860	23 860	36 900	40 700

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table IV * (Continued). Standard Beams

DIMENSIONS—FUNCTIONS



I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.

	15 in					
Depth = d in	15	15	15	15	15	15
Weight per foot	50.0	55.0	60.8	65.0	70.0	75.0
Area, sq in	14.59	16.06	17.68	18.91	20.38	21.85
b in	5.64	5.74	6.00	6.08	6.18	6.28
t in	.550	.648	.590	.672	.770	.868
c in	12.468	12.468	11.749	11.749	11.749	11.749
g in	3½	3½	3½	3½	3½	3½
u in	⅝	⅝	⅞	⅞	⅞	⅞
AXES X-X I	481.1	508.7	609.0	632.1	659.6	687.2
AXES X-X S	64.15	67.83	81.20	84.28	87.95	91.63
AXES X-X r	5.74	5.63	5.87	5.78	5.69	5.61
AXES Y-Y I	16.0	17.0	26.0	27.2	28.8	30.6
AXES Y-Y S	5.7	5.9	8.7	8.9	9.3	9.8
AXES Y-Y r	1.05	1.03	1.21	1.20	1.19	1.18
Max Mom., in-lb	1 154 600	1 220 900	1 461 600	1 517 000	1 583 000	1 649 300
V	99 000	116 600	106 200	121 000	138 600	156 200
R	59 800	70 500	64 200	73 100	83 700	94 400
W	47 700	47 700	47 700	47 700	47 700	47 700

	18 in					
Depth = d in	18	18	18	18	18	18
Weight per foot	54.7	60.0	65.0	70.0	75.6	80.0
Area, sq in	15.94	17.50	18.98	20.46	22.04	23.34
b in	6.00	6.09	6.17	6.25	7.00	7.07
t in	.460	.547	.629	.711	.560	.632
c in	15.207	15.207	15.207	15.207	14.492	14.492
g in	3¾	3¾	3¾	3¾	4	4
u in	¾	¾	¾	¾	1	1
AXES X-X I	795.5	837.8	877.7	917.5	1141.8	1176.8
AXES X-X S	88.39	93.09	97.53	101.94	126.87	130.76
AXES X-X r	7.07	6.92	6.80	6.70	7.20	7.10
AXES Y-Y I	21.2	22.3	23.4	24.5	46.3	47.9
AXES Y-Y S	7.1	7.3	7.6	7.8	13.2	13.6
AXES Y-Y r	1.15	1.13	1.11	1.09	1.45	1.43
Max. Mom., in-lb	1 591 000	1 675 600	1 755 400	1 835 000	2 283 600	2 353 600
V	99 400	118 200	135 900	153 600	121 000	136 500
R	52 800	65 600	75 500	85 300	67 200	75 800
W	41 400	47 700	47 700	47 700	47 700	47 700

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

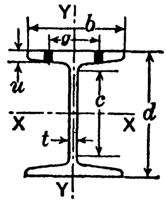
Table IV * (Continued). Standard Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



	18 in		20 in		
Depth = *d* in	18	18	20	20	20
Weight per foot	85.0	90.0	65 4	70 0	75.0
Area, sq in	24 81	26 29	19 08	20 42	21 90
b in	7.15	7.24	6.25	6.32	6 39
t in	714	.796	.500	.567	.641
c in	14.492	14 492	16 925	16 925	16 925
g in	4	4	4	4	4
u in	1	1	¾	¾	¾
AXES X–X *I*	1216.6	1 256 5	1 169 5	1 214.2	1 263 5
AXES X–X *S*	135.18	139 61	116.95	121 42	126.35
AXES X–X *r*	7.00	6 91	7 83	7.71	7 60
AXES Y–Y *I*	49.8	51 9	27 9	28 9	30 1
AXES Y–Y *S*	14.0	14 3	8 9	9.2	9 4
AXES Y–Y *r*	1.42	1 40	1 21	1 19	1.17
Max. Mom., in-lb.	2 433 200	2 513 000	2 105 100	2 185 600	2 274 300
V	154 200	171 900	120 000	136 100	153 800
R	85 700	95 500	60 400	71 900	81 700
W	47 700	47 700	56 250	59 600	59 600

	20 in				
Depth = *d* in	20	20	20	20	20
Weight per foot	81.4	85 0	90.0	95.0	100 0
Area, sq in	23.74	24.8	26.26	27.74	29 20
b in	7.00	7.05	7.13	7.20	7.27
t in	.600	.653	.726	.800	873
c in	16.448	16.448	16.448	16.448	16.448
g in	4	4	4	4	4
u in	1	1	1	1	1
AXES X–X *I*	1466.3	1501 7	1550 3	1599 7	1648.3
AXES X–X *S*	146 63	150.17	155 03	159 97	164 83
AXES X–X *r*	7 86	7 78	7 68	7 59	7.51
AXES Y–Y *I*	45 8	47 0	48.7	50 5	52.4
AXES Y–Y *S*	13.1	13 3	13.7	14.0	14.4
AXES Y–Y *r*	1 39	1 38	1 36	1 35	1.34
Max. Mom., in-lb.	2 639 300	2,703,100	2 790 500	2 879 500	2 966 900
V	144 000	156 700	174 200	192 000	209 500
R	76 500	83 300	92 600	102 000	111 300
W	59 600	59 600	59 600	59 600	59 600

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table IV * (Continued). Standard Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

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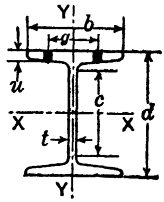

			24 in				
Depth = *d* in			24	24	24	24	24
Weight per foot			79 9	85 0	90.0	95 0	100.0
Area, sq in			23 33	24.84	26 30	27 79	29 25
b in			7.000	7.063	7.124	7 186	7 247
t in			.500	.563	.624	.686	.747
c in			20.699	20 699	20.699	20 699	20.699
g in.			4	4	4	4	4
u in			⅞	⅞	⅞	⅞	⅞
AXES	X–X	*I*	2087.2	2 159 8	2 230 1	2 301 5	2 371 8
		S	173 93	180 00	185.84	191 80	197 65
		r	9 46	9 33	9 21	9 08	9 05
	Y–Y	*I*	42 9	44 2	45 5	47 0	48.4
		S	12 2	12 5	12 8	13 0	13 4
		r	1 36	1 33	1 32	1 30	1.29
Max. Mom., in-lb			3 130 000	3 240 000	3 345 000	3 452 000	3 557 700
V			144 000	162 100	179 700	197 600	215 000
R			61 800	73 900	85 700	97 400	106 400
W			67 500	71 600	71 600	71 600	71 600

			24 in				
Depth = *d* in			24	24	24	24	
Weight per foot			105 9	110 0	115.0	120 0	.
Area, sq in			30 98	32 18	33.67	35 13	.
b in			7 875	7 925	7.987	8 048	. .
t in			.625	675	.737	798	. . . .
c in			20 175	20 175	20.175	20 175	. .
g in			5	5	5	5	. . .
u in			1$\frac{1}{16}$	1$\frac{1}{16}$	1$\frac{1}{16}$	1$\frac{1}{16}$	
AXES	X–X	*I*	2811 5	2 869 1	2 940 5	3 010 8	. .
		S	234 30	239 10	245 04	250 90	. . . .
		r	9 53	9 44	9 35	9 26	.
	Y–Y	*I*	78 9	80 6	82 8	84 9	
		S	20 0	20 3	20.7	21 1	. . . . .
		r	1 60	1 58	1 57	1.56	. . . . .
Max. Mom., in-lb			4 217 000	4 304 000	4 410 800	4 516 000	. .
V			180 000	194 400	212 300	229 800	. .
R			85 800	95 400	105 000	113 700	. . . .
W			71 600	71 600	71 600	71 600	. . . .

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Table V.* Bethlehem Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
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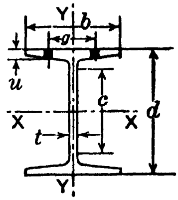

			8 in		9 in		10 in	
Depth = *d* in			8 00	8 00	9 00	9 06	9 90	10 00
Weight per foot			17 5	19 0	20 5	22.0	21.0	23 5
Area, sq in			5 20	5 68	6 09	6.51	6 28	6 96
b in			5 250	5 270	5 500	5 510	5 740	5 750
t in			.250	270	.250	260	240	250
c in			6 625	6 625	7 500	7 500	8 375	8 375
g in			2¼	2¼	2½	2½	2¾	2¾
u in			11/32	3/8	3/8	13/32	11/32	13/32
AXES	X–X	*I*	57 7	63 7	86 5	93 9	108 1	123 2
		S	14 43	15 81	19 22	20 73	21 81	24 64
		r	3 33	3 35	3 77	3 80	4 15	4 21
	Y–Y	*I*	6 39	7.20	8 54	9 42	9 30	10 9
		S	2 44	2 73	3 10	3 42	3 24	3 80
		r	1 11	1 13	1 18	1 20	1 22	1.25
Max. Mom., in-lb.			259 700	284 500	346 000	373 100	393·100	443 500
V			24 000	26 100	27 000	28 300	28 500	30 000
R			20 600	22 300	21 300	22 400	20 200	21 300
W			22 500	23 900	22 500	23 400	21 600	22 500

			10 in		12 in			
Depth = *d* in			10 09	10 19	11 88	12 00	12 12	12.25
Weight per foot			26 0	28 5	25 0	28 0	31.5	36 0
Area, sq in			7 68	8.41	7 44	8 28	9 36	10 58
b in			5 770	5.785	6 495	6.500	6 525	6 555
t in			270	285	.240	.245	.270	300
c in			8 375	8 375	10 250	10.250	10 250	10 250
g in			2¾	2¾	3	3	3	3
u in			15/32	1/2	3/8	7/16	1/2	9/16
AXES	X–X	*I*	137 9	154 1	185 1	213 6	245 7	281 8
		S	27 33	30 25	31 16	35 60	40 54	46 01
		r	4 24	4 28	4 99	5 08	5 12	5 16
	Y–Y	*I*	12 5	14 2	13 6	16 4	19 4	22.7
		S	4 33	4 92	4 19	5 04	5 93	6 93
		r	1 28	1 30	1 35	1 41	1 44	1 46
Max. Mom., in-lb.			492 000	544 400	560 900	640 800	729 800	828 100
V			32 700	34 800	34 200	35 300	39 300	44 100
R			23 700	25 400	19 900	20 500	23 800	27 300
W			23 900	23 900	21 600	22 100	23 860	23 860

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
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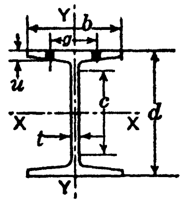

			12 in			14 in		
Depth = *d* in			12.00	12.12	12.25	13 88	14 00	14.12
Weight per foot			40 0	44 0	48.5	30 0	33 0	37.5
Area, sq in			11.80	12.97	14 28	8 89	9 70	11.07
b in			6 750	6 780	6.815	6.750	6.750	6.790
t in			.330	.360	.395	.265	.265	.305
c in			9 750	9 750	9 750	12.125	12.125	12.125
g in			3	3	3	3½	3½	3½
u in			23/32	25/32	27/32	13/32	15/32	½
AXES	X–X	*I*	301 2	335 1	373 2	294 9	334.3	383.7
		S	50 20	55 30	60 93	42.49	47.76	54.35
		r	5 05	5 08	5 11	5 76	5 87	5 89
	Y–Y	*I*	27 6	31 1	35 1	16 9	19 9	23.4
		S	8.18	9.18	10 29	4 99	5.90	6.91
		r	1.53	1.55	1.57	1 38	1.43	1.46
Max. Mom., in-lb			903 600	995 300	1 096 700	764 900	859 600	978 300
V			47 500	52 400	58 100	44 100	44 500	51 700
R			31 700	35 300	38 900	22 900	22 800	28 300
W			23 860	23 860	23 860	23 900	23 900	27 500

			14 in	15 in				
Depth = *d* in			14 25	14.91	15.00	15 03	15.09	14.75
Weight per foot			42.0	36 0	38 5	40 0	42.5	46.0
Area, sq in			12.46	10 61	11 37	11 80	12.50	13 63
b in			6 825	6.740	6.750	6.765	6.785	6.955
t in			.340	.280	.290	.305	.325	.365
c in			12 125	12 875	12 875	12 875	12 875	12.375
g in			3½	3½	3½	3½	3½	3½
u in			19/32	½	17/32	9/16	19/32	5/8
AXES	X–X	*I*	436.5	410 9	447.6	463.3	492.0	508.2
		S	61 26	55.12	59.68	61.65	65 21	68.91
		r	5.92	6 22	6 27	6 27	6 27	6 11
	Y–Y	*I*	27.3	21.7	24.1	25.1	26 9	30.8
		S	8.00	6.45	7.15	7.42	7.93	8.85
		r	1.48	1.43	1 46	1.46	1 47	1.50
Max. Mom., in-lb.			1 102 700	992 100	1 074 200	1 109 700	1 173 700	1 240 300
V			58 100	50 100	52 200	55 000	58 900	64 600
R			33 100	24 800	26 200	28 400	31 300	37 400
W			30 600	25 200	26 100	27 500	29 300	32 900

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

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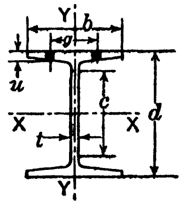

	15 in				16 in	
Depth = *d* in	14.88	15 00	15.12	15.00	15.81	16 00
Weight per foot	50.5	54.5	59.5	71.5	35.0	40.0
Area, sq in	14 84	16 05	17.49	21.04	10 29	11 83
b in	6.975	7.000	7.040	7.500	7.240	7.250
t in	.385	.410	.450	.520	.285	.295
c in	12.375	12.375	12.375	11.750	14.000	14.000
g in	3½	3½	3½	3¾	3¾	3¾
u in	1 1/16	3/4	13/16	15/16	13/32	1/2
AXES X–X *I*	563.3	617.0	676.2	799 5	435.8	526.2
AXES X–X *S*	75.71	82.27	89.44	106 60	55.13	65.78
AXES X–X *r*	6.16	6.20	6 22	6 16	6.51	6 67
AXES Y–Y *I*	34 7	38.6	42 8	60.9	21.4	27 6
AXES Y–Y *S*	9.96	11 0	12 2	16.2	5.92	7 61
AXES Y–Y *r*	1 53	1 55	1.56	1.70	1 44	1 53
Max. Mom., in-lb.	1 362 800	1 480 800	1 610 000	1 918 800	992 300	1 184 000
V	68 700	73 800	81 700	93 600	54 100	56 600
R	40 100	43 800	48 900	56 600	25 400	26 700
W	34 700	36 900	40 500	46 800	25 700	25 700

	16 in					
Depth = *d* inch	16.12	16.25	15.88	16 00	16 12	16.25
Weight per foot	45.0	50.0	56 5	60.5	66 0	71.5
Area, sq in	13.26	14.78	16.63	17.89	19 40	21.07
b in	7.285	7.320	8.485	8.500	8 530	8 565
t in	.330	.365	.375	.390	.420	.455
c in	14 000	14 000	13.375	13 375	13.375	13.375
g in	3¾	3¾	3¾	3¾	3¾	3¾
u in	9/16	5/8	11/16	3/4	13/16	7/8
AXES X–X *I*	594.5	669.0	742.3	812.1	888.4	973.5
AXES X–X *S*	73 76	82.34	93.49	101.51	110 22	119 82
AXES X–X *r*	6 69	6 73	6.68	6.74	6 77	6 80
AXES Y–Y *I*	31 9	36.6	57.8	64 3	71.2	79.0
AXES Y–Y *S*	8 75	10.01	13.6	15.1	16.7	18.4
AXES Y–Y *r*	1 55	1 57	1.86	1.90	1 92	1.94
Max. Mom., in-lb.	1 327 700	1 482 100	1 682 800	1 827 200	1 984 000	2 156 700
V	63 800	71 200	71 500	74 900	81 200	88 700
R	31 900	37 100	39 000	41 100	45 500	50 600
W	28 800	32 400	33 800	35 100	37 800	41 000

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
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r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
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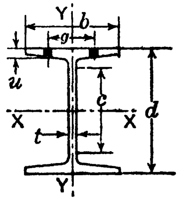

			18 in		
Depth = d in		17 94	18 00	18.06	18.12	17.75
Weight per foot		47 0	49 0	52.0	54 5	59.0
Area, sq in		13 90	14 44	15 34	16 06	17 48
b in		7.495	7.500	7.525	7.540	8 710
t in		325	.330	.355	.370	.380
c in		15 750	15 750	15.750	15 750	15 250
g in		3¾	3¾	3¾	3¾	4
u in		9/16	19/32	5/8	21/32	21/32
AXES X-X	I	764 1	802 8	851 7	896 1	960.3
	S	85 18	89 20	94 32	98 91	108 20
	r	7 42	7 46	7 45	7 47	7 41
AXES Y-Y	I	34 0	36.1	38 7	41 1	60 7
	S	9 06	9 64	10 3	10 9	13 9
	r	1 56	1.58	1 59	1 60	1 86
Max. Mom., in-lb		1 533 300	1 605 600	1 697 700	1 780 300	1 947 600
V		70 000	71 300	76 900	80 500	80 900
R		31 000	31 800	35 800	38 200	40 200
W		36 600	37 100	39 900	41 600	42 800

		18 in			20 in	
Depth = d in		17 88	18 00	18.12	19 88	20.00
Weight per foot		64 5	69.0	74.0	56 0	59.5
Area, sq in		18.97	20 38	21 79	16 51	17.47
b in		8 730	8.750	8 770	8.000	8.000
t in		.400	.420	.440	.375	375
c in		15 250	15.250	15 250	17 625	17 625
g in		4	4	4	4	4
u in		23/32	25/32	27/32	19/32	21/32
AXES X-X	I	1 059 7	1 153 7	1 249 2	1 086 1	1 181.5
	S	118 53	128 19	137.88	109 27	118.15
	r	7 47	7 53	7 57	8 11	8.22
AXES Y-Y	I	68 4	75 6	82 9	43 5	48.6
	S	15 7	17 3	18 9	10 9	12.2
	r	1.90	1 93	1.95	1 62	1.67
Max. Mom., in-lb.		2 133 600	2 307 400	2 481 800	1 966 800	2 126 700
V		85 800	90 700	95 700	89 500	90 000
R		43 100	46 300	49 500	39 100	38 900
W		45 000	47 300	49 500	42 200	42 200

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

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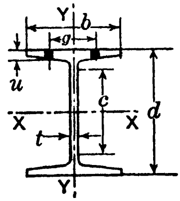

			20 in				
Depth = *d* in			20.06	20.12	19.88	20.00	20.09
Weight per foot			62.0	64.5	68.5	73.0	78.0
Area, sq in			18.25	18.93	20.12	21.58	22.98
b in			8.015	8.025	8.855	8.875	8.905
t in			.390	.400	.410	.430	.460
c in			17.625	17.625	17.125	17.125	17.125
g in			4	4	4	4	4
u in			11/16	23/32	23/32	25/32	27/32
AXES	X–X	*I*	1 239.8	1 295.1	1 366.0	1 485.0	1 585.5
		S	123.61	128.74	137.42	148.50	157.84
		r	8.24	8.27	8.24	8.30	8.31
	Y–Y	*I*	51.5	54.3	71.0	78.5	84.7
		S	12.9	13.5	16.0	17.7	19.0
		r	1.68	1.69	1.88	1.91	1.92
Max. Mom., in-lb.			2 225 000	2 317 300	2 473 600	2 673 000	2 841 100
V			93 900	96 600	97 800	103 200	110 900
R			41 500	43 200	45 100	48 400	53 500
W			43 900	45 000	46 200	48 400	51 800

			22 in				
Depth = *d* in			21.75	21.88	22.00	22.12	22.25
Weight per foot			54.5	58.0	62.5	67.5	73.0
Area, sq in			16.04	17.14	18.38	19.84	21.51
b in			8.490	8.490	8.500	8.520	8.545
t in			.360	.360	.370	.390	.415
c in			19.500	19.500	19.500	19.500	19.500
g in			4	4	4	4	4
u in			½	9/16	5/8	11/16	¾
AXES	X–X	*I*	1 232.6	1 363.9	1 495.4	1 637.5	1 796.7
		S	113.34	124.67	135.95	148.06	161.50
		r	8.77	8.92	9.02	9.08	9.14
	Y–Y	*I*	42.2	48.9	55.2	61.8	69.1
		S	9.95	11.5	13.0	14.5	16.2
		r	1.62	1.69	1.73	1.76	1.79
Max. Mom., in-lb.			2 040 200	2 244 100	2 447 000	2 665 000	2 907 000
V			94 000	94 500	97 700	103 500	110 800
R			36 000	36 000	37 700	41 300	45 800
W			48 600	48 600	49 980	52 680	56 060

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

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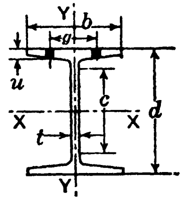

			22 in				24 in
Depth = *d* in			21 88	22 00	22 12	22 25	23.88
Weight per foot			77 0	83 0	89 0	96.5	70.0
Area, sq in			22.74	24.51	26.28	28.38	20.62
b in			9 220	9.250	9.280	9 320	9.000
t in			.425	.455	.485	.525	.395
c in			19 000	19 000	19.000	19 000	21 375
g in			4	4	4	4	4
u in			25/32	27/32	29/32	3 1/32	2 1/32
AXES	X–X	*I*	1 866 7	2 026 5	2 188 6	2 373 7	1 954.1
		S	170 63	184 23	197 88	213 37	163.66
		r	9 06	9 09	9.13	9 15	9.74
	Y–Y	*I*	87 0	95 8	104 8	115 1	67 4
		S	18 9	20 7	22.6	24 7	15.0
		r	1.96	1 98	2 00	2 01	1 81
Max. Mom., in-lb			3 071 300	3 316 100	3 561 900	3 840 600	2 945 900
V			111 600	120 100	128 700	140 200	113 200
R			47 700	53 000	58 400	65 400	42 000
W			57 400	61 400	65 500	70 900	53 300

			24 in				
Depth = *d* in			24 00	24 09	24 00	24 12	23.91
Weight per foot			73 5	79 5	84.5	90 5	95 5
Area, sq in			21.70	23.35	24.97	26 47	28.05
b in			9 000	9 035	9 500	9 515	9.730
t in			395	.430	460	.475	.505
c in			21 375	21 375	21 125	21 125	20 750
g in			4	4	4	4	5½
u in			23/32	25/32	1 3/16	27/32	1 3/16
AXES	X–X	*I*	2 108 8	2 266 7	2 405 7	2 588 2	2 692.7
		S	175 73	188.19	200.48	214 61	225.24
		r	9 86	9 85	9 82	9 89	9.80
	Y–Y	*I*	74 7	81 2	95 8	104 9	117.1
		S	16 6	18.0	20.2	22 1	24.1
		r	1 86	1 87	1 96	1 99	2 04
Max. Mom., in-lb.			3 163 200	3 387 300	3 608 600	3 863 000	4 054 200
V			113 800	124 300	132 500	137 500	144 900
R			41 800	48 400	54 100	56 800	62 700
W			53 300	58 100	62 100	64 100	68 200

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

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S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for $3\frac{1}{2}$ in bearing.
W is Maximum Load on one Standard Connection.

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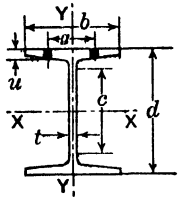

			24 in		26 in		
Depth = *d* in			24 00	24 09	25.78	25.88	26 00
Weight per foot			99.5	104.5	81.0	85.5	91 0
Area, sq in			29.40	30 88	23 90	25.11	26 76
b in			9 750	9.775	9.470	9 480	9 500
t in			.525	.550	.440	.450	470
c in			20.750	20.750	23.00	23 00	23 00
g in			$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$
u in			$\frac{7}{8}$	$\frac{29}{32}$	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{3}{4}$
AXES	X–X	*I*	2 841.3	2 997 3	2 600 1	2 772.5	2 993.1
		S	236.78	248.84	201.71	214.26	230 24
		r	9 83	9.85	10.43	10 51	10.58
	Y–Y	*I*	124 9	132 9	84 3	91 7	100.9
		S	25 6	27 2	17.8	19.3	21.2
		r	2.06	2.07	1.88	1 91	1 94
Max. Mom., in-lb			4 262 000	4 479 100	3 630 900	3 856 600	4 144 300
V			151 200	159 000	136 100	139 700	146 600
R			66 600	71 400	50 400	52 100	56 000
W			70 900	71 600	69 300	70 900	74 000

			26 in	28 in			
Depth = *d* in			26.12	27.69	27.88	28 00	28 12
Weight per foot			98.0	85.0	91.0	97 0	104 0
Area, sq in			28.69	24.96	26.86	28.61	30 66
b in			9.530	9.980	9.980	10.000	10 030
t in			.500	.450	.450	470	.500
c in			23.00	24.875	24.875	24 875	24.875
g in			$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$
u in			$\frac{13}{16}$	$\frac{5}{8}$	$\frac{23}{32}$	$\frac{25}{32}$	$\frac{27}{32}$
AXES	X–X	*I*	3 231.2	3 075.2	3 441.1	3 711.5	4 003.3
		S	247.4	222.12	246 85	265 11	284.73
		r	10.61	11 10	11.32	11 39	11.43
	Y–Y	*I*	110.6	91.0	106.7	117.4	128.7
		S	23.2	18.2	21.4	23 5	25.7
		r	1.96	1.91	1.99	2 03	2.05
Max. Mom., in-lb.			4 453 400	3 998 100	4 443 300	4 771 900	5 125 100
V			156 800	149 500	150 600	157 900	168 700
R			62 100	51 800	51 700	55 900	62 100
W			78 800	81 000	81 000	84 600	90 000

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus
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V is Maximum Web Shear in Pounds.
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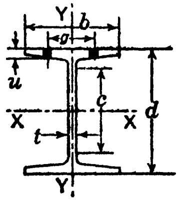

			28 in			30 in	
Depth = *d* in			28.25	28.38	28.59	29.78	29.88
Weight per foot			112 0	119 0	133 0	110.0	115.0
Area, sq in			32.95	35.11	39 09	32.45	33.80
b in			10.065	10.095	10.160	10.470	10.480
t in			.535	.565	.630	.520	.530
c in			24 875	24 875	24 875	26.50	26.50
g in			5½	5½	5½	5½	5½
u in			29/32	31/32	1 1/32	27/32	7/8
AXES	X–X	*I*	4 328 0	4 647 4	5 204 0	4 687 7	4 942.9
		S	306.41	327 51	364 04	314 82	330 85
		r	11 46	11 50	11 54	12 02	12 09
	Y–Y	*I*	141 2	153 7	175 3	141.8	151.8
		S	28 1	30 5	34 5	27.1	29.0
		r	2 07	2 09	2 12	2 09	2 12
Max. Mom., in-lb			5 515 300	5 895 200	6 552 800	5 666 800	5 955 300
V			181 400	192 400	216 100	185 800	190 000
R			69 500	75 900	89 900	66 600	68 400
W			95 400	95 400	95 400	105 300	107 400

			30 in				
Depth = *d* in			30 00	30 12	30 25	30 44	30.65
Weight per foot			121 0	129 0	137 0	149 0	163.0
Area, sq in			35 65	37 82	40 40	43.93	48 00
b in			10 500	10 530	10.570	10.620	10.680
t in			550	.580	620	.670	730
c in			26.50	26 50	26 50	26 50	26 50
g in			5½	5½	5½	5½	5½
u in			15/16	1	1 1/16	1 3/16	1 9/32
AXES	X–X	*I*	5 269 7	5 622 7	6 026 7	6 606 6	7 270 7
		S	351 31	373 35	398.46	434.07	474.43
		r	12 16	12 19	12 21	12 26	12 31
	Y–Y	*I*	164 3	177 6	192 6	214 5	239 8
		S	31 3	33 7	36 4	40 4	44 9
		r	2 15	2 17	2 18	2 21	2 24
Max. Mom., in-lb			6 323 600	6 720 400	7 172 300	7 813 300	8 539 800
V			198 000	209 700	225 100	244 700	268 500
R			72 800	79 400	88 400	99 700	113 400
W			107 400	107 400	107 400	107 400	107 400

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Table V * (Continued). Bethlehem Beams

DIMENSIONS—FUNCTIONS

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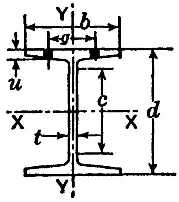

			33 in				
Depth = *d* in			32 89	33 00	33 12	33.27	33 50
Weight per foot..			125.0	135 0	143.0	152.0	165 0
Area, sq in			36.83	39.55	42.05	44.65	48.52
b in....... ...			11.205	11.250	11.285	11.312	11 350
t in........			.535	.580	.615	.642	.680
c in....... . .			29.375	29.375	29.375	29.375	29.375
g in			7½	7½	7½	7½	7½
u in			13/16	7/8	15/16	1	1⅛
AXES	X–X	*I* .	6 498 2	6 967 4	7 442 2	7 991 4	8 835 4
		S. .	395.15	422 27	449 41	480 40	527.49
		r .	13 28	13.27	13 30	13 38	13 49
	Y–Y	*I* . . .	183.2	198.7	215 1	234 9	265 5
		S .	32 7	35 3	38 1	41.5	46 8
		r....	2 23	2 24	2 26	2.29	2.34
Max. Mom., in-lb.			7 112 700	7 600 800	8 089 300	8 647 100	9 494 800
V...			211 200	229 700	244 400	256 300	273 400
R			69 300	79 700	87 900	94 300	103 500
W			107 370	107 370	107 370	107 370	107 370

			36 in				
Depth = *d* in			35 9	36.00	36 12	36 25	36 52
Weight per foot			147.0	155 0	164 0	173.0	190 0
Area, sq in			43.23	45 58	48 10	50 94	55 87
b in..........			11.968	12.000	12.030	12.065	12.111
t in.........			.583	.615	.645	.680	.726
c in....... .			32 125	32 125	32 125	32 125	32 125
g in... .			7½	7½	7½	7½	7½
u in			29/32	31/32	1 1/32	1 3/32	1 7/32
AXES	X–X	*I*.. .	9 036.3	9 547 4	10 133	10 784.	12 049.
		S .	503 42	530 41	561 07	594.98	659 86
		r	14 46	14 47	14 51	14 55	14 68
	Y–Y	*I*.	243 3	259.9	279.4	301 1	344 9.
		S. ..	40.7	43.	46.	49.	57.
		r .	2 37	2.39	2 41	2 43	2 48
Max. Mom., in-lb.			9 061 500	9 547 400	10 099 300	10 709 600	11 877 400
V...			251 200	265 700	279 600	295 800	318 200
R...........			80 200	88 100	95 500	104 300	116 100
W			107 370	107 370	107 370	107 370	107 370

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Table VI.* Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS

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V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for $3\frac{1}{2}$ in bearing.
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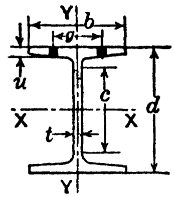

			8 in			9 in		
Depth = *d* in.			7 88	8 00	8 12	8 94	9 00	9.12
Weight per foot.			29 5	33 0	36 5	36 0	38 5	43 5
Area, sq in			8 69	9 69	10 81	10 66	11 35	12.73
b in			7 995	8 000	8 020	8 480	8 500	8 540
t in			285	290	310	290	310	350
c in.			5 938	5 938	5 938	6 875	6 875	6 875
g in			$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$
u in			$\frac{11}{32}$	$\frac{13}{32}$	$\frac{15}{32}$	$\frac{15}{32}$	$\frac{15}{32}$	$\frac{9}{16}$
AXES	X–X	*I*	100 7	116 1	132 6	160 5	171 9	195 4
		S.	25 56	29 03	32 66	35 91	38 20	42 85
		r	3 41	3 46	3 50	3 88	3 89	3 92
	Y–Y	*I*	28 4	33 6	39 0	41 0	44 4	51 3
		S	7 10	8 39	9 72	9 67	10 4	12 0
		r	1 81	1 86	1 90	1 96	1 98	2 01
Max. Mom., in-lb			460 000	522 500	587 900	646 300	687 600	771 300
V			26 900	27 800	30 200	31 100	33 500	38 300
R			23 400	23 900	25 700	25 000	26 700	30 200
W			25 650	26 100	27 920	26 100	27 900	31 500

			10 in			12 in		
Depth = *d* in			9 91	10 00	10 12	11 91	12 00	12 12
Weight per foot			41 5	44 5	50 0	51 5	55 5	61 0
Area, sq in			12 23	13 14	14 62	15 21	16 35	17 92
b in			8 990	9 000	9 040	9 980	10 000	10 030
t in			310	.320	.360	360	380	.410
c in			7 750	7.750	7 750	9 5	9 5	9 5
g in			$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$
u in			$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$
AXES	X–X	*I*	225 8	246 7	277 5	400 6	435 6	483 6
		S	45 57	49 34	54 81	67 27	72 60	79 80
		r	4 30	4 33	4 36	5 13	5 16	5 20
	Y–Y	*I*	52 6	58 2	66 4	76 9	84 9	95 9
		S	11 7	12 9	14 7	15 4	17 0	19.1
		r	2 07	2 10	2 13	2 25	2 28	2 31
Max. Mom., in-lb.			820 300	888 100	987 100	1 210 900	1 305 800	1436 400
V			36 900	38 400	43 700	51 500	54 700	59 700
R			27 900	28 800	32 400	35 100	37 100	40 000
W			27 900	28 800	32 400	48 600	51 300	55 350

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Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
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V is Maximum Web Shear in Pounds.
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W is Maximum Load on one Standard Connection.

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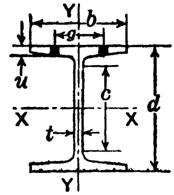

			12 in			15 in		
Depth = *d* in			11 88	12 00	12 12	14 82	14.88	15 00
Weight per foot			66 0	70 5	76 5	64 5	69 0	74 0
Area, sq in			19.32	20 79	22 50	19 09	20 18	21.76
b in			10 230	10 250	10 290	10.700	10.730	10.750
t in			450	470	510	390	.420	440
c in			9 0	9 0	9 0	12.125	12 125	12.125
g in			5½	5½	5½	7½	7½	7½
u in			23/32	25/32	13/16	9/16	19/32	21/32
AXES	X–X	*I*	496 9	543 6	594 2	771 6	815 3	892 7
		S	83 65	90 60	98 05	104 13	109 58	119 03
		r	5.07	5 11	5 14	6.36	6.36	6.40
	Y–Y	*I*	108 3	119 7	132 1	108 6	115 8	128 9
		S	21 2	23 4	25 7	20 3	21 6	24 0
		r	2.37	2 40	2 42	2.39	2.40	2.43
Max. Mom., in-lb.			1 505 800	1 630 800	1 764 900	1 874 300	1 972 500	2 142 500
V			64 100	67 700	74 200	69 400	75 000	79 200
R			43 900	45 800	49 700	41 000	45 200	47 900
W			60 750	63 450	68 850	70 200	75 600	79 200

			15 in					
Depth = *d* in			15 12	14 80	14 88	15 00	15 12	14 75
Weight per foot			80 5	94 0	99 0	105 0	111 0	127.0
Area, sq in			23 66	27 66	29 00	30 80	32 75	37 47
b in			10 790	11 190	11 220	11.250	11 290	11 680
t in			480	.540	.570	600	640	730
c in			12 125	11 250	11 250	11 250	11 250	10 250
g in			7½	7½	7½	7½	7½	7½
u in			23/32	27/32	7/8	15/16	1	1⅛
AXES	X–X	*I*	977 4	1090 2	1147.7	1231.3	1319.3	1415.6
		S	129.29	147 32	154 26	164 17	174 51	191 95
		r	6 43	6.28	6.29	6.32	6 35	6.15
	Y–Y	*I*	143 1	187 4	198 5	214 4	231.3	289 1
		S	26 5	33 5	35.4	38 1	41 0	49 5
		r	2 46	2 60	2 62	2 64	2 66	2 78
Max. Mom., in-lb.			2 327 100	2 651 800	2 776 700	2 955 100	3 141 200	3 455 000
V			87 100	95 900	101 700	108 000	116 200	129 200
R			52 200	58 700	62 000	65 300	69 600	79 400
W			86 400	95 440	95 440	95 440	95 440	95 440

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Table VI * (Continued). Bethlehem Girder Beams

Dimensions—Functions

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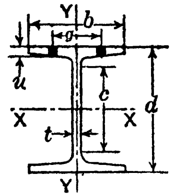

			15 in			16 in	
Depth = *d* in			14 88	15.00	15 12	15.88	16 00	16.12
Weight per foot			135 0	141.0	147.0	74 5	81.0	87.0
Area, sq in			39 58	41 44	43.30	21.96	23 82	25.68
b in			11.720	11.750	11.780	11.470	11 500	11.530
t in			770	800	830	390	.420	450
c in.........			10 250	10 250	10 250	12.875	12 875	12 875
g in . .			$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$
u in			$1\frac{7}{32}$	$1\frac{9}{32}$	$1\frac{5}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
AXES	X–X	*I* .	1 509 9	1 596 8	1 685.4	1 033.6	1 131.3	1 230 8
		S .	202.94	212.91	222.94	130.18	141.41	152.70
		r	6 18	6.21	6.24	6 86	6 89	6.92
	Y–Y	*I*	309 7	328.5	347.5	148.1	164.6	181.3
		S	52.9	55.9	59.0	25.8	28.6	31.5
		r	2 80	2 82	2 83	2 60	2 63	2 66
Max. Mom., in-lb.			3 653 000	3 832 300	4 012 800	2 343 200	2 545 400	2 748 700
V.			137 400	144 000	150 600	74 300	80 600	87 000
R.			83 700	87 000	90 300	41 300	45 700	50 000
W			95 440	95 440	95 440	70 200	75 600	81 000

			16 in	18 in				20 in
Depth = *d* in ..			16.25	17.88	18 00	18.12	18 25	19 75
Weight per foot.			94.0	80 0	86.0	92.0	99.0	99.0
Area, sq in			27.75	23 59	25 35	27 13	29 11	29 21
b in			11.565	11.730	11.750	11 770	11.795	11.950
t in			485	.420	.440	.460	.485	510
c in			12.875	14.875	14.875	14 875	14 875	16 375
g in			$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$
u in . .			$\frac{13}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{25}{32}$
AXES	X–X	*I* .	1 341.4	1 380.7	1 503.6	1 628.5	1 767 7	2 034.4
		S ..	165.10	154.44	167.07	179.75	193.72	206.02
		r .	6 95	7.65	7 70	7.75	7 79	8.35
	Y–Y	*I*. ..	199.9	157.8	174.9	192.2	211.3	202.1
		S...	34.6	26.9	29.8	32.7	35.8	33.8
		r .. .	2.68	2.59	2.63	2.66	2 69	2.63
Max. Mom., in-lb.			2 971 700	2 779 900	3 007 200	3 235 400	3 487 000	3 708 300
V...			94 600	90 100	95 000	100 000	106 200	120 900
R			54 600	46 200	49 500	52 800	56 900	62 400
W .			87 300	94 500	99 000	103 500	109 100	114 800

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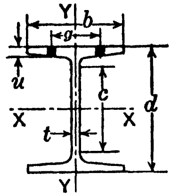
Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS



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			20 in					
Depth = *d* in			19.88	20 00	20 12	19 75	19 88	20 00
Weight per foot			107.0	113 0	120.0	127.0	135.0	142.0
Area, sq in			31 36	33 20	35 24	37 33	39.58	41.71
b in			11.980	12.000	12.030	12.690	12 720	12 750
t in			540	.560	.590	600	630	660
c in			16.375	16.375	16.375	15 750	15.750	15.750
g in			7½	7½	7½	7½	7½	7½
u in			13/16	7/8	15/16	1"	1 1/16	1 1/32
AXES	X-X	*I*	2 206.5	2 362 8	2 528.0	2 607.3	2 783.9	2 960 6
		S	221.98	236 28	251.29	264.03	280.57	296 06
		r	8.39	8.44	8.47	8.36	8.39	8 43
	Y-Y	*I*	222.4	240.8	260.2	313.0	337.7	361 0
		S	37.1	40.1	43.3	49.3	53.1	56.6
		r	2.66	2 69	2.72	2 90	2.92	2.94
Max. Mom., in-lb.			3 995 700	4 253 000	4 523 300	4 752 500	5 050 300	5 329 100
V			128 800	134 400	142 500	142 200	150 300	158 400
R			67 200	70 700	75 200	76 500	80 300	84 200
W			119 300	119 300	119 300	119 300	119 300	119 300

			20 in	22 in				
Depth = *d* in			20.12	21 88	22 00	22.12	22.25	22 38
Weight per foot			149 0	101.0	108.0	116.0	124.0	132 0
Area, sq in			43 84	29 68	31 89	34 12	36 59	38 96
b in			12.780	12.970	13.000	13.030	13.065	13.095
t in			.690	.450	.480	.510	.545	.575
c in			15 750	18.625	18.625	18.625	18.625	10.625
g in			7½	7½	7½	7½	7½	7½
u in			1 5/32	3/4	13/16	7/8	15/16	1"
AXES	X-X	*I*	3 134.9	2 590.4	2 804.3	3 021.2	3 261.7	3 501.2
		S	311.62	236.78	254.94	273 16	293 19	312.89
		r	8 46	9 34	9.38	9.41	9 44	9 48
	Y-Y	*I*	384.6	238 1	261.9	286 0	312 6	339.3
		S	60.2	36.7	40.3	43.9	47.9	51.8
		r	2 96	2.83	2.87	2.90	2 92	2.95
Max. Mom., in-lb.			5 609 200	4 262 100	4 588 800	4 917 000	5 277 300	5 632 000
V			166 600	118 200	126 700	135 400	145 500	154 400
R			88 000	52 300	57 600	62 900	69 100	75 200
W			119 300	121 600	129 600	137 800	143 160	143 160

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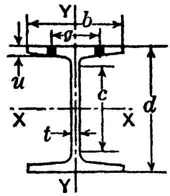
Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS



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	24 in				
Depth = *d* in	23 78	23.88	24 00	24 12	23.88
Weight per foot	107 0	113 0	120.0	128.0	132.0
Area, sq in	31 60	33.18	35.36	37.79	38 82
b in	12 195	12.210	12.240	12.280	13.210
t in	485	.500	530	.570	570
c in	20.375	20.375	20 375	20 375	20.125
g in	7½	7½	7½	7½	7½
u in	25/32	13/16	7/8	15/16	15/16
AXES X-X *I*	3 173 1	3 363 3	3 607.8	3 867 1	3 939 6
AXES X-X *S*	266 87	281 68	300 65	320 66	329 95
AXES X-X *r*	10 02	10 07	10 10	10 12	10.07
AXES Y-Y *I*	220 0	236.1	256 3	277 5	329.9
AXES Y-Y *S*	36 1	38 7	41 9	45 2	50 0
AXES Y-Y *r*	2 64	2 67	2 69	2 71	2 92
Max. Mom., in-lb.	4 803 700	5 070 300	5 411 700	5 771 800	5 939 000
V	138 400	143 300	152 600	165 000	163 300
R	58 800	61 700	67 500	75 300	75 200
W	130 980	135 000	143 160	143 160	143 160

	24 in		26 in		
Depth = *d* in	24.00	24 12	25 81	25 88	26 00
Weight per foot	140 0	143 0	138 0	144 0	151.0
Area, sq in	41 13	43.68	40 65	42 38	44 55
b in	13 240	13.280	13 700	13.730	13.750
t in	.600	.640	.580	.610	.630
c in	20 125	20 125	22.0	22.0	22.0
g in	7½	7½	10	10	10
u in	1	1 1/16	27/32	7/8	31/32
AXES X-X *I*	4 201.3	4 478.0	4 779 9	4 983 4	5 289.8
AXES X-X *S*	350 11	371.31	370 39	385 12	406.91
AXES X-X *r*	10 11	10 13	10 84	10 84	10.90
AXES Y-Y *I*	355 6	382 5	357.4	375 0	402 8
AXES Y-Y *S*	53.7	57 6	52 2	54.6	58.6
AXES Y-Y *r*	2 94	2.96	2.97	2 97	3.01
Max. Mom., in-lb.	5 301 900	6 683 600	6 667 000	6 932 100	7 324 300
V	172 800	185 200	179 600	189 400	196 600
R	8 100	88 800	78 500	84 300	88 300
W	143 160	143 160	167 000	167 000	167 000

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

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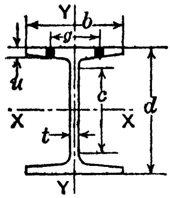


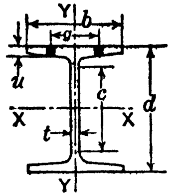
	26 in	28 in		
Depth = *d* in	26 12	27 75	27 88	28 00
Weight per foot	160 0	145 0	156 0	165.0
Area, sq in	47 25	42 69	45 93	48 75
b in	13 790	14 160	14 210	14 250
t in	.670	.585	.635	.675
c in	22.0	23 750	23 750	23.750
g in	10	10	10	10
u in	1 1/32	27/32	29/32	31/32
AXES X–X *I*	5 629 4	5 772 3	6 218 6	6 624.6
AXES X–X *S*	431 04	416 02	446 10	473 19
AXES X–X *r*	10 92	11 63	11 64	11 66
AXES Y–Y *I*	432 8	389.8	425 4	458.3
AXES Y–Y *S*	62 8	55 1	59 9	64 3
AXES Y–Y *r*	3 03	3 02	3 04	3 07
Max. Mom, in-lb.	7 758 700	7 488 400	8 029 700	8 517 300
V	210 000	194 800	212 400	226 800
R	96 400	80 000	90 600	99 100
W	167 000	167 000	167 000	167 000

	28 in		30 in	
Depth = *d* in	28 12	28 31	29 88	30 00
Weight per foot	175 0	186 0	173 0	180 0
Area, sq in	51 45	54.73	50 80	53 20
b in	14 285	14.305	14 980	15.000
t in	.710	.730	660	.680
c in	23 750	23.750	25 50	25.50
g in	10	10	10	10
u in	1 1/32	1 1/8	31/32	1 1/32
AXES X–X *I*	7 026 0	7 604 0	7 895 2	8 343.1
AXES X–X *S*	499 72	537 20	528 46	556 21
AXES X–X *r*	11 69	11 79	12 47	12 52
AXES Y–Y *I*	491 1	539 7	519 1	555 1
AXES Y–Y *S*	68 8	75 5	69 3	74 0
AXES Y–Y *r*	3 09	3 14	3 20	3 23
Max. Mom., in-lb.	8 994 900	9 669 500	9 512 300	10 011 700
V	239 600	248 000	236 600	244 800
R	106 700	111 100	97 200	101 600
W	167 000	167 000	190 880	190 880

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS



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	30 in			
Depth = *d* in	30.12	30.25	30.50	30.75
Weight per foot	190.0	200.0	220.0	240.0
Area, sq in	55.90	58.92	64.82	70.60
b in	15.030	15.065	15.135	15.200
t in	.710	.745	.815	.880
c in	25.50	25.50	25.50	25.50
g in	10	10	10	10
u in	1 3/32	1 5/32	1 9/32	1 11/32
AXES X-X *I*	8 818.0	9 343.8	10 378.0	11 423.0
AXES X-X *S*	585.52	617.77	680.52	742.96
AXES X-X *r*	12.56	12.59	12.65	12.72
AXES Y-Y *I*	592.7	634.2	716.1	799.2
AXES Y-Y *S*	78.9	84.2	94.6	105.2
AXES Y-Y *r*	3.26	3.28	3.32	3.36
Max. Mom., in-lb.	10 539 400	11 119 900	12 249 400	13 373 300
V	256 600	270 400	298 300	324 700
R	108 400	116 400	132 300	147 200
W	190 880	190 880	190 880	190 880

	33 in			
Depth = *d* in	32.88	33.00	33.12	33.25
Weight per foot	200.0	210.0	220.0	230.0
Area, sq in	58.87	61.91	64.80	67.85
b in	15.715	15.750	15.780	15.810
t in	.700	.735	.765	.795
c in	28.250	28.250	28.250	28.250
g in	10	10	10	10
u in	1 3/32	1 5/32	1 7/32	1 9/32
AXES X-X *I*	11 055	11 671	12 278	12 935
AXES X-X *S*	672.45	707.33	741.43	778.05
AXES X-X *r*	13.70	13.73	13.77	13.81
AXES Y-Y *I*	664.6	708.5	752.2	799.6
AXES Y-Y *S*	84.6	90.0	95.3	101.2
AXES Y-Y *r*	3.36	3.38	3.41	3.43
Max. Mom., in-lb.	12 104 000	12 732 000	13 345 700	14 004 800
V	276 200	291 100	304 000	317 200
R	108 000	116 300	123 600	130 900
W	190 880	190 880	190 880	190 880

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VI * (Continued). Bethlehem Girder Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
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V is Maximum Web Shear in Pounds.
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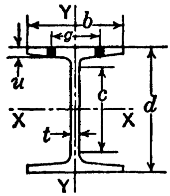

			33 in		36 in	
Depth = *d* in			33 44	33.63	35.88	36.00
Weight per foot			245 0	260 0	230 0	240 0
Area, sq in			72 19	76.54	67 67	70 55
b in			15 850	15 890	16 475	16.500
t in			.835	875	.765	790
c in			28 250	28 250	31 000	31 000
g in			10	10	10	10
u in			1⅜	1 15/32	1 1/16	1¼
AXES	X–X	*I*	13 895	14 868	14 960	15 696
		S	831 04	884.21	833.89	872 00
		r	13 87	13 94	14 87	14 92
	Y–Y	*I*	869 2	939 8	824 5	873 5
		S	109 7	118 3	100 1	105 9
		r	3 47	3 50	3 49	3 52
Max. Mom., in-lb			14 958 700	15 915 800	15 010 000	15 696 000
V			335 100	353 100	329 400	341 300
R			140 700	150 500	125 600	132 000
W			190 880	190 880	190 880	190 880

			36 in			
Depth = *d* in			36 12	36 24	36.50	36 72
Weight per foot			250 0	260 0	280 0	300 0
Area, sq in			73 61	76 50	82 45	88.12
b in			16.530	16 555	16 600	16.655
t in			.820	.845	.890	.945
c in			31 000	31 000	31.000	31.000
g in			10	10	10	10
u in			1 5/16	1⅜	1½	1⅝
AXES	X–X	*I*	16 457	17 205	18 811	20 262
		S	911.24	949 50	1 030.74	1 103 59
		r	14 95	15 00	15 10	15 16
	Y–Y	*I*	923.8	973.7	1 081.4	1 177.7
		S	111.8	117 6	130.3	141 4
		r	3 54	3 57	3 62	3 66
Max. Mom., in-lb			16 402 300	17 091 100	18 553 300	19 864 700
V			355 400	367 500	389 800	416 400
R			139 700	146 000	158 000	172 300
W			190 880	190 880	190 880	190 880

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VII.* Carnegie Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
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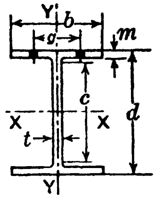
Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



			8 in					
Depth = d in			8 000	8 098	8 196	8 060	8.198	8 360
Weight per foot			24 0	27 0	30 0	31 0	36 0	42.0
Area, sq in			7 06	7 93	8 81	9 10	10 58	12 34
b in			6 500	6 529	6 559	8 000	8 046	8 100
t in			239	268	.298	290	336	390
m in			400	449	498	430	499	.580
c in			6¼	6¼	6¼	6¼	6¼	6¼
g in			3	3	3	5½	5½	5½
AXES	X–X	I	84 3	95 9	107 8	110 9	131 3	156 2
		S	21 08	23 68	26 31	27 52	32 03	37 37
		r	3 46	3 48	3 50	3 49	3 52	3 56
	Y–Y	I	18 3	20 8	23 4	36 7	43 4	51 4
		S	5 6	6 4	7.1	9 2	10 8	12.7
		r	1 61	1 62	1 63	2 01	2 02	2 04
Max. Mom., in-lb.			379 400	426 300	473 500	495 300	576 600	672 600
V			22 900	26 000	29 300	28 100	33 100	39 100
R			19 720	22 210	24 800	23 990	27 970	32 700
W			21 510	23 860	23 860	26 100	30 240	35 100

			9 in					
Depth = d in			9.000	9.096	9.192	9 000	9 122	9.242
Weight per foot			29 0	32 0	35.0	38 0	43 0	48 0
Area, sq in			8 53	9 40	10 29	11 17	12 65	14 11
b in			6 500	6 528	6 556	9 000	9 041	9.082
t in			279	307	335	316	357	.398
m in			470	.518	566	470	531	591
c in			7	7	7	7	7	7
g in			3	3	3	5½	5½	5½
AXES	X–X	I	126 0	140 5	155.4	170 4	195 5	221 1
		S	28 00	30 89	33 81	37 87	42 86	47 85
		r	3 84	3 87	3 89	3 91	3 93	3 96
	Y–Y	I	21.5	24 0	26.6	57 1	65 4	73 8
		S	6 6	7.4	8 1	12 7	14 5	16 3
		r	1 59	1 60	1 61	2 26	2 28	2 29
Max. Mom., in-lb			504 000	556 100	608 600	681 600	771 500	861 200
V			30 100	33 500	37 000	34 100	39 100	44 100
R			24 060	26 590	29 130	27 250	30 950	34 690
W			23 860	23 860	23 860	28 440	32 130	35 820

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Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS

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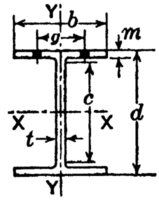


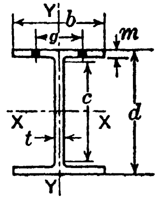
		10 in					
Depth = *d* in		9.902	10.000	10 098	10.228	10 000	10.000
Weight per foot		21.0	23.0	26 0	30.0	31 0	36 0
Area, sq in		6 17	6.76	7.64	8 82	9 11	10.58
b in		6.000	6 000	6.029	6.068	8.000	8 147
t in		.230	.230	.259	298	320	.467
m in		.332	.381	430	495	381	381
c in		8⅝	8⅝	8⅝	8⅝	8⅝	8⅝
g in		3	3	3	3	4	4
AXES X-X	*I*	107.6	122 2	139 5	163 2	163 4	175.6
	S	21.73	24 44	27 63	31 91	32 68	35.12
	r	4 18	4 25	4 27	4 30	4 23	4 07
AXES Y-Y	*I*	12 0	13 7	15 7	18 5	32 5	34 4
	S	4.0	4 6	5 2	6 1	8 1	8.5
	r	1 39	1 43	1 43	1 45	1 89	1 80
Max. Mom., in-lb.		391 200	439 900	497 300	574 400	588 200	632 200
V		27 300	27 600	31 400	36 600	38 400	56 000
R		18 900	18 890	22 410	27 070	28 800	42 030
W		20 700	20 700	23 310	23 860	28 800	42 030

		10 in					12 in
Depth = *d* in		10 000	10 000	10 000	10 000	10 000	11 924
Weight per foot		42 0	49 0	54 0	59 0	64 0	25.0
Area, sq in		12 35	14 40	15 87	17 34	18 81	7 34
b in		8 324	10 000	10 147	10 294	10 441	6 000
t in		644	350	.497	644	791	240
m in		381	558	558	558	558	382
c in		8⅝	715/16	7 15/16	7 15/16	7 15/16	10⅜
g in		4	5½	5½	5½	5½	3
AXES X-X	*I*	190 4	272 0	284.3	296 5	308 8	183 0
	S	38 08	54.40	56 86	59 30	61 76	30.69
	r	3 93	4 35	4 23	4 13	4 05	4 99
AXES Y-Y	*I*	36 8	93 0	97 3	101 7	106 3	13 8
	S	8.9	18 6	19 2	19 8	20 4	4 6
	r	1 73	2 54	2 48	2 42	2 38	1 37
Max. Mom., in-lb.		685 400	979 200	1 023 500	1 067 400	1 111 700	552 500
V		77 300	42 000	59 600	77 300	94 900	34 300
R		57 960	31 500	44 730	57 960	71 190	19 840
W		47 720	31 520	44 730	47 720	47 720	21 600

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS



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			12 in					
Depth = *d* in			12.000	12.118	†12.022	12.236	12.000	12.130
Weight per foot			28.0	32.0	34.0	36.0	40.0	45.0
Area, sq in			8.22	9.40	9.99	10.59	11.76	13.23
b in			6.500	6.534	6.635	6.568	8.000	8.036
t in			.240	.274	.375	.308	.290	.326
m in			.420	.479	.431	.538	.526	.591
c in			10⅜	10⅜	10⅜	10⅜	9⅞	9⅞
g in			3½	3½	3½	3½	4	4
AXES	X–X	*I*	213.4	246.3	238.1	280.1	313.7	356.9
		S	35.57	40.65	39.61	45.78	52.28	58.85
		r	5.10	5.12	4.88	5.14	5.17	5.19
	Y–Y	*I*	19.2	22.3	21.0	25.4	44.9	51.2
		S	5.9	6.8	6.3	7.7	11.2	12.7
		r	1.53	1.54	1.45	1.55	1.95	1.97
Max. Mom., in-lb			640 200	731 700	713 000	824 100	941 100	1 059 200
V			34 600	39 800	45 100	45 200	41 800	47 500
R			19 820	24 290	36 590	28 790	26 400	31 140
W			21 600	23 860	23 860	23 860	23 860	23 860

			12 in					
Depth = *d* in			12.258	12.000	12.118	12.260	12.000	12.000
Weight per foot			50.0	55.0	60.0	66.0	65.0	70.0
Area, sq in			14.69	16.17	17.65	19.41	19.11	20.58
b in			8.071	9.000	9.034	9.073	12.000	12.123
t in			.361	.375	.409	.448	.400	.523
m in			.655	.665	.724	.795	.608	.608
c in			9⅞	9½	9½	9½	9⅝	9⅝
g in			4	5½	5½	5½	7½	7½
AXES	X–X	*I*	400.5	428.4	472.0	525.7	521.3	539.0
		S	65.35	71.40	77.90	85.76	86.88	89.83
		r	5.22	5.15	5.17	5.20	5.22	5.12
	Y–Y	*I*	57.5	80.9	89.0	99.1	175.2	180.7
		S	14.2	18.0	19.7	21.8	29.2	29.8
		r	1.98	2.24	2.25	2.26	3.03	2.96
Max. Mom., in-lb.			1 176 200	1 285 200	1 402 200	1 543 700	1 563 900	1 617 000
V			53 100	54 000	59 500	65 900	57 600	75 300
R			35 550	36 560	40 060	44 120	39 000	50 990
W			23 860	50 640	55 220	60 480	54 000	70 610

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

† Special section.

Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS

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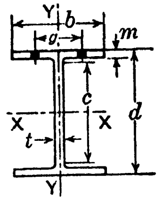


	12 in	14 in				
Depth = d in.	12.000	13 964	14 000	14.080	†14.000	14 160
Weight per foot.	76 0	30.0	33.0	36.0	38.0	39 0
Area, sq in	22.35	8.82	9.71	10.58	11.18	11.47
b in.	12 270	6.000	6.750	6.774	6.855	6 798
t in.	.670	.270	.270	.294	.375	.318
m in	.608	.431	.449	.489	.449	.529
c in	9⅝	12¼	12¼	12¼	12¼	12¼
g in	7½	3	3½	3½	3½	3½
AXES X-X I	560.2	292.0	333.4	365 6	357 5	398 3
AXES X-X S	93.36	41.82	47.63	51 93	51 07	56 26
AXES X-X r	5 01	5 75	5 86	5 88	5 66	5 89
AXES Y-Y I	187.5	15 5	23 0	25.4	24 2	27 7
AXES Y-Y S	30 6	5.2	6.8	7.5	7 1	8.2
AXES Y-Y r	2 90	1 33	1 54	1 55	1 47	1 56
Max. Mom., in-lb	1 680 600	752 800	857 300	934 800	919 300	1 012 600
V	96 500	45 200	45 400	49 700	63 000	54 000
R	65 300	23 500	23 490	26 880	38 340	30 290
W	71 580	24 300	24 300	26 460	33 750	28 620

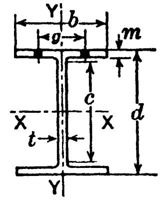
	14 in					
Depth = d in	14.240	14.000	14 122	14 242	14.094	14 238
Weight per foot	42.0	48.0	53.0	58.0	61 0	68 0
Area, sq in	12 35	14.12	15.59	17 05	17 94	19 99
b in.	6.822	8.000	8.035	8.070	10.000	10.043
t in.	.342	.343	.378	.413	.382	.425
m in.	.569	.595	.656	.716	.642	.714
c in	12¼	11⅝	11⅝	11⅝	11⅝	11⅝
g in	3½	4	4	4	5½	5½
AXES X-X I	431.5	496.0	552.5	609.4	656.2	733.8
AXES X-X S	60.60	70.86	78.25	85.58	93.12	103.78
AXES X-X r	5 91	5 93	5 95	5.98	6 05	6 08
AXES Y-Y I	30.2	50.8	56.8	62.8	107.1	120.6
AXES Y-Y S	8 8	12 7	14 1	15.6	21 4	24.0
AXES Y-Y r	1 56	1.90	1.91	1 92	2 44	2 46
Max. Mom., in-lb.	1 090 900	1 275 400	1 408 400	1 540 400	1 676 100	1 863 000
V	58 400	57 600	64 100	70 600	64 600	72 600
R	33 720	33 820	38 810	43 740	39 360	45 000
W	30 780	46 300	51 030	55 760	51 570	57 370

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

† Special section.

Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS



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S is Section-Modulus.
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For Allowable Total Loads, see Table V, Chapter XV

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.

	14 in				16 in	
Depth = *d* in	14 382	14 000	14 186	14.370	15 930	16 012
Weight per foot.	75.0	85.0	95.0	105 0	35.0	38.0
Area, sq in	22 05	24.99	27 93	30 88	10 29	11.17
b in.....	10 086	12.000	12 050	12 101	6 000	6.024
t in.	.468	.435	.485	536	.290	.314
m in......	.786	805	898	990	485	.526
c in....	11⅝	11	11	11	14	14
g in.. .	5½	7½	7½	7½	3	3
AXES X–X *I*	823 5	921 3	1 044 0	1 169.6	435 5	475 1
AXES X–X *S*	114 52	131.61	147.19	162.78	54 68	59.34
AXES X–X *r*	6 11	6 07	6 11	6 15	6 50	6 52
AXES Y–Y *I*	134 5	232 0	262.0	292 6	17.5	19 2
AXES Y–Y *S*	26 7	38 7	43.5	48.4	5 8	6 4
AXES Y–Y *r* ...	2 47	3 05	3 06	3 08	1 30	1.31
Max. Mom., in-lb.	2 061 300	2 369 100	2 649 400	2 930 100	984 200	1 068 200
V........	80 800	73 100	82 600	92 400	55 400	60 300
R.. . .	49 810	45 670	51 260	57 020	25 990	29 590
W .	63 180	58 730	65 470	71 580	26 100	28 260

	16 in					
Depth = *d* in .	16.000	†15.934	16 128	16 254	16 000	16 114
Weight per foot..	40.0	43 0	45 0	50.0	58 0	63.0
Area, sq in ...	11 75	12.65	13 23	14.70	17 06	18.52
b in	7 000	7.085	7.036	7.072	8.500	8 531
t in	290	.375	.326	362	375	.406
m in . .	.520	.487	.584	.647	.663	.720
c in	14	14	14	14	13⅜	13⅜
g in	4	4	4	4	5½	5½
AXES X–X *I*...	524 6	523 8	595.0	666.0	776.6	849.9
AXES X–X *S* ..	65.58	65 75	73 78	81.95	97.08	105.49
AXES X–X *r*	6.68	6.44	6 71	6.73	6 75	6 77
AXES Y–Y *I*. .	29 8	28.9	34 0	38 2	68 0	74.6
AXES Y–Y *S*...	8 5	8.2	9.7	10 8	16.0	17.5
AXES Y–Y *r*... .	1 59	1.51	1.60	1 61	2.00	2.01
Max. Mom., in-lb.	1 180 400	1 183 400	1 328 100	1 475 100	1 747 400	1 898 700
V.......... ..	55 700	71 700	63 100	70 600	72 000	78 500
R	25 970	38 830	31 390	36 890	38 840	43 580
W	26 100	33 750	29 340	32 580	67 500	73 080

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† Special section.

Table VII* (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS

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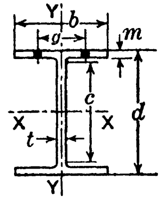

	16 in					
Depth = *d* in	16 226	16 000	16.120	16 240	16 000	16.110
Weight per foot	68.0	76 0	83.0	90.0	100 0	107.0
Area, sq in	20.00	22.34	24.41	26.46	29 41	31.46
b in	8.563	12.000	12.039	12.076	14 000	14.032
t in	.438	419	.458	.495	464	.496
m in	776	663	.723	.783	800	.855
c in	13⅝	13⅝	13⅝	13⅝	13	13
g in	5½	7½	7½	7½	10	10
AXES X-X *I*	923 7	1 061.3	1 167.7	1 275.5	1 426.8	1 537.2
AXES X-X *S*	113 85	132 66	144.88	157.08	178 35	190.84
AXES X-X *r*	6.80	6 89	6 92	6.94	6 97	6.99
AXES Y-Y *I*	81 3	191.1	210 4	230.0	366 0	393.9
AXES Y-Y *S*	19 0	31.8	35.0	38.1	52 3	56.1
AXES Y-Y *r*	2 02	2.92	2.94	2 95	3 53	3.54
Max. Mom., in-lb.	2 049 400	2 387 900	2 607 800	2 827 500	3 210 300	3 435 100
V	85 300	80 500	88 600	96 500	89 100	95 900
R	48 480	45 510	51 450	56 130	52 200	56 000
W	78 840	75 420	82 440	89 100	83 520	89 280

	16 in	18 in				
Depth = *d* in	16.236	18 000	†18 024	18 114	18 252	18.000
Weight per foot	115 0	47 0	51.0	52.0	58 0	67 0
Area, sq in	33.82	13 82	15.00	15.30	17.05	19.69
b in	14 068	7.500	7.555	7.534	7 573	8.500
t in	532	320	.375	354	393	406
m in	918	.550	562	607	676	.745
c in	13	15⅞	15⅞	15⅞	15⅞	15⅞
g in	10	3¾	3¾	3¾	3¾	5½
AXES X-X *I*	1 665.6	768 6	810 0	855 1	960 8	1 117.1
AXES X-X *S*	205.17	85.4	89 88	94 41	105.28	124.12
AXES X-X *r*	7.02	7.46	7.35	7 48	7 51	7.53
AXES Y-Y *I*	426.2	38 7	40 5	43 3	49.0	76.4
AXES Y-Y *S*	60.6	10.3	10.7	11.5	13.0	18.0
AXES Y-Y *r*	3.55	1.67	1 64	1.68	1 70	1.97
Max. Mom., in-lb.	3 693 100	1 537 200	1 617 800	1 699 400	1 895 100	2 234 200
V	103 700	69 100	81 100	77 000	86 100	87 700
R	60 320	30 170	39 020	35 610	41 950	44 040
W	95 440	36 000	42 190	39 830	44 220	91 350

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Table VII* (Continued). Carnegie Beams

Dimensions—Functions

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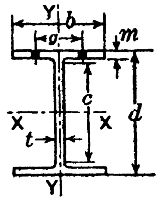

			18 in				
Depth = *d* in			18 110	18 242	18.000	18.120	18.238
Weight per foot			72.0	78 0	86 0	93.0	100 0
Area, sq in			21 17	22 94	25 29	27.35	29 40
b in			8 530	8 565	12 000	12 034	12 069
t in			.436	471	429	463	.498
m in			800	866	745	805	864
c in			15⅛	15⅛	15⅛	15⅛	15⅛
g in			5½	5½	7½	7½	7½
AXES	X–X	*I*	1 208 1	1 318 8	1 514 1	1 648.4	1 783.4
		S	133 42	144 59	168.23	181 94	195 57
		r	7 55	7 58	7 74	7 76	7 79
	Y–Y	*I*	82 9	90 9	214.7	234 0	253.4
		S	19 4	21 2	35 8	38 9	42 0
		r	1 98	1 99	2 91	2 93	2 94
Max. Mom., in-lb.			2 401 500	2 602 600	3 028 200	3 275 000	3 520 200
V			94 800	103 100	92 700	100 700	109 000
R			48 930	54 670	47 760	53 310	59 040
W			98 100	105 980	96 530	104 180	112 050

			21 in				
Depth = *d* in			20 890	21.000	21.126	21 248	21 370
Weight per foot			55 0	58.0	64 0	70 0	76.0
Area, sq in			16 17	17 05	18.82	20 59	22.34
b in			8.000	8 000	8.036	8 073	8 109
t in			.360	.360	.396	.433	469
m in			.553	.608	.671	.732	793
c in			18⅝	18⅝	18⅝	18⅝	18⅝
g in			4	4	4	4	4
AXES	X–X	*I*	1 166.7	1 263 2	1 403 3	1 542 9	1 684.0
		S	111.70	120.30	132.85	145 23	157.60
		r	8 49	8 61	8 64	8 66	8 68
	Y–Y	*I*	47.29	52 0	58.2	64 3	70.67
		S	11.8	13.0	14 5	15 9	17.4
		r	1 71	1 75	1 76	1.77	1 78
Max. Mom., in-lb.			2 010 600	2 165 500	2 391 300	2 614 100	2 836 900
V			90 240	90 700	100 400	110 400	120 300
R			36 210	36 180	42 460	49 010	55 460
W			48 600	48 600	53 460	58 460	63 320

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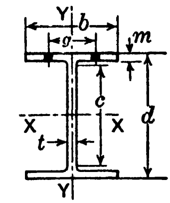
Table VII* (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS



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	21 in				
Depth = *d* in.	21.000	21.120	21.240	21.358	21 000
Weight per foot	80.0	86 0	92.0	98.0	104 0
Area, sq in	23.53	25.28	27.05	28.82	30 57
b in.	9.000	9.032	9.064	9.097	13 000
t in.	.438	.470	.502	.535	.465
m in.	.815	.875	935	.994	815
c in.	17⅞	17⅞	17⅞	17⅞	17⅞
g in.	5½	5½	5½	5½	7½
AXES X–X *I*	1794.4	1 939.3	2 086.4	2 234.5	2 475 3
AXES X–X *S*	170.90	183 65	196 46	209.24	235 74
AXES X–X *r*	8.73	8.76	8 78	8 80	9 00
AXES Y–Y *I*	99 2	107 7	116.3	125.0	298 7
AXES Y–Y *S*	22 0	23.8	25 7	27 5	45 9
AXES Y–Y *r*	2.05	2 06	2 07	2 08	3 13
Max. Mom., in-lb.	3 076 100	3 305 600	3 536 300	3 766 400	4 243 400
V	110 400	119 100	128 000	137 100	117 200
R	49 880	55 580	61 320	67 260	54 660
W	59 130	63 450	67 770	71 580	104 630

	21 in				24 in
Depth = *d* in.	21.126	21.248	21.372	21 492	24 000
Weight per foot	112 0	120.0	128.0	136.0	70.0
Area, sq in	32.93	35.28	37.65	40.00	20.58
b in.	13 034	13.070	13.105	13.141	8 500
t in.	.499	.535	.570	.606	.400
m in.	.878	.939	1.001	1.061	.666
c in.	17⅞	17⅞	17⅞	17⅞	21⅜
g in.	7½	7½	7½	7½	4
AXES X–X *I*	2 683.7	2 890 9	3 103.4	3 313.7	1 953 8
AXES X–X *S*	254.07	272.11	290.42	308.37	162.82
AXES X–X *r*	9.03	9 05	9 08	9 10	9 74
AXES Y–Y *I*	324.3	349.7	375.9	401.7	68.0
AXES Y–Y *S*	49.8	53.5	57.4	61.1	16 0
AXES Y–Y *r*	3 14	3 15	3 16	3 17	1 82
Max. Mom., in-lb.	4 573 200	4 898 000	5 227 500	5 550 600	2 930 700
V	126 500	136 400	146 200	156 300	115 200
R	60 730	67 190	73 500	80 020	42 750
W	112 280	119 300	119 300	119 300	54 000

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Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS

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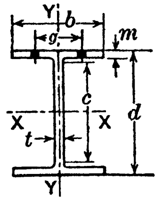

			24 in				
Depth = *d* in.			24 000	24 154	24 308	24 000	24.156
Weight per foot			76 0	85 0	94 0	100 0	110 0
Area, sq in			22.35	24.99	27 64	29 41	32.34
b in			9.750	9.797	9.844	12.000	12.044
t in			.405	452	499	.450	.494
m in			.663	740	.817	787	865
c in			21⅛	21⅛	21⅛	20¾	20¾
g in			5½	5½	5½	7½	7½
AXES	X–X	*I*	2 184 4	2 457 2	2 734 9	3 020 5	3 343 5
		S	182 03	203 46	225 02	251 71	276 83
		r	9 89	9 92	9 95	10 14	10 17
	Y–Y	*I*	102 6	116 2	130 2	226 9	252.2
		S	21 0	23 7	26 4	37.8	41 9
		r	2 14	2 16	2 17	2 78	2 79
Max. Mom., in-lb.			3 276 600	3 662 300	4 050 400	4 530 700	4 982 900
V			116 600	131 000	145 600	129 600	143 200
R			43 680	52 580	61 640	52 200	60 650
W			54 680	61 020	67 370	121 500	133 380

			24 in				
Depth = *d* in			24.310	24 250	24 388	24 526	24.664
Weight per foot			120 0	130 0	140 0	150 0	160.0
Area, sq in			35 29	38 23	41 16	44 10	47.06
b in			12.089	14 000	14.041	14.082	14.123
t in			.539	.547	588	629	.670
m in			.942	.912	981	1 050	1.119
c in			20¾	20¾	20¾	20¾	20¾
g in			7½	10	10	10	10
AXES	X–X	*I*	3 669 7	4 045 1	4 380 4	4 720 5	5 065 7
		S	301 91	333 62	359 23	384 94	410.78
		r	10 20	10 29	10 32	10.35	10 38
	Y–Y	*I*	277 8	417 5	453 1	489 3	526 0
		S	46 0	59 6	64 5	69 5	74.5
		r	2 81	3 31	3 32	3 33	3 34
Max. Mom., in-lb.			5 434 300	6 005 100	6 466 100	6 928 900	7 394 000
V			157 200	159 200	172 100	185 100	198 300
R			69 400	70 920	78 940	87 000	95 100
W			143 160	143 160	143 160	143 160	143 160

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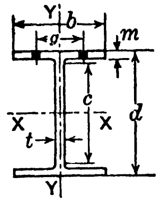
Table VII* (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS



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	27 in				
Depth = *d* in.	26.820	27.000	27 166	27.340	27.536
Weight per foot	85.0	91 0	101.0	112.0	124 0
Area, sq in	25 00	26.76	29.70	32 94	36 47
b in.	9 750	9 750	9 799	9.855	9 913
t in	.461	.461	.510	.566	624
m in	.665	.755	838	.925	1 023
c in	24⅛	24⅛	24⅛	24⅛	24⅛
g in	5½	5½	5½	5½	5½
AXES X–X *I*	2 899.3	3 217.0	3 595.7	4 007.6	4 472.1
AXES X–X *S*	216 20	238 30	264.72	293 17	324.82
AXES X–X *r*	10 77	10 97	11 00	11 03	11 07
AXES Y–Y *I*	103 0	116 9	131 7	148 0	166.7
AXES Y–Y *S*	21 1	24 0	26 9	30 0	33 6
AXES Y–Y *r*	2 03	2 09	2 11	2 12	2 14
Max. Mom., in-lb.	3 891 700	4 289 300	4 765 000	5 277 000	5 846 700
V	148 400	149 400	166 300	185 700	206 200
R	54 140	54 120	64 140	75 810	88 050
W	82 980	82 980	91 800	95 440	95 440

	27 in				
Depth = *d* in.	27.742	27.000	27.200	27.400	27.598
Weight per foot	137 0	145.0	160 0	175.0	190.0
Area, sq in	40.29	42 64	47 04	51 47	55 87
b in.	9 977	14.000	14 059	14 118	14 176
t in.	.688	.580	.639	698	.756
m in.	1.126	.985	1.085	1.185	1.284
c in.	24⅛	23¼	23¼	23¼	23¼
g in	5½	10	10	10	10
AXES X–X *I*	4975 9	5 508.7	6 121.8	6 746.8	7 376.9
AXES X–X *S*	358.73	408.05	450.13	492.47	534.60
AXES X–X *r*	11.11	11.37	11.41	11 45	11.49
AXES Y–Y *I*	187.1	451.0	503.2	556.6	610.7
AXES Y–Y *S*	37 5	64.4	71.6	78 9	86 2
AXES Y–Y *r*	2 16	3.25	3 27	3.29	3 31
Max. Mom., in-lb.	6 457 100	7 344 900	8 102 400	8 864 400	9 622 700
V	229 000	187 900	208 600	229 500	250 400
R	101 660	78 620	90 990	103 470	115 800
W	95 440	167 020	167 020	167 020	167 020

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Table VII * (Continued). Carnegie Beams

DIMENSIONS—FUNCTIONS

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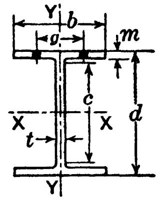


			30 in				
Depth = *d* in			30 000	30 162	30 344	30 538	30 742
Weight per foot			115 0	126 0	138 0	151 0	165 0
Area, sq in			33 81	37 05	40 58	44 41	48.52
b in			10 500	10 551	10 604	10 662	10 725
t in			530	581	634	692	755
m in			882	963	1 054	1 151	1 253
c in			26¾	26¾	26¾	26¾	26¾
g in			5½	5½	5½	5½	5½
AXES	X–X	*I*	4 985 3	5 486 7	6 049 5	6 663 7	7 326 7
		S	332 35	363 82	398 73	436 42	476 66
		r	12 14	12 17	12 21	12 25	12 29
	Y–Y	*I*	170 6	189 0	210 1	233 4	258 7
		S	32 5	35 8	39 6	43 8	48 2
		r	2 25	2 26	2 28	2 29	2 31
Max. Mom., in-lb.			5 982 400	6 548 700	7 177 100	7 855 600	8 579 900
V			190 800	210 300	230 900	253 600	278 500
R			68 410	79 670	91 550	104 700	119 090
W			107 330	107 370	107 370	107 370	107 370

			30 in				33 in
Depth = *d* in			30 000	30 263	30 522	30 781	33 000
Weight per foot			180 0	200 0	220 0	240 0	125.0
Area, sq in			52 93	58 82	64 70	70 58	36.75
b in			14 000	14 073	14 146	14.218	12.000
t in			.670	743	.816	.888	.540
m in			1 207	1 338	1 468	1.597	.797
c in			25½	25½	25½	25½	29½
g in			10	10	10	10	7½
AXES	X–X	*I*	8 301 4	9 305 7	10 320 4	11 356 0	6 514 3
		S	553 43	614 99	676.26	737 86	394.81
		r	12 52	12 58	12 63	12 69	13 31
	Y–Y	*I*	552 7	622 7	693.9	766 9	230 1
		S	79.0	88.5	98 1	107 9	38 4
		r	3 23	3 25	3 28	3 30	2 50
Max. Mom., in-lb.			9 961 700	11 069 800	12 172 700	13 281 400	7 106 500
V			241 200	269 800	298 900	328 000	213 800
R			99 440	115 940	132 570	149 090	70 370
W			190 880	190 880	190 880	190 880	107 370

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction

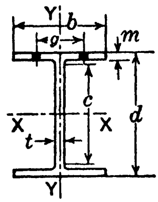
Table VII * (Continued). Carnegie Beams

Dimensions—Functions

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



			33 in			
Depth = *d* in			33 164	33.342	33 530	33 000
Weight per foot.			138 0	152.0	167.0	200.0
Area, sq in			40 58	44 69	49.12	58 82
b in			12.056	12 115	12.179	16 000
t in			596	.655	.719	.720
m in			879	.968	1 062	1 1255
c in			29½	29½	29½	28¾
g in			7½	7½	7½	10
AXES	X–X	*I*	7 223 0	7 998 5	8 836 1	11 049 6
		S	435 59	479 79	527 06	669.67
		r	13 34	13 38	13.41	13 71
	Y–Y	*I*	257 5	287 8	321 0	769 5
		S	42 7	47.5	52.7	96 2
		r	2 52	2 54	2.56	3 62
Max. Mom., in-lb.			7 840 700	8 636 100	9 487 000	12 054 100
V			237 200	262 100	289 300	285 100
R			83 420	97 450	112 860	112 770
W			107 370	107 370	107 370	190 880

			33 in			36 in
Depth = *d* in			33 272	33.546	33.786	36 000
Weight per foot.			220 0	240 0	260 0	147.0
Area, sq in			64.70	70 58	76.47	43 23
b in			16.046	16 090	16.150	12.000
t in			766	.810	.870	.590
m in			1 2615	1 3985	1.5185	.9345
c in			28¾	28¾	28¾	32¼
g in			10	10	10	7½
AXES	X–X	*I*	12 385 5	13 750 6	15 037 7	9 040 4
		S	744.50	819 81	890.17	502 24
		r	13 84	13 96	14 02	14 46
	Y–Y	*I*	870 0	972 5	1 068 0	269 9
		S	108.4	120 9	132 3	45 0
		r	3 67	3 71	3 74	2 50
Max. Mom., in-lb.			13 401 000	14 756 500	16 023 100	9 040 400
V			305 800	326 100	352 700	254 900
R			123 930	134 790	149 460	81 940
W			190 880	190 880	190 880	107 370

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

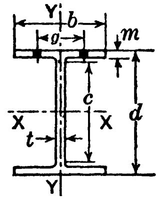
Table VII * (Continued). Carnegie Beams

Dimensions—Functions

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.

Note: The attention of the reader is especially called to Remarks on Stock Sizes, Chapter XV.



	36 in			
Depth = *d* in	36 183	36 395	36 645	36.000
Weight per foot	160 0	175.0	192 0	230.0
Area, sq in	47 06	51 47	56 47	67.65
b in	12 045	12 096	12 150	16.000
t in	635	.686	.740	769
m in	1 026	1 132	1 257	1.290
c in	32¼	32¼	32¼	31½
g in	7½	7½	7½	10
AXES X-X *I*	9 933 2	10 978 8	12 208 5	15 012 9
AXES X-X *S*	549 05	603 31	666 31	834 05
AXES X-X *r*	14 53	14 61	14 70	14 90
AXES Y-Y *I*	299 8	335 0	377 2	882.2
AXES Y-Y *S*	49 8	55 4	62 1	110 3
AXES Y-Y *r*	2 52	2 55	2 58	3 61
Max. Mom., in-lb.	9 883 000	10 859 600	11 993 600	15 012 900
V	275 700	299 600	325 400	332 200
R	93 050	105 870	119 740	126 690
W	107 370	107 370	107 370	190 880

	36 in			
Depth = *d* in	36 243	36 550	36 851	
Weight per foot	250.0	275.0	300 0	
Area, sq in	73 53	80.87	88 23	
b in	16 055	16.121	16 189	
t in	.824	.890	958	
m in	1 4115	1 565	1 7155	
c in	31½	31½	31½	
g in	10	10	10	
AXES X-X *I*	16 499 3	18 400.2	20 317.7	
AXES X-X *S*	910 48	1 006 85	1 102 69	
AXES X-X *r*	14 98	15 08	15 18	
AXES Y-Y *I*	975.4	1 095 1	1 215 9	
AXES Y-Y *S*	121 5	135 9	150 2	
AXES Y-Y *r*	3 64	3 68	3 71	
Max. Mom., in-lb.	16 388 700	18 123 300	19 848 500	
V	358 400	390 400	423 600	
R	140 860	158 030	175 860	
W	190 880	190 880	190 880	

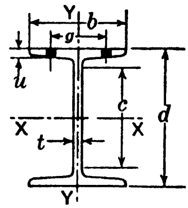
* Compiled from data in Steel Construction, published by the American Institute of Steel Construction

Table VIII.* Miscellaneous Beams

Carnegie Mill Sections. Carnegie H Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.
For Allowable Total Loads, see Table V, Chapter XV.



Carnegie Mill Sections

Depth = *d* in			8 00	8 00	9 00	9 00
Weight per foot			17 5	21 0	20 5	25 0
Area, sq in			5 14	6 17	6 02	7 34
b in			4.981	5 110	5 234	5.380
t in			231	360	234	380
c in			6⅝	6⅝	7½	7½
g in			2¼	2¼	2½	2½
u in			⅜	⅜	13/32	13/32
AXES	X–X	*I*	57 4	63 4	86 6	95 5
		S	14.35	15 85	19 24	21 22
		r	3 36	3 21	3 79	3 61
	Y–Y	*I*	6 0	6 6	8 0	8 8
		S	2 4	2 6	3 1	3 3
		r	1.08	1 03	1 15	1 09
Max. Mom., in-lb			258 300	285 300	346 400	382 000
V			22 180	34 560	25 270	41 040
R			19 060	29 700	19 430	32 770
W			20 800	23 860	21 060	23 860

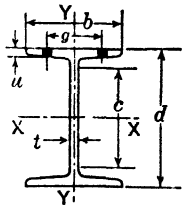
Carnegie H Beams

Depth = *d* in			**4 in**	**5 in**	**6 in**	**6 in**
Weight per foot			13 8	18 9	20 0	22 5
Area, sq in			3 99	5 47	5 86	6 61
b in			4 000	5 000	5 938	6.063
t in			313	313	250	.375
c in			2 522	3 413	4 458	4 458
g in			2¼	2¾	3½	3½
u in			⅜	7/16	⅜	⅜
AXES	X–X	*I*	10.7	23 8	38 8	41.0
		S	5 35	9 52	12 93	13.67
		r	1 64	2 08	2 57	2 49
	Y–Y	*I*	3 6	7 8	11 4	12 2.
		S	1 8	3 1	3 8	4.0
		r	0 95	1 20	1 39	1 36
Max. Mom., in-lb.			96 300	173 160	232 740	246 060
V			15 020	18 780	18 000	27 000
R			21 130	22 300	18 750	28 130
W			11 930	11 930	11 250	11 930

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table VIII* (Continued). Miscellaneous Beams

Carnegie H Beams. Special Phoenix I Beams



DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 3½ in bearing.
W is Maximum Load on one Standard Connection.
For Allowable Total Loads, see Table V, Chapter XV

Carnegie H Beams

			6 in	6 in	8 in	8 in	8 in
Depth = d in...			**6 in**	**6 in**	**8 in**	**8 in**	**8 in**
Weight per foot.			25.0	27.5	32.6	34.3	37.7
Area, sq in			7.33	8.08	9.50	10.00	11.00
b in			5.938	6.063	7.938	8.000	8.125
t in			.313	.438	.313	.375	.500
c in			4.256	4.256	6.287	6.287	6.287
g in			3½	3½	4	4	4
u in			½	½	½	½	½
AXES	X–X	I	47.0	49.3	112.8	115.5	120.8
		S	15.67	16.43	28.20	28.88	30.20
		r	2.53	2.47	3.45	3.40	3.31
	Y–Y	I	14.9	16.0	34.2	35.1	36.9
		S	5.0	5.3	8.6	8.8	9.1
		r	1.43	1.41	1.90	1.87	1.83
Max. Mom, in-lb			282 000	295 740	507 600	519 840	543 600
V			22 540	31 540	30 050	36 000	48 000
R			23 480	32 850	25 820	30 940	41 250
W			11 930	11 930	23 860	23 860	23 860

Special Phoenix I Beams

			12 in	15 in
Depth = d in			**12 in**	**15 in**
Weight per foot			27.5	36.0
Area, sq in			8.09	10.59
b in			5.00	5.50
t in			.255	.289
c in			9¾	12⅝
g in			3	3½
u in			½	9/16
AXES	X–X	I	199.6	405.1
		S	33.27	54.01
		r	4.98	6.17
	Y–Y	I	8.70	13.50
		S	3.48	4.91
		r	1.04	1.13
Maximum Moment, in-lb			598 800	972 240
V			36 700	52 000
R			21 800	26 000
W			22 950	26 010

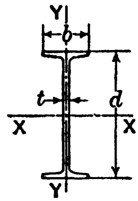
* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

Table IX.* Steel Joists

Bethlehem Steel Joists. J & L Junior Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 2½ in bearing.
W is Maximum Load on one Standard Connection.
For Allowable Total Loads, see Table V, Chapter XV.



		Depth = *d* in	6 in	8 in	10 in	12 in
Bethlehem Steel Joists		Weight per foot	11 0	14.5	16 5	18 5
		Area, sq in	3 25	4.28	4.86	5 44
		b in.. . . .	3 33	3 875	4 000	4 125
		t in	230	240	240	240
	AXES X–X	*I*	19 3	44 9	77.4	121 5
		S..	6 43	11 23	15 48	20 25
		r.	2 44	3 24	3 99	4 73
	AXES Y–Y	*I*.	1 64	2 73	3 02	3 33
		S..	98	1 41	1 51	1 61
		r. .	71	80	79	78
		Max. Mom., in-lb..	115 800	202 050	278 640	364 500
		V.	16 560	23 040	28 800	34 560
		R	13 800	16 200	16 750	16 770
		W .	10 360	10 800	10 800	10 800
.			6 in	7 in	8 in	9 in
J & L Junior Beams		Depth = *d* in	6.028	7 028	8.028	9 018
		Weight per foot.	4 41	5 48	6 54	7 48
		Area, sq in.	1 30	1 61	1 92	2 20
		b in	1 84	2 08	2 28	2 38
		t in	114	126	135	145
	AXES X–X	*I*	7.30	12 13	18 67	26 20
		S ..	2 42	3.45	4 65	5 81
		r	2 372	2 744	3 116	3 450
	AXES Y–Y	*I*	.1654	2482	3434	3939
		S. . .	1794	2389	.3011	.3317
		r .	3569	3925	4226	4230
		Max. Mom., in-lb.	43 610	62 137	83 726	104 591
		V..	8 210	10 580	12 960	15 660
		R.	5 620	6 370	6 900	7 550
		W .	5 130	5 670	6 075	6 525

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

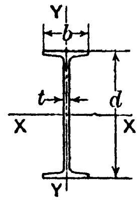
Table IX * (Continued). Steel Joists

J & L Junior Beams

DIMENSIONS—FUNCTIONS

I is Moment of Inertia.
S is Section-Modulus.
r is Radius of Gyration.
V is Maximum Web Shear in Pounds.
R is Allowable End Reaction for 2½ in bearing.
W is Maximum Load on one Standard Connection.

For Allowable Total Loads, see Table V, Chapter XV.



				10 in	11 in	12 in
J & L Junior Beams	Depth = *d* in			10 026	11 024	12 028
	Weight per foot			8 96	10.23	11 74
	Area, sq in			2 64	3 01	3.45
	b in			2 69	2.84	3 06
	t in			155	.165	175
	AXES	X–X	*I*	39 01	53 08	72.21
			S	7 78	9 63	12 01
			r	3 847	4 200	4 573
		Y–Y	*I*	.6078	7459	.9776
			S	4523	5246	.6385
			r	.4802	4979	.5320
	Max. Mom., in-lb			140 054	173 332	216 131
	V			18 330	20 930	23 700
	R			8 240	8 960	9 710
	W			6 975	7 425	7 875

* Compiled from data in Steel Construction, published by the American Institute of Steel Construction.

CHAPTER XI

RESISTANCE TO TENSION. PROPERTIES OF IRON AND STEEL

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1. Definitions, Working Stresses and Examples

The Ultimate Tensile Strength of a material is the amount of internal stress which a section one square inch in area is capable of exerting against an external axial force. It is the UNIT STRESS or INTENSITY OF STRESS, expressed in pounds per square inch, which the material can withstand. It is often called the ULTIMATE STRENGTH or ULTIMATE STRESS of the material. Its value for any material depends on the tenacity of the fibers or the cohesion of the particles of which the material is composed.

An Axial Force is one which acts uniformly over the section of a prismatic body so that the resultant of the distributed forces coincides with the axis of the body. Hence the total AXIAL FORCE which any cross-section of a body will resist is the product of the ultimate strength of the material and the area of the cross-section, in square inches.

Safe Working Stress. The ultimate strength of different building materials has been found by pulling apart bars of known dimensions and dividing the maximum load each sustained by the area of the bar before testing. This ultimate strength, however, must not be used to proportion the size of members of structures, because of variations in material, hidden defects and imperfect workmanship; and, especially, because of indefiniteness as to the maximum load that may be imposed on the structure. To provide safety against the rupture of a member and the consequent failure of the structure from any of these causes, the proportions of the members must be based on SAFE WORKING STRESSES which are usually some fractional part of the ultimate strength found by experiment to provide proper security against failure.

The Factor of Safety is the ratio of the ultimate strength to this safe working stress for that material. Its value ranges generally from 2 to 10, depending upon the nature of the material and the service to which it is applied.

Safe Working Stress in Tension. The SAFE WORKING STRESS on structural steel is usually taken at 16 000 lb per sq in. Philadelphia Code permits 18 000 on reinforcing steel. There is a tendency toward increasing these stresses; 20 000 is recommended by the American Institute of Steel Construction, Inc. On wrought iron, which is only occasionally used, the stress is 14 000. It is not considered good practice to use cast iron and timber in tension. In case these materials should be used in tension, safe values are suggested in Chapter XX. The total SAFE LOAD that may be applied to a piece of material of uniform section is found by multiplying the cross-section of the piece, in square inches, by the safe working stress opposite the name of the material of which the piece is composed.

Then if P = the safe load in lb,
S_t = the allowable safe working stress in tension,
b = the width of a rectangular bar,
h = the depth of a rectangular bar,
d = the diameter of a round bar,

there results, for a rectangular bar,

$$P = bhS_t \tag{1}$$

and for a round bar,

$$P = 0.7854\, d^2S_t \tag{2}$$

The area of cross-section to support a load P is, for a rectangular bar,

$$A = \frac{P}{S_t} \tag{3}$$

and for a round bar

$$d = \sqrt{\frac{P}{0.7854\, S_t}} \tag{4}$$

Example 1. What size of medium-steel angle should be used to sustain a tensile force of 64 000 lb?

Answer. By Formula (3),

$$\text{the net sectional area} = \frac{64\,000}{16\,000} = 4\ 00 \text{ sq in}$$

From the Table of the Properties of Angles (Chapter X) we find that a 4 by 4 by ⅝-in angle has an area of 4 61 sq in, which is to be reduced by a ⅞-in hole for a ¾-in rivet, leaving 4 61 − (⅞ × ⅝) = 4 06 sq in, net area. This is slightly in excess of the required amount

The SAFE LOAD for angles commonly used in roof-trusses is given in Table X; and the REDUCTION IN SECTIONAL AREA caused by rivet-holes in Table XI, this chapter.

2. Wrought Iron

Manufacture. Wrought iron is a mixture of pure iron and slag, about 96% iron and 3% slag, together with from ½ to ¾% of other elements including carbon, phosphorus, sulphur and manganese. It is made from pig iron and iron oxide, or mill-scale, in a reverberatory furnace consisting of a firebox, a hearth or working-chamber, and the necessary dampers and flues. The impurities are removed from the iron at different stages in the process, silicon and manganese during the melting-down stage, part of the phosphorus and sulphur during the clearing-stage and the carbon and remainder of the phosphorus and sulphur during the boiling-stage. The iron is then in a pasty condition ready for a thorough stirring by the workman, who collects it into balls of about 80 lb weight and takes it to a squeezer or forge where the greater part of the slag is removed. It is then rolled out into MUCK-BARS. These bars are cut into pieces which are piled into bundles suited to the size of the finished bar. The piles are heated and rolled again. The rolling reduces the amount of slag and makes the material denser. The process of rerolling may be repeated a number of times to produce double or triple-refined MERCHANT-BAR IRON.

The Appearance of Wrought Iron is very much like that of steel. It may be distinguished from steel by nicking one side of the bar and bending it away from the nick. Iron will split along the slag-laminations and show the

COARSELY FIBROUS nature of the material; while steel will bend or rupture at the nick without splitting, any fracture being FINELY FIBROUS or CRYSTALLINE. When ruptured in a tension-test wrought iron shows a dark fibrous fracture. If the specimen is grooved before testing or broken in impact the fracture will be coarsely crystalline.

Welds. Wrought iron is more easily welded than steel because the work may be accomplished through a wider range of temperature than with steel. A weld may develop the full strength of the bar, but tests on hand-forged welds on rough tie-bars reported by Kirkaldy gave average values of about 60% of the strength of the bar.

Use. Wrought iron is no longer used for the manufacture of structural shapes, such as angles, channels and beams, its use for structural work being practically limited to bars, rods and bolts. It can be worked more easily than steel in threading-machines; and on this account, unless steel is specified, some companies will furnish truss-rods, bolts, etc , in wrought iron.

Specifications * for Wrought Iron. Wrought iron may be purchased under the Specifications of the American Society for Testing Materials.

Material Covered. 1. These specifications cover two classes of wrought-iron plates, as determined by the kind of material used in their manufacture, namely:

Class A, as defined in Section 2 (*b*);
Class B, as defined in Section 2 (*c*).

I. Manufacture

Process. 2. (*a*) All plates shall be rolled from piles entirely free from any admixture of steel.

(*b*) Piles for Class A plates shall be made from puddle-bars made wholly from pig iron and such scrap as emanates from rolling the plates.

(*c*) Piles for Class B plates shall be made from puddle-bars made wholly from pig iron or from a mixture of pig iron and cast-iron scrap, together with wrought-iron scrap.

II. Physical Properties and Tests

Tension-Tests. 3. (*a*) The plates shall conform to the following minimum requirements as to tensile properties:

Table I

Properties considered	Class A		Class B	
	6 in to 24 in incl, in width	Over 24 in to 90 in incl, in width	6 in to 24 in incl, in width	Over 24 in to 90 in incl,. in width
Tensile strength, lb per sq in	49 000	48 000	48 000	47 000
Yield-point, lb per sq in....	26 000	26 000	26 000	26 000
Elongation in 8 in, per cent	16	12	14	10

* These Specifications for Wrought-Iron Plates are issued by the Society under the fixed designation A 42. They were adopted in 1913 and revised in 1918. There are also A.S.T.M. Standard Specifications for Staybolt Iron, Refined Wrought-Iron Bars, Iron and Steel Chain, etc.

(*b*) The yield-point shall be determined by the drop of the beam of the testing-machine. The speed of the cross-head of the machine shall not exceed $\frac{3}{4}$ in per minute.

Modifications in Elongation. 4. For plates under $\frac{7}{16}$ in in thickness, a deduction of 1 from the percentages of elongation specified in Section 3 shall be made for each decrease of $\frac{1}{16}$ in in thickness below $\frac{7}{16}$ in.

Bend Tests. 5. (*a*) COLD-BEND TESTS. The test-specimen shall bend cold through 90° without fracture on the outside of the bent portion, as follows: For Class A plates, around a pin the diameter of which is equal to $1\frac{1}{2}$ times the thickness of the specimen; and for Class B plates, around a pin the diameter of which is equal to three times the thickness of the specimen.

(*b*) NICK-BEND TESTS. The test-specimen, when nicked on one side and broken, shall show for Class A plates a wholly fibrous fracture, and for Class B plates, not more than 10% of the fractured surface to be crystalline.

Test-Specimens. 6. Tension and bend-test specimens shall be taken from the finished plates and shall be of the full thickness of plates as rolled. The longitudinal axis of the specimen shall be parallel to the direction in which the plates are rolled.

Number of Tests. 7. (*a*) One tension, one cold-bend and one nick-bend test shall be made for each variation in thickness of $\frac{1}{8}$ in and not less than one test for every ten plates as rolled.

(*b*) If any test-specimen fails to conform to the requirements specified by reason of an apparent local defect, a retest shall be made. If the retest also fails, the plates represented by such test will be rejected.

III. Finish

Finish. 8. The plates shall be straight, smooth, and free from cinder-spots and holes, injurious flaws, buckles, blisters, seams, and laminations.

IV. Marking

Marking. 9. The plates shall be stamped or otherwise marked as designated by the purchaser.

V. Inspection and Rejection

Inspection. 10. (*a*) The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the plates ordered. (See complete Specifications for Sections 10, 11 and 12.)

3. Cast Iron

Cast Iron has been defined as a saturated solution of carbon in iron, the carbon-content varying from $1\frac{1}{2}$ to 4% according to the other impurities contained. It is hard, brittle, non-malleable and very fluid when melted, so that it is well adapted for casting into complex forms.

Manufacture. It is produced in the blast-furnace, which is essentially a closed refractory-lined stack, with a valve-charging device at the top, tuyeres or openings in the lower part for the introduction of the air-blast, and a hearth at the bottom with a tap-hole for the periodic withdrawal of the iron and slag. The FURNACE-IRON is cast into PIGS about 3 ft long and weighing about 100 lb each. FOUNDRY-CASTINGS are made from PIG IRON and SCRAP melted in a cupola and poured into green-sand molds. The charge is made up of different

quantities of the different grades of pig so as to control the physical properties of the castings, principally through control of the silicon-content.

Appearance. CASTINGS have a gray or white fracture according to the condition of the contained carbon, the gray fracture indicating graphitic or separated carbon and the white the combined carbon. GRAY IRON is softer and tougher and is specified for ordinary CASTINGS.

Strength. Cast iron does not have a definite ELASTIC LIMIT. A relatively small stress will produce some permanent deformation. Its ULTIMATE TENSILE STRENGTH varies from 15 000 to 20 000 lb per sq in; and in some iron is as high as 30 000 lb per sq in. Its COMPRESSIVE STRENGTH varies over a wide range, 80 000 lb per sq in being a fair average value.

Defects. Castings are liable to several common DEFECTS, the chief of which are blow-holes due to the formation of steam from the damp molds, sand-holes due to misplaced sand, rough surfaces, cold shuts due to chilling of the iron and failure to fill the parts of the mold, shrinkage-cracks due to uneven cooling of the castings in parts of different thickness. In cored castings, also, the walls are frequently of variable thickness because of the shifting of the cores. This is especially frequent in case of hollow columns cast in a horizontal position. Because of these defects and on account of the low ULTIMATE STRENGTH, cast iron should never be used where it is subjected to any great tensile stress.

Specifications * for Cast Iron. The specifications of the American Society for Testing Materials, for GRAY-IRON CASTINGS, include the following requirements:

1. Unless FURNACE-IRON is specified, all GRAY CASTINGS are understood to be made by the CUPOLA-PROCESS.

2. The SULPHUR-CONTENTS are to be:

For light castings, not over 0.10 per cent.
For medium castings, not over 0 10 per cent.
For heavy castings, not over 0 12 per cent.

3. In dividing castings into LIGHT, MEDIUM and HEAVY classes, the following standards have been adopted:

Castings having any section less than ½ in thick shall be known as LIGHT CASTINGS.

Castings in which no section is less than 2 in thick shall be known as HEAVY CASTINGS.

MEDIUM CASTINGS are those not included in the above classification.

4. TRANSVERSE TEST. The minimum BREAKING STRENGTH of the ARBITRATION-BAR under transverse load on 18 in span shall be:

For light castings, not under 1 500 lb.
For medium castings, not under 1 750 lb.
For heavy castings, not under 2 000 lb.

In no case shall the DEFLECTION be under 0.20 in.

TENSION-TEST. Where specified this shall be:

For light castings, not less than 18 000 lb per sq in.
For medium castings, not less than 21 000 lb per sq in.
For heavy castings, not less than 24 000 lb per sq in.

* These specifications are issued under the fixed designation A 48. They were adopted in 1905 and revised in 1929. The complete specification can be obtained from the Society.

The specifications give explicit directions for casting the ARBITRATION-BAR which is 1.20 in in diameter and 21 in long. Two of these are cast for each twenty tons of castings. One of each pair must fulfill the requirements to permit acceptance of the castings. The bar is loaded at the middle at a rate that will cause a 0.10-in deflection in from twenty to forty seconds. The tension-test is not recommended.

5. CASTINGS shall be true to pattern, free from cracks, flaws and excessive shrinkage. In other respects they shall conform to whatever points shall be specially agreed upon.

4. Steel

Steel is a mixture of compounds of iron and carbon with small quantities of other elements, including manganese, phosphorus, sulphur, silicon, etc. The carbon-content controls the hardness and strength of the steel. Less than 0 10% of carbon is present in the soft steels, which have most of the characteristics of wrought iron; while steel with more than 0.40% carbon is capable of being tempered, cannot be welded and is very much stronger. Manganese acts as a cleanser during the process of manufacture, and increases the forgeability of the steel. Phosphorus and sulphur are harmful in their effects, phosphorus making steel brittle under sudden loading and sulphur making it hot-short or brittle when heated.

Stainless Steel is a recent development of FERROUS ALLOYS. An IRON-CHROMIUM ALLOY from 15 to 20% chrome is of almost silver-white appearance and is rust-resistant. By control of proportions of carbon, manganese, silicon, as well as limiting the sulphur and phosphorus, the mechanical properties of the alloy may be altered over wide ranges of hardness and workability. Some stainless steels are used in the manufacture of table cutlery, surgical instruments, food utensils, hospital equipment, etc., while others are suitable for deep drawn work like radiator covers and lamps for automobiles. The tower of the Chrysler Building in New York is sheathed in a stainless steel. Stainless steel may be worked by all the processes applied to common sheet steel.

Manufacture. STEEL is manufactured by the BESSEMER and the OPEN-HEARTH PROCESSES.

Bessemer Process. Molten cast iron is charged into a Bessemer converter, an air-blast is driven through the charge from perforations in the false bottom of the converter and the silicon, sulphur and carbon burned out. Carbon in the form of ferro-manganese is then added to deoxidize the charge and give the proper content of carbon in the finished steel, which is quickly drawn off and poured into ingots. Phosphorus is not removed ordinarily by the Bessemer process; but if the lining of the converter is made of basic material, such as dolomite limestone, and if lime is added with the charge, the phosphorus will unite with it and be poured off with the slag.

The Open-Hearth Process. In this process scrap-steel, pig-iron or molten furnace-iron and limestone flux are charged on the hearth of a Siemens furnace. A reducing gas-flame is directed onto the charge and the carbon and other impurities are gradually removed. When the reduction is about completed samples are taken and are analyzed for carbon content so that the charge may be withdrawn at the proper time. The process thus permits of much more accurate control of the product. It requires from four to ten hours to produce a heat as compared with fifteen to twenty minutes by the

Bessemer process. Consequently, OPEN-HEARTH STEEL is more uniform and dependable in service than BESSEMER STEEL, and is used in structural work.

Phosphorus is removed when it occurs in the ore in excessive amounts by the basic process, which eliminates it in a basic slag. This slag requires a basic lining to the furnace to avoid undue wear. Brick for basic furnaces do not have the physical strength to resist the action of the bath, like the silica brick in the acid process. Since in America low-phosphorous ores may be obtained, most furnaces are acid.

The Effect of Carbon and Phosphorus on the STATIC STRENGTH of steel for the limits of carbon included in structural steel is an increase in strength of about 1 000 lb per sq in for each 0.01% increase in either element.

The Percentage of Elongation decreases as the carbon-content and ULTIMATE STRENGTH increase. An approximate relation is

$$\text{percentage of elongation in 8 in} = \frac{1\ 500\ 000}{\text{tensile strength}}$$

Since the TOTAL ELONGATION of a ruptured specimen is due to the local stretching at the point of rupture and the uniform elongation over the whole gauge-length, it is necessary to report the gauge-length when reporting this result. Since the LOCAL ELONGATION is the same for a 2- or an 8-in length the PERCENTAGE OF ELONGATION for the same material, tested on a 2-in gauge-length, is greater than if measured on an 8-in length.

The Elastic Behavior of a specimen of steel loaded to rupture is best shown by a STRESS-STRAIN DIAGRAM on which the stresses are plotted as vertical ordinates and the elongations or strains as abscissas, as in Fig. 1. Five significant results are shown:

(1) **The Modulus of Elasticity** (E). The relation between the stress and the strain or elongation is called the MODULUS OF ELASTICITY. It is equal to the unit stress divided by the unit strain or deformation and is represented graphically by the tangent of the angle of the initial line with the horizontal. Its value for steel for tension is about 30 000 000 lb per sq in.

(2) **The Elastic Limit** ($E.L.$) is that unit stress beyond which the ratio of stress to strain ceases to be constant, or beyond which the curve ceases to be a straight line.

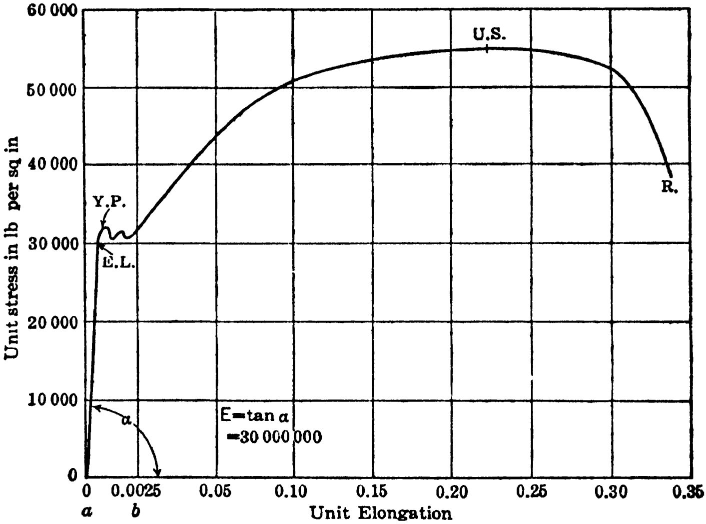
(3) **The Yield-Point** ($Y.P.$), slightly above or beyond the ELASTIC LIMIT, is that unit stress at which the specimen begins to stretch without increase in the load. This stress may be determined from a test without the use of delicate measuring-apparatus by the DROP OF THE BEAM of lever-type testing-machines or halt in the gauge of hydraulic-type testing-machines.

(4) **The Ultimate Strength** ($U.S.$) is the greatest unit stress the specimen can sustain. (Total load divided by the original area.)

(5) **The Rupture-Stress** (R) is the unit stress at the time of failure. This is the unit stress at the point of failure after the area of the cross-section of the specimen has been reduced; and because of the rapid dropping off of the load it is difficult to determine. It is not regularly observed in testing, attention being called to it merely to emphasize the fact that the ULTIMATE STRENGTH of steel is not the stress at the time of failure of the specimen. This is true, also, for wrought iron and ductile materials in general.

Effect of Punching and Shearing. Structural steel is hardened by the action of the punch and shear in the process of manufacture in the shop. On the die-side the metal is forced to flow from the tool and this cold working

hardens and injures it as may be shown by a cold-bend test. The effect may be removed by annealing; but in the best structural shop work it is usually specified that rivet-holes shall be reamed during the assembling of the parts. This removes the injured metal and brings the parts into better alinement for



The horizontal scale for the distance *ab* is ten times greater than for the remaining distance.

Fig. 1. Stress-strain Diagram of Test on Steel Specimens

the insertion of the rivets. The injury from shearing may be removed by milling the sheared edges.

The Coefficient of Expansion of steel is 0.000 006 5 per degree Fahrenheit. The ELONGATION in a length l, due to a change in temperature of t degrees, is then

$$e = 0.000\,006\,5\,lt$$

in which l and e are expressed in inches and t in degrees Fahrenheit.

The Weight of Steel is taken at 489 6 lb per cu ft. The sectional area of a member in square inches multiplied by 3.4 equals the weight in pounds per linear foot.

The Working Stress for structural steel in tension in buildings and bridges is 16 000 lb per sq in in most specifications and building laws (Philadelphia Code: 18 000 lb.) For members subject to constant load some designers use a WORKING STRESS of 20 000 lb per sq in.

5. Standard Specifications for Structural Steel for Buildings

Specifications. These specifications are issued by the American Society for Testing Materials under the fixed designation A 9. They were adopted in

1901 and revised in 1909, 1913, 1914, 1916, 1921, 1924 and 1929. Extracts from these specifications follow:

I. Manufacture

Process. 1. (*a*) Structural steel, except as noted in Paragraph (*b*), shall be made by either or both of the following processes: Bessemer or open-hearth.

(*b*) Rivet steel, and steel for plates or angles over ¾ in in thickness which are to be punched, shall be made by the open-hearth process.

II. Chemical Properties and Tests

Chemical Composition. 2. The steel shall conform to the following requirements as to chemical composition:

Properties considered	Structural steel	Rivet steel
Phosphorus, per cent { Bessemer	Not over 0 10	....
Phosphorus, per cent { Open-hearth	Not over 0 06	Not over 0 06
Sulphur, per cent	....	Not over 0.045
Copper, when copper steel is specified, per cent	Not under 0 20	Not under 0 20

Ladle Analyses. 3. (*a*) A carbon determination, and a copper determination when copper steel is specified, shall be made of each melt of Bessemer steel, and determinations for manganese, phosphorus and sulphur representing the average of the melts applied for each 12-hour period of operation of the plant.

(*b*) An analysis of each melt of open-hearth steel shall be made to determine carbon, manganese, phosphorus and sulphur; also copper when copper steel is specified.

(*c*) These analyses shall be made by the manufacturer from test ingots taken during the pouring of each melt. The chemical composition thus determined shall be reported to the purchaser or his representative and the percentages of phosphorus and sulphur, and also copper when copper steel is specified, shall conform to the requirements specified in Section 2.

Check Analyses. 4. Analyses may be made by the purchaser from finished material representing each melt. The phosphorus and sulphur content thus determined shall not exceed that specified in Section 2 by more than 25%.

III. Physical Properties and Tests

Tension Tests. 5. (*a*) The material shall conform to the following requirements as to tensile properties:

Properties considered	Structural steel	Rivet steel
Tensile strength, lb per sq in	55 000–65 000	46 000–56 000
Yield point, min, lb per sq in	0 5 tens. str	0 5 tens. str.
But in no case less than	30 000	25 000
Elongation in 8 in, min., per cent	$\frac{1\ 400\ 000^*}{\text{Tens str.}}$	$\frac{1\ 400\ 000}{\text{Tens str.}}$
Elongation in 2 in. min., per cent	22	....

* See Section 6.

(*b*) The yield-point shall be determined by the drop of the beam of the testing-machine.

Modifications in Elongation. 6. (*a*) For structural steel over ¾ in in thickness, a deduction from the percentage of elongation in 8 in specified in Section 5 (*a*) of 0.25% shall be made for each increase of ⅓₂ in of the specified thickness above ¾ in, to a minimum of 18%.

(*b*) For structural steel under 5⁄16 in in thickness, a deduction from the percentage of elongation in 8 in specified in Section 5 (*a*) of 1.25% shall be made for each decrease of 1⁄32 in of the specified thickness below 5⁄16 in.

Bend Tests. 7. (*a*) Bend test specimens, except as specified in Paragraph (*b*), shall stand being bent cold through 180° without cracking on the outside of the bent portion, as follows: For material ¾ in or under in thickness, flat on itself; for material over ¾ in to and including 1¼ in in thickness, around a pin the diameter of which is equal to the thickness of the specimen: and for material over 1¼ in in thickness, around a pin the diameter of which is equal to twice the thickness of the specimen.

(*b*) Bend test specimens for rivet steel shall stand being bent cold through 180° flat on themselves without cracking on the outside of the bent portion.

Test Specimens. 8. (*a*) Test specimens shall be prepared for testing from the material in its rolled or forged condition, except as specified in Paragraphs (*b*) and (*c*).

(*b*) Test specimens for annealed material shall be prepared from the material as annealed for use, or from a short length of a full section similarly treated.

(*c*) Test specimens for rivet bars which have been cold-drawn shall be normalized before testing.

(*d*) Test specimens shall be taken longitudinally and, except as specified in Paragraphs (*f*), (*g*), and (*h*), shall be of the full thickness or section of material as rolled.

(*e*) Test specimens for plates, shapes and flats may be machined to the form and dimensions shown in Fig 2, or with both edges parallel.

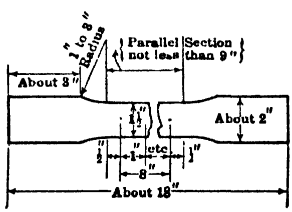


Fig. 2. Form of Specimen for Steel-test

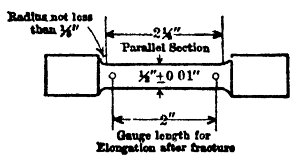


Fig. 2A. Form of Specimen for Pins, Rollers, Bars, etc , Over 1½ Inch Thick

(*f*) Tension test specimens for material over 1½ in in thickness or diameter, except pins and rollers, may be machined to a thickness or diameter of at least ¾ in for a length of at least 9 in, or they may conform to the dimensions shown in Fig. 2A.

(*g*) Bend test specimens for material over 1½ in in thickness or diameter, except pins and rollers, may be machined to a thickness or diameter of at least ¾ in or to 1 by ½ in in section.

(*h*) Tension test specimens for pins and rollers shall conform to the dimensions shown in Fig. 2-A, and bend test specimens shall be 1 by 1½ in in section.

(*i*) Test specimens for pins and rollers shall be taken so that the axis is 1 in from the surface.

(*j*) The machined sides of rectangular bend test specimens may have the corners rounded to a radius not over 1/16 in.

Number of Tests. 9. (*a*) One tension and one bend-test shall be made from each melt; except that if material from one melt differs 3/8 in or more in thickness, one tension and one bend-test shall be made for both the thickest and the thinnest material rolled.

(*b*) If any test-specimen shows defective machining or develops flaws, it may be discarded and another specimen substituted.

(*c*) If the percentage of elongation of any tension-test specimen is less than that specified in Section 5 (*a*) and any part of the fracture is more than 3/4 in from the center of the gauge-length of a 2-in specimen or is outside the middle third of the gauge-length of an 8-in specimen, as indicated by scribe-scratches marked on the specimen before testing, a retest shall be allowed.

IV. Permissible Variations in Weight and Thickness

Permissible Variations. 10. The cross-section or weight of each piece of steel shall not vary more than 2 5% from that specified; except in case of sheared plates, which shall be covered by the following permissible variations. One cubic inch of rolled steel is assumed to weigh 0.2833 lb.

(*a*) WHEN ORDERED TO WEIGHT PER SQUARE FOOT: The weight of each lot in each shipment shall not vary from the weight ordered more than the amount given in Table I.*

(*b*) WHEN ORDERED TO THICKNESS: The thickness of each plate shall not vary more than 0.01 in under that order.

The overweight of each lot in each shipment shall not exceed the amount given in Table II.*

V. Finish

Finish. 11. The finished material shall be free from injurious defects and shall have a workmanlike finish.

VI. Marking

Marking. 12. The name or brand of the manufacturer and the melt-number shall be legibly stamped or rolled on all finished material, except that rivet and lattice-bars and other small sections shall, when loaded for shipment, be property separated and marked for identification. The identification-marks shall be legibly stamped on the end of each pin and roller. The melt-number shall be legibly marked, by stamping if practical, on each test-specimen.

VII. Inspection and Rejection

Inspection. 13. (See complete Specifications for Sections 13, 14 and 15.)

6. Tension-Members

Angles. The best section for tension-members of relatively small size depends greatly on the kind of end-connections used. Angles or channels are generally used for riveted connections. For very small members rectangular bars, such as lacing-bars, may be used. The strength of such members is computed on the net area through the rivet-holes. Angles used in tension should have lugs riveted to the outstanding legs and the tie-plate for the better distribution of the stress over the section. Tests on angles with riveted

* Tables I and II are omitted here for lack of space. The complete specifications can be obtained from the Society.

connections reported by F. P. McKibben * gave from 77 to 86% of the strength of the material as shown by tension-tests on standard specimens cut from these angles. Lugs increased the strength from 4.7 to 8.7%. It was also shown that a connection giving the center of the pull on the center of gravity of the section gave considerably higher strengths than when the center of pull was in line with the gauge-line of the rivets In computing the NET SECTIONAL AREA as reduced by rivet and bolt-holes Table XI will be found very convenient.

Eye-Bars are used for the main tension-members of pin-connected trusses. They are rectangular in section with a forged head upset in dies and of the same thickness as the bar. The eye is accurately drilled in position in the axis of the bar, true to diameter and exact central distance. Because of its advantages for forging, soft steel is used in making eye-bars. They are also carefully annealed before drilling. Table VI gives the dimensions of STANDARD EYE-BARS manufactured by the mills of the American Bridge Company. These bars are of practically the same dimensions as the standard bars of other companies. There is from 34 to 42% excess material in the section through the eye to insure in the forged part the development of the full strength of the body of the bar. Standard bars should be used in design to avoid the expense of making special dies in which to form the heads. Bars of less than the given minimum thickness are liable to fail, when loaded, by buckling in the head. Thick bars increase the BENDING-STRESSES in the pins and thus, indirectly, the necessary size of the eye. Except for very large structures they are limited to about 2 in.

Tests of Full-Size Eye-Bars are generally required when a great number of them are to be used in a structure, one in every fifty bars being usually tested. The specifications for carbon-steel bars require that an ULTIMATE TENSILE STRENGTH of 55 000 lb per sq in shall be developed, that the ELONGATION in 10 ft, including fracture, shall not be less than 15% and that failure shall

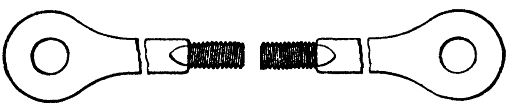


Fig. 3. Eye-bar with Screw-ends for Sleeve-nut or Turn-buckle

occur in the body of the bar. Nickel steel has been used for tension-members on a few long-span bridges. The WORKING STRESS on the eye-bars was increased about one-half over that used for carbon steel, and the requirements of the test-bars made correspondingly severe. The eye is made 1/50 in greater than the diameter of the pin. Bars packed on the same pins are

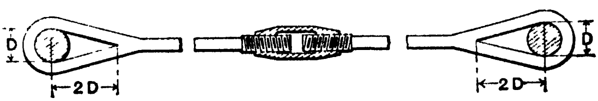


Fig. 4. Loop-eyes and Sleeve-nuts

drilled at the same setting so as to be of exactly the same length. Bars must be true to length within 1/32 in. Small eye-bars are sometimes made with UPSET SCREW-ENDS and SLEEVE-NUTS or TURNBUCKLES in the middle for adjustment, as shown in Fig. 3 and Table VI.

* Proceedings of the American Society for Testing Materials, Vol. VI, 1906.

Loop-Rods (Fig. 4, and Table VII) of round or square section with welded loop-ends are used for counterties and bracing. Because of the weld they are not so dependable as other types of tension-members, but, because of the adjustment, are well adapted for this service as secondary members.

A Forked-Loop Rod, Fig. 5, may be used for one of two tension-rods so as to avoid eccentricity where two rods balance each other on a pin. A CLEVIS at each end of one of the rods accomplishes the same object.

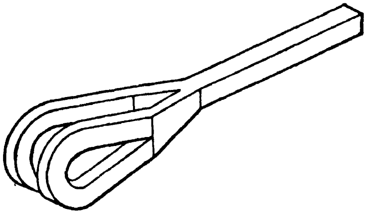


Fig 5. Forked Loop

Turnbuckles and Sleeve-Nuts. The dimensions of these for adjusting the lengths and initial stress in ties are given in Table VIII. The open turnbuckle has the advantage of being easily inspected to note that the thread has sufficient bearing and that the ends of the rods do not butt together

Upset Screw-Ends are threaded enlargements on the ends of rods or bolts designed to give to the threaded portions a strength as great as that of the body of the bar. Because of effects of forging it is necessary to make the area of the cross-section of the upset end at the root of the thread a little larger than that of the rod itself. A standard upset rod will fail in the body of the bar without damaging the threaded portion enough to prevent the turning of the nuts. The dimensions given are nearly the same with all manufacturers. If upset rods can not be obtained the section-area at the root of the thread must be used in computing the safe load.

Clevises. Table IX gives the dimensions and other details for clevises according to the latest standards of the American Bridge Company.

Tables. The following tables will be found useful in designing tension-members, or for drawing turnbuckles, sleeve-nuts, clevises, etc. The strength of plain rods in Table II is based on the area at the root of the thread.

Table II. Safe Loads in Tension in Pounds on Round Rods

Diameter in inches	Plain rods Load in pounds based on area at root of thread U. S. Standard			Upset rods Load in pounds based on full area of rod		
	Stress in lb per sq in			Stress in lb per sq in		
	10 000	16 000	18 000	10 000	16 000	18 000
1/4	270	432	486	491	785	885
5/16	450	720	810	767	1 230	1 380
3/8	680	1 088	1 224	1 104	1 770	1 980
7/16	930	1 488	1 674	1 503	2 400	2 700
1/2	1 260	2 016	2 270	1 963	3 140	3 540
9/16	1 620	2 592	2 916	2 485	3 960	4 455
5/8	2 020	3 232	3 636	3 068	4 910	5 520
3/4	3 020	4 832	5 436	4 418	7 070	7 950
7/8	4 200	6 720	7 560	6 013	9 620	10 815
1	5 500	8 800	9 900	7 854	12 570	14 130
1 1/8	6 940	11 104	12 492	9 940	15 900	17 895
1 1/4	8 930	14 288	16 074	12 270	19 630	22 080
1 3/8	10 570	16 910	19 020	14 840	23 750	26 715
1 1/2	12 950	20 720	23 310	17 670	28 270	31 800
1 5/8	15 150	24 240	27 270	20 730	33 170	37 320
1 3/4	17 440	27 900	31 395	24 050	38 480	43 290
1 7/8	20 480	32 760	36 870	27 610	44 180	49 695
2	23 020	36 830	41 430	31 420	50 270	56 550
2 1/8	26 490	42 380	47 700	35 460	56 640	63 825
2 1/4	30 230	48 370	54 420	39 760	63 600	71 565
2 3/8	34 190	54 600	59 400	44 300	70 880	79 740
2 1/2	37 150	59 440	66 945	49 080	78 530	88 350
2 3/4	46 190	73 900	83 145	59 390	95 020	106 905
3	54 280	86 850	97 710	70 680	113 090	127 230
3 1/4	65 100	104 160	117 180	82 950	132 720	149 310
3 1/2	75 480	120 770	135 855	96 210	153 840	173 175
3 3/4	86 410	138 250	155 535	110 450	176 690	198 810
4	99 930	159 890	179 880	125 660	201 050	226 185
4 1/4	113 290	181 300	203 850	141 800	226 880	255 240
4 1/2	127 430	203 900	229 350	159 000	254 400	286 200
4 3/4	142 200	227 500	255 900	177 200	283 520	318 960
5	157 630	252 200	283 650	196 300	314 080	353 340
5 1/4	175 720	281 100	316 200	216 400	346 200	389 520
5 1/2	192 670	308 300	346 800	237 500	380 000	427 500
5 3/4	212 620	340 200	382 650	259 600	414 700	466 500
6	230 980	369 600	415 800	282 700	452 300	508 800

Table III. Safe Loads in Tension in Pounds for Flat Rolled Bars

Computed for a stress of 16 000 lb per sq in

Thickness in inches	Width in inches									
	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3	3 1/4
1/16	1 000	1 250	1 500	1 750	2 000	2 250	2 500	2 750	3 000	3 250
1/8	2 000	2 500	3 000	3 500	4 000	4 500	5 000	5 500	6 000	6 500
3/16	3 000	3 750	4 500	5 250	6 000	6 750	7 500	8 250	9 000	9 750
1/4	4 000	5 000	6 000	7 000	8 000	9 000	10 000	11 000	12 000	13 000
5/16	5 000	6 250	7 500	8 750	10 000	11 250	12 500	13 750	15 000	16 250
3/8	6 000	7 500	9 000	10 500	12 000	13 500	15 000	16 500	18 000	19 500
7/16	7 000	8 750	10 500	12 250	14 000	15 750	17 500	19 250	21 000	22 750
1/2	8 000	10 000	12 000	14 000	16 000	18 000	20 000	22 000	24 000	26 000
9/16	9 000	11 250	13 500	15 750	18 000	20 250	22 500	24 750	27 000	29 250
5/8	10 000	12 500	15 000	17 500	20 000	22 500	25 000	27 500	30 000	32 500
11/16	11 000	13 750	16 500	19 250	22 000	24 750	27 500	30 250	33 000	36 750
3/4	12 000	15 000	18 000	21 000	24 000	27 000	30 000	33 000	36 000	39 000
13/16	13 000	16 250	19 500	22 750	26 000	29 250	32 500	35 750	39 000	42 250
7/8	14 000	17 500	21 000	24 500	28 000	31 500	35 000	38 500	42 000	45 500
15/16	15 000	18 750	22 500	26 250	30 000	33 750	37 500	41 250	45 000	48 750
1	16 000	20 000	24 000	28 000	32 000	36 000	40 000	44 000	48 000	52 000
1 1/16	17 000	21 250	25 500	29 750	34 000	38 250	42 500	46 750	51 000	55 250
1 1/8	18 000	22 500	27 000	31 500	36 000	40 500	45 000	49 500	54 000	58 500
1 3/16	19 000	23 750	28 500	33 250	38 000	42 750	47 500	52 250	57 000	61 750
1 1/4	20 000	25 000	30 000	35 000	40 000	45 000	50 000	55 000	60 000	65 000
1 3/8	22 000	27 500	33 000	38 500	44 000	49 500	55 000	60 500	66 000	73 500
1 1/2	24 000	30 000	36 000	42 000	48 000	54 000	60 000	66 000	72 000	78 000
1 5/8	26 000	32 500	39 000	45 500	52 000	58 500	65 000	71 500	78 000	84 500
1 3/4	28 000	35 000	42 000	49 000	56 000	63 000	70 000	77 000	84 000	91 000
1 7/8	30 000	37 500	45 000	52 500	60 000	67 500	75 000	82 500	90 000	97 500
2	32 000	40 000	48 000	56 000	64 000	72 000	80 000	88 000	96 000	104 000

Table III (Continued). Safe Loads in Tension in Pounds for Flat Rolled Bars

Computed for a stress of 16 000 lb per sq in

Thickness in inches	Width in inches									
	3½	3¾	4	4¼	4½	4¾	5	5½	6	6½
1/16	3 500	3 750	4 000	4 250	4 500	4 750	5 000	5 500	6 000	6 500
⅛	7 000	7 500	8 000	8 500	9 000	9 500	10 000	11 000	12 000	13 000
3/16	10 500	11 250	12 000	12 750	13 500	14 250	15 000	16 500	18 000	19 500
¼	14 000	15 000	16 000	17 000	18 000	19 000	20 000	22 000	24 000	26 000
5/16	17 500	18 750	20 000	21 250	22 500	23 750	25 000	27 500	30 000	32 500
⅜	21 000	22 500	24 000	25 500	27 000	28 500	30 000	33 000	36 000	39 000
7/16	24 500	26 250	28 000	29 750	31 500	33 250	35 000	38 500	42 000	45 500
½	28 000	30 000	32 000	34 000	36 000	38 000	40 000	44 000	48 000	52 000
9/16	31 500	33 750	36 000	38 250	40 500	42 750	45 000	49 500	54 000	58 500
⅝	35 000	37 500	40 000	42 500	45 000	47 500	50 000	55 000	60 000	65 000
11/16	38 500	41 250	44 000	46 750	49 500	52 250	55 000	60 500	66 000	71 500
¾	42 000	45 000	48 000	51 000	54 000	57 000	60 000	66 000	72 000	78 000
13/16	45 500	48 750	52 000	55 250	58 500	61 750	65 000	71 500	78 000	84 500
⅞	49 000	52 500	56 000	59 500	63 000	66 500	70 000	77 000	84 000	91 000
15/16	52 500	56 250	60 000	63 750	67 500	71 250	75 000	82 500	90 000	97 500
1	56 000	60 000	64 000	68 000	72 000	76 000	80 000	88 000	96 000	104 000
1 1/16	59 500	63 750	68 000	72 250	76 500	80 750	85 000	93 500	102 000	110 500
1⅛	63 000	67 500	72 000	76 500	81 000	85 500	90 000	99 000	108 000	117 000
1 3/16	66 500	71 250	76 000	80 750	85 500	90 250	95 000	104 500	114 000	123 500
1¼	70 000	75 000	80 000	85 000	90 000	95 000	100 000	110 000	120 000	130 000
1⅜	77 000	82 500	88 000	93 500	99 000	104 500	110 000	121 000	132 000	143 000
1½	84 000	90 000	96 000	102 000	108 000	114 000	120 000	132 000	114 000	156 000
1⅝	91 000	97 500	104 000	110 500	117 000	123 500	130 000	143 000	156 000	169 000
1¾	98 000	105 000	112 000	119 000	126 000	133 000	140 000	154 000	168 000	182 000
1⅞	105 000	112 500	120 000	127 500	135 000	142 500	150 000	165 000	180 000	195 000
2	112 000	120 000	128 000	136 000	144 000	152 000	160 000	176 000	192 000	208 000

Table IV. Safe Loads in Tension in Pounds for Flat Rolled Bars

Computed for a stress of 10 000 lb per sq in *

Thickness in inches	Width in inches									
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
1/16	630	780	940	1 090	1 250	1 410	1 560	1 720	1 880	2 030
⅛	1 250	1 560	1 880	2 190	2 500	2 810	3 130	3 440	3 750	4 060
3/16	1 880	2 340	2 810	3 280	3 750	4 220	4 690	5 160	5 630	6 090
¼	2 500	3 130	3 750	4 380	5 000	5 630	6 250	6 880	7 500	8 130
5/16	3 130	3 910	4 690	5 470	6 250	7 030	7 810	8 590	9 380	10 200
⅜	3 750	4 690	5 630	6 560	7 500	8 440	9 380	10 300	11 300	12 200
7/16	4 380	5 470	6 560	7 660	8 750	9 840	10 900	12 000	13 100	14 200
½	5 000	6 250	7 500	8 750	10 000	11 300	12 500	13 800	15 000	16 300
9/16	5 630	7 030	8 440	9 840	11 300	12 700	14 100	15 500	16 900	18 300
⅝	6 250	7 810	9 380	10 900	12 500	14 100	15 600	17 200	18 800	20 300
11/16	6 880	8 590	10 300	12 000	13 800	15 500	17 200	18 900	20 600	22 300
¾	7 500	9 380	11 300	13 100	15 000	16 900	18 800	20 600	22 500	24 400
13/16	8 130	10 200	12 200	14 200	16 300	18 300	20 300	22 300	24 400	26 400
⅞	8 750	10 900	13 100	15 300	17 500	19 700	21 900	24 100	26 300	28 400
15/16	9 380	11 700	14 100	16 400	18 800	21 100	23 400	25 800	28 100	30 500
1	10 000	12 500	15 000	17 500	20 000	22 500	25 000	27 500	30 000	32 500
1 1/16	10 600	13 300	15 900	18 600	21 300	23 900	26 600	29 200	31 900	34 500
1⅛	11 300	14 100	16 900	19 700	22 500	25 300	28 100	30 900	33 800	36 600
1 3/16	11 900	14 800	17 800	20 800	23 800	26 700	29 700	32 700	35 600	38 600
1¼	12 500	15 600	18 800	21 900	25 000	28 100	31 300	34 400	37 500	40 600
1⅜	13 800	17 200	20 600	24 100	27 500	30 900	34 400	37 800	41 300	44 700
1½	15 000	18 800	22 500	26 300	30 000	33 800	37 500	41 300	45 000	48 800
1⅝	16 300	20 300	24 400	28 400	32 500	36 600	40 600	44 700	48 800	52 800
1¾	17 500	21 900	26 300	30 600	35 000	39 400	43 800	48 100	52 500	56 900
1⅞	18 800	23 400	28 100	32 800	37 500	42 200	46 900	51 600	56 300	60 900
2	20 000	25 000	30 000	35 000	40 000	45 000	50 000	55 000	60 000	65 000

* For unit stresses of 12 000, 12 500, and 15 000 lb increase by ⅕, ¼, and ½ respectively.

Table IV (Continued). Safe Loads in Tension in Pounds for Flat Rolled Bars

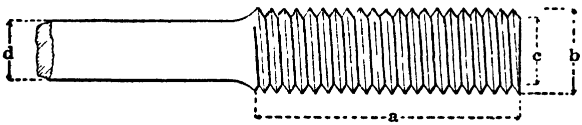
Computed for a stress of 10 000 lb per sq in *

Thickness in inches	Width in inches									
	3½	3¾	4	4¼	4½	4¾	5	5½	6	6½
1/16	2 190	2 340	2 500	2 660	2 810	2 970	3 130	3 440	3 750	4 060
⅛	4 380	4 690	5 000	5 310	5 630	5 940	6 250	6 880	7 500	8 130
3/16	6 560	7 030	7 500	7 970	8 440	8 910	9 380	10 300	11 300	12 200
¼	8 750	9 380	10 000	10 600	11 300	11 900	12 500	13 800	15 000	16 300
5/16	10 900	11 700	12 500	13 300	14 100	14 800	15 600	17 200	18 800	20 300
⅜	13 100	14 100	15 000	15 900	16 900	17 800	18 800	20 600	22 500	24 400
7/16	15 300	16 400	17 500	18 600	19 700	20 800	21 90[illegible]	24 100	26 300	28 400
½	17 500	18 800	20 000	21 300	22 500	23 800	25 000	27 500	30 000	32 500
9/16	19 700	21 100	22 500	23 900	25 300	26 700	28 100	30 900	33 800	36 600
⅝	21 900	23 400	25 000	26 600	28 100	29 700	31 300	34 400	37 500	40 600
11/16	24 100	25 800	27 500	29 200	30 900	32 700	34 400	37 800	41 300	44 700
¾	26 300	28 100	30 000	31 900	33 800	35 600	37 500	41 300	45 000	48 800
13/16	28 400	30 500	32 500	34 500	36 600	38 600	40 600	44 700	48 800	52 800
⅞	30 600	32 800	35 000	37 200	39 400	41 600	43 800	48 100	52 500	56 900
15/16	32 800	35 200	37 500	39 800	42 200	44 500	46 900	51 600	56 300	60 900
1	35 000	37 500	40 000	42 500	45 000	47 500	50 000	55 000	60 000	65 000
1 1/16	37 200	39 800	42 500	45 200	47 800	50 500	53 100	58 400	63 800	69 100
1⅛	39 400	42 200	45 000	47 800	50 600	53 400	56 300	61 900	67 500	73 100
1 3/16	41 600	44 500	47 500	50 500	53 400	56 400	59 400	65 300	71 300	77 200
1¼	43 800	46 900	50 000	53 100	56 300	59 400	62 500	68 800	75 000	81 300
1⅜	48 100	51 600	55 000	58 400	61 900	65 300	68 800	75 600	82 500	89 400
1½	52 500	56 300	60 000	63 800	67 500	71 300	75 000	82 500	90 000	97 500
1⅝	56 900	60 900	65 000	69 100	73 100	77 200	81 300	89 400	97 500	105 600
1¾	61 300	65 600	70 000	74 400	78 800	83 100	87 500	96 300	105 000	113 800
1⅞	65 600	70 300	75 000	79 700	84 400	89 100	93 800	103 100	112 500	121 900
2	70 000	75 000	80 000	85 000	90 000	95 000	100 000	110 000	120 000	130 000

* See foot-note, preceding table.

Table V. Upset Screw Ends for Square Bars *

American Bridge Company Standard



Pitch and shape of thread A B Co standard

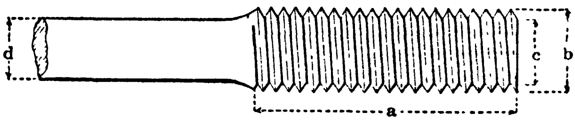
Bar			Upset					
							Area	
Side of square *d*, inches	Area, sq inches	Weight per foot, pounds	Diameter *b*, inches	Length *a*, inches	Additional length for upset +10% inches	Diameter at root of thread *c*, inches	At root of thread, sq inches	Excess over area of bar, per cent
†¾	0.563	1 91	1⅛	4	4	0 939	0 693	23.2
†⅞	0 766	2 60	1¼	4	3½	1 064	0 890	16 2
1	1 000	3 40	1½	4	4	1 283	1 294	29.4
1⅛	1 266	4 30	1⅝	4	3½	1 389	1 515	19.7
1¼	1 563	5 31	1⅞	4½	4½	1 615	2 049	31 1
1⅜	1.891	6 43	2	4½	4	1 711	2.300	21.7
1½	2 250	7 65	2¼	5	5	1 961	3 021	34.3
1⅝	2 641	8 98	2⅜	5	4½	2 086	3 419	29.5
1¾	3 063	10 41	2½	5½	4½	2 175	3 716	21 3
1⅞	3.516	11 95	2¾	5½	5	2 425	4 619	31 4
2	4 000	13 60	2⅞	6	5	2 550	5.108	27.7
2⅛	4 516	15 35	3	6	4½	2 629	5 428	20 2
2¼	5 063	17 21	3¼	6½	5½	2 879	6 509	28.6
2⅜	5 641	19.18	3½	7	6½	3 100	7 549	33 8
2½	6 250	21 25	3¾	7	7	3 317	8 641	38 3
2⅝	6 891	23 43	3¾	7	5½	3 317	8 641	25 4
2¾	7.563	25.71	4	7½	6½	3 567	9 993	32.1
2⅞	8.266	28.10	4¼	8	7½	3 798	11 330	37 1
3	9.000	30 60	4¼	8	6	3 798	11 330	25 9
3⅛	9.766	33 20	4½	8½	7	4 028	12 741	30 5
3¼	10 563	35 91	4¾	8½	7½	4 255	14.221	34 6

Upsets marked † are special

* From Pocket Companion, Carnegie Steel Co , Pittsburgh, Pa.

Table V (Continued). Upset Screw Ends for Round Bars *

American Bridge Company Standard



Pitch and shape of thread A B Co standard

Bar			Upset					
							Area	
Diameter, d inches	Area, sq inches	Weight per foot, pounds	Diameter b, inches	Length a, inches	Additional length for upset + 10%, inches	Diameter at root of thread c, inches	At root of thread, sq inches	Excess over area of bar, per cent
†¾	0 442	1 50	1	4	4	0 838	0 551	24 7
†⅞	0.601	2 04	1¼	4	5	1 064	0 890	48.0
1	0 785	2 67	1⅜	4	4	1.158	1 054	34.2
1⅛	0.994	3 38	1½	4	4	1 283	1.294	30.2
1¼	1.227	4 17	1⅝	4	4	1 389	1 515	23 5
1⅜	1.485	5.05	1¾	4	4	1.490	1.744	17.5
1½	1.767	6 01	2	4½	4½	1.711	2.300	30.2
1⅝	2 074	7.05	2⅛	4½	4	1 836	2 649	27 7
1¾	2 405	8.18	2¼	5	4	1 961	3 021	25.6
1⅞	2.761	9.39	2⅜	5	4	2.086	3.419	23.8
2	3.142	10 68	2½	5½	4	2.175	3 716	18.3
2⅛	3 547	12 06	5⅝	5½	3½	2 300	4 156	17.2
2¼	3.976	13 52	2⅞	6	4½	2 550	5 108	28.4
2⅜	4.430	15.06	3	6	4½	2.629	5.428	22.5
2½	4.909	16 69	3¼	6½	5½	2 879	6.509	32.6
2⅝	5.412	18 40	3¼	6½	4½	2 879	6.509	20.3
2¾	5.940	20 19	3½	7	5½	3 100	7.549	27.1
2⅞	6.492	22.07	3¾	7	6	3.317	8.641	33.1
3	7.069	24.03	3¾	7	5	3.317	8.641	22.2
3⅛	7.670	26 08	4	7½	6	3.567	9 993	30.3
3¼	8.296	28.21	4	7½	5	3.567	9.993	20.5
3⅜	8.946	30.42	4¼	8	5½	3.798	11.330	26.6
3½	9 621	32.71	4¼	8	5	3.798	11.330	17.8
3⅝	10.321	35 09	4½	8½	5½	4 028	12.741	23.4
3¾	11.045	37.55	4¾	8½	6	4.255	14.221	28.8
3⅞	11.793	40.10	4¾	8½	5½	4.255	14.221	20.6

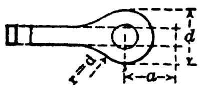
Upsets marked † are special.

*** From Pocket Companion, Carnegie Steel Co., Pittsburgh, Pa.**

Table VI.* Steel Eye-Bars

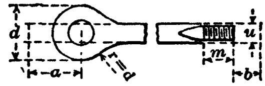
American Bridge Company Standard

Ordinary Eye-Bar



Bar: Width, in	Bar: Thickness: Maximum, in	Bar: Thickness: Minimum, in	Head: Diameter d, in	Head: Maximum pin: Diameter, in	Head: Maximum pin: Excess head over bar, %	Head: Additional material, a, ft and in: For ordering bar	Head: Additional material, a, ft and in: For figuring weight
2	1	½	4½	1¾	37 5	1-0	0-7
			5½	2¾		1-4	0-11
			† 6½	3¾		1-9	1-4
2½	1	⅝	6	2½	40 0	1-3	0-10
			7	3½		1-7	1-2
			† 8	4½		2-0	1-7
3	1½	⅝	7½	3¼	41.7	1-6	1-1
			8½	4¼		1-11	1-5
			† 9½	5¼		2-4	1-10
4	1¾	¾	10	4½	37.5	1-11	1-6
		⅞	11	5½		2-3	1-10
		1	†12	6½		2-8	2-2
5	2	¾	12	5¼	35.0	2-1	1-8
		1	13½	6¾		2-8	2-2
		1	†15	8¼		3-3	2-9
6	2	¾	14	5¾	37.5	2-4	1-10
		1	14¾	6½		2-6	2-1
		1	†16½	8¼		3-2	2-8
7	2	1	16½	7	35.7	2-7	2-2
		1⅛	17½	8		2-11	2-6
		1⅛	†18½	9		3-4	2-11
8	2	1	18	7	37.5	2-8	2-3
		1⅛	19	8		3-0	2-6
		1¼	†20	9		3-4	2-11
9	2	1⅛	20	7½	38.9	2-11	2-6
		1¼	22	9½		3-7	3-1
10	2	1⅛	22½	9	35.0	3-5	2-10
		1¼	24	10½		3-9	3-3
		1⅜	†25	11½		4-1	3-7
12	2	1¼	26½	10	37.5	3-8	3-3
		1⅜	28	11½		4-2	3-8
		1½	†29½	13		4-8	4-1
14	2	1⅜	31	12	35.7	4-3	3-9
		1½	33	14		4-7	4-4
		1⅝	†34	15		5-5	4-8
16	2	1¾	36	14	37.5	4-7	4-5
		1⅞	†37½	16	34.4	4-11	4-10

Adjustable Eye-Bar



Minimum length of short end from center of pin to end of screw, 6 ft, preferably 7 ft.

Thread on short end be to left hand

Pitch and shape of thread A B Co. standard

Bar: Width, in	Bar: Min thickness in	Screw-end: Diameter u, in	Screw-end: Excess upset over bar, %	Screw-end: Length m, in	Screw-end: Additional material, b, ft and in: For ordering bar	Screw-end: Additional material, b, ft and in: For figuring weight
2	† ⅝	1¾	39.6	4	1-0	8
	¾	1⅞	36.6	4½	1-0	7½
	⅞	2	31 4	4½	0-11	7½
2½	† ¾	2⅛	41 2	4½	1-0	8
	⅞	2¼	38.1	5	1-0	8
	1	2⅜	36 7	5	1-0	7½
3	† ¾	2¼	34.3	5	1-0	7½
	⅞	2½	41.6	5½	1-1	9½
	1	2½	23 9	5½	1-1	8½
4	† ¾	2½	23 9	5½	1-1	8½
	⅞	2¾	32.0	5½	0-11	7½
	1	3	35 7	6	1-1	8½
	1⅛	3¼	44 6	6½	1-2	9½
5	† ¾	2⅞	36 2	6	1-0	8
	⅞	3	24 1	6	0-11	7
	1	3¼	30.2	6½	1-0	8
	1⅛	3½	34.2	7	1-1	8½
	1¼	3¾	38.3	7	1-2	9
6	†1	3½	25.8	7	1-0	7½
	1⅛	3¾	28.0	7	1-0	8
	1¼	4	33.2	7½	1-1	8½
	1⅜	4¼	37 3	8	1-2	9½
7	†1⅛	4	26.9	7½	1-0	8
	1¼	4¼	29.5	8	1-1	8½
	1⅜	4½	32.4	8½	1-2	9
	1½	4¾	35.4	8½	1-2	9½
8	†1⅛	4¼	25.9	8	1-0	8
	1¼	4½	27.4	8½	1-1	8½
	1⅜	4¾	29.3	8½	1-1	8½
	1½	5	31.4	9	1-2	9
	1⅝	5¼	35.2	9½	1-3	10

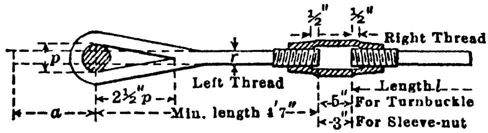
Bars marked † should be used only when absolutely unavoidable

Deduct pin-hole when figuring weight

*From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.

Table VII.* Loop-Rods

American Bridge Company Standard



Pitch and shape of thread A. B. Co. standard

Additional length A, in feet and inches, for one loop. $A = 4.17p + 5.89r$

Diam. of pin, p, in	Diameter or side r of rod in inches										
	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2
1⅛	0- 9½	0-10	0-11	0-11½	...	....				.	
1¼	0-10	0-10½	0-11½	1- 0	1- 1	....	....	...	.	..	
1½	0-11	0-11½	1- 0½	1- 1	1- 2	1- 2½	....	....	....	....	
1¾	1- 0	1- 0½	1- 1½	1- 2	1- 3	1- 3½	1- 4½	1- 5	1- 6	....	....
2	1- 1	1- 1½	1- 2½	1- 3	1- 4	1- 4½	1- 5½	1- 6	1- 7	1- 7½	1- 8½
2¼	1- 2	1- 3	1- 3½	1- 4¼	1- 5	1- 5½	1- 6½	1- 7	1- 8	1- 8½	1- 9½
2½	1- 3	1- 4	1- 4½	1- 5½	1- 6	1- 7	1- 7½	1- 9	1- 9	1- 9½	1-10½
2¾	1- 4	1- 5	1- 5½	1- 6½	1- 7	1- 8	1- 8½	1- 9½	1-10	1-11	1-11½
3	1- 5	1- 6	1- 6½	1- 7½	1- 8	1- 9	1- 9½	1-10½	1-11	2- 0	2- 0½
†3¼	1- 6	1- 7	1- 7½	1- 8½	1- 9	1-10	1-10½	1-11½	2- 0	2- 1	2- 1½
3½	1- 7½	1- 8	1- 8½	1- 9½	1-10	1-11	1-11½	2- 0½	2- 1	2- 2	2- 2½
†3¾	1- 8½	1- 9	1-10	1-10½	1-11	2- 0	2- 0½	2- 1½	2- 2	2- 3	2- 3½
4	1- 9½	1-10	1-11	1-11½	2- 0½	2- 1	2- 2	2- 2½	2- 3	2- 4	2- 4½
†4¼	.	1-11	2- 0	2- 0½	2- 1½	2- 2	2- 3	2- 3½	2- 4½	2- 5	2- 6
4½		2- 0	2- 1	2- 1½	2- 2½	2- 3	2- 4	2- 4½	2- 5½	2- 6	2- 7
†4¾		2- 1	2- 2	2- 2½	2- 3½	2- 4	2- 5	2- 5½	2- 6½	2- 7	2- 8
5		2- 2½	2- 3	2- 3½	2- 4½	2- 5	2- 6	2- 6½	2- 7½	2- 8	2- 9
†5¼	.	.	2- 4	2- 5	2- 5½	2- 6	2- 7	2- 7½	2- 8½	2- 9	2-10
5½	..	....	2- 5	2- 6	2- 6½	2- 7½	2- 8	2- 9	2- 9½	2-10	2-11
†5¾	...	....	2- 6	2- 7	2- 7½	2- 8½	2- 9	2-10	2-10½	2-11½	3- 0
6	...	....	2- 7	2- 8	2- 8½	2- 9½	2-10	2-11	2-11½	3- 0½	3- 1
†6¼	....	....	....	2- 9	2- 9½	2-10½	2-11	3- 0	3- 0½	3- 1½	3- 2
6½	..	....	....	2-10	2-10½	2-11½	3- 0	3- 1	3- 1½	3- 2½	3- 3
†6¾	....	...	..	2-11	3- 0	3- 0½	3- 1	3- 2	3- 2½	3- 3½	3- 4
7	....	..	...	3- 0	3- 1	3- 1½	3- 2½	3- 3	3- 3½	3- 4½	3- 5

Pins marked † are special. Maximum shipping length of l = 35 ft

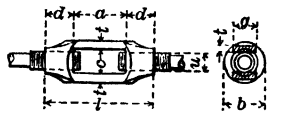
*** From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.**

Table VIII.* Turnbuckles and Sleeve-Nuts

American Bridge Company Standard

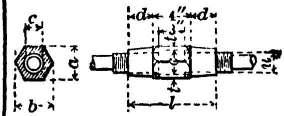
All dimensions in inches

Turnbuckles



$a = 6''$; $a = 9''$ for turnbuckles marked †
Pitch and shape of thread, A B Co standard

Sleeve-Nuts



Pitch and shape of thread, A. B. Co. standard

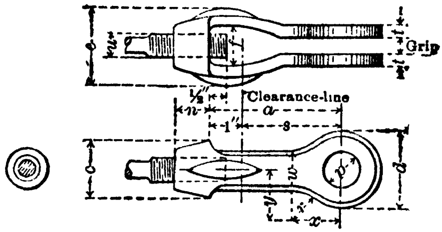
Dia. of screw u	Standard dimensions						W't, lb	Dia. of screw u	Standard dimensions						W't lb
	d	l	c	t	g	b			d	l	a	b	c	t	
3/8	9/16	7 1/8	9/16	3/16	1/2	1 1/16	1				.			.	..
7/16	21/32	7 5/16	5/8	1/4	5/8	1 3/8	1	.		..	.			..	....
1/2	3/4	7 1/2	5/8	1/4	5/8	1 3/8	1								..
9/16	27/32	7 11/16	13/16	5/16	3/4	1 9/16	1 1/2	.			.				...
5/8	15/16	7 7/8	13/16	5/16	3/4	1 9/16	1 1/2	.		.	.			.	...
3/4	1 1/8	8 1/4	1 1/16	11/32	7/8	2	2			..	..	.		.	..
7/8	1 5/16	8 5/8	1 1/4	3/8	1	2 1/4	3	7/8	1 1/2	7	1 5/8	1 7/8	1 1/8	1/4	3
1	1 1/2	9	1 5/16	7/16	1 1/4	2 7/16	4	1	1 1/2	7	1 5/8	1 7/8	1 1/8	1/4	3
1 1/8	1 11/16	9 3/8	1 7/16	1/2	1 1/4	2 9/16	5	1 1/8	1 3/4	7 1/2	2	2 5/16	1 3/8	5/16	4
1 1/4	1 7/8	9 3/4	1 9/16	1/2	1 1/2	2 3/4	6	1 1/4	1 3/4	7 1/2	2	2 5/16	1 3/8	5/16	4
1 3/8	2 1/16	10 1/8	1 11/16	1/2	1 5/8	3 1/16	7	1 3/8	2	8	2 3/8	2 3/4	1 5/8	3/8	5
1 1/2	2 1/4	10 1/2	1 3/4	5/8	1 3/4	3 3/16	8	1 1/2	2	8	2 3/8	2 3/4	1 5/8	3/8	6
1 5/8	2 7/16	10 7/8	2	5/8	1 7/8	3 1/2	10	1 5/8	2 1/4	8 1/2	2 3/4	3 3/16	1 7/8	7/16	8
1 3/4	2 5/8	11 1/4	2 1/8	5/8	2	3 3/4	11	1 3/4	2 1/4	8 1/2	2 3/4	3 3/16	1 7/8	7/16	9
1 7/8	2 13/16	11 5/8	2 3/16	11/16	2 1/8	3 7/8	12	1 7/8	2 1/2	9	3 1/8	3 5/8	2 1/8	1/2	10
2	3	12	2 3/8	11/16	2 1/4	4 1/4	14	2	2 1/2	9	3 1/8	3 5/8	2 1/8	1/2	11
2 1/8	3 3/16	12 3/8	2 1/2	23/32	2 1/2	4 1/2	17	2 1/8	2 3/4	9 1/2	3 1/2	4 1/16	2 3/8	9/16	14
2 1/4	3 3/8	12 3/4	2 11/16	13/16	2 1/2	4 3/4	20	2 1/4	2 3/4	9 1/2	3 1/2	4 1/16	2 3/8	9/16	15
2 3/8	3 9/16	13 1/8	2 3/4	13/16	2 3/4	4 7/8	22	2 3/8	3	10	3 7/8	4 1/2	2 5/8	5/8	18
2 1/2	3 3/4	13 1/2	3 1/16	27/32	3	5 3/8	25	2 1/2	3	10	3 7/8	4 1/2	2 5/8	5/8	19
2 3/4	4 1/8	14 1/4	3 1/4	15/16	3 1/4	5 3/4	33	2 3/4	3 1/4	10 1/2	4 1/4	4 15/16	2 7/8	1 1/16	23
2 7/8	4 5/16	14 5/8	3 7/16	1 1/32	3 1/4	6 1/16	36	2 7/8	3 1/2	11	4 5/8	5 3/8	3 1/8	3/4	27
3	4 1/2	15	3 5/8	1 1/32	3 1/2	6 3/8	40	3	3 1/2	11	4 5/8	5 3/8	3 1/8	3/4	28
3 1/4	4 7/8	15 3/4	3 7/8	1 1/16	4	6 3/4	50	3 1/4	3 3/4	11 1/2	5	5 13/16	3 3/8	13/16	35
3 1/2	5 1/4	16 1/2	4 1/4	1 7/32	4	7 1/4	65	3 1/2	4	12	5 3/8	6 1/4	3 5/8	7/8	40
3 3/4	5 5/8	17 1/4	4 7/16	1 5/16	5	8 1/4	95	3 3/4	4 1/4	12 1/2	5 3/4	6 11/16	3 7/8	15/16	47
4	6	18	4 5/8	1 7/16	5	8 3/4	108	4	4 1/2	13	6 1/8	7 1/16	4 1/8	1	55
†4 1/4	6 1/4	21 1/2	4 5/8	1 5/8	5 5/32	9 1/4	140	4 1/4	4 3/4	13 1/2	6 1/2	7 1/2	4 3/8	1 1/16	65
†4 1/2	6 3/4	22 1/2	5 1/2	1 3/4	6 1/2	10 3/4	195	4 1/2	5	14	6 7/8	7 15/16	4 3/4	1 1/16	75
†4 3/4	7 1/4	23 1/2	5 5/8	2	6 1/2	11 1/4	205	...	..	..	...	...	.	...	...
†5	7 1/2	24	6	2 1/4	6 1/2	11 7/8	250	...	...	....	.	...	...	...	....

* From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.

Table IX.* Clevises

AMERICAN BRIDGE COMPANY STANDARD

All dimensions in inches



Grip = thickness of plate + 1/4 in, but must not exceed dimension *f*

Clevis No.	Head								Nut						Fork			W't, lb
	d	w	t	Max p	Min p	r	x	y	n	c	Max u	Min u	e	f	a	s	
3	3	1½	½	1½	1	2¼	2¼	3	1½	2¼	1⅛	1	$3\frac{1}{16}$	1¼	5	4	4
4	4	2	½	2	1¼	3	3	4	1¾	2⅞	1⅝	1⅛	3⅝	1¾	6	5	8
5	5	2½	⅝	2½	1½	3¾	3¾	5	2¼	3¾	2⅛	1½	4½	2¼	7	6	16
6	6	3	¾	3	2	4½	4½	6	2½	4⅛	2⅝	2	5⅜	2¾	8	7	26
7	7	3½	⅞	3½	2½	5¼	5¼	7	3	5	3	2¼	$6\frac{3}{16}$	3¼	9	8	36

CLEVIS-NUMBERS FOR VARIOUS RODS AND PINS

Rods			Pins										
Round	Square	Upset	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½
¾	.	1	3	3	3								
. . .	¾	1⅛	3	3	3	4	4						
⅞	⅞	1¼	..	4	4	4	4						
1	...	1⅜	..	4	4	4	4						
1⅛	1	1½		4	4	4	4	5	5	.	.	.	..
1¼	1⅛	1⅝		4	4	4	4	5	5				
1⅜	. . .	1¾			5	5	5	5	5				
..	1¼	1⅞			5	5	5	5	5			.	
1½	1⅜	2			5	5	5	5	5	6	6		
1⅝	. ..	2⅛			5	5	5	5	5	6	6		
1¾	1½	2¼	. .	.	..		6	6	6	6	6	7	7
1⅞	1⅝	2⅜					6	6	6	6	6	7	7
2	1¾	2½		.			6	6	6	6	6	7	7
2⅛	. .	2⅝					6	6	6	6	6	7	7
..	1⅞	2¾						...	7	7	7	7	7
2¼	2	2⅞						.	7	7	7	7	7
2⅜	2⅛	3							7	7	7	7	7

Clevises above and to right of zigzag line may be used with forks straight, those below and to left of this line should have forks closed so as not to overstress the pin.

* From Pocket Companion, Carnegie Steel Company Pittsburgh, Pa.

Table X. Safe Loads in Tension for Common Sizes of Angles with One $\frac{7}{8}$-Inch Rivet-Hole for a $\frac{3}{4}$-Inch Rivet

Load in pounds for a stress of 16 000 lb per sq in

Size of angle	Load	Size of angle	Load
$6 \times 4 \times \frac{3}{4}$	100 500	$3\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{8}$	45 000
$\frac{5}{8}$	85 000	$\frac{9}{16}$	41 100
$\frac{1}{2}$	68 900	$\frac{1}{2}$	37 000
		$\frac{3}{8}$	28 500
$5 \times 3\frac{1}{2} \times \frac{3}{4}$	82 500	$\frac{1}{4}$	19 500
$\frac{5}{8}$	69 900		
$\frac{1}{2}$	57 000	$3 \times 3 \times \frac{5}{8}$	45 000
		$\frac{1}{2}$	37 000
$5 \times 3 \times \frac{3}{4}$	76 500	$\frac{3}{8}$	28 500
$\frac{5}{8}$	64 900	$\frac{1}{4}$	19 500
$\frac{1}{2}$	53 000		
$\frac{3}{8}$	40 500	$3 \times 2\frac{1}{2} \times \frac{1}{2}$	33 000
		$\frac{3}{8}$	25 400
$4 \times 4 \times \frac{3}{4}$	76 500	$\frac{5}{16}$	21 600
$\frac{3}{8}$	40 500	$\frac{1}{4}$	17 400
$4 \times 3\frac{1}{2} \times \frac{5}{8}$	60 000		
$\frac{3}{8}$	37 600	$3 \times 2 \times \frac{7}{16}$*	25 900
		$\frac{3}{8}$*	22 500
$4 \times 3 \times \frac{5}{8}$	55 000	$\frac{5}{16}$*	19 200
$\frac{1}{2}$	45 000	$\frac{1}{4}$*	15 600
$\frac{3}{8}$	34 400		
		$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{7}{16}$	25 900
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{4}$	64 500	$\frac{3}{8}$	22 400
$\frac{5}{8}$	54 900	$\frac{5}{16}$	19 200
$\frac{1}{2}$	45 000	$\frac{1}{4}$	15 500
$\frac{3}{8}$	34 400		
		$2\frac{1}{2} \times 2 \times \frac{7}{16}$	22 400
$3\frac{1}{2} \times 3 \times \frac{5}{8}$	50 100	$\frac{3}{8}$	19 500
$\frac{1}{2}$	41 000	$\frac{5}{16}$	16 600
$\frac{3}{8}$	31 500	$\frac{1}{4}$	13 400

* These are special angles. It is better not to use them in ordinary work because of risk of delay in delivery.

The End-Connections often determine the strength of ANGLE TENSION-MEMBERS. Some specifications for structural work require angles subject to direct tension to be connected by both legs if the section of both legs is considered; and if connected by one leg, the section of one leg only is considered effective. Reliable tests show this requirement to be needlessly severe. For single angles connected by one leg, the Specifications for the Structural Steelwork of Buildings, Chapter XXVIII, Article 5, allow the net area of the connected leg and one-half that of the outstanding leg to be considered effective.

Table XI. Sectional Area to be Deducted from Plates and Angles for One Round Hole

NOTE. Bolt-holes should be 1/16 in larger than the diameter of the bolt; rivet-holes are usually 1/8 in larger than the diameter of the rivet.

Thickness of Plate	Diameter of hole in fractions of an inch and inches																
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16	1 3/16	1 7/16	1 9/16
3/16	0 05	0 06	0 07	0.08	0.09	0 11	0 12	0 13	0 14	0 15	0.16	0.18	0.19	0 20	0 23	0 27	0.30
1/4	0 06	0.08	0 09	0.11	0.13	0.14	0 16	0 17	0 19	0 20	0 22	0.23	0 25	0.27	0.30	0 36	0 39
5/16	0 08	0 10	0 12	0.14	0 16	0 18	0 20	0 21	0 23	0 25	0 27	0 29	0 31	0 33	0 37	0 45	0 49
3/8	0 09	0 12	0 14	0.16	0 19	0 21	0 23	0.26	0 28	0 30	0 33	0 35	0 38	0 40	0 45	0 54	0 59
7/16	0 11	0 14	0 16	0.19	0.22	0 25	0 27	0 30	0 33	0 36	0 38	0 41	0 44	0 46	0 52	0.63	0 69
1/2	0.13	0 16	0 19	0.22	0.25	0 28	0 31	0 34	0 38	0 41	0 44	0 47	0 50	0 53	0 59	0 72	0 78
9/16	0.14	0 18	0 21	0 25	0.28	0 32	0.35	0 39	0 42	0 46	0 49	0 53	0 56	0 60	0 67	0.81	0 88
5/8	0 16	0 20	0 23	0 27	0 31	0 35	0 39	0 43	0 47	0 51	0 55	0 59	0 63	0.66	0 74	0 90	0 98
11/16	0.17	0 21	0 26	0 30	0 34	0 39	0 43	0 47	0 52	0 56	0 60	0 64	0.69	0 73	0 82	0.99	1 08
3/4	0 19	0 23	0 28	0 33	0 38	0 42	0 47	0 52	0 56	0 61	0 66	0 70	0 75	0 80	0 89	1 08	1 17
13/16	0 20	0 25	0 30	0 36	0 41	0 46	0 51	0 56	0 61	0 66	0 71	0 76	0 81	0 86	0 97	1 17	1 27
7/8	0 22	0 27	0 33	0 38	0 44	0 49	0 55	0 60	0 66	0 71	0 77	0 82	0 88	0 93	1.04	1 26	1 37
15/16	0 23	0 29	0 35	0 41	0 47	0 53	0 59	0 64	0 70	0 76	0 82	0 88	0 94	1 00	1 11	1.35	1 47
1	0 25	0 31	0 38	0.44	0 50	0 56	0 63	0 69	0 75	0 81	0 88	0 94	1 00	1 06	1 19	1 44	1 56

7. Wire.

Manufacture. Iron and steel wires are made from BILLETS about 4 in square. These are rolled into long rods which are dipped in acid to remove the scale and furnish lubrication for the drawing process. This consists in pulling the rods while cold through steel dies having a series of holes of gradually decreasing diameters. The cold working of the metal hardens it and makes it brittle so that it is necessary to anneal it at intervals during the process. The drawing increases the strength of the material, so that wires of different sizes, although made of the same material, differ greatly in ULTIMATE STRENGTH.

Finish. The common grades of iron and steel wire are furnished in several different finishes: plain black, bright tinned, copper-coated, japanned and with single and double coats of zinc galvanizing. This is applied by passing the wire through a bath of melted zinc which adheres to it and forms a coating, the most common effective protection against corrosion.

Wire-Gauges. Table XIII gives, according to several GAUGES, the diameters of the different numbers of wire that have come into use for different purposes and have been brought out by different manufacturers. In ordering wire by number it is necessary to specify which GAUGE is meant.

Strength. Table XIV gives the sizes according to the J. A. Roebling's Sons Company gauge, with the weight and length and the strength on an assumed basis of 100 000 lb per sq in. The different kinds of wire vary so widely in ULTIMATE STRENGTH, on account of both the difference in quality of the

material and the effect of the drawing, that in order to obtain the approximate strength of a wire, reference must be made to Table XII in connection with the foot-note to Table XIV. The following table is arranged from values which were published in the Catalogue of the J. A. Roebling's Sons Company:

Table XII. Approximate Ultimate Strength of Different Sizes of Iron and Steel Wire

Kind of wire	Ultimate strength	
	Large size lb per sq in	Small size lb per sq in
Soft iron	45 000	60 000
Telegraph and telephone (steel)	60 000	80 000
Special aviator	247 000	285 000
Piano wire	307 000	340 000
Plow steel wire	200 000	345 000
Hard-drawn copper trolley wire	50 000	Not used
Hard-drawn telegraph and telephone copper	56 000	66 000

The American Steel and Wire Company's Gauge is almost universally followed throughout the United States for steel wire. The Birmingham gauge, an English gauge, is the only wire-gauge recognized in successive Acts of Congress establishing tariffs, and for many years has been used as the basis for duties assessed on imported wire.

Table XIII. Comparison of Standard Gauges for Wire and Sheet Metal

Number of gauge	Diameter or thickness in decimals of an inch						
	Birmingham or Stubs iron-wire-gauge	American or Brown & Sharpe wire-gauge	United States standard gauge for sheet and plate iron and steel	Washburn & Moen, Roebling, American Steel & Wire Co., steel-wire gauge	Stubs steel-wire-gauge	American Screw Co. wire-gauge	British Imperial or English legal standard wire-gauge
0000000	.	. . .	0 5	0 4900	....	.....	0 500
000000	. .	0 580000	0 46875	0 4615	....	.. ..	0.464
00000	0 500	0.516500	0.4375	0.4305	....	.....	0.432
0000	0 454	0 460000	0 40625	0.3938	....	.....	0.400
000	0 425	0 409642	0 375	0 3625	.. .	0 0315	0.372
00	0 380	0 364796	0 34375	0 3310	.....	0 0447	0 348
0	0 340	0 324861	0 3125	0 3065	.	0.0578	0.324
1	0 300	0 289297	0 28125	0 2830	0 227	0 0710	0 300
2	0 284	0 257627	0 265625	0 2625	0 219	0 0842	0 276
3	0 259	0 229423	0 25	0 2437	0 212	0 0973	0 252
4	0 238	0 204307	0 234375	0 2253	0.207	0 1105	0 232
5	0 220	0 181940	0 21875	0 2070	0.204	0.1236	0.212
6	0 203	0 162023	0 203125	0 1920	0.201	0 1368	0 192
7	0 180	0 144285	0.1875	0 1770	0.199	0 1500	0 176
8	0 165	0 128490	0 171875	0 1620	0 197	0.1631	0.160
9	0 148	0 114423	0 15625	0 1483	0 194	0 1763	0 144
10	0 134	0 101897	0 140625	0 1350	0 191	0.1894	0.128
11	0 120	0 090742	0 125	0 1205	0 188	0.2026	0.116
12	0 109	0 080808	0 109375	0 1055	0 185	0 2158	0.104
13	0 095	0 071962	0 09375	0 0915	0 182	0 2289	0 092
14	0 083	0 064084	0 078125	0 0800	0 180	0 2421	0.080
15	0 072	0.057068	0 0703125	0 0720	0.178	0 2552	0.072
16	0 065	0.050821	0 0625	0 0625	0.175	0.2684	0.064
17	0 058	0 045257	0.05625	0 0540	0 172	0 2816	0 056
18	0 049	0 040303	0 05	0 0475	0.168	0 2947	0.048
19	0 042	0.035890	0 04375	0.0410	0.164	0.3079	0.040
20	0 035	0 031961	0 0375	0 0348	0.161	0.3210	0.036
21	0 032	0.028462	0 034375	0 0317	0 157	0.3342	0.032
22	0 028	0 025346	0 03125	0 0286	0 155	0.3474	0.028
23	0 025	0 022572	0 028125	0 0258	0 153	0.3605	0 024
24	0 022	0 020101	0 025	0.0230	0.151	0.3737	0.022
25	0 020	0 017900	0 021875	0 0204	0.148	0.3868	0.020
26	0 018	0 015941	0.01875	0 0181	0 146	0.4000	0 018
27	0 016	0.014195	0.0171875	0 0173	0.143	0.4132	0.0164
28	0 014	0 012641	0 015625	0 0162	0 139	0.4263	0.0148
29	0 013	0.011257	0.0140625	0.0150	0.134	0.4395	0.0136
30	0.012	0.010025	0.0125	0 0140	0.127	0.4526	0.0124
31	0.010	0 008928	0.0109375	0 0132	0.120	0.4658	0.0116
32	0.009	0.007950	0.01015625	0.0128	0.115	0.4790	0.0108
33	0 008	0 007080	0.009375	0 0118	0.112	0.4921	0.0100
34	0 007	0 006305	0.00859375	0.0104	0.110	0.5053	0.0092
35	0 005	0 005615	0.0078125	0 0095	0.108	0.5184	0.0084
36	0.004	0 005000	0.00703125	0.0090	0.106	0.5316	0.0076
37	.....	0 004453	0.006640625	0 0085	0 103	0.5448	0.0068
38	.....	0 003965	0 00625	0.0080	0 101	0.5579	0.0060
39	.....	0.003531	....	0.0075	0.099	0.5711	0.0052
40	...	0 003144	.. ..	0.0070	0 097	0 5842	0.0048

The United States Standard Gauge was legalized by Act of Congress, March 3, 1893, as a standard gauge for sheet and plate iron and steel, and is used by the Custom House Department and by sheet-plate and tin-plate manufacturers.

Table XIV. Weight, Length and Strength of Steel Wire

Gauge of J. A. Roebling's Sons Company *

Number of gauge	Diameter, in	Area, sq in	Breaking-load in pounds at rate of 100 000 lb per sq in	Weight in pounds		Number of feet in 2 000 lb
				Per 1 000 ft	Per mile	
000000	0 460	0 166191	16 619	558.4	2 948	3 582
00000	0 430	0 145221	14 522	487 9	2 576	4 099
0000	0 394	0 121304	12 130	407.6	2 152	4 907
000	0 362	0 102922	10 292	345.8	1 826	5 783
00	0.331	0 086049	8 605	289 1	1 527	6 917
0	0.307	0.074023	7 402	248.7	1 313	8 041
1	0.283	0 062902	6 290	211.4	1 116	9 463
2	0.263	0 054325	5 433	182 5	964	10 957
3	0.244	0.046760	4 676	157 1	830	12 730
4	0.225	0 039761	3 976	133.6	705	14 970
5	0.207	0.033654	3 365	113.1	597	17 687
6	0.192	0 028953	2 895	97.3	514	20 559
7	0.177	0 024606	2 461	82 7	437	24 191
8	0.162	0 020612	2 061	69 3	366	28 878
9	0.148	0 017203	1 720	57 8	305	34 600
10	0.135	0.014314	1 431	48.1	254	41 584
11	0.120	0 011310	1 131	38.0	201	52 631
12	0.105	0.008659	866	29 1	154	68 752
13	0 092	0 006648	665	22 3	118	89 525
14	0.080	0 005027	503	16 9	89 2	118 413
15	0.072	0.004071	407	13.7	72.2	146 198
16	0.063	0 003117	312	10 5	55.3	191 022
17	0 054	0 002290	229	7 70	40 6	259 909
18	0.047	0 001735	174	5 83	30 8	343 112
19	0 041	0 001320	132	4 44	23 4	450 856
20	0 035	0 000962	96	3 23	17 1	618 670

* Also American Steel & Wire Company, etc.

This table was calculated on a basis of 483 84 lb per cu ft for steel wire. Iron wire is a trifle lighter.

The breaking strengths were calculated for 100 000 lb per sq in throughout, simply for convenience, so that the breaking strengths per square inch of wires of any strength may be quickly determined by multiplying the values given in the table by the ratio between the strength per square inch and 100 000. Thus, a No. 15 wire, with a strength per square inch of 150 000 pounds, has a breaking strength of

$$407 \times \frac{150\,000}{100\,000} = 610\ 5 \text{ lb.}$$

It must not be inferred from this table that steel wire invariably has a strength of 100 000 lb per sq in. As a matter of fact its strength ranges from 45 000 lb per sq in for soft, annealed wire to over 400 000 lb per sq in for hard wire.

8. Wire Rope

Kinds of Wire Rope. There are several kinds of WIRE ROPE in common use. In each there are three or more qualities depending on the kind of wire used and the kind of core about which the strands are laid. The following are found in standard catalogues:

(1) **Haulage or Transmission-Rope,** composed of six strands of seven wires each, laid about a hemp core It is used for haulage, transmission of power, in places where surface-wear is of chief consideration and where sheaves of sufficient diameter may be used.

(2) **Hoisting-Rope,** composed of six strands of nineteen wires each. It is used for elevator service, shafts and derricks, and in places where it is not subject to abrasion and where flexibility is of chief consideration.

(3) **Seale Rope,** composed of six strands of nineteen wires each, the inner coils of the strands being of finer wire. It is intermediate in flexibility between the first and second kinds of rope.

(4) **Non-Spinning Hoisting-Rope,** having eighteen strands of seven wires each. Twelve of the strands are laid in reverse direction to the inner six, making it well adapted for hoisting in free suspension without untwisting and turning the load.

(5) **Extra-Pliable Hoisting-Rope,** having eight strands of nineteen wires each.

(6) **Special Flexible Hoisting-Rope,** having six strands of thirty-seven wires each.

(7) **Hawser-Rope and Flexible Running-Rope,** having six strands of twelve galvanized wires each, laid about a hemp core

(8) **Tiller-Rope,** composed of six small seven-strand ropes laid about a hemp core. It is the most flexible of wire ropes and is used to operate tillers and for hand-ropes in elevators.

The Lay of Wire Rope is the twist of the wires in the strands relatively to the strands in the rope. In the ORDINARY LAY the twist of the strands is the reverse of that of the wires, while in the LANG LAY the strands are laid in the same direction as the twist of the wires. This latter gives a greater distribution of the wearing surface and a somewhat greater flexibility; but it has the disadvantage of a tendency to untwist and for this reason should not be used for hoisting weights in free suspension. Wire rope is also made up in FLAT or RIBBON FORM. For large sizes it is more flexible than standard rope and may be run over smaller drums.

Materials for Rope. Nearly all of the above kinds of rope are made up in the following materials:

(1) **Best Grade of Wrought Iron.** This is used in high-speed PASSENGER-ELEVATOR SERVICE as it seems to suffer less from the effects of the stresses due to the starting and stopping of the cars.

(2) **Cast-Steel Wire,** with an ultimate strength of from 160 000 to 210 000 lb per sq in, according to the size used.

(3) **Extra-Strong Cast-Steel Wire,** with an ultimate strength of from 190 000 to 230 000 lb per sq in.

(4) **Plow-Steel Wire** with an ultimate strength of from 200 000 to 230 000 lb per sq in.

Table XV. Strength of Wire Rope

Arranged from the list of John A. Roebling's Sons Company

Trade number	Diameter inches	Weight per foot, hemp core	Approximate breaking-load in pounds		Minimum diameter of drum or sheave in feet	
			Iron	Cast steel	Iron	Cast steel
HOISTING-ROPE Six strands of nineteen wires each, about a hemp core						
1	2¼	8.00	144 000	266 000	14	9
2	2	6.30	110 000	212 000	12	8
2½	1⅞	5.55	100 000	192 000	12	8
3	1¾	4 85	88 000	170 000	11	7
4	1⅝	4.15	76 000	144 000	10	6 5
5	1½	3.55	66 000	128 000	9	6
5½	1⅜	3	56 000	112 000	8 5	5 5
6	1¼	2.45	45 600	94 000	7.5	5
7	1⅛	2	37 200	76 000	7	4 5
8	1	1.58	29 000	60 000	6	4
9	⅞	1 20	23 600	46 000	5.5	3 5
10	¾	0 89	17 000	35 000	4 5	3
10¼	⅝	0 62	12 000	25 000	4	2 5
10½	9/16	0.50	9 400	20 000	3 5	2 25
10¾	½	0.39	7 800	16 800	3	2
10a	7/16	0 30	5 800	13 000	2.75	1 75
10b	⅜	0 22	4 800	9 600	2 25	1.5
STANDING ROPE Six strands of seven wires each						
11	1½	3.55	64 000	126 000	16	11
12	1⅜	3	56 000	106 000	15	10
13	1¼	2.45	46 000	92 000	13	9
14	1⅛	2	38 000	74 000	12	8
15	1	1.58	30 000	62 000	10 5	7
16	⅞	1 20	24 000	48 000	9	6
17	¾	0 89	17 600	37 200	7 5	5
18	11/16	0.75	14 600	30 800	7 25	4 75
19	⅝	0.62	12 000	26 000	7	4 50
20	9/16	0.50	9 600	20 000	6	4
21	½	0 39	7 400	15 400	5 5	3 5
22	7/16	0 30	5 200	11 000	4.5	3
23	⅜	0.22	4 400	9 200	4	2.75
24	5/16	0.15	3 400	7 000	3.5	2.25
25	9/32	0.125	2 400	5 000	3	1.75

The working load is to be taken at one-fifth the breaking-load. This is assumed in calculating the diameter of the sheaves.

Ordinary GALVANIZED-WIRE ROPE should not be used for other than standing rope, that is, rope fixed in place. A short service running through sheaves will break the coating and permit rapid corrosion of the rope Because of the many kinds and qualities of rope it is well to consult the manufacturers as to which kind will best suit the conditions for any particular service. The John A. Roebling's Sons Company, Trenton, N. J., and A. Leschen & Sons Rope Company, St. Louis, Mo., are among the largest manufacturers of full lines of ropes.

Coils. Wire rope should not be coiled like hemp rope, and in order to avoid kinking, should be taken from the reels without twisting. If it is not shipped on a reel, to avoid injury it must be rolled over the ground like a wheel.

Lubrication. It is very important that running ropes be properly lubricated, since, if proper care is not taken, the wear on the interior parts, between the wires, may be almost as great as the outside abrasion. The oil should penetrate to the core of the rope and yet not drip off a few days after application. Information as to the care of rope may be obtained from the manufacturers.

Sheaves. The size of sheaves recommended in the tables are calculated for a working-load of one-fifth the given breaking-load. If smaller sheaves are used the life of the rope will be greatly shortened, because of the excessive bending of the outer wires

Table XVI. Galvanized, Steel-Wire Strands

For smokestack guys, electric light plants, street railways, signal cord, fencing, etc.

A. Leschen & Sons Rope Company list *

Diameter in inches	Breaking strength in pounds	Approximate weight per 1 000 ft
$\frac{1}{8}$	540	31 8
$\frac{5}{32}$	870	51 3
$\frac{3}{16}$	1 150	72 9
$\frac{7}{32}$	1 540	98 3
$\frac{1}{4}$	1 900	121
$\frac{5}{16}$	3 200	205
$\frac{3}{8}$	4 250	296
$\frac{7}{16}$	5 700	399
$\frac{1}{2}$	7 400	517

* Catalogue No 37.

9. Cotton, Hemp and Manila Rope

Rope is made of cotton, hemp, and Manila fiber. Cotton is used for small sizes only. Sash cord is woven from cotton thread.

Manufacture. In the manufacture of rope the fiber is first spun into YARN. From twenty to eighty threads are twisted together into STRANDS and the strands, three or four, are laid together, opposite in direction to the TWIST in the strands, but in the same direction as the THREADS. This causes the fibers to be twisted as the rope untwists and produces a balancing of forces that tends to keep the rope in shape.

Cables and Hawsers are made up of strands of rope.

Rope used for Hoisting wears rapidly from the action of the pulleys and also from the bending which causes a slight internal motion between the fibers and a chafing and grinding away of the interior.

Stevedore-Rope is filled with a tallow and plumbago lubricant which decreases the internal friction, lubricates the outside of the rope and thus greatly prolongs its life.

Strength. The values of the strength of new rope, given in Table XVII, are taken from the Specifications of the United States Navy Department, issued in October, 1929. Manufacturers generally adopt these sizes and weights and claim a strength equal to or a little greater than the values given. The UNIT STRENGTH for the different sizes varies, being about 14 000 lb per sq in for the smaller and about 10 000 for the largest size. The approximate formula, offered by C. W. Hunt, of 720 times the square of the circumference in inches, is equivalent to about 9 000 lb per sq in. Four-strand, medium-laid rope shall not be over 7% heavier than three-strand rope of the same size, and shall have at least 95% of the strength required for three-strand rope of the same size. Other requirements are the same as for three-strand rope.

Table XVII. Strength and Weight of Rope

Specifications of the United States Navy, October 1, 1929
(Standard rope three-strand medium lay)

Circumference, in	Diameter (nominal), in	Weight per foot (maximum) in pounds	Breaking strength (minimum) in pounds	Load $P = 200\ D^2$, lb
1/2	3/16 (6 yarns)	0 015	590	7
3/4	1/4 (6 yarns)	0 020	700	12 5
1	5/16 (9 yarns)	0 029	1 200	19 5
1 1/8	3/8 (12 yarns)	0 041	1 450	28 2
1 1/4	7/16 (15 yarns)	0 054	1 750	38 2
1 1/2	1/2 (21 yarns)	0 074	2 450	50
1 3/4	9/16	0.103	3 150	63 4
2	5/8	0 131	4 000	78 2
2 1/4	3/4	0 162	4 900	112.5
2 1/2	13/16	0 191	5 900	132
2 3/4	7/8	0.221	7 000	153
3	1	0.265	8 200	200
3 1/4	1 1/16	0 309	9 500	226
3 1/2	1 1/8	0 353	11 000	252
3 3/4	1 1/4	0 412	12 500	312
4	1 5/16	0.470	14 200	345
4 1/2	1 1/2	0.588	17 500	450
5	1 5/8	0.735	21 500	528
5 1/2	1 3/4	0 882	25 500	612
6	2	1 06	30 000	800
7	2 1/4	1.44	38 500	1 012
8	2 5/8	1.88	49 000	1 380
9	3	2.38	61 000	1 800
10	3 1/4	2.94	73 000	2 120
11	3 1/2	3.57	86 400	2 450
12	3 3/4	4 24	101 000	2 812

Working Load. The WORKING LOAD for slow-speed derrick and hoisting-service is usually taken at one-seventh the BREAKING-LOAD. This makes some allowance for the loss of strength at splices and connections. The deterioration of rope exposed to the weather is very rapid. For Manila rope from 1 to 1¾ in in diameter, running over sheaves of the diameters given, C. W. Hunt in Trans. Am. Soc. M. E., Vol. XXIII, gives a table embodying approximately the following results of experience:

Table XVIII. Working Loads for Manila Rope

Working load = C × breaking-load of new rope

D = minimum diameter of sheave in inches

Speed	Feet per minute	Kind of work	Value of C	D for rope of diameter of 1 in	D for rope of diameter of 1¾ in
Slow	50 to 100	Derrick, crane, quarry, etc	0 014	8	14
Medium	150 to 300	Wharf, cargo, etc	0 056	12	18
Rapid	400 to 800	.	0 028	40	70

The wear in such service is very rapid, a 1½-in rope wearing out in lifting from 7 000 to 10 000 tons of coal On the other hand, a 1½-in transmission-rope running at 5 000 ft per min. and carrying 1 000 horse-power over sheaves 5 ft and 17 ft in diameter, lasts for years, the difference being due to the smaller stress and larger sheaves.

10. Chains

Manufacture. There are two types of welded chain, fire-welded and electric-welded. The first is a hand process. Mild open-hearth steel is used for the most part. Wrought iron bar is used for some large sizes. The bending and welding reduce the strength so that the chain is not twice but only from 1.55 to 1.70 times as strong as the original bar from which it was made. STUD CHAIN having a bar welded across each link to stiffen it and prevent fouling in handling, is not as strong as OPEN-LINK CHAIN, but has a higher elastic limit and working strength. G. A. Goodenough, in a Bulletin from the Illinois Engineering Experiment-Station, finds the maximum stresses at the elastic limit of the material to be as follows: If P is the load, d the diameter of the bar, and S the stress, the formulas are.

$$P = 0.5\, d^2 S \text{ for stud-link, and}$$

$$P = 0.4\, d^2 S \text{ for open link.}$$

Proof-Tests. A proof-test is applied to chains by the manufacturers The load applied is one-half the average BREAKING-LOAD. It serves to detect bad welds and gives a chain a slight permanent set, so that for working loads thereafter there will be little stretching of the chain.

Care of Chains. Chains in constant use require lubrication and frequent annealing. They harden in service and are liable to unexpected failure if not annealed. It is recommended that hoisting chains and sling chains be annealed at least once a year. If subject to shock or occasional overload, it is better to anneal twice during the year. (Bradlee & Company, Philadelphia, Pa.)

Table XIX. Sizes, Weights, Proof-Tests and Average Breaking-Loads for Chains

Bradlee and Company, Philadelphia

Size of chain	Dist. from cen. of one link to cen. of next	Weight per ft in lb, approximately	Outside width	D. B. G. Special Crane			Crane		
				Proof test, lb	Average breaking strain, lb	Ordinary safe load General use, lb	Proof test, lb	Average breaking strain, lb.	Ordinary safe load. General use, lb
1/4	25/32	3/4	15/16	1 932	3 864	1 288	1 680	3 360	1 120
5/16	27/32	1	1 1/8	2 898	5 796	1 932	2 520	5 040	1 680
3/8	31/32	1 1/2	1 5/16	4 186	8 372	2 790	3 640	7 280	2 427
7/16	1 5/32	2	1 1/2	5 796	11 592	3 864	5 040	10 080	3 360
1/2	1 11/32	2 1/2	1 13/16	7 728	15 456	5 152	6 720	13 440	4 480
9/16	1 15/32	3 1/10	2	9 660	19 320	6 440	8 400	16 800	5 600
5/8	1 23/32	4 1/10	2 3/16	11 914	23 828	7 942	10 360	20 720	6 907
11/16	1 13/16	5	2 3/8	14 490	28 980	9 660	12 600	25 200	8 400
3/4	1 15/16	6 2/10	2 9/16	17 388	34 776	11 592	15 120	30 240	10 080
13/16	2 1/16	6 7/10	2 3/4	20 286	40 572	13 524	17 640	35 280	11 760
7/8	2 3/16	8 3/8	2 15/16	22 484	44 968	14 989	20 440	40 880	13 627
15/16	2 7/16	9	3 1/16	25 872	51 744	17 248	23 520	47 040	15 680
1	2 1/2	10 1/2	3 3/8	29 568	59 136	19 712	26 880	53 760	17 920
1 1/16	2 5/8	12	3 9/16	33 264	66 538	22 176	30 240	60 480	20 160
1 1/8	2 3/4	13 5/8	3 13/16	37 576	75 152	25 050	34 160	68 320	22 773
1 3/16	3 1/16	13 7/10	4	41 888	83 776	27 925	38 080	76 160	25 387
1 1/4	3 1/8	16	4 3/16	46 200	92 400	30 800	42 000	84 000	28 000
1 5/16	3 3/8	16 1/2	4 3/8	50 512	101 024	33 674	45 920	91 840	30 613
1 3/8	3 9/16	19 1/4	4 9/16	55 748	111 496	37 165	50 680	101 360	33 787
1 7/16	3 11/16	19 7/10	4 3/4	60 368	120 736	40 245	54 880	109 760	36 587
1 1/2	3 7/8	23	5 1/8	66 528	133 056	44 352	60 480	120 960	40 320
1 9/16	4	25	5 5/16	70 762	141 524	47 174	65 520	131 140	43 180
1 5/8	4 1/4	28	5 1/2	74 382	148 764	49 588	..	.....	.
1 11/16	4 1/2	30	5 11/16	78 733	157 466	52 488	....	..	..
1 3/4	4 3/4	31	5 7/8	82 320	164 640	54 880	...	.... .	. .
1 13/16	5	33	6 1/16	88 256	176 512	55 504		. . .	.
1 7/8	5 1/4	35	6 3/8	94 360	188 720	62 906	.	.	
1 15/16	5 1/2	38	6 9/16	100 800	201 600	67 200		.	
2	5 3/4	40	6 3/4	107 520	215 040	71 680	..		
2 1/16	6	43	6 15/16	114 240	228 480	76 160	. .	. .	.
2 1/8	6 1/4	46 1/2	7 1/8	121 240	242 480	80 823	.	. . .	
2 3/16	6 1/2	49 1/2	7 5/16	128 576	257 152	85 750	....	....	...
2 1/4	6 3/4	52 3/4	7 5/8	136 080	272 160	90 720	....	.. ..	..
2 3/8	6 7/8	58 1/4	8	151 580	303 160	101 053	.	..	
2 1/2	7	64 1/2	8 3/8	168 000	336 000	112 000	....	. .	..
2 5/8	7 1/8	70	8 3/4	180 544	361 088	120 362	. .	. ..	...
2 3/4	7 1/4	73	9 1/8	193 088	386 176	128 725	....	...	....
2 7/8	7 1/2	76	9 1/2	205 408	410 816	136 938	....	. .	....
3	7 3/4	86	9 7/8	217 728	435 456	145 152	.		. .

The specifications of the United States Navy Department require the same proof-test as is given above for crane-chain and a breaking-strength 10% greater than that given for special crane-chain.

Factors of Safety. For dead loads the FACTOR OF SAFETY may be as low as four provided the breaking of the chain would not imperil life. This is the factor generally quoted in catalogues, but is too low for most purposes as the MAXIMUM FIBER-STRESS is then well above the ELASTIC LIMIT of chain-iron. Where loads are to be raised repeatedly with machinery which can be operated without jerks or sudden change of speed, the use of a factor of six is good practice. If a chain must be used where shocks occur, the INSTANTANEOUS LOAD should be calculated, and a high factor of safety employed.

Grades of Chain. Chains up to 1¼ in are usually made in three grades, called PROOF, BB, and BBB. The proof is the cheapest grade, and is made in longer links than the others. This is not ordinarily proof-tested, BB is the next grade, somewhat shorter linked, and is proof-tested. It is made of high-grade open-hearth steel. BBB is of still shorter link and more carefully made. Only the highest-grade open-hearth chain steel is used in its manufacture.

Crane Chain is finished in such a way as to be without twist when hanging with one end free, so that hooks and fittings are always facing their proper direction.

Dredge Chain is straightened as is Crane Chain, and made with uniform links to run over a wheel. It is of BBB grade.

Steel Loading Chain is made mostly in small sizes for use where the weight compared to the strength is to be a maximum. It is the highest-grade hand-made chain. Some companies catalogue an electric-welded loading chain, made on individual welding-machines, not automatic.

Block Chain is fitted to the pocket-wheel in which it is to run. In small sizes it is usually electric-welded.

Electric-Welded Chain is made in small sizes and is rapidly replacing the hand-made below ½ in. It is stronger and more uniform.

Sizes of Chain. Chain is ordinarily made of wire or rod, 1/32 in larger than the NOMINAL DIAMETER, by which it is called If chain is desired made of wire of the size by which it is called, it must be specified as EXACT SIZE. Steel Loading Chain, Block Chain, and frequently Dredge Chain, are made EXACT. Stud-Link Anchor Chain is made of wire, 3/64 in above its NOMINAL DIAMETER.

CHAPTER XII

RESISTANCE TO SHEAR. RIVETED JOINTS. PINS AND BOLTS

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1. Shear

Shear is the internal stress in a body which resists the tendency of two adjacent parts to slide on each other, due to the action of two equal and parallel external forces, called SHEARING-FORCES, acting on opposite sides of the plane of shear.

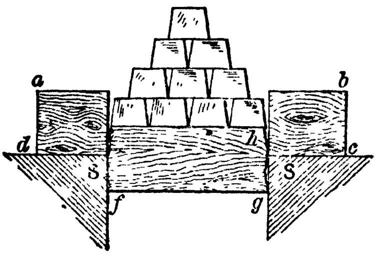


Fig. 1. Shearing-failure of Beam

If the piece *abcd* of Fig. 1 represents a short simple beam of brittle material on which a sufficient load is applied, it will fail in VERTICAL SHEAR at *f* and *g*, as shown, by a sliding on the sections of the beam at these points, because the upward force of the reaction at *S* and the downward force of the load adjacent to *S*, against which it acts across the section at *S*, is greater than the total SHEARING RESISTANCE of the section. Shear is present over the entire length of the beam, and at any section is equal to the reaction at *S* minus the weight of the load between the reaction and the section in question. In general, the VERTICAL SHEAR at any section of a beam subjected to vertical loads is equal to the algebraic sum of all the vertical forces acting on the beam on either side of the section.

Single and Double Shear. A rivet connecting two bars under tension (Fig. 2) is subjected to a SHEARING-STRESS. If one section of the rivet transmits the force the rivet is said to be in SINGLE SHEAR; if two sections, it is in DOUBLE SHEAR.

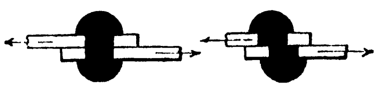


Fig. 2. Example of Single Shear

Distribution of Shear. Shear is considered to be UNIFORMLY DISTRIBUTED over the section except in cases of torsion and of complex stresses.

For the ordinary cases of shear in rivets, etc., if

S_s = the allowable unit stress in shear,
A = the area under stress,
and P = the safe shearing-load;

then
$$P = AS_s \tag{1}$$

The Ultimate Strength in Shear has been determined for building materials by testing suitably prepared specimens and dividing the maximum load

observed by the area under stress. For material like wood, in which there are planes of weakness, tests must be made which take these into account. The direction of the force with respect to these planes must be considered in choosing the SAFE WORKING STRESS from the tables.

Safe Working Stresses in Shear. Table I gives SAFE WORKING STRESSES in SHEAR for those building materials usually subjected to such stresses.

Table I. Safe Working Stress in Shear for Building Materials *

Material	Safe stress in lb per sq in.	
Cast iron (New York)	3 000	
Wrought iron	7 500	
Steel, bolts, rivets	10 000 (average)	
	With the grain	Across the grain
White oak	200	1 000
White pine	100	500
Long-leaf yellow pine	150	1 250
Short-leaf yellow pine	130	1 000
Douglas fir	100	900
Hemlock	100	600
Spruce	100	750

* NOTE For woods, these values may be increased up to 30% for selected, perfectly protected, commercially dry timber, not subject to impact, that is, for ideal conditions. (See, also, Chapter XX.)

Shear in Wooden Tie-Beams. There are a few cases in architectural construction in which the weakness of wood in shear must be provided for. The one most frequently arising is the framing of the end of the tie-beams in wooden trusses.

Fig. 3 was made from a photograph of a SHEARING-FAILURE of a tie-beam from the thrust of the rafter.

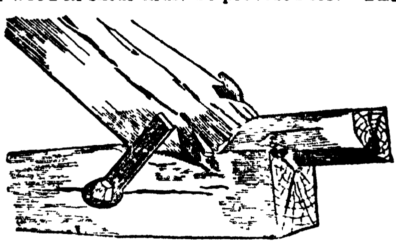


Fig. 3 Shearing-failure in Wood

Horizontal Shear in Wooden Beams. Failure like that shown in Fig. 1 rarely occurs in wood; but rectangular wooden beams, the length of which is less than about twenty times the depth, are liable to fail by HORIZONTAL SHEAR along the middle, under about the same loads that cause the allowable working stresses in bending.

Shear at the End of a Tie-Beam. In the case of the truss-joint (Fig. 4), the thrust S of the rafter tends to shear off the part $ABCD$ along the plane of which CD is the trace. This area under stress must offer a SHEARING RESISTANCE equal to the horizontal component H of the thrust S. The width of the beam b being fixed, formula (1) gives

$$H = (CD \times b)\, S_s \quad \text{or} \quad CD = H/bS_s$$

The shear being in the same direction as the grain of the wood, the lower value in Table I must be used.

Example 1 (Fig. 4). The horizontal component of the thrust of a rafter is 20 000 lb. The long-leaf yellow pine tie-beam is 10 in wide. How far should the beam extend beyond the point D?

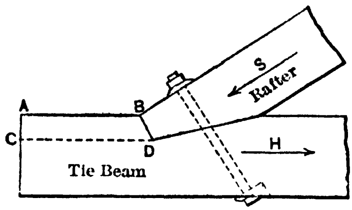


Fig. 4. Truss-joint

Solution. In this case H = 20 000 lb. From Table I, $S_s = 150$ lb per sq in. Then $CD = \frac{20\,000}{10 \times 150} = 13.3$ in and should be made at least $13\frac{1}{2}$ in.

As actually constructed a large part of the thrust is generally taken up by a bolt or strap at the foot of the rafter to hold it in place. As the bolt and shoulder seldom act together, either the length CD on the tie-beam should be made long enough to resist the entire thrust, or the bolt or strap designed to do so without relying on the shearing resistance in the plane of CD. The design of such joints is more fully considered under Subdivision 4 of this chapter.

2. Riveted Joints

Use of Rivets. Rivets almost exclusively are used in connecting the plates and shapes which make up the members of framed steel construction.

Rivet-Definitions. A rivet is a piece of cylindrical rod with a HEAD forged on one end and usually with a slight taper at the other end of the SHANK. The grip (Table III) of the rivet is the length between the under sides of the heads after driving, or the thickness of the parts joined. The LENGTH (Table III) of the rivet is equal to the grip plus enough of the stock to form a head, and is measured from the end of the shank to the under surface of the head. The DIAMETER OF THE SHANK of a rivet is made equal to its NOMINAL DIAMETER, but rivets are driven into holes $\frac{1}{8}$ in larger in diameter and upset by the driving so as to completely fill the holes. The shearing values and bearing values are based upon the NOMINAL AREA and not upon the area of the hole.

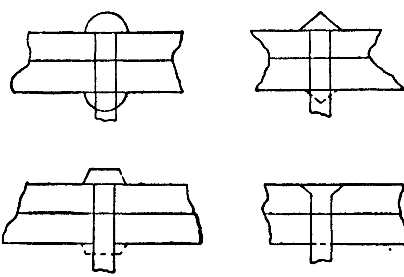


Fig. 5. Forms of Rivet-heads

Riveting consists in heating the rivet to a welding-heat, passing it through holes in the parts to be joined and forging another head out of the projecting shank. This may be done by hand-hammering; but shops use compressed-air-operated hand-hammers or large riveting-machines which form the head and cause the shank to completely fill the hole by heavy pressure on a die.

Material of Rivets. Rivets are made of soft steel and of wrought iron. Rivet-steel is generally used. The head may have any of the forms shown in Fig. 5, although the first, called the BUTTON-HEAD, is the standard for struc-

tural work. The fourth or COUNTERSUNK HEAD is used where it is necessary to have a flat surface, as over a bearing-plate.

The Sizes of Rivet-Heads differ slightly at different mills. The Standards of the American Bridge Company give for the DIAMETER OF THE HEAD, one and one-half times the diameter of the shank plus ⅛ in, and for the HEIGHT of the head, 0.425 times the diameter of the head. Countersunk heads have a SLOPE of 30° and a DEPTH equal to one-half the diameter of the shank.

The Pitch of Rivets. By this is meant the center-to-center distance between them in a line of riveting. The distance between lines of rivets, or from the back of an angle or channel to a rivet-line is called the GAUGE-DISTANCE. By STAGGERED PITCH is meant the arrangement of rivets midway between others on successive rivet-lines in order to decrease the section less than when they are arranged in rectangular rows, and at the same time to place a greater number of rivets in a definite area. The PITCH should not be made less than three diameters of the rivet and the DISTANCE FROM THE EDGE of the plate not less than one and one-half diameters, although it may be necessary to make the distance less when small angles are used. The pitch of countersunk rivets must be greater than that of button-head rivets because of the greater amount of material removed.

Punching Rivet-Holes. Rivet-holes are made with power-punches. The SPACING is marked on the different parts to be fastened together by means of wooden templates with holes drilled to locate the position of the rivets. When the different parts are assembled, the holes are laid out by the same TEMPLATE-REGISTER, so that the rivets may be inserted without difficulty. PUNCHING makes a ragged hole. The flow of the metal under the great pressure hardens it and causes a loss in strength of from 11 to 33% as reported by W. C. Unwin for soft steel. The injury may be removed by ANNEALING or by REAMING away the injured part of the metal. Enlarging a ⅞-in hole by reaming to 1⅛ in has been found to remove all the injurious effects of punching. One method practiced in the best work is to punch the holes 1/16 in less in diameter than the diameter of the rivets, and to ream them to a diameter 1/16 in greater, after the parts are assembled and bolted together. This removes the greater part of the injury from punching and corrects the alinement of the holes. (See Table XI, Chapter XI.)

Drift-Pins. When the alinement of a hole is such as to prevent the insertion of the rivet, it is the practice in some shops to drive in a tapered DRIFT-PIN and distort the holes in some of the plates sufficiently to set the rivet. This causes LOCAL STRESSES and injury to the plates and should not be permitted.

Shop-Riveting is done with powerful air or hydraulic riveting-machines which may exert a pressure of from 30 to 50 tons, sufficient to upset a perfect head on the projecting end of the shank and to completely fill the hole even though the alinement is imperfect. Contraction on cooling causes great pressure between the parts, so that it is probable that in good work the rivet is under little or no shearing-stress, the force being transmitted through the frictional resistance of the plates.

Clearance. It is important that the designer place the rivets so they may be inserted from one side and pounded on the other for HAND-WORK, or so that the machine may reach them for MACHINE-RIVETED WORK. For example, the minimum distance from the inside face of the leg of one angle to a line of rivets in the other leg must not be less than 1⅛ in for ⅞-in rivets, 1 in for ¾-in rivets, etc. In general, a distance ⅜ in greater than the diameter of the head should be allowed for CLEARANCE.

Inspection. The common imperfections in riveting are LOOSE RIVETS and ECCENTRIC HEADS. Loose rivets may be detected by holding the hand against one side of the rivet-head and tapping the other side with a light hammer. If loose, a slight slip may be felt. The loose rivets should be marked to be cut out and replaced. The inspector should also carefully check open holes left for field-connections, and see that flattened and countersunk rivets are as called for, because such work may be done at less expense in the shop than in the field, where it may cause delay.

The Failure of Riveted Joints may occur

(1) IN TENSION, by the tearing of the plate through the line of rivets (Fig. 8).

(2) IN SHEAR, by the cutting of the rivets (Fig. 7).

(3) IN BEARING, by the crushing of the plate in front of the rivets, the splitting of the plate, or, in some cases, by the shearing out of the sections in front of the rivets. In a careful design of a joint the strength against failure by each of these methods must be investigated (Fig. 6 and Fig. 9).

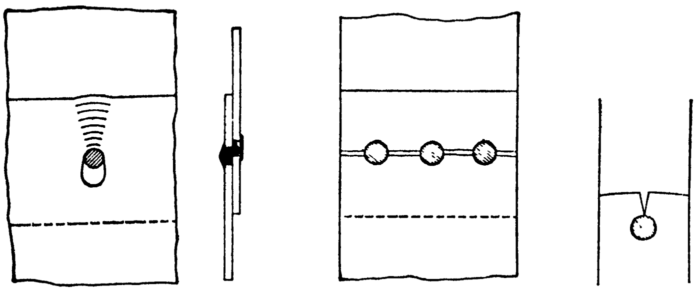


Fig. 6 Fig. 7 Fig 8 Fig. 9

Figs. 6 to 9. Methods of Failure in Riveted Joints

The Steps in the Design of any type of riveted joint are, (1) the selection of the size of the rivet to be used, (2) the determination of its shearing and bearing strength and the use of the smaller value of the two to divide into the total load to be transmitted and thus determine the number of rivets, (3) the arrangement of the rivets in the plate and the investigation of its strength in tension at the dangerous section.

The Size of Rivets is determined in part by SHOP-PRACTICE. Holes cannot be punched in plates which are thicker than the diameter of the punch. The following table gives the size of rivets used with plates of different thickness. Some specifications for structural work require all rivets to be ¾ in, except where thick plates require larger ones.

Thickness of plates	Size of rivets
¼ to 7/16 in	⅝ in
½ to ⅝ in	¾ in
11/16 to 13/16 in	⅞ in
⅞ to 1 in	1 in

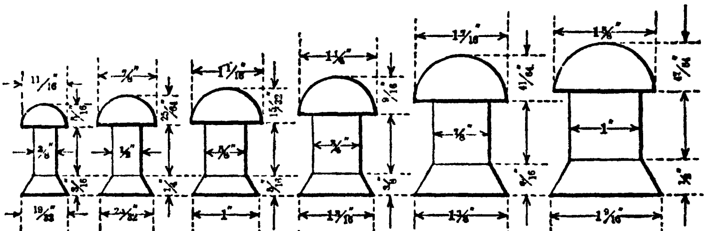
Table II gives the SHEARING and BEARING VALUES for different sizes of rivets in plates of different thickness for six values of working stresses each. The lower stresses should be used in parts subject to live loads, and the higher stresses in parts subject to constant or dead loads only. The SHEAR-

ING VALUE is equal to the area of the rivet multiplied by the working stress; the BEARING VALUE is equal to the area of the projected surface under pressure multiplied by the working stress in bearing, or, if

t = the thickness of the plate;
d = the diameter of the rivet;
and S_b = the working stress in bearing;
then the bearing value $P = dtS_b$ (2)

The Shearing and Bearing Values may be taken directly from the tables. Care must be taken to follow the footnotes.

Rivet-Proportions.* The following diagrams show various rivet-proportions, including the dimensions of shanks and of finished and countersunk heads.



FINISHED HEADS

Diam head = 1½ diam of shank + ⅛" depth of head = 45/100 diam of head

COUNTERSUNK HEADS

Depth of head = ½ diam of shank. Bevel of head = 60°

Conventional Signs for Riveting. The following diagrams show some conventional signs for riveting:

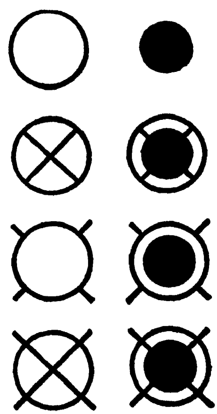
Shop — Field

Two Full Heads

Countersunk Inside (Farside) and Chipped

Countersunk Outside (Nearside) and Chipped

Countersunk both Sides and Chipped



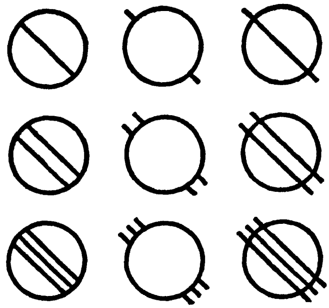
* These proportions vary slightly at different mills and in different handbooks. American Bridge Company and Carnegie standards permit a depth of the head 1/64 less for the first three, 1/32 less for the next two, and 3/64 for the 1-in rivet.

INSIDE (FARSIDE) | OUTSIDE (NEARSIDE) | BOTH SIDES

Flattened to 1/8 in high or Countersunk and not Chipped

Flattened to 1/4 in high

Flattened 3/8 in high



This system, designed by F C. Osborn, has for its foundation a diagonal cross to represent a countersink, a blackened circle for a field-rivet and a diagonal stroke for a flattened head. The position of the cross with respect to the circle, inside, outside, or on both sides, indicates the location of the countersink; and similarly, the number and position of the diagonal strokes indicate the height and position of the flattened heads. Any combination of field, countersunk and flattened-head rivets liable to be used may be readily indicated by the proper combination of the above signs.

Table II. Rivets *

Shearing and Bearing Values

Values in Pounds, all Dimensions in Inches

3/8-Inch Rivets—Area .1104 Square Inch

Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
Shear	Single Shear per Rivet	770	880	990	1 100	1 210	1 320
Shear	Double Shear per Rivet	1 540	1 760	1 980	2 200	2 420	2 640
Bearing	Unit, Lb per Sq In	14 000	16 000	18 000	20 000	22 000	24 000
Bearing — Thickness in Inches	1/8	660	750	840	940	1 030	1 130
	3/16	980	1 130	1 270	1 410	1 550	1 690
	1/4	1 310	1 500	1 690	1 880	2 060	2 250
	....	....	....	....	....	....	....
	5/16	1 640	1 880	2 110	2 340	2 580	2 810
	3/8	1 910	2 250	2 530	2 810	3 090	3 380

1/2-Inch Rivets—Area .1963 Square Inch

Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
Shear	Single Shear per Rivet	1 370	1 570	1 770	1 960	2 160	2 360
Shear	Double Shear per Rivet	2 750	3 140	3 530	3 930	4 320	4 710
Bearing	Unit, Lb per Sq In ..	14 000	16 000	18 000	20 000	22 000	24 000
Bearing — Thickness in Inches	3/16	1 310	1 500	1 690	1 880	2 060	2 250
	1/4	1 750	2 000	2 250	2 500	2 750	3 000
	5/16	2 190	2 500	2 810	3 130	3 440	3 750
	3/8	2 630	3 000	3 380	3 750	4 130	4 500
	....	....	....	....	....	....	....
	7/16	3 060	3 500	3 940	4 380	4 810	5 250
	1/2	3 500	4 000	4 500	5 000	5 500	6 000

5/8-Inch Rivets—Area .3068 Square Inch

Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
Shear	Single Shear per Rivet	2 150	2 450	2 760	3 070	3 370	3 680
Shear	Double Shear per Rivet	4 300	4 910	5 520	6 140	6 750	7 360
Bearing	Unit, Lb per Sq In .	14 000	16 000	18 000	20 000	22 000	24 000
Bearing — Thickness in Inches	3/16	1 640	1 880	2 110	2 340	2 580	2 810
	1/4	2 190	2 500	2 810	3 130	3 440	3 750
	5/16	2 730	3 130	3 520	3 910	4 300	4 690
	3/8	3 280	3 750	4 220	4 690	5 160	5 630
	7/16	3 830	4 380	4 920	5 470	6 020	6 560
	....	....	....	....	....	....	....
	1/2	4 380	5 000	5 630	6 250	6 880	7 500
	9/16	4 920	5 630	6 330	7 030	7 730	8 440
	5/8	5 470	6 250	7 040	7 810	8 590	9 380

Values below dotted lines are greater than double shear.

* From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.

Table II (Continued). Rivets *

Shearing and Bearing Values

Values in Pounds, all Dimensions in Inches

¾-Inch Rivets—Area .4418 Square Inch

Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
	Single Shear per Rivet	3 090	3 530	3 980	4 420	4 860	5 300
	Double Shear per Rivet	6 190	7 070	7 950	8 840	9 720	10 600
Bearing	Unit, Lb per Sq In	14 000	16 000	18 000	20 000	22 000	24 000
Thickness in Inches	¼	2 630	3 000	3 380	3 750	4 130	4 500
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	5/16	3 280	3 750	4 220	4 690	5 160	5 630
	⅜	3 940	4 500	5 060	5 630	6 190	6 750
	7/16	4 590	5 250	5 910	6 560	7 220	7 880
	½	5 250	6 000	6 750	7 500	8 250	9 000
	9/16	5 910	6 750	7 590	8 440	9 280	10 130
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	⅝	6 560	7 500	8 440	9 380	10 310	11 250

⅞-Inch Rivets—Area .6013 Square Inch

Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
	Single Shear per Rivet	4 210	4 810	5 410	6 010	6 610	7 220
	Double Shear per Rivet	8 420	9 620	10 820	12 030	13 230	14 430
Bearing	Unit, Lb per Sq In	14 000	16 000	18 000	20 000	22 000	24 000
Thickness in Inches	¼	3 060	3 500	3 940	4 380	4 810	5 250
	5/16	3 830	4 380	4 920	5 470	6 020	6 560
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	⅜	4 590	5 250	5 910	6 560	7 220	7 880
	7/16	5 360	6 130	6 890	7 660	8 420	9 190
	½	6 130	7 000	7 880	8 750	9 630	10 500
	9/16	6 890	7 880	8 860	9 840	10 830	11 810
	⅝	7 660	8 750	9 840	10 940	12 030	13 130
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	11/16	8 420	9 630	10 830	12 030	13 230	14 430

1-Inch Rivets—Area .7854 Square Inch

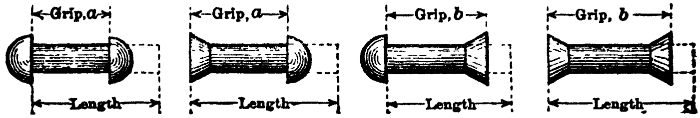
Shear	Unit, Lb per Sq In	7 000	8 000	9 000	10 000	11 000	12 000
	Single Shear per Rivet	5 500	6 280	7 070	7 850	8 640	9 420
	Double Shear per Rivet	11 000	12 570	14 140	15 710	17 280	18 850
Bearing	Unit, Lb per Sq In	14 000	16 000	18 000	20 000	22 000	24 000
Thickness in Inches	¼	3 500	4 000	4 500	5 000	5 500	6 000
	5/16	4 380	5 000	5 630	6 250	6 880	7 500
	⅜	5 250	6 000	6 750	7 500	8 250	9 000
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	7/16	6 130	7 000	7 880	8 750	9 630	10 500
	½	7 000	8000	9 000	10 000	11 000	12 000
	9/16	7 880	9 000	10 130	11 250	12 380	13 500
	⅝	8 750	10 000	11 250	12 500	13 750	15 000
	11/16	9 630	11 000	12 380	13 750	15 130	16 500
	¾	10 500	12 000	13 500	15 000	16 500	18 000
	· · · · · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
	13/16	11 380	13 000	14 630	16 250	17 880	19 500

Values above upper dotted lines are less than single shear.
Values below lower dotted lines are greater than double shear.

* **From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.**

Table III. Length of Field-Rivets for Various Grips. Length of Rivet-Shank to Form Head

American Bridge Company Standard. Dimensions in Inches



Grip a	Diameter 1/2	5/8	3/4	7/8	1	Grip b	Diameter 1/2	5/8	3/4	7/8	1
1/2	1 1/2	1 3/4	1 7/8	2	2 1/8	1/2	1 1/8	1 1/4	1 1/4	1 3/8	1 3/8
5/8	1 5/8	1 7/8	2	2 1/8	2 1/4	5/8	1 1/4	1 3/8	1 3/8	1 1/2	1 1/2
3/4	1 3/4	2	2 1/8	2 1/4	2 3/8	3/4	1 3/8	1 1/2	1 1/2	1 5/8	1 5/8
7/8	1 7/8	2 1/8	2 1/4	2 3/8	2 1/2	7/8	1 1/2	1 5/8	1 5/8	1 3/4	1 3/4
1	2	2 1/4	2 3/8	2 1/2	2 5/8	1	1 5/8	1 3/4	1 3/4	1 7/8	1 7/8
1/8	2 1/8	2 3/8	2 1/2	2 5/8	2 3/4	1/8	1 3/4	1 7/8	1 7/8	2	2
1/4	2 1/4	2 1/2	2 5/8	2 3/4	2 7/8	1/4	1 7/8	2	2	2 1/8	2 1/8
3/8	2 3/8	2 5/8	2 3/4	2 7/8	3	3/8	2	2 1/8	2 1/8	2 1/4	2 1/4
1/2	2 1/2	2 7/8	3	3 1/8	3 1/4	1/2	2 1/8	2 1/4	2 3/8	2 3/8	2 1/2
5/8	2 3/4	3	3 1/8	3 1/4	3 3/8	5/8	2 1/4	2 3/8	2 1/2	2 1/2	2 5/8
3/4	3	3 1/4	3 3/8	3 1/2	3 5/8	3/4	2 1/2	2 5/8	2 3/4	2 3/4	2 7/8
7/8	3 1/8	3 3/8	3 1/2	3 5/8	3 3/4	7/8	2 5/8	2 3/4	2 7/8	2 7/8	3
2	3 1/4	3 1/2	3 5/8	3 3/4	3 7/8	2	2 3/4	2 7/8	3	3	3 1/8
1/8	3 3/8	3 5/8	3 3/4	3 7/8	4	1/8	2 7/8	3	3 1/8	3 1/8	3 1/4
1/4	3 1/2	3 3/4	3 7/8	4	4 1/8	1/4	3	3 1/8	3 1/4	3 1/4	3 3/8
3/8	3 5/8	3 7/8	4	4 1/8	4 1/4	3/8	3 1/8	3 1/4	3 3/8	3 3/8	3 1/2
1/2	3 3/4	4	4 1/8	4 1/4	4 3/8	1/2	3 1/4	3 3/8	3 1/2	3 1/2	3 5/8
5/8	3 7/8	4 1/8	4 1/4	4 3/8	4 1/2	5/8	3 3/8	3 1/2	3 5/8	3 5/8	3 3/4
3/4	4	4 1/4	4 3/8	4 1/2	4 5/8	3/4	3 1/2	3 5/8	3 3/4	3 3/4	3 7/8
7/8	4 1/8	4 3/8	4 1/2	4 5/8	4 3/4	7/8	3 5/8	3 3/4	3 7/8	3 7/8	4
3	4 3/8	4 5/8	4 3/4	4 7/8	5	3	3 7/8	4	4	4 1/8	4 1/4
1/8	4 1/2	4 3/4	4 7/8	5	5 1/8	1/8	4	4 1/8	4 1/8	4 1/4	4 3/8
1/4	4 5/8	4 7/8	5	5 1/8	5 1/4	1/4	4 1/8	4 1/4	4 1/4	4 3/8	4 1/2
3/8	4 3/4	5	5 1/8	5 1/4	5 3/8	3/8	4 1/4	4 3/8	4 3/8	4 1/2	4 5/8
1/2	4 7/8	5 1/8	5 1/4	5 3/8	5 1/2	1/2	4 3/8	4 1/2	4 1/2	4 5/8	4 3/4
5/8	5	5 1/4	5 3/8	5 1/2	5 5/8	5/8	4 1/2	4 5/8	4 5/8	4 3/4	4 7/8
3/4	5 1/8	5 3/8	5 1/2	5 5/8	5 3/4	3/4	4 5/8	4 3/4	4 3/4	4 7/8	5
7/8	5 1/4	5 1/2	5 5/8	5 3/4	5 7/8	7/8	4 3/4	4 7/8	4 7/8	5	5 1/8
4	5 3/8	5 5/8	5 3/4	5 7/8	6	4	4 7/8	5	5	5 1/8	5 1/4
1/8	5 5/8	5 7/8	6	6 1/8	6 1/4	1/8	5 1/8	5 1/4	5 1/4	5 3/8	5 3/8
1/4	5 3/4	6	6 1/8	6 1/4	6 3/8	1/4	5 1/4	5 3/8	5 3/8	5 1/2	5 1/2
3/8	6	6 1/4	6 3/8	6 1/2	6 5/8	3/8	5 1/2	5 5/8	5 5/8	5 5/8	5 3/4
1/2	6 1/8	6 3/8	6 1/2	6 5/8	6 3/4	1/2	5 5/8	5 3/4	5 3/4	5 3/4	5 7/8
5/8	6 1/4	6 1/2	6 5/8	6 3/4	6 7/8	5/8	5 3/4	5 7/8	5 7/8	5 7/8	6
3/4	6 3/8	6 5/8	6 3/4	6 7/8	7	3/4	5 7/8	6	6	6	6 1/8
7/8	6 1/2	6 3/4	6 7/8	7	7 1/8	7/8	6	6 1/8	6 1/8	6 1/8	6 1/4
5	6 5/8	6 7/8	7	7 1/8	7 1/4	5	6 1/8	6 1/4	6 1/4	6 1/4	6 3/8
1/8	...	...	7 1/8	7 1/4	7 3/8	1/8	..	..	6 3/8	6 3/8	6 1/2
1/4	...	...	7 1/4	7 3/8	7 1/2	1/4	.	.	6 1/2	6 1/2	6 5/8
3/8	...	...	7 3/8	7 1/2	7 5/8	3/8	..	..	6 5/8	6 5/8	6 3/4
1/2	...	...	7 5/8	7 3/4	7 7/8	1/2	.	..	6 7/8	6 7/8	7
5/8	...	..	7 3/4	7 7/8	8	5/8	..	..	7	7	7 1/8
3/4	...	...	7 7/8	8	8 1/8	3/4	..	..	7 1/8	7 1/8	7 1/4
7/8	...	.	8	8 1/8	8 1/4	7/8	..	.	7 1/4	7 1/4	7 3/8

Use of Riveted Joints. Riveted joints are used in building-construction: (1) in tie-bar splices, (2) in floor-beam connections, (3) in the joints of trusses, (4) in riveted girders, and (5) in column-connections.

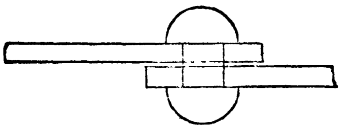


Fig. 10. Lap-joint

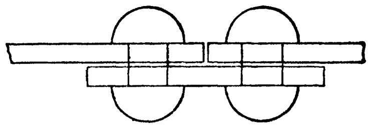


Fig. 11. Butt-joint with Single Cover-plate

Splicing of Tie-Bars. Tie-bars may be spliced by a LAP-JOINT (Fig. 10); by a BUTT-JOINT with a single cover-plate (Fig 11); or by a BUTT-JOINT with two cover-plates (Fig. 12).

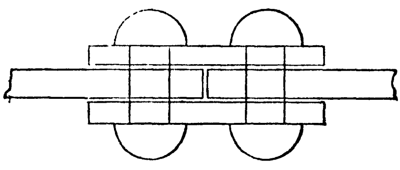


Fig. 12. Butt-joint with Two Cover-plates

The Butt-Joint is symmetrical and more efficient than the others because of the absence of any tendency to bend when under a load. The net area of the cover-plates at the section through the rivets at the end of the main plate must be equal to the net area of the main plate through the rivets at the end of the cover-plate. Fig. 14 shows a better arrangement of rivets than that in Fig. 13, because less area is removed at the critical section of the cover-plates. In some cases it may be necessary to make the aggregate thickness of the cover-plates greater than the thickness of the main plates.

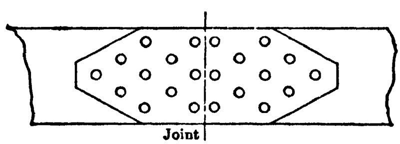


Fig 13. Cover-plate. Six Rivets at Critical Section

A joint with one line of rivets is said to be SINGLE-RIVETED, one with two lines DOUBLE-RIVETED, and one with more than two lines, CHAIN-RIVETED.

Example 2. It is required to determine the number of rivets in the splice of a 12 by ½-in tie-bar which is subject to a tensile force of 65 000 lb, allowing 10 000 lb per sq in in shear and 18 000 lb per sq in in bearing.

Solution. Assume the load to be constant and a lap-joint like that in Fig. 15; also, assume ¾-in rivets. The value in shear of one rivet is found to be 4 420 lb in Table II, and the bearing value against a ½-in plate 6 750 lb. The number of rivets is determined by the shear to be equal to 65 000 divided by 4 420, or fifteen. Since sixteen rivets are required to complete a figure smaller but similar in arrangement to that shown in Fig. 15, this number is used. There is some latitude possible in the spacing of the rivets, but with a width of 12 in, the horizontal gauge-lines are placed 1½ in apart for symmetry. If the pitch P, as shown in Fig. 15, is required to be three times the diameter

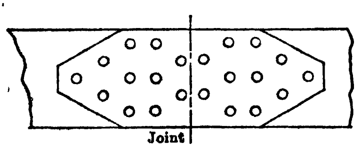


Fig. 14. Cover-plate Four Rivets at Critical Section

of the rivet, this diagonal pitch across the rivet-spacing must be 2.25 in, or greater. The length of the horizontal or third side of the right-angled triangle, having an hypotenuse of 2.25 in and a vertical altitude of 1.5 in, is 1 68 in, which requires that this distance *ED*, etc., be 1.75 in, if measured in multiples of $\frac{1}{4}$ in.

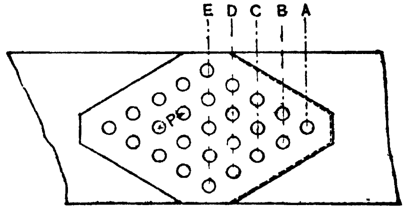


Fig. 15. Rivet-spacing in Cover-plate

Floor-Beam Connections. The two following examples illustrate common types of floor-beam connections.

Example 3. It is required to determine the number of $\frac{3}{4}$-in rivets to connect a 10-in 25.4-lb I beam supporting 24 000 lb to a 15-in 42.9-lb I beam, using a shearing-stress of 10 000 lb per sq in and a bearing-stress of 18 000 lb per sq in.

Solution. From the table of properties of standard I beams, the thickness of the web of the 10-in 25 4-lb beam is found to be 0.31 in, say $\frac{5}{16}$ in, and of the 15-in 42-lb beam, 0.41 in, say $\frac{7}{16}$ in. Referring to Table II, the bearing values for a $\frac{3}{4}$-in rivet for these thicknesses of webs are respectively 4 220 lb and 5 910 lb. The shearing value of the rivet is 4 420 lb. The rivets in the 10-in beam are in double shear; hence the bearing value governs. The number of rivets, then, is 12 000, the end-reaction divided by 4 220, or 3. For the 15-in beam the shearing value is less, and the number of rivets required is 12 000 divided by 4 420, or 3 Hence two standard connection-angles, 6 by 4 by $\frac{3}{8}$ in and 5 in long, may be used Each has three holes in one leg and two in the other. The leg with three holes is placed on the 10-in beam with the rivets in double shear, and the leg with two holes is connected to the 15-in beam; thus, in the latter case there are four rivets where only three are required for strength. They are driven in the field during the erection of the structure and the working stress is accordingly made less in most specifications because of the better work possible with the heavy machines used in shop-work, than with the tools available in the field.

Example 4. It is required to determine the number of $\frac{3}{4}$-in rivets in a 4 by 4 by $\frac{1}{2}$-in angle-bracket attached to an 18-in 54.7-lb beam and supporting a 10 by 12-in wooden beam on which there is a load of 18 000 lb.

Solution. The rivets are in single shear with a shearing-resistance of 4 420 lb, taken from Table II The thickness of the web of the I beam is $\frac{7}{16}$ in, giving a bearing value of 5 910 lb Dividing 9 000 lb, the end-reaction, by 4 420 lb, the controlling value, we find that two rivets are insufficient. The bracket may be fastened with three $\frac{3}{4}$-in rivets with a spacing of 4 in. Two $\frac{7}{8}$-in rivets are sufficient to hold the bracket.

Rivets in Plate Girders. The methods of determining the rivets in plate and box girders are given in Chapter XIX.

Bending Stress in Rivets. While the BENDING STRENGTH OF PINS at the joints of articulated trusses is always investigated, this is never done in the case of RIVETS. A hot rivet properly driven is, when cold, under a tensile stress which is nearly equal to the elastic limit of the material. This causes great pressure between the plates and a consequent frictional resistance to movement, which, under the usual conditions, equals the allowed shearing-force on the rivet; and so, until an INITIAL SLIP occurs, there can be no BEND-

ING STRESSES in the rivet. In the case of very long rivets driven in holes where there is an imperfect alinement of the plates and a consequent difficulty in making the rivets fill the holes completely, it is not probable that any large bending stresses can occur in the rivets of a structure This has been avoided in a few structures for which long TAPER RIVETS were specified to be used in holes REAMED with TAPERED REAMERS, thus insuring a perfect filling of the holes.

Working Stresses. Table II includes the stresses used in good practice for various conditions. The American Railway Engineering Association specifications for Steel Railway Bridges give a shearing value of 12 000 lb per sq in on shop-driven rivets and 10 000 lb per sq in for field-driven rivets; they give a bearing value of 24 000 lb per sq in for shop-driven rivets and 20 000 lb per sq in for field-driven rivets. M. S. Ketchum's specifications for Steel Highway Bridges give the same values.

In the Philadelphia Building Code (1929) the shear allowed on power-driven rivets is 13 500 lb per sq in, and on hand-driven rivets 10 000 lb per sq in, while the bearing on power-driven rivets in single shear is 24 000 lb per sq in, and on hand-driven rivets 16 000 lb per sq in.

3. Strength of Pins in Trusses *

Truss-Pins. In the design of the PINS at the joints of trusses the stresses in SHEAR, BEARING, and FLEXURE or BENDING must be investigated.

The Shearing-Force at any section of the pin is the algebraic sum of all the forces acting on the pin on either side of the section. The stress is considered to be uniformly distributed over the cross-section of the pin. When the forces do not act in the same plane they must be resolved into vertical and horizontal components and the resultant of these components taken as the shear at any desired section. This may be done by the principles of GRAPHIC STATICS, or by TRIGONOMETRICAL and ALGEBRAICAL METHODS, the graphic method being, for some, the more rapid.

The Bearing Area on the pin is taken as the PROJECTION OF THE AREA OF CONTACT, the area of this projection being equal to the diameter of the pin multiplied by the thickness of the plate. The bearing is assumed to be uniformly distributed; hence for any load the intensity of the pressure may decreased by increasing the thickness of the plate or the diameter of the pin.

The Bending Moments on the pin may be found by the PRINCIPLE OF MOMENTS or by methods involving the principles of GRAPHIC STATICS explained in Chapter IX in finding the bending moments of beams. The forces are considered to be concentrated at the middle of the bearing-plates. If they do not lie in a plane with the pin they must be resolved into their vertical and horizontal components and these component forces in the two planes treated separately. The resultants in both planes at any section may be combined and a single resultant force acting on the section obtained, and also the consequent stresses due to it.

In the Method of Moments a section is taken at each force in succession and the moment of the forces about a point in the section found, due consideration being given to the direction of turning. This is done at each force on one side of the pin, if the bars are arranged symmetrically, and in both the

* Since the introduction of rolled-steel shapes and riveted joints, pin-joints for trusses of moderate span in buildings have fallen into disuse. The general principles of their design, however, are given here.

Table IV. Pins *

Bearing Values in Pounds on Metal One Inch Thick

Bearing Value = Diameter of Pin × Bearing Stress per Square Inch

Pin		Bearing Stresses in Pounds per Square Inch				
Diameter, Inches	Area, Sq In	12 000	15 000	20 000	22 000	24 000
1	.785	12 000	15 000	20 000	22 000	24 000
1¼	1 227	15 000	18 800	25 000	27 500	30 000
1½	1.767	18 000	22 500	30 000	33 000	36 000
1¾	2.405	21 000	26 300	35 000	38 500	42 000
2	3.142	24 000	30 000	40 000	44 000	48 000
2¼	3 976	27 000	33 800	45 000	49 500	54 000
2½	4.909	30 000	37 500	50 000	55 000	60 000
2¾	5.940	33 000	41 300	55 000	60 500	66 000
3	7.069	36 000	45 000	60 000	66 000	72 000
3¼	8 296	39 000	48 800	65 000	71 500	78 000
3½	9.621	42 000	52 500	70 000	77 000	84 000
3¾	11.045	45 000	56 300	75 000	82 500	90 000
4	12 566	48 000	60 000	80 000	88 000	96 000
4¼	14 186	51 000	63 800	85 000	93 500	102 000
4½	15.904	54 000	67 500	90 000	99 000	108 000
4¾	17.721	57 000	71 300	95 000	104 500	114 000
5	19 635	60 000	75 000	100 000	110 000	120 000
5¼	21.648	63 000	78 800	105 000	115 500	126 000
5½	23.758	66 000	82 500	110 000	121 000	132 000
5¾	25.967	69 000	86 300	115 000	126 500	138 000
6	28.274	72 000	90 000	120 000	132 000	144 000
6¼	30.680	75 000	93 800	125 000	137 500	150 000
6½	33.183	78 000	97 500	130 000	143 000	156 000
6¾	35.785	81 000	101 300	135 000	148 500	162 000
7	38.485	84 000	105 000	140 000	154 000	168 000
7¼	41.282	87 000	108 800	145 000	159 500	174 000
7½	44.179	90 000	112 500	150 000	165 000	180 000
7¾	47.173	93 000	116 300	155 000	170 500	186 000
8	50.265	96 000	120 000	160 000	176 000	192 000
8¼	53.456	99 000	123 800	165 000	181 500	198 000
8½	56.745	102 000	127 500	170 000	187 000	204 000
8¾	60.132	105 000	131 300	175 000	192 500	210 000
9	63.617	108 000	135 000	180 000	198 000	216 000
9¼	67.201	111 000	138 800	185 000	203 500	222 000
9½	70.882	114 000	142 500	190 000	209 000	228 000
9¾	74.662	117 000	146 300	195 000	214 500	234 000
10	78.540	120 000	150 000	200 000	220 000	240 000
10¼	82.516	123 000	153 800	205 000	225 500	246 000
10½	86.590	126 000	157 500	210 000	231 000	252 000
10¾	90.763	129 000	161 300	215 000	236 500	258 000
11	95.033	132 000	165 000	220 000	242 000	264 000
11¼	99.402	135 000	168 800	225 000	247 500	270 000
11½	103.869	138 000	172 500	230 000	253 000	276 000
11¾	108.434	141 000	176 300	235 000	258 500	282 000
12	113.097	144 000	180 000	240 000	264 000	288 000

* From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.

Table V. Pins *

BENDING MOMENTS IN INCH-POUNDS

Bending Moment = (Diameter of Pin)³ × 0.098175 × Stress per Square Inch

Pin		Fiber Stress in Pounds per Square Inch						
Diameter, Inches	Area, Sq. In.	15 000	18 000	20 000	22 000	22 500	24 000	25 000
1	785	1 500	1 800	2 000	2 200	2 200	2 400	2 500
1¼	1 227	2 900	3 500	3 800	4 200	4 300	4 600	4 800
1½	1 767	5 000	6 000	6 600	7 300	7 500	8 000	8 300
1¾	2.405	7 900	9 500	10 500	11 600	11 800	12 600	13 200
2	3 142	11 800	14 100	15 700	17 300	17 700	18 800	19 600
2¼	3 976	16 800	20 100	22 400	24 600	25 200	26 800	28 000
2½	4 909	23 000	27 600	30 700	33 700	34 500	36 800	38 300
2¾	5 940	30 600	36 800	40 800	44 900	45 900	49 000	51 000
3	7 069	39 800	47 700	53 000	58 300	59 600	63 600	66 300
3¼	8 296	50 600	60 700	67 400	74 100	75 800	80 900	84 300
3½	9 621	63 100	75 800	84 200	92 600	94 700	101 000	105 200
3¾	11 045	77 700	93 200	103 500	113 900	116 500	124 300	129 400
4	12 566	94 200	113 100	125 700	138 200	141 400	150 800	157 100
4¼	14 186	113 000	135 700	150 700	165 800	169 600	180 900	188 400
4½	15 904	134 200	161 000	178 900	196 800	201 300	214 700	223 700
4¾	17 721	157 800	189 400	210 400	231 500	236 700	252 500	263 000
5	19 635	184 100	220 900	245 400	270 000	276 100	294 500	306 800
5¼	21 648	213 100	255 700	284 100	312 500	319 600	340 900	355 200
5½	23 758	245 000	294 000	326 700	359 300	367 500	392 000	408 300
5¾	25 967	280 000	336 000	373 300	410 600	419 900	447 900	466 600
6	28 274	318 100	381 700	424 100	466 500	477 100	508 900	530 100
6¼	30 680	359 500	431 400	479 400	527 300	539 300	575 200	599 200
6½	33 183	404 400	485 300	539 200	593 100	606 600	647 100	674 000
6¾	35 785	452 900	543 500	603 900	664 300	679 400	724 600	754 800
7	38 485	505 100	606 100	673 500	740 800	757 700	808 200	841 800
7¼	41 282	561 200	673 400	748 200	823 100	841 800	897 900	935 300
7½	44 179	621 300	745 500	828 400	911 200	931 900	994 000	1 035 400
7¾	47 173	685 500	822 600	914 000	1 005 400	1 028 200	1 096 800	1 142 500
8	50 265	754 000	904 800	1 005 300	1 105 800	1 131 000	1 206 400	1 256 600
8¼	53 456	826 900	992 300	1 102 500	1 212 800	1 240 400	1 323 000	1 378 200
8½	56 745	904 400	1 085 300	1 205 800	1 326 400	1 356 600	1 447 000	1 507 300
8¾	60 132	986 500	1 183 900	1 315 400	1 446 900	1 479 800	1 578 500	1 644 200
9	63 617	1 073 500	1 288 300	1 431 400	1 574 500	1 610 300	1 717 700	1 789 200
9¼	67 201	1 165 500	1 398 600	1 554 000	1 709 400	1 748 300	1 864 800	1 942 500
9½	70 882	1 262 600	1 515 100	1 683 500	1 851 800	1 893 900	2 020 100	2 104 300
9¾	74 662	1 364 900	1 637 900	1 819 900	2 001 900	2 047 400	2 183 900	2 274 900
10	78 540	1 472 600	1 767 100	1 963 500	2 159 800	2 208 900	2 356 200	2 454 400
10¼	82 516	1 585 900	1 903 000	2 114 500	2 325 900	2 378 800	2 537 400	2 643 100
10½	86 590	1 704 700	2 045 700	2 273 000	2 500 300	2 557 100	2 727 600	2 841 200
10¾	90 763	1 829 400	2 195 300	2 439 200	2 683 200	2 744 100	2 927 100	3 049 100
11	95 033	1 960 100	2 352 100	2 613 400	2 874 800	2 940 100	3 136 100	3 266 800
11¼	99 402	2 096 800	2 516 100	2 795 700	3 075 200	3 145 100	3 354 800	3 494 600
11½	103 869	2 239 700	2 687 600	2 986 200	3 284 900	3 359 500	3 583 500	3 732 800
11¾	108 434	2 388 900	2 866 700	3 185 300	3 503 800	3 583 400	3 822 300	3 981 600
12	113 097	2 544 700	3 053 600	3 392 900	3 732 200	3 817 000	4 071 500	4 241 200

Remarks. The following is the formula for flexure, $M = SI/c$, with the reductions made to adapt it to a beam of circular section:

$$M = S\pi d^3/32 = SAd/8$$

M = the moment of forces for any section through the pin;
S = the stress per sq in in extreme fibers of pin at that section;
A = the area of the section;
d = the diameter;
π = 3.14159.

The forces are assumed to act in a plane passing through the axis of the pin.

The above table gives the values of M for different diameters of pin, and for seven values of S.

If the maximum value of M is known, an inspection of the table will show what the diameter of the pin must be so that S will not exceed 15 000, 18 000, or 20 000 lb, as the requirements of the case may be.

* From Pocket Companion, Carnegie Steel Company, Pittsburgh, Pa.

vertical and horizontal planes. Inspection of the results will usually indicate which section has the GREATEST RESULTANT MOMENT when the horizontal and vertical components, H and V, are combined. This is done by using the formula $R^2 = H^2 + V^2$ since, graphically, the resultant R is the diagonal of the rectangle on H and V. Example 6 illustrates the method for the condition of INCLINED FORCES acting on the pin. In Example 5 the same method is employed to determine the size of the pin in a simple joint.

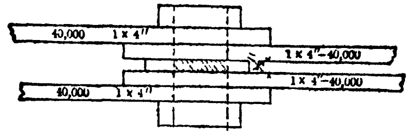


Fig 16 Pin-joint

Example 5. It is required to determine the size of the pin for the joint shown in Fig. 16 in the lower chord of a steel truss. The middle bar is a vertical suspension-rod to hold the chord in place.

Solution. Beginning at the section between the outer bars, the algebraic sum of the forces on either side of the section is 40 000 lb, hence this is the shear. At the section next to the suspender the sum is zero; therefore there is no shear at the middle of the pin. The bearing pressure is 40 000 lb. Its intensity depends on the diameter of the pin and the thickness of the bars. To find the bending moment on the pin the forces are considered concentrated at the middle of the bars and moments taken about sections through the forces. The moment at the section through the second bar is 40 000 lb × 1 in, equal to 40 000 in-lb. If moments are taken about a point between the inner forces the same result is obtained. From Table V it is found that a 2¾-in pin at 24 000 lb per sq in is sufficient. From Table IV the bearing value of a 2¾-in pin is found to be 66 000 lb at a stress of 24 000 lb per sq in. The shearing value of this pin is 5 940 × 12 000 = 71 300 lb. In this case the diameter of the pin is determined by the bending stress, but it is necessary to investigate the other stresses to be sure of the correct size, especially in case of heavy bearing-plates.

Bending Moments on Pins. The finding of the BENDING MOMENT due to the forces acting on a pin is usually the most difficult part of the work of determining its proper size. In the case of a simple pin, properly packed and lying in the plane of the forces acting on it, the GREATEST MOMENT is usually the product obtained by multiplying the outer force by the central distance between the outer bars; but when the forces act in several planes the work is more complicated. The GRAPHICAL METHOD illustrated in the solution of the two following examples has some advantages; but the METHOD OF MOMENTS applied at the end of the solution of the first example is equally rapid in practiced hands and capable of greater refinement in the results.

Example 6. It is required to find the bending moment on the pin of the joint, one-half of which is shown in Fig. 17. The bars are each 1 in thick, the channel of the vertical member ½ in thick and the center of the hanger is ¾ in from the center of the channel.

Solution. Since the joint is symmetrical it is necessary to construct but one-half of the force-diagram and equilibrium-polygon which really apply to the joint. From the conditions of equilibrium of forces, the vertical component of the inclined force is upward, and equal to the sum of the downward forces, 34 000 lb; and its horizontal component acts with the 60 000-lb force, to the amount of 17 000 lb, a sufficient amount to close the force-diagram. The following construction is special, in that but one-half of the entire graph-

ical diagram is shown. This is made possible because of the symmetry of the joint, the bending moment being constant over the middle of the pin.

In the diagram (Fig. 18) AB is drawn at an angle of 45° with the horizontal, and commencing at c, the distances are laid off to scale between the bars, and the lines 1–2, 2–3, etc., drawn parallel to the forces they represent at the joint. The oblique force is resolved into its components 1–4 and 1–5.

The stress-diagram (Fig. 19) is drawn as follows: On a horizontal line the forces are laid off to scale in the order they occur on the pin, 1–2, 2–3, 3–4 and 4–1, the closing of the diagram being a check on the correctness of the value of the forces. Beginning at 1, 1–5, 5–6 and 6–1 are laid off to scale, parallel to

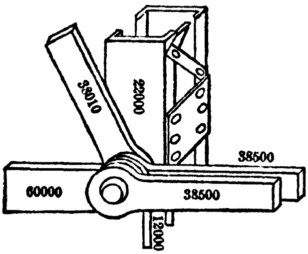


Fig. 17

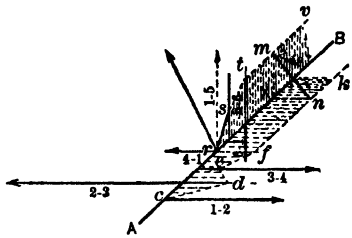


Fig. 18

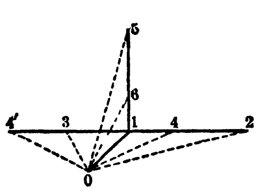


Fig 19

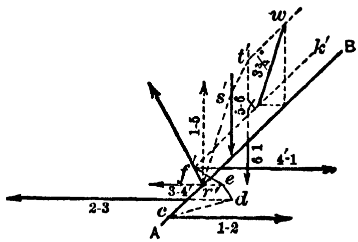


Fig. 20

Figs 17 to 20 Pin-joint and Moment-diagrams

the forces in the vertical plane. From 1 the line 1–0 is drawn at an angle of 45°, for convenience in making good intersections, and equal to a convenient number, say 20 000 lb, in the same scale to which the loads are drawn. The point O is the pole of the stress-diagram, the pole-distance being 20 000 lb. From the principles of graphics the bending moment at any point on the pin is equal to the intercept between the proper ray of the equilibrium-polygon and the closing line, multiplied by the pole-distance. To complete the figures, 0–2, 0–3 and 0–4 are drawn from O, and from c cd is drawn parallel to 0–2, de parallel to 0–3, ef parallel to 0–4 and fk parallel to 0–1. In the same way rs is drawn parallel to 0–5, st to 0–6 and tv to 0–1. Then according to the above principles, the moment at any section due to the forces in the horizontal plane is proportional to the ordinate at that section drawn from the line AB to the line $cdefk$ bounding the equilibrium-polygon; and the moments due to the vertical forces are proportional to the ordinates drawn to the line $rstv$, the numerical value being the length of the ordinate times 20 000, the pole-distance. Where both moments are present, the resultant or true moment is proportional to the hypotenuse of the right-angled triangle having for its sides

the ordinates in the two planes at the point in question. At X this is shown by the line mn. This measures 2.42 in, and being the longest diagonal or hypotenuse that can be drawn in the figure, it follows that the maximum bending moment on the pin is $2.42 \times 20\,000 = 48\,400$ in-lb.

To find the effect of changing the arrangement of the members on the pin, it may be assumed that the inclined bar is placed outside the inner chord-bar. The horizontal stress-diagram then becomes 1–2, 2–3, 3–4′, 4′–1. The equilibrium-polygons become $cdef'k'$ and $r's't'w$, as shown in Fig. 20. In these polygons the longest diagonal measures 3¾ in, which gives a bending moment of 3¾ in × 20 000 lb = 75 000 in-lb, showing that the arrangement of the eye-bars in Fig. 17 is better. As a rule the bending moment is less when those forces that oppose each other are placed together. It may be further reduced by making the outside bar one-half the thickness of the main horizontal bars.

To check by the METHOD OF MOMENTS the value of the maximum bending moment obtained by the GRAPHIC METHOD for the first arrangement, the moments of the forces in the horizontal plane are taken about r. This gives

$$M_h = 38\,500 \text{ lb} \times 3.0 \text{ in} + 38\,500 \text{ lb} \times 1.0 \text{ in} - 60\,000 \text{ lb} \times 2.0 \text{ in} = 34\,000 \text{ in-lb},$$

which is the value of the moment in the horizontal plane across the middle of the pin.

In the vertical plane moments are taken about a point t, giving

$$M_v = 34\,000 \text{ lb} \times 1.5 \text{ in} - 22\,000 \text{ lb} \times 0.75 \text{ in} = 34\,500 \text{ in-lb}$$

From these component moments the resultant maximum bending moment is

$$M = \sqrt{34\,000^2 + 34\,500^2} = 48\,400 \text{ in-lb}$$

Example 7. Another illustration of the GRAPHICAL METHOD of finding the bending moment on a pin is given for the joint A of the truss-diagram shown in Fig 21. Fig. 22 shows the arrangement and size of the members. The stresses given in Fig 21 are for one-half the number of members at the joint. As in Example 6, the symmetrical arrangement makes it unnecessary to draw more than one-half of the force-polygon and equilibrium-polygon. The web of the channel is reinforced to make it ⅝ in thick.

Solution. The line AB (Fig 24) is drawn at an angle of 45° and ah, etc, are laid off to scale, equal to the distances between the members. At each point of application of a force a line is drawn parallel and to scale, to represent that force. The inclined forces are then resolved into their horizontal and vertical components. The force-diagram (Fig. 23) is then drawn, the horizontal forces being laid off to scale in the order in which they occur, 1–2, 2–3, 3–4 and 4–1. The pole-distance is then laid off at an angle of 45° and equal to 20 000 lb to the same scale of forces. The pole 0 is then joined with 2, 3, 4 and 1. Then in Fig. 24, ab is drawn parallel to 0–2, bc to 0–3, cd to 0–4 and de to 0–1. In the same way the line $hjkB$ is drawn. From inspection it is seen that hb is the longest intercept, even longer than any diagonal that may be drawn from the extremities of the horizontal and vertical intercepts at any point along AB. To the same scale that makes 0–1 represent 20 000 lb, hb represents 31 800 in-lb; therefore the bending moment on the pin is 31 800 in-lb. In Table V a pin 2¾ in in diameter, at a fiber-stress of 20 000 lb per sq in, has an allowable moment of 40 800 in-lb, and in Table IV a bearing value on 1 in of 33 000 lb. A force of 31 800 lb on ¾ in is equal to 42 400 for a 1-in bar; so it is necessary to use a larger pin to accommodate the bearing requirement. From Table IV a pin 3½ in in diameter is found to be about 1% scant. The shearing value of this pin is $9.621 \times 12\,000$

= 115450, which is more than three times the load, so, again, it is the bearing that controls the size of the pin. If the thickness of the bars is increased the diameter of the pin may be reduced to 3 in.

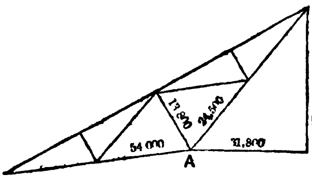


Fig. 21

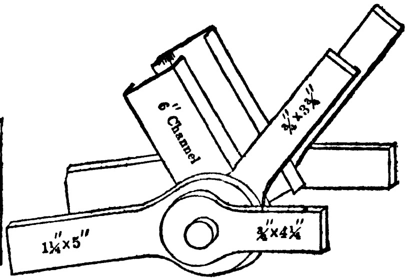


Fig. 22

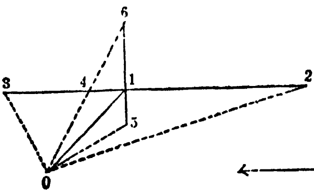


Fig. 23

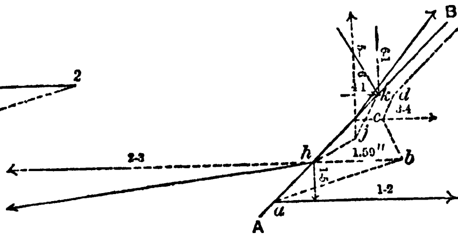


Fig. 24

Figs. 21 to 24. Force-polygons and Equilibrium-polygons for Bending Moments on a Pin

4. Strength of Bolts in Wooden Trusses and Girders

The Working Stresses for Bolts on which Table VI and Table VII are computed are based on a FACTOR OF SAFETY of five applied to the average of many tests on dry timber. In some specifications it is permitted to increase the BEARING PRESSURE between timber and bolts as much as 50% above that permitted for short struts. The values in the tables are somewhat less than the tests on large trusses made at the Massachusetts Institute of Technology. Table VIII gives the allowable maximum tension, shear and bending moments for wrought-iron and steel bolts.

Kinds of Stress in Bolts. BOLTS in wooden trusses are subject to the same kinds of stress as the RIVETS and PINS in steel structures. When the pieces joined are less than 2 in thick and the bolts are tightly drawn up so as to develop considerable frictional resistance between the pieces, the bolts are proportioned to resist the total force in SHEAR and in BEARING. When the pieces are more than 2 in thick the BENDING is taken into account and the bolts must be investigated for stresses in SHEAR, in BEARING and in BENDING. The SHEAR is assumed to be uniformly distributed over the cross-section of the bolt, and the BEARING AREA is the area of the projection of the bolt on the timber, which area is equal to the diameter of the bolt multiplied by the length in contact. The BEARING STRENGTH is given as a property of the bolt although

its value depends upon the crushing strength of the timber. The BENDING MOMENT on the bolt is found in the same manner as for pins in steel trusses, although the cases are usually less complicated.

Illustrations of the Use of Bolts. The principles involved in the use of bolts in wooden trusses and girders and in the use of the tables may be best illustrated by the solution of examples in each of the following cases:

(1) Bolts in tie-beams, thin pieces.
(2) Bolts in girders to support brackets.
(3) Bolts as pins in the joints of trusses.
(4) Bolt-and-strap joints in trusses.
(5) Bolts under tension to hold the foot of a rafter.

See, also, "Joints in Wooden Trusses," Chapter XXVI.

Case 1. Bolts in Tie-Beams, Thin Pieces. Tie-beams of wooden trusses, when longer than 30 ft, are usually made up of a number of pieces. This construction is cheaper than the use of a single stick. Two-inch planks bolted together are generally used. The location of the joints in the courses of planks and the number and size of the bolts are the special considerations in the design of such a joint. In general, the joints in adjacent courses are placed as far apart as possible and not more than two joints are placed opposite each other in the same section. The simplest case is that of a plain FISH-PLATE JOINT like a common BUTT-JOINT with two cover-plates as shown in Fig. 12. The number of BOLTS for such a joint is found in the same way as the number of RIVETS in steel tie-bars. The bolts must be spaced as required in the second column under each timber in Table VI to provide against shearing in front of the bolt.

Table VI.* Safe Bearing Value of Bolts per Inch of Length Parallel to the Grain in Timber and Distance from Center to Center of Bolts or to End of Timber

Diameter of bolt, in	Long-leaf yellow pine		White pine and short-leaf yellow pine		Douglas fir		White oak	
	Bearing at 1 400 lb per sq in, lb	Distance, in	Bearing at 1 100 lb per sq in, lb	Distance, in	Bearing at 1 200 lb per sq in, lb	Distance, in	Bearing at 1 400 lb per sq in, lb	Distance, in
¾	1 050	4½	825	5¼	900	4¼	1 050	3½
⅞	1 225	5	960	5¾	1 050	5	1 225	4
1	1 400	5¾	1 100	6½	1 200	5½	1 400	4½
1⅛	1 575	6½	1 237	7½	1 350	6¼	1 575	5
1¼	1 750	7	1 375	8	1 500	7	1 750	5⅛
1⅜	1 925	7¾	1 512	9	1 650	7¾	1 925	6¼
1½	2 100	8½	1 650	9¾	1 800	8½	2 100	6¾
1¾	2 450	10	1 925	11½	1 950	9¼	2 450	7¾
2	2 800	11½	2 200	13	2 400	11¼	2 800	9
2¼	3 150	12¾	2 475	14¾	2 700	12½	3 150	10
2½	3 500	14¼	2 750	16¼	3 000	14	3 500	11¼
2¾	3 850	15¼	3 025	18	3 300	15½	3 850	12¼
3	4 200	17	3 300	19	3 600	17	4 200	13⅛

The distance from the end is equal to the diameter of the bolt plus the length on which twice the SHEAR is equal to the BEARING VALUE of the bolt against the end-fibers.

* See, also, Chapter XXI.

Table VII. Safe Bearing Value of Bolts per Inch of Length Across the Grain in Timber

Diameter of bolt, in	Long-leaf yellow pine, lb	Short-leaf yellow pine and Douglas fir, lb	White pine, lb	White oak, lb
¾	262	187	150	375
⅞	306	218	175	437
1	350	250	200	500
1⅛	394	281	225	562
1¼	437	312	250	625
1⅜	482	343	275	687
1½	525	375	300	750
1¾	612	437	350	875
2	700	500	400	1 000

Table VIII. Maximum Allowable Tension, Shear and Bending Moment for Wrought-Iron and Steel Bolts

(Shear is computed on the section under Shear, which is the full area.)

Diameter of bolt, in	Net area, sq in	Wrought iron			Steel		
		Tension at 12 000 lb per sq in, lb	Shear at 7 500 lb per sq in, lb	Bending moment at 15 000 lb per sq in, in-lb	Tension at 16 000 lb per sq in, lb	Shear at 10 000 lb per sq in, lb	Bending moment at 20 000 lb per sq in, in-lb
¾	0.302	3 620	3 310	620	4 830	4 420	830
⅞	0 420	5 040	4 510	980	6 720	6 010	1 310
1	0.550	6 600	5 890	1 470	8 800	7 850	1 960
1⅛	0.694	8 328	7 460	2 100	11 100	9 940	2 800
1¼	0.893	10 716	9 200	2 880	14 290	12 270	3 830
1⅜	1.057	12 680	11 140	3 830	16 910	14 850	5 100
1½	1.295	15 540	13 250	4 970	20 720	17 670	6 630
1¾	1 746	20 930	18 040	7 890	27 910	24 050	10 500
2	2.302	27 620	23 560	11 800	36 830	31 420	15 700
2¼	3 023	36 280	29 820	16 800	48 370	39 760	22 400
2½	3.719	44 630	36 820	23 000	59 510	49 090	30 700
2¾	4.620	55 430	44 550	30 600	73 910	59 400	40 800
3	5.428	65 140	53 010	39 800	86 850	70 690	53 000
3¼	6 510	78 120	62 220	50 600	104 160	82 960	67 400

Example 8. A typical tie-beam used as a lower chord of a Howe truss is shown in Fig. 25. It is 50 ft long, of Douglas fir and subject to the tension in the different panels shown in the figure.

Solution. The thickness of the plank is drawn out of scale in the figure to show the joints more clearly. The black circles show the vertical tension-rods, which so nearly cut the middle plank in two that it is not considered a part of the tensile member. The arrangement of the planks and the lengths to be used must be determined for each case. In the one shown there is but one splice in the middle panels where there is the greatest tension. The dis-

tance XY is 12 ft, which is about as small as will serve for the transfer of the tension from A' to B. In this beam the two outer planks, A and A', must be large enough to resist the whole tensile stress in the middle panels because of

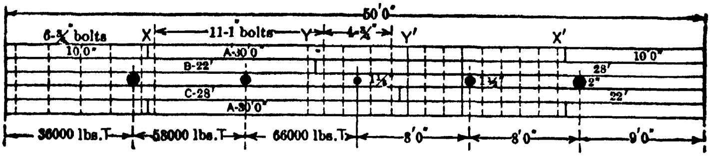


Fig. 25. Plan of Built-up Tie-beam

the joints in B and C. At the inner end of the second panel there is 58 000 lb tension which must be carried to the end of the first panel. Because of the joints in A and A' this must be transmitted to B and C in order to pass the point X.

Assuming that 29 000 lb, one-half the tension, is carried on plank A to be transmitted to C by the shear and bearing on the bolts, and dividing this by 7 850 lb, the allowable shear on a 1-in bolt, four bolts are found to be necessary. But the bearing value of a 1-in bolt in Douglas fir 2 in thick, is only 2 400 lb, which makes twelve bolts necessary. These are required in the distance XY, 12 ft.

From the distances in Table VI it is found that the end-bolts must be 5½ in from the ends of the planks, say 6 in; this leaves 11 ft, in which distance eight bolts are to be arranged. If four bolts are placed in pairs, two at each

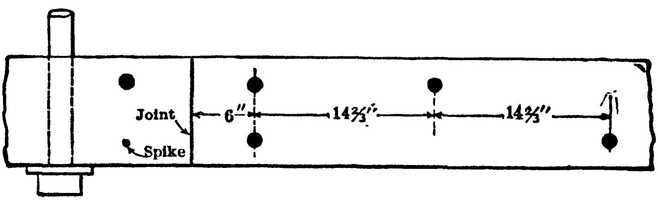


Fig. 26. Elevation of Beam Opposite X of Fig. 25

end, as shown in Fig. 26, the intermediate spaces are 14⅔ in. The bolts bind the beam together better if they are staggered, as indicated in Fig. 26, and not placed on the middle line.

The number of bolts mentioned is sufficient to make the splice, but there should be bolts in the distance YY', and between the ends and X and X', to bind the planks together. These need not be as large or as close together as the others; ¾-in bolts spaced 2 ft are sufficient. There should be two bolts at the end of the beam. Each bolt should be driven through a hole of the same size as the bolt and the nuts should be screwed up tight.

Case II. Bolts in Girders to Support Brackets. The construction shown in Figs 27 and 28 is commonly used in cases in which the requirements do not allow the girder to project its full depth below the joists. The BOLTS shown in Fig. 27 must be investigated for BEARING and SHEAR, and those shown in Fig. 28 for BEARING, SHEAR and BENDING. In either case the SHEARING VALUE of the bolt in single shear must equal or be greater than the greater of the forces S or S'.

The BEARING per inch on the wood of the girder, when B is in inches, is

$$(S + S')/B$$

This must be kept within the values given in Table VI for the timber used. For the case shown in Fig. 28 the BENDING MOMENT in pound-inches is

$$M = SL/2 \quad \text{or} \quad M = S'L/2$$

whichever is the larger. B and L are measured in inches and S in pounds.

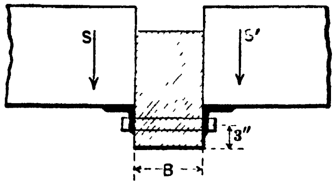


Fig. 27

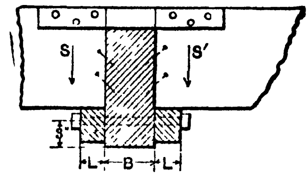


Fig. 28

Figs. 27 and 28. Bolts Supporting Brackets on Girders

Example 9. For the construction shown in Fig. 27 it is required to determine the number and size of bolts, the Douglas fir girder being 8 by 14 in, with a span of 14 ft, and the Douglas fir joists 3 by 12 in, with a span of 20 ft, center to center of girders. The floor-load, including the floor, is 60 lb per sq ft. The angles are 4 by $3\frac{1}{2}$ by $\frac{3}{8}$ in.

Solution. The floor-area supported by the girder is 14 by 20 ft. At 60 lb per sq ft, the load is $14 \times 20 \times 60 = 16\,800$ lb. The load S, on one side, is 8 400 lb.

A $\frac{3}{4}$-in bolt has a shearing-value of 4 420 lb. Hence two bolts are necessary to satisfy the shearing condition. The bearing value of the bolt in the wood, across the grain, is, from Table VII, 187 lb per inch of length, or 1 496 lb for the width of the girder. The number of bolts required, then, is 16 800 divided by 1 496 or approximately 11, which gives a spacing of about 15 in.

Example 10. In the construction shown in Fig. 28, the girder is 6 by 14 in, of Douglas fir and has a span of 12 ft. The joists are 2 by 12 in and have an 18-ft span, center to center of girders. The floor-load is 65 lb per sq ft. There are 3 by 4-in strips on the sides of the girder. The distance L is 3 in. It is required to find the number and size of bolts to be used.

Solution. The total load on the girder is

$$12 \times 18 \times 65 = 14\,040 \text{ lb}$$

$$S = 7\,020 \text{ lb}$$

The bearing load per inch of thickness of the girder is

$$\frac{14\,040}{6} = 2\,340 \text{ lb}$$

The bending moment on one side of the girder is

$$\frac{7\,020 \times 3}{2} = 10\,530 \text{ in-lb}$$

since the force S acts at the center of pressure on the bracket-strip, $1\frac{1}{2}$ in from the edge of the girder.

The shear is 7 020 lb, which requires two $\frac{3}{4}$-in steel bolts at 4 420 lb for one as given in Table VIII.

The bearing (Table VII) on a $\frac{3}{4}$-in bolt is 187 lb per inch of length; therefore it requires thirteen bolts for bearing.

The allowable bending moment on a $\frac{3}{4}$-in steel bolt is 830 in-lb, from Table VIII. To take care of the 10 530 in-lb requires thirteen bolts. A $\frac{7}{8}$-in steel bolt has an allowable bending moment of 1 310 lb-in, making eight of them sufficient. The 3 by 4-in pieces may be held in place by thirteen $\frac{3}{4}$-in bolts spaced 11 in on centers, if two of them are placed 6 in from the ends.

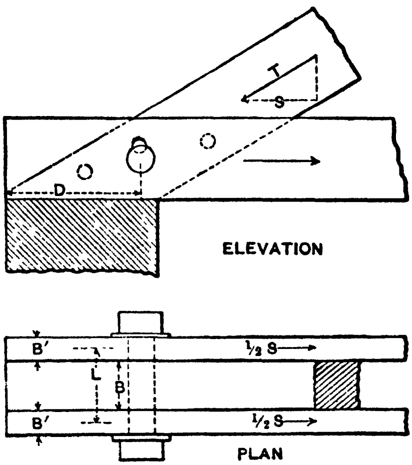


Fig. 29. Bolt through Rafter and Tie-beam

Case III. Bolts as Pins in the Joints of Trusses. For TIES or STRUTS joined by BOLTS in the manner indicated in Figs. 29, 30 and 31 and having the thickness B exceeding 2 in, the diameter of the bolt or the number of bolts must be computed for SHEARING, BEARING and FLEXURE.

For any of these joints the forces are as follows:

The single shear $= S/2$

On the sections between B and B' (Fig. 30)

The bearing on the pin per inch of length $= S/B$ or S'/B'

The greater is to be used.

The bending moment $= SL/12$ on the assumption of a CONTINUOUS BEAM, uniformly loaded.

If there are more bolts than one, the quantities obtained by the above formulas are to be divided by the number of bolts to find the part to be taken care of by one bolt.

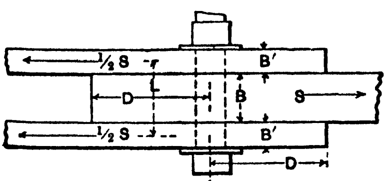


Fig. 30. Bolt in Wooden Tie-beam

In Fig. 29, S is the horizontal component of the thrust T.

Example 11. It is required to determine the diameter of a bolt for a joint like that shown in Fig. 29. The rafter is 6 by 10 in, of Douglas fir, the tie beams 3 by 10 in, of the same material, the thrust in the rafter 30 000 lb, and its inclination 30°.

Solution. The horizontal component of 30 000 lb at 30° is practically 26 000 lb. Then $S = 26\,000$ lb and the shear $= 13\,000$ lb. $B = 6$ in and $L = 9$ in.

Bearing per inch of length on the bolt $= 26\,000/6 = 4\,333$ lb

Bending moment $= 26\,000 \times 9/12 = 19\,500$ in-lb

In Table VIII, a 1⅜-in steel bolt is found to be necessary to resist a shear of 13 000 lb, and a 2¼-in bolt for a bending moment of 19 500 in-lb. To resist 4 333 lb end-bearing pressure on 1 in a larger bolt is required than is given in Table VI. Dividing 4 333 by 1 200, the allowable bearing on Douglas fir, a 3⅝-in bolt is found to be necessary. This is larger than it is desirable to use, so the joint must be redesigned with a view to reduce the bearing pressure on the bolt. If an 8 by 8-in strut and 4 by 8-in tie-beams are used, B becomes 8 in and L 12 in. This gives

Bearing pressure = 26 000/8 = 3 250 lb per inch of length of the bolt
Bending moment = 26 000 × 12/12 = 26 000 in-lb

The total shear at the section on one side of the strut is the same as before.

From Table VI it is found that a 2¾-in bolt is large enough to provide for the bearing and that a 2½-in bolt is sufficient for the bending as given in Table VIII. Hence if an 8 by 8-in strut is used, there must be a 2¾-in bolt and the distance D must be 15½ in (Table VI).

Example 12. For the same construction as in Fig. 29 and the same conditions as in the first part of Example 11, it is required to determine the size of the bolts when it is necessary to use three.

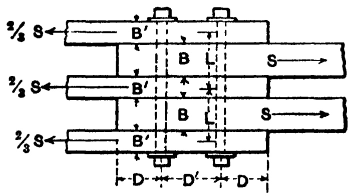


Fig. 31. Bolts in Wooden Tie-beam

Solution. The shear, bearing, and bending moment are the same as in Example 11, but because there are three bolts each quantity is divided by 3 to determine the force resisted by each.

Shear = 13 000/3 = 4 333 lb and requires a ¾-in steel bolt (Table VIII)

Bearing = 4 333/3 = 1 444 lb and requires a 1¼-in bolt (Table VI)

Bending moment = 19 500/3 = 6 500 in-lb, and requires a 1½-in steel bolt (Table VIII).

In this case the bending moment determines the size of the bolts, which may be arranged as shown by the dotted circles in Fig. 29.

Example 13. It is required to determine the diameter of the bolt for the construction shown in Fig. 30, in which the inner beam is of Douglas fir and 6 by 8 in in section, and the outer beams 3 by 8 in, the tension being 24 000 lb.

Solution. $S = 24\,000$; $B = 6$ in; $L = 9$ in.

Single shear on the bolt = 24 000/2 = 12 000 lb
Bearing-pressure per inch of length of bolt = 24 000/6 = 4 000 lb
Bending moment = 24 000 × 9/12 = 18 000 in-lb

From Table VIII a 1¼-in steel bolt is found sufficient to resist the shear, and a 2¼-in bolt large enough to resist the bending. In Table VI the largest bolt considered, 3 in, is too small in bearing value. Dividing the load to be resisted by 1 200 gives 3⅓ in as the diameter necessary to resist the bearing. The distance D must be 4 000/(2 × 130) + 3⅓ in or 18¾ in.

Example 14. If two bolts are used, one behind the other, it is required to determine the diameter of the bolt that should be used, the conditions and loading being the same as in Example 13.

Solution. Dividing the quantities obtained in Example 13 by 2,

Single shear = 6 000 lb and requires a ⅞-in steel bolt
Bearing = 2 000 lb and requires a 2-in bolt
Bending moment = 9 000 in-lb and requires a 1¾-in steel bolt

The allowable bearing on a 1¾-in bolt is (2½%) less than the required amount, so that in general, since the other requirements are more than satisfied, the smaller bolt would be used. For the 1¾-in bolt, the distance D is 9¼ in. The space between the bolts may be increased somewhat beyond the value given in Table VI and they may be located out of the same line as a further precaution against splitting.

Case IV. Bolt-and-Strap Joints in Trusses. The construction shown in Fig. 32 is sometimes used to connect the foot of the rafter of a wooden truss to the tie-beam. When the distance D is sufficient to resist the shear due to the thrust of the rafter, the strap is of value only in holding the rafter in place, and there are no greater pressures brought upon the BOLT. When it is impossible to make D the necessary length, the BOLT and STRAP must be designed to resist the full force in the direction of the STRAP.

As the STRAP is usually not more than from ½ to ¾ in thick, its width is such that the bearing between it and the rafter is small compared with that between the BOLT and rafter. The forces acting on the BOLT are the only ones that need consideration. These are.

SINGLE SHEAR = $S/2$ = the tension in the strap on one side

BEARING PRESSURE per inch of length = S/B, where B is the width of the tie-beam in inches

BEARING PRESSURE per inch of length between strap and bolt = $S/2t$.

To find the value of S, the force-polygon is drawn as shown at the right in Fig. 32. T is drawn parallel to the rafter and with a length, to a convenient scale, equal to the thrust. From the end a an indefinite line is drawn parallel to the axis of the strap, and from b another line perpendicular to the SEAT of the rafter. These intersect at c, so that ac, measured by the same scale used in laying off T, is the magnitude of the force S in the strap. If the rafter rests on top of the beam, bc is vertical, but if the tie-beam is dapped, as shown by the dotted line, the line from b is drawn perpendicular to the bottom of the notch, making the intersection at c'. It is seen that notching the tiebeam in this way increases the stress in the strap.

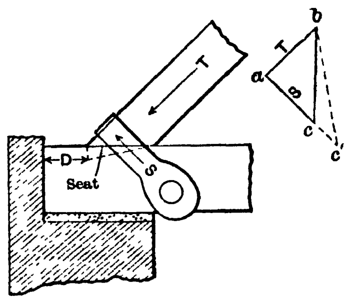


Fig. 32. Strap and Bolt at Foot of Rafter

Example 15. It is required to determine the size of a strap and pin-bolt to hold the rafter without notching into the tie-beam of a long-leaf yellow-pine king-post truss. The rafter is 6 by 6 in, is inclined at an angle of 45° and is under a compressive stress of 18 000 lb. The tie-beam is 6 by 8 in in section.

Solution. Since the inclination is 45°, a consideration of the force-polygon in Fig. 32 shows ab equal to ac, so that

The force S = the thrust T = 18 000 lb
Single shear on bolt = 18 000/2 = 9 000 lb

Tension in strap on one side = 9 000 lb
Bearing pressure per inch of bolt against wood = 18 000/6 = 3 000 lb
Bearing pressure in pounds per inch between strap and bolt = 9 000/t

in which t equals the thickness of the strap.

The allowable pressure between the strap and the top of the rafter is 350 lb per sq in (Table VII) which, on the 6-in rafter, gives

Allowable load per inch of width of strap = 6 × 350 = 2 100 lb

The strap then must be 18 000/2 100 or 8.6 in wide. At 10 000 per sq in in tension the necessary section of the strap is 0.9 sq in, requiring a thickness of about 0.1 in, a sufficient thickness if the strap were strong enough to develop a uniform pressure over the rafter. It is not good practice, however, to use such thin material, because of the danger of loss of strength due to corrosion. No metal less than 3/8 in thick should be used in such places.

The bearing-pressure per inch, between the strap and the bolt, for a 3/8-in strap = 9 000/3/8 = 24 000 lb.

The bolt, then, must take a single shear of 9 000 lb, a bearing pressure of 3 000 lb against the wood for each inch of length, and a bearing of 24 000 lb per inch of length against the strap. From Table VIII a 1⅛-in steel bolt is sufficient to resist the shear, from Table IV a 2-in bolt is large enough to resist the bearing from the strap, and from Table VI a 2¼-in bolt is found necessary to resist the 3 000-lb bearing from the wood per inch of length of bolt. This makes the 2¼-in bolt satisfactory for the joint.

The pressure from the bolt to the wood, however, is not parallel with the grain but inclined at 45°. The allowable pressure against wood across the grain is about one-fourth of that with the grain. According to the empirical formula, ($r = q + (p - q)\,(\theta/90)^2$, where r equals the permissible normal unit stress on the inclined surface, q that across the fibers, p that on the end of the fibers and θ the angle the inclined surface makes with the direction of the grain), the allowable pressure per square inch for this case is 612 lb instead of the 1 400 per sq in allowed for direct compression with the grain. The reduced allowable pressure makes it necessary to use a 4.9-in bolt, say a 5-in bolt, which would be impracticable, for it would almost cut the tie-beam in two. It thus appears that this form of joint is not good design for a truss of this span. For shorter spans the joint may be made in accordance with the requirements given. It has the advantage of not presenting any projections below the tie-beam.

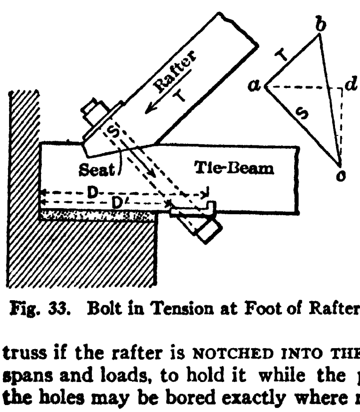


Fig. 33. Bolt in Tension at Foot of Rafter

Case V. Bolts in Tension to Hold the Foot of a Rafter. In the joint shown in Fig. 33 the bolt is subject to DIRECT TENSION only. The amount of the tension S is found by the construction explained in Case IV. The rafter may be let into the tie-beam or rest on top of it, the tension in the bolt being less in the latter case; but it is easier to erect the truss if the rafter is NOTCHED INTO THE BEAM from 1¼ to 1½ in for ordinary spans and loads, to hold it while the pieces are fitted. After this is done, the holes may be bored exactly where required.

Whenever S exceeds about 10 000 lb for trusses made of timber for which the highest bearing stresses are allowed, a CAST PLATE, as shown in Fig. 34 and made to fit the inclination of the bolt, should be let into the tie-beam at the head of the bolt to distribute the pressure. The diameter of the hole for the bolt should be ⅛ in larger than the diameter of the bolt. The distance D must be made sufficient to provide for the horizontal component of S, at the allowed working stress of the material for shear with the grain.

The horizontal component is found by drawing a vertical line from c and a horizontal line from a and measuring ad to the scale of the diagram. For safety, this force must be less than the product of the distance D, the width of the beam and the allowed shearing-stress given in Table I.

Example 16. For the same conditions as in Example 15, for the size of the members and the thrust in the rafter, it is required to determine the diameter of the bolt and the distance D for a joint of the type shown in Fig. 33.

Solution. To find S, draw T equal to 18 000 lb, at a convenient scale, and parallel to the rafter. At a, draw an indefinite line perpendicular to the rafters and at b a line perpendicular to THE SEAT of the rafter. This makes S greater than in Example 15, as ac now scales 27 000 lb. From Table VIII a 1¾-in steel bolt is sufficient to take this in direct tension. The horizontal component found as directed above, scales 19 000 lb. The width of the tie-beam is 6 in, which at the allowed shearing-stress, 150 lb per sq in, gives 900 lb as the stress that must be cared for by each inch of D. 19 000 lb divided by 900 gives 21 in, the required distance D. (See, also, Chapter XXVI, Joints in Wooden Trusses.)

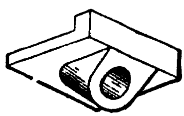


Fig. 34. Special Washer

The compression against the grain on the end of the cast-iron washer must also be investigated. 19 000 lb divided by the width, 6 in, gives 3 166 lb that must be resisted per inch of width of beam. At 1 400 lb per sq in, as an allowable working stress, this makes it necessary to set the casting 2¼ in into the lower side of the beam, which exceeds the depth usual in ordinary practice. Some tests made at the Massachusetts Institute of Technology on large trusses, and reported in 1897, indicated that for a TEST CARRIED TO RUPTURE the stresses prescribed for usual designs might safely be more than doubled. Tests on timber under LONG-CONTINUED LOADING indicate that rupture finally occurs for stresses approximating one-half of those developed in TESTS CARRIED TO IMMEDIATE FAILURE. This, and the fact that decay may affect the strength of the members, emphasizes the wisdom of using CONSERVATIVE WORKING-STRESSES in this material.

Example 17. It is required to determine the size of bolts for the joint shown in Fig. 35, the thrust being 65 500 lb and the truss-members being made of long-leaf yellow pine.

Solution. The tension in the bolts is found first by drawing the force-polygon as shown at the right in the figure. To the same scale that ab represents 65 500, ac represents 96 500 lb. If the load is equally divided between the bolts, each has a tension of 48 250 lb. From Table VIII this force requires a 2¼-in steel bolt.

The horizontal component ad is 68 350 lb, which must be resisted by the shearing strength of the wood between the end of the cast-iron washer on the under side of the tie-beam and the end of the beam resting on the wall. At 150 lb per sq in, this requires 68 350/150, or 455 sq in. If the beam is

8 in wide, this requires a length of 57 in along the beam from the washer to the end.

The bearing of the cast-iron washer against the end-fibers of the tie-beam is also 68 350 lb. At an allowable pressure of 1 400 lb per sq in the depth of the washer should be 68 350/(8 × 1 400) = 6.1 in. This would almost cut the beam in two. The ultimate strength of the wood in compression is about five times the working stress, and since a considerable part of the horizontal force may be resisted by the body of the bolt as well as by the friction of the washer, it is probable that with washers ¾ in thick there would be little sign of weakness at the joint even when the truss is fully loaded.

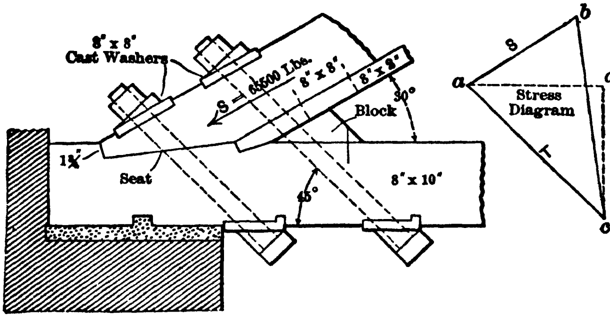


Fig. 35. Joint with Two Bolts in Direct Tension

Theoretically the washers on the top surface of the rafter should be determined by the allowable working stress in compression across the fibers. This for long-leaf pine is taken at 350 lb per sq in (see Table IV, Chapter XIV). The area, then, is 48 250/350, or 138 sq in. This requires a washer 11¾ in square. The 8 by 8-in washer used, assumes a pressure of 755 lb per sq in, but as the Forest Service of the United States Department of Agriculture recommends a working stress from 793 to 1 027 for common grade yellow pine, depending on the conditions as to moisture, it is very likely that there would be no signs of injury at this point, other than a SLIGHT INDENTATION, when the truss is fully loaded.

CHAPTER XIII

BEARING-PLATES AND BASES FOR COLUMNS, BEAMS AND GIRDERS. BRACKETS ON CAST-IRON COLUMNS *

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1. Bearing-Plates and Bases

The Purpose of Bearing-Plates or Bases. When a heavily loaded column, beam or girder is supported on a masonry wall or pier, a BEARING-PLATE or BASE of suitable dimensions must be used to distribute the load so that the pressure will not exceed the safe BEARING STRENGTH of the masonry (Table I).

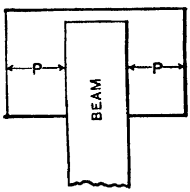


Fig. 1. Simple Bearing-plate

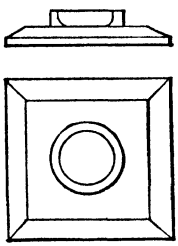
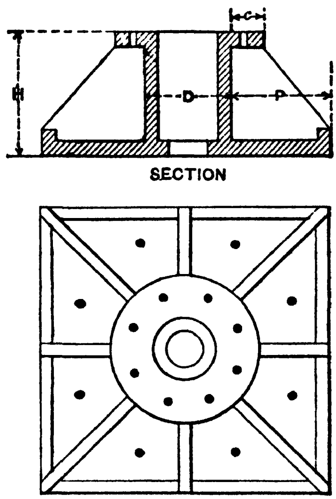


Fig. 2. Beveled Cast-iron Plate with Pin



PLAN

Fig 3. Ribbed Cast-iron Plate

The BEARING-PLATE is designed to be stiff enough to distribute the pressure under it uniformly, and its area is determined by dividing the load on it by the allowable pressure per unit of area (Table II).

Simple Bearing-Plates. Fig. 1 shows a SIMPLE BEARING-PLATE under a beam. It may be a steel or cast-iron rectangular plate of sufficient thickness to prevent its bending at the edge of the beam from the pressure of the masonry below.

* See, also, Chapter XIV, Subdivision 5.

Cast-Iron Plates with Pin. Fig. 2 is a cast-iron PLATE WITH A DOWEL-PIN to fit inside the shell of a cast-iron column, or into a recess cut in the bottom of a wooden one. The pin holds the base-plate in position.

Cast-Iron Ribbed Bases. Fig. 3 is a cast-iron RIBBED BASE for a large cylindrical cast-iron column, capable of supporting a load heavy enough to break a plate similar to the one shown in Fig. 2, at the edges of the column, unless the plate were made unduly thick.

Table I. Allowable Bearing Pressure on Different Kinds of Masonry

Kind of masonry	Allowable pressures	
	Lb per sq in	Tons per sq ft
From the building laws of New York, 1925		
Brick, in lime mortar	110	8
in lime-and-cement mortar	160	11½
in Portland-cement mortar	250	18
Rubble masonry, in Portland-cement mortar	140	10
Concrete, Portland cement, 1 : 2 : 4	500	36
From the building laws of Kansas City, 1927		
Rubble, in lime mortar	70	4 9
in Portland-cement mortar	170	12 2
Ashlar, limestone, in Portland-cement mortar	600	43 2
granite, " " "	600	43 2
Concrete, Portland-cement, 1 : 2 : 4	500	36 0

The Bases of the Steel Cores of Composite Columns used in reinforced-concrete construction have areas sufficient to distribute the loads of the columns over the concrete in the footings at the allowable working stress of the concrete.

Example 1. The basement-columns of a warehouse are designed for a load of 212 000 lb each. It is required to determine the size of the base-plates to rest on the concrete foundations. (Table II used.)

Solution. At an allowable pressure of 208 lb per sq in, the required area is 212 000/208 or 1 020 sq in, or about 32½ in square. The plan and section of the base-plate is shown in Fig. 3.

Forms of Base-Plates. For small columns and wooden posts with light loads, PLAIN FLAT PLATES of cast iron or steel are generally used. The cast-iron plates may have a raised ring or cross to fit inside a hollow metal column, or a dowel, from 1½ to 2 in in height for a wooden one. If the plate is very thick the outer edges may be beveled to save weight, as shown in Fig. 2, but no part of it should be less than about ¾ in thick.

Ribbed Bases. If the calculated size of a bearing-plate is so large that its projection beyond the edge of the column would be more than about 6 in, a RIBBED BASE similar to that shown (Fig. 3) for a cylindrical column is used. For such bases it is unnecessary to consider the transverse stresses. When

these bases are bolted to the columns they add greatly to the general stability of the supporting members because of the greater width of such bases.

Table II. Allowable Loads on Standard, Steel Bearing-Plates on Walls

Bearing on wall, in	Size of plate, in	Safe bearing value of plate in pounds — Bricks laid in mortar of: Lime 110* lb per sq in	Lime and cement 160* lb per sq in	Cement 250* lb per sq in
6	6× 6	3 960	5 760	9 000
	6× 8	5 280	7 680	12 000
	6×10	6 600	9 600	15 000
8	8× 8	7 040	10 240	16 000
	8×10	8 800	12 800	20 000
	8×12	10 560	15 360	24 000
10	10×10	11 000	16 000	25 000
	10×12	13 200	19 200	30 000
	10×14	15 400	22 400	35 000
12	12×12	15 840	23 040	36 000
	12×14	18 480	26 880	42 000
	12×16	21 120	30 720	48 000
	12×18	23 760	34 560	54 000
14	14×14	21 560	31 360	49 000
	14×16	24 640	35 840	56 000
	14×18	27 720	40 320	63 000
	14×20	30 800	44 800	70 000
16	16×16	28 160	40 960	64 000
	16×18	31 680	46 080	72 000
	16×20	35 200	51 200	80 000
	16×22	38 720	56 320	88 000

* These values are taken from the New York building laws, 1925.

Proportions of Ribbed Bases. The HEIGHT H of this type of base should be approximately equal to the PROJECTION P, and the DIAMETER D equal to the diameter of the column. The projection C should be at least 3 in to permit the bolting of the column to the base. The THICKNESS of all parts of the casting should be the same and approximately equal to the thickness of the column-shell. There must be no thin webs, as they result in breakage from shrinkage-stresses.

Base-Plates for Steel Columns are usually made of STEEL PLATES and SHAPES. Cast-iron bases are sometimes used for very heavy columns. If conditions are favorable to the action of corrosion the cast iron is to be preferred.

The Area of Bearing-Plates under Beams and Girders is found in the same manner as the area of plates under columns. If the load on the beam is uniformly distributed over the beam or concentrated at its middle, the

required area of the plate is one-half the total load on the beam divided by the allowable bearing per unit of area on the masonry; but if the load is a moving load, the greatest possible end-reaction must be divided by the allowable bearing. For example, a heavily loaded truck standing near the end of the beam causes a pressure on the bearing-plate much greater than one-half its weight. The true reaction for the actual conditions must be found by the methods explained in Chapter IX.

The Thickness of the Bearing-Plate is found by the formula used to determine the flexure of beams. It must be determined in each case. For a typical case the forces acting are shown in Fig. 5, which represents a transverse

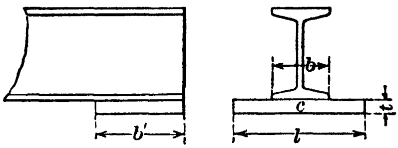


Fig. 4. Simple Bearing-plate under I Beam

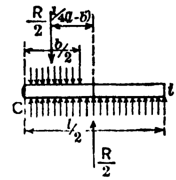


Fig. 5. Forces Acting on Half of Bearing-plate

vertical section through one-half the plate. The vertical section at C, and through and parallel with the web of the I beam, is taken through the center of the plate, which is the dangerous section, or section of maximum bending moment.

In Figs. 4 and 5, b' is the bearing depth on the wall;

l is the length of the plate, parallel with the wall;
b is the width of the flange of the beam;
R is the load on the bearing-plate.

Replacing the uniform loads by the equivalent forces at the center of gravity of each, these forces are represented by the longer arrows. The bending moment at the section at c is the same as the moment of the concentrated forces, giving,

$$M = (R/2 \times l/4) - (R/2 \times b/4)$$

or

$$M = R/2 \times (l - b)/4$$

This is equal to the resisting moment at the same section c, or, at stress S, SI/c, in which I/c is the section-factor. (See Chapter XV) This reduces to $St^2b'/6$. Equating the bending moment and the resisting moment there results

$$St^2b'/6 = R(l - b)/8$$

and

$$t = 0.866 \sqrt{R(l - b)/Sb'}$$

For $S = 3\,000$ for cast iron, this reduces to

$$t = 0.0158 \sqrt{R(l - b)/b'} \qquad (1)$$

For $S = 16\,000$ for steel plates, it becomes

$$t = 0.00685 \sqrt{R(l - b)/b'} \qquad (2)$$

Example 2. It is required to determine the length and thickness of a cast-iron bearing-plate under a wooden beam which is 10 in wide and supports a

load of 24 000 lb. The plate is 8 in wide and bears that width on a brick wall laid up in lime mortar.

Solution. The load on the plate is 24 000/2 = 12 000 lb From Table II, the area of the plate is 12 000/110 = 109 sq in. Hence, if the width of the plate is 8 in, its length must be $13\frac{5}{8}$ in. Then, from Formula (1)

$$t = 0.0158\sqrt{12\,000\,(13\tfrac{5}{8} - 10)/8} = 1.162 \text{ in}$$

A plate $1\frac{1}{4}$ in thick would be used.

Example 3. It is required to determine the length and thickness of a steel bearing-plate under the end of a 24-in 79.9-lb I beam supported on a 12-in brick wall laid up in lime-and-cement mortar and carrying a load of 60 000 lb. The width of the flange of the beam is 7 in. (See Table I)

Solution. The load on the plate is 60 000/2 = 30 000 lb
The area of the plate = 30 000/160 = $187\frac{1}{2}$ sq in
The length of the plate is 187.5/12 = 15.6 in

Then, from Formula (2)

$$t = 0.00685\sqrt{30\,000\,(15.5 - 7)/12} = 1 \text{ in}$$

Standard Sizes of Steel, Wall Bearing-Plates. These are given in Table II, and are based upon ALLOWABLE PRESSURES of 110, 160 and 250 lb per sq in. These UNIT PRESSURES are based upon the ALLOWABLE PRESSURES of the New York building laws, Table I. Because of the complicated formula on which the thickness depends it is best to compute the thickness for each case.

Bearing-Plates under Columns. The general rules already given for the proportions of RIBBED BASES similar to that shown in Fig. 3 are a sufficient guide for detailing such bases; but in case simple FLAT PLATES are used under columns, their thickness must be computed according to the principles governing bending. The stress in a FLAT PLATE supported at the middle and subjected to a uniform load cannot be determined by the ordinary methods of mechanics. The approximate solution here given is generally used in the design of BASE-PLATES and COLUMN-FOOTINGS. It gives values found to be safe in practice.

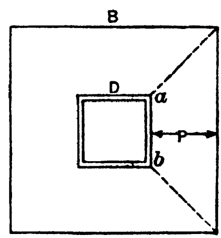


Fig 6. Flat Bearing-plate for Column

In Fig. 6, let B = the length of the side of the plate as determined by the allowable pressure on the supporting masonry;
D = the side or diameter of the column;
P = $(B - D)/2$ = the projection of the plate;
t = the thickness of the plate;
A' = the area of the plate outside the column;
w = the allowable bearing pressure on the masonry due to the load on the column.

Then in Fig. 6, the pressure on one-fourth of A', shown enclosed by the dotted lines in the figure, causes shearing and bending stresses in the section of the plate along the line ab. Considering the part enclosed and taking moments about the section ab, the following equation is obtained from the usual bending-moment formula (assuming the center of pressure at the outer edge instead of between $\frac{1}{2}$ and $\frac{2}{3}$ of P, which is true for a trapezoid). This

errs on the side of safety almost 100%. (See Chapter XV.) That is, the resisting moment equals the bending moment, or

$$SI/c = \tfrac{1}{4} A'Pw$$

For the rectangular section at ab, this may be written

$$St^2D/6 = \tfrac{1}{4} A'Pw$$

whence $t = \sqrt{3 A'Pw/2 SD}$

which becomes for $S = 3\,000$

$$t = 0.0224 \sqrt{A'Pw/D}$$

and for $S = 16\,000$

$$t = 0.0097 \sqrt{A'Pw/D}$$

Example 4. It is required to determine the size and thickness of a cast-iron bearing-plate to be used under a wooden post 12 in square in cross-section and designed for a load of 115 200 lb. The plate is to be set on brickwork laid in cement mortar in New York. (See Table I.)

Solution. The required area of the base is $115\,200/250 = 461$ sq in. $\sqrt{461} = 21.47$ and a 22-in square plate would be used.

Then $A' = 461 - 144 = 317$ sq in

$P = (22 - 12)/2 = 5$ in

$D = 12$ in

$w = 250$ lb per sq in

Hence $t = 0.0224 \sqrt{317 \times 5 \times 250/12} = 4$ in. The exact value is 2.96

This thickness may be beveled to $1\frac{1}{2}$ in at the edge. The computed thickness is greater than is usual for such plates; some formulas, having more practical constants, really assume a stress of about 10 000 lb per sq in in cast iron in bending.

If the plate is made of steel

$$t = 0.0097 \sqrt{317 \times 5 \times 250/12} = 1\tfrac{3}{4} \text{ in}$$

2. Bearing-Brackets on Cast-Iron Columns

The Usual Column-Connections for fastening beams and girders to cast-iron columns are shown in Fig. 7. The end of the beam or girder is set on a SHELF P, under which is a BRACKET-SUPPORT C, cast on the side of the column. For a single beam, one bracket is sufficient; for wide beams or girders there should be two ribs. The ends of the beams are fastened to the column by bolting to LUGS L, cast on the column above the bracket. Sometimes a column is fastened by bolts passing through the bottom flange of the beam and through the shelf-plate. This connection greatly decreases the lateral stability of a building and should not be used.

The Shelf and Brackets, when loaded, are subject to SHEARING and BENDING-STRESSES. The SHEAR at the outer surface of the column-shell is equal to the end-reaction of the beam it supports. The BENDING-STRESS is due to the application of the load on the shelf-plate at some distance from the surface of the column. It causes a tension at the top of the bracket which tends to tear out the shell of the column, and causes, also, a compression at the foot of the rib. The THICKNESS OF THE RIB must be great enough to withstand the compression from the load above; and since the stress is variable along a section, as along the line X, a rough approximation may be made by assuming

the stress at the extreme edge to be twice the average stress, and by further assuming that the section in the rib takes care of all the compression. This makes it unnecessary to find the CENTER OF GRAVITY and the MOMENT OF INERTIA of the section at X, both of which must be known if the FLEXURE-FORMULA is used. This procedure, also, makes unnecessary any assumption as to the true position of the CENTER OF PRESSURE on the top surface of the

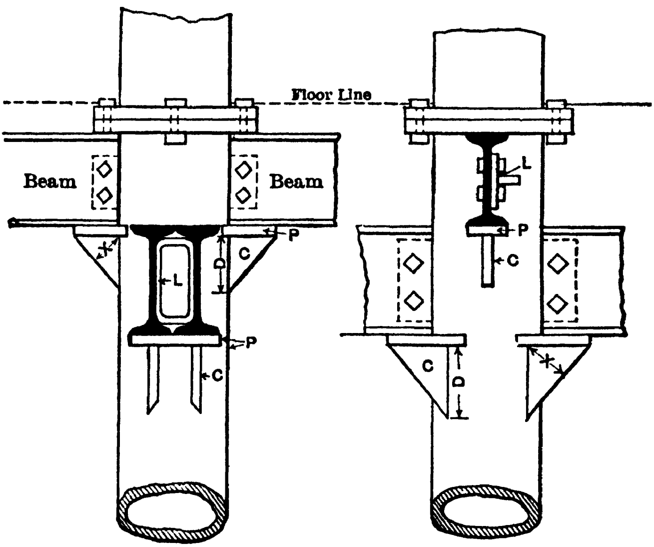


Fig 7. Cast-iron Columns with Bearing-brackets

bracket. With the thickness of rib given in the tables there is an ample FACTOR OF SAFETY for any load that may be applied through a beam. The double ribs are required when wide beams are used, not for strength, but to prevent the failure of the shelf from ECCENTRIC LOADING.

Tests of Cast-Iron Brackets. Brackets of cast-iron columns tested by the New York Building Department gave a SHEARING STRENGTH of 4 200 lb per sq in on the section at the column when the load was applied at the end of the bracket, and an average of 8 000 lb per sq in when the load was distributed over the bracket-shelf. The RANGE OF STRESS in the first case was from 2 450 to 5 600 and in the second from 4 100 to 10 900 lb per sq in. In seventeen out of twenty-two tests the MANNER OF FAILURE was the tearing out of a hole in the body of the column. It appears that when the thickness of the rib and shelf is the same as that of the shell of the column, there is generally ample strength for the support of beams and girders; but that in the case of very heavily loaded beams, the SHEARING and CRUSHING STRENGTH should be investigated. From the results of the tests mentioned, a low WORKING STRESS FOR SHEAR must be assumed.

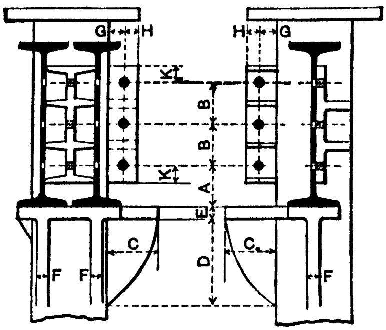
The Bevel of Brackets. If the shelf P (Fig. 7), on which the beam rests, is cast SQUARE with the column, when the beam deflects, the load is brought on the extreme end of the bracket, causing an increased bending-stress in the bracket and connections and tending to tear a hole in the column-shell. To

avoid this the bracket-shelf should be sloped downward, away from the column, and should have a BEVEL of $\frac{1}{8}$ in to the foot.

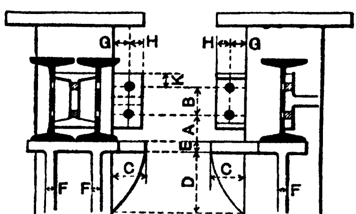
Standard Connections for Cast-Iron Columns. Table III, published originally in the Passaic Rolling Mill Handbook, and widely used by other manufacturers, will be found useful when detailing cast-iron columns.

Table III. Standard Connections for Cast-Iron Columns

All dimensions are in inches



Depth of beam	A	B	C	D	E	F	G	H	K	Thickness of lugs	Holes cored for $\frac{3}{4}$-in bolts
20	5	5	6	$10\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$1\frac{1}{2}$	2	1	
18	4	5	6	$10\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$1\frac{1}{2}$	2	1	
15	4	$3\frac{1}{2}$	$5\frac{1}{2}$	$9\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{3}{4}$	1	
12	3	3	$4\frac{1}{2}$	$7\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	



Depth of beam	A	B	C	D	E	F	G	H	K	Thickness of lugs	Holes cored for $\frac{3}{4}$-in bolts
10	$3\frac{1}{4}$	$3\frac{1}{2}$	4	7	$1\frac{1}{4}$	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	
9	3	3	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	
8	$2\frac{1}{2}$	3	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{4}$	
7	$2\frac{1}{4}$	$2\frac{1}{2}$	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	

CHAPTER XIV

STRENGTH OF COLUMNS, POSTS AND STRUTS

BY

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1. General Principles and Definitions

Compression members in structures are called COLUMNS, POSTS or STRUTS. In timber frame construction, light vertical compression members spaced close together in walls are called STUDS The term STANCHION, used in England to denote a column or post, has been adopted by the Bethlehem Steel Company to designate a series of special rolled columns of H-shaped section.

The ALLOWABLE LOAD which may be supported by a column depends upon the material in the column, the area of a cross-section and the slenderness-ratio of the column.

The SLENDERNESS-RATIO, L/r, of a column is the ratio of the length of the column between adjacent points of lateral support to the least radius of gyration of the cross-sectional area of the column with respect to the centroidal axis about which the column can bend. The RADIUS OF GYRATION * is equal to the square root of the quotient obtained by dividing the moment of inertia of a plane cross-section of the column about a centroidal axis about which the column can bend, by the cross-sectional area of the column.

A compression member whose slenderness-ratio, L/r, is less than about 30 is called a SHORT COLUMN, while one whose slenderness-ratio is greater than about 150 is called a LONG COLUMN. The slenderness-ratio of practically every column used in building-construction lies between these limits.

SHORT COLUMNS (L/r less than about 30) under axial load fail at nearly a constant unit stress which corresponds to the yield-point of ductile materials like steel or wrought iron, or to the crushing strength of brittle materials like concrete, cast iron or timber. The critical load at which a short column will fail depends only upon the area of the cross-section and the STRENGTH of the material in the member.

LONG COLUMNS (L/r greater than about 150) fail by buckling under a critical load which, uniformly distributed over the column cross-section, would not stress the material to its yield-point or its crushing strength. The critical load for a long column depends upon the STIFFNESS of the member, its LENGTH and the CONDITIONS OF LATERAL RESTRAINT at the ends or along the member. When the critical load is reached, the column is in unstable equilibrium and any slight displacement of the column axis laterally will increase progressively until the material at some point fails under the combined direct stress and bending.

* See Chapter X.

BUILDING COLUMNS seldom have a slenderness-ratio smaller than 30 and they should never have a slenderness-ratio greater than 120.* Classified according to their slenderness-ratios, building columns form a transition between short columns and long columns and may have some of the characteristics of one or the other. Building columns may fail by direct crushing of the material if they have the characteristics of short columns and by lateral buckling if they have the characteristics of long columns, or by local crippling followed by lateral buckling.

2. Column Formulas

Column formulas give a relation between the slenderness-ratio, L/r,† of a column of a given material and either the allowable working unit stress in the column, the axial load under which the column may be expected to fail, or the unit fiber-stress at which failure is likely to occur.

Formulas for Short Columns. The SAFE LOAD on a short column, the length of which does not exceed 30 times the least radius of gyration of its cross-sectional area, may be computed by the formula

$$P = f \times A \tag{1}$$

in which P is the safe axial load in pounds;
A is the area of cross-section in square inches;
f is the safe working stress in pounds per square inch.

Values of safe working stress, f, for various materials are given in Table I. Table I also gives values for the fiber-stresses, f_u, at which short axially loaded columns of various materials may be expected to fail. In order to determine the CRITICAL LOAD in pounds which, on the average, will cause failure of a short column of a given material, multiply the cross-sectional area of the column in square inches by the ultimate fiber-stress, f_u, given in Table I.

Example 1. What is the safe load which can be supported by a 10 in $\times$ 10 in long-leaf yellow pine column 8 ft long?

Solution. $L/d = 96/10 = 9.6$. Since this is less than 10, the values given in Table I may be used. The cross-sectional area $= 10 \times 10 = 100$ sq in;* the safe working stress (from Table I) $= 960$ lb per sq in; the safe load $= 960 \times 100 = 96\,000$ lb.

Example 2. What load will cause the above column to fail?

Solution. The critical load, f_u, which, on the average, will cause failure of short columns of long-leaf yellow pine is 5 000 lb per sq in (Table I). The cross-sectional area $= 100$ sq in;‡ the critical load, to cause failure, $= 5\,000 \times 100 = 500\,000$ lb.

Formulas for Long Columns which fail by pure buckling are called EULER FORMULAS. In structural design they are to be used only for timber columns

* Compression members which carry no calculated stress due to loads on a structure and those members which carry wind-stresses only are sometimes permitted to have slenderness-ratios as large as 150 to 200.

† Formulas for allowable stresses in timber columns are usually expressed in terms of L/d, in which L is the unsupported length of the column in inches, and d is its diameter or least width. The radius of gyration, $r = 0.29\,d$ for a solid square or rectangular section; $r = 0.25\,d$ for a solid circular section; and $r =$ (approximately) $0.32\,d$ for a hollow circular section whose shell thickness is 1/10 the outside diameter of the column section.

‡ The nominal section is here used for purposes of illustration. See Article 4, Timber, Columns.

whose length exceeds about 25 times the least width or about 80 times the least radius of gyration. The formula for safe loads on timber columns proposed by the Forest Products Laboratory of the United States Department of Agriculture uses the Euler formula for a column free to rotate at each end if the Euler formula gives fiber-stresses less than ⅔ of the safe fiber-stress on a short block of the same material.

Table I. Average Critical Loads and Safe Working Stresses in Pounds per Square Inch, for Building Materials

Materials*	Critical load f_u, pounds per square inch	Safe working stress, f, pounds per square inch
Cast iron	80 000	9 000
Wrought iron	25 000	10 000
Steel, rolled shapes (structural grade)	35 000	14 000
Steel alloy—3¼% nickel	50 000	20 000
Douglas fir	4 500	960†
Long-leaf yellow pine	5 000	960†
Short-leaf yellow pine	4 000	700†
Oak	5 000	790†
Norway pine	3 500	610†
White pine	4 000	610†
Hemlock	4 000	440†

* For stone, brick, concrete and masonry, see Chapter V

† Working stresses for wooden columns are for members whose length does not exceed 10 times its least width. For detailed information concerning stresses for timber, see Chapter XX.

Table II gives theoretical formulas for the ultimate strength of ideal long columns which fail by pure buckling. The safe working load for long columns is equal to the ultimate load calculated by the formulas of Table II divided by a factor of safety which depends upon the nature of the loading, the quality of the material in the column and other factors. The FACTOR OF SAFETY for sound timber columns should be at least 3 for best quality timbers under static loads and 6 to 8 or even higher for cross-grained columns or for columns which contain bad knots, checks or other defects.

Table III gives average values of E, the MODULUS OF ELASTICITY* of various materials, to be used in formulas for buckling loads on ideal long columns.

Example 3. What load can be carried on a long-leaf yellow pine column 4 in square and 10 ft long, using a factor of safety of 3? The column is free to rotate at each end.

Solution. $L/d = 120$ in $/4$ in $= 30$. Since L/d is greater than 25 the formulas given in Table II may be used for computing ultimate buckling loads. $r = 0.29 \times 4$ in $= 1.16$ in. $L = 120$ in. $A = 16$ sq in.† $E =$ 1 600 000 lb/sq in (Table III). The formula for the load P, in pounds, which will cause the column to buckle, is

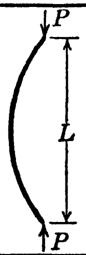
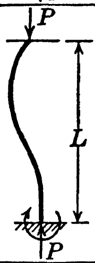
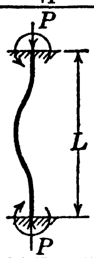
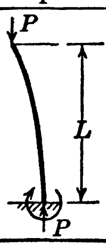
$$P = 9.87\, AE/(L/r)^2$$

$$P = \frac{9.87 \times 16 \times 1\,600\,000}{(120/1.16)^2} = 23\,600 \text{ lb}$$

* See Chapter I.

† Nominal section used for purposes of illustration.

Table II. Ultimate Loads for Ideal Long Columns

Condition of loading		Critical load
	Column free to rotate at each end	$P = 9.87\ AE/(L/r)^2$
	Column fixed at one end and free to rotate at the other end Ends restrained against lateral movement	$P = 20.1\ AE/(L/r)^2$
	Column fixed at both ends	$P = 39.5\ AE/(L/r)^2$
	Column fixed at one end Other end free to move in any direction	$P = 2.47\ AE/(L/r)^2$

A = cross-sectional area of column (gross section) in square inches;
r = radius of gyration of column cross-section with respect to axis about which column buckles, in inches;
E = modulus of elasticity of material in column, pounds per square inch;
L = length of column in inches;
P = critical or buckling load, in pounds.

Table III. Modulus of Elasticity of Building Materials

Materials	E in pounds per square inch
Cast iron	12 000 000
Wrought iron	28 000 000
Steel, all grades	29 000 000
Douglas fir	1 200 000–1 600 000
Southern (long-leaf) yellow pine	1 600 000
Western yellow pine	1 000 000
Oak	1 500 000
Norway pine	1 200 000
Hemlock	1 100 000–1 400 000

The ALLOWABLE LOAD on the column, using a factor of safety of 3, is equal to $P/3 = 23\,600$ lb $/3 = 7\,870$ lb.

Timber columns whose length exceeds 30 times the least width are not permitted by most building codes.

Formulas for Building Columns. The slenderness-ratio of building columns is seldom less than $L/r = 30$ and it is generally limited by building codes to a maximum value of $L/r = 120$ for main compression-members. Unimportant steel compression-members, and those carrying wind-loads only, sometimes have slenderness-ratios as great as $L/r = 200$, but this is about the maximum value allowed by existing building codes or design specifications.

There is no satisfactory theory by which the ultimate strength of building columns can be determined from known physical or elastic properties of the material of which the column is made. The relation between the strength of columns and the slenderness-ratio is, within the range of slenderness-ratios mentioned, strictly an empirical one based on laboratory tests of columns. Even the results obtained by laboratory tests of full-sized columns are far from satisfactory for determining a reasonably precise relation between the slenderness-ratio and the ultimate strength or allowable working strength of the column. The variations in the test data have led to many column formulas, each presumably in agreement with the data considered, giving wide variations in allowable working stresses. A number of the column formulas in general use for steel columns are given in Table XIII and are represented graphically in Fig. 7.

The greater number of column formulas used in practice are either (1) STRAIGHT-LINE FORMULAS or (2) FORMULAS OF THE RANKINE TYPE. A few design specifications give formulas of a more complicated type such as (3) the SECANT FORMULA, and one or two specify the use of (4) PARABOLIC COLUMN FORMULAS.

The Straight-Line Formulas are the simplest of the column formulas. Any curve can, within proper limits of one of the variables, be replaced by a straight line which is very nearly equivalent to the curve. If the curve happens to be a reversed or ogee curve, such as those representing the Rankine or secant column formulas, it can be very nearly replaced by a straight line over a considerable range of one of the variables. Straight-line column formulas are based on no theoretical considerations whatever, but it would be difficult to show any valid argument that, within properly selected limits of slenderness-ratios, the straight-line formulas are in less satisfactory agreement with the results of tests than any other column formulas.

The Rankine Column Formulas are based on the following quasi-theoretical considerations: An axially loaded column which was originally straight is deflected laterally under load as shown in Fig. 1. The stresses in the outermost fibers of the column on its concave side are made up of a direct stress, P/A, and a flexual stress, Pyc/I, in which A is the cross-sectional area of the column; P the applied axial load; y, the deflection of the axis of the column from its initial (straight) position; c, the distance from the axis about which the column bends to the outermost fiber of the column; and I, the moment of inertia of a cross-section of the column with respect to the axis about which the column bends. The fiber-stress will be a maximum at the section which has the greatest deflection (assuming the column has the same section throughout its length), and this stress will be:

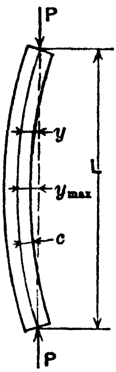


Fig. 1. Axially Loaded Column

$$f_{max} = P/A + Py_{max}\,c/I \;*$$

and since $I = Ar^2$, r being the radius of gyration of the area,

$$f_{max} = P/A\,(1 + y_{max}\,c/r^2)$$

This formula may be written

$$\frac{P}{A} = \frac{f_{max}}{1 + k\left(\frac{L}{r}\right)^2} \tag{2}$$

where k is a constant which is to be determined empirically on the basis of column tests. Column formulas of this type are called Rankine column formulas and are specified by various building codes and design specifications. See Table XIII.

The Secant Formula is said to have been helpful in interpreting certain test data on steel and wrought-iron columns. It is based on the assumption that the load is eccentric a known amount, e, at the ends of the column, that the column has the same section throughout, and that there is no external restraint against bending either at the ends or along the length of the member. Under these conditions the maximum fiber-stress on the concave side of the bent column at mid-height will be

$$f_{max} = P/A\left[1 + \frac{ec}{r^2}\sec\frac{L}{2r}\sqrt{\frac{P}{AE}}\right] \tag{3}$$

Since the last term in the parenthesis vanishes if the load P is axial, e being equal to zero in this case, a small arbitrary eccentricity (say 0.3 in) is sometimes assumed which gives the secant formula the following form:

$$f_{max} = P/A\left[1 + \left(\frac{ec}{r^2} + 0.3\right)\sec\frac{L}{2r}\sqrt{\frac{P}{AE}}\right] \tag{4}$$

Where this formula is specified for design of columns, the maximum allowable fiber-stress f_{max} is specified and the average fiber-stress P/A must be calculated by trial. Curves representing the secant formula have the same general shape as curves representing formulas of the Rankine type, and the difference between the two types of formulas from the designer's standpoint is of no importance.

The secant formula is given in the Cleveland Building Code.

* See Chapter XV.

Parabolic Column Formulas are of the type

$$P/A = f_{max} - k(L/r)^2 \qquad (5)$$

and are strictly empirical. At the present time this type of column formula is not generally specified as a basis for designing building columns.*

3. Stresses in Columns

Axially Loaded Columns. If a column is axially loaded, that is, if the resultant load coincides with the axis of the unbent column, the fiber-stress in the column, in pounds per square inch, is obtained by dividing the axial load, P, in pounds, by the cross-sectional area of the column, A, in square inches.

$$f = \frac{P(\text{axial})}{A} \qquad (6)$$

Eccentrically Loaded Columns. If the resultant load acting on a column does not coincide with the axis of the column, the column will bend slightly and the load is said to be eccentric. Girders and beams which frame into the flange or web of a column actually cause eccentric loads on the column, but in steel design this eccentricity of loading is usually neglected. Crane girder loads or other heavy loads which are supported on brackets connected to a column are eccentric, and this eccentricity should be taken into consideration in calculating the maximum fiber-stresses in the column.†

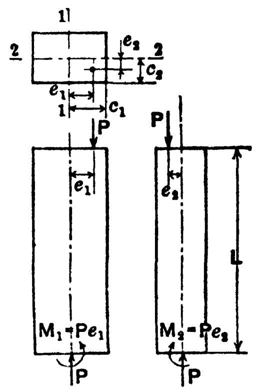


Fig. 2. Eccentrically Loaded Column

Suppose a column is loaded as shown in Fig. 2. The two axes 1–1 and 2–2 are principal axes of the column cross-section. If the column section is symmetrical about one axis, the other axis is at right-angles thereto. Both axes pass through the centroid of the area of cross-section. Let e_1 and e_2 be the eccentricity, in inches, of the load P, in pounds, with respect to the axes 1–1 and 2–2, respectively, A the area of cross-section, in square inches, I_{1-1} and I_{2-2} the moment of inertia ‡ of the cross-section about the two axes, in inches^4, and c_1 and c_2 the distances, in inches, from the axes to the outermost fibers on the compression side of the column. The maximum fiberstress in the column, in pounds per square inch,

$$f_{max} = (P/A) + (Pe_1c_1/I_{1-1}) + (Pe_2c_2/I_{2-2}) \qquad (7)$$

* Formulas of this type are given in the Progress Report of the Special Committee on Steel Column Research. Proceedings Am. Soc. C. E., March, 1926, pages 146–209. The formula $P_{max} = S\left[1 - \frac{1}{3}\left(\frac{L}{Kd}\right)^4\right]$ recommended for timber columns by the Forest Products Laboratory, United States Department of Agriculture, is of the same general form.

† See Chapter XV for method of calculating bending moments due to transverse loads.

‡ See Chapter X.

However, Pe_1 and Pe_2 are the bending moments, in inch-pounds, of the load about the axes 1–1 and 2–2, respectively, and are hereafter designated by M_1 and M_2. This equation may be rewritten as follows:

$$Af_{max} = P + (M_1/k_1) + (M_2/k_2) \tag{8}$$

in which $k_1 = I_1/Ac_1$ and $k_2 = I_2/Ac_2$, the "kern distances" * in inches with respect to axes 1–1 and 2–2. For a rectangular section, the kern distance $k = d/6$ and for a circular section $k = d/8$, where d is the diameter or dimension of the section, in inches, perpendicular to the axis considered. The value k is also called the BENDING FACTOR, and values of this factor for some column sections are given in the tables. If f_{max} is the allowable fiber-stress in the column, then Af_{max} is the maximum allowable axial load, in pounds, which the column can support. It is evident, therefore, that M_1/k_1 and M_2/k_2 are EQUIVALENT AXIAL LOADS which would produce the same maximum fiber-stresses as the given bending moments.

If the Column Supports Several Loads, either Axial or Eccentric, this equation may be written in its general terms as follows:

$$Af_{max} = \Sigma P + (\Sigma M_1/k_1) + (\Sigma M_2/k_2) \tag{9}$$

in which ΣP is the sum of all the loads on the column, in pounds;
ΣM_1 is the sum of the moments of loads about axis 1–1, in inch-pounds;
ΣM_2 is the sum of the moments of loads about axis 2–2, in inch-pounds;
k_1 and k_2 are the bending factors for the axis 1–1 and axis 2–2, respectively, in inches;
f_{max} is the allowable fiber-stress in the column, in pounds per square inch, calculated from the column formula given in the specifications used, and
A is the area of the column cross-section in square inches.

Example 4. A short hollow circular column 10 in in outside diameter has a shell thickness of 1 in and supports an axial load of 165 000 pounds. What is the fiber-stress in the column?

Solution. The area of a hollow circular section of uniform wall thickness is equal to $\pi\left(\frac{d_i + d_o}{2}\right)t$, in which $\left(\frac{d_i + d_o}{2}\right)$ is the average of the inside and outside diameters, in inches, and t is the shell thickness in inches. Hence, the area is $3.1416 \times 9 \times 1 = 28.3$ sq in. The fiber-stress in the column, $f = 165\,000/28.3 = 5\,830$ lbs per sq in.

Example 5. A round timber column 12 in in diameter supports an axial load of 60 000 lb and a load of 5 000 lb on a bracket. The eccentricity of the 5 000-lb load is 9 in. What is the fiber-stress in the column?

Solution. The area of the column, $A = 0.7854 \times 12^2 = 113.2$ sq in. The kern distance or bending factor, $k = d/8 = 12/8 = 1.5$ in. The bending moment of the eccentric load $= 5\,000 \times 9 = 45\,000$ in-lb.

* See Chapter X. The kern distance, k, in inches, with respect to a given axis is equal to the section-modulus of the area with respect to that axis divided by the area in square inches.

Total load on column:	
Axial load	60 000 lb
Eccentric load	5 000
Bending moment, $M = 45\ 000$ in-lb:	
$M/k = 45\ 000/1.5$	30 000*
Total equivalent axial load	95 000 lb
Maximum fiber-stress = 95 000/113.2 =	840 lb per sq in

Example 6. An 8 × 12-in timber column supports a total load of 50 000 lb. The bending moment about an axis perpendicular to the 12-in face is 20 000 in-lb and the bending moment about an axis perpendicular to the 8-in face is 1 000 ft-lb. What is the maximum fiber-stress in the column?

Solution. Area = 8 × 12 = 96 sq in.† Kern distance or bending factor for bending about axis perpendicular to 12-in face = 12/6 = 2.0 in; for bending about axis perpendicular to 8-in face = 8/6 = 1.333 in.

Total load on column	50 000 lb
Bending about axis perpendicular to 12-in face:	
$M = 20\ 000$ in-lb	
$k = 2.0$ in	
$M/k = 20\ 000/2.0$	10 000
Bending about axis perpendicular to 8-in face:	
$M = 1\ 000$ ft-lb $= 12\ 000$ in-lb	
$k = 1.333$	
$M/k = 12\ 000/1.333$	9 000
Total equivalent axial load	69 000 lb
Maximum fiber-stress = 69 000/96 =	719 lb per sq in

4. Timber Columns

A. General. The NOMINAL SIZES of commercial timbers used for posts and columns range from 6 × 6 in to about 18 × 18 in, intermediate sizes being in multiples of 2 in in each lateral dimension. The ACTUAL DIMENSIONS of dressed timbers are ½ in less than their nominal dimensions.

In designing timber compression-members, three FACTORS must be kept in mind: (1) The ratio of the length of the member to its least lateral dimension should not exceed 30 (Chicago Code).‡ (2) The maximum fiber-stress in the member parallel to the grain should not exceed the allowable stress given in the design specifications (see Table IV). (3) If other timber members, such as bolsters, beams or truss members, are bearing on the ends of the column the bearing stress perpendicular to the grain in the connecting members must not exceed the allowable stresses given in Table IV, Column 6. If metal column caps and column bases are used the last factor mentioned will ordinarily have no effect on the size of the compression member.

B. Formulas for Timber Columns. The formulas for maximum allowable fiber-stresses in timber columns as specified by various building codes are given in Table IV. The differences between these formulas for a given

* An axial load of 30 000 lb produces a fiber-stress of 30 000/113.2 = 265 lb per sq in in the column. A bending moment of 45 000 in-lb produces a maximum fiber-stress of 45 000/169.8 = 265 lb per sq in in a circular section 12 in in diameter. The section-modulus of a circular section, $I/c = \frac{1}{8} Ad = 169.8\ in^3$. See Chapter X.

† This assumes the actual size of timber to be 8 × 12 in. If the nominal size is 8 × 12 in, the actual size is 7½ × 11½ in, as explained in the next section.

‡ Boston Code (1926) and the Philadelphia Code (1929) allow a maximum unsupported length of 40 diameters while Seattle Code (1927) limits the length to 24 diameters.

species of timber are of more apparent than real importance in design. In general, one will find that, for a given load to be supported by a column of a given material, the same commercial size of timber column will be required regardless of the particular formula used in the design. It may be said, however, that the allowable fiber-stresses according to the Chicago Building Code are slightly less over the working range than the allowable stresses according to the formula recommended by the Forest Products Laboratory of the United States Department of Agriculture.

In the formulas given in Table IV,

f is the maximum allowable fiber-stress in the column, in pounds per square inch; f may not exceed the value given in Column 4 of the table;

L is the unsupported length of the column, in inches, and

d is the least width or diameter of the column, in inches.

Table IV also gives maximum allowable compressive stresses on short blocks loaded parallel to the grain (Column 5), and on timbers loaded perpendicular to the grain (Column 6). The last column in the table (Column 7) gives maximum allowable shearing-stresses parallel to the grain. All stresses given in the table are in pounds per square inch.

C. Tables of Safe Loads for Timber Columns. Table V gives the safe load in kips * for square timber columns of various sizes and lengths and of different kinds of wood. Table VI gives safe loads for rectangular timber columns of the same materials. Both tables are computed on the basis of the formulas specified by the Chicago Building Code. Table VII gives the safe axial load in kips for square yellow pine columns of various sizes and lengths calculated on the basis of the formulas recommended by the Forest Products Laboratory of the United States Department of Agriculture. The values 900, 1 000, 1 200 and 1 300 at the top of the last four columns of the table are the maximum unit stresses, in pounds per square inch, allowed by the formula for short blocks loaded parallel to the grain.

Table IV. Maximum Allowable Fiber-Stresses in Timber Columns

Building code	Timber	Maximum allowable fiber stress, f, pounds per square inch	Not to exceed	Maximum stresses in pounds per square inch		
				Bearing with grain	Bearing across grain	Shear with grain
Boston (1926)	Southern yellow pine, dense	$1\,200-20\frac{L}{d}$	1 000	1 200	350	150
	Southern yellow pine, sound	$900-15\frac{L}{d}$	750	900	250	100
	Douglas fir, sound	$1\,000-17\frac{L}{d}$ (approx)	840	1 000	200	100
	Spruce	$740-12\frac{L}{d}$	620	750	200	100
	White pine	$700-12\frac{L}{d}$ (approx)	585	700	200	80
	White oak	$900-15\frac{L}{d}$	750	900	500	200
		Max. $\frac{L}{d}=40$				

* One kip is equal to 1 000 lb.

Table IV (Continued). Maximum Allowable Fiber-Stresses in Timber Columns

Building code	Timber	Maximum allowable fiber stress, f, pounds per square inch	Not to exceed	Maximum stresses in pounds per square inch		
				Bearing with grain	Bearing across grain	Shear with grain
Chicago (1924)	Douglas fir and long-leaf yellow pine	$1\,100\left(1-\frac{L}{80\,d}\right)$		1 100	250	130
	Oak	$900\left(1-\frac{L}{80\,d}\right)$		900	500	200
	Short-leaf yellow pine	$800\left(1-\frac{L}{80\,d}\right)$		800	250	120
	Norway pine	$700\left(1-\frac{L}{80\,d}\right)$		700	200	80
	White pine	$700\left(1-\frac{L}{80\,d}\right)$		700	200	80
	Hemlock	$500\left(1-\frac{L}{80\,d}\right)$		500	150	60
Philadelphia (1925)	Long-leaf yellow pine	$750\left(1-\frac{L}{100\,d}\right)$		750	92	67
	Spruce	$500\left(1-\frac{L}{100\,d}\right)$		500	50	50
Factor of Safety = 6	Hemlock	$350\left(1-\frac{L}{100\,d}\right)$		350	42	42
New York (1925)	Long-leaf yellow pine, white oak	$1\,200\left(1-\frac{L}{60\,d}\right)$		1 600 1 400	1 000 1 000	150 200
	Short-leaf yellow pine, N C pine, Douglas fir	$900\left(1-\frac{L}{60\,d}\right)$		1 000 1 000 1 200	800 800 800	100 100 100
	Spruce, White pine, Fir	$900-17\frac{L}{d}$		1 200 1 000 1 000	800 800 800	100 100 100
Seattle (1927)	Douglas fir	$1\,600\left(1-\frac{L}{70\,d}\right)$ $\frac{L}{d}$ less than 24	1 200	1 600	400	200
	Spruce	$800\left(1-\frac{L}{70\,d}\right)$	600	800	300	130
	Western hemlock	$1\,400\left(1-\frac{L}{70\,d}\right)$	1 150	1 400	350	180
San Francisco (1926)	Oregon pine	$1\,300-20\frac{L}{d}$	1 000	...	...	...
	White Pine, Spruce	Cols under $\frac{L}{d}=15$ " $f=700$	700	800	200	100
	Douglas Oregon yellow fir	" $f=1\,000$	1 000	1 600	300	150
	Washington or Red fir	" $f=800$	800	900	250	125
	Redwood	" $f=700$	700	800	200	100

Table V. Safe Axial Load in Kips for Square Timber Columns

Chicago Building Code (1924) $f = C\left(1 - \frac{L}{80\,d}\right)$

Note: Safe load on round columns is 0.785 times the safe load on square columns of the same diameter.

Nominal size, inches	Actual size, inches	Area, square inches	Length, feet	Douglas fir and long-leaf yellow pine $C = 1\,100$	Oak $C = 900$	Short-leaf yellow pine $C = 800$	Norway and white pine $C = 700$	Hemlock $C = 500$
6×6	5½×5½	30.3	8	26.0	21.2	18.9	16.5	11.8
			10	24.2	19.8	17.6	15.4	11.0
			12	22.4	18.4	16.3	14.3	10.2
			14*	20.6*	16.8*	15.0*	13.1*	9.3*
8×8	7½×7½	56.3	8	52.0	42.5	37.8	33.0	23.6
			10	49.5	40.5	36.0	31.5	22.5
			12	47.0	38.4	34.2	29.9	21.4
			14	44.5	36.4	32.4	28.4	20.2
			16	42.0	34.4	30.6	26.8	19.1
			18	39.6	32.4	28.8	25.2	18.0
			20†	37.2†	30.4†	27.1†	23.6†	16.9†
10×10	9½×9½	90.3	8	86.8	71.0	63.1	55.2	39.4
			10	83.6	68.4	60.8	53.2	38.0
			12	80.1	65.5	58.3	51.0	36.4
			14	77.2	63.3	56.2	49.2	35.1
			16	74.3	60.8	54.0	47.3	33.8
			18	71.0	58.1	51.6	45.2	32.2
			20	67.8	55.5	49.4	43.2	30.9
12×12	11½×11½	132.3	8	130.5	106.9	95.0	83.1	59.4
			10	127.0	103.8	92.2	80.7	57.6
			12	123.0	100.7	89.4	78.2	55.9
			14	119.0	97.4	86.5	75.7	54.1
			16	115.2	94.4	83.9	73.4	52.4
			18	111.3	91.1	81.0	70.9	50.6
			20	109.5	89.6	79.6	69.7	49.8
14×14	13½×13½	182.3	8	183	149.8	133.0	116.3	83.1
			10	178	145.9	129.7	113.3	81.0
			12	174	142.1	126.3	110.6	79.0
			14	169	138.5	123.0	107.8	76.9
			16	165	135.0	120.0	105.0	75.0
			18	161	131.5	116.9	102.2	73.0
			20	156	127.8	113.7	99.4	71.0
16×16	15½×15½	240.3	8	244	200	178	155.5	111.0
			10	238	195	174	152.0	108.5
			12	233	191	170	148.2	106.0
			14	229	187	167	145.8	104.0
			16	224	184	163	142.9	102.0
			18	219	179	159	139.4	99.5
			20	214	175	155	135.8	97.0

$^{*}\frac{L}{d} = 30.6$ $\quad ^{\dagger}\frac{L}{d} = 32.0$

Table VI. Safe Axial Load in Kips for Rectangular Timber Columns

Chicago Building Code (1924) $f = C\left(1 - \frac{L}{80\,d}\right)$

Nominal size, inches	Actual size, inches	Area, square inches	Length, feet	Douglas fir and long-leaf yellow pine $C = 1\,100$	Oak $C = 900$	Short-leaf yellow pine $C = 800$	Norway and white pine $C = 700$	Hemlock $C = 500$
6×8	5½×7½	41.3	8	35.4	29.0	25.8	22.6	16.1
			10	33.0	27.0	24.0	21.0	15.0
			12	30.5	25.0	22.2	19.4	13.9
			14	28.1	23.0	20.4	17.9	12.8
6×10	5½×9½	52.3	8	45.0	36.7	32.6	28.6	20.4
			10	41.9	34.2	30.4	26.6	19.0
			12	38.6	31.6	28.1	24.6	17.5
			14	35.5	29.0	25.8	22.6	16.1
6×12	5½×11½	63.3	8	54.4	44.5	39.5	34.6	24.7
			10	50.6	41.4	36.8	32.2	23.0
			12	46.8	38.2	34.0	29.8	21.2
			14	43.0	35.2	31.2	27.4	19.5
8×10	7½×9½	71.3	8	65.9	53.9	47.9	41.9	29.9
			10	62.8	51.3	45.6	39.9	28.5
			12	59.5	48.7	43.3	37.9	27.0
			14	56.5	46.2	41.0	35.9	25.6
			16	53.3	43.6	38.8	33.9	24.2
			18	50.2	41.0	36.5	31.9	22.8
			20	47.0	38.5	34.2	29.9	21.4
8×12	7½×11½	86.3	8	79.8	65.2	58.0	50.7	36.2
			10	76.0	62.1	55.2	48.3	34.5
			12	72.1	59.0	52.5	45.9	32.8
			14	68.4	55.9	49.7	43.5	31.0
			16	64.5	52.8	46.9	41.0	29.3
			18	60.8	49.7	44.2	38.6	27.6
			20	57.0	46.6	41.4	36.2	25.9
10×12	9½×11½	109.3	8	105.0	86.0	76.4	66.9	47.8
			10	101.2	82.9	73.6	64.5	46.0
			12	97.1	79.5	70.6	61.8	44.1
			14	93.6	76.6	68.1	59.6	42.5
			16	90.0	73.5	65.4	57.2	40.8
			18	86.0	70.4	62.5	54.7	39.1
			20	82.3	67.3	59.8	52.4	37.4
10×16	9½×15½	147.3	8	141.8	116.0	103.0	90.1	64.4
			10	136.5	111.7	99.3	86.9	62.0
			12	131.0	107.1	95.2	83.3	59.5
			14	126.3	103.3	91.8	80.4	57.4
			16	121.2	99.2	88.1	77.1	55.1
			18	116.0	94.8	84.3	73.8	52.6
			20	111.0	90.7	80.6	70.5	50.4

Table VII.* Safe Axial Load in Kips for Square Yellow-Pine Columns

Based on formula of the Forest Products Laboratory, United States Department of Agriculture, Division of Forestry. Square end bearing and symmetrically loaded.

$$f_{max} = C\left[1-\frac{1}{3}\left(\frac{L}{Kd}\right)^4\right] \qquad f_{Euler} = 0.274\, E/\left(\frac{L}{d}\right)^2$$

Nominal size, inches	Actual size, inches	Area, square inches	$\frac{L}{d}$	Length feet	Safe axial load in kips for given values of C			
					$C =$ 900	$C =$ 1 000	$C =$ 1 200	$C =$ 1 300
6×6	5½×5½	30 3	17.5	8	24 6	28 1	32 7	34 6
			21 8	10	23 4	25.1	27.2	27.9
			26 2	12	19 2	19 4†	19 4†	19 4†
			30.5	14	14 2†	14 2†	14.2†	14 2†
8×8	7½×7½	56 3	12 8	8	49 8	55 1	65 5	70 6
			16 0	10	48 5	53 4	62.8	66 9
			19 2	12	46 3	50 3	57 4	60 0
			22 4	14	42 5	45 6	48.6	49 0
			25 6	16	37 0	37 7	37.7†	37.7†
			28 8	18	29 7†	29 7†	29 7†	29.7†
			32 0	20	24.1†	24.1†	24 1†	24.1†
10×10	9½×9½	90.3	10 1	8	80 7	89 5	108 3	115 8
			12 6	10	80 0	88.4	105 0	113 6
			15 2	12	78 6	86 6	101 8	109.5
			17 7	14	76 4	83 5	96.4	102.4
			20 2	16	72 7	78.5	88 8	96.7
			22 7	18	67.8	71 3	76 9	76.5†
			25 3	20	60 5	62.3	62.0†	62.0†
12×12	11½×11½	132.3	8 3	8	118 7	131.7	157.9	170 9
			10 4	10	118 2	131 1	156 6	169 4
			12 5	12	117 2	129 7	154 4	166 4
			14 6	14	115 7	127 8	150 8	161 6
			16 7	16	113 3	124.3	145 2	154.7
			18 8	18	109 5	119.7	137.3	144 4
			20 9	20	104.7	113.1	125 4	128 9
14×14	13½×13½	182 3	7 1	8	163 9	181 9	218 0	236 2
			8 9	10	163 4	181.3	217 2	235 0
			10.7	12	162 7	180 4	215.6	232.9
			12 4	14	161.6	179 8	213.0	229.3
			14.2	16	159 8	176.8	209.1	224.6
			16 0	18	157 1	173 1	203 4	216.8
			17 8	20	153 7	168 6	194 6	206 1
16×16	15½×15½	240 3	6 2	8	216 0	240 0	288 0	311.7
			7 7	10	215 8	239 5	287.1	311 1
			9 3	12	215 4	238 8	286 0	309.2
			10 8	14	214 5	237 8	284 0	306.7
			12 4	16	213 0	235.9	280.8	303.0
			14 0	18	211 0	233 0	276.2	297.3
			15 5	20	208 4	229 7	269 6	288 9

* Adapted from Southern Yellow Pine Manual (1929).

† Determined by Euler's formula.

Use of Tables of Safe Loads. (1) **Axially Loaded Columns.** GIVEN: The length of the column, in feet, the load in kips to be carried and the kind of timber. *To find* the dimensions of the column required to carry the given load. In Table V (for square columns) or Table VI (for rectangular columns) note at the top of the last five columns in the table the kind of timber to be used. Follow the values down this column until the load to be carried, in kips, is found in the horizontal row which shows the given column length in the fourth column of the table. Find the dimensions of the section required in the first column of the table.

Example 7. What must be the size of a Douglas-fir column 14 ft long if it is to support an axial load of 50 000 lb?

Solution. Safe axial loads on Douglas-fir columns are shown in the fifth column of Tables V and VI. Note (Table V) that an 8 × 8-in column 14 ft long can support a load of 44 5 kips, a 10 × 10-in column 14 ft long can support a load of 77.2 kips, and (Table VI) that an 8 × 10-in column can support a load of 56.5 kips. Either an 8 × 10-in or a 10 × 10-in column is satisfactory.

(2) **Eccentrically Loaded Columns.** GIVEN: The length of the column, in feet; the kind of timber, the actual load in kips to be carried by the column, the maximum bending moments M_1 and M_2 about axes through the center of gravity of a column cross-section perpendicular to the faces 1 and 2 of the column. To FIND the dimensions of the column required to carry the given load.

Example 8. A post supports a total load of 60 000 lb including the reaction of 12 000 lb from a girder, this reaction being applied 7 in from the center line of the column. What should be the size of the column, complying with the Chicago Building Code, if it is made of Douglas fir and is 12 ft long?

Solution. The girder reaction, 12 000 lb, is eccentric to the axis of the column 7 in, producing a bending moment in the column of 84 000 in-lb. The total load on the column is 60 000 lb but, assuming the column to be 12 in wide, the equivalent axial load which will produce the same fiber-stress as the bending moment of 84 000 in-lb will be $M_1/k_1 = 84{,}000 \div (12/6) = 42\,000$ lb and the total equivalent axial load will be 42 000 + 60 000 lb = 102 000 lb. Referring to Table V it will be found that a 12 × 12-in column of Douglas fir, 12 ft long, will support an axial load of 123 000 lb. Hence, the column is satisfactory. (Strictly, $k_1 = d/6 = 11.5/6$ based on actual dimensions of the post. The difference, about 4%, is not important in this case since the bending stress is relatively small.)

Example 9. Fig. 3 shows a panel of a 60-ft Howe roof truss to be made of long-leaf yellow pine and to be designed according to the Chicago Building Code. The top chord member, 10 ft in length, carries a direct compressive stress of 30.0 kips and also supports a uniformly distributed load of 500 lb/ft over its length. Design the member.

Solution. The maximum bending moment due to the uniformly distributed load is equal to $WL/8 = \dfrac{10 \times 500 \times (10 \times 12)}{8} = 75\,000$ in-lb. The direct stress is 30 000 lb. Try a member 12 in deep. Then the equivalent axial load which would produce the same fiber-stress as the bending moment is equal to $M/(d/6) = 75\,000/(12/6) = 37\,500$ lb and the total equivalent axial load is 30 000 + 37 500 = 67 500 lb. Referring to Table VI it will be found that an 8 × 12-in post 10 ft long will support an axial load of 76 000 lb.

An 8 × 10 or a 6 × 12 would carry 62 800 lb and 50 600 lb, respectively, both of which would be too small for the member in question. An 8 × 12-in chord member is required.

D. Crushing of Wood Perpendicular to the Grain. If columns bear on timber girders or bolsters, the bearing should be proportioned so that the quotient obtained by dividing the load on the column by the area of contact between the column and the girder or bolster does not exceed the safe unit stresses given in the sixth column of Table IV headed Bearing across Grain.

E. Timber Column Details and Connections are shown in Chapter XXI. Whenever timber columns are used in tiers, one above another, metal post-caps should be provided between the ends of two abutting columns. The upper column should bear directly on the post-cap on the lower column

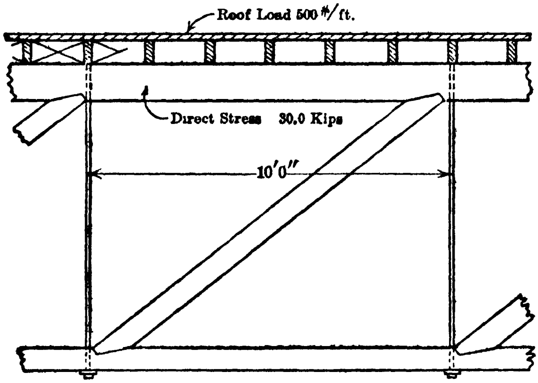


Fig. 3. Panel of a Howe Roof Truss

and not on the girder framing in at the joint. If a column extends two stories in height and supports girders at or near its middle, these girders may be supported on steel post-caps for continuous columns or on vertical bolsters keyed to or set in notches in the column. Bolsters and caps must be connected to the column by bolts. Wooden bolsters may be used on the top of columns which carry only beam or girder loads. Details of post-caps, bolsters and connections are shown in Chapter XXI.

5. Cast-Iron Columns

A. General. The use of CAST-IRON COLUMNS is generally restricted to buildings of which the height does not exceed twice the least width and to buildings less than 100 ft in height. Cast-iron columns should never be used in garages or other buildings where they may be subjected to heavy lateral forces or sudden loads. At the present time their use is generally limited to supporting balconies in auditoriums and as columns in low, heavy timber framed structures.

The Main Disadvantage of Cast Iron is that, although it has a very high crushing strength, 80 000 lb per sq in and even more, its dependable strength in tension and in shear is very low and uncertain. Thorough inspection of castings may fail to reveal internal defects, honeycomb or blowholes, cinders, foreign matter, etc., which may seriously affect the

strength of the member. Flaws in the metal and initial stresses due to shrinkage or unequal cooling of the metal may cause the material to fail under small suddenly applied loads. Cast iron is very brittle and cannot be punched or riveted; consequently all connections must be bolted. It is extensively used for making bases for steel and timber columns, for column caps and other details in heavy timber construction, but at the present time it is much less extensively used than steel as a material for columns in buildings.

The Advantages of Cast-Iron Columns are that they are relatively inexpensive; they can be obtained readily, and can be fitted with suitable brackets and flanges so that erection costs are reduced to a low figure. Further, tests and observations show that cast-iron columns resist fire better and corrode less easily than steel columns.

Cast-iron columns are usually of hollow, circular cross-section and are either cast vertically in sand molds or are cast centrifugally in metal molds.

B. Formulas for Cast-Iron Columns. Table VIII gives formulas for the maximum allowable compressive fiber-stress, f, in pounds per square inch, in cast-iron columns as specified by various building codes. Besides limiting the compressive stress in a column, specifications provide that cast-iron columns shall not be subjected to such eccentric loads or to such transverse loads that there is tension in any fiber of the column. Tab'e VIII also shows limiting values of the slenderness ratio, L/r, or the ratio L/d, which may not be exceeded for main members and for secondary members.

C. Tables of Safe Loads for Cast-Iron Columns. Table IX gives allowable axial loads in kips, on hollow round cast-iron columns of various sizes and lengths calculated on the basis of the Chicago Building Code formula,

$$f = 10\,000 - 60\,L/r \tag{10}$$

in which f is the maximum allowable compressive fiber-stress, in pounds per square inch;
L is the unsupported length of the column in inches;
r is the radius of gyration of the cross-sectional area of the column, in inches.

This table also shows values of the kern distance or bending factor, k, in inches, for each column section. The bending moment in the column due to eccentric or transverse loads, in inch-pounds, divided by the bending factor, k, in inches, gives the equivalent axial load which would produce the same fiber-stress as the given bending moment. In order that there shall be no tension in any fiber of the column due to the loads, the quotient obtained by dividing the bending moment by the bending factor must be less than the axial load.

Example 10. What must be the size of a round cast-iron column 12 ft in length to carry an axial load of 150 kips?

Solution. Referring to Table IX it will be found that a column 12 ft long, 8 in in outside diameter with a shell thickness of $1\frac{1}{8}$ in will carry an axial load of 158 kips, while one 12 ft long, 9 in in outside diameter with a $\frac{7}{8}$-in shell will carry 157 kips. Either column is satisfactory. The 9-in column is the stiffer of the two and weighs about 10% less than the 8-in column. From the standpoint of weight and rigidity, the 9-in column is preferable.

Example 11. A cast-iron column 16 ft long supports a total load of 110 kips. This is made up of an axial load of 80 kips and a girder reaction of 30 kips which has an eccentricity of 9 in. What must be the dimensions of the column according to the Chicago Building Code requirements?

Solution. Total load supported by the column = 110 kips. Bending moment of eccentric load = 30 × 9 = 270 kip-in. The bending moment in kip-inches divided by the bending factor, k, in inches, for the column to be used gives an equivalent axial load which would produce the same fiber-stress in that column as the given bending moment.

Table VIII. Maximum Allowable Fiber-Stresses in Building Columns

Wrought-Iron Columns

Building code	Max. fiber stress in lb per sq in (f)	f not to exceed	Limiting values of L/r	
			Main members	Secondary members
Chicago (1924)	$12\,000 - 60\,L/r$	10 000	....	....
Philadelphia (1925)	$\dfrac{12\,500}{1+\dfrac{L^2}{15\,000\,r^2}}$	.. ..	140 $L/d = 45$ max	. .... .

Cast-Iron Columns

Building code	Max. fiber stress in lb per sq in (f)	f not to exceed	Limiting values of L/r	
			Main members	Secondary members
Chicago (1924)	$10\,000 - 60\,L/r$	10 000	70	......
Boston (1926)	$9\,000 - 60\,L/r$	. .	70 $d = 6''$ min. $L/d = 24$	96 $L/d = 30$
New York (1925)	$9\,000 - 40\,L/r$	.	70	70
Philadelphia (1925)	$9\,000 - 40\,L/r$		70	.... .
Seattle (1927)	$10\,000 - 60\,L/r$	..	70	70
San Francisco (1926)	Round: $\dfrac{8\,000}{1+\dfrac{L^2}{800\,d^2}}$ Square: $\dfrac{8\,000}{1+\dfrac{L^2}{1\,067\,d^2}}$ Min. diameter or side, 5 in.	......	$L/d = 20$ $L/d = 20$	.. ..

f = maximum allowable compressive fiber-stress, in pounds per square inch;
L = unsupported length of the column, in inches;
r = radius of gyration of the cross-sectional area of the column, in inches;
d = outside diameter or least dimension of column cross-section, in inches.

Table IX. Safe Axial Loads in Kips for Round Cast-Iron Columns

Chicago Building Code (1924) $f = 10\,000 - 60\,\frac{L}{r}$

Outer diam, in	Thickness, in	Area, sq in	Weight, lb per ft	Least r in	Effective length in ft: 8	10	12	14	16	18	20	22	24	κ bending factor in in
6	¾	12.37	38.7	1.88	86	76	....	....	....	....	....	....	....	1.18
	⅞	14.09	44.0	1.84	97	86	....	....	....	....	....	....	....	1.13
7	¾	14.73	46.0	2.23	109	100	90	....	....	....	....	....	....	1.42
	⅞	16.84	52.6	2.19	124	113	102	....	....	....	....	....	....	1.37
	1	18.85	58.9	2.15	138	125	113	....	....	....	....	....	....	1.32
8	¾	17.08	53.4	2.58	133	123	114	104	....	....	....	....	....	1.66
	⅞	19.59	61.2	2.54	151	140	129	118	....	....	....	....	....	1.61
	1	21.99	68.7	2.50	169	157	144	131	....	....	....	....	....	1.56
	1⅛	24.30	75.9	2.46	186	172	158	143	....	....	....	....	....	1.51
9	⅞	22.34	69.8	2.89	179	168	157	145	134	....	....	....	....	1.86
	1	25.13	78.5	2.85	201	188	175	162	150	....	....	....	....	1.80
	1⅛	27.83	87.0	2.81	222	207	193	179	164	....	....	....	....	1.76
	1¼	30.43	95.1	2.78	241	226	210	194	178	....	....	....	....	1.72
10	1	28.28	88.4	3.20	232	219	206	194	181	169	....	....	....	2.04
	1⅛	31.37	98.0	3.16	256	242	228	214	199	185	....	....	....	2.00
	1¼	34.36	107.4	3.13	280	264	248	233	217	201	....	....	....	1.96
	1⅜	37.26	116.4	3.09	303	284	268	251	234	216	....	....	....	1.91
11	1⅛	34.90	109.1	3.51	292	278	263	249	235	221	206	....	....	2.24
	1¼	38.29	119.7	3.48	319	304	288	272	256	240	224	....	....	2.20
	1⅜	41.58	129.9	3.44	338	321	304	287	270	253	236	....	....	2.15
	1½	44.77	139.9	3.40	372	353	334	315	296	277	258*	....	....	2.10

* L/r greater than 70.

Table IX (Continued). Safe Axial Loads in Kips for Round Cast-Iron Columns

Chicago Building Code (1924) $f = 10\,000 - 60\frac{L}{r}$

Outer diam, in	Thickness, in	Area, sq in	Weight, lb per ft	Least r in	Effective length in ft									k bending factor in in
					8	10	12	14	16	18	20	22	24	
12	1¼	42.22	131 9	3.83	359	343	327	311	296	280	264	247	.	2.44
	1⅜	45.90	143.4	3 79	389	371	354	336	320	302	284	267	..	2.40
	1½	49.48	154 6	3.75	419	400	381	362	343	323	305	286*		2 34
	1⅝	52.97	165.5	3.71	448	427	406	386	366	345	324	304*	. .	2.30
13	1⅜	50.22	156.9	4.14	433	415	398	380	363	346	328	310	292	2 64
	1½	54 19	169 4	4.10	466	446	428	409	390	371	352	333	314*	2.58
	1⅝	58.07	181.5	4 06	498	478	458	437	416	396	376	354	334*	2 54
	1¾	61.85	193.3	4.03	530	508	486	464	441	416	398	376	354*	2 50
14	1½	58.91	184.1	4.45	512	494	475	455	437	417	398	379	361	2 83
	1⅝	63.18	197.4	4 41	549	529	508	488	466	446	426	405	384	2 78
	1¾	67.35	210 5	4 38	585	562	540	518	496	474	452	430	408	2 74
	1⅞	71.42	223 2	4.34	619	595	571	548	524	501	477	453	429	2 69
15	1⅝	68 29	213 4	4 76	600	580	559	538	517	497	476	456	436	3 02
	1¾	72.85	227 6	4 73	639	618	596	574	551	529	507	484	462	2.99
	1⅞	77.31	241.6	4 69	678	654	631	616	582	559	536	512	488	2.93
	2	81.68	255 3	4.65	716	690	665	640	615	590	564	539	514	2.88
16	1¾	78 34	244.8	5.08	695	672	650	628	605	584	561	539	517	3 22
	1⅞	83.20	260 0	5 04	737	713	690	666	641	618	594	571	547	3.18
	2	87.97	274 9	5 00	778	754	727	702	678	652	626	601	575	3.12
	2⅛	92 63	289 5	4 96	819	792	766	739	711	684	658	631	604	3 07

* L/r greater than 70.

Referring to Table IX it will be found that a 10-in column has a bending factor of approximately 2 0 in. The allowable axial loads on the 10-in columns listed which are 16 ft in length vary from 181 kips to 234 kips.

Try a 10-in column:

Total load on column	110 kips
Bending moment = 270 kip-in:	
Assume $k = 2.0$ in	
$M/k = 270/2.0$	135
	245
Total equivalent axial load $\left(\text{not less than } 2\dfrac{M}{k}\right) =$	270 kips

Note that $M/k = 135$ kips is greater than the total load on the column, which is only 110 kips. In order that there will be no tension in the column section, the load M/k must be less than 110 kips if the bending factor, k, was correctly assumed. A column 12 in in diameter with a $1\frac{1}{4}$-in shell will support a load of 296 kips and the bending factor for this column section is 2.44: $\dfrac{M}{k} = 110$.

This column section is satisfactory.

Table X shows dimensions and safe loads on Clow-National cast-iron columns calculated in accordance with the New York Building Code formula,

$$f = 9\,000 - 40\,L/r \tag{11}$$

The column sections marked S– are cast in vertical sand molds while those marked C– are centrifugally cast in metal molds by the de Lavaud process. Loads to the right of the heavy line are for columns whose slenderness-ratio, L/r, exceeds 70, which is the maximum slenderness-ratio allowed by the New York Building Code and others. The last column in the table shows values of the bending factor, k, in inches.

Example 12. Select from Table X a column, 16 ft in length, which will support an axial load of 80 kips in addition to a load of 30 kips on a bracket with an eccentricity of 9 in.

Solution. Try a 12-in column:

Total load on column:		110 kips
Axial load	80 kips	
Eccentric load	30	
Bending moment:		
$30 \times 9 = 270$ kip-in		
Assume $k = 2.54$ in (Section No. S–16)		
$M/k = 270/2.54 =$		106
Total equivalent axial load (not less than $2M/k$)		216 kips

Column No. S–16, which has an outside diameter of 12 in and a shell thickness of 1.00 in, has a bending factor $k = 2.54$ (as assumed) and can support a total axial load of 242 kips on a column 16 ft in length. The column section is satisfactory.

D. Design of Cast-Iron Columns. The outside diameter of cast-iron building columns should never be less than 5 in nor less than about one-twentieth * of the unsupported length of the column. The thickness of

* This is slightly more conservative than the requirement that the unsupported length of a cast-iron column may not be greater than 70 times its least radius of gyration.

Table X.* Safe Loads in Kips for Clow-National Cast-Iron Columns

New York Building Code: $f = 9\,000 - 40\frac{L}{r}$

Column Section No.	Inside diameter	Outside diameter	Thickness	Wt. per ft	Area, sq in	Radius of gyration	Effective column lengths in feet and safe loads in thousands of pounds									Bending factor k inches
							6′	8′	10′	11′	12′	13′	14′	15′	16′	
S- 1	4	4.9	.45	19.6	6.29	1.58	45.2	41.3	37.5	....	....	....	....	....	....	1.02
S- 2	4	5.0	.45	19.7	6.30	1.62	45.4	41.6	37.8	....	....	....	....	....	....	1.05
S- 3	4	5.0	.48	21.4	7.04	1.61	50.7	46.6	42.4	....	....	....	....	....	....	1.04
S- 4	4	5.0	.52	23.1	7.46	1.60	53.7	49.2	44.8	....	....	....	....	....	....	1.03
S- 5	6	7.0	.48	30.4	9.83	2.31	76.2	72.2	68.0	66.0	64.0	61.9	59.8	....	....	1.52
S- 6	6	7.1	.48	30.6	10.41	2.34	80.9	76.6	72.3	69.2	68.1	65.6	65.0	....	....	1.54
S- 7	6	7.1	.51	33.1	12.18	2.33	94.2	89.5	84.5	82.0	79.5	76.6	76.2	....	....	1.53
S- 8	6	7.1	.55	35.6	12.76	2.32	99.0	93.7	88.5	85.8	83.2	80.0	79.6	....	....	1.52
S- 9	8	9.0	.51	43.9	14.91	3.02	120.0	115.2	110.5	108.1	105.8	103.0	102.6	100.2	97.9	2.02
S-10	8	9.3	.60	51.5	20.32	3.08	163.9	157.6	151.2	148.1	144.9	141.2	140.7	137.5	134.3	2.04
S-11	8	9.4	.70	61.0	19.13	3.10	154.0	148.0	142.0	139.0	136.0	134.0	131.0	128.0	125.0	2.04
S-12	8	9.6	.80	69.0	22.12	3.12	178.6	171.8	165.0	161.6	158.2	154.8	151.4	148.0	144.6	2.03
S-13	8	10.0	1.00	88.2	28.70	3.20	229.0	221.0	212.0	208.0	203.0	199.0	195.0	191.0	187.0	2.05
S-14	10	11.1	.57	58.9	20.72	3.72	170.5	165.1	159.8	157.1	154.4	151.3	150.8	148.2	145.5	2.49
S-15	10	11.4	.68	71.2	29.96	3.79	222.2	215.3	208.5	205.1	201.7	197.7	197.1	193.7	190.3	2.52
S-16	10	12.0	1.00	108.0	34.45	3.90	284.0	276.0	268.0	263.0	259.0	255.0	250.0	246.0	242.0	2.54
S-17	12	13.2	.62	76.2	27.14	4.45	226.7	220.9	215.0	214.8	209.1	205.7	205.2	202.3	199.4	3.00
S-18	12	13.5	.75	92.7	30.04	4.51	251.1	244.8	238.4	235.2	232.0	228.3	227.7	224.5	221.4	3.01
C-19	4	4.8	.30	13.5	4.24	1.59	30.5	27.9	25.4	...	....	..	....	....	....	1.05
C-20	6	6.9	.31	20.4	6.42	2.33	49.8	47.2	44.6	41.9	....	....	....	....	....	1.57
C-21	6	6.9	.39	25.4	7.98	2.31	61.8	58.6	55.2	51.9	....	....	..	....	....	1.55
C-22	8	9.05	.46	39.5	12.41	3.04	99.9	96.0	92.1	88.2	....	...	..	....	....	2.04
C-23	10	11.1	.51	54.0	16.97	3.75	139.6	135.3	131.0	126.7	...			...	....	2.53

S numbers = sand cast. *C* numbers = centrifugally cast.

Column lengths to right of heavy line exceed $70r$.

* From Cast Iron Columns, published by James B. Clow & Sons, Chicago; National Cast Iron Pipe Co., Birmingham.

metal should not be less than one-twelfth the outside diameter of the column and many specifications provide that the shell thickness should never be less than ¾ in for columns less than 9 in in outside diameter.

Cast-iron columns should be so designed that eccentric loads or transverse loads on the column will not cause tensile stresses in any part of the section. They should not be used in places where they might possibly be rammed by vehicles or heavy machinery.

The method of determining the size of a cast-iron column necessary to support a given axial or eccentric load is explained in Article C of this section.

E. Connections and Details. Details of caps, bases and connections for cast-iron columns are shown in Chapter XIII.

Careful INSPECTION of cast-iron columns is an important and difficult matter. The material used in the columns should be of a good quality of gray iron free from blow holes, or other serious imperfections. The column should be faced to a plane surface at right-angles to the axis of the column.

6. Lally Columns

A. General. LALLY COLUMNS are patented columns made up of a cylindrical steel pipe shell filled with 1 : 1½ : 3 Portland-cement concrete. The light-weight column is 4 in in outside diameter with a shell thickness of 0.134 in, while the heavy-weight columns are from 3½ to 12¾ in in outside diameter with shell thicknesses of 0 216 to 0.375 in. The STANDARD TYPE of Lally columns is reinforced with only the steel pipe shell. SPECIAL TYPES of columns are obtainable with additional reinforcement consisting of steel pipe, reinforcing bars or structural steel shapes.

"Lally (steel pipe, concrete-filled) columns built of not smaller than 7-in standard steel or wrought-iron pipe with interior filled with not less than 1 : 4½ mixture of concrete have the same FIRE RESISTANCE period as cast-iron columns. They should be fitted with cast-iron caps. When reinforced with structural steel shapes embedded in the concrete filling, their fire resistance is nearly doubled." *

B. The Formula for Safe Loads on Lally Columns, as given in the manufacturers' catalogue, is

$$P = (A_c + 12\ A_s)\ (1\ 600 - 24\ L/d)\ \dagger \tag{12}$$

in which P is the safe axial load on the column, in pounds;
A_c is the cross-sectional area of concrete, in square inches;
A_s is the cross-sectional area of steel, in square inches;
L is the length of the column, in inches;
d is the outside diameter of the column, in inches.

C. Table of Safe Loads for Lally Columns adapted from the manufacturers' catalogue is given in Table XI. Table XI for Lally columns is analogous to Tables IX and X for cast-iron columns. The examples given in Section C, Article 5, for cast-iron columns serve to explain the use of Table XI.

D. Typical Connections and Details for Lally columns are shown in Fig. 4. The dimensions of standard steel base plates and gauges for plate-caps and bracket-caps are shown in Table XII.

* Fire Tests of Building Columns, Inspection Department of the Associated Factory Mutual Fire Insurance Companies, Boston.

† Lally Catalog, 10th Edition, (1921) Lally Column Company of Chicago.

Table XI. Safe Load in Kips for Lally Concrete-Filled Columns*

Based on formula $P = (A_c + 12\,A_s)(1\,600 - 24\,L/d)$

Diameter of columns, inches	Weight lb per ft filled	Area of steel, sq in	Area of concrete, sq in	Thickness of steel, inches	Limiting length, feet	Unbraced length, feet							Bending factor k, inches
						8	10	12	14	16	18	20	
4 Lt. Wt.	17	1.63	10.94	0.134	13.71	31.2	26.8	22.4	....	...	...	...	..
3½	15	2.23	7.39	0.216	11.64	32.3	26.7	....	....	....	..	...	0.83
4	20	2.68	9.89	0.226	13.37	43.1	37.0	30.9	...	....	..	.	0 92
4½	24	3.17	12.73	0.237	15.10	55.3	48.8	42.3	35.8	....	...	..	1.00
5	29	3.69	15.95	0.247	16.83	68.6	61.7	54.7	47.8	40.9	...	.	1.10
5½	36	4.30	20.01	0.258	18.78	84.6	77.1	69.6	62.1	54.6	47.1	....	1 19
6⅝	49	5.58	28.89	0.280	22.45	120.0	111.7	103.4	95.0	86.7	78.4	70.1	1.39
7⅝	64	6.92	38.74	0.301	25.92	156.9	147.8	138.6	129.7	120.5	111.4	102.3	1.56
8⅝	81	8.40	50.03	0.322	29.38	201.1	191.0	181.0	170.9	160.8	150.8	140.7	1.79
9⅝	100	9.97	62.79	0.342	32.84	248.3	237.4	226.5	215.6	204.6	193.7	182.8	1.89
10¾	123	11.91	78.86	0.365	36.74	307.2	295.4	283.5	271.6	259.7	247.9	236.0	2 08
12¾	169	14.58	113 10	0 375	43 77	408.8	395 8	382 8	369 7	356.7	343.6	330 6	2 38

*From Lally Column Company, Chicago.

Table XII. Standard Steel Bases and Gauges for Lally Columns*

Diameter of column, inches	g for plate caps, inches (See Fig. 4)	g for bracket caps, inches (See Fig. 4)	Standard steel base plates	
			Dimensions, inches	Weight, pounds
4 Lt Wt	...	...	8× 8×½	9
3½	3	4¼	8× 8×½	9
4	3	4½	8× 8×⅝	12
4½	3½	4¾	10×10×⅝	18
5	3¾	5	10×10×¾	21
5½	4	5¼	12×12×⅞	36
6⅝	4½	6¼	16×16×1	72
7⅝	5	6¾	18×18×1	92
8⅝	5½	7¼	18×18×1⅛	104
9⅝	6	7¾	20×20×1¼	142
10¾	6½	8¼	22×22×1¼	171
12¾	7	9	24×24×1½	245

* From Lally Column Company, Chicago.

7. Composite Columns

A. General. A column in which a concrete core enclosed by spiral reinforcement is further reinforced with a steel or a cast-iron core designed to support a part of the load is called a COMPOSITE COLUMN. Cast-iron cores may be either solid shafts or hollow pipe sections. Steel cores are usually made up of four angles latticed or battened. Typical sections of composite columns are shown in Fig. 5.

B. Factors in Design of Composite Columns. If a solid cast-iron core or a metal pipe core is used in a composite column, the OUTSIDE DIAMETER OF THE METAL CORE should not be greater than ½ the diameter of the spiral reinforcement in the column, and the outside surface of the core should never be less than 3 in from the spiral steel. If a steel core made up of latticed or battened angles is used in a composite column, the corners of the metal core should be at least 3 in from the spiral steel. The CROSS-SECTIONAL AREA OF THE METAL CORE should not be greater than 12% of the cross-sectional area of the column within the spiral reinforcement. The VOLUME OF THE STEEL COMPRISING THE SPIRAL should not be less than 1% of the volume of the column itself contained within the spiral reinforcement. The PITCH OF THE SPIRAL should not exceed ⅙ the diameter of the column inside the spiral reinforcement and should never be greater than 3 in. Not less than 6 LONGITUDINAL BARS (running lengthwise of the column) having a total

cross-sectional area of from 2 to 4% of the area of the column within the spiral should be spaced equidistantly immediately inside the spiral and should be wired to the spiral steel.

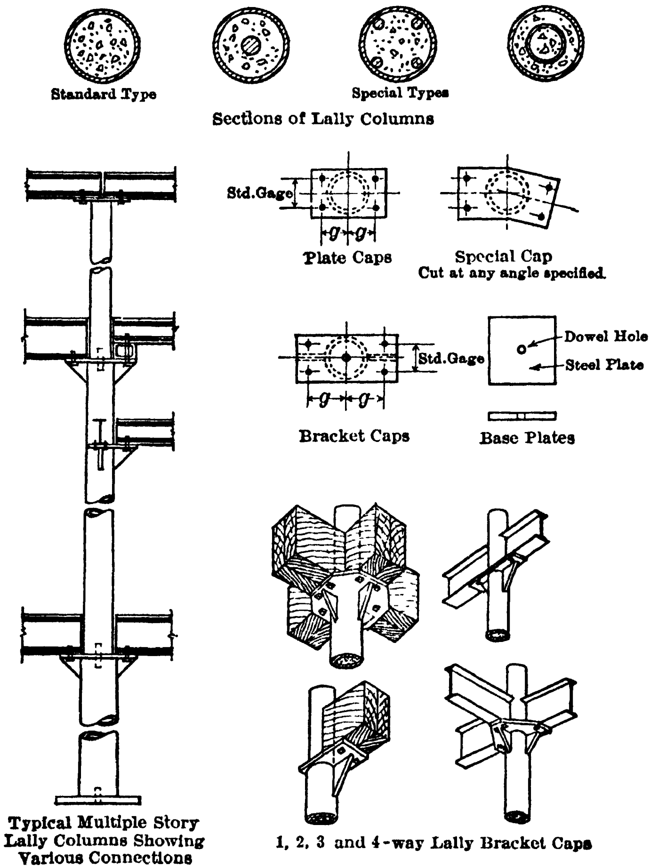


Fig. 4. Details and Typical Connections for Lally Columns

C. Regulations for the Design of Composite Columns are not entirely standardized. For data on the subject of reinforced-concrete columns see Chapter XXIII. A problem requiring special attention in the design of such columns is the detail of splices. This may be said also of the splices between reinforced-concrete and steel columns.

8. Steel Columns

A. Use and Advantages of Steel Columns. Steel columns have many advantages over cast-iron and other metal columns and have largely supplanted cast-iron columns even in the construction of small buildings.

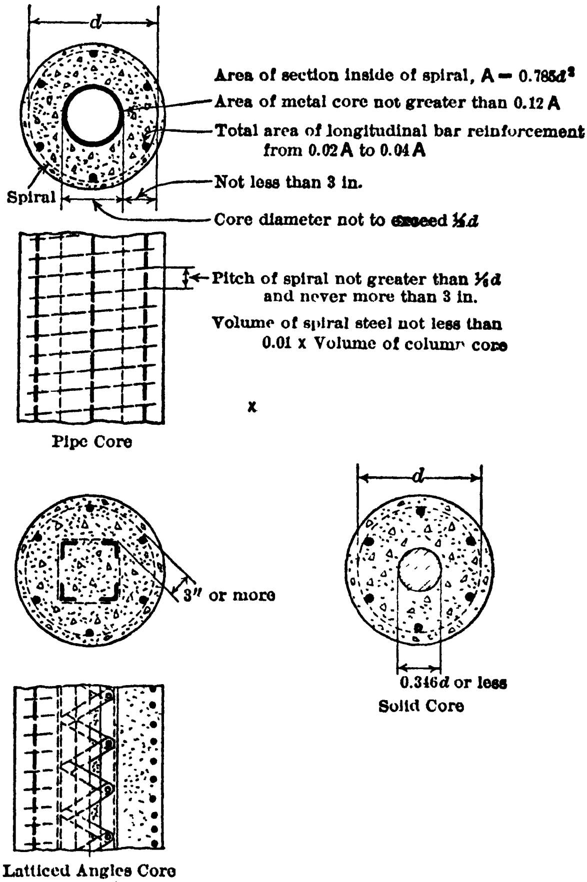


Fig. 5. Typical Sections of Composite Columns

Considered from the standpoint of DEPENDABILITY and safety, steel columns are unquestionably superior to cast-iron columns. A steel column may be damaged in a collision, badly bent out of line, and still support its load without collapsing IF THAT LOAD DOES NOT APPROACH THE CRITICAL LOAD FOR BUCKLING.* The steel column, by virtue of the toughness and ductility

* See Article 2, Chapter XIV.

of the material, has a very large RESERVE STRENGTH against ultimate collapse even if the yield-point of the material has been exceeded and a permanent deformation has taken place. Under the same conditions of lateral impact loading, a cast-iron column would be liable to snap before any appreciable deformation could take place and the member would be liable to fail suddenly. Suddenly applied loads which are eccentric enough to produce significant tensile stresses in the outer fibers of a column have much the same effect as lateral impact loads. Steel should be used for metal columns which may be subjected to heavy eccentric loads or which may be rammed by moving vehicles or heavy machinery.

B. General Notes on Steel Columns. In the following sections, types of steel columns, formulas for allowable stresses in them and methods for their design are discussed, and at the end of the chapter tables for allowable loads are given. As these are studied, the reader will realize that in order to carry a given load many different types of columns are available, that the formulas used to determine the required area vary greatly, that for the type chosen many different combinations may be used to give the required area and finally that the required area depends on the radius of gyration of the section as designed.

In spite of all these variables the tables will be found to be very helpful guides. They are, however, not intended to be exhaustive. In general they have been restricted to the lighter sections, to the rolled sections and to the simpler types of built sections.

Usually the type and approximate overall dimensions of column are first determined. A column can then usually be tentatively selected from the tables. If the allowable stress differs from that given by Formula (13), the section must be increased in inverse ratio to the stresses permitted. The type chosen depends on many factors. Thus a small local fabricator often prefers to build up a section from stock. Details of connections must always be considered. The designer, however, can usually select pretty closely the type, general dimensions, approximate make-up and approximate radius of gyration before going into refinements of design. This should be remembered in reading the discussion which follows.

C. Types of Steel Columns are shown in Fig. 6. Struts of one or two angles (*a*) are used for compression members in roof trusses, light towers and lattice girders. The two angles of a double angle strut are riveted together by rivets driven through washers placed between the two angles at intervals of 4 to 6 ft, or, more accurately, at such intervals that the unsupported $\frac{L}{r}$ of one angle does not exceed $\frac{L}{r}$ for the whole strut. Two or four angles starred (*b*) and connected by batten plates spaced at intervals of 3 to 4 ft are sometimes used to support light loads. LATTICED COLUMNS (*c*) made up of channels or angles connected by lattice bars (*d*) are often used where light loads are to be supported on long columns. ROLLED H-COLUMNS (*e*) are obtainable with depths ranging from 6 in to 16 in and are now commonly used instead of built-up columns in steel skeleton construction. BUILT-UP COLUMNS are usually of H-shaped section as shown in (*f*) although BOX COLUMNS with two or more webs (*g*) are not uncommonly used in heavy building frames. TOP CHORD SECTIONS of heavy trusses are usually unsymmetrical as shown in (*h*) and are made up of two rolled or built-up channel sections and a cover-plate. The open (bottom) side of the section is latticed. COLUMNS FOR BENTS (*i*) are sometimes made up of a pair of channels and an I-beam with

batten plates at intervals of 3 or 4 ft connecting the flanges of the channels. Columns made up of four angles and a web-plate are commonly used in mill building bents. BATTENED COLUMNS (*j*) in which two component parts of

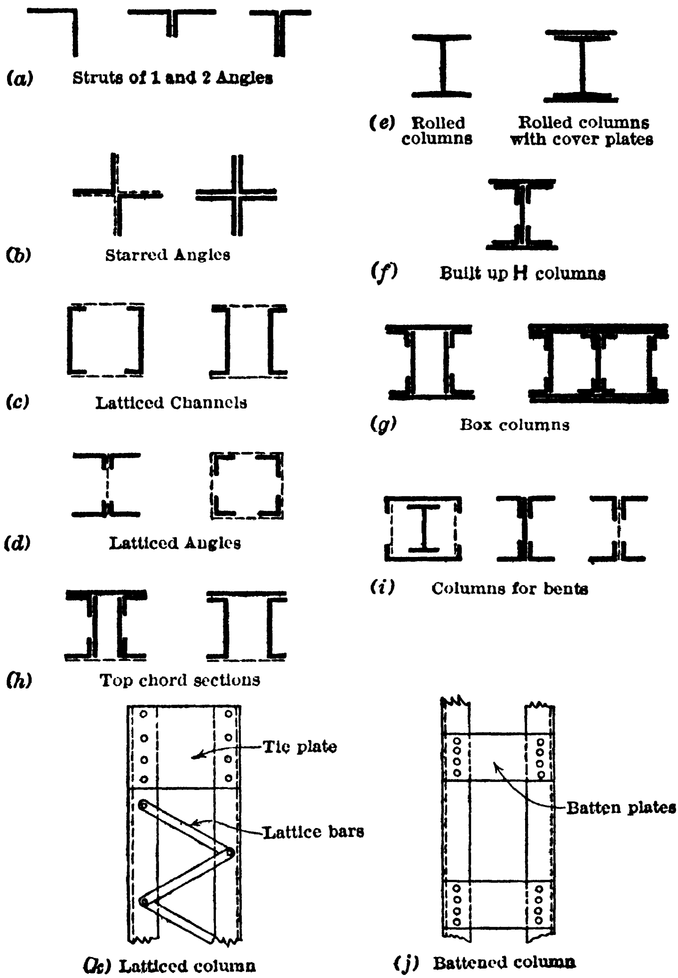


Fig. 6. Types of Steel Columns

a column are connected only by batten plates are decidedly inferior to LATTICED COLUMNS (*k*) and should be avoided if a continuous plate or latticing can be used instead.

D. Formulas for Steel Columns. Table XIII gives formulas for maximum allowable fiber-stresses in steel columns according to a number of building codes and design specifications. In these formulas:

f is the maximum allowable fiber-stress, in pounds per square inch;

L is the unsupported length of the column, in inches;

r is the least radius of gyration of the area of cross-section of the column with respect to an axis about which the column can bend, in inches.

The formula of the American Institute of Steel Construction:

$$f = \frac{18\,000}{1 + \dfrac{L^2}{18\,000\,r^2}} \tag{13}$$

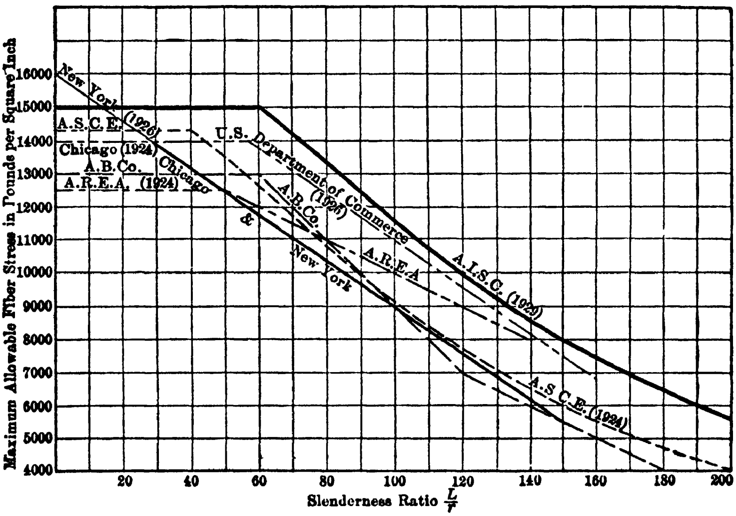


Fig. 7. Comparative Diagram of Formulas for Maximum Allowable Fiber Stresses in Steel Columns

has been adopted by a number of building codes recently. For slenderness-ratios less than $L/r = 60$ the maximum allowable fiber-stress is 15 000 lb per sq in. Slenderness-ratios of main members must not be greater than $L/r = 120$, and slenderness-ratios of secondary members must not exceed $L/r = 200$.

Approximate Rule for Design of Columns. The following APPROXIMATE RULE for determining the allowable fiber-stress in a column gives values of allowable fiber-stress which are less than 0.8 of one per cent in excess of those calculated according to the above formula for main members (L/r less than 120):

The allowable fiber-stress in pounds per square inch, for a steel column whose slenderness ratio, L/r, does not exceed 120, shall not be greater than 15 000 lb per sq in, or

$$20\,000 - 1\,000 \times \left\{\frac{\text{the length of the column in feet}}{\text{its least radius of gyration in inches}}\right\} \tag{14}$$

whichever gives the lower value.

Table XIII. Maximum Allowable Fiber-Stresses in Steel Columns

Building code	Maximum fiber-stress in lb per sq in	Not to exceed	Limiting values of $\frac{L}{r}$	
			Main members	Secondary members
American Institute of Steel Constr. (1929) San Francisco (1926) Philadelphia (1929)	$\dfrac{18\,000}{1+\dfrac{L^2}{18\,000\,r^2}}$	15 000	120	200
Boston (1926)	$\dfrac{18\,000}{1+\dfrac{L^2}{18\,000\,r^2}}$	13 500	160	200
Boston (1921)	$20\,000-100\dfrac{L}{r}$	12 000	160	...
American Bridge Co :				
$\frac{L}{r}$ less than 120	$19\,000-100\dfrac{L}{r}$	13 000	120	...
$\frac{L}{r}$ greater than 120	$13\,000-50\dfrac{L}{r}$	7 000	...	200
Chicago (1924) Seattle (1927)	$16\,000-70\dfrac{L}{r}$	14 000	120	150
New York (1926)	$16\,000-70\dfrac{L}{r}$	16 000	120	120
U.S. Dept. Commerce (1926)	$18\,000-70\dfrac{L}{r}$	14 000	160	160
A.R.E A. Railway Bridges (1920)	$15\,000-50\dfrac{L}{r}$	12 500	100	120
A.R E.A. Highway Bridges (1924)	$15\,000-50\dfrac{L}{r}$	12 500	120	140
A S.C.E. Railway Bridges (1923)	$\dfrac{16\,000}{1+\dfrac{L^2}{13\,500\,r^2}}$	14 300	100	120
A.S C.E Highway Bridges (1924)	$\dfrac{16\,000}{1+\dfrac{L^2}{13\,500\,r^2}}$	14 300	120	140

Table XIII (Continued). Maximum Allowable Fiber-Stresses in Pounds per Square Inch for Steel Columns

L = unsupported length of column, in inches
r = corresponding least radius of gyration, in inches

	A.I.S.C. San Francisco Philadelphia	Chicago* and New York†	U.S. Dept. of Commerce	A.S.C.E.	A. B. Co.	A.R.E.A.
L/r	$\dfrac{18\,000}{1+\dfrac{L^2}{18\,000\,r^2}}$	$16\,000-70\dfrac{L}{r}$	$18\,000-70\dfrac{L}{r}$	$\dfrac{16\,000}{1+\dfrac{L^2}{13\,500\,r^2}}$	$\dfrac{L}{r}<120$: $19\,000-100\dfrac{L}{r}$; $\dfrac{L}{r}>120$: $13\,000-50\dfrac{L}{r}$	$15\,000-50\dfrac{L}{r}$
30	15 000	13 900	14 000	14 300	13 000	12 500
32	15 000	13 760,	14 000	14 300	13 000	12 500
34	15 000	13 620	14 000	14 300	13 000	12 500
36	15 000	13 480	14 000	14 300	13 000	12 500
38	15 000	13 340	14 000	14 300	13 000	12 500
40	15 000	13 200	14 000	14 300	13 000	12 500
42	15 000	13 060	14 000	14 150	13 000	12 500
44	15 000	12 920	14 000	13 990	13 000	12 500
46	15 000	12 780	14 000	13 830	13 000	12 500
48	15 000	12 640	14 000	13 670	13 000	12 500
50	15 000	12 500	14 000	13 500	13 000	12 500
52	15 000	12 360	14 000	13 330	13 000	12 400
54	15 000	12 220	14 000	13 160	13 000	12 300
56	15 000	12 080	14 000	12 980	13 000	12 200
58	15 000	11 940	13 940	12 810	13 000	12 100
60	15 000	11 800	13 800	12 630	13 000	12 000
62	14 832	11 660	13 660	12 450	12 800	11 900
64	14 663	11 520	13 520	12 270	12 600	11 800
66	14 493	11 380	13 380	12 100	12 400	11 700
68	14 321	11 240	13 240	11 920	12 200	11 600
70	14 148	11 100	13 100	11 740	12 000	11 500
72	13 975	10 960	12 960	11 560	11 800	11 400
74	13 801	10 820	12 820	11 380	11 600	11 300
76	13 627	10 680	12 680	11 210	11 400	11 200
78	13 453	10 540	12 540	11 030	11 200	11 100
80	13 279	10 400	12 400	10 860	11 000	11 000
82	13 105	10 260	12 260	10 680	10 800	10 900
84	12 931	10 120	12 120	10 510	10 600	10 800
86	12 758	9 980	11 980	10 340	10 400	10 700
88	12 585	9 840	11 840	10 170	10 200	10 600
90	12 414	9 700	11 700	10 000	10 000	10 500

* Maximum f = 14 000 pounds per square inch

† Maximum $\dfrac{L}{r}$ = 120

Table XIII (Continued). Maximum Allowable Fiber-Stresses in Pounds per Square Inch for Steel Columns

L = unsupported length of column, in inches
r = corresponding least radius of gyration, in inches

	A.I.S.C. San Francisco Philadelphia	Chicago* and New York†	U.S. Dept. of Commerce	A.S.C.E.	A. B. Co.	A.R.E.A.
L/r	$\dfrac{18\,000}{1+\dfrac{L^2}{18\,000\,r^2}}$	$16\,000-70\dfrac{L}{r}$	$18\,000-70\dfrac{L}{r}$	$\dfrac{16\,000}{1+\dfrac{L^2}{13\,500\,r^2}}$	$\dfrac{L}{r}<120$ $19000-100\dfrac{L}{r}$ $\dfrac{L}{r}>120$ $13\,000-50\dfrac{L}{r}$	$15\,000-50\dfrac{L}{r}$
92	12 243	9 560	11 560	9 830	9 800	10 400
94	12 073	9 420	11 420	9 670	9 600	10 300
96	11 905	9 280	11 280	9 510	9 400	10 200
98	11 737	9 140	11 140	9 350	9 200	10 100
100	11 571	9 000	11 000	9 190	9 000	10 000
102	11 407	8 860	10 860	9 040	8 800	9 900
104	11 244	8 720	10 720	8 880	8 600	9 800
106	11 082	8 580	10 580	8 730	8 400	9 700
108	10 922	8 440	10 440	8 580	8 200	9 600
110	10 764	8 300	10 300	8 430	8 000	9 500
112	10 608	8 160	10 160	8 290	7 800	9 400
114	10 453	8 020	10 020	8 150	7 600	9 300
116	10 300	7 880	9 880	8 010	7 400	9 200
118	10 149	7 740	9 740	7 870	7 200	9 100
120	10 000	7 600	9 600	7 740	7 000	9 000
125	9 636	7 250	9 250	7 420	6 750	8 750
130	9 284	6 900	8 900	7 110	6 500	8 500
135	8 944	6 550	8 550	6 810	6 250	8 250
140	8 617	6 200	8 200	6 520	6 000	8 000
145	8 302	5 850	7 850	6 250	5 750	...
150	8 000	5 500	7 500	6 000	5 500	....
155	7 710	..	7 150	5 760	5 250	....
160	7 431	..	6 800	5 520	5 000	.....
165	7 164	...	.....	5 300	4 750	....
170	6 908			5 090	4 500	....
175	6 663		..	4 890	4 250	...
180	6 429	......	..	4 710	4 000	...
185	6 204	....	..	4 530	3 750	....
190	5 989	..	....	4 350	3 500	.....
195	5 783	...	....	4 190	3 250	.....
200	5 586	.....	....	4 040	3 000	.....

* Maximum f = 14 000 pounds per square inch

† Maximum $\dfrac{L}{r}$ = 120

E. Tables of Safe Loads for Steel Columns. Tables XIV to XX were taken from the handbook of the American Institute of Steel Construction either in the form there given or with some rearrangement. The values of the bending factors, k, shown for built-up sections were added by the writers. The tables show the SAFE LOAD on steel column sections of given lengths calculated according to the formula of the American Institute of Steel Construction, Formula (13). with a maximum value of $f = 15\,000$ lb per sq in.

The tables are arranged as follows:

Tables XIV and XV	Allowable axial loads in kips for struts of one or two angles. Note that these tables may also be used for latticed H struts of four angles of the type shown in Fig. 6 (d) by doubling the values tabulated about axis YY providing the spacing of angles is $\frac{3}{8}$ in
Table XVI	Allowable axial loads in kips on small Standard beams used as columns
Table XVII	Allowable axial loads in kips for Bethlehem rolled column sections
Table XVIII	Allowable axial loads in kips for Carnegie rolled column sections
Table XIX	Allowable axial loads in kips for plate and angle columns
Table XX	Allowable axial loads in kips for channel columns with cover-plates

Example 13. What must be the size of a steel column 16 ft in length to support an axial load of 827 000 lb (827 kips)? The member is an interior column in the first story of an office-building Fiber-stresses are not to exceed those given by the A.I.S C. Specification.

Solution. Enter the tables with the approximate area required $\dfrac{827\,000}{15\,000} = 55$ sq in. Referring to Table XVII, note that a 12-in Bethlehem column section weighing 190.0 lb per ft will support an axial load of 839 kips if the column has an unsupported length of 16 ft. Other sections are given in the following tabulation·

Referring to	note that a	weighing	will support an axial load of	if the unsupported length is
Table XVII	12-in Bethlehem H–	190.0 lb	839 kips	16 ft 0 in
Table XVII	14-in Bethlehem H–	194 0 lb	855 "	16 ft 0 in
Table XVII	16-in Bethlehem H–	195.0 lb	860 "	16 ft 0 in
Table XVIII	12-in Carnegie H–	190.0 lb	838 "	16 ft 0 in
Table XVIII	14-in Carnegie H–	195.0 lb	860 "	16 ft 0 in
Table XX	15-in Channel Column 2 Channels 15 in × 50 lb 2 Cov. 16 × $\frac{7}{8}$	195.2 lb	859 "	16 ft 0 in
Table XX	15-in Channel Column 2 Channels 15 in × 50 lb 2 Cov. 18 × $\frac{3}{4}$	191 8 lb	844 "	16 ft 0 in

Any one of the columns listed in the above tabulation can support an axial load of 827 kips without exceeding the maximum fiber-stress allowed by the

A.I.S.C. specifications. Rolled columns or plate and angle columns with cover plates would be preferable to channel columns in this case.

Example 14. A double angle strut in a roof truss is 9 ft 6 in long and carries a compressive stress of 35 000 lb. Select a strut of two unequal angles, with short legs out, from Table XV to support this load.

Solution. Referring to Table XV, note that two angles $3 \times 2\frac{1}{2} \times \frac{3}{8}$, governed by bending about axis $X-X$, will support an axial load of 38 kips (interpolated between values 40 kips for 9 ft-length and 36 kips for 10 ft-length) and governed by bending about axis $Y-Y$ it will support a load of 45 kips. The slenderness-ratio, L/r, for the member is greater than 120 as shown by the position of the heavy line in the table. A stiffer section must be selected.

From the same table, it will be found that a strut of two $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$-in angles may carry 40.5 kips if it is governed by bending about axis $X-X$ and 40.5 kips if it is governed by bending about axis $Y-Y$. These values were interpolated between tabulated values for the given section for lengths of 9 ft and 10 ft, respectively. This section is satisfactory.

F. Design of Steel Columns. 1. Rules for Design. (a) Loading. Specifications for loads to be considered in designing steel columns are given in Chapter XXVIII. DEAD LOAD in a building includes the weight of the roof, floors, walls, partitions, and other stationary construction. LIVE LOAD includes all gravity loads other than dead loads. WIND-LOADS, transverse and longitudinal loads and DYNAMIC OR IMPACT LOAD INCREMENTS are usually kept separate from live loads in arranging the computations for stresses.

Except in buildings for storage purposes and in buildings which are likely to be loaded fully on all floors at the same time REDUCTIONS IN THE FULL LIVE FLOOR-LOADS are commonly made in designing the lower columns of a building and in designing the piers and foundations. These live-load reductions are governed by building codes.

(b) Length of Columns. The unsupported length of main compression-members shall not be greater than 120 times the least radius of gyration of the cross-sectional area of the member with respect to a centroidal axis about which the column can bend. The unsupported length of secondary compression-members and members used for wind-bracing only shall not exceed 200 times the corresponding least radius of gyration of the section.

(c) Working Stresses. All parts of the structure shall be so proportioned that the sum of the maximum static stresses in pounds per square inch shall not exceed in compression

$$f = \frac{18\,000}{1 + \dfrac{L^2}{18\,000\,r^2}} \tag{13}$$

(with a maximum of 15,000 lb per sq in) on the gross section of columns; in which L is the unsupported length of the column and r is the corresponding least radius of gyration, both in inches. Full provision shall be made for stresses caused by eccentric loads. For COMBINED STRESSES due to wind and other loads, and for stresses due to wind alone, the permissible working stresses may be increased $33\frac{1}{3}\%$ over those given above. (Maximum allowable fiber-stresses in steel building columns according to a number of building codes are given in Table XIII.)

(d) Design. No part of a steel column shall be less than $\frac{1}{4}$ in thick. No material, whether in the body of the column or used as a lattice bar or stay plate, shall be of less thickness than $\frac{1}{32}$ of its unsupported width,

measured between centers of rivets transversely, or $\frac{1}{16}$ of the distance between centers of rivets in the direction of stress. TIE-PLATES are to have not less than 4 rivets and are to be spaced so that the ratio of length to the least radius of gyration of the parts connected does not exceed 40, the distance between nearest rivets of two stay plates in this case being considered as length. In built-up columns the THICKNESS OF ANY OUTSTANDING MEMBER (for example, the outstanding legs of angles) shall not be less than $\frac{1}{12}$ of the width of the outstanding portion.

(e) **Joints.** The ends of all building columns which bear on metal shall be FACED to a plane surface. Compression members when faced for bearing shall be spliced sufficiently to hold the connecting members accurately in place. Other joints in riveted work, whether in tension or compression, shall be fully spliced. Where any part of the section of a column projects beyond that of the column above or below, FILLER-PLATES shall be provided between the column and the splice-plate. Where stress is transmitted from one piece to another, through a loose filler, the number of rivets shall be properly increased, tight-fitting fillers shall be preferred.

(f) **Lattice.*** (1) "The open sides of compression-members shall be provided with lattice having tie-plates at each end and at intermediate points if the lattice is interrupted. TIE-PLATES shall be as near the ends as practicable. In main members carrying calculated stresses the end tie-plates shall have a length of not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones of not less than $\frac{1}{2}$ of this distance. The THICKNESS OF TIE-PLATES shall not be less than $\frac{1}{50}$ of the distance between the lines of rivets connecting them to the segments of the members, and the rivet pitch shall not be more than four diameters. Tie-plates shall be sufficient in size and number to equalize the stress in the parts of the members.

(2) "LATTICE-BARS shall have neatly finished ends. The THICKNESS OF LATTICE-BARS shall be not less than $\frac{1}{40}$, for single lattice, and $\frac{1}{60}$ for double lattice, of the distance between end rivets. Their MINIMUM WIDTH shall be as follows:

"For 15-in channels, or built-up sections with $3\frac{1}{2}$-in and 4-in angles—$2\frac{1}{4}$ ($\frac{3}{4}$-in rivets), or $2\frac{1}{2}$ in ($\frac{7}{8}$-in rivets).

"For 12-in, 10-in, and 9-in channels, or built-up sections with 3-in angles—$2\frac{1}{4}$ in ($\frac{3}{4}$-in rivets).

"For 8-in and 7-in channels, or built-up sections with $2\frac{1}{2}$-in angles—2 in ($\frac{5}{8}$-in rivets), or $2\frac{1}{4}$ in ($\frac{3}{4}$-in rivets).

"For 6-in and 5-in channels, or built-up sections with 2-in angles—$1\frac{1}{2}$ in ($\frac{1}{2}$-in rivets), or $1\frac{3}{4}$ in ($\frac{5}{8}$-in rivets).

(3) "THE INCLINATION OF LATTICE-BARS to the axis of the members shall generally be not less than 45° but when the distance between the rivet lines in the flanges is more than 15 in, the lattice shall be double and riveted at the intersection if bars are used, or else shall be made of angles.

(4) "Lattice-bars shall be so spaced that the ratio L/r of the flange included between their connections shall be not over $\frac{3}{4}$ of that of the member as a whole."

2. Column Sheets, showing in tabular form the loads which must be supported at each story in building columns, are essential in preparing designs for a multi-story building. A form of column sheet is herewith shown.

* **Standard Specification for Structural Steel for Buildings, American Institute of Steel Construction.**

Form of Column Sheet

Story	Character of loading	Column A1 Items	Column A1 Sums	Column C6 Items	Column C6 Sums
	Dead Load:				
	Tanks				
	Masonry walls				
	Roof and ceiling				
	Spandrels, cornice				
	Elevators				
	Column and casing				
	Total..........				
	Live Load:				
18th	Roof load				
(top)	Elevators				
	Total..................				
	Sum of dead and live load..				
	Wind-Load:				
	Bending moment				
	Axial load				
	B M./k				
	Total....				
	Sum of dead, live and wind.				
	Section required:				
	Dead Load: (from column above)				
	Floor				
	Safes, vaults, etc.				
	Masonry walls				
	Spandrels				
	Column and casing				
	Total.......				
	Live Load: (from column above)				
	Floor load				
	Moving loads				
	Total................. . ..				
	Wind-Load:				
	Bending moment				
	Axial load				
	B.M./k				
17th	Total......				
	Sum of dead and live load..... .				
	Reduction factor				
	Effective live load..............				
	Sum of dead load and effective live load......................				
	Wind-Load:				
	Bending moment				
	Axial load				
	B.M./k				
	Total.......				

Form Column Sheet (Continued)

Story	Character of loading	Column A1 Items	Column A1 Sums	Column C6 Items	Column C6 Sums
17th	Sum of dead load, effective live load and wind-load...............				
	Section required:				
Basement	Dead Load: (from column above)				
	Floor				
	Safes, vaults, etc.				
	Masonry, walls				
	Spandrels				
	Column and casing				
	Sidewalk				
	Total..............				
	Live Load: (from column above)				
	Floor load				
	Sidewalk load				
	Moving loads				
	Total.................... ...				
	Reduction factor				
	Effective live load.............				
	Sum of dead load and effective live load				
	Wind-Load:				
	Bending moment				
	Axial load				
	B.M./k				
	Total...........				
	Sum of dead load, effective live load and wind-load..........				
	Section required:				

The load in kips supported by the column at any story is divided into three parts: DEAD LOAD, which includes the weight of the structure and the weight of fixed equipment which is supported by the given column; LIVE LOAD, which includes distributed and concentrated loads other than dead and wind-loads, applied on the floors, the roof and other parts of the structure which is supported by the given column; and WIND-LOAD, which includes axial load and bending moment in the column if it is a member of the wind-bracing structure. Axial loads, shears and bending moments due to wind forces which are resisted by members in the wind bents are determined by a separate analysis of wind stresses.

The item in the table showing live loads carried down from the column above (at 17th-floor level, for instance) should be the total live load from the upper column before it has been reduced by the REDUCTION FACTOR allowed in the lower stories of multi-story buildings. The EFFECTIVE LIVE LOAD at any story is the total live load reduced in accordance with specifications for live-load reduction.

The last term under wind loads, B.M./k, is the bending moment in inch-kips divided by the bending factor* in inches for the section of column used,

* The bending factor = the section-modulus, I/c, of the section of the column, divided by its cross-sectional area. See Chapter X.

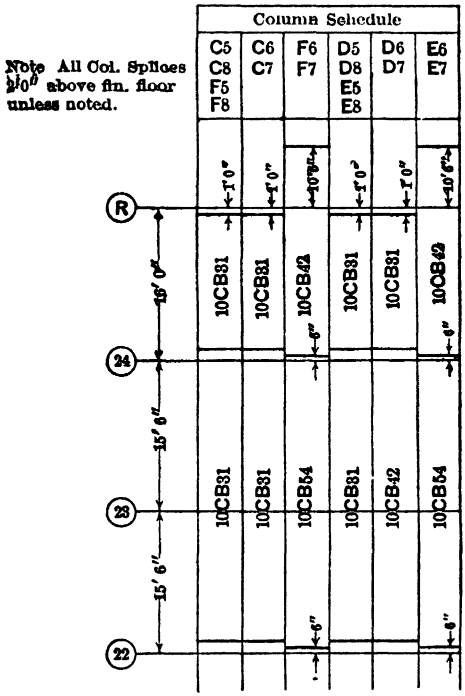
and it represents an equivalent axial load which would produce the same maximum fiber-stress in the column section as is produced by the given bending moment.

The sum of the dead load and effective live load, divided by the cross-sectional area of the column, must not exceed the fiber-stress allowed by the specifications.

The sum of the dead load, effective live load and wind-load (axial load plus B.M./k), divided by the cross-sectional area of the column, must not exceed the fiber-stresses allowed by the specifications for stresses due to a combination of wind and other loads.

A COLUMN SCHEDULE showing column sections and location of column splices should be prepared for every large building summarizing the data given in the column sheet, described above, in suitable form for detailing the columns. After the column has been detailed the sheet number of the drawing which shows the column details should be entered in the column schedule.

A form of column schedule follows:



Form of Column Schedule

3. Design of Built-Up Columns Not Given in Tables of Safe Loads. The load which a column may support depends upon the cross-sectional area of the column and upon the allowable fiber-stress, which is limited by the design specifications. The allowable fiber-stress depends upon the slenderness-

ratio of the column which in turn depends upon the radius of gyration of the cross-sectional area of the column. There is no direct way to determine the section of a column of a given length to support a given load, and the following steps must be followed in designing a column whose section is not given in the tables of safe loads:

(1) Assume the general proportions of the column section, and its overall width and depth.

(2) Estimate the allowable fiber-stress in the column section.

(3) Determine the cross-sectional area required by dividing the load to be supported by the estimated allowable fiber-stress (2).

(4) Select suitable rolled sections which put together, have the estimated required area (3), and suitable proportions conforming to those assumed in (1).

(5) Check the column selected (4) to see if it will support the given load without exceeding the allowable fiber-stresses.

(*a*) **General Proportions of Built-Up Columns.** It is desirable that columns in a tall building have the same overall depth throughout as many stories as practicable. This allows greater duplication of beams and girders in the building, simplifies column splices, and reduces both shop and erection costs.

The American Bridge Company has adopted CONSTANT DIMENSION COLUMNS of H-shaped sections which have different cross-sectional area but have the same overall depth, out to out, of cover-plates. These depths vary by increments of 2 in from a minimum column $10\frac{1}{2}$ in deep to a maximum column $26\frac{1}{2}$ in deep. The columns $10\frac{1}{2}$ to $16\frac{1}{2}$ in in depth, inclusive, have either 4×3-in angles, $5 \times 3\frac{1}{2}$-in angles or 6×4-in angles, while columns $18\frac{1}{2}$ in in depth, and larger, have 8×6-in angles. The heavier sections, $16\frac{1}{2}$ in and less in depth, have 6×4-in angles and cover-plates either 14 in or 16 in in width. The heavier sections, $18\frac{1}{2}$ in and greater in overall depth, have 8×6-in angles and cover-plates whose width is $\frac{1}{2}$ in less than the depth of the section.

Various types of steel columns are shown in Figs. 6 and 8.

(*b*) **Approximate Radii of Gyration of Column Sections** are very useful in estimating the allowable fiber-stresses on a column of given general proportions. A table of approximate radii of gyration of common column sections, which is satisfactory for preliminary design only, is given in Fig. 8. Knowing the unsupported length of the column to be designed and the approximate radius of gyration of the column of the size and general proportions assumed, the allowable fiber-stress can be calculated, for preliminary design, from the column formula given in the particular specifications which are being used. (See Table XIII.)

(*c*) **Checking Column Section.** After a preliminary design of a column section has been made as indicated in (1), (2), (3), and (4), the radii of gyration of the column selected should be calculated and the allowable fiber-stress should be calculated from the known slenderness-ratio, L/r, of the section found in (4). If the product of this allowable fiber-stress and the actual area of the column differs materially from the load to be supported by the column, the area of the column section should be increased or decreased as required and the steps in the design should be repeated for the revised section.

Example 16. Using the formula of the American Institute of Steel Construction for allowable stresses, design a column 19 ft 6 in long to support an axial load of 740 000 lb. The column is in the first story of a 15-story office-building.

Solution. (*a*) General proportions of column: An H-shaped built-up column $12\frac{1}{2}$ in in overall depth with 14-in cover-plates will be assumed. From Fig. 8 it will be seen that the approximate radius of gyration of an H-shaped section made up of a web, 4 angles and 2 cover-plates is 0.45 *h* for

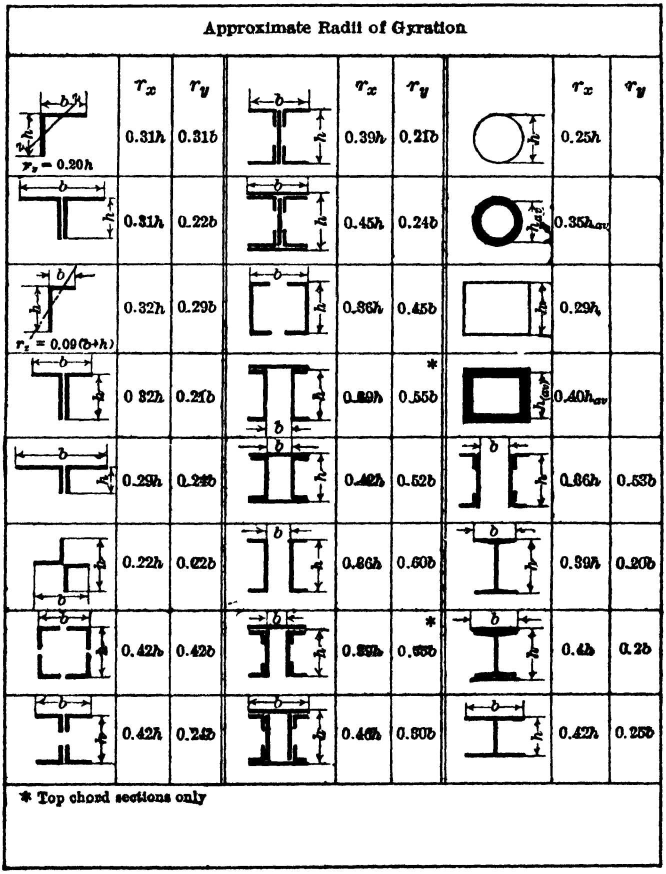


Fig 8. Approximate Radii of Gyration

bending about an axis perpendicular to the web and 0.24 *b* for bending about an axis coinciding with the web. The least radius of gyration of the section is approximately $0.24 \times 14 \text{ in} = 3.36$ in.

(*b*) Estimate of allowable fiber-stress: Using the approximate rule for the allowable fiber stress *

* See Approximate Rule under 8, D, of this chapter.

$$f = 20\,000 - 1\,000 \times \frac{\text{length in feet}}{\text{radius of gyration in inches}}$$

the allowable fiber-stress is estimated to be equal to

$$20\,000 - 1\,000 \times 19.5/3.36 = 14\,200 \text{ lb per sq in}$$

(*c*) Area required = 740 000/14 200 = 52.1 sq in.

(*d*) Section used (preliminary design): 6 × 4-in angles and a 10-in web-plate will be used in the first trial section;

4 L's 6 × 4 × 5/8 @ 5.86 sq in	= 23.44 sq in
Web 10 × 5/8	= 6.25
	29.69
Total area required (*c*)	= 52.10
Required area of cover plates	= 22.41 sq in
Use 2 covers 14 × 13/16	= 22.75 sq in

SECTION
Web – 10 × 5/8
4 ∠ 6 × 4 × 5/8
2 Cov. 14 × 13/16
Area = 52.44 sq in
I_x = 1315.5 in^4
K_x = 4.01″
I_y = 584.5 in^4
K_y = 1.59″

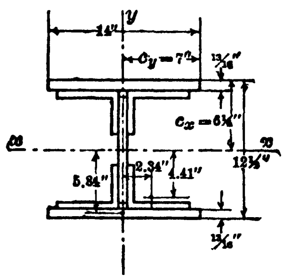


Fig. 9

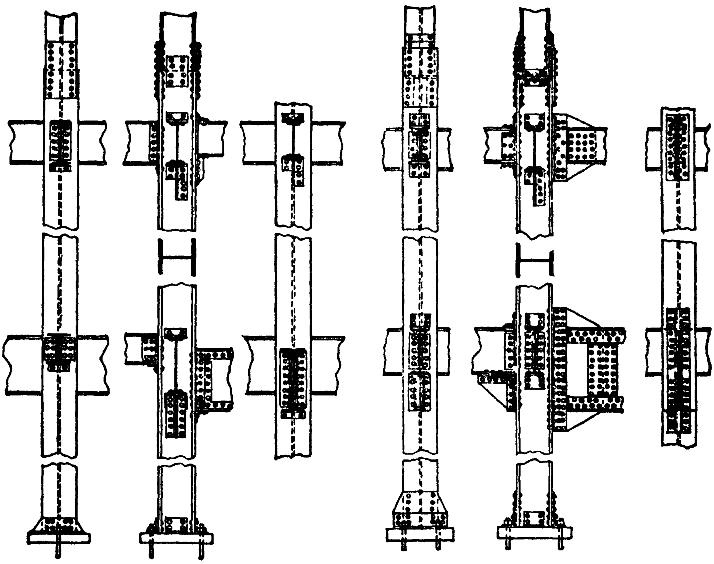
(*e*) Check: The method of calculating the moment of inertia and radius of gyration of a section is shown in Chapter X. Since the section is symmetrical about the axes *X–X* and *Y–Y* (Fig. 9), columns headed *Ax* and *Ay* in Chapter X are not shown in the following tabulation. The least radius of gyration is obviously about the *YY* axis.

Section	A	x	Ax²
Web 10 × 5/8	6.25	0.00	0.0 0.2
4 L's 6 × 4 × 5/8	23.44	2.34	128.4 84.3
2 Cov. 14 × 13/16	22.75	0.00	0.0 371.6
	52.44		584.5

Least radius of gyration = $\sqrt{584.5/52.44}$ = 3.34 in
L/r = 12 × 19.5/3.34 = 70.1.
Allowable fiber-stress = 18 000/(1 + 70.1²/18 000) = 14 140 lb per sq in
Allowable axial load = 14 140 × 52.44 = 741 500 lb. Load to be supported = 740 000 lb. Therefore the column is satisfactory.

G. Connections and Details. TYPICAL DETAILS of steel building columns, connections and splices are shown in Figs. 10 to 12.

1. Bases. Steel columns are supported on CAST-IRON or CAST-STEEL BASES, on BUILT-UP BASES which are usually shop riveted to the column itself, or on ROLLED STEEL SLABS. The width of steel slabs generally varies in increments of 4 in from 14 in to 56 in, inclusive; the thickness varies in increments of $\frac{1}{2}$ in from $1\frac{1}{4}$ in to 7 in, inclusive.* Slabs which are more than 2 in thick are likely to be bowed and it is usually necessary to plane the top surface which is in contact with the steel column. If these slabs are



Typical Bethlehem H Column. Floor Framing

Typical Bethlehem H Column. Wind Bracing Connections

Fig. 10.* Typical Details of Rolled H Columns

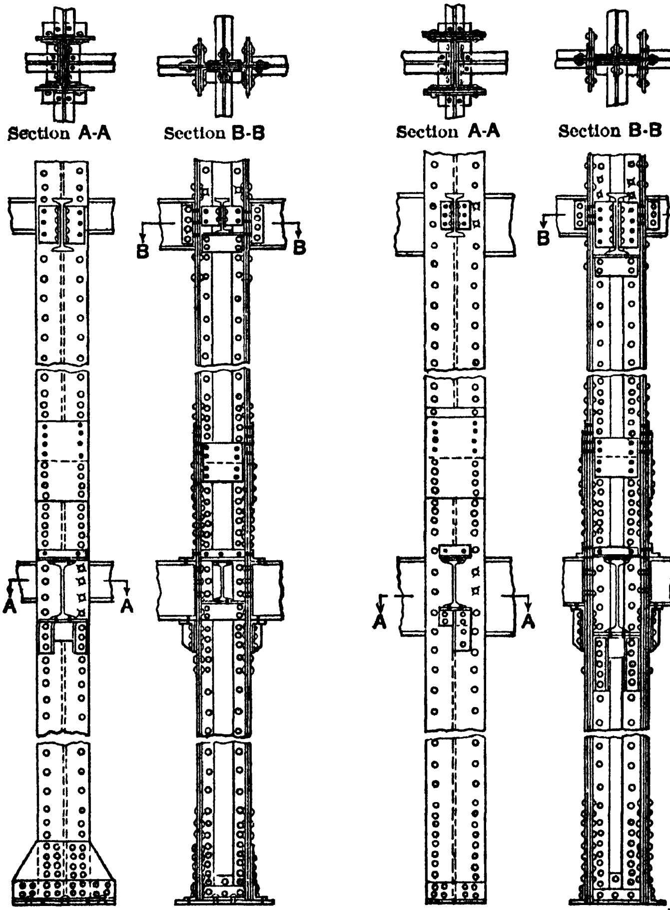
* From Bethlehem Structural Shapes, Bethlehem Steel Co.

set on fresh cement grout on concrete foundations it is not necessary to plane the bottom surface of the slab. Details and methods of proportioning column bases are shown in Chapter XIII.

2. Splices. Building columns are usually made in lengths of two or more stories and are spliced about 2 ft or $2\frac{1}{2}$ ft above a floor level. The ends of two columns which meet at a joint should be faced to a plane surface and the columns should be held firmly in place by SPLICE PLATES riveted to the flanges of the columns. If the widths of the two connected members differ,

* "The size of slabs that can now be rolled is limited by the size of the ingot produced and the capacity of the rolling mill. Slabs have been rolled up to 120 × 120×9 in thick, weighing 36 000 lb, and can be rolled up to 12 in thick, weighing 40 000 lb." Steel Construction, 1st edition, 6th printing, page 432.

FILLER-PLATES should be placed between the flange of the smaller column and the splice-plate. These fillers should preferably extend beyond the end of the splice-plate and should be riveted to the smaller column with a



Typical Angle-Column. Bearing on Masonry

Typical Angle-Column. Bearing on Steel

Fig. 11.* Connections for Steel Plate-and-angle Columns

* From Pocket Companion, Carnegie Steel Co., Pittsburgh, Pa.

sufficient number of shop rivets to develop the strength value of the filler. The lower end of the filler should be faced to the same plane as the lower end of the column. The flange splices should be strong enough to hold the columns accurately in place and to resist bending moments in the column due to eccen-

tric or transverse loads. SMALL COMPRESSION-MEMBERS which are not faced for bearing must be spliced sufficiently to develop the full strength of the member.

The WEBS of two columns at a splice are usually connected by angles as shown in Fig. 12. The standard SPLICE ANGLES * on web faces of the constant dimension columns of the American Bridge Company are as follows:

$12\frac{1}{2}$, $14\frac{1}{2}$ and $16\frac{1}{2}$-in width	$3\frac{1}{2} \times 3 \times \frac{3}{8}$
$18\frac{1}{2}$, $20\frac{1}{2}$ and $22\frac{1}{2}$-in width	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$
$24\frac{1}{2}$ and $26\frac{1}{2}$-in width	$4 \times 4 \times \frac{1}{2}$

Typical column splices are shown in Fig. 12.

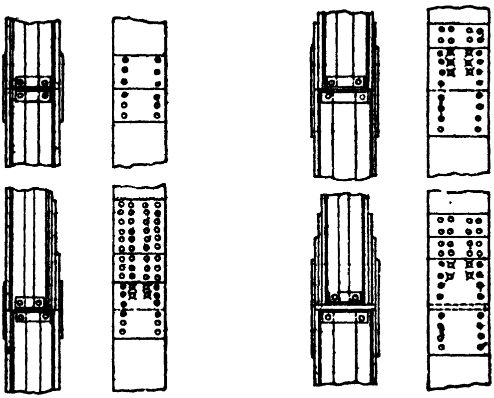


Fig. 12. Typical Column Splices

3. Connections. Beams and girders may be connected to columns by either SEATED CONNECTIONS or FRAMED CONNECTIONS.

In a SEATED CONNECTION the bottom flange of the beam rests on and is field-riveted to an angle shop-riveted to the flange or web of the column. Another angle is field-riveted to the column and to the top flange of the beam.

In a FRAMED CONNECTION the web of the beam or girder is connected by angles or by gusset-plates to the column. In framed connections, angle seats are usually shop-riveted to the columns to hold the beam or girder during erection. Details of framed connections are shown in Chapter XV.

* American Bridge Company Engineering Standards (1926).

Table XIV.* Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY

Size in Inches	Thickness	Weight per foot	Area in sq in	Least Radius Gyration	Unsupported length in feet: 2	3	4	5	6	7	8	9	10	11	12
1¾ × 1¾	1/8	1.44	0.42	0.35	6.0	4 8	3.7	2 9	...	.	. .	..	.	. .	
	3/16	2 12	0.62	0.34	8.7	6 9	5.3	4 1	...	..	.	...		. .	.
	1/4	2 77	0.81	0.34	11.4	9 0	6.9	5 3	...	. .	.			.	
2 × 2	1/8	1.65	0.48	0.40	7.2	6 0	4 8	3 8	3 1	...	...	..	..	...	.
	3/16	2.44	0.72	0.39	10.7	8 8	7 0	5 7	4 5	...	....	..	...	. .	.
	1/4	3.19	0 94	0.39	14 0	11 5	9 2	7 4	5 8	....	....	...	..	. .	. .
	5/16	3.92	1.15	0.39	17.1	14.1	11 2	9 1	7 2	.		. .	. .	....	
2½ × 2	1/8	1.86	0.55	0 43	8 3	7.1	5 8	4 8	3.9	3 2	..	...	...	...	
	3/16	2.75	0 81	0 43	12 2	10 5	8 6	7 0	5 7	4 7	...	...	..		.
	1/4	3 62	1.06	0.42	15 9	13 5	11 1	8 9	7.2	5.9	...	...	.	.	
	5/16	4 50	1.31	0.42	19.7	16.7	13.7	11 0	8 9	7.3	. .	....	.	...	.
2½ × 2½	1/8	2 08	0.61	0.50	9 2	8 5	7 3	6 1	5 1	4 3	3 6	...	...	.	...
	3/16	3 07	0.90	0.49	13 5	12 5	10.6	8 8	7.4	6.1	5.2	..	...	.	..
	1/4	4 1	1.19	0.49	17.9	16.5	14 0	11 7	9.7	8.1	6.8	...	....	.. .	. .
	5/16	5 0	1.47	0.49	22.1	20.4	17 3	14.4	12.0	10.0	8.5	..	.	...	.
	3/8	5 9	1 73	0 48	26 0	23 7	20 0	16 7	13 8	11 5	.	..			

LOADS BY A. I. S C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY

Size in Inches	Thickness	Weight per foot	Area in sq in	Least Radius Gyration	Unsupported length in feet										
					2	3	4	5	6	7	8	9	10	11	12
3 × 2½	¼	4 5	1 31	0 53	19 7	18 8	16 2	13 8	11 6	9 8	8 4	..	.	....	..
	5/16	5 6	1 62	0 53	24 3	23 2	20 0	17 0	14 4	12 2	10 3	. .	...	.	. .
	⅜	6 6	1 92	0 52	28 8	27 3	23.4	19 9	16 7	14.1	11 9	. .	....	.	.
	7/16	7 6	2 22	0 52	33 3	31 6	27.1	23 0	19 4	16 3	13 8	.	. .	.	..
3 × 3	¼	4 9	1 44	0 59	21 6	21 5	19 0	16 5	14 2	12 2	10 5	9 1		.	
	5/16	6 1	1 78	0.59	26 7	26.5	23 4	20 3	17 5	15 1	13 0	11 2	..		
	⅜	7 2	2 11	0 58	31 7	31 3	27.5	23 8	20 5	17 5	15 1	13 0	. .		
	7/16	8 3	2 43	0 58	36 5	36 0	31 7	27 4	23 6	20 2	17 4	14 9			
	½	9 4	2 75	0 58	41 3	40 8	35.9	31 0	26 7	22 9	19.6	16 9	.		
3½ × 2½	¼	4 9	1 44	0 54	21 6	20 8	18 0	15 4	13 0	11 1	9.4	8 0	.		
	5/16	6 1	1 78	0 54	26 7	26 7	22 3	19 0	16 1	13 7	11 6	9 9			
	⅜	7 2	2 11	0 54	31 7	30.4	26 4	22 5	19.1	16 2	13 8	11 8	.		
	7/16	8 3	2 43	0 54	36 5	35 1	30 4	26 0	22 0	18 7	15 9	13 6			
	½	9 4	2 75	0 53	41 3	39 4	34 0	28 9	24 4	20 7	17 5				
3½ × 3½	5/16	7 2	2 09	0 69	31 4	31 4	29 6	26 5	23 4	20 6	18 1	15 9	14 0	12 4	
	⅜	8 5	2 48	0 68	37 2	37 2	35 0	31 2	27 5	24 2	21 2	18 6	16 3	14 4	
	7/16	9 8	2 87	0 68	43 1	43 1	40 5	36 1	31 8	28 0	24 5	21 5	18 9	16 7	.
	½	11 1	3 25	0 68	48 8	48 8	45 8	40 9	36 0	31 7	27 8	24 4	21 4	18 9	.
	9/16	12 4	3 62	0 68	54 3	54 3	51 0	45 5	40 1	35 3	30 9	27 2	23 9	21 1	
	⅝	13 6	3 98	0 68	59 7	59 7	56 1	50 0	44 1	38 8	34 0	29 9	26 2	23 2	

LOADS BY A. I. S. C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY.

Size in Inches	Thickness	Weight per foot	Area in sq in	Least Radius Gyration	Unsupported length in feet										
					2	3	4	5	6	7	8	9	10	11	12
4 × 3	5/16	7.2	2.09	0.65	31.4	31.4	28.9	25.5	22.4	19.5	17.0	14.8	13.0	...	
	3/8	8.5	2.48	0.64	37.2	37.2	34.0	30.0	26.2	22.8	19.8	17.3	15.1	...	
	7/16	9.8	2.87	0.64	43.1	43.1	39.4	34.7	30.3	26.4	23.0	20.0	17.5	...	.
	1/2	11.1	3.25	0.64	48.8	48.8	44.6	39.3	34.4	29.9	26.0	22.7	19.8	....	..
	9/16	12.4	3.62	0.64	54.3	54.3	49.6	43.8	38.3	33.3	29.0	25.2	22.1	..	. .
	5/8	13.6	3.98	0.64	59.7	59.7	54.6	48.1	42.1	36.6	31.8	27.7	24.3	...	...
4 × 3½	5/16	7.7	2.25	0.73	33.8	33.8	32.6	29.5	26.3	23.3	20.7	18.3	16.2	14.4	12.8
	3/8	9.1	2.67	0.73	40.1	40.1	38.7	35.0	31.2	27.7	24.5	21.7	19.2	17.1	15.2
	7/16	10.6	3.09	0.72	46.4	46.4	44.6	40.1	35.8	31.7	28.0	24.7	21.9	19.4	17.2
	1/2	11.9	3.50	0.72	52.5	52.5	50.5	45.5	40.5	35.9	31.7	28.0	24.8	22.0	19.5
	9/16	13.3	3.90	0.72	58.5	58.5	56.3	50.7	45.1	40.0	35.3	31.2	27.6	24.5	21.8
	5/8	14.7	4.30	0.72	64.5	64.5	62.1	55.9	49.8	44.1	39.0	34.4	30.4	27.0	24.0
	11/16	16.0	4.68	0.72	70.2	70.2	67.6	60.8	54.1	48.0	42.4	37.4	33.1	29.4	26.1
	3/4	17.3	5.06	0.72	75.9	75.9	73.1	65.7	58.5	51.9	45.8	40.5	35.8	31.8	28.2

LOADS BY A. I. S. C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z
Loads to right of heavy vertical line are for Secondary Members ONLY

Size in Inches	Thickness	Weight per foot	Area in sq in	Least Radius Gyration	Unsupported length in feet										
					2	3	4	5	6	7	8	9	10	11	12
4 × 4	5/16	8.2	2.40	0.79	36.0	36.0	35.9	32.7	29.6	26.5	23.7	21.2	18.9	16.9	15.2
	3/8	9.8	2.86	0.79	42.9	42.9	42.7	39.0	35.2	31.6	28.3	25.3	22.6	20.2	18.1
	7/16	11.3	3.31	0.78	49.7	49.7	49.2	44.9	40.4	36.2	32.3	28.9	25.7	23.0	20.6
	1/2	12.8	3.75	0.78	56.3	56.3	55.8	50.8	45.8	41.1	36.6	32.7	29.1	26.1	23.3
	9/16	14.3	4.18	0.78	62.7	62.7	62.2	56.6	51.1	45.8	40.8	36.4	32.5	29.1	26.0
	5/8	15.7	4.61	0.77	69.2	69.2	68.2	62.1	55.9	50.0	44.5	39.6	35.3	31.5	28.2
	11/16	17.1	5.03	0.77	75.5	75.5	74.4	67.7	61.0	54.5	48.6	43.3	38.5	34.4	30.8
	3/4	18.5	5.44	0.77	81.6	81.6	80.5	73.2	65.9	59.0	52.6	46.8	41.7	37.2	33.3
5 × 3	5/16	8.2	2.40	0.66	36.0	36.0	33.4	29.6	26.0	22.8	19.8	17.4	15.2	13.4	...
	3/8	9.8	2.86	0.65	42.9	42.9	39.5	34.9	30.6	26.7	23.3	20.3	17.8	....	...
	7/16	11.3	3.31	0.65	49.7	49.7	45.7	40.4	35.4	30.9	26.9	23.5	20.6	....	..
	1/2	12.8	3.75	0.65	56.3	56.3	51.8	45.8	40.1	35.0	30.5	26.6	23.3	.	. .
	9/16	14.3	4.18	0.65	62.7	62.7	57.8	51.1	44.7	39.0	34.0	29.7	26.0	....	
	5/8	15.7	4.61	0.64	69.2	69.2	63.2	55.7	48.7	42.4	36.9	32.1	28.1	...	. .
	11/16	17.1	5.03	0.64	75.5	75.5	69.0	60.8	53.2	46.3	40.2	35.1	30.7	....	.
	3/4	18.5	5.44	0.64	81.6	81.6	74.6	65.8	57.5	50.0	43.5	37.9	33.2	....	....

LOADS BY A I S. C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV* (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY

Size in inches	Thickness	Weight per foot	Area in sq in	Least radius gyration	Unsupported length in feet												
					3	4	5	6	7	8	10	12	14	16	18	20	22
5 × 3½	5/16	8.7	2.56	0.77	38	38	34	31	28	25	20	16					
	3/8	10.4	3.05	0 76	46	45	41	37	33	29	23	18					
	7/16	12 0	3.53	0 76	53	52	47	42	38	34	27	21					
	½	13.6	4.00	0 75	60	59	53	48	42	38	30	24					
	9/16	15.2	4 47	0.75	67	66	59	53	47	42	33	26					
	5/8	16 8	4 92	0 75	74	72	65	59	52	46	37	29					
	11/16	18.3	5.37	0.75	81	79	71	64	57	51	40	32					
	¾	19.8	5 81	0.75	87	85	77	69	62	55	43	34					
6 × 3½	3/8	11.7	3 42	0.77	51	51	46	41	37	33	26	21					
	7/16	13.5	3.97	0.76	60	58	53	48	43	38	30	24					
	½	15 3	4 50	0.76	68	66	60	54	48	43	34	27					
	9/16	17.1	5.03	0.75	75	74	67	60	53	47	37	30					
	5/8	18.9	5.55	0.75	83	81	74	66	59	52	41	33					
	11/16	20.6	6.06	0 75	91	89	80	72	64	57	45	36					
	¾	22 4	6.56	0 75	98	96	87	78	70	62	49	39					
	13/16	24.0	7 06	0.75	106	103	94	84	75	67	52	42					
	7/8	25.7	7.55	0 75	113	111	100	90	80	71	56	45					

LOADS BY A I S. C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY.

Size in inches	Thickness	Weight per foot	Area in sq in	Least radius gyration	Unsupported length in feet												
					3	4	5	6	7	8	10	12	14	16	18	20	22
6 × 4	3/8	12.3	3.61	0.88	54	54	52	47	43	39	32	26	.	.		.	
	7/16	14.3	4.18	0.87	63	63	60	55	50	45	37	30	24			.	.
	1/2	16.2	4.75	0.87	71	71	68	62	56	51	42	34	28				.
	9/16	18.1	5.31	0.87	80	80	76	69	63	57	46	38	31		.		..
	5/8	20.0	5.86	0.86	88	88	83	76	69	62	51	41	34				
	11/16	21.8	6.40	0.86	96	96	91	83	75	68	55	45	37				.
	3/4	23.6	6.94	0.86	104	104	98	90	82	74	60	49	40				.
	13/16	25.4	7.47	0.86	112	112	106	97	88	79	65	53	43	...	..		..
	7/8	27.2	7.98	0.86	120	120	113	103	94	85	69	56	46	...	..	.	..
6 × 6	3/8	14.9	4.36	1.19	65	65	65	65	61	58	50	43	37	32	28	.	
	7/16	17.2	5.06	1.19	76	76	76	76	71	67	58	51	43	37	32	..	
	1/2	19.6	5.75	1.18	86	86	86	86	81	76	66	57	49	42	36	.	
	9/16	21.9	6.43	1.18	96	96	96	96	90	85	73	63	54	47	40	.	..
	5/8	24.2	7.11	1.17	107	107	107	106	99	93	81	69	60	51	44		.
	11/16	26.5	7.78	1.17	117	117	117	116	109	102	88	76	65	56	48		.
	3/4	28.7	8.44	1.17	127	127	127	126	118	111	96	82	71	61	52		
	13/16	31.0	9.09	1.17	136	136	136	135	127	119	103	89	76	66	57		
	7/8	33.1	9.73	1.16	146	146	146	144	136	127	110	94	81	69	60		..
	15/16	35.3	10.37	1.16	156	156	156	154	144	135	117	100	86	74	64	.	....
	1	37.4	11.00	1.16	165	165	165	163	153	143	124	107	91	78	68		

LOADS BY A I S C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY.

Size in inches	Thick-ness	Weight per foot	Area in sq in	Least radius gyra-tion	Unsupported length in feet												
					3	4	5	6	7	8	10	12	14	16	18	20	22
7 × 3½	3/8	13.0	3.80	0.76	57	56	51	46	41	36	29	23	....	...	...	...	....
	7/16	15.0	4.40	0.76	66	65	59	53	47	42	33	26	....	...	...	....	....
	1/2	17.0	5.00	0.75	75	73	66	60	53	47	37	30	...	...	...	....	....
	9/16	19.1	5.59	0.75	84	82	74	67	59	53	42	33	...	...	...	...	....
	5/8	21.0	6.17	0.75	93	90	82	73	65	58	46	36	...	....	...	...	.....
	11/16	23.0	6.75	0.74	101	98	89	80	71	63	49	39	...	...	..	...	...
	3/4	24.9	7.31	0 74	110	107	96	86	77	68	54	42	...	...	..	..	..
	13/16	26.8	7.87	0.74	118	115	104	93	83	73	58	46	....	...	..	....	..
	7/8	28.7	8.42	0.74	126	123	111	99	88	78	62	49	...	...	..	...	..
	15/16	30.5	8.97	0.74	135	131	118	106	94	84	66	52	...	...	....	....	..
	1	32 3	9 50	0 74	143	139	125	112	100	88	70	55	..	..	.	.	
8 × 6	7/16	20.2	5.93	1.30	89	89	89	89	87	82	72	63	55	48	42	37	.
	1/2	23.0	6.75	1.30	101	101	101	101	99	93	82	72	63	55	48	42	...
	9/16	25.7	7.56	1.30	113	113	113	113	110	104	92	81	71	61	54	47	...
	5/8	28.5	8.36	1.29	125	125	125	125	122	115	102	89	77	67	59	51	...
	11/16	31.2	9.15	1.29	137	137	137	137	133	126	111	97	85	74	64	56	..
	3/4	33.8	9.94	1.28	149	149	149	149	144	136	120	105	91	80	69	61	..
	13/16	36 5	10.72	1.28	161	161	161	161	156	147	130	113	99	86	75	65	...
	7/8	39.1	11.48	1.28	172	172	172	172	167	157	139	121	106	92	80	70	..
	15/16	41.7	12.25	1.28	184	184	184	184	178	168	148	129	113	98	85	75	...
	1	44.2	13 00	1 28	195	195	195	195	189	178	157	137	120	104	91	79	...

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XIV * (Continued). Struts of One Angle

Allowable Concentric Loads in Kips

Values given are for Least Radius of Gyration which is about Axis Z–Z.
Loads to right of heavy vertical line are for Secondary Members ONLY.

Size in inches	Thick-ness	Weight per foot	Area in sq in	Least radius gyra-tion	Unsupported length in feet												
					3	4	5	6	7	8	10	12	14	16	18	20	22
8×8	1/2	26.4	7.75	1.59	116	116	116	116	116	116	106	96	86	77	69	62	55
	9/16	29.6	8.68	1.58	130	130	130	130	130	130	118	107	96	86	77	68	61
	5/8	32.7	9.61	1.58	144	144	144	144	144	144	131	118	106	95	85	76	68
	11/16	35.8	10.53	1.58	158	158	158	158	158	157	144	130	116	104	93	83	74
	3/4	38.9	11.44	1.57	172	172	172	172	172	170	155	140	126	112	100	90	80
	13/16	42.0	12.34	1 57	185	185	185	185	185	184	168	151	136	121	108	97	86
	7/8	45.0	13.23	1.56	198	198	198	198	198	197	179	162	145	129	115	103	92
	15/16	48.1	14.12	1.56	212	212	212	212	212	210	191	173	155	138	123	110	98
	1	51 0	15.00	1.56	225	225	225	225	225	223	203	183	164	147	131	117	104
	1 1/16	54.0	15.87	1.56	238	238	238	238	238	236	215	194	174	155	138	123	110
	1 1/8	56.9	16.73	1 55	251	251	251	251	251	248	226	203	182	162	145	129	115

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV.* Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Equal Angles—⅜ in Back to Back

Loads to right of heavy vertical lines are for Secondary Members ONLY

3/8″ Back to Back

Size	Thickness	Two Angles Area	Two Angles Wt.	Radius of gyration X-X	Radius of gyration Y-Y	Axis X-X 3	4	5	6	7	8	9	10	12	14	16	18	20	Axis Y-Y 6	7	8	9	10	12	14	16	18	20	22	24	26
2½ × 2½	3/16	1.80	6.1	.78	1.18	27	27	24	22	20	18	16	14	11	.	.			27	25	24	22	21	18	15	13	11	.	..		.
	¼	2.38	8.2	.77	1.19	36	35	32	29	26	23	21	18	15		...	.		36	34	31	29	27	24	20	18	15	...	..		.
	5/16	2.94	10.0	.76	1.20	44	43	39	35	32	28	25	22	18	...	.			44	42	39	36	34	29	25	22	19	16	..		.
	⅜	3.46	11.8	.75	1.21	52	51	46	41	37	33	29	26	20	.		.		52	49	46	43	40	35	30	26	23	..	..		
3 × 3	¼	2.88	9.8	.93	1.38	43	43	42	39	36	33	30	27	22	18		.		43	43	41	39	37	32	28	25	22	19	17		.
	5/16	3.56	12.2	.92	1.40	53	53	52	48	44	40	36	33	27	23	...	.		53	53	51	48	46	40	36	31	28	24	22	.	.
	⅜	4.22	14.4	.91	1.41	63	63	61	56	52	47	43	39	32	26	..	...		63	63	60	57	54	48	42	37	33	29	26	..	..
	7/16	4.86	16.6	.91	1.42	73	73	71	65	59	54	49	45	37	30	.			73	73	70	66	63	56	49	43	38	34	30	..	.
3½ × 3½	5/16	4.18	14.4	1.08	1.60	63	63	63	60	56	52	48	45	38	32	27	..	.	63	63	63	60	57	52	47	42	37	33	30	27	24
	⅜	4.96	17.0	1.07	1.61	74	74	74	71	67	62	57	53	45	38	32		.	74	74	74	71	68	62	56	50	45	40	36	32	29
	7/16	5.74	19.6	1.07	1.62	86	86	86	83	77	71	66	61	52	44	37	...	.	86	86	86	83	79	72	65	58	52	47	42	37	34
	½	6.50	22.2	1.06	1.63	98	98	98	93	87	80	74	68	58	49	41	...	.	98	98	98	94	90	82	74	66	59	53	48	43	39
	9/16	7.24	24.8	1.05	1.64	109	109	109	103	96	89	82	76	64	54	46	...	.	109	109	109	105	100	91	82	74	66	59	53	48	43
	⅝	7.96	27.2	1.04	1.66	119	119	119	113	105	97	90	82	69	59	50	...		119	119	119	116	111	101	91	82	74	66	60	54	48
4 × 4	5/16	4.80	16.4	1.24	1.80	72	72	72	72	69	65	61	57	49	43	37	32	.	72	72	72	72	69	64	58	53	48	44	39	36	32
	⅜	5.72	19.6	1.23	1.81	86	86	86	86	82	77	72	67	59	51	44	38	..	86	86	86	86	83	76	70	63	57	52	47	43	39
	7/16	6.62	22.6	1.23	1.82	99	99	99	99	95	89	83	78	68	59	51	44	..	99	99	99	99	96	88	81	74	67	61	55	50	45
	½	7.50	25.6	1.22	1.83	113	113	113	113	107	100	94	88	76	66	57	49	...	113	113	113	113	109	100	92	84	76	69	63	57	52
	9/16	8.36	28.6	1.21	1.85	125	125	125	125	119	111	104	97	84	73	63	54		125	125	125	125	122	113	103	94	86	78	71	64	58
	⅝	9.22	31.4	1.20	1.86	138	138	138	138	131	122	114	107	92	79	69	59		138	138	138	138	135	124	114	104	95	86	78	71	65

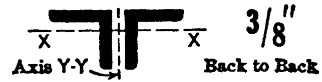
Loads by A. I. S. C. Specification

* From Steel Construction, published by American Institute of Steel Construction.

Table XV* (Continued). Struts of Two Angles.

Allowable Concentric Loads in Kips for Struts of Two Equal Angles—3/8 in Back to Back

Loads to right of heavy vertical lines are for Secondary Members ONLY



Size	Thickness	Two Angles		Radius of gyration		Axis X-X													Axis Y-Y												
						Unsupported length in feet													Unsupported length in feet												
		Area	Wt.	X-X	Y-Y	3	4	5	6	7	8	9	10	12	14	16	18	20	6	7	8	9	10	12	14	16	18	20	22	24	26
6 × 6	3/8	8.72	29.8	1.88	2.62	131	131	131	131	131	131	131	128	118	109	99	91	82	131	131	131	131	131	131	128	121	114	107	100	94	88
	7/16	10.12	34.4	1.87	2.63	152	152	152	152	152	152	152	148	137	126	115	105	95	152	152	152	152	152	152	148	140	132	125	117	109	102
	1/2	11.50	39.2	1.86	2.64	173	173	173	173	173	173	173	168	155	143	130	118	108	173	173	173	173	173	173	169	160	151	142	133	125	117
	9/16	12.86	43.8	1.85	2.65	193	193	193	193	193	193	193	188	173	159	145	132	120	193	193	193	193	193	193	189	179	169	159	149	140	131
	5/8	14.22	48.4	1.84	2.66	213	213	213	213	213	213	213	207	191	175	160	145	132	213	213	213	213	213	213	210	198	187	176	165	155	145
	11/16	15.56	53.0	1.83	2.67	233	233	233	233	233	233	233	226	208	191	174	158	143	233	233	233	233	233	233	230	218	205	193	182	170	159
	3/4	16.88	57.4	1.83	2.68	253	253	253	253	253	253	253	245	226	207	189	171	156	253	253	253	253	253	253	249	236	223	210	197	185	173
	13/16	18.18	62.0	1.82	2.69	273	273	273	273	273	273	273	264	243	222	202	184	167	273	273	273	273	273	273	269	255	241	227	213	200	187
	7/8	19.46	66.2	1.81	2.70	292	292	292	292	292	292	292	282	259	237	216	196	177	292	292	292	292	292	292	288	273	258	243	229	215	201
	15/16	20.74	70.6	1.80	2.71	311	311	311	311	311	311	311	300	275	252	229	207	188	311	311	311	311	311	311	308	292	276	260	245	229	215
	1	22.00	74.8	1.80	2.72	330	330	330	330	330	330	330	318	292	267	243	220	199	330	330	330	330	330	330	327	310	293	277	260	244	229
8 × 8	1/2	15.50	52.8	2.51	3.44	233	233	233	233	233	233	233	233	233	223	211	198	185	233	233	233	233	233	233	233	233	229	219	210	201	191
	9/16	17.36	59.2	2.50	3.46	260	260	260	260	260	260	260	260	260	250	235	221	207	260	260	260	260	260	260	260	260	257	247	236	226	215
	5/8	19.22	65.4	2.49	3.47	288	288	288	288	288	288	288	288	288	276	260	244	228	288	288	288	288	288	288	288	288	285	273	262	250	239
	11/16	21.06	71.6	2.48	3.48	316	316	316	316	316	316	316	316	316	302	284	267	249	316	316	316	316	316	316	316	316	312	300	287	275	262
	3/4	22.88	77.8	2.47	3.49	343	343	343	343	343	343	343	343	343	328	308	289	270	343	343	343	343	343	343	343	343	340	326	313	299	285
	13/16	24.68	84.0	2.46	3.50	370	370	370	370	370	370	370	370	370	353	332	311	291	370	370	370	370	370	370	370	370	367	352	338	323	308
	7/8	26.46	90.0	2.45	3.51	397	397	397	397	397	397	397	397	397	378	355	333	311	397	397	397	397	397	397	397	397	393	378	362	347	331
	15/16	28.24	96.2	2.44	3.52	424	424	424	424	424	424	424	424	424	402	378	354	331	424	424	424	424	424	424	424	424	420	404	387	371	354
	1	30.00	102.0	2.44	3.53	450	450	450	450	450	450	450	450	450	428	402	376	351	450	450	450	450	450	450	450	450	447	430	412	394	377
	1 1/16	31.74	108.0	2.43	3.54	476	476	476	476	476	476	476	476	476	451	424	397	370	476	476	476	476	476	476	476	476	473	455	436	418	399
	1 1/8	33.46	113.8	2.42	3.55	502	502	502	502	502	502	502	502	502	475	446	417	389	502	502	502	502	502	502	502	502	499	480	461	441	421

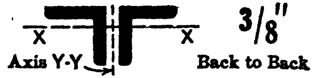
LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8 in Back to Back

Short Legs Back to Back   Loads to right of heavy vertical lines for are Secondary Members ONLY



Size	Thickness	Two Angles		Radius of gyration		Axis X-X Unsupported length in feet											Axis Y-Y Unsupported length in feet														
		Area	Wt.	X-X	Y-Y	3	4	5	6	7	8	9	10	12	14	16	6	7	8	9	10	12	14	16	18	20	22	24	26	28	30
2½ × 2	3/16	1.62	5.5	.60	1.24	24	22	19	16	14	12	10	9	.		.	24	23	22	21	19	17	14	13	11	9	..	..	..	.	
	1/4	2.12	7.2	.59	1.25	32	28	24	21	18	15	13	.	..	.		32	30	29	27	25	22	19	17	14	13	..	..			
	5/16	2.62	9.0	.58	1.26	39	34	30	25	22	19	16					39	38	36	33	31	27	24	21	18	16					
3 × 2½	1/4	2.62	9.0	.75	1.45	39	38	35	31	28	25	22	20	16	...		39	39	38	36	34	31	27	24	21	19	17	15	.	..	..
	5/16	3.24	11.2	.74	1.46	49	47	43	38	34	30	27	24	19	..	.	49	49	47	45	42	38	34	30	26	23	21	19	..	.	.
	3/8	3.84	13.2	.74	1.48	58	56	51	45	40	36	32	28	22	...	.	58	58	56	53	51	45	40	36	32	28	25	22	...	.	..
	7/16	4.42	15.2	.73	1.49	66	64	58	52	46	41	36	32	25	.		66	66	65	62	58	52	47	41	37	33	29	26	.	.	.
3½ × 2½	1/4	2.88	9.8	.74	1.71	43	42	38	34	30	27	24	21	17		.	43	43	43	43	41	37	34	31	27	25	22	20	18	17	..
	5/16	3.56	12.2	.73	1.73	53	52	46	42	37	33	29	26	20	.	.	53	53	53	53	51	46	42	38	34	31	28	25	23	21	...
	3/8	4.22	14.4	.72	1.74	63	61	55	49	43	38	34	30	24	.	.	63	63	63	63	60	55	50	45	41	37	33	30	27	25	...
	7/16	4.86	16.6	.71	1.75	73	70	63	56	49	43	38	34	..	.	.	73	73	73	72	69	64	58	52	47	43	39	35	32	29	...
	1/2	5.50	18.8	.70	1.76	83	78	70	62	55	48	43	38	...			83	83	83	82	79	72	66	60	54	49	44	40	36	33	..
4 × 3	5/16	4.18	14.4	.89	1.93	63	63	60	55	51	46	42	37	31	25	. .	63	63	63	63	62	57	53	49	44	41	37	34	31	28	. .
	3/8	4.96	17.0	.88	1.94	74	74	71	65	60	54	49	44	36	30	.	74	74	74	74	73	68	63	58	53	48	44	40	37	33	..
	7/16	5.74	19.6	.87	1.95	86	86	82	75	68	62	56	50	41	34		86	86	86	86	85	79	73	67	61	56	51	47	43	39	36
	1/2	6.50	22.2	.86	1.96	98	98	92	84	76	69	62	56	46	38	.	98	98	98	98	97	90	83	76	70	64	58	53	49	44	41
	9/16	7.24	24.8	.86	1.97	109	109	103	94	85	77	69	63	51	42	..	109	109	109	109	108	100	93	85	78	71	65	60	54	50	46
	5/8	7.96	27.2	.85	1.98	119	119	112	102	93	84	75	68	55	45		119	119	119	119	119	111	102	94	86	79	72	66	60	55	51

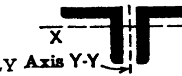
Loads by A.I.S.C. Specification

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—⅜ in Back to Back

Short Legs Back to Back Loads to right of heavy vertical lines are for Secondary Members ONLY



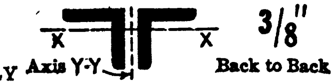
Size	Thickness	Two Angles		Radius of gyration		Axis X-X											Axis Y-Y														
						Unsupported length in feet											Unsupported length in feet														
		Area	Wt	X-X	Y-Y	3	4	5	6	7	8	9	10	12	14	16	6	7	8	9	10	12	14	16	18	20	22	24	26	28	30
4 × 3½	5/16	4.50	15.4	1.07	1.86	68	68	68	65	60	56	52	46	40	34	29	68	68	68	68	66	61	56	51	46	42	38	35	32	29	.
	3/8	5.34	18.2	1.06	1.88	80	80	80	76	71	66	61	56	47	40	34	80	80	80	80	78	73	67	61	55	50	46	42	38	35	...
	7/16	6.18	21.2	1.05	1.88	93	93	93	88	82	76	70	64	54	46	39	93	93	93	93	91	84	77	70	64	58	53	48	41	40	.
	1/2	7.00	23.8	1.04	1.89	105	105	105	100	92	86	79	72	61	51	44	105	105	105	105	103	95	88	80	73	66	60	55	50	46	. .
	9/16	7.80	26.6	1.03	1.90	117	117	117	110	102	95	87	80	67	57	48	117	117	117	117	115	106	98	90	82	74	68	62	56	51	..
	5/8	8.60	29.4	1.02	1.91	129	129	129	121	112	104	95	88	73	62	52	129	129	129	129	127	118	108	99	91	82	75	68	62	57	...
	11/16	9.36	32.0	1.02	1.93	140	140	140	132	122	113	104	95	80	67	57	140	140	140	140	139	129	119	109	99	91	83	75	69	63	...
	3/4	10.12	34.6	1.01	1.94	152	152	152	142	132	121	111	102	86	72	61	152	152	152	152	150	139	129	118	108	98	90	82	75	68	.
5 × 3	5/16	4.80	16.4	.85	2.47	72	72	67	62	56	51	46	41	33	27		72	72	72	72	72	72	69	65	61	57	53	49	46	43	40
	3/8	5.72	19.6	.84	2.48	86	86	80	73	66	60	53	48	39	32		86	86	86	86	86	86	82	77	72	68	63	59	55	51	47
	7/16	6.62	22.6	.84	2.49	99	99	93	85	77	69	62	56	45	37		99	99	99	99	99	99	95	90	84	79	73	68	64	59	55
	1/2	7.50	25.6	.83	2.50	113	113	105	95	86	77	70	62	50	..		113	113	113	113	113	113	108	102	95	89	83	78	72	67	63
	9/16	8.36	28.6	.82	2.52	125	125	116	105	95	85	77	69	55	.		125	125	125	125	125	125	121	114	107	100	93	87	81	76	70
	5/8	9.22	31.4	.82	2.53	138	138	128	116	105	94	85	76	61	..		138	138	138	138	138	138	133	126	118	111	103	97	90	84	78
	11/16	10.06	34.2	.81	2.54	151	151	139	126	113	102	91	82	66	..		151	151	151	151	151	151	146	137	129	121	113	106	98	92	86
	3/4	10.88	37.0	.80	2.55	163	163	149	135	121	109	97	87	70	.	.	163	163	163	163	163	163	158	150	140	131	123	115	107	100	93
5 × 3½	5/16	5.12	17.4	1.03	2.39	77	77	77	72	67	62	57	52	44	37	31	77	77	77	77	77	77	72	68	63	59	55	51	47	44	41
	3/8	6.10	20.8	1.02	2.40	91	91	91	86	80	74	68	62	52	44	37	92	92	92	92	92	92	86	81	76	71	66	61	57	53	49
	7/16	7.06	24.0	1.01	2.41	106	106	106	99	92	85	78	71	60	50	42	106	106	106	106	106	106	100	94	88	82	76	71	66	61	57
	1/2	8.00	27.2	1.01	2.43	120	120	120	112	104	96	88	80	67	56	48	120	120	120	120	120	120	114	107	100	93	87	81	75	70	65
	9/16	8.94	30.4	1.00	2.44	134	134	134	125	116	106	98	89	75	63	53	134	134	134	134	134	134	127	120	112	105	98	91	84	78	73
	5/8	9.84	33.6	.99	2.45	148	148	147	137	127	116	107	98	81	68	57	148	148	148	148	148	148	140	132	124	116	108	100	93	87	80
	11/16	10.74	36.6	.98	2.46	161	161	160	149	137	126	115	105	88	73	62	161	161	161	161	161	161	154	144	135	126	118	110	102	95	88
	3/4	11.62	39.6	.98	2.48	174	174	173	161	149	136	125	114	95	79	67	174	174	174	174	174	174	167	157	147	138	128	120	111	104	96

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8 in Back to Back



Short Legs Back to Back Loads to right of heavy vertical lines are for Secondary Members ONLY

Size	Thickness	Two Angles		Radius of gyration		Axis X-X												Axis Y-Y														
						Unsupported length in feet												Unsupported length in feet														
		Area	Wt.	X-X	Y-Y	4	5	6	7	8	9	10	12	14	16	18	20	14	16	18	20	22	24	26	28	30	32	34	36	38	40
6 × 3½	3/8	6.84	23 4	.99	2 95	103	102	95	88	81	74	68	57	47	40			103	100	95	90	85	81	76	72	67	63	60	56	53	50
	7/16	7.94	27 0	.98	2 96	119	118	110	102	93	85	78	65	54	46			119	116	110	105	99	94	88	83	78	74	70	65	62	58
	1/2	9.00	30 6	.97	2.97	135	133	124	114	105	98	87	73	61	51			135	131	125	119	113	106	100	95	89	84	79	74	70	66
	9/16	10 06	34 2	.96	2.99	151	149	138	127	116	106	97	80	67	56			151	147	140	133	126	119	113	106	100	94	89	84	79	74
	5/8	11.10	37.8	.96	3 00	167	164	152	140	128	117	107	89	74	62			167	163	155	147	140	132	125	118	111	105	99	93	87	82
	11/16	12.12	41.2	.95	3.01	182	179	165	152	139	127	116	96	80				182	178	170	161	153	145	137	129	122	115	108	102	96	90
	3/4	13.12	44.8	.94	3 03	197	193	178	164	149	136	124	102	85				197	193	184	175	166	157	149	140	132	125	118	111	105	99
	13/16	14.12	48 0	.94	3 04	212	207	192	176	161	147	133	110	92				212	208	199	189	179	169	160	152	143	135	127	120	113	107
	7/8	15.10	51 4	93	3 05	227	221	204	187	171	155	141	117	97				227	223	213	202	192	182	172	162	153	145	136	129	121	114
6 × 4	3/8	7.22	24.6	1 17	2.87	108	108	107	101	95	88	82	71	60	52	45		108	104	99	94	88	83	78	74	69	65	61	58	54	51
	7/16	8.36	28 6	1.16	2.88	125	125	124	117	109	102	95	81	69	59	50		125	121	115	109	103	97	91	86	81	76	71	67	63	59
	1/2	9 50	32.4	1.15	2 90	142	142	140	132	124	115	107	92	78	67	58		143	138	131	124	117	110	104	98	92	87	81	77	72	68
	9/16	10.62	36 2	1.14	2.91	159	159	157	147	137	127	119	102	87	74	64		159	154	146	139	131	124	117	110	103	97	91	86	81	76
	5/8	11.72	40 0	1 13	2.92	176	176	172	162	150	139	130	111	94	81	70		176	170	162	153	145	137	129	122	114	108	101	95	90	84
	11/16	12.80	43.6	1 13	2.94	192	192	188	176	164	153	142	121	103	88	76		192	186	177	168	159	150	142	134	126	118	111	105	99	93
	3/4	13 88	47.2	1 12	2.95	208	208	203	190	177	165	153	130	111	95	81		208	202	193	183	173	163	154	145	137	129	121	114	107	101
	13/16	14.94	50 8	1.11	2 96	224	224	218	204	190	176	163	139	118	101	87		224	218	208	197	186	176	166	157	148	139	131	123	116	109
	7/8	15.96	54 4	1.11	2 97	239	239	233	218	203	188	174	148	126	108	93		239	233	222	211	200	189	178	168	158	149	140	132	124	117

LOADS BY A I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8 in Back to Back

Short Legs Back to Back Loads to right of heavy vertical lines are for Secondary Members ONLY

3/8" Back to Back

Size	Thickness	Two Angles		Radius of gyration		Axis X-X Unsupported length in feet												Axis Y-Y Unsupported length in feet													
		Area	Wt.	X-X	Y-Y	4	5	6	7	8	9	10	12	14	16	18	20	14	16	18	20	22	24	26	28	30	32	34	36	38	40
7 × 3½	3/8	7 60	26.0	.96	3 50	114	112	104	96	88	80	73	61	51	42	..	..	114	114	113	108	104	99	95	91	86	82	78	74	70	67
	7/16	8 80	30 0	.95	3.51	132	130	120	111	101	92	84	69	58	..	.	...	132	132	131	126	120	115	110	105	100	95	90	86	82	78
	1/2	10.00	34 0	.94	3.53	150	147	135	125	114	104	94	78	65	..	..	..	150	150	149	143	137	131	126	120	114	109	103	98	93	89
	9/16	11 18	38 2	.93	3 54	168	163	151	139	126	115	105	86	72	....	. .		168	168	167	160	154	147	141	134	128	122	116	110	105	100
	5/8	12.34	42 0	.93	3.55	185	180	167	153	140	127	115	95	79	..			185	185	184	177	170	163	155	148	141	135	128	122	116	110
	11/16	13.50	46 0	.92	3 57	203	197	181	166	152	138	125	103	85	.	.		203	203	202	194	186	178	171	163	155	148	141	134	127	121
	3/4	14.62	49 8	91	3 58	219	212	195	179	163	148	134	110	91	.		..	219	219	219	211	202	194	185	177	169	161	153	145	138	132
	13/16	15.74	53 6	91	3 59	236	228	210	192	175	159	144	119	98	.			236	236	236	227	218	209	200	190	182	173	165	157	149	142
	7/8	16 84	57 4	90	3 60	253	243	224	204	186	168	153	125	103	..			253	253	253	243	233	224	214	204	195	186	177	168	160	153
	15/16	17 94	61 0	.89	3 62	269	258	237	216	196	178	161	132	108	..	..		269	269	269	260	249	239	229	218	208	199	189	180	172	163
	1	19 00	64 6	89	3 63	285	273	251	229	208	188	170	139	115			.	285	285	285	275	264	253	242	232	221	211	201	191	182	173
8 × 6	7/16	11.86	40 4	1 80	3 68	178	178	178	178	178	178	171	158	144	130	119	108	178	178	178	173	166	159	153	146	139	133	127	121	115	110
	1/2	13.50	46 0	1 79	3 69	202	202	202	202	202	202	195	179	163	148	134	122	203	203	203	197	189	182	174	166	159	152	145	138	131	125
	9/16	15 12	51 4	1.78	3.71	227	227	227	227	227	226	217	200	182	165	150	135	227	227	227	221	212	204	195	187	179	171	163	155	148	141
	5/8	16 72	57.0	1 77	3 71	251	251	251	251	251	249	239	221	200	183	165	148	251	251	251	244	235	226	216	207	198	189	180	172	164	156
	11/16	18 30	62 4	1 77	3 72	275	275	275	275	275	273	262	241	219	199	180	163	'75	275	275	268	257	247	237	227	217	207	197	188	180	171
	3/4	19 88	67 6	1 76	3 73	298	298	298	298	298	297	285	260	238	215	194	177	298	298	298	291	280	269	258	247	236	225	215	205	195	186
	13/16	21 44	73 0	1 75	3.75	322	322	322	322	322	319	306	280	255	231	209	189	322	322	322	314	303	291	279	267	255	244	233	222	212	202
	7/8	22 96	78 2	1 74	3.76	344	344	344	344	344	340	327	299	272	247	223	201	344	344	344	337	324	312	299	286	274	262	250	238	227	217
	15/16	24 50	83 4	1 73	3.77	368	368	368	368	368	363	348	319	289	262	236	213	368	368	368	360	347	333	319	306	293	280	267	255	243	232
	1	26 00	88 4	1 73	3 78	390	390	390	390	390	385	369	338	307	278	251	226	390	390	390	382	368	354	340	325	311	297	284	271	259	247

LOADS BY A.I.S C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8 in Back to Back

Long Legs Back to Back   Loads to right of heavy vertical lines are for Secondary Members ONLY

Size	Thickness	Two Angles Area	Two Angles Wt.	Radius of gyration X-X	Radius of gyration Y-Y	Axis X-X, Unsupported length in feet: 4	5	6	7	8	9	10	12	14	16	18	20	22	Axis Y-Y, Unsupported length in feet: 4	5	6	7	8	9	10	12	14	16	18	20	22
2½ × 2	3/16	1.62	5.5	.79	.92	24	22	20	18	16	14	13	10						24	24	22	20	18	17	15	12	10				
	1/4	2.12	7.2	.78	.93	32	29	26	23	21	19	17	13						32	31	29	26	24	22	20	16	14				
	5/16	2.62	9.0	.78	.95	39	36	32	29	26	23	20	16						39	39	36	33	30	27	25	21	17				
3 × 2½	1/4	2.62	9.0	.95	1.13	39	39	36	33	30	28	25	20	17					39	39	38	36	34	31	29	25	21	18	16		
	5/16	3.24	11.2	.94	1.14	49	48	44	40	37	34	31	25	21					49	49	48	45	42	39	36	31	26	23	19		
	3/8	3.84	13.2	.93	1.16	58	56	52	48	43	40	36	30	25					58	58	57	53	50	47	43	37	32	27	24		
	7/16	4.42	15.2	.92	1.17	66	64	59	54	50	45	41	34	28					66	66	66	62	58	54	50	43	37	32	27		
3½ × 2½	1/4	2.88	9.8	1.12	1.09	43	43	42	39	37	34	32	27	23	20	17			43	43	42	39	36	34	31	26	22	19	16		
	5/16	3.56	12.2	1.11	1.10	53	53	52	49	45	42	39	33	28	24	21			53	53	52	48	45	42	39	33	28	24	20		
	3/8	4.22	14.4	1.10	1.11	63	63	61	57	53	50	46	39	33	28	24			63	63	62	58	54	50	46	39	33	29	24		
	7/16	4.86	16.6	1.09	1.12	73	73	70	66	61	57	52	44	38	32	26			73	73	71	67	62	58	53	46	39	33	29		
	1/2	5.50	18.8	1.09	1.13	83	83	80	75	69	64	59	50	43	36	31			83	83	81	76	71	66	61	52	44	38	33		
4 × 3	5/16	4.18	14.4	1.27	1.30	63	63	63	61	57	54	50	44	38	33	29	25		63	63	63	61	58	54	51	45	39	34	30	26	
	3/8	4.96	17.0	1.26	1.31	74	74	74	72	68	63	59	52	45	39	31	30		74	74	74	73	69	65	61	53	47	41	36	31	
	7/16	5.74	19.6	1.25	1.32	86	86	86	83	78	73	68	60	52	45	39	34		86	86	86	84	80	75	71	62	54	47	42	36	32
	1/2	6.50	22.2	1.25	1.33	98	98	98	94	88	83	77	67	58	51	44	38		98	98	98	96	91	86	81	71	62	54	47	42	37
	9/16	7.24	24.8	1.24	1.35	109	109	109	104	98	92	86	75	65	56	49	42		109	109	109	107	102	96	91	80	70	61	54	47	42
	5/8	7.96	27.2	1.23	1.36	119	119	119	114	107	100	94	81	70	61	53	46		119	119	119	118	112	106	100	88	78	68	60	52	46

Loads by A.I.S.C. Specification

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—⅜ in Back to Back

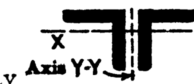
Long Legs Back to Back   Loads to right of heavy vertical lines are for Secondary Members ONLY

Size	Thickness	Two Angles		Radius of gyration		Axis X-X: Unsupported length in feet													Axis Y-Y: Unsupported length in feet												
		Area	Wt.	X-X	Y-Y	4	5	6	7	8	9	10	12	14	16	18	20	22	4	5	6	7	8	9	10	12	14	16	18	20	22
4 × 3½	5/16	4.50	15.4	1.26	1.55	68	68	68	65	61	58	54	47	41	35	31	27	.	68	68	68	68	67	64	61	55	49	44	39	35	31
	3/8	5.34	18.2	1.25	1.56	80	80	80	77	72	68	64	55	48	42	36	32	.	80	80	80	80	79	76	72	65	58	52	47	41	37
	7/16	6.18	21.2	1.24	1.57	93	93	93	89	83	78	73	64	55	48	41	36		93	93	93	93	92	88	84	76	68	61	54	48	43
	1/2	7.00	23.8	1.23	1.58	105	105	105	100	94	88	82	72	62	54	46	40	.	105	105	105	105	105	100	95	86	77	69	62	55	49
	9/16	7.80	26.6	1.23	1.59	117	117	117	112	105	98	92	80	69	60	52	45	.	117	117	117	117	117	112	107	96	87	78	69	62	55
	5/8	8.60	29.4	1.22	1.60	129	129	129	123	115	108	101	87	75	65	56	49	.	129	129	129	129	129	123	118	107	96	86	77	69	62
	11/16	9.36	32.0	1.21	1.62	140	140	140	133	125	117	109	94	81	70	61	53	.	140	140	140	140	140	135	129	117	105	95	85	76	68
	3/4	10.12	34.6	1.20	1.63	152	152	152	143	134	126	117	101	87	75	65	57		152	152	152	152	152	146	140	127	115	103	92	83	74
× 3	5/16	4.80	16.4	1.61	1.22	72	72	72	72	72	69	66	60	54	48	43	39	35	72	72	72	68	64	60	56	49	42	36	32	27	..
	3/8	5.72	19.6	1.61	1.23	86	86	86	86	86	82	79	71	64	57	51	46	41	86	86	86	82	77	72	67	59	51	44	38	33	.
	7/16	6.62	22.6	1.60	1.24	99	99	99	99	99	95	91	82	74	66	59	53	47	99	99	99	95	89	84	78	68	59	51	44	39	..
	1/2	7.50	25.6	1.59	1.25	113	113	113	113	112	107	103	93	83	75	67	60	53	113	113	113	108	102	95	89	78	67	58	51	44	..
	9/16	8.36	28.6	1.58	1.26	125	125	125	125	125	119	114	103	92	83	74	66	59	125	125	125	121	114	107	100	87	76	66	57	50	..
	5/8	9.22	31.4	1.57	1.28	138	138	138	138	137	131	125	113	101	91	81	72	64	138	138	138	134	126	119	112	98	85	74	64	56	
	11/16	10.06	34.2	1.56	1.29	151	151	151	151	150	143	136	123	110	98	88	78	70	151	151	151	147	138	130	122	107	93	81	71	62	.
	3/4	10.88	37.0	1.55	1.31	163	163	163	163	161	154	147	132	118	106	94	84	75	163	163	163	159	151	142	134	117	102	89	78	68	.
5 × 3½	5/16	5.12	17.4	1.61	1.45	77	77	77	77	77	74	70	64	57	51	46	41	37	77	77	77	77	74	71	67	60	53	47	41	37	..
	3/8	6.10	20.8	1.60	1.46	92	92	92	92	92	88	84	76	68	61	55	49	44	92	92	92	92	89	84	80	71	63	56	50	44	
	7/16	7.06	24.0	1.59	1.47	106	106	106	106	106	101	97	87	78	70	63	56	50	106	106	106	106	103	98	93	83	74	65	58	51	.
	1/2	8.00	27.2	1.58	1.49	120	120	120	120	120	114	109	99	88	79	71	63	56	120	120	120	120	117	112	106	95	84	75	66	59	...
	9/16	8.94	30.4	1.57	1.50	134	134	134	134	133	127	121	110	98	88	78	70	62	134	134	134	134	131	125	119	107	95	84	75	66	..
	5/8	9.84	33.6	1.56	1.51	148	148	148	148	146	140	133	120	108	96	86	76	68	148	148	148	148	145	138	131	118	105	93	83	74	..
	11/16	10.74	36.6	1.56	1.52	161	161	161	161	160	153	146	131	118	105	94	83	75	161	161	161	161	158	151	144	129	115	103	91	81	...
	3/4	11.62	39.6	1.55	1.54	174	174	174	174	172	165	157	141	127	113	101	90	80	174	174	174	174	172	164	156	141	126	112	100	89	.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV.* (Continued). Struts of Two Angles



Allowable Concentric Loads of Kips for Struts of Two Unequal Angles—⅜ in Back to Back

Long Legs Back to Back Loads to right of heavy vertical lines are for Secondary Members ONLY

Size	Thickness	Two Angles		Radius of gyration		Axis X-X Unsupported length in feet												Axis Y-Y Unsupported length in feet														
		Area	Wt.	X-X	Y-Y	9	10	12	14	16	18	20	22	24	26	28	30	6	7	8	9	10	12	14	16	18	20	22	24	26	28
6 × 3½	⅜	6 84	23 4	1 94	1 39	103	102	94	87	80	73	67	61	55	50	46		103	102	97	92	87	77	68	60	53	46	41			
	7/16	7.94	27 0	1 93	1 40	119	118	109	101	92	84	77	70	64	58	53		119	119	113	107	102	90	79	70	62	54	48			
	½	9.00	30.6	1 92	1.41	135	133	123	114	104	95	87	79	72	66	60		135	135	129	122	116	103	91	80	70	62	55			
	9/16	10.06	34 2	1.91	1 42	151	148	138	127	116	106	96	88	80	73	67		151	151	144	137	130	115	102	90	79	70	62			
	⅝	11.10	37 8	1 90	1 43	167	164	152	139	127	116	106	96	88	80	73		167	167	160	152	144	128	113	100	88	78	69			
	11/16	12.12	41 2	1 89	1 45	182	178	165	152	139	126	115	105	95	87	79		182	182	175	167	158	141	125	110	98	87	77			
	¾	13.12	44 8	1 89	1 46	197	193	179	164	150	137	125	113	103	94	86		197	197	190	181	172	153	136	120	107	94	84			
	13/16	14 12	48 0	1 88	1 48	212	207	192	176	161	147	133	121	110	101	92		212	212	206	196	186	167	148	131	116	103	92			
	⅞	15 10	51 4	1 87	1 49	227	221	205	188	172	156	142	129	117	107	97		227	227	221	211	200	179	159	141	125	111	99			
6 × 4	⅜	7 22	24 6	1 93	1 62	108	107	99	91	84	77	70	64	58	53	48		108	108	108	104	100	90	81	73	65	59	53	47		
	7/16	8 36	28 6	1 92	1 63	125	124	115	106	97	88	81	73	67	61	56		125	125	125	121	116	105	95	85	76	68	61	55		
	½	9 50	32 4	1 91	1 65	143	140	130	120	110	100	91	83	76	69	63		143	143	143	138	132	120	109	98	88	79	71	64		
	9/16	10.62	36 2	1 90	1 66	159	156	145	133	122	111	101	92	84	77	70		159	159	159	155	148	135	122	110	98	88	80	72		
	⅝	11.72	40.0	1.90	1 67	176	173	160	147	135	123	112	102	93	85	77		176	176	176	171	164	149	135	122	109	98	88	80		
	11/16	12.80	43 6	1 89	1 68	192	188	174	160	146	134	121	111	101	92	84		192	192	192	187	180	164	148	134	120	108	97	87		
	¾	13.88	47 2	1 88	1 69	208	204	188	173	158	144	131	119	108	99	90		208	208	208	204	195	178	161	146	131	118	106	96		
	13/16	14 94	50 8	1 87	1 70	224	219	202	186	170	155	141	128	116	106	96		224	224	224	220	211	192	174	158	142	128	115	104		
	⅞	15 96	54 4	1 86	1 71	239	233	215	198	181	164	149	136	123	112	102		239	239	239	235	226	206	187	169	152	137	124	112		

LOADS BY A I S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XV * (Continued). Struts of Two Angles

3/8" Back to Back

Allowable Concentric Loads of Kips for Struts of Two Unequal Angles—3/8 in Back to Back

Long Legs Back to Back Loads to right of heavy vertical lines are for Secondary Members ONLY

Size	Thickness	Two Angles: Area	Two Angles: Wt.	Radius of gyration: X-X	Radius of gyration: Y-Y	Axis X-X, Unsupported length in feet: 9	10	12	14	16	18	20	22	24	26	28	30	Axis Y-Y, Unsupported length in feet: 6	7	8	9	10	12	14	16	18	20	22	24	26	28
7 × 3½	3/8	7.60	26.0	2.27	1.33	114	114	112	105	98	91	84	78	72	67	62	57	114	112	106	100	94	83	72	63	55	49	43	..		.
	7/16	8.80	30.0	2.26	1.34	132	132	129	121	113	105	97	90	83	77	71	66	132	130	123	116	110	96	85	74	65	57	50			.
	1/2	10.00	34.0	2.25	1.35	150	150	147	137	128	119	110	102	94	87	80	74	150	148	141	133	125	110	97	85	74	65	58			
	9/16	11.18	38.2	2.25	1.36	168	168	164	154	143	133	123	114	105	97	90	83	168	166	158	149	140	124	109	96	84	74	65	.	..	
	5/8	12.34	42.0	2.24	1.37	185	185	181	169	158	146	136	125	116	107	99	91	185	184	175	165	156	138	121	106	93	82	72	.		..
	11/16	13.50	46.0	2.23	1.39	203	203	197	185	172	160	148	137	126	116	107	99	203	202	192	182	172	152	134	118	104	92	81	...		...
	3/4	14.62	49.8	2.22	1.40	219	219	213	200	186	173	160	147	136	125	116	107	219	219	209	198	187	166	146	129	113	100	88	..		..
	13/16	15.74	53.6	2.21	1.41	236	236	229	215	200	185	171	158	146	134	124	115	236	236	225	214	202	179	158	140	123	109	96	..	.	
	7/8	16.84	57.4	2.20	1.42	253	253	245	229	213	197	183	168	155	143	132	122	253	253	242	229	217	193	170	150	133	117	104		.	.
	15/16	17.94	61.0	2.19	1.44	269	269	260	243	226	210	194	179	165	152	140	129	269	269	259	246	233	208	184	162	144	127	113	..		..
	1	19.00	64.6	2.19	1.45	285	285	276	258	240	222	205	189	174	161	148	137	285	285	275	261	248	221	196	173	153	136	121	...		.
8 × 6	7/16	11.86	40.4	2.57	2.43	178	178	178	173	163	153	144	135	126	117	110	102	178	178	178	178	178	178	169	158	148	138	129	120	111	104
	1/2	13.50	46.0	2.56	2.44	203	203	203	196	185	174	163	153	143	133	124	116	203	203	203	203	203	203	192	181	169	158	147	137	127	118
	9/16	15.12	51.4	2.55	2.46	227	227	227	219	208	195	182	171	159	148	138	129	227	227	227	227	227	227	216	203	191	178	166	155	144	134
	5/8	16.72	57.0	2.54	2.46	251	251	251	242	228	215	201	188	176	164	153	142	251	251	251	251	251	251	239	225	211	197	184	171	159	148
	11/16	18.30	62.4	2.54	2.47	275	275	275	265	250	235	220	206	192	179	167	156	275	275	275	275	275	275	262	247	231	216	202	188	175	162
	3/4	19.88	67.6	2.53	2.48	298	298	298	287	271	255	239	223	208	194	181	168	298	298	298	298	298	298	285	268	252	235	220	205	191	177
	13/16	21.44	73.0	2.52	2.49	322	322	322	309	292	274	257	240	224	208	194	181	322	322	322	322	322	322	308	290	272	255	238	222	206	192
	7/8	22.96	78.2	2.51	2.51	344	344	344	331	312	293	274	256	239	223	207	193	344	344	344	344	344	344	331	312	293	274	256	239	223	207
	15/16	24.50	83.4	2.50	2.51	368	368	368	353	332	311	292	272	254	236	220	205	368	368	368	368	368	368	353	333	312	293	273	255	237	221
	1	26.00	88.4	2.49	2.52	390	390	390	374	352	330	309	288	269	250	233	217	390	390	390	390	390	390	375	354	332	311	291	271	253	235

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XVI.* Standard Beams as Columns

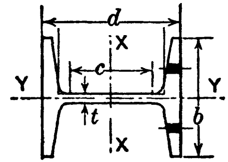
Allowable Concentric Loads in Kips for 3-in to 8-in Rolled Columns. American Standard Beams

Weight per foot	Area sq in	Least radius gyration	Unsupported Length in Feet												
			3	4	5	6	7	8	9	10	11	12	13	14	
3-in American Standard Beams															
5.7	1.64	0.53	23.5	20.3	17.2	14.6	12.3	10.5	......	...	.....	...	.	...	.....
6.5	1.88	0.52	26.7	23.0	19.5	16.4	13.8	11.7	......	...	....	...	.	...	......
7.5	2.17	0.52	30.9	26.5	22.5	18.9	15.9	13.5	......	...	...	.			..
4-in American Standard Beams															
7.7	2.21	0.59	33.0	29.1	25.3	21.8	18.7	16.1	13.9	.....	....	.	..	.....	.....
8.5	2.46	0.58	36.5	32.1	27.8	23.9	20.4	17.6	15.1	......	...	.....	.		.....
9.5	2.76	0.58	40.9	36.0	31.2	26.8	22.9	19.7	17.0	.....	.....	...	..		...
10.5	3.05	0.57	44.9	39.4	34.0	29.1	24.9	21.3	18.3	...	.				
5-in American Standard Beams															
10.0	2.87	0.65	43.1	39.7	35.1	30.7	26.8	23.4	20.4	17.9	.....	....	...	..	
12.25	3.56	0.63	53.4	48.5	42.6	37.1	32.2	28.0	24.4	21.3	.....	..	..	..	....
14.75	4.29	0.63	64.4	53.4	51.4	44.7	38.8	33.7	29.3	25.6	..	.	.		
6-in American Standard Beams															
12.5	3.61	0.72	54.2	52.1	45.9	41.8	37.0	32.7	28.9	25.6	22.7	20.1	.	..	....
14.75	4.29	0.69	64.4	60.8	54.4	48.1	42.3	37.2	32.7	28.8	25.5	..	.	...	.....
17.25	5.02	0.68	75.3	70.8	63.1	55.7	48.9	42.9	37.7	33.1	29.2			..	.
7-in American Standard Beams															
15.3	4.43	0.78	66.5	65.9	60.0	54.1	48.5	43.3	38.6	34.4	30.8	27.6	24.7	...	....
17.5	5.09	0.76	76.4	75.0	68.1	61.1	54.6	48.6	43.2	38.4	34.3	30.6	....	...	......
20.0	5.83	0.74	87.5	85.1	76.9	68.8	61.2	54.3	48.1	42.7	38.0	33.8			.
8-in American Standard Beams															
18.4	5.34	0.84	80.1	80.1	74.9	68.2	61.8	55.7	50.1	45.0	40.5	36.5	32.9	29.6	....
20.5	5.97	0.82	89.6	89.6	82.8	75.2	67.9	61.0	54.7	49.1	44.1	39.6	35.7	.....	......
23.0	6.71	0.81	100.6	100.6	92.5	83.9	75.6	67.8	60.8	54.4	48.8	43.8	39.5	....	.....
25.5	7.43	0.80	111.5	111.2	101.9	92.2	82.9	71.3	66.4	59.4	53.2	47.8	42.9	...	..

Loads to right of heavy vertical lines are for Secondary Members Only. LOADS BY A.I S C SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction.

Table XVII.* Bethlehem Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

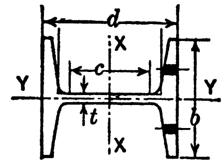
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 4	6	8	9	10	11	12	13	14	15	16	17
6-in Bethlehem Stanchions																			
15.5	6.000	6.00	.240	4.813	4.61	2 56	1.41	69	69	66	63	59	56	53	49	46	44	41	38
18 0	6.094	6.03	.270	4.813	5.33	2 59	1.43	80	80	77	73	69	65	61	58	54	51	48	45
20 5	6.188	6.06	300	4 813	6 06	2 62	1 45	91	91	88	83	79	75	70	66	62	59	55	49
6-in Bethlehem H Columns																			
20 0	6 000	6.000	.250	4.625	5 89	2 58	1 49	88	88	86	82	78	74	70	66	62	59	55	52
23.0	6.125	6.020	.270	4.625	6.76	2 62	1 51	101	101	99	95	90	85	81	76	72	68	64	60
26.5	6.250	6.060	.310	4 625	7.76	2 65	1 53	116	116	115	109	104	99	94	89	84	79	74	70
30 0	6.375	6.100	.350	4.625	8 77	2 68	1 54	132	132	130	124	118	112	106	101	95	90	85	80
33.5	6.500	6.140	.390	4.625	9 80	2 70	1 55	147	147	145	139	132	126	119	113	107	101	95	90
37.0	6.625	6.180	.430	4 625	10 83	2 73	1 57	162	162	161	154	147	140	133	126	119	113	106	101
40 5	6 750	6.220	470	4 625	11 87	2 76	1 58	178	178	177	170	162	154	146	139	131	124	117	111
8-in Bethlehem Beams																			
17 5	8 000	5 250	250	6 625	5 20	3 33	1 11	78	76	66	61	57	52	48	45	41	38	35	33
19.0	8 060	5 270	270	6 625	5 68	3 35	1 13	85	83	73	68	63	58	54	50	46	42	39	36
8-in Bethlehem Girder Beams																			
29.5	7.880	7.995	285	6.000	8 69	3 41	1 81	130	130	130	130	126	121	116	111	106	101	96	92
33.0	8.000	8.000	.290	6 000	9 69	3.46	1 86	145	145	145	145	141	136	131	125	120	115	110	105
36.5	8 120	8 020	310	6 000	10 81	3 50	1 90	162	162	162	162	159	154	148	142	136	130	124	119

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



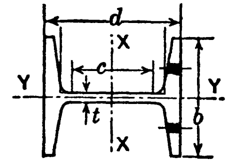
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 4	6	8	9	10	11	12	13	14	15	16	17
8-in Bethlehem H Columns																			
23.5	7.750	6.500	.250	6.125	6.85	3.33	1.57	103	103	102	98	93	89	84	80	75	71	67	64
27.0	7.875	6.530	.280	6.125	7.89	3.37	1.59	118	118	118	113	108	103	98	93	88	83	78	74
30.5	8.000	6.560	.310	6.125	8.95	3.41	1.61	134	134	134	129	123	117	112	106	100	95	90	85
34.5	8.125	6.600	.350	6.125	10.10	3.43	1.62	152	152	152	146	139	133	126	120	114	108	102	97
32.0	7.875	8.000	.310	6.125	9.30	3.40	1.96	140	140	140	140	139	134	129	124	119	114	109	105
35.0	8.000	8.000	.310	6.125	10.30	3.46	2.00	155	155	155	155	155	149	144	139	133	128	123	117
39.5	8.125	8.040	.350	6.125	11.63	3.48	2.01	174	174	174	174	174	169	163	157	151	145	139	133
44.0	8.250	8.080	.390	6.125	12.96	3.51	2.03	194	194	194	194	194	189	182	176	169	162	156	149
48.5	8.375	8.120	.430	6.125	14.31	3.54	2.04	215	215	215	215	215	209	202	194	187	180	173	166
53.0	8.500	8.160	.470	6.125	15.66	3.57	2.06	235	235	235	235	235	230	222	214	206	198	190	182
58.0	8.625	8.200	.510	6.125	17.03	3.60	2.07	255	255	255	255	255	250	242	233	224	216	207	199
62.5	8.750	8.240	.550	6.125	18.40	3.62	2.09	276	276	276	276	276	271	262	253	244	235	225	217
67.5	8.875	8.280	.590	6.125	19.79	3.65	2.10	297	297	297	297	297	292	282	273	263	253	243	234
72.0	9.000	8.320	.630	6.125	21.18	3.68	2.11	318	318	318	318	318	313	303	292	282	271	261	251
77.0	9.125	8.360	.670	6.125	22.59	3.71	2.12	339	339	339	339	339	335	324	313	301	290	279	269
81.5	9.250	8.390	.700	6.125	23.91	3.74	2.14	359	359	359	359	359	355	344	332	321	309	297	286
86.0	9.375	8.430	.740	6.125	25.33	3.77	2.15	380	380	380	380	380	377	365	353	340	328	316	304
91.0	9.500	8.470	.780	6.125	26.77	3.80	2.16	402	402	402	402	402	399	386	374	361	348	335	322

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



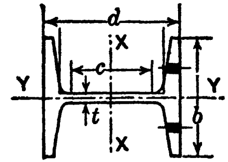
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet: 6	7	8	9	10	12	14	16	18	20	22	24
10-in Bethlehem Bs																			
21.0	9.90	5.740	.240	8.375	6.28	4 15	1.22	94	89	84	79	73	64	55	48	41	36	...	...
23 5	10 00	5.750	.250	8.375	6.96	4.21	1.25	104	100	94	89	83	72	63	54	47	41	..	....
26.0	10.09	5.770	.270	8.375	7.68	4.24	1.28	115	112	105	99	93	81	71	61	54	47	...	...
28 5	10 19	5.785	.285	8 375	8 41	4.28	1.30	126	123	116	109	103	90	78	68	60	52	..	..
10-in Bethlehem Gs																			
41.5	9.91	8 990	.310	7.750	12.23	4.30	2.07	183	183	183	183	183	174	161	149	137	126	116	106
44.5	10.00	9.000	.320	7 750	13.14	4.33	2.10	197	197	197	197	197	188	174	161	149	137	126	116
50.0	10.12	9 040	.360	7.750	14.62	4 36	2 13	219	219	219	219	219	210	196	181	168	154	142	131
10-in Bethlehem H Columns																			
33.5	9.625	8.00	.28	7.688	9 80	4.16	1.93	147	147	147	147	145	135	124	114	104	95	86	79
38.0	9.750	8.03	.31	7.688	11.09	4.20	1.96	166	166	166	166	165	154	142	130	119	109	99	91
42.5	9.875	8.07	.35	7.688	12.49	4.23	1.97	187	187	187	187	186	173	160	147	135	123	113	103
47.5	10.000	8.11	.39	7.688	13.90	4.25	1.99	209	209	209	209	208	194	179	165	151	138	126	116
49.5	9.875	9.97	.36	7.688	14.57	4.28	2.47	219	219	219	219	219	219	209	196	184	172	160	149
55.0	10.000	10.00	.39	7.688	16.12	4.32	2.50	242	242	242	242	242	242	232	219	205	192	179	167
60.5	10.125	10.04	.43	7.688	17.77	4 34	2 51	267	267	267	267	267	267	256	241	227	212	198	185
66.0	10.250	10.08	.47	7.688	19.44	4 37	2.53	292	292	292	292	292	292	281	265	249	233	218	203
72.0	10.375	10.12	.51	7 688	21.11	4.40	2.54	317	317	317	317	317	317	306	288	271	254	237	222

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



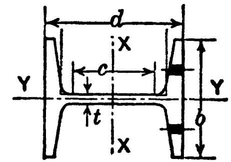
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 6	7	8	9	10	12	14	16	18	20	22	24
10-in Bethlehem H Columns																			
77.5	10.500	10.16	.55	7.688	22.80	4.43	2.56	342	342	342	342	342	342	331	313	294	276	258	241
83.5	10.625	10.20	.59	7.688	24.49	4.46	2.57	367	367	367	367	367	367	356	336	317	297	278	260
89.0	10.750	10.24	.63	7.688	26.20	4.49	2.59	393	393	393	393	393	393	382	361	340	319	299	280
95.0	10.875	10.28	.67	7.688	27.91	4.51	2.60	419	419	419	419	419	419	408	386	363	341	319	299
100.5	11.000	10.31	.70	7.688	29.53	4.55	2.61	443	443	443	443	443	443	432	409	385	362	339	317
106.5	11.125	10.35	.74	7.688	31.26	4.58	2.63	469	469	469	469	469	469	459	434	409	385	361	338
112.0	11.250	10.39	.78	7.688	33.00	4.60	2.64	495	495	495	495	495	495	485	459	427	407	382	358
118.0	11.375	10.43	.82	7.688	34.76	4.63	2.65	521	521	521	521	521	521	511	484	457	430	403	378
124.0	11.500	10.47	.86	7.688	36.52	4.66	2.66	548	548	548	548	548	548	538	510	481	453	425	398
130.0	11.625	10.51	.90	7.688	38.30	4.69	2.68	575	575	575	575	575	575	566	536	507	477	448	420
136.5	11.750	10.55	.94	7.688	40.08	4.72	2.69	601	601	601	601	601	601	593	562	531	500	470	441
								12	14	16	18	20	22	24	26	28	30	32	34
62.0	10.000	11.99	.38	7.688	18.29	4.38	3.04	274	274	270	258	245	232	220	208	196	185	174	164
68.0	10.125	12.03	.42	7.688	20.13	4.40	3.05	302	302	297	283	270	256	242	229	216	204	193	182
75.0	10.250	12.06	.45	7.688	22.00	4.44	3.08	330	330	326	311	296	281	267	252	238	225	212	200
82.0	10.375	12.10	.49	7.688	23.98	4.47	3.09	360	360	355	340	323	307	291	276	261	246	232	219
88.0	10.500	12.14	.53	7.688	25.86	4.49	3.11	388	388	384	367	350	332	315	298	282	267	252	238
94.0	10.625	12.17	.56	7.688	27.63	4.52	3.12	414	414	411	393	374	356	338	320	302	286	270	255
100.0	10.750	12.20	.59	7.688	29.54	4.56	3.14	443	443	440	421	401	382	362	343	325	307	290	274
107.0	10.875	12.23	.62	7.688	31.45	4.59	3.16	472	472	470	449	429	408	387	367	348	329	311	294
113.0	11.000	12.26	.65	7.688	33.25	4.62	3.17	499	499	497	476	454	432	410	389	368	348	330	312

Loads to right of heavy vertical lines are for Secondary Members ONLY

LOADS BY A. I. S. C. SPECIFICATION

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

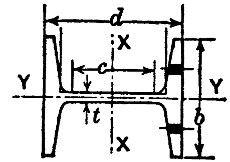
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 12	14	16	18	20	22	24	26	28	30	32	34
10-in Bethlehem H Columns																			
125.0	11.000	14.00	.65	7.688	36.89	4.66	3.65	553	553	553	553	535	514	493	472	451	431	411	392
133.0	11.125	14.04	.69	7.688	39.02	4.69	3.66	585	585	585	585	567	545	523	500	478	457	436	416
140.0	11.250	14.08	.73	7.688	41.29	4 72	3.68	619	619	619	619	601	578	555	531	508	486	463	442
148.0	11.375	14.11	.76	7.688	43.46	4.75	3.69	652	652	652	652	633	609	585	560	535	512	488	466
155.0	11.500	14.15	.80	7.688	45 62	4.78	3 70	684	684	684	684	666	640	614	588	563	538	514	490
162.0	11.625	14.19	.84	7.688	47.78	4.81	3 72	717	717	717	717	699	672	645	618	592	566	540	516
170.0	11.750	14.22	.87	7.688	49.98	4.84	3.73	750	750	750	750	731	704	676	648	620	593	566	540
177.0	11.875	14.25	.90	7.688	52.18	4 87	3.74	783	783	783	783	764	736	706	677	648	620	592	566
185.0	12.000	14.29	.94	7.688	54.37	4 90	3.76	816	816	816	816	798	768	738	708	678	648	620	592
192.0	12.125	14.32	.97	7.688	56.45	4.93	3.77	847	847	847	847	829	799	767	736	705	675	645	615
200.0	12.250	14.36	1.01	7.688	58.80	4.96	3.78	882	882	882	882	864	833	800	768	736	704	673	642
208.0	12.375	14.40	1 05	7.688	61 17	4.99	3 79	918	918	918	918	900	867	834	800	766	733	701	670
215.0	12.500	14.43	1.08	7.688	63.27	5.02	3.80	949	949	949	949	932	898	863	828	794	760	727	694
222 0	12.625	14.46	1 11	7.688	65 38	5.05	3.81	981	981	981	981	964	929	893	858	822	787	752	719
230.0	12.750	14.50	1.15	7.688	67.77	5.09	3.83	1 017	1 017	1 017	1 017	1 002	965	928	891	855	818	783	748
238.0	12.875	14.53	1.18	7.688	70.04	5.12	3.84	1 051	1 051	1 051	1 051	1 036	999	961	922	885	847	810	775
246.0	13.000	14 57	1.22	7.688	72 30	5.15	3 85	1 085	1 085	1 085	1 085	1 070	1 032	993	954	915	876	838	801

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 4	8	12	14	16	18	20	22	24	26	28	30
								12-in Bethlehem Bs											
28.0	12.00	6.500	.245	10.250	8.28	5.08	1.41	124	118	94	83	73	65	57	51	.	.	...	..
36.0	12.25	6.555	.300	10.250	10.58	5.16	1.46	159	154	124	110	97	86	76	68	60	..	...	...
40.0	12.00	6.750	.330	9.750	11.80	5.05	1.53	177	174	142	127	113	101	90	80	72	.	....	...
48.5	12.25	6.815	.395	9.750	14.28	5.11	1.57	214	213	175	157	140	125	112	100	90		..	..
								12-in Bethlehem Gs											
55.5	12.00	10.000	.380	9.500	16.35	5.16	2.28	245	245	241	226	211	196	182	169	156	144	134	123
61.0	12.12	10.030	.410	9.500	17.92	5.20	2.31	269	269	265	249	233	217	202	187	173	160	148	137
70.5	12.00	10.250	.470	9.000	20.79	5.11	2.40	312	312	312	294	276	258	241	224	208	193	179	166
76.5	12.12	10.290	.510	9.000	22.50	5.14	2.42	338	338	338	319	300	281	262	244	227	210	196	182
								12-in Bethlehem H Columns											
40.5	11.500	8.00	.31	9.188	11.85	4.93	1.90	178	178	162	149	136	124	113	103	94	85	78	.
45.5	11.625	8.04	.35	9.188	13.31	4.95	1.92	200	200	183	168	154	141	128	117	106	97	89	...
50.5	11.750	8.08	.39	9.188	14.79	4.98	1.93	222	222	203	187	172	157	143	131	119	109	99	....
55.0	11.875	8.12	.43	9.188	16.27	5.00	1.94	244	244	224	207	190	173	158	144	132	120	110	...
52.5	11.625	10.00	.35	9.188	15.40	5.04	2.44	231	231	231	219	206	193	180	168	156	145	135	126
58.0	11.750	10.04	.39	9.188	17.12	5.06	2.45	257	257	257	244	230	215	201	187	174	162	151	140
64.0	11.875	10.08	.43	9.188	18.85	5.09	2.47	283	283	283	270	254	238	223	208	193	180	167	156
70.0	12.000	10.12	.47	9.188	20.59	5.12	2.49	309	309	309	296	279	261	244	228	213	198	184	172

Loads to right of heavy vertical lines are for Secondary Members Only — Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

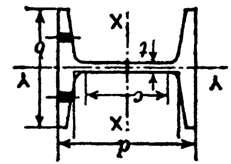


Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions

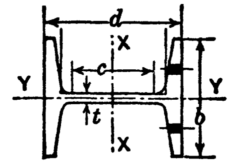
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 4	8	12	14	16	18	20	22	24	26	28	30
								12-in Bethlehem H Columns											
65.5	11.750	11.92	.39	9.188	19.29	5.12	2.96	289	239	289	289	281	268	254	241	228	215	202	191
72.5	11.875	11.96	.43	9.188	21.25	5.15	2.98	319	319	319	319	311	296	281	266	252	238	224	211
79.0	12.000	12.00	.47	9.188	23.23	5.18	2.99	348	348	348	348	340	324	308	292	276	261	246	232
85.5	12.125	12.04	.51	9.188	25.21	5.21	3.01	378	378	378	378	370	353	335	318	301	284	268	253
92.5	12.250	12.08	.55	9.188	27.21	5.23	3.03	408	408	408	408	400	382	363	345	326	308	291	275
99.5	12 375	12 12	.59	9.188	29.21	5 26	3 04	438	438	438	438	430	411	391	371	351	332	313	296
106.0	12.500	12.16	.63	9.188	31.23	5.29	3.06	468	468	468	468	461	440	419	398	377	356	337	318
113.0	12.625	12.20	.67	9.188	33 25	5.32	3 07	499	499	499	499	492	469	447	424	402	380	359	339
119.5	12.750	12 23	.70	9 188	35.16	5.35	3.09	527	527	527	527	521	498	474	450	427	404	382	361
126.5	12.875	12.27	.74	9.188	37 21	5.38	3.10	558	558	558	558	552	528	502	477	453	429	405	383
133.5	13.000	12.31	.78	9.188	39 26	5.41	3.11	589	589	589	589	583	557	531	505	479	453	429	405
140.5	13.125	12.35	.82	9.188	41 32	5 44	3.13	620	620	620	620	615	588	561	533	506	479	453	428
147.5	13.250	12.39	.86	9.188	43 40	5 47	3.14	651	651	651	651	647	619	590	561	532	504	477	451
154.5	13.375	12.43	.90	9.188	45.48	5.50	3.15	682	682	682	682	679	649	619	589	559	530	502	474
162.0	13.500	12.47	.94	9.188	47 57	5.52	3.17	714	714	714	714	711	681	649	618	587	557	527	499
169.0	13.625	12 51	.98	9.188	49 68	5.55	3.18	745	745	745	745	744	712	679	647	614	583	552	522
176.0	13.750	12.55	1.02	9.188	51 79	5 58	3.19	777	777	777	777	776	743	709	675	642	609	577	546
183.0	13.875	12 58	1.05	9.188	53.78	5.61	3.20	807	807	807	807	807	773	738	702	668	633	600	568
190.0	14.000	12.62	1 09	9 188	55 91	5 64	3.22	839	839	839	839	839	805	769	733	697	661	627	594

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



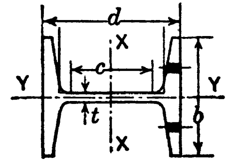
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	9	10	12	14	16	18	20	22	24	26	28	30
								Unsupported Length in Feet											
14-in Bethlehem H Columns (8, 10 and 12-in Nominal Flange Widths)																			
43.0	13.375	8.00	31	11.063	12.58	5 70	1.86	189	184	170	156	142	129	11[illegible]	107	97	88	81	...
48.0	13.500	8 04	.35	11 063	14 12	5.72	1.88	212	207	192	176	161	147	133	121	110	101	92	...
53.5	13.625	8.08	39	11 063	15.67	5.74	1.89	235	231	213	196	179	163	149	135	123	112	102	.....
58.5	13.750	8.12	.43	11.063	17.23	5.76	1 90	258	254	235	216	198	181	164	150	136	124	113	...
55.0	13.500	10.00	.35	11 063	16.25	5.82	2.39	244	244	243	229	215	201	188	174	162	150	139	....
61.5	13.625	10.04	.39	11 063	18.04	5.85	2.41	271	271	271	256	240	224	209	195	181	168	156	145
67.5	13.750	10.08	.43	11.063	19.85	5 87	2 43	298	298	298	282	265	248	232	216	201	186	173	161
73.5	13 875	10.12	.47	11 063	21.66	5.90	2.44	325	325	325	309	290	272	254	236	220	204	190	177
69.0	13.625	12.00	.39	11.063	20.34	5.93	2.93	305	305	305	305	296	281	267	252	238	225	212	199
76.0	13.750	12 04	.43	11.063	22.39	5 95	2 95	336	336	336	336	326	311	295	279	264	249	234	221
83.0	13 875	12.08	.47	11 063	24.45	5.98	2.97	367	367	367	367	357	340	323	306	289	273	257	242
90.0	14 000	12 12	51	11 063	26 52	6 01	2.98	398	398	398	398	388	369	351	333	314	297	280	264
14-in Bethlehem H Columns (14-in Nominal Flange Width)																			
84 0	13.750	13.92	.43	11 063	24.76	6 01	3.45	371	371	371	371	371	366	351	336	321	306	292	278
92.0	13.875	13.96	.47	11 063	27.05	6 04	3.47	406	406	406	406	406	401	385	368	352	336	320	305
100.0	14.000	14.00	.51	11.063	29.36	6.07	3.49	440	440	440	440	440	436	418	401	383	366	349	332
107.5	14.125	14.04	.55	11.063	31.67	6.10	3.50	475	475	475	475	475	471	452	433	414	396	377	359
115.5	14 250	14 08	.59	11.063	34.00	6.12	3.52	510	510	510	510	510	506	486	466	446	426	407	387
123 5	14 375	14 12	63	11 063	36 33	6 15	3 54	545	545	545	545	545	542	521	500	478	457	436	415

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A I S C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions

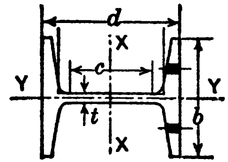


Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 9	10	12	14	16	18	20	22	24	26	28	30
14-in Bethlehem H Columns (14-in Nominal Flange Width)																			
131 5	14.500	14.16	.67	11.063	38 68	6 18	3 55	580	580	580	580	580	577	555	533	510	487	465	443
139.0	14.625	14 19	.70	11.063	40 88	6.21	3 57	613	613	613	613	613	612	588	565	540	517	493	470
147.0	14.750	14.23	.74	11 063	43 25	6 24	3.58	649	649	649	649	649	647	623	598	573	548	523	499
155.0	14.875	14.27	.78	11.063	45.62	6.27	3 60	684	684	684	684	684	684	659	632	606	579	553	528
161.0	14.875	15.00	.78	11.063	47.33	6.29	3 80	710	710	710	710	710	710	697	672	646	620	594	568
168.0	15.000	15.02	.80	11.063	49.51	6 33	3 82	743	743	743	743	743	743	731	705	677	650	623	597
177.0	15.125	15.06	.84	11 063	51 99	6 36	3 83	780	780	780	780	780	780	768	740	712	684	656	628
185.0	15.250	15.10	.88	11 063	54 48	6 39	3 84	817	817	817	817	817	817	806	777	747	718	688	659
194.0	15.375	15.14	.92	11 063	56.99	6.42	3.86	855	855	855	855	855	855	845	814	784	753	722	692
202.0	15.500	15.18	.96	11.063	59.50	6.44	3.87	893	893	893	893	893	893	882	851	819	787	755	724
210.0	15.625	15.21	.99	11.063	61.86	6.48	3 88	928	928	928	928	928	928	918	886	852	819	786	753
219.0	15.750	15.25	1.03	11.063	64.40	6.51	3.89	966	966	966	966	966	966	957	923	888	854	820	785
227.0	15.875	15.29	1.07	11.063	66.94	6.53	3.91	1 004	1 004	1 004	1 004	1 004	1 004	996	961	926	890	854	819
236.0	16.000	15.33	1.11	11.063	69.49	6.56	3.92	1 042	1 042	1 042	1 042	1 042	1 042	1 035	999	962	925	888	852
245.0	16.125	15.37	1.15	11.063	72.05	6 59	3.93	1 081	1 081	1 081	1 081	1 081	1 081	1 074	1 038	999	960	922	885
254.0	16.250	15.41	1.19	11.063	74.62	6 62	3.94	1 119	1 119	1 119	1 119	1 119	1 119	1 113	1 075	1 036	996	957	918
262 0	16.375	15.45	1.23	11.063	77.20	6 65	3.96	1 158	1 158	1 158	1 158	1 158	1 158	1 154	1 115	1 075	1 033	993	953
271.0	16.500	15.49	1.27	11.063	79 79	6 68	3.97	1 197	1 197	1 197	1 197	1 197	1 197	1 194	1 153	1 111	1 069	1 028	986
280.0	16.625	15.53	1.31	11.063	82 39	6 71	3.98	1 236	1 236	1 236	1 236	1 236	1 236	1 233	1 191	1 149	1 106	1 062	1 020
289.0	16.750	15.57	1.35	11.063	85 01	6 74	3 99	1 275	1 275	1 275	1 275	1 275	1 275	1 274	1 231	1 187	1 142	1 097	1 053
298.0	16 875	15 61	1.39	11 063	87 63	6 77	4 01	1 314	1 314	1 314	1 314	1 314	1 314	1 314	1 272	1 226	1 180	1 135	1 089

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

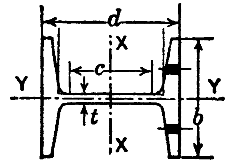
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 16	18	20	22	24	26	28	30	32	34	36	38
								16-in Bethlehem H Columns											
143.0	14.500	15 54	.72	11.063	42 03	6.19	3.90	630	630	625	603	580	558	536	514	492	470	450	430
151.0	14.625	15 58	.76	11.063	44 56	6 22	3.92	668	668	664	641	617	593	569	546	523	501	479	458
160.0	14 750	15 62	.80	11.063	47.10	6 25	3.93	707	707	702	678	653	628	603	578	554	530	507	485
169.0	14.875	15.66	.84	11.063	49.65	6.28	3.95	745	745	742	716	690	664	538	612	586	561	537	513
177.0	15.000	15.70	.88	11.063	52.20	6 31	3 96	783	783	780	754	727	698	671	644	617	591	565	541
186.0	15.125	15.74	.92	11.063	54.77	6.34	3.98	822	822	820	792	763	735	706	678	650	622	596	570
195.0	15.250	15.78	96	11.063	57 35	6.37	4 00	860	860	860	832	801	771	742	712	683	654	626	599
203.0	15.375	15.82	1.00	11.063	59 94	6 40	4.01	899	899	899	870	839	807	776	745	715	685	656	628
212.0	15.500	15.86	1.04	11.063	62.53	6.43	4.02	938	938	938	909	875	844	811	778	747	716	685	657
221.0	15.625	15.90	1.08	11.063	65 14	6.46	4.04	977	977	977	948	914	881	847	814	780	748	717	687
230.0	15.750	15.93	1.11	11.063	67.60	6.49	4.05	1 014	1 014	1 014	984	950	915	880	846	811	778	746	714
238.0	15.875	15.96	1.14	11.063	70.07	6.53	4.07	1 051	1 051	1 051	1 022	987	951	914	879	844	809	776	743
247.0	16.000	16 00	1.18	11 063	72.70	6 55	4.08	1 091	1 091	1 091	1 061	1 025	988	950	913	877	841	806	773
256.0	16.125	16.04	1.22	11.063	75.35	6.58	4.10	1 130	1 130	1 130	1 102	1 065	1 026	988	949	912	875	839	804
265.0	16.250	16 08	1.26	11.063	78 00	6.61	4.11	1 170	1 170	1 170	1 142	1 103	1 064	1 024	984	945	907	870	834
274 0	16.375	16.12	1 30	11.063	80.67	6.64	4.12	1 210	1 210	1 210	1 183	1 142	1 101	1 060	1 020	979	940	901	864

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVII * (Continued). Bethlehem Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



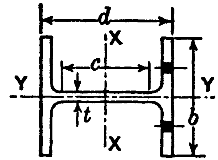
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported Length in Feet 16	18	20	22	24	26	28	30	32	34	36	38
16-in Bethlehem H Columns																			
288.0	16.563	16 18	1 36	11 063	84.69	6.69	4.14	1 270	1 270	1 270	1 243	1 201	1 159	1 116	1 073	1 032	991	950	910
301.0	16.750	16.23	1.41	11.063	88.56	6.74	4 16	1 328	1 328	1 328	1 303	1 259	1 214	1 170	1 126	1 082	1 039	997	956
314.0	16.938	16.28	1.46	11.063	92 45	6.78	4.18	1 387	1 387	1 387	1 362	1 317	1 270	1 225	1 179	1 133	1 088	1 045	1 002
328.0	17.125	16.34	1.52	11.063	96.53	6.83	4.20	1 448	1 448	1 448	1 425	1 377	1 330	1 282	1 234	1 186	1 140	1 095	1 050
342.0	17.313	16 40	1.58	11.063	100.63	6.87	4 22	1 509	1 509	1 509	1 487	1 439	1 390	1 339	1 290	1 241	1 192	1 145	1 099
356.0	17.500	16.46	1.64	11.063	104.75	6.92	4 24	1 571	1 571	1 571	1 551	1 501	1 450	1 397	1 346	1 296	1 245	1 196	1 148
370 0	17.688	16.52	1.70	11 063	108.90	6 96	4 26	1 634	1 634	1 634	1 616	1 563	1 509	1 456	1 403	1 350	1 299	1 247	1 198
384.0	17.875	16.58	1.76	11.063	113.07	7.01	4.27	1 696	1 696	1 696	1 679	1 625	1 569	1 514	1 459	1 404	1 350	1 297	1 246
399.0	18 063	16.64	1.82	11.063	117.26	7.05	4 29	1 759	1 759	1 759	1 744	1 687	1 631	1 574	1 517	1 460	1 405	1 350	1 297
413 0	18 250	16 70	1 88	11.063	121.48	7.10	4 31	1 822	1 822	1 822	1 810	1 753	1 695	1 635	1 577	1 517	1 460	1 403	1 348
427.0	18.438	16 76	1 94	11.063	125 72	7 14	4 33	1 886	1 886	1 886	1 876	1 817	1 756	1 696	1 636	1 575	1 515	1 457	1 401

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII.* Carnegie Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	4	6	8	9	10	11	12	13	14	15	16	17
								Unsupported length in feet											
4-in Carnegie H Beams																			
13 8	4 000	4 000	313	2 522	3 99	1 64	0 95	60	54	46	42	38	35	32	29	26	24		
5-in Carnegie H Beams																			
18.9	5 000	5.000	.313	3.413	5 47	2 08	1 20	82	82	73	68	63	59	55	51	47	44	41	38
6-in Carnegie H Beams																			
20.0	6 000	5.938	.250	4.458	5 86	2 57	1.39	88	88	83	79	75	70	66	62	58	55	51	48
22.5	6.000	6.063	.375	4 458	6.61	2.49	1.36	99	99	93	88	83	78	73	69	64	60	56	53
25.0	6 000	5 938	313	4 256	7 33	2 53	1 43	110	110	106	100	95	90	84	79	75	70	66	62
27 5	6 000	6 063	438	4 256	8 08	2 47	1 41	121	121	116	110	104	98	92	87	81	76	72	67
6-in Carnegie Special Beam Sections																			
40 0	5.750	9 500	.489	3 872	11.76	2 43	2 44	176	176	176	176	176	176	176	172	167	162	157	152
50.0	5 986	9 617	606	3 872	14.70	2.49	2 48	221	221	221	221	221	221	221	217	211	205	198	192
60.0	6.216	9.733	722	3.872	17.63	2.54	2.51	264	264	264	264	264	264	264	261	254	247	239	232
70.0	6 444	9 846	835	3.872	20.58	2.60	2.54	309	309	309	309	309	309	309	306	298	290	281	273
80.0	6.666	9.959	.948	3 872	23.52	2.65	2 58	353	353	353	353	353	353	353	352	343	333	324	314
88.0	6 842	10 046	1 035	3 872	25 87	2 69	2 60	388	388	388	388	388	388	389	388	378	368	357	347

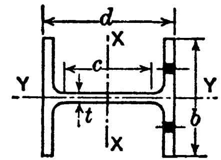
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	Unsupported length in feet 4	6	8	9	10	11	12	13	14	15	16	17
8-in Carnegie Mill Sections																			
17.5	8.000	4.981	.231	6.608	5.14	3.36	1 08	77	74	64	59	55	51	47	43	39	36	34	31
21.0	8.000	5.110	360	6.608	6.17	3 21	1 03	93	87	75	69	63	58	53	49	45	41	38	35
8-in Carnegie H Beams																			
32.6	8.000	7.938	.313	6 287	9.50	3 45	1.90	143	143	143	143	140	135	130	124	119	114	109	104
34.3	8.000	8.000	.375	6.287	10.00	3.40	1.87	150	150	150	150	146	141	136	130	124	119	114	108
37.7	8.000	8.125	.500	6 287	11.00	3 31	1.83	165	165	165	165	160	154	147	141	135	129	123	117
9-in Carnegie Mill Sections																			
20.5	9.000	5.234	.234	7.458	6.02	3.79	1.15	90	89	78	73	68	63	58	54	50	46	43	39
25.0	9.000	5.380	.380	7.458	7 34	3 61	1 09	110	106	92	85	79	73	67	62	57	53	49	45
8-in Carnegie H Beams																			
32.6	8.000	7 938	.313	6.287	9.50	3 45	1.90	143	143	143	143	140	135	130	124	119	114	109	104
34.3	8.000	8 000	.375	6.287	10.00	3.40	1.87	150	150	150	150	146	141	136	130	124	119	114	108
37.7	8.000	8.125	.500	6.287	11.00	3.31	1.83	165	165	165	165	160	154	147	141	135	129	123	117

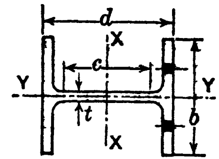
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions

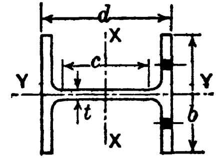


Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	Unsupported length in feet 4	6	8	9	10	11	12	13	14	15	16	17
8-in Carnegie Beam Sections																			
24.0	8.000	6.500	.239	6.300	7.06	3.46	1.61	106	106	106	102	97	93	88	84	79	75	71	67
27.0	8.098	6.529	.268	6.300	7.93	3.48	1.62	119	119	119	114	109	104	99	94	89	85	80	76
30.0	8.196	6.559	.298	6.300	8.81	3.50	1.63	132	132	132	127	122	116	111	105	100	95	90	85
31.0	8.060	8.000	.290	6.300	9.10	3.49	2.01	137	137	137	137	137	132	127	123	118	113	109	104
36.0	8.198	8.046	.336	6.300	10.58	3.52	2.02	159	159	159	159	159	154	149	143	138	132	127	122
42.0	8.360	8.100	.390	6.300	12.34	3.56	2.04	185	185	185	185	185	180	174	168	161	155	149	143
48.0	8.520	8.155	.445	6.300	14.10	3.59	2.06	212	212	212	212	212	207	200	192	185	178	171	164
54.0	8.680	8.208	.498	6.300	15.87	3.63	2.07	238	238	238	238	238	233	225	217	209	201	193	186
60.0	8.838	8.261	.551	6.300	17.63	3.67	2.09	264	264	264	264	264	260	251	242	234	225	216	208
66.0	8.994	8.314	.604	6.300	19.40	3.70	2.11	291	291	291	291	291	287	277	268	258	249	239	230
72.0	9.150	8.366	.656	6.300	21.17	3.74	2.12	318	318	318	318	318	314	303	293	282	272	262	252
78.0	9.302	8.418	.708	6.300	22.93	3.77	2.14	344	344	344	344	344	341	330	319	307	296	285	274
84.0	9.456	8.469	.759	6.300	24.71	3.81	2.15	371	371	371	371	371	368	356	344	332	320	308	296
90.0	9.606	8.520	.810	6.300	26.47	3.84	2.17	397	397	397	397	397	395	383	370	357	345	332	320

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections



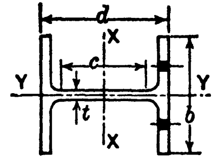
Allowable Concentric Loads in Kips, Dimensions and Functions

Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	Unsupported length in feet 4	6	8	9	10	11	12	13	14	15	16	17
								9-in Carnegie Beam Sections											
29 0	9 000	6.500	.279	7 060	8 53	3 84	1 59	128	128	128	122	117	111	105	100	95	90	85	80
32.0	9 096	6 528	307	7 060	9 40	3 87	1.60	141	141	141	135	129	123	117	111	105	99	94	89
35.0	9.192	6 556	.335	7 060	10 29	3 89	1.61	154	154	154	148	142	135	128	122	115	109	103	98
38.0	9.000	9 000	.316	7.060	11 17	3 91	2 26	168	168	168	168	168	168	164	159	154	149	144	138
43.0	9.122	9 041	.357	7 060	12 65	3 93	2.28	190	190	190	190	190	190	186	181	175	169	163	157
48 0	9 242	9 082	398	7 060	14 11	3 96	2 29	212	212	212	212	212	212	208	202	196	189	183	176
								8	10	12	13	14	15	16	17	18	19	20	22
								10-in Carnegie Beam Sections											
21	9 902	6 000	230	8 638	6 17	4.18	1.39	88	79	70	65	61	57	54	51	47	45	42	37
23	10 000	6.000	.230	8 638	6.76	4.25	1.43	97	87	78	73	69	65	61	57	54	50	47	42
26	10.098	6 029	259	8.638	7 64	4 27	1.43	110	99	88	83	78	73	69	65	61	57	54	48
30	10 228	6 068	298	8.638	8.82	4 30	1.45	128	115	103	97	91	86	80	76	71	69	63	56
31	10.000	8.000	.320	8.638	9.11	4 23	1.89	137	134	124	119	114	109	104	100	95	91	86	79
36	10.000	8 147	467	8.638	10 58	4 07	1.80	159	153	140	134	128	122	117	111	106	101	96	87
42	10 000	8.324	644	8 638	12 35	3 93	1 73	185	175	161	153	146	139	132	125	119	113	107	97
49	10 000	10 000	350	7.984	14 40	4 35	2 54	216	216	216	214	209	203	197	191	185	179	173	162
54	10.000	10.147	497	7.984	15 87	4 23	2 48	238	238	238	234	228	221	214	208	201	194	188	175
59	10.000	10 294	.644	7.984	17.34	4.13	2.42	260	260	260	254	246	239	231	224	216	209	202	188
64	10 000	10 441	791	7 984	18 81	4 05	2 38	282	282	281	273	265	257	249	240	232	224	216	201

Loads to right of heavy vertical lines are for Secondary Members Only. Loads by A I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	Unsupported length in feet 8	10	12	13	14	15	16	17	18	19	20	22
10-in Carnegie Beam Sections																			
70	10.000	10.000	.515	7.390	20.59	4.24	2.55	309	309	309	307	299	290	282	273	265	257	248	232
77	10.000	10.206	.721	7.390	22.65	4.13	2.51	340	340	340	336	326	317	308	298	289	280	270	253
84	10.000	10.411	.926	7.390	24.70	4.04	2.48	371	371	371	364	354	344	334	323	313	303	292	273
92	10.000	10.647	1.162	7.390	27.06	3.96	2.45	406	406	406	398	386	375	363	352	340	329	318	296
100	10.000	12.000	.600	6.768	29.40	4.23	3.16	441	441	441	441	441	441	439	430	420	410	401	381
108	10.000	12.236	.836	6.768	31.76	4.14	3.13	476	476	476	476	476	476	473	463	452	442	431	410
116	10.000	12.471	1.071	6.768	34.11	4.07	3.11	512	512	512	512	512	512	507	496	484	473	461	438
124	10.000	12.706	1.306	6.768	36.46	4.00	3.09	547	547	547	547	547	547	540	528	516	504	492	467
132	10.000	12.941	1.541	6.768	38.81	3.94	3.09	582	582	582	582	582	582	575	562	549	536	523	497
140	10.000	13.177	1.777	6.768	41.17	3.89	3.08	618	618	618	618	618	618	609	596	582	568	554	526
								8	10	12	14	15	16	17	18	19	20	22	24
12-in Carnegie Beam Sections																			
25	11.924	6.000	.240	10.460	7.34	4.99	1.37	104	93	82	72	67	63	59	55	52	49	43	
28	12.000	6.500	.240	10.460	8.22	5.10	1.53	121	110	99	89	84	79	74	70	66	63	56	50
32	12.118	6.534	.274	10.460	9.40	5.12	1.54	139	127	114	102	96	91	86	81	76	72	64	57
†34	12.022	6.635	.375	10.460	9.99	4.88	1.45	145	130	116	103	97	91	86	81	76	71	63	56
36	12.236	6.568	.308	10.460	10.59	5.14	1.55	157	143	129	115	109	103	97	92	87	82	73	65

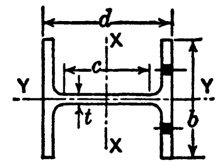
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

† Special section. Web thickness ⅜ in.

Table XVIII * (Continued). Carnegie Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

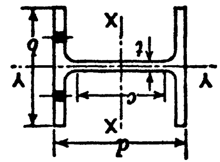
Weight per foot	d	b	t	c	Area sq in	r, in X–X	r, in Y–Y	Unsupported length in feet 8	10	12	14	15	16	17	18	19	20	22	24
12-in Carnegie Beam Sections																			
40	12 000	8.000	290	9 948	11 76	5 17	1 95	176	175	163	150	144	138	132	125	121	115	105	96
45	12.130	8.036	.326	9 948	13 23	5.19	1 97	198	197	184	170	163	156	149	143	137	131	119	109
50	12.258	8.071	.361	9.948	14 69	5.22	1 98	220	220	204	189	181	174	166	159	152	146	133	122
55	12.000	9.000	.375	9.570	16.17	5.15	2 24	243	243	237	222	214	207	199	192	185	178	164	152
60	12.118	9.034	.409	9.570	17.65	5 17	2 25	265	265	259	243	234	226	218	210	202	195	180	166
66	12.260	9.073	.448	9.570	19 41	5.20	2 26	291	291	285	267	258	249	241	232	223	215	199	184
65	12.000	12.000	.400	9.684	19 11	5 22	3.03	287	287	287	287	287	281	275	268	262	255	242	229
70	12.000	12.123	.523	9 684	20 58	5 12	2 96	309	309	309	309	307	300	293	286	279	271	257	243
76	12.000	12.270	.670	9.684	22 35	5 01	2 90	335	335	335	335	331	324	316	308	299	291	275	260
82	12.000	12.000	.453	9.300	24.11	5 20	3 09	362	362	362	362	362	357	349	341	333	325	309	293
88	12.000	12 147	.600	9.300	25 88	5.10	3 04	388	388	388	388	388	381	373	364	355	346	328	311
95	12 000	12.318	.771	9.300	27.93	4.99	2 99	419	419	419	419	418	409	399	390	380	370	351	332
102	12.000	12.490	.943	9.300	29 99	4.90	2 95	450	450	450	450	447	437	427	416	405	395	374	353
110	12.000	12 000	.640	8 650	32 34	5 06	3.10	485	485	485	485	485	480	469	458	448	437	415	393
120	12.000	12.245	.885	8 650	35 28	4 95	3 06	529	529	529	529	529	521	509	497	485	473	449	426
130	12.000	12.491	1.131	8 650	38 24	4 85	3 03	574	574	574	574	574	563	550	537	524	510	484	458
140	12 000	12 736	1 376	8 650	41 18	4 76	3 01	618	618	618	618	618	605	591	576	562	548	519	491

Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

Weight per foot	d	b	t	c	Area, sq in	r, in X–X	r, in Y–Y	8	10	12	14	15	16	17	18	19	20	22	24
								Unsupported length in feet											
12-IN CARNEGIE BEAM SECTIONS																			
150	12.000	14.000	.757	8 076	44 12	5 02	3 69	662	662	662	662	662	662	662	662	655	643	618	593
160	12.000	14.245	1.002	8.076	47.06	4.94	3.67	706	706	706	706	706	706	706	706	698	684	658	631
170	12.000	14 490	1.247	8.076	50.00	4.86	3.65	750	750	750	750	750	750	750	750	740	726	697	669
180	12.000	14.735	1.492	8.076	52.94	4 80	3.64	794	794	794	794	794	794	794	794	782	768	737	707
190	12.000	14.000	1.000	7.346	55.88	4 86	3.71	838	838	838	838	838	838	838	838	831	816	785	754
200	12.000	14.245	1.245	7.346	58.82	4.80	3.71	882	882	882	882	882	882	882	882	875	859	826	793
210	12.000	14 490	1.490	7.346	61.76	4.75	3.72	926	926	926	926	926	926	926	926	920	903	869	834
220	12.000	14.735	1.735	7.346	64.70	4 70	3.73	971	971	971	971	971	971	971	971	964	947	911	875
230	12.000	14.980	1 980	7 346	67 64	4 65	3 74	1 015	1 015	1 015	1 015	1 015	1 015	1 015	1 015	1 009	991	954	916
14-IN CARNEGIE BEAM SECTIONS																			
30	13.964	6 000	270	12 302	8 82	5 75	1.33	123	109	96	84	79	74	69	64	60	57	50	
33	14.000	6 750	.270	12.302	9.71	5.86	1.54	144	131	118	105	99	94	88	84	79	74	66	59
36	14.080	6 744	.294	12.302	10 58	5 88	1 55	157	143	129	115	109	103	97	92	86	82	73	65
†38	14.000	6 855	.375	12 302	11 18	5 66	1 47	163	147	131	117	110	103	97	91	86	81	72	64
39	14.160	6 798	318	12 302	11 47	5 89	1 56	171	155	140	126	119	112	106	100	94	89	80	71

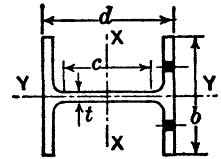
Loads to right of heavy vertical lines are for Secondary Members ONLY.

LOADS BY A.I.S C. SPECIFICATION

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

† Special Section. Web Thickness ⅜ in.

Table XVIII * (Continued). Carnegie Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

Weight per foot	d	b	t	c	Area, sq in	r, in		Unsupported length in feet											
						X–X	Y–Y	8	10	12	14	15	16	17	18	19	20	22	24
14-in Carnegie Beam Sections																			
42	14 240	6 822	342	12 302	12 35	5 91	1.56	184	167	151	135	128	121	114	108	102	96	86	77
48	14 000	8 000	.343	11 710	14 12	5 93	1.90	212	208	193	177	170	162	155	148	141	135	123	112
53	14 122	8 035	.378	11.710	15 59	5 95	1.91	234	230	213	196	188	180	172	164	157	149	136	124
58	14 242	8.070	.413	11.710	17.05	5.98	1.92	256	252	234	215	206	197	189	180	172	164	150	136
61	14.094	10 000	.382	11.710	17.94	6.05	2.44	269	269	269	256	248	240	233	225	217	210	196	182
68	14.238	10.043	.425	11.710	19.99	6.08	2.46	300	300	300	286	277	269	260	252	244	235	219	204
75	14 382	10 086	.468	11.710	22 05	6 11	2 47	331	331	331	316	306	297	288	279	269	260	243	226
85	14.000	12.000	.435	11.090	24.99	6 07	3.05	375	375	375	375	375	369	360	352	343	335	318	301
95	14 186	12.050	.485	11 090	27.93	6.11	3 06	419	419	419	419	419	413	403	394	384	375	356	337
105	14.370	12.101	.536	11.090	30.88	6 15	3.08	463	463	463	463	463	457	447	437	426	416	395	374
86	13 714	15.008	.414	11 090	25 28	6.04	3 84	379	379	379	379	379	379	379	379	379	374	360	347
96	13.866	15 056	.462	11 090	28.23	6 08	3.86	423	423	423	423	423	423	423	423	423	418	403	388
106	14.018	15.103	.509	11 090	31.18	6.11	3 87	468	468	468	468	468	468	468	468	468	462	446	429
115	14.154	15.145	.551	11 090	33 82	6.14	3 89	507	507	507	507	507	507	507	507	507	502	485	467
125	14.304	15.191	.597	11 090	36 75	6.18	3 90	551	551	551	551	551	551	551	551	551	547	527	508

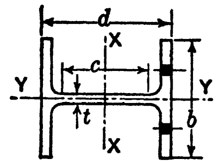
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XVIII * (Continued). Carnegie Column Sections

Allowable Concentric Loads in Kips, Dimensions and Functions



Weight per foot	d	b	t	c	Area, sq in	r, in X-X	r, in Y-Y	8	10	12	14	15	16	17	18	19	20	22	24
								Unsupported length in feet											
14-in Carnegie Beam Sections																			
‡131	14.162	15.468	.874	11 090	38.52	5 94	3 77	578	578	578	578	578	578	578	578	576	566	545	524
135	14.452	15.239	.645	11.090	39 70	6 21	3 92	596	596	596	596	596	596	596	596	596	591	570	549
145	14.602	15 284	.690	11.090	42.64	6 24	3.93	640	640	640	640	640	640	640	640	640	636	614	591
155	14.750	15.330	.736	11.090	45.58	6 28	3 94	684	684	684	684	684	684	684	684	684	680	657	633
165	14.896	15.377	.783	11.090	48.52	6.31	3 96	728	728	728	728	728	728	728	728	728	725	700	675
175	15.042	15.424	.830	11.090	51.47	6.34	3.97	772	772	772	772	772	772	772	772	772	770	744	717
185	15.188	15.469	.875	11 090	54 41	6 38	3 98	816	816	816	816	816	816	826	816	816	815	787	759
195	15.334	15.513	.919	11.090	57.34	6.41	4 00	860	860	860	860	860	860	860	860	860	860	831	801
205	15.478	15.559	.965	11.090	60.28	6.45	4 01	904	904	904	904	904	904	904	904	904	904	874	843
215	15.622	15.604	1.010	11 090	63 23	6.48	4.03	949	949	949	949	949	949	949	949	949	949	919	886
225	15.764	15.650	1 056	11.090	66.17	6.51	4 04	993	993	993	993	993	993	993	993	993	993	963	929
235	15.908	15.693	1 099	11.090	69.11	6.55	4 05	1 037	1 037	1 037	1 037	1 037	1 037	1 037	1 037	1 037	1 037	1 006	971
245	16 050	15.738	1.144	11 090	72.06	6 58	4 06	1 081	1 081	1 081	1 081	1 081	1 081	1 081	1 081	1 081	1 081	1 051	1 014
255	16 192	15.781	1 187	11 090	74 99	6 61	4 08	1 125	1 125	1 125	1 125	1 125	1 125	1 125	1 125	1 125	1 125	1 095	1 057
265	16 332	15.826	1.232	11.090	77 93	6 65	4 09	1 169	1 169	1 169	1 169	1 169	1 169	1 169	1 169	1 169	1 169	1 139	1 100

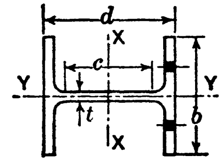
Loads to right of heavy vertical lines are for Secondary Members Only

Loads by A I S C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

‡ Special Section for Column Core.

Table XVIII * (Continued). Carnegie Column Sections



Allowable Concentric Loads in Kips, Dimensions and Functions

Weight per foot	d	b	t	c	Area, sq in	r, in		Unsupported length in feet											
						X-X	Y-Y	8	10	12	14	15	16	17	18	19	20	22	24
14-in Carnegie Beam Sections																			
275	16.472	15 870	1.276	11.090	80 87	6.68	4.10	1 213	1 213	1 213	1 213	1 213	1 213	1 213	1 213	1 213	1 213	1 183	1 143
285	16.614	15.912	1.318	11 090	83.82	6.71	4.12	1 257	1 257	1 257	1 257	1 257	1 257	1 257	1 257	1 257	1 257	1 229	1 187
295	16.752	15.956	1.362	11.090	86.76	6 75	4.13	1 301	1 301	1 301	1 301	1 301	1 301	1 301	1 301	1 301	1 301	1 273	1 229
305	16.890	16.000	1.406	11.090	89.70	6 78	4.14	1 346	1 346	1 346	1 346	1 346	1 346	1 346	1 346	1 346	1 346	1 317	1 272
325	17.164	16.087	1 493	11.090	95.58	6.84	4.17	1 434	1 434	1 434	1 434	1 434	1 434	1 434	1 434	1 434	1 434	1 407	1 360
345	17.438	16 172	1 578	11.090	101.47	6.91	4.19	1 522	1 522	1 522	1 522	1 522	1 522	1 522	1 522	1 522	1 522	1 496	1 447
365	17.710	16.255	1.661	11.090	107.34	6.97	4.22	1 610	1 610	1 610	1 610	1 610	1 610	1 610	1 610	1 610	1 610	1 587	1 535
385	17.978	16.340	1.746	11.090	113.22	7.04	4.24	1 698	1 698	1 698	1 698	1 698	1 698	1 698	1 698	1 698	1 698	1 677	1 622
405	18.246	16.423	1 829	11 090	119.12	7.10	4.27	1 787	1 787	1 787	1 787	1 787	1 787	1 787	1 787	1 787	1 787	1 769	1 712
425	18.510	16.506	1.912	11.090	124.99	7 17	4 29	1 875	1 875	1 875	1 875	1 875	1 875	1 875	1 875	1 875	1 875	1 859	1 799

Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I S.C. Specification

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table XIX.* Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-Plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported Length in Feet												Kx	Ky
					4	8	10	11	12	13	14	15	16	17	18	20		
8 × 1/4	2 1/2 × 2 1/2 × 1/4	23 2	6.76	.96	101	78	65	59	54	49	45	41	38	..	.	..	2.53	.355
	3 × 2 1/2 × 1/4	24.8	7.24	1.19	109	96	83	77	72	66	62	57	53	..		..	2.65	.456
	5/16	29.2	8.48	1.23	127	114	100	93	87	81	75	70	65	...	...		2.69	.483
	3 1/2 × 2 1/2 × 1/4	26.4	7.76	1.44	116	112	101	95	90	85	80	75	70	66	62	55	2.72	.567
	5/16	31.2	9.12	1.49	137	133	120	114	108	102	96	90	85	80	76	67	2.78	.614
	3/8	35.6	10.44	1.52	157	154	140	132	125	118	112	106	100	94	89	79	2.79	.642
	3 1/2 × 3 × 1/4	28.4	8.24	1.40	124	118	105	99	93	87	82	77	73	68	64	56	2.58	.534
	5/16	33.2	9.72	1.44	146	140	126	119	112	106	100	94	88	83	78	69	2.62	.576
	3/8	38.4	11.20	1.47	168	163	147	139	131	124	117	110	103	97	92	81	2.64	.598
	4 × 3 × 1/4	30.0	8.76	1.64	131	131	122	116	110	105	100	95	90	85	80	72	2.68	.662
	5/16	35.6	10.36	1.69	155	155	146	139	133	127	120	113	109	103	98	88	2.70	.695
	3/8	40.8	11.92	1.72	179	179	169	162	154	147	140	133	127	120	114	103	2.72	.721
	4 × 3 1/2 × 3/8	43.2	12.68	1.68	190	190	178	170	162	154	147	139	132	125	119	107	2.57	.686
	5 × 3 1/2 × 5/16	41.6	12.24	2.15	184	184	184	182	176	171	165	159	153	147	141	130	2.70	.899
	3/8	48 4	14 20	2.19	213	213	213	213	206	199	193	186	179	172	166	153	2.72	.930

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.

Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I S.C. Specification.

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet: 4	8	10	11	12	13	14	15	16	17	18	20	Kx	Ky
8 × 5/16	2½×2½×5/16	28.5	8.38	.98	126	98	82	75	69	63	57	52	48	...	...	...	2.51	.420
	3 ×2½×5/16	30.9	8.98	1.22	135	120	105	98	91	85	79	73	68	...	...	...	2.61	.468
	3/8	34.9	10.18	1.26	153	139	122	114	106	99	92	86	80	...	...	...	2.62	.501
	3½×2½×5/16	32.9	9.62	1.47	144	140	126	119	113	106	100	94	89	84	79	70	2.64	.582
	3/8	37.3	10.94	1.51	164	161	146	138	131	124	117	110	104	98	92	82	2.71	.622
	3½×3 ×5/16	34.9	10.22	1.42	153	147	132	124	117	110	103	97	91	85	80	71	2.55	.558
	3/8	40.1	11.70	1.46	176	170	153	144	137	129	121	114	107	101	95	84	2.57	.581
	4 ×3 ×5/16	37.3	10.86	1.67	163	163	152	145	138	132	125	119	113	107	101	91	2.65	.572
	3/8	42.5	12.42	1.71	186	186	175	168	160	153	145	138	132	125	118	107	2.66	.700
	4 ×3½×3/8	44.9	13.18	1.67	198	198	184	176	168	160	152	144	137	130	123	110	2.52	.668
	5 ×3½×5/16	43.3	12.74	2.13	191	191	191	189	183	177	170	164	158	152	146	135	2.64	.879
	3/8	50.1	14.70	2.17	221	221	221	219	213	206	198	191	184	177	171	158	2.67	.912
8 × 3/8	3 ×2½×3/8	36.6	10.68	1.25	160	145	127	119	111	103	96	89	83	77	72		2.56	.496
	3 ×3 ×3/8	39.0	11.44	1.21	172	152	133	124	115	107	99	92	86	80	74	...	2.43	.463
	3½×2½×3/8	39.0	11.44	1.50	172	168	152	144	136	128	121	114	108	102	96	...	2.66	.603
	3½×3 ×3/8	41.8	12.20	1.45	183	177	159	150	142	134	126	118	111	104	98	...	2.52	.573
	7/16	46.6	13.60	1.49	204	199	180	170	161	152	143	135	127	120	113	100	2.54	.595
	4 ×3 ×3/8	44.2	12.92	1.70	194	194	182	174	166	158	151	143	136	129	123	110	2.61	.689
	7/16	49.4	14.48	1.73	217	217	206	197	188	180	171	163	155	147	140	126	2.62	.718
	4 ×3½×3/8	46.6	13.68	1.66	205	205	191	182	174	165	157	149	141	134	127	114	2.47	.658
	5 ×3½×3/8	51.8	15.20	2.15	228	228	228	226	219	212	204	197	190	182	175	162	2.62	.895
	7/16	58.2	17.12	2.19	257	257	257	256	248	240	232	224	216	208	200	185	2.63	.923

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members Only.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet												Kx	Ky
					6	7	8	9	10	12	14	16	18	20	22	24		
10 × ¼	3½×2½×¼	28.1	8 26	1.39	124	123	118	111	105	93	82	72	63	56	.		3 41	532
	5/16	32 9	9.62	1.45	144	144	139	132	125	112	99	88	78	69			3.48	.582
	⅜	37.3	10.94	1 49	164	164	160	153	145	130	115	102	91	81			3 53	.612
	3½×3 ×¼	30.1	8 74	1.36	131	130	123	117	110	97	85	75	66	58			3 27	.504
	5/16	34.9	10 22	1.41	153	153	146	139	131	117	103	91	80	71			3 35	.548
	⅜	40.1	11.70	1.44	176	176	169	161	152	135	120	106	94	83	..	...	3 38	.573
	4 ×3 ×¼	31.7	9 26	1.60	139	139	139	133	127	115	103	93	83	74	66	..	3 37	626
	5/16	37.3	10 86	1 65	163	163	163	158	151	137	124	112	100	90	81	. .	3 43	.663
	⅜	42.5	12 42	1 69	186	186	186	182	175	159	144	130	117	105	95	....	3.48	.692
	4 ×3½×⅜	44.9	13.18	1.65	198	198	198	192	184	167	151	135	122	109	98	...	3.32	.660
	5 ×3½×5/16	43.3	12.74	2.11	191	191	191	191	191	182	170	157	145	133	'23	113	3 45	.864
	⅜	50.1	14.70	2 15	221	221	221	221	221	212	198	183	170	156	144	133	3 47	.905
10 × 5/16	3½×2½×5/16	35.0	10.25	1.42	154	154	147	140	132	117	104	92	81	71	63	..	3.36	.556
	⅜	39.4	11.57	1.47	174	174	168	160	152	136	121	107	95	84	75	...	3 42	.588
	3½×3 ×5/16	37.0	10.85	1.38	163	162	154	146	138	122	107	94	83	73	64	.	3.24	.525
	⅜	42.2	12.33	1.42	185	185	177	168	159	141	125	110	97	86	76	....	3 28	.551
	4 ×3 ×5/16	39.4	11.49	1.62	172	172	172	166	159	144	129	116	104	93	84	....	3.33	.635
	⅜	44.6	13.05	1.67	196	196	196	191	183	166	150	135	122	109	98	...	3 39	.667
	4 ×3½×⅜	47.0	13.81	1.63	207	207	207	200	191	173	156	140	126	113	101	....	3.24	.644
	5 ×3½×5/16	45.4	13.37	2.08	201	201	201	201	201	190	177	163	151	138	127	117	3.37	.837
	⅜	52 2	15 33	2 12	230	230	230	230	230	220	205	190	175	161	148	136	3 40	814

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members ONLY.

LOADS BY A I.S C. SPECIFICATION

*** From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.**

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-Plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet: 6	7	8	9	10	12	14	16	18	20	22	24	Kx	Ky
10 × 3/8	3½×3 ×3/8	44 4	12 95	1 41	194	194	185	176	166	148	130	115	101	89	79	.	3 21	540
	4 ×3 ×3/8	46 8	13.67	1 65	205	205	205	199	190	173	156	140	126	113	102	.	3.30	.651
	4 ×3½×3/8	49 2	14.43	1.62	217	217	217	208	199	181	163	146	131	117	105	...	3.17	623
	5 ×3½×3/8	54.4	15 95	2.10	239	239	239	239	239	228	212	196	181	166	153	140	3 34	.853
	7/16	60.8	17.87	2 14	268	268	268	268	268	257	240	222	206	189	174	160	3.36	.884
	½	67 2	19.75	2 19	296	296	296	296	296	287	268	249	231	213	197	181	3.37	.922
	6 ×4 ×3/8	62.0	18.19	2 56	273	273	273	273	273	273	264	249	235	220	206	192	3 34	1 06
	7/16	70 0	20.47	2 61	307	307	307	307	307	307	299	283	267	251	235	220	3 36	1 10
	½	77.6	22.75	2 65	341	341	341	341	341	341	335	317	299	281	264	247	3.36	1.13
10 × ½	6 ×4 ×½	81.8	24.00	2 62	360	360	360	360	360	360	352	333	314	295	276	258	3.27	1.10
	9/16	89.4	26.24	2 66	394	394	394	394	394	394	387	366	346	325	305	286	3.27	1.13
	5/8	97.0	28.44	2 69	427	427	427	427	427	427	421	399	377	355	333	313	3.28	1.16
10 × 5/8	6 ×4 ×5/8	101.3	29 69	2 68	445	445	445	445	445	445	438	416	393	370	347	325	3.21	1.14
	¾	115.7	34.01	2.75	510	510	510	510	510	510	507	482	456	430	405	380	3.19	1.20
10 × ¾	6 ×4 ×¾	119 9	35 26	2.74	529	529	529	529	529	529	525	499	472	445	419	393	3.13	1.18
	7/8	134 3	39.42	2.80	591	591	591	591	591	591	591	563	533	504	475	447	3.12	1.23

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members ONLY.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-Plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet: 6	8	10	12	14	16	18	20	22	24	26	28	Kx	Ky
12 × ¼	3½×2½×¼	29.8	8.76	1 36	131	124	110	97	86	75	66	58	51				4.12	.502
	⅜	39.0	11 44	1.45	172	165	149	133	118	104	92	82	73			.	4 28	.586
	3½×3 ×¼	31.8	9.24	1 32	139	129	114	100	88	76	67	59	52				3.89	.476
	⅜	41.8	12.20	1.41	183	175	157	139	123	108	95	84	74	.		.	4.10	.549
	4 ×3 ×¼	33.4	9.76	1.56	146	145	132	119	107	95	85	76	68	.	..	..	4.10	.594
	⅜	44 2	12.92	1 66	194	194	180	164	148	133	120	108	97	..	.	..	4.26	.665
	4 ×3½×⅜	46.6	13.68	1.62	205	205	189	171	154	138	124	111	100	...	..	.	4.09	.636
	5 ×3½×5/16	45 0	13.24	2.07	199	199	199	188	175	161	149	137	125	115	105	96	4 23	.831
	⅜	51.8	15.20	2.11	228	228	228	217	202	187	173	159	146	134	124	114	4.28	875
12 × 5/16	3½×2½×5/16	37.2	10.87	1 38	163	154	138	122	107	94	83	73	64	.	.		4.05	.525
	⅜	41.6	12.19	1.43	183	176	158	140	124	110	97	86	76	..	.		4 10	.588
	3½×3 ×5/16	39.2	11.47	1 34	172	161	143	126	110	96	84	74	65	..	...	.	3 93	.497
	⅜	44.4	12.95	1.39	194	184	165	146	129	113	99	88	78	.			4.02	.525
	4 ×3 ×5/16	41.6	12.11	1.58	182	181	165	149	134	120	107	96	85	.			4.05	.603
	⅜	46.8	13.67	1.63	205	205	189	172	155	139	125	112	100	..	...	.	4.10	.636
	4 ×3½×⅜	49.2	14.43	1.59	217	216	197	178	160	143	128	115	102	....	..	..	3.95	.617
	5 ×3½×5/16	47.6	13.99	2.03	210	210	210	197	182	168	155	142	130	119	109	100	4 09	.801
	⅜	54.4	15.95	2.08	239	239	239	227	211	195	180	165	152	139	128	118	4 14	.840

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A I S C. Specification

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-Plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet												Kx	Ky
					6	8	10	12	14	16	18	20	22	24	26	28		
12 × 3/8	4 ×3 ×3/8	49.3	14 42	1 61	216	216	198	180	162	145	130	116	104	.	.		4 02	617
	4 ×3½×3/8	51.7	15.18	1.57	228	226	206	186	167	149	133	119	106	.		.	3.82	593
	5 ×3½×3/8	56.9	16.70	2.05	251	251	251	236	219	202	186	171	156	143	131	120	4.01	.814
	7/16	63.3	18.62	2.10	279	279	279	266	247	229	211	194	178	164	150	138	4.08	.849
	½	69.7	20.50	2.15	308	308	308	295	275	256	237	218	201	185	170	156	4.10	.888
	6 ×4 ×3/8	64.5	18 94	2.51	284	284	284	284	273	257	241	226	211	197	184	171	4 07	1 02
	7/16	72.5	21.22	2.56	318	318	318	318	309	291	274	257	240	224	209	195	4.10	1 06
	½	80 1	23.50	2.61	353	353	353	353	344	325	306	288	270	252	236	220	4.13	1.10
12 × ½	5 ×3½×½	74.8	22.00	2.11	330	330	330	315	293	271	250	230	212	194	179	164	3.95	.856
	5/8	87.6	25.68	2.19	385	385	385	372	348	324	300	277	256	236	217	200	4.07	.930
	6 ×4 ×½	85.2	25.00	2.57	375	375	375	375	364	344	323	303	284	265	248	231	4.06	1.06
	9/16	92.8	27.24	2.61	409	409	409	409	399	377	355	334	313	293	273	255	4 06	1.09
	5/8	100 4	29 44	2.65	442	442	442	442	433	410	387	364	342	320	300	280	4 04	1 12
12 × 5/8	6 ×4 ×5/8	105.5	30.94	2.62	464	464	464	464	453	429	404	380	356	333	312	291	4.04	1.09
	¾	119.9	35 26	2 70	529	529	529	529	522	495	468	441	415	389	364	341	3 95	1 15
12 × ¾	6 ×4 ×¾	125.0	36.76	2.69	551	551	551	551	544	516	487	459	431	404	379	354	3 87	1.13
	7/8	139.4	40.92	2.75	614	614	614	614	611	579	549	518	487	457	430	403	3.86	1.19
	6 ×6 ×¾	145.4	42.76	2.51	641	641	641	641	616	581	545	510	477	445	414	386	3 34	.989
	7/8	163.0	47.92	2.57	719	719	719	719	697	658	620	581	544	508	474	443	3.34	1 03

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

*From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX * (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-Plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	6	8	10	12	14	16	18	20	22	24	26	28	Kx	Ky
					Unsupported length in feet													
14 × ¼	3½×3 ×¼	33.5	9 74	1.29	146	134	118	104	90	79	68	60		.			4 57	.463
	⅜	43.5	12.70	1.38	191	180	161	142	125	110	97	85	...				4.82	.527
	4 ×3 ×¼	35.1	10.26	1.52	154	151	137	123	110	98	87	78	69	.	..		4 35	.565
	⅜	45.9	13.42	1.62	201	201	185	168	151	136	122	109	98				4 93	.641
	4 ×3½×⅜	48.3	14.18	1.59	213	212	194	175	157	141	126	113	101	.	.	..	4.78	.614
	5 ×3½×5/16	46.7	13.74	2.03	206	206	206	194	179	165	152	139	127	117	107	98	4 91	.800
	⅜	53 5	15.70	2.08	236	236	236	223	207	192	177	162	149	137	126	116	4 99	.841
14 × 5/16	4 ×3 ×5/16	43.7	12.74	1.54	191	189	171	154	138	123	110	98	87	.	.		4 66	.573
	⅜	48 9	14.30	1.59	215	214	195	177	159	142	127	114	102		.	.	4.77	.608
	4 ×3½×⅜	51.3	15.06	1.56	226	224	204	184	165	147	131	117	105	.			4 65	.584
	5 ×3½×5/16	49 7	14 62	1 98	219	219	219	204	188	173	158	145	132	121	111	101	4 75	.766
	⅜	56.5	16.58	2.04	249	249	249	234	217	201	184	169	155	142	130	119	4 85	.808
	6 ×4 ×⅜	64.1	18.82	2.50	282	282	282	282	271	255	239	224	209	195	182	169	4 89	1 01
	7/16	72.1	21.10	2.55	317	317	317	317	306	291	272	254	238	222	207	193	4 94	1 05
14 × ⅜	4 ×3 ×⅜	51 9	15.17	1.57	228	226	206	186	167	149	133	119	106	.	..	.	4 63	.587
	4 ×3½×⅜	54.3	15 93	1.54	239	236	214	193	173	154	137	122	109		.		4 51	.565
	5 ×3½×⅜	59 5	17.45	2.01	262	262	262	244	226	209	192	175	160	147	134	123	4 71	.779
	7/16	65 9	19 37	2.06	291	291	291	274	255	235	217	199	182	167	153	140	4 79	.821
	½	72 3	21 25	2 11	319	319	319	304	283	262	242	223	204	188	173	159	4.83	861

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members Only.

Loads by A.I.S.C. Specification

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XIX* (Continued). Plate and Angle Columns

Allowable Concentric Loads in Kips for Plate and Angle Columns

1 Web-plate	4 Angles	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet: 6	8	10	12	14	16	18	20	22	24	26	28	Kx	Ky
14 × 3/8	6 × 4 × 3/8	67 1	19.69	2.46	295	295	295	295	282	265	248	232	216	201	187	174	4.78	.981
	7/16	75.1	21.97	2.52	330	330	330	330	317	299	281	263	246	229	214	199	4.84	1.02
	1/2	82 7	24 25	2.57	364	364	364	364	353	333	314	294	275	257	240	224	4 85	1.06
	6 × 6 × 3/8	77.5	22.69	2.30	340	340	340	335	315	294	274	255	236	218	202	187	4.23	.855
	7/16	86 7	25.49	2.34	382	382	382	379	357	334	311	290	269	249	231	214	4 27	.875
	1/2	96 3	28 25	2.38	424	424	424	423	398	373	349	325	302	280	260	241	4 29	.886
14 × 1/2	5 × 3½ × 1/2	78 2	23.00	2.07	345	345	345	326	303	280	258	237	218	199	183	168	4 63	.818
	5/8	91.0	26 68	2.15	400	400	400	384	358	333	308	284	261	241	221	204	4 73	.881
	6 × 4 × 1/2	88.6	26.00	2.52	390	390	390	390	375	354	332	311	291	271	253	235	4.69	1.02
	9/16	96 2	28.24	2.56	424	424	424	424	410	387	364	341	320	298	278	260	4.73	1.05
	5/8	103.8	30.44	2.60	457	457	457	457	445	420	396	372	348	326	304	284	4.79	1.08
	6 × 6 × 1/2	102.2	30.00	2.35	450	450	450	447	420	394	368	342	317	294	273	253	4.17	.883
	9/16	111.4	32 72	2.39	491	491	491	490	462	434	405	378	351	326	303	281	4 18	.916
	5/8	120.6	35 44	2 42	532	532	532	532	503	472	442	412	384	357	331	308	4 19	.940
14 × 5/8	6 × 6 × 5/8	126 6	37 19	2 41	558	558	558	558	527	495	463	431	402	373	347	322	4 11	917
	3/4	144 6	42 51	2 47	638	638	638	638	609	573	537	502	468	436	406	377	4 11	968
14 × 3/4	6 × 6 × 3/4	150 5	44 26	2 47	664	664	664	664	634	596	559	522	487	454	422	393	4 03	955
	7/8	168 1	49 42	2.53	741	741	741	741	714	674	633	593	554	517	482	449	4 04	1 00

Unequal Angles have short leg against web-plate. Weights given do not include rivets or other details.
Loads to right of heavy vertical lines are for Secondary Members ONLY.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX.* Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet												Kx	Ky
					14	16	18	20	22	24	26	28	30	32	34	36		
10 in×15.3 lb	12×¼	51.0	14.19	3.62	213	213	213	205	197	189	181	173	165	157	150	143	3.65	2.18
	5/16	56.1	16.44	3.60	247	247	247	237	228	218	209	199	190	181	173	164	3.81	2.16
	3/8	61.2	17.94	3.59	269	269	269	259	248	238	228	217	207	197	188	179	3.90	2.15
	7/16	66.3	19 44	3.58	292	292	291	280	269	257	246	235	224	214	203	193	3 97	2 13
	½	71.4	20.94	3.57	314	314	313	301	289	277	265	253	241	230	218	208	4.04	2.12
10 in×20.0 lb	12×¼	60.4	16.97	3.53	255	255	253	243	233	223	213	203	194	184	175	167	3 31	2 07
	5/16	65.5	19.22	3.52	288	288	286	275	264	252	241	230	219	208	198	188	3 49	2.06
	3/8	70.6	20.72	3.51	311	311	308	296	284	271	259	247	235	224	213	202	3.58	2.06
	7/16	75.7	22.22	3.51	333	333	330	317	304	291	278	265	252	240	228	217	3.66	2.05
	½	80.8	23.72	3.51	356	356	353	339	325	311	297	283	269	256	244	232	3.74	2.05
10 in×25.0 lb	12×¼	70.4	19.91	3.45	299	299	294	282	270	258	246	235	223	212	202	192	3.05	1.98
	5/16	75.5	22.16	3.45	332	332	328	314	301	287	274	261	248	236	225	213	3 24	1.98
	3/8	80.6	23.66	3.45	355	355	350	336	321	307	293	279	265	252	240	228	3.33	1.98
	½	90 8	26.66	3.45	400	400	394	378	362	346	330	314	299	284	270	257	3 49	1.98
	5/8	101 0	29.66	3.45	445	445	438	421	403	385	367	349	333	316	301	285	3.62	1.98

The ¼-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars and batten-plates; 87½% of their area is included in the functions and column areas. Weights given do not include rivets or other details.

Loads to right of heavy vertical lines are for secondary members ONLY.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX * (Continued). Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet												Kx	Ky
					14	16	18	20	22	24	26	28	30	32	34	36		
10 in×30.0 lb	12×¼	80.4	22.85	3 33	343	343	333	319	305	290	276	263	249	237	224	213	2 87	1.84
	5/16	85.5	25.10	3.34	377	377	367	351	335	320	304	289	275	261	250	234	3 04	1 86
	⅜	90.6	26.60	3 35	399	399	389	373	356	339	323	307	292	277	265	249	3.13	1.86
	12×½	100.8	29.60	3 36	444	·444	433	415	397	378	360	343	325	309	293	278	3.30	1 88
	⅝	111.0	32.60	3.37	489	489	478	458	438	418	398	378	359	341	323	307	3 43	1 89
	¾	121.2	35.60	3 38	534	534	522	501	479	457	435	414	393	373	354	336	3 55	1 90
10 in×35.0 lb	12×¼	90 4	25.79	3 25	387	387	373	356	340	323	307	291	276	261	247	234	2 72	1 78
	5/16	95 5	28.04	3 27	421	421	406	389	370	353	335	318	302	286	271	256	2 88	1.78
	⅜	100.6	29.54	3 28	443	443	429	410	391	372	354	336	318	302	286	271	2 97	1.79
	12×½	110 8	32.54	3 30	488	488	473	453	432	412	392	372	353	334	317	300	3 14	1.81
	⅝	121.0	35 54	3 31	533	533	518	495	473	450	428	407	386	366	347	329	3 27	1 83
	¾	131.2	38 54	3 33	578	578	562	539	514	490	466	443	421	399	378	358	3 39	1 84

The ¼-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars and batten-plates; 87½% of their area is included in the functions and column areas. Weights given do not include Rivets or other details.
Loads to right of heavy vertical lines are for secondary members ONLY.

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX * (Continued). Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet: 18	20	22	24	26	28	30	32	34	36	38	40	Kx	Ky
12 in×20.7 lb	14×1/4	65.2	17.31	4.38	260	260	259	251	243	235	227	218	210	202	195	187	4.19	2.74
	5/16	71.2	20.81	4.33	312	312	310	301	291	281	271	261	251	241	232	223	4.47	2.67
	3/8	77.1	22.56	4.30	338	338	336	325	314	303	292	281	271	260	250	240	4.58	2.64
	14×7/16	83.1	24.31	4.29	365	365	361	350	338	326	315	303	291	280	269	258	4.67	2.62
	1/2	89.0	26.06	4.27	391	391	387	374	362	349	336	324	311	299	287	275	4.75	2.60
	5/8	100.9	29.56	4.24	443	443	438	424	409	394	380	366	351	338	324	311	4.87	2.57
12 in×25.0 lb	14×1/4	73.8	19.89	4.29	298	298	296	286	277	267	257	248	238	229	220	211	3.89	2.63
	5/16	79.8	23.39	4.25	351	351	347	335	324	312	301	290	279	268	257	246	4.19	2.59
	3/8	85.7	25.14	4.24	377	377	372	360	348	335	323	311	299	287	276	265	4.30	2.56
	14×7/16	91.7	26.89	4.23	404	404	398	385	372	358	345	332	319	307	294	282	4.40	2.55
	1/2	97.6	28.64	4.22	430	430	423	410	396	381	367	353	339	326	313	300	4.48	2.54
	5/8	109.5	32.14	4.20	482	482	474	459	443	427	411	395	380	365	350	335	4.62	2.52
12 in×30.0 lb	14×1/4	83.8	22.83	4.20	343	343	337	326	315	303	292	281	270	259	248	238	3.64	2.52
	5/16	89.8	26.33	4.18	395	395	388	375	362	349	336	323	310	298	285	273	3.93	2.50
	3/8	95.7	28.08	4.17	421	421	413	400	386	372	358	344	330	317	304	291	4.05	2.49
	14×1/2	107.6	31.58	4.16	474	474	465	449	433	417	401	386	370	356	341	327	4.24	2.47
	5/8	119.5	35.08	4.15	526	526	516	498	481	463	445	428	411	394	378	362	4.39	2.46
	3/4	131.4	38.58	4.14	579	579	566	547	528	508	489	470	451	433	415	398	4.52	2.45

The 1/4-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars and batten-plates; 75% of their area is included in the functions and column areas. Weights given do not include Rivets or other details.

Loads to right of heavy vertical lines are for secondary members ONLY.

LOADS BY A.I S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX * (Continued). Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet												Kx	Ky
					18	20	22	24	26	28	30	32	34	36	38	40		
12 in×35.0 lb	14×¼	93 8	25 77	4.12	387	387	378	365	352	339	326	313	300	288	276	264	3.44	2.42
	5/16	99 8	29.27	4.11	439	439	429	414	399	384	369	355	341	326	313	300	3.73	2 41
	⅜	105.7	31 02	4.10	465	465	454	439	422	407	391	375	360	345	331	317	3 84	2 40
	½	117.6	34.52	4.10	518	518	505	488	470	453	435	418	401	384	368	353	4.03	2.40
	14×⅝	129.5	38.02	4 09	570	570	556	536	517	498	478	459	441	422	405	388	4 18	2 39
	¾	141 4	41.52	4 09	623	623	607	586	565	543	522	502	481	461	442	424	4.32	2 38
	⅞	153 4	45.02	4.08	675	675	657	635	612	588	565	543	521	499	479	458	4.44	2.38
12 in×40.0 lb	14×¼	103 8	28 71	4 04	431	431	418	403	388	373	359	344	330	316	303	289	3 28	2 33
	5/16	109 8	32.21	4.04	483	483	469	452	436	419	402	386	370	355	340	325	3 56	2 33
	⅜	115.7	33.96	4.04	509	509	494	477	459	441	424	407	390	374	358	342	3.67	2 33
	½	127.6	37.46	4.04	562	562	545	526	506	487	468	449	430	412	395	378	3 86	2.33
	14×⅝	139.5	40 96	4 04	614	614	596	575	554	532	512	491	471	451	432	413	4.02	2.33
	¾	151 4	44 46	4 04	667	667	647	624	601	578	555	533	511	490	469	448	4 15	2.33
	⅞	163.4	47.96	4.04	719	719	698	673	648	623	599	575	551	528	505	483	4.27	2.33

The ¼-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars as batten-plates; 75% of their area is included in the functions and column areas. Weights given do not include Rivets or other details.

Loads to right of heavy vertical lines are for secondary members ONLY.

LOADS BY A.I S C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX * (Continued). Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet														Kx	Ky
					20	22	24	26	28	30	32	34	36	38	40	42	44	46		
15 in×33.9 lb	16×¼	95.0	25.05	5.12	376	376	376	374	364	354	343	333	323	313	303	293	283	274	4.80	3.27
	⅜	108.6	31.80	5.02	477	477	477	471	458	445	432	419	405	392	379	367	355	342	5.32	3.15
	7/16	115.4	33.80	5.00	507	507	507	500	486	473	458	444	430	416	402	389	376	363	5.44	3.12
	16×½	122.2	35.80	4.98	537	537	537	529	514	499	484	469	454	439	425	411	397	383	5.54	3.09
	⅝	135.8	39.80	4.94	597	597	597	586	570	553	537	520	503	486	470	454	438	423	5.71	3.05
	¾	149.4	43.80	4.91	657	657	657	644	625	607	589	570	551	533	515	498	480	463	5.85	3.02
15 in×35.0 lb	16×¼	97.2	25.71	5.07	386	386	386	382	372	361	351	340	330	319	309	299	289	279	4.73	3.22
	⅜	110.8	32.46	4.98	487	487	487	480	467	453	439	426	412	398	385	372	360	348	5.27	3.10
	½	124.4	36.46	4.94	547	547	547	537	522	507	491	476	461	446	430	416	401	387	5.48	3.05
	16×⅝	138.0	40.46	4.91	607	607	607	595	578	561	544	526	509	492	475	460	443	428	5.65	3.01
	¾	151.6	44.46	4.89	667	667	667	653	634	615	596	577	558	540	521	503	486	469	5.79	3.98
	⅞	165.2	48.46	4.87	727	727	727	710	690	669	648	628	607	586	566	547	528	509	5.92	3.96
15 in×40.0 lb	16×¼	107.2	28.65	5.02	430	430	430	425	413	401	389	377	365	354	342	331	319	308	4.49	3.15
	⅜	120.8	35.40	4.95	531	531	531	522	507	492	478	463	448	433	418	404	391	377	5.03	3.06
	½	134.4	39.40	4.92	591	591	591	580	563	546	530	513	496	480	464	448	433	417	5.24	3.02
	16×⅝	148.0	43.40	4.89	651	651	651	637	619	600	582	563	545	527	509	491	474	457	5.43	3.99
	¾	161.6	47.40	4.87	711	711	711	695	675	655	634	614	593	574	554	535	516	498	5.88	3.96
	⅞	175.2	51.40	4.85	771	771	771	752	730	708	686	664	642	620	599	578	558	538	5.71	3.94

The ¼-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars and batten-plates; 65⅝% of the 16×¼ in. and 58⅓% of the 18×¼-in plates are included in the functions and column areas

Weights given do not include Rivets or other details.

Loads by A.I.S.C. Specification

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

Table XX * (Continued). Channel and Plate Columns

Allowable Concentric Loads in Kips for Channel Columns with Cover Plates

2 Channels	2 Cover-plates	Weight per foot	Area, sq in	Least radius gyration	Unsupported length in feet														Kx	Ky
					20	22	24	26	28	30	32	34	36	38	40	42	44	46		
15 in×45.0 lb	16×¼	117.2	31 59	4 93	474	474	474	465	452	439	426	412	399	385	372	360	347	335	4.30	3 04
	½	144.4	42.34	4.86	635	635	635	620	602	584	566	548	530	512	495	477	460	444	5.04	2 95
	⅝	158.0	46.34	4 84	695	695	695	677	658	638	618	598	578	558	539	520	502	484	5 23	2 92
	¾	171.6	50.34	4.82	755	755	755	735	713	692	670	648	627	605	584	564	544	524	5 39	2.90
	16×⅞	185.2	54.34	4.80	815	815	815	792	769	745	722	698	674	652	629	606	585	564	5 52	2.89
15 in×50.0 lb	16×¼	127.2	34 53	4 89	518	518	518	507	492	478	463	448	434	419	405	391	377	364	4.13	2.99
	½	154.4	45.28	4 83	679	679	679	662	643	623	603	584	564	545	526	508	490	472	4.86	2 91
	⅝	168.0	49.28	4 81	739	739	739	719	698	677	655	634	613	592	571	551	531	512	5.06	2 90
	¾	181.6	53.28	4.80	799	799	799	777	754	731	708	684	661	639	616	595	573	553	5.21	2.88
	16×⅞	195.2	57.28	4.79	859	859	859	834	809	785	760	735	710	686	662	638	616	593	5 35	2 86
15 in×50.0 lb	18×¼	130.6	34.53	5.67	518	518	518	518	518	507	495	483	470	457	444	432	420	407	4.14	3 77
	¾	191 8	56.28	5.59	844	844	844	844	844	823	803	782	761	740	719	698	677	657	5.33	3 47
	⅞	207.2	60.78	5.56	912	912	912	912	909	887	865	842	819	796	774	751	729	707	5 48	3.44

The ¼-in Plates are tabulated with all weights of Channels, as adding some sectional area, without costing appreciably more than lattice-bars and batten-plates; 65⅝% of the 16×¼ in and 58⅓% of the 18×¼-in plates are included in the functions and column areas.
Weights given do not include Rivets or other details

LOADS BY A.I.S.C. SPECIFICATION

* From Steel Construction, published by American Institute of Steel Construction, with the addition of bending factors.

CHAPTER XV

STRENGTH OF STEEL BEAMS AND BEAM GIRDERS

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1. General Remarks on Beams

Definitions. A BEAM may be defined as a member which is in bending. In general, the term is restricted to members with solid webs and is not applied to trusses, though trusses with webs made up of several systems of bars are sometimes called LATTICED BEAMS or LATTICED GIRDERS. Special terms are used to describe particular beams. A GIRDER is a heavy beam which carries loads from other beams. A JOIST is a light beam which takes directly the load from the floor or deck. A LINTEL is a beam bridging an opening over a window or doorway. A RAFTER is a beam which carries the load of a roof, the rafters running transversely across the roof. PURLINS are members in roof framing which run longitudinally in the roof and carry to the roof trusses or girders either the loads of the rafters or of the sheathing. A GIRT is a horizontal beam in the walls of a building intended to furnish resistance of the walls against horizontal loads such as wind.

Beams are made up of the material at the top and bottom, called FLANGES, and of a WEB which connects these flanges. They rest at their ends on BEARINGS which are often strengthened by BEARING STIFFENERS, or angles riveted to the web.

Usually the most important property of a beam is its MOMENT OF RESISTANCE or ability to resist bending moment. This is measured by its SECTION-MODULUS, which is equal to its moment of inertia divided by its depth. Another property important in a beam is its STIFFNESS or resistance to deflection under load.

Loads. Beams are loaded with fixed, or DEAD, load, with movable, or LIVE, load, with IMPACT loads from moving vehicles or machinery and by WIND and other causes. In building construction the dead load on a beam consists of the weight of the floor, of partitions and other fixtures, and of the beam itself. The live load consists of the weight of people, furniture and equipment, of machinery in factories, of goods stored on the floors in warehouses. Impact occurs where moving loads produce vibration in the beams, as in the case of moving machinery, cranes or trucks. Beams in building construction are also subjected to wind-loads on the roofs and walls of the building. Rafters, purlins on sloping roofs and girts are subject to wind-load, and roof beams are also subject to loads from snow in cold climates.

Loads may be DISTRIBUTED or spread over the beam either UNIFORMLY or NON-UNIFORMLY or they may be CONCENTRATED at certain points on a beam. On a beam which carries directly the load from a floor the load is usually assumed to be uniformly distributed, but where a beam carries the load of a brick wall the intensity of the load is assumed to be greater near the center of the beam and hence is distributed, but not uniformly. Where the load is brought to a beam by joists, the reactions of these joists are concentrated.

Walls of Brick or Masonry impose on beams loads which are necessarily somewhat uncertain. There will usually be considerable arching action if the masonry is laid above a certain height. It is customary to assume that the beam under a solid brick wall carries a triangular loading as indicated in Fig. 1. Some discretion should be used, however, in applying this standard. Beams carrying such walls should be made quite stiff; otherwise cracks will appear in the brick wall. Where, as is frequently the case, openings occur in the brickwork, special judgment is required in

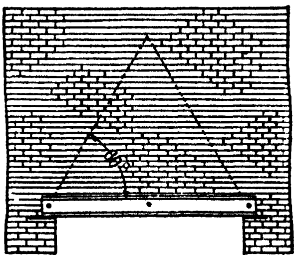


Fig. 1. Triangular Loading of Beams under Brick Walls

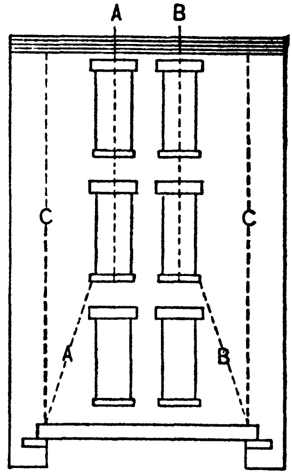


Fig. 2. Loading of Beams under Walls with Openings

determining what total load shall be assumed as carried by the beam and what distribution shall be assumed for such loading. It is not difficult to make sufficiently accurate assumptions as to the distribution in such cases. In Fig. 2, for example, the load between lines AA and BB may be assumed to be carried by the girder.

Supports. Beams may be supported in several ways. A beam which extends between two supports which do not restrain its ends is referred to as simply supported, or as a SIMPLE BEAM. Beams which extend beyond their supports are said to be CANTILEVERED. Where a beam rests on more than two supports without hinges between the supports, it is said to be CONTINUOUS. Beams which are restrained against any rotation at their ends are called FIXED BEAMS. The condition of perfect fixation at both ends of a beam, however, can scarcely exist in reality because any deflection of the supporting walls relieves the end moments which produce fixation.

Mechanics of Flexure. Flexure means bending. On one side of any imaginary transverse section of a beam the external forces tend to produce rotation of the part of the beam on that side of the section. This rotation is resisted by internal stresses of tension and compression in the beam. These are known as FIBER-STRESSES. For gravity loads on simply supported beams these stresses will be COMPRESSIVE at the top and TENSILE at the bottom of the beam. The laws of statics demand that the total compressive stress be equal to the total tensile stress wherever pure bending exists. This is not true where the bending is accompanied by longitudinal forces along the beam. These equal and opposite stresses of tension and compression constitute what is known in mechanics as a couple. The moment of this couple must equal the moment of the external forces on one side of the section

under consideration. This moment of the external forces is called the BENDING MOMENT on the section. The moment of the couple produced by the fiber-stresses is called the RESISTING MOMENT.

Assumptions. It is common to assume that the fiber-stresses normal to any cross-section of a beam vary directly as the distance from the line of zero stress intensity on the section. This line of zero stress is called the NEUTRAL AXIS of the section. The assumption of linear variation of stress is based on two other assumptions. It is assumed that cross-sections of a beam which are plane before the beam is bent remain plane after the beam is bent; this is referred to as the theory of CONSERVATION OF PLANE SECTIONS. It is also commonly assumed that the intensity of stress in any fiber is proportional to the amount of strain or deformation in that fiber. This assumption is known as HOOKE'S LAW. The stress intensity for any material above which the stress is no longer proportional to the strain is called the ELASTIC LIMIT, PROPORTIONAL LIMIT or YIELD-POINT * of the material. The assumption implied in Hooke's Law is applicable very imperfectly to materials at stresses above their yield-point. It is applicable to steel quite accurately for working loads, but rather imperfectly to cast iron at any loads, because cast iron does not have a well-defined yield-point. It is also imperfectly applicable to timber, stone and concrete.

Material for Beams. Several materials are used for structural beams. TIMBER BEAMS are discussed in Chapter XX. CAST-IRON BEAMS were at one time extensively used, but are now restricted largely to lintels and to other minor uses. BEAMS OF STONE or of PLAIN CONCRETE are used occasionally, usually as slabs over areaways and sometimes as lintels, but they are not so common as they were years ago. Many beams of REINFORCED CONCRETE are used, steel being introduced on the tension side of the beam to prevent failure of the concrete in tension. STEEL BEAMS of many types are now used in building-construction. Steel beams are often encased in concrete to protect them against rust and against the action of fire. There has been some tendency to reinforce this encasing concrete and depend upon action together of the steel beam and its reinforced-concrete encasement to form a COMPOSITE BEAM. Largely because of doubt as to the effectiveness of the combination at ultimate loads, the practice has received little encouragement from building departments in this country.

Failure of Beams. The engineer or architect is interested primarily in the type and cause of failure which may occur. Probability of failure is indicated by what is called the FACTOR OF SAFETY of a structure. The term is not entirely definite since it may be used to indicate either the ratio of the working stress used in design to the yield-point of the material, or the ratio of the working stress used in design to the ultimate strength of the material, or it may mean the ratio of the total load at failure to the total load used in the design of the structure. Whether the factor of safety should be based on the ultimate strength of the material or upon its yield-point has been debated at times. In the case of steel beams and of beams of reinforced concrete, failure usually occurs by secondary effects soon after the yield-point of the steel is exceeded. The term factor of safety as used by structural engineers usually indicates the ratio of total load which would produce failure to total load used in design.

Beams may fail in several different ways, depending upon the material used in the beam and upon the type of loading. Failure may occur by rupture

* These terms are not strictly interchangeable, but are sufficiently so for ordinary purposes.

of the TENSION FLANGE. This rupture is more likely to occur at those sections where rivet-holes, if any, are present. Failure may occur from too high compression in the COMPRESSION FLANGE. Such failure usually occurs through buckling of the flange. This buckling may take place either by lateral deflection of the flange or by rotation of the flange, the flange folding down onto the web. In order to provide against buckling of the flange it is customary to specify that the flange-stress shall be reduced unless the unsupported length of flange is less than fifteen times its width, and that in no case shall the ratio of unsupported length to width exceed 40. In other cases the allowable lateral stress in the flange is reduced according to some formula derived from the column formula. To be dependable, lateral restraint must be secure and definite. Failure may take place through VERTICAL OR LONGITUDINAL SHEARING, depending upon the characteristics of the material used. The material of a beam may be CRUSHED or may BUCKLE at reaction-points or at points of concentrated loading. Failure may occur through INCLINED TENSION in the web in some cases, by INCLINED COMPRESSION in the web in other cases. In the case of built-up members special attention must be given to prevent failure in DETAILS.

Different Materials are peculiarly liable to certain types of failure when used as beams. Thus CAST IRON is peculiarly inclined to failure in TENSION. TIMBER BEAMS may fail in TENSION or in LONGITUDINAL SHEAR. Longitudinal shear should always be investigated in timber beams and will frequently control the design. ROLLED-STEEL BEAMS are designed on the assumption that under normal conditions they will fail by TENSION or by COMPRESSION in the flanges. In SHORT BEAMS or those carrying heavy concentrated load, the SHEAR in the web may be dangerous. PLATE GIRDERS may fail in the same way as rolled beams, but are particularly liable to DIAGONAL COMPRESSION failure in the web which should be provided against by stiffeners, and to failure in the RIVETS connecting the flange to the web. BEAMS OF STONE OR OF PLANE CONCRETE are more liable to failure in TENSION. Beams of REINFORCED CONCRETE may fail in several ways but special consideration must be given to the details of EMBEDMENT of reinforcing steel in the concrete and to the danger of failure from DIAGONAL TENSION in the web.

2. Bending Stresses

Bending Moment is the sum of the products obtained by multiplying the forces on one side of a cross-section of a beam by their distance from that section. Bending moments which produce compression in the top fiber of a beam are called POSITIVE, and those which produce tension in the top fibers are called NEGATIVE bending moments. In the case of plate girders it is sometimes necessary to draw a complete curve of maximum moments for the beam, but in designing beams of timber or rolled-steel beams, it is necessary to compute only the maximum bending moment. The maximum bending moment in a beam, in building-construction, for loads uniformly distributed either with joists equally spaced or without joists is nearly the same, irrespective of the spacing of the joists.

Since the loads are given either in pounds or in thousands of pounds * the moments will in general be computed first in terms of foot-pounds or foot-kips, the foot-kip being one thousand foot-pounds. The properties of sections, however, are given in inch-units and the bending moment must be reduced to inch-pounds before applying the flexure formula. Failure to reduce foot-

* One thousand pounds is called a kip, abbreviated to K.

pounds to inch-pounds is the most common source of error in beam computations.

The Beam Formula. Stresses produced by flexure in beams are computed by what is known as the beam formula or flexure formula. This is

$$M = \frac{sI}{c} \tag{1}$$

in which M is the external bending moment in inch-pounds;
s is the intensity of stress in an outermost fiber in pounds per square inch;
I is the moment of inertia of the section in inch-units;
c is the distance from the centroid of the section to the outermost fiber in inches.

The quotient $\frac{I}{c}$ is called the SECTION-MODULUS of the section. The allowable moment equals the section-modulus multiplied by the allowable fiber-stress. There may be some difference of opinion as to whether on cross-sections of a beam containing rivet-holes the moment of inertia should be computed on sections without rivet-holes or on sections from which rivet-holes are deducted. The former is called the GROSS SECTION and the latter the NET SECTION. By some, the moment of inertia is computed for a section in which rivet-holes are deducted on the tension side of the section and are not deducted on the compression side, on the theory that on the compression side the rivets fill their holes completely, so that failure could not occur at the rivets. If bolts instead of rivets are used in holes in the compression flange, these holes will not be completely filled. The best practice is to compute the moment of inertia for the net section assuming rivet-holes deducted from both flanges wherever there are holes in the tension flange or unfilled holes in the compression flange.

For RECTANGULAR BEAMS the beam formula just given takes the form

$$M = \tfrac{1}{6} sAd \tag{2}$$

or

$$M = \tfrac{1}{6} sbd^2 \tag{3}$$

in which A is the area of the cross-section in square inches;
b is the width of the section in inches;
d is the depth of the section in inches;
M and s are as previously defined.

This is the proper form for use in the design of timber beams.

Modulus of Rupture. It has been stated above that Hooke's Law, on which the beam formula is based, is not applicable beyond the yield-point of the materials, and is imperfectly applicable to cast iron, timber, stone and concrete in any case. Consequently the stresses in the outer fibers of steel beams which are near failure or of beams made of materials which have no clearly defined yield-point will not be correctly given by the beam formula. It is nevertheless customary in interpreting breaking tests of beams to compute these stresses from the beam formula. The false stress thus computed in the outer fiber at failure by application of the beam formula is called the MODULUS OF RUPTURE of the beam. Obviously it is not particularly important that the assumptions on which the beam formula is based be correct at failure, provided the error is recognized in interpreting the modulus of rupture and in fixing the working stresses for the materials.

Beams Sharply Curved occur in machine parts and sometimes, though rarely, in structural work. Where the radius of curvature is small compared with the depth of the beam, the beam formula needs modification. Reference should be had in such cases to special treatments of sharply curved beams.

Relative Strength of Beams. It is very useful to keep in mind the relations of the different elements in the beam formula. It will be seen that the fiber-stress for a given moment and moment of inertia is inversely proportional to the depth. In general the deeper the beam the stronger it is in resisting moment. In the case of rectangular beams the moment of resistance varies directly as the width and directly as the square of the depth of the beam.

Direct Estimate of Stress. It has been explained that the moment of the outer forces on one side of a section is resisted by the couple made up of the compressive stress and the tensile stress in the beam. Beams may be analyzed by estimating or computing the lever arm of this couple. If the moment is divided by this lever arm the quotient is the total tension or compression in the beam. If, then, it is known over what area this tension or compression acts, the average intensity can be computed and from the known variation in stress intensity, the maximum can be deduced from the average. This conception is valuable in clearly understanding the action of ordinary beams and in quickly estimating stresses in beams of irregular shape. As an illustration of the method, in the case of a rectangular beam, the lever arm of the couple is two-thirds of the depth of the beam. That half of the beam above the neutral axis is in compression. The maximum intensity over this area is twice the average. From this we deduce

$$s = 2\frac{M/\frac{2}{3}d}{\frac{1}{2}A} = \frac{6M}{Ad}$$

$$M = \tfrac{1}{6}sAd$$

as before. In the case of rolled-steel beams the lever arm of the couple will be somewhat less than the depth of the beam. The stress intensity over the flange is nearly uniform. If, then, we neglect the moment of resistance of the web, we may write

$$M = sA_f d \text{ (approximately)}$$

Where A_f is the area of one flange. This method of analysis is frequently used in the design of plate girders, as discussed in Chapter XX.

Combined Stresses. Members are sometimes subjected to a combination of bending and compression or to a combination of bending and tension. The MAXIMUM STRESS in a member may be computed in such cases by adding the stress due to bending to that due to the tension or compression. This problem occurs, for example, in the design of chords of roof trusses which carry purlin loads between panel points. Members in tension or in compression are also sometimes subject to considerable flexure from the bending of other members which are connected to them. Columns in the side walls of buildings are subject to bending stress from wind load on the walls. Columns carrying crane girders on brackets or carrying jib cranes are subject to considerable bending. For use of the so-called bending factor in eccentrically loaded columns, see Chapters X and XIV.

Bending about Two Axes sometimes occurs. Thus a column may carry bracket loads on two adjacent faces. In these cases also stresses from the

two sources may be added. It should be noted that the beam formula as given above applies only where the bending is in the same plane as one of the principal axes of the section. In any case the bending should be resolved into components in the planes of the principal axes of the section.*

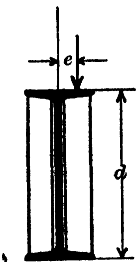


Fig. 3. Beam Eccentrically Loaded

Beams Eccentrically Loaded. Beams are sometimes eccentrically loaded, as where the load is not applied centrally on the web of an I beam. (See Fig. 3.) Where this load is concentrated it is evident that STIFFENERS must be provided to prevent crippling of the web and also to prevent tipping and buckling of the loaded flange. If the flanges of the beam are free to bend laterally and the load is free to move laterally, the couple represented by the shear in the web and by the load will be resisted by shears set up in the flanges of the beam. The flexural stresses resulting from the transverse bending of the flanges may be quite high. They may be computed by the following approximate formula:

$$f_t = 2 f_b \frac{S}{S'} \frac{e}{d} \tag{4}$$

in which f_t is the additional fiber-stress due to the eccentricity of the load;
f_b is the fiber-stress as ordinarily computed in the beam for the transverse load;
S is the section-modulus for the axis about which bending takes place;
S' is the section-modulus about the other axis of the section;
$\frac{e}{d}$ is the ratio of the eccentricity of the load to the depth of the beam. (See Fig. 3.)

This formula, while approximate, is sufficiently accurate to indicate the dangers from such eccentric loads.

If either flange is restrained against lateral movement, lateral stress will be eliminated from that flange. If the load is restrained against lateral movement the eccentricity will be eliminated. Consequently, there is little danger from this source in ordinary spandrel beams because the walls are not free to move laterally and the top flange is also restrained against lateral movement. Cases occur, though, in which the walls or top flanges are not adequately restrained. The type of failure is so sudden and so threatening in such cases that great caution should be exercised where eccentric loading is probable. In general such eccentric load should be avoided.

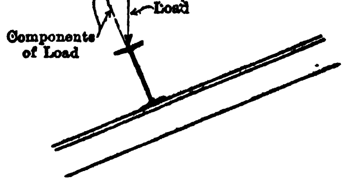


Fig. 4. Purlin on Sloping Roof

Purlins on Sloping Roofs are set normal to the rafters or to the roof trusses, as shown in Fig. 4. The vertical load, then, is not normal to the principal axis of the purlin. The load may be resolved into a component normal to the purlin and a component parallel to the roof. The former is resisted by

* For a definition of principal axes, see Chapter X. See, also, the discussion of columns in flexure in Chapter XIV.

bending about the major axis of the purlin. The latter will be resisted by the stiffness of the sheathing if the roof is sheathed and if the sheathing is connected to the ridge of the roof. If, as with corrugated-iron roofing, the roof covering is flexible, the purlins should be supported by sag rods which are securely fastened at the ridge. The ridge purlin must then be designed for the vertical component of the stress in these sag rods. Lateral failure of purlins on sloping roofs is a real danger which should be avoided by proper methods of construction.

Working Stresses in Steel Beams have increased in recent years. It was formerly customary to design beams and girders in buildings for a working stress of 16 000 lb per sq in. It became customary, even before generally sanctioned by code requirements, to use as high as 18 000 lb per sq in. It is now the practice in some offices, where not restricted by code requirements, to design for stresses as high as 20 000 lb per sq in. The specifications of the American Institute of Steel Construction permit 18 000 lb per sq in. This is safe in the hands of a competent designer. Higher stresses than this should be used with caution.

3. Shearing-Stresses

Shear is defined as the algebraic sum of the loads and reactions on one side of a section. In designing a beam of timber or metal, the sign or direction of the shear is not commonly a matter of importance. Except in plate girders, variation of the shear along the length of the beam is also not a matter of importance, so that it is usually sufficient to design a beam for the maximum shear coming upon it.

The shear on any cross-section is resisted by shearing-stresses in the beam. These transverse shearing-stresses must equal the external shear. Their distribution over the cross-section is dependent upon the intensity of the longitudinal stresses normal to the cross-section. The intensity of shearing-stress at any point on the section may be computed from the formula

$$v = \frac{VQ}{It}\,^{*} \tag{5}$$

in which v is the intensity of vertical shear at any point on the cross-section, in pounds per square inch;

V is the total external shear on the cross-section, in pounds;

Q is the statical moment about the gravity axis of that part of the section above or below the fiber in question;

I is the moment of inertia of the section;

t is the thickness of the web of the beam at the point being considered.

* This formula may be derived as follows. The stress intensity in any fiber equals $\frac{Mc}{I}$. The change in the total stress in any fiber in a unit length of beam, then, equals the change in moment per unit length times the area of the fiber times $\frac{c}{I}$. This equals $\frac{V}{I}$ times the statical moment of the area of the fiber about the neutral axis. The total change in stress above or below any longitudinal section one unit long, then, equals $\frac{VQ}{I}$. This unbalanced longitudinal stress must be balanced by longitudinal shear. The longitudinal shear per unit of length, then, equals $\frac{VQ}{I}$ and the longitudinal shear per unit of area, $v = \frac{VQ}{It}$. The equality of the intensity of vertical and longitudinal shear at any point is a well established principle of mechanics.

The shearing intensity on rectangular beams will, according to this formula, vary as a parabola, as shown in Fig. 5. The intensity of shearing-stress has maximum intensity at the neutral axis, equal to one and one-half times the average intensity.

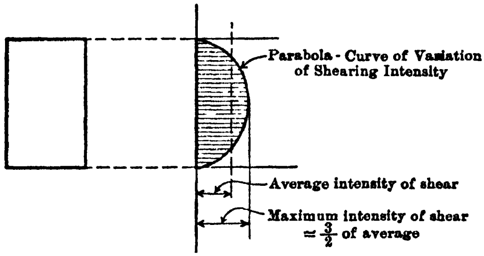


Fig. 5. Distribution of Shearing Stresses on a Rectangular Section

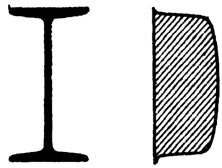


Fig. 6. Distribution of Shearing Stresses on an I-Beam

In rolled-steel beams the shearing intensity varies as shown in Fig. 6. This distribution is sometimes computed as uniform over an area of web equal to the thickness of web times the total depth of the beam. Sometimes the net depth of the web between fillets of flanges is used instead of the total depth. This is more nearly correct.

It is convenient to use the average intensity of shear on a cross-section as a measure of the maximum intensity, but it should be recognized that this is merely a conventional process and in special cases an analysis should be made according to the formula given above to determine the maximum intensity.

Longitudinal Shearing-Stresses. Since the longitudinal fiber-stresses vary from section to section because of variation in bending moment along the beam, there will be set up longitudinal shearing-stresses equal in intensity at any point to the vertical shearing-stresses. They may be critical in the design of timber beams.

Inclined Web-Stresses. Shearing-stresses at the neutral axis produce, on planes at 45° to this axis, tensions and compressions equal in intensity to the shears. These inclined stresses of tension and compression are known as INCLINED OR DIAGONAL WEB-STRESSES. Such stresses also exist at all points in the beam but are modified in intensity, except at the neutral axis, by the direct stresses and do not have a maximum value on 45° planes. These stresses are important in the design of some types of beams but they are provided against by general methods of construction and are not usually computed in detail. They are important in plate girders and beams of reinforced concrete but not in rolled-steel beams or in timber beams, because in these failure will occur first in other ways.

Bearing. At the bearing-points and at points of concentrated loading the webs of beams are subject to direct compression for a part of their depth. It is customary to assume that the reaction acts on a vertical columnar section of web, the cross-section of which has a thickness equal to that of the web and a depth equal to the length of bearing-block plus one-fourth the beam depth for an end reaction or plus one-half the beam depth for an interior load. (See Fig. 7.) The allowable stress is computed by the column formula for a length of column equal to half the beam depth.

Bearing Stiffeners. If the buckling load on the web as computed by the method just indicated is excessive, stiffener angles, fitted to the flanges, are riveted to the web. These stiffeners are usually computed as columns which may buckle about an axis in the beam web, the length of column being taken as one-half the beam depth.

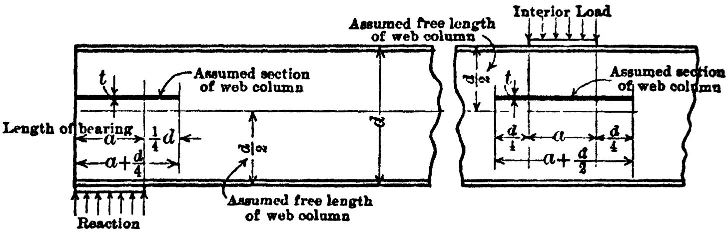


Fig. 7. Assumed Distribution of Reaction and Concentrated Load Stresses over Beam Webs

A very much simpler, more convenient and a somewhat more conservative practice is to assume that the load is taken entirely by the outstanding legs of the bearing angles allowing the stress permitted for bearing of steel on steel, 18 000 lb per sq in.

Shearing Failure. It is important to note that the type of failure in beams is largely dependent upon the ratio of maximum shear to maximum moment. Where the ratio of shear to moment is high, shearing failure is particularly liable to occur. High shear may produce either rupture of the web by vertical or longitudinal shear or inclined buckling of the web, or, in built-up girders, failure of rivets connecting flange to web. These types of failure need special consideration in beams carrying heavy concentrated loads near the support, or in beams of short span, or in girders in which the depth is unusually great compared with the span length.

In the case of rolled-steel beams, failure by diagonal buckling of the web may, for practical purposes, be neglected in design. Shear failure of the web is also not commonly to be feared in such beams. For uniformly distributed loading, web-shear is never critical if the beam depth in inches is equal to or less than the span in feet, and for most beams it is not critical if the depth in inches is less than twice the span in feet.

4. Deflection of Beams

Consideration must often be given to the DEFLECTION OF BEAMS. This is especially true where the beams carry plastered ceilings, because excessive deflection will lead to objectionable cracking of the plaster. It is usually specified that beams carrying plastered ceilings shall not have deflections greater than 1/360 of the span. There has been some dispute as to whether this deflection shall be computed for the total load or for the live load alone. The latter seems more logical and is commonly used; the former is more conservative and is usually not difficult to comply with. For beams that are not continuous this condition is generally satisfied closely enough if the depth of the beam in inches is greater than one-half its span length in feet. Where moving live loads are encountered as, for example, in dance-halls, drill-halls or where machinery is to be carried, deflection becomes a matter of special importance, and should be limited to perhaps one-half the above value to prevent objectionable vibration.

The deflection of a beam symmetrical about its gravity axis may be computed from the formula

$$\text{Deflection in inches} = K\frac{sL^2}{Ed} \qquad (6)$$

in which s is the maximum fiber-stress produced by the loads in pounds per square inch;
L is the span length in inches;
E is the modulus of elasticity of the material;
d is the depth of the beam in inches.

The value of E for steel may be taken as 29 000 000 lb per sq in, and for timber as 2 000 000 lb per sq in for temporary loads and 1 000 000 lb per sq in for permanent loads. The value K is a constant depending for its value on the type of loading and the method of support. It may be taken accurately enough as $^2/_{10}$ for simple beams and as $^2/_3$ for beams cantilevered from a fixed end.*

If s is 18 000 lb per sq in, the deflection of rolled-steel beams may be taken as

$$\text{Approximate deflection in inches} = \frac{1}{60}\,\frac{(\text{span in feet})^2}{\text{depth in inches}} \qquad (7)$$

for beams simply supported,

$$\text{Approximate deflection in inches} = \frac{1}{18}\,\frac{(\text{span in feet})^2}{\text{depth in inches}} \qquad (8)$$

for beams cantilevered from a fixed end.

5. Rolled Steel Beams

Steel Beams may be rolled beams having the shape of a letter **I**, channel beams, or angles. Beams of T shape were formerly used but are now restricted to minor uses. The rolled beams may have REINFORCING PLATES riveted to their flanges. GIRDERS may be built up of angles and plates riveted together. These are called PLATE GIRDERS where they have a single web-plate, and BOX GIRDERS where more than one web-plate is used. Built-up girders are now being made in some cases by welding together the constituent parts instead of using rivets. At present the practice in this regard is not completely standardized to the satisfaction of all structural engineers.

CHANNELS are sections having this shape: [. They vary in depth from 3 in to 15 in, and in weight from 4.1 lb per ft to 55 lb per ft. Of each depth several different weights are rolled. The flanges are beveled. For sections heavier than the minimum, the width of flange and thickness of web are increased, the thickness of flange remaining unchanged.

I Beams are sections rolled into the shape of the letter "I." The horizontal portions of the I are the flanges and the vertical portion is the web. The I beam is the ideal type of steel beam.

The American Standard I Sections vary in depth from 3 in to 24 in, and in weight from 5.7 lb per ft to 120 lb per ft. Of each depth there are several

* The deflection of a fixed beam loaded at the center is 17% less than $\frac{2}{10}\frac{sL^2}{Ed}$. In other cases this formula is within 10% of the true theoretical deflection. The coefficient for cantilever beams varies from $^2/_3$ to $^1/_2$, but should be taken conservatively, since the end is never completely fixed. The designer does not need to compute deflections with great accuracy, nor is it possible to predict them with great accuracy in building design because of various uncertainties.

weights rolled. The flanges are beveled. For sections heavier than the minimum for each depth, the thickness of web and the width of flange are increased, the thickness of flange remaining unchanged.

Bethlehem I Beams are rolled by the Bethlehem Steel Company. They vary in depth from 8 in to 36 in, and in weight from 17.5 lb per ft to 190.0 lb per ft. They have wider flanges than the standard I beams, but thinner webs. The weights are increased by increasing the web and flange thickness, the width of the flange remaining the same. This changes slightly the depth of beam as the weight of section is increased or decreased.

Bethlehem Girder Beams differ from Bethlehem I beams in that they have thicker and wider flanges. They also vary in nominal depth from 8 in to 36 in. In weight they vary from 29.5 lb per ft to 300.0 lb per ft.

Carnegie Beams are rolled by the Carnegie Steel Company. They vary in depth from 8 in to 36 in, and in weight from 24.0 lb per ft to 300.0 lb per ft. They are similar to the Bethlehem beams except that there is no bevel of the flange. No distinction is made between beams and girders, but in general the Carnegie beams cover the range of weights and strengths covered by Bethlehem beams and Bethlehem girders. Of each nominal depth there are two or more groups of flange widths which differ materially. Thus of beams nominally 18 in deep, one group has flange widths from 7.5 in to 8.5 in and another group has flange widths of about 12 in. Of the 36-in beams, one group has flange widths of about 12 in and the other of about 16 in. The beams with wide flanges correspond to the Bethlehem girder sections.

Jones and Laughlin Junior Beams, rolled by Jones and Laughlin Steel Company, vary in depth from 6 in to 12 in. They have thin webs varying from .114 in to .175 in. There is only one size of each nominal depth. The flanges are beveled.

Bethlehem Joists, rolled by the Bethlehem Steel Company, are light-weight beams varying in depth from 6 in to 12 in. There is only one section of each nominal depth. The web thickness is about $\frac{1}{4}$ in for all depths. The flanges are beveled.

Metal Lumber has recently come into use for light floor joists. This is made by bending sheet-metal into the form of channels and spot-welding two channels together to form an I beam. The metal is about $\frac{1}{12}$ in thick and the sections vary in depth from 6 in to 12 in. Obviously, in these sections the thickness of the web is twice that of the flanges.

Trussed Steel Joists of various types are now on the market, the particular types varying in detail. These are excellent for floor construction when their details are properly designed.

Welded Joists are also coming into use.

Where a single beam is not adequate, TWO BEAMS JOINED TOGETHER by separators are sometimes used. The practice is not desirable except where it is possible to be reasonably sure of equal distribution of load between the two beams. Diaphragms are sometimes used in such cases in place of separators.

Rolled Beams with Cover-Plates. Beams are sometimes reinforced with cover-plates. This practice is less common than formerly because of the heavier sections now available. Where cover-plates are used, they may be designed by using the moment of inertia of the net section in the beam formula. (See Chapter X.) A more convenient approximate method is to add to the section-modulus of the beam the product of the added net area due to the

Table I. Comparative Data on Rolled Sections *

Type of beam	Weight of stock size	Section-modulus	Section-modulus per pound per foot of depth	Web thickness	Percentage of total weight in web	Flange thickness at rivet-hole	Flange width
12-in standard channel	20.7	21.35	1.03	0.280	56	½	2.94
12-in standard I beam	31.8	35.97	1.13	0.350	45	9/16	5.00
18-in standard I beam	54.7	88.39	1.08	0.460	52	¾	6.00
12-in Bethlehem I beam	28.0	35.60	1.27	0.245	36	7/16	6.50
18-in Bethlehem I beam	49.0	89.20	1.21	0.330	42	19/32	7.50
36-in Bethlehem I beam	155.0	530.41	1.14	0.615	49	31/32	12.00
12-in Bethlehem girder beam	55.5	72.60	1.31	0.380	28	⅝	10.00
18-in Bethlehem girder beam	86.0	167.07	1.30	0.440	32	11/16	11.75
36-in Bethlehem girder beam	240.0	872.00	1.21	0.790	41	1¼	16.50
12-in Carnegie beam	25.0	30.69	1.23	0.240	40	⅜	6.00
12-in Carnegie beam	65.0	86.88	1.33	0.400	25	19/32	12.00
18-in Carnegie beam	47.0	85.40	1.21	0.320	42	17/32	7.50
18-in Carnegie beam	86.0	168.23	1.30	0.429	31	¾	12.00
36-in Carnegie beam	147.0	502.24	1.14	0.590	50	15/16	12.00
36-in Carnegie beam	230.0	834.05	1.21	0.769	41	1 9/32	16.00
12-in Jones & Laughlin Junior	11.74	12.01	1.02	0.175	61	...	3.06
12-in Bethlehem joist	18.5	20.25	1.10	0.240	53	...	4.125

* Compiled from data in the handbook Steel Construction, published by American Institute of Steel Construction.

plates times one-half the beam depth. (See Examples 1 and 9 at the end of the chapter.) This method is correct within a small per cent. Evidently where plates are used to reinforce the flanges, the danger of overstressing the web in shear is increased.

Sometimes a plate is added to only one flange of a beam in order to support a wall or for some other reason. Because of the shift of the neutral axis in such cases, there is relatively little gain in strength. If it seems worth while, the section-modulus of the section may be computed as explained in Chapter X. See Example 2 at the end of the chapter. The gross or the net section should be used in such computations according to whether the plate is added to the compression or to the tension flange.

Comparison of Steel Beam Sections. In Table I is shown a comparison of beams of various types. Note the general uniformity of the section-modulus per pound of weight per foot of depth. This measures the efficiency of the section and varies from about 1 to 1.3 for different beams. The greater efficiency of the modern Bethlehem and Carnegie sections should be noted. Note also the wider flanges and the relatively smaller percentage of metal in the webs of these beams as compared with the standard sections.

6. Design of Steel Beams

Formulas for the Strength of Ordinary Steel Beams include the ordinary beam formula, or formula for flexural stresses, the formula for web buckling due to concentrated loads and the formula for reduction of stress in the compression flange due to lateral deflection.

THE BEAM FORMULA is most conveniently written, for use in steel beams, in the form

$$S = \frac{M}{s} \tag{1}$$

or in words, the required section-modulus equals the quotient of the bending moment divided by the allowable stress in the extreme fiber.

THE FORMULA FOR WEB BUCKLING from concentrated loads or from reactions is based on the column formula and varies with the column formula used. Using the column formula recommended by the American Institute of Steel Construction and the assumptions already explained, we may derive

$$R = \frac{18\,000}{1 + \dfrac{d^2}{6\,000\,t^2}}(a + \tfrac{1}{4}\,d)t \tag{9}$$

in which R is the allowable reaction in pounds if no bearing stiffeners are used;
d is the depth of the beam in inches;
t is the thickness of the web in inches;
a is the length of the bearing plate in inches.

Data are given in the tables for computing the strength of the web according to this formula.

For loads applied to the beam this formula becomes

$$P = \frac{18\,000}{1 + \dfrac{d^2}{6\,000\,t^2}}(a + \tfrac{1}{2}\,d)t \tag{10}$$

in which P is the allowable load in pounds if no bearing stiffeners are used. The other quantities have the same meaning as above.

In cases where greater reactions or loads than are permitted by these formulas are to be carried, stiffeners fitted to the flanges must be used.

WHERE THE COMPRESSION FLANGE IS UNSUPPORTED for a length greater than 15 times its width, the allowable stress is reduced by use of the column formula, the reduction depending somewhat on the type of column formula used. The American Institute of Steel Construction recommends

$$f_c = \frac{20\,000}{1 + \dfrac{L^2}{2\,000\,b^2}} \tag{11}$$

in which f_c is the allowable compressive stress in the flange;
L is the distance between points of lateral support;
b is the flange width.

The unsupported length of flange should never exceed 40 times the width. Table II gives values of the allowable flange stress for various values of $\frac{L}{b}$ from 15 to 40 and also the allowable load that can be carried in percentage of that allowed where the flange has full lateral support.

Fig. 7A shows a NOMOGRAPHIC CHART for determining the allowable load on beams with unsupported compression flanges in percentage of the load which the same beam could support if the compression flange were fixed against lateral buckling. To use the diagram draw a straight line connecting the flange width, b, in in, on the vertical line at the left of the diagram, with the unsupported length of flange, in ft, on the right, and read on the diagonal scale the allowable load on the given beam in per cent of the load which would be permitted if the compression flange were fixed. Flange widths of various Standard I-beam, Carnegie beam and Bethlehem beam sections are shown as graduations on the line at the left of the diagram.

For example, a 24 in Carnegie beam weighing 70 lb per ft will support a load of 75 kips uniformly distributed over a span length of 26 ft if the compression flange is laterally fixed (Table V). A straight line on Fig. 7A connecting the point 24 in C 70 on the flange width scale at the left with the point 26 ft span length on the scale at the right intersects the diagonal scale at approximately 67 per cent. The allowable load on the given beam if the compression flange is unsupported for its full length is

$$0.67 \times 75 \text{ kips} = 50 \text{ kips.}$$

If a straight line connecting the beam size and the unsupported length of flange does not intersect the diagonal line above the point marked 61.72, a beam with a wider flange must be used or the compression flange must be supported laterally at closer intervals.

Steel Beams in Buildings usually carry loads which are uniformly distributed and either directly applied to the beam itself or brought to the beam through joists or columns. If the load is applied directly to the beam, the curve of moments is a parabola having a center ordinate $\frac{1}{8}\,WL$, and the curve of shears is a straight line varying from $+W/2$ at the left end of the beam to $-W/2$ at the right end of the beam. These curves are shown in Fig. 8.

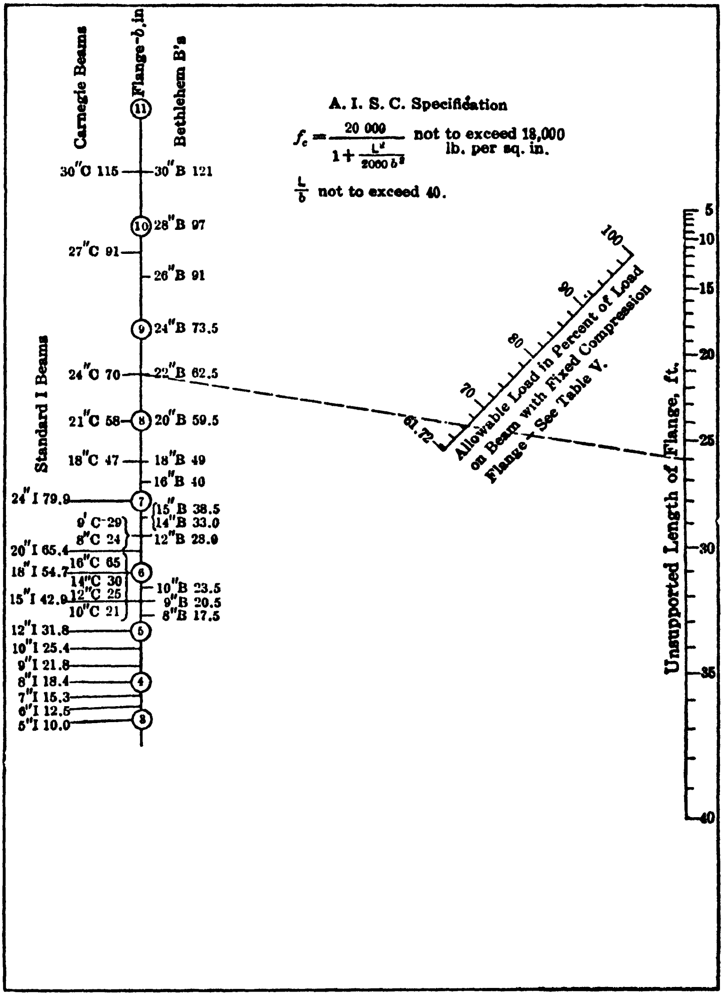


Fig. 7A

The curve of moments shows ordinates of the parabola at each tenth point of the span.

If the load is applied to the beam through joists or columns the curve of moments will be a polygon having vertices at the load-points, and the curve of shears will be a series of steps rising at the load-points. These curves are related to the curve for uniform load directly applied as shown in Fig. 9.

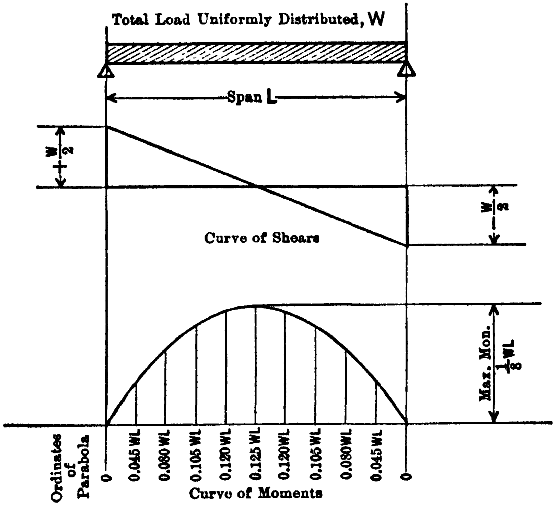


Fig. 8. Curves of Shear and Moment for a Beam Uniformly Loaded

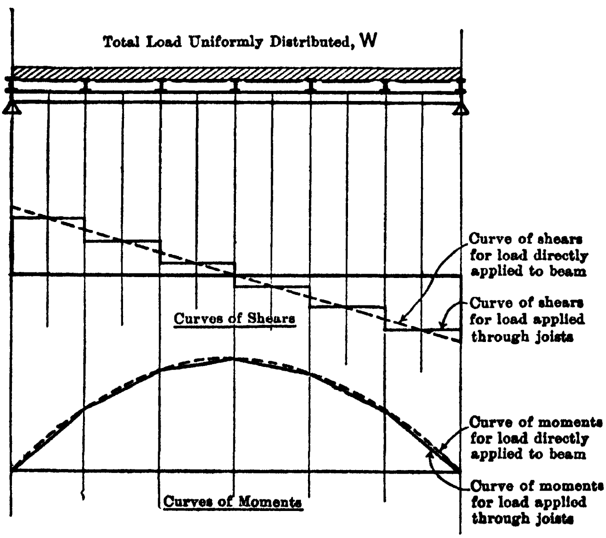


Fig. 9. Comparison of Shears and Moments for Loads Applied Directly and for Loads Applied through Joists

It will be seen that the curve of moments is a polygon inscribed in the parabola and that the curve of shears is a series of steps which intersects at the centers of the panels the curve of shears for loads directly applied. These relations hold whatever the spacing of joists or columns, provided the load applied to them is uniformly distributed.

This gives a convenient method of drawing curves of shears and moments for any case in which the load coming to the beam either directly or through joists or columns is uniformly distributed. The maximum shear is $\frac{W}{2}$ less the load over the support, and the maximum moment equals $\frac{1}{8}WL$ if there is a load at the center and in other cases is less than $\frac{1}{8}WL$. Hence it is always safe to design a girder which carries a load uniformly distributed as if the load were applied directly to the girder. For joists equally spaced the moment is exact if the number of panels is even. If the number of panels is odd, the moment may be reduced by the ratio $\frac{1}{(\text{number of panels})^2}$. Thus for three equal panels it may be reduced by $\frac{1}{9}$ of its value, for five equal panels by $\frac{1}{25}$ of its value.

Irregular Loading. Loadings which are not uniformly distributed either directly to the beam or to joists or columns carried by the beam sometimes occur in building-construction In such cases the maximum shears and bending moments should either be computed or found graphically by the methods explained in Chapter IX.

Design of Steel Beams. Steel beams may be designed in one of four ways as follows:

(1) Compute the maximum bending moment and find in the tables of beams in Chapter X a beam which will carry this bending moment.

(2) Divide the bending moment by the allowable fiber-stress and so determine the section-modulus required and choose from the tables of Chapter X or from the Beam Summary, Table V, at the end of the present chapter, a beam having this required section-modulus.

(3) If the load is uniform, compute the required value of what is usually called the COEFFICIENT OF STRENGTH, which is the uniformly distributed load which a beam 1 ft long would theoretically carry. If we multiply the uniformly distributed load on the beam by the span and choose a beam having this coefficient of strength, we have satisfied the flexural requirement. This coefficient of strength is given in many handbooks but has been omitted here.

(4) If the load is uniformly distributed, choose from the tables at the end of this chapter a beam section corresponding to the given load and span. If the span is very short or the moment is due to a load very near the support, see that the value of the shearing-strength of the beam, given in the tables of Chapter X, is adequate.

7. Tables for Steel Beams

A Summary of Beam Strengths is given in Table V, taken from the handbook Steel Construction of the American Institute of Steel Construction. It gives for each different kind of beam the section-modulus and the allowable total load in thousands of pounds uniformly distributed for various span lengths for beams simply supported. The beams are listed in the order of their section-moduli. The following abbreviations have been used in describ-

ing the beams: I for standard I beam; B for Bethlehem beam; G for Bethlehem girder beam; BJ for Bethlehem steel joist; C for Carnegie beam; J for Jones & Laughlin Junior beam; Cm for Carnegie Mill sections; P for special Phoenix I beams. These tables contain all information needed for the selection of steel beams for the problems ordinarily occurring in building construction. The following limitations should be noted:

(*a*) They may be used SAFELY in all problems of building-construction involving either uniform or trapezoidal loads, provided the beams are simply supported, except in the case in which the joists framing into the beam are loaded by loads which lie beyond the beam span or in the very rare case where the load is trapezoidal and the beam size is determined by shear.

(*b*) The load is to be taken as the total load between the ends of the beam.

(*c*) Where joists are equally spaced and the number of panels is odd, the load may be reduced by the ratio $\frac{1}{(\text{number of panels})^2}$.

(*d*) On very short spans where shear controls the section, they are over-conservative unless the load is directly applied. This is rarely important.

(*e*) For cantilever beams the beam size for the negative moment at the support may be selected after multiplying both load and loaded length by two.

(*f*) For triangular loads, such as those from a brick wall as shown in Fig. 1, increase the load by $\frac{1}{3}$. This is slightly conservative if the beam size is determined by shear.

Table IV gives, for angles used as beams, data similar to that given in Table V for beam sections.

Table II gives data for computing the allowable load where the compression flange is unsupported.

Tables of Beam Strengths. The size of beam may ordinarily be selected directly from the BEAM SUMMARY just described. In cases of very irregular loading or of high shear, reference should be made to the tables of BEAM PROPERTIES in Chapter X. These give the allowable bending moment at 18 000 lb per sq in, and the allowable shears at an allowable shearing stress of 12 000 lb per sq in.

The tables in Chapter X give also the allowable reaction without stiffeners for a length of bearing-plate of $3\frac{1}{2}$ in. To obtain the allowable reaction for other lengths of bearing-plates, multiply the value of R given in the table by

$$\frac{\frac{d}{4} + x}{\frac{d}{4} + 3\frac{1}{2}} = \frac{d + 4x}{d + 14}$$

in which d is the depth of beam in inches;
x is the length of bearing plate in inches.

For concentrated loads along the beam, multiply the value of R given in the table by

$$\frac{\frac{d}{2} + x}{\frac{d}{4} + 3\frac{1}{2}} = \frac{2d + 4x}{d + 14}$$

in which d and x have the meaning just explained.

End Connections for Beams (Table III), vary somewhat in different shops.

The standards recommended for different beams by the American Institute of Steel Construction are shown in Table III at the end of this chapter. In the tables of Properties of Beams at the end of Chapter X, are shown values (w) of the strengths of these connections computed as follows:

Power-driven rivets in bearing on the web of the beam at 30 000 lb per sq in.

Power-driven rivets in single shear in the outstanding legs at 13 500 lb per sq in.

Table III at the end of this chapter gives the required thickness of the web of the connecting beam to develop shear value for power-driven rivets in the outstanding leg of the connection angles.

End Connections May Fail either in the rivets which connect the angles to the beam with which they are shipped or in the rivets connecting the outstanding legs. The first-named group is almost invariably power-driven and is in double shear and in bearing on the web. The connection of the outstanding legs may be by field bolts and is in single shear and in bearing on the web of the beam to which connection is being made. For other stresses than given for the values of the Institute of Steel Construction, the strength of the connections should be investigated.

8. Examples

Example 1. It is desired to reinforce a 20-in Standard I Beam weighing 65.4 lb per ft to carry a load of 5 500 lb per ft uniformly distributed on a 20-ft span. What thickness of cover-plates 8 in wide should be added to keep the stress to 18 000 lb per sq in on the net section?

Bending moment $\frac{1}{8} WL = \frac{1}{8} \times 5\,500 \times 20 \times 20 \times 12 = 3\,300\,000$ in-lb.

$$\text{Section-modulus required} = \frac{3\,300\,000}{18\,000} = 183.33$$

$$S \text{ for 20-in I (from Table IV, Chapter X)} = 116.95$$

$$\text{Section-modulus required in plates} = 66.38$$

Dividing by the beam depth, net area to be added by plate = 3.37 sq in.

The plate and flange will each have two rivet-holes out.

Assuming the same thickness in plate and flange, the net width of plate added is $8 - (4 \times \frac{7}{8}) = 4\frac{1}{2}$ in.

$$\text{Thickness required} = \frac{3.37}{4.5} = 0.75 \text{ in}$$

Use $\frac{3}{4}$-in plate

This is an approximate solution. More accurately we compute

$$I \text{ of beam} = 1\,169.5$$

$$I \text{ of plates} = 8 \times \tfrac{3}{4}\,\frac{(20.75)^2}{2} = 1\,300$$

$$2\,470$$

$$\text{Total depth} = 21.5 \text{ in} \quad S = \frac{2\,470}{\frac{21.5}{2}} = 230$$

Rivet-holes in one flange =
(See Equation (7) Chapter X)

Value of u in Table IV, Chapter X ¾ in
Thickness of plate ¾ in

$$2 \times \tfrac{7}{8} \times 1\tfrac{1}{2} \text{ in} = 2.62 \text{ sq in}$$

Section-modulus to be deducted for holes

$$2.62 \times \left(\frac{20}{21.5}\right)^2 21.5 = 49$$

The total section-modulus = 181

The section assumed is close enough.

Example 2. An 18-in I 54.7 lb Standard I beam has an 8 × ¾-in plate riveted to the top flange. What concentrated load will it safely carry at the center on a 8-ft span at a fiber-stress of 16 000 lb per sq in in compression or 18 000 lb per sq in in tension?

I of beam = 795.5
Area of beam = 15.94 sq in
Area of plate = 8 × ¾ = 6.00 sq in

Distance between centroids = 9.375 in

$$\text{Added } I \text{ for plate} = \frac{6.00 \times 15.94}{6.00 + 15.94}(9.375)^2 = 382$$

Total I (See Equation (3), Chapter X) = 1 177

Distance of centroid above center of beam,

$$\frac{6.00}{6.00 + 15.94} 9.375 = 2.57 \text{ in}$$

From centroid to extreme compression fiber = 9.75 − 2.57 = 7.18

$$S = \frac{1\,177}{7.18} = 164$$

$$M = 164 \times 16\,000 = 2\,630\,000 \text{ in lb}$$

From centroid to extreme tension fiber = 9.00 + 2 57 = 11.57

$$S = \frac{1\,177}{11\,57} = 101.8$$

$$M = 101\,8 \times 18\,000 = 1\,830\,000 \text{ in lb}$$

Bending moment for 1 kip center load

$$= \tfrac{1}{4} \times 1\,000 \times 8 \times 12 = 24\,000 \text{ in lb}$$

$$\text{Allowable center load } \frac{1\,830\,000}{24\,000} = 76.5 \text{ kips}$$

Since the beam can carry 99.4 kips shear, shear will not govern.

Example 3. Choose a rolled-steel beam to carry 1 250 lb per ft uniformly distributed on an 18-ft span. The total load carried is 18 × 1.25 = 22.5 K. Table V gives in the column headed 18 the following beams for a load of 23 K; a 9-in Carnegie Beam 35.0 lb, 10-in Carnegie Beam 36.0 lb; and for a load 24 K, 12-in Carnegie Beam 28.0 lb, 12-in Bethlehem Beam 28.0 lb, 9-in

Bethlehem Girder 36.0 lb, 12 Standard I Beam 31.8 lb. Obviously some of these beams are more economical than others. Which beam will be used depends on what is available in stock. The weight of the 28-lb beam itself (without fireproofing) is $18 \times \frac{28}{1\,000} = 0.5$ K and the total load is 23 K for which the section chosen is adequate. If the beam is fireproofed, add also the weight of fireproofing.

Example 4. In reconstructing a building, Girder A is to take column loads as shown in Fig. 10. The columns are 20 ft on centers in the direction

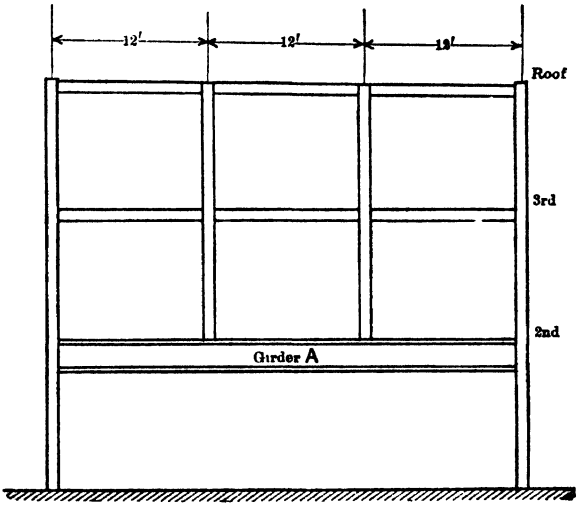


Fig. 10. Girder with Column Loads. Example 4

normal to that shown. The floor loads, live plus dead, are as follows: roof, 60 lb per sq ft; 3rd floor, 90 lb per sq ft; 2nd floor, 90 lb per sq ft. Total load * above 2nd floor = $(60 + 90)\, 36 \times 20 = 108$ K. Because there is no column at the center (see (*c*) under A Summary of Beam Strengths) this may be reduced by $\frac{1}{3^2}$

then	$\frac{8}{9} \times 108 =$	96 K
Load from 2nd floor	$= 90 \times 36 \times 20 =$	64 K
		160 K

Table V shows that this requires a 33-in Carnegie or a 33-in Bethlehem beam at 152 lb per ft and that no beam of lighter weight will carry the load.

* For certain types of occupancy, some reduction might be permitted in the live load to be carried by this girder on account of the large area required to be loaded. See Minimum Live Load Requirements Recommended by the Uniform Building Code Committee of the Bureau of Standards, or local building codes.

Example 5. What is the approximate deflection of the beam in Example 4? From Equation (6),

$$\text{Deflection} = \frac{2}{10}\,\frac{18\,000 \times (36 \times 12)^2}{29\,000\,000 \times 33} = 0.7 \text{ in}$$

Example 6. A beam carries a load of 30 K from a column at a point 8 ft from one end of a 22-ft span.

If we design by computing the maximum moment,

$$\text{Bending moment} = 30 \times \frac{14}{22} \times 8 \times 12 = 1\,830 \text{ in K}$$

$$\text{Required section-modulus} = \frac{1\,830\,000}{18\,000} = 101.7$$

Table V gives quite a range of choice with section-moduli greater than the value of 101.94 given for an 18-in Standard I, 70.0 lb. Assume that it is desired to use a Standard I Beam of stock size. The first stock size having a section-modulus greater than that required is 20-in I 65.4 lb, which is lighter than the stock 24-in I. The 20-in I has a section-modulus of 116.95 which gives ample margin for the weight of the beam itself.

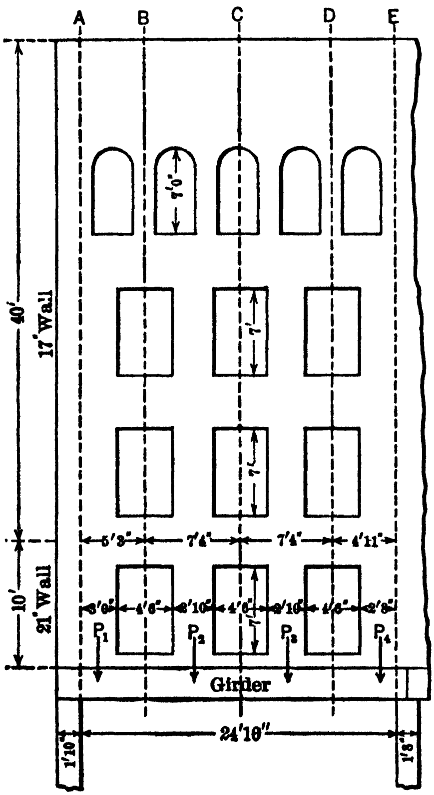


Fig. 11. Girder Supporting Brick Wall. Example 9

Example 7. Is the beam in Example 6 strong enough in shear? Can a standard connection be used?

A glance at the values of the maximum web-shear (V) and maximum load on one end connection (W) in Table IV, Chapter X, shows them to be greater than the total load on this beam. Consequently, the web strength and end connection are satisfactory.

Example 8. A beam cantilevered 8 ft carries a load of 400 lb per ft on the cantilever end. Select a stock size of Standard I.

Total load $8 \times 400 = 3.2$ K.

Double both load and span (see (e) under A Summary of Beam Strengths), and we find in Table V that a 6-in I 17.25 lb will carry 7 K on a 16-ft span.

Example 9. The wall shown in Fig. 11 is to be carried by two rolled beams with cover-plates at a fiber-stress of 16 000 lb per sq in on the net section. It is desired to use two 20-in I 65.4 lb which are available. Because of the thickness of wall, 20-in covers will be used. What should be the thickness of cover-plates?

Assume that a 21-in wall weighs 200 lb per sq ft and that a 17-in wall weighs 165 lb per sq ft.

Total load without openings

$$\begin{array}{l} 200 \times 10 = 2\,000 \\ 165 \times 40 = 6\,600 \\ \hline \quad 8\,600 \times 24.9 \end{array} \qquad 214\,000 \text{ lb}$$

Openings

$$\begin{array}{l} 3\,(4.5 \times 7) \times 200 = 18\,900 \\ 6\,(4.5 \times 7) \times 165 = 30\,000 \\ 5\,(2.75 \times 7) \times 165 = 15\,900 \end{array} \qquad 64\,800 \text{ lb}$$

	149 200 lb
Weight of girder—say	3 000
Total load =	152 200 lb

Span center to center of bearings—26.7 ft.

Load is approximately concentrated at points P_1, P_2, P_3, P_4.

If these were three equal spans, we could use an equivalent reduced load of $\frac{8}{9} \times 152\,200 = 135\,000$ lb. (See (*e*) under A Summary of Beam Strengths.) Exact computation of the bending moment is scarcely possible here, nor is it important. Take the equivalent uniform load as 140 K.

$$S = \frac{M}{s} = \frac{\frac{1}{8} \times 140\,000 \times 26.7 \times 12}{16\,000} = \qquad 350$$

$$S \text{ for 2–20-in I 65.4 lb} = 2 \times 116.95 = \qquad 234$$

$$\text{Plates must furnish} \qquad 116$$

Net width added by plate (2 holes in each plate, 2 holes in each flange and assuming plate and flanges same thickness)

$$20 - (4 \times \tfrac{7}{8}) = 16.5$$

$$\text{Thickness required} = \frac{116}{20 \times 16.5} = 0.35 \text{ in}$$

The thickness of plate should not be less than ½ in, which is evidently adequate.

9. Tables

The tables following have been taken from the printing dated January, 1930, of the handbook Steel Construction, published by the American Institute of Steel Construction. The use of these tables has been discussed elsewhere in this chapter. The attention of the reader is called especially to the Remarks on Stock Sizes following.

Remarks on Stock Sizes. Not all sizes of beams are carried in stock.

For beams of STANDARD SECTION, the MINIMUM WEIGHT only is usually carried in stock. Of the Bethlehem and Carnegie sections, no particular

section can be designated as the usual stock size, though certain sizes are so listed in the handbook of the American Institute of Steel Construction.

The question of sizes available for immediate delivery is, however, one of actual warehouse stock, which will depend on the size of warehouse and condition of the market. Local conditions must, therefore, be considered.

Table II.* Beams and Girders with Laterally Unsupported Flanges †

Allowable Stress, in Pounds per Sq In, in Extreme Fiber for Various Ratios of l/b

$\frac{l}{b}$	Fiber-stress	$\frac{l}{b}$	Fiber-stress	$\frac{l}{b}$	Fiber-stress	$\frac{l}{b}$	Fiber-stress	$\frac{l}{b}$	Fiber-stress
15.0	18 000	20 0	16 667	25 0	15 238	30.0	13 793	35.0	12 403
16 0	17 730	21.0	16 387	26 0	14 948	31.0	13 509	36 0	12 136
17.0	17 475	22.0	16 103	27 0	14 657	32.0	13 228	37.0	11 873
18 0	17 212	23.0	15 817	28.0	14 368	33.0	12 949	38.0	11 614
19 0	16 942	24.0	15 528	29.0	14 080	34 0	12 674	39 0	11 360
.	.....		.	..	. . .	. . .	.	40 0	11 111

l = the unsupported length, in inches, of the compression flange. b = the width of the flange in inches. * From Steel Construction, published by the American Institute of Steel Construction. † See Equation (11).

Table II* (Continued). Beams and Girders with Laterally Unsupported Flanges †

Percentage of Fixed Beam Loads for Various Ratios of l/b

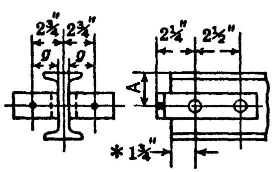
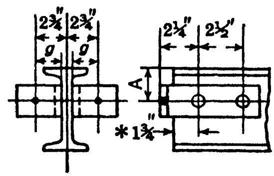
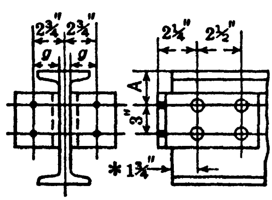
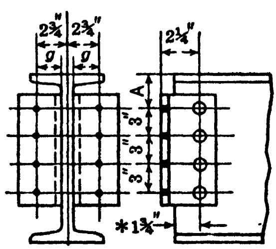
$\frac{l}{b}$	%	$\frac{l}{b}$	%	$\frac{l}{b}$	%	$\frac{l}{b}$	%	$\frac{l}{b}$	%
15.0	100.00	20.0	92.58	25.0	84.65	30.0	76.63	35.0	68.90
15.1	99.74	20.1	92.43	25.1	84.49	30.1	76.47	35.1	68.76
15.2	99.61	20.2	92.28	25.2	84.33	30.2	76.31	35.2	68.61
15.3	99.47	20.3	92.13	25.3	84.17	30.3	76.15	35.3	68.46
15.4	99.33	20.4	91.97	25.4	84.01	30.4	75.99	35.4	68.31
15.5	99.19	20.5	91.82	25.5	83.85	30.5	75.84	35.5	68.16
15.6	99.06	20.6	91.66	25.6	83.69	30.6	75.68	35.6	68.01
15.7	98.93	20.7	91.50	25.7	83.53	30.7	75.52	35.7	67.86
15.8	98.78	20.8	91.35	25.8	83.37	30.8	75.36	35.8	67.72
15.9	98.64	20.9	91.19	25.9	83.20	30.9	75.21	35.9	67.57
16.0	98.50	21.0	91.04	26.0	83.04	31.0	75.05	36.0	67.42
16.1	98.36	21.1	90.88	26.1	82.88	31.1	74.89	36.1	67.27
16.2	98.22	21.2	90.72	26.2	82.72	31.2	74.74	36.2	67.13
16.3	98.08	21.3	90.56	26.3	82.55	31.3	74.58	36.3	66.98
16.4	97.94	21.4	90.41	26.4	82.39	31.4	74.42	36.4	66.83
16.5	97.80	21.5	90.25	26.5	82.23	31.5	74.26	36.5	66.69
16.6	97.65	21.6	90.09	26.6	82.07	31.6	74.11	36.6	66.54
16.7	97.51	21.7	89.93	26.7	81.91	31.7	73.95	36.7	66.39
16.8	97.37	21.8	89.78	26.8	81.75	31.8	73.79	36.8	66.25
16.9	97.23	21.9	89.62	26.9	81.59	31.9	73.64	36.9	66.11
17.0	97.08	22.0	89.46	27.0	81.43	32.0	73.48	37.0	65.96
17.1	96.94	22.1	89.30	27.1	81.27	32.1	73.33	37.1	65.82
17.2	96.80	22.2	89.14	27.2	81.11	32.2	73.17	37.2	65.67
17.3	96.65	22.3	88.98	27.3	80.94	32.3	73.02	37.3	65.53
17.4	96.50	22.4	88.82	27.4	80.78	32.4	72.86	37.4	65.38
17.5	96.35	22.5	88.66	27.5	80.62	32.5	72.71	37.5	65.24
17.6	96.21	22.6	88.51	27.6	80.46	32.6	72.55	37.6	65.09
17.7	96.06	22.7	88.35	27.7	80.30	32.7	72.40	37.7	64.95
17.8	95.92	22.8	88.19	27.8	80.14	32.8	72.25	37.8	64.81
17.9	95.77	22.9	88.03	27.9	79.98	32.9	72.09	37.9	64.67
18.0	95.62	23.0	87.87	28.0	79.82	33.0	71.94	38.0	64.52
18.1	95.47	23.1	87.71	28.1	79.66	33.1	71.78	38.1	64.38
18.2	95.33	23.2	87.55	28.2	79.50	33.2	71.63	38.2	64.24
18.3	95.17	23.3	87.39	28.3	79.34	33.3	71.48	38.3	64.09
18.4	95.02	23.4	87.23	28.4	79.18	33.4	71.32	38.4	63.96
18.5	94.87	23.5	87.07	28.5	79.02	33.5	71.17	38.5	63.81
18.6	94.72	23.6	86.91	28.6	78.86	33.6	71.02	38.6	63.67
18.7	94.57	23.7	86.75	28.7	78.69	33.7	70.87	38.7	63.53
18.8	94.43	23.8	86.59	28.8	78.53	33.8	70.72	38.8	63.39
18.9	94.27	23.9	86.43	28.9	78.38	33.9	70.56	38.9	63.25
19.0	94.12	24.0	86.27	29.0	78.22	34.0	70.41	39.0	63.11
19.1	93.97	24.1	86.11	29.1	78.06	34.1	70.26	39.1	62.97
19.2	93.82	24.2	85.94	29.2	77.90	34.2	70.11	39.2	62.83
19.3	93.66	24.3	85.78	29.3	77.74	34.3	69.96	39.3	62.69
19.4	93.51	24.4	85.62	29.4	77.58	34.4	69.80	39.4	62.55
19.5	93.36	24.5	85.46	29.5	77.42	34.5	69.65	39.5	62.42
19.6	93.20	24.6	85.30	29.6	77.26	34.6	69.50	39.6	62.27
19.7	93.05	24.7	85.14	29.7	77.10	34.7	69.35	39.7	62.14
19.8	92.90	24.8	84.98	29.8	76.94	34.8	69.21	39.8	62.00
19.9	92.75	24.9	84.81	29.9	76.78	34.9	69.05	40.0	61.72

l = the unsupported length, in inches, of the compression flange. b = the width of the flange in inches. * From Steel Construction, published by American Institute of Steel Construction. † See Equation (11).

Table III. Beam Connections *

Minimum web in connecting beam

To develop single shear in rivets throughout standing angle legs 0.33 in.
To develop double shear in rivets throughout standing angle legs 0 53 in.

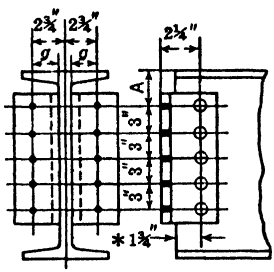
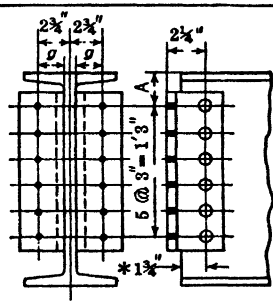
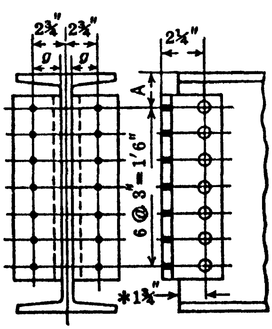
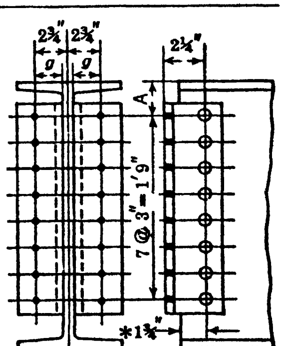
Size and length	**6 × 4 × 3/8 in 0 ft 2 in long**	**6 × 4 × 3/8 in 0 ft 2½ in long**
Weight inc. web rivets	5 lb	6 lb
Connection details		
	3 in, 4 in [and I	5 in, 6 in, 7 in [and I
Size and length	**6 × 4 × 3/8 in 0 ft 5½ in long**	**4 × 3½ × 3/8 in 0 ft 11½ in long**
Weight inc. web rivets	13 lb	19 lb
Connection details		
	8 in, 9 in, 10 in, 12 in [, I and B 8 in C 30 lb and under 9 in C 35 lb " " 10 in C 30 lb " " 12 in C 50 lb " "	15 in [15 in, 18 in I 14 in, 15 in, 16 in B 14 in C 42 lb and under 16 in C 50 lb " "

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table III (Continued). Beam Connections *

Minimum web in connecting beam

To develop single shear in rivets through outstanding angle legs 0.33 in.
To develop double shear in rivets through outstanding angle legs 0.53 in.

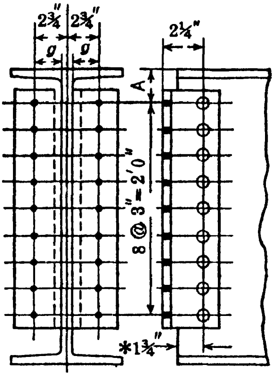
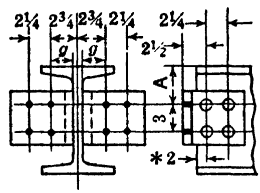
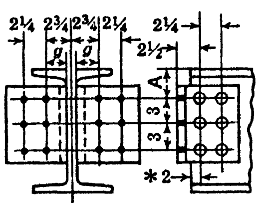
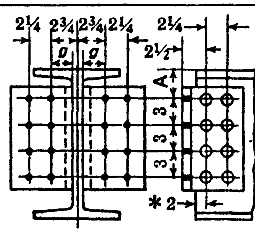
Size and length	4 × 3½ × ⅜ in 1 ft 2½ in long	4 × 3½ × ⅜ in 1 ft 5½ in long
Weight inc. web rivets	25 lb	30 lb
Connection details		
	20 in I, 18 in, 20 in B 18 in C 58 lb and under	24 in I, 22 in, 24 in B 21 in C 98 lb and under 24 in C 94 lb " "
Size and length	4 × 3½ × ⅜ in 1 ft 8½ in long	4 × 3½ × ⅜ in 1 ft 11½ in long
Weight inc. web rivets	35 lb	40 lb
Connection details		
	26 in B	28 in B 27 in C 137 lb and under

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table III (Continued). Beam Connections *

Minimum web in connecting beam

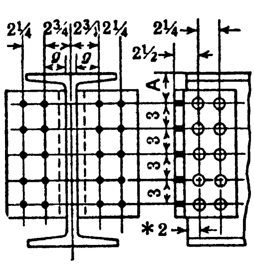
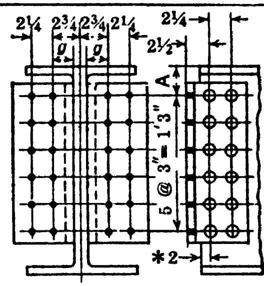
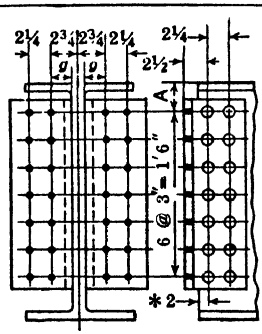
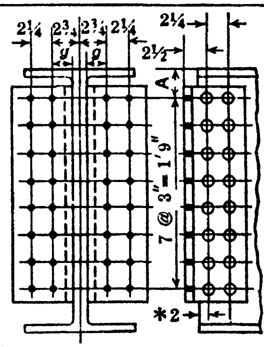
To develop single shear in rivets through outstanding angle legs 0.33 in
To develop double shear in rivets through outstanding angle legs 0 53 in

<table>
<tr><td>Size and length</td><td>4 × 3½ × ⅜ in
2 ft 2½ in long</td><td>6 × 6 × ⅜ in
0 ft 5½ in long</td></tr>
<tr><td>Weight inc. web rivets</td><td>45 lb</td><td>16 lb</td></tr>
<tr><td>Connection details</td><td></td><td></td></tr>
<tr><td></td><td>30 in, 33 in, 36 in B
30 in C 165 lb and under
33 in C 167 lb " "
36 in C 192 lb " "</td><td>8 in, 9 in, 10 in G
8 in C over 30 lb
9 in C " 35 lb
10 in C " 30 lb</td></tr>
<tr><td>Size and length</td><td>6 × 6 × ⅜ in
0 ft 8½ in long</td><td>6 × 6 × ⅜ in
0 ft 11½ in long</td></tr>
<tr><td>Weight inc. web rivets</td><td>24 lb</td><td>33 lb</td></tr>
<tr><td>Connection details</td><td></td><td></td></tr>
<tr><td></td><td>12 in G
14 in C over 42 lb
12 in C " 50 lb</td><td>15 in G 111 lb and under
16 in G
16 in C over 50 lb</td></tr>
</table>

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table III (Continued). Beam Connections *

Minimum web in connecting beam
To develop single shear in rivets through outstanding angle legs 0.33 in.
To develop double shear in rivets through outstanding angle legs 0.53 in.

Size and length	6 × 6 × ⅜ in 1 ft 2½ in long	6 × 6 × ⅜ in 1 ft 5½ in long
Weight inc. web rivets	41 lb	49 lb
Connection details		
	18 in, 20 in G 18 in C over 58 lb 21 in C " 98 lb	22 in, 24 in G 24 in C over 94 lb
Size and length	6 in × 6 in × ⅜ in 1 ft 8½ in long	6 in × 6 in × ⅜ in 1 ft 11½ in long
Weight inc. web rivets	58 lb	66 lb
Connection details		
	26 in, 28 in G 27 in C over 137 lb	30 in, 33 in, 36 in G 30 in C over 165 lb 33 in C " 167 lb 36 in C " 192 lb

* Compiled from data in Steel Construction, published by American Institute of Steel Construction.

Table IV.* Allowable Uniform Load in Kips for Standard Angles Used as Beams

Position	Angle: Size	Angle: Thickness	Angle: Weight per foot	Length in span in feet. Laterally supported: 3	4	5	6	7	8	9	10	11	12	13	14
Equal Legs	2½ × 2½	¼	4 1	1.57	1.18	.94	.79	.67	.59	.52	47	.43	.	. . .	. . .
		5/16	5 0	1.93	1.45	1.16	.97	.83	.72	.64	.58	.53	. .	. .	. . .
	3 × 3	¼	4 9	2 30	1 72	1.38	1.15	.98	.86	.77	.69	.63	.57	....	. . .
		5/16	6.1	2.84	2.13	1.70	1.42	1.22	1.06	.95	.85	.77	.71	. . .	....
	3½ × 3½	5/16	7.2	3.90	2 93	2.34	1.95	1.67	1 46	1.30	1.17	1.06	.98	....	. . .
		⅜	8.5	4.61	3.46	2.77	2.31	1.98	1 73	1.54	1.38	1 26	1.15	....	....
	4 × 4	5/16	8.2	5.15	3.86	3.09	2.58	2.21	1.93	1.72	1.55	1.41	1 29	1.19	....
		⅜	9.8	6.10	4.57	3.66	3.05	2.61	2 29	2.03	1.83	1.66	1.52	1.41	....
	6 × 6	⅜	14.9	14.12	10.59	8.47	7.06	6.05	5.29	4.71	4.24	3.85	3.53	3 26	3.03
		½	19.6	18.44	13.83	11.06	9.22	7.90	6.91	6.15	5.53	5.03	4.62	4.25	3.95
Long Leg Up	3 × 2½	¼	4.5	2 24	1.68	1.34	1.12	.96	.84	.75	.67	.61	....	....	....
		5/16	5.6	2 74	2.06	1.65	1 37	1.18	1.03	.91	.82	.75	. .	. . .	....
	3½ × 2½	¼	4.9	3.01	2.26	1.81	1.51	1 29	1.13	1.00	.90	.82	.75	....	....
		5/16	6 1	3.71	2.78	2.23	1.86	1 59	1.39	1.24	1.11	1.01	.93	....	....
	4 × 3	5/16	7.2	4.93	3.70	2.96	2.47	2.11	1.85	1 64	1.48	1.35	1.23	1.14	....
		⅜	8.5	5.82	4.37	3.49	2.91	2.50	2.18	1 94	1.75	1.59	1.46	1.34	....
	4 × 3½	5/16	7.7	5 05	3.79	3.03	2.52	2.16	1.89	1.68	1.51	1.38	1.26	1.17	....
		⅜	9.1	5.99	4.49	3.60	3.00	2.57	2.25	2.00	1.80	1.63	1.50	1.38	....
	5 × 3	5/16	8.2	7.54	5.66	4.53	3.77	3 23	2.83	2.51	2.26	2 06	1.89	1.74	1.62
		⅜	9.8	8.93	6.70	5.36	4.47	3.83	3.35	2.98	2.68	2.44	2.23	2.06	1.91
	5 × 3½	5/16	8.7	7.74	5.81	4 65	3.87	3.32	2.90	2.58	2.32	2.11	1.94	1.79	1.66
		⅜	10.4	9.18	6.89	5.51	4 59	3.93	3.44	3.06	2.75	2.50	2.30	2.12	1.97
	6 × 4	⅜	12.3	13.27	9.95	7.96	6.64	5.69	4.98	4 42	3 98	3.62	3.32	3.06	2.84
		½	16.2	17 36	13 02	10 41	8 68	7.44	6 51	5 79	5 21	4.73	4 34	4.01	3 72

* From Steel Construction, published by the American Institute of Steel Construction.
For Dimensions and Technical Functions of Angles, see Table II, Chapter X.

Table IV* (Continued). Allowable Uniform Load in Kips for Standard Angles Used as Beams

Position	Angle			Length in span in feet. Laterally supported											
	Size	Thickness	Weight per foot	3	4	5	6	7	8	9	10	11	12	13	14
Short Leg Up	3 × 2½	¼	4.5	1.61	1.21	96	.80	.69	60	.54	.48	.44	....	..	...
		5/16	5.6	1.98	1 48	1.19	.99	.85	.74	.66	.59	.54	.	...	..
	3½ × 2½	¼	4.9	1.65	1 24	.99	.83	.71	.62	.55	.50	.45	41	..	...
		5/16	6.1	2.02	1.52	1.21	1.01	.87	.76	67	.61	.55	51	.	
	4 × 3	5/16	7.2	2 95	2.21	1 77	1.47	1.26	1 10	.98	.88	80	.74	.68	....
		⅜	8 5	3.46	2 59	2.08	1.73	1.48	1 30	1 15	1 04	.94	86	80	...
	4 × 3½	5/16	7.7	3.97	2 98	2 38	1 98	1.70	1.49	1 32	1.19	1 08	99	.92	....
		⅜	9.1	4.71	3 53	2.83	2.35	2.02	1.77	1.57	1.41	1.28	1 18	1 09	....
	5 × 3	5/16	8 2	3 02	2.26	1.81	1.51	1.29	1.13	1.01	91	.82	75	.70	....
		⅜	9.8	3.55	2.66	2.13	1.77	1.52	1.33	1.18	1.06	.97	.89	.82	..
	5 × 3½	⅜	10 4	4.09	3.07	2.45	2.05	1.75	1 53	1.36	1.23	1.12	1 02	.94	..
		7/16	12 0	4.82	3.61	2.89	2.41	2.06	1.81	1.61	1.45	1.31	1.20	1.11	...
	6 × 4	⅜	12 3	6.41	4 80	3.84	3 20	2.75	2 40	2 14	1.92	1.75	1.60	1 48	1 37
		½	16.2	8.33	6 25	5 00	4 17	3 57	3.12	2.78	2.50	2 27	2 08	1.92	1.79

* From Steel Construction, published by the American Institute of Steel Construction.
For Dimensions and Technical Functions of Angles, see Table II, Chapter X.

Table V.* **Summary of Beams for Various Uniform Loads and Spans**

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	22	24
3 in I	5.7	1.67	5.0	4.0	3.3	2.9	2.5	2.2	2.0	1.8	1.7	1.5	1.4	1.3					
3 in I	6.5	1.80	5.4	4.3	3.6	3.1	2.7	2.4	2.2	2.0	1.8	1.7	1.5	1.4					
3 in I	7.5	1.93	5.8	4.6	3.9	3.3	2.9	2.6	2.3	2.1	1.9	1.8	1.7	1.5					
6 in J	4.41	2.42	7.3	5.8	4.9	4.2	3.6	3.2	2.9	2.6	2.4	2.2	2.1	1.9					
4 in I	7.7	3.0	9.0	7.2	6.0	5.1	4.5	4.0	3.6	3.3	3.0	2.8	2.0	2.4					
4 in I	8.5	3.15	10	8	6	5	5	4	4	3	3	3	3	3					
4 in I	9.5	3.35	10	8	7	6	5	5	4	4	3	3	3	3					
7 in J	5.48	3.45	10	8	7	6	5	5	4	4	3	3	3	3					
4 in I	10.5	3.55	11	9	7	6	5	5	4	4	4	3	3	3					
8 in J	6.54	4.65	14	11	9	8	7	6	6	5	5	4	4	4	3	3	3		
5 in I	10.0	4.84	15	12	10	8	7	7	6	5	5	5	4	4	4	3	3		
5 in I	12.25	5.40	16	13	11	9	8	7	7	6	5	5	5	4	4	4	3		
9 in J	7.48	5.81	17	14	12	10	9	8	7	6	6	5	5	5	4	4	3		
5 in I	14.75	6.00	18	14	12	10	9	8	7	7	6	6	5	5	5	4	4		
6 in BJ	11.0	6.43	19	15	13	11	10	9	8	7	6	6	6	5	5	4	4	4	3
6 in I	12.5	7.27	22	17	15	13	11	10	9	8	7	7	6	6	6	5	4	4	4
10 in J	8.96	7.78	23	19	16	13	12	10	9	8	8	7	7	6	6	5	5	4	4
6 in I	14.75	7.93	24	19	16	14	12	11	10	9	8	7	7	6	6	5	5	4	4
6 in I	17.25	8.67	26	21	17	15	13	12	10	10	9	8	7	7	7	6	5	5	4
11 in J	10.23	9.63	29	23	19	17	14	13	12	11	10	9	8	8	7	6	6	5	5

* From Steel Construction, published by the American Institute of Steel Construction.

For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed 4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	22	24
7 in I	15 3	10.34	31	25	21	18	16	14	12	11	10	10	9	8	8	7	6	6	5
7 in I	17 5	11.11	33	27	22	19	17	15	13	12	11	10	10	9	8	7	7	6	6
8 in BJ	14.5	11.23	34	27	22	19	17	15	13	12	11	10	10	9	8	7	7	6	6
7 in I	20 0	11 97	36	29	24	21	18	16	14	13	12	11	10	10	9	8	7	7	6
12 in J	11.74	12.01	36	29	24	21	18	16	14	13	12	11	10	10	9	8	7	7	6
8 in I	18.4	14 22	43	34	29	24	21	19	17	16	14	13	12	11	11	9	9	8	7
8 in Cm	17.5	14 35	43	34	29	25	22	19	17	16	14	13	12	11	11	...	..	..	..
8 in B	17.5	14.43	43	35	29	25	22	19	17	16	14	13	12	12	11	10	9	..	..
8 in I	20.5	15 05	45	36	30	26	23	20	18	16	15	14	13	12	11	10	9	8	8
10 in BJ	16.5	15.48	46	37	31	27	23	21	19	17	15	14	13	12	12	10	9	8	8
8 in B	19.0	15.81	47	38	32	27	24	21	19	17	16	15	14	13	12	11	9	...	....
8 in Cm	21 0	15.85	48	38	32	27	24	21	19	17	16	15	14	13	12	11	10	...	...
8 in I	23 0	16 05	48	39	32	28	24	21	19	18	16	15	14	13	12	11	10	9	8
8 in I	25.5	17.02	51	41	34	29	26	23	20	19	17	16	15	14	13	11	10	9	9
9 in I	21 8	18 87	57	45	38	32	28	25	23	21	19	17	16	15	14	13	11	10	9
9 in B	20 5	19 22	54	46	39	33	29	26	23	2.	19	18	17	15	14	13	12	.	...
9 in Cm	20 5	19 24	51	46	39	33	29	26	23	21	19	18	17	15	14	13	12	.	..
12 in BJ	18 5	20.25	61	49	41	35	30	27	24	22	20	19	17	16	15	14	12	11	10
9 in I	25 0	20.31	61	49	41	35	31	27	24	22	20	19	17	16	15	14	12	11	10
9 in B	22.0	20.73	57	50	42	36	31	28	25	23	21	19	18	17	16	14	12	..	...

* From Steel Construction, published by the American Institute of Steel Construction.
For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	22	24
8 in C	24.0	21.08	46	46	42	36	32	28	25	23	21	19	18	17	16	14	13	12	11
9 in Cm	25.0	21.22	64	51	42	36	32	28	26	23	21	20	18	17	16	14	13	..	...
10 in C	21.0	21.73	55	52	44	37	33	29	26	24	22	20	19	17	16	15	13	12	11
10 in B	21.0	21.84	57	52	44	38	33	29	26	24	22	20	19	18	16	15	13	12	11
9 in I	30.0	22.53	68	54	45	39	34	30	27	25	23	21	19	18	17	15	14	12	11
8 in C	27.0	23.68	52	52	47	41	36	32	28	26	24	22	20	19	18	16	14	13	12
10 in I	25.4	24.42	73	59	49	42	37	33	29	26	24	23	21	20	18	16	15	13	12
10 in C	23.0	24.44	55	55	49	42	37	33	29	27	24	23	21	20	18	16	15	13	12
10 in B	23.5	24.64	60	59	49	42	37	33	30	27	25	23	21	20	19	16	15	13	12
9 in I	35.0	24.73	74	59	50	42	37	33	30	27	25	23	21	20	19	17	15	14	12
8 in G	29.5	25.56	54	54	51	44	38	34	31	28	26	24	22	20	19	17	15	14	13
8 in C	30.0	26 31	59	59	53	45	40	35	32	29	26	24	23	21	20	18	16	14	13
10 in I	30.0	26.70	80	64	53	46	40	36	32	29	27	25	23	21	20	18	16	15	13
10 in B	26.0	27.33	65	65	55	47	41	36	33	30	27	25	23	22	21	18	16	15	14
8 in C	31.0	27.52	56	56	55	47	41	37	33	30	28	25	24	22	21	18	17	15	14
10 in C	26.0	27.63	63	63	55	47	41	37	33	30	28	26	24	22	21	18	17	15	14
9 in C	29.0	28.00	60	60	56	48	42	37	34	31	28	26	24	22	21	19	17	15	14
8 in G	33.0	29.03	56	56	56	50	44	39	35	32	29	27	25	23	22	19	17	16	15
10 in I	35.0	29.16	88	70	58	50	44	39	35	32	29	27	25	23	22	19	18	16	15
10 in B	28.5	30.25	70	70	61	52	45	40	36	33	30	28	26	24	23	20	18	16	15
12 in C	25.0	30.69	69	69	61	53	46	41	37	34	31	28	26	25	23	21	18	17	15
9 in C	32 0	30.89	67	67	62	53	46	41	37	34	31	29	26	25	23	21	19	17	15
12 in B	25.0	31.16	68	68	62	53	47	42	37	34	31	29	27	25	23	21	19	17	16
10 in I	40.0	31.60	95	76	63	54	47	42	38	35	32	29	27	25	24	21	19	17	16
10 in C	30 0	31 91	73	73	64	55	48	43	38	35	32	30	27	26	24	21	19	17	16

* From Steel Construction, published by the American Institute of Steel Construction.

For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			4	6	8	10	11	12	13	14	15	16	18	20	22	24	26	28	30
8 in C	36.0	32.03	66	64	48	38	35	32	30	28	26	24	21	19	18	16	.	....	..
8 in G	36.5	32.66	60	60	49	39	36	33	30	28	26	24	22	20	18	16	...	..	...
10 in C	31.0	32.68	77	65	49	39	36	33	30	28	26	25	22	20	18	16	...	...	..
12 in P	27.5	33.27	73	67	50	40	36	33	31	29	27	25	22	20	18	..	...	..	...
9 in C	35.0	33.81	74	68	51	41	37	34	31	29	27	25	23	20	18	17	..	.	..
10 in C	36.0	35.12	105	70	53	42	38	35	32	30	28	26	23	21	19	18	.		...
12 in C	28.0	35.57	69	69	53	43	39	36	33	31	28	27	24	21	19	18	..		..
12 in B	28.0	35 60	71	71	53	43	39	36	33	31	28	27	24	21	19	18	..	.	.
9 in G	36.0	35.91	62	62	54	43	39	36	33	31	29	27	24	22	20	18	.	.	..
12 in I	31.8	35.97	101	72	54	43	39	36	33	31	29	27	24	22	20	18	...	.	..
8 in C	42.0	37 37	78	75	56	45	41	37	35	32	30	28	25	22	20	19	...	.	.
12 in I	35.0	37.83	114	76	57	45	41	38	35	32	30	28	25	23	21	19	..	.	..
9 in C	38.0	37.87	68	68	57	45	41	38	35	33	30	28	25	23	21	19	..	.	..
10 in C	42.0	38 08	114	76	57	46	42	38	35	33	31	29	25	23	21	19	.	.	
9 in G	38.5	38.20	67	67	57	46	42	38	35	33	31	29	25	23	21	19			....
12 in C	34.0	39.61	108	79	59	48	43	40	37	3'	32	30	26	24	22	20			..
12 in B	31.5	40.54	79	79	61	49	44	41	37	35	32	30	27	24	22	20	.		.
12 in C	32.0	40.65	80	80	61	49	44	41	38	35	33	31	27	24	22	20		...	
14 in C	30.0	41.82	91	84	63	50	46	42	39	36	34	31	28	25	23	21	19	18	17
14 in B	30.0	42.49	88	85	64	51	46	43	39	36	34	32	28	26	23	21	20	18	17

* From Steel Construction, published by the American Institute of Steel Construction.
For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			4	6	8	10	11	12	13	14	15	16	18	20	22	24	26	28	30
9 in G	43.5	42.85	77	77	64	51	47	43	40	37	34	32	29	26	23	21			
9 in C	43.0	42.86	78	78	64	51	47	43	40	37	34	32	29	26	23	21			
12 in I	40.8	44 82	132	90	67	54	49	45	41	38	36	34	30	27	24	22			
10 in G	41.5	45.57	74	74	68	55	50	46	42	39	36	34	30	27	25	23			
12 in C	36 0	45.78	90	90	69	55	50	46	42	39	37	34	31	28	25	23			
12 in B	36.0	46 01	88	88	69	55	50	46	42	39	37	35	31	28	25	23			
12 in I	45.0	47 35	142	95	71	57	52	47	44	41	38	36	32	28	26	24			
14 in C	33.0	47.63	91	91	72	57	52	48	44	41	38	36	32	29	26	24	22	20	19
14 in B	33 0	47.76	89	89	72	57	52	48	44	41	38	36	32	29	26	24	22	21	19
9 in C	48 0	47.85	88	88	72	57	52	48	44	41	38	36	32	29	26	24			
10 in G	44 5	49.34	77	77	74	59	54	49	46	42	39	37	33	30	27	25			
12 in B	40.0	50.20	95	95	75	60	55	50	46	43	40	38	33	30	27	25			
12 in I	50.0	50.27	151	101	75	60	55	50	46	43	40	38	34	30	27	25			
14 in C	38.0	51.07	126	102	77	61	56	51	47	44	41	38	34	31	28	26	24	22	20
14 in C	36.0	51.93	99	99	78	62	57	52	48	45	42	39	35	31	28	26	24	22	21
12 in C	40.0	52.28	84	84	78	63	57	52	48	45	42	39	35	31	29	26			
12 in I	55 0	53.22	160	106	80	64	58	53	49	46	43	40	36	32	29	27			
15 in P	36.0	54 01	104	104	81	65	59	54	50	46	43	41	36	32	30	27			
14 in B	37.5	54 35	103	103	82	65	59	54	50	47	44	41	36	33	30	27	25	23	22
10 in C	49.0	54 40	84	84	82	65	59	54	50	47	44	41	36	33	30	27			

* From Steel Construction, published by the American Institute of Steel Construction.

For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			4	6	8	10	11	12	13	14	15	16	18	20	22	24	26	28	30
16 in C	35.0	54 68	111	109	82	66	60	55	51	47	44	41	37	33	30	27	25	24	22
10 in G	50.0	54 84	87	87	82	66	60	55	51	47	44	41	37	33	30	27	.	.	
15 in B	36.0	55 12	100	100	83	66	60	55	51	47	44	41	37	33	30	28	25	24	22
16 in B	35 0	55.13	108	108	83	66	60	55	51	47	44	41	37	33	30	28	25	24	22
12 in B	44.0	55.30	105	105	83	66	60	55	51	47	44	41	37	33	30	28			...
14 in C	39.0	56.26	108	108	84	68	61	56	52	48	45	42	38	34	31	28	26	24	23
10 in C	54.0	56.86	119	114	85	68	62	57	52	49	45	43	38	34	31	28	.	...	...
12 in C	45 0	58 85	95	95	88	71	64	59	54	50	47	44	39	35	32	29		.	...
15 in I	42.9	58.91	148	118	88	71	64	59	54	50	47	44	39	35	32	29	27	25	24
10 in C	59 0	59.30	155	119	89	71	65	59	55	51	47	44	40	36	32	30		. .	.
16 in C	38.0	59.34	121	118	89	71	65	59	55	51	47	44	40	36	32	30	27	25	24
15 in B	38 5	59 68	104	104	90	72	65	60	55	51	48	45	40	36	33	30	28	26	24
15 in I	45.0	60 48	163	121	91	73	66	60	56	52	48	45	40	36	33	30	28	26	24
14 in C	42 0	60.60	117	117	91	73	66	61	56	52	49	46	40	36	33	30	28	26	24
12 in B	48 5	60 93	116	116	91	73	66	61	56	52	49	46	41	37	33	30	.		
14 in B	12.0	61 26	116	116	92	74	67	61	57	53	49	46	41	37	33	31	28	26	25
15 in B	40 0	61 65	110	110	92	74	67	62	57	53	49	46	41	37	34	31	28	26	25
10 in C	64 0	61.76	185	124	93	74	67	62	57	53	49	46	41	37	34	31			.
15 in I	50 0	64 15	192	128	96	77	70	64	59	55	51	48	43	38	35	32	30	27	26
15 in B	42 5	65 21	118	118	98	78	71	65	60	56	52	49	43	39	36	33	30	28	26
12 in C	50 0	65.35	106	106	98	78	71	65	60	56	52	49	44	39	36	33			
16 in C	40.0	65 58	111	111	98	79	72	66	61	56	52	49	44	39	36	33	30	28	26
16 in C	43 0	65.75	143	132	99	79	72	66	61	56	53	49	44	40	36	33	30	28	26
16 in B	40 0	65 78	113	113	99	79	72	66	61	56	53	49	44	40	36	33	30	28	26
12 in G	51 5	67.27	103	103	101	81	73	67	62	58	54	50	45	40	37	34			

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed															
			6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
15 in I	55 0	67 83	136	102	81	68	58	51	45	41	37	34	31	29	27	..		..	..
15 in B	46.0	68.91	129	103	83	69	59	52	46	41	38	35	32	30	28	..			.
14 in C	48.0	70.86	115	106	85	71	61	53	47	43	39	35	33	30	28		.		...
12 in C	55.0	71 40	108	107	86	71	61	54	48	43	39	36	33	...	..	..	.	.	..
12 in G	55.5	72.60	109	109	87	73	62	54	48	44	40	36	33	....	..		.	.	...
16 in B	45.0	73.76	128	111	89	74	63	55	49	44	40	37	34	32	30	..	..	..	..
16 in C	45.0	73.78	126	111	89	74	63	55	49	44	40	37	34	32	30	.	..	..	...
15 in B	50.5	75.71	137	114	91	76	65	57	50	45	41	38	35	32	30	.	.	...	...
12 in C	60.0	77.90	119	117	93	78	67	58	52	47	43	39	36	..	.	.	.	.	...
14 in C	53.0	78.25	128	117	94	78	67	59	52	47	43	39	36	34	31	.		..	...
12 in G	61.0	79.80	119	119	96	80	68	60	53	48	44	40	37	....	.	....	.	..	.
15 in I	60.8	81.20	162	122	97	81	70	61	54	49	44	41	37	35	32	.		...	...
16 in C	50.0	81.95	141	123	98	82	70	61	55	49	45	41	38	35	33			.	...
15 in B	54.5	82.27	148	123	99	82	71	62	55	49	45	41	38	35	33	.	..	.	..
16 in B	50.0	82.34	142	124	99	82	71	62	55	49	45	41	38	35	33	31	..	.	..
12 in G	66.0	83.65	128	126	100	84	72	63	56	50	46	42	39	...	...	..	.	.	...
15 in I	65.0	84.28	169	126	101	84	72	63	56	51	46	42	39	36	34	.		..	..
18 in B	47.0	85.18	140	128	102	85	73	64	57	51	47	43	39	37	34	32	30	28	..
18 in C	47.0	85.40	138	128	102	85	73	64	57	51	47	43	39	37	34	32	30	28	...
14 in C	58.0	85.58	141	128	103	86	73	64	57	51	47	43	40	37	34				

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes. Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
12 in C	66.0	85.76	132	129	103	86	74	64	57	51	47	43	40	.		.	.		
12 in C	65.0	86.88	115	115	104	87	75	65	58	52	47	43	40	..					.
15 in I	70.0	87 95	176	132	106	88	75	66	59	53	48	44	41	33	35				..
18 in I	54.7	88.39	177	133	106	88	76	66	59	53	48	44	41	38	35	33	31	29	...
18 in B	49.0	89.20	143	134	107	89	76	67	59	54	49	45	41	38	36	33	31	30	..
15 in B	59.5	89.44	163	134	107	89	77	67	60	54	49	45	41	38	36				.
12 in C	70.0	89.83	151	135	108	90	77	67	60	54	49	45	41	.		.			
18 in C	51.0	89.88	162	135	108	90	77	67	60	54	49	45	41	39	36	34	32	30	..
12 in G	70.5	90.60	135	135	109	91	78	68	60	54	49	45	42	...	..	..			..
15 in I	75.0	91.63	183	137	110	92	79	69	61	55	50	46	42	39	37	..			. .
18 in I	60.0	93 09	186	140	112	93	80	70	62	56	51	47	43	40	37	35	33	31	..
14 in C	61.0	93.12	129	129	112	93	80	70	62	56	51	47	43	40	37		..	..	.
12 in C	76.0	93.36	187	140	112	93	80	70	62	56	51	47	43	..	...	.	..	.	.
16 in B	56.5	93.49	143	140	112	94	80	70	62	56	51	47	43	40	37	35		.	.
18 in B	52 0	94.32	154	141	113	94	81	71	63	57	51	47	44	40	38	35	33	31	.
18 in C	52.0	94.41	154	142	113	94	81	71	63	57	52	47	44	40	38	35	33	31	..
16 in C	58 0	97.08	144	144	117	97	83	73	65	58	53	49	45	42	39	36	34		..
18 in I	65.0	97.53	195	146	117	98	84	73	65	59	53	49	45	42	39	37	34	32	...
12 in G	76.5	98 05	148	147	118	98	84	74	65	59	54	49	45	.		.	..	.	..
18 in B	54.5	98.91	161	148	119	99	85	74	66	59	54	49	46	42	40	37	35	33	.

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
16 in B	60.5	101.51	150	150	122	102	87	76	68	61	55	51	47	44	41	38			.
18 in I	70.0	101.94	204	153	122	102	87	76	68	61	56	51	47	44	41	38	36	34	.
14 in C	68.0	103.78	145	145	125	104	89	78	69	62	57	52	48	45	42	...	.	.	.
15 in G	64.5	104.13	139	139	125	104	89	78	69	63	57	52	48	45	42	39	37	.	..
18 in C	58.0	105.28	172	158	126	105	90	79	70	63	57	53	49	45	42	39	37	35	..
16 in C	63.0	105.49	157	157	127	106	90	79	70	63	58	53	49	45	42	40	37	..	..
15 in B	71.5	106.60	187	160	128	107	91	80	71	64	58	53	49	46	43	.	..	.	..
18 in B	59.0	108.20	162	162	130	108	93	81	72	65	59	54	50	46	43	41	38	36	.
20 in B	56.0	109.27	179	164	131	109	94	82	73	66	60	55	50	47	44	41	39	36	35
15 in G	69.0	109.58	150	150	132	110	94	82	73	66	60	55	51	47	44	41	39	37	..
16 in B	66.0	110.22	162	162	132	110	95	83	74	66	60	55	51	47	44	41	..	.	...
22 in B	54.5	113.34	188	170	136	113	97	85	76	68	62	57	52	49	45	43	40	38	36
16 in C	68.0	113.85	171	171	137	114	98	85	76	68	62	57	53	49	46	43	40	..	..
14 in C	75.0	114.52	162	162	137	115	98	86	76	69	63	57	53	49	46	.		.	.
20 in I	65.4	116.95	234	175	140	117	100	88	78	70	64	58	54	50	47	44	41	39	37
20 in B	59.5	118.15	180	177	142	118	101	89	79	71	64	59	55	51	47	44	42	39	37
18 in B	64.5	118.53	172	172	142	119	102	89	79	71	65	59	55	51	47	44	42	40	.
15 in G	74.0	119.03	158	158	143	119	102	89	79	71	65	60	55	51	48	45	42	40	..
16 in B	71.5	119.82	177	177	144	120	103	90	80	72	65	60	55	51	48	45	.	.	.
21 in C	58.0	120.30	181	180	144	120	103	90	80	72	66	60	56	52	48	45	42	40	38
20 in I	70.0	121.42	243	182	146	121	104	91	81	73	66	61	56	52	49	46	43	40	38
20 in B	62.0	123.61	188	185	148	124	106	93	82	74	67	62	57	53	49	46	44	41	39
18 in C	67.0	124.12	175	175	149	124	106	93	83	74	68	62	57	53	50	47	44	41	.
22 in B	58.0	124.67	189	187	150	125	107	94	83	75	68	62	58	53	50	47	44	42	39
20 in I	75.0	126.35	253	190	152	126	103	95	84	76	69	63	58	54	51	47	45	42	40
18 in I	75.6	126.87	242	190	152	127	109	95	85	76	69	63	59	54	51	48	45	42	

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
18 in B	69.0	128.19	181	154	128	110	96	85	77	70	64	59	55	51	48	45	43	.	.
20 in B	64.5	128 74	193	154	129	110	97	86	77	70	64	59	55	51	48	45	43	41	..
15 in G	80 5	129 29	174	155	129	111	97	86	78	71	65	60	55	52	48	46	43	.	..
16 in G	74 5	130.18	149	149	130	112	98	87	78	71	65	60	56	52	49	46	43	.	
18 in I	80 0	130.76	196	157	131	112	98	87	78	71	65	60	56	52	49	46	44	.	.
14 in C	85 0	131.61	146	146	132	113	99	88	79	72	66	61	56	53	...	...	...		
16 in C	76.0	132.66	161	159	133	114	100	88	80	72	66	61	57	53	50	47	.		.
21 in C	64 0	132.85	199	159	133	114	100	89	80	72	66	61	57	53	50	47	44	42	.
18 in C	72 0	133.42	190	160	133	114	100	89	80	73	67	62	57	53	50	47	44	..	.
18 in I	85 0	135.18	203	162	135	116	101	90	81	74	68	62	58	54	51	48	45	..	
22 in B	62 5	135.95	195	163	136	117	102	91	82	74	68	63	58	54	51	48	45	43	...
20 in B	68 5	137.42	196	165	137	118	103	92	83	75	69	63	59	55	52	49	46	43	
18 in B	74 0	137.88	191	165	138	118	103	92	83	75	69	64	59	55	52	49	46	..	.
18 in I	90 0	139.61	209	168	140	120	105	93	84	76	70	64	60	56	52	49	47	..	.
16 in G	81 0	141.41	161	161	141	121	106	94	85	77	71	65	61	57	53	50	47	.	.
18 in C	78 0	144 59	206	174	145	124	108	96	87	79	72	67	62	58	54	51	48		
16 in C	83 0	144 88	177	174	145	124	109	97	87	79	72	67	62	58	54	51	..		
21 in C	70.0	145 23	218	174	145	124	109	97	87	79	73	67	62	58	54	51	48	46	44
20 in I	81.4	146 63	220	176	147	126	110	98	88	80	73	68	63	59	55	52	49	46	44
14 in C	95.0	147.19	165	165	147	126	110	98	88	80	74	68	63	59	.	.		.	

* From Steel Construction, published by the American Institute of Steel Construction
For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V* (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
15 in G	94 0	147.32	192	177	147	126	111	98	88	80	74	68	63	59	55	52	.	.	.
22 in B	67.5	148.06	207	178	148	127	111	99	89	81	74	68	63	59	56	52	49	46	44
20 in B	73.0	148.50	206	178	149	127	111	99	89	81	74	69	64	59	56	52	50	47	45
20 in I	85.0	150.17	225	180	150	129	113	100	90	82	75	69	64	60	56	53	50	47	45
16 in G	87.0	152 70	174	174	153	131	115	102	92	83	76	71	65	61	57	54	51	...	...
15 in G	99 0	154.26	203	185	154	132	116	103	93	84	77	71	66	62	58	54	51	..	...
18 in G	80 0	154.44	180	180	154	132	116	103	93	84	77	71	66	62	58	55	51	49	46
20 in I	90.0	155.03	233	186	155	133	116	103	93	85	78	72	66	62	58	55	52	49	47
16 in C	90.0	157.08	193	189	157	135	118	105	94	86	79	73	67	63	59	55		..	...
21 in C	76.0	157.60	236	189	158	135	118	105	95	86	79	73	68	63	59	56	53	50	47
20 in B	78.0	157.84	222	189	158	135	118	105	95	86	79	73	68	63	59	56	53	50	47
20 in I	95 0	159.97	240	192	160	137	120	107	96	87	80	74	69	64	60	56	53	50	48
22 in B	73.0	161.50	222	194	162	138	121	108	97	88	81	75	69	65	61	57	54	51	48
14 in C	105.0	162.78	185	185	163	140	122	109	98	89	81	75	70	65	..	...	..	...	..
24 in C	70.0	162.82	230	195	163	140	122	109	98	89	81	75	70	65	61	57	54	51	49
24 in B	70 0	163.66	226	196	164	140	123	109	98	89	82	76	70	66	61	58	55	52	49
15 in G	105.0	164.17	216	197	164	141	123	109	99	90	82	76	70	66	62	58	55	..	...
20 in I	100.0	164.83	247	198	165	141	124	110	99	90	82	76	71	66	62	58	55	52	49
16 in G	94 0	165.10	189	189	165	142	124	110	99	90	83	76	71	66	62	58	55	....	..
18 in G	86.0	167.07	190	190	167	143	125	111	100	91	84	77	72	67	63	59	56	53	50

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
18 in C	86.0	168.23	185	185	168	144	126	112	101	92	84	78	72	67	63	59	56	..	..
22 in B	77.0	170.63	223	205	171	146	128	114	102	93	85	79	73	68	64	60	57	54	51
21 in C	80.0	170.90	221	205	171	146	128	114	103	93	85	79	73	68	64	60	57	54	51
24 in I	79.9	173.93	261	209	174	149	130	116	104	95	87	80	74	70	65	61	58	55	52
15 in G	111.0	174.51	232	209	175	150	131	116	105	95	87	81	75	70	65	62	58	..	..
24 in B	73.5	175.73	228	211	176	151	132	117	105	96	88	81	75	70	66	62	59	55	53
16 in C	100.0	178.35	178	178	178	153	134	119	107	97	89	82	76	71	67	63	..		..
18 in G	92.0	179.75	200	200	180	154	135	120	108	98	90	83	77	72	67	63	60	57	54
24 in I	85.0	180.00	270	216	180	154	135	120	108	98	90	83	77	72	68	63	60	57	54
18 in C	93.0	181.94	201	201	182	156	136	121	109	99	91	84	78	73	68	64	61	.	..
24 in C	76.0	182.03	233	218	182	156	137	121	109	99	91	84	78	73	68	64	61	57	55
21 in C	86.0	183.65	238	220	184	157	138	122	110	100	92	85	79	73	69	65	61	58	55
22 in B	83.0	184.23	240	221	184	158	138	123	111	101	92	85	79	74	69	65	61	58	55
24 in I	90.0	185.84	279	223	186	159	139	124	111	101	93	86	80	74	70	66	62	59	56
24 in B	79.5	188.19	249	226	188	161	141	125	113	103	94	87	81	75	71	66	63	59	56
16 in C	107.0	190.84	192	192	191	164	143	127	114	104	95	88	82	76	72	67	..	..	..
24 in I	95.0	191.80	288	230	192	164	144	128	115	105	96	89	82	77	72	68	64	61	58
15 in G	127.0	191.95	258	230	192	165	144	128	115	105	96	89	82	77	72	68	.	..	.
18 in G	99.0	193.72	212	212	194	166	145	129	116	106	97	89	83	77	73	68	65	61	58
18 in C	100.0	195.57	218	218	196	168	147	130	117	107	98	90	84	78	73	69	65		...
21 in C	92.0	196.46	256	236	196	168	147	131	118	107	98	91	84	79	74	69	65	62	59
24 in I	100.0	197.65	296	237	198	169	148	132	119	108	99	91	85	79	74	70	66	62	59
22 in B	89.0	197.88	257	237	198	170	148	132	119	108	99	91	85	79	74	70	66	63	59
24 in B	84.5	200.48	265	241	201	172	150	134	120	109	100	93	86	80	75	71	67	63	60
36 in B	81.0	201.71	272	242	202	173	151	135	121	110	101	93	87	81	76	71	67	64	61

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
15 in G	135.0	202.94	203	174	152	135	122	111	101	94	87	81	76	72	68	...	...	...	
24 in C	85.0	203.46	203	174	153	136	122	111	102	94	87	81	76	72	68	64	61	...	...
16 in C	115.0	205.17	205	176	154	137	123	112	103	95	88	82	77	72	...	...	...	...	...
20 in G	99.0	206.02	206	177	155	137	124	112	103	95	88	82	77	73	69	65	62	...	...
21 in C	98.0	209.24	209	179	157	139	126	114	105	97	90	84	78	74	70	66	63	...	...
15 in G	141.0	212.91	213	182	160	142	128	116	106	98	91	85	80	75	71	...	...	...	...
22 in B	96.5	213.37	213	183	160	142	128	116	107	99	91	85	80	75	71	67	64	...	...
26 in B	85.5	214 26	214	184	161	143	129	117	107	99	92	86	80	76	72	68	64	57	...
24 in B	90.5	214.61	215	184	161	143	129	117	107	99	92	86	81	76	72	68	64	...	...
27 in C	85.0	216.20	216	185	162	144	130	118	108	100	93	86	81	76	72	68	65	58	52
20 in G	107.0	221.98	222	190	167	148	133	121	111	102	95	89	83	78	74	70	67	...	...
28 in B	85.0	222.12	222	190	167	148	133	121	111	103	95	89	83	78	74	70	67	59	53
15 in G	147.0	222 94	223	191	167	149	134	122	111	103	96	89	84	79	74	...	...	...	...
24 in C	94 0	225.02	225	193	169	150	135	123	113	104	96	90	84	79	75	71	68	...	...
24 in B	95.5	225.24	225	193	169	150	135	123	113	104	97	90	84	79	75	71	68	...	...
26 in B	91.0	230.24	230	197	173	153	138	126	115	106	99	92	86	81	77	73	69	61	...
24 in I	105.9	234.30	234	201	176	156	141	128	117	108	100	94	88	83	78	74	70	...	
21 in C	104.0	235.74	234	202	177	157	141	129	118	109	101	94	88	83	79	74	71	...	...
20 in G	113.0	236.28	236	203	177	158	142	129	118	109	101	95	89	83	79	75	71	...	...
22 in G	101.0	236.78	236	203	178	158	142	129	118	109	101	95	89	84	79	75	71	...	...

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
24 in B	99.5	236.78	237	203	178	158	142	129	118	109	101	95	89	84	79	75	71		
27 in C	91.0	238.30	238	204	179	159	143	130	119	110	102	95	89	84	79	75	71	64	57
24 in I	110.0	239.10	239	205	179	159	143	130	120	110	102	96	90	84	80	76	72	..	..
24 in I	115.0	245.04	245	210	184	163	147	134	123	113	105	98	92	86	82	77	74		.
28 in B	91.0	246 85	247	212	185	165	148	135	123	114	106	99	93	87	82	78	74	66	59
26 in B	98.0	247.41	247	212	186	165	148	135	124	114	106	99	93	87	82	78	74	66	..
24 in B	104.5	248.84	249	213	187	166	149	136	124	115	107	100	93	88	83	79	75	..	.
24 in I	120.0	250.90	251	215	188	167	151	137	125	116	108	100	94	89	84	79	75		
20 in G	120.0	251.29	251	215	188	168	151	137	126	116	108	100	94	89	84	79	75	..	.
24 in C	100.0	251.71	252	216	189	168	151	137	126	116	108	101	94	89	84	79	76	67	60
21 in C	112.0	254.07	253	218	191	169	152	139	127	117	109	102	95	90	85	80	76	...	..
22 in G	108.0	254.94	253	219	191	170	153	139	128	118	109	102	96	90	85	81	77	...	...
20 in G	127.0	264.03	264	226	198	176	158	144	132	122	113	106	99	93	88	83	79	..	..
27 in C	101.0	264.72	265	227	199	176	159	144	132	122	113	106	99	93	88	84	79	71	64
28 in B	97.0	265.11	265	227	199	177	159	145	133	122	114	106	99	94	88	84	80	71	64
24 in G	107.0	266.87	267	229	200	178	160	146	133	123	114	107	100	94	89	84	80	71	64
21 in C	120.0	272.11	272	233	204	181	163	148	136	126	117	109	102	96	91	86	82	..	..
22 in G	116.0	273.16	271	234	205	182	164	149	137	126	117	109	102	96	91	86	82		..
24 in C	110.0	276.83	277	237	208	185	166	151	138	128	119	111	104	98	92	87	83	74	66
20 in G	135.0	280.57	281	240	210	187	168	153	140	130	120	112	105	99	94	89	84	..	...
24 in G	113.0	281.68	282	241	211	188	169	154	141	130	121	113	106	99	94	89	85	75	68
28 in B	104.0	284.73	285	244	214	190	171	155	142	131	122	114	107	100	95	90	85	76	68
21 in C	128.0	290.42	290	249	218	194	174	158	145	134	124	116	109	102	97	92	87	...	..
27 in C	112.0	293.17	293	251	220	195	176	160	147	135	126	117	110	103	98	93	88	78	70
22 in G	124 0	293.19	291	251	220	195	176	160	147	135	126	117	110	104	98	93	88	...	..

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For Dimensions and Technical Functions of Beam Sections, see Tables III to VIII, Chapter X.

Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
20 in G	142.0	296.06	296	254	222	197	178	162	148	137	127	118	111	105	99	94	89	..	...
24 in G	120.0	300.65	301	258	225	200	180	164	150	139	129	120	113	106	100	95	90	80	72
24 in C	120.0	301.91	302	259	226	201	181	165	151	139	129	121	113	107	101	95	91	81	72
28 in B	112.0	306.41	306	263	230	204	184	167	153	141	131	123	115	108	102	97	92	82	74
21 in C	136.0	308.37	308	264	231	206	185	168	154	142	132	123	116	109	103	97	93	...	..
20 in G	149.0	311.62	312	267	234	208	187	170	156	144	134	125	117	110	104	98	93	..	..
22 in G	132.0	312.88	309	268	235	209	188	171	157	144	134	125	117	110	104	99	94	..	..
30 in B	110.0	314.82	315	270	236	210	189	172	157	145	135	126	118	111	105	99	95	84	76
24 in G	128.0	320.66	321	275	240	214	192	175	160	148	137	128	120	113	107	101	96	86	77
27 in C	124.0	324.82	325	278	244	217	195	177	162	150	139	130	122	115	108	103	97	87	78
28 in B	119.0	327.51	328	281	246	218	197	179	164	151	140	131	123	116	109	103	98	87	79
24 in G	132.0	329.95	327	283	247	220	198	180	165	152	141	132	124	116	110	104	99	88	79
30 in B	115.0	330.85	331	284	248	221	198	180	165	153	142	132	124	117	110	104	99	88	79
30 in C	115.0	332.35	332	285	249	222	199	181	166	153	142	133	125	117	111	105	100	89	80
24 in C	130.0	333.62	318	286	250	222	200	182	167	154	143	133	125	118	111	105	100	89	80
24 in G	140.0	350.11	346	300	263	233	210	191	175	162	150	140	131	124	117	111	105	93	84
30 in B	121.0	351.31	351	301	263	234	211	192	176	162	151	141	132	124	117	111	105	94	85
27 in C	137.0	358.73	352	302	264	235	211	192	176	163	151	141	132	124	117	111	106	96	85
24 in C	140.0	359.23	344	308	269	239	216	196	180	166	154	144	135	127	120	113	108	96	86
30 in C	126.0	363.82	364	312	273	243	218	198	182	168	156	145	136	128	121	115	109	97	87
28 in B	133.0	364.04	364	312	273	243	219	199	182	168	156	146	137	129	121	115	109	97	87
26 in G	138.0	370.39	359	317	278	247	222	202	185	171	159	148	139	131	124	117	111	99	89
24 in G	148.0	371.31	370	318	278	248	223	203	186	171	159	149	139	131	124	117	111	99	89
30 in B	129.0	373.35	373	320	280	249	224	204	187	172	160	149	140	132	124	118	112	100	90

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Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
24 in C	150 0	384.94	370	330	289	257	231	210	192	178	165	154	144	136	128	122	115	103	92
26 in G	144.0	385.12	379	330	289	257	231	210	193	178	165	154	144	136	128	122	116	103	92
33 in C	125.0	394.81	395	338	296	263	237	215	197	182	169	158	148	139	132	125	118	105	95
33 in B	125.0	395.15	395	339	296	263	237	216	198	182	169	158	148	140	132	125	119	105	95
30 in B	137.0	398.46	398	342	299	266	239	217	199	184	171	159	149	141	133	126	120	106	96
30 in C	138.0	398.73	399	342	299	266	239	217	199	184	171	159	150	141	133	126	120	106	96
26 in G	151.0	406.91	393	349	305	271	244	222	203	188	174	163	153	144	136	128	122	109	98
27 in C	145.0	408.05	376	350	306	272	245	223	204	188	175	163	153	144	136	129	122	109	98
24 in C	160.0	410.78	397	352	308	274	246	224	205	190	176	164	154	145	137	130	123	110	99
28 in G	145.0	416.02	390	357	312	277	250	227	208	192	178	166	156	147	139	131	125	111	100
33 in B	135.0	422.27	422	362	317	282	253	230	211	195	181	169	158	149	141	133	127	113	101
26 in G	160.0	431.04	420	369	323	287	259	235	215	199	185	172	162	152	144	136	129	115	103
30 in B	149.0	434.07	434	372	326	289	261	237	217	200	186	174	163	153	145	137	130	116	104
33 in C	138.0	435.59	436	373	327	290	261	238	218	201	187	174	163	154	145	138	131	116	105
30 in C	151.0	436.42	436	374	327	291	262	238	218	201	187	175	164	154	145	138	131	116	105
28 in G	156 0	446.10	425	382	335	297	268	243	223	206	191	178	167	157	149	141	134	119	107
33 in B	143 0	449.41	449	385	337	300	279	245	225	207	193	180	169	159	150	142	135	120	108
27 in C	160.0	450.13	417	386	338	300	270	246	225	208	193	180	169	159	150	142	135	120	108
28 in G	165.0	473.19	454	406	355	315	284	258	237	218	203	189	177	167	158	149	142	126	114
30 in B	163.0	474.43	474	407	356	316	285	259	237	219	203	190	178	167	158	150	142	127	114

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Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9.

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
30 in C	165.0	476 66	477	409	358	318	286	260	238	220	204	191	179	168	159	151	143	127	114
33 in C	152.0	479 79	480	411	360	320	288	262	240	221	206	192	180	169	160	152	144	128	115
33 in B	152.0	480.40	480	412	360	320	288	262	240	222	206	192	180	170	160	152	144	128	115
27 in C	175.0	492.47	459	422	369	328	295	269	246	227	211	197	185	174	164	156	148	131	118
28 in G	175.0	499.72	479	428	375	333	300	273	250	231	214	200	187	176	167	158	150	133	120
36 in C	147.0	502 24	502	430	377	335	301	274	251	232	215	201	188	177	167	159	151	134	121
36 in B	147.0	503.42	502	432	378	336	302	275	252	232	216	201	189	178	168	159	151	134	121
33 in C	167.0	527.06	527	452	395	351	316	288	264	243	226	211	198	186	176	166	158	141	127
33 in B	165.0	527.49	527	452	396	352	317	288	264	244	226	211	198	186	176	167	158	141	127
30 in G	173.0	528 46	473	453	396	352	317	288	264	244	226	211	198	187	176	167	159	141	127
36 in B	155.0	530 41	530	455	398	354	318	289	265	245	227	212	199	187	177	167	159	141	127
27 in C	190 0	534.60	501	458	401	356	321	292	267	247	229	214	200	189	178	169	160	143	128
28 in G	186.0	537.19	496	460	402	358	322	293	269	248	230	215	201	190	179	170	161	143	128
36 in C	160.0	549 05	549	471	412	366	329	300	275	253	235	220	206	194	183	173	165	146	132
30 in C	180.0	553.43	482	474	415	369	332	302	277	255	237	221	208	195	184	175	166	148	133
30 in G	180.0	556 21	490	477	417	371	334	303	278	257	238	222	209	196	185	176	167	148	133
36 in B	164.0	561.07	559	481	421	374	337	306	281	259	240	224	210	198	187	177	168	150	135
30 in G	190.0	585 52	513	502	439	390	351	319	293	270	251	234	220	207	195	185	176	156	141
36 in B	173.0	594.98	592	510	446	397	357	325	297	275	255	238	223	210	198	188	178	159	143
36 in C	175.0	603.31	599	517	452	402	362	329	302	278	259	241	226	213	201	191	181	161	145
30 in C	200.0	614.99	540	527	461	410	369	335	307	284	264	246	231	217	205	194	184	164	148
30 in G	200.0	617.77	541	530	463	412	371	337	309	285	265	247	232	218	206	195	185	165	148
36 in B	190.0	659.86	636	566	495	440	396	360	330	305	283	264	247	233	220	208	198	176	158
36 in C	192.0	666 31	651	571	500	444	400	363	333	308	286	267	250	235	222	210	200	178	160
33 in C	200.0	669 67	570	570	502	446	402	365	335	309	287	268	251	236	223	211	201	179	161

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Table V * (Continued). Summary of Beams for Various Uniform Loads and Spans

Note. The attention of the reader is especially called to the Remarks on Stock Sizes, Article 9

Size and kind	Weight per foot	Section-modulus	Span in feet—beams laterally fixed																
			12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	45	50
33 in G	200.0	672.45	552	552	504	448	403	367	336	310	288	269	252	237	224	212	202	179	161
30 in C	220 0	676.26	598	580	507	451	406	369	338	312	290	271	254	239	225	214	203	180	162
30 in G	220.0	680.52	597	583	510	454	408	371	340	314	292	272	255	240	227	215	204	181	163
33 in G	210.0	707.33	582	582	531	472	424	386	354	326	303	283	265	250	236	223	212	189	170
30 in C	240.0	737.86	656	632	553	492	443	402	369	341	316	295	277	260	246	233	221	197	177
33 in G	220.0	741 43	608	608	556	494	445	404	371	342	318	297	278	262	247	234	222	198	178
30 in G	240.0	742.95	649	636	557	495	446	405	372	343	318	297	279	262	248	235	223	198	178
33 in C	220.0	744 50	612	612	558	496	447	406	372	344	319	298	279	263	248	235	223	199	179
33 in G	230 0	778 05	634	634	584	519	467	424	389	359	333	311	292	275	259	246	233	207	187
33 in C	240.0	819.81	652	652	615	547	492	447	410	378	351	328	307	289	273	259	246	219	197
33 in G	245.0	831.04	670	670	623	554	499	453	416	384	356	332	312	293	277	262	249	221	199
36 in G	230 0	833.89	659	659	625	556	500	455	417	385	358	334	313	294	278	263	250	222	200
36 in C	230.0	834 05	664	664	626	556	500	455	417	385	358	334	313	294	278	263	250	222	200
36 in G	240 0	872 00	683	683	654	581	523	476	436	402	374	349	327	308	291	275	262	233	209
33 in G	260.0	884.21	706	706	663	589	531	482	442	408	379	354	332	312	295	279	265	236	212
33 in C	260.0	890.17	705	705	668	593	534	486	445	411	382	356	334	314	297	281	267	237	214
36 in C	250 0	910.48	717	717	683	607	546	497	455	420	390	364	341	321	304	288	273	243	219
36 in G	250.0	911.24	711	711	683	607	547	497	455	420	390	364	342	322	304	288	273	243	219
36 in G	260.0	949.50	735	735	712	633	570	518	475	438	407	380	356	335	316	300	285	253	228
36 in C	275.0	1 006.85	781	781	755	671	604	549	503	465	431	403	378	355	336	318	302	268	242
36 in G	280.0	1 030.74	780	780	773	687	618	562	515	476	442	412	387	364	344	326	309	275	247
36 in C	300.0	1 102 69	847	847	827	735	662	601	551	509	473	441	414	389	368	348	331	294	265
36 in G	300.0	1 103.59	833	833	828	736	662	602	552	509	473	441	414	389	368	348	331	294	265

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CHAPTER XVI

STRENGTH OF BUILT-UP, FLITCHED AND TRUSSED WOODEN GIRDERS

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1. Built-Up Wooden Girders

Built-Up Wooden Beams. Wooden beams or girders, built up of planks, spiked or bolted together side by side, are generally considered to be somewhat stronger than solid girders of the same dimensions, because the planks will be better seasoned and freer from check-cracks, knots and other defects if properly inspected. This type of girder is widely used in small buildings where planks cost less and are more easily obtained than heavy timbers. Where used, care should be taken that the spiking and bolting is adequate, that the planking does not under-run in thickness and that the depth is uniform. For beams or girders 10 in or less in depth, spikes will usually be sufficient to bind the planks together, but for deeper beams, bolts should be used in addition to the spikes to prevent the planks from separating and the outer planks from warping and curling away from the others.

Bolts. Two bolts should be placed at each end of the beam and every 3 ft of its length, with spikes between. These bolts should be $\frac{5}{8}$ in or $\frac{3}{4}$ in and should have full-size washers.

Length of Planks. When a beam is thus built up, each plank should extend the full length of the beam. In a CONTINUOUS BEAM, the planks should break joints over the supports. The planks of BUILT-UP BEAMS should always be set on edge, never flatwise.

Compound Wooden Beams. It is sometimes necessary to use a wooden beam for a longer span or greater load than is safe for the deepest SINGLE BEAM that can be obtained, or for a beam built up of planks. In such cases COMPOUND WOODEN BEAMS may be used.

Definition. By a COMPOUND WOODEN BEAM OR GIRDER is meant a beam built up by placing two or more single beams over one another, with the view of having them act as a SINGLE BEAM having the depth of the combined beams.

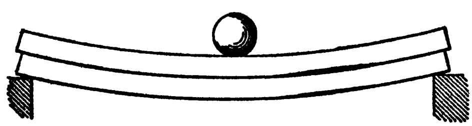


Fig. 1. Two Simple Wooden Beams, One over the Other, Loaded in Middle

Strength of Compound Beams. If two 10 by 10-in beams were placed one on top of the other, and the upper one loaded at the middle, the beams would act as two separate beams (Fig. 1) and their combined strength would be no greater than if the two beams were placed side by side. If, however, the two beams can be joined so that there is no slipping of their adjacent surfaces, the COMPOUND OR DEEPENED BEAM will have four times the strength of the SINGLE BEAM.

Tests of Compound Beams. Various methods have been used to join beams thus placed so as to prevent the two parts slipping on each other, and during the years 1896–7, Edgar Kidwell, of the Michigan College of Mines, made an extended series of tests of the efficiency of COMPOUND BEAMS of different patterns. From these tests many valuable data were obtained. A full description of the tests, accompanied by the conclusions of the author, and the rules and data for proportioning the bolts and keys, of KEYED BEAMS, was published in Trans. Am. Inst. of Mining Eng eers, vol. 27. An abstract was published in Eng. News, vol. 37, page 149, and vol. 39, pages 76 and 182.

Simple Form of Compound Beams. A form of compound beam which has been used in American building-construction is shown in Fig. 2, diagonal

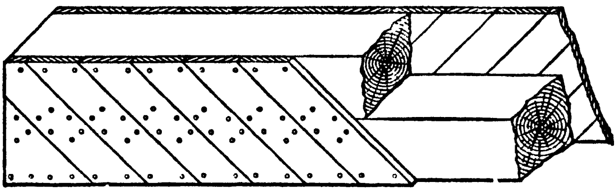


Fig. 2. Simple Form of Compound Wooden Beam

boards in opposite directions being nailed to each side of the two timbers to prevent their slipping on each other. Professor Kidwell made nine tests of this type of beam. In six of the beams the ratio of span to depth was as 12 to 1, and in three of the beams, as 24 to 1. The shorter beams gave an average efficiency, without much variation, of 71.4%, and the longer beams an efficiency of 80.7%. It was found that the beams failed by the splitting of the diagonal pieces or the drawing of the nails. When built with diagonal boards of 1¼ to 2-in thickness, running at 45°, the WORKING STRENGTH of such a beam may be taken at 65% of the strength of a solid beam of the same depth and of a breadth equal to the breadth of the timbers. The DEFLECTION of the beam, however, will be about double that of a solid beam of the same size, and on that account, this type of beam is not to be recommended for supporting floors with plastered ceilings or for carrying plastered partitions. Ten-penny nails should be used for 1¼-in sheathing and twenty-penny nails for 2-in sheathing. The nails should be concentrated near the ends of the diagonals and near the junction of the timbers. For uniform loading, the diagonals near the ends require more spikes than those near the center of the girder.

Keyed Beams. The most effective type of compound beam is that in which the two timbers are thoroughly keyed and bolted together as in Fig. 3. In tests made by Kidwell on this type of fastening it was found that with oak keys it was possible to obtain an efficiency for spruce beams of 95%, while the deflection was about 20% more than would be expected in a solid beam.

Keys. The keys may be of hard wood or cast iron and should be slightly wedge-shaped in order to insure a tight fit. The number, size and spacing of keys depend on the horizontal shear in the girder as illustrated hereafter.

Bolts. The bolts are designed to take only tension and not to resist any of the lateral force. The horizontal forces on the keys result in a tendency

toward rotation, with a consequent separation of the upper and lower beams. The bolts are designed to resist this separation.

Example 1. It is required to design a deepened beam for indoor use with a span of 20 ft to support a concentrated load at the center of 15 000 lb with

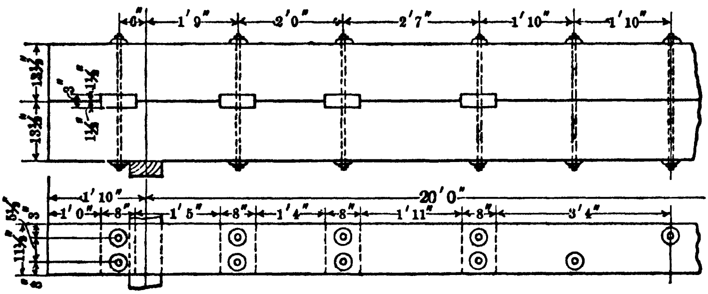


Fig. 3. Keyed Beam

also an uniform load of 2 000 lb per ft including the weight of the beam.

The following working unit stresses are specified:

Dense select Douglas fir	Lb per sq in
Extreme fiber-stress in bending	1 750
Longitudinal shear	210
Compression on the sides of fibers	380
Compression on the ends of fibers	1 710

The loading is shown in Fig. 4, the moment diagram being plotted directly below. The maximum moment is $(15\,000 \times 20/4) + (2\,000 \times 20 \times 20/8)$

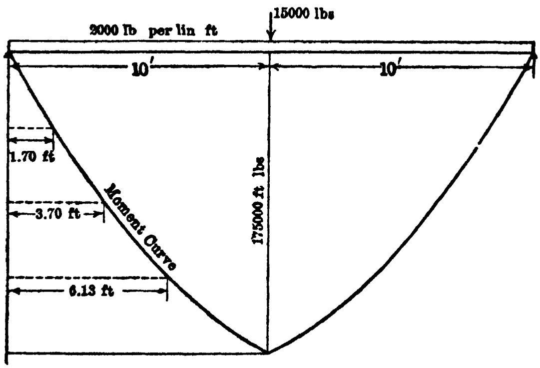


Fig. 4. Moment Curve for Simple Beam having a Uniformly Distributed and a Concentrated Load. Example 1

$= 175\,000$ ft-lb. The resisting moment of a beam of breadth b and depth d is $1\,750\, bd^2/6$. Equating the resisting moment to the bending moment, $1\,750\, bd^2/6 = 175\,000 \times 12$, or $bd^2 = 7\,200$. As the breadth of a compound

beam should be not less than two-fifths of the depth, let d be assumed as 27 in. The required value of b is then found to be 9.87 in. A value of 11.5 in will be used to allow for bolt-holes and also for the fact that the efficiency of the keyed beam is somewhat less than that of a solid beam of the same section area.

The total horizontal shear along the neutral axis from the end to any vertical section equals $3\ M/2d$, where M is the bending moment at the section. Hence, in this problem the total horizontal shear from the end of the beam to the center of the span is $(3 \times 175\,000 \times 12)/(2 \times 27) = 116\,670$ lb. This shear must be resisted by the brace blocks. Assuming four brace blocks are used on either side of the center, the force resisted by each block is $116\,700/4 = 29\,170$ lb.

Since the total horizontal shear from the end of the beam to any vertical section is proportional to the moment at that section, the positions of the blocks may be found by locating those ordinates on the moment diagram having values one-quarter, one-half and three-quarters of the maximum ordinate. These distances are found to be, respectively, 1.70, 3.70 and 6.13 ft from the supports. Since the brace block at the support may be moved farther to the left, if necessary, without changing the pressure which it must resist, the critical spacing is between blocks 2 and 3. The distance between these sections is $3.70 - 1.70 = 2$ ft, or 24 in. For an allowable unit stress of 210 lb per sq in, the required net length of shearing surface on the timbers is $29\,170/(210 \times 11.5) = 12.1$ in.

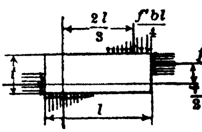


Fig. 5. Stresses on Block. Example 1

The required depth of brace block is $(2 \times 29\,170)/(11.5 \times 1\,710) = 2.97$, or 3 in. To prevent crushing of the block into the main timbers on a horizontal plane, the length of brace blocks must not be less than that found by the following method.

In Fig. 5 let f equal allowable unit stress on end of fibers; f', the allowable unit stress on side of fibers, b, breadth of block; t, thickness of block and l, length of block.

Equating the two couple moments

$$fbt^2/4 = f'bl^2/6, \quad \text{or} \quad l = 1.225\, t \sqrt{f/f'}$$

In this problem,

$$\sqrt{f/f'} = \sqrt{1\,710/380} = 2.12$$

hence

$$l = 1.225 \times 3 \times 2.12 = 7.8, \quad \text{or} \quad 8 \text{ in}$$

Therefore, if cast-iron brace blocks are used, the minimum allowable spacing is $12.1 + 8 = 20.1$ in. If timber brace blocks are used the material should have a high longitudinal shearing-resistance and the fibers should be parallel to those in the main timbers. Using white-oak blocks, and an allowable longitudinal shear of 315 lb per sq in, the required length of block based on shearing is $29\,170/(315 \times 11.5) = 8.05$ in. As this is only slightly over the value of 8 in previously determined based on moments, the value of 8 in will be used.

The bolts will be placed at the center of the keys. Using two bolts for each key, the stress in each bolt equals one-half the compressive force on the side of the brace block, which is $\left(\frac{1}{2} \times 29\,170 \times \frac{t}{2}\right) \div \frac{2l}{3} = 4\,100$ lb. Using an allowable tension of 18 000 lb per sq in. the required area at the root of

the thread is 4 100/18 000 = 0.228 sq in. This requires ¾-in bolts, whose area at the root of thread is 0.302 sq in. The required bearing area under washer is 4 100/380 = 10.8 sq in. Use 4-in ogee washers, which furnishes a bearing area of 11.97 sq in.

The detail of the left half of the deepened girder is shown in Fig. 3.

2. Flitched Beams or Flitch-Plate Girders

Flitched Beams (Fig. 6) consisting of two timbers with a steel plate between or three timbers with two plates between, were widely used previous to the

Fig. 6. Flitch-Plate Girder

development of steel structural shapes. At the present time the low price of structural shapes has made the combination beam practically obsolete.

The design of a flitch beam is based on equal deflection of its parts. The deflection of a beam varies as W/EI, in which W is the total load; E, the modulus of elasticity of the material; and I the moment of inertia of the cross-section of the material. Hence $W_w/E_wI_w = W_s/E_sI_s$, in which the subscripts w refers to wood and s to steel. For rectangular cross-sections of equal depth, this equation becomes

$$W_w/E_wb_w = W_s/E_sb_s$$

also
$$S_w/S_s = W_wb_s/W_sb_w$$

from which
$$S_w/S_s = E_w/E_s$$

where S represents the unit stress in the outer fibers.

For example, if Southern pine and steel are combined in a beam, with values of modulus of elasticity of 1 600 000 and 30 000 000 lb per sq in, respectively, the timber can be stressed only to 960 lb per sq in, when the steel is stressed to 18 000 lb per sq in. Hence the efficiency of the timber is only 55% based on a working stress of 1 750 lb per sq in. Furthermore, a steel plate has only about 50% of the strength of an I beam having the same depth and sectional area; hence it is not economical to use a flitched beam where I beams are available.

Formulas for Flitch-Plate Girders. The following formulas have been derived for the strength of FLITCH-PLATE GIRDERS on the basis of a stress in the steel of 18 000 lb per sq in and moduli of elasticity as given above. These values are applicable to Southern pine and Douglas fir. The thickness of the timber is usually about 16 times that of the steel.

Let d = depth of beam in inches;
b = total thickness of wood in inches;
l = clear span in feet;
t = thickness of steel plate in inches;
P = total load at center of span in pounds;
W = distributed load in pounds

Then for beams supported at both ends,

$$\text{Safe load at middle in pounds} = \frac{d^2}{l}(53.3b + 1\,000\,t) \qquad (1)$$

$$\text{Safe distributed load in pounds} = \frac{2\,d^2}{l}(53.3b + 1\,000\,t) \qquad (2)$$

For load at middle, $$d = \sqrt{\frac{Pl}{53.3\,b + 1\,000\,t}} \tag{3}$$

For distributed load, $$d = \sqrt{\frac{Wl}{106.6\,b + 2\,000\,t}} \tag{4}$$

The bolts should be ¾ in in diameter and spaced 2 ft on centers. Each end should have two bolts, as in Fig. 6.

Example 2. What is the safe load, uniformly distributed, for a girder composed of three 4-in by 14-in Douglas fir timbers (3⅝-in by 13½-in dressed dimensions) and two ⅜-in by 14-in flitch-plates, with a span of 25 ft?

Solution. By Formula (2),

$$\text{Safe load} = \frac{2 \times \overline{13.5}^2}{25}\,[(53.3 \times 10.87) + (1\,000 \times .75)] = 19\,370\ \text{lb}$$

3. Trussed Beams and Girders

Use of Trussed Beams and Girders. For spans of over 20 ft or for heavy loads, beams may be trussed as in Figs. 7, 8, 9, and 10, where conditions permit a sufficient depth to be occupied. Types 7 and 9 are used where ample headroom is below the beam supports, and types 8 and 10 where it is above. For spans over 25 ft, types 9 and 10 are preferable to types 7 and 8, especially where the head room is small.

Depth of Trussed Girder. It is desirable to give the girders as much depth as the conditions allow, since the deeper the girder, the smaller the stresses in the members.

Composition of Trussed Girders. The beam may be composed of a single timber or of two or more timbers placed side by side. These timbers should be in one continuous length for the whole span. In types 7 and 9 the diagonal pieces, T, are steel rods. Where a single rod is used with a single-timber beam, the rod passes through oblique holes in the ends of the beam. Where two rods are used they pass outside the beam, or if multiple-timber beams are used, they are separated and the rods pass between them.

Design. The required dimensions of the tie-rods, struts and beams, in any given case, must be determined by first finding the stresses developed in these pieces, and then the areas of cross-sections required to resist these stresses. The determination of the correct stresses requires the application of the theorem of least work, but the following approximate formulas may be used satisfactorily. All loads and stresses are in pounds, all distances in feet, and all bending moments in foot-pounds.

For a Single-Strut Truss (Fig. 7), the stresses in the members are as follows:

For a Uniformly Distributed Load W over the Girder (Fig. 7)

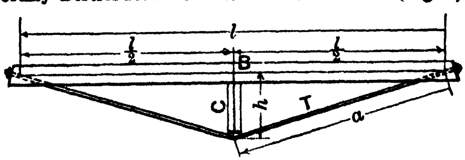


Fig. 7. Trussed Girder

Tension in T	$= 5\,Wa/16\,h$	(5)
Compression in C	$= 5\,W/8$	(6)
Compression in B	$= 5\,Wl/32\,h$	(7)
Bending Moment in B	$= Wl/32$	(8)

For a Concentrated Load P at Center (Fig. 7)

Tension in T	$= Pa/2\ h$	(9)
Compression in C	$= P$	(10)
Compression in B	$= Pl/4\ h$	(11)

For a Girder Trussed as in Fig. 8, under a Uniformly Distributed Load W over the Girder

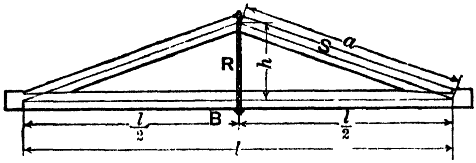


Fig. 8. Trussed Girder

Compression in S	$= 5\ Wa/16\ h$	(12)
Tension in R	$= 5\ W/8$	(13)
Tension in B	$= 5\ Wl/32\ h$	(14)
Bending Moment in B	$= Wl/32$	(15)

For a Concentrated Load P at Center (Fig. 8)

Compression in S	$= Pa/2\ h$	(16)
Tension in R	$= P$	(17)
Tension in B	$= Pl/4\ h$	(18)

For a Double-Strut Trussed Beam (Fig. 9) with a Uniformly Distributed Load W over the Girder

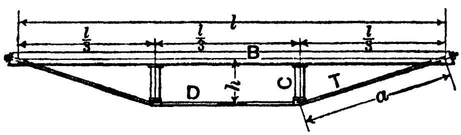


Fig. 9. Double-Strut Trussed Beam

Tension in T	$= 3\ Wa/8\ h$	(19)
Compression in C	$= 3\ W/8$	(20)
Compression in B or tension in D	$= Wl/8\ h$	(21)
Bending Moment in B	$= Wl/72$	(22)

For a Concentrated Load P over Each of the Struts C (Fig. 9)

Tension in T	$= Pa/h$	(23)
Compression in C	$= P$	(24)
Compression in B or tension in D	$= Pl/3\ h$	(25)

For a Girder Trussed as in Fig. 10 and under a Uniformly Distributed Load over the Girder

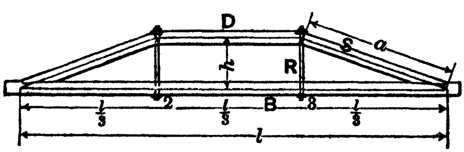


Fig. 10. Trussed Girder

Compression in S	$= 3\ Wa/8\ h$	(26)
Tension in R	$= 3\ W/8$	(27)
Tension in B or compression in D	$= Wl/8\ h$	(28)
Bending Moment in B	$= Wl/72$	(29)

For Concentrated Loads P Applied at Joints 2 and 3 (Fig. 10)

Compression in S	$= Pa/h$	(30)
Tension in R	$= P$	(31)
Tension in B and Compression in D	$= Pl/3\ h$	(32)

Trusses constructed as shown in Figs. 9 and 10 should be divided so that the rods R, or the struts C, shall divide the length of the girder into three equal or nearly equal parts. The distances l, h and a are measured along the axial lines of the members.

After determining the stresses in the members by these formulas, the areas of the cross-sections may be computed by the following rules:

$$\text{Area of cross-section of short strut } C = \frac{\text{Compression in strut}}{S_c} \tag{33}$$

in which S_c for cast iron may be taken at 14 000 lb per sq in and for wood as given in Table I, Chapter XX.

The size of the long struts S and D (Figs. 8 and 10) should be determined by means of Table XLVII, Chapter XX, and Tables V and VI, Chapter XIV.

The diameter of the tie-rods may be obtained from Table II, Chapter XI.

For the beam B, when the load is distributed, compute its necessary cross-section area as a STRUT (Figs. 7 and 9) or a TIE (Figs 8 and 10), and also the area of cross-section, as a BEAM, required to support its load, and use a beam with a section equal to the sum of the two sections thus obtained.

$$\left.\begin{array}{l}\text{Area of cross-section of } B \text{ to resist}\\ \text{tension or compression}\end{array}\right\} = \frac{\text{tension}}{S_t} \text{ or } \frac{\text{compression}}{S_c} \tag{34}$$

In the trusses shown in Figs. 7 and 8 with distributed loads,

$$\text{Breadth of } B \text{ (as a beam), } b = 9\ Wl/4\ S_b d^2 \tag{35}$$

In the trusses shown in Figs. 9 and 10, with distributed loads,

$$\text{Breadth of } B \text{ (as a beam), } b = Wl/S_b d^2 \tag{36}$$

W denotes the total distributed load in pounds on the girder; l, the span of the girder in feet; d, the depth of the girder in inches; b, the breadth in inches; and S_b, S_t and S_c the allowable bending, tension and compression stresses, respectively, in lb per sq in. (See Table I, Chapter XX.) The value of S_t may be taken equal to S_b.

When the loads are concentrated over the struts C or at the joints R, the TRANSVERSE STRESS in the beams B may be neglected, but in proportioning for the compressive or tensile stress, the required area of cross-section should be increased about 20% to provide for the transverse stress.

The allowable unit stress on a surface of timber oblique to the grain may be obtained from the Hankinson formula:

$$s = \frac{p}{\frac{p}{q}\cos^2\theta + \sin^2\theta}$$

s = allowable compression unit on the inclined plane;
p = allowable compression unit on the ends of the fibers;
q = allowable compression unit on the sides of the fibers (perpendicular to the grain);
θ = angle made by the plane with the direction of the fibers.

All dimensions are expressed in inches. The value of p may be taken one-third greater than that given for short columns.

Example 3. It is required to design a trussed girder of the form shown in Fig. 7, for a span of 22 ft. The girders are to be 10 ft on centers and are to carry a floor load of 100 lb per sq ft. We can allow the rod T to come 3 ft 4 in below the center line of the beam B. By computation a is found to be 11.54 ft.

Solution. The total load on the girder equals 100 lb, times the span, times the distance of the girders on centers: or, $100 \times 22 \times 10 = 22\,000$ lb.

From Formula (5), tension in $T = (5 \times 22\,000 \times 11.54) \div (16 \times 3.33) = 23\,820$ lb. Using an upset rod and allowing 16 000 lb per sq in for steel rods, we find from Table II, Chapter XI, that we must use a diameter of $1\frac{1}{2}$ in. If a unit stress of 18 000 lb is used, the diameter will be $1\frac{3}{8}$ in.

The beam B will be made of dense select Southern pine. From Formula (7) we find the compressive stress $= (5 \times 22\,000 \times 22) \div (32 \times 3.33) = 22\,710$ lb. With the beam braced laterally by the floor, assuming the allowable unit compression to be 1 285, the required area to resist compression $= 22\,710/1\,285 = 17.67$ sq in. For a depth of 11.5 in the required breadth $= 17.67/11.5 = 1.54$ in.

From Formula (35), the breadth required to resist the transverse loading, assuming a dressed depth of 11.5 in and using an allowable bending unit stress of 1 750, is $(9 \times 22\,000 \times 22)/(4 \times 1\,750 \times 11.5^2) = 4.70$ in. The total required breadth is therefore $1.54 + 4.70 = 6.24$ in, hence an 8 by 12-in beam, dressed to $7\frac{1}{2}$ by $11\frac{1}{2}$ in, will be used.

By Formula (6) the stress in $C = (5 \times 22\,000)/8 = 13\,750$ lb. The theoretical sectional area in square inches necessary to resist this stress $= 13\,750/14\,000 = 0.982$ sq in for cast iron, and $13\,750/1\,285 = 10.70$ sq in for timber. The strut bears on the sides of the fibers on the beam B, hence the required bearing area, based on a unit of 380, is $13\,750/380 = 36.2$ sq in. A 6 by 8-in strut, dressed to $5\frac{5}{8}$ by $7\frac{1}{2}$ in, furnishes an area of 42.2 sq in. A casting will be used at the bottom of the strut.

The angle between the plane of end washers and the direction of the fibers of the wood is about 73°, hence the allowable unit pressure is 1 330, based on an allowable unit bearing on the ends of the fibers of 1 713 and on the sides of the fibers of 380 lb per sq in. The bearing area required is $23\,820/1\,330 = 17.9$ sq in. A $5\frac{1}{4}$-in ogee washer, with a $1\frac{7}{8}$-in diameter hole, furnishes a net bearing area of 18.9 sq in.

Example 4. It is required to design a trussed girder of the form shown in Fig. 9 for a span of 33 ft. The girders are to be 12 ft on centers, and are to carry a floor load of 150 lb per sq ft. The girder consists of three strut beams, side by side, and two rods. We will allow the rods TDT to come 2 ft below the beam B, and we will assume that the depth of the beam B will be $11\frac{1}{2}$ in. By computation a is found to be 11.28 ft.

Solution. The total load on the girder equals 150 lb times the span, times the distance of the girders on centers, or, $150 \times 33 \times 12 = 59\,400$ lb. The size of TDT will be governed by the stress in T, as this is greater than that in D. From Formula (19), tension in $T = (3 \times 59\,400 \times 11.28)/(8 \times 2.48)$

= 101 300 lb. Using two upset rods, with an allowable unit fiber-stress of 18 000 lb per sq in, we find that a diameter of 2 in is required.

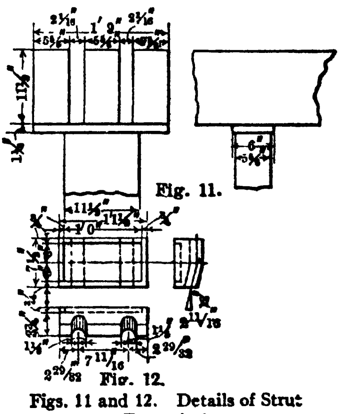
The beams B will be made of dense select Douglas fir. From Formula (21) we find the compressive stress = (59 400 × 33)/(8 × 2.48) = 98 800 lb. With the beam braced laterally, the allowable unit compression is 1 285, hence the required area to resist compression = 98 800/1 285 = 76.9 sq in. For a depth of 11.5 in the required breadth = 76.9/11.5 = 6 69 in.

From Formula (36), the breadth required to resist the transverse loading, on the basis of a dressed depth of 11.5 in, and using an allowable bending unit stress of 1 750 = 59 400 × 33)/(1 750 × 11.5^2) = 8.48 in. The total breadth = 6.69 + 8.48 = 15.17 in, hence the strut beams will each be 6 by 12 in, dressed to $5\frac{5}{8}$ by $11\frac{1}{2}$ in.

By Formula (20) the stress in C = 3 × 59 400/8 = 22 300 lb. The theoretical sectional area in square inches necessary to resist this stress = 22 300/14 000 = 1.59 for cast iron and 22 300/1 285 = 17.35 for dense select Douglas fir. The bearing on the beam B is on the sides of the fibers, hence the required bearing area, based on a unit of 345, is 22 300/345 = 64.6 sq in. In order to provide for proper end bearings a timber strut 6 by 12 in, dressed to $5\frac{5}{8}$ by $11\frac{1}{2}$ in, will be used. A 6- by 21-in steel plate will be used at the top. The required thickness based on an allowable unit stress of 18 000, is $t = \sqrt{6 \times 22\,300\,(21 - 11.5)/(8 \times 18\,000 \times 6)}$ = 1 21, say $1\frac{3}{8}$ in. (Fig. 11.)

The angle between the plane of the end washer and the direction of the fibers of the wood is about 77°, hence the allowable unit pressure is 1 430, based on an allowable unit bearing on the ends of the fibers of 1 713 and on the sides of the fibers of 345. The bearing area required is 101 300/1 430 = 70.8 sq in. Using a washer 21 in long, the required width is 70.8/[21 − (2 × 2.06)] = 4.19 in. A width of 8 in will be used.

The maximum bending moment in the washer is at the center of the rod and equals (101 300 × 3.84/3) − (101 300 × .97/4) = 105 200 in-lb. But net width of the plate is 8 − 2.62 = 5.38 in. Using a flexural stress of 24 000 lb per sq in, the required thickness, t, of the plate is 24 000 × 5.38 t^2/6 = 105 200 or t = $2\frac{1}{4}$ in.



Figs. 11 and 12. Details of Strut Example 3

A detail drawing of the washer under the struts C is shown in Fig. 12.

CHAPTER XVII

STIFFNESS AND DEFLECTION OF SIMPLE AND CANTILEVER BEAMS

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1. General Principles of the Deflection of Beams

Elastic Deflection of Beams. When a beam is subjected to the action of a system of loads the horizontal fibers on one side of the neutral axis are shortened (compression) and those on the opposite side are elongated (tension). The beam therefore bends between supports, and all points, excepting those over the supports, deflect below their original position in the unloaded beam.

The curve described by the neutral plane of the beam, called the ELASTIC CURVE, follows very definite laws for all loading conditions which do not produce flexural stress beyond the elastic limit of the material.

The vertical ordinates to the elastic curve, measured from the unloaded or horizontal position of the neutral axis, are the vertical deflections of the beam.

General Equation for Deflection. The DEFLECTION of any beam, under a given system of loading, varies directly with the magnitude of the applied load or loads, and the cube of the span; and inversely as the modulus of elasticity of the material of which the beam is composed and the moment of inertia of the section of the beam referred to the axis about which bending takes place.

The general equation for deflection, as derived in all standard works on Mechanics, may be expressed as follows:

$$\Delta = \frac{WL^3}{\alpha EI} \tag{1}$$

in which W = total load on beam span;
L = length of span in units in which deflection is desired;
α = a constant depending upon the conditions of loading;
E = modulus of elasticity in the same units as W and L;
I = the moment of inertia of the beam section in the same units as L.

Since for any condition of loading, W can be expressed in terms of the bending moment and span length, the deflection may also be expressed as follows:

$$\Delta = \frac{ML^2}{\beta EI} \tag{2}$$

in which M = the bending moment (at the point of span where deflection is to be determined) in the same units as W and L, Equation (1);
β = a constant depending upon the condition of loading and the point of span where deflection is to be determined;
L, E, and I are as given above.

From the theory of flexure of beams, $M = \frac{fI}{c}$; hence, Equation (2) may be expressed in terms of the maximum fiber-stress on the section of the beam at that point of span where the deflection is to be determined, as follows:

$$\Delta = \frac{fL^2}{c\beta E} \tag{3}$$

in which c = distance from neutral axis of beam section to extreme fiber on which the unit stress is f;

f = the maximum unit fiber-stress at c distance from the neutral axis in units determined by units of M, L and $\frac{I}{c}$;

β, E and L are as given above.

It is evident, from Equation (3), that for symmetrical beam sections, where $c = \frac{d}{2}$, the deflection varies directly as the maximum unit fiber-stress and the square of the length of span, and inversely as the depth of the beam section and the modulus of elasticity

The values of β, Equation (3), for maximum deflection in the more common cases of loading conditions, are given in the following table.

Table I. Values of β in Equation (3)

Type of beam and condition of loading	β
Simple Beam; Concentrated Load at Center	12 00
Simple Beam; Uniform Load over Entire Span	9 60
Simple Beam; Triangular Load, Apex at Center Line of Span	10.00
Simple Beam; Two Equal Concentrated Loads at ⅓ points of Span	9 39
Simple Beam; Any Irregular System of Loads (approx)	10 50
Cantilever Beam; Concentrated Load at Free End . . .	3.00
Cantilever Beam; Uniform Load over Span	4.00
Cantilever Beam; Concentrated Load at $L/2$ (Free End). .	4.80
Cantilever Beam; Any Irregular System of loads (approx.) at free end.	3 50

Stiffness of Beams. The ratio of the deflection to the span, Δ/L, is called the STIFFNESS FACTOR of the beam Where beams are to support plastered ceilings, experience has established a maximum value for the stiffness factor of 1/360.

In this connection it should be pointed out that the deflection due to dead loads, with the exception of the plaster work itself, has taken place before the plaster is applied, hence a correct statement of deflection limitation to avoid plaster cracks, is: The DEFLECTION DUE TO LIVE-LOAD must not exceed 1/360 of the span length in inches.

In first-class work it is desirable to limit the deflection not only to provide absolute safety against cracking of plaster work but also to provide a certain degree of stiffness for the sake of general appearance, and in such cases a stiffness factor of 1/360 for combined dead and live load is adopted.

In structures where stiffness limitations are to be considered, deflections must be calculated since in many cases the DEFLECTION rather than FLEXURAL STRENGTH of the beam may be the governing factor.

2. Deflection of Steel Beams

Equations (2) and (3) are applicable to beams of any material for which the value of the modulus of elasticity, E, is known and to any system of loading for which the value of β is known.

It is expedient, however, to reduce Equation (3) to more convenient terms for beams of any given material and for assumed loading conditions. For steel of the structural grade the value of the modulus of elasticity and the allowable working unit stress in flexure are·

$$E = 29\,000\,000 \text{ lb per sq in}$$
$$f = 18\,000 \text{ lb per sq in}$$

The deflection of steel beams under full allowable uniformly distributed load, as given by Equation (3), is:

$$\Delta_{max} = \frac{fL^2}{c\beta E} = \frac{2\,(18\,000)\,L^2 \times 144}{9.6\,(d)\,29\,000\,000}$$

or

$$\Delta_{max} = 0.01862\,\frac{L^2}{d} \tag{4}$$

in which L = span of beam in feet;
d = depth of beam in inches for sections symmetrical about the N.A. (neutral axis), or equal to two times the greater distance from the N.A. to the extreme fiber ($2 \times c$) for unsymmetrical sections;
Δ_{max} = the maximum vertical deflection in inches.

For any fiber-stress other than 18 000 lb per sq in, the coefficient of L^2/d in Equation (4) may be found by direct proportion, thus for a maximum flexural stress of 16 000 lb per sq in the maximum deflection is

$$\frac{16}{18}\,(0.01862)\,\frac{L^2}{d} = 0.01655\,\frac{L^2}{d}$$

and for $f = 10\,000$

$$\Delta_{max} = 0.0103\,\frac{L^2}{d}$$

It should be noted that deflections are direct functions of the flexural stress and that when smaller stresses than the maximum allowable unit values are developed under any system of loading the resulting deflections are correspondingly smaller than those obtaining for maximum allowable stress values.

From the values given above the following table of deflection coefficients is readily obtained by adopting values of L, the span in feet.

Table II. Coefficients of Deflection for Uniform Load *

Span, feet	f(lb per sq in)			Span, feet	f(lb per sq in)		
	10 000	16 000	18 000		10 000	16 000	18 000
3	0 093	0 149	0 168	27	7 533	12.066	13.574
4	0 165	0.265	0.298	28	8 102	12.977	14.599
5	0 259	0 414	0.466	29	8.691	13.920	15.660
6	0 372	0.596	0.670	30	9.301	14.897	16.759
7	0 502	0.811	0.912	31	9.931	15.906	17.894
8	0 661	1.059	1.191	32	10 582	16 949	19.067
9	0 837	1.341	1.508	33	11.254	18.025	20.278
10	1 034	1.655	1.862	34	11 947	19 134	21.526
11	1 250	2.003	2.253	35	12.693	20.276	22.870
12	1 488	2.383	2.681	36	13 393	21.451	24.132
13	1 746	2.797	3.146	37	14.161	22.659	25 516
14	2 025	3 244	3.649	38	14.923	23.901	26.888
15	2 325	3.724	4.189	39	15 717	25.175	28.319
16	2 646	4 237	4.767	40	16 535	26.483	29.793
17	2 987	4 783	5.381	41	17 372	27 823	31.301
18	3 348	5 363	6.033	42	18 229	29.197	32.846
19	3 730	5.975	6.722	43	19.108	30.604	34.429
20	4 134	6.621	7.449	44	20 007	32 044	36.049
21	4 557	7.299	8 211	45	20.926	33 517	37.706
22	5 001	8 011	9.012	46	21 868	35.023	39 401
23	5 467	8.756	9.850	47	22 828	36 562	41.132
24	5.952	9.534	10 725	48	23 811	38.135	42.902
25	6 459	10 345	11 638	49	24 813	39.741	44.709
26	6.986	11.189	12.588	50	25 941	41.379	46.541

* For Concentrated Center Load Coefficient = 0.80 of above values. For Triangular Loading, apex at C.L. Coefficient = 0.96 of above values. For Equal Concentrated Loads at ⅓ points Coefficient = 1.02 of above values. For Irregular Loading (approx.) Coefficient = 0.92 of above values.

To Find the Deflection in Inches of any Structural Steel Section symmetrical about the neutral axis, divide the coefficient in Table II corresponding to the span and fiber-stress by the depth of the section in inches. For unsymmetrical sections divide the coefficient corresponding to the span and the fiber-stress by twice the distance from the neutral axis to the extreme fiber. When the maximum unit stress is a value other than one of the values given in Table II, the deflection may be taken for one of the given values and reduced to the correct value by the proper ratio of actual stress to tabular stress, as explained in Example I.

Span Limits for Steel Beams. It has been pointed out in the discussion of Stiffness of Beams that the deflection in first-class work should be limited to 1/360 of the span length. Equation (4), in which $f = 18\,000$, and the load uniformly distributed, may be expressed as follows:

$$\frac{L \times 12}{360} \leq 0.01862 \frac{L^2}{d}$$

or

$$\frac{L}{d} = 1.79$$

or

$$L = 1.79d \qquad (5)$$

in which L = the span length in feet;
d = depth of the section (or $2 \times c_{max}$) in inches.

In a similar manner the relation between the span and depth for maximum allowable deflection and existing maximum fiber-stress can be found for any condition of loading.

These relations, for the more common cases of loading, are given in Table III.

Table III. Ratios of L/d for Deflections = 1/360 Span

Beam and loading	L (feet) ÷ d (inches)	
	f = 16 000	f = 18 000
Simple Beam; Concentrated Load at Center	2.52	2.24
Simple Beam; Uniform Load over Span. . .	2.01	1.79
Simple Beam; Triangular Load, Apex at Center Line of Span...	2.09	1.86
Simple Beam; Two equal concentrated loads at one-third points of span.......	1.97	1.75
Simple Beam; Any irregular loading (approx)....	2 19	1 95
Cantilever Beam; Concentrated Load at Free End.	0.63	0 56
Cantilever Beam; Uniform Load over Span.... .	0.84	0.75
Cantilever Beam; Concentrated Load at $L/2$ (at Free End)	1.09	0.90
Cantilever Beam; Any Irregular Loading (approx)	0 73	0.65

Approximate Deflections of Steel Beams. In practice it frequently occurs that a beam is subjected to a system of unsymmetrical or irregular loads for which no coefficient of deflection is readily available. An analytical solution of such a case requires considerable labor, and often an APPROXIMATE VALUE of the maximum deflection (within 10 to 15%) is sufficiently accurate for practical purposes.

It will be noted that in the preceding tables a value is given for approximate deflections of irregular systems of combined concentrated and uniform loads.

The given load system may also be reduced to an equivalent uniform load, or a uniform load which produces the same maximum bending moment, and the coefficients of Table II corresponding to the maximum unit stress divided by the depth of the section (or by $2\,c_{max}$) will give the approximate deflection. The method of equivalent uniform loads gives an excess of from 20 to 30% for single concentrated loads at the center and between the center and quarter point of span, but when concentrated loads occur to each side of the center and when the beam carries uniform load together with a series of concentrated loads the method of equivalent uniform load deflection gives much less excess values than the above extreme cases.

Examples of Deflections of Steel Beams

Example 1. A 12-in—CB—40.0 lb supports a uniform load of 29 000 lb. over a span of 20 ft 4 in. Required: the maximum vertical deflection.

$$M_{max} = \frac{29\,000}{8} \times 20.33 \times 12 = 885\,000 \text{ lb-in}$$

The section-modulus, axis 1—1 = 52.28 in^3

$$f = \frac{885\ 000}{52.28} = 16\ 950 \text{ lb per sq in}$$

The deflection coefficient (Table II) for $L = 20.33$ ft and for a fiber-stress of 16 000 is, by interpolation, 6.847. Then $\Delta_{max} = \frac{16\ 950}{16\ 000} \times \frac{6.847}{12} = 0.605$ **in**

Example 2. A 20-in—Beth. B—73 0 lb of a span = 24 ft 0 in is loaded as follows: (*a*) Uniform load over span of 40 000 lb; (*b*) concentrated load at each one-third point of span of 12 000 lb. Required. the maximum vertical deflection.

$$M_a = \frac{40\ 000 \times 24 \times 12}{8} = 1\ 440\ 000 \text{ lb-in}$$

$$M_b = \frac{12\ 000 \times 24 \times 12}{3} = 1\ 152\ 000 \text{ lb-in}$$

The section-modulus of the beam, axis 1 — 1 = 148.5 in^3

$$f_a = \frac{1\ 440\ 000}{148.5} = 9\ 700 \text{ lb per sq in}$$

$$f_b = \frac{1\ 152\ 000}{148\ 5} = 7\ 780 \text{ lb per sq in}$$

The deflection coefficient, for uniform load, corresponding to a span of 24 ft and a fiber-stress of 10 000 lb per sq in (Table II), is 5.952. For two equal concentrated loads at the one-third points of span this coefficient must be multiplied by 1.02 (see note below Table II). Then:

$$\Delta_a = \frac{9\ 700}{10\ 000} \times \frac{5.952}{20} = 0.288 \text{ in}$$

$$\Delta_b = \frac{7\ 780}{10\ 000} \times \frac{5.952}{20} \times 1.02 = 0.237 \text{ in}$$

$$\text{Total maximum } \Delta = 0.525 \text{ in}$$

Example 3. Let it be required to calculate the approximate vertical deflection of the 14 in—CB—58.0 lb, loaded over the span of 24 ft 0 in, as shown in Fig. 6.

The maximum moment occurs at 11.20 ft from the left reaction, and is:

$$M_{max} = 129\ 750 \text{ lb-ft}$$

The section-modulus, axis 1—1, is 85.58 in^3

$$f_{max} = \frac{129\ 750 \times 12}{85.58} = 18\ 150 \text{ lb per sq in}$$

The approximate maximum deflection may be determined by using the correction factor for irregular loading given below Table II and the deflection coefficient corresponding to the span and fiber-stress given in Table II, or:

$$\Delta_{max} \text{ (approx.)} = \frac{18\ 150}{18\ 000} \times \frac{10.725}{14} \times 0.92 = 0.715 \text{ in}$$

The approximate deflection by the equivalent uniform load method, that is, assuming a substitute uniform load which will produce the same maximum

fiber-stress as the given loading, is found by using the proper coefficient for uniform loads as given in Table II:

$$\Delta_{max}\ (\text{approx.}) = \frac{18\ 150}{18\ 000} \times \frac{10.725}{14} = 0.775 \text{ in}$$

The vertical deflection as determined by the graphical method illustrated in Fig. 6 is 0.775 in.

Example 4. A lintel supporting a 13-in brick wall over a span of 10 ft 0 in is composed of two 6 × 4 × ⅝-in angles, long legs back to back. The total triangular load, assuming a base angle of 45°, and a unit weight of wall (plastered one side) at 126 lb per sq ft, is W = 4 536 lb.

$$M_{max} = \frac{4\ 536 \times 10 \times 12}{6} = 90\ 720 \text{ lb-in}$$

The section-modulus, axis parallel to short legs, is 5.31 in³, and N.A. is at 2.03 in from back of short legs, or c_{max} = 3 97 in.

$$f_{max} = \frac{90\ 720}{5.31} = 17\ 100 \text{ lb per sq in}$$

The coefficient of deflection for uniform load for a span of 10 ft and f = 18 000 is, 1.862. The correction factor for triangular loading as given below Table II, is 0 96. Then $\Delta_{max} = \dfrac{17\ 100}{18\ 000} \times \dfrac{1.862}{2 \times 3.97} \times 0.96 = 0.214$ in

Lateral Deflection of Beams. The compression flange of a beam is subject to column action, hence, if such a flange is unsupported laterally, LATERAL DEFLECTION will take place. The amount of such deflection is indeterminate since the flange area in compression is partially supported by the area in tension and since the compressive stress throughout the flange is variable. The important consideration is to provide an EXCESS SECTION for laterally unsupported beams such that the combined compressive stresses due to vertical bending and lateral bending do not exceed the safe working unit stress.

This result is accomplished by decreasing the allowable working unit stress as the laterally unsupported length of the compression flange increases. Through experience it has been established that no reduction in the working unit stress is necessary until the unsupported length of the compression flange exceeds 15 times its breadth.

The following table gives the permissible percentage of the maximum working unit stress for various ratios of the unsupported length of the compression flange to its breadth:

Table IV. Allowable Percentages of Maximum Working Unit Stress for Various Ratios of Length to Breadth of Unsupported Compression Flange

Ratio $\frac{L}{b}$	% of f_{max}	Ratio $\frac{L}{b}$	% of f_{max}	Ratio $\frac{L}{b}$	% of f_{max}	Ratio $\frac{L}{b}$	% of f_{max}	Ratio $\frac{L}{b}$	% of f_{max}
16	98.5	21	91.0	26	83.0	31	75.1	36	67 4
17	97.1	22	89.5	27	81.4	32	73.5	37	66.0
18	95.6	23	87.9	28	79.8	33	71.9	38	64.5
19	94.1	24	86.3	29	78 2	34	70.4	39	63.1
20	92.6	25	84.7	30	76.6	35	68.9	40	61.7

In addition to the lateral deflection resulting from vertical loads, additional lateral deflection may result from the thrust of floor arches or from other loads acting normal to the plane of principal bending.

The THRUST OF FLOOR ARCHES should be reduced by the introduction of intermediate lateral supports in the form of TIE-RODS. If tie-rods are provided at the one-third points of span (as in the usual procedure),

$$f_{\max} = \frac{M_v c_1}{I_1} + \frac{8}{90}\frac{M_h c_2}{I_2} \tag{6}$$

in which M_v = moment of vertical loads;

M_h = moment of horizontal loads (no supports);

$\frac{c_1}{I_1}$ = reciprocal of section-modulus, axis normal to vertical plane;

$\frac{c_2}{I_2}$ = reciprocal of section-modulus, axis normal to horizontal plane.

Examples of Lateral Deflection

Example 1. Let it be required to determine the maximum allowable uniform load for a 15-in Beth. Girder Bm —74.0 lb over an unsupported span of 24 ft 0 in and a maximum allowable working unit stress for laterally supported beams of 18 000 lb per sq in.

The properties of the given beam section are:

$$b = 10.75 \text{ in}$$

$$S_{1-1} = 119.03 \text{ in}^3 \text{ (section-modulus)}$$

$$M_{\max} = \frac{W \times 24 \times 12}{8} = 119.03 \times 18\,000$$

or

$$W_{(\text{fixed})} = 59\,515 \text{ lb (beam supported laterally)}$$

$$\frac{L}{b} = \frac{24 \times 12}{10.75} = 26.79$$

From Table IV, the percentage of maximum stress allowable is 81.74, hence, the allowable safe load is

$$W = 59\,515 \times 0.8174 = 48\,648 \text{ lb}$$

Example 2. A typical interior floor panel of flat tile arch floor construction is 18 ft 0 in × 18 ft 0 in. The beams are spaced 6 ft 0 in on centers (span of arch); the design loads are: dead load = 100 lb per sq ft and live load = 75 lb per sq ft.

The arch thrust due to live load on one side of the beam only is $H = \left(\frac{3}{2} \times 75 \times \frac{6^2}{7.6}\right) \times 18 = 9\,600$ lb for a 10-in tile arch. The total vertical load (live load on one side only) is $(6 \times 18 \times 100) + (3 \times 18 \times 75) = 14\,850$ lb.

Tie rods are to be placed at the one-third points of the beam span providing intermediate supports, and laterally the beam is, therefore, a continuous beam of three spans, but with somewhat yielding supports. A bending

moment coefficient of 1/10 may be employed for the lateral moments, then:

$$M_v = \frac{14\,850}{8} \times 18 \times 12 = 400\,000 \text{ lb-in}$$

$$M_h = \frac{8}{90} \times \frac{9\,600}{8} \times 18 \times 12 = 32\,000 \text{ lb-in}$$

A 9 in—CB—29.0 lb will be assumed, giving:

$$S_{1-1} = 28\,0 \text{ in}^3$$
$$S_{2-2} = 6.6 \text{ in}^3$$
$$\text{Width of flange} = 6.50 \text{ in}$$

The maximum, combined unit fiber-stress is:

$$f_{\max} = \frac{400\,000}{28\,0} + \frac{23\,000}{6.6} = 17\,800 \text{ lb per sq in.}$$

If the tile arch is properly constructed the compression flange of the beam is supported laterally throughout its length. However, without this support the L/b ratio between tie-rods is $\frac{72}{6.5} = 11\,1$ and full maximum unit stress is allowable.

3. Deflection of Timber Beams

Allowable Unit Stresses. The allowable unit stresses for the different structural timbers, as specified by building ordinances, vary widely, due to such factors as: kind of wood; age of tree, time of year felled, method of sawing, proportion of heart to sap wood, and percentage of knots.

The recent tendency is to increase allowable unit stresses, especially in flexure, as a result of the more careful attention which lumber manufacturers' organizations are giving to the grading rules, thereby insuring timber of uniformly higher quality.

Horizontal Shear. Timber beams of relatively short spans and correspondingly large loads must be investigated for horizontal shearing tendency. All timber is comparatively weak in horizontal shear resistance and often, in the case of short spans, beams which are apparently satisfactory for extreme fiber-stress are overstressed in horizontal shear. The ratio of allowable flexural stress to allowable horizontal shear is not a constant for various kinds of timber, hence no general relation between limiting span length and depth of section can be given excepting for a given kind of timber. The following table gives values of the minimum limiting ratios of the length in feet to the depth of beam in inches for flexural stress as a governing factor. Spans less than those given in Table V must be investigated for horizontal shear.

In standard works on mechanics it is shown that the maximum unit horizontal shear on a rectangular section is:

$$v_{\max} = \frac{3}{2}\frac{V}{bd} \tag{7}$$

in which V = the maximum vertical shear in pounds (in simple beams, the greatest reaction at support);
b = breadth of beam in inches;
d = depth of beam in inches.

Table V. Ratios Less than which Horizontal Shear Governs

$L(\text{ft})/d(\text{in})$

Kind of timber	Values of $\frac{L}{d}$ beyond which Horizontal Shear does not govern
Western hemlock Redwood	17 0
Eastern hemlock White fir Douglas fir (Oregon and Washington)	16.0
Southern yellow pine Red and white spruce Norway pine Douglas fir (Rocky Mountain) Southern cypress	13 0
White and red oak Northern white pine Western yellow pine	11 0

Lateral Deflection of Timber Beams. In building-construction timber beams are generally supported laterally by cross-beams or by continuous floor construction. When, however, lateral support is not provided for the compression side of a timber beam the allowable unit stress for design should be reduced as the laterally unsupported distance increases, in the ratio given in the following table:

Table VI. Allowable Percentages of Maximum Working Unit Stress for Various Ratios of Length to Breadth of Laterally Unsupported Timber Beams

Ratio $\frac{L}{b}$	Per cent of f_{max}
10	100 0
15	85.0
20	80.0
25	75 0
30	70 0
35	65.0
40	60 0

Example of Design and Investigation of Unit Stresses in a Timber Beam. A timber beam is to be designed to support a uniformly distributed load of 19 600 lb over a span of 9 ft 0 in. Procedure:

$$M = \frac{19\,600 \times 9 \times 12}{8} = 264\,000 \text{ lb-in}$$

$$V = \frac{19\,600}{2} = 9\,800 \text{ lb}$$

For an allowable working unit stress in flexure of 1 600 lb per sq in and an allowable horizontal shearing unit stress of 110 lb per sq in:

$$S_{1-1} = \frac{264\ 000}{1\ 600} = 165.2$$

A nominal 8 × 12-in section (actual dimensions 7½ × 11½ in) provides:

$$A = 86.25 \text{ sq in}$$
$$I_{1-1} = 950.55 \text{ in}^4$$
$$S_{1-1} = 160.31 \text{ in}^3$$

Reference to Table V indicates that the section should be investigated for horizontal shear:

$$v_{max} = \frac{3}{2} \times \frac{V}{bd} = \frac{3}{2} \times \frac{9\ 800}{86.25} = 170 \text{ lb per sq in}$$

Horizontal shear is, therefore, the governing factor, and a larger section than that required for flexural stress must be adopted.

Select a nominal 12 × 12-in section (11½ × 11½ in).

$$A = 132.25 \text{ sq in, and } S_{1-1} = 253.5 \text{ in}^3$$
$$v = \frac{3}{2} \times \frac{9\ 800}{132.25} = 111 \text{ lb. per sq in}$$

The maximum fiber-stress for which the deflection shall be computed is, therefore:

$$f = \frac{M}{S_{1-1}}$$

or

$$f = \frac{264\ 000}{253.5} = 1\ 045 \text{ lb per sq in}$$

Modulus of Elasticity of Timber. The values of the modulus of elasticity for the usual kinds of timber employed in building construction are given in Table VII.

Table VII. Modulus of Elasticity of Structural Timber *

Kinds of timber	Modulus of elasticity
Yellow, pine, Southern Douglas, fir, Oregon and Washington	1 600 000
White oak Red oak	1 500 000
Hemlock, Western	1 400 000
Spruce, Red and White Norway pine Redwood Douglas fir, Rocky Mountain Region Cypress, Southern	1 200 000
White Pine, Northern Hemlock, Eastern White fir Yellow pine, Western	1 000 000

* See Table I, Chapter XX.

Timber differs from most other building materials, in that, if beam members are loaded to the capacity of safe working limits with permanent loads, under long-continued loading, vertical deflection will increase with time with no increase of load. For this reason most design specifications require a reduction of the modulus of elasticity values as given in Table VII for fixed total loading.

In consideration of the character of the loads usually encountered in building-construction and the relatively high proportion of design live load to total loads, a limit of vertical deflection to 1/360 of the span under full dead plus live load, based on the modulus of elasticity values given in Table VII, is sufficient when the timber is used in dry and sheltered locations and is free from excessive impact stresses.

Deflection of Timber Beams. Equations (1), (2), and (3) are general and are applicable to beams of any material and any form of cross-section where the proper values of f, c, and E are introduced.

From Equation (3) the deflection coefficients for the various kinds of structural timber listed in Table VII have been derived for a fixed value of $f = 1\,000$. These values are given in Table VIII. For values of flexural unit stress other than $f = 1\,000$, the deflection can be readily determined by direct proportion since, for any given set of conditions, deflections up to the elastic limit of the material vary directly as the flexural unit stress.

To Find the Deflection of a Uniformly Loaded Timber Beam, take the value of the deflection coefficient corresponding to the length of span in feet and the kind of timber from Table VIII. This coefficient divided by the actual depth of the beam in inches gives the deflection for a maximum fiber-stress of 1 000 lb per sq in and when multiplied by the actual fiber-stress in thousands (kips) the result is the maximum vertical deflection of the beam in inches. For conditions of loading other than uniform load multiply the result as found above by the corrective coefficient given in the notes below Table VIII.

Example 1. A 12 × 12-in ($11\frac{1}{2} \times 11\frac{1}{2}$ in) timber beam of Southern yellow pine supports a uniform load of 19 600 lb over a span of 9 ft 0 in. The area of the section is 132.25 sq in and $S_{1-1} = 253.5$ in³. The limiting stress in design of the beam is horizontal shear,

$$v = \frac{3}{2} \times \frac{V}{bd}$$

or

$$v = \frac{3}{2} \times \frac{9\,800}{132.25} = 111 \text{ lb per sq in}$$

$$f = \frac{M}{S_{1-1}}$$

or

$$f_{max} = \frac{19\,600 \times 9 \times 12}{8\,(253.5)} = 1\,045 \text{ lb per sq in}$$

The deflection coefficient for Southern yellow pine, span 9 ft and $f = 1\,000$, is (Table VIII), 1.519 and the maximum vertical deflection is, therefore:

$$\Delta_{max} = \frac{1\,045}{1\,000} \times \frac{1.519}{11.5} = 0.138 \text{ in}$$

Table VIII. Deflection Coefficients for Uniformly * Loaded Timber Beams

$f = 1\,000$ lb per sq in

Span in feet	Southern Yellow pine, Douglas fir (W)	White oak, Red oak	Western hemlock	Spruce (R.S.W.), Norway pine, Redwood, Douglas fir (RM) Cypress	White pine, Hemlock (E), Western yellow pine, White fir
4	0 300	0 320	0 343	0 400	0 480
5	0 469	0 500	0 536	0 625	0 750
6	0 675	0 720	0.771	0 900	1 080
7	0 919	0 980	1.050	1.225	1 470
8	1.200	1 280	1 370	1 600	1 920
9	1.519	1 620	1 735	2 025	2 430
10	1.875	2.000	2 142	2 500	3 000
11	2.269	2 420	2 592	3 025	3.630
12	2.700	2 880	3 084	3 600	4 320
13	3.169	3 380	3 620	4 225	5.070
14	3.675	3 920	4 198	4 900	5 880
15	4.219	4 500	4.820	5 625	6 725
16	4.800	5 120	5.484	6 150	7 680
17	5 419	5 780	6.190	7 225	8 670
18	6 075	6 480	6 940	8 100	9 720
19	6 769	7 220	7 733	9 025	10 830
20	7 500	8 000	8 568	10 000	12 000
21	8.269	8 820	9.446	11.025	13 230
22	9 075	9.680	10.367	12 100	14 520
23	9.919	10 580	11.331	13 225	15 870
24	10 800	11 520	12 338	14 400	17 280
25	11.719	12.500	13.388	15.625	18 750
26	12 675	13.520	14.480	16.900	20 280
27	13.669	14 580	15 615	18 225	21 870
28	14.700	15.680	16.793	19 600	23.520
29	15.769	16 820	18.014	21 025	25 230
30	16.875	18 000	19 278	22.500	27 000
31	18 019	19 220	20 585	24.025	28 830
32	19 200	20.480	21.934	25.600	30 720
33	20 419	21 780	23 326	27.225	32 670
34	21 675	23 120	24 762	28 900	34 680
35	22 969	24.500	26 240	30 625	36.750

* For Concentrated Center Load, Coefficient = 0.80 of above values. For Triangular Loading, apex at C.L., Coefficient = 0.96 of above values. For Equal Concentrated Loads at ⅓ Points, Coefficient = 1.02 of above values. For Irregular Loading (approx.), Coefficient = 0.92 of above values.

Example 2. A 12 × 18-in (nominal) timber beam of Western hemlock has been designed to carry a uniformly distributed load of 26 000 lb over a span of 18 ft 0 in. The section-modulus, axis 1—1 of the actual section of $11\frac{1}{2} \times 17\frac{1}{2}$ in, is $S_{1-1} = 586.98$ in³. Required: The maximum vertical deflection.

$$f_{max} = \frac{26\,000 \times 18 \times 12}{8\,(586.98)} = 1\,197 \text{ lb per sq in}$$

The deflection coefficient, from Table VIII, for Western hemlock, span = 18 ft and f = 1 000 for uniform load, is 6.940. The maximum vertical deflection is, therefore:

$$\Delta_{max} = \frac{1\,197}{1\,000} \times \frac{6\,940}{17.5} = 0.475 \text{ in}$$

Example 3. Design, and calculate the maximum vertical deflection of, a Western Douglas fir girder for a span of 21 ft 0 in and loaded as follows:

(*a*) Uniform load over span of 6 000 lb
(*b*) Equal third point loads of 5 000 lb

$$f_{max} \text{ allowable} = 1\,600 \text{ lb per sq in}$$
$$v_{max} \text{ (horizontal)} = 105 \text{ lb per sq in}$$

Procedure:

$$\text{at C.L. } M_{(a)} = \frac{6\,000 \times 21 \times 12}{8} = 189\,000 \text{ lb-in}$$

$$\text{at C.L. } M_{(b)} = \frac{5\,000 \times 21 \times 12}{3} = 420\,000 \text{ lb-in}$$

$$M_{max} \text{ at C.L. span} = 609\,000 \text{ lb-in}$$

The required section-modulus is:

$$S_{1-1} = \frac{609\,000}{1\,600} = 380.0 \text{ in}^3$$

A nominal 10 × 16 in provides a section-modulus of 380.39 and a nominal 8 × 18 in provides 382.81 with respective areas of 147.25 sq in and 131.25 sq in.

Since the girder is to be laterally supported at each one-third point by a cross-beam the 8 × 18 in (width 7.5) will be sufficient to provide all necessary lateral stiffness between cross-beam connections, and this section will be adopted pending an investigation of horizontal shear.

$$v_{max} = \frac{3}{2} \cdot \frac{8\,000}{131.25} = 91.5 \text{ lb per sq in}$$

which is satisfactory.

The maximum fiber stresses due to uniform and concentrated load, are:

$$f_{(a)} = \frac{189\,000}{382.81} = 494 \text{ lb per sq in}$$

$$f_{(b)} = \frac{420\,000}{382.81} = 1\,096 \text{ lb per sq in}$$

The maximum vertical deflection at the center of span is equal to the sum of the deflections due to the two systems of loading. From Table VIII the deflection coefficient for uniform load, f = 1 000 and Western Douglas fir on a span of 21 ft, is 8.269. The correction factor for equal third-point loads is 1.02, whence:

$$\Delta_{(a)} = \frac{494}{1\,000} \times \frac{8.269}{17.5} = 0.233 \text{ in}$$

$$\Delta_{(b)} = \frac{1\,096}{1\,000} \times \frac{8.269}{17.5} \times 1.02 = 0.529 \text{ in}$$

$$\text{Total} = 0.762 \text{ in}$$

Deflection of Beams by the Moment Area Method. The interpretation of the laws of deflection as given by the method of Moment Areas is exceedingly valuable in that it provides a simplified point of view and establishes

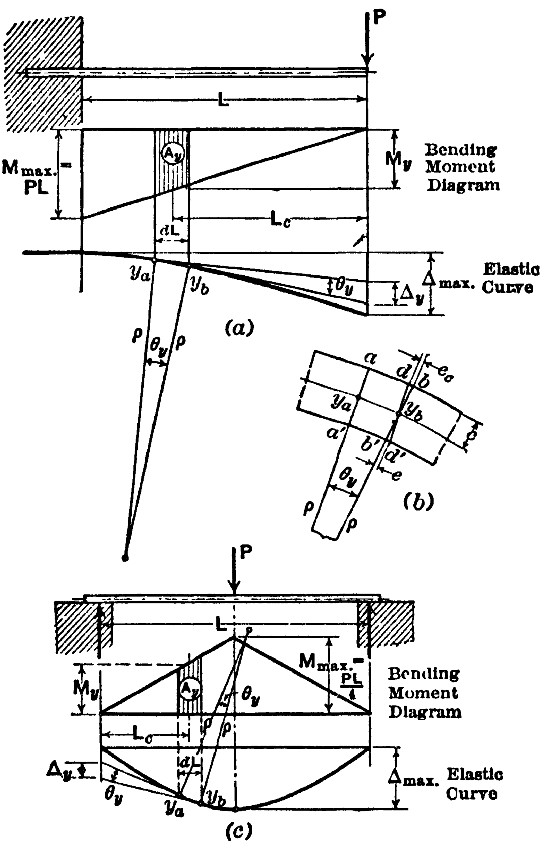


Fig. 1. Deflection of Beams by the Moment Area Method

the fundamental principles upon which graphical methods of deflections may be developed. See Fig. 1.

In Fig. 1(a), let:

$y_a\, y_b = dL$ = a short length of the neutral axis of a beam subjected to bending. The arc and its horizontal projection may be taken equal without appreciable error.

aa' and bb' = two parallel cross-sections before bending; after bending they intersect at O, the center of curvature.

Let section dd' be drawn through y_b parallel to rotated position of aa'.

$$e = b'd' = \text{shortening of } a'b'$$

$$e_c = bd = \text{lengthening of } ab$$

Let the fiber-stress corresponding to e_c be f_c and the modulus of elasticity $= E$. Then

$$e_c = \frac{f_c \cdot dL}{E}$$

but

$$f_c = \frac{M_y \cdot c}{I}$$

From similar triangles

$$\frac{\rho}{dL} = \frac{c}{e_c} = \frac{cE}{f_c\ dL}$$

whence

$$\rho = \frac{cE}{f_c} = \frac{EI}{M_y} \tag{8}$$

In Figs. 1(*a*) and 1(*c*), let the angle subtending dL (taken as horizontal projection of $y_a\ y_b$) be θ_y, in circular measure, then the angle between the tangents at y_a and y_b will also be y. The vertical deviation of the two tangents at the free end of the span, Figs. 1(*a*) or 1(*c*) is.

$$d_y = \theta_y \cdot L_c = \frac{dL}{\rho} \cdot L_c \tag{9}$$

where L_c is the distance from the centroid of the arc $y_a\ y_b$ to the free end of the span where d_y is measured

Substituting the value of the radius of curvature, ρ, from Equation (8), in Equation (9):

$$\theta_y = \frac{M_y\ dL}{EI} \tag{10}$$

or the angle between the tangents at any two points along the elastic curve of a beam of any type is equal to the area under the moment curve, between those points, divided by EI.

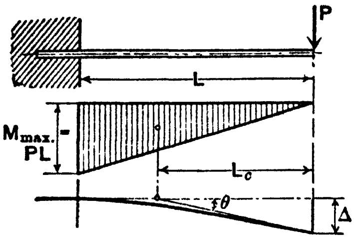
Also from Equation (9):

$$d_y = \theta_y\ L_c = \frac{M_y \cdot dL}{EI} \cdot L_c \tag{11}$$

or: the vertical deflection, measured at the free end of a beam span, due to the bending between any two points along the span, so long as the tangents at these points have the same sign of slope, is equal to the moment of the area under the bending moment curve between the two points taken about the adjacent free end, and divided by EI.

The total deflection at the free end is numerically equal to the sum of the moments of each elementary area of the bending-moment diagram between the free end of reference and the point at which the deflection is desired, taken about the free end and divided by EI. Instead of dealing with the moments of each element of bending moment area, the entire moment area between the free end and the point of tangency may be taken so long as the centroidal distance, from the free end, of the area so taken is known.

Figs. 2 and 3 showing the more common loading conditions of cantilever and simple beams, respectively, illustrate clearly the general application of



Case I. Cantilever Beam. Concentrated Load of P at the Free End

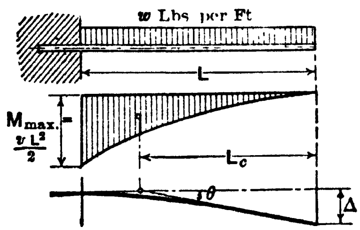
Bending Moment Diagram

$$A = PL \times \frac{L}{2} = \frac{PL^2}{2}$$

$$L_c = \tfrac{2}{3} L$$

Elastic Curve

$$\Delta = \frac{PL^2}{2} \times \tfrac{2}{3} L \times \frac{1}{EI} = \frac{PL^3}{3EI}$$



Case II. Cantilever Beam. Uniform Load Over Span of w Lb per Lineal Ft

$$W = wL$$

Bending Moment Diagram

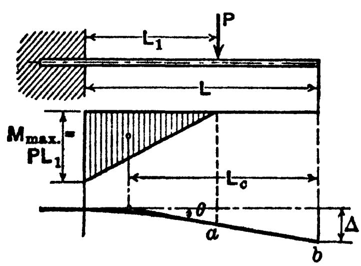
$$A = \tfrac{1}{3}\left(\frac{wL^2}{2} \times L\right)$$

$$L_c = \tfrac{3}{4} L$$

Elastic Curve

$$\Delta = \tfrac{1}{3}\left(\frac{wL^2}{2} \times L\right) \times \tfrac{3}{4} L \times \frac{1}{EI}$$

$$= \frac{WL^3}{8\,EI}$$



Note. Portion a–b is a right Line

Case III. Cantilever Beam. Concentrated Load P at Any Point of Span—L_1 from Support

Bending Moment Diagram

$$A = PL_1 \times \frac{L_1}{2} = \frac{PL_1^2}{2}$$

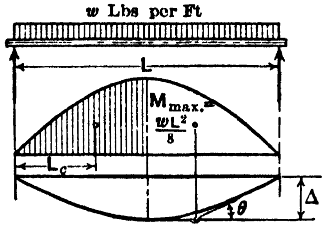
$$L_c = L - \frac{L_1}{3}$$

Elastic Curve

$$\Delta = \frac{PL_1^2}{2\,EI}\left(L - \frac{L_1}{3}\right)$$

$$\Delta_a = \frac{PL_1^3}{3\,EI}$$

Fig. 2. Deflection of Cantilever Beams



Case IV. Simple Beam. Uniform Load Over Span of w Lb per Lineal Ft.

Bending Moment Diagram

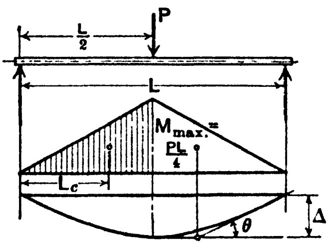
$$A = \tfrac{2}{3}\left(\frac{wL^2}{8} \times \frac{L}{2}\right)$$

$$L_c = \tfrac{5}{16} L$$

Elastic Curve

$$\Delta = \tfrac{2}{3}\left(\frac{wL^2}{8} \times \frac{L}{2}\right) \times \tfrac{5}{16} L \times \frac{1}{EI}$$

$$= \tfrac{5}{384} \frac{WL^3}{LI}$$



Case V. Simple Beam. Single Concentrated Load of P at Center of Span.

Bending Moment Diagram

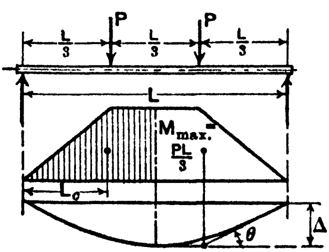
$$A = \frac{PL}{4} \times \frac{L}{4}$$

$$L_c = \tfrac{2}{3} \times \frac{L}{2} = \frac{L}{3}$$

Elastic Curve

$$\Delta = \frac{PL^2}{16} \times \frac{L}{3} \times \frac{1}{EI}$$

$$= \frac{PL^3}{48\,EI}$$



Case VI. Simple Beam. Equal Concentrated Loads of P at Third Points of Span.

Bending Moment Diagram

$$A = 2\left(\frac{PL}{3} \times \frac{L}{6}\right) = \frac{PL^2}{9}$$

$$L_c = \tfrac{2}{3}\frac{L}{3} + \tfrac{1}{2}\left(\frac{L}{12} + \frac{L}{9}\right) = \tfrac{23}{72} L$$

Elastic Curve

$$\Delta = \frac{PL^2}{9} \times \tfrac{23}{72} L \times \frac{1}{EI}$$

$$= \tfrac{23}{648} \frac{PL^3}{EI}$$

Fig 3. Deflection of Simple Beams

the moment area method of deflections. It should be noted that the tangents drawn at any two points along the elastic curve intersect at the centroid of the moment area included between these two points. This fact is clearly illustrated in Fig. 4.

Unsymmetrical Loading. It should be noted that the point of maximum deflection does not necessarily occur at the section of maximum bending moment.

If a second shear diagram is constructed, using the area of the original bending moment diagram as load, the maximum deflection occurs at that point where the second shear diagram passes through zero.

Referring to Fig. 4, the second shear value at any distance l_x from R_1 is:

$$R_1 - \frac{M}{L_1} \times \frac{l_x^2}{2}$$

but

$$R_1 = \frac{ML_1}{2L}\left(L_2 + \frac{L_1}{3}\right) + \frac{ML_2^2}{3L} \tag{12}$$

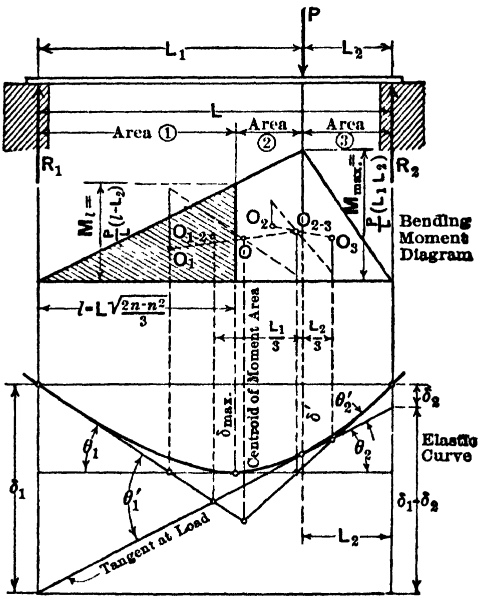


Fig. 4. Deflection of Simple Beam

Therefore, the second shear diagram passes through zero at l distance from R_1, when:

$$\frac{ML_1}{2L}\left(L_2 + \frac{L_1}{3}\right) + \frac{ML_2^2}{3L} = \frac{Ml^2}{2L_1}$$

or

$$l = \sqrt{\frac{2L_1L^2 - L_1^2L}{3L}} = L\sqrt{\frac{2n - n^2}{3}} \tag{13}$$

where $n = L_1/L$.

The deflection at l is:

$$\Delta_{max} = \frac{M_l \cdot l}{2} \times \frac{2}{3} l \times \frac{1}{EI}$$

$$= \frac{PL^3}{3\,EI}(1 - n)\left(\frac{2\,n - n^2}{3}\right)^{3/2} \tag{14}$$

The deflection under the concentrated load may be found as follows. Draw the tangent to the elastic curve under the load, and at both free ends, then:

$$\delta_1 = \left(R_1L_1 \times \frac{L_1}{2} \times \frac{2}{3} L_1\right)\frac{1}{EI} = \frac{R_1L_1^3}{3\,EI}$$

$$\delta_2 = \left(R_2L_2 \times \frac{L_2}{2} \times \frac{2}{3} L_2\right)\frac{1}{EI} = \frac{R_2L_2^3}{3\,EI}$$

The slope of the tangent to the elastic curve under the load is

$$\frac{\delta_1 - \delta_2}{L}$$

and the deflection under the load is

$$\delta_2 + \text{Slope} \times L_2$$

or

$$\delta = \frac{R_2L_2^3}{3\,EI} + \frac{R_1L_1^3 - R_2L_2^3}{3\,EI} \times \frac{L_2}{L} \tag{15}$$

The value of R_1 is given by Equation (12) and the value of R_2 is found in a similar manner. Substituting these values of R_1 and R_2 in Equation (15), the deflection under the load is

$$\delta = \frac{PL_1^2L_2^2}{3\,EI \cdot L} \tag{16}$$

If $L_1 = L_2 = \frac{L}{2}$ it is seen that Equation (16) reduced to Case V, Fig. 3.

Graphical Method of the Deflection of Beams. The relation between vertical deflection and the bending moment area as established by the moment area method of deflection, can be readily adapted to the methods of graphical analysis.

The principal advantages of a graphical solution are: (*a*) the result shows at once the deflection of all points throughout the span; (*b*) the point of maximum deflection is given directly; (*c*) regardless of the irregularity and combinations of loading systems, so long as the moment diagram can be constructed, the vertical deflections throughout can be found as easily as for the case of simple symmetrical loads; (*d*) the method is applicable with equal facility to beams with overhanging ends.

It is demonstrated in standard works on graphic statics that the bending moment at any point in the span of a beam is the product of the ordinate of the equilibrium polygon, at that point, and the pole distance from which the equilibrium polygon was constructed. The ordinate, or vertical intercept is measured to the same scale as the space drawing of the beam, while the pole distance is a force measured to the same scale as the load line.

Since it has been shown by the moment area method of deflections that

the deflection of a given point with respect to a free end or support is numerically equal to the moment of the included area of the bending moment diagram taken about the free end or support and all divided by EI, it follows that a second moment diagram constructed from the first bending moment diagram areas, taken as loads, will be the elastic curve, to some scale, for the given beam and loading, since for any element of length the deflection increment, $\delta_y = \frac{M_y l_y \cdot dl}{EI}$ and the total deflection between points distant x and O from the free end is:

$$\delta = \frac{1}{EI} \sum_0^x M \cdot dl \cdot l$$

Procedure for Graphical Method of Deflections. Referring to Fig. 5: The uniform loads are divided into one- or two-foot sections along the span and the vertical vectors representing their respective magnitude are drawn through the centers of each division.

The load line a—n is constructed at any convenient scale, and the pole distance H of any magnitude and location desired is measured to the same scale as the load line.

The equilibrium polygon (moment diagram) is drawn, remembering that the various rays oa, ob, oc, etc., cross the spaces between load vectors, designated, respectively, by a, b, c, etc. The numerical value of the bending moment at any point in the span is the product of the pole distance H and the vertical intercept of the bending moment diagram, at that point, measured in units of the space scale of the beam span.

The given span is considered to be reloaded with the area of the first bending moment diagram. This area is divided into convenient elements, and the vertical vector representing the area magnitude of each division drawn through its center of gravity.

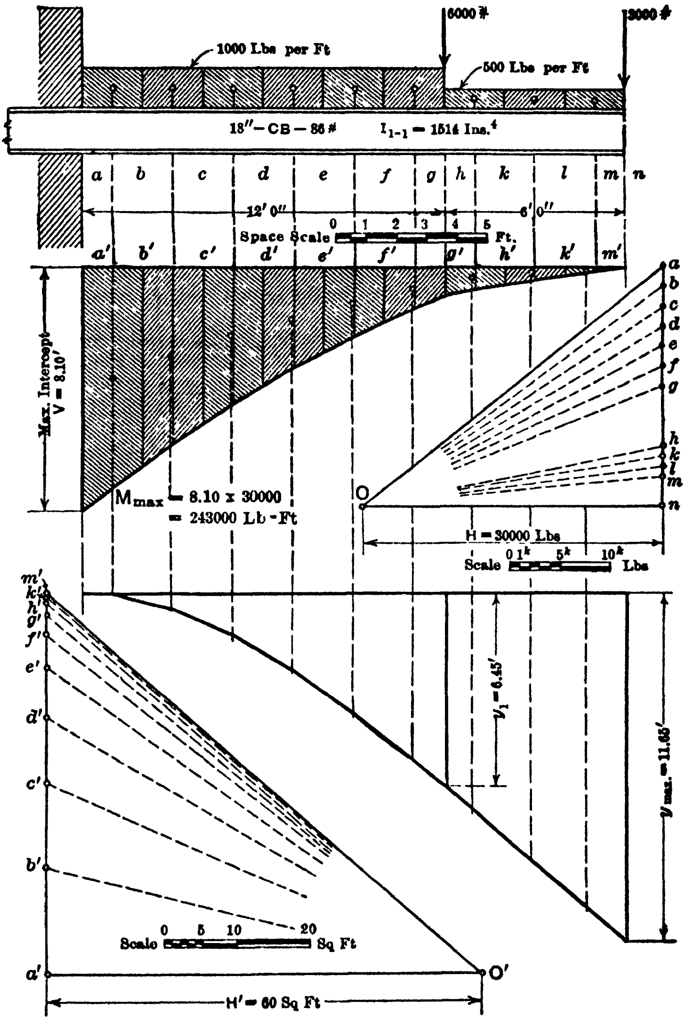
The area of each division is: average vertical intercept $\times H \times$ horizontal length of division $= H$ ft², all lineal measurements on the moment diagram being taken to the space scale of span.

The second load line (area vectors) is constructed at a'—m', the magnitudes being taken in ft² or each being $\frac{1}{H}$ the true value. The second pole distance H' is taken at any convenient magnitude to the same scale in ft² as the second load line, then the vertical deflections, or moments of moment-areas, are shown relatively by the second equilibrium polygon, drawn in the usual manner from the rays of the second load line.

The deflection at any point in the span is, therefore, equal to the product of the vertical intercept of the second equilibrium polygon into the true value of H' divided by EI. If the intercept y is also measured in feet to the space scale of span, and the deflection is desired in inches:

$$\Delta = \frac{H \times H' \times y \times 1\,728}{EI} \tag{17}$$

Example 1. Fig. 5 illustrates a cantilever beam of a span of 18 ft loaded with a uniform load of 1 000 lb per ft over the first 12 ft from the support and 500 lb per ft over the outer 6 ft. In addition to the uniform loads, concentrated loads of 6 000 lb and 3 000 lb are located 12 ft and 18 ft, respectively, from the support. The bending moment diagram is constructed from the



$$\Delta_{max} = \frac{11.65 \times 30\,000 \times 60 \times 1\,728}{29\,000\,000 \times 1\,514} = 0.825 \text{ in}$$

$$\Delta_1 = \frac{6.45 \times 30\,000 \times 60 \times 1\,728}{29\,000\,000 \times 1\,514} = 0.457 \text{ in}$$

Fig. 5. Graphical Method of Determining Deflection. Example 1

load line a—n with a pole distance H = 30 000 lb. In this example the ray On was taken horizontal in order to make the closing string nO horizontal in the bending moment diagram. Any other position of O and string nO would have given the same numerical results but with the bending moment diagram turned in a different position. For any position of the bending moment diagram the intercepts are always measured vertically (parallel to the section about which moments are desired).

The maximum bending moment, at the support, is

$$M = v_{max} \cdot H = 8.1 \times 30\,000 = 243\,000 \text{ lb-ft}$$

The moment diagram is divided into 9 sections as shown, and the area of each section measured in square feet to the space scale of span and the vector representing each area magnitude placed at the center of gravity of its respective area. These area vectors comprise the second load system and the area vector load line m'—a' constructed.

In this example the area vector load line is inverted in order to obtain the second moment or deflection diagram in its true position. If the area vector load line had not been inverted the deflection diagram would have been inverted, but the numerical results as shown in Fig. 5 would not have been altered.

Referring to Equation (3) and Table I, the values of β for approximate deflection values for an irregularly loaded cantilever beam, the following results are obtained:

S_{1-1} for given beam = 168.2 in³

$$f_{max} = \frac{243\,000 \times 12}{168.2} = 17\,300 \text{ lb per sq in}$$

then:

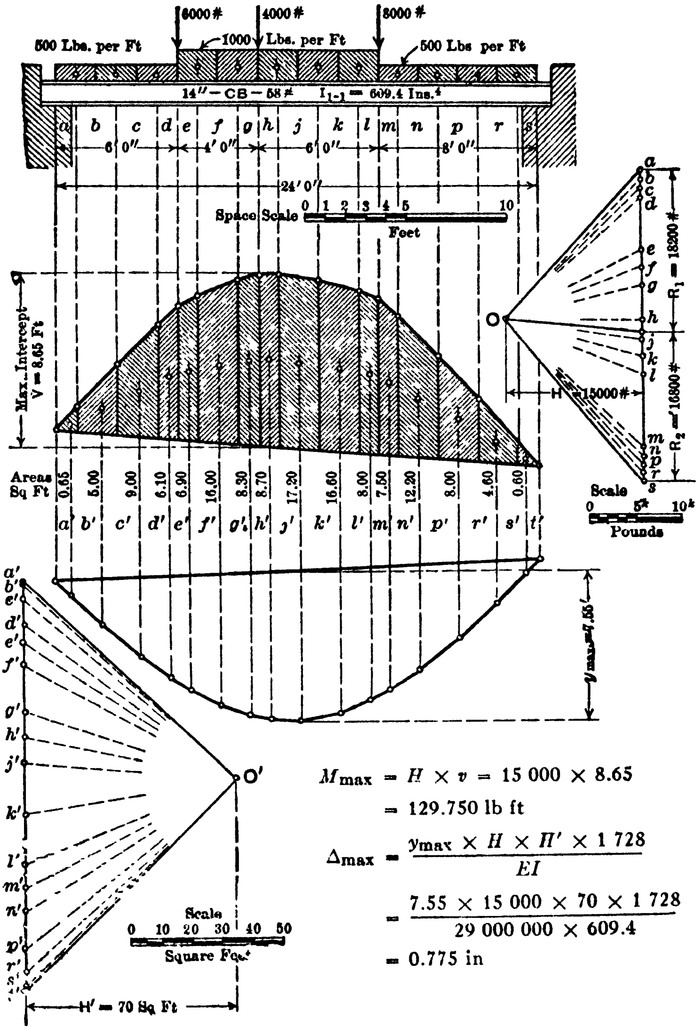
$$\Delta_{max} = \frac{17\,300 \times \overline{18}^2 \times 144}{9\,(3.5)\,29\,000\,000} = 0.830 \text{ in}$$

Example 2. Let it be required to determine the deflection of the beam loaded as shown in Fig. 6. The bending moment diagram for the given load system is constructed in the usual manner, described above, from the load line a—s and pole O, with a pole distance (taken in this case) equal to 15 000 lb. The position of O, in this case, taken at random produces a moment diagram in which the base or closing line is rotated, as shown.

The intercepts are measured vertically regardless of the position of the moment diagram, and the procedure is the same as outlined in the preceding example.

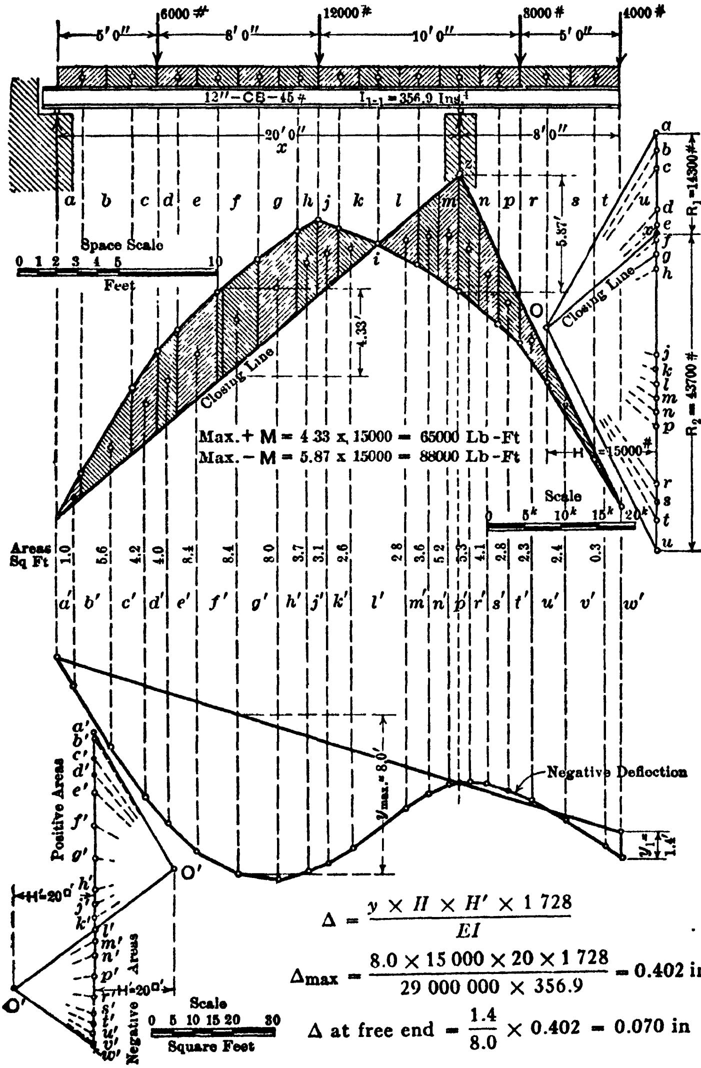
The position of O' with respect to the moment-area-vector load line, also selected at random, produces a deflection diagram which is rotated from its true horizontal position. If a ray be drawn through O' parallel to the closing string (or base line) of the deflection diagram to intersect the area load line, a horizontal ray through this intersection is the locus of poles O' such as would produce deflection diagrams with horizontal base lines. This construction is unnecessary since the vertical intercepts at any point are equal for all positions of O' as long as H' is a constant value.

Example 3. The beam illustrated in Fig. 7 is a typical example of a span with an overhanging end. The equilibrium polygon is constructed from the load line a—u and pole O in the usual manner. The ray Ou is a component of the end load and also of R_2, hence it is drawn across the space between the end load and R_2 to intersect R_2, thus locating point Z on the closing line.



$$M_{max} = H \times v = 15\,000 \times 8.65$$
$$= 129.750 \text{ lb ft}$$
$$\Delta_{max} = \frac{y_{max} \times H \times H' \times 1\,728}{EI}$$
$$= \frac{7.55 \times 15\,000 \times 70 \times 1\,728}{29\,000\,000 \times 609.4}$$
$$= 0.775 \text{ in}$$

Fig. 6. Graphical Method of Determining Deflection. Example 2



$$\Delta = \frac{y \times H \times H' \times 1\,728}{EI}$$

$$\Delta_{max} = \frac{8.0 \times 15\,000 \times 20 \times 1\,728}{29\,000\,000 \times 356.9} = 0.402 \text{ in}$$

$$\Delta \text{ at free end} = \frac{1.4}{8.0} \times 0.402 = 0.070 \text{ in}$$

Fig. 7. Graphical Method of Determining Deflection. Example 3

The correct position for O in order to produce a moment diagram with a horizontal closing line is given in the preceding example; however, this additional construction is unnecessary. It should be noted that the moment diagram lies partly on one side of the closing line and partly on the other and passes through zero at i, the inflection point.

The areas of the moment diagram above and to the left of the closing line will be considered positive and those below and to the right negative.

Where the sign of the moment areas change, the slope of the deflection diagram will change in sign. This condition is readily provided for by transposing the pole position O' from the side of the moment-area-vector load line where it is placed for positive areas, to the opposite side for negative areas.

The ray corresponding to the string of the equilibrium polygon which crosses the space in which the inflection point is located (in the example ol') is then common to both pole positions.

Two points of zero deflection, namely, the supports, define the closing line of the deflection diagram.

It should be noted that the loading on the overhanging end may not be sufficient in magnitude to cause downward deflection throughout the overhang, or in other words, the magnitude of the loads between the supports may be sufficient to cause the overhanging end to deflect upwards. In the example a portion of the overhanging end does deflect upward as noted.

CHAPTER XVIII

STRENGTH AND STIFFNESS OF RESTRAINED, FIXED AND CONTINUOUS BEAMS

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1. Definitions

Restrained Beams. In the case of a simple beam, the shortening of the fibers on the compressive side and the lengthening of the fibers on the tension side of the neutral plane result in a curvature of the same character throughout the span, generally convex downward.

A RESTRAINED BEAM SPAN is one in which a constraint is introduced, at or by a support, of sufficient magnitude to reverse the character of the curvature of the beam at that support from that which would exist if the beam were simply supported.

A beam span may be restrained at one end and simply supported at the other or it may be restrained at both supports. Beams overhanging a support are restrained at that support when the length of, and load on, the overhanging portion are sufficient to produce a reversal in the character of the curvature from that produced by the loads between supports.

At a support where restraint exists there is, therefore, a reversal in the characters of fiber-stress on the two sides of the neutral plane. That portion of the beam which was subjected to compression in the central zone of the span is subjected to tension at and near the restrained support.

The point where the curvature changes from positive to negative, and where the fiber-stresses change from compression to tension, is called the INFLECTION POINT. At this point the BENDING MOMENT passes through a zero value, since at this point where the curvature changes character, there the bending moment must do likewise.

Fixed Beams. A FIXED BEAM SPAN is one that is fully restrained, or one in which the slope of the elastic curve, at the fixed support, remains unchanged from its original position in the unloaded beam (usually horizontal). Beams may be fixed at one or both supports.

When a beam is fixed at both supports the angular change between the tangents to the elastic curve at the two supports must equal zero. In Chapter XVII the angular change between any two tangents to the elastic curve was shown to be

$$\Theta = \frac{A_m}{EI}$$

For a fully restrained, or fixed beam, therefore:

$$\frac{\sum_0^L A_m}{EI} = 0 \tag{1}$$

in which $\sum_0^L A_m$ represents the net sum of the positive and negative moment areas throughout the span.

An isolated fixed beam span is not a common occurrence in practice; however, partially fixed or restrained beams are often encountered, especially in reinforced-concrete structures.

Continuous Beams. A CONTINUOUS BEAM is one which has three or more supports, usually at the same level. The end spans of continuous beams are said to be OVERHANGING when they project beyond the first interior support and are without support at their outer ends; SIMPLY SUPPORTED when the outer end rests upon the support without restraint, and RESTRAINED when the outer support provides constraint.

A single-span beam with fixed ends may be regarded as a special case of a three-span continous beam with the two end spans indefinitely shortened.

2. General Considerations of Restrained and Continuous Beams

The MAGNITUDES of the moments and shears throughout a restrained or continuous span depend upon the magnitude of the restraining moment at each support at which the span is restrained or over which it is continuous. The existence of an end moment in any span prevents a direct determination of the moments and shears throughout the span by the usual methods of statics. For a single span, fully restrained at both supports and symmetrically loaded, the reactions and hence the shears may be determined by SYMMETRY; however, the moments still remain indeterminate statically.

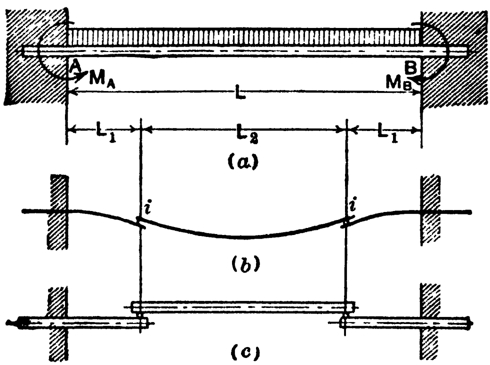


Fig. 1. Fully Restrained Beam Uniformly Loaded

When a single span is fully restrained at both supports and loaded with a uniformly distributed load as in Fig. 1(a), the bending moment at any section at a distance x from the left support is:

$$M_x = R_1 \cdot x - M_A - \frac{wx^2}{2}$$

in which M_A, the unknown fixing moment, is represented as a negative moment since its rotational tendency is counter-clockwise at the left support. From the general definition of restrained beams it is apparent that a certain

portion of the beam span adjacent to each of the fixed supports will possess the general properties and characteristics of a CANTILEVER beam as shown at (*b*), Fig. 1; the beam may, therefore, be regarded as a short central span supported at its ends *i—i* (the inflection points) by short cantilevers.

In order to determine the unknown FIXING MOMENTS, the positions of the inflection points (length of end cantilevers) must first be determined.

It has been shown in the preceding discussion, Equation (1), that the change in slope of the elastic curve between any two points of any span is equal to the moment area between these points divided by EI, and this relation will provide the additional equation necessary for the determination of the unknown end moments.

3. Shears, Moments and Deflections for Fully Restrained Beams

By definition, the FULLY RESTRAINED BEAM is one in which the tangents to the elastic curve at the supports remain unchanged in position as the beam deflects under load, or as stated by Equation (1) the total negative moment area equals the total positive moment area. When the loading on a fully restrained span is symmetrical with respect to the center line of the span, it is evident therefore, that the negative moment area over half the span equals the positive moment area over half the span.

Case I

Fully Restrained Beam, Concentrated Load at Center of Span.

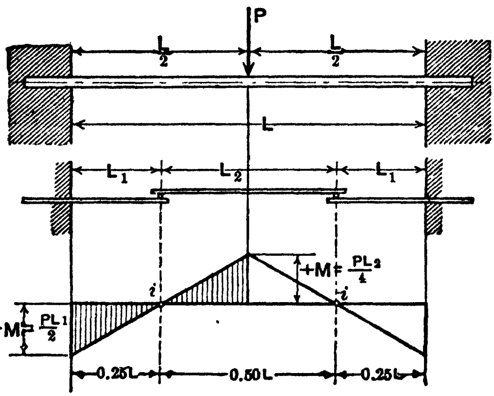


Fig. 2. Fully Restrained Beam, Concentrated Load at Center of Span
Case I

$$\text{Moment area for end cantilever} = \frac{P}{2}\cdot L_1 \times \frac{L_1}{2} = \frac{PL_1^2}{4}$$

$$\text{Moment area for half central span} = \frac{P}{2}\cdot\frac{L_2}{2} \times \frac{L_2}{4} = \frac{PL_2^2}{16}$$

and since these areas must be equal:

$$\frac{PL_1^2}{4} = \frac{PL_2^2}{16} \quad \text{or} \quad L_1 = \frac{L_2}{2}$$

whence:
$$L_1 = \frac{L}{4} = \frac{L_2}{2} \tag{2}$$

The maximum positive bending moment at the center of the central span is:
$$+M = \frac{PL_2}{4} = \frac{PL}{8} \tag{3}$$

The maximum negative bending moment or the cantilever moment is:
$$-M = \frac{PL_1}{2} = \frac{PL}{8} \tag{4}$$

The maximum vertical deflection of the central span is:
$$\Delta_2 = \frac{PL_2^3}{48\,EI} = \frac{P\left(\frac{L}{2}\right)^3}{48\,EI} = \frac{PL^3}{384\,EI}$$

with respect to the free end of the cantilever.

The maximum vertical deflection of the cantilever is:
$$\Delta_1 = \frac{\frac{P}{2}L_1^3}{3\,EI} = \frac{\frac{P}{2} \times \left(\frac{L}{4}\right)^3}{3\,EI} = \frac{PL^3}{384\,EI}$$

The total maximum deflection at the center of the beam, with respect to the support, is:
$$\Delta_{max} = \Delta_1 + \Delta_2 = \frac{PL^3}{192\,EI} \tag{5}$$

From a comparison of these values with those for a free end beam similarly loaded (Fig. 3, Case V, Chapter XVII) it will be seen that the maximum bending moment is reduced one-half and the deflection one-fourth when the ends are fully restrained.

Case II

Fully Restrained Beam, Uniform Load throughout the Span.

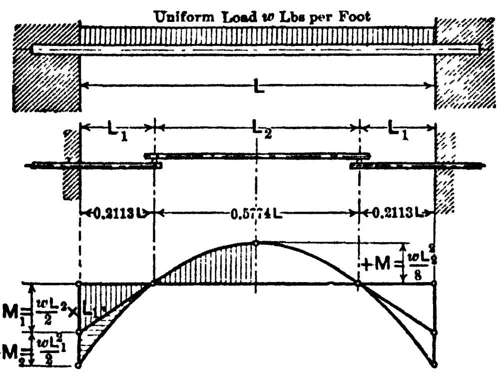


Fig. 3. Fully Restrained Beam with Uniform Load. Case II

$$\text{Moment area for end cantilevers} = \frac{wL_2}{2} \cdot \frac{L_1^2}{2} + \frac{1}{3}\,\frac{wL_1^2}{2} \cdot L_1$$

(The second term is the area of the ex-parabolic portion.)

$$\text{Moment area for half central span} = \left(\frac{2}{3}\cdot\frac{wL_2^2}{8}\cdot L_2\right)\frac{1}{2}$$

Since these areas must be equal as explained above:

$$\frac{wL_2L_1^2}{4} + \frac{wL_1^3}{6} = \frac{wL_2^3}{24}$$

or

$$6\,L_2L_1^2 + 4\,L_1^3 = L_2^3$$

But

$$L_2 = L - 2L_1$$

Whence, by substitution and reduction:

$$L^2 - 6LL_1 + 6L_1 = 0$$

or

$$L = 4.73\,L_1 \quad \text{and} \quad L_1 = 0.211\,L \tag{6}$$

then

$$L_2 = L - 2\,(0.211\,L) \quad \text{or} \quad L_2 = 0.577\,L \tag{7}$$

The maximum positive bending moment at the center of the central span is:

$$+M = \frac{wL_2^2}{8} = \frac{w\,(0.577\,L)^2}{8} = \frac{wL^2}{24} \tag{8}$$

The maximum negative bending moment, or the cantilever moment, is:

$$-M = \frac{wL_2}{2}\cdot L_1 + \frac{wL_1^2}{2}$$

$$= \frac{w}{2}(0.577\,L \times 0.211\,L + \overline{0.211\,L}^2) = \frac{wL^2}{12} \tag{9}$$

The maximum vertical deflection of the central span is:

$$\Delta_2 = \frac{5}{384}\,\frac{(0\ 577\ L)\ w\ (0\ 577\ L)^3}{EI} = \frac{wL^4}{689\ EI}$$

The deflection of the end cantilever due to end load of one-half central span load is:

$$\Delta_1 = \frac{(0.289)\ wL\ (0.211\ L)^3}{3\ EI} = \frac{wL^4}{1\ 105}$$

The deflection of the end cantilever due to uniform load is:

$$\Delta_1' = \frac{(0.211\ L)\ w\ (0.211)^3}{8\ EI} = \frac{wL^4}{4\ 038\ EI}$$

The total vertical deflection at the center of the center span is:

$$\Delta_{max} = \Delta_1 + \Delta_1' + \Delta_2 = \frac{wL^4}{384\ EI} = \frac{WL^3}{384\ EI} \tag{10}$$

Thus, when the ends of a uniformly loaded beam are fully restrained, the maximum positive bending moment is ⅓, and the maximum vertical deflection is ⅕ of the corresponding values for a similarly loaded span with free ends. The maximum negative moment at the supports is ⅔ the value of the maximum positive moment for a similarly loaded span with free ends.

Case III

Fully Restrained Beam, with Irregular Loading.

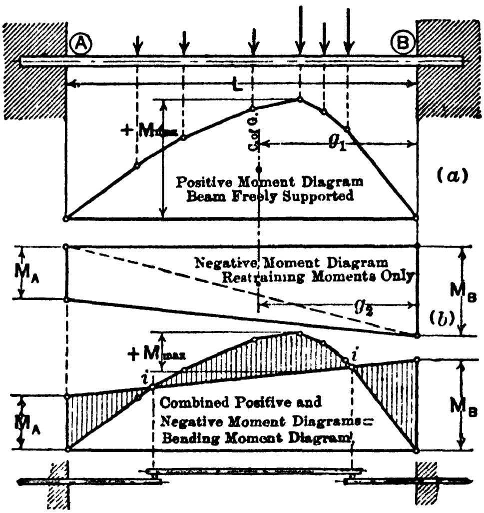


Fig. 4. Fully Restrained Beam with Irregular Loading. Case III

In this, as in the preceding cases, the plus and minus area of the bending moment diagram, from A to B, must equal zero, since the tangential rotation at B with respect to A is zero for all fully restrained beams. Moreover, for any condition of loading and with unyielding supports, the deflection of B with respect to A is equal to zero. When the bending moment area is symmetrical with respect to the center line of span it is unnecessary to employ the second consideration, since the net moment areas in such cases are zero for each half of the span, as in Cases I and II.

The two essential conditions, as outlined above, may be stated as follows:

$$(1) \quad \frac{Am\,(A \text{ to } B)}{EI} = 0 \tag{11}$$

$$(2) \quad \frac{Am\,(A \text{ to } B) \times Lc}{EI} = 0 \tag{12}$$

The resultant moment diagram for any restrained beam may be divided into two parts: (*a*) the simple bending moment diagram, as shown in Fig. 4(*a*), and (*b*) the negative bending moment diagram, as shown in Fig. 4(*b*). From Equation (11) it is evident that the positive moment area equals the negative moment area, and also from Equation (12) it is seen that the center of gravity of the positive and negative moment areas must lie on the same vertical line, or $g_1 = g_2$ (Fig. 4).

The negative moment area (*b*), Fig. 4, may always be divided into two triangular areas with bases M_A and M_B and if $+A_m$ represents the area of the

bending moment diagram for a simple span, as at (*b*), then Equation (12), for the deflection of B with respect to A, gives:

$$-\left(\frac{M_B L}{2} \times \frac{L}{3} + \frac{M_A L}{2} \times \frac{2}{3} L\right) + A g_1 = 0$$

or

$$\frac{M_B L^2}{6} + \frac{M_A L^2}{3} = A g_1 \tag{13}$$

From Equation (11):

$$A = \frac{M_B + M_A}{2} \cdot L \quad \text{or,} \quad M_A + M_B = \frac{2\,A}{L} \tag{14}$$

whence

$$M_B = \frac{2\,A}{L} - M_A \tag{15}$$

and

$$M_A = \frac{2\,A}{L} - M_B \tag{16}$$

Substituting the value of M_B from (14) in (13):

$$M_A = \frac{2\,A}{L^2}(3g_1 - L) \tag{17}$$

In like manner Equations (13) and (15) give:

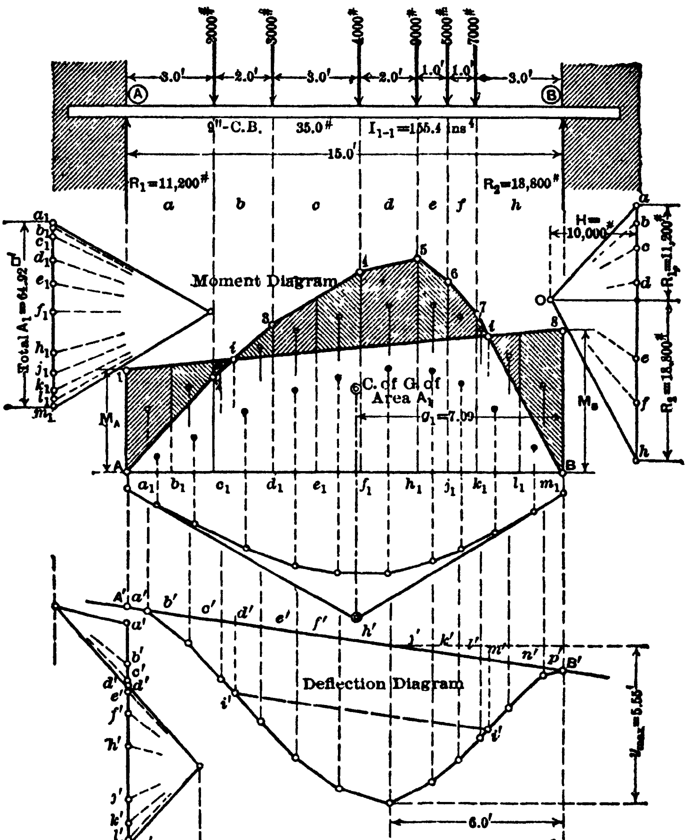
$$M_B = \frac{2\,A}{L^2}(2\,L - 3g_1) \tag{18}$$

When the load (hence the simple span moment area) is symmetrical with respect to the center line of the span, $g_1 = g_2 = \frac{L}{2}$ and $M_A = M_B = \frac{A}{L}$, or the end moments are equal to the average bending moment for a freely supported span, and the maximum positive bending moment is equal to the maximum bending moment minus the average moment for the freely supported span. The change in the moment conditions, from those existing when the beam is freely supported, amounts to a shifting of the base or reference line, to occupy the position determined by M_A and M_B, the end moments.

The new base line is the right line connecting the ordinates M_A and M_B, as in Fig. 4, and its position, in the case of a fully restrained beam, is such that the area included between it and the original base line is equal to, and with its centroid on the same vertical line as, the moment diagram for the freely supported beam.

Deflection of Fully Restrained Beams with Unsymmetrical Loading. The mathematical procedure involved in the determination of deflections for restrained beams with unsymmetrical loading is quite complex; however, the graphical method explained in Chapter XVII provides a simple and reasonably accurate method for the solution of such problems after the moment diagrams have been determined. The values of M_A and M_B can readily be determined by Equations (17) and (18) after the bending moment diagram for the freely supported span has been constructed. The deflection diagram may then be drawn, as the second moment diagram, in the usual manner.

Example 1. Consider the fully restrained span, loaded as shown in Fig. 5. The moment diagram for the unrestrained span (A, 2—7, B) is constructed from the load line A—h with the pole distance H = 10 000 lb.



Summary of Results:

$$M_A = \frac{2\,A_1}{L^2}(3g_1 - L) = \frac{129\ 84}{225}(21.27 - 15)$$

$= 3.617'$ for intercept on moment diagram

$= 3.617 \times 10\ 000 = 36\ 170$ lb-ft

$$M_B = \frac{2\,A_1}{L^2}(2\,L - 3g_1) \times 10\ 000$$

$$= \frac{129.84}{225}(30 - 21.27) = 50\ 363 \text{ lb-ft}$$

$$\Delta_{max} = \frac{y_{max}\,HH' \times 1\ 728}{EI}$$

$$= \frac{5.55 \times 10\ 000 \times 7.5 \times 1\ 728}{29\ 000\ 000 \times 155.4}$$

$= 0.160$ in at $6.0'$ from right end

Fig. 5. Fully Restrained Beam with Irregular Loading. Example 1

In order to determine the position of the vertical line through the centroid of the unrestrained moment area this moment diagram is divided into a number of convenient areas as shown and the area-vector load line drawn to any scale as at a_1—m_1. The equilibrium polygon immediately below the moment diagram, drawn from this area-vector load line with any pole, O_1, locates the resultant, or vertical line through the centroid of the simple moment area, thus determining g_1, and since ΣA ($= a_1$ to m_1) has been determined, the necessary data for the solution of Equations (17) and (18) are known; referring to Fig. 5, the relative magnitudes of M_A and M_B in feet to the space scale of span, are:

$$M_A = \frac{2 \times 64.92}{15^2}(3 \times 7.09 - 15) = 3.6170 \text{ ft}$$

$$M_B = \frac{2 \times 64.92}{15^2}(2 \times 15 - 3 \times 7.09) = 5.0363 \text{ ft}$$

It should be noted that the values of L and g_1 are measured in feet to the space scale of span and A is measured in square feet to the same scale.

The values of M_A and M_B are laid off on the ordinates at A and B, respectively, to the space scale, and the true base line 1—8 drawn to complete the moment diagram for the fully restrained beam. The inflection points i and i are thus automatically obtained.

The true moment diagram, thus found, is subdivided into elementary areas and the moment-area vector load line a'—p', with the pole distance $H' = 7.5$ sq ft, is constructed as shown. From this load line the deflection polygon is drawn as explained in Chapter XVII.

The base or reference line $A'B'$ for the deflection diagram is a right line drawn through points of support, A' and B', at which the deflection is zero.

It should be noted that the broken line connecting the deflected position of the inflection points is parallel to the base line $A'B'$. Thus, in a fully restrained beam, the deflections of the inflection points are equal.

4. Comparison of Freely Supported and Restrained Beams

The MAXIMUM BENDING MOMENTS for beams which are fully restrained at both supports are considerably less, and the strength correspondingly greater, than the same beam freely supported.

Thus, for a uniform load of W, the bending moment for a freely supported beam is $\frac{WL}{8}$ and for a fully restrained beam the maximum (negative) bending moment is $\frac{WL}{12}$. The strength of the fully fixed beam is, therefore, 50% greater than that of the freely supported beam.

For a single concentrated load of P at the center of the span, $M = \frac{PL}{4}$ for the freely supported beam, and for a fully restrained beam $M = \frac{PL}{8}$ (both positive and negative); hence the strength of the latter is 100% greater than that of the former.

With respect to STIFFNESS, the advantage of the restrained beams is much greater than the advantage in strength. The deflection of a freely supported

span under uniform load is five times as great as for the same span and load when the ends of the beam are fully restrained.

For a single concentrated load at the center of span the DEFLECTION of a freely supported beam is four times as great as that of the same beam when the ends are fully restrained.

5. Shears and Moments for Continuous Beams

The theory of CONTINUOUS BEAMS presented in this article includes only those of constant cross-section, with all supports at the same level and the outer ends of ends spans freely supported. At each intermediate support there exists a negative moment and for each interior span, where the loading and span length of adjacent spans do not vary greatly, there are two points of inflection. When an END SPAN is relatively short, the reaction at the outer

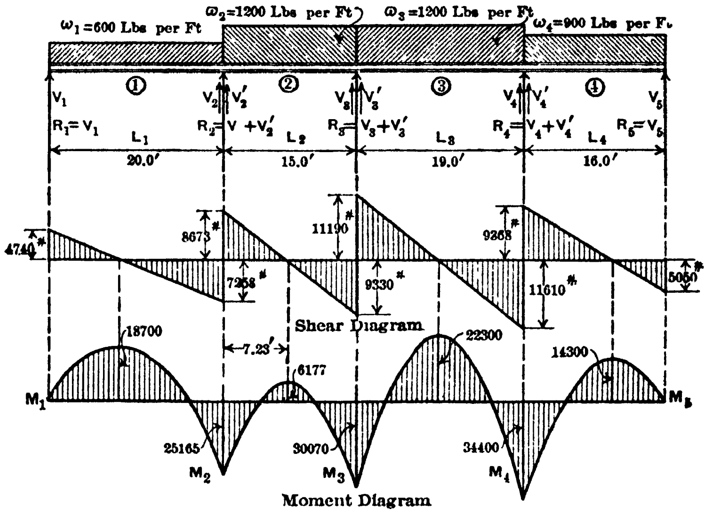


Fig. 6. Continuous Beam with Uniformly Distributed Loads

end may become negative, and when an INTERIOR SPAN is relatively short the bending moment may become negative throughout that span.

The end spans of a continuous beam are somewhat similar to a simple beam with one overhanging end, while the intermediate spans are somewhat similar to restrained beams.

The SHEARS, and, therefore, the REACTIONS, cannot be determined until the VALUES OF MOMENTS are known. The fundamental problem involved in the design and investigation of continuous beams consists of the determination of the negative moments at the supports. The solution of the problem requires, in addition to the equations of STATIC EQUILIBRIUM, the general equation of the ELASTIC CURVE, and the result, known as CLAPEYRON'S THEOREM OF THREE MOMENTS, is as follows, for uniform loading:

Referring to Fig. 6 for notation:

$$M_1L_1 + 2\,M_2\,(L_1 + L_2) + M_3L_2 = -\,\frac{w_1L_1^3}{4} - \frac{w_2L_2^3}{4} \qquad (19)$$

For concentrated loads the same theorem is as follows:

Referring to Fig. 7 for notation:

$$M_1L_1 + 2\,M_2\,(L_1 + L_2) + M_3L_2$$
$$= -\,\Sigma P_1L_1^2[n(1-n)\,(1+n)] - \Sigma P_2L_2^2\,[n(1-n)\,(2-n)] \quad (20)$$

where ΣP_1 and ΣP_2 indicate summation of values of all concentrated loads in the given span, each of which is multiplied by the bracket quantity containing the proper n value for each concentrated load.

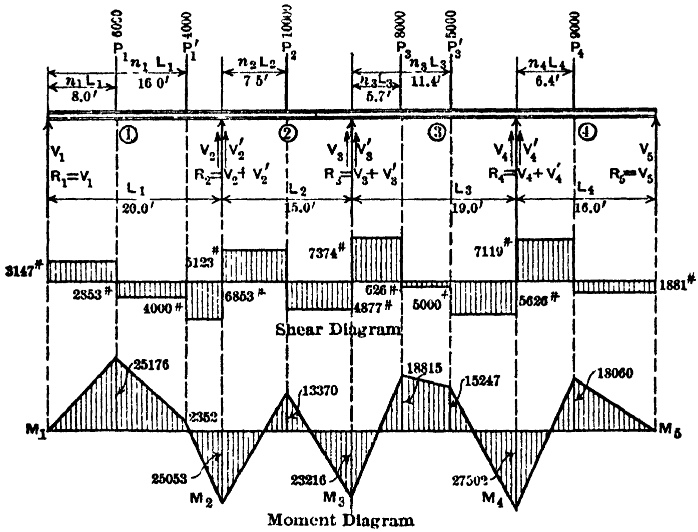


Fig. 7. Continuous Beam with Concentrated Loads

Example of the Application of the Three Moment Theorem. Uniform Loads. In Fig. 6, let the following numerical values be assigned:

$$w_1 = 600 \text{ lb per ft; } L_1 = 20.0 \text{ ft}$$
$$w_2 = 1\,200 \text{ lb per ft; } L_2 = 15.0 \text{ ft}$$
$$w_3 = 1\,200 \text{ lb per ft; } L_3 = 19.0 \text{ ft}$$
$$w_4 = 900 \text{ lb per ft; } L_4 = 16.0 \text{ ft}$$

and $M_1 = M_5 = 0$.

Equation (19) applied to spans (1) and (2) gives:

$$20\,M_1 + 70\,M_2 + 15\,M_3 = -\frac{600 \times \overline{20}^3}{4} - \frac{1\,200 \times \overline{15}^3}{4}$$

or

$$0 + 70\,M_2 + 15\,M_3 = -\,2\,212\,500 \quad (a)$$

Equation (19 applied to spans (2) and (3) gives:

$$15\,M_2 + 68\,M_3 + 19\,M_4 = -\frac{1\,200 \times \overline{15}^3}{4} - \frac{1\,200 \times \overline{19}^3}{4}$$

or

$$15\,M_2 + 68\,M_3 + 19\,M_4 = -\,3\,070\,200 \qquad (b)$$

Equation (19) applied to spans (3) and (4) gives:

$$19\,M_3 + 70\,M_4 + 16\,M_5 = -\frac{1\,200 \times \overline{19}^3}{4} - \frac{900 \times \overline{16}^3}{4}$$

or

$$19\,M_3 + 70\,M_4 + 0 = -\,2\,979\,300 \qquad (c)$$

The three Equations (*a*), (*b*) and (*c*) can be solved for the three unknowns, M_2, M_3 and M_4. Multiply Equation (*b*) by 3 684 and subtract from Equation (*c*), eliminating M_4, whence:

$$-55\cdot 4\,M_2 - 232\,M_3 = +\,8\,370\,700 \qquad (d)$$

Multiply Equation (*a*) by 15.47 and subtract from Equation (*d*), eliminating M_3, whence:

$$-1\,027\cdot 5\,M_2 = +\,25\,856\,700$$

or

$$M_2 = -\,25\,165 \text{ lb-ft}$$

Substituting the value of M_2 in Equation (*a*):

$$M_3 = -\,30\,070 \text{ lb-ft}$$

Substituting the value of M_3 in Equation (*c*):

$$M_4 = -\,34\,400 \text{ lb-ft}$$

The values of the reactions can be found by the method of moments since the moments at the supports are now known:

To find R_1, take the center of moments at R_2, then:

$$20\,R_1 - (600 \times 20)\,10 = -\,25{,}165$$

or

$$R_1 = 4\,742 \text{ lb}$$

To find R_2, take the center of moments at R_3, then:

$$(4\,742)\,35 + 15\,R_2 - (600 \times 20)\,25 - (1\,200 \times 15)\,7.5 = -\,30\,070$$

or

$$R_2 = 15\,931 \text{ lb}$$

The value of R_2 can be found more easily by assuming the beam to be cut just to the right of the support at R_2. On the free end there exists a counterclockwise moment M_2 and the shear V'_2, then by taking moments about R_3:

$$15\,V'_2 - (1\,200 \times 15)\,7.5 - 25\,165 = -\,30\,070$$

whence

$$V'_2 = 8\,673 \text{ lb}$$

and since

$$V_2 = (600 \times 20) - R_1 = 12\,000 - 4\,742 = 7\,258 \text{ lb}$$

$$R_2 = V_2 + V'_2 = 7\,258 + 8\,673 = 15\,931 \text{ lb}$$

The values of R_3, R_4 and R_5 were calculated in a similar manner and a summary of the results is:

$$\begin{aligned} R_1 &= 4\,742 \text{ lb} \\ R_2 &= 15\,931 \text{ lb} \\ R_3 &= 20\,525 \text{ lb} \\ R_4 &= 20\,978 \text{ lb} \\ R_5 &= 5\,050 \text{ lb} \\ \hline \Sigma R &= 67\,226 \text{ lb} \end{aligned}$$

The sum of reactions should just equal the total sum of loads or 67 200 lb The slight error is due to the omission of significant figures beyond the decimal point throughout the calculations.

The shear diagram, Fig. 6, may now be constructed and the points of maximum positive moments determined from the positions where the shears pass through zero values.

Thus, in span (2) the shear just to the right of support R_2 is 8 673 and the point of maximum moment is at 8 673/1 200 = 7.23 ft. from R_2.

The maximum positive moment at this section is:

$$+M_{\max} \text{ (span 2)} = 7.23\ V'_2 + M_2 - 1\,200 \left(\frac{\overline{7\ 23}^2}{2}\right)$$

$$= 62\,703 - 25\,165 - 31\,364$$

or

$$+M_{\max} = 6\,177 \text{ lb-ft}$$

The values of positive moments at all other points of any span can be determined in a similar manner, or may be found by taking the moments of the loads and reactions, as:

$$+M_{\max} \text{ (span 2)} = R_1\,(20 + 7.23) + R_2\,(7.23) - (600 \times 20)\ 17.23$$

$$-1\,200 \times \frac{\overline{7.23}^2}{2} = 6\,180 \text{ ft-lb}$$

Example of the Application of the Three Moment Theorem. Concentrated Loads. In Fig 7 let the following numerical values be assigned.

$n_1 = 0.4$	$P_1 = 6\,000$ lb	$L_1 = 20$ ft
$n'_1 = 0\ 8$	$P'_1 = 4\,000$ lb	$L_2 = 15$ ft
$n_2 = 0.5$	$P_2 = 10\,000$ lb	$L_3 = 19$ ft
$n_3 = 0.3$	$P_3 = 8\,000$ lb	$L_4 = 16$ ft
$n'_3 = 0.6$	$P'_3 = 5\,000$ lb	
$n_4 = 0.4$	$P_4 = 9\,000$ lb	
$M_1 = M_5 = 0$		

Equation (20) applied to spans (1) and (2) gives.

$$20\,M_1 + 70\,M_2 + 15\,M_3 = -\ 6\,000 \times \overline{20}^2\,(0.4 \times 0.6 \times 1\ 4)$$

$$-4\,000 \times \overline{20}^2\,(0.8 \times 0.2 \times 1.8) - 1\,000 \times \overline{15}^2\,(0.5 \times 0.5 \times 1.5)$$

or, Equation (20) applied to spans (2) and (3) gives:

$$0 + 70\,M_2 + 15\,M_3 + -\ 2\,101\,950 \qquad (a)$$

$$15 M_2 + 68 M_3 + 19 M_4 = -10\,000 \times \overline{15}^2 (0.5 \times 0.5 \times 1.5)$$
$$-8\,000 \times \overline{19}^2 (0.3 \times 0.7 \times 1.7) - 5\,000 \times \overline{19}^2 (0.6 \times 0.4 \times 1.4)$$

or

$$15 M_2 + 68 M_3 + 19 M_4 = -2\,481\,246 \qquad (b)$$

Equation (20) applied to spans (3) and (4) gives:

$$19 M_3 + 70 M_4 + 16 M_5 = -8\,000 \times \overline{19}^2 (0.3 \times 0.7 \times 1.3)$$
$$-5\,000 \times \overline{19}^2 (0.6 \times 0.4 \times 1.6) - 9\,000 \times \overline{16}^2 (0.4 \times 0.6 \times 1.6)$$

or

$$19 M_3 + 70 M_4 + 0 = -2\,366\,280 \qquad (c)$$

Equations (*a*), (*b*) and (*c*) can be solved to determine the values of the unknown moments M_2, M_3 and M_4, as follows:

Multiply (*b*) by 3.684 and subtract from (*c*), whence:

$$-55.4 M_2 - 232 M_3 = +6\,774\,620 \qquad (d)$$

Multiply (*a*) by 15.47 and subtract from (*d*), whence:

$$M_2 = -25\,053 \text{ lb-ft}$$

Substituting the value of M_2 in (*a*):

$$M_3 = -23\,216 \text{ lb-ft}$$

Substituting the value of M_3 in (*c*):

$$M_4 = -27\,502 \text{ lb-ft}$$

The values of the various reactions may be found as in the preceding example by the method of moments, thus:

With the center of moments at R_2;

$$20 R_1 - 6\,000 \times 12 - 4\,000 \times 4 = -25\,053$$

whence

$$R_1 = 3\,147 \text{ lb}$$

and

$$V_2 = 10\,000 - 3\,147 = 6\,853 \text{ lb}$$

Taking the center of moments at R_3:

$$15 V'_2 + M_2 - 10\,000 \times 7.5 + M_3 = 0$$

or

$$15 V'_2 = +25\,053 + 75\,000 - 23\,216$$
$$V'_2 = 5\,123 \text{ lb}$$

then

$$R_2 = V_2 + V'_2 = 6\,853 + 5\,123 = 11\,976 \text{ lb}$$

The remaining reactions are found in a similar manner and a summary of the results follows:

$$R_1 = 3\,147 \text{ lb}$$
$$R_2 = 11\,976 \text{ lb}$$
$$R_3 = 12\,251 \text{ lb}$$
$$R_4 = 12\,745 \text{ lb}$$
$$R_5 = 1\,881 \text{ lb}$$

$$\Sigma R = 42\,000 \text{ lb} = \Sigma \text{ Loads}$$

The complete shear diagram and the moment diagram calculated from the preceding results in the manner explained in the foregoing example are shown in Fig. 7.

6. Continuous Beams with Outer Ends of End Spans Restrained

In the preceeding examples the outer ends of the end spans were assumed to be simply supported. If, however, the outer ends are restrained, Equations (19) and (20) are still applicable by assuming additional spans of infinitely short lengths beyond the restrained outer ends.

As an example, consider a continuous beam of two equal spans, loaded throughout with a uniform load of w lb per ft, and let the outer ends be restrained. Indefinitely short additional spans of L_0 will be assumed beyond each outer end and the end moments of these spans noted as $M_0 = 0$.

Then from Equation (19) for spans (0) and (1):

$$M_0L_0 + 2\,M_1\,(L + L_0) + M_2L = -\frac{wL_0^3}{4} - \frac{wL^3}{4}$$

or

$$2\,M_1L + M_2L = -\frac{wL^3}{4} \tag{a}$$

For spans (1) and (2):

$$M_1L + 2\,M_2\,(L + L) + M_3L = -\frac{wL^3}{4} - \frac{wL^3}{4}$$

or

$$M_1L + 4\,M_2L + M_3L = -\frac{wL^3}{2} \tag{b}$$

For spans (3) and (0)

$$M_2L + 2\,M_3\,(L + L_0) + M_0L = -\frac{wL^3}{4} - \frac{wL_0^3}{4}$$

or

$$M_2L + 2\,M_3L = -\frac{wL^3}{4} \tag{c}$$

Multiply (*b*) by 2 and subtract (*a*), then

$$7\,M_2L + 2\,M_3L = -\tfrac{3}{4}\,wL^3 \tag{d}$$

Multiply (*c*) by 7 and substract (*d*), then

$$12\,M_3L = -\,wL^3$$

or

$$M_3 = -\frac{wL^2}{12} = M_1$$

and from (*c*) by substituting the value of M_3

$$M_2 = -\frac{wL^2}{12}$$

Then, taking moments about the center support:

$$R_1L - \frac{wL^2}{2} = -\frac{wL^2}{12}$$

or

$$R_1 = \tfrac{5}{12}\,wL$$

and

$$R_2 = V_2 + V'_2 = 2 \times \tfrac{7}{12}\,wL = \tfrac{7}{6}\,wL$$

In reinforced-concrete construction the outer ends of end spans generally frame into and are poured integrally with a column or a girder. The outer end is, therefore, restrained to some extent depending upon the stiffness of the supporting member. The end moment may be as great as for a fully fixed span, as in the preceding example, or it may be some intermediate value between this and zero.

When the stiffness of the supporting member is great as compared to the stiffness of the beam the end moment will have a value varying from $\frac{1}{12}\,wL^2$ to about $\frac{1}{20}\,wL^2$, depending upon the stiffness ratio. When the conditions are reversed the value of the end moment will vary from about $\frac{1}{20}\,wL^2$ to zero.

7. Moment and Shear Coefficients for Continuous Beams of Equal Spans and Symmetrical Loading

Case I

Uniform Loads. The coefficient of wL^2 for positive and negative moments and the coefficients of wL for shears given in Fig. 8 were derived from Equation (19).

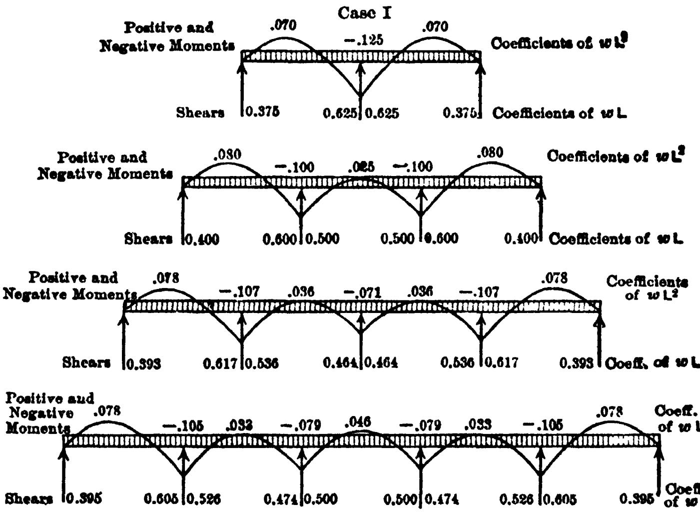


Fig. 8. Continuous Beams. Moment and Shear Coefficients, Uniform Loads, Equal Spans. Case I

Case II

Concentrated Center Loads of Equal Magnitude. In Fig. 9 the coefficients of PL for positive and negative moments and the coefficients of P for shears were derived from Equation (20).

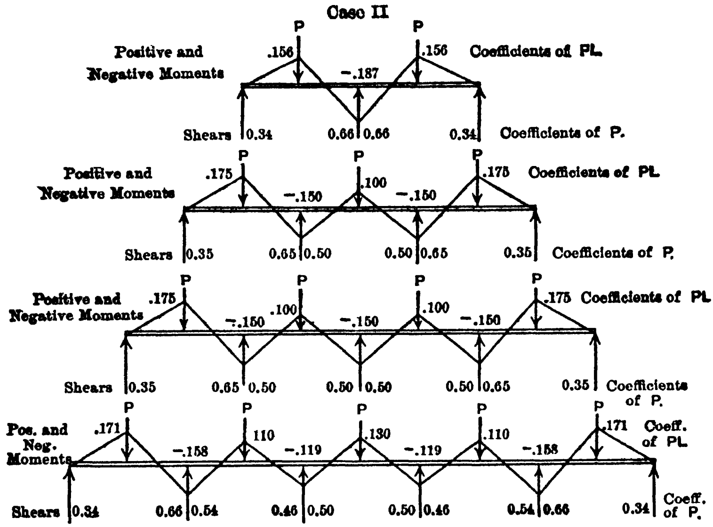


Fig. 9. Continuous Beams. Moment and Shear Coefficients, Concentrated Loads at Centers, Equal Spans. Case II

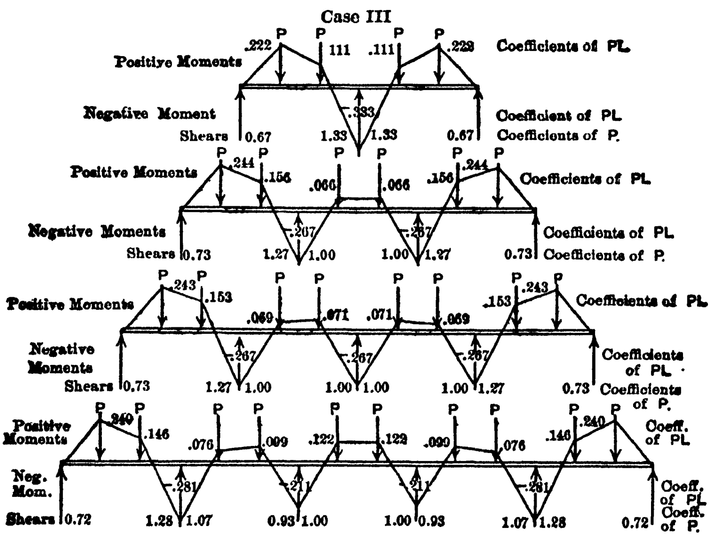


Fig. 10. Continuous Beams. Moment and Shear Coefficients, Equal Loads at Third Points, Equal Spans. Case III

Case III

Equal Concentrated Loads at the Third Points of Each Span. The coefficients of PL for positive and negative moments and of P for shears, shown in Fig. 10, were calculated as in the preceding case, from Equation (20).

Case IV

Equal Concentrated Loads at the Quarter Points of Each Span. The coefficients of PL for positive and negative moments and the coefficients of P for shears, from Equation (20), are given in Fig. 11.

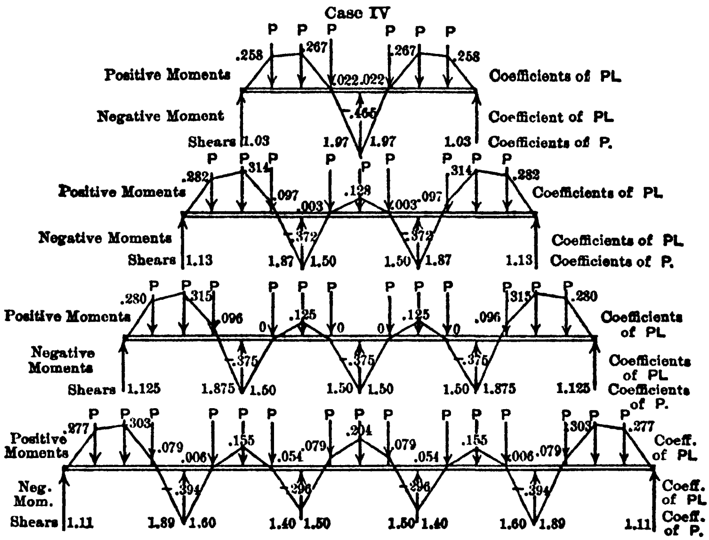


Fig. 11. Continuous Beams. Moment and Shear Coefficients, Equal Loads at Quarter Points, Equal Spans. Case IV

8. Continuous Beams with Variable Moment of Inertia

When, as sometimes happens in the case of continuous beams in reinforced-concrete construction, the MOMENTS OF INERTIA of the beam sections in the various spans are not equal, Equations (19) and (20) may be adjusted to take account of this variation as follows:

$$M_1L_1I_2+2M_2(L_1I_2+L_2I_1)+M_3L_2I_1=-\frac{w_1L_1^3I_2}{4}-\frac{w_2L_2^3I_1}{4} \qquad (19')$$

$$M_1L_1I_2+2M_2(L_1I_2+L_2I_1)+M_3L_2I_1=-\Sigma P_1L_1^2I_2[n(1-n)(1+n)]-$$

$$\Sigma P_2L_2^2I_1[n(1-n)(2-n)] \qquad (20')$$

9. Graphical Methods of Shears and Moments for Continuous Beams

The GRAPHICAL METHOD for the solution of moments in continuous beams, illustrated in Figs. 12 and 13, was originated by Professor T. Claxton Fidler, University College, Dundee, Scotland.*

The addition of the HOMOLOGY DIAGRAM, SS', which eliminates Professor Fidler's CUT-AND-TRY procedure, was developed by Professor A. Ostenfeld, University of Copenhagen.†

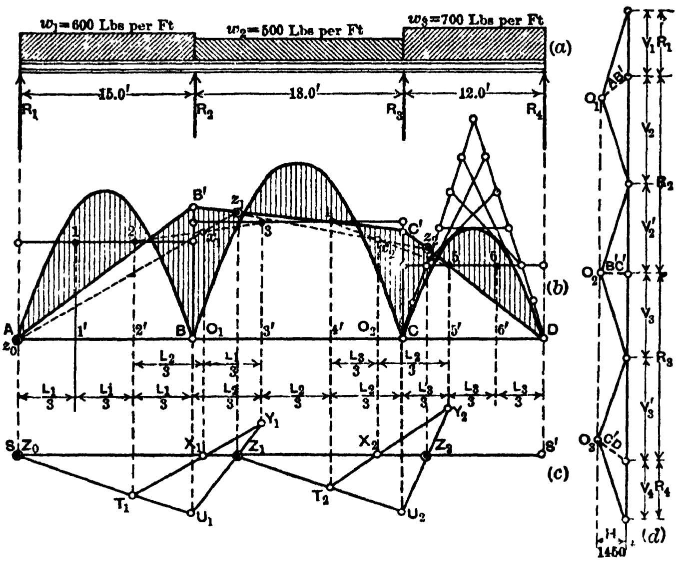


Fig. 12. Graphical Method of Determining Shears and Moments. Uniformly Distributed Loads

The discussion of the theory involved cannot, because of its length, be discussed here. For a detailed presentation of the subject, the reader is referred to A Practical Treatise on Bridge Construction by T. Claxton Fidler (Griffin's, London, 1909) and to the references given below.

The construction illustrated in Figs. 12 and 13 is a direct graphical solution of CAPEYRON'S THEOREM and the simple procedure required for the complete solution of both moments and shears for any continuous beam, loaded in any manner whatever, is as follows:

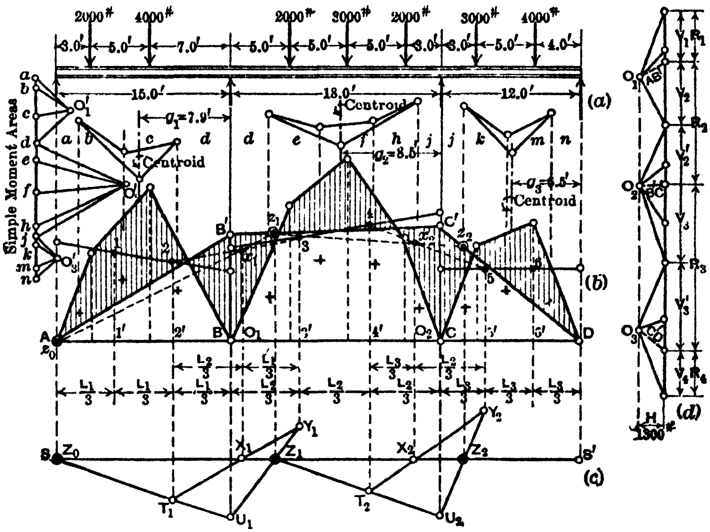
(1) Referring to Figs. 12 and 13: On any horizontal reference axis, AD, construct, at a convenient scale, the bending moment diagrams for each span, taken separately as an independent simple span. These bending moment diagrams can be conveniently constructed from a load line and ray diagram as illustrated at (*d*). When the load over any span if uniformly

* See Proceedings of Institution of Civil Engineers, Vol. 74 (1883), p. 196.

† See Teknisk Statik, Vol. 2, Second Edition, 1913.

distributed, the parabolic bending moment diagram can be constructed by the simple method illustrated in the third span of Fig. 12, where the tangents at C and D are the first and last rays, respectively, in the corresponding ray diagram.

(2) Consider that each span is fully restrained at both ends and construct the negative moment diagram using Equations (17) and (18) to determine the end moments.



AREAS IN SQ FT. SIMPLE M DIAGRAM

Span (1)	1	11 6		Span (2)	1	29 0		Span (3)	1	12.0	
	2	52 5			2	66.0			2	45 0	
	3	46 5	110.6		3	61 5			3	18.0	75.0
					4	13 2	170 7				

Fig. 13. Graphical Method of Determining Shears and Moments. Concentrated Loads

When the loading is uniform, the moment curve is a parabola and the moment area $= \frac{2}{3} \cdot M \cdot L$, and from Equations (17) and (18):

$$M_A = M_B = \frac{2\left(\frac{2}{3} ML\right)}{L^2} \cdot \left(3\frac{L}{2} - L\right) = \frac{2}{3} M$$

When the loading is unsymmetrical, as in Fig. 13, the centroid of the moment area must be located as explained for Case III, Article 3, and illustrated in Fig. 5.

(3) Divide each span into three equal lengths and locate points 1, 2; 3, 4; 5, 6; etc., on the base line of reference for fully restrained ends These third-point positions are called, by Professor Fidler, the CHARACTERISTIC POINTS.

(4) Connect the characteristic points adjacent to each support, such as 2, 3; 4, 5: etc.

(5) From each one-third point division of each span lay off along axis AD one-third the nearest adjacent span length, locating the points O_1, O_2, etc. That is,

$$2'O_1 = \frac{L_2}{3}; \quad 3'O_1 = \frac{L_1}{3}, \text{ etc.}$$

(6) Select any axis of homology SS' parallel to axis AD. Now the points Z along the axis of homology are the horizontal projections of points z on the true closing, or reference base line of the final moment diagram.

In Figs. 12 and 13, the outer ends of the end spans are assumed as unrestrained, hence the true base line will contain points A and D. At S, locate Z_0 and draw Z_0U_1 at any convenient angle, to intersect the line of action of reaction R_2. Project the second characteristic point, 2, vertically onto line Z_0U_1 locating point T_1. Project point O_1 (found as explained in 5) vertically to the axis of homology to locate point X_1. Connect T_1 and X_1 and produce to intersect the vertical through the first characteristic point, 3, of the next adjacent span, locating point Y_1. Connect Y_1 and U_1 locating Z_1 on the axis of homology.

To find the projection of Z_1 on the true base line: Project X_1 vertically onto line 2,3 locating x_1. Connect x_1 and the projection on the true base line of Z_0, or z_0, and prolong z_0x_1 to intersect the vertical through Z_1 in z_1 locating the desired point on the true base line.

From Z_1, which is homologous to Z_0, draw Z_1U_2 at any convenient angle to intersect the action line of R_3, and as before project the second characteristic point, 4, to locate T_2 on Z_1U_2; project O_2 to locate X_2 on the axis of homology. Prolong T_2X_2 to intersect a vertical through the first characteristic point of the next span in Y_2. Draw Y_2U_2 and locate Z_2.

Project O_2 vertically to intersect line 4, 5 in x_2; draw z_1x_2 (homologous to z_0x_1) and prolong to intersect the vertical through Z_2 in z_2.

Since the end moment at D is zero, connect D and z_2, two points on the true base line DC' of the moment diagram for span CD. Draw $C'z_1$ and prolong to B' to determine the true base line $C'B'$ for span BC. Draw $B'z_0$ which is the true base line for span AB.

These base lines, thus located, are the closing strings for the moment diagram polygons and as such will divide the load lines into the true end shears for each span. In (*d*) draw AB' parallel to the true base line AB'; $B'C'$ parallel to the true base line of span BC, and $C'D$ parallel to the true base line of span CD, thus dividing the load line into V_1 V_2 V'_2, etc., giving both shear and reaction values, as shown in (*d*).

Outer Ends of End Spans Restrained. The method outlined in the preceding article can be readily applied to continuous beams in which the outer ends of the end spans are fully restrained. Referring to Fig. 14, in which the outer ends A and E of end spans AB and DE, respectively, are assumed to be fully restrained:

(1) Assume imaginary spans of indefinitely short lengths extending outward from A and E.

(2) Carry out the same procedure for these imaginary spans as outlined in the preceding article.

(3) It is then evident that x_0 and z_0 are coincident with characteristic point No. 1, hence Z_0 is located on the axis of homology at the point where this axis is intersected by a vertical from characteristic point No. 1.

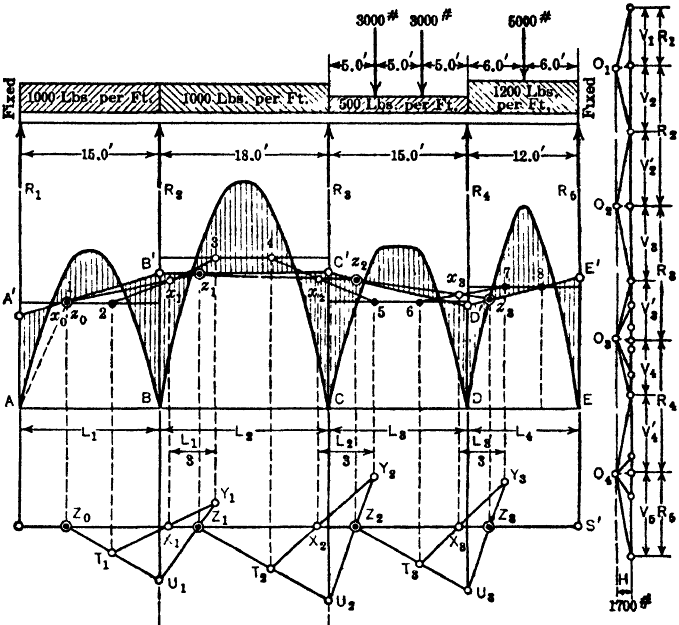
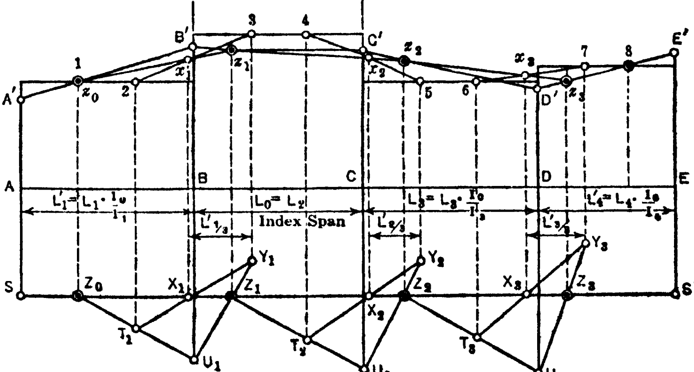


Figure 14



Construction for Variable Moment of Inertia

Figure 15

Fig. 14

Fig. 15

Figs. 14 and 15. Continuous Beams. Irregular Loading, Fully Fixed Ends

(4) Having located Z_0, the construction is the same as previously described until z_3 is located.

(5) The true base line for the last span is drawn through z_3 and the last characteristic point, No. 8, thus locating $E'D'$.

(6) The true base line $D'C'$ is the prolongation of $D'z_2$, etc.

Variable Moments of Inertia. When a difference in span lengths or loading result in the selection of different sections for the various spans of a continuous beam, as frequently occurs in the case of reinforced-concrete construction, the DISTRIBUTION OF MOMENTS will depend upon the relative stiffness of the several spans, and a corresponding alteration is necessary. Referring to Figs. 14 and 15:

In order to take into account unequal values of moment of inertia in the different spans of a continuous beam, the true span lengths must be altered in such a manner that a virtual condition equivalent to equal moments of inertia is obtained. Any one of the several spans may be selected as a reference span, and for comparative rigidity, spans having smaller values of I must be lengthened, and those having larger values of I must be shortened.

Let L_x = true length of span X;
I_x = moment of inertia of span X;
L_0 and I_0 = true length and moment of inertia of any other span;
L'_x and L'_0 = comparative span lengths, then:

$$L'_x = \frac{L_x}{I_x} \quad \text{and} \quad L'_0 = \frac{L_0}{I_0}$$

If I_0 is taken as the index value of moment of inertia and each comparative span length multiplied by the constant I_0, the relative stiffness of the various spans remains unaltered, whence:

$$L'_x = L_x \frac{I_0}{I_x} \quad \text{and} \quad L'_0 = L_0$$

The comparative span length L'_0 is equal to the true length of L_0, and is called the index span. Other span lengths altered to conform in relative rigidity to the index span may be designated as equivalent transformed spans.

After the equivalent transformed span lengths have been found, as illustrated in Fig. 15, the graphical solution is the same as for equal moments of inertia. From the skeleton construction shown in Fig. 15, the negative moments AA', BB', etc., are given in their true magnitudes and if positive moments, inflection points, and reactions are desired, the true negative moment values may be transferred to the diagram of moments for simple independent spans.

10. Deflection of Continuous Beams

Continuous Girder of Two Equal Spans. Uniformly Distributed Load over Each Span. The greatest deflection of a continuous girder of two equal spans loaded with a uniformly distributed load of w lb per unit of length is

$$\text{Maximum deflection} = 0.005416\, wl^4/EI \tag{21}$$

in which E is the MODULUS OF ELASTICITY and I the MOMENT OF INERTIA of the cross-section of the beam. The greatest deflection of a similar beam supported at both ends and uniformly loaded is

$$\text{Maximum deflection} = 0.013020\, wl^4/EI$$

Hence the deflection of the continuous girder is only about $\frac{2}{5}$ that of a non-continuous girder. The greatest deflection of a continuous girder of two spans is not at the middle of either span, but between the middle point of a span and one of the abutments. The greatest deflection of a continuous girder of two equal spans, loaded at the middle of one span with a load of P lb, and at the middle of the other with P_1 lb, is, for the span with the load, P

$$\text{Maximum deflection} = \frac{(23\,P - 9\,P_1)\,l^3}{1\,536\,EI} \tag{22}$$

for the span with load P_1

$$\text{Maximum deflection} = \frac{(23\,P_1 - 9\,P)\,l^3}{1\,536\,EI} \tag{22a}$$

When both spans have the same load

$$\text{Maximum deflection} = \tfrac{7}{768}\,Pl^3/EI \tag{22b}$$

The greatest deflection of a simple beam supported at both ends and loaded at the middle with P lb is

$$\text{Maximum deflection} = Pl^3/48\,EI$$

or the deflection of the continuous girder is only $\frac{7}{16}$ that of a non-continuous one.

Continuous Girder of Three Equal Spans. Uniformly Distributed Load over Each Span.

The load per unit of length is w lb.

$$\text{Greatest deflection at the middle of middle span} = 0\,00052\,wl^4/EI \tag{23}$$

The load per unit of length is w lb.

$$\text{Greatest deflection at the middle of middle span} = 0.00052\,wl^4/EI \tag{23}$$

$$\text{Greatest deflection in the end-spans} = 0.006884\,wl^4/EI \tag{24}$$

Hence the maximum deflection of the continuous girder is only about $\frac{1}{2}$ that of a non-continuous girder.

Continuous Girder of Three Equal Spans. Concentrated Load P at the Middle of Each Span.

$$\text{Greatest deflection at the middle span} = \tfrac{1}{480}\,Pl^3/EI \tag{25}$$

$$\text{Greatest deflection at the middle of end-spans} = \tfrac{11}{960}\,Pl^3/EI \tag{26}$$

Hence the maximum deflection of the continuous girder is only $\frac{11}{20}$ of that of the non-continuous girder.

11. Beams of One Span, Restrained at One End Only

A BEAM OF ONE SPAN, with ONE END RESTRAINED may be considered as a special case of a TWO-SPAN CONTINUOUS BEAM in which the second span, adjacent to the restrained end is indefinitely shortened.

The method of solution outlined in Article 9 may be readily applied to all such beams under any condition of loading.

Case I

Uniform Load (Fig. 16). The bending moment diagram for the span considered as a simple span is constructed to any convenient scale and a load line

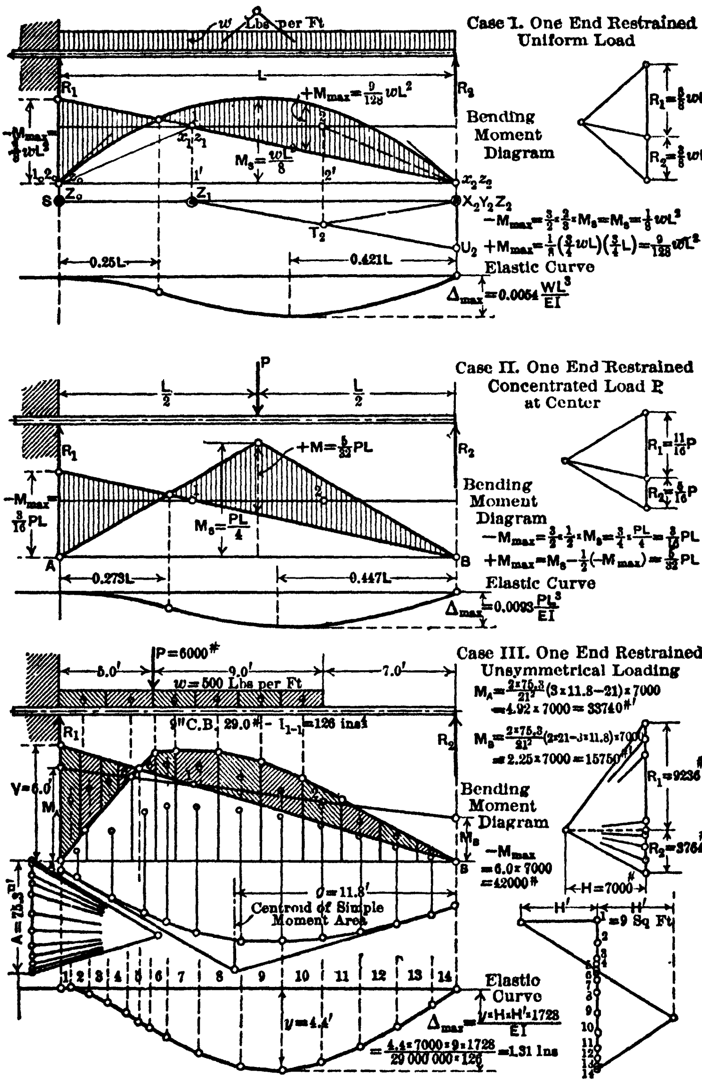


Fig. 16. Beams Restrained at One End. Cases I, II and III

drawn; the pole position corresponding to the parabolic moment diagram is located by drawing, from the extremities of the load line, lines parallel to the base tangents of the parabola. The homology diagram is constructed on the horizontal axis through S as before. The characteristic points for the imaginary span are 1_0 and 2_0, while 1 and 2 are the characteristic points for the given span and are located as shown on the base line for a fully restrained span which is at $2/3\ M_s$ above the axis of moments for unrestrained ends.

Following the usual procedure: connect the characteristic points (2_0 1) adjacent to the restrained support and lay off one-third the imaginary span from 1′ to coincide with 1′, then x_1 coincides with 1. Draw z_0x_1 and prolong to intersect the vertical through Z_1, whence z_1 also coincides with 1. Since there is no span and no restraint at or beyond R_2, x_2 and z_2 coincide with the zero moment point at this support. The true base line is, therefore, $z_2\ z_1$ prolonged to intersect the action line of R_1.

For any beam of one span, loaded in any manner, having one end restrained, the following theorem is applicable:

The true base line of the moment diagram, constructed upon the diagram of moments for the span as a simple beam, is a line drawn from the zero moment point at the free end and passing through the characteristic point adjacent to the restrained end.

From the properties of the parabola the maximum negative and positive moments and the position of the inflection point can be readily determined, and in a manner similar to that employed in Article 3 the position and magnitude of the maximum deflection can be found.

Case II

Concentrated Center Load (Fig. 16). The negative moment diagram for the case of a single concentrated load at the center of the span is a rectangle with an altitude equal to one-half the altitude of the triangular bending moment diagram, and the true base line, from the general theorem stated in the discussion of Case I, may be drawn directly from B through characteristic point No. 1 and prolonged to intersect the action line of the reaction at the restrained support.

The values of the maximum negative and positive moments and the reactions are readily determined, as shown in Fig. 16. The position of the inflection point is determined as follows:

$$(M_s \cdot x) \div \frac{L}{2} = \frac{M_s/2}{2\ L/3} \cdot (L - x)$$

where x = distance from fixed support to the inflection point.

Then, x from the preceding equation, based on the slopes of the two intersecting lines, is:

$$x\left(\frac{2}{L} + \frac{3}{4\ L}\right) = \frac{3}{4}$$

or

$$x = \frac{3}{4} \div \left(2 + \frac{3}{4}\right) \cdot \frac{1}{L} = 0.2727\ L$$

The position of the maximum deflection is calculated by locating the section of zero shear for the moment area loading:

Taking moments of moment areas about A:

$$R'_2L = \left(\frac{PL}{4}\cdot\frac{L}{2}\right)\frac{L}{2} - \left(\frac{3}{16}PL\cdot\frac{L}{2}\right)\frac{L}{3}$$

whence
$$R'_2 = \frac{PL^2}{32}$$

where R'_2 is the moment area reaction at B.

The rate of increase of moment from B to the center line is:

$$\frac{5}{32}PL \div \frac{L}{2} = \frac{10}{32}P$$

Then if x_1 equals the distance from B to the zero shear section of moment areas:

$$\left(\frac{10}{32}Px_1\right)\frac{x_1}{2} = \frac{PL^2}{32}$$

or

$$x_1 = \frac{L}{\sqrt{5}} = 0.447\ L$$

The maximum deflection is:

$$\Delta_{\max} = \left(\frac{10}{32}P\cdot\frac{\overline{0.447\ L}^2}{2}\right)\frac{2}{3}\cdot 0.447\ L \div EI = 0.0093\frac{PL^3}{EI}$$

Case III

Unsymmetrical Loading (Fig. 16). For any unsymmetrical loading the moment diagram may be constructed graphically or by calculating and plotting the moment values for various points throughout the span.

The values of M_A and M_B are calculated from Equations (17) and (18) after the centroid of the moment area for the unrestrained span has been located. The location of the vertical line through this centroid is readily determined by the graphical method as illustrated.

The negative moment diagram, or base line for the condition of fully restrained ends, is drawn and the characteristic points 1 and 2 located.

The true base line is then drawn from B through characteristic point No. 1 and prolonged to intersect the action line of R_1, as in the preceding cases.

The true moment area thus obtained is divided into elementary strips and the area of each strip measured to the scale of the space scale of span.

The area-vector load-line 1—14 is drawn to a convenient scale, the pole distances chosen, and the deflection polygon constructed and the deflection found as illustrated in Fig. 16.

CHAPTER XIX

RIVETED STEEL PLATE AND BOX GIRDERS

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1. General Considerations on Plate and Box Girders

Girders Built of Angles and Plates are required for long spans and for unusual cases of very heavy loading. The rolled beam is adequate for most cases in the design of steel buildings and is usually cheaper than a built-up section. It was formerly common to reinforce rolled beams by the use of cover-plates riveted to the back as explained in Chapter XV. There is little occasion for this now because of the heavier beams which have become available in the Bethlehem beam and girder sections and in the Carnegie sections. The addition of covers to rolled beams involves a good deal of expensive shop work due to drilling holes in the heavy flanges.

Definitions. A PLATE GIRDER is a beam composed of a wide plate, known as a web, at the top and bottom of which are riveted angles or angles and plates. The WEB may be and frequently is reinforced against BUCKLING by angles riveted to its sides, known as STIFFENER ANGLES. The stiffener angles which occur at points of concentrated loading, as at reactions or where the girder carries a column load, are known as BEARING STIFFENERS. Other stiffeners are known as INTERMEDIATE STIFFENERS. Stiffeners may be either CRIMPED by bending them around the flange angles or the stiffeners may be STRAIGHT between the outstanding legs of the flange angles with plates inserted between the stiffeners and the web, in which case they are said to be FILLED. The parts of the girder connected at top and bottom of the web are known as the FLANGES and are made up of FLANGE ANGLES, which are directly riveted to the web, and FLANGE-PLATES or COVER-PLATES which are connected to the outside of these angles. In the case of very heavy girders, SIDE PLATES under the down-standing legs of the flange angles may be added to secure additional area in the flange. By the DEPTH of a girder is usually meant the distance back to back of flange angles. The distance between the centers of gravity of the flanges is called the EFFECTIVE DEPTH of the girder.

Types. The simplest type of plate girder consists of a web and four flange angles as shown in Fig. 1. If this does not give adequate flange area, cover-plates are added as shown in Figs. 2 and 3. Fig. 4 shows a heavy

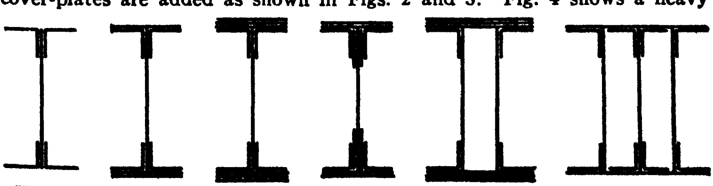

Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5 Fig. 6

Types of Riveted Girders

plate girder in which side plates have been used in addition to covers. A BOX GIRDER is a plate girder having more than one web. Fig 5 shows a typical box girder consisting of two webs, four angles and cover-plates. Fig. 6 shows a very heavy box girder consisting of three webs, eight flange angles and cover-plates.

2. The Web

THE FUNCTION OF THE WEB is to resist the shear in the girder. The distribution of this shear over the web is nearly uniform along the depth of the web.* The depth of web is taken as one of the stock widths of plate, usually in even inches, and because the edge of this plate is somewhat irregular the backs of the flange angles are set out from the edge of the plate so that the total distance back to back of flange angles is usually $\frac{1}{2}$ in greater than the depth of the web plate.

THE DEPTH OF THE GIRDER is usually chosen as $\frac{1}{12}$ to $\frac{1}{15}$ of the span length, if possible. Plate girders having a depth less than $\frac{1}{20}$ of the span should in general have their flange stress reduced in order to provide against excessive deflection.

THE WEBS OF PLATE GIRDERS MAY FAIL in one of three ways:

(a) By shearing of the web longitudinally or vertically.

(b) By buckling of the web due to inclined compression set up by the shearing-stresses.

(c) By buckling of the web vertically at points of concentrated loading.

In rolled beams, failure in the web is not usually to be feared except in special cases, whereas in the plate girder the types of failure indicated are real sources of danger in design. It is usually specified that the thickness of the web shall not be less than $\frac{1}{60}$ of the clear depth between the flanges in any case.

WEB STIFFENERS. If the thickness of web is less than $\frac{1}{60}$ times the clear distance between flanges INTERMEDIATE STIFFENERS are required to prevent diagonal buckling of the web. In ordinary cases it is well to add intermediate stiffeners not more than 5 ft on centers in any case. Intermediate stiffeners should not be less than $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$-in angles. It is customary to make the outstanding leg about 1 in less in width than the width of the outstanding leg of the flange angles. Thus, where the outstanding leg of the flange angle is 6 in, it is customary to use a 5-in outstanding leg for intermediate stiffeners. Intermediate stiffeners may be either CRIMPED or FILLED. There is no general practice favoring either method. It is usually specified that the outstanding legs of intermediate stiffeners shall bear against their flange angles. This is perhaps not absolutely necessary but it has the advantage that it prevents folding down of the flange on to the web, a type of buckling failure which is obviously possible in the compression flange.

The type of formula used to compute STIFFENER SPACING depends upon the type of column formula in use. The formula recommended by the American Institute of Steel Construction is

$$v = \frac{18\,000}{1 + \dfrac{h^2}{7\,200\,t^2}} \tag{1}$$

* This may be seen to be a result of the fact that most of the bending stress is carried by the flanges. Consideration of the theory of longitudinal stress given in Chapter XV shows that if all the stress were carried by the flanges, the change, in any length of girder, of the total stress above any horizontal section would be the same irrespective of the location of the longitudinal section with reference to the neutral axis.

where v is the intensity of shear to be permitted in the web in pounds per square inch;

h is the clear distance between flange angles or between stiffeners, whichever is the smaller;

t is the thickness of the web.

Values of v as given by this formula are given in Table I at the end of this chapter. This is a more complicated type of formula than would be derived by the use of a straight-line column formula and the added complication does not seem to be justified by any accurate data. The type of formula used, however, is not very important to the designer because all formulas for spacing intermediate stiffeners are adjusted to give practically the same results. The basic fact from experience is that where the thickness of web is greater than $\frac{1}{60}$ of the clear distance between flange angles or between stiffeners, whichever is the smaller, full shearing value may be used in the web. The practice of not exceeding this spacing of $60 \times t$ is convenient, conservative and of long standing, and is sufficient for the ordinary problems of building-construction. The specifications of the American Institute of Steel Construction permit a shearing stress of 12 000 lb per sq in on the GROSS SECTION of the web.

Bearing Stiffeners. It is usually required that BEARING STIFFENERS be ground to fit the flange angles both in bearing of the outstanding legs against the flanges and in bearing of the legs which rest against the fillets. It is then customary to assume the whole angle in bearing and to compute the strength of the angles as that of a column having a length equal to half the girder depth. A practice which is more conservative and much simpler is to compute the area required in the outstanding legs at the allowable stress used in a short column. This practice is recommended here. The additional weight due to the slightly heavier stiffener angles is trivial.

Bearing stiffeners should be connected by enough rivets to transmit the load or reaction to the web and should either have tight fillers or should have the number of rivets increased by 50%. Tight fillers should be connected to the web by half as many additional rivets as are used in the stiffener angles themselves.

Holes in Girder Webs. Holes sometimes occur in the web of girders for pipe runs and for other reasons. They are obviously objectionable but they can be safely used if proper attention is given to the reinforcement of the web where the hole occurs. The net section of the web should be maintained through the hole. The sides of the hole in general should be reinforced by angles to prevent buckling along the unsupported edges. Holes are particularly liable to tearing of the metal at those corners of the holes which lie on a diagonal running from the load to the reaction, and special thought should be given to reinforcement at these corners.

Diaphragms in Box Girders. Diaphragms are used in box girders to equalize the deflections of the two webs and prevent twisting of the girder. The webs of such diaphragms should be capable of transmitting in shear $\frac{1}{2}$ of the concentrated load occurring at the diaphragm. The rivets connecting the diaphragm to the web should also be adequate to transmit $\frac{1}{2}$ of the load. Diaphragms are commonly used only at points of concentrated loading, which means usually under columns which rest upon the girder.

3. The Flanges

The Function of the Flanges is to resist the COUPLE OF INTERNAL STRESS which is equal and opposite to the couple produced by the external loads, that is, the BENDING MOMENT. Since the couple due to the internal stresses equals the bending moment, we may get the TOTAL STRESS IN EITHER FLANGE by dividing the bending moment by the distances between centers of gravity of the flanges. The total stress in the flange and the allowable fiber-stress being known, the AREA OF THE FLANGE may be computed. We may write then a general formula

$$A_f = \frac{M}{s_a d} \quad \text{or} \quad M = A_f s_a d \tag{2}$$

in which M is the moment at any section in inch-pounds;

s_a is the allowable fiber stress in pounds per square inch;

d is the effective depth of the girder, or distance between centers of gravity of flanges, in inches;

A_f is the area required in the flange, in square inches.

This should be the net area required in the tension flange or the gross area required in the compression flange. This assumes that none of the bending is carried by the web and also that the stress is equally distributed over the flange. The former assumption is obviously incorrect since the stress in the web at its connection to the flange must be the same as the flange stress. THE WEB WILL CARRY as a rectangular beam a moment of $M = \frac{1}{6} SA_w d$. This is the same as if $\frac{1}{6}$ of the area of the web were treated as flange-area in Formula (1). In order, however, to allow for the reduction of strength produced by the holes in the web, it is customary to use $\frac{1}{8}$ of the web as flange-area in each flange.

Plate-Girder Method of Design. The effect of bending of the web is then included if we write

$$A_f = \frac{M}{s_a d} - \frac{1}{8} A_w \tag{3}$$

in which A_w is the gross area of a cross-section of the web and the other terms are as previously defined.

The Net-Section Method of Design. There has been some difference in opinion among engineers as to the DISTRIBUTION OF STRESS over the flanges. In the case of very shallow girders, it is certainly not correct to assume it as uniform. In the case of deep girders, it is probably as nearly correct as any assumption. The method of analysis just indicated is usually called the PLATE-GIRDER METHOD, and is very simple to apply and forms a convenient and reasonably accurate basis for all computation in the design of plate girders. The NET-SECTION METHOD of designing the flanges uses the BEAM FORMULA as applied to a section of the girder in which rivet-holes are deducted from both flanges. This gives a design somewhat more conservative than does the use of the plate-girder method. It is, however, more difficult to design by this method, and it is recommended that the design be made by the plate-girder method and the stresses checked by the net method in cases where the latter is required. It should be noted that the American Institute of Steel Construction in their specifications require the use of the net section and that the stresses of 18 000 lb per sq in permitted in their specification should not be used for girders which are relatively shallow where the plate-girder method of design is used.

The Gross-Section Method of Design is used by many engineers and was the basis of the tables of plate girders of the Carnegie Steel Company. Section-moduli of the gross sections as given in these tables are given at the end of this chapter.

Other Methods of Flange Design. A method of design has sometimes been used in which the moment of inertia of the section is computed on the assumption that the net section is effective in the tension flange and the gross section in the compression flange. A variation of this method is to assume that the neutral axis is determined in position by the gross section and then the design is carried through by the beam formula $s = \frac{Mc}{I}$, using for c the distance to the outer fiber from the gravity axis of the gross section and for I the moment of inertia determined about this axis. These variations in the method of design have not had very wide use but they are perhaps in some ways more logical. There do not seem to be adequate experimental data to justify either of them, however. Of the two variations of the method, that which assumes the neutral axis to be determined by the gross section is almost certainly more nearly correct, because the neutral axis will not shift in position abruptly on those sections where rivet-holes occur.

Design of the Flanges. Usually the thickness of metal in the flange should not be over $\frac{3}{4}$ in. It is usually specified that not over one-half of the TOTAL FLANGE-AREA should be in the cover-plates and also that the center of gravity of the flange should not lie outside of the back of the angles.

In computing the NET SECTION, the diameter of the rivet-holes is invariably taken $\frac{1}{8}$ in greater than the diameter of the rivet. In general, either two or four holes must be deducted in each flange, depending upon the number of rivet-lines used in the outstanding legs. If, as is customary, four rows of rivets are used in the cover-plates, those in the outer row in general fall in line with rivets in the down-standing legs and two holes must be deducted in the down-standing legs and two holes in the outstanding legs and cover-plates.

Where FLANGE-PLATES are used, they are usually given a width of 2 in greater than the width of the two flange angles so that with 5-in legs outstanding, the width of cover used would be 12 in. The cover should not project beyond the outer row of rivets a distance greater than 6 in or 12 times the thickness of the thinnest plate. In general, covers should not be less than $\frac{5}{16}$ in thick in any case. The use of cover-plates entails some additional shop work in punching and riveting but they need not extend to the end of the girder and some metal is saved in consequence. It is quite customary to extend the cover-plate next to the angles for the full length of the girder, but this custom has been derived largely from bridge practice and has no point in interior girders for building-construction. It is often specified that no cover shall be used of a thickness less than that of the flange angles, nor less than that of any plate which lies between it and the flange angles. There seems to be little basis for this recommendation, but there is no particular difficulty in complying with it.

Lengths of Cover-Plates. Where the covers are no longer needed to furnish the area in the flanges required by the bending moment they may be cut off. In general, the cover should extend beyond the point of theoretical cut-off by two or three lines of rivets plus an edge distance of $2\frac{3}{4}$ in necessary to grip the cover in the multiple punches used in most shops. The total overrun at each end, then, will be about 1 ft.

For uniform loads applied directly to the girder the curve of maximum moments is a parabola.

Where the uniform load is applied to the girder through joists or floor beams the curve of maximum moments is a polygon inscribed at the points of loading within this parabola. The theoretical points of cut-off for cover-plates may be determined either analytically or graphically. Using the parabolic relation just indicated we may deduce the following formula which gives the length required for the cover-plates where the uniform load is directly applied to the girder

$$L' = L\sqrt{\frac{A'}{A}} \tag{4}$$

in which L' is the length of cover-plate theoretically required;
L is the span length;
A is the total flange area;
A' is the combined area of the plates under consideration and of any covers outside of it.

Where the load is uniformly distributed, but is applied to the girder through joists or columns equally spaced, the same formula may be used if the areas in the formula are defined as explained below, but will give values for the theoretical length required slightly too great. The formula is, then, conservative in such cases. In such cases, in the formula,

A is the area of flange, required for a moment equal to $\frac{1}{8}WL$;
A' is the difference between area just defined and the area of flange remaining after the cover is cut off.

Graphical Determination of Lengths of Cover-Plates. A graphical construction is convenient where columns or other heavy concentrated loads occur

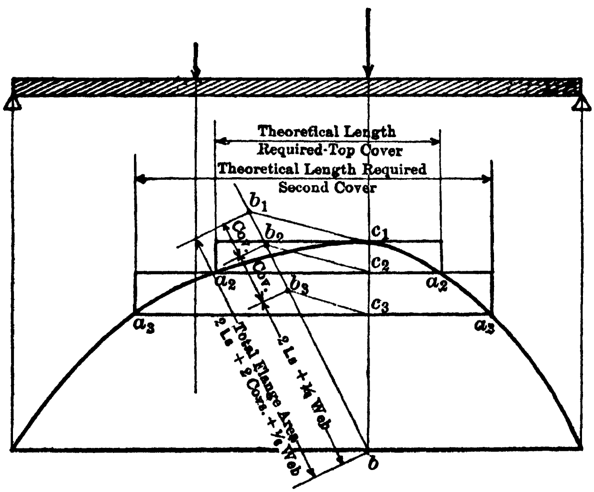

Fig. 7. Graphical Determination of Required Lengths of Cover Plates

with unequal spacing or where, for other reasons, the curve of moments is irregular. The following graphical construction as shown in Fig. 7 is cus-

tomary and convenient. Draw the curve of moments. At the point of maximum moments lay off to a convenient scale on a diagonal line bb the areas of the different parts which make up the flange section. Lay off first the area of the flange angles and $\frac{1}{6}$ of the area of the web (bb_3),* then successively the areas of each of the cover-plates (b_3b_2, b_2b_1). Draw a line from the end of bb_1 to the point of maximum moment, and project parallel to this line the component parts of the total area giving successively points C_1, C_2, C_3, as indicated in the figure. Through these points draw horizontals to intersect the curve of maximum moments at points a_2 a_2, a_3 a_3, as indicated. The top cover-plate then must extend from a_2 on the right to a_2 on the left, the next cover must extend from a_3 to a_3. Several rows of rivets should be added beyond the points of theoretical cut-off. Special consideration must be given to such development of the covers in designing girders subject to heavy concentrated loads near the supports, because in this case the end shear is high and the moment curve drops off rapidly

Design of Flange Riveting. Special attention must be given to the PITCH of the rivets connecting the flange to the web. The stress per running inch of girder which the rivets are called upon to resist may be computed from the formula,

$$v = \frac{VQ}{I} \tag{5}$$

given in Chapter XV. In this formula,

v is the stress on the flange rivets per inch of length of girder in pounds;
V is the total shear at the section under consideration;
Q is the statical moment, about the neutral axis of the girder, of the area of the flange connected by these rivets;
I is the moment of inertia of the girder.

This formula should be used in cases of shallow girders carrying heavy shears.

In ordinary girders it is sufficiently accurate to use the formula

$$p = \frac{rd}{V}\dagger \tag{6}$$

in which p is the required pitch of the rivets in inches;
d is the effective depth of the girder in inches;
r is the value of one row of rivets in pounds;
V is the total shear at the section in pounds.

The value of the rivets may be determined either in bearing on the web or in shear. In the case of plate girders the rivets connecting the angles to the web are in DOUBLE SHEAR, but in the outside webs of box girders they are in SINGLE SHEAR.

The formula just given is somewhat conservative because the moment carried by the web was neglected in its derivation. If somewhat more exact

* We may lay off either the gross area of the flange plus $\frac{1}{6}$ of the web or the net area plus $\frac{1}{6}$ of the web. The difference in result is insignificant.

† This formula may be derived as follows: The stress in the flange is equal to the bending moment divided by the effective depth. The change of stress per running inch is equal to the shear divided by the effective depth. The value of one rivet divided by the rivet pitch is the resisting shear in the rivets per running inch. Since the resistance of the rivets per inch must equal the shear on the rivets per inch, we write $\frac{r}{p} = \frac{V}{d}$ or $p = \frac{rd}{V}$.

values are wanted, the pitch thus obtained may be multiplied by the ratio of the total flange-area including $\frac{1}{8}$ of the web-area to the total flange-area omitting $\frac{1}{8}$ of the web-area.

The pitch of the rivets connecting cover-plates to flange angles is usually made the same as that connecting the flange angles to the web. Sometimes it is found convenient, however, to increase this pitch. The required pitch may be computed from the pitch required to connect the flanges to the web by multiplying by the ratio of the total area of flange to the total area of covers.

In box girders having three webs, as much shear is transmitted to the center web by the rivets as to the side webs. If there is a center web it is therefore usually made twice as thick as the side webs and is assumed to carry the same shear as the two side webs together. Consequently, the required pitch of rivets required is the same in both webs if the strength of the rivet is limited by bearing on the web.

Splices. It is rarely necessary to SPLICE flanges in girders used in ordinary building work. Flange splices may occur in the cantilever girders of theater balconies, but such girders constitute a rather special case. Splices in the web are determined by the length of web-plate rolled. They are rarely required in girders less than 60 ft in length, and greater spans than this are not common in building-construction. The maximum length of plates and angles rolled may be found in handbooks of the various steel companies. Ordinarily it is sufficient to splice the web for SHEAR only, preferably at a point of low shear. This is done by adding on each side of the web one plate having a thickness equal to that of the web and connecting this plate on each side of the cut in the web by sufficient rivets to transmit the total shear which the web is capable of carrying. It should be said that differences in opinion exist among engineers as to the proper design of splices for the web. Some require that the web be developed in MOMENT as well as in SHEAR. This does not seem to be necessary unless the splice occurs at a point where all the flange section is needed to develop the bending moment. In any other case, the angles and flange cover may be depended on to splice the web for its moment resistance.

4. Method of Design

The Net-Section Method of Design with an allowable extreme fiber-stress will probably become a general standard in the future, together with the specifications of the American Institute of Steel Construction. In accordance with these specifications, the following method of design for plate and box girders to be used in building construction is recommended:

(1) Determine the curves of maximum shear, in pounds, and of maximum bending moment, in inch-pounds, on the girder.

(2) Assume a depth of web-plate, preferably about $\frac{1}{12}$ of the span length, unless otherwise limited by architectural construction. If the depth must be less than $\frac{1}{20}$ of the span length, the allowable flange stress should be reduced in direct proportion as the depth used is to a depth equal to $\frac{1}{20}$ of the span.

(3) Determine the web thickness needed to give a gross area of web equal to the quotient of the maximum shear divided by 12 000 lb per sq in.

$$A_w = \frac{V}{12\,000} \tag{7}$$

Use a web thickness not less than $\frac{5}{16}$ in.

(4) (a) Assume an effective depth (d) about 1 in less than the depth of web-plate.

(b) Estimate the total depth of girder, out to out (d').

(c) Assume an allowable stress of 15 000 lb per sq in on the gross section as equivalent to 18 000 lb per sq in on the net section.

(d) Compute the approximate gross area of flange required from

$$A_f \text{ gross} = \frac{M}{15\,000\,d'}\left(\frac{d'}{d}\right)^2 \tag{8}$$

(5) Proportion the flanges counting ⅙ of the gross area of the web effective in the gross flange-area. Use two angles of appropriate size and add cover-plates necessary to give the required total gross area

(6) (a) Determine the net section-modulus of the section thus designed as explained in Chapter X. Determine the section-modulus of the gross area and deduct the section-modulus of the rivet-holes.

(b) Compare with the required section-modulus as determined from

$$S = \frac{M}{18\,000} \tag{9}$$

If the section-modulus used differs appreciably from that required, modify slightly the thickness of cover-plates used or, if no covers are used, change slightly the size of the angles.

(7) If cover-plates have been used, determine their required length.

(8) Decide what spacing to use for intermediate stiffeners.

(9) Determine the necessary rivet-pitch at the end and at several intermediate points, from the formula,

$$p = \frac{rd}{V} \tag{6}$$

(10) Design the end bearing or end connection as the case may be.

5. Deflection of Plate Girders

Deflection. Attention has been called to the importance of limiting the DEFLECTION OF GIRDERS. The same considerations in regard to this apply to PLATE GIRDERS as are explained in connection with ROLLED BEAMS in Chapter XV. The formulas there given for deflections are applicable to plate girders, provided the flange-section of the girder is nearly constant. Where, however, cover-plates are cut off at frequent intervals, the flange-stress in the girder for maximum moments would be nearly constant, and the deflection will be $\frac{1}{4} \times \frac{sL^2}{Ed}$. In general, the deflection of a plate girder with cover-plates and resting on rigid supports at the ends can be computed closely enough by the formula

$$\text{Deflection in inches} = \frac{1}{4.5}\,\frac{sL^2}{Ed} \tag{10}$$

in which s is the fiber-stress used in design in pounds per square inch;
L is the span length in inches;
E is the modulus of elasticity of steel, 29 000 000 lb per sq in;
d is the overall depth of the girder in inches.

If the design stress, s, is 18 000 lb per sq in this reduces to

$$\text{Deflection in inches} = \frac{1}{50}\frac{L^2}{d} \tag{11}$$

Where L and d are as defined above.

For girders without cover-plates the deflection will be about 10% less than this.

Girders in building construction should not have a depth less than $\frac{1}{20}$ of the span. If shallower girders must be used, the section-modulus should be increased in inverse proportion to the depth.

6. Examples

To design a reasonably economical and perfectly safe girder is not difficult. True economy in special cases or where large tonnages are involved may require a good deal of judgment. The following examples illustrate typical problems:

Example 1. Select from the tables at the end of the chapter a girder section to resist a bending moment of 1 200 000 ft-lb, allowing a fiber-stress of 18 000 lb per sq in on the net section.

Either a 36-in Carnegie Beam 230 lb per ft, or a 36-in Bethlehem Beam 230 lb per ft, will give the required resisting moment. See Tables VI and VII, Chapter X. It is, nevertheless, desired to use a plate girder.

Assume a fiber-stress of 15 000 lb per sq in on the gross section as equivalent to 18 000 lb per sq in on the net section. (See notes preceding tables at end of Chapter.)

Gross section-modulus, approximately, $\dfrac{1\,200\,000 \times 12}{15\,000} = 960$

Net section-modulus required $\dfrac{1\,200\,000 \times 12}{18\,000} = 800$

Try Web $36 \times \frac{5}{8}$ in
$4 \angle 6 \times 4 \times \frac{3}{4}$ in
2 plates $14 \times \frac{3}{4}$ in

Gross section-modulus (See Table II) 943.9

Deduct holes $1\frac{7}{8} \times \left(\dfrac{32.0}{38.0}\right)^2 = 1.50$

$2 \times \frac{3}{2} \times \left(\dfrac{36.5}{38.0}\right)^2 = 2.78$

$4.28 \times \frac{7}{8} \times 38.0 = 142$

Net section-modulus 802

This is satisfactory.

Example 2. Design this girder assuming that the total depth is limited to 31 in.

This limited depth for the section-modulus required is out of the range given in the tables.

Assume a web-plate $28 \times \frac{1}{2}$ in = 14.0 sq in.

Effective depth about 27 in.

Total depth about 31 in.

Gross area of flange required:

$\frac{1\,200\,000 \times 12}{15\,000 \times 31}\left(\frac{31}{27}\right)^2$ (Equation 8)	= 40.90
⅛ Web (See (5) under The Net-Section Method of Design)	= 2.50
	38.40
2 ∠ 6 × 6 × ⅞ in = 2 × 9.73 (Table II, Chapter X)	= 19.46
Area required in covers	18.94
Use covers 14 in × 1⅜ in, total	= 19.30

This section will now be checked by the use of the section-modulus of the net section. Compute first the section-modulus of the gross section. Tabulate successively the areas of each part of the section, the distances from the center of the section to the centroids of those parts, and the centroidal moments of inertia. From these compute the areas times the squares of the distances.

Member	Area A	Distance to center of section	Area × (dist.)² + moment of inertia about centroid	
Web 28 × ½ in .	14 0	.	$\frac{1}{12} \times 14\,0 \times 28^2 =$	915
4 ∠ 6 × 6 × ⅞ in (Table II, Chapter X)	38 92	14 25 1 82 12 43	$38\,92 \times 12\,43^2 =$ $4 \times 31.92 =$	6 010 130
2 cover-plates 14 × 1⅜ in	38 50	14 25 69 14 94	$38.50 \times 14\,94^2 =$ Centroidal I negligible	8 610
				15 665

Total depth = 28 + ½ + 2¾ = 31.25 in

Section-modulus of gross section $\dfrac{15\,665}{\frac{31.25}{2}}$ = 1 000

Deduct for rivet-holes

$$\frac{9}{4} \times \left(\frac{24\,00}{31.25}\right)^2 = 1.32$$

$$2 \times 2.25 \times \left(\frac{29.00}{31.25}\right)^2 = 3.86$$

$5.18 \times \frac{7}{8} \times 31.25$ = 142

Section-modulus of net section 858

Section-modulus required $\dfrac{1\,200\,000 \times 12}{18\,000}$ = 800

Reducing the thickness of cover-plates to $1\frac{1}{4}$ in and recomputing gives S net = 800.

Use 2 cover-plates 14 × $\frac{5}{8}$ in.

Example 3. Assuming the span to be 46 ft, and the load approximately uniformly distributed, what is the deflection of this girder?

From Equation (10),

$$\text{Deflection} = \frac{1}{4.5}\frac{sL^2}{Ed} = \frac{1}{4.5} \times \frac{18\,000 \times (46 \times 12)^2}{29\,000\,000 \times 31} = 1.36 \text{ in}$$

Example 4. Are intermediate stiffeners required for a total web-shear of 140 000 lb?

Since the clear distance between flange angles is $\dfrac{28.5 - (2 \times 6)}{\frac{1}{2}} = 33$ times the web thickness, which is less than 60, intermediate stiffeners are not required for web-shear. Use 5 × $3\frac{1}{2}$ × $\frac{5}{16}$-in angles in pairs at 5-ft centers.

Example 5. What stiffener spacing would be required by the formula of the American Institute of Steel Construction (Equation (1)) in a girder whose section is as follows:

Web, 38 × $\frac{5}{16}$ in
2 ∠s 6 × 4 × $\frac{3}{4}$ in
1 Cov. 14 × $\frac{3}{4}$ in

to carry a total shear of 120 000 lb?

The intensity of web-shear is

$$\frac{120\,000}{38 \times \frac{5}{16}} = 10\,100 \text{ lb per sq in}$$

Referring to Table I, this requires $\dfrac{h}{t} = 75$. Web must be stiffened at intervals of 75 × $\frac{5}{16}$ = 23.4 in, which is less than the clear height 38.5 − 12 = 26.5 in. Hence stiffeners are needed about 22 in clear or 24 in on centers.

Fig. 8 shows a NOMOGRAPHIC CHART for determining the spacing of web stiffeners for plate girders and rolled beams if the average shearing stress in the web, V/A, and the web thickness, t, are known. Draw a straight line connecting the thickness of web, t, and the average shearing stress, V/A, and note where this straight line intersects the scale of stiffener spacing, h.

To solve Example 5 by the diagram, draw a straight line through the web thickness, $t = \frac{5}{16}$ in and the average shearing stress, $V/A = 10\,100$ lb per sq in. Project this straight line until it intersects the scale of stiffener spacing at $h = 23.5$ in which is less than the clear depth of web, $d' = 26.5$ in. In case h were greater than d' and the average shearing stress were greater than 3 951 lb per sq in, the maximum allowable spacing of intermediate stiffeners according to A.I.S.C. Specifications would be 72 in.

Example 6. What end stiffeners are required in bearing for an end reaction of 180 000 lb for a girder 48 in deep? Web, $\frac{3}{8}$ in.

Required area of outstanding leg $\dfrac{180\,000}{18\,000} = 10$ sq in.

Use 5-in outstanding leg

4 stiffeners 5 × 3½ × ½ in gives 10 sq in in outstanding legs.

Rivets ¾ in are in double shear and bearing on web and tight fillers.

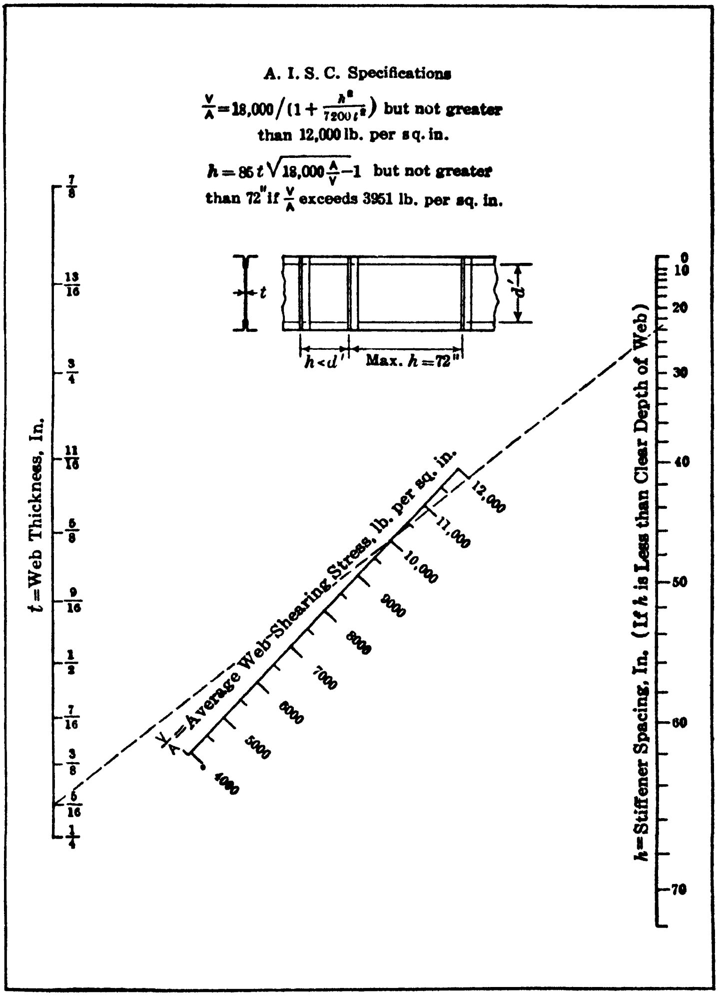

Fig. 8. Chart for Determining Spacing of Web Stiffeners

Value of one rivet 11 930 lb in double shear using the specifications of the American Institute of Steel Construction.

$$\text{Required, } \frac{180\,000}{11\,930} = 15 \text{ rivets}$$

If we compute the area of these stiffeners as required by the column formula

$$r = 2.4 \text{ in approximately}; \quad l = 24 \text{ in}; \quad \frac{l}{r} = 10$$

$$f = 15\,000 \text{ lb per sq in}$$

$$\text{Area required, } \frac{180\,000}{15\,000} = 12 \text{ sq in}$$

$$4 \angle 5 \times 3\tfrac{1}{2} \times \tfrac{3}{8} \text{ in} = 12.2 \text{ sq in}$$

Example 7. Given the girder designed in Example 2, what is the required pitch of $\frac{3}{4}$ in rivets to connect the flange angles to the web for a shear of 100 000 lb?

The value of these rivets is determined by bearing on the $\frac{1}{2}$-in web and is 11 250 lb.

Hence $p = \dfrac{rd}{V} = \dfrac{11\,250 \times 27}{100\,000} = 3$ in (Equation 6)

Example 8. What is the required length of the cover-plates in Example 2 if the load is uniformly distributed on a 50-ft span?

Area of outer cover 14 $\times$ $\frac{5}{8}$ in = 8.75 sq in
Area of both covers 14 $\times$ $1\frac{1}{4}$ in = 17.50 sq in
Total area of flange = 39.46 sq in
From Equation 4,

Theoretical length required for outer cover

$$50 \times \sqrt{\frac{8.75}{39.46}} = 23.5 \text{ ft}$$

Theoretical length required for second cover

$$50 \times \sqrt{\frac{17.50}{39.46}} = 33.5 \text{ ft}$$

7. Tables

Table I gives the allowable intensity of shear on the web for values of $\frac{h}{t}$,

Table I.* Allowable Intensity of Shear for Unstiffened Webs

$\frac{h}{t}$	$\frac{V}{A}$	$\frac{h}{t}$	$\frac{V}{A}$	$\frac{h}{t}$	$\frac{V}{A}$	$\frac{h}{t}$	$\frac{V}{A}$
60	12 000	74	10 224	87	8 775	100	7 535
61	11 868	75	10 105	88	8 672	105	7 111
62	11 734	76	9 988	89	8 571	110	6 722
63	11 604	77	9 871	90	8 471	115	6 345
64	11 473	78	9 756	91	8 372	120	6 000
65	11 343	79	9 642	92	8 274	125	5 678
66	11 215	80	9 529	93	8 177	130	5 378
67	11 087	81	9 418	94	8 082	135	5 097
68	10 961	82	9 308	95	7 988	140	4 836
69	10 835	83	9 199	96	7 895	145	4 592
70	10 711	84	9 091	97	7 803	150	4 364
71	10 587	85	8 984	98	7 712	155	4 151
72	10 465	86	8 880	99	7 623	160	3 951
73	10 344						

* From Steel Construction, published by the American Institute of Steel Construction.

where h is the smaller unsupported length of web horizontally or vertically and t is the web thickness.

Table II gives the gross section-modulus of girders of various make-up. These may be used directly to select a girder having a stress intensity of 15 000 lb per sq in on the gross section. By subtracting the section-modulus of the rivet-holes, they may also be used in designing by the section-modulus of the net section.

Table II.* Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches			Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
136.6		4×3 ×3/8	..	428.4		6×4 ×1/2	14×1/2
168.6		4×3 ×1/2	..	447.9		5×3½×5/8	12×5/8
198.7		5×3½×1/2	.	472.7	26×5/16†	6×4 ×1/2	14×5/8
236.1	24×5/16†	5×3½×5/8	. .	519.5		6×4 ×5/8	14×5/8
238.0		5×3½×1/2	12×1/2	563.4		6×4 ×5/8	14×3/4
372.9		5×3½×1/2	12×5/8				
408.5		5×3½×5/8	12×5/8	158.5		4×3 ×3/8	. ...
				183.8		5×3½×3/8	.. ....
142 5		4×3 ×3/8	. .	193 5		4×3 ×1/2	. ....
165.5		5×3½×3/8	.	208.1		6×4 ×3/8	.. ....
174 5		4×3 ×1/2		226.5		4×3 ×5/8	. ....
204.5		4×3 ×5/8	..	226.6		5×3½×1/2	. ... ..
204.6		5×3½×1/2	. ..	258.9		6×4 ×1/2	.. ...
242.0	24×3/8†	5×3½×5/8	......	267.6		5×3½×5/8	.. ...
270.9		5×3½×3/8	12×3/8	298.0		5×3½×3/8	12×3/8
306.1		5×3½×3/8	12×1/2	307.9		6×4 ×5/8	....
343.6		5×3½×1/2	12×1/2	336.2		5×3½×3/8	12×1/2
378.5		5×3½×1/2	12×5/8	341.5	26×3/8†	6×4 ×3/8	14×3/8
414.1		5×3½×5/8	12×5/8	354.4		6×4 ×3/4	. ....
				377.4		5×3½×1/2	12×1/2
151.5		4×3 ×3/8		386.1		6×4 ×3/8	14×1/2
176.8		5×3½×3/8		415.2		5×3½×1/2	12×5/8
186.6		4×3 ×1/2	. ...	435.1		6×4 ×1/2	14×1/2
201.2		6×4 ×3/8	. .	454.5		5×3½×5/8	12×5/8
219.6		5×3½×1/2	. ...	479.3		6×4 ×1/2	14×5/8
252.0		6×4 ×1/2	. ...	526.1		6×4 ×5/8	14×5/8
260.7		5×3½×5/8	. ...	569.9		6×4 ×5/8	14×3/4
291.3	26×5/16†	5×3½×3/8	12×3/8	613.9		6×4 ×3/4	14×3/4
301.0		6×4 ×5/8	......				
329.5		5×3½×3/8	12×1/2	200.4		4×3 ×1/2	......
334.8		6×4 ×3/8	14×3/8	233.4		4×3 ×5/8	....
370.7		5×3½×1/2	12×1/2	233.5	26×7/16†	5×3½×1/2	....
379.4		6×4 ×3/8	14×1/2	265.8		6×4 ×1/2	.. ....
408.6		5×3½×1/2	12×5/8	274 5		5×3½×5/8	....

* From Pocket Companion, Carnegie Steel Company.

† For additional sections with these web-plates, but with 6 × 6-in ∠s and no covers, see tabulation at end of this table.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches			Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
314 8		6×4 ×⅝	.	500.5		6×4 ×½	14×⅝
361 3		6×4 ×¾	. .	549.5	27×⅜†	6×4 ×⅝	14×⅝
384 0		5×3½×½	12×½	595.1		6×4 ×⅝	14×¾
421.8		5×3½×½	12×⅝	641.2		6×4 ×¾	14×¾
441 7	26×7/16†	6×4 ×½	14×½				
461.1		5×3½×⅝	12×⅝	245.2		5×3½×½	
485.9		6×4 ×½	14×⅝	279.0	27×7/16†	6×4 ×½	
532 7		6×4 ×⅝	14×⅝	288.1		5×3½×⅝	
576.5		6×4 ×⅝	14×¾	330 2		6×4 ×⅝	
620.5		6×4 ×¾	14×¾				
				474.3		6×4 ×½	14×½
185.6		5×3½×⅜		495 3		5×3½×⅝	12×⅝
211.0		6×4 ×⅜		521.9	28×⅜	6×4 ×½	14×⅝
230.3		5×3½×½		573 1		6×4 ×⅝	14×⅝
264.1		6×4 ×½	. .	620 4		6×4 ×⅝	14×¾
273.2		5×3½×⅝		668.6		6×4 ×¾	14×¾
304.5		5×3½×⅜	12×⅜				
315.3		6×4 ×⅝	....	257.1		5×3½×½	.
344.2		5×3½×⅜	12×½	292 4		6×4 ×½	.
349.8	27×5/16†	6×4 ×⅜	14×⅜	301 8		5×3½×⅝	.
387 3		5×3½×½	12×½	345 8		6×4 ×⅝	
396.2		6×4 ×⅜	14×½	396.5		6×4 ×¾	
426.7		5×3½×½	12×⅝	419.5		5×3½×½	12×½
447.4		6×4 ×½	14×½	445.1	28×7/16	6×4 ×⅞	. ....
467.7		5×3½×⅝	12×⅝	460 2		5×3½×½	12×⅝
493.4		6×4 ×½	14×⅝	482.0		6×4 ×½	14×½
542.4		6×4 ×⅝	14×⅝	503 0		5×3½×⅝	12×⅝
588.0		6×4 ×⅝	14×¾	529 6		6×4 ×½	14×⅝
				580.8		6×4 ×⅝	14×⅝
193.1		5×3½×⅜	. .	628 0		6×4 ×⅝	14×¾
218.5		6×4 ×⅜	. .	676.2		6×4 ×¾	14×¾
237.8		5×3½×½	..				
271.5		6×4 ×½	.	221 8		5×3½×⅜	
280.6		5×3½×⅝		250.5		6×4 ×⅜	
311.7		5×3½×⅜	12×⅜	272 1		5×3½×½	..
322.7		6×4 ×⅝	.....	310 3		6×4 ×½	. . .
351.4	27×⅜†	5×3½×⅜	12×½	320.5		5×3½×⅝	. .
357.1		6×4 ×⅜	14×⅜	353.8		5×3½×⅜	12×⅜
371.4		6×4 ×¾	.....	366.2	30×⅜†	5×3½×¾	.....
394.5		5×3½×½	12×½	368.1		6×4 ×⅝	......
403.4		6×4 ×⅜	14×½	397.8		5×3½×⅜	12×½
417.9		6×4 ×⅞	....	404.7		6×4 ×⅜	14×⅜
433.8		5×3½×½	12×⅝	423.1		6×4 ×¾	....
454.6		6×4 ×½	14×½	446.6		5×3½×½	12×½
474.8		5×3½×⅝	12×⅝	456.1		6×4 ×⅜	14×½

* From Pocket Companion, Carnegie Steel Company.

† For additional sections with these web-plates, but with 6 × 6-in ∠s and no covers, see tabulation at end of this table.

Table II * (Continued) Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates
475.8		6×4 ×7/8	
490.3		5×3½×½	12×5/8
514.0		6×4 ×½	14×½
536.7	30×3/8†	5×3½×5/8	12×5/8
565.1		6×4 ×½	14×5/8
620.6		6×4 ×5/8	14×5/8
671.3		6×4 ×5/8	14×3/4
723.8		6×4 ×3/4	14×3/4
281.4		5×3½×½	
319.5		6×4 ×½	
329.7		5×3½×5/8	
375.5		5×3½×3/4	
377.3		6×4 ×5/8	
432.3		6×4 ×3/4	
455.5		5×3½×½	12×½
485.0	30×7/16†	6×4 ×7/8	...
499.2		5×3½×½	12×5/8
523.0		6×4 ×½	14×½
545.6		5×3½×5/8	12×5/8
574.0		6×4 ×½	14×5/8
629.5		6×4 ×5/8	14×5/8
680.1		6×4 ×5/8	14×3/4
732.6		6×4 ×3/4	14×3/4
290.6		5×3½×½	
328.8		6×4 ×½	
338.9		5×3½×5/8	
384.7		5×3½×3/4	
386.5		6×4 ×5/8	.
441.5		6×4 ×3/4	.
464.4		5×3½×½	12×½
494.2	30×½†	6×4 ×7/8	. .
508.0		5×3½×½	12×5/8
531.9		6×4 ×½	14×½
554.5		5×3½×5/8	12×5/8
582.8		6×4 ×½	14×5/8
638.3		6×4 ×5/8	14×5/8
688.9		6×4 ×5/8	14×3/4
741.3		6×4 ×3/4	14×3/4
251.7		5×3½×3/8	
283.7	33×3/8	6×4 ×3/8	
307.7		5×3½×½	......
308.4		6×6 ×3/8	. ...

Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates
350.3		6×4 ×½	.
361.5		5×3½×5/8	.
383.6		6×6 ×½	.
396.9		5×3½×3/8	12×3/8
412.5		5×3½×3/4	......
414.7		6×4 ×5/8	....
445.5		5×3½×3/8	12×½
453.4		6×4 ×3/8	14×3/8
455.9		6×6 ×5/8	......
476.1		6×4 ×3/4	......
477.6		6×6 ×3/8	14×3/8
499.8		5×3½×½	12×½
510.0		6×4 ×3/8	14×½
525.4		6×6 ×3/4	.....
534.1	33×3/8	6×6 ×3/8	14×½
548.0		5×3½×½	12×5/8
574.7		6×4 ×½	14×½
590.6		6×6 ×3/8	14×5/8
592.6		6×6 ×7/8	. ...
599.9		5×3½×5/8	12×5/8
607.1		6×6 ×½	14×½
630.9		6×4 ×½	14×5/8
663.1		6×6 ×½	14×5/8
693.0		6×4 ×5/8	14×5/8
719.2		6×6 ×½	14×3/4
732.7		6×6 ×5/8	14×5/8
748.9		6×4 ×5/8	14×3/4
788.3		6×6 ×5/8	14×3/4
807.6		6×4 ×3/4	14×3/4
854.9		6×6 ×3/4	14×3/4
318.9		5×3½×½	
361.5		6×4 ×½	
372.7		5×3½×5/8	.
394.8		6×6 ×½	.
423.7		5×3½×3/4	
425.8		6×4 ×5/8	....
467.0	33×7/16	6×6 ×5/8	....
487.2		6×4 ×3/4	..
510.7		5×3½×½	12×½
536.6		6×6 ×3/4	.....
558.8		5×3½×½	12×5/8
585.6		6×4 ×½	14×½
603.8		6×6 ×7/8	......
610.6		5×3½×5/8	12×5/8

* From Pocket Companion, Carnegie Steel Company.

† For additional sections with these web-plates, but with 6 × 6-in ∠s and no covers, see tabulation at end of this table.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches			Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
617.9		6×6 ×½	14×½	531.6		6×6 ×⅜	14×⅜
641.7		6×4 ×½	14×⅝	554.3		5×3½×½	12×½
673.9		6×6 ×½	14×⅝	565 1		6×4 ×⅜	14×½
703.8		6×4 ×⅝	14×⅝	593 2		6×6 ×⅜	14×½
729.9	33×7/16	6×6 ×½	14×¾	595.3		6×4 ×⅞	. ..
743.5		6×6 ×⅝	14×⅝	606 8		5×3½×½	12×⅝
759.6		6×4 ×⅝	14×¾	636.5		6×4 ×½	14×½
799.0		6×6 ×⅝	14×¾	654 9		6×6 ×⅜	14×⅝
818.3		6×4 ×¾	14×¾	664 2	36×⅜	5×3½×⅝	12×⅝
865.6		6×6 ×¾	14×¾	674 4		6×6 ×½	14×½
				698.0		6×4 ×½	14×⅝
330.0		5×3½×½		735.5		6×6 ×½	14×⅝
372.6		6×4 ×½	.	766.6		6×4 ×⅝	14×⅝
383.9		5×3½×⅝	. .	796 8		6×6 ×½	14×¾
406.0		6×6 ×½	.	813 1		6×6 ×⅝	14×⅝
434.9		5×3½×¾	..	827.6		6×4 ×⅝	14×¾
437.0		6×4 ×⅝		873 8		6×6 ×⅝	14×¾
478.2		6×6 ×⅝	.	892.8		6×4 ×¾	14×¾
498.4		6×4 ×¾	.				
521.5		5×3½×½	12×½	357.7		5×3½×½	
547 8		6×6 ×¾	. . . .	404 7		6×4 ×½	.
569.5		5×3½×½	12×⅝	417.0		5×3½×⅝	..
596.4	33×½	6×4 ×½	14×½	443 6		6×6 ×½	.
615.0		6×6 ×⅞	.....	473 3		5×3½×¾	. .
621.4		5×3½×⅝	12×⅝	475.7		6×4 ×⅝	
628.8		6×6 ×½	14×½	523.8		6×6 ×⅝	..
652.5		6×4 ×½	14×⅝	543.5		6×4 ×¾	. . . .
684 6		6×6 ×½	14×⅝	567 2		5×3½×½	12×½
714.5		6×4 ×⅝	14×⅝	608 6		6×4 ×⅞	. . . .
740.6		6×6 ×½	14×¾	619.7	36×7/16	5×3½×½	12×⅝
754 3		6×6 ×⅝	14×⅝	649 5		6×4 ×½	14×½
770.3		6×4 ×⅝	14×¾	677.1		5×3½×⅝	12×⅝
809.7		6×6 ×⅝	14×¾	687.3		6×6 ×½	14×½
829.0		6×4 ×¾	14×¾	710.8		6×4 ×½	14×⅝
876.3		6×6 ×¾	14×¾	748.4		6×6 ×½	14×⅝
				779.5		6×4 ×⅝	14×⅝
318.0		6×4 ×⅜	..	809.5		6×6 ×½	14×¾
344.4		5×3½×½	..	825.9		6×6 ×⅝	14×⅝
346.9		6×6 ×⅜	. ..	840 4		6×4 ×⅝	14×¾
391.4		6×4 ×½	..	886 6		6×6 ×⅝	14×¾
403.7		5×3½×⅝	..	905.5		6×4 ×¾	14×¾
430 3	36×⅜	6×6 ×½	. .				
469.0		5×3½×¾		418.0		6×4 ×½	.. .
462.4		6×4 ×⅝		456.9		6×6 ×½	. . .
503.3		6×4 ×⅜	14×⅜	489.0	36×½	6×4 ×⅝	. .....
510.5		6×6 ×⅝	.	537.1		6×6 ×⅝	....
530.2		6×4 ×¾	.	556 9		6×4 ×¾	.....

* From Pocket Companion, Carnegie Steel Company.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1-1, inches³	Size in inches			Section-modulus, axis 1-1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
614.5		6×6 ×3/4	.	527.2		6×6 ×1/2	. ..
621.9		6×4 ×7/8	.	561.4		6×4 ×5/8	..
662.5		6×4 ×1/2	14×1/2	606.6		6×4 ×3/8	14×3/8
689.2		6×6 ×7/8	. .	623.5		6×6 ×5/8	...
700.3		6×6 ×1/2	14×1/2	638.3		6×4 ×3/8	16×3/8
723.7		6×4 ×1/2	14×5/8	642.1		6×4 ×3/4	. ...
761.3		6×6 ×1/2	14×5/8	643.2		6×6 ×3/8	14×3/8
792.3		6×4 ×5/8	14×5/8	675.1		6×6 ×3/8	16×3/8
822.3	36×1/2	6×6 ×1/2	14×3/4	678.6		6×4 ×3/8	14×1/2
838.8		6×6 ×5/8	14×5/8	715.2		6×6 ×3/8	14×1/2
853.2		6×4 ×5/8	14×3/4	716.5		6×6 ×3/4	.. . .
899.4		6×6 ×5/8	14×3/4	719.5		6×4 ×7/8	.....
918.3		6×4 ×3/4	14×3/4	757.7		6×6 ×3/8	16×1/2
973.7		6×6 ×3/4	14×3/4	763.7		6×4 ×1/2	14×1/2
1039.4		6×4 ×3/4	14×1	787.2		6×6 ×3/8	14×5/8
1094.1		6×6 ×3/4	14×1	806.2		6×4 ×1/2	16×1/2
1101.1		6×4 ×7/8	14×1	806.4		6×6 ×7/8	. .
1164.9		6×6 ×7/8	14×1	812.7		6×6 ×1/2	14×1/2
				835.5	42×3/8	6×4 ×1/2	14×5/8
444.7		6×4 ×1/2		855.2		6×6 ×1/2	16×1/2
483.5		6×6 ×1/2		884.2		6×6 ×1/2	14×5/8
515.7		6×4 ×5/8		917.3		6×4 ×5/8	14×5/8
563.7		6×6 ×5/8	.	937.3		6×6 ×1/2	16×5/8
583.5		6×4 ×3/4	.	955.7		6×6 ×1/2	14×3/4
641.2		6×6 ×3/4	...	970.4		6×4 ×5/8	16×5/8
648.5		6×4 ×7/8	.	977.6		6×6 ×5/8	14×5/8
688.4		6×4 ×1/2	14×1/2	988.7		6×4 ×5/8	14×3/4
715.8		6×6 ×7/8	.....	1030.8		6×6 ×5/8	16×5/8
726.2		6×6 ×1/2	14×1/2	1048.6		6×6 ×5/8	14×3/4
749.4		6×4 ×1/2	14×5/8	1066.6		6×4 ×3/4	14×3/4
787.0	36×5/8	6×6 ×1/2	14×5/8	1112.4		6×6 ×5/8	16×3/4
818.1		6×4 ×5/8	14×5/8	1130.4		6×4 ×3/4	16×3/4
847.9		6×6 ×1/2	14×3/4	1138.5		6×6 ×3/4	14×3/4
864.6		6×6 ×5/8	14×5/8	1194.1		6×6 ×5/8	16×7/8
878.8		6×4 ×5/8	14×3/4	1202.3		6×6 ×3/4	16×3/4
924.9		6×6 ×5/8	14×3/4	1283.5		6×6 ×3/4	16×7/8
943.9		6×4 ×3/4	14×3/4	1286.4		6×4 ×7/8	16×7/8
999.3		6×6 ×3/4	14×3/4	1369.9		6×6 ×7/8	16×7/8
1045.9		6×6 ×5/8	14×1				
1064.7		6×4 ×3/4	14×1	495.3		6×4 ×1/2	.. ...
1119.3		6×6 ×3/4	14×1	545.4		6×6 ×1/2	. .....
1126.3		6×4 ×7/8	14×1	579.5		6×4 ×5/8	....
1190.1		6×6 ×7/8	14×1	641.6	42×7/16	6×6 ×5/8	....
				660.2		6×4 ×3/4	......
390.2		6×4 ×3/8	. . ...	734.7		6×6 ×3/4	. .....
427.5	42×3/8	6×6 ×3/8	. .....	737.6		6×4 ×7/8	... ..
477.2		6×4 ×1/2	....	781.5		6×4 ×1/2	14×1/2

* From Pocket Companion, Carnegie Steel Company.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches			Section modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
824.0		6×4 ×½	16×½	1165 4		6×4 ×¾	16×¾
824.6		6×6 ×⅞	. . . . .	1173 6		6×6 ×¾	14×¾
830.4		6×6 ×½	14×½	1229.0		6×6 ×⅝	16×⅞
853.1		6×4 ×½	14×⅝	1237.4	42×½	6×6 ×¾	16×¾
872 9		6×6 ×½	16×½	1318.4		6×6 ×¾	16×⅞
901.8		6×6 ×½	14×⅝	1321.2		6×4 ×⅞	16×⅞
934 9		6×4 ×⅝	14×⅝	1404.7		6×6 ×⅞	16×⅞
954 9		6×6 ×½	16×⅝				
973 2		6×6 ×½	14×¾	466.9		6×4 ×⅜	. . . . .
988.1		6×4 ×⅝	16×⅝	512.7		6×6 ×⅜	. . . . .
995.3	42×7/16	6×6 ×⅝	14×⅝	567.4		6×4 ×½	. .
1006.2		6×4 ×⅝	14×¾	628 9		6×6 ×½	. . . .
1048.4		6×6 ×⅞	16×⅝	664.9		6×4 ×⅝	.
1066.2		6×6 ×⅝	14×¾	714.4		6×4 ×⅜	14×⅜
1084.1		6×4 ×¾	14×¾	741 3		6×6 ×⅝	.
1129.9		6×6 ×⅝	16×¾	750 8		6×4 ×⅜	16×⅜
1147.9		6×4 ×¾	16×¾	758 5		6×4 ×¾	
1156.0		6×6 ×¾	14×¾	759.5		6×6 ×⅜	14×⅜
1211 6		6×6 ×⅝	16×⅞	795.9		6×6 ×⅜	16×⅜
1219 8		6×6 ×¾	16×¾	797 0		6×4 ×⅜	14×½
1300.9		6×6 ×¾	16×⅞	841.9		6×6 ×⅜	14×½
1387.3		6×6 ×⅞	16×⅞	848 3		6×4 ×⅞	. . .
				850 1		6×6 ×¾	. .
513.5		6×4 ×½	. . . . . .	890.4		6×6 ×⅜	16×½
563.5		6×6 ×½	. . . .	895.5		6×4 ×½	14×½
597.7		6×4 ×⅝	.	924 3		6×6 ×⅜	14×⅝
659.8		6×6 ×⅝	. . .	944.0		6×4 ×½	16×½
678.4		6×4 ×¾	. . .	955.2	48×⅜	6×6 ×⅞	. . . .
752.8		6×6 ×¾	. .	955 8		6×6 ×½	14×½
755.8		6×4 ×⅞	. . . .	977 7		6×4 ×½	14×⅝
799.2		6×4 ×½	14×½	1004.3		6×6 ×½	16×½
841.7		6×4 ×½	16×½	1037.6		6×6 ×½	14×⅝
842.7		6×6 ×⅞	. . . . . .	1072.7		6×4 ×⅝	14×⅝
848.1		6×6 ×½	14×½	1098.2		6×6 ×½	16×⅝
870 8	42×½	6×4 ×½	14×⅝	1119.5		6×6 ×½	14×¾
890.6		6×6 ×½	16×½	1133.3		6×4 ×⅝	16×⅝
919.4		6×6 ×½	14×⅝	1147.1		6×6 ×⅝	14×⅝
952.6		6×4 ×⅝	14×⅝	1154.4		6×4 ×⅝	14×¾
972.6		6×6 ×½	16×⅝	1207.8		6×6 ×⅝	16×⅝
990.8		6×6 ×½	14×¾	1228.4		6×6 ×⅝	14×¾
1005.7		6×4 ×⅝	16×⅝	1245.2		6×4 ×¾	14×¾
1012.9		6×6 ×⅝	14×⅝	1301.2		6×6 ×⅝	16×¾
1023.7		6×4 ×⅝	14×¾	1317.9		6×4 ×¾	16×¾
1066.0		6×6 ×⅝	16×⅝	1334.0		6×6 ×¾	14×¾
1083.7		6×6 ×⅝	14×¾	1394.7		6×6 ×⅝	16×⅞
1101.7		6×4 ×¾	14×¾	1406.7		6×6 ×¾	16×¾
1147.5		6×6 ×⅝	16×¾	1498.1		6×4 ×⅞	16×⅞

* From Pocket Companion, Carnegie Steel Company.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches: Web plates	Size in inches: Flange angles	Size in inches: Flange plates
1499.7	48×3/8	6×6 ×3/4	16×7/8
1601.3		6×6 ×7/8	16×7/8
591.2		6×4 ×1/2	
652.7		6×6 ×1/2	
688.7		6×4 ×5/8	
765.0		6×6 ×5/8	
782.3		6×4 ×3/4	.
872.1		6×4 ×7/8	...
873.8		6×6 ×3/4	..
918.8		6×4 ×1/2	14×1/2
967.3		6×4 ×1/2	16×1/2
979.0		6×6 ×7/8	.
979.0		6×6 ×1/2	14×1/2
1000.8		6×4 ×1/2	14×5/8
1027.6		6×6 ×1/2	16×1/2
1060.8		6×6 ×1/2	14×5/8
1095.8	48×7/16	6×4 ×5/8	14×5/8
1121.4		6×6 ×1/2	16×5/8
1142.5		6×6 ×1/2	14×3/4
1156.5		6×4 ×5/8	16×5/8
1170.3		6×6 ×5/8	14×5/8
1177.4		6×4 ×5/8	14×3/4
1230.9		6×6 ×5/8	16×5/8
1251.5		6×6 ×5/8	14×3/4
1268.2		6×4 ×3/4	14×3/4
1324.3		6×6 ×5/8	16×3/4
1341.0		6×4 ×3/4	16×3/4
1357.0		6×6 ×3/4	14×3/4
1417.7		6×6 ×5/8	16×7/8
1429.8		6×6 ×3/4	16×3/4
1521.0		6×4 ×7/8	16×7/8
1522.7		6×6 ×3/4	16×7/8
1624.2		6×6 ×7/8	16×7/8
615.0		6×4 ×1/2	
676.4		6×6 ×1/2	. .
712.4		6×4 ×5/8	.
788.8		6×6 ×5/8	.. ..
806.0		6×4 ×3/4	. .
895.8	48×1/2	6×4 ×7/8	......
897.6		6×6 ×3/4	....
942.1		6×4 ×1/2	14×1/2
990.6		6×4 ×1/2	16×1/2
1002.3		6×6 ×1/2	14×1/2
1002.7		6×6 ×7/8	... ..
1024.0		6×4 ×1/2	14×5/8
1050.8		6×6 ×1/2	16×1/2
1083.9		6×6 ×1/2	14×5/8
1119.0		6×4 ×5/8	14×5/8
1144.5		6×6 ×1/2	16×5/8
1165.6		6×6 ×1/2	14×3/4
1179.6		6×4 ×5/8	16×5/8
1193.4		6×6 ×5/8	14×5/8
1200.5		6×4 ×5/8	14×3/4
1254.1	48×1/2	6×6 ×5/8	16×5/8
1274.5		6×6 ×5/8	14×3/4
1291.2		6×4 ×3/4	14×3/4
1347.3		6×6 ×5/8	16×3/4
1364.0		6×4 ×3/4	16×3/4
1380.0		6×6 ×3/4	14×3/4
1440.6		6×6 ×5/8	16×7/8
1452.8		6×6 ×3/4	16×3/4
1543.9		6×4 ×7/8	16×7/8
1545.6		6×6 ×3/4	16×7/8
1647.1		6×6 ×7/8	16×7/8
194.7		6×6 ×3/8	.. ...
245.7	24×5/16	6×6 ×1/2	.
294.2		6×6 ×5/8	
340.7		6×6 ×3/4	.
200.6		6×6 ×3/8	.
251.5		6×6 ×1/2	....
300.1	24×3/8	6×6 ×5/8	..
346.6		6×6 ×3/4	. . .
216.6		6×6 ×3/8	. ...
272.9		6×6 ×1/2	
326.7	26×5/16	6×6 ×5/8	. .
378.2		6×6 ×3/4	.. .
223.5		6×6 ×3/8	. .
279.8	26×3/8	6×6 ×1/2	.. ...
333.6		6×6 ×5/8	.
385.2		6×6 ×3/4	.....
230.4		6×6 ×3/8	.....
286.7	26×7/16	6×6 ×1/2	.. ..
340.5		6×6 ×5/8	....
392.1		6×6 ×3/4	... .

* From Pocket Companion, Carnegie Steel Company.

Table II * (Continued). Gross Section-Modulus of Riveted Girders

Section-modulus, axis 1–1, inches³	Size in inches			Section-modulus, axis 1–1, inches³	Size in inches		
	Web plates	Flange angles	Flange plates		Web plates	Flange angles	Flange plates
227.8		6×6 ×3/8		271.2		6×6 ×3/8	...
286.8	27×5/16	6×6 ×1/2	.	338.3	30×3/8	6×6 ×1/2	...
343.1		6×6 ×5/8	. .	402.6		6×6 ×5/8	
397.3		6×6 ×3/4		464.4		6×6 ×3/4	
235.2		6×6 ×3/8		280.4		6×6 ×3/8	..
294.2	27×3/8	6×6 ×1/2	.	347.5	30×7/16	6×6 ×1/2	
350.6		6×6 ×5/8		411.8		6×6 ×5/8	. .
404.7		6×6 ×3/4		473.6		6×6 ×3/4	.
242.7		6×6 ×3/8		289.6		6×6 ×3/8	
301.7	27×7/16	6×6 ×1/2		356.7	30×1/2	6×6 ×1/2	
358.1		6×6 ×5/8	. .	421.0		6×6 ×5/8	
412.2		6×6 ×3/4		482.8		6×6 ×3/4	

* From Pocket Companion, Carnegie Steel Company.

CHAPTER XX

WOOD FRAMING

By

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1. General Considerations

General Considerations. Among the considerations influencing the selection of timber may be mentioned the following as having the most general importance: DENSITY, DEFECTS, DRYNESS and ECONOMY. The first three have direct influence upon the strength of the material, and the last consideration controls the success of the entire project.

Density is now considered as the important factor in determining the efficiency of lumber, since the strength of wood both in bending and in compression increases rapidly as the density increases. It is, therefore, used as a measure of quality in formulating the rules for the grading of lumber. The more dense grades are then selected for the heavy load bearing timbers such as girders or posts and the open grain lumber for light framing and studs. The summer growth, called summer wood, has thicker cell walls than the spring growth and is therefore denser and stronger. Likewise wood with narrow annual rings is denser and stronger than wood with wide rings. For these reasons timber selected for density must have at least six annual rings to a radial inch, each ring containing at least one-third summer wood.

Defects. Large or loose knots, excess of sapwood, checks or decay naturally reduce the strength of timber both as beams and posts Defects in the under side or bottom quarter of a beam are particularly weakening since the fibers in maximum tension are at the lower edge. Defects near the neutral plane at the center of a beam reduce the area resisting horizontal shear. The grading rules, therefore, recognize such defects as being vital factors in determining the class to which a timber belongs. It should be remembered, however, that grades of wood which cannot be used for interior finish or heavy load bearing may be perfectly permissible for studding, boarding or light framing.

Dryness. Extensive tests have shown that the strength of timber is increased as its moisture content is decreased, that is to say, dry or seasoned wood is much stronger and stiffer than green wood. The amount of consideration which should be given to this increased strength depends upon the purpose for which the timber is used. If in positions exposed to the weather, as in bridges, trestles or open sheds, the wood will absorb moisture and consequently its strength is only that of green wood. On the other hand, if the timber be protected, as in building-construction, advantage may be taken of the increased strength gained by seasoning. This increase is more evident in clear moderately small pieces than in pieces of large dimensions, because the checking due to shrinkage in the drying process is more severe in the larger timbers and consequently reduces the strength of the piece.

Economy. The denser and more perfect grades, though stronger, are more expensive as well as heavier to handle and harder to cut. In light building-construction they should be used, therefore, only where especial strength is required, as in girders, posts or heavy joists, the less dense and less perfect grades or lighter species of woods being specified for studding, light joists and rafters. Wood and sizes of members easily obtainable in local markets should be selected as far as possible, and the lengths should be chosen for least waste in cutting.

Seasoning. In wood frame construction it is very important that the minimum amount of shrinking, twisting, checking and warping should take place after the wood is in place in the building. This is true in the case of joists, studs, girders and framing timber as well as for wood trim, sash and doors, and quite irrespective of the relative strength of the wood before and after drying. All timber should, therefore, be shrunk by drying before being delivered at the building, the drying or seasoning being done by either the AIR-DRYING or the KILN-DRYING process. More moisture, however, is permissible for structural than for decorative uses such as interior trim and paneling.

Seasoning of structural timber is usually done by the AIR-DRYING process, which consists of piling the material in sheds and allowing currents of air to circulate freely between and around the pieces. Air-dried lumber averages from 12 to 20% moisture content. The KILN-DRYING process consists of piling the material in heated kilns and is used most generally for wood intended for interior trim, flooring, etc. It reduces the moisture content to from 6 to 12%.

Kinds of Wood. In general, heavy dense woods are stronger than the lighter woods of coarser fiber and are used in beams and girders where especial strength is required. The lighter woods are easier and quicker to handle and are largely employed for joists, studding, etc., especially in the frame construction of residences and buildings of moderate size. In the West, Douglas fir is a most satisfactory wood, being very strong for its weight, and it is largely used for both heavy and light framing. In the East dense Southern yellow pine, formerly specified as long-leaf yellow pine, or L.L.Y.P., is used for girders and posts and wherever heavy loads are to be carried, while the joists and lighter pieces are cut from common Southern yellow pine, spruce and hemlock.

Grading. Of late years much study has been given to the grading of lumber according to its quality by the lumber associations, such as the National Lumber Manufacturers' Association, the Southern Pine Association and the West Coast Lumberman's Association together with the Departments of Commerce and Agriculture at Washington, with the result that lumber grades are now well standardized and the product purchased as belonging to a certain grade is fairly reliable. Higher allowable working stresses for good structural material are consequently being permitted by the building codes of the various cities of the country.

The General Central Committee on Lumber Standards constituted from the Department of Commerce and from the lumber associations arrived at a general agreement on national standards for softwood, which the lumber associations have used as a basis for preparing and coordinating their grading rules.

SOFTWOOD as it comes from the sawmill is divided into three main classes (*a*) YARD LUMBER, (*b*) STRUCTURAL MATERIAL, and (*c*) FACTORY and SHOP LUMBER.

YARD LUMBER constitutes 80% of the lumber produced. It is the type carried most generally in retail lumber-yards throughout the country and is of most interest to architects, engineers, contractors and builders for general utility purposes. It includes boards and siding less than 2 in thick and of any width; planks less than 4 in thick and over 8 in wide; scantlings less than 5 in thick and less than 8 in wide; and heavy joists 4 in thick and 8 in or more in width. The grading is largely on the basis of appearance and the presence of knots, sap and other blemishes. It will be seen that the ordinary 2 × 4-in, 3 × 4-in, 4 × 4-in, 2 × 6-in, and 3 × 6-in studs are included in this class. Where joists, rafters and posts are required to meet definite working stresses they should be chosen, however, from structural material rather than from the yard lumber.

STRUCTURAL MATERIAL is heavy material for load-bearing and is graded almost wholly on the basis of its density, strength and stiffness It comprises such items as beams, girders, posts and sills which are 5 in and thicker in least dimensions, and also joists, rafters, plank and heavy flooring 2 to 4 in thick. Typical uses for structural materials are light frame and heavy timber construction, definite stress values being assigned to the various grades.

FACTORY and SHOP LUMBER is graded wholly for further manufacture into doors, sash, millwork, patterns, toys and many other industrial commodities. It is, therefore, of most importance to manufacturers and has very little relation to the structure of light wood framing. Hence it will not be further considered in this chapter.

Yard lumber is generally graded in six grades, Grades A, B, and C and Grades No. 1 common, No. 2 common and No. 3 common, Grades A and B often being combined into one grade called Grade B and Better. The first three grades are for trim and fittings either natural finish or painted. The last three are for use where appearance is not so important. Ordinary studding, joists and rafters, unless subjected to special loads, are usually taken from No. 2 common grade.

Structural material is now generally graded by the lumber associations according to the basic classifications and standards agreed upon by the General Central Committee. These basic classifications are as follows:

(*a*) Dense select—Douglas fir and Southern pine
(*b*) Select—Douglas fir
(*c*) Select—Other softwood species except Southern pine
(*d*) Dense Common—Douglas fir and Southern pine
(*e*) Common—All softwood species

Following as far as possible these classifications as a basis the most important lumber associations have determined their grades and prepared their grading rules for structural material.

Manufacturers' Association Standard Grades

SPECIES OF TIMBER	GRADE
Douglas fir, Coast Region	Dense Super Structural Super Structural Dense Structural Structural Common Structural
Douglas fir, Inland Empire	Dense Super Structural Dense Structural No. 1—Common Dimension and Timbers

Larch, Western	No. 1 Common Dimension and Timbers
Pine, Southern yellow	Extra Dense Select Structural Select Structural Extra Dense Heart Dense Heart Structural Square Edge and Sound No. 1—Common
Redwood	Super Structural Prime Structural Select Structural Heart Structural

The following woods are classified according to the Basic Provisions for Structural Material into two grades, select and common.

Cedar, Alaska
Cedar, North and South White
Cedar, Port Orford
Cedar, Western red
Cypress, Southern
Tamarack, Eastern
Douglas fir, Rocky Mt. Region
Fir, balsam
Fir, golden, noble, silver, white
Hemlock, Eastern
Hemlock, West Coast
Oak, white and red
Pine, Calif., Idaho,
Northern white, lodgepole
pondosa, sugar, Norway
Spruce, Englemann
Spruce, red, white, Sitka

The allowable unit stresses for each grade in the two lists above will be found in Table I.

In general it is recommended that the minimum quality to be used for framing and structural purposes in the dry and protected locations usual in buildings shall be as follows:

(*a*) For lumber less than 2 in thick and for all studding, No. 1 common, yard lumber.

(*b*) For joists and rafters, common structural.

(*c*) For girders, posts and heavy beams, structural, prime structural or structural square edge and sound, depending upon the species.

The more dense and select grades are generally used only for trestles, bridges and exposed positions under heavy loads, but should be chosen also building-construction wherever the stresses are sufficiently high to require absolutely dependable quality of material.

Certain changes in the specifications for Southern pine should be noted. Southern pine is a general term for yellow pine as distinguished from white California pine. The yellow pine was formerly again divided upon a commercial basis into long-leaf yellow pine and short-leaf yellow pine, known as L.L.Y.P. and S.L.Y.P. While it is true the short-leaf and long-leaf pines are distinct species, the terms have largely come to be a description of their strength, weight and density. The denser wood is heavier and stronger, and a larger percentage of the true long-leaf pine than of the short-leaf pine is heavy, strong and dense. Commercially, then, it is allowable to furnish either long-leaf or short-leaf pine if the specifications for strength and density be fulfilled. The strongest quality is graded simply as Southern yellow pine, dense select structural, and this is the wood usually desired when specified as L.L.Y.P. If a lighter, softer, less strong but more easily worked yellow pine were desired a S.L.Y.P. might readily be specified but the grade would

be Southern pine No. 1 common. The pines formerly known commercially as North Carolina and Georgia pines are included in the general term Southern yellow pine and are now graded for density as are the long-leaf and short-leaf pines.

Size Standards. It is commercially allowable to furnish lumber the actual dimensions of which are somewhat less than the nominal dimensions. Such allowances are due to the shrinkage of wood in the seasoning process after sawing and because lumber is often shipped surfaced or planed on one or two sides and on one or two edges, S1S1E, S2S2E, S1E, etc. Nominal thickness of 1 in and 2 in must be actually at least $^{25}/_{32}$ in and $1^5/_8$ in, respectively. Corresponding allowances varying from $^3/_8$ to $^3/_4$ in are made in the widths, a smaller allowance being made in the narrower pieces than in the wider. Rough lumber is wider and thicker than dressed lumber of the same nominal dimensions by the amount removed by the surfacing. Such deductions in width and thickness naturally affect the strength of the piece and the actual dimensions only must be used in all calculations.

2. Light Wood Framing

Light Wood Framing is the type of framing employed in wood construction, such as residences or the smaller types of buildings, where the loads are fairly light. It also usually refers to the use of wood for the construction of the walls and partitions as well as of the floors, although wood floor joists and wood stud partitions are often used with enclosing walls of masonry. HEAVY WOOD FRAMING, or the beam and girder framing used in mill construction, garages, hangars and buildings involving heavy loads, necessitates different and special methods which are treated in Chapter XXI.

There are two methods of framing the exterior walls in light wood construction which are in general use, the BALLOON FRAME and the COMBINATION FRAME. In various parts of the country modifications and variations in detail of these two general methods may be found, but the fundamental principles readily fall under these two heads

A wood frame consists in general of a piece called a SILL laid level on top of the masonry cellar wall and bedded in cement mortar. Vertical pieces, the CORNER-POSTS and the STUDS, are fastened to the sill and support the PLATE, a horizontal piece at the top of the exterior walls carrying the RAFTERS or ROOF-BEAMS. The FLOOR BEAMS or JOISTS for the ground floor are supported on the SILL, and for the second and third floors on GIRTS and RIBBONS, which are horizontal pieces supported by the studs or fastened to them at the floor-levels.

The Balloon Frame (see Fig. 1) is the cheaper and lighter method and the quicker to construct. It is, however, far less rigid and permanent and much more readily consumed by fire. In this method the sills are rarely more than 4 × 4 in and are sometimes only 2 × 6-in plank. They are laid on the basement wall bedded in cement and halved at the corners. The first-story joists are spiked in place on the sill, and the corner-posts, 4 × 4 in, or two 2 × 4 in, are set in position and held by temporary braces The studs, which run from sill to plate, are spiked in position to the sill and held near their upper ends by temporary boards nailed across them. A 1 × 6-in, or 1 × 8-in board, called a RIBBON, is notched and nailed into the studs and corner-posts at the proper height to support the second-story joists, which are next nailed in place. A joist is brought against a stud wherever possible. The tops of the studs and corner-posts are then sawed off level and the plate, consisting of a

4 × 4-in piece, or two 2 × 4's, is nailed on top of the studs and halved together over the corner-posts. The outside boarding should be nailed on diagonally to brace the frame. It may be seen that the long slender studs, the light sill, plate and corner-posts, the thin ribbon and the omission of bracing, except that derived from the outside sheathing, all tend to produce a frame lacking in rigidity and liable to sway, creak and tremble in heavy winds. Unless fire stops of brick or concrete are introduced at the floor levels, the long unencumbered spaces between the studs extending from sill to roof-eaves provide excellent flues for the passage of flames and render the balloon frame easily consumed by fire.

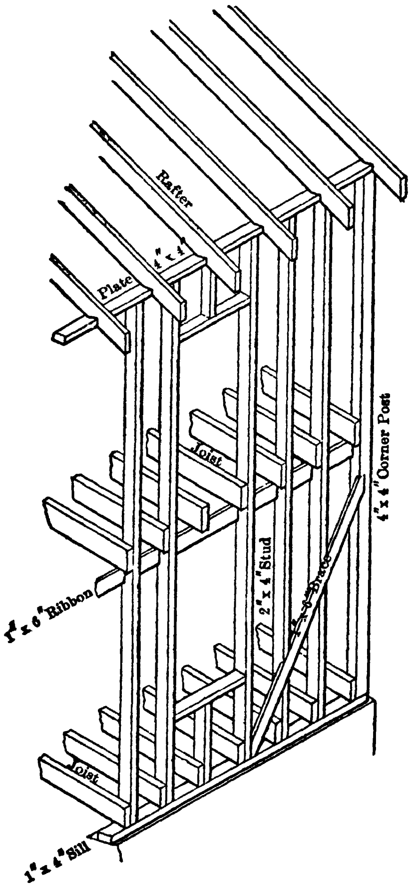

Fig. 1. Balloon Frame

Modifications to obtain greater rigidity consist in using 3 × 4-in studs and 4 × 6-in sills, in introducing horizontal bridging between the studs of the outer walls and in cutting 1 × 6-in boards diagonally into the studs from sill to corner-posts to act as braces.

The Combination Frame (see Fig. 2) is a modification of the old braced frame which was the original stoutly constructed frame of heavy timbers mortised and pinned together commonly used in this country until 1860 or 1870. The modifications consist of somewhat lighter timbers and less mortising and pinning to save material and labor. The method employs × 6-in sills, corner-posts and girts and 4 × 4-in, or 4 × 6-in plates. The girts are framed into the corner-posts at the second-story level and brace the whole frame as well as acting as support for the second-story floor joists. Diagonal 4 × 4-in braces are run from the sill to the corner-posts and from the corner-posts to the plate. Strips, 1 in × 4 in, let into the stud faces under the sheathing likewise make very efficient braces. The result is a very solid rigid framework for the entire body of the building quite irrespective of the bracing effects of studding, floor-joists or outside sheathing. The wall studs run from sill to girt and from girt to plate in separate lengths. The girts which support the floor-joists are called DROP GIRTS, but those

parallel to the joists are framed with their top edges level with the top of the joists to receive the flooring and are called RAISED OR FLUSH GIRTS. The studs should always be carried down to the sill and should never rest upon

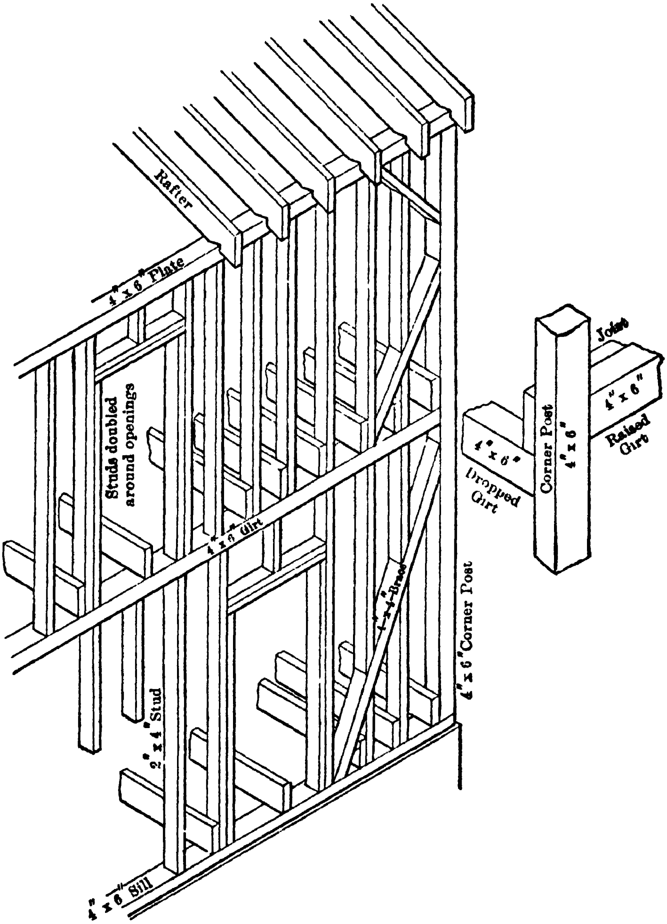

Fig. 2. Combination Frame

soles on top of the flooring over the joists. A somewhat stiffer frame may be constructed by mortise and tenon joints connected with hardwood pins, but to avoid excessive labor costs the pieces are now usually spiked together. Very much cutting for joints also weakens the members and larger dimensions were formerly used partly on this account.

3. Stiffness of Framed Buildings

Recently completed tests * (July, 1929) at the United States Forest Products Laboratory, Madison, Wisconsin, answer definitely many long-standing questions about the design of light-framed buildings and show the very great importance of a few simple precautions such as the use of DIAGONAL SHEATHING, effective LET-IN BRACING, intelligent NAILING, WOOD LATH and PLASTER and reasonably DRY LUMBER. The government engineers decided from investigation that typical wood-framed and sheathed walls are sufficiently strong to resist any pressure of wind blowing directly against them, but that wall resistance to END THRUST, caused when the pressure against the front is transmitted to the side walls, is very questionable unless the side walls are properly constructed. It is this end thrust which causes vibration and trembling of the building under moderate winds, and distortion and possible collapse under severe winds or earthquakes.

With this in mind, the laboratory subjected to test nearly fifty frame walls, 8 ft and 9 ft high and 12 ft and 14 ft long to show how a real wall would act under stress conditions. They were framed with 2 × 4-in studs spaced 16 in on centers and 4 × 6-in corner-posts, the sill was bolted to a fixed base and pressure was applied horizontally at the top plate in the plane of the wall surface. Both the rigidity of the walls, as shown by the end thrust necessary to cause a given movement of the end-posts from the perpendicular, and the strength, as evidenced by the end thrust required to cause failure of the whole panel, were measured. The results are measured in pounds and also upon the basis of stiffness and strength factors, the values for a horizontally sheathed panel, with two nails in each board at each stud, taken as unity. When it is considered that four-fifths of the houses in this country are of wood-frame construction, without including garages, shops, barns and other farm buildings, the significance of the test becomes apparent. The tabulation of the results of the tests follows.

The significant conclusions derived from the tests may be summarized as follows:

Inclination of Sheathing. When SHEATHED DIAGONALLY the walls are from 4 to 7 times as stiff and 7 to 8 times as strong as if HORIZONTALLY SHEATHED.

Frequency of Nailing. Three or four nails instead of two in 1 × 8-in horizontal sheathing improve the wall but little. In diagonally sheathed walls they add from 30% to 100% to the stiffness.

Size of Nails. Tenpenny nails instead of eights for horizontal sheathing increase the stiffness 50% and strength 40%, but do not improve diagonal sheathing.

Effect of Matching. Side- and end-matched sheathing not butting over the studs is as stiff and strong as sheathing which butts over the studs.

Effect of Green Lumber. A wall horizontally sheathed with green lumber and allowed to dry before testing lost about 50% in stiffness and 30% in strength compared to a dry-sheathed panel.

Types of Bracing. (*a*) HERRINGBONE OR BRIDGE BRACING between the studs has little value.

(*b*) Two by 4-in DIAGONAL CORNER BRACES cut in between studs add 60% to stiffness and 40% to strength.

* The following table and information are taken by permission from a pamphlet published by the National Lumber Manufacturers' Association incorporating the tests of the Forest Products Laboratory.

Results of Tests of Panels without Window and Door Openings

Size of panel, Height by length	Description of panel — Panel frames consisted of 2 by 4-in upper and lower plates, vertical studs spaced 16 in, and triple end-posts	Stiffness factor	Maximum load	Strength factor	Remarks
Ft In Ft In			Pounds		
9 ×14	8-in horizontal sheathing, two 8d nails, no braces				
7 4×12 1⅝	8-in horizontal sheathing, two 8d nails, no braces				
7 4×12 1⅝	8-in horizontal sheathing, two 8d nails, no braces . .	1.0	2 538	1.0	No 20 vibrated 50 000 cycles
9 ×14	8-in horizontal sheathing, two 8d nails, no braces				
9 ×14	8-in diagonal sheathing, two 8d nails, no braces, boards in tension	4.3	.	Over 8	Test stopped at 20 000 lb load
7 4×12 1⅝	8-in diagonal sheathing, two 8d nails, no braces, boards in tension	4.3	17 100	6.6	
9 ×14	8-in diagonal sheathing, two 8d nails, no braces, boards in tension	2.8		Over 8	Test stopped at 20 000 lb load
9 ×14	8-in diagonal sheathing, two 8d nails, no braces, boards in compression	7.3	20 100	7.8	
9 ×14	8-in horizontal sheathing, two 8d nails, herringbone or bridge 2 by 4-in braces	1.3	2 800	1.1	
9 ×14	8-in horizontal sheathing, two 8d nails, cut-in 2 by 4-in braces	1.6	3 700	1.4	
9 ×14	8-in horizontal sheathing, two 8d nails, let-in 1 by 4-in braces, first arrangement	2.6	9 250	3.6	
9 ×14	8-in horizontal sheathing, two 8d nails, let-in 1 by 4-in braces, second arrangement	4.2	9 000	3.5	
9 ×14	8-in horizontal sheathing, three 8d nails, no braces	1.0	2 300	0.9	
9 ×14	8-in horizontal sheathing, four 8d nails, no braces	1.4	3 550	1.4	
9 ×14	8-in diagonal sheathing, three 8d nails, no braces, boards in tension	5.2	. . .	Over 8	Test stopped at 20 000 lb load
9 ×14	8-in diagonal sheathing, four 8d nails, no braces, boards in tension	7.5	.	Over 8	Test stopped at 20 000 lb load

Results of Tests of Panels without Window and Door Openings (Continued)

Size of panel, Height by length	Description of panel: Panel frames consisted of 2 by 4-in upper and lower plates, vertical studs spaced 16 in, and triple end-posts	Stiffness factor	Maximum load	Strength factor	Remarks
Ft In Ft In			Pounds		
9 ×14	8-in horizontal sheathing, two 10d nails, no braces	1.5	3 500	1.4	
9 ×14	8-in horizontal sheathing, two 12d nails, no braces	1.3	2 800	1.1	
9 ×14	8-in diagonal sheathing, two 10d nails, no braces, boards in tension	5.1	.	Over 8	Test stopped at 20 000 lb load
9 ×14	6-in horizontal sheathing, two 8d nails, end and side matched, no braces	1.0	2 550	1.0	
9 ×14	Plaster on wood lath, no sheathing	7.2	11 400	4.4	First plaster crack at 10 800 lb
9 ×14	Plaster on wood lath, 8-in horizontal sheathing, two 8d nails, no braces	7.9	14 500	5.6	First plaster crack at 9 900 lb
9 ×14	Plaster on wood lath, 8-in diagonal sheathing, two 8d nails, no braces	9.2	20 300	7.8	First plaster crack at 12 200 lb
9 ×14	Plaster on wood lath, studs and horizontal sheathing, green lumber then seasoned one month	6.0	12 700	4.9	First plaster crack at 8 200 lb
9 ×14	8-in horizontal green sheathing, two 8d nails, no braces, panel seasoned one month	0.5	1 700	0.7	
7 4×12 1⅝	8-in horizontal green sheathing, two 8d nails, no braces, panel seasoned one month	0.7	1 800	0.7	Vibrated one million cycles
9 ×14	8-in diagonal green sheathing, two 8d nails, no braces, panel seasoned one month	1.7	..	.	
7 4×12 1⅝	8-in diagonal green sheathing, two 8d nails, no braces, panel seasoned one month	1.7	.	...	
9 ×14	8-in horizontal sheathing, two 8d nails, no braces, alternate sunshine and rain one month	0.7	2 175	0.8	

Results of Tests of Panels with Window and Door Openings

Size of panel, Height by length (Ft In Ft In)	Openings	Description of panel: Panel frames consisted of 2 by 4-in upper and lower plates, vertical studs spaced 16 inches, and triple end-posts	Stiffness factor	Maximum load (Pounds)	Strength factor	Remarks
9 × 14	window	8-in horizontal sheathing, 1 by 4-in let-in braces	3.0	6 500	2.5	
9 × 14	window	8-in diagonal sheathing, no braces, boards in tension	3.1	13 000	5.0	
9 × 14	window and door	8-in horizontal sheathing, no braces	0.7	2 100	0.8	
9 × 14	window and door	8-in diagonal sheathing, no braces, boards in tension	1.4	10 240	4.0	
9 × 14	window and door	8-in diagonal sheathing, no braces, boards in tension	1.4	10 150	3.9	
9 × 14	window and door	8-in diagonal sheathing, no braces, boards in compression	0.8	3 250	1.3	
9 × 14	window and door	8-in diagonal sheathing, no braces, boards in compression	1.2	3 400	1.3	
9 × 14	window and door	8-in horizontal sheathing, 1 by 4-in let-in braces	1.5	5 650	2.2	
9 × 14	window and door	8-in horizontal sheathing, no braces, 6-in bevel siding	1.1	3 400	1.3	
9 × 14	window and door	8-in diagonal sheathing, boards in compression, 6-in bevel siding	2.0	8 500	3.3	
9 × 14	window and door	8-in diagonal sheathing, boards in tension, 6-in bevel siding	3.3	13 900	5.4	
9 × 14	window and door	8-in horizontal sheathing, 1 by 4-in let-in braces, 6-in bevel siding	2.7	8 880	3.4	
9 × 14	window and door	Plaster on wood lath, no sheathing	2.3	4 200	1.6	First plaster crack at 1 300 lb
9 × 14	window and door	Plaster on wood lath, 8-in horizontal sheathing, no braces	2.4	5 800	2.2	First plaster crack at 800 lb
9 × 14	window and door	Plaster on wood lath, 8-in diagonal sheathing, no braces	2.8	11 300	4.4	First plaster crack at 800 lb
9 × 14	window and door	Plaster on wood lath, 8-in horizontal sheathing, 1 by 4-in let-in braces	4.1	9 360	3.6	First plaster crack at 1 500 lb

(*c*) One by 4-in STRIPS, let into the stud faces diagonally under the sheathing, make a horizontally sheathed wall 2½ to 4 times stiffer and 3½ to 4 times stronger.

Effect of Wall Openings. Window and door openings closely spaced reduce the stiffness of horizontally sheathed walls 30% and their strength 20%. Diagonally sheathed walls lose 63% in stiffness and 50% in strength, but are still much better than horizontally sheathed walls without openings.

Effect of Lath and Plaster. PLASTER ON WOOD LATH makes an unsheathed wall 90% stiffer and about half as strong as though diagonally sheathed. It increases the stiffness of a horizontally sheathed panel with window and door openings over 200%. However, if the plaster begins to crack from shrinkage, settlement or other causes, the rigidity of sheathing comes into play and is all-important in violent winds. On this account and because of insulation and the distribution of concentrated loads, sheathing should never be omitted.

4. Framing Details

Sills (see Figs. 3, 4 and 5) should be at least 4 in × 6 in, and for heavy buildings or where wide openings are spanned, as over cellar window and doors, they should be 6 in × 8 in. They are well bedded to a level position with cement mortar and should set back one inch from the outside face of the cellar wall to allow room for the sheathing. In the best work they are bolted to the cellar wall with ¾-in bolts 24 in long set not more than 8 ft 0 in apart. Sills should run in one piece from angle to angle of the building with no splicing and are usually halved together and pinned or spiked at the corners, but never butted or mitered. BOX SILLS allow greater shrinkage possibilities and are less rigid than SOLID SILLS and should never be used.

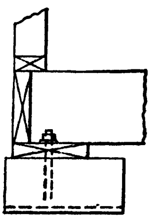

Fig. 3. Box Sill

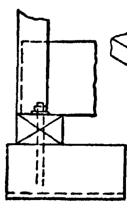

Fig. 4. Solid Sill

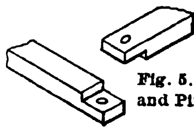

Fig. 5. Sills Halved and Pinned at Corners

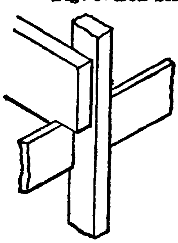

Fig. 6. Joist and Ribbon

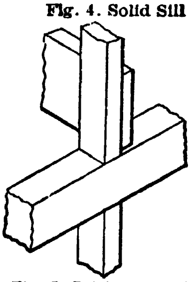

Fig. 7. Joist and Girt

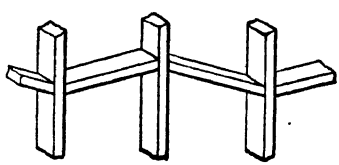

Fig. 8. Stud Bridging

Studs. (See Figs. 6, 7 and 8.) Two by 4-in and 3 × 4-in studs are almost always used, though where special thickness of wall is required for strength

or to conceal plumbing or heating pipes, etc., 2 × 6 in can be used. Studs are very generally set 16 in on centers as this spacing provides sufficient strength and stiffness for the walls and furnishes a suitable nailing for wood or metal lath and for outside sheathing. A 12-in centering is likewise suitable for the nailing of lath and may be used where special strength or rigidity of frame is desired.

In BALLOON-FRAME construction studs extend in one piece from the sill to the roof-plate and are notched out at the proper height to receive the 1 × 6-in or 1 × 8-in ribbon which carries the second-floor joist.

In BRACED or COMBINATION-FRAME construction the first-floor studs reach only from the sill to the girt, the second-floor studs resting on the girt and extending to the plate. They are cut in above and below the diagonal braces of the corner-posts.

Studs should be brought to an even plane on the inside by FURRING or DRESSING to furnish a proper surface for the lathing and plastering. They should be doubled at the heads, sills and sides of all openings such as windows or doors to give rigidity to the frames. Horizontal or slightly diagonal 2 × 4-in bridging should be cut between the studs in the middle of each story height to give additional rigidity. This is particularly essential in the balloon-frame method.

Posts, Girts, and Braces. (See Figs. 9 and 10.) One dimension of the CORNER-POST should be the same as the studs. For 2 × 4-in studs corner-posts should be 4 in × 6 in or 4 in × 8 in. A 2 × 4-in stud should be set against the corner-post to give a nailing for the plaster lath.

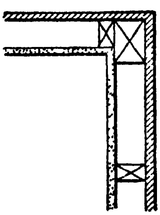

Fig. 9. Furring Stud at Corner

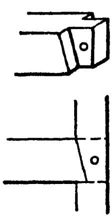

Fig. 10. Girt and Corner Post

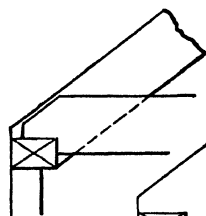

Fig. 11. Rafter Plate and Attic Joist

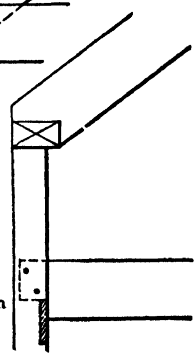

Fig. 12. Attic Joist Supported on Ribbon below Plate

GIRTS should be the same dimensions as the corner-posts. The connection between the girt and the corner-post is the one case where a MORTISE and TENON JOINT should still be preserved to give proper support to the girt and for the satisfactory stiffening of the entire frame. There should be a bevel on the shoulder of the tenon, and the joint should be pinned together with an oak pin. If a mortise and tenon joint cannot be made, the girt should be supported on a block of wood spiked to the corner-post or by a steel angle

lag-screwed in place. When built-up corner-posts are used, the girt may rest on one of the component studs.

DIAGONAL BRACES should be 3 in × 4 in or 4 in × 4 in and are most effective at an angle of 45°. They often run from the sill to the intersection of the girt and corner-post, omitting the brace from the corner-post to the under side of the girt. Strips 1 in × 4 in let into the stud faces under the sheathing also make very efficient diagonal braces.

Plates. It is better to use PLATES built up of two pieces each 2 in thick, spiked together, than to use solid timber, for built-up pieces warp and twist to a less degree and have their joints more firmly spliced. Unless the extra width projects undesirably into the finished walls or ceiling of a room it is better to use a 4 × 6-in plate rather than a 4 × 4-in, as it is stiffer and offers more resistance to the thrust of the rafters.

Joists. (See Figs. 11 and 12.) The FLOOR-JOISTS of wooden buildings are generally planks 2 in thick. In buildings with masonry walls within the fire limits of cities the building codes often call for 3-in planks. The joists are set on edge and are spaced 12 in or 16 in apart on centers depending on the required stiffness and the loads. For the first story the outer ends of the joists are supported on the sill and the inner ends on girders or masonry walls, which in turn are usually placed under the bearing partitions of the first story. At the second story the outer ends of the joists rest on the girts or on the ribbons and the inner ends on the first-story bearing partitions. In the attic the outer ends of the joists rest either directly on the plate or on ledger boards depending upon the RELATIVE levels of the plate and the attic-joists. The inner ends rest upon the second-story partitions.

JOISTS (see Fig. 13) should be framed to sills without weakening either one more than necessary. This may be done by using IRON HANGERS which allow the full dimensions and strength of the pieces to be available. The method most generally employed, however, is to cut a TENON in the floor-beam and a

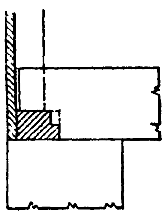

Fig. 13. Joist let down on Sill

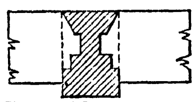

Fig. 14. Mortised Joint Joists to Girder

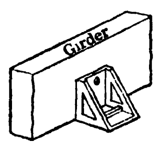

Fig. 16. Iron Joist Hanger

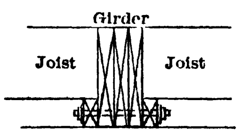

Fig. 15. Wood Ledger Strip

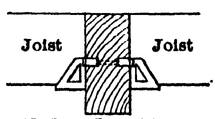

Fig. 17. Iron Joist Hanger

MORTISE in the sill. By this means the joist obtains a bearing upon the cellar wall as well as upon the sill, and the joist and sill are each weakened very little. If the design calls for a higher first-floor level the joist may rest entirely on top of the sill and be spiked to the studs for rigidity. When the inner ends of the joists rest on cross walls in the cellar they are either continuous over the wall, or they are cut with a lap of one or two feet and are spiked to another corresponding joist running to the outside wall on the other

side of the building. When the joists are supported on girders, the girder is either dropped so that the joist rests upon it, or the top of the joist is framed flush with top of the girder. This is best done by means of iron hangers, which do not weaken the girder or the joist and permit only a minimum of settling in the joists through shrinkage of the girders.

Joists are also mortised into the girders or rest upon wood strips nailed or bolted to the sides of the girders. The first method is laborious and weakens very much the girder and the second is a poor method for the joists. The use of steel hangers is very little more expensive than the cutting of the mortise holes and tenons. (See Figs. 14, 15, 16 and 17.)

FLOOR-JOISTS should be leveled when set so that the tops form a level plane to receive the flooring. This is done when setting the ends or by dressing the tops of those joists requiring it.

Framing around stair-wells or chimneys should be done with heavier beams

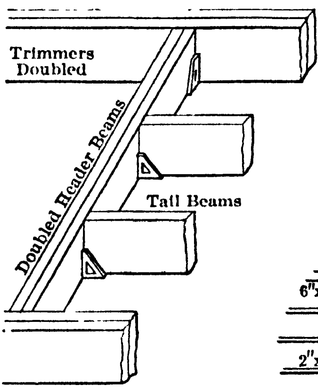

Fig. 18. Header and Trimmer Beams

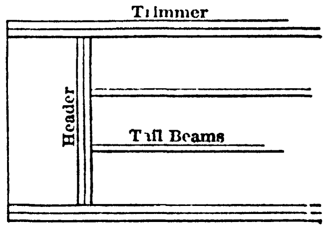

Fig. 19. Header and Trimmer Framing at Fireplace

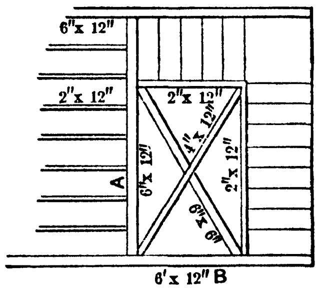

Fig. 20. Framing at Stairwell

or doubled joists called TRIMMERS supporting the cross-beams or HEADERS which in turn support the short joists or TAIL-BEAMS. Headers, trimmers and tail-beams should rest on steel hangers. Joists should never be built into a chimney but should be supported by a header set away from the chimney breast. (See Figs. 18 and 19.)

Where a projecting corner of floor occurs without vertical support from below, as in a stair-well or stair-landing, a CANTILEVER GIRDER may be introduced. Assuming the joists in Fig. 20 to be 2 in $\times$ 12 in, the 6 $\times$ 12-in header and trimmer beams are framed in the usual manner for stair-wells. A 6 $\times$ 6-in piece, half the depth of the joists, is framed diagonally across from beam *B* to header *A* with its lower edge flush with the lower edge of the joists and beams. A 4 $\times$ 12-in beam is then notched down 6 in over the 6 $\times$ 6-in piece and cantilevers over to support the corner.

Joists carrying non-bearing and discontinuous partitions should be doubled

when running parallel with the partitions. They should not touch but be sufficiently separated to allow pipes and wires to pass between them and to give nailing for floor. Pieces of 2 × 6-in plank are inserted between the joists to carry the partition. To support discontinuous cross partitions the joists must be made heavy enough to carry the concentrated load of the partition. (See Fig. 21.)

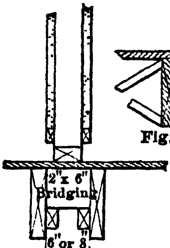

Fig. 21. Partition Parallel to Joists

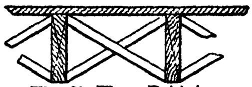

Fig. 22. Floor Bridging

Bridging. After the joists are set and leveled they should be BRIDGED by cutting in between the joists a line of diagonal braces of 1 × 3-in or 2 × 3-in pieces for each 5 to 8 ft of span. The pieces should be cut on a miter to the exact length and fastened by nails to the joists. This bridging stiffens the floor and prevents vibration, as well as helping to spread a concentrated load on one joist to the adjoining beams. It does not increase the strength of the floor to carry distributed loads. (See Fig. 22.)

Framing around Openings. In the case of narrow openings it is sufficient to double the studs at the sides and over the head of the opening. Where the openings are wider, however, TRUSSING is used to give the required strength. The header may consist of two 2 × 4-in set flat-wise, with a 2-in space between the upper and lower pieces, so that any sag developed in the upper piece by the weight above will not be transmitted to the lower

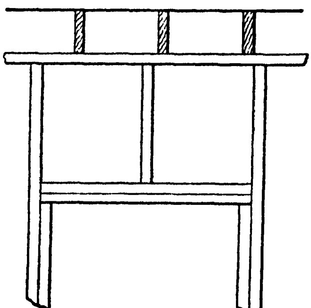

Fig. 23. Studs Doubled around Openings

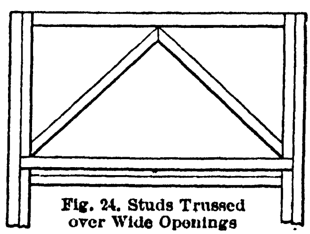

Fig. 24. Studs Trussed over Wide Openings

piece, causing the window or door to bind. The two head-pieces may also be set side by side with the larger dimension vertical, thereby forming a much stiffer head-piece. All these methods are considered good practice. (See Figs. 23 and 24.)

Wood-Girders should be of long-leaf yellow pine or Douglas fir and may be solid or built up of several pieces. The joists are best supported by steel hangers, but for light loads they may rest on bearing strips bolted along the lower part of the girder. This arrangement should not be used unless the

girder is deeper than the joists, for the joists should never be notched out to permit a bearing. The strips should be of hardwood 3 in thick and from 4 in to 6 in deep depending upon the depth of the joists. The bolts should be ¾ in to ⅞ in spaced 16 in to 20 in apart. (See Fig. 15.)

Built-up girders consist of several planks bolted together side by side with ⅝-in bolts 20 in apart in staggered rows. They are as strong as solid girders and are less likely to warp or to contain decayed wood.

Steel Girders. Joists may be attached to steel girders by stirrups, hangers or shelf angles, and should always be set with their tops ⅝ in above the top flange of the steel girder to allow for shrinkage. Stirrups can be hooked over the top flange of the girder and can be furnished in sizes to carry the joist at any height desired. Another type of hanger is bolted to the web of the girder and rests on the lower flange with a shelf to carry the joist. Shelf angles are riveted to the web of the girder in the shop and are the most commonly used. (See Fig. 25.) It is not considered good practice to rest the

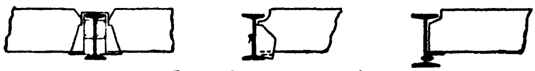

Fig. 25. Connections of Wood Joists to Steel I-Beams

joist directly on the lower flange of the steel girder because the narrow and sloping surface of the flange does not give sufficient bearing unless Bethlehem or Carnegie shapes are used.

Supports for Girders. First-story girders may be supported in the basement at intervals as required by wood posts, pipe columns, steel columns or masonry piers. Wood posts should be set up above the cellar floor on a

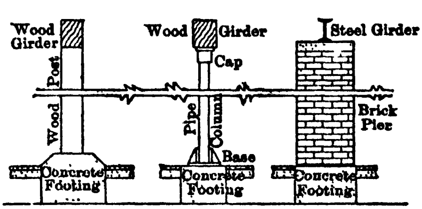

Fig. 26 Girder Supports

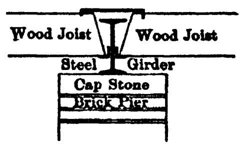

Fig. 27. Joists Supported by Shelf Angles Riveted to Girder

concrete base to avoid dampness. Pipes, steel columns or masonry piers are preferable to wood posts because of freedom from shrinkage. (See Figs. 26 and 27.)

5. Partitions

Partitions are generally built of 2 × 4-in or 3 × 4-in studs 12 in or 16 in on centers. For bearing partitions studs are set 12 in on centers and when the loads are heavy or the ceiling is over 9 ft 6 in high 2 × 6-in studs should be used. It is sometimes also necessary to use 2 × 6-in studs to provide sufficient space when plumbing or heating pipes are contained in the partitions. Wood and metal lath work out evenly for both 12-in and 16-in spacing. For bearing partitions the studs should run through between the joists and should rest on the girder or on the cap of the partition of the story below in order to avoid all shrinkage possible. If the studs rest on soles placed over the flooring across the joists, the shrinking of the joists will cause the

studs to settle sufficiently to crack plastering and prevent doors from closing. For this reason, too, it is much better to rest the first-story joists in beam hangers with tops flush with top of girder than to rest the joists on top of the girders. It is well to fill the spaces between the studs, where they pass between the joists, with brick or concrete to act as a fire stop. With steel girders there is naturally no shrinkage and they are often used instead of wood girders especially when the exterior walls are of masonry. In this way an equally unyielding bearing is obtained for the outer and inner ends of the joists. (See Fig. 28)

Partitions should be braced with 2 × 4-in horizontal bridging cut in between the studs, there being at least one row of bridging in each story height.

For CLOSET PARTITIONS, where space must be saved and the wall area is small, the studs may be set the 2-in way, well bridged.

The top of WOOD-FRAMED PARTITIONS generally consists of a horizontal stud called a CAP. When partitions run at right-angles to the joists the cap is spiked to the under side of the joists and the studs nailed to the cap. When the partition runs parallel to the joists it is well to arrange the joists so that

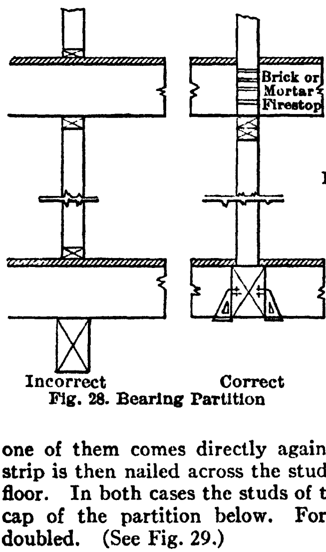

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Fig. 28. Bearing Partition

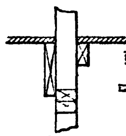


Fig. 29. Bearing Partition Parallel to Joists

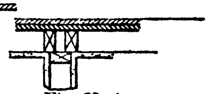


Fig. 30. Arrangement of Studs at Corners

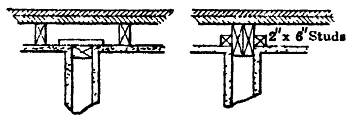


Fig. 31. Alternate Arrangements of Studs at Corners

one of them comes directly against the studs for nailing and bracing. A strip is then nailed across the studs on the other side to give nailing for the floor. In both cases the studs of the partition of the floor above rest on the cap of the partition below. For greater security this cap is sometimes doubled. (See Fig. 29.)

Corners. INTERSECTIONS of partitions with each other or with the outside wall should be made solid and give a nailing for the lath. The studs may be doubled and kept far enough apart to give nailing, or horizontal bridging may be cut in between two studs with regular 12-in or 16-in spacing. This bridging should be spaced about 3 ft apart vertically and the stud of the abutting partition is spiked to the bridging. A very solid construction is also obtained by using two 2 × 6-in studs at the angle with nailing strips for the lath (See Figs. 30 and 31.)

Trussed Partitions. Where a partition of wide span is parallel to the joists with no support below, it may be TRUSSED to prevent sagging. This is done by a stout bottom member, diagonal braces and vertical steel tension

rods. The cost of trussing is very small and the truss may be used to support floors or partitions above. Such a truss should be slightly crowned

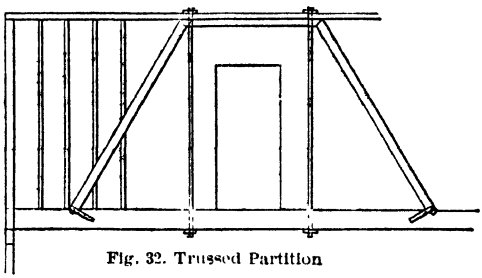


Fig. 32. Trussed Partition

and care must be taken that the supports under its ends are ample. (See Fig. 32)

6. Roof Construction

Roof Construction. (See Fig. 33 (*a*) (*b*), (*c*), (*d*), (*e*), and (*f*)) The PITCH of a roof is the inclination of the rafters with a horizontal plane and is expressed in degrees or in inches to the foot. The pitch of the roof is a matter of design depending upon appearance or upon the amount of space required in the top story. As a matter of practical construction shingles require a pitch of at least 6 in to the foot, slate 5 in and tiles 7 in to avoid leaking. The most economical pitch for trussed roofs is about 30°. Roofs may in general be divided into six types. (1) LEAN-TO ROOFS having one slope. (2) GABLE or PITCHED ROOFS having two slopes and a triangular or gable end. (3) GAMBREL ROOF with a break in the roof surface near the middle on each side of the building with the portion of roof below the break steeper than the part above it. (4) MANSARD or FRENCH ROOF with breaks in the roof surface like a gambrel roof but with straighter sides and slopes from all sides of the building. (5) DECK ROOF having sloping sides below and a flat deck on top. (6) HIPPED ROOF with slopes running back from the eaves at the ends of the building as well as at the sides.

Roof boarding is supported on rafters which are in turn supported at their lower ends by the wall plate and at their upper ends by the ridge-pole or by HIP and VALLEY RAFTERS. The hips and valleys mark the intersection of roof planes and they should be framed with heavier and deeper timbers than the rafters to give solidity and stiffness to the roof. When rafters are over 18 ft long they should be supported near their center spans by partitions or collar beams or by cross-timbers called PURLINS resting on trusses or posts. (See Figs. 34 and 35.)

The type of roof is often a matter of design, but it is of assistance in determining the intersections to use the following method. Construct the largest rectangle which is contained in the outline of the building, as *ABCD*. This gives the main body of the roof to which the various projections are subordinated. Draw 45° lines from each corner of the rectangle until they intersect and connect their intersections, obtaining the main hip lines and the main ridge. Then draw in the 45° hip and valley lines for the ells and other projections and their ridges. A hip roof throughout is thus obtained but if gable ends are desired the hip lines may be erased and the ridge extended to

the exterior wall lines. If a gambrel roof is desired the longitudinal lines may be drawn in by taking off their positions from the elevations. A deck can be introduced where wanted. (See Fig. 36.)

Rafters are spaced from 16 in to 24 in apart. If a roof be less than 30 ft in span and the plates are securely tied by the attic joists no interior supports are needed. The proper sizes and spacing of rafters for the various spans and loads may be found by consulting Tables XXXVII to XLIV.

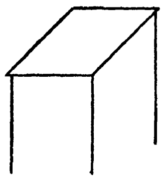


Fig. 33.(a) Lean to Roof

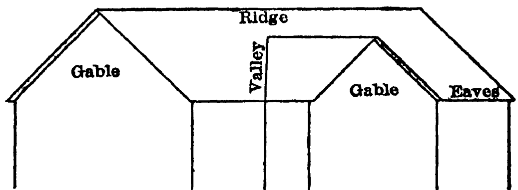


Fig. 33.(b) Gable or Pitched Roof

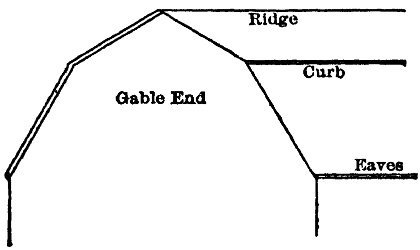


Fig. 33.(c) Gambrel Roof

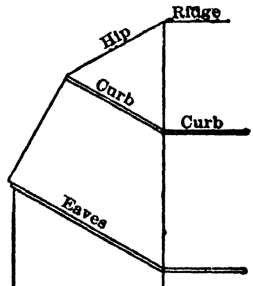


Fig. 33.(d) Mansard Roof

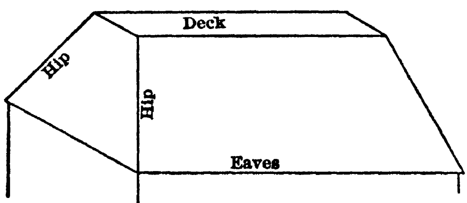


Fig. 33.(e) Deck Roof

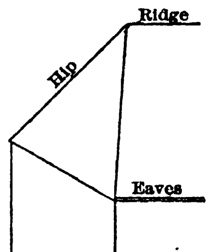


Fig. 33.(f) Hipped Roof

As a rule it is more economical to reduce the lengths of rafters by using purlins, trusses or partitions for supports than to use rafters heavier than 3 in × 12 in. Each rafter should have a firm bearing of from 2½ to 4 in depending upon the depth of the rafter. They should be accurately cut to fit the plate and be securely spiked to it. At the ridge rafters are spiked to a longitudinal plank, generally 1 in thick, called the RIDGE or RIDGE-POLE. In simple roofs of moderate span and rigid plate the HIP-RAFTER need not be

more than 2 in thick but should be 2 in deeper than the common rafters to give space for nailing and for beveling the top edge. For heavy roofs with

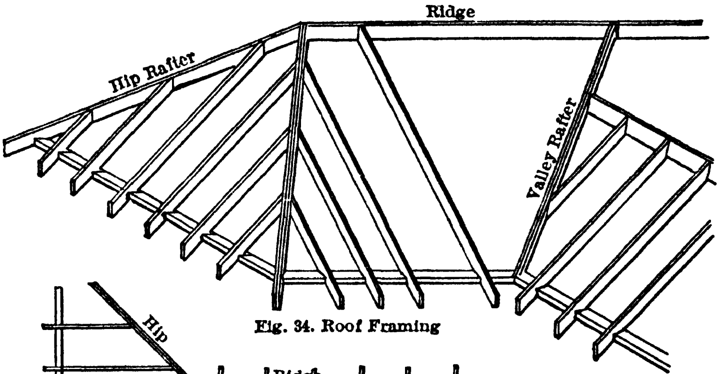


Fig. 34. Roof Framing

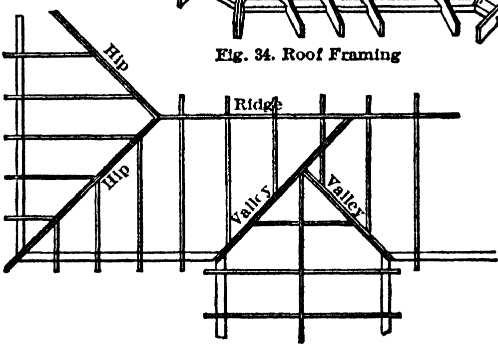


Fig. 35. Framing Plan

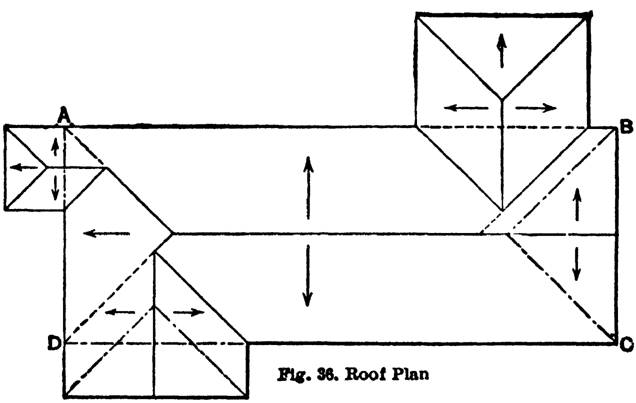


Fig. 36. Roof Plan

large spans the hip rafters should be 3 in thick or consist of two pieces each 2 in thick.

VALLEYS and HEADERS should be computed in each case by the same formulas as if they were horizontal beams. Valley rafters have to support nearly all tbe roof above them and one of each pair should extend to the main ridge or to a hip-rafter.

Rafters should be doubled or tripled at sides and heads of dormers, depending on the size of opening and expanse of roof to be supported.

Collar-Beams and Ties. A pitched roof is much strengthened by horizontal TIE-BEAMS across each pair of rafters 8 or 9 ft from the attic floor. They serve to prevent the rafters from sagging or separating and to relieve the thrust on the plate. They also act as ceiling-beams for the attic. (See Fig. 37.)

Ties are often used when the attic joists are below the plate and so cannot tie it across the building. They are set at an angle and spiked to rafter and attic joist. (See Fig. 38.)

Gambrel and Mansard Roofs. The lower rafters are framed at the curb into a purlin or plate at least 4 in × 6 in which carries the lower ends of

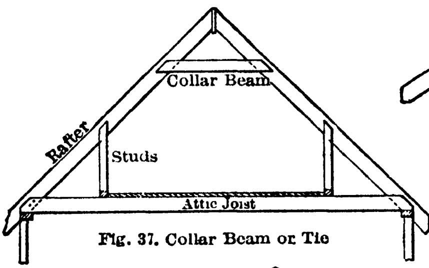


Fig. 37. Collar Beam or Tie

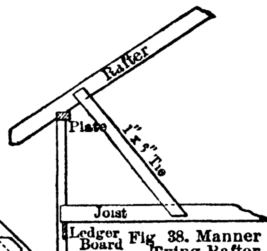


Fig. 38. Manner of Tying Rafter

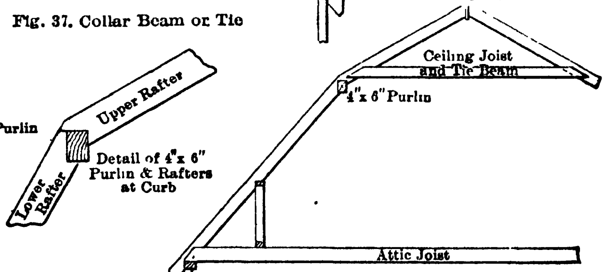


Fig. 39. Gambrel Roof

the upper rafters. Such roofs should be tied together by ceiling joists as near the curb as possible and by attic joists at the main plate. Mansard roofs sometimes have curved fronts which are formed with 2-in furring pieces cut to the curve and spiked to the straight main rafters, which are the true structural members. (See Fig. 39.)

Dormers. The opening is framed in the roof w th doubled side or trimmer-rafters and header. Studs at side of dormer are notched out 1 in over the roof boarding and under the trimmer-rafter and extend to the floor, thus

preventing the roof from sagging or breaking away from the dormer. (See Figs. 40, 41 and 42.)

Roofs of Wide Span. When the span of the roof exceeds 30 ft either TRUSSES OR INTERIOR SUPPORTS must be used. In residences partitions or

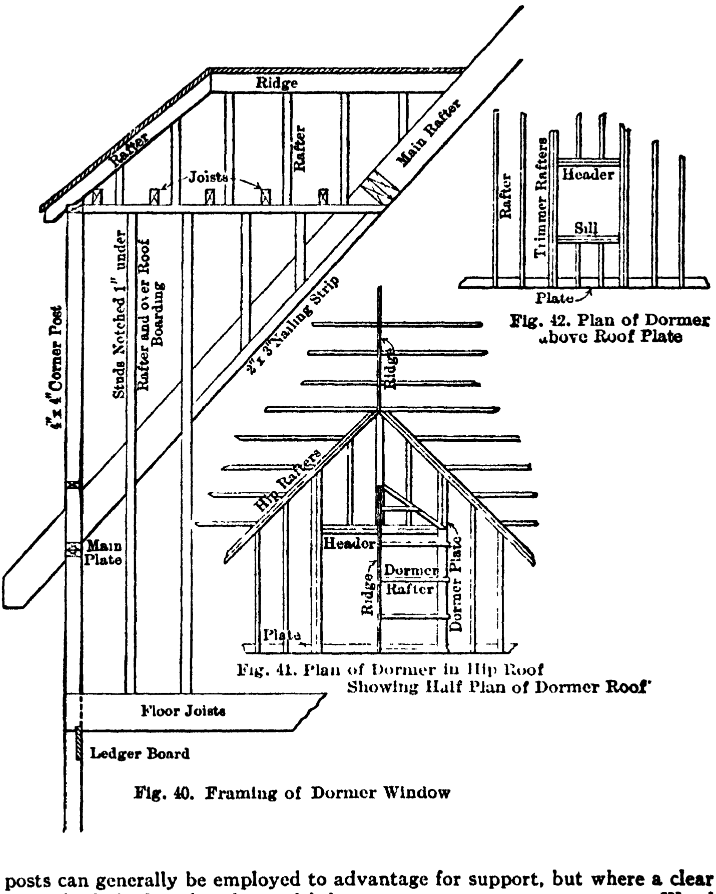


Fig. 40. Framing of Dormer Window

Fig. 41. Plan of Dormer in Hip Roof Showing Half Plan of Dormer Roof

Fig. 42. Plan of Dormer above Roof Plate

posts can generally be employed to advantage for support, but where a clear space is desired under the roof it is necessary to resort to trusses. Wood roof-trusses, except for special cases, are triangular in shape and consist of top chords, bottom chords, purlins and diagonal members in wood and vertical tension members of steel rods. The purlins are placed horizontally across the top chords from truss to truss to support the rafters at one or more points depending upon the number of purlins.

7. Design

In selecting the joists and girders for given spans and loads the following requirements must be remembered.

(1) The allowable working stresses for bending and for shear must not be exceeded.

(2) The allowable limit for deflection must not be exceeded.

(3) The depth of the member must be confined to the limits set by the requirements of design as to floor to ceiling space, or as to clear height of the story below.

(4) The sizes of the joists and girders should be easily obtainable in the local markets.

(5) The passage of pipes and necessary cutting or connections may influence the cross-section of the joists.

Framing Plans. The safest and most economical method of running the floor-joists is determined by laying them out on the working floor-plan of the building or, if the conditions are at all complicated, by making a separate framing plan. By this means are learned the spans of the joists themselves, the concentrated loads such as partitions or header-beams which may come upon them and the position of any girders, trimmer-beams or posts which may be required. In general the joists should run parallel to the shortest dimension of the area to be floored over to avoid waste both in material and in depth of beam. It is not practical, however, to employ greater spans for joists than 24 ft 0 in. When the distance between the supports exceed this amount girders may be introduced running parallel to the short dimension of the area and the joists then laid running in the opposite direction and supported upon the girders. It may, however, be found more practical to run the joists over the shorter span and merely support them by a longitudinal girder at their center. The girders are so spaced that the span of the joists will not exceed 20 ft to 24 ft for light buildings and 16 ft to 18 ft for warehouses. If the design permit, the spans of the girders may be reduced by the use of wood posts, concrete-filled pipe columns, steel columns or brick or concrete piers. If such supports interfere with the design and the loads on the girders are too great for single timbers, trussed wood beams or steel beams may be used as girders in connection with the wood joists. Steel beams are very useful in reducing the depth of the girder and thereby increasing the ceiling height of the story below. (See Figs. 43, 44 and 45.)

Joists are usually spaced either 12 in or 16 in on centers to give even nailing to plaster lath without cutting the lath, both wood and metal lath being furnished in proper widths to take these nailings. The most usual spacing of joists is 16 in on centers. The 12-in spacing, however, is often used when the loads are too heavy for a 16-in spacing and the ceiling height of the floor below will not permit a deeper joist. Framing lumber is furnished by the mills in lengths which are multiples of 2 ft, and much waste both in labor and material may be saved by planning the rooms and the story heights so that the entire length of a timber without cutting may be utilized for joists, studs, rafters and girders. A change of a few inches will often effect this saving without seriously altering the architectural design.

Economy may often likewise be effected by a careful choice of the grade of the wood. While a dense or select grade of wood may be stronger against bending, it is also heavier and more costly. The strength of a piece of timber varies as the square of its depth, and its stiffness or resistance to deflection varies as the cube of its depth. Its stiffness also is little influenced by the

grade of the material. Consequently, it often happens in light framing that when No. 2 common grade is used instead of No. 1 common, a deeper joist will be required to carry the load but that this joist will be much stiffer than the lighter one required for No. 1 common. The cost of No. 2 common being much less than No. 1 common, a joist of the former grade will not only be cheaper but stiffer also, which is important when ceilings are plastered.

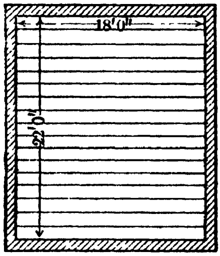


Fig. 43. Joists Parallel to Shorter Dimension

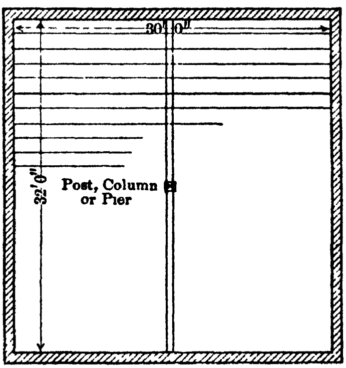


Fig. 44. Joists Parallel to Shorter Dimension and Supported by Girder at Center of Span

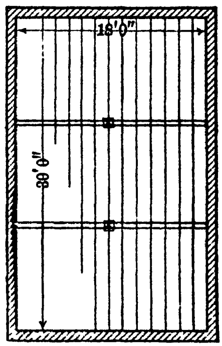


Fig. 45. Joists Parallel to Longer Dimension and Supported by Two Girders at Third Points of Span

On the other hand, where girders with fairly heavy loads are required, it may be found that economy will be attained if the best dense grades are used at a higher price but containing less material.

Cutting. The possibility of holes or notches being cut in the joists, beams and girders by electricians, plumbers and steam-fitters for the passage of pipes, conduits, etc., should be borne in mind and the general run of such pipes considered in designing the members.

Calculations of Floor-Loads. Tables have recently been prepared by the various lumber associations and by the Federal Government giving the allowable sizes of joists, beams and girders for the various loads, spans and kinds of timber. These tables are based upon the latest tests on wood and

also upon the latest methods of grading. They are likewise derived by the use of the recognized formulas of mechanics so that there is no question as to their accuracy. In many cases two values of allowable loads are given for the same span and size of timber, the one value being allowable in cases where it is necessary to consider bending only, as in warehouses, factories, etc., with unplastered ceilings, the other and smaller value where deflection of the joist or beam must also be considered. The allowable maximum deflection is taken as 1/360 of the span or 1/30 in for each foot of span.

The tables are based upon uniformly distributed loads over the entire member. In cases where a concentrated load exists, this load must be transformed into an equivalent distributed load by the use of a factor depending upon the point of application of the concentrated load.

Factors for equivalent distributed loads are given in Table V.

Allowable sizes of joists and rafters are given in Tables XXI to XLIV.

In using these tables it should be remembered that only a small part of the factor of safety for wood is available for taking care of overloads, most of it being required to adjust for the known variability in the strength of clear wood, the effect of defects and the duration of the load. Timbers should not be subjected, therefore, to long-time loads much greater than the design load.

8. Sections, Stresses, Buckling and Deflection of Wood Beams

Sections and Fiber-Stresses. The cross-sections of wood beams are almost invariably square or rectangular and those shapes only are considered in the following rules and formulas. Beams should have such a cross-section that the maximum fiber-stress due to transverse bending, the maximum horizontal shear and the maximum compression across the grain at the end bearings do not exceed the allowable working unit stresses as set forth in Table I.

Buckling. Wood girders should be braced laterally to prevent buckling when the ratio of length to breadth exceeds twenty, or designed with a reduced fiber-stress from that allowable, where this ratio is exceeded. The tables contained herein assume such bracing which, in building-construction, is nearly always furnished by the beams or joists framing into the girder. Joists should have cross-bridging not over 8 ft on centers. The percentage of reduction of fiber-stress for girders where no bracing occurs should be as follows:

Ratio of length to width.......	20 to 30	30 to 40	40 to 50	50 to 60
Percentage of reduction.......	25	34	42	50

Deflection. It is also important that beams carry their loads without deflection beyond a limit fixed by the use to which the structure is applied. This limit is generally taken for plastered ceilings at 1/30 of an inch per foot of span or 1/360 of the span.

9. Working Unit Stresses for Commercially Dry Lumber

Table I gives the allowable working unit stresses for commercially seasoned wood in sheltered and continuously dry locations such as are present generally in building-construction. For timber occasionally wet or continuously wet,

as in bridge, trestle or pier construction, reductions should be made in the values for bending, compression and shear. Tables for timber in such conditions may be obtained from the Forest Products Laboratory, Madison, Wisconsin. Since this construction comes rather in the province of the civil and railroad engineer the tables are not included here.

The grades of lumber are for dimension lumber and timber equivalent in quality to the American Standards for Structural Material as recommended by the Bureau of Standards of the United States Department of Commerce in Simplified Practice Recommendation No. 16. For lumber not equivalent to American Standard Structural grades suitable lower stresses should be used.

The stresses in Table I are those recommended by the Forest Products Laboratory of the United States Department of Agriculture at Madison, Wisconsin, after many series of tests. These grades and stresses have been adopted by the American Railway Engineering Association and by several cities in their revised building codes, they have been accepted by the American Society for Testing Materials and they are recommended by the Building Code Committee of the United States Department of Commerce. When buildings are constructed in communities whose building codes prescribe other allowable working stresses, the stresses of the code must of necessity be followed. The tendency is, however, to accept the stresses as recommended by the Forest Products Laboratory on the part of most communities as they successively amend or standardize their codes.

For DIRECT TENSION the same values as for extreme fiber-stress in bending may be used.

SHEARING STRESS for JOINT DETAILS may, for all grades, be taken as 50% greater than the horizontal shear values otherwise permitted.

In the case of JOISTS supported on a ribbon board and spiked to the studding, the allowable stress in compression across the grain may be increased 50% above that specified.

Table I. Allowable Working Stresses for Structural Lumber and Timber

All computations to determine the required sizes of lumber members should be based on the net cross-sectional area or actual size. The size of members should be sufficient to carry the imposed load safely and without exceeding the allowable working stresses given in Table I.

Allowable Unit Stresses for Structural Lumber and Timber

All Sizes, Dry Locations

Species of timber	Grade	Allowable unit stress in pounds per square inch: Extreme fiber in bending: Joist and plank sizes; 4 in and less in thickness	Extreme fiber in bending: Beam and stringer sizes; 5 in and thicker	Maximum horizontal shear	Compression: Parallel to grain (short columns)	Compression: Perpendicular to grain	Modulus of Elasticity
Table I–A. Working Stresses for Manufacturers' Association Standard Commercial Grades							
Douglas Fir, Coast Region	Dense Super-Structural	2 000	2 000	120	1 466	380	1 600 000
	Super-Structural and Dense Structural	1 800	1 800	105	1 300	345	1 600 000
	Structural	1 600	1 600	90	1 200	345	1 600 000
	Common Structural	1 200	1 400	84	1 100	325	1 600 000
Douglas Fir, Inland Empire	*Dense Super-Structural	2 000	2 000	120	1 466	380	1 600 000
	*Dense Structural	1 800	1 800	105	1 300	345	1 600 000
	No 1 Common Dimension and Timbers	1 135	1 135	70	1 010	315	1 500 000
Larch, Western	No. 1 Common Dimension and Timbers	1 135	1 135	70	1 010	325	1 300 000

* When graded the same as corresponding grade of Coast Region Douglas Fir.

Table I (Continued). Allowable Working Stresses for Structural Lumber and Timber

All computations to determine the required sizes of lumber members should be based on the net cross-sectional area or actual size. The size of members should be sufficient to carry the imposed load safely and without exceeding the allowable working stresses given in Table I.

Allowable Unit Stresses for Structural Lumber and Timber

All Sizes, Dry Locations

Species of timber	Grade	Allowable unit stress in pounds per square inch					
		Extreme fiber in bending		Maximum horizontal shear	Compression		Modulus of Elasticity
		Joist and plank sizes; 4 in and less in thickness	Beam and stringer sizes; 5 in and thicker		Parallel to grain (short columns)	Perpendicular to grain	
Table I–A Working Stresses for Manufacturers' Association Standard Commercial Grades							
Pine, Southern Yellow	Extra Dense Select Structural	2 300	2 300	200	1 600	475	1 600 000
	Select Structural	2 000	2 000	175	1 450	375	1 600 000
	Extra Dense Heart	2 000	2 000	175	1 450	475	1 600 000
	Dense Heart	1 800	1 800	150	1 300	375	1 600 000
	Structural Square Edge and Sound	1 600	1 600	125	1 200	375	1 600 000
	No. 1 Common	1 200	1 200	100	1 000	325	1 600 000
Redwood	Super-Structural	2 133	1 707	93	1 422	267	1 200 000
	Prime Structural	1 707	1 494	82	1 245	267	1 200 000
	Select Structural	1 280	1 322	70	1 100	267	1 200 000
	Heart Structural	1 024	1 150	56	1 000	267	1 200 000

Table I (Continued). Allowable Working Stresses for Structural Lumber and Timber

All computations to determine the required sizes of lumber members should be based on the net cross-sectional area or actual size. The size of members should be sufficient to carry the imposed load safely and without exceeding the allowable working stresses given in Table I.

Allowable Unit Stresses for Structural Lumber and Timber

All Sizes, Dry Locations

Species of timber	Grade	Allowable unit stress in pounds per square inch: Extreme fiber in bending: Joist and plank sizes; 4 in and less in thickness	Extreme fiber in bending: Beam and stringer sizes; 5 in and thicker	Maximum horizontal shear	Compression: Parallel to grain (short columns)	Compression: Perpendicular to grain	Modulus of Elasticity
Table I–B. Working Stresses for Structural Lumber and Timber Graded under the Structural Grade Examples of the American Lumber Standards							
Cedar, Alaska	Select Structural	1 100	1 100	90	800	250	1 200 000
	Common Structural	880	880	72	640	250	1 200 000
Cedar, Northern and Southern White	Select Structural	750	750	70	550	175	800 000
	Common Structural	600	600	56	440	175	800 000
Cedar, Port Orford	Select Structural	1 100	1 100	90	900	250	1 200 000
	Common Structural	880	880	72	720	250	1 200 000
Cedar, Western Red	Select Structural	900	900	80	700	200	1 000 000
	Common Structural	720	720	64	560	200	1 000 000
Cypress, Southern	Select Structural	1 300	1 300	100	1 100	350	1 200 000
	Common Structural	1 040	1 040	80	880	350	1 200 000
Douglas Fir, Rocky Mountain Region	Select Structural	1 100	1 100	85	800	275	1 200 000
	Common Structural	880	880	68	640	275	1 200 000
Fir, Balsam	Select Structural	900	900	70	700	150	1 000 000
	Common Structural	720	720	56	560	150	1 000 000
Fir, Golden, Noble, Silver, White (Commercial White)	Select Structural	1 100	1 100	70	700	300	1 100 000
	Common Structural	880	880	56	560	300	1 100 000

Table I (Continued). Allowable Working Stresses for Structural Lumber and Timber

All computations to determine the required sizes of lumber members should be based on the net cross-sectional area or actual size. The size of members should be sufficient to carry the imposed load safely and without exceeding the allowable working stresses given in Table I.

Allowable Unit Stresses for Structural Lumber and Timber

All Sizes, Dry Locations

Species of timber	Grade	Allowable unit stress in pounds per square inch: Extreme fiber in bending: Joist and plank sizes; 4 in and less in thickness	Extreme fiber in bending: Beam and stringer sizes; 5 in and thicker	Maximum horizontal shear	Compression: Parallel to grain (short columns)	Compression: Perpendicular to grain	Modulus of Elasticity
Table I–B. Working Stresses for Structural Lumber and Timber Graded under the Structural Grade Examples of the American Lumber Standards							
Hemlock, Eastern	Select Structural	1 100	1 100	70	700	300	1 100 000
	Common Structural	880	880	56	560	300	1 100 000
Hemlock, West Coast	Select Structural	1 300	1 300	75	900	300	1 400 000
	Common Structural	1 040	1 040	60	720	300	1 400 000
Oak, Commercial White and Red	Select Structural	1 400	1 400	125	1 000	500	1 500 000
	Common Structural	1 120	1 120	100	800	500	1 500 000
Pine, California, Idaho and No. White, Lodgepole, Pondosa, Sugar	Select Structural	900	900	85	750	250	1 000 000
	Common Structural	720	720	68	600	250	1 000 000
Pine, Norway	Select Structural	1 100	1 100	85	800	300	1 200 000
	Common Structural	880	880	68	640	300	1 000 000
Spruce, Englemann	Select Structural	750	750	70	600	175	800 000
	Common Structural	600	600	56	480	175	800 000
Spruce, Red, White, Sitka	Select Structural	1 100	1 100	85	800	250	1 200 000
	Common Structural	880	880	68	640	250	1 200 000
Tamarack, Eastern	Select Structural	1 200	1 200	95	1 000	300	1 300 000
	Common Structural	960	960	76	800	300	1 300 000

Table II gives the nominal and dressed sizes, the weight per linear foot, area of cross-section, moment of inertia, and section-modulus of American Standard Yard Lumber and Timber. These values are recommended by the Forest Products Laboratory and are furnished here for use with the usual formulas of mechanics employed in designing structural members in wood.

Table II. Properties of American Standard Yard Lumber and Timber Sizes

Size (Nominal in inches)	American standard dressed size, in	Area of section $A = bd$, sq in	Weight per lin ft *, lb	Moment of inertia $I = \frac{bd^3}{12}$	Section-modulus $S = \frac{bd^2}{6}$
2×4	1⅝×3⅝	5.89	1.6	6.45	3.56
2×6	1⅝×5⅝	9.14	2.5	24.10	8.57
2×8	1⅝×7½	12.19	3.4	57.13	15.32
2×10	1⅝×9½	15.44	4.3	116.09	24.44
2×12	1⅝×11½	18.69	5.2	205.94	35.82
2×14	1⅝×13½	23.62	6.5	333.15	49.36
2×16	1⅝×15½	25.18	7.0	504.24	65.07
2×18	1⅝×17½	28.43	7.9	725.71	82.94
2×20	1⅝×19½	31.69	8.8	1 004.05	102.98
3×4	2⅝×3⅝	9.51	2.6	10.42	5.75
3×6	2⅝×5⅝	14.76	4.2	38.93	13.84
3×8	2⅝×7½	19.68	5.7	92.28	24.60
3×10	2⅝×9½	24.93	7.2	187.55	39.48
3×12	2⅝×11½	30.18	8.8	332.69	57.86
3×14	2⅝×13½	35.43	10.3	538.21	79.73
3×16	2⅝×15½	40.68	11.3	814.60	105.11
3×18	2⅝×17½	45.94	12.8	1 172.36	133.98
3×20	2⅝×19½	51.19	14.21	1 622.00	166.36
4×4	3⅝×3⅝	13.14	3.6	14.38	7.94
4×6	3⅝×5⅝	20.39	5.7	53.76	19.11
4×8	3⅝×7½	27.18	7.5	127.44	33.98
4×10	3⅝×9½	34.43	9.6	258.99	54.52
4×12	3⅝×11½	41.68	11.6	459.42	79.90
4×14	3⅝×13½	48.93	13.6	743.23	110.11
4×16	3⅝×15½	56.18	15.6	1 124.90	145.15
4×18	3⅝×17½	63.43	17.6	1 618.96	185.02
4×20	3⅝×19½	70.69	19.6	2 239.88	229.73
6×6	5½×5½	30.25	8.4	76.25	27.73
6×8	5½×7½	41.25	11.4	193.35	51.56
6×10	5½×9½	52.25	14.5	392.96	82.73
6×12	5½×11½	63.25	17.5	697.06	121.23
6×14	5½×13½	74.25	20.6	1 127.66	167.06
6×16	5½×15½	85.25	23.6	1 706.76	220.22

* Based on assumed average weight of 40 lb per cu ft.

Table II (Continued). Properties of American Standard Yard Lumber and Timber Sizes

Size (Nominal in inches)	American standard dressed size (in)	Area of section $A = bd$ (sq in)	Weight per lin ft * (lb)	Moment of inertia $I = \frac{bd^3}{12}$	Section modulus $S = \frac{bd^2}{6}$
6×18	5½×17½	96.25	26.7	2 456.36	280.73
6×20	5½×19½	107.25	29.8	3 398.46	348.53
6×22	5½×21½	118.25	32.8	4 555.05	423.76
8×8	7½×7½	56.25	15.6	263.67	70.31
8×10	7½×9½	71.25	19.8	535.85	112.81
8×12	7½×11½	86.25	23.9	950.55	165.31
8×14	7½×13½	101.25	28.0	1 537.73	227.81
8×16	7½×15½	116.25	32.0	2 327.42	300.31
8×18	7½×17½	131.25	36.4	3 349.60	382.81
8×20	7½×19½	146.25	40.6	4 634.30	475.31
8×22	7½×21½	161.25	44.8	6 211.48	577.81
8×24	7½×23½	176.25	48.9	8 111.17	690.31
10×10	9½×9½	90.25	25.0	678.75	142.89
10×12	9½×11½	109.25	30.3	1 204.01	209.39
10×14	9½×13½	128.25	35.6	1 947.78	288.56
10×16	9½×15½	147.25	40.9	2 948.04	380.39
10×18	9½×17½	166.25	46.1	4 242.80	484.89
10×20	9½×19½	185.25	51.4	5 870.05	602.06
10×22	9½×21½	204.25	56.7	7 867.81	731.89
10×24	9½×23½	223.25	62.0	10 274.06	874.39
10×26	9½×25½	242.25	67.3	13 126.81	1 029.56
10×28	9½×27½	261.25	72.5	16 465.24	1 197.39
10×30	9½×29½	280.25	77.8	20 323.79	1 377.89
12×12	11½×11½	132.25	36.7	1 457.50	253.47
12×14	11½×13½	155.25	43.1	2 357.85	349.31
12×16	11½×15½	178.25	49.5	3 568.70	460.48
12×18	11½×17½	201.25	55.9	5 136.49	586.98
12×20	11½×19½	224.25	62.3	7 105.90	728.81
12×22	11½×21½	247.25	68.7	9 524.21	885.98
12×24	11½×23½	270.25	75.0	12 437.08	1 058.47
12×26	11½×25½	293.25	81.4	15 890.42	1 246.31
12×28	11½×27½	316.25	87.8	19 932.58	1 449.47
12×30	11½×29½	339.25	94.2	24 602.61	1 667.97
14×14	13½×13½	182.25	50.6	2 767.92	410.06
14×16	13½×15½	209.25	58.1	4 189.36	540.56
14×18	13½×17½	236.25	65.6	6 029.29	689.06
14×20	13½×19½	263.25	73.1	8 311.73	855.56
14×22	13½×21½	290.25	80.6	11 180.67	1 040.06

* Based on assumed average weight of 40 lb per cu ft.

Table II (Continued). Properties of American Standard Yard Lumber and Timber Sizes

Size	American standard dressed size	Area of section	Weight per lin ft *	Moment of inertia	Section modulus
(Nominal in inches)		$A = bd$		$I = \frac{bd^3}{12}$	$S = \frac{bd^2}{6}$
	in	sq in	lb		
14×24	13½×23½	317 25	88 1	14 600.10	1 242.56
14×26	13½×25½	344.25	95.6	18 654.04	1 463.06
14×28	13½×27½	371.25	103 1	23 398.73	1 701.56
14×30	13½×29½	398.25	110.6	28 881.42	1 958.06
16×16	15½×15½	240.25	66.7	4 809.98	620.64
16×18	15½×17½	271 25	75.3	6 922.49	791.14
16×20	15½×19½	302.25	83.9	9 577.50	982.31
16×22	15½×21½	333.25	92.5	12 837.00	1 194.14
16×24	15½×23½	364.25	101.2	16 763.00	1 426.64
16×26	15½×25½	395 25	109.8	21 417.50	1 679.81
16×28	15½×27½	426.25	118.4	26 863.78	1 953.64
16×30	15½×29½	457.25	127.0	33 159.98	2 248.14
18×18	17½×17½	306.25	85.0	7 815.73	893.23
18×20	17½×19½	341.25	94.8	10 813.33	1 109.06
18×22	17½×21½	376.25	104.5	14 493 43	1 348.23
18×24	17½×23½	411.25	114 2	18 926.02	1 610.72
18×26	17½×25½	446 25	123.9	24 181.11	1 896.56
18×28	17½×27½	481.25	133.7	30 331.62	2 205 72
18×30	17½×29½	516 25	143.4	37 438.79	2 538 22
20×20	19½×19½	380 25	105.6	12 049.49	1 235.81
20×22	19½×21½	419.25	116.4	16 149 86	1 502.31
20×24	19½×23½	458.25	127.3	21 089.04	1 794.81
20×26	19½×25½	497.25	138.1	26 944.73	2 113.31
20×28	19½×27½	536.25	148.9	33 798.17	2 457.81
20×30	19½×29½	575.25	159.8	41 717.61	2 828.31
24×24	23½×23½	552 25	153 4	25 414.96	2 162.97
24×26	23½×25½	599.25	166 4	32 471.80	2 546 81
24×28	23½×27½	646 25	179 5	40 731.06	2 916.97
24×30	23½×29½	693 25	192 5	50 274.98	3 408.47

* Based on assumed average weight of 40 lb per cu ft.

Table III gives the unit working stresses allowed for various kinds of wood by the current building codes of six representative cities, New York, Chicago, Boston, Philadelphia, Denver and San Francisco. A constant effort is being made to standardize the allowable stresses in all the cities of the country and as the codes are revised the tendency is to adopt the values based upon the government tests and recommended by the government bureaus.

Table III. Unit Working Stresses in Pounds per Square Inch

Allowed in Various Cities

Kind of stress	Kind of wood	New York, 1927	Chicago, 1928	Boston, 1926	Philadelphia, 1929	Denver, 1927	San Francisco, 1928
Transverse Bending	Yellow pine—L.-L	1 600	1 300	1 600	1 600	1 600	
	Yellow pine—S.-L	1 000	1 000	1 200	1 200	1 300	
	Douglas fir. Dense Sound	1 200	1 300	1 100	1 600 1 200	1 600 1 300	1 600
	Oak	1 200	1 200	1 400	1 400	1 400	.
	Spruce .	1 200	800	1 000	1 100	1 100	700
	Eastern hemlock	800	600	. .	1 100	1 100	.
	W. Coast hemlock	.	. . .	. .	1 300	1 300	
	White pine	1 200	800	1 000			700
Compression with the Grain	Yellow pine—L.-L	1 600	1 100	1 200	1 175	1 200	
	Yellow pine—S.-L.	1 000	800	900	880	1 000	. . . . .
	Douglas fir. Dense Sound	1 200	1 100	1 000	1 175 880	1 200 1 000	1 600
	Oak . . .	1 400	900	900	1 000	1 000	.
	Spruce . .	1 200	700	750	800	800	800
	Eastern hemlock	800	500	. . .	700	700	.
	W. Coast hemlock	.	. . . . .	. .	900	900	.
	White pine	1 000	700	700			800
Compression across the Grain	Yellow pine—L.-L	1 000	250	350	345	350	
	Yellow pine—S.-L	800	250	250	325	300	. .
	Douglas fir. Dense Sound	800	.	200	345 325	350 300	300
	Oak	1 000	500	500	500	500	. .
	Spruce.	800	200	250	250	250	200
	Eastern hemlock .	800	150	. . . . .	300	300	. . .
	W. Coast hemlock	. . . .	. . .	. . . .	. . .	300	. . . .
	White pine	800	200	200			200
Shear with the Grain	Yellow pine—L.-L	150	130	150	110	125	
	Yellow pine—S.-L	100	120	100	88	105	
	Douglas fir. Dense Sound	100	130	100	90 72	100 90	150
	Oak	200	200	200	125	125	. . .
	Spruce. .	100	80	100	85	85	100
	Eastern hemlock	100	60		70	70	. . .
	W. Coast hemlock .			.	75	75	. . .
	White pine	100	80	80			100
Shear across the Grain	Yellow pine—L.-L	1 000					
	Yellow pine—S.-L.	1 000					
	Douglas fir . .	1 000	.				750
	Oak	1 000	.			.	. .
	Spruce.	500		.			500
	Eastern hemlock . . .	600				. .	
	W. Coast hemlock	.		.		.	
	White pine . . .	500		. .		. .	500

10. Constants and Coefficients for Beams

Value of Constant A. The letter A in the following formulas (1) to (13) denotes the Safe Load for a Unit Beam 1 in square in cross-section and 1 ft in span, having a concentrated load, W, at the middle of the span.

$$M = \frac{SI}{c} = \frac{Sbd^2}{6}$$

In this case

$$M = \frac{Wl}{4}$$

then

$$\frac{Wl}{4} = \frac{Sbd^2}{6}$$

or

$$W = \frac{4\,Sbd^2}{6\,l}$$

when l is in inches.

When l is in feet,

$$W = \frac{4\,Sbd^2}{72l}$$

Table IV. Coefficient for Wood Beams Values for A in Formulas

Commercially dry timber protected as in building construction

Materials	New York	Philadelphia	Boston	Chicago	Denver	San Francisco	Recommended
L.-L. yellow pine	88.9	88.9	88.9	72.2	88.9	.	88.9
S.-L. yellow pine	55.6	66.7	66.7	55.6	72.2		66.7
Douglas fir—Select		88.9	..	72.2	88.9	88.9	88.9
Douglas fir	66.7	66.7	61.1		72.2	.	66.7
Oak: Select	66.7	77.8	77.8	66.7	77.8	.	77.8
Oak: Common		....	.		.		61.1
W. Coast hemlock: Select	.	72.2		....	72.2	...	72.2
W. Coast hemlock: Common		.		.	..	..	55.6
Eastern hemlock: Select	44	61.1	.	33	61.1	..	61.1
Eastern hemlock: Common	. .	...	...	...	....	..	44
White pine	66.7	..	55.6	44	50	...	61.1
Spruce: Select	66.7	61.1	55.6	44	61.1	38.9	61.1
Spruce: Common	.	..	...	....	....	....	44
Redwood		.	.	...	...	41.6	55.6

Therefore, when b (the breadth of the beam), and d (the depth of the beam), each equal 1 in, and l (the span) equals 1 ft,

$$W = \frac{S}{18}$$

A is therefore also one-eighteenth of the allowable bending fiber-stress in pounds per square inch. The following are the values of A, obtained by dividing by eighteen the Recommended Unit Stresses for Transverse Bending, and those given in the building laws of New York, Philadelphia, Boston, Chicago, Denver and San Francisco. The value of A for other woods may be found by dividing by eighteen the allowable working unit stresses for bending as given in Table III.

11. Flexural Strength of Wooden Beams

Section-Modulus. For beams with a rectangular cross-section, the formulas for strength can be simplified by substituting for the SECTION-MODULUS its value $\frac{1}{6}bd^2$, where b is the breadth and d the depth of the section.

Substituting this value in the general formulas for beams with rectangular cross-sections and of any material, the following formulas result:

Beams Fixed at One End and Loaded at the Other (Fig. 46).

$$\text{Safe load, in pounds} = \frac{\text{breadth} \times \text{square of depth} \times A\ ^*}{4 \times \text{length in feet}} \qquad (1)$$

or

$$\text{Breadth, in inches} = \frac{4 \times \text{load} \times \text{length in feet}}{\text{square of depth} \times A\ ^*} \qquad (2)$$

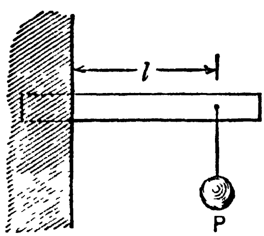


Fig. 46. Cantilever Beam. Load near Free End

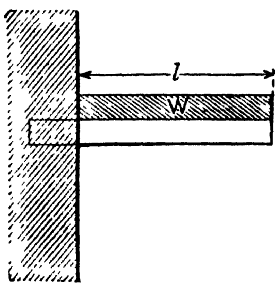


Fig. 47. Cantilever Beam. Distributed Load over Entire Span

Beams Fixed at One End and Loaded with a Uniformly Distributed Load (Fig. 47).

$$\text{Safe load, in pounds} = \frac{\text{breadth} \times \text{square of depth} \times A\ ^*}{2 \times \text{length in feet}} \qquad (3)$$

or

$$\text{Breadth, in inches} = \frac{2 \times \text{load} \times \text{length in feet}}{\text{square of depth} \times A\ ^*} \qquad (4)$$

* For values of A. see Table IV.

Beams Supported at Both Ends and Loaded at the Middle (Fig. 48).

$$\text{Safe load, in pounds} = \frac{\text{breadth} \times \text{square of depth} \times A\ ^*}{\text{span in feet}} \qquad (5)$$

$$\text{Breadth, in inches} = \frac{\text{span in feet} \times \text{load}}{\text{square of depth} \times A\ ^*} \qquad (6)$$

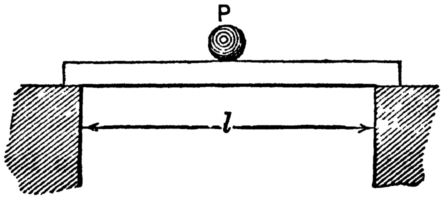


Fig. 48. Simple Beam. Load at Middle of Span

Beams Supported at Both Ends and Loaded with a Uniformly Distributed Load over Entire Span (Fig. 49).

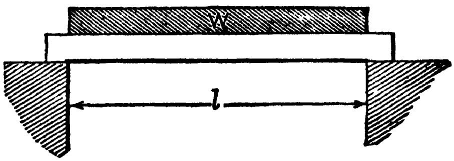


Fig. 49. Simple Beam. Distributed Load over Entire Span

$$\text{Safe load, in pounds} = \frac{2 \times \text{breadth} \times \text{square of depth} \times A\ ^*}{\text{span in feet}} \qquad (7)$$

or

$$\text{Breadth, in inches} = \frac{\text{span in feet} \times \text{load}}{2 \times \text{square of depth} \times A\ ^*} \qquad (8)$$

Beams Supported at Both Ends and Loaded with a Uniformly Distributed Load over only a Portion of the Span (Fig. 50).

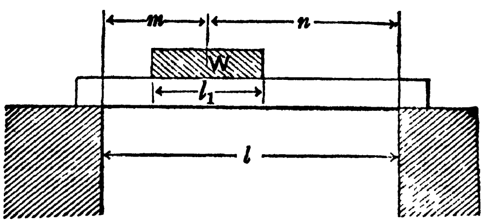


Fig. 50. Simple Beam. Distributed Load over Part of Span

* For values of *A*, see Table IV.

In this case the dimensions of the beam required to carry the load can be accurately determined only by computing the MAXIMUM BENDING MOMENT, as explained in Chapter IX, and substituting the value thus found in Formula (13), following. If, however, the length l_1 be very short in comparison with l, and near the middle, then the load may be considered as CONCENTRATED at the middle of the span and the breadth of the beam may be found by Formula (6). Formula (10) is used if the load be at one side of the middle. The error will be on the safe side.

Beams Supported at Both Ends and Loaded with Concentrated Load, not at the Middle of the Span (Fig. 51).

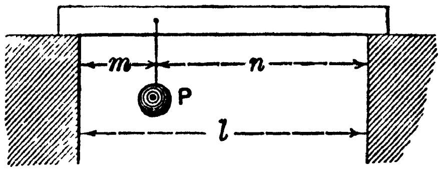


Fig. 51. Simple Beam. Concentrated Load at Any Point

$$\text{Safe load, in pounds} = \frac{\text{breadth} \times \text{square of depth} \times \text{span} \times A\,^*}{4 \times m \times n} \quad (9)$$

or

$$\text{Breadth, in inches} = \frac{4 \times \text{load} \times m \times n}{\text{square of depth} \times \text{span} \times A\,^*} \quad (10)$$

Beams Supported at Both Ends and Loaded with P Pounds at a Distance *m*, from each End (Fig. 52).

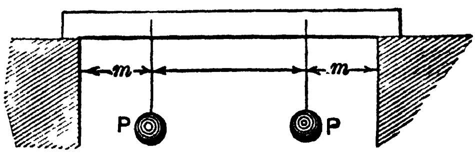


Fig. 52. Simple Beam. Two Equal Concentrated Loads Symmetrically Placed

$$\left.\begin{array}{l}\text{Safe load, } P\text{, in pounds} \\ \text{at each point}\end{array}\right\} = \frac{\text{breadth} \times \text{square of depth} \times A\,^*}{4 \times m} \quad (11)$$

or

$$\text{Breadth, in inches} = \frac{4 \times \text{load at one point} \times m}{\text{square of depth} \times A\,^*} \quad (12)$$

Note. In the last two cases the lengths denoted by *m* and *n* should be in feet, as the spans are in feet.

12. Application of Formulas for Flexural Strength of Wooden Beams

Example 1. What load, 6 ft out from the wall, will an 8 by 14-in long-leaf yellow pine beam, securely fastened at one end into a brick wall, sustain with safety?

* For values of *A*, see Table IV.

Solution. The safe load in pounds (Formula 1) $= \dfrac{8 \times 195 \times 88.9}{4 \times 6}$ $= 5\,808$ lb

Example 2. It is desired to suspend two loads of 10 000 lb each, 4 ft from each end of an oak beam, 20 ft long. What should be the size of the beam?

Solution. Let the depth of the beam be assumed to be 16 in. Then (Formula 12)

$$\text{The breadth} = \frac{4 \times 10\,000 \times 4}{256 \times 77.8} = 8 \text{ in, nearly}$$

The beam, therefore, should be 10 by 16 in in cross-section.

Beam with Several Loads. It is required, next, to determine the size of a beam which is supported at both ends, and which will safely support several concentrated loads, or a distributed load and one or more concentrated loads. The correct method of finding the least size of a beam that will safely support a combination of loads, is first to find the MAXIMUM BENDING MOMENT, as explained in Chapter IX, and then substitute the value thus found for the BENDING MOMENT in the following formula:

$$\text{Breadth, in inches} = \frac{4 \times \text{maximum bending moment in ft-lb}}{\text{square of depth} \times A} \tag{13}$$

A shorter and easier method is to find the EQUIVALENT DISTRIBUTED LOAD for each concentrated load, and then find the size of a beam required to support the total equivalent distributed load thus found. The equivalent distributed loads for concentrated loads applied at different proportions of the span from either end, may be obtained by multiplying the concentrated loads by the following FACTORS:

Table V. Factors for Equivalent Distributed Loads

	Position of load	Factor
For a concentrated load	Applied at middle span	Multiply by 2
For a concentrated load	Applied at one-third the span	Multiply by 1.78
For a concentrated load	Applied at one-fourth the span	Multiply by 1.5
For a concentrated load	Applied at one-fifth the span	Multiply by 1.28
For a concentrated load	Applied at one-sixth the span	Multiply by $1\frac{1}{9}$
For a concentrated load	Applied at one-seventh the span	Multiply by 0.98
For a concentrated load	Applied at one-eighth the span	Multiply by $\frac{7}{8}$
For a concentrated load	Applied at one-ninth the span	Multiply by 0.79
For a concentrated load	Applied at one-tenth the span	Multiply by 0.72

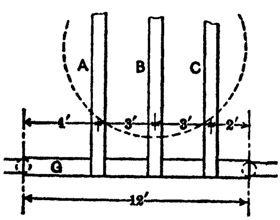


Fig. 52A. Girder with Three Concentrated Loads

Thus, a concentrated load of 900 lb, applied at one-sixth the span from one support, will result in the same maximum bending moment as a distributed load of $900 \times 1\frac{1}{9}$, or 1 000 lb.

The above method for finding the size of a beam for a combination of several loads gives a larger beam than the correct method, by Formula (13), for the reason that the maximum bending moment will not be equal to the sum of the individual bending moments. Hence, when there are several heavy loads to be supported, it is economical to compute the maximum bending moment by the GRAPHIC METHOD explained in Chapter IX.

Example 3. The girder G, Fig. 52A, supports the rafters of a flat roof, and also three heavy beams, A, B and C, blocked up above the roof and supporting a large tank filled with water. The timber is to be long-leaf yellow pine. The weight of the roof and allowance for snow is 7 500 lb. Each of the beams A, B and C, impose a load on the girder, due to the weight of the tank and its contents, of 3 000 lb. What should be the size of the girder?

Solution. The roof-load may be considered to be uniformly distributed. The load from beam A is applied at one-third the span from one end; the load from B, five-twelfths the span from the other end; and the load from C, one-sixth the span. The fraction five-twelfths is the mean of one-half and one-third; hence the load from B should be multiplied by 1.89. Multiplying the concentrated loads by their proper factors, the equivalent distributed load is found to be as follows:

Roof-load, distributed,	=	7 500
Load from A, 3 000 × 1 78	=	5 340
Load from B, 3 000 × 1 89	=	5 670
Load from C, 3 000 × 1⅑	=	3 333
Equivalent distributed load	=	21 843 lb

Assuming 16 in as the depth of the beam, and using Formula (8),

$$\text{The breadth} = \frac{12 \times 21\,843}{2 \times 256 \times 88\,9} = 5.7 \text{ in}$$

Assuming 14 in for the depth, 8 in is obtained for the breadth; hence, the girder must be 8 by 14 in, or 6 by 16 in in cross-section.

Strut-Beams and Tie-Beams. A STRUT-BEAM is a beam that is subject to both a transverse and a compressive stress. A TIE-BEAM is one that is subject to direct tension in addition to the transverse stress. To find the strength of either, first find the size of a beam required to resist the transverse stress, and then the size of a timber, of the same depth as the beam, to resist the direct tension or compression, and add the two breadths together.

Example 4. A spruce tie-beam, 10 ft long between joints, sustains a ceiling-load of 2 000 lb and a direct tensile stress of 40 000 lb. What should be the dimensions of the beam?

Solution. As a ceiling-load is uniformly distributed, the size of the beam is determined by Formula (8). Assuming the depth to be 10 in

$$\text{The breadth} = \frac{10 \times 2\,000}{2 \times 100 \times 61.1}, \quad \text{or} \quad 1.6 \text{ in, nearly}$$

The resistance of spruce to tension is 800 lb per sq in. 40 000/800 = 50 sq in, which is equivalent to a 5 by 10-in section. It will require, therefore, a beam 7 by 10 in in cross-section to resist both the transverse stress and the direct tension. If the tie-beam be cut in any way so as to reduce the section, except over a support, the dimensions must be increased accordingly.

Example 5. A strut-beam of white pine, 10 ft long, supports a distributed roof-load of 6 000 lb, and is also subject to a direct compression of 64 000 lb. What should be the size of the beam?

Solution. Assuming 14 in for the depth, the breadth for the transverse load is found by Formula (8).

$$\text{The breadth} = \frac{10 \times 6\,000}{2 \times 196 \times 61.1} = 2.5 \text{ in, nearly}$$

It is found that a $7\frac{1}{2}$ by 14-in post, 10 ft long, will safely carry the compressive stress, 64 000 lb. Hence it will require a $7\frac{1}{2}$ by 14-in beam to resist the compressive stress, and a $2\frac{1}{2}$ by 14-in beam to resist the transverse load. The beam, therefore, should be 10 by 14 in in cross-section to resist them both.

13. Relative Strengths of Beams

Relative Strengths of Rectangular Beams. From an inspection of the foregoing formulas it is found that the RELATIVE STRENGTHS of beams of rectangular cross-sections, for the different cases, is as shown in Table V-a.

Strengths of Beams of Any Constant Cross-Section. The STRENGTH-RATIOS given in Table V-a are true for beams of any constant cross-section of whatever form.

Beam on Edge. When a beam of square cross-section is supported on its edge, that is, when one of its diagonals is vertical, it will bear about seven-tenths as great a breaking-load as it will when it is supported on one side.

Table V-a. Relative Strengths of Rectangular Beams

Kind of load	Position of load	Strength ratios
Beam supported at both ends		
Uniformly distributed	Over entire span	1
Concentrated	At middle of span	1/2
Concentrated	At one-third the span	9/16
Concentrated	At one-fourth the span	2/3
Concentrated	At one-fifth the span	25/32
Concentrated	At one-sixth the span	9/10
Concentrated	At one-seventh the span	49/48
Concentrated	At one-eighth the span	8/7
Concentrated	At one-ninth the span	81/64
Concentrated	At one-tenth the span	25/18
Beam fixed at one end, or cantilever beams		
Uniformly distributed	Over entire span	1/4
Concentrated	At the free end	1/8
Beam supported at one end and fixed at the other end		
Uniformly distributed	Over entire span	1
Concentrated	Near the middle of span	13/20
Beam fixed at both ends		
Uniformly distributed	Over entire span	1 1/2
Concentrated	At middle of span	1

The Strongest Beam Cut from a Cylindrical Log is one in which the breadth is to the depth as 5 is to 7, very nearly, and the dimensions of such a beam

can be found graphically, as shown in Fig. 53. Any diagonal, as *ab*, is drawn and divided into three equal parts by the points *c* and *d*; from these points lines perpendicular to *ab* are drawn, and the points *e* and *f* connected with *a* and *b*, as shown.

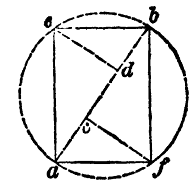


Fig. 53. Strongest Beam of Rectangular Section Cut from Log

Cylindrical Beams. A CYLINDRICAL BEAM is only ten-seventeenths as strong as a beam with a square cross-section, the side of the square being equal to the diameter of the circular section of the cylindrical beam. Hence, to find the safe load for a cylindrical beam, first find the proper load for the corresponding square-section beam, and divide this load by 1.7.

The Bearing of the Ends of a Beam on a wall beyond a certain distance does not strengthen the beam. In general, a oeam should have a bearing of 4 in, or if it is very long, 6 in.

The Weight of the Beam Itself. The formulas given for the strength of beams do not take into account the WEIGHT OF THE BEAMS THEMSELVES, and hence the safe loads of the formulas include both the external loads and the weights of the material in the beams. In small wooden beams, the weight of each beam is generally so small, compared with the external load, that it need not be taken into account. But for larger wooden beams the weight of the beam should be subtracted from the safe load if the load be distributed; and if the load be applied at the middle, one-half the weight of the beam should be subtracted.

The Weight of Timber. The weight per cubic foot for different kinds of timber may be found in the table in Part III, giving the Weights of Various Substances.

14. Tables for Strength and Stiffness of Wooden Beams

Tables VII to XX Give the Strength and Stiffness of Wooden Beams for BEAMS ONE INCH IN BREADTH. To find the strength for any other breadth, multiply the proper tabular value by the breadth of the beam in inches. To obtain the required breadth for any load, divide the given load in pounds by the proper tabular value. The building laws of some cities specify different values of S for the various woods. The values here given follow the recommendations of the Forest Products Laboratory of the Department of Agriculture. In heading the tables, prominence has been given to the values used for S, and the corresponding values of A, so that those who prefer to use for any wood a value different from that recommended, need only to look up the table based on the value they desire to employ. For certain cases and in some cities, the building laws specify 1 500 pounds as the value of S, hence Table XVII, based on this value, is added.

Since timber is weak in HORIZONTAL SHEAR compared with its strength in TENSION and COMPRESSION, the safe load a beam of short span can carry is governed, not by its resistance to CROSS-BREAKING, but by its RESISTANCE to SHEARING along the NEUTRAL SURFACE. Wooden beams and joists, therefore, should be dimensioned to withstand safely this SHEARING ACTION. The ratio of the SHEARING to the FLEXURAL STRENGTH is not exactly the same for different kinds of wood, but for practical use and in the tables it has been assumed to be one-twelfth of the WORKING UNIT FIBER-STRESS. As it can be shown that the ratio of the span to the depth of a rectangular beam, uniformly loaded, is directly proportional to its CROSS-BREAKING STRESS and SHEARING WORKING STRESS, the tabular loads are figured for the PERMISSIBLE UNIT FIBER-STRESS,

where the length of the span is twelve or more times the depth of the beam; while for shorter lengths the tabular loads are governed by the SHEAR. To determine the safe load on beams for a deflection not exceeding $\frac{1}{360}$ of the span, tabular values have been placed directly underneath the safe loads for strength. These values are based on the MODULUS OF ELASTICITY, E, given in the tables.

The FORMULA FOR FLEXURE used in determining the safe uniformly distributed loads in the tables is

$$M = \frac{SI}{c} = \frac{Sbd^2}{6} = \frac{Wl}{8}$$

Hence $$W = \frac{4\,bd^2S}{3\,l},$$ in which l is the span in inches

The FORMULA FOR SHEAR is $W = \dfrac{4\,bdS_s}{3}$ or $S_s = \dfrac{3\,W}{4\,bd}$

The FORMULA FOR DEFLECTION is

$W = \dfrac{Ed^3}{8\,100\,l^2}$ in which l is the span in feet;

M = maximum bending moment in inch-pounds;

I = moment of inertia of the cross-section of the beam in biquadratic inches;

$c = d/2$ = one-half the depth of the beam in inches;

SI/c = resisting moment of the cross-section in inch-pounds;

W = total safe load in pounds, uniformly distributed;

b = breadth of the beam in inches;

d = depth of the beam in inches;

l = span, in feet or inches, as noted for the different formulas;

S = unit flexural fiber-stress in pounds per square inch;

$S_s = S/12$ = horizontal unit shearing-stress, in pounds per square inch, along the neutral surface;

E = modulus of elasticity in pounds per square inch.

Example 6. What is the safe, uniformly distributed load, corresponding to a fiber-stress of 1 500 lb per sq in, for an 8 by 14-in beam supported at both ends, and having a 24-ft clear span?

Solution. From Table XVII, the load for a 1-in thickness is 1 362 lb. Hence, 1 362 × 8 = 10 896 lb, the total load for the beam. If the deflection of this beam should not be more than $\frac{1}{360}$ of the span, the safe load for 1-in thickness should not exceed 882 lb. Hence, 882 × 8 = 7 056 lb, is the maximum load to be used in this case. It is assumed that 1 500 lb per sq in is allowed for S.

Example 7. What should be the size of a common grade Norway-pine beam required to carry a distributed load of 6 400 lb over a clear span of 18 ft?

Solution. From Table X, it is found that a beam 12 in deep and 1 in thick and with an 18-ft span, will support 711 lb. Dividing the load, 6 400 lb, by 711, the result is 9 for the breadth of the beam in inches. Hence the beam should be 9 by 12 in, to carry a distributed load of 6 400 lb over a span of 18 ft. As the deflection-load is more than 711 lb for an 18-ft span, the beam is safe for deflection.

Different Positions of Loads. To find the safe load, concentrated at the middle of the span of a given beam, find the safe distributed load, as in Example 6, and divide this load by 2. To find the safe load concentrated at some point other than the middle of the span, find the safe distributed load for the given span, and divide this load by the proper factor taken from Table

V. To find the size of a beam to support a given concentrated load, multiply the given load by the factor corresponding to the position of the load, as given in Table V, and then proceed as in Example 7.

Use of Formulas. If in doubt as to the application of the tables, in special cases, use one of the formulas, from (1) to (13), applying to the case.

Nominal and Actual Sizes of Beams. The tables may be used for beams the dimensions of which are less than the NOMINAL DIMENSIONS. Dressed beams, and, in many localities, floor-joists carried in stock, are more or less scant of the nominal dimensions, and for such beams and joists a reduction in the safe load must be made to correspond with the reduction in size. The DRESSED SIZES are generally 1/4 in scant, up to 4 in in breadth, above which they are 1/2 in scant; while in depth they are all generally 1/2 in less than the nominal size. The safe loads may be obtained by multiplying the safe loads for the corresponding nominal sizes, as given in Tables VII to XX, by the factors given in the following table.

Example 8. What is the safe load for a 2 3/4 by 13 1/2-in common grade spruce beam, with an 18-ft span?

Solution. From Table X, the safe load for a 1 by 14-in beam is 968 lb. Multiplying this by 2.56, we have 2 478 lb as the safe distributed load for a beam 2 3/4 by 13 1/2 in in cross-section. For a full 3 by 14-in cross-section, the safe load would be 2 904 lb.

Table VI. Conversion Factors for Actual Sizes of Wooden Beams

Cross-sections of beams in inches	Factors	Cross-sections of beams in inches	Factors
1 3/4 × 5 1/2	1.47	1 3/4 × 11 1/2	1.61
2 3/4 × 5 1/2	2.31	2 3/4 × 11 1/2	2.53
1 3/4 × 6 1/2	1.51	1 3/4 × 13 1/2	1.63
2 3/4 × 6 1/2	2.51	2 3/4 × 13 1/2	2.56
1 3/4 × 7 1/2	1.54	1 3/4 × 15 1/2	1.65
2 3/4 × 7 1/2	2.42	2 3/4 × 15 1/2	2.58
1 3/4 × 9 1/2	1.58	1 3/4 × 17 1/2	1.65
2 3/4 × 9 1/2	2.48	2 3/4 × 17 1/2	2.60

Use of Tables VII to XX. The safe loads given in Tables VII to XX are correct for the fiber-stresses indicated; but for greater convenience in using the tables, each figure in the units-place of each value may be made a cipher, and each figure in the tens-place may be increased by one when the unit-figure is six or greater. Thus, 505 would be 500, 506 would be 510, etc.

Loads above zigzag lines in tables are calculated for horizontal shear. Where two loads are given, the upper is calculated for STRENGTH, the lower for DEFLECTION not to exceed 1/360 the span.

These values for the various woods are based upon the allowable unit stresses recommended by the Forest Products Laboratory and given in Table I, for commercially seasoned timber of standard grades in dry locations.

Important Notes on Stresses and Loads for Wooden Beams. In compiling and using the tables of safe loads for wooden beams, the following important considerations must be kept in mind:

(1) Unseasoned timber is very much weaker than commercially dry timber, that is, timber containing from 10 to 15% of moisture.

(2) Timber containing large or loose knots is much weakened.

(3) When impact has to be considered, the stresses should be reduced.

(4) For continuous, heavy loading, relatively low stresses should be used.

(5) Commercial dimensions are smaller than nominal dimensions.

(6) Timbers deteriorate and the factors of safety for strength grow smaller with time.

(7) The modulus of elasticity, E, for unseasoned timber, should be reduced 50% from its value given for thoroughly seasoned timber.

Table VII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Common Grade White Cedar

Maximum fiber-stress, $S = 600$ lb per sq in., $E = 800\,000$ lb per sq in. $A = 33$

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	400	533	666	800	933	1 066	1 200
7	343	533	666	800	933	1 066	1 200
8	300	533	666	800	933	1 066	1 200
9	266 263	474	666	800	933	1 066	1 200
10	240 213	427	666	800	933	1 066	1 200
11	218 176	388	605	800	933	1 066	1 200
12	200 148	356 351	555	800	933	1 066	1 200
13	185 126	328 300	513	738	933	1 066	1 200
14	171 108	305 258	477	686	933	1 066	1 200
15	160 95	285 224	445 439	640	871	1 066	1 200
16	150 83	267 197	417 385	600	817	1 066	1 200
17	......	251 175	392 346	565	762	1 003	1 200
18	.....	237 156	371 304	534 526	726	948	1 200
19		225 140	351 273	505 472	688	898	1 137
20		213 126	333 247	480 426	653	854	1 080
21			317 224	462 387	623	813	1 029
22	.....	.	303 204	436 352	594 560	776	982
23	. .		290 186	417 322	568 512	742	939
24			278 171	400 296	545 470	712 702	900
25	....	. . .	......	384 273	523 433	683 647	864
26			.	369 252	503 400	657 597	831
27	. . .			356 234	482 371	633 554	800 790
28	.....	...	. . . .	343 217	467 345	609 516	772 734
29	......	. . . .	.....	.....	451 322	589 481	745 684
30	.	..		. . . .	436 301	569 449	720 640

Loads above zigzag lines calculated for horizontal shear. Where two loads are given, the upper is calculated for strength, the lower for deflection not to exceed 1/360 the span.

Table VIII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Common Western Soft Pine and Red Cedar

Maximum fiber-stress, $S = 700$ lb per sq in. $E = 1\ 000\ 000$ lb per sq in. $A = 39$

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	467	622	777	933	1 089	1 244	1 400
7	400	622	777	933	1 089	1 244	1 400
8	350	622	777	933	1 089	1 244	1 400
9	311	552	777	933	1 089	1 244	1 400
10	280 267	497	777	933	1 089	1 244	1 400
11	255 221	453	707	933	1 089	1 244	1 400
12	233 185	415	648	933	1 089	1 244	1 400
13	216 158	383 374	598	861	1 089	1 244	1 400
14	200 136	356 323	556	800	1 089	1 244	1 400
15	187 119	332 281	518	747	1 016	1 244	1 400
16	175 104	311 247	486 482	700	952	1 244	1 400
17	.	293 219	458 427	660	897	1 172	1 400
18		276 195	433 381	623	847	1 107	1 400
19		262 175	410 342	590	802	1 048	1 326
20		.	389 308	560 534	762	996	1 260
21			370 280	534 484	726	948	1 200
22	...		354 255	509 441	692	906	1 144
23	..		338 234	487 403	662 641	866	1 096
24	.		324 215	468 371	635 588	830	1 050
25	.			448 342	610 542	796	1 008
26	.		.. .	430 316	586 502	766 750	970
27	.	..	.... .	415 293	565 465	738 695	934
28	...		. .	400 272	544 432	711 646	900
29	..	.	.. .	. .	526 403	687 602	868 856
30	...	. .	. .	.	508 377	664 562	840 800

Table IX. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade White Cedar

Maximum fiber-stress, S = 750 lb per sq in. E = 800 000 lb per sq in. A = 41.7

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	500	667	833	1 000	1 167	1 333	1 500
7	428	667	833	1 000	1 167	1 333	1 500
8	375 333	667	833	1 000	1 167	1 333	1 500
9	333 263	592	833	1 000	1 167	1 333	1 500
10	300 213	533 505	833	1 000	1 167	1 333	1 500
11	274 176	485 388	757	1 000	1 167	1 333	1 500
12	250 148	445 351	695 685	1 000	1 167	1 333	1 500
13	231 126	410 300	641 584	923	1 167	1 333	1 500
14	214 108	382 258	595 503	857	1 167	1 333	1 500
15	....	356 224	556 439	800 758	1 088	1 333	1 500
16	.	333 197	521 385	750 666	1 020	1 333	1 500
17	...	..	491 346	706 590	961 937	1 254	1 500
18	...	...	463 304	667 526	908 836	1 184	1 500
19	.		439 273	[illegible] 472	[illegible] 750	1 122	1 421
20	...	...	....	600 426	816 677	1 066 1 011	1 350
21	.....	..	...	572 387	778 614	1 016 917	1 286
22	....	...	....	547 352	742 560	970 835	1 227 1 190
23	......	.....	.....	522 322	710 512	928 764	1 174 1 088
24	.....	....	......	500 296	681 470	890 702	1 125 1 000
25	......	......	......	480 273	653 433	854 647	1 080 921
26	......	......	......	463 252	628 400	821 597	1 038 852
27	......	......	......	444 234	605 371	791 554	1 000 790
28	......	......	.....	428 217	583 345	762 516	965 734
29	.....	...	.....	.....	563 322	736 481	931 684
30	......	......	.....	.....	544 301	712 449	900 640

Table X. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Common Grades, Eastern Hemlock, Spruce and Norway Pine

Fiber-stress, $S = 800$ lb per sq in. $E = 1\,000\,000$ lb per sq in. $A = 44$

For spruce use deflection loads in Table XIII.

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	533	711	889	1 066	1 244	1 422	1 600
7	457	711	889	1 066	1 244	1 422	1 600
8	400	711	889	1 066	1 244	1 422	1 600
9	356 329	632	889	1 066	1 244	1 422	1 600
10	320 267	569	889	1 066	1 244	1 422	1 600
11	291 221	517	809	1 066	1 244	1 422	1 600
12	267 185	474 439	742	1 066	1 244	1 422	1 600
13	246 158	438 374	684	985	1 244	1 422	1 600
14	229 136	407 323	635 629	914	1 244	1 422	1 600
15	214 119	379 281	593 548	854	1 161	1 422	1 600
16	200 104	356 247	556 482	800	1 089	1 422	1 600
17	...	335 219	524 427	754	1 025	1 339	1 600
18	...	316 195	494 381	711	968	1 264	1 600
19	...	300 175	468 342	674 657	917	1 198	1 517
20	...	284 159	445 308	640 534	871 847	1 138	1 441
21	...		423 280	609 484	830 768	1 084	1 372
22	...	...	404 255	582 441	792 700	1 035	1 309
23	...	...	387 234	557 403	758 641	990 955	1 253
24	...	...	371 215	534 371	726 588	949 876	1 200
25	...	...	...	512 342	697 542	911 809	1 152 1 152
26	...	...	...	492 316	670 502	876 750	1 108 1 065
27	...	...	...	474 293	646 465	843 695	1 068 987
28	...	...	...	457 272	622 432	813 646	1 029 918
29	...	...	...	...	601 403	785 602	993 856
30	...	...	...	...	581 377	759 562	960 800

Table XI. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade Red Cedar and Western Soft Pine

Fiber stress, S = 900 lb per sq in. E = 1 000 000 lb per sq in. A = 50

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	600	800	1 000	1 200	1 400	1 600	1 800
7	514	800	1 000	1 200	1 400	1 600	1 800
8	450 416	800	1 000	1 200	1 400	1 600	1 800
9	400 329	713	1 000	1 200	1 400	1 600	1 800
10	360 267	640 632	1 000	1 200	1 400	1 600	1 800
11	327 221	581 522	909	1 200	1 400	1 600	1 800
12	300 185	533 439	833	1 200	1 400	1 600	1 800
13	277 158	492 374	769 730	1 108	1 400	1 600	1 800
14	257 136	457 323	714 629	1 028	1 400	1 600	1 800
15	240 119	427 281	666 548	960	1 306	1 600	1 800
16	225 104	400 247	625 482	900 833	1 225	1 600	1 800
17	.	376 219	588 427	847 738	1 153	1 506	1 800
18	..	356 195	555 381	800 657	1 088 1 045	1 422	1 800
19	..	337 175	526 342	757 591	1 031 938	1 348	1 705
20		320	500 308	720 534	980 847	1 280 1 264	1 620
21	. . . .	.. .	476 280	686 484	933 768	1 219 1 146	1 543
22	.. .	. . .	455 255	654 441	891 700	1 163 1 044	1 473
23	. ..	..	435 234	626 403	852 641	1 113 955	1 409 1 361
24	... .	.	417 215	600 371	817 588	1 066 876	1 350 1 250
25	.. .	.	.. .	576 342	784 542	1 024 809	1 296 1 152
26	. . . .	.	.. .	554 316	754 502	984 750	1 246 1 065
27	. . . .	..	.. ...	533 293	726 465	948 695	1 200 987
28	. . . .	. .	..	514 272	700 432	914 646	1 157 918
29	..	.. .	.	. . .	676 403	882 602	1 117 856
30	.. .	.	. .	. . .	653 377	853 562	1 080 800

Table XII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Common Grade West Coast Hemlock and Southern Cypress; Structural Grade Redwood

Fiber-stress, S = 1 000 lb per sq in. E = 1 200 000 lb per sq in. A = 55.6

For west coast hemlock use deflection loads in Table XV.

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	667	889	1 111	1 333	1 556	1 778	2 000
7	571	889	1 111	1 333	1 556	1 778	2 000
8	{500 500}	889	1 111	1 333	1 556	1 778	2 000
9	{444 395}	790	1 111	1 333	1 556	1 778	2 000
10	{400 320}	711	1 111	1 333	1 556	1 778	2 000
11	{364 265}	{647 628}	1 010	1 333	1 556	1 778	2 000
12	{333 222}	{593 527}	926	1 333	1 556	1 778	2 000
13	{308 190}	{547 449}	855	1 231	1 556	1 778	2 000
14	{286 163}	508 388	{794 757}	1 143	1 556	1 778	2 000
15	{267 143}	474 337	{741 659}	1 067	1 452	1 778	2 000
16	{250 125}	445 296	695 578	{1 000 1 000}	1 361	1 778	2 000
17	.	{419 263}	654 512	{942 886}	1 281	1 674	2 000
18	. . .	{395 234}	618 457	{890 790}	1 210	1 581	2 000
19		{374 210}	585 410	843 710	{1 146 1 126}	1 498	1 895
20	. . .	{356 190}	556 370	800 641	{1 088 1 016}	1 423	1 800
21			{528 336}	762 581	{1 037 922}	1 355	1 714
22	. .		{505 306}	727 529	990 841	{1 293 1 254}	1 636
23		.	{483 281}	696 484	947 770	{1 237 1 147}	1 565
24	. .	. .	{463 258}	667 445	908 706	1 186 1 053	{1 500 1 500}
25	. . . . .	.	. . . .	{640 410}	871 650	1 138 972	{1 440 1 384}
26	. .	.	. . . .	{615 380}	838 602	1 094 900	{1 385 1 280}
27	. .	.	. . .	{593 352}	807 558	1 054 834	{1 334 1 186}
28	. . .	. . . . . .	. . . .	{572 327}	778 518	1 016 776	{1 286 1 103}
29	. . .	. . .	. . . .	.	{751 484}	982 725	{1 241 1 027}
30	. . . . .	. . .	. . . .	. .	{726 452}	949 674	{1 200 960}

Table XIII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade Eastern Hemlock, Spruce and Norway Pine; Common Grade Oak

Fiber-stress, $S = 1\,100$ lb per sq in. $E = 1\,200\,000$ lb per sq in. $A = 61.1$

For **eastern hemlock,** use deflection loads in Table X.
For **oak,** use deflection loads in Table XVI.

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	733	977	1 222	1 466	1 711	1 955	2 200
7	628	977	1 222	1 466	1 711	1 955	2 200
8	550 500	977	1 222	1 466	1 711	1 955	2 200
9	466 400	869	1 222	1 466	1 711	1 955	2 200
10	440 320	782 758	1 222	1 466	1 711	1 955	2 200
11	400 265	715 626	1 110	1 466	1 711	1 955	2 200
12	366 222	652 527	1 018	1 466	1 711	1 955	2 200
13	338 190	602 449	940 876	1 354	1 711	1 955	2 200
14	314 163	558 388	873 757	1 257	1 711	1 955	2 200
15	293 143	521 337	815 659	1 173	1 597	1 955	2 200
16	275 125	489 296	764 578	1 100 1 000	1 497	1 955	2 200
17	....	460 263	719 512	1 035 886	1 409 1 406	1 840	2 200
18	. .	434 234	679 457	978 790	1 331 1 254	1 738	2 200
19	.	412 210	643 410	926 710	1 260 1 125	1 646	2 084
20	. . .	391 190	611 370	880 641	1 197 1 016	1 564 1 517	1 980
21	. . .	....	582 336	838 581	1 140 922	1 490 1 376	1 885
22	.	. .	555 306	800 529	1 089 841	1 422 1 253	1 799 1 785
23			531 281	765 484	1 041 770	1 360 1 147	1 721 1 633
24			509 258	733 445	998 706	1 303 1 053	1 650 1 500
25	.		.....	704 410	958 650	1 251 972	1 583 1 384
26			.... .	677 380	921 602	1 203 900	1 523 1 280
27			.	652 352	887 558	1 158 834	1 466 1 186
28				628 327	855 518	1 117 776	1 414 1 103
29				.....	826 484	1 078 725	1 365 1 027
30	.		. .	.....	798 452	1 042 674	1 320 960

Table XIV. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade Redwood, Common Grade Douglas Fir and Southern Yellow Pine

Maximum fiber-stress, S = 1 200 lb per sq in. E = 1 600 000 lb per sq in. A = 66.7

For **redwood,** use deflection loads in Table XII.

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	800	1 067	1 333	1 600	1 867	2 133	2 400
7	686	1 067	1 333	1 600	1 867	2 133	2 400
8	600	1 067	1 333	1 600	1 867	2 133	2 400
9	533 527	949	1 333	1 600	1 867	2 133	2 400
10	480 426	854	1 333	1 600	1 867	2 133	2 400
11	437 352	776	1 212	1 600	1 867	2 133	2 400
12	400 296	711 702	1 111	1 600	1 867	2 133	2 400
13	369 252	656 599	1 026	1 477	1 867	2 133	2 400
14	343 217	610 523	953	1 371	1 867	2 133	2 400
15	320 190	569 449	890 877	1 280	1 741	2 133	2 400
16	300 166	533 396	834 771	1 200	1 633	2 133	2 400
17		502 350	785 683	1 130	1 537	2 009	2 400
18		474 312	741 609	1 067 1 053	1 452	1 898	2 400
19		449 280	702 547	1 010 945	1 375	1 795	2 274
20		426 252	666 493	960 853	1 306 1 355	1 708	2 160
21			634 447	914 774	1 245 1 229	1 626	2 057
22			606 408	872 705	1 188 1 199	1 552	1 968
23			579 373	835 645	1 136 1 024	1 484	1 878
24			556 342	800 592	1 090 941	1 423 1 404	1 800
25				768 546	1 045 867	1 366 1 294	1 728
26				738 505	1 006 801	1 313 1 197	1 662
27				711 468	969 743	1 265 1 109	1 600 1 580
28				686 435	933 691	1 218 1 032	1 543 1 469
29					902 644	1 178 962	1 489 1 369
30					871 602	1 138 898	1 440 1 279

Table XV. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade West Coast Hemlock and Southern Cypress

Fiber-stress, S = 1 300 lb per sq in. E = 1 400 000 lb per sq in. A = 72.2

For **cypress**, use deflection loads in Table XII.

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	867	1 155	1 444	1 733	2 022	2 311	2 600
7	743	1 155	1 444	1 733	2 022	2 311	2 600
8	650 583	1 115	1 144	1 733	2 022	2 311	2 600
9	567 460	1 027	1 444	1 733	2 022	2 311	2 600
10	520 373	924 885	1 444	1 733	2 022	2 311	2 600
11	473 308	840 731	1 311	1 733	2 022	2 311	2 600
12	433 259	770 613	1 200 1 200	1 733	2 022	2 311	2 600
13	400 221	711 523	1 111 1 022	1 600	2 022	2 311	2 600
14	371 190	660 451	1 032 881	1 486	2 022	2 311	2 600
15	347 166	616 393	963 768	1 387 1 327	1 887	2 311	2 600
16	325 145	578 343	903 675	1 300 1 166	1 770	2 311	2 600
17		544 306	849 598	1 224 1 033	1 664 1 641	2 175	2 600
18		514 273	802 533	1 156 921	1 572 1 463	2 054	2 600
19		487 245	760 478	1 095 827	1 400 1 313	1 946	2 463
20		462 221	722 432	1 040 746	1 415 1 185	1 849 1 769	2 340
21			688 391	990 677	1 348 1 075	1 761 1 605	2 229
22			657 357	945 617	1 286 980	1 681 1 462	2 127 2 082
23			628 326	904 564	1 230 896	1 608 1 338	2 035 1 905
24			602 300	867 518	1 179 823	1 541 1 229	1 950 1 750
25				832 471	1 132 759	1 479 1 132	1 872 1 613
26				800 441	1 088 701	1 422 1 048	1 800 1 491
27				770 409	1 048 650	1 369 971	1 733 1 382
28				743 380	1 011 605	1 321 903	1 671 1 285
29					976 562	1 275 841	1 614 1 198
30					943 526	1 232 786	1 560 1 120

Table XVI. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Select Grade Oak

Fiber-stress, S = 1 400 lb per sq in. E = 1 500 000 lb per sq in. A = 77.8

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	933	1 244	1 555	1 866	2 177	2 488	2 800
7	800	1 244	1 555	1 866	2 177	2 488	2 800
8	700 625	1 244	1 555	1 866	2 177	2 488	2 800
9	622 495	1 106	1 555	1 866	2 177	2 488	2 800
10	560 400	996 948	1 555	1 866	2 177	2 488	2 800
11	509 332	905 783	1 414	1 866	2 177	2 488	2 800
12	467 278	840 658	1 297	1 866	2 177	2 488	2 800
13	430 247	766 561	1 197	1 723	2 177	2 488	2 800
14	400 204	711 485	1 111	1 600	2 177	2 488	2 800
15	373 179	664 422	1 037 823	1 493 1 422	2 033	2 488	2 800
16	350 156	622 371	972 714	1 400 1 250	1 906	2 488	2 800
17		586 329	915 642	1 318 1 108	1 800 1 757	2 343	2 800
18		553 293	864 572	1 244 990	1 694 1 568	2 213	2 800
19		524 263	820 513	1 180 886	1 605 1 410	2 096	2 653
20		498 237	778 462	1 120 802	1 525 1 272	1 991	2 520
21			721 420	1 066 726	1 452 1 154	1 896 1 719	2 400
22			707 383	1 018 662	1 386 1 051	1 810 1 560	2 291 2 231
23			676 351	974 605	1 326 962	1 731 1 435	2 191 2 041
24			648 322	933 557	1 271 882	1 659 1 318	2 100 1 875
25				896 513	1 220 813	1 593 1 215	2 016 1 727
26				861 473	1 173 753	1 532 1 125	1 939 1 596
27				830 440	1 130 698	1 475 1 043	1 867 1 480
28				800 410	1 090 648	1 422 970	1 800 1 377
29					1 050 605	1 373 903	1 738 1 284
30					1 017 566	1 327 843	1 680 1 200

Table XVII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams

Maximum fiber-stress, S = 1 500 lb per sq in. A = 83.3

Span in feet	The first horizontal line gives the depth of the beam in inches The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	1 000	1 333	1 667	2 000	2 333	2 667	3 000
7	857	1 333	1 667	2 000	2 333	2 667	3 000
8	750	1 333	1 667	2 000	2 333	2 667	3 000
9	667	1 185	1 667	2 000	2 333	2 667	3 000
10	600	1 067	1 667	2 000	2 333	2 667	3 000
11	548	970	1 515	2 000	2 333	2 667	3 000
12	500	890	1 390	2 000	2 333	2 667	3 000
13	462	820	1 282	1 846	2 333	2 667	3 000
14	428	764	1 190	1 714	2 333	2 667	3 000
15	.	712	1 112	1 600	2 178	2 667	3 000
16	. ...	667	1 042	1 500	2 042	2 667	3 000
17	.	.... .	982	1 412	1 974	2 510	3 000
18	...	...	926	1 334	1 815	2 370	3 000
19	...	...	878	1 264	1 720	2 246	2 842
20	....		...	1 200	1 632	2 133	2 700
21	.	.	....	1 144	1 556	2 032	2 571
22	.	.	......	1 094	1 484	1 940	2 455
23	.	...	.....	1 044	1 420	1 856	2 348
24	....	...	....	1 000	1 362	1 780	2 250
25	..	....	......	960	1 306	1 708	2 160
26	....	...	......	926	1 256	1 642	2 076
27	.....	..	......	888	1 210	1 582	2 000
28	..	....	.	856	1 166	1 524	1 930
29	.		....	....	1 126	1 472	1 862
30	....		....	....	1 088	1 422	1 800

Loads above the heavy, black zigzag lines are calculated for resistance to horizontal shear.

Table XVIII. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Structural Grade Douglas Fir and Southern Yellow Pine

Fiber-stress, S = 1 600 lb per sq in. E = 1 600 000 lb per sq in. A = 88.9

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	1 067	1 422	1 778	2 133	2 489	2 844	3 200
7	914 870	1 422	1 778	2 133	2 489	2 844	3 200
8	800 666	1 422	1 778	2 133	2 489	2 844	3 200
9	711 527	1 265 1 248	1 778	2 133	2 489	2 844	3 200
10	640 426	1 138 1 011	1 778	2 133	2 489	2 844	3 200
11	582 352	1 034 825	1 616	2 133	2 489	2 844	3 200
12	534 296	948 702	1 482 1 371	2 133	2 489	2 844	3 200
13	492 252	875 599	1 368 1 168	1 970	2 489	2 844	3 200
14	457 217	813 523	1 270 1 007	1 830 1 741	2 489	2 844	3 200
15	426 190	759 449	1 185 877	1 707 1 517	2 323	2 844	3 200
16	400 166	711 396	1 111 771	1 600 1 333	2 178 2 117	2 844	3 200
17	...	669 350	1 046 683	1 506 1 181	2 050 1 832	2 677	3 200
18	.....	632 312	988 609	1 422 1 053	1 936 1 672	2 527 2 497	3 200
19	.	600 280	936 547	1 348 945	1 834 1 536	2 396 2 207	3 032
20	..	569 252	889 493	1 280 853	1 743 1 355	2 276 2 022	2 880 2 880
21	.	...	847 447	1 220 774	1 660 1 229	2 168 1 831	2 743 2 612
22	. .	. . .	808 408	1 163 705	1 584 1 119	2 069 1 671	2 618 2 380
23	....	.....	773 373	1 113 645	1 515 1 024	1 979 1 531	2 505 2 177
24	.....	......	741 342	1 066 592	1 452 941	1 897 1 404	2 400 2 000
25	......	......	......	1 024 546	1 394 867	1 821 1 294	2 304 1 843
26	.. ..	......	......	985 505	1 340 801	1 751 1 197	2 216 1 763
27	....	......	......	950 468	1 291 743	1 682 1 109	2 134 1 580
28	......	......	......	914 435	1 244 691	1 625 1 032	2 057 1 469
29	....	.. . .	......	....	1 200 644	1 570 962	1 987 1 369
30	... .	. . .	.....	...	1 162 602	1 517 838	1 920 1 279

Table XIX. Safe Distributed Loads in Pounds for Rectangular Wood Beams for Prime Grade Redwood

Fiber-stress, S = 1 700 lb per sq in. E = 1 200 000 lb per sq in. A = 94.4

For strength of beams 5 in and over in width use Table XVII

Span in feet	The first horizontal line gives the depth of the beam in inches. The loads are for beams one inch wide and supported at both ends						
	6	8	10	12	14	16	18
6	1 133 889	1 511	1 889	2 266	2 644	3 022	3 400
7	972 653	1 511 1 511	1 889	2 266	2 644	3 022	3 400
8	850 500	1 511 1 185	1 889	2 266	2 644	3 022	3 400
9	755 400	1 342 936	1 889 1 829	2 266	2 644	3 022	3 400
10	680 320	1 208 758	1 889 1 481	2 266	2 644	3 022	3 400
11	618 265	1 098 626	1 716 1 222	2 266 2 115	2 644	3 022	3 400
12	566 222	1 007 527	1 573 1 028	2 266 1 777	2 644	3 022	3 400
13	523 190	929 449	1 452 876	2 091 1 515	2 644 2 405	3 022	3 400
14	486 163	863 388	1 348 757	1 941 1 306	2 644 2 074	3 022	3 400
15	453 143	805 337	1 258 659	1 812 1 138	2 467 1 806	3 022 2 754	3 400
16	425 125	755 296	1 180 578	1 699 1 000	2 313 1 588	3 022 2 370	3 400
17		711 263	1 111 512	1 599 886	2 182 1 406	2 844 2 099	3 400 2 989
18		671 234	1 049 457	1 510 790	2 055 1 254	2 685 1 872	3 400 2 666
19		636 210	993 410	1 431 710	1 947 1 125	2 544 1 681	3 220 2 393
20		604 190	944 370	1 359 641	1 850 1 016	2 416 1 517	3 058 2 160
21			899 336	1 294 581	1 762 922	2 301 1 376	2 913 1 959
22			858 306	1 236 529	1 682 841	2 197 1 253	2 780 1 785
23			821 281	1 182 484	1 608 770	2 101 1 147	2 660 1 633
24			786 258	1 133 445	1 542 706	2 013 1 053	2 518 1 500
25				1 087 410	1 480 650	1 933 972	2 446 1 384
26				1 045 380	1 423 602	1 859 900	2 352 1 280
27				1 003 352	1 370 558	1 790 834	2,265 1 186
28				971 327	1 321 518	1 726 776	2 184 1 103
29					1 276 484	1 666 725	2 109 1 027
30					1 233 452	1 611 674	2 039 960

Table XX. Safe Distributed Loads in Pounds for Rectangular Wooden Beams for Dense Structural Grade Douglas Fir and Dense Heart Southern Yellow Pine

Fiber-stress, $S = 1\,800$ lb per sq in. $E = 1\,600\,000$ lb per sq in. $A = 100$

Span in feet	The first horizontal line gives the depth of the beam in inches The loads are for beams one inch wide and supported at both ends 6	8	10	12	14	16	18
6	1 200 1 185	1 600	2 000	2 400	2 800	3 200	3 600
7	1 030 870	1 600	2 000	2 400	2 800	3 200	3 600
8	900 666	1 600 1 580	2 000	2 400	2 800	3 200	3 600
9	800 527	1 422 1 248	2 000	2 400	2 800	3 200	3 600
10	720 426	1 280 1 011	2 000 1 975	2 400	2 800	3 200	3 600
11	655 352	1 164 825	1 818 1 616	2 400	2 800	3 200	3 600
12	600 296	1 067 702	1 667 1 371	2 400 2 370	2 800	3 200	3 600
13	554 252	985 599	1 539 1 168	2 215 1 970	2 800	3 200	3 600
14	514 217	914 523	1 428 1 007	2 057 1 741	2 800 2 765	3 200	3 600
15	480 190	853 449	1 333 877	1 920 1 517	2 613 2 323	3 200	3 600
16	450 166	800 396	1 250 771	1 800 1 333	2 450 2 117	3 200 3 160	3 600
17		753 350	1 176 683	1 694 1 181	2 306 1 832	3 012 2 677	3 600
18		711 312	1 111 609	1 600 1 053	2 178 1 672	2 844 2 497	3 600 3 555
19		674 280	1 053 547	1 516 945	2 063 1 536	2 695 2 207	3 411 3 191
20		640 252	1 000 493	1 440 853	1 960 1 355	2 560 2 022	3 240 2 880
21				1 371 774	1 867 1 229	2 438 1 831	3 086 2 612
22				1 309 705	1 782 1 199	2 327 1 671	2 945 2 380
23				1 252 645	1 704 1 024	2 226 1 531	2 817 2 177
24				1 200 592	1 633 941	2 133 1 404	2 700 2 000
25				1 152 546	1 568 867	2 048 1 294	2 592 1 843
26				1 108 505	1 508 801	1 969 1 197	2 492 1 763
27				1 067 468	1 452 743	1 896 1 109	2 400 1 580
28				1 029 435	1 400 691	1 829 1 032	2 314 1 469
29					1 352 644	1 766 962	2 235 1 369
30					1 307 602	1 707 898	2 160 1 279

15. Maximum Spans for Floor-Joists and Rafters Uniformly Loaded

Tables XXI to XXXVI * give the minimum sizes of floor-joists allowable for various spans with live loads varying from 40 lb to 160 lb per sq ft. Tables XXXVI to XLIV * give the minimum sizes for rafters and roof-joists allowable for various spans and with live loads varying from 15 lb to 50 lb per sq ft.

In all these tables the weights of the member itself and of the flooring, the plastered ceiling or the roofing have been included in determining the allowable size of the joist or rafter. Such structural weights are known as dead loads.

Knowing the live load in pounds per square foot which is to be supported, the span and the species and grade of lumber to be used, and having ascertained the allowable working stress for that species and grade by reference either to the local building code or to Table I, it is easy to determine the size and spacing of the joists by turning to the table for that particular live load. Reading across from the given span under the column headed by the allowable working stress, the size and spacing of the joists will be found

In cases where the deflection of the joist is to be limited to 1/360 of the span the allowable modulus of elasticity in pounds per square inch for the species of timber used must be determined from the local building code or from Table I. The allowable size of joist will then be found by reading across from the given span under the column headed by the required modulus of elasticity.

For live loads of 80 lb and over per square foot, two tables are given for each live load, one for safe joist size as determined by bending and the other as determined by horizontal shear. Both tables should be consulted and the larger joist selected. Where deflection is a consideration the sizes as required by bending, shear and deflection should be checked and the largest of the three selected.

The dead loads used in these tables are as follows:

For joists

Plaster	10 lb per sq ft
Finished floor	2.5 lb per sq ft
Rough floor	2.5 lb per sq ft
Weight of joist	Based on assumed average of 40 lb per cu ft

For rafters

Wood roof sheathing	2.5 lb per sq ft

Group I—Assumed as 2.5 lb per sq ft, including

Shingles	2.5 lb per sq ft
Copper sheets	1.5 lb per sq ft
Copper tile	1.75 lb per sq ft
Three-ply ready roofing	1.00 lb per sq ft

Group II—Assumed as 8 lb per sq ft

Five-ply felt and gravel	7 lb per sq ft
Slate 3/16 in.	7¼ lb per sq ft
Roman tile—new style—1 part	8 lb per sq ft
Spanish tile—new style—1 part	8 lb per sq ft
Ludowici tile	8 lb per sq ft

* These tables were prepared by Mr. Richard G. Kimbell for the National Lumber Manufacturers' Association and are reprinted here with the permission of the Association.

Table XXI. Floor-Joist Spans (30-Pound Load)

MAXIMUM SPANS FOR FLOOR JOISTS—UNIFORMLY LOADED

Live load 30 pounds per square foot with plastered ceiling Live load 40 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span): Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	9	1	9	9	10	2	10	8	10	4	10	11	11	6	12	0	12	6	13	0	13	5	13	10	14	3	14	8
	16	8	4	8	11	9	3	9	9	9	1	9	6	10	0	10	6	10	10	11	3	11	8	12	1	12	5	12	10
	24	7	3	7	9	8	2	8	6	7	5	7	10	8	3	8	7	9	0	9	4	9	7	10	0	10	2	10	6
2×8	12	12	0	12	10	13	6	14	0	13	9	14	6	15	3	15	11	16	6	17	2	17	9	18	4	18	10	19	5
	16	11	0	11	9	12	4	12	11	12	0	12	8	13	4	13	11	14	5	15	0	15	6	16	0	16	6	17	0
	24	9	8	10	3	10	10	11	4	9	11	10	5	10	11	11	5	11	11	12	4	12	9	13	3	13	7	14	0
2×10	12	15	2	16	1	17	0	17	9	17	3	18	3	19	1	19	11	20	8	21	6	22	3	23	0	23	8	24	4
	16	13	11	14	10	15	6	16	3	15	1	15	11	16	8	17	4	18	2	18	10	19	6	20	1	20	9	21	4
	24	12	2	13	0	13	8	14	4	12	6	13	3	13	10	14	5	15	0	15	7	16	1	16	7	17	1	17	8
2×12	12	18	3	19	5	20	5	21	4	20	8	21	10	22	11	23	11	24	10	25	9	26	8	27	7	28	5	29	3
	16	16	9	17	10	18	8	19	6	18	1	19	3	20	1	20	11	21	10	22	8	23	5	24	3	24	11	25	8
	24	14	9	15	8	16	6	17	3	15	0	15	10	16	7	17	5	18	1	18	9	19	5	20	0	20	8	21	3
2×14	12	21	3	22	7	23	9	24	9	24	0	25	3	26	6	27	8	28	9	29	10	30	0	...		..		.....	
	16	19	6	20	9	21	9	22	9	21	0	22	3	23	4	24	4	25	3	26	4	27	3	28	1	29	0	29	10
	24	17	3	18	4	19	4	20	2	17	6	18	6	19	4	20	3	21	0	21	10	22	7	23	4	24	1	24	9

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—30 pounds per square foot of floor area with plastered ceiling, or 40 pounds per square foot with ceiling unplastered.

Table XXI (Continued). Floor-Joist Spans (30-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 30 pounds per square foot with plastered ceiling. Live load 40 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span): Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3× 6	12	10	6	11	2	11	10	12	4	13	0	13	8	14	4	15	0	15	7	16	2	16	9	17	4	17	10	18	4
	16	9	7	10	2	10	10	11	4	11	4	12	0	12	6	13	1	13	8	14	2	14	8	15	2	15	8	16	1
	24	8	6	9	0	9	6	10	0	9	4	9	11	10	4	10	10	11	3	11	7	12	1	12	6	12	11	13	3
3× 8	12	13	11	14	10	15	7	16	4	17	1	17	11	18	10	19	8	20	6	21	3	22	0	22	9	23	5	24	2
	16	12	10	13	8	14	4	14	11	15	0	15	9	16	7	17	4	18	0	18	8	19	4	20	0	20	7	21	2
	24	11	3	12	0	12	8	13	2	12	5	13	1	13	9	14	4	14	11	15	6	16	0	16	7	17	1	17	6
3×10	12	17	6	18	7	19	6	20	5	21	3	22	5	23	6	24	7	25	7	26	7	27	6	28	5	29	3	30	0
	16	16	0	17	0	17	11	18	9	18	9	19	9	20	9	21	8	22	7	23	5	24	3	25	0	25	9	26	6
	24	13	2	15	1	15	11	16	8	15	7	16	5	17	3	18	0	18	9	19	5	20	2	20	10	21	5	22	1
3×12	12	20	11	22	3	23	5	24	5	25	5	26	9	28	1	29	4	30	0			...		...		...		...	
	16	19	4	20	6	21	7	22	7	22	5	23	8	24	10	25	11	27	0	28	0	29	0	30	0	...		...	
	24	17	1	18	2	19	2	19	11	18	9	19	9	20	9	21	8	22	6	23	4	24	2	25	0	25	9	26	6
3×14	12	23	4	25	11	27	3	28	6	29	5	30	0			...				...				...		...		...	
	16	22	6	23	10	25	2	26	4	26	1	27	6	28	10	30	0	...		...		...		...		...		...	
	24	20	0	21	3	22	4	23	4	21	8	23	0	24	3	25	2	26	3	27	3	28	2	29	1	30	0	...	

Note: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot)
Double thickness of flooring (5 pounds per square foot)

Live load—30 pounds per square foot of floor area with plastered ceiling, or 40 pounds per square foot with ceiling unplastered.

Table XXII. Floor-Joist Spans (40-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 40 pounds per square foot with plastered ceiling Live load 50 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	8	6	9	1	9	6	10	0	9	6	10	0	10	5	10	11	11	4	11	10	12	2	12	7	13	0	13	5
	16	7	9	8	3	8	8	9	1	8	3	8	8	9	1	9	6	9	10	10	3	10	8	11	0	11	4	11	7
	24	6	10	7	3	7	7	8	0	6	9	7	1	7	6	7	10	8	1	8	5	8	9	9	0	9	3	9	6
2×8	12	11	5	12	0	12	8	13	3	12	6	13	2	13	10	14	5	15	0	15	7	16	1	16	8	17	2	17	8
	16	10	5	11	0	11	7	12	1	10	11	11	6	12	0	12	7	13	1	13	8	14	1	14	6	15	0	15	5
	24	9	2	9	8	10	2	10	8	8	11	9	5	9	11	10	4	10	9	11	2	11	7	12	0	12	4	12	8
2×10	12	14	4	15	2	16	0	16	8	15	9	16	7	17	4	18	2	18	11	19	7	20	3	21	0	21	8	22	3
	16	13	1	13	10	14	7	15	3	13	9	14	6	15	2	15	10	16	6	17	1	17	9	18	4	18	10	19	5
	24	11	6	12	2	12	10	13	5	11	4	11	11	12	6	13	1	13	8	14	1	14	7	15	1	15	7	16	0
2×12	12	17	3	18	3	19	3	20	1	18	11	19	11	20	11	21	11	22	8	23	6	24	5	25	2	26	0	26	9
	16	15	9	16	9	17	7	18	5	16	6	17	5	18	3	19	1	19	11	20	7	21	4	22	0	22	9	23	5
	24	13	10	14	9	15	6	16	2	13	8	14	4	15	1	15	9	16	5	17	0	17	7	18	2	18	9	19	4
2×14	12	20	0	21	2	22	6	23	5	21	11	23	2	24	3	25	4	26	4	27	4	28	4	29	4	30	0		
	16	18	4	19	6	20	6	21	5	19	3	20	3	21	3	22	3	23	2	24	0	24	10	25	8	26	6	27	3
	24	16	3	17	3	18	1	18	11	15	11	16	9	17	7	18	5	19	2	19	10	20	7	21	3	21	11	22	6

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot)
Double thickness of flooring (5 pounds per square foot).
Live load—40 pounds per square foot of floor area with plastered ceiling, or 50 pounds per square foot with ceiling unplastered.

Table XXII (Continued). Floor-Joist Spans (40-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 40 pounds per square foot with plastered ceiling. Live load 50 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending.** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	9	11	10	6	11	2	11	8	11	10	12	5	13	1	13	8	14	3	14	9	15	3	15	9	16	3	16	9
	16	9	1	9	8	10	2	10	8	10	4	10	11	11	5	11	11	12	5	12	10	13	4	13	9	14	3	14	9
	24	7	11	8	5	8	11	9	4	8	6	8	11	9	5	9	9	10	3	10	8	11	0	11	4	11	9	12	0
3×8	12	13	1	13	11	14	8	15	4	15	7	16	5	17	3	18	0	18	9	19	5	20	1	20	9	21	5	22	0
	16	12	0	12	9	13	5	14	0	13	8	14	5	15	1	15	9	16	5	17	0	17	8	18	3	18	9	19	4
	24	10	8	11	3	11	10	12	4	11	3	11	10	12	5	13	0	13	7	14	1	14	7	15	0	15	6	15	11
3×10	12	16	5	17	6	18	5	19	3	19	6	20	7	21	7	22	6	23	3	24	4	25	2	26	0	26	9	27	7
	16	15	1	16	1	16	11	17	8	17	2	18	1	18	11	19	9	20	8	21	4	22	2	22	10	23	7	24	3
	24	13	4	14	2	14	11	15	7	14	3	15	0	15	9	16	5	17	1	17	9	18	4	18	11	19	6	20	1
3×12	12	19	9	20	11	22	1	23	1	23	4	24	7	25	9	26	11	28	0	29	1	30	0	...	...	...	...	...	...
	16	18	2	19	4	20	4	21	3	20	6	21	8	22	9	23	9	24	8	25	8	26	6	27	5	28	3	29	1
	24	16	1	17	1	18	0	18	9	17	1	18	0	18	10	19	9	20	6	21	4	22	1	22	9	23	6	24	2
3×14	12	23	1	24	5	25	9	26	11	27	0	28	6	30	0	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	16	21	3	22	6	23	9	24	10	23	11	25	2	26	5	27	7	28	9	29	9	30	0	...	...	...	...	...	...
	24	18	9	20	0	21	0	22	1	19	11	21	0	22	0	23	0	23	11	24	10	25	9	26	7	27	5	28	2

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—40 pounds per square foot of floor area with plastered ceiling, or 50 pounds per square foot with ceiling unplastered.

Table XXIII. Floor-Joist Spans (50-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 50 pounds per square foot with plastered ceiling Live load 60 pounds per square foot with unplastered ceiling

Maximum allowable lengths between supports (clear span)

Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span

Size of joists (nominal) in inches	Spacing of joists center to center in inches	E = 1 000 000		E = 1 200 000		E = 1 400 000		E = 1 600 000	
		Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	8	1	8	7	9	0	9	6
	16	7	4	7	10	8	3	8	7
	24	6	6	6	10	7	3	7	7
2×8	12	10	9	11	5	12	0	12	7
	16	9	9	10	5	11	0	11	6
	24	8	7	9	2	9	8	10	1
2×10	12	13	7	14	5	15	2	15	10
	16	12	5	13	2	13	10	14	7
	24	10	0	11	7	12	2	12	9
2×12	12	16	4	17	5	18	3	19	1
	16	14	11	15	10	16	9	17	6
	24	13	1	14	0	14	9	15	5
2×14	12	19	1	20	3	21	4	22	4
	16	17	6	18	6	19	6	20	5
	24	15	5	16	4	17	3	18	0

Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span

Size of joists (nominal) in inches	Spacing of joists center to center in inches	f = 900		f = 1 000		f = 1 100		f = 1 200		f = 1 300		f = 1 400		f = 1 500		f = 1 600		f = 1 700		f = 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	8	9	9	3	9	8	10	0	10	6	10	11	11	3	11	7	11	11	12	4
	16	7	8	8	0	8	4	8	9	9	1	9	5	9	9	10	1	10	5	10	9
	24	6	3	6	7	6	10	7	3	7	6	7	10	8	1	8	4	8	6	8	10
2×8	12	11	7	12	2	12	9	13	4	13	10	14	5	14	11	15	5	15	10	16	3
	16	10	1	10	8	11	2	11	8	12	2	12	7	13	0	13	5	13	10	14	3
	24	8	3	8	9	9	2	9	6	9	11	10	4	10	8	11	0	11	4	11	8
2×10	12	14	7	15	4	16	1	16	10	17	6	18	2	18	9	19	5	19	11	20	7
	16	12	8	13	4	14	0	14	8	15	3	15	10	16	5	16	11	17	6	18	0
	24	10	5	11	0	11	6	12	1	12	7	13	1	13	6	13	11	14	4	14	9
2×12	12	17	6	18	5	19	4	20	2	21	0	21	10	22	7	23	4	24	0	24	9
	16	15	3	16	2	16	11	17	8	18	5	19	1	19	9	20	5	21	0	21	8
	24	12	7	13	4	13	11	14	7	15	2	15	9	16	3	16	10	17	4	17	10
2×14	12	20	4	21	5	22	6	23	6	24	6	25	4	26	3	27	1	28	0	28	9
	16	17	10	18	9	19	8	20	7	21	5	22	3	23	0	23	9	24	6	25	3
	24	14	8	15	6	16	3	17	0	17	8	18	4	19	0	19	8	20	3	20	10

Note: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—50 pounds per square foot of floor area with plastered ceiling, or 60 pounds per square foot with ceiling unplastered.

Table XXIII (Continued). Floor-Joist Spans (50-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 50 pounds per square foot with plastered ceiling. Live load 60 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	9	5	9	8	10	6	11	0	10	11	11	6	12	1	12	7	13	2	13	8	14	1	14	7	15	0	15	6
	16	8	7	9	1	9	7	10	0	9	6	10	0	10	6	11	0	11	6	11	11	12	3	12	8	13	1	13	5
	24	7	6	8	0	8	5	8	10	7	10	8	3	8	8	9	0	9	5	9	10	10	1	10	5	10	9	11	1
3×8	12	12	6	13	3	14	0	14	7	14	5	15	2	16	0	16	8	17	4	18	0	18	8	19	3	19	10	20	5
	16	11	5	12	1	12	9	13	4	12	7	13	4	14	0	14	6	15	2	15	9	16	4	16	10	17	4	17	10
	24	10	1	10	8	11	3	11	9	10	5	11	0	11	6	12	0	12	6	13	0	13	6	13	11	14	4	14	9
3×10	12	15	9	16	8	17	7	18	4	18	1	19	1	20	0	20	11	21	9	22	7	23	4	24	2	24	9	25	7
	16	14	5	15	3	16	1	16	10	15	11	16	9	17	6	18	4	19	1	19	10	20	6	21	2	21	10	22	5
	24	12	8	13	5	14	2	14	10	13	1	13	10	14	6	15	2	15	9	16	4	16	11	17	6	18	1	18	7
3×12	12	18	10	20	0	21	1	22	0	21	8	22	10	24	0	25	0	26	0	27	0	28	0	29	0	29	10	30	0
	16	17	4	18	4	19	4	20	3	19	0	20	1	21	1	22	0	22	11	23	10	24	8	25	5	26	2	27	0
	24	15	3	16	3	17	1	17	10	15	10	16	8	17	6	18	3	19	0	19	9	20	5	21	1	21	9	22	4
3×14	12	21	11	23	5	24	7	25	8	25	2	26	7	27	10	29	1	30	0	.....		.....		.....		.....		.....	
	16	20	2	21	5	22	7	23	8	22	2	23	5	24	6	25	8	26	9	27	8	28	8	29	8	30	0	.....	
	24	17	10	18	11	20	0	20	10	18	5	19	5	20	5	21	4	22	2	23	0	23	10	24	8	25	4	26	1

Note: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—50 pounds per square foot of floor area with plastered ceiling, or 60 pounds per square foot with ceiling unplastered.

Table XXIV. Floor-Joist Spans (60-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 60 pounds per square foot with plastered ceiling. Live load 70 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending**. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	7	9	8	2	8	8	9	0	8	1	8	6	9	0	9	5	9	9	10	2	10	6	10	10	11	2	11	6
	16	7	0	7	6	7	11	8	3	7	1	7	5	7	10	8	1	8	6	8	10	9	1	9	5	9	9	10	0
	24	6	2	6	6	6	11	7	2	5	9	6	1	6	5	6	8	6	11	7	2	7	6	7	8	7	11	8	2
2×8	12	10	2	10	11	11	6	12	0	10	9	11	4	11	11	12	5	13	0	13	6	13	11	14	5	14	10	15	3
	16	9	4	9	11	10	6	11	0	9	5	9	11	10	4	10	10	11	3	11	8	12	1	12	6	12	11	13	3
	24	8	2	8	9	9	2	9	7	7	8	8	1	8	6	8	11	9	3	9	7	9	11	10	3	10	6	10	11
2×10	12	13	0	13	9	14	6	15	2	13	7	14	4	15	0	15	8	16	4	16	11	17	6	18	1	18	8	19	2
	16	11	10	12	7	13	3	13	10	11	10	12	6	13	1	13	8	14	3	14	9	15	3	15	9	16	3	16	9
	24	10	5	11	0	11	8	12	1	9	9	10	3	10	9	11	3	11	9	12	2	12	7	13	0	13	4	13	9
2×12	12	15	8	16	8	17	6	18	3	16	4	17	3	18	1	18	11	19	8	20	5	21	1	21	10	22	6	23	2
	16	14	4	15	2	16	0	16	8	14	3	15	1	15	9	16	6	17	2	17	9	18	5	19	0	19	8	20	2
	24	12	6	13	4	14	0	14	8	11	9	12	4	13	0	13	7	14	2	14	8	15	2	15	8	16	2	16	7
2×14	12	18	3	19	5	20	5	21	4	19	1	20	1	21	1	22	0	22	11	23	10	24	7	25	5	26	3	26	11
	16	16	7	17	9	18	7	19	6	16	7	17	6	18	5	19	3	20	0	20	10	21	6	22	3	22	11	23	7
	24	14	9	15	7	16	5	17	3	13	9	14	6	15	3	15	11	16	6	17	1	17	9	18	4	18	11	19	5

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—60 pounds per square foot of floor area with plastered ceiling, or 70 pounds per square foot with ceiling unplastered.

Table XXIV (Continued). Floor-Joist Spans (60-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 60 pounds per square foot with plastered ceiling. Live load 70 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	9	0	9	7	10	1	10	6	10	3	10	10	11	4	11	10	12	4	12	10	13	3	13	7	14	0	14	6
	16	8	3	8	9	9	3	9	7	8	11	9	5	9	10	10	4	10	9	11	1	11	6	11	11	12	3	12	7
	24	7	3	7	7	8	1	8	5	7	4	7	9	8	1	8	5	8	10	9	1	9	5	9	9	10	0	10	4
3×8	12	11	11	12	9	13	4	14	0	13	6	14	3	14	11	15	7	16	3	16	11	17	5	18	0	18	7	19	1
	16	10	11	11	7	12	3	12	10	11	10	12	5	13	0	13	7	14	3	14	9	15	3	15	9	16	3	16	9
	24	9	7	10	1	10	9	11	3	9	9	10	3	10	9	11	3	11	9	12	1	12	6	13	0	13	4	13	9
3×10	12	15	0	15	11	16	10	17	7	17	0	17	11	18	10	19	7	20	5	21	3	21	11	22	7	23	4	24	0
	16	13	9	14	8	15	5	16	1	14	10	15	9	16	5	17	1	17	11	18	6	19	3	19	10	20	5	21	0
	24	12	1	12	11	13	6	14	1	12	3	12	11	13	7	14	1	14	9	15	4	15	10	16	4	16	11	17	4
3×12	12	18	0	19	3	20	3	21	1	20	4	21	5	22	6	23	6	24	6	25	5	26	4	27	1	28	0	28	10
	16	16	7	17	7	18	6	19	4	17	11	18	10	19	9	20	7	21	5	22	4	23	0	23	10	24	6	25	4
	24	14	7	15	6	16	4	17	0	14	10	15	7	16	4	17	1	17	9	18	5	19	1	19	9	20	4	20	11

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—60 pounds per square foot of floor area with plastered ceiling, or 70 pounds per square foot with ceiling unplastered.

Table XXIV (Continued). Floor-Joist Spans (60-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 60 pounds per square foot with plastered ceiling. Live load 70 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	21	1	22	5	23	7	24	7	23	9	25	0	26	3	27	4	28	6	29	6	30	0	......		......		......	
	16	19	5	20	6	21	7	22	7	20	10	21	11	23	0	24	0	25	0	26	0	26	11	27	10	28	7	29	5
	24	17	0	18	1	19	1	20	0	17	4	18	5	19	1	19	11	20	10	21	6	22	4	23	0	23	9	24	5
4×6	12	9	11	10	7	11	1	11	7	11	11	12	6	13	3	13	9	14	4	14	11	15	5	15	11	16	5	16	11
	16	9	1	9	9	10	3	10	9	10	5	11	0	11	6	12	0	12	6	13	0	13	6	13	11	14	4	14	9
	24	8	0	8	6	9	0	9	4	8	6	9	0	9	6	9	11	10	4	10	9	11	0	11	5	11	10	12	1
4×8	12	13	3	14	0	14	10	15	5	15	9	16	7	17	5	18	1	18	11	19	7	20	4	21	0	21	7	22	3
	16	12	1	12	11	13	6	14	1	13	9	14	6	15	3	15	11	16	6	17	1	17	9	18	5	18	11	19	6
	24	10	7	11	4	11	10	12	5	11	5	12	0	12	7	13	1	13	7	14	3	14	9	15	3	15	7	16	1
4×10	12	16	7	17	7	18	6	19	5	19	7	20	9	21	9	22	9	23	7	24	6	25	5	26	3	27	0	27	10
	16	15	3	16	1	17	0	17	10	17	4	18	3	19	1	19	11	20	9	21	6	22	4	23	0	23	9	24	5
	24	13	5	14	3	15	0	15	9	14	4	15	1	15	10	16	6	17	3	17	10	18	6	19	1	19	9	20	3

Note: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—60 pounds per square foot of floor area with plastered ceiling, or 70 pounds per square foot with ceiling unplastered.

Table XXV. Floor-Joist Spans (70-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 70 pounds per square foot with plastered ceiling. Live load 80 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 6	12	7	5	7	9	8	2	8	7	7	8	8	1	8	6	8	9	9	2	9	6	9	10	10	2	10	6	10	9
	16	6	8	7	1	7	6	7	10	6	8	7	0	7	5	7	8	8	0	8	4	8	7	8	10	9	2	9	5
	24	5	10	6	2	6	7	6	10	5	6	5	9	6	0	6	5	6	7	6	10	7	1	7	4	7	6	7	9
2× 8	12	9	9	10	5	11	0	11	6	10	1	10	8	11	2	11	8	12	2	12	7	13	2	13	7	13	10	14	5
	16	8	10	9	6	10	0	10	6	8	9	9	4	9	9	10	2	10	7	11	0	11	5	11	9	12	2	12	6
	24	7	9	8	4	8	9	9	2	7	4	7	8	8	0	8	5	8	8	9	1	9	5	9	8	10	0	10	2
2×10	12	12	5	13	1	13	10	14	6	12	9	13	6	14	2	14	9	15	5	16	0	16	6	17	1	17	7	18	1
	16	11	4	12	0	12	8	13	2	11	2	11	9	12	4	12	10	13	5	13	10	14	5	14	10	15	4	15	9
	24	9	10	10	6	11	1	11	7	9	2	9	7	10	1	10	7	11	0	11	5	11	9	12	2	12	7	12	10
2×12	12	15	0	15	10	16	8	17	6	15	5	16	4	17	0	17	9	18	6	19	2	19	10	20	7	21	2	21	9
	16	13	8	14	6	15	4	16	0	13	5	14	2	14	10	15	6	16	2	16	8	17	5	17	10	18	6	19	0
	24	12	0	12	8	13	5	14	0	11	1	11	8	12	2	12	9	13	4	13	9	14	4	14	9	15	2	15	7
2×14	12	17	6	18	7	19	7	20	6	18	0	19	0	19	11	20	10	21	7	22	5	23	3	24	0	24	9	25	5
	16	16	1	17	0	17	11	18	10	15	9	16	7	17	5	18	3	18	11	19	7	20	4	21	0	21	7	22	4
	24	14	1	15	0	15	10	16	6	13	0	13	7	14	4	15	0	15	7	16	3	16	9	17	4	17	10	18	4

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—70 pounds per square foot of floor area with plastered ceiling, or 80 pounds per square foot with ceiling unplastered.

Table XXV (Continued). Floor-Joist Spans (70-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 70 pounds per square foot with plastered ceiling. Live load 80 pounds per square foot with unplastered ceiling

Maximum allowable lengths between supports (clear span)

Limited by deflection of 1/360 of the span

Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span

Size of joists (nominal) in inches	Spacing of joists center to center in inches	$E =$ 1 000 000 Ft	In	$E =$ 1 200 000 Ft	In	$E =$ 1 400 000 Ft	In	$E =$ 1 600 000 Ft	In
3× 6	12	8	7	9	3	9	9	10	1
	16	7	11	8	5	8	10	9	3
	24	6	11	7	5	7	9	8	1
3× 8	12	11	6	12	3	12	10	13	5
	16	10	6	11	1	11	9	12	3
	24	9	3	9	10	10	4	10	10
3×10	12	14	6	15	5	16	3	16	11
	16	13	3	14	0	14	10	15	5
	24	11	7	12	4	13	0	13	7
3×12	12	17	5	18	6	19	5	20	5
	16	15	11	16	11	17	10	18	7
	24	14	0	14	11	15	9	16	5

Determined by bending

Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span

Size of joists (nominal) in inches	Spacing of joists center to center in inches	$f =$ 900 Ft	In	$f =$ 1 000 Ft	In	$f =$ 1 100 Ft	In	$f =$ 1 200 Ft	In	$f =$ 1 300 Ft	In	$f =$ 1 400 Ft	In	$f =$ 1 500 Ft	In	$f =$ 1 600 Ft	In	$f =$ 1 700 Ft	In	$f =$ 1 800 Ft	In
3× 6	12	9	7	10	1	10	9	11	1	11	7	12	0	12	5	12	11	13	4	13	7
	16	8	5	8	11	9	4	9	9	10	1	10	6	10	11	11	3	11	7	11	11
	24	6	11	7	4	7	7	8	0	8	4	8	7	8	11	9	3	9	6	9	9
3× 8	12	12	10	13	6	14	1	14	9	15	5	15	11	16	6	17	0	17	6	18	0
	16	11	1	11	9	12	4	12	10	13	6	13	11	14	5	14	10	15	4	15	9
	24	9	3	9	7	10	1	10	7	11	1	11	5	11	10	12	3	12	7	13	0
3×10	12	16	0	16	11	17	9	18	6	19	4	20	0	20	9	21	5	22	0	22	9
	16	14	0	14	10	15	6	16	3	16	11	17	6	18	1	18	9	19	4	19	10
	24	11	7	12	3	12	10	13	4	13	11	14	5	14	11	15	5	15	11	16	4
3×12	12	19	3	20	3	21	4	22	3	23	1	24	0	24	10	25	9	26	5	27	3
	16	16	11	17	10	18	9	19	6	20	4	21	0	21	10	22	6	23	3	23	11
	24	13	11	14	9	15	5	16	1	16	9	17	5	18	0	18	7	19	3	19	9

Note: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—70 pounds per square foot of floor area with plastered ceiling, or 80 pounds per square foot with ceiling unplastered.

Table XXV (Continued). Floor-Joist Spans (70-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 70 pounds per square foot with plastered ceiling. Live load 80 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	20	4	21	7	22	7	23	9	22	5	23	7	24	10	25	11	26	11	27	11	28	11	29	11	30	0	...	
	16	18	7	19	10	20	11	21	10	19	9	20	9	21	9	22	9	23	7	24	6	25	5	26	3	27	0	27	10
	24	16	5	17	6	18	5	19	3	16	4	17	3	18	0	18	10	19	7	20	4	21	0	21	9	22	5	23	0
4× 6	12	9	7	10	3	10	9	11	3	11	3	11	11	12	5	13	0	13	6	14	0	14	6	15	0	15	6	15	11
	16	8	9	9	4	9	10	10	3	9	10	10	4	10	11	11	4	11	10	12	3	12	9	13	1	13	6	13	11
	24	7	9	8	1	8	7	9	0	8	1	8	6	8	11	9	4	9	9	10	1	10	5	10	10	11	1	11	5
4× 8	12	12	9	13	6	14	3	14	10	14	10	15	9	16	5	17	1	17	10	18	6	19	3	19	10	20	5	21	0
	16	11	7	12	4	13	0	13	7	13	0	13	9	14	5	15	0	15	7	16	3	16	10	17	4	17	11	18	5
	24	10	3	10	10	11	5	12	0	10	9	11	4	11	10	12	5	12	11	13	5	13	10	14	4	14	9	15	3
4×10	12	15	11	16	11	17	10	18	7	18	7	19	7	20	7	21	6	22	4	23	3	24	0	24	10	25	7	26	4
	16	14	7	15	6	16	5	17	1	16	4	17	3	18	1	18	11	19	7	20	5	21	1	21	10	22	5	23	1
	24	12	11	13	9	14	5	15	1	13	6	14	3	14	11	15	7	16	3	16	10	17	5	18	0	18	6	19	1

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot)
Double thickness of flooring (5 pounds per square foot).
Live load—70 pounds per square foot of floor area with plastered ceiling, or 80 pounds per square foot with ceiling unplastered.

Table XXVI. Floor-Joist Spans (80-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 80 pounds per square foot with plastered ceiling. Live load 90 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXVII.)** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXVII and in the same manner determine the span of the joist limited by **horizontal shear** and use the **shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 6	12	7	1	7	7	8	0	8	4	7	2	7	7	8	0	8	4	8	8	9	0	9	4	9	8	10	0	10	2
	16	6	6	6	10	7	2	7	7	6	4	6	7	6	10	7	2	7	6	7	9	8	1	8	5	8	7	8	9
2× 8	12	9	6	10	1	10	7	11	1	9	7	10	1	10	7	11	1	11	6	12	0	12	5	12	9	13	2	13	7
	16	8	7	9	3	9	7	10	1	8	4	8	9	9	2	9	8	10	1	10	5	10	9	11	1	11	6	11	9
	24	7	6	8	0	8	5	8	9	6	9	7	2	7	7	7	10	8	2	8	6	8	9	9	1	9	5	9	8
2×10	14	12	0	12	8	13	5	14	0	12	1	12	9	13	5	14	0	14	7	15	1	15	7	16	2	16	8	17	1
	14	10	10	11	7	12	2	12	9	10	6	11	1	11	8	12	2	12	8	13	2	13	7	14	1	14	6	14	10
	24	9	7	10	2	10	8	11	2	8	7	9	1	9	6	10	0	10	5	10	9	11	2	11	6	11	10	12	2
2×12	12	14	5	15	5	16	2	16	10	14	7	15	5	16	1	16	10	17	7	18	2	18	10	19	6	20	1	20	8
	16	13	2	14	0	14	9	15	5	12	8	13	5	14	1	14	8	15	4	15	10	16	5	17	0	17	6	18	0
	24	11	7	12	4	12	10	13	6	10	5	11	0	11	7	12	1	12	7	13	1	13	6	14	0	14	5	14	9
2×14	12	16	11	18	0	19	0	19	10	17	1	18	0	18	11	19	9	20	6	21	4	22	0	22	10	23	6	24	3
	16	15	5	16	5	17	4	18	1	14	11	15	9	16	6	17	3	17	11	18	7	19	3	19	10	20	6	21	1
	24	13	7	14	5	15	3	15	11	12	4	12	11	13	7	14	1	14	10	15	4	15	10	16	5	16	11	17	4

NOTE: The lengths are based on

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—80 pounds per square foot of floor area with plastered ceiling, or 90 pounds per square foot with ceiling unplastered.

Table XXVI (Continued). Floor-Joist Spans (80-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 80 pounds per square foot with plastered ceiling. Live load 90 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXVII.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. Then refer to Table XXVII and in the same manner determine the span of the joist limited by **horizontal shear** and use the shorter of the two spans																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	8	4	8	11	9	4	9	9	9	1	9	7	10	1	10	7	11	0	11	5	11	10	12	3	12	7	12	11
	16	7	7	8	1	8	6	8	11	7	11	8	5	8	10	9	3	9	7	9	11	10	4	10	7	11	0	11	4
	24	6	9	7	1	7	6	7	10	6	7	6	11	7	3	7	6	7	11	8	3	8	5	8	9	9	0	9	4
3×8	12	11	1	11	10	12	5	12	11	12	1	12	10	13	5	14	0	14	9	15	1	15	7	16	3	16	7	17	1
	16	10	1	10	10	11	4	11	10	10	7	11	1	11	9	12	3	12	10	13	3	13	7	14	1	14	6	14	11
	24	8	11	9	5	9	11	10	5	8	9	9	3	9	7	10	0	10	6	10	10	11	3	11	7	11	11	12	4
3×10	12	14	0	14	10	15	7	16	4	15	3	16	1	16	10	17	7	18	4	19	0	19	7	20	4	20	11	21	6
	16	12	10	13	6	14	4	14	11	13	4	14	0	14	9	15	5	16	0	16	7	17	3	17	9	18	4	18	10
	24	11	3	11	11	12	6	13	1	11	0	11	6	12	1	12	7	13	3	13	9	14	1	14	7	15	1	15	6
3×12	12	16	10	17	10	18	10	19	9	18	4	19	4	20	3	21	1	22	0	22	10	23	7	24	5	25	1	25	11
	16	15	5	16	4	17	3	18	0	16	0	16	11	17	9	18	6	19	3	20	0	20	9	21	5	22	0	22	7
	24	13	6	14	5	15	1	15	10	13	3	13	11	14	7	15	4	15	11	16	6	17	1	17	7	18	3	18	9

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—80 pounds per square foot of floor area with plastered ceiling, or 90 pounds per square foot with ceiling unplastered.

Table XXVI (Continued). Floor-Joist Spans (80-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 80 pounds per square foot with plastered ceiling   Live load 90 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXVII.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXVII and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	19	7	20	10	21	11	23	0	21	4	22	6	23	7	24	7	25	7	26	7	27	6	28	5	29	4	30	0
	16	18	0	19	1	20	1	21	1	18	9	19	9	20	9	21	7	22	6	23	4	24	1	24	11	25	9	26	5
	24	15	10	16	10	17	9	18	6	15	6	16	4	17	1	17	10	18	7	19	4	19	11	20	7	21	3	21	11
4× 6	12	9	4	9	10	10	5	10	10	10	9	11	4	11	10	12	4	12	10	13	4	13	10	14	3	14	9	15	1
	16	8	6	9	0	9	5	9	11	9	4	9	10	10	4	10	10	11	3	11	7	12	0	12	5	12	10	13	3
	24	7	5	7	11	8	4	8	9	7	9	8	1	8	6	8	10	9	3	9	6	9	11	10	3	10	6	10	10
4× 8	12	12	3	13	0	13	9	14	4	14	1	14	11	15	7	16	4	16	11	17	7	18	3	18	10	19	5	19	11
	16	11	3	11	11	12	6	13	1	12	4	13	0	13	7	14	3	14	10	15	5	15	11	16	5	16	11	17	5
	24	9	10	10	6	11	0	11	6	10	1	10	9	11	3	11	9	12	3	12	9	13	1	13	7	14	0	14	5
4×10	12	15	5	16	5	17	4	18	0	17	9	18	7	19	6	20	5	21	4	22	0	22	10	23	7	24	4	25	0
	16	14	1	15	0	15	10	16	6	15	6	16	4	17	1	17	11	18	7	19	4	20	0	20	9	21	4	21	11
	24	12	5	13	3	13	11	14	6	12	10	13	6	14	1	14	10	15	5	16	0	16	6	17	1	17	7	18	1

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load— 80 pounds per square foot of floor area with plastered ceiling, or 90 pounds per square foot with ceiling unplastered.

Table XXVII. Floor-Joist Spans (80-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 80 pounds per square foot with plastered ceiling. Live load 90 pounds per square foot with unplastered ceiling

Maximum allowable lengths between supports (clear span)

Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXVI.)

Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXVI and in the same manner determine the span of the joist limited by its **bending strength and use the shorter of the two spans**

Size of joists (nominal) in inches	Spacing of joists center to center in inches	$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 6	12	8	10	9	5	10	0	10	7	11	2	11	10	12	6	13	1	13	8	15	0	15	7
	16	6	7	7	1	7	6	8	0	8	6	9	0	9	5	9	10	10	5	11	4	11	9
2× 8	12	11	7	12	5	13	2	14	0	14	10	15	8	16	6	17	5	18	2	19	9	20	7
	16	8	9	9	5	10	0	10	7	11	2	11	10	12	6	13	1	13	9	15	0	15	7
	24	5	10	6	4	6	8	7	1	7	7	8	0	8	5	8	9	9	2	10	1	10	6
2×10	12	14	6	15	6	16	7	17	7	18	8	19	8	20	8	21	9	22	9	24	10	25	10
	16	11	0	11	9	12	7	13	5	14	2	14	10	15	8	16	6	17	4	18	10	19	8
	24	7	5	7	10	8	6	9	0	9	6	10	1	10	7	11	1	11	8	12	8	13	2
2×12	12	17	5	18	8	19	10	21	1	22	5	23	7	24	10	26	1	27	5	29	9	30	0
	16	13	2	14	2	15	1	16	1	17	0	18	0	18	10	19	10	20	9	22	8	23	7
	24	8	10	9	7	10	2	10	10	11	6	12	1	12	9	13	5	14	0	15	4	16	0
2×14	12	20	2	21	7	23	0	24	6	25	10	27	5	28	9	30	0						
	16	15	5	16	6	17	7	18	8	19	9	20	10	22	0	23	1	24	2	26	5	27	6
	24	10	5	11	2	11	10	12	8	13	5	14	1	14	10	15	7	16	5	17	10	18	7
3× 6	12	13	10	14	10	15	10	16	10	17	10	18	10	19	9	20	9	21	9	23	9	24	9
	16	10	6	11	4	12	0	12	9	13	6	14	4	15	1	15	9	16	7	18	1	18	9
	24	7	1	7	7	8	1	8	7	9	1	9	7	10	1	10	7	11	2	12	2	12	8
3× 8	12	18	2	19	7	20	10	22	2	23	6	24	9	26	1	27	5	28	8	30	0		
	16	13	10	14	10	15	10	16	10	17	10	18	9	19	9	20	9	21	9	23	9	24	9
	24	9	5	10	1	10	8	11	5	12	1	12	8	13	5	14	1	14	9	16	1	16	9

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—80 pounds per square foot of floor area with plastered ceiling, or 90 pounds per square foot with ceiling unplastered.

Table XXVII (Continued). Floor-Joist Spans (80-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 80 pounds per square foot with plastered ceiling. Live load 90 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists Center to Center in inches	Maximum allowable lengths between supports (clear span). Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXVI.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXVI and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×10	12	22	9	24	5	26	0	27	8	29	4	30	0										
	16	17	5	18	7	19	10	21	1	22	5	23	7	24	9	26	1	27	4	29	9	30	0
	24	11	9	12	7	13	6	14	4	15	2	16	0	16	10	17	8	18	6	20	2	21	1
3×12	12	27	2	29	1	30	0																
	16	20	9	22	4	23	9	25	4	26	9	28	4	29	8	30	0						
	24	14	2	15	2	16	2	17	2	18	2	19	2	20	2	21	4	22	4	24	4	25	4
3×14	12	30	0																				
	16	24	2	25	10	27	7	29	4	30	0												
	24	16	6	17	8	18	10	20	1	21	2	22	5	23	7	24	9	26	0	28	4	29	6
4× 6	12	18	10	20	4	21	7	23	0	24	4	25	8	27	0	28	5	29	8	30	0		
	16	14	5	15	5	16	5	17	6	18	6	19	6	20	7	21	7	22	7	24	8	25	8
	24	9	8	10	5	11	1	11	9	12	6	13	2	13	10	14	7	15	4	16	8	17	5
4× 8	12	24	8	26	6	28	2	30	0														
	16	18	10	20	2	21	7	22	10	24	4	25	7	27	0	28	4	29	8	30	0		
	24	12	9	13	8	14	8	15	7	16	6	17	5	18	4	19	2	20	2	22	0	22	10
4×10	12	30	0																				
	16	23	7	25	2	26	10	28	7	30	0												
	24	16	1	17	2	18	5	19	6	20	8	21	9	23	0	24	1	25	4	27	7	28	8

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.

Dead load—Weight of joist
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—80 pounds per square foot of floor area with plastered ceiling, or 90 pounds per square foot with ceiling unplastered.

Table XXVIII. Floor-Joist Spans (90-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 90 pounds per square foot with plastered ceiling. Live load 100 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXIX.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXIX and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 100		$f=$ 1 000		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 6	12	6	10	7	4	7	8	8	1	6	10	7	2	7	7	8	0	8	4	8	7	8	10	9	2	9	5	9	9
	16	6	4	6	8	7	0	7	4	6	0	6	4	6	7	6	10	7	2	7	6	7	8	8	0	8	2	8	5
2× 8	12	9	2	9	8	10	4	10	8	9	1	9	8	10	1	10	6	11	0	11	5	11	9	12	2	12	7	13	0
	16	8	4	8	10	9	4	9	9	8	0	8	5	8	9	9	2	9	6	9	10	10	4	10	7	10	10	11	2
2×10	12	11	7	12	4	13	0	13	7	11	6	12	2	12	9	13	4	13	10	14	5	14	10	15	5	15	10	16	4
	16	10	7	11	2	11	9	12	5	10	1	10	7	11	1	11	7	12	1	12	7	13	0	13	5	13	9	14	2
	24	9	2	9	9	10	5	10	9	8	2	8	8	9	1	9	6	9	10	10	4	10	7	11	0	11	4	11	8
2×12	12	14	0	14	10	15	8	16	5	13	10	14	8	15	5	16	1	16	8	17	5	18	0	18	7	19	1	19	8
	16	12	9	13	7	14	3	14	10	12	1	12	9	13	5	14	0	14	7	15	2	15	8	16	2	16	8	17	2
	24	11	2	11	10	12	6	13	1	10	0	10	6	11	0	11	6	12	0	12	5	12	9	13	4	13	8	14	1
2×14	12	16	5	17	6	18	5	19	3	16	4	17	3	18	0	18	10	19	7	20	4	21	0	21	9	22	5	23	0
	16	15	0	15	11	16	10	17	6	14	3	15	0	15	9	16	5	17	1	17	9	18	5	18	11	19	6	20	1
	24	13	3	14	0	14	9	15	5	11	9	12	4	12	11	13	6	14	1	14	7	15	1	15	7	16	1	16	7

Note: The lengths are based on:

When limited by deflection—
- Maximum allowable deflection of 1/360 of span length.
- Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
- Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
- Weight of lath and plaster ceiling (10 pounds per square foot)
- Double thickness of flooring (5 pounds per square foot).

Live load—90 pounds per square foot of floor area with plastered ceiling, or 100 pounds per square foot with ceiling unplastered.

Table XXVIII (Continued). Floor-Joist Spans (90-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 90 pounds per square foot with plastered ceiling. Live load 100 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXIX.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXIX and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	8	1	8	7	9	0	9	6	8	9	9	3	9	7	10	0	10	6	10	11	11	4	11	7	12	0	12	4
	16	7	5	7	10	8	3	8	7	7	7	8	0	8	5	8	10	9	1	9	6	9	10	10	1	10	5	10	9
3×8	12	10	9	11	5	12	0	12	6	11	6	12	3	12	10	13	4	14	0	14	5	14	11	15	5	15	11	16	4
	16	9	10	10	5	11	0	11	6	10	0	10	7	11	1	11	7	12	3	12	7	13	0	13	5	13	10	14	3
	24	8	7	9	1	9	7	10	0	8	3	8	9	9	3	9	6	10	0	10	4	10	9	11	0	11	5	11	9
3×10	12	13	6	14	5	15	1	15	10	14	6	15	4	16	1	16	10	17	6	18	1	18	10	19	5	20	0	20	6
	16	12	5	13	3	13	10	14	6	12	9	13	4	14	0	14	9	15	3	15	10	16	5	16	11	17	5	17	11
	24	10	11	11	6	12	1	12	9	10	5	11	0	11	6	12	0	12	7	13	0	13	6	13	11	14	5	14	10
3×12	12	16	4	17	4	18	3	19	1	17	6	18	5	19	4	20	3	21	0	21	10	22	6	23	4	24	0	24	9
	16	14	11	15	10	16	9	17	3	15	4	16	1	16	11	17	7	18	5	19	1	19	9	20	5	21	0	21	7
	24	13	1	13	11	14	7	15	4	12	7	13	4	13	11	14	7	15	1	15	9	16	3	16	10	17	4	17	10

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—90 pounds per square foot of floor area with plastered ceiling, or 100 pounds per square foot with ceiling unplastered.

Table XXVIII (Continued). Floor-Joist Spans (90-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 90 pounds per square foot with plastered ceiling. Live load 100 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXIX.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. Then refer to Table XXIX and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	19	1	20	3	21	4	22	4	20	5	21	6	22	6	23	6	24	6	25	5	26	4	27	3	28	0	28	10
	16	17	6	18	6	19	6	20	5	17	10	18	10	19	9	20	7	21	5	22	3	23	0	23	10	24	6	25	3
	24	15	5	16	4	17	3	17	11	14	9	15	6	16	4	17	0	17	9	18	5	19	0	19	7	20	3	20	10
4× 6	12	9	0	9	6	10	0	10	6	10	3	10	9	11	4	11	9	12	3	12	9	13	3	13	7	14	0	14	5
	16	8	3	8	9	9	1	9	7	8	11	9	4	9	10	10	3	10	9	11	0	11	5	11	10	12	2	12	6
	24	7	3	7	7	8	0	8	5	7	4	7	9	8	1	8	5	8	10	9	1	9	5	9	9	10	0	10	4
4× 8	12	11	11	12	7	13	4	13	11	13	6	14	3	14	11	15	7	16	3	16	10	17	5	17	11	18	6	19	0
	16	10	10	11	6	12	1	12	9	11	9	12	5	13	0	13	7	14	1	14	9	15	3	15	9	16	3	16	7
	24	9	6	10	1	10	9	11	1	9	9	10	3	10	9	11	3	11	7	12	1	12	6	12	11	13	4	13	9
4×10	12	15	0	15	11	16	9	17	6	16	11	17	10	18	9	19	6	20	4	21	1	21	10	22	6	23	3	23	11
	16	13	9	14	6	15	4	16	1	14	10	15	7	16	4	17	1	17	10	18	5	19	1	19	9	20	4	20	11
	24	12	0	12	10	13	6	14	0	12	3	12	11	13	6	14	1	14	9	15	3	15	9	16	4	16	10	17	4

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—90 pounds per square foot of floor area with plastered ceiling, or 100 pounds per square foot with ceiling unplastered.

Table XXIX. Floor-Joist Spans (90-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 90 pounds per square foot with plastered ceiling. Live load 100 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span). Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXVIII.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress.** **Then** refer to Table XXVIII and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 6	12	7	10	8	6	9	1	9	7	10	2	10	9	11	4	11	10	12	6	13	7	14	2
	16	6	0	6	5	6	9	7	4	7	8	8	1	8	6	9	0	9	5	10	4	10	8
2× 8	12	10	6	11	2	12	0	12	8	13	6	14	2	15	0	15	8	16	6	18	0	18	8
	16	7	10	8	6	9	1	9	7	10	2	10	9	11	4	11	10	12	6	13	7	14	2
2×10	12	13	2	14	1	15	1	16	0	17	0	17	10	18	9	19	9	20	8	22	7	23	6
	16	10	0	10	8	11	5	12	1	12	9	13	7	14	4	15	0	15	8	17	1	17	9
	24	6	8	7	2	7	8	8	2	8	7	9	1	9	7	10	1	10	7	11	6	12	0
2×12	12	15	9	17	0	18	1	19	2	20	5	21	6	22	7	23	9	24	10	27	1	28	4
	16	12	0	12	10	13	8	14	7	15	5	16	4	17	2	18	0	18	10	20	7	21	6
	24	8	1	8	8	9	4	9	9	10	5	11	0	11	7	12	2	12	8	13	10	14	6
2×14	12	18	5	19	8	21	0	22	4	23	7	24	10	26	2	27	6	28	9	30	0		
	16	14	0	15	0	16	0	17	0	18	0	19	0	20	0	21	0	22	0	24	0	25	0
	24	9	6	10	1	10	9	11	6	12	2	12	9	13	6	14	2	14	10	16	2	16	10
3× 6	12	12	7	13	6	14	5	15	4	16	2	17	1	18	0	18	10	19	9	21	7	22	6
	16	9	7	10	2	10	10	11	7	12	4	13	0	13	8	14	4	15	0	16	5	17	1
3× 8	12	16	7	17	9	19	0	20	2	21	4	22	6	23	8	24	10	26	1	28	6	29	7
	16	12	7	13	6	14	5	15	4	16	2	17	1	18	0	18	10	19	9	21	7	22	6
	24	8	6	9	1	9	8	10	5	11	0	11	7	12	2	12	9	13	5	14	7	15	2

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—90 pounds per square foot of floor area with plastered ceiling, or 100 pounds per square foot with ceiling unplastered.

Table XXIX (Continued). Floor-Joist Spans (90-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 90 pounds per square foot with plastered ceiling   Live load 100 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center in center in inches	Maximum allowable lengths between supports (clear span) — Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXVIII.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXVIII and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×10	12	20	8	22	2	23	8	25	2	26	8	28	1	29	7	30	0						
	16	15	9	16	10	18	1	19	2	20	4	21	6	22	7	23	8	24	9	27	1	28	2
	24	10	8	11	6	12	2	13	0	13	9	14	6	15	4	16	1	16	9	18	5	19	1
3×12	12	24	9	26	6	28	4	30	0														
	16	18	10	20	4	21	7	23	0	24	5	25	8	27	1	28	5	29	9	30	0		
	24	12	10	13	9	14	8	15	7	16	7	17	6	18	5	19	4	20	2	22	1	23	0
3×14	12	28	8	30	0																		
	16	22	0	23	7	25	2	26	8	28	4	29	10	30	0								
	24	15	0	16	1	17	2	18	2	19	4	20	5	21	6	22	6	23	7	25	8	26	9
4× 6	12	17	2	18	5	19	8	20	10	22	1	23	4	24	7	25	9	27	0	29	6	30	0
	16	13	1	14	0	14	10	15	10	16	9	17	8	18	8	19	7	20	6	22	5	23	4
	24	8	9	9	6	10	1	10	8	11	4	12	0	12	7	13	2	13	10	15	1	15	9
4× 8	12	22	6	24	1	25	8	27	4	28	10	30	0										
	16	17	2	18	5	19	7	20	10	22	1	23	4	24	6	25	9	27	0	29	5	30	0
	24	11	8	12	6	13	4	14	1	15	0	15	9	16	7	17	6	18	4	20	0	20	9
4×10	12	28	0	30	0																		
	16	21	5	23	0	24	6	26	0	27	7	29	1	30	0								
	24	14	7	15	8	16	8	17	9	18	9	19	9	20	10	21	10	23	0	25	1	26	1

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—90 pounds per square foot of floor area with plastered ceiling, or 100 pounds per square foot with ceiling unplastered.

Table XXX. Floor-Joist Spans (100-Pound Load)

MAXIMUM SPANS FOR FLOOR JOISTS—UNIFORMLY LOADED

Live load 100 pounds per square foot with plastered ceiling. Live load 110 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXI.)** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXXI and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×8	12	8	10	9	5	10	0	10	5	8	8	9	2	9	7	10	1	10	6	10	10	11	4	11	8	12	0	12	5
	16	8	1	8	7	9	1	9	6	7	7	8	0	8	5	8	9	9	1	9	6	9	9	10	1	10	5	10	9
2×10	12	11	2	12	0	12	7	13	2	11	0	11	8	12	2	12	9	13	4	13	9	14	4	14	8	15	2	15	7
	16	10	2	10	10	11	6	12	0	9	7	10	1	10	7	11	1	11	6	12	0	12	5	12	9	13	2	13	7
	24	9	0	9	6	10	0	10	6	7	10	8	4	8	8	9	1	9	5	9	9	10	1	10	6	10	9	11	1
2×12	12	13	7	14	5	15	2	15	10	13	4	14	1	14	8	15	5	16	0	16	7	17	2	17	9	18	4	18	10
	16	12	5	13	2	13	10	14	6	11	7	12	2	12	9	13	5	14	0	14	6	15	0	15	6	16	0	16	5
	24	10	10	11	6	12	2	12	8	9	6	10	1	10	6	11	0	11	5	11	10	12	4	12	8	13	1	13	6
2×14	12	16	0	16	11	17	10	18	9	15	7	16	6	17	4	18	0	18	10	19	6	20	1	20	10	21	5	22	1
	16	14	6	15	6	16	4	17	0	13	7	14	5	15	1	15	9	16	5	17	0	17	7	18	1	18	9	19	4
	24	12	10	13	7	14	4	14	11	11	3	11	10	12	5	12	10	13	6	14	0	14	6	14	11	15	5	15	10

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—100 pounds per square foot of floor area with plastered ceiling, or 110 pounds per square foot with ceiling unplastered.

Table XXX (Continued). Floor-Joist Spans (100-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 100 pounds per square foot with plastered ceiling Live load 110 pounds per square foot with unplastered ceiling

Maximum allowable lengths between supports (clear span)

Limited by deflection of 1/360 of the span
Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span

Size of joists (nominal) in inches	Spacing of joists center to center in inches	$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000	
		Ft	In	Ft	In	Ft	In	Ft	In
3× 6	12	7	11	8	4	8	10	9	3
	16	7	3	7	7	8	0	8	5
3× 8	12	10	6	11	1	11	7	12	3
	16	9	6	10	1	10	9	11	1
	24	8	4	8	11	9	4	9	9
3×10	12	13	3	14	0	14	9	15	5
	16	12	0	12	9	13	5	14	0
	24	10	6	11	3	11	10	12	4
3×12	12	15	10	16	10	17	9	18	6
	16	14	6	15	5	16	3	16	11
	24	12	9	13	6	14	3	14	11

Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXI.)
Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXXI and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**

Size of joists (nominal) in inches	Spacing of joists center to center in inches	$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3× 6	12	8	5	8	10	9	3	9	7	10	0	10	5	10	10	11	1	11	6	11	10
	16	7	3	7	9	8	0	8	5	8	9	9	1	9	5	9	9	10	0	10	3
3× 8	12	11	0	11	9	12	3	12	10	13	5	13	10	14	4	14	9	15	3	15	9
	16	9	7	10	1	10	7	11	1	11	9	12	0	12	5	12	10	13	3	13	7
	24	7	11	8	5	8	10	9	3	9	7	9	10	10	3	10	6	10	11	11	3
3×10	12	13	11	14	9	15	5	16	1	16	9	17	5	18	0	18	7	19	1	19	9
	16	12	3	12	10	13	3	14	0	14	7	15	3	15	9	16	3	16	9	17	3
	24	10	0	10	6	11	0	11	7	12	0	12	5	12	11	13	4	13	9	14	1
3×12	12	16	9	17	9	18	6	19	4	20	1	20	11	21	7	22	4	23	0	23	9
	16	14	7	15	5	16	3	16	11	17	7	18	3	18	11	19	6	20	1	20	9
	24	12	0	12	9	13	4	13	11	13	6	15	0	15	7	16	1	16	7	17	1

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—100 pounds per square foot of floor area with plastered ceiling, or 110 pounds per square foot with ceiling unplastered.

Table XXX (Continued). Floor-Joist Spans (100-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 100 pounds per square foot with plastered ceiling Live load 110 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (**Read carefully. These spans must be compared and checked with those in Table XXXI.**) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist **Then** refer to Table XXXI and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	18	6	19	9	20	9	21	9	19	6	20	7	21	7	22	7	23	6	24	5	25	3	26	1	26	11	27	7
	16	16	11	18	0	19	0	19	10	17	1	18	0	18	11	19	9	20	6	21	4	22	1	22	10	23	6	24	3
	24	14	11	15	10	16	9	17	6	14	1	14	11	15	7	16	4	17	0	17	7	18	3	18	10	19	5	19	11
4× 6	12	8	9	9	4	9	9	10	3	9	9	10	4	10	10	11	4	11	9	12	3	12	7	13	0	13	5	13	10
	16	8	0	8	5	8	11	9	4	8	6	8	11	9	5	9	10	10	3	10	7	11	0	11	4	11	9	12	0
	24	7	0	7	5	7	10	8	1	7	0	7	4	7	9	8	1	8	5	8	9	9	0	9	4	9	7	9	10
4× 8	12	11	6	12	3	12	11	13	6	12	11	13	7	14	3	14	11	15	6	16	1	16	7	17	3	17	9	18	3
	16	10	6	11	3	11	10	12	4	11	4	11	11	12	5	13	0	13	6	14	0	14	6	15	0	15	6	15	11
	24	9	4	9	10	10	5	10	10	9	4	9	9	10	3	10	9	11	1	11	7	12	0	12	5	12	9	13	1
4×10	12	14	6	15	5	16	4	17	0	16	3	17	1	17	11	13	9	19	6	20	3	20	11	21	7	22	4	22	11
	16	13	4	14	1	14	11	15	7	14	3	14	11	15	9	16	5	17	0	17	9	18	4	18	11	19	6	20	0
	24	11	9	12	5	13	1	13	9	11	9	12	4	12	11	13	6	14	0	14	7	15	1	15	7	16	1	16	6

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—100 pounds per square foot of floor area with plastered ceiling, or
110 pounds per square foot with ceiling unplastered.

Table XXXI. Floor-Joist Spans (100-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 100 pounds per square foot with plastered ceiling. Live load 110 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																					
		Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXX.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXX and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 8	12	9	7	10	4	11	0	11	8	12	5	13	0	13	8	14	5	15	1	16	6	17	2
	16	7	4	7	9	8	4	8	9	9	4	9	10	10	5	10	10	11	5	12	5	13	0
2×10	12	12	1	12	10	13	9	14	8	15	6	16	5	17	4	18	1	19	0	20	8	21	7
	16	9	1	9	9	10	6	11	1	11	9	12	5	13	1	13	8	14	5	15	8	16	4
	24	6	2	6	7	7	0	7	6	7	10	8	4	8	9	9	2	9	8	10	6	11	0
2×12	12	14	6	15	7	16	7	17	7	18	8	19	8	20	8	21	9	22	9	24	10	25	10
	16	11	0	11	9	12	7	13	5	14	2	14	10	15	8	16	6	17	4	18	10	19	8
	24	7	5	7	10	8	6	9	0	9	6	10	1	10	7	11	1	11	8	12	8	13	2
2×14	12	16	9	18	1	19	4	20	6	21	8	22	10	24	1	25	4	26	6	28	10	30	0
	16	12	9	13	8	14	7	15	7	16	6	17	5	18	4	19	2	20	1	22	0	22	10
	24	8	8	9	4	9	10	10	6	11	1	11	8	12	5	13	0	13	7	14	9	15	6
3× 6	12	11	7	12	5	13	2	14	0	14	10	15	8	16	6	17	4	18	2	19	9	20	8
	16	8	9	9	5	10	0	10	7	11	4	11	10	12	6	13	1	13	9	15	0	15	7
3× 8	12	15	2	16	4	17	5	18	6	19	7	20	8	21	8	22	9	23	10	26	1	27	2
	16	11	7	12	5	13	2	14	0	14	10	15	8	16	6	17	4	18	2	19	9	20	7
	24	7	9	8	5	8	10	9	6	10	0	10	7	11	1	11	8	12	4	13	5	13	10
3×10	12	19	0	20	5	21	9	23	1	24	6	25	9	27	2	28	7	29	10	30	0		
	16	14	6	15	6	16	7	17	7	18	7	19	8	20	8	21	9	22	9	24	10	25	10
	24	9	9	10	6	11	2	11	10	12	7	13	4	14	0	14	8	15	5	16	9	17	6

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.
Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—100 pounds per square foot of floor area with plastered ceiling, or 110 pounds per square foot with ceiling unplastered.

Table XXXI (Continued). Floor-Joist Spans (100-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 100 pounds per square foot with plastered ceiling Live load 110 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																					
		Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXX.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXX and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×12	12	22	9	24	5	26	0	27	7	29	4	30	0										
	16	17	5	18	7	19	10	21	7	22	5	23	7	24	9	26	1	27	4	29	9	31	1
	24	11	9	12	7	13	6	14	4	15	2	16	0	16	10	17	8	18	6	20	2	21	1
3×14	12	26	5	28	4	30	0																
	16	20	2	21	8	23	1	24	7	26	0	27	5	28	10	30	0						
	24	13	9	14	9	15	8	16	8	17	8	18	8	19	8	20	8	21	7	23	7	24	7
4× 6	12	15	9	16	10	18	0	19	2	20	4	21	5	22	6	23	8	24	9	27	0	28	2
	16	12	0	12	9	13	8	14	6	15	5	16	2	17	1	18	0	18	9	20	6	21	5
	24	8	1	8	8	9	2	9	9	10	5	11	0	11	6	12	1	12	8	13	9	14	5
4× 8	12	20	8	22	1	23	7	25	1	26	7	28	0	29	6	30	0						
	16	15	9	16	10	18	0	19	1	20	4	21	5	22	6	23	7	24	9	27	0	28	1
	24	10	8	11	5	12	2	13	0	13	8	14	6	15	2	16	0	16	9	18	4	19	1
4×10	12	25	8	27	7	29	5	30	0														
	16	19	8	21	1	22	6	23	10	25	4	26	8	28	1	29	7	30	0				
	24	13	5	14	5	15	4	16	4	17	2	18	2	19	1	20	1	21	1	23	0	23	10

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—100 pounds per square foot of floor area with plastered ceiling, or 110 pounds per square foot with ceiling unplastered.

Table XXXII. Floor-Joist Spans (125-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 125 pounds per square foot with plastered ceiling. Live load 135 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXIII.)** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength of the joist. Then** refer to Table XXXIII and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 12 00		$f =$ 1 100		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 8	12	8	4	8	10	9	4	9	9	8	0	8	5	8	9	9	2	9	6	9	10	10	2	10	7	10	10	11	2
	16	7	7	8	1	8	6	9	0	6	10	7	2	7	7	8	0	8	4	8	7	8	10	9	2	9	6	9	9
2×10	12	10	7	11	2	11	9	12	5	10	1	10	7	11	1	11	7	12	1	12	7	13	0	13	5	13	9	14	2
	16	9	7	10	6	10	9	11	4	8	8	9	2	9	8	10	1	10	6	10	10	11	4	11	8	12	0	12	5
2×12	12	12	9	13	7	14	4	14	10	12	1	12	9	13	5	14	0	14	7	15	2	15	8	16	2	16	8	17	2
	16	11	7	12	5	13	0	13	7	10	6	11	1	11	8	12	2	12	8	13	2	13	7	14	1	14	6	14	10
2×14	12	15	0	15	11	16	10	17	6	14	3	15	0	15	9	16	5	17	1	17	9	18	5	18	11	19	6	20	1
	16	13	9	14	6	15	4	16	0	12	5	13	0	13	9	14	4	14	11	15	0	16	0	16	6	17	0	17	0
	24	12	0	12	10	13	5	14	1	10	1	10	9	11	3	11	9	12	3	12	9	13	1	13	7	14	0	14	5

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—125 pounds per square foot of floor area with plastered ceiling, or 135 pounds per square foot with ceiling unplastered

Table XXXII (Continued). Floor-Joist Spans (125-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 125 pounds per square foot with plastered ceiling Live load 135 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXIII.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist **Then** refer to Table XXXIII and in the same manner determine the span of the joist limited by **horizontal shear** and use the shorter of the two spans																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	7	5	7	10	8	3	8	7	7	6	8	0	8	5	8	10	9	1	9	6	9	10	10	1	10	5	10	9
	16	6	9	7	1	7	6	7	10	6	7	6	11	7	4	7	7	7	11	8	3	8	6	8	10	9	1	9	4
3×8	12	9	10	10	5	11	0	11	6	10	0	10	7	11	1	11	7	12	2	12	7	13	0	13	5	13	10	14	3
	16	8	11	9	6	10	0	10	5	8	10	9	3	9	9	10	1	10	7	10	11	11	4	11	9	12	0	12	5
	24	7	10	8	4	8	10	9	3	7	3	7	6	7	6	8	4	8	9	9	0	9	4	9	7	9	11	10	1
3×10	12	12	5	13	1	13	10	14	6	12	9	13	4	14	0	14	9	15	3	15	10	16	5	16	11	17	5	17	11
	16	11	4	12	0	12	7	13	3	11	0	11	9	12	3	12	10	13	4	13	10	14	4	14	9	15	3	15	7
	24	9	11	10	6	11	1	11	7	9	1	9	7	10	0	10	6	10	11	11	4	11	9	12	1	12	6	12	10
3×12	12	14	11	15	10	16	9	17	6	15	4	16	1	16	11	17	7	18	5	19	1	19	9	20	5	21	0	21	7
	16	13	7	14	6	15	3	15	11	13	4	14	0	14	9	15	5	16	0	16	7	17	3	17	10	18	4	18	10
	24	12	0	12	9	13	5	14	0	11	0	11	7	12	1	12	7	13	3	13	9	14	1	14	7	15	1	15	6

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—125 pounds per square foot of floor area with plastered ceiling, or 135 pounds per square foot with ceiling unplastered.

Table XXXII (Continued). Floor-Joist Spans (125-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 125 pounds per square foot with plastered ceiling. Live load 135 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span): Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the corresponding column below with the corresponding value to determine span								Spans determined by bending. (**Read carefully. These spans must be compared and checked with those in Table XXXIII.**) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXXIII and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×14	12	17	6	18	6	19	6	20	5	17	10	18	10	19	9	20	7	21	5	22	3	23	0	23	10	24	6	25	3
	16	15	11	16	11	17	10	18	7	15	7	16	5	17	3	18	0	18	9	19	5	20	1	20	10	21	5	22	0
	24	14	0	14	10	15	9	16	4	12	10	13	6	14	3	14	10	15	5	16	0	16	7	17	1	17	7	18	3
4× 6	12	8	3	8	9	9	1	9	7	8	11	9	4	9	10	10	3	10	9	11	0	11	5	11	10	12	3	12	6
	16	7	5	7	11	8	4	8	9	7	9	8	1	8	6	8	11	9	4	9	7	9	11	10	4	10	7	10	11
	24	6	6	7	0	7	4	7	7	6	4	6	9	7	0	7	4	7	7	7	11	8	3	8	5	8	9	8	11
4× 8	12	10	11	11	6	12	3	12	9	11	9	12	5	13	0	13	7	14	1	14	9	15	3	15	9	16	3	16	7
	16	9	11	10	6	11	1	11	7	10	3	10	10	11	4	11	10	12	4	12	10	13	3	13	9	14	1	14	6
	24	8	9	9	3	9	9	10	3	8	5	8	11	9	4	9	9	10	1	10	6	10	11	11	3	11	7	11	11
4×10	12	13	9	14	7	15	4	16	0	14	10	15	7	16	4	17	1	17	10	18	5	19	1	19	9	20	4	20	11
	16	12	6	13	4	14	0	14	7	12	11	13	7	14	4	14	11	15	6	16	1	16	9	17	3	17	9	18	3
	24	11	0	11	9	12	4	12	11	10	7	11	3	11	9	12	4	12	10	13	4	13	9	14	1	14	7	15	0

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).
Live load—125 pounds per square foot of floor area with plastered ceiling, or 135 pounds per square foot with ceiling unplastered.

Table XXXIII. Floor-Joist Spans (125-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 125 pounds per square foot with plastered ceiling Live load 135 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span) — Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXXII.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXXII and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 8	12	7	10	8	6	9	1	9	7	10	2	10	9	11	4	11	10	12	6	13	7	14	2
	16	6	0	6	5	6	9	7	4	7	8	8	1	8	7	9	0	9	5	10	4	10	8
2×10	12	10	0	10	8	11	5	12	1	12	9	13	7	14	4	15	0	15	8	17	1	17	9
	16	7	6	8	1	8	7	9	2	9	8	10	2	10	9	11	4	11	10	12	10	13	6
2×12	12	12	0	12	10	13	8	14	7	15	5	16	4	17	2	18	0	18	10	20	7	21	6
	16	9	1	9	8	10	5	11	0	11	8	12	4	13	0	13	7	14	4	15	7	16	2
2×14	12	14	0	15	0	16	0	17	0	18	0	19	0	20	0	21	0	22	0	24	0	25	0
	16	10	7	11	5	12	1	12	10	13	7	14	5	15	1	15	10	16	8	18	2	18	10
	24	7	1	7	8	8	2	8	8	9	2	9	8	10	2	10	8	11	2	12	2	12	9
3× 6	12	9	7	10	2	10	10	11	7	12	4	13	0	13	8	14	4	15	0	16	5	17	1
	16	7	2	7	8	8	4	8	9	9	4	9	9	10	4	10	9	11	5	12	5	12	10
3× 8	12	12	7	13	6	14	5	15	4	16	2	17	1	18	0	18	10	19	9	21	7	22	6
	16	9	7	10	2	10	10	11	7	12	4	13	0	13	7	14	4	15	0	16	5	17	1
	24	6	5	6	10	7	5	7	9	8	4	8	8	9	2	9	7	10	1	11	0	11	6
3×10	12	15	9	16	10	18	1	19	2	20	4	21	6	22	7	23	8	24	9	27	1	28	2
	16	12	0	12	10	13	8	14	7	15	5	16	4	17	2	18	0	18	10	20	7	21	5
	24	8	1	8	8	9	4	9	9	10	5	11	0	11	7	12	2	12	8	13	10	14	6

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for *S*.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—125 pounds per square foot of floor area with plastered ceiling, or 135 pounds per square foot with ceiling unplastered.

Table XXXIII (Continued). Floor-Joist Spans (125-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 125 pounds per square foot with plastered ceiling. Live load 135 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																					
		Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXXII.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXXII and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×12	12	18	10	20	4	21	7	23	0	24	5	25	8	27	1	28	5	29	9	30	0		
	16	14	5	15	6	16	6	17	6	18	6	19	7	20	7	21	7	22	8	24	8	25	9
	24	9	9	10	6	11	2	11	10	12	6	13	2	13	10	14	7	15	4	16	8	17	5
3×14	12	22	0	23	7	25	2	26	8	28	4	29	10	30	0								
	16	16	9	18	0	19	2	20	5	21	7	22	9	24	0	25	2	26	5	28	9	30	0
	24	11	5	12	2	13	0	13	9	14	8	15	6	16	4	17	1	17	10	19	6	20	4
4× 6	12	13	1	14	0	14	10	15	10	16	9	17	8	18	8	19	7	20	6	22	5	23	4
	16	9	10	10	7	11	4	12	0	12	8	13	5	14	1	14	9	15	7	17	0	17	8
	24	6	8	7	1	7	7	8	1	8	7	9	0	9	6	10	0	10	6	11	5	11	10
4× 8	12	17	2	18	5	19	7	20	10	22	1	23	4	24	6	25	9	27	0	29	5	30	0
	16	13	1	14	0	14	10	15	10	16	9	17	8	18	7	19	7	20	6	22	5	23	4
	24	8	9	9	5	10	1	10	8	11	4	12	0	12	7	13	2	13	10	15	1	15	8
4×10	12	21	5	23	0	24	6	26	0	27	7	29	1	30	0								
	16	16	4	17	6	18	8	19	9	21	0	22	2	23	4	24	6	25	8	28	0	29	2
	24	11	1	11	10	12	8	13	6	14	4	15	0	15	9	16	7	17	5	19	0	19	9

Note: The lengths are based on:
Allowable horizontal shear stress as noted for S

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—125 pounds per square foot of floor area with plastered ceiling, or 135 pounds per square foot with ceiling unplastered.

Table XXXIV. Floor-Joist Spans (150-Pound Load)

Maximum Spans for Floor-Joists—Uniformly Loaded

Live load 150 pounds per square foot with plastered ceiling Live load 160 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table 1 the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXV.)** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXXV and in the same manner determine the span of the joist limited by **horizontal shear** and **use the shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 8	12	7	10	8	5	8	9	9	2	7	4	7	8	8	1	8	5	8	9	9	1	9	5	9	8	10	1	10	4
	16	7	2	7	7	8	0	8	5	6	5	6	8	7	0	7	4	7	7	7	10	8	2	8	6	8	8	9	0
2×10	12	10	0	10	7	11	2	11	8	9	4	9	8	10	2	10	8	11	1	11	7	12	0	12	5	12	9	13	1
	16	9	1	9	8	10	2	10	8	8	0	8	6	8	10	9	4	9	7	10	0	10	5	10	8	11	0	11	5
2×12	12	12	1	12	10	13	6	14	2	11	2	11	9	12	5	12	10	13	5	13	10	14	5	15	0	15	5	15	10
	16	11	0	11	8	12	4	12	10	9	8	10	2	10	9	11	2	11	8	12	2	12	7	13	0	13	5	13	9
2×14	12	14	3	15	1	15	11	16	7	13	3	13	11	14	6	15	3	15	10	16	5	16	11	17	6	18	1	18	7
	16	13	0	13	10	14	6	15	7	11	5	12	0	12	7	13	3	13	9	14	3	14	10	15	3	15	9	16	3
3× 6	12	7	0	7	5	7	10	8	3	7	0	7	5	7	9	8	1	8	5	8	9	9	1	9	4	9	7	9	11
	16	6	4	6	9	7	1	7	6	6	1	6	5	6	9	7	0	7	4	7	7	7	11	8	1	8	5	8	7
3× 8	12	9	4	9	11	10	5	10	11	9	4	9	10	10	4	10	9	11	4	11	7	12	0	12	5	12	10	13	3
	16	8	5	9	0	9	6	9	11	8	1	8	6	8	11	9	4	9	10	10	1	10	5	10	10	11	1	11	5
3×10	12	11	9	12	6	13	1	13	9	11	9	12	5	13	0	13	6	14	1	14	7	15	1	15	9	16	1	16	7
	16	10	9	11	5	12	0	12	6	10	3	10	10	11	4	11	10	12	4	12	9	13	3	13	7	14	0	14	6
	24	9	5	10	0	10	6	11	0	8	5	8	10	9	4	9	9	10	0	10	5	10	10	11	3	11	6	11	10

Note: The lengths are based on:

When limited by deflection—
- Maximum allowable deflection of 1/360 of span length.
- Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
- Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
- Weight of lath and plaster ceiling (10 pounds per square foot).
- Double thickness of flooring (5 pounds per square foot).

Live load—150 pounds per square foot of floor area with plastered ceiling, or 160 pounds per square foot with ceiling unplastered.

Table XXXIV (Continued). Floor-Joist Spans (150-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 150 pounds per square foot with plastered ceiling. Live load 160 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Spans determined by bending. (Read carefully. These spans must be compared and checked with those in Table XXXV.) Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of timber used, refer to the column below with the corresponding value to determine the span limited by the **bending strength** of the joist. **Then** refer to Table XXXV and in the same manner determine the span of the joist limited by **horizontal shear** and use the **shorter of the two spans**																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×12	12	14	3	15	0	15	10	16	7	14	1	14	11	15	7	16	4	17	0	17	7	18	3	18	10	19	5	20	0
	16	12	11	13	9	14	6	15	1	12	4	13	0	13	7	14	3	14	10	15	5	15	11	16	5	16	11	17	5
	24	11	4	12	0	12	9	13	4	10	1	10	9	11	3	11	9	12	3	12	7	13	1	13	6	13	11	14	4
3×14	12	16	7	17	7	18	6	19	5	16	6	17	5	18	4	19	1	19	10	20	7	21	4	22	0	22	9	23	5
	16	15	1	16	1	16	11	17	9	14	5	15	3	15	11	16	7	17	4	18	0	18	7	19	3	19	10	20	5
	24	13	4	14	1	14	11	15	6	11	11	12	6	13	1	13	9	14	3	14	10	15	4	15	10	16	4	16	10
4×6	12	7	10	8	3	8	9	9	1	8	3	8	9	9	1	9	6	9	10	10	3	10	7	10	11	11	4	11	7
	16	7	1	7	6	7	11	8	4	7	1	7	6	7	11	8	3	8	6	8	11	9	3	9	6	9	10	10	1
	24	6	3	6	7	6	11	7	3	5	10	6	3	6	6	6	10	7	0	7	4	7	6	7	10	8	1	8	3
4×8	12	10	4	10	11	11	6	12	0	10	11	11	5	12	0	12	7	13	1	13	7	14	0	14	6	14	11	15	5
	16	9	5	10	0	10	6	11	0	9	6	10	0	10	6	10	11	11	5	11	10	12	3	12	7	13	0	13	5
	24	8	3	8	10	9	3	9	7	7	9	8	3	8	7	9	0	9	4	9	9	10	0	10	4	10	9	11	0
4×10	12	13	0	13	10	14	6	15	3	13	9	14	5	15	1	15	10	16	6	17	1	17	9	18	3	18	10	19	5
	16	11	11	12	7	13	4	13	11	11	11	12	7	13	3	13	10	14	5	14	11	15	5	15	11	16	5	16	11
	24	10	5	11	1	11	9	12	3	9	10	10	4	10	11	11	4	11	10	12	3	12	9	13	1	13	6	13	11

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—150 pounds per square foot of floor area with plastered ceiling, or 160 pounds per square foot with ceiling unplastered.

Table XXXV. Floor-Joist Spans (150-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 150 pounds per square foot with plastered ceiling Live load 160 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																					
		Spans limited by horizontal shear. (Read carefully. **These spans must be compared and checked with those in Table XXXIV.**) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXXIV and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 8	12	6	9	7	2	7	8	8	2	8	8	9	2	9	8	10	1	10	7	11	7	12	1
	16	5	1	5	6	5	9	6	2	6	6	6	10	7	4	7	7	8	0	8	8	9	1
2×10	12	8	6	9	1	9	8	10	4	10	10	11	7	12	2	12	9	13	5	14	7	15	2
	16	6	5	6	10	7	4	7	9	8	4	8	8	9	2	9	7	10	1	11	0	11	6
2×12	12	10	2	11	0	11	8	12	5	13	2	13	10	14	7	15	5	16	1	17	7	18	4
	16	7	8	8	4	8	10	9	5	10	0	10	6	11	1	11	7	12	2	13	4	13	9
2×14	12	11	10	12	9	13	7	14	6	15	4	16	2	17	0	17	10	18	9	20	6	21	4
	16	9	0	9	8	10	4	11	0	11	7	12	4	12	10	13	7	14	2	15	6	16	1
3× 6	12	8	1	8	8	9	4	9	10	10	6	11	1	11	7	12	2	12	9	14	0	14	7
	16	6	2	6	7	7	0	7	6	7	10	8	4	8	9	9	2	9	8	10	6	11	0
3× 8	12	10	9	11	6	12	4	13	1	13	9	14	7	15	5	16	2	16	10	18	6	19	2
	16	8	1	8	8	9	4	9	10	10	6	11	0	11	7	12	2	12	9	14	0	14	6
3×10	12	13	6	14	6	15	5	16	5	17	5	18	4	19	4	20	4	21	2	23	2	24	1
	16	10	2	11	0	11	8	12	5	13	2	13	10	14	7	15	5	16	1	17	7	18	4
	24	6	10	7	5	7	10	8	5	8	10	9	5	9	10	10	5	10	9	11	9	12	4

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for S.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot).

Live load—150 pounds per square foot of floor area with plastered ceiling, or 160 pounds per square foot with ceiling unplastered.

Table XXXV (Continued). Floor-Joist Spans (150-Pound Load)

MAXIMUM SPANS FOR FLOOR-JOISTS—UNIFORMLY LOADED

Live load 150 pounds per square foot with plastered ceiling. Live load 160 pounds per square foot with unplastered ceiling

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span)																					
		Spans limited by horizontal shear. (Read carefully. These spans must be compared and checked with those in Table XXXIV.) Having determined by reference to the building code or Table I the **horizontal shear stress** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine the span limited by the **horizontal shear stress. Then** refer to Table XXXIV and in the same manner determine the span of the joist limited by its **bending strength** and **use the shorter of the two spans**																					
		$S=70$		$S=75$		$S=80$		$S=85$		$S=90$		$S=95$		$S=100$		$S=105$		$S=110$		$S=120$		$S=125$	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×12	12	16	2	17	5	18	6	19	8	20	10	22	0	23	2	24	4	25	6	27	9	29	0
	16	12	4	13	2	14	1	15	0	15	9	16	8	17	7	18	6	19	5	21	1	22	0
	24	8	4	8	10	9	6	10	1	10	8	11	4	11	10	12	6	13	1	14	4	14	10
3×14	12	18	10	20	2	21	7	22	10	24	4	25	7	27	0	28	4	29	8	30	0	. .	. .
	16	14	5	15	5	16	5	17	5	18	6	19	6	20	6	21	6	22	7	24	7	25	8
	24	9	8	10	5	11	1	11	9	12	6	13	2	13	10	14	7	15	4	16	8	17	5
4× 6	12	11	2	11	10	12	8	13	6	14	4	15	1	15	10	16	8	17	6	19	1	19	10
	16	8	5	9	0	9	7	10	2	10	9	11	5	12	1	12	8	13	4	14	6	15	1
	24	5	8	6	1	6	6	6	10	7	4	7	8	8	1	8	6	8	10	9	8	10	1
4× 8	12	14	8	15	8	16	9	17	9	18	10	19	10	21	0	22	0	23	1	25	2	26	2
	16	11	1	11	10	12	8	13	6	14	4	15	1	15	10	16	8	17	6	19	1	19	10
	24	7	6	8	0	8	7	9	1	9	8	10	2	10	8	11	4	11	9	12	10	13	5
4×10	12	18	5	19	8	21	0	22	4	23	7	24	10	26	4	27	7	28	10	30	0	. .	. .
	16	14	0	15	0	16	0	17	0	18	0	19	0	20	0	21	0	22	0	24	0	25	0
	24	9	6	10	1	10	9	11	6	12	2	12	9	13	6	14	2	14	10	16	2	16	10

NOTE: The lengths are based on:
Allowable horizontal shear stress as noted for *S*.

Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Double thickness of flooring (5 pounds per square foot)

Live load—150 pounds per square foot of floor area with plastered ceiling, or 160 pounds per square foot with ceiling unplastered.

Table XXXVI. Ceiling and Attic Joist Spans

MAXIMUM SPANS FOR CEILING-JOISTS AND ATTIC FLOOR-JOISTS—UNIFORMLY LOADED

Size of joists (nominal) in inches	Spacing of joists center to center in inches	Maximum allowable lengths between supports (clear span). Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine maximum safe span															
		Ceiling joists								Attic floor-joists—Live load 20 lb per sq ft							
		E = 1 000 000		E = 1 200 000		E = 1 400 000		E = 1 600 000		E = 1 000 000		E = 1 200 000		E = 1 400 000		E = 1 600 000	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 4	12	9	4	10	0	10	6	11	0	6	6	7	0	7	4	7	8
	16	8	7	9	2	9	8	10	0	5	11	6	3	6	8	6	11
	24	7	7	8	1	8	6	8	11	5	3	5	7	5	10	6	1
2× 6	12	14	2	15	5	15	10	16	7	10	0	10	9	11	3	11	0
	16	13	3	14	0	14	8	15	4	9	1	9	8	10	2	10	8
	24	11	8	12	5	13	0	13	8	8	1	8	7	9	0	9	7
2× 8	12	18	6	19	8	20	0	21	8	13	4	14	2	14	11	15	7
	16	17	2	18	3	19	3	20	2	12	1	12	10	13	6	14	2
	24	15	4	16	4	17	2	18	0	10	9	11	5	12	0	12	7
2×10	12	23	0	24	5	25	8	26	10	15	9	17	9	18	9	19	7
	16	21	4	22	9	24	0	25	0	15	3	16	2	17	0	17	9
	24	19	3	20	5	21	6	22	6	13	7	14	5	15	2	15	10
2×12	12	27	2	28	11	30	0			20	0	21	4	22	6	23	6
	16	25	6	27	0	28	6	29	9	18	4	19	5	20	6	21	5
	24	23	0	24	5	25	9	26	10	16	4	17	4	18	3	19	1

NOTE: The lengths are based on.
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
Ceiling joists:
Dead load—Weight of joists plus plaster ceiling (10 pounds per square foot).
Live load—None.

Attic floor joists:
Dead load—Weight of joist.
Weight of lath and plaster ceiling (10 pounds per square foot).
Single thickness of flooring (2 5 pounds per square foot).
Live load—20 pounds per square foot of floor area.

Table XXXVII. Rafter Spans (15-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more. Live load 15 pounds per square foot.

Size of rafters (nominal) in inches	Spacing of joists center to center in inches	Maximum available unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×4	12	7	7	8	0	8	6	8	11	9	11	10	4	10	11	11	5	11	11	12	4	12	9	13	2	13	7	14	0
	16	6	11	7	5	7	8	8	1	8	8	9	1	9	6	10	0	10	5	10	9	11	2	11	7	11	10	12	4
	24	6	1	6	6	6	9	7	1	7	1	7	6	7	11	8	2	8	7	8	10	9	2	9	6	9	9	10	1
2×6	12	11	7	12	5	13	0	13	7	15	1	15	10	16	8	17	5	18	1	18	9	19	0	20	1	20	8	21	4
	16	10	8	11	4	12	0	12	6	13	2	14	0	14	8	15	4	15	10	16	6	17	1	17	8	18	2	18	9
	24	9	5	10	0	10	6	11	0	11	0	11	7	12	2	12	8	13	2	13	8	14	2	14	7	15	1	15	6
2×8	12	15	4	16	4	17	1	17	10	19	8	20	9	21	9	22	9	23	8	24	7	25	6	26	4	27	1	27	10
	16	14	1	15	0	15	9	16	6	17	5	18	5	19	4	20	1	20	10	21	8	22	6	23	4	24	0	24	7
	24	12	6	13	2	13	10	14	7	14	6	15	4	16	0	16	8	17	5	18	0	18	8	19	4	19	10	20	6
2×10	12	19	2	20	5	21	6	22	5	24	6	25	10	27	1	28	4	29	0	30	0	..	..	..	..	..	..	..	..
	16	17	8	18	9	19	9	20	8	21	9	22	10	24	0	25	1	26	1	27	1	28	1	29	0	29	10	30	0
	24	15	8	16	8	17	6	18	4	18	2	19	1	20	1	21	0	21	9	22	7	23	5	24	2	25	0	25	8

Note: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof coverings (2.5 pounds per square foot). (Group I)
Live load—15 pounds per square foot of roof surface considered as acting normal to the surface

Table XXXVII (Continued). Rafter Spans (15-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more. Live load 15 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	22	10	24	5	25	7	26	9	29	2	30	0																
	16	21	2	22	6	23	8	24	9	26	0	27	5	28	8	30	0												
	24	18	10	20	0	21	1	22	1	21	9	23	0	24	1	25	1	26	2	27	2	28	1	29	0	29	10	30	0
2×14	12	26	6	28	1	29	9	30	0																				
	16	24	7	26	3	27	6	28	10	29	11	30	0																
	24	22	0	23	5	24	7	25	9	25	3	26	7	27	11	29	1	30	0										
3×6	12	13	5	14	3	14	11	15	7	18	6	19	6	20	6	21	5	22	4	23	1	23	11	24	9	25	6	26	3
	16	12	4	13	1	13	10	14	5	16	5	17	4	18	3	18	11	19	9	20	6	21	3	21	11	22	7	23	3
	24	10	11	11	7	12	3	12	10	13	9	14	6	15	3	15	10	16	6	17	1	17	9	18	4	18	10	19	5
3×8	12	17	6	18	7	19	6	20	5	24	0	25	4	26	6	27	9	29	0	30	0								
	16	16	3	17	3	18	1	18	11	21	5	22	6	23	7	24	9	25	11	26	9	27	7	28	6	29	5	30	0
	24	14	5	15	4	16	1	16	10	18	0	18	11	19	11	20	10	21	9	22	5	23	3	24	0	24	9	25	5
3×10	12	21	9	23	1	24	4	25	5	29	6	30	0																
	16	20	3	21	5	22	7	23	7	26	6	27	11	29	4	30	0												
	24	18	0	19	3	20	3	21	1	22	5	23	7	24	10	25	11	26	11	27	11	28	11	30	0				

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist
Weight of roof sheathing (2.5 pounds per square foot)
Weight of roof coverings (2.5 pounds per square foot). (Group I.)
Live load—15 pounds per square foot of roof surface considered as acting normal to the surface.

Table XXXVIII. Rafter Spans (20-Pound Load—Group I Covering)

Maximum Spans for Rafters—Uniformly Loaded

Slope of 20° or more   Live load 20 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable modulus of elasticity in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable extreme fiber-stress in bending in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 4	12	7	1	7	6	7	10	8	4	8	10	9	5	9	9	10	4	10	8	11	1	11	6	11	10	12	3	12	7
	16	6	6	6	9	7	2	7	6	7	9	8	4	8	7	9	0	9	4	9	7	10	1	10	5	10	8	11	0
	24	5	7	6	0	6	4	6	7	6	5	6	8	7	0	7	5	7	8	8	0	8	4	8	6	8	9	9	1
2× 6	12	10	9	11	6	13	5	12	8	13	7	14	5	15	0	15	8	16	5	17	0	17	7	18	1	18	8	19	2
	16	9	10	10	7	11	1	11	7	11	10	12	7	13	2	13	9	14	5	14	9	15	5	15	10	16	5	16	10
	24	8	8	9	8	9	10	10	4	9	9	10	2	10	10	11	5	11	10	12	3	12	8	13	1	13	7	14	0
2× 8	12	14	4	15	2	16	1	16	9	17	10	18	10	19	9	20	8	21	6	22	3	23	1	23	10	24	7	25	4
	16	13	2	14	0	14	8	15	5	15	8	16	7	17	5	18	2	18	10	19	6	20	4	21	0	21	8	22	4
	24	11	7	12	4	13	0	13	7	13	0	13	8	14	5	15	1	15	8	16	3	16	9	17	5	17	10	18	5
2×10	12	18	0	19	1	20	1	21	0	22	4	23	6	24	8	25	9	26	9	27	8	28	9	29	8	30	0	. . .	
	16	16	7	17	7	18	6	19	5	19	8	20	9	21	9	22	9	23	8	24	6	25	6	26	4	27	1	27	10
	24	14	7	15	7	16	5	17	1	16	5	17	4	18	1	18	10	19	8	20	5	21	2	21	10	22	6	23	2

Note: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2 5 pounds per square foot).
Weight of roof coverings (2 5 pounds per square foot). (Group I.)
Live load—20 pounds per square foot of roof surface considered as acting normal to the surface

Table XXXVIII (Continued). Rafter Spans (20-Pound Load—Group I Covering)

Maximum Spans for Rafters—Uniformly Loaded

Slope of 20° or more Live load 20 pounds per square foot

Maximum allowable unsupported lengths from plate to ridge (without collar-beams)

Limited by deflection of 1/360 of the span

Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000	
		Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	21	7	22	10	24	1	25	2
	16	19	10	21	1	22	4	23	4
	24	17	7	18	8	19	8	20	7
2×14	12	25	0	26	7	28	0	29	4
	16	23	3	24	7	25	11	27	1
	24	20	7	21	11	23	1	24	1
3×6	12	12	6	13	4	14	0	14	9
	16	11	6	12	4	12	11	13	6
	24	10	3	10	10	11	5	11	11
3×8	12	16	5	17	6	18	5	19	3
	16	15	3	16	3	17	0	17	10
	24	13	6	14	4	15	1	15	10
3×10	12	20	6	21	10	22	11	24	0
	16	19	0	20	3	21	4	22	3
	24	16	11	18	0	18	11	19	10

Determined by bending

Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	26	7	28	1	29	6	30	0	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
	16	23	7	24	10	26	1	27	2	28	5	29	5	30	0	. .	. .	. .	. .	. .	. .
	24	19	8	20	9	21	9	22	8	23	8	24	6	25	5	26	4	27	1	27	10
2×14	12	30	0									.				. .		. .	. .	. .	
	16	27	4	28	10	30	0	.		. .		.		.		. .				. .	
	24	23	1	24	1	25	4	26	5	27	6	28	6	29	7	30	0				
3×6	12	16	11	17	10	18	9	19	6	20	4	21	0	21	10	22	6	23	3	23	11
	16	14	11	15	9	16	6	17	3	17	10	18	6	19	3	19	10	20	5	21	0
	24	12	5	13	0	13	9	14	4	14	11	15	5	16	0	16	6	17	0	17	6
3×8	12	21	11	23	1	24	3	25	4	26	6	27	4	28	4	29	3	30	0		
	16	19	5	20	6	21	6	22	6	23	6	24	4	25	1	25	11	26	9	27	6
	24	16	4	17	1	18	0	18	10	19	9	20	4	21	0	21	9	22	5	23	0
3×10	12	27	1	28	7	30	0							.				. .		.	.
	16	24	3	25	6	26	9	27	11	29	1	30	0	.				.	.	. .	
	24	20	4	21	5	22	6	23	6	24	6	25	5	26	4	27	1	28	0	28	10

Note: The lengths are based on:

When limited by deflection—
 Maximum allowable deflection of 1/360 of span length.
 Modulus of elasticity as noted for E
When determined by bending strength of the piece—
 Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
 Weight of roof sheathing (2.5 pounds per square foot).
 Weight of roof coverings (2.5 pounds per square foot) (Group I.)
Live load—20 pounds per square foot of roof surface considered as acting normal to the surface.

Table XXXIX. Rafter Spans (30-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more. Live load 30 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table 1 the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		= 1 400		$f =$ 1 500		= 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×4	12	6	5	6	9	7	2	7	6	7	8	8	0	8	5	8	9	9	2	9	7	9	11	10	2	10	5	10	9
	16	5	10	6	2	6	6	6	10	6	8	7	0	7	4	7	8	7	11	8	3	8	6	8	10	9	1	9	5
	24	5	1	5	5	5	8	6	0	5	5	5	9	6	0	6	3	6	7	6	10	7	0	7	3	7	6	7	8
2×6	12	9	10	10	5	11	0	11	6	11	8	12	4	12	11	13	6	14	0	14	6	15	1	15	7	16	1	16	6
	16	9	0	9	7	10	1	10	6	10	2	10	9	11	3	11	9	12	3	12	9	13	2	13	8	14	0	14	6
	24	7	11	8	5	8	10	9	3	8	4	8	10	9	3	9	8	10	1	10	6	10	10	11	2	11	6	11	11
2×8	12	13	1	13	10	14	7	15	3	15	5	16	3	17	0	17	10	18	6	19	2	19	11	20	7	21	2	21	10
	16	11	11	12	9	13	4	14	0	13	6	14	3	15	0	15	7	16	3	16	10	17	5	18	0	18	7	19	1
	24	10	6	11	2	11	9	12	3	11	2	11	9	12	4	12	10	13	4	13	10	14	5	14	10	15	4	15	9
2×10	12	16	4	17	5	18	4	19	2	19	4	20	4	21	4	22	3	23	3	24	1	24	11	25	9	26	6	27	4
	16	15	0	15	11	16	10	17	6	17	0	17	11	18	9	19	7	20	5	21	2	21	11	22	8	23	4	24	0
	24	13	4	14	0	14	10	15	6	14	0	14	10	15	0	16	3	16	11	17	6	18	1	18	8	19	3	19	10

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof coverings (2.5 pounds per square foot). (Group I.).
Live load—30 pounds per square foot of roof surface considered as acting normal to the surface.

Table XXXIX (Continued). Rafter Spans (30-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more. Live load 30 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	19	8	20	11	22	0	23	0	23	1	24	5	25	7	26	8	27	9	28	10	29	10	30	0	. .	.	. .	
	16	18	1	19	3	20	3	21	2	20	4	21	6	22	6	23	6	24	6	25	4	26	3	27	2	28	0	28	10
	24	15	11	17	0	17	10	18	8	16	11	17	10	18	8	19	6	20	3	21	1	21	10	22	6	23	3	23	11
2×14	12	22	10	24	4	25	6	26	9	26	9	28	3	29	6	30	0							.		. . . .		. . .	
	16	21	3	22	6	23	9	24	10	23	10	25	1	26	5	27	6	28	9	29	9	30	0	. .		.	.		
	24	18	7	19	10	20	10	21	10	19	9	20	9	21	9	22	9	23	7	24	6	25	5	26	3	27	0	27	10
3×6	12	11	5	12	1	12	9	13	4	14	7	15	4	16	1	16	10	17	6	18	3	18	10	19	5	20	0	20	7
	16	10	5	11	1	11	9	12	3	12	10	13	6	14	1	14	10	15	5	15	11	16	6	17	1	17	7	18	1
	24	9	3	9	10	10	4	10	10	10	7	11	1	11	9	12	3	12	9	13	3	13	9	14	1	14	6	15	0
3×8	12	15	0	15	11	16	10	17	6	19	1	20	1	21	0	22	0	23	0	23	9	24	7	25	5	26	3	26	11
	16	13	10	14	7	15	5	16	1	16	10	17	9	18	7	19	5	20	4	21	0	21	9	22	5	23	1	23	9
	24	12	3	12	11	13	7	14	3	14	0	14	9	15	5	16	1	16	11	17	5	18	0	18	7	19	3	19	[illegible]
3×10	12	18	9	20	0	21	0	21	11	23	9	25	0	26	3	27	4	28	6	29	6	30	0						
	16	17	4	18	4	19	4	20	3	21	0	22	1	23	3	24	3	25	3	26	3	27	1	28	0	28	10	29	9
	24	15	4	16	4	17	1	17	11	17	6	18	6	19	5	20	3	21	0	21	10	22	7	23	4	24	1	24	10

NOTE: The lengths are based on.

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof coverings (2.5 pounds per square foot). (Group I.)

Live load—30 pounds per square foot of roof surface considered as acting normal to the surface.

Table XL. Rafter Spans (40-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more. Live load 40 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 4	12	5	9	6	2	6	6	6	9	6	8	7	1	7	5	7	9	8	1	8	5	8	8	9	0	9	2	9	6
	16	5	4	5	8	6	0	6	2	5	10	6	1	6	5	6	8	7	0	7	4	7	7	7	9	8	0	8	4
	24	4	8	5	0	5	2	5	5	4	9	5	0	5	2	5	6	5	9	6	0	6	2	6	5	6	7	6	9
2× 6	12	9	0	9	7	10	1	10	7	10	2	10	10	11	5	12	0	12	5	12	10	13	5	13	9	14	2	14	7
	16	8	2	8	9	8	10	9	8	9	0	9	6	10	0	10	5	10	10	11	4	11	8	12	0	12	5	12	9
	24	7	2	7	8	8	1	8	6	7	5	7	9	8	2	8	7	8	10	9	3	9	7	9	10	10	2	10	6
2× 8	12	12	0	12	9	13	5	14	0	13	8	14	5	15	1	15	9	16	6	17	1	17	8	18	4	18	9	19	5
	16	11	0	11	8	12	4	12	9	12	0	12	7	13	2	13	9	14	5	14	10	15	5	16	0	16	5	16	10
	24	9	7	10	2	10	9	11	4	9	10	10	5	10	10	11	5	11	9	12	4	12	8	13	1	13	7	14	0
2×10	12	15	1	16	1	16	10	17	8	17	2	18	1	19	0	19	10	20	8	21	6	22	2	23	0	23	8	24	4
	16	13	9	14	8	15	6	16	2	15	1	15	10	16	8	17	5	18	1	18	9	19	5	20	1	20	8	21	4
	24	12	2	12	10	13	7	14	2	12	5	13	1	13	8	14	4	15	0	15	6	16	0	16	7	17	1	17	7

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (2.5 pounds per square foot). (Group I.)
Live load—40 pounds per square foot of roof surface considered as acting normal to the surface

Table XL (Continued). Rafter Spans (40-Pound Load—Group I Covering)

MAXIMUM SPANS FOR RAFTERS—UNIFORMLY LOADED

Slope of 20° or more Live load 40 pounds per square foot

Size of rafters (nominal) in inches	Spacing of rafters center to center in inches	Maximum allowable unsupported lengths from plate to ridge (without collar-beams) — Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	18	2	19	4	20	5	21	4	20	7	21	9	22	9	23	9	24	9	25	8	26	8	27	6	28	5	29	2
	16	16	8	17	8	18	8	19	6	18	1	19	1	20	0	20	10	21	9	22	7	23	5	24	2	24	10	25	7
	24	14	8	15	7	16	5	17	2	15	0	15	9	16	7	17	4	18	0	18	8	19	4	20	0	20	7	21	2
2×14	12	21	3	22	7	23	10	24	10	24	0	25	4	26	6	27	9	28	10	30	0								
	16	19	6	20	10	21	10	22	10	21	1	22	3	23	4	24	5	25	5	26	4	27	3	28	1	29	0	29	10
	24	17	3	18	4	19	4	20	1	17	6	18	6	19	5	20	3	21	0	21	10	22	7	23	5	24	1	24	10
3× 6	12	10	6	11	3	11	10	12	4	13	0	13	9	14	5	15	0	15	7	16	3	16	10	17	4	17	11	18	5
	16	9	7	10	4	10	10	11	4	11	5	12	0	12	7	13	1	13	9	14	3	14	9	15	3	15	7	16	1
	24	8	6	9	0	9	6	9	11	9	5	9	11	10	5	10	11	11	4	11	9	12	1	12	6	12	11	13	4
3× 8	12	13	11	14	10	15	7	16	4	17	1	18	0	18	11	19	9	20	7	21	4	22	0	22	10	23	6	24	1
	16	12	10	13	6	14	4	14	11	15	0	15	10	16	7	17	4	18	1	18	9	19	4	20	0	20	7	21	3
	24	11	3	12	0	12	7	13	3	12	5	13	1	13	9	14	5	15	0	15	6	16	0	16	7	17	1	17	6
3×10	12	17	6	18	6	19	6	20	5	21	4	22	5	23	7	24	7	25	7	26	7	27	6	28	5	29	4	30	0
	16	16	1	17	0	17	11	18	10	18	10	19	10	20	10	21	9	22	7	23	5	24	3	25	0	25	10	26	7
	24	14	3	15	1	15	10	16	7	15	7	16	6	17	4	18	0	18	10	19	6	20	1	20	10	21	5	22	1

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (2.5 pounds per square foot). (Group I.)
Live load—40 pounds per square foot of roof surface considered as acting normal to the surface

Table XLI. Rafter and Roof-Joist Spans (20-Pound Load—Group II Covering)

MAXIMUM SPANS FOR RAFTERS AND ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 20 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2× 4	12	6	7	7	0	7	5	7	9	8	1	8	7	9	0	9	5	9	8	10	1	10	6	10	9	11	1	11	6
	16	6	0	6	5	6	9	7	1	7	1	7	5	7	9	8	1	8	6	8	9	9	1	9	5	9	8	10	0
	24	5	4	5	7	5	10	6	2	5	9	6	2	6	5	6	8	7	0	7	2	7	6	7	9	8	0	8	2
2× 6	12	10	2	10	10	11	5	12	0	12	5	13	1	13	9	14	4	15	0	15	6	16	0	16	7	17	1	17	7
	16	9	4	9	10	10	6	10	10	10	10	11	5	12	0	12	7	13	1	13	7	14	1	14	6	15	0	15	5
	24	8	2	8	8	9	2	9	7	9	0	9	5	9	10	10	4	10	9	11	2	11	7	12	0	12	4	12	8
2× 8	12	13	6	14	5	15	1	15	9	16	5	17	4	18	1	18	10	19	8	20	5	21	1	21	9	22	6	23	2
	16	12	5	13	2	13	10	14	6	14	5	15	2	15	10	16	7	17	4	17	10	18	7	19	2	19	9	20	4
	24	10	10	11	7	12	2	12	9	11	10	12	6	13	1	13	8	14	4	14	9	15	4	15	9	16	4	16	9
2×10	12	17	0	18	1	19	0	19	10	20	6	21	7	22	7	23	8	24	7	25	6	26	6	27	4	28	2	29	0
	16	15	7	16	7	17	6	18	4	18	4	19	4	20	2	21	1	21	10	22	8	23	6	24	4	25	1	25	9
	24	13	9	14	7	15	5	16	1	15	0	15	9	16	0	17	2	18	0	18	8	19	4	19	10	20	6	21	1
2×12	12	20	5	21	8	22	9	23	10	24	6	25	9	27	1	28	4	29	5	30	0								
	16	18	9	20	6	21	0	22	0	21	7	22	9	23	10	25	0	26	0	27	0	27	10	28	9	29	8	30	0
	24	16	7	17	7	18	7	19	5	18	0	19	0	19	10	20	9	21	7	22	5	23	2	24	0	24	8	25	5

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)

Live load—20 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLI (Continued). Rafter and Roof-Joist Spans (20-Pound Load—Group II Covering)

MAXIMUM SPANS FOR RAFTERS AND ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 20 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																									
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×14	12	23	9	25	3	26	6	27	9	28	4	29	10	30	0							...		...		...			
	16	22	11	24	4	25	7	26	10	26	11	28	4	29	9	30	0	...		...				...					
	24	19	5	20	7	21	9	22	9	21	0	22	1	23	3	24	3	25	3	26	1	27	0	27	11	28	10	29	7
3×6	12	11	10	12	7	13	3	13	10	15	6	16	4	17	1	17	11	18	7	19	4	20	0	20	7	21	4	21	11
	16	10	11	11	7	12	3	12	9	13	7	14	5	15	1	15	8	16	5	17	0	17	7	18	3	18	9	19	4
	24	9	7	10	3	10	9	11	3	11	4	11	11	12	6	13	0	13	7	14	1	14	7	15	0	15	[illegible]	16	10
3×8	12	15	7	16	7	17	5	18	3	20	3	21	4	22	4	23	4	24	5	25	3	26	1	26	11	27	9	28	6
	16	14	4	15	3	16	1	16	10	17	11	18	10	19	9	20	7	21	7	22	4	23	1	23	10	24	[illegible]	25	4
	24	12	9	13	6	14	3	14	11	14	11	15	9	16	5	17	3	18	0	18	6	19	3	19	10	20	5	21	0
3×10	12	19	5	20	9	21	10	22	10	25	1	26	5	27	9	28	11	30	0										
	16	18	0	19	1	20	1	21	1	22	4	23	6	24	7	25	9	26	10	27	10	28	9	29	9	30	0		
	24	16	0	17	0	17	11	18	9	18	7	19	7	20	7	21	[illegible]	22	5	23	3	24	1	24	10	25	7	26	5
3×12	12	23	3	24	9	26	0	27	3	29	9	30	0																
	16	21	6	22	11	24	1	25	3	26	6	27	11	29	4	30	0												
	24	19	3	20	5	21	5	22	5	22	4	23	6	24	9	25	9	26	10	27	10	28	10	29	9	30	0		

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)
Live load—20 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLII. Rafter and Roof-Joist Spans (30-Pound Load—Group II Covering)

MAXIMUM SPANS FOR RAFTERS AND ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 30 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	**Maximum allowable lengths between supports or from plate to ridge (without collar-beams)**																											
		Limited by deflection of 1/360 of the span Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								**Determined by bending** Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×4	12	6	1	6	6	6	10	7	1	7	1	7	6	7	10	8	2	8	6	8	10	9	2	9	6	9	9	10	0
	16	5	7	5	11	6	2	6	6	6	2	6	6	6	9	7	2	7	5	7	8	7	11	8	3	8	6	8	9
	24	4	10	5	1	5	5	5	8	5	0	5	3	5	7	5	10	6	1	6	4	6	7	6	9	6	11	7	2
2×6	12	9	5	10	0	10	6	11	0	10	11	11	6	12	0	12	7	13	1	13	7	14	1	14	7	15	0	15	5
	16	8	7	9	1	9	7	10	0	9	5	10	0	10	6	11	0	11	5	11	10	12	3	12	8	13	1	13	6
	24	7	6	8	0	8	5	8	10	7	10	8	4	8	8	9	0	9	5	9	9	10	1	10	5	10	9	11	1
2×8	12	12	6	13	3	14	0	14	6	14	5	15	2	15	11	16	8	17	4	17	11	18	7	19	3	19	10	20	5
	16	11	6	12	1	12	9	13	4	12	7	13	3	13	11	14	7	15	1	15	8	16	3	16	10	17	4	17	10
	24	10	0	10	8	11	3	11	9	10	5	11	0	11	6	12	0	12	6	12	11	13	5	13	10	14	3	14	8
2×10	12	15	8	16	8	17	7	18	4	18	1	19	0	20	0	20	11	21	9	22	7	23	4	24	1	24	10	25	7
	16	14	4	15	3	16	1	16	9	15	10	16	8	17	6	18	4	19	0	19	9	20	6	21	2	21	10	22	5
	24	12	9	13	6	14	3	14	10	13	2	13	11	14	7	15	3	15	11	16	6	17	0	17	7	18	1	18	8

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II)

Live load—30 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLII (Continued). Rafter and Roof-Joist Spans (30-Pound Load—Group II Covering)

MAXIMUM SPANS FOR RAFTERS AND ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 30 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span.								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×12	12	18	10	20	0	21	1	22	0	21	8	22	10	24	0	25	0	26	0	27	0	28	0	28	11	29	10	30	0
	16	17	5	18	4	19	4	20	3	19	0	20	1	21	0	22	0	22	11	23	9	24	7	25	5	26	2	26	11
	24	15	3	16	2	17	0	17	11	15	9	16	8	17	5	18	3	18	10	19	9	20	4	21	0	21	8	22	4
2×14	12	21	11	23	4	24	6	25	7	25	1	26	6	27	9	29	0	30	0										
	16	20	1	21	5	22	6	23	7	22	1	23	4	24	6	25	7	26	7	27	7	28	7	29	6	30	0		
	24	17	10	18	11	19	11	20	10	18	5	19	5	20	4	21	3	22	1	23	0	23	9	24	6	25	4	26	0
3×6	12	10	11	11	7	12	3	12	9	13	7	14	5	15	1	15	9	16	5	17	0	17	7	18	3	18	9	19	4
	16	10	0	10	7	11	1	11	7	12	0	12	7	13	3	13	10	14	5	14	11	15	5	15	11	16	5	16	11
	24	8	10	9	4	9	10	10	4	9	11	10	5	10	11	11	5	11	11	12	4	12	10	13	3	13	7	14	0
3×8	12	14	4	15	3	16	1	16	10	17	11	18	10	19	9	20	7	21	7	22	4	23	1	23	10	24	7	25	4
	16	13	3	14	0	14	10	15	5	15	9	16	7	17	5	18	3	19	0	19	7	20	4	21	0	21	7	22	3
	24	11	7	12	4	15	0	13	7	13	0	13	7	14	5	15	1	15	10	16	4	16	11	17	5	17	11	18	5

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)

Live load—30 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLII (Continued). Rafter and Roof-Joist Spans (30-Pound Load—Group II Covering)

MAXIMUM SPANS FOR RAFTERS AND ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 30 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×10	12	18	0	19	1	20	1	21	1	22	4	23	6	24	7	25	9	26	10	27	10	28	10	29	9	30	0		
	16	16	7	17	7	18	6	19	5	19	9	20	9	21	9	22	9	23	7	24	6	25	5	26	3	27	0	27	10
	24	14	9	15	7	16	5	17	1	16	5	17	4	18	1	18	11	19	9	20	5	21	3	21	10	22	6	23	3
3×12	12	21	10	23	1	24	4	25	6	27	0	28	5	29	10	30	0			..		..		. .		.		. . .	
	16	20	0	21	4	22	5	23	5	23	10	25	1	26	4	27	6	28	7	29	9	30	0	.				. . .	
	24	17	9	18	11	19	11	20	10	19	10	20	11	21	11	22	11	23	11	24	9	25	7	26	6	27	4	28	1
3×14	12	25	0	26	7	28	0	29	4	30	0	.				..		..		.		....		. . .				. . .	
	16	23	3	24	7	25	11	27	1	27	4	28	10	30	0											..		..	
	24	20	7	21	11	23	0	24	1	22	11	24	1	25	4	26	5	27	6	28	6	29	6	30	0				

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)
Live load—30 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLIII. Rafter and Roof-Joist Spans (40-Pound Load—Group II Covering)

MAXIMUM SPANS FOR ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 40 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
2×6	12	8	8	9	4	9	9	10	2	9	9	10	4	10	9	11	4	11	9	12	2	12	8	13	1	13	6	13	10
	16	7	10	8	5	8	10	9	4	8	7	9	0	9	5	9	10	10	2	10	7	11	0	11	5	11	8	12	1
	24	7	0	7	5	7	9	8	2	7	0	7	5	7	9	8	1	8	5	8	8	9	0	9	4	9	7	9	10
2×8	12	11	7	12	4	13	0	13	6	13	0	13	8	14	4	15	0	15	7	16	2	16	9	17	4	17	9	18	5
	16	10	7	11	2	11	9	12	5	11	4	12	0	12	6	13	1	13	7	14	1	14	7	15	1	15	7	16	0
	24	9	4	9	10	10	5	10	10	9	4	9	9	10	4	10	9	11	2	11	7	12	0	12	5	12	9	13	2
2×10	12	14	7	15	6	16	4	17	1	16	4	17	2	18	0	18	9	19	7	20	4	21	1	21	9	22	5	23	1
	16	13	4	14	2	14	10	15	7	14	4	15	1	15	9	16	6	17	2	17	9	18	5	19	0	19	7	20	2
	24	11	8	12	6	13	1	13	8	11	8	12	5	13	0	13	7	14	1	14	8	15	2	15	8	16	2	16	8
2×12	12	17	7	18	8	19	8	20	7	19	7	20	8	21	8	22	7	23	7	24	5	25	4	26	1	27	0	27	8
	16	16	1	17	1	18	0	18	9	17	2	18	1	19	0	19	9	20	7	21	5	22	2	22	10	23	7	24	4
	24	14	2	15	1	15	10	16	7	14	2	15	0	15	9	16	5	17	1	17	8	18	4	18	10	19	6	20	1
2×14	12	20	6	21	10	23	0	24	0	22	10	24	0	25	3	26	4	27	5	28	5	29	5	30	0				
	16	18	10	20	0	21	1	22	1	20	0	21	1	22	1	23	1	24	1	25	0	25	11	26	9	27	6	28	4
	24	16	7	17	9	18	7	19	5	16	7	17	6	18	5	19	5	19	11	20	9	21	5	22	1	22	10	23	6

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)
Live load—40 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLIII (Continued). Rafter and Roof-Joist Spans (40-Pound Load—Group II Covering)

MAXIMUM SPANS FOR ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 40 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E =$ 1 000 000		$E =$ 1 200 000		$E =$ 1 400 000		$E =$ 1 600 000		$f =$ 900		$f =$ 1 000		$f =$ 1 100		$f =$ 1 200		$f =$ 1 300		$f =$ 1 400		$f =$ 1 500		$f =$ 1 600		$f =$ 1 700		$f =$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3× 6	12	10	3	10	10	11	5	11	11	12	4	13	0	13	7	14	3	14	10	15	5	15	11	16	5	16	11	17	5
	16	9	4	9	11	10	5	10	11	10	10	11	5	11	11	12	5	13	0	13	6	13	11	14	5	14	10	15	3
	24	8	3	8	9	9	3	9	7	8	11	9	5	9	10	10	3	10	9	11	1	11	6	11	10	12	3	12	7
3× 8	12	13	5	14	4	15	1	15	9	16	3	17	1	17	11	18	9	19	7	20	3	20	11	21	7	22	4	22	11
	16	12	4	13	1	13	10	14	5	14	3	15	0	15	9	16	5	17	3	17	9	18	5	18	11	19	6	20	1
	24	10	11	11	6	12	1	12	9	11	9	12	5	13	0	13	7	14	3	14	9	15	3	15	9	16	3	16	7
3×10	12	16	11	17	11	18	11	19	10	20	4	21	5	22	5	23	5	24	4	25	4	26	3	27	0	27	10	28	9
	16	15	6	16	6	17	5	18	1	17	10	18	10	19	9	20	7	21	5	22	3	23	0	23	10	24	6	25	3
	24	13	9	14	6	15	4	16	0	14	10	15	7	16	5	17	1	17	10	18	5	19	1	19	9	20	4	20	11
3×12	12	20	4	21	6	22	9	23	9	24	3	25	6	26	10	27	11	29	1	30	0	...	...	...	...	...	...	...	...
	16	18	7	19	10	20	10	21	10	21	5	22	6	23	7	24	9	25	9	26	7	27	7	28	6	29	5	30	0
	24	16	6	17	6	18	5	19	4	17	10	18	9	19	7	20	6	21	5	22	3	23	0	23	9	24	5	25	1
3×14	12	23	7	25	1	26	5	27	7	28	1	29	7	30	0	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	16	21	9	23	1	24	4	25	5	24	10	26	3	27	5	28	9	29	10	30	0	...	...	...	...	...	...	...	...
	24	19	4	20	6	21	7	22	6	20	9	21	11	22	11	23	11	24	11	25	11	26	10	27	7	28	6	29	4

NOTE: The lengths are based on:

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)

Live load—40 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLIV. Rafter and Roof-Joist Spans (50-Pound Load—Group II Covering)

MAXIMUM SPANS FOR ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 50 pounds per square foot

Maximum allowable lengths between supports or from plate to ridge (without collar-beams)

Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	$E =$ 1 000 000 Ft	In	$E =$ 1 200 000 Ft	In	$E =$ 1 400 000 Ft	In	$E =$ 1 600 000 Ft	In
2× 6	12	8	4	8	10	9	4	9	9
	16	7	7	8	0	8	5	8	10
	24	6	8	7	1	7	5	7	9
2× 8	12	11	0	11	9	12	4	12	11
	16	10	1	10	8	11	4	11	9
	24	8	10	9	5	9	11	10	5
2×10	12	14	0	14	9	15	7	16	4
	16	12	9	13	6	14	3	14	10
	24	11	2	11	11	12	6	13	1
2×12	12	16	9	17	10	18	9	19	8
	16	15	4	16	4	17	2	18	0
	24	13	6	14	4	15	1	15	10
2×14	12	19	5	20	9	21	10	22	10
	16	17	10	19	0	19	11	20	11
	24	15	9	16	9	17	5	18	4

Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span

Size	Spacing	$f =$ 900 Ft	In	$f =$ 1 000 Ft	In	$f =$ 1 100 Ft	In	$f =$ 1 200 Ft	In	$f =$ 1 300 Ft	In	$f =$ 1 400 Ft	In	$f =$ 1 500 Ft	In	$f =$ 1 600 Ft	In	$f =$ 1 700 Ft	In	$f =$ 1 800 Ft	In
2× 6	12	9	0	9	7	10	0	10	6	10	11	11	4	11	9	12	1	12	6	12	10
	16	7	11	8	3	8	8	9	0	9	5	9	10	10	2	10	6	10	10	11	1
	24	6	6	6	9	7	2	7	6	7	10	8	1	8	4	8	8	8	11	9	2
2× 8	12	12	0	12	8	13	4	13	11	14	6	15	0	15	7	16	1	16	7	17	0
	16	10	6	11	0	11	8	12	1	12	7	13	1	13	7	14	0	14	5	14	10
	24	8	7	9	0	9	6	9	11	10	4	10	9	11	1	11	6	11	10	12	2
2×10	12	15	2	16	0	16	9	17	6	18	3	18	11	19	7	20	2	20	10	21	5
	16	13	3	13	11	14	8	15	3	15	11	16	6	17	1	17	8	18	2	18	9
	24	10	10	11	6	12	0	12	7	13	1	13	7	14	1	14	6	15	0	15	5
2×12	12	18	3	19	2	20	2	21	0	21	11	22	9	23	6	24	3	25	0	25	9
	16	15	11	16	10	17	7	18	5	19	2	19	9	20	7	21	3	21	11	22	6
	24	13	2	13	10	14	6	15	2	15	9	16	5	17	0	17	6	18	0	18	7
2×14	12	21	0	22	3	23	3	24	4	25	4	26	3	27	1	28	0	28	11	29	9
	16	18	5	19	5	20	5	21	4	22	3	23	0	23	10	24	7	25	4	26	1
	24	15	3	16	1	16	11	17	7	18	4	19	0	19	9	20	4	21	0	21	7

NOTE: The lengths are based on

When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.

When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)

Live load—50 pounds per square foot of roof surface considered as acting normal to the surface.

Table XLIV (Continued). Rafter and Roof-Joist Spans (50-Pound Load—Group II Covering)

MAXIMUM SPANS FOR ROOF-JOISTS—UNIFORMLY LOADED

Any slope. Live load 50 pounds per square foot

Size of joists or rafters (nominal) in inches	Spacing of joists or rafters center to center in inches	Maximum allowable lengths between supports or from plate to ridge (without collar-beams)																											
		Limited by deflection of 1/360 of the span. Having determined by reference to the building code or Table I the allowable **modulus of elasticity** in pounds per square inch for the species of timber used, refer to the column below with the corresponding value to determine span								Determined by bending. Having determined by reference to the building code or Table I the allowable **extreme fiber-stress in bending** in pounds per square inch for the species and grade of lumber used, refer to the column below with the corresponding value to determine maximum safe span																			
		$E=$ 1 000 000		$E=$ 1 200 000		$E=$ 1 400 000		$E=$ 1 600 000		$f=$ 900		$f=$ 1 000		$f=$ 1 100		$f=$ 1 200		$f=$ 1 300		$f=$ 1 400		$f=$ 1 500		$f=$ 1 600		$f=$ 1 700		$f=$ 1 800	
		Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In	Ft	In
3×6	12	9	7	10	3	10	10	11	3	11	4	12	0	12	6	13	1	13	7	14	1	14	7	15	1	15	7	16	0
	16	8	10	9	4	9	10	10	4	9	11	10	5	11	0	11	5	11	11	12	4	12	10	13	3	13	7	14	0
	24	7	9	8	3	8	7	9	0	8	1	8	7	9	0	9	5	9	10	10	0	10	6	10	11	11	3	11	6
3×8	12	12	9	13	6	14	3	14	11	14	11	15	9	16	6	17	3	18	1	18	7	19	4	19	11	20	(	21	1
	16	11	9	12	5	13	0	13	7	13	1	13	10	14	6	15	1	15	10	16	4	16	11	17	5	18	0	18	6
	24	10	3	10	11	11	6	12	0	10	10	11	5	11	11	12	5	13	1	13	6	13	11	14	5	14	10	15	4
3×10	12	16	1	17	0	17	11	18	10	18	10	19	10	20	10	21	9	22	7	23	5	24	3	25	0	25	10	26	7
	16	14	9	15	7	16	5	17	3	16	5	17	4	18	3	19	0	19	9	20	6	21	3	21	11	22	7	23	3
	24	12	11	13	10	14	6	15	1	13	7	14	4	15	0	15	9	16	4	16	11	17	6	18	1	18	9	19	3
3×12	12	19	3	20	6	21	6	22	6	22	5	23	7	24	9	25	10	26	11	27	11	28	11	29	10	30	0	...	...
	16	17	9	18	10	19	10	20	9	19	9	20	10	21	10	22	10	23	9	24	7	25	5	26	4	27	1	27	11
	24	15	7	16	7	17	6	13	3	16	4	17	3	18	1	18	11	19	7	20	5	21	1	21	10	22	6	23	1
3×14	12	22	5	23	10	25	1	26	3	26	0	27	5	28	9	30	0			.	.	..		..		.	.	.	..
	16	20	7	21	11	23	1	24	1	22	11	24	3	25	5	26	6	27	7	28	7	29	7	30	0		.	.	.
	24	18	3	19	5	20	5	21	5	19	1	20	1	21	1	22	0	22	11	23	10	24	7	25	6	26	3	27	0

NOTE: The lengths are based on:
When limited by deflection—
Maximum allowable deflection of 1/360 of span length.
Modulus of elasticity as noted for E.
When determined by bending strength of the piece—
Allowable stress in extreme fiber in bending as noted for f.

Dead load—Weight of roof joist.
Weight of roof sheathing (2.5 pounds per square foot).
Weight of roof covering (8 pounds per square foot). (Group II.)
Live load—50 pounds per square foot of roof surface considered as acting normal to the surface.

16. Determination of Size and Spacing of Wood Joists and Rafters

In general, the loadings on floor-joists, beams, rafters and girders in light wood construction may be reduced to the four following cases:

(1) Simple uniformly distributed floor or roof load
(2) Uniformly distributed floor or roof-load plus a concentrated load
(3) Header and trimmer-beams
(4) Girders.

Dead and Live Loads. In all cases the weights or loads must first be determined. These consist of the DEAD LOADS and the LIVE LOADS. By DEAD LOAD is meant the weight of the construction coming upon the beams, such as the weight of the flooring or roofing, the weight of the ceiling and the weights of partitions or of other floor-beams supported by the beam in question By LIVE LOAD is meant the weight of the furniture, merchandise, machinery, occupants and other superimposed loads upon the floors, not included in the constructive features of the building. Most building laws fix this live load for residences at 40 lb per sq ft for finished and occupied floors, and at 20 lb per sq ft for unfinished attics. Table XLVI gives the minimum live or superimposed loads required by various building codes and as recommended by the United States Department of Commerce.

Two layers of $1\frac{3}{16}$-in dressed flooring weigh about 5 lb per sq ft, or 2.5 lb for the rough floor and 2 5 lb for the finished floor.

A plastered ceiling weighs 10 lb per sq ft.

A partition constructed of 2 × 4-in studs plastered on both sides weighs 20 lb per sq ft of face.

The weight of the beam itself must be assumed and added to the dead loads. Table II gives the weight per linear foot of many sizes of wood beams based upon an average weight of 40 lb per cu ft. Table XLV gives the weights of various sizes of wood joists and rafters per square foot of floor or roof-area supported by each member for both 12-in and 16-in spacing.

In preparing Tables XXI to XLIV the dead load has been allowed for in determining the sizes and spans, so that in using these tables only the live load need be considered. In using Tables VII to XX for girders, however, the total distributed dead and live load must first be computed before entering the tables.

In 1921 the Building Code Committee of the United States Department of Commerce was organized to study greater economy and uniformity in building-code requirements. Early in the committee's work efforts were made to collect data on actual superimposed loads on the floors of buildings of different types to be used in recommending standard live loads to be used in building codes. Their report on minimum allowable live loads was published in 1925, and the following data are taken by permission from this report. "It was found that live loads assumed in designing many types of buildings were largely matters of tradition and had scant scientific basis. The result was that accuracy in stress computations was defeated because of ignorance of the loads causing stresses. The building professions for years have busied themselves with tests of materials, but have given little attention to the complementary factor of loads.

(1) Human Occupancy Floor Loads. "For rooms of private dwellings, hospital-rooms and wards, guest-rooms in hotels, lodging and tenement-houses and for similar occupancies, the minimum live load shall be taken as

40 lb per sq ft uniformly distributed, except that where floors of one and two family dwellings are of monolithic type or of solid or ribbed slabs the live load may be taken as 30 lb per sq ft."

The heaviest furniture loads were found to be pianos weighing up to 55 lb per sq ft and bookcases weighing up to 170 lb per lin ft, but in both cases the distribution was such as to bring the equivalent uniform load well below that specified above. The furniture in typical hotel guest-rooms weighed about 4.1 lb per sq ft for a 11 by 18-ft room.

The loads in hospital wards averaged in several institutions from 6.9 lb to 9.0 lb per sq ft.

It was considered wise, however, to fix the minimum loads at 40 lb for this class of building, to provide for contingencies, such as the congregation of an unusual number of occupants in one room.

(2) Floor-Loads in Offices. "For floors for office purposes and for rooms with fixed seats, as in churches, school class-rooms, reading-rooms, museums, art galleries and theaters, the minimum live load shall be taken as 50 lb per sq ft uniformly distributed. Provision shall be made, however, in designing office floors for a load of 2 000 lb placed upon any space 2½ ft sq wherever this load upon an otherwise unloaded floor would produce stresses greater than the 50-lb distributed load."

Many careful tests and examinations of actual conditions in offices have been made, particularly by Mr. C. T. Coley in the Equitable Building, New York, Mr. M. W. McIntire in the Union Central Life Insurance Company's Building, Cincinnati, and by Mr. C. H. Blackall and Mr. A. C. Everett in four office-buildings in Boston. The heaviest loads were from filing-cabinets, safes and bookcases. The average loads per square foot ranged from 7 to 16 lb with maximum loads not exceeding 40 lb. Steel filing-cases filled with maps weighed 21.7 lb per cu ft, with correspondence 26.4 lb, and with cards 47 lb. Wood filing-cases filled with correspondence weighed 21.2 lb per cu ft, and filled with cards 31.8 lb.

(3) Class-Rooms. Standard class-rooms investigated by Mr. N. M. Stineman contained about 740 sq ft of floor-area and accommodate a teacher and 45 to 48 pupils. The total weight of furniture and inmates was about 7 500 lb or 10 lb per sq ft. If filled with adults, as in night school, the weight amounted to 12.9 lb per sq ft. When crowded with 258 pupils, two in each seat and all aisles and open spaces filled, the resulting weight including desks was 41.7 lb per sq ft. Other investigators put the live load in class-rooms normally filled at 14 lb per sq ft and at 22 lb if the aisles are crowded.

Even with due allowance for increased weight of adults the Committee believed that the minimum load limit set for schools should apply also to theaters and other indoor places of assembly with fixed seats.

(4) Miscellaneous Floor-Loads. "For aisles, corridors, lobbies, public spaces in hotels and public buildings, banquet rooms, assembly halls without fixed seats, grandstands, theater stages, gymnasiums, stairways, fire-escapes or exit passageways, and other spaces where crowds of people are likely to assemble, the minimum live load shall be taken as 100 lb per sq ft uniformly distributed. This requirement shall not apply, however, to such spaces in private dwellings, for which the minimum live load shall be taken as in Paragraph 1 of this section.

(5) Industrial and Commercial Occupancy Floor-Loads. "In designing floors used for industrial or commercial purposes or purposes other than previously mentioned, the live load shall be assumed as the maximum caused by the use which the building or part of the building is to serve. The following

loads shall be taken as the minimum live loads permissible for the occupancies listed, and loads at least equal shall be assumed for uses similar in nature to those listed in this section.

Floors used for:	Minimum live load, lb per sq ft
Storage purposes (general)	250
Storage purposes (special)	100
Manufacturing (light)	75
Printing plants	100
Wholesale stores (light merchandise)	100
Retail salesrooms (light merchandise)	75
Stables	75
Garages	
All types of vehicles	100
Passenger cars only	80

Sidewalks—250 lb per sq ft or 8 000 lb concentrated, whichever gives the largest moment or shear."

For buildings of the above-mentioned types the Committee considered it more fair and economical to the builders to recommend minimum live load requirements which would permit design of buildings in each case for the purposes intended, rather than to recommend high loads to cover a broad general type of building, which would be safe under all circumstances. In this connection, however, the Committee observes that the designer must bear in mind that the building may be used in the future for purposes involving greater loads and that by employing the minimum loads for one purpose the

Table XLV. Weight of Joists and Rafters per Square Foot of Floor or Roof

Sizes of joists	Spruce, Hemlock		Yellow pine, Oak	
	Spacing in inches, center to center		Spacing in inches, center to center	
	12	16	12	16
inches	lb	lb	lb	lb
2×6	3	2¼	4	3
2×8	4	3	5⅓	4
3×8	6	4½	8	6
2×10	5	3¾	6⅔	5
3×10	7½	5½	10	7½
2×12	6	4½	8	6
3×12	9	6¾	12	9
2×14	7	5¼	9⅓	7
3×14	10½	8½	14	10½

future adaptability of the structure may be impaired. The minimum allowable loads recommended have been derived from the study of very complete investigations of actual loads resulting in storage buildings, warehouses, manufacturing plants, wholesale and retail stores and garages. The load for sidewalks of 250 lb was taken in view of the possibility that coal, paving materials or delivered goods might be piled upon them temporarily, which would approximate conditions in a general storage warehouse.

Table XLVI. Minimum Live Loads Required by Building Codes

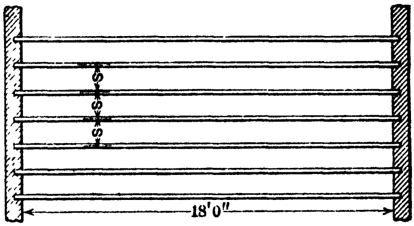
Classes of buildings	Minimum live loads per square foot of floor						
	New York 1927	Phila-delphia 1929	Boston 1926	Chicago 1928	Den-ver 1927	San Fran-cisco 1928	Dept. of Com-merce
Dwellings.	40	40	50	40	60 and 40	40	40
Hotels, Tenements, Lodging-houses, Hospitals	40	40	50	40	90 and 70	40	40
Office-buildings: First floor	100	100	125	125	120	125	100
Other floors	60	60	60	40	70	40	50
School Class-rooms	75	50	50	75	75	75	50
Buildings or rooms for public assembly: With fixed seats	100	60	100	75	90	75	50
Without fixed seats	100	100	100	125	120	125	100
Aisles and corridors	100	100	100	125	120	125	100
Garages: Public	120	100	150	100	150	100	100
Private	120	100	75	100	150	100	80
Warehouses	120	150	125–250	125–250	200	125–250	100–250
Manufacturing: Heavy	120	200	250	250	250	250	100
Light	120	120	125	125	120	125	75
Stores: Wholesale	120	110	250	250	120	125	100
Retail	120	110	125	125	120	100	75
Sidewalks	300	120	250	150	150	150	250

17. Examples

Case I

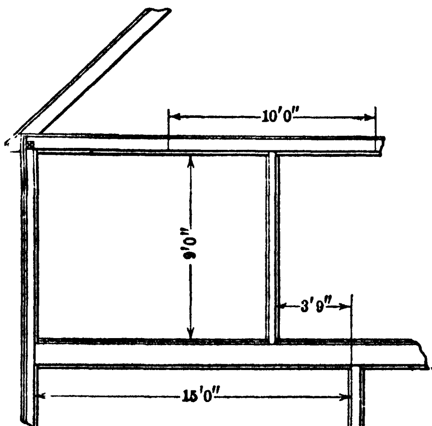
Uniformly Distributed Load. See Diagram, Case I. The simplest case where all the joists are of the same span and support equal floor-areas. S = the spacing of the joists on center and L = the span. Assume the building

to be a residence, the span to be 18 ft and that select-grade spruce joists will be used. Ascertain the allowable working stresses for spruce and the live load per square foot for residences from the local building code, or find the extreme fiber-stress and modulus of elasticity from Table I, and assume the live load to be 40 lb per sq ft. In the latter case Table I gives 1 100 lb per sq in as the allowable fiber-stress for spruce and 1 200 000 as the modulus of elasticity. Then from Table XXII for live load of 40 lb with plastered ceiling, under column headed by 1 100, it is found that for a span of 18 ft 3 in, spruce joists 2 in × 12 in in size spaced 16 in on centers are sufficient to withstand the bending moment. If, however, it be desired to limit the deflection to $\frac{1}{360}$ of the span, the column headed $E = 1\ 200\ 000$ should be entered and it is found that for a span of 18 ft 3 in, joists 2 in × 12 in in size and 12 in on centers are required.



Case I. Plan of Floor Joists

If it were decided to use a stronger wood such as Southern yellow pine, select grade, which has an allowable fiber-stress of 1 600 lb and a modulus of elasticity of 1 600 000 lb, then from Table XXII it is found that 2 in × 10 in, 16 in on centers or 3 in × 8 in, 16 in on centers, would be sufficient to withstand bending, and 2 in × 12 in or 3 in × 10 in, both 16 in on centers, would be required if limited by deflection.



Case II. Section through Floors and Partitions

Case II

Distributed Load and Concentrated Partition Load. See Diagram, Case II. Second-floor joists supporting a partition which in turn supports the attic joists. Assume that the second-floor joists are of spruce, that the attic joists are 2 in × 8 in, 16 in on centers, and that the width of attic floor supported by the partition is 10 ft 0 in. Assume also that the second floor has a double flooring, the attic a single flooring and that both first- and second-story ceilings are plastered.

The concentrated load due to the partition is first found and transformed into an equivalent uniformly distributed load. From Table II it is seen that a 2 × 8-in joist weighs 3.4 lb a lin ft of joist, or with 16-in spacing about 3 lb per lin ft of partition. The ceiling weighs 10 lb per sq ft and the single flooring 2.5 lb per sq ft. The live load in the attic will be assumed at 20 lb per sq ft.

Then the weight on the partition from the attic joists will equal per square foot.

2.5	lb	weight of flooring
10.	lb	weight of ceiling
12 5	lb	dead load
20.	lb	live load
32.5	lb	total per square foot
10		= span in feet
325	lb	
30	lb	weight of attic joists (3×10 ft)
355	lb	= total load per linear foot on partition

Weight of 2×4-in stud partition plastered both sides = 20 lb per sq ft of face; $20 \times 9 = 180$ lb per lin ft; $180 + 355 = 535$ lb per lin ft = weight of concentrated load on each second-story joist at a point $\frac{1}{4}$ of span from bearing of joist. From Table V it is found that this load must be multiplied by the factor 1.5 to be transformed into an equivalent uniformly distributed load, $535 \times 1.5 = 802\ 5$ lb or $\frac{802\ 5}{15} = 54$ lb per sq ft.

The uniformly distributed live load on the second-story joists is taken at 40 lb per sq ft. Therefore the total uniformly distributed load on each second-story joist will be $54 + 40 = 94$ lb per sq ft.

From Table XXX it is found that 2×12-in or 3×10-in spruce joists 12 in on centers will be sufficient to withstand the bending and that 2×14-in or 3×12-in joists 16 in on centers will not deflect over 1/360 of the span. The 2×12-in and the 2×14-in pieces are more economical unless headroom under the joists must be considered.

Case III

Header and Trimmer-Beams and Girders. See Diagram, Case III. Data: Height of story = 10 ft. Double floor, ceiling and partition plastered. Floor-joists 16 in on centers. Use structural-grade yellow-pine timber. Span 24 ft 0 in. Live load = 40 lb per sq ft.

Common Joists A. From Table I we find the allowable stress in extreme fiber for structural yellow-pine timber to be 1 600 lb per sq in and modulus of elasticity 1 600 000 lb. From Table XXII for a live load of 40 lb. per sq ft and plastered ceiling we find under the column headed 1 600 lb fiber-stress that for a 24 ft span, 2×14-in joists 16 in on centers are required to resist the bending, and under the column headed $E = 1\ 600\ 000$ that 3×14-in joists 16 in on centers are required to limit the deflection to 1/360 of the span.

Joist B. Required to support a partition as well as the floor-load but since the span is only 18 ft and the partition is near the bearing of the joist it will be safe to make these joists the same size as joist A, or 3 in $\times$ 14 in, 16 in on centers.

Joist C has the same floor-load as joist A and in addition the concentrated load of a partition at third point of span. Partition is 10 ft high, therefore 10 ft $\times 1\frac{1}{3}$ or $13\frac{1}{3}$ sq ft will be supported on each joist with 16 in

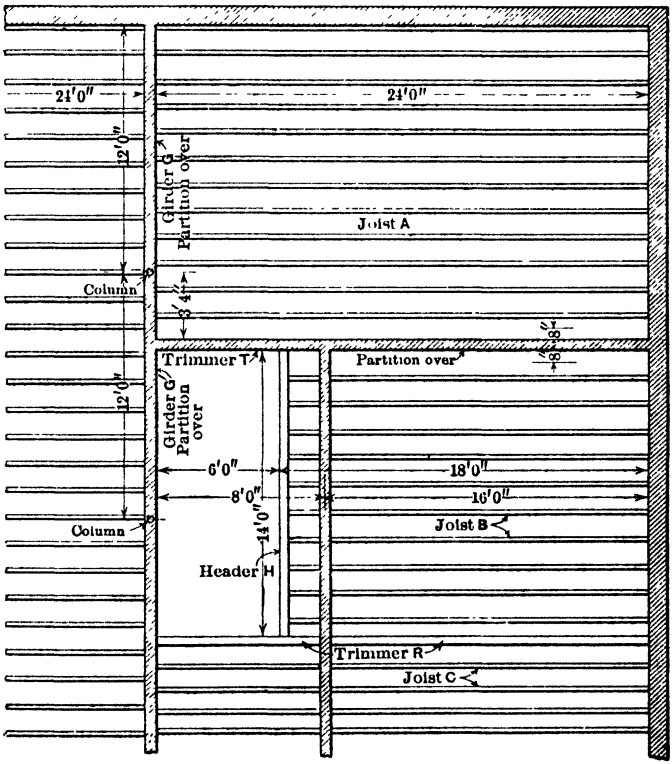
spacing. If a plastered partition weigh 20 lb per sq ft, $13.33 \times 20 = 267$ lb. To reduce to equivalent distributed load multiply by 1.78. (See Table V.)

$$267 \times 1.78 = 468 \text{ lb}$$

$$\frac{468}{24} = 19\tfrac{1}{2} \text{ lb per lin ft of joist}$$

$$\underline{40} \quad \text{lb live load}$$

$$59\tfrac{1}{2}, \text{ say } 60 \text{ lb} = \text{total load}$$



Case III. Plan of Floor Framing showing Partitions above

From Table XXIV it is found that 3×14-in joists are sufficient to limit the deflection but that they must be spaced 12 in on centers instead of 16 in. For bending, 2×14 in spaced 12 in on centers would be sufficient. If the load of the partition on each joist were refigured using a 12-in spacing instead of a 16-in no difference would be found in the size of joist required in this case.

It is generally best if possible to select joists of the same depth throughout, varying the thickness and spacing according to the varying loads, so that the ceilings may all be on the same level.

Header H. We will use 14 in as the depth of header, the same as the joists, and we must find the thickness. For single beams and girders Table XVIII must be used and the total live and dead loads determined. Span = 14 ft 0 in.

Floor-loads per square foot

Dead	$10\frac{1}{2}$ lb	– weight of joist (Table XLV)
	5 lb	– weight of floor
	10 lb	– weight of ceiling
	$25\frac{1}{2}$ lb	
Live.	40	
Total.	$65\frac{1}{2}$ lb per sq ft	

Floor area = 14 × 9 = 126 sq ft, 126 × $65\frac{1}{2}$ = 8 253 lb = floor-load.

Partition 14 ft 0 in – 1 ft 4 in = 12 ft 8 in by 10 ft 0 in = 126.6 sq ft

Multiplying by 20 lb, 126.6 × 20 = 2 532 lb.

Since the partition is situated $\frac{1}{9}$ of span from header, $\frac{8}{9}$ of load will rest on the header. $\frac{2\,532}{9} \times 8 = 2\,251$ lb

2 251 lb	= partition load
8 253 lb	= floor-load
10 504 lb	= total load on header

From Table XVIII we find that for a 14 ft 0 in span the determining stress is that of horizontal shear rather than bending and that a 1 by 14 in beam will support 2 489 lb of distributed load on a 14 ft 0 in span. Therefore a load of 10 504 lb would require a beam $\frac{10\,504}{2\,489}$ or 4 22 in wide. We will slect a 5 × 14 beam.

Trimmer T. This beam has four loads:

(1) Distributed floor-load
(2) Distributed load from partition
(3) One-half load from header
(4) Small load from longitudinal partition

The span is 24 ft 0 in.

(1) The strip of floor is 12 in wide and 24 ft 0 in long, or 24 sq ft. 24 × $65\frac{1}{2}$ = 1 572 lb.

(2) The partition is 10 ft 0 in high by 24 ft 0 in long and has 240 sq ft of surface. 240 × 20 lb = 4 800 lb.

(3) One-half load from header is $\frac{10\,504}{2}$ = 5 252 lb. This is a concentrated load at $\frac{1}{4}$ span. Multiplying by 1 5 to find equivalent distributed load we have 7 878 lb.

(4) The end section about 8 in long of the longitudinal partition is supported by the trimmer.

10 × $\frac{2}{3}$ × 20 = 133 lb, concentrated at $\frac{1}{3}$ span.

Equivalent distributed load = 133 × 1.78 = 237 lb.

1 572 lb	floor-load
4 800 lb	partition-load
7 878 lb	header-load
237 lb	longitudinal partition-load
14 487 lb	total load

The trimmer should be the same depth as the joists to give an even ceiling underneath. From Table XVIII we find that a 1 by 14-in beam will support a load of 1 452 lb on a 24 ft 0 in span. Therefore a load of 14 487 lb will require a beam $\frac{14\ 487}{1\ 452}$ or 10 in wide.

We therefore select a 10 × 14-in beam.

Trimmer R. The load will be the same as for trimmer T except for the cross partition. Deducting 4 800 from 14 487 we have 9 687 lb. Dividing by 1 452 we find a width of 7 in is required. We therefore select a 7 × 14-in beam.

Girder G. The floor area supported is 12 ft 0 in × 24 ft 0 in or 288 sq ft. We may estimate that only 85% of the live load on the floor-joists will at any one time be supported by the girder. We therefore take 34 lb per sq ft instead of 40 lb as live load. The 3 × 14-in joists 16 in on centers framing into the girder will weigh 10½ lb per sq ft of floor and the floor and ceiling 15 lb. The girder itself will weigh about 40 lb per lin ft or 40 × 12 = 480 lb for the entire girder; 480 divided by 288 = 1½ lb = the weight of girder per square foot of floor.

10½	lb joists
5	lb floor
10	lb ceiling
1½	lb girder
27	lb dead load
34	lb live load
61	lb total floor-load per square foot

(1) Total floor-load = 288 × 61 lb = 17 568 lb
(2) Partition-load = 12 × 10 × 20 lb = 2 400 lb
Total load 19 968 lb

Assuming 14 in as depth of girder we find from Table XVIII that a 1 × 14-in beam will support 2 489 lb on a 12 ft 0 in span. We will therefore use a beam $\frac{19\ 968}{2\ 489}$ or 8 in in width and 14 in in depth.

Girder G¹. (1) Floor-load = 12 × 12 = 144 sq ft; 144 × 61 lb = 8 784 lb floor-load on left of girder. Strip of floor 12 ft 0 in long by 40 in wide = 40 sq ft; 40 × 61 lb = 2 240 lb floor-load on right of girder, 8 in of floor being included in load on T.

This strip may be considered as concentrated at a point 20 in or 1/7 span from end of girder. The factor is 0.98, which is practically the same as if the load were distributed over the whole span.

(2) Trimmer-load equals ½ distributed load on T plus ⅜ (½ of ¾) of the load on H. The actual distributed load on T was found to be 1 572 + 4 800 = 6 372 and one-half of this is 3 186 lb. The load from H is applied at a distance from G equal to ¼ span of T, therefore ¾ of the load would fall upon G. But only ½ total load on H is transmitted to trimmer T, the other half falling on trimmer R. Therefore, the load transmitted to girder G from header H will be ⅜ × 10 504 or 3 939 lb. The total load from T will therefore be 3 186 + 3 939 or 7 125 lb. This load is concentrated at one-third of span from support, therefore it must be multiplied by 1.78 to obtain the equivalent distributed load: 7 125 lb × 1.78 = 12 682.5 lb, equivalent distributed load.

(3) Partition-load = 12 × 10 × 20 lb = 2 400 lb

The total load on girder G will be

8 784 lb	floor on left side
2 240 lb	floor on right side
12 682 lb	trimmer T
2 400 lb	partition
26 106	

Assuming 14 in as depth of girder we find from Table XVIII that a 1×14-in beam will support 2 489 lb on a 12 ft 0 in span. We will therefore use a beam $\frac{26\ 106}{2\ 489}$ or 11 in wide by 14 in deep.

RECAPITULATION. All timber structural-grade Southern pine.

Joist A	3 in $\times$ 14 in — 16 in on centers
Joist B	3 in $\times$ 14 in — 16 in on centers
Joist C	3 in $\times$ 14 in — 12 in on centers
Header H	5 in $\times$ 14 in
Trimmer T	10 in $\times$ 14 in
Trimmer R	7 in $\times$ 14 in
Girder G	8 in $\times$ 14 in
Girder G'	11 in $\times$ 14 in

The sizes shown here are sufficiently large to support the given loads, but in practice the timbers used would be the nearest stock sizes The 5 in $\times$ 14 in, 7 in $\times$ 14 in and 11 in $\times$ 14 in would be respectively 6 in $\times$ 14 in, 8 in $\times$ 14 in and 12 in $\times$ 14 in.

The three cases of this example illustrate nearly all the computations necessary to figure the sizes of the joists, special beams and girders in any ordinary floor-construction. As will be seen, the most laborious calculations are those for beams receiving loads from different sources, and it will generally be found that the weakest portions of any particular floor are the headers, trimmers and girders and the beams which support partitions.

18. Timber Columns

Table XLVII * gives the safe loads in pounds per square inch of cross-sectional area for timber columns of various ratio of length to least cross-section dimension, L/d. After finding the ratio of length to least dimension of a given timber column it is a simple process to determine its safe load by finding from the table the safe load in pounds per square inch for that L/d for the species and grade used, and then to multiply it by the cross-sectional area of the column. For short columns for which L/d does not exceed 11, the full working compression value of the material is used. This table was published by the National Lumber Manufacturers' Association and is based upon conclusions drawn by the Forest Products Laboratory from its extensive series of tests on timber columns. The same data were utilized by the Building Code Committee of the Department of Commerce in its recommendations.

To calculate the safe load on a round column it is only necessary to find the L/d of a square timber of the same cross-sectional area, then having ascertained the safe load per square inch for that L/d, multiply it by the cross-sectional area in square inches. The value of d (the side of an equivalent square) for a round column is a little more than $\frac{7}{8}$ of its diameter.

For tapered columns the diameter should be measured at a section $\frac{1}{3}$ the length from the small end. In no case should the stress at the small end of a tapered column exceed the allowable stress for a column with an L/d of 10 or less.

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Table XLVII. Working Stresses in Pounds per Square Inch for Posts and Timbers 6 In × 6 In and Larger

(Continuously dry)

Ratio of Length to Least Dimension L/D

Species	American standard grade	10 and less	12	14	16	18	20	25	30	35	40	50	Modulus of elasticity
Cedar, Alaska	Select	793	786	774	753	726	688	526	36	268	206	132	1 200 000
	Common	636	632	627	617	602	582	500					
Northern and Southern white	Select	545	540	530	516	496	468	351	244	179	137	88	800 000
	Common	437	435	430	423	412	398	338					
Port Orford	Select	890	879	861	834	796	741	526	365	268	206	132	1 200 000
	Common	715	709	700	686	667	639	521					
Western red	Select	693	686	674	656	629	592	438	304	224	171	110	1 100 000
	Common	557	553	547	538	524	505	425					
Cypress, Southern	Select	1 082	1 063	1 030	981	909	810	526	365	268	206	132	1 200 000
	Common	870	861	843	818	781	729						
Douglas fir, Coast type	Dense Select	1 268	1 250	1 221	1 177	1 113	1 024	702	487	358	274	175	1 600 000
	Select	1 162	1 149	1 127	1 093	1 045	975	702					
	Dense Common	1 025	1 017	996	965	935	893	698					
	Common	876	870	861	847	826	796	675					
Rocky Mountain type	Select	793	786	774	753	726	688	526	365	268	206	132	1 200 000
	Common	636	632	627	617	602	582	500					
Fir, balsam	Select	693	686	674	656	629	592	438	304	224	171	110	1 000 000
	Common	557	553	547	538	524	505	425					
Commercial white	Select	694	689	678	664	641	611	482	335	246	188	121	1 100 000
	Common	557	554	549	542	530	515	449					

Table XLVII* (Continued). Working Stresses in Pounds per Square Inch for Posts and Timbers 6 In × 6 In and Larger

(Continuously dry)

Species	American standard grade	10 and less	12	14	16	18	20	25	30	35	40	50	Modulus of elasticity
		Ratio of Length to Least Dimension L/D											
Hemlock: Eastern	Select	694	689	678	664	641	611	c 482	335	246	188	121	1 100 000
	Common	557	554	549	542	530	515	449					
West Coast	Select	893	885	872	852	823	783	614	426	313	240	153	1 400 000
	Common	716	712	706	696	680	660	573					
Larch: Western	Select	1 085	1 068	1 041	999	937	851	570	396	291	223	142	1 300 000
	Common	872	863	849	828	798	752						
Pine: Calif, Idaho and Northern white, pondosa and sugar	Select	742	733	718	695	663	617	438	304	224	171	110	1 100 000
	Common	596	591	583	572	556	532	434					
Norway	Select	793	786	774	753	726	688	526	365	268	206	132	1 200 000
	Common	636	632	627	617	602	582	500					
Southern yellow	Dense Select	1 268	1 250	1 221	1 177	1 113	1 024	702	487	358	274	175	1 600 000
	Dense Common	1 025	1 017	996	965	935	893	698					
	Common	876	870	861	847	826	796	675					
Redwood	Select	986	972	947	910	856	781	526	365	268	206	132	1 200 000
	Common	793	786	773	754	726	638						
Spruce, Engelmann	Select	594	586	574	556	530	494	351	244	179	137	88	800 000
	Common	476	473	466	457	444	426	347					
Red, white and Sitka	Select	793	786	774	753	726	688	526	365	268	206	132	1 200 000
	Common	636	632	627	617	602	582	500					
Tamarack	Select	988	976	955	923	877	817	570	396	291	223	142	1 300 000
	Common	794	788	777	761	737	706	566					

19. Stud Partitions

In Table XLVIII is shown the strength of stud partitions of Southern yellow pine. In using this table the weight of plaster and ceiling should be added to the values given.

Table XLVIII.* Stud Partitions—Southern Yellow Pine

Weight and strength based on actual size. Board measure based on nominal size. Add weight of plaster and ceiling: single plate at top and bottom included, same size as studs:

Safe load based on studs being bridged at center

Nominal size, inches	Actual size, inches	Distance on centers, inches	Height, feet	Per linear foot of partition		
				Safe load, pounds	Weight, pounds	Board feet
2×4	1⅝×3⅝	12	8	3 723	16.30	6.66
2×4	1⅝×3⅝	12	10	3 180	19.56	8.00
2×4	1⅝×3⅝	12	12	2 631	22.82	9.33
2×4	1⅝×3⅝	16	8	2 793	13.04	5.33
2×4	1⅝×3⅝	16	10	2 385	15.50	6.33
2×4	1⅝×3⅝	16	12	1 974	18.00	7.33
2×6	1⅝×5⅝	12	8	5 767	25.30	10.00
2×6	1⅝×5⅝	12	10	4 926	30.56	12.00
2×6	1⅝×5⅝	12	12	4 076	35.42	14.00
2×6	1⅝×5⅝	16	8	4 326	20.24	8.00
2×6	1⅝×5⅝	16	10	3 699	24.03	9.50
2×6	1⅝×5⅝	16	12	3 057	27.83	11.00
2½×6	2⅛×5⅝	12	8	8 568	33.20	12.50
2½×6	2⅛×5⅝	12	10	7 767	39.84	15.00
2½×6	2⅛×5⅝	12	12	6 871	46.48	17.50
2½×6	2⅛×5⅝	16	8	6 426	26.56	10.00
2½×6	2⅛×5⅝	16	10	5 825	31.56	12.00
2½×6	2⅛×5⅝	16	12	5 153	36.52	13.75
3×6	2⅝×5⅝	12	8	11 388	41.00	15.00
3×6	2⅝×5⅝	12	10	10 516	49.20	18.00
3×6	2⅝×5⅝	12	12	9 689	57.20	21.00
3×6	2⅝×5⅝	16	8	8 541	32.80	12.00
3×6	2⅝×5⅝	16	10	7 887	38.95	14.25
3×6	2⅝×5⅝	16	12	7 267	45.10	16.50
2×8	1⅝×7½	12	8	7 692	33.80	13.33
2×8	1⅝×7½	12	10	6 570	40.56	16.00
2×8	1⅝×7½	12	12	5 436	47.32	18.66
2×8	1⅝×7½	12	14	4 315	54.08	21.33
2×8	1⅝×7½	16	8	5 769	27.04	10.66
2×8	1⅝×7½	16	10	4 927	32.11	12.66
2×8	1⅝×7½	16	12	4 077	37.18	14.66
2×8	1⅝×7½	16	14	3 236	42.25	16.66
2½×8	2⅛×7½	12	8	11 429	44.30	16.66
2½×8	2⅛×7½	12	10	10 361	53.16	20.00
2½×8	2⅛×7½	12	12	9 165	62.02	23.33
2½×8	2⅛×7½	12	14	8 066	70.88	26.66
2½×8	2⅛×7½	16	8	8 572	35.44	13.33
2½×8	2⅛×7½	16	10	7 771	42.08	15.83
2½×8	2⅛×7½	16	12	6 874	48.73	18.33
2½×8	2⅛×7½	16	14	6 049	55.37	20.83
3×8	2⅝×7½	12	8	15 181	54.70	20.00
3×8	2⅝×7½	12	10	14 019	65.64	24.00
3×8	2⅝×7½	12	12	12 916	76.58	28.00
3×8	2⅝×7½	12	14	11 814	87.52	32.00
3×8	2⅝×7½	16	8	11 386	43.76	16.00
3×8	2⅝×7½	16	10	10 514	51.96	19.00
3×8	2⅝×7½	16	12	9 687	60.17	22.00
3×8	2⅝×7½	16	14	8 860	68.37	25.00

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20. Mill Floors

Strength of Mill Floors. The beams and girders for mill floors should be computed by the same general method illustrated in the foregoing examples or by the formulas given at the beginning of the chapter, involving the determination, first, of the loads on the beams and girders and, secondly, the sizes of beams and girders required to support such loads.

In designing the flooring in mills Table XLIX may be consulted for flooring laid flat, generally known as MILL FLOORS, and Table L for flooring laid on edge, or LAMINATED FLOORS. Fiber-stresses are given from 1 200 to 1 800 lb per sq. in and loads from 50 to 400 lb. In both tables the spans are shown having a deflection of $\frac{1}{30}$ in per ft of span. These tables are taken by permission from the Southern Pine Manual of the Southern Pine Association.

Table XLIX. Maximum Spans for Matched and Dressed Mill Floors

Nominal thickness, inches	Actual thickness, inches	Fiber-stress pounds per square inch	Span in feet — Live loads in pounds per square foot: 50		100		125		150		175		200	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
2	1⅝	1 200	8	9	6	4	5	8	5	3	4	10	4	6
2	1⅝	1 300	9	1	6	7	5	11	5	5	5	10	4	9
2	1⅝	1 500	9	9	7	1	6	4	5	10	5	5	5	1
2	1⅝	1 600	10	1	7	4	6	7	6	0	5	7	5	3
2	1⅝	1 800	10	8	7	9	7	0	6	5	5	11	5	7
2	1⅝	Deflection	5	8½	4	7	4	3	4	0½	3	10	3	8
2½	2⅛	1 200	11	3	8	2	7	5	6	9	6	4	5	11
2½	2⅛	1 300	11	9	8	7	7	9	7	1	6	7	6	2
2½	2⅛	1 500	12	7	9	2	8	3	7	7	7	0	6	7
2½	2⅛	1 600	13	0	9	6	8	6	7	10	7	3	6	10
2½	2⅛	1 800	13	10	10	1	9	1	8	4	7	8	7	3
2½	2⅛	Deflection	7	4½	5	11½	5	7	5	3	5	0	4	9½
3	2⅝	1 200	13	8	10	1	9	1	8	4	7	9	7	3
3	2⅝	1 300	14	3	10	6	9	6	8	8	8	1	7	7
3	2⅝	1 500	15	4	11	3	10	2	9	4	8	8	8	2
3	2⅝	1 600	15	10	11	8	10	6	9	7	8	11	8	4
3	2⅝	1 800	16	9	12	4	11	2	10	3	9	6	8	11
3	2⅝	Deflection	9	0	7	4	6	10	6	5½	6	1½	5	10½
4	3⅝	1 200	18	5	13	8	12	4	11	5	10	7	10	0
4	3⅝	1 300	19	2	14	3	12	11	11	10	11	0	10	4
4	3⅝	1 500	20	7	15	4	13	10	12	9	11	10	11	2
4	3⅝	1 600	21	3	15	10	14	4	13	2	12	3	11	6
4	3⅝	1 800	22	7	16	9	15	2	13	11	13	0	12	2
4	3⅝	Deflection	12	2½	10	0	9	4½	8	10½	8	5½	8	1⅜
5	4⅝	1 200	22	10	17	8	15	7	14	5	13	5	12	7
5	4⅝	1 300	23	10	17	11	16	3	14	11	13	11	13	1
5	4⅝	1 500	25	7	19	3	17	5	16	1	15	0	14	1
5	4⅝	1 600	26	5	19	11	18	0	16	7	15	6	14	7
5	4⅝	1 800	28	0	21	1	19	1	17	7	16	5	15	5
5	4⅝	Deflection	15	3½	12	8	11	10½	11	2½	10	8½	10	3½
6	5½	1 200	.....		20	3	18	4	16	11	15	10	15	1
6	5½	1 300	....		21	1	19	1	17	8	16	5	15	8
6	5½	1 500	.....		22	7	20	9	18	11	17	8	16	10
6	5½	1 600	...		23	4	21	3	19	7	18	3	17	5
6	5½	1 800	.....		24	9	22	6	20	9	19	4	18	5
6	5½	Deflection	...		14	11	14	0	13	3	12	8	12	2

Table XLIX (Continued). Maximum Spans for Matched and Dressed Mill Floors

Nominal thickness, inches	Actual thickness, inches	Fiber-stress pounds per square inch	Span in feet											
			Live loads in pounds per square foot											
			225		250		275		300		350		400	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
2½	2⅛	1 200	5	7	5	4	.		.		. .		.	
2½	2⅛	1 300	5	10	5	6	.		.		.		.	
2½	2⅛	1 500	6	3	5	11	.		.		.		.	
2½	2⅛	1 600	6	5	6	1	.				.		...	
2½	2⅛	1 800	6	10	6	6					.		.	
2½	2⅛	Deflection	4	7½	4	5½	.		.		.		..	
3	2⅝	1 200	6	12	6	6	6	3	6	0	5	7	5	2
3	2⅝	1 300	7	2	6	10	6	6	6	3	5	9	5	5
3	2⅝	1 500	7	8	7	4	7	0	6	8	6	2	5	10
3	2⅝	1 600	7	11	7	7	7	2	6	11	6	5	6	0
3	2⅝	1 800	8	5	8	0	7	8	7	4	6	9	6	4
3	2⅝	Deflection	5	8	5	6	5	4	5	2	4	11	4	8½
4	3⅝	1 200	9	5	9	0	8	7	8	3	7	7	7	2
4	3⅝	1 300	9	10	9	4	8	11	8	7	7	11	7	5
4	3⅝	1 500	10	6	10	0	9	7	9	2	8	6	8	0
4	3⅝	1 600	10	11	10	4	9	11	9	6	8	10	8	3
4	3⅝	1 800	11	7	11	0	10	6	10	1	9	4	8	9
4	3⅝	Deflection	7	7½	7	6½	7	4	7	1½	6	9½	6	6
5	4⅝	1 200	11	11	11	4	10	10	10	5	9	8	9	1
5	4⅝	1 300	12	5	11	10	11	4	10	10	10	1	9	5
5	4⅝	1 500	13	4	12	8	12	2	11	8	10	10	10	2
5	4⅝	1 600	13	9	13	1	12	6	12	0	11	2	10	6
5	4⅝	1 800	14	7	13	11	13	4	12	9	11	10	11	1
5	4⅝	Deflection	9	1	9	7	9	4	9	1	8	7½	8	3
6	5½	1 200	14	1	13	5	12	10	12	4	11	6	10	9
6	5½	1 300	14	8	14	0	13	4	12	10	11	11	11	2
6	5½	1 500	15	9	15	0	14	4	13	9	12	10	12	0
6	5½	1 600	16	4	15	6	14	10	14	3	13	3	12	5
6	5½	1 800	17	3	16	5	15	9	15	1	14	0	13	2
6	5½	Deflection	11	9	11	4½	11	0½	10	9	10	2½	9	9½

Table XLIX (Continued). Maximum Spans for Matched and Dressed Mill Floors

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet											
			Live loads in pounds per square foot											
			450		500		550		600		650		700	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
4	3⅝	1 200	6	9	6	5	6	1	5	10	5	8	5	5
4	3⅝	1 300	7	0	6	8	6	4	6	1	5	10	5	8
4	3⅝	1 500	7	6	7	2	6	10	6	7	6	4	6	1
4	3⅝	1 600	7	9	7	5	7	1	6	9	6	6	6	3
4	3⅝	1 800	8	3	7	10	7	6	7	2	6	11	6	8
4	3⅝	Deflection	6	3	6	0½	5	10	5	8	5	6½	5	5
5	4⅝	1 200	8	7	8	2	7	9	7	6	7	2	6	11
5	4⅝	1 300	8	11	8	6	8	1	7	9	7	6	7	3
5	4⅝	1 500	9	7	9	1	8	8	8	4	8	0	7	9
5	4⅝	1 600	9	11	9	5	9	0	8	7	8	3	8	0
5	4⅝	1 800	10	6	10	0	9	6	9	2	8	9	8	6
5	4⅝	Deflection	7	11½	7	8	7	5½	7	3	7	0¾	6	10¼
6	5½	1 200	10	2	9	8	9	3	8	10	8	6	8	2
6	5½	1 300	10	7	10	1	9	7	9	3	8	10	8	6
6	5½	1 500	11	4	10	10	10	4	9	11	9	6	9	2
6	5½	1 600	11	9	11	2	10	8	10	3	9	10	9	6
6	5½	1 800	12	6	11	10	11	4	10	10	10	5	10	0
6	5½	Deflection	9	5	9	1½	8	10	8	7	8	4½	8	2

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet											
			Live loads in pounds per square foot											
			750		800		850		900		950		1 000	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
4	3⅝	1 200	5	3	5	1	...		...		...		...	
4	3⅝	1 300	5	6	5	4	...		...		...		...	
4	3⅝	1 500	5	11	5	8	...		...		...		...	
4	3⅝	1 600	6	1	5	11	...		...		...		...	
4	3⅝	1 800	6	5	6	3	...		...		...		...	
4	3⅝	Deflection	5	3½	5	2	...		...		...		...	
5	4⅝	1 200	6	8	6	6	...		...		...		...	
5	4⅝	1 300	7	0	6	9	...		...		...		...	
5	4⅝	1 500	7	6	7	3	...		...		...		...	
5	4⅝	1 600	7	9	7	6	...		...		...		...	
5	4⅝	1 800	8	2	7	11	...		...		...		...	
5	4⅝	Deflection	5	3½	6	7	...		...		...		...	
6	5½	1 200	7	11	7	8	7	6	7	3	7	1	6	11
6	5½	1 300	8	3	8	1	7	9	7	7	7	4	7	2
6	5½	1 500	8	11	8	7	8	4	8	1	7	11	7	9
6	5½	1 600	9	2	8	11	8	8	8	5	8	2	8	0
6	5½	1 800	9	9	9	5	9	2	8	11	8	8	8	5
6	5½	Deflection	8	0	7	10	7	8	7	6½	7	4½	7	3

Table L. Maximum Spans for Laminated Floors—Planks on Edge Laid Close

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet — Live loads in pounds per square foot: 100		125		150		175		200		225	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
6	5⅝	1 200	20	8	18	9	17	4	16	2	15	3	14	5
6	5⅝	1 300	21	6	19	6	18	0	16	10	15	10	15	0
6	5⅝	1 500	23	1	21	0	19	4	18	1	17	0	16	1
6	5⅝	1 600	23	10	21	8	20	0	18	8	17	7	16	7
6	5⅝	1 800	25	3	23	0	21	2	19	10	18	8	17	8
6	5⅝	Deflection	15	3	14	3½	13	6½	12	11½	12	5½	12	0
8	7½	1 200	26	10	24	6	22	8	21	2	20	0	19	0
8	7½	1 300	27	11	25	6	23	7	22	1	20	10	19	9
8	7½	1 500	30	0	27	5	25	4	23	9	22	4	21	2
8	7½	1 600	31	0	28	3	26	2	24	6	23	1	21	11
8	7½	1 800	32	10	30	0	27	9	26	0	24	6	23	3
8	7½	Deflection	20	0	18	10	17	10½	71	1	16	5¼	15	10½

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet — Live loads in pounds per square foot: 250		275		300		350		400		450	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
6	5⅝	1 200	13	9	13	2	12	8	11	9	11	0	10	5
6	5⅝	1 300	14	3	13	8	13	1	12	2	11	5	10	10
6	5⅝	1 500	15	4	14	8	14	1	13	1	12	3	11	7
6	5⅝	1 600	15	10	15	2	14	7	13	6	12	8	12	0
6	5⅝	1 800	16	10	16	1	15	5	14	4	13	6	12	9
6	5⅝	Deflection	11	7½	11	3½	11	0	10	5½	10	0½	9	8
8	7½	1 200	18	1	17	4	16	7	15	6	14	7	13	9
8	7½	1 300	18	10	18	0	17	4	16	1	15	2	14	4
8	7½	1 500	20	3	19	4	18	7	17	4	16	3	15	5
8	7½	1 600	20	10	20	0	19	2	17	10	16	9	15	11
8	7½	1 800	22	2	21	2	20	4	19	0	17	10	16	10
8	7½	Deflection	15	4½	14	11	14	6½	13	10	13	3½	12	9½
10	9½	1 200	. .		. . .		20	10	19	5	18	3	17	4
10	9½	1 300	. . .		.		21	9	20	3	19	1	18	0
10	9½	1 500	.		.		23	4	21	9	20	5	19	4
10	9½	1 600	. .		. . .		24	1	22	5	21	2	20	0
10	9½	1 800	. . .		. . . . .		25	7	23	10	22	5	21	2
10	9½	Deflection	. . .		. . .		18	3½	17	5½	16	9½	16	2
12	11½	1 200	.		. . .		. .		. .		22	0	20	10
12	11½	1 300	. .		. . .		. . .		. . . .		22	11	21	8
12	11½	1 500	.		.		. .		. .		24	7	23	3
12	11½	1 600	.		. . .		.		. .		25	4	24	0
12	11½	1 800	. .		. .		. . . .		. .		26	11	25	6
12	11½	Deflection			. . .		.		. . .		20	2¼	19	5½

Table L (Continued). Maximum Spans for Laminated Floors— Planks on Edge Laid Close

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet — Live loads in pounds per square foot: 500		550		600		650		700		750	
			ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
6	5⅝	1 200	9	10	9	5	9	0	8	8	8	5	8	1
6	5⅝	1 300	10	3	9	10	9	5	9	1	8	9	8	5
6	5⅝	1 500	11	0	10	7	10	2	9	9	9	3	9	1
6	5⅝	1 600	11	5	10	11	10	5	10	1	9	8	9	4
6	5⅝	1 800	12	1	11	7	11	1	10	8	10	3	9	11
6	5⅝	Deflection	9	4	9	0½	8	9½	8	7	8	4½	8	2
8	7½	1 200	13	1	12	6	12	0	11	7	11	2	10	10
8	7½	1 300	13	8	13	0	12	6	12	0	11	7	11	3
8	7½	1 500	14	8	14	0	13	5	12	11	12	6	12	1
8	7½	1 600	15	2	14	5	13	10	13	4	12	10	12	5
8	7½	1 800	16	0	15	4	14	8	14	2	13	5	13	2
8	7½	Deflection	12	4½	12	0	11	8	11	4½	11	1½	10	10½
10	9½	1 200	16	6	15	9	15	2	14	7	14	1	13	7
10	9½	1 300	17	2	16	5	15	8	15	2	14	7	14	2
10	9½	1 500	18	5	17	7	16	11	16	3	15	8	15	2
10	9½	1 600	19	0	18	2	17	5	16	10	16	2	15	8
10	9½	1 800	20	2	19	3	18	6	17	10	17	3	16	8
10	9½	Deflection	15	7½	15	2	14	9	14	4½	14	0½	13	9
12	11½	1 200	19	10	19	0	18	2	17	6	16	11	16	4
12	11½	1 300	20	8	19	9	19	0	18	3	17	7	17	0
12	11½	1 500	22	2	21	2	20	4	19	7	18	11	18	4
12	11½	1 600	22	11	21	11	21	0	20	3	19	7	18	17
12	11½	1 800	24	4	23	3	22	4	21	6	20	9	20	1
12	11½	Deflection	18	10¼	18	3½	17	9½	17	4	16	11½	16	1

Table L (Continued). Maximum Spans for Laminated Floors—Planks on Edge Laid Close

Nominal thickness, inches	Actual thickness, inches	Fiber-stress, pounds per square inch	Span in feet — Live loads in pounds per square foot: 800		850		900		950		1 000	
			ft	in	ft	in	ft	in	ft	in	ft	in
6	5⅝	1 200	7	10	7	7	7	5	7	3	7	1
6	5⅝	1 300	8	2	7	11	7	9	7	6	7	4
6	5⅝	1 500	8	10	8	6	8	4	8	1	7	11
6	5⅝	1 600	9	1	8	10	8	7	8	4	8	2
6	5⅝	1 800	9	8	9	4	9	1	8	10	8	8
6	5⅝	Deflection	8	0	7	10	7	8½	7	6½	7	5½
8	7½	1 200	10	6	10	2	9	10	9	7	9	4
8	7½	1 300	10	10	10	7	10	3	10	0	9	9
8	7½	1 500	11	8	11	4	11	0	10	9	10	6
8	7½	1 600	12	1	11	8	11	5	11	1	10	10
8	7½	1 800	12	9	12	5	12	1	11	9	11	6
8	7½	Deflection	10	7½	10	5	10	3	10	1	9	11
10	9½	1 200	13	2	12	9	12	5	12	1	11	10
10	9½	1 300	13	8	13	4	12	11	12	7	12	4
10	9½	1 500	14	9	14	4	13	11	13	7	13	3
10	9½	1 600	15	3	14	9	14	5	14	0	13	8
10	9½	1 800	16	2	15	8	15	3	14	10	14	6
10	9½	Deflection	13	5½	13	2	12	11½	12	8½	12	6½
12	11½	1 200	15	11	15	5	15	0	14	8	14	4
12	11½	1 300	16	6	16	1	15	8	15	3	14	10
12	11½	1 500	17	9	17	3	16	9	16	4	16	0
12	11½	1 600	18	4	17	10	17	4	16	11	16	6
12	11½	1 800	19	6	18	11	18	5	17	11	17	6
12	11½	Deflection	16	3	15	11	15	8	15	4½	15	1½

21. Strength of Existing Floors

To Determine the Strength of an Existing Floor. When a building is leased for mercantile or manufacturing purposes the tenant will generally desire to know the greatest load which it will be safe to put upon the floors, and some building laws require that the safe load for the floors in certain classes of buildings shall be computed and posted in a conspicuous place in each story. It is therefore important that every architect should know how to compute the safe strength of any existing floor. The problem is practically the reverse of that of proportioning a floor to a given load. In speaking of the strength of a floor a distinction should be made between the safe strength and the safe load. The SAFE STRENGTH should mean the maximum safe load for the beams, including the weight of the construction, flooring and ceiling, while the SAFE LOAD refers to the maximum live load which may safely be placed upon the floor. The safe load is found by first computing the safe strength and then subtracting the weight of the materials forming the floor, including the ceiling

below, if there be one. The most convenient measurement for either the SAFE STRENGTH or the SAFE LOAD of a floor is in pounds per square foot. The following examples will serve to show the method of determining the safe load for an ordinary warehouse-floor.

Example 1. It is required to determine the safe load per square foot for a floor framed as shown in Fig. 54, the building being in a city the laws of which allow 1 200 lb per sq in for the safe flexure fiber-stress for the wood of which

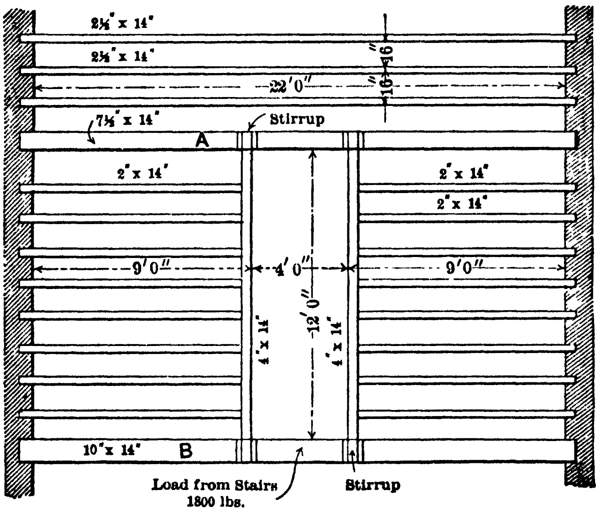

Fig. 54. Plan of a Warehouse-floor

the joists and girders are made. The joists are covered with two thicknesses of $\frac{7}{8}$-in flooring and the ceiling below is corrugated iron.

Solution. The first step will be to find the SAFE STRENGTH of the 22-ft joists. As this is a warehouse-floor we will use the tables for strength throughout. From Table XIV, for S = 1 200 lb per sq in, we find the safe strength of a 1 by 14-in joist of 22-ft span to be 1 188 lb; hence the strength of a $2\frac{1}{2}$ by 14-in joist will be 1 188 $\times$ $2\frac{1}{2}$ = 2 970 lb. As the joists are 16 in on centers, each joist supports a floor-area of $1\frac{1}{3}$ $\times$ 22 ft = $29\frac{1}{3}$ sq ft. The SAFE STRENGTH PER SQUARE FOOT of this portion of the floor will therefore be 2 970/29.3 = 101 lb. Suppose the estimated weight of the floor per square foot is 8 lb for the joists, 6 lb for the flooring and 1 lb for the corrugated-iron ceiling, or, say, 15 lb in all. Then the SAFE LOAD PER SQUARE FOOT for the 22-ft joists will be 101 − 15 = 86 lb.

The strength of the joists may also be investigated by the use of Tables XXV and XXVI. We find that for a 90-lb load and 3 $\times$ 14-in joists, 16 in on center, the safe span is 21 ft 7 in, and for an 80-lb load 22 ft 9 in. By interpolating it is found that a 22 ft 0-in span is safe for a load of 86.39 lb.

The Headers. We will next find the safe load for the 4 by 14-in headers at each side of the stair-well. As the tail-beams are framed into the headers,

we should deduct one inch from the thickness of each header for the loss of strength in framing, leaving 3 by 14 in for the effective dimension of each. From Table XIV we find the safe strength of a 1 by 14-in beam of 12-ft span to be 1 867-lb. Hence the strength of the 3 by 14 will be 1 867 × 3 = 5 601 lb. The floor-area supported by each header is $4\frac{1}{2} \times 12$ ft = 54 sq ft; hence the SAFE STRENGTH of the header per square foot of floor is 5 601/54 = 104 lb. Deducting the weight of the floor per square foot, we have 104 − 15 = 89 lb for the SAFE LOAD.

Trimmer A. Trimmer A (Fig. 54) supports about the same amount of flooring as one of the common joists, and supports, also, the ends of the headers. Deducting $2\frac{1}{2}$ in, the thickness of the common joists, we have a 5 by 14-in beam left to support the headers. As the headers are supported in iron stirrups, or beam-hangers, no deduction in strength need be made for framing. To find the safe strength of a beam loaded with two concentrated loads, equidistant from the supports, we must use Formula (11), Fig. 52. In this case m = 8 ft 10 in, or $8\frac{5}{6}$ ft and A = 1 200/18 = 66.7 (Table XIV).

Applying the formula, the safe load at each joint = 5 × 14 × 14 × 66.7/4 × $8\frac{5}{6}$ = 1 848 lb.

The floor-area supported by one stirrup is equal to one-half of the area supported by the header, or 27 sq ft; hence the safe strength per square foot of the 5 by 14-in header is 1 848/27 = 68 lb, and deducting 15 lb per sq ft for the weight of the floor, we have 53 lb per sq ft as the safe load that the trimmer will support on the floor at each side of the stairs Considering, as found above, that the safe load for the $2\frac{1}{2}$ in, which we deducted to take the place of a common joist, is 86 lb per sq ft, we might consider the safe load for the trimmer as the average of 86 and 53, or about 70 lb per sq ft.

Trimmer B. This 10 by 14-in timber (Fig. 54) has to support the same floor loads as trimmer A, and also the lower end of a flight of stairs for which an allowance of at least 1 800 lb should be made. This stair-load being practically concentrated at the middle of the trimmer is equivalent to a distributed load of 3 600 lb. As the safe load for a 1 by 14-in joist of 22-ft span is 1 188 lb (Table XIV), it will require a thickness of 3 600/1 188 = 3 in to support the stairs, leaving 7 in to support the floor loads. As this is $\frac{1}{2}$ in less than the thickness of trimmer A, it is evident that the strength of the floor at B will be a little less than at A; but as it is improbable that the entire floor-space will be loaded at any given time, it would be safe to rate the strength of the floor at each side of the stairway at 70 lb per sq ft, LIVE LOAD, and beyond the stairway at 86 lb.

Partitions. When the floor supports partitions, the weight of the latter and any load resting upon them must be taken into account in determining the safe load for the floor. If a partition runs the same way as the joists, then only the joist directly under the partition, and the joists at each side will be affected; but if a partition runs across the joists, then it affects the safe load of the entire floor.

Example 2. Suppose that the 22-ft joists in the floor shown in Fig. 54 have to support a plastered partition 12 ft high, running across the joists halfway between the walls. What will be the safe load for the floor?

Solution. A plastered partition with 2 by 4 or 2 by 6-in studs, set 16 in on centers, weighs about 20 lb per sq ft of partition-face; hence a partition 12 ft high will weigh 240 lb per lin ft of partition. As the joists are 16 in on centers, each joist supports $1\frac{1}{3}$ lin ft of partition, weighing 320 lb. As this load is concentrated at the middle span of the joists it is equivalent to a distributed

load of 640 lb. In the previous example we found the safe distributed load for the $2\frac{1}{2}$ by 14-in joists of 22-ft span to be 2 970 lb. Subtracting 640 lb from this we have 2 330 lb, which may be used for the floor. As the floor-area supported by one joist is $29\frac{1}{3}$ sq ft, the safe strength of the floor per square foot is $2\,330/29\frac{1}{3} = 79$ lb, and the safe load is $79 - 15 = 64$ lb per sq ft. Hence the partition decreases the safe load by $86 - 64 = 22$ lb per sq. ft. Whenever the upper-floor joists are supported by a partition carried by a floor below, the effect of the partition and its load upon the strength of the lower floor should be very carefully computed.

Fig. 55. Floor-joists with Bridging

22. Bridging and Framing

Bridging of Floor-Joists. By BRIDGING is meant a system of bracing for floor-joists, either by means of small struts, as in Fig 55, or by means of single pieces of boards set at right-angles to the joists and fitting in between them. The effect of this bracing is of decided advantage in sustaining any CONCENTRATED LOAD upon a floor; but it does not materially strengthen a floor to resist a UNIFORMLY DISTRIBUTED LOAD. The bridging also stiffens the joists, and prevents them from turning sidewise. It is customary to insert rows of cross-bridging from 5 to 8 ft apart; and to be effective the rows of bridging should be in straight lines along the floor, so that each bridging-strut may abut directly opposite those adjacent to it The method of bridging shown in Fig. 55 and known as CROSS-BRIDGING is considered to be by far the best, as it allows the thrust to act parallel to the axis of the strut, and not across the grain, as must be the case where single pieces of boards are used. The bridging should be of $1\frac{1}{4}$ by 3-in stock, for 2 by 10-in and smaller joists, and of 2 by 3-in stock for 12- and 14-in joists.

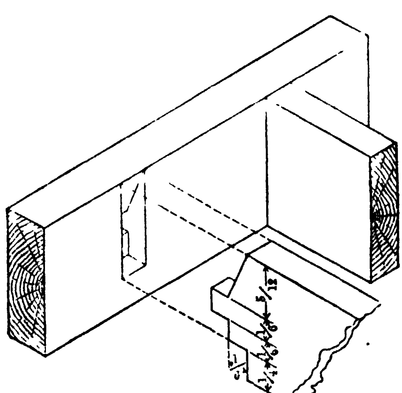

Fig. 56. Framing of Joists into Header

Framing of Wooden Floor-Beams. In dwellings, tenements and lodging-houses it is frequently necessary to frame the timbers so that they are flush with one another. The old methods of framing the tail-beams and headers or headers and trimmers by mortise-and-tenon joints are now generally superseded by hanging the timbers in stirrups or malleable-iron joist-hangers. In this construction the entire strength of the timbers is

retained, while the cost of the hangers is often less than the labor-cost in preparing the mortise-and-tenon joints. All headers 6 ft or more in length should be carried in joist-hangers or stirrups and this is usually required in the building codes of the large cities. In warehouses and all first-class buildings the framing should be done by means of joist-hangers. For light floors, with moderate spans, it is generally safe to frame the tail-beams into a header, provided the latter is strong enough to carry the load and allow 1 in in thickness for the mortising. Headers, also, carrying not more than two tail-beams are often framed into the trimmers. In case the old methods of

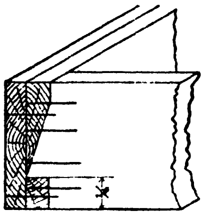

Fig. 57. Alternate Method of Framing Joists into Header

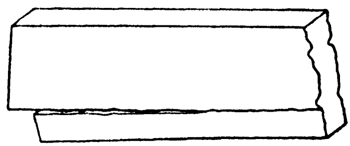

Fig. 58. Framed Joist Split by Load

framing are used instead of the superior methods with joist-hangers, the best shape and proportions for the tenons and ends of the tail-beams or headers are those shown in Fig. 56. This form of framing probably offers as large a proportion of the strength of the timbers as it is possible to utilize, although for tail-beams it was the opinion of Mr. Kidder that a single tenon like that shown in Fig. 57 is fully as strong, especially when the header is built up of 2-in planks spiked together. In either case, if the floor be loaded to its full strength, the tail-beam will split at the bottom of the tenon, as shown in Fig. 58, which illustrates the weakening effect of the mortise-and-tenon framing.

CHAPTER XXI

WOODEN MILL AND WAREHOUSE-CONSTRUCTION

By

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1. Mill-Construction

Definition. The term MILL-CONSTRUCTION is commonly used to designate a method of construction brought about largely through the influence of the Boston Manufacturers' Mutual Fire Insurance Company of Boston, Mass., and especially through the efforts of Mr. Wm. B. Whiting, whose judgment in mechanical matters, and experience and skill as a manufacturer were for many years devoted to the interests of insurance companies, and to the improvement of factories of all kinds. The extended use of this system and the improvements that have been made in it during recent years are probably due more to the influence of Mr. Edward Atkinson, President of the Boston Manufacturers' Mutual Insurance Company and Director of the Insurance Engineering Experiment Station at Boston, than to that of any other individual.

Cost. The purpose of mill-construction is to reduce the fire-risk to its lowest point without going to the expense of fire-proof construction. The increasing cost of heavy timber, however, and in fact of all lumber, together with the lessened cost of the erection of the so-called FIRE-PROOF TYPES, constructed entirely of reinforced concrete, or built with protected steel frames and incombustible floors, and the recognition, also, of the obvious advantages of more FIRE-RESISTING CONSTRUCTION, especially in the congested sections of cities, are bringing these types into more general use. The cost of these latter types of construction is, in many instances, no more than the cost of various types of mill-construction.

The Slow-Burning or Mill-Construction Type. The experience of years has entirely justified the use of this type. It renders possible a somewhat less costly, and at the same time, what is of great importance, a more effective system of fire-protection than can be installed in buildings of light construction, with the so-called JOISTED FLOORS and with the roofs made of boards supported on 2-in, 3-in, or 4-in joists. The entire subject of SLOW-BURNING or MILL-CONSTRUCTION as applied to factories is most admirably described and illustrated in Report No. 5 of the Insurance Engineering Station of the Boston Manufacturers' Insurance Company, No. 31 Milk Street, Boston, Mass., from which the author has, by permission, taken and adapted many of the following illustrations and descriptions.

2. What Mill-Construction Is *

(1) Heavy Timbers. MILL-CONSTRUCTION consists in so disposing the timbers and planks in heavy, solid masses as to expose the least number of corners or ignitable projections to fire; and to the end, also, that when fire occurs it may be most readily reached by water from sprinklers or hose.

* From Report No. 5 of the Insurance Engineering Station of the Boston Manufacturers' Insurance Company, No. 31 Milk Street, Boston, Mass.

(2) **Fire-Stops.** It consists in separating every floor from every other floor by incombustible STOPS, by installing automatically closing hatchways and by encasing stairways either in brick or other incombustible partitions, so that a fire will be retarded to the utmost in passing from floor to floor consistent with the use of wood or any material not absolutely fire-proof.

(3) **Fire-Retardants.** It consists in guarding the ceilings over all specially hazardous stock or processes with FIRE-RETARDANT MATERIALS, such as plastering laid over wire lath or expanded metal, or over wooden dovetailed lath, following the lines of the ceilings and of the timbers and leaving no interspaces between the plastering and the wood; or else in protecting the ceilings over hazardous places with asbestos, air-cell boards, sheet metal, Sackett Plaster Board, or other fire-retardant.

(4) **Fire-Safeguards.** It consists not only in so constructing the mill, work-shop, or warehouse, that fire will pass as slowly as possible from one part of the building to another, but also in providing all suitable SAFEGUARDS AGAINST FIRE.

3. What Mill-Construction Is Not

(1) **Concealed Spaces.** Mill-construction does not consist in so disposing a given quantity of materials that the whole interior of a building becomes a SERIES OF WOODEN CELLS, or concealed spaces, connected with each other directly or by cracks through which fire may freely pass where it cannot be reached by water.

(2) **Size of Timbers, Fire-Stops, etc.** It does not consist of an open-timber construction of floors and roofs which resembles mill-construction, but which is built with light timber of insufficient size and with thin planks, without fire-stops or fire-guards from floor to floor.

(3) **Stairways.** It does not consist in connecting floor with floor by COMBUSTIBLE WOODEN STAIRWAYS encased in wood less than two inches thick.

(4) **Partitions.** It does not consist in putting in very numerous LIGHT, WOODEN DIVISIONS or partitions.

(5) **Sheathing and Furring.** It does not consist in SHEATHING brick walls with wood, especially when the wood is set off from the walls by FURRING, and even if there are stops behind the furring.

(6) **Varnish.** It does not consist in permitting the use of VARNISH on woodwork over which a fire will pass rapidly.

(7) **Glass, Fire-Shutters and Wire-Glass.** It does not consist in leaving windows exposed to adjacent buildings and unguarded by FIRE-SHUTTERS or WIRE-GLASS.

(8) **Painting and Dry-Rot.** It does not consist in painting, varnishing, filling or encasing heavy timbers and thick planks, as they are customarily delivered, and thus making possible what is called DRY-ROT, caused by a lack of ventilation or opportunity to season.

(9) **Sprinklers, Pumps, Pipes, Hydrants, etc.** It does not consist in leaving even the best-constructed building in which dangerous occupations are followed without AUTOMATIC SPRINKLERS, and without a complete and adequate equipment of PUMPS, PIPES and HYDRANTS.

(10) **Finishing in Wood and Other Materials.** It does not consist in using more WOOD IN FINISHING a building after the floors and roof are laid than is absolutely necessary, since there are now many safe methods available at low cost for finishing walls and constructing partitions with slow-burning or

incombustible materials. Accordingly if plaster is to be put on a ceiling and is to follow the line of the underside of the flooring and the flooring-timbers, it should be PLAIN LIME-MORTAR PLASTER, which is sufficiently porous to permit seasoning. The addition of a skim-coat of lime-putty is hazardous, especially if the overflooring is laid over rosin-sized or asphalt paper. This rule applies to almost all timber as now delivered. Examples of all of the faulty methods of construction above mentioned have been found in various buildings purporting to be of mill-construction, and they all form parts of what has sometimes been called COMBUSTIBLE CONSTRUCTION.

4. Standard Mill-Construction

Example of Standard Mill-Construction. Fig. 1 shows a cross-section through a mill of the customary or STANDARD TYPE recommended by the Boston Manufacturers' Mutual Insurance Company.

Walls. If additional stories are required, the walls may be increased in thickness according to the number of stories added, after a computation has been made of the loads which a STANDARD FACTORY may be called upon to sustain. Walls should be of brick and at least 13 in thick in the upper story, and their thickness should be increased in the lower stories to support additional loads. Plastered walls are often to be preferred to unplastered walls. Window-arches and door-arches should be of brick, and window-sills, outside door-sills and under-pinning of granite or concrete.

Roofs and Floors. The roofs should be of 3 in pine planks spiked directly to the heavy roof-timbers, and covered with five-ply tar-and-gravel roofing. Roofs should incline from ½ to ¾ in per ft, and incombustible cornices are recommended when there is exposure from neighboring buildings. Floors should be of spruce planks, 4 in or more in thickness according to the floor-loads, spiked directly to the floor-timbers, and kept at least ½ in away from the face of the brick walls. In order to obviate the danger of cracking the walls, which sometimes results from the swelling of planks laid close against them, these spaces left between walls and floor-planks must be covered by strips or battens both above and below. In floors and roofs, the bays should be from 8 to 10½ ft wide, and all planks two bays in length should be laid to break joints every 4 ft, and grooved for hard-wood splines. Usually an overfloor of birch or maple is laid at right-angles to the planking, but the best mills have a double overfloor, a lower one of soft wood, laid diagonally upon the planks and an upper one laid lengthwise. This latter method allows boards in alleys or passageways to be easily replaced when worn, while the diagonal boards brace the floors, reduce the vibration, and distribute the floor-loads more uniformly than the former method. Between the planking and the overfloor should be two or three layers of heavy, hard paper, laid to break joints, and each mopped with hot tar or similar material to make a reasonably water-tight as well as dust-tight floor. The usually rapid decay of the basement or lower floors of mills makes it desirable, whenever wood is not absolutely necessary, to make such floors of cement. If wooden floors are required, crushed stone, cinders, or furnace slag should be spread evenly over the surface, and covered with a thick layer of hot-tar concrete. On this tarred felt is often laid, well mopped with hot-tar asphalt, and over it a flooring of 2 in seasoned planks, well pressed down and nailed on edge without perforating the water-proofing under it. The hard-wood boards of the overfloor are then nailed across the planks. Cement concretes promote decay of wood in contact with them. If extra supports are required for heavy machinery, independent foundations of masonry should be provided. In

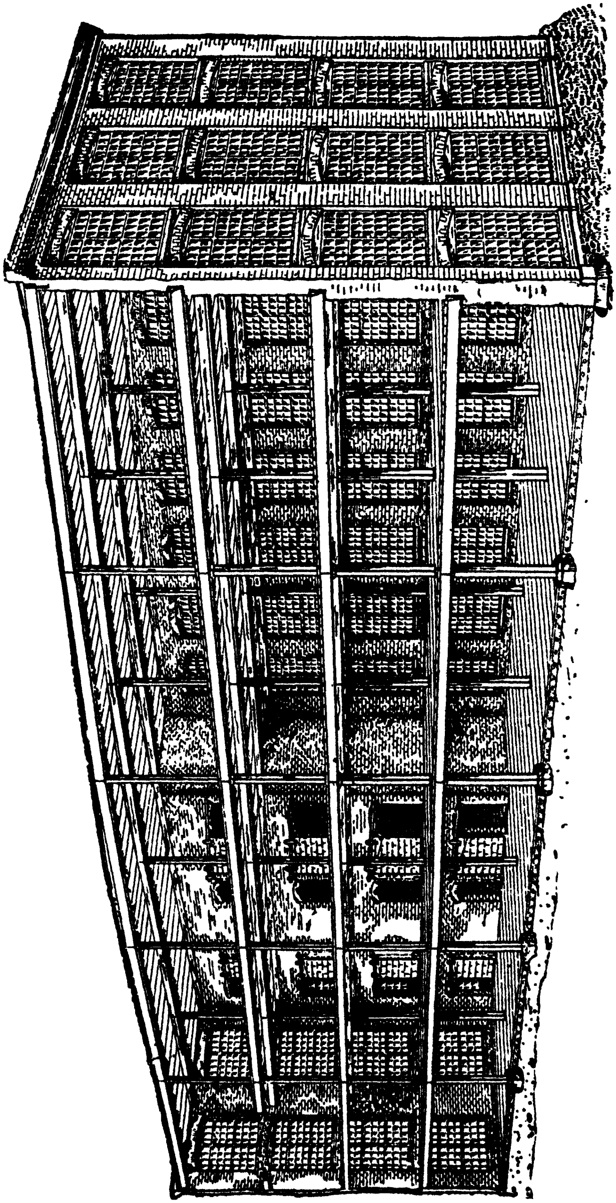


Fig. 1. Modern Mill-building of the Standard Type

view of the difficulties frequently met with in preserving basement floors of the ordinary timber construction, because of the lack of suitable ventilation underneath, and also in view of the rapid decay of timber and plank floors in bleacheries, dye-works, print-works, and the like, in which the floors quickly become saturated with moisture, artificial-stone floors are being laid in many of the modern plants.

Sizes and Kinds of Timbers. All woodwork, not STANDARD-CONSTRUCTION, in order to be SLOW-BURNING, must be in LARGE MASSES which present the least surface possible to a fire. No pieces less than 6 in in width should be used for the lightest roofs, and for substantial roofs and floors much wider ones are needed. Timbers should be of sound, long-leaf, yellow pine, and for sizes up to 14 by 16 in, single pieces are preferred; or, timbers 7 to 8 by 16 in, are often used in pairs bolted together, without air-spaces between. They should not be painted, varnished or filled for three years because of the danger of dry rot, and for the same reason, an air-space should be left in the masonry around the ends.

Beam-Boxes, Column-Caps, etc. Timbers should rest on CAST-IRON PLATES or BEAM-BOXES in the walls and on cast-iron caps on the columns. BEAM-BOXES are of value as they strengthen the walls when the floor loads are heavy and the distance between windows small; they facilitate the laying of the bricks and the handling of the beams; and there is less danger of breaking the bricks in putting the beams in place. They also insure proper air-spaces around the ends of the beams. Fig. 2 shows a floor-timber resting on a CAST-

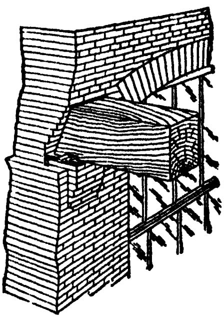


Fig. 2. Floor-timber on Wall-plate

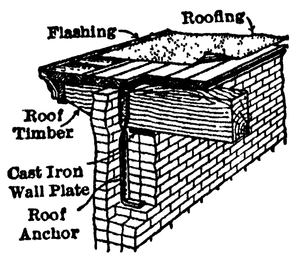


Fig 3. Roof-timber on Wall-plate

IRON WALL-PLATE with a lug for anchoring the timber to the wall. Fig. 3 shows a roof-timber resting on a CAST-IRON WALL-PLATE, an overhanging, open, wooden cornice and a wrought-iron joist-anchor. Fig. 4 shows a CAST-IRON CAP and PINTLE for columns, and dogs for holding the floor-timbers together. Fig. 5 shows a roof-timber resting on a COLUMN-CAP cast to fit the slope of the roof; the timbers are held together by 1 in wrought-iron dogs. These diagrams are intended only as general illustrations of SLOW-BURNING or MILL-CONSTRUCTION. The details should always be adapted to the special conditions of the site and to the purposes for which the buildings are used.

Columns of yellow pine should be bored through the axis, making a 1½ in diameter hole, and should have ½ in lateral vent-holes near the top and bottom. The ends should be carefully squared. To prevent dry-rot, WOODEN COLUMNS should not be painted until they are thoroughly seasoned. They

should be set on PINTLES which may be cast in one piece with the cap, or separately. CAST-IRON COLUMNS are preferred by some engineers, and when a building is equipped with automatic sprinklers, such columns have proved satisfactory; but they are not as fire-resisting as wooden columns. WROUGHT-

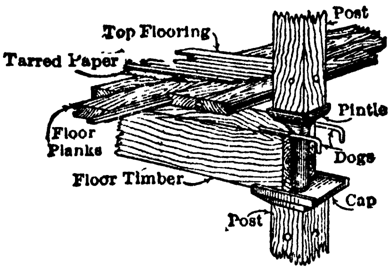


Fig. 4. Post-cap and Pintle for Floor-timber and Columns

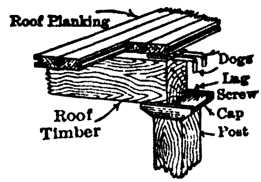


Fig. 5 Roof-timbers on Column-cap

IRON or STEEL COLUMNS should not be used unless encased with at least 3 in of fireproofing.

Windows should be placed as high and made as wide as possible to obtain the greatest amount of light, and the use of RIBBED GLASS is recommended for the upper sashes.

Weight, Deflection and Vibration. In computing the size of the timbers as a ratio to the working-load, consideration must be given not only to the weights which are to be carried, but also to the CHARACTER OF THE MACHINERY which is to be operated on the floors. Beams of sufficient strength to support the weights may vibrate or deflect under the weight and action of the machinery; and there are, therefore, three factors, WEIGHT, DEFLECTION and VIBRATION, which must be considered in determining the width and depth of the beams that are to be used in the structure.

Objectionable Types of Construction. "We do not approve what has been sometimes miscalled MILL-CONSTRUCTION, that is, longitudinal girders resting upon posts and supporting floor-beams spaced 4 ft, more or less, on centers. This mode of construction not only adds to the quantity of wood used, but the disposal of the timbers obstructs the action of the sprinklers, prevents the sweeping of a hose-stream from one side of the mill to the other, and the girders also obstruct the most important light, that from the top of the windows."

Timber, Ventilation, Painting, etc. Timbers, unless known to be thoroughly seasoned, should not be encased in any kind of air-proof plastering nor painted with oil-paints; white-wash, calcimine and water-paints may be used, as they are porous. As a rule, timbers should be LEFT UNPROTECTED, since a fire which will seriously impair and destroy heavy timbers will already have done its work upon other parts of the structure.

Single and Compound Beams. While, in general, SINGLE BEAMS should be used, in some instances it may be desirable to substitute COMPOUND BEAMS, made by fastening two or more beams or thick planks side by side. It is often easier to obtain well-seasoned lumber in small dimensions. Such COMPOUND BEAMS should be tightly bolted together without air-spaces, and owing to the danger of dry-rot, should not be painted or varnished for three years.

Steam-Pipes. If a mill is to be heated by conveying steam through pipes, such pipes should be hung overhead.

Cornices. Wherever buildings are exposed or are liable to be exposed to fire in the near future, the cornices should be of non-combustible construction or, preferably, the walls should extend above the roof-timbers.

Glass, Frames and Shutters. All openings in walls should be protected either by approved wire-glass in approved metal frames or by standard fire-shutters.

5. Belts, Stairways and Elevator-Towers

Continuous Floors. One of the most important features of SLOW-BURNING CONSTRUCTION is to make each and every floor CONTINUOUS from wall to wall,

BELT, STAIRWAY AND ELEVATOR TOWERS

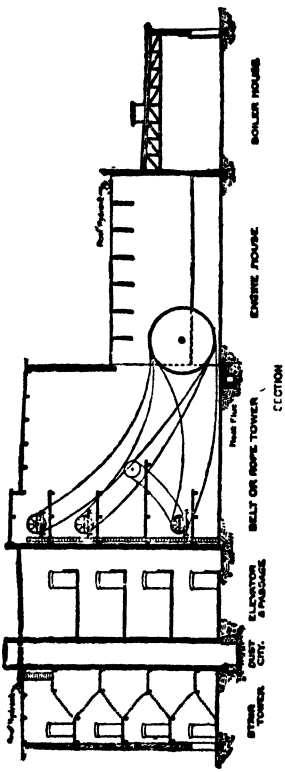


Fig. 6. Section through Tower for Elevators, Stairs and Belts

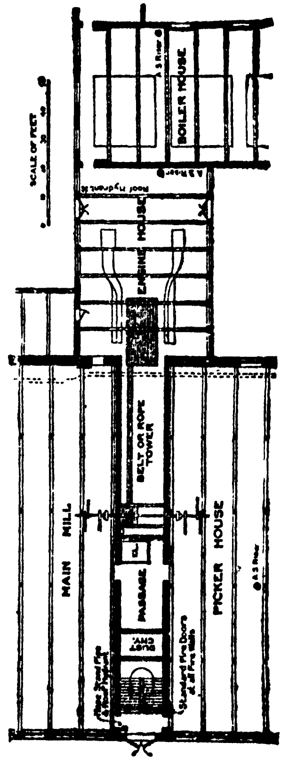


Fig. 7. Plan of Tower for Elevators, Stairs and Belts

avoiding, as far as possible, holes for belts, stairways, or elevators so that a fire may be confined to the story in which it starts. No well-informed mill-owner, engineer or builder will, therefore, fail to locate elevators, stairs, and main belts, in BRICK TOWERS or in sections of the building cut off from all rooms by incombustible walls. All openings in these walls should be pro-

tected by STANDARD FIRE-DOORS, preferably self-closing. In modern practice all belts and ropes which may be used for the transmission of power to the various rooms, are placed in INCOMBUSTIBLE VERTICAL BELT-CHAMBERS, from which the power is transmitted by shafts through the walls into the several rooms of the factory. There should be no unprotected openings in the inner walls of this BELT-CHAMBER.

Shafts above Roof. Skylights. All SHAFTS for STAIRS, ELEVATORS BELTS, etc., should extend at least 36 in above the roof, and all such shafts should be, if possible, on the outside of the building. Elevator and belt-shafts should be covered with thin glass skylights in metal frames, protected underneath with wire-netting. Figs. 6 and 7 illustrate a section and plan of a COTTON-MILL, showing elevator, stair and belt-shafts arranged on the above principle. CLOSETS should be in a separate tower rather than in manufacturing rooms.

The Boiler-Plant should be in a separate building cut off from the engine-room by a brick wall, and the openings in this wall should be protected by AUTOMATIC, SLIDING, STANDARD FIRE-DOORS.

6. Standard Storehouse-Construction

Example of Storehouse-Construction. Fig. 8 shows a cross-section through the fire-tower and Fig. 9 the first-story plan, including the elevator and stair-tower of a four-story storehouse.

Area. Buildings for this purpose should not, in general exceed 5 000 sq ft in AREA. When used, however, for storage of non-hazardous goods, the area may be increased to 10 000 sq ft.

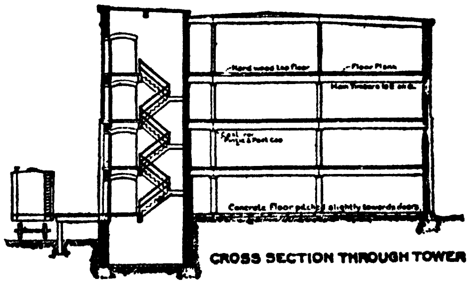


Fig. 8. Four-story Storehouse. Section through Fire-tower

Height of Stories. In storehouses, the stories should be made low enough (Fig. 10) to prevent overloading, and when designed for case-goods, the HEIGHT OF STORIES should be sufficient to take two cases, with a 12 in, clear space under the beams to allow for the distribution of water from the sprinklers.

Fire-Walls. For convenience, as well as to separate the different hazards of raw materials and finished goods, the building should be divided into sections by FIRE-WALLS extending at least 36 in above the roof.

One-Story Storehouses. A ONE-STORY STOREHOUSE is recommended in preference to the design just described, whenever there is a sufficient quantity of level land at disposal for this purpose. The one-story building is cheaper, more convenient, and, when separated into small divisions by fire-walls, represents the safest method of storehouse-construction.

Timbers and Framing. The FLOOR-TIMBERS and ROOF-TIMBERS should be of long-leaf yellow pine, in single pieces, if possible. If necessary to use double beams, they should be bolted together without air-spaces between them. Tim-

bers should rest on cast-iron plates or beam-boxes in the walls, and on cast-iron caps on the columns. At least ½ in air-spaces should be left around all beams built into the masonry, allowing free ventilation and preventing dry rot. COL-

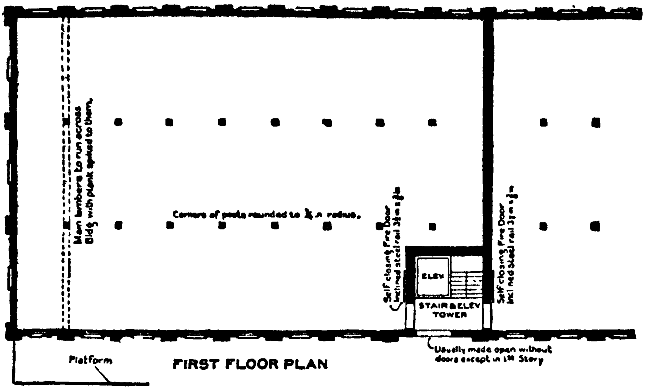
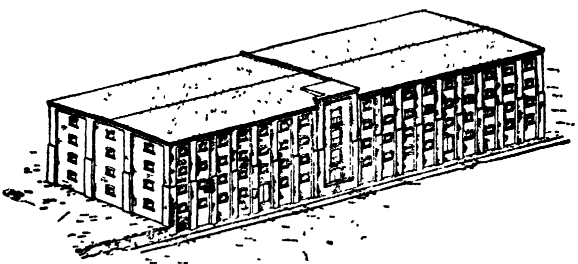


Fig. 9. Four-story Storehouse. First-story Plan

UMNS of yellow pine should have their end-surfaces cut square with the column-axis.

Floors. The FLOORS of such buildings should be continuous, without openings, and of the standard slow-burning construction, described under STANDARD MILL-CONSTRUCTION.



ISOMETRIC VIEW

Fig. 10. Four-story Storehouse. Isometric View

The flooring should be constructed as called for under STANDARD MILL-CONSTRUCTION. In order that the floors may be as nearly water-proof as possible, tarred paper, mopped with tar, should be applied, as previously suggested. The floors in each story of the tower should be at least 1 in lower than the floor in the adjoining compartment, and the sills of the door-openings to the tower should be inclined to make up the difference in levels. The sill, also, of the outside door of the tower should be lower than the tower-floor.

Scuppers. Water on the floors of the tower will ordinarily flow down the tower-stairs, and the arrangement of the floor-levels indicated above will

ordinarily prevent water from an upper story from flowing into one of the lower compartments, if it is escaping through the tower. Cast-iron SCUPPERS are advised, and they should be set in the brickwork at frequent intervals, and so designed that they will carry away rapidly a maximum quantity of water from the floors of each compartment. To further the drainage of water, the floors should be inclined from the middle of the compartments to the scuppers. Fig. 11 shows the WIND-SHIELD SCUPPER * which embodies the latest improve-

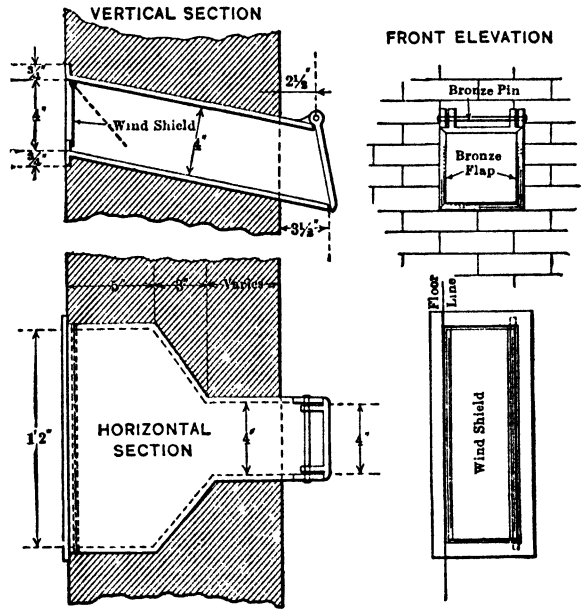


Fig. 11. Detail of Wind-shield Scupper

ments. In the old-style scupper only one flap is provided on the outside of the building. During winter and windy weather, this flap blows open and sometimes freezes open. This results in a continuous draft through the scupper and over the working floor of the factory or warehouse and necessitates an increase in the amount of heat furnished. The scupper shown in Fig. 11 corrects this condition by providing the light wind-shield on the floor-level of the scupper. When the outer flap blows open the wind-shield shuts off the draft from the outside. This scupper, in addition, acts as a fire-retardant when an adjoining building is burning, and when there is a tendency for the flames to communicate through an open scupper and ignite merchandise on the floor. The wind-shield, by shutting off the drafts and fire, acts as a retardant or shield to keep out the flames.

* Manufactured by the Wind-Shield Scupper Company, 1 Madison Avenue, New York City.

Tower for Stairways, Elevators, etc. Access to the various stories is obtained by means of a BRICK TOWER outside the main building, extending 36 in above the roof, and containing STAIRWAYS, ELEVATORS, ETC., access to

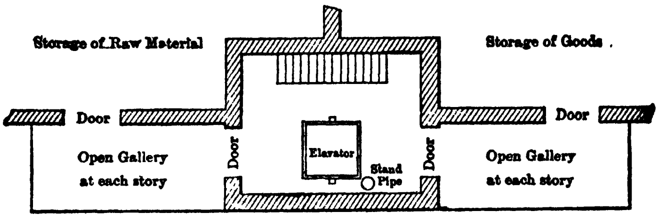


Fig. 12. Stairway-tower and Galleries at Side of Storehouse

which is obtained by open galleries at each floor-level. (See Fig. 12.) A doorway from the upper story of the tower affords a ready means of reaching the roof. AUTOMATIC HATCHES are not necessary for the elevator, as GUARD-GATES serve every purpose. If it is necessary to construct the towei for the

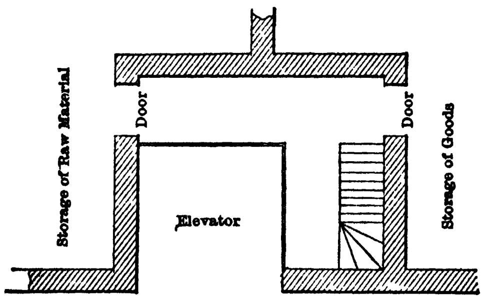


Fig. 13. Stairway-tower Inside of Storehouse

elevator and stairs inside of the building, access to it should be as shown in Fig. 13. This construction serves, also, as a FIRE-TOWER, part of the outside wall being omitted.

Roof Walls and Parapets. The WALLS should extend 36 in above the roof and the PARAPET should be laid in cement, because the moisture readily absorbed by the bricks would otherwise pass downward and make the walls of the top story damp. In some instances a course of bricks dipped in coal-tar is laid above the roof-level.

Sprinklers, Standpipes and Hose. Mills and storehouses should be protected throughout by AUTOMATIC SPRINKLERS and by inside STANDPIPE and HOSE-EQUIPMENTS. Dry-pipe sprinklers should never be used unless it is impracticable to heat the building. These systems should be planned and supervised by a thoroughly reliable fire-protection engineer.

7. Example of One-Story Work-Shop

Economy. For work-shops on cheap, level land, and especially for buildings in which the stock is heavy, ONE-STORY BUILDINGS have proved to be more economical than higher buildings, in cost of floor-area, supervision, moving stock in process of manufacture and repairs to machinery, much of which can be run at greater speeds than when it is in high buildings.

Warming and Ventilating. Window-Area. Such buildings are readily warmed and ventilated, and heavy-plank roofs are free from condensation in cold weather. Window-areas should be as large as practicable, as a large window-area reduces the hours of artificial illumination. If the building is exposed to fire from another building or buildings of hazardous occupancy, the windows should be of the Fenestra, Lupton or other equally good, steel construction, glazed with wire-glass. The forced circulation of heated air is a very desirable method of heating mills, and should be used in connection with overhead steampipes.

Floors. As wooden floors are subject to rot, the general floor-construction, if possible, should be of concrete or earth or some other non-combustible material. But as the dust rising from floors of such materials injures machinery, and as the dripping of oils weakens such floors and seems to make a WOODEN FLOORING-SURFACE necessary, the following construction is recommended. Broken slag or stone, several inches in thickness and thoroughly rolled, is first put down, and over this a 4-in layer of tar-concrete. On this is laid a 1-in thickness of asphalt, evenly rolled. Over this, 2 or 3 in hemlock planks, bedded in hot pitch, are laid and over them a 7/8 or 1 1/8-in maple floor, at right-angles to the planks.

Column and Beam-Construction. Figs. 14 and 15 show clearly the mode of COLUMN AND BEAM-CONSTRUCTION. No beams or other structural timbers should be painted or varnished until thoroughly seasoned.

The Roofs should be as called for under STANDARD MILL-CONSTRUCTION. TRUSSES in roofs are ordinarily from 8 to 20 ft on centers, the 3-in planks spanning the distance between the trusses as shown in Fig. 14, or resting on PURLINS not less than 8 ft on centers, and running longitudinally, as in Fig. 15.

Cornices and Gutters. In Fig. 14, the overhanging OPEN CORNICE is shown, with a drip to the outside and without gutters. Roofs sloping back to inside gutters, as shown in Fig. 15, are preferable. Projecting BRICK CORNICES, which protect the woodwork from outside fires, are shown in Fig. 15. If the building is exposed to other buildings of hazardousconstruction and occupancy, PARAPETTED BRICK WALLS and cornices are needed.

Roof-Construction. The roof-planks should be at least two bays in length, breaking joints every 3 ft; or, if purlins are used, the planks should cover at least two spaces between the purlins, and break joints as above. Roof-timbers should be well anchored to walls in a safe and suitable manner. While the SAW-TOOTH form of roof may be used with this type of building, it may not be always necessary or advisable; and the types shown in Figs. 14 and 15 are types common for machine-shops, foundries, and similar buildings, in which increased head-room is required for traveling cranes. The middle section over the crane is often provided with SAW-TOOTH SKYLIGHTS with excellent results, and the side bays and others are made higher for galleries.

Steel Structural Members. In ordinary one-story machine-shops, or in buildings of similar nature, where wide spans or trusses are necessary, the use of STEEL STRUCTURAL MEMBERS is not objectionable.

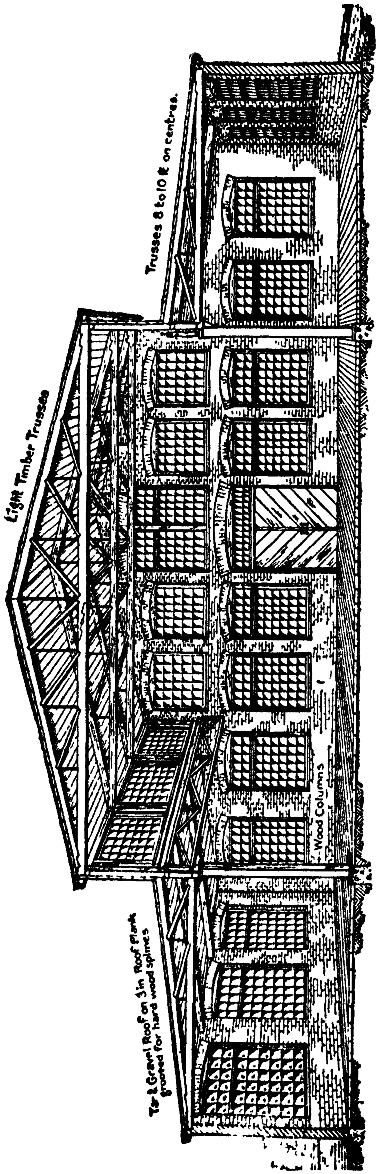


Fig. 14 One-story Work-shop Roof-boards on Trusses.

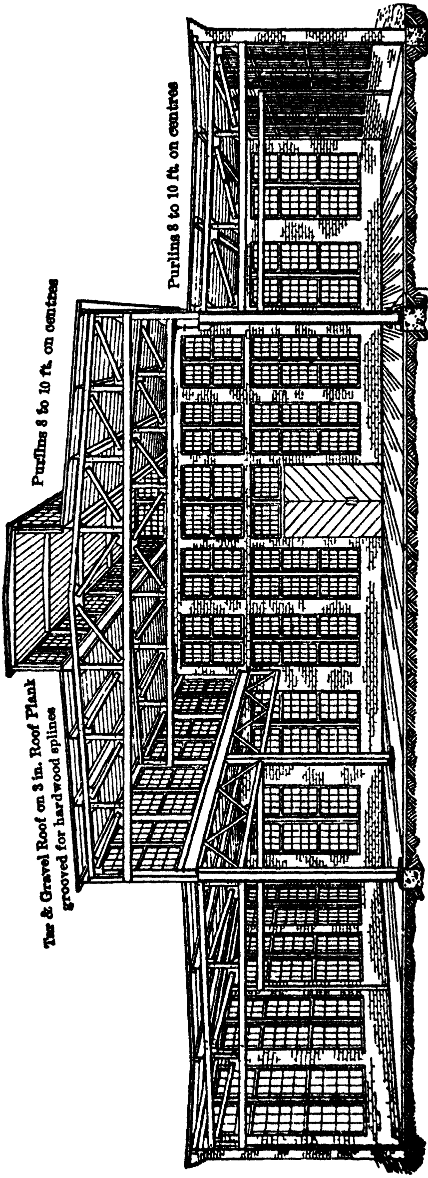


Fig. 15. One-story Work-shop. Roof-boards on Purlins

8. Saw-Tooth Roof-Construction *

The Great Advantages and the increasing use of SAW-TOOTH roof-construction, and the lack of familiarity with it at many factories, make it desirable to outline important features.

Two Typical Designs are illustrated, Fig. 16, a TEXTILE WEAVE-SHED with a good basement for the shafting for driving the looms on the main floor above, thus dispensing with the overhead shafting and belting in the weave-room; and Fig. 17, a design for a light MACHINE-SHOP or FOUNDRY. Other designs, using light wooden trusses or reinforced-concrete walls, are applicable.

Roof-Types. It may be well to state here that while light roofs with 2 in and 3 in joists and with light boards should never be used, and while the principles of SLOW-BURNING or MILL-CONSTRUCTION, with its heavy timbers, are preferred, the increasing difficulty of promptly obtaining yellow-pine lumber of good dimensions, and its increasing cost, often necessitate the use of trusses and rather light timbers; but in no case should these timbers be less than 6 in in width nor of insufficient depth to carry the load. This, also, is in order that they may be SLOW-BURNING. The roofs in all cases should be constructed of planks and have wide bays.

Steel Roof-Trusses. The adaptability of the light forms of STEEL FOR FRAMING TRUSSES, especially when wide spans are needed, often compels their use; and in plants having a safe occupancy, such as that of metal-workers, steel trusses are not objectionable, providing adequate sprinkler-protection with a good water-supply is available to prevent quick failure of the steel work, due to heat from the combustion of the contents of the building or from the burning of the roof. Similar protection is, of course, needed in shops with WOODEN TRUSSES, if disastrous fires are to be prevented; but experience has shown that the STEEL-TRUSSED ROOF will fail much more rapidly than one of wood under similar conditions.

Wooden versus Steel Columns. WOODEN POSTS are nearly always available and should be given preference; but if light STEEL COLUMNS are necessary they should be well protected by insulating materials if they are in rooms containing combustibles, as the column is the vital part of the roof-support.

Advantages of Saw-Tooth Roofs may be outlined as follows:

(1) **Uniform Diffusion of Light** throughout the room, thus making all space in it available. With all interior surfaces painted white and with ribbed glass in the sashes, the DIFFUSION OF LIGHT is almost perfect.

(2) **Better and Cheaper Lighting.** Greater adaptability for lighting large floor-areas in wide buildings with low head-room when compared with what is necessary in wide buildings with the ordinary form of monitor-skylights. Saw-tooth roofs furnish the true solution of the problem of excluding the direct rays of the sun and obtaining the very desirable north light. They result in greater ECONOMY IN LIGHTING, as they lower the fixed charges due to the smaller number of hours per day during which artificial light is necessary.

(3) **Better Working-Conditions,** especially in textile-mills, thereby increasing production and encouraging permanency of employees

(4) **Special Adaptability to Many Industries.** The SAW-TOOTH form is especially adapted to weaving and similar processes in textile-factories, to

* Taken and adapted by permission from the Boston Manufacturers' Mutual Insurance Company's specifications for the construction of saw-tooth roofs.

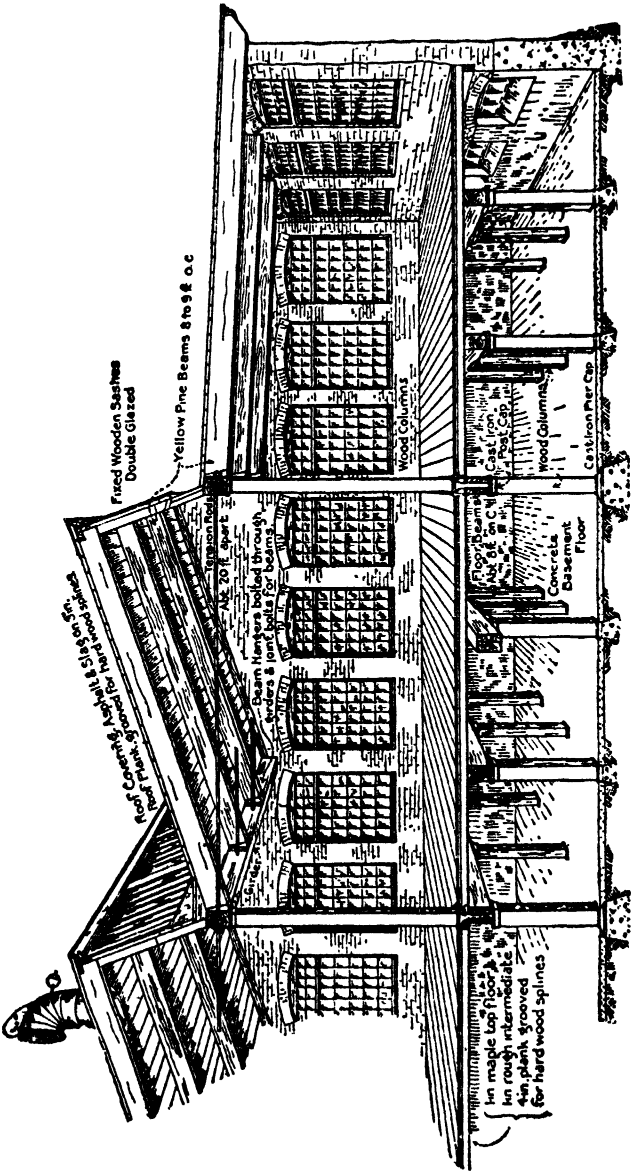


Fig. 16. Saw-tooth Roof for Textile Weave-shed

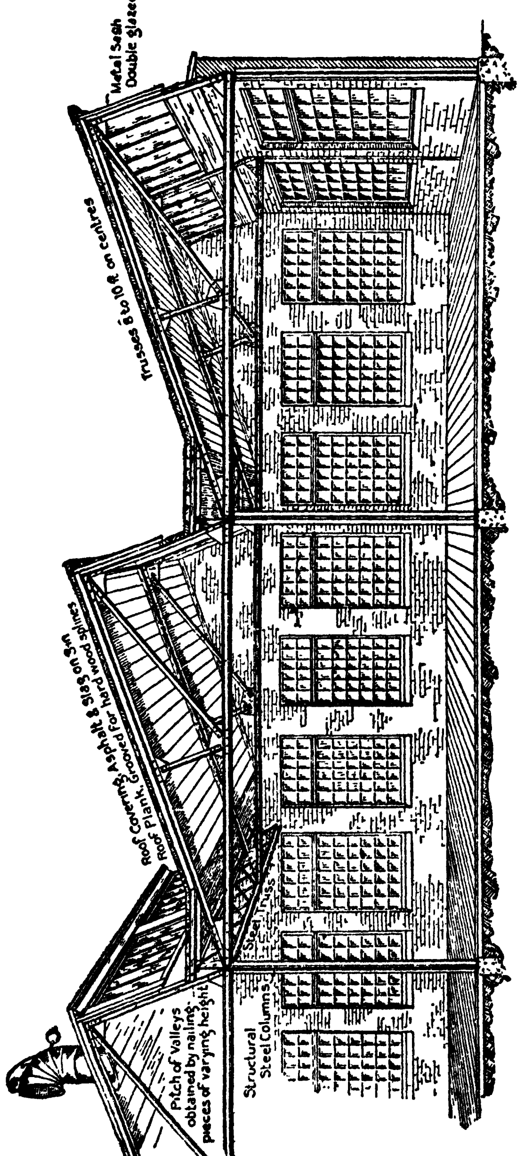


Fig. 17. Saw-tooth Roof for Machine-shop

machine-shops, foundries doing light work, and similar processes, such as assembling and drafting, and to some dye-houses where careful matching of colors is necessary.

Disadvantages of Saw-Tooth Roofs. While the testimony of those who have had experience with SAW-TOOTH ROOFS is almost uniformly favorable, some difficulties have been experienced, practically all of which may be summed up as due to either faulty design or poor workmanship. The difficulties in general are caused by

(1) **Leaks,** due to severe conditions during winter in our northern climates.

(2) **Poor Ventilation.**

(3) **Excessive Heat** when roofs are thin.

(4) **Excessive Condensation** on the underside of roof and glass when the temperature outside is low and there is considerable moisture in the rooms.

Approved Methods of Construction. The following suggestions show how the difficulties mentioned may be obviated if the APPROVED METHODS are applied to special cases by competent engineers or architects. What is good ENGINEERING from the view-point of the manufacturer can also be good FIRE-PROTECTION ENGINEERING, and any design should be adapted to both if the best interests of the manufacturer are to be served:

(1) **Diffused Indirect Sunlight.** As it is desirable to avoid direct sunlight and at the same time obtain an abundance of light, perfectly diffused, the SAW-TEETH should face approximately north and the glass should be inclined to the vertical to take advantage of the brighter light in the upper sky and to prevent cutting off the light by the saw-tooth immediately in front; and, above all, to assure the DIFFUSION OF THE LIGHT over the floor rather than on the under side of the roof-planking.

(2) **Angle of Glass.** For the glass an angle of from 20° to 25° from the vertical and an angle of approximately 90° at the top of the SAW-TOOTH will be about right, the variations depending upon the amount of light required and the latitude. A sharper angle at the top is not needed, as it increases the cost, and makes more roof to be covered and larger spans; more glass, also, is required in proportion, and the light is not as good, as more light from the sky is lost and too much light is thrown on the under side of the roof.

(3) **Glazing-Details.** DOUBLE GLAZING with a space left between the lights of glass is preferred on account of its conducting qualities; but it is not always necessary, except in the more northerly countries. The inside glazing should be done with factory-ribbed glass, set with the ribs vertical and facing in. Shadows cast by trusses are then almost unnoticeable.

(4) **Gutters and Conductors.** CONDENSATION-GUTTERS are needed inside, at the bottom of the sashes, and they should be drained through INSIDE CONDUCTORS and not to the outside under the bottom of the sashes, as these latter admit cold air and are liable to freeze.

(5) **Valleys** between the SAW-TEETH should be flat, from 14 in to 2 ft in width and pitched ½ in per ft towards the conductors, which should be of ample size, and not much over 50 ft apart, and preferably less. The necessary PITCH may be obtained by cross-pieces of varying heights set on top of the trusses, and thus avoiding hollow spaces.

(6) **Prevention of Leaks.** LEAKS, which are common faults, may ordinarily be prevented by a careful design of the gutters, valleys and sashes, and by insisting on good workmanship and materials. The roof-covering of

asphalt or pitch should be continuous through the valleys and extend up to the glass. One form of construction understood to have been very satisfactory is shown in Fig. 18.

(7) **Warming and Ventilation.** Experience has demonstrated the advantage of a combination of DIRECT RADIATION with a FAN sufficient only for VENTILATION and TEMPERING the heat of the room. Heating-pipes should usually be placed overhead and directly under the front of the SAW-TEETH, and run the entire length, and in this position assist in preventing condensation. Where there is no moving shafting, some forced circulation is necessary, and it is best obtained by a fan, which drives the air from either a dry basement or from outside as may be required, and discharges it over heating-coils to the story above. In weaving and similar rooms this is especially necessary and advantageous in promoting the health and comfort of the employees, and in making their working-efficiency greater. Ventilation and cooling of these large areas with comparatively low stories must not be neglected. Ample vents are needed at the top in the form of large metal ventilators with double walls and tight dampers. They are recommended in place of pivoted or swinging sash, which are apt to leak in driving storms, and when open, allow dirt to blow in from the roof. Good windows are advised in side walls and experience has shown their value.

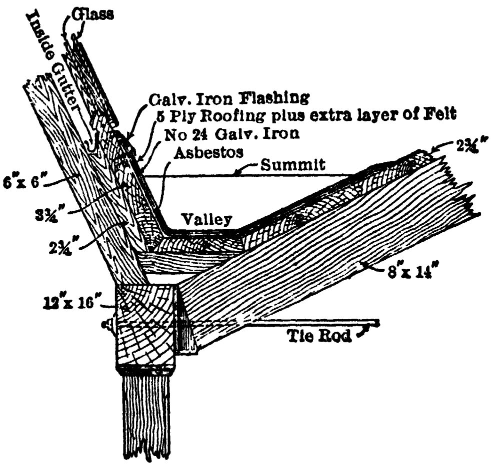


Fig. 18. Detail of Valley of Saw-tooth Roof

(8) **Details of Framing and Construction.** The FRAMING of the SAW-TEETH may be of timber, steel or reinforced concrete. The design should be such as will obstruct the light as little as possible, strong enough to hold wet snow without sagging, and stiff enough to carry shafting motors, etc., when they are to be overhead. When wood or steel is used the roof-planking should be 3 in or more in thickness spanning bays from 8 to 10 ft in width. HOLLOW SPACES in roofs should not be permitted. They are very undesirable from a fire-standpoint, and any condensation which may take place in them during cold weather soon rots both planks and sheathing. SHEATHING, even without spaces behind it, is a more or less objectionable feature, as it is readily combustible; but if it is used it should be applied directly to the under side of the roof-planks, with only a layer of some insulating material between, so that there will be no concealed spaces. If 3 in planks are sufficient for a flat roof, they should be, also, for a SAW-TOOTH roof; and with a good circulation of air there should be no trouble, except in wet rooms. In such rooms there is

bound to be condensation, whether they are under a roof or under the floor of a room above, unless large quantities of dry air are discharged into them.

(9) **Cost.** SAW-TOOTH ROOFS necessarily cost more than FLAT ROOFS, as there is practically the same amount of roofing as in flat roofs and, in addition, the cost of windows, glazing, flashing, conductors, condensation-gutters for skylights, and a somewhat larger cost for heating. The additional cost of these items does not, however, fairly represent the comparative cost, as there should be considered the total cost of the building compared with that of an ordinary one with sufficiently high stories and with a width narrow enough to give the required light. When this is done the slight additional cost is far outweighed by the advantages gained for work requiring very good light.

9. Mill-Construction as Applied to Warehouses

Cost. Owing to the increasing cost of heavy timbers for wooden construction, to the lower cost of the so-called FIRE-PROOF CONSTRUCTION, and also to the better FIRE-RESISTING qualities of the latter, owners, architects and builders should carefully compare the cost of construction, and also the cost of insurance of the two types, before deciding on the one to be used. The difference in the cost of construction between these two types is so small, that in many localities the lower cost will be in favor of the REINFORCED CONCRETE or other type of FIRE-PROOF CONSTRUCTION. The cost of construction is also in favor of the FIRE-PROOF TYPE, where both long spans and strength are required

Timber-Spacing for Sprinklers. Warehouses of MILL-CONSTRUCTION should be built so as to allow the best possible distribution of water from AUTOMATIC SPRINKLERS, with the least possible obstructions, and floor-timbers, therefore, should be as few as the floor-loads will allow. There should be no concealed spaces of any kind in the building. To insure the greatest efficiency for sprinkler-systems, it is better to adapt the timber-spacing to suit the sprinklers, rather than to arrange the sprinklers to suit the timber-spacing.

Mill-Construction Adapted to Warehouses. The features of bad construction mentioned under WHAT MILL-CONSTRUCTION IS NOT are as objectionable in warehouses as in factories, while the construction advocated for mills may be used with almost equal advantage in the erection of warehouses. But as the latter are usually erected in the more thickly settled portions of a city, they are more subject to the dangers of a conflagration; and it should be understood that even the best SLOW-BURNING CONSTRUCTION will stand but a short time after a fire has obtained a good headway, the main object of MILL-CONSTRUCTION being to retard the spreading of fire by the use of heavy timbers and the absence of concealed spaces. In applying the principles of MILL-CONSTRUCTION to warehouses, therefore, the general principle of using large timbers placed as far apart as the loads will permit, and of avoiding all concealed spaces, should be constantly kept in mind.

Warehouse-Floors, however, are generally required to sustain heavier loads than are found in woolen and cotton-mills, and hence require heavier construction. While WAREHOUSE-FLOORS are quite often built with transverse girders, 8 or 10 ft apart, the spaces being spanned by flooring from 4 to 6 in thick, the more common method of construction is to use one or more lines of longitudinal girders supporting floor-beams spaced as far apart as possible, preferably not less than 8 ft on centers.

Area and Height. The AREA of buildings of this type should be, preferably, not over 7 500 sq ft, and in no case should it exceed 15 000 sq ft between fire-

walls. If buildings of LARGE AREA are required, it is advisable to divide them into separate sections by fire-walls, thus reducing the liability to one fire, and affording an opportunity of storing hazardous goods in one or more sections, and non-hazardous or less hazardous goods in the remaining sections. Where ground is available, it is better to have a building of LARGE AREA AND LOWER HEIGHT divided into fire-sections, than to have a building of LESSER AREA AND GREATER HEIGHT, as the former construction affords a more economical handling of goods, and less concentration of values. Buildings of this type should be limited to 65 ft in height, and to six stories, thus discouraging the overloading of floors. Piled goods should be kept at least 18 in away from beams, thus allowing for the distribution of water from the sprinklers.

Walls should be of brick, and not less than 13 in thick in the upper story, and they should be increased in thickness on the lower floors to take care of additional loads. PARTY WALLS should be increased at least 4 in in thickness, and all walls should be laid in cement mortar, should extend above the roof at least 36 in and be coped with stone, salt-glazed terra-cotta, or similar noncombustible materials. OPENINGS IN DIVISION WALLS should be limited to as few as possible, not over three in each story, they should not exceed 80 sq ft each in area, and should be protected by double, automatic, sliding fire-doors, as specified elsewhere.

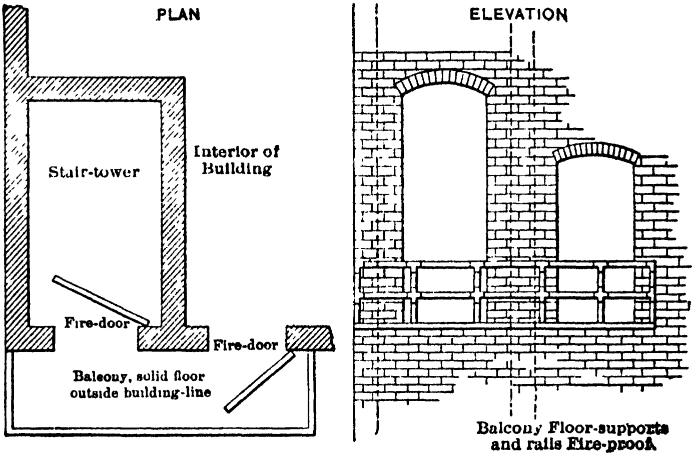
Openings in Walls. As a protection against fires from surrounding properties, OPENINGS IN OUTER WALLS should be small, limited to as few as possible, and protected by standard fire-shutters and doors, or standard wire-glass windows. If the surrounding buildings are of hazardous occupancy or inferior construction, and the distance between the warehouse and the latter but a few feet, shutters are preferable, as wire-glass windows are recommended only where the exposures are moderate. Even though the building is not exposed to fire from other buildings, the protection of WINDOW-OPENINGS may prevent the spread of fire from story to story through the windows.

Girders and Beams which support the floors and roof should be SINGLE PIECES, not less than 6 in in least dimension, and with a sectional area of not less than 72 sq in; while columns should be not less than 8 by 8 in in cross-section in the upper story, and should be increased in size in the other stories to take care of any additional loads. The beams and girders should be SELF RELEASING (Fig. 2), and the floors should be built as outlined under STANDARD MILL-CONSTRUCTION, inclined at least 1 in in 20 ft, made as nearly water-proof as possible, and scuppered to the outside of the building. These scuppers should be set in brick-work at frequent intervals, of sufficient size to carry off the maximum amount of water from each floor, and so constructed that they will prevent the admission of cold air to the building. (See Fig. 11.)

Towers. The floors should be continuous from wall to wall, avoiding holes for belts, stairways, elevators, etc. All such openings should be enclosed in a BRICK TOWER or in TOWERS extending not less than 36 in above the roof, coped as above, and accessible from each story by means of an outside balcony (Fig. 19). Where it is impossible, owing to the location or otherwise, to have these openings on the outside, they should be placed in BRICK TOWERS constructed inside the building and connecting with an entrance to a fire-proof vestibule, open to the weather. There should be openings from each story to the vestibule, each protected by standard fire-doors (Fig. 20).

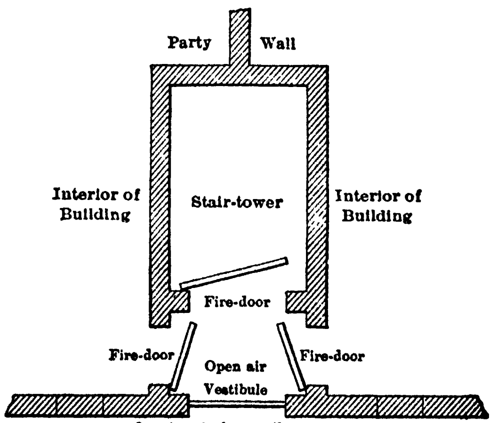
Gravity-Tanks for Automatic Sprinklers are usually placed on extensions of such towers, and they should be built to carry the additional load imposed. Easy access to the roof of the building may be had from a window or windows

placed in the tower, and such opening or openings should be protected by fire-shutters, especially where the tower is elevated a sufficient distance to allow



Note: Walls of brick or other approved material, built solidly from foundations to at least 36 inches above roof. Stair-treads, etc., of fire-proof material.

Fig. 19. Tower Fire-escape. Outside-balcony Entrance



Openings in face-wall extend from floor to ceiling, Vestibule-floor of fire-proof construction, Railing at opening.

Fig. 20. Tower Fire-escape for Adjoining Buildings

the tank to be placed inside of the tower, thus preventing flames from gaining access to the tower and destroying the tank and tank-supports.

Boilers should be, preferably, in a separate building, cut off by standard fire-doors from the warehouse; or, if in the main building, should be located in a room of FIRE-PROOF CONSTRUCTION, access to which should be from outside the building only.

Structural Steel Members should never be used in this type of construction, as they will not resist even a moderate fire. If used, they should be protected with fire-proof material. The lintels should be brick arches and not steel sections.

10. Steel and Iron Structural Members in Warehouse-Construction

Metal versus Wooden Standard Members. Owing to the fact that a beam or column of STEEL OR WROUGHT IRON when heated will fail by buckling or bending very much sooner than an equivalent beam or post of WOOD, it is important that such members be of WOOD, provided that the WOODEN BEAMS have a sectional area of at least 72 sq in, and are not less than 6 in in least dimension, and that WOODEN COLUMNS have a sectional area of not less than 8 by 8 in. CAST-IRON COLUMNS, also, will generally fail in fire and water sooner than wooden columns.

Fireproofing Steel Beams and Girders. When STEEL BEAMS and COLUMNS are used, fireproofing is necessary to make them as FIRE-RESISTING as the floors. Such beams and girders may be FIREPROOFED as shown in Fig. 21. Metal-wire mesh should be placed as shown, and tied to the beams and girders with metal clips; and to insure rigidity during the pouring of the concrete and to keep the mesh in alignment, forms should be used. The concrete should be poured before the floors are laid, and after the wooden beams are in position. After completion, the insulation should be at least 1 in at the edges of the flanges, 2 in under the lower flange of the beam and 3 in under the lower flange of the girder. The webs should be filled solid. Where there is little storage of a combustible nature in the building the beams may be protected as shown in Fig. 22.

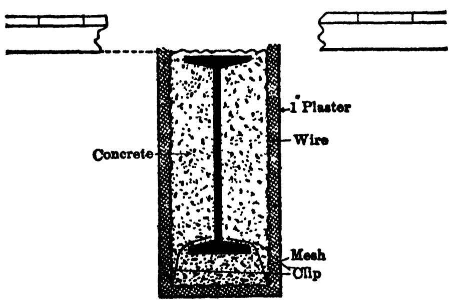


Fig. 21. Fireproofing of Steel Beam with Concrete and Plaster

Fireproofing Metal Columns. COLUMNS, either STEEL, WROUGHT-IRON, or CAST-IRON, should be protected even to a greater extent than girders and beams, and should have at least 3 in of concrete at the flanges, at least $1\frac{1}{2}$ in at the edges, and be filled solidly against the webs. Fig. 23 shows two columns protected by concrete held by wire mesh on $\frac{5}{8}$ in rods, and all securely held to the column by metal clips. Forms should be used and the concrete should be poured as the girders and beams are protected. Steel beams, girders and columns are difficult to protect, especially at the intersections of steel and wood, and this insulating material can best be applied before the floors are laid. The fireproofing of these members will be of little avail, unless the materials

are good, well tied to the metal members, and applied by workmen who understand such work.

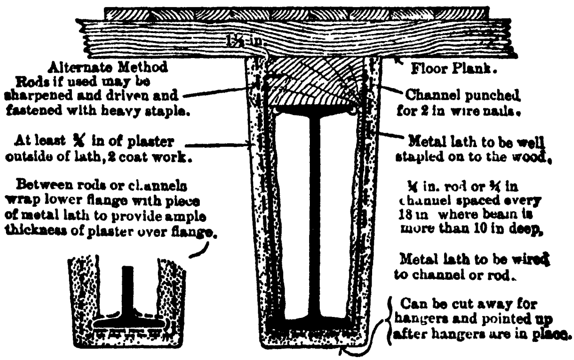


Fig. 22 Fireproofing of Steel Beam with Metal-lath and Plaster

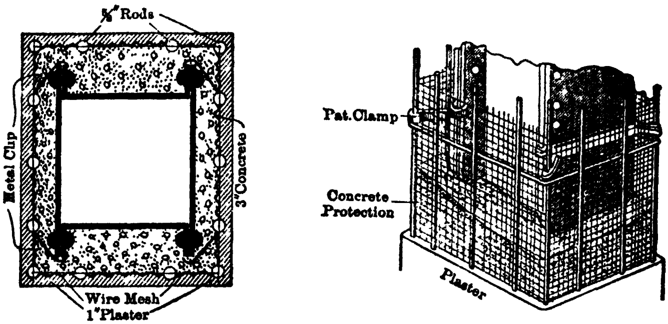


Fig. 23. Fireproofing of Steel Columns with Concrete and Plaster.

Fig. 24 illustrates the PROTECTION OF A ROUND COLUMN by reinforced concrete. Here the concrete is held in position by wire mesh on metal furring, held in position by metal clips or ties. The fireproofing should be at least 4 in thick, and forms should be used in surrounding the columns. In addition to the above reinforcements for these columns, lateral reinforcement should be added by means of iron rods wound spirally around mesh, and placed 12 in on centers. After the forms are removed, and the wooden floors are laid, the columns and girders should be finished with a 1 in thickness of hard plaster, filling all interstices between the woodwork

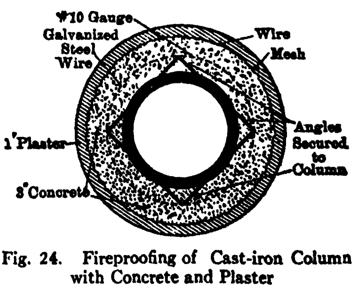


Fig. 24. Fireproofing of Cast-iron Column with Concrete and Plaster

and the insulation. Tile, owing to the difficulty of properly bonding it, is not as effective as concrete; but if securely bonded by means of metal, it is quite satisfactory. Fig. 25 illustrates the PROTECTION OF A GIRDER AND A COLUMN by means of tile. There are other equally efficient methods of beam

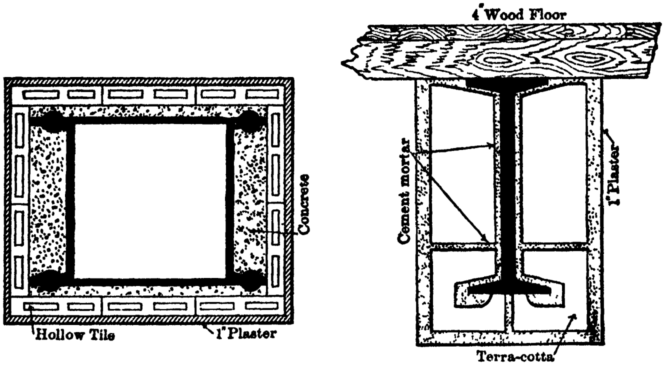


Fig 25. Fireproofing of Steel Columns and Beam with Tile

and column-protection, described in Chapter XXII. In buildings of warehouse-construction, heavy goods are handled, and it may be advisable to protect the base of each column with sheet metal to a height of 36 in above the floor, to prevent any weakening of the fireproofing.

Pipes for Gas, Water, etc., should not be enclosed in column or girder-insulation.

11. Structural Details of Mill-Construction as Applied to Factories and Warehouses

Column, Girder and Joist-Framing. Fig. 26 illustrates the method of carrying the girders from the walls, posts, etc., the bottom post resting on a steel POST-BASE. The first floor above the basement is shown with longitudinal girders only, and heavy mill-flooring set on them. The girders are framed at the post in a steel POST-CAP, and are hung clear of the wall in an approved steel WALL-HANGER. The next floor above shows the construction in which the joists are framed into the girders by means of JOIST-HANGERS. The framing at the post, also, is done by means of a DUPLEX FOUR-WAY POST-CAP, while the girder is built into the wall in a DUPLEX WALL-BOX. The JOIST-HANGERS are used singly or opposite each other as required and are bolted to the girder, thus tying the building laterally. The upper floor shows the joists resting on the girder. This construction, however, does not comform to STRICT MILL-CONSTRUCTION, as it exposes a larger amount of timber-surface. The girder is shown built into the wall and resting on a WALL-PLATE. This distributes the load over the masonry but is not as effective in preventing dry-rot as the WALL-BOX or WALL-HANGER.

Steel and Malleable-Iron Post-Caps and Bases. Fig. 27 illustrates other details of construction which may be used. The bottom post rests on a steel POST-BASE. The POST-CAP shown on the bottom post is a DUPLEX FOUR-WAY

STEEL POST-CAP, while the POST-CAP above it is one of the malleable-iron type, approved by the National Board of Fire Underwriters. The POST-CAP shown at the top, also, is of malleable iron and intended for lighter construction or for girders which run across the post as shown. The girders in every case are

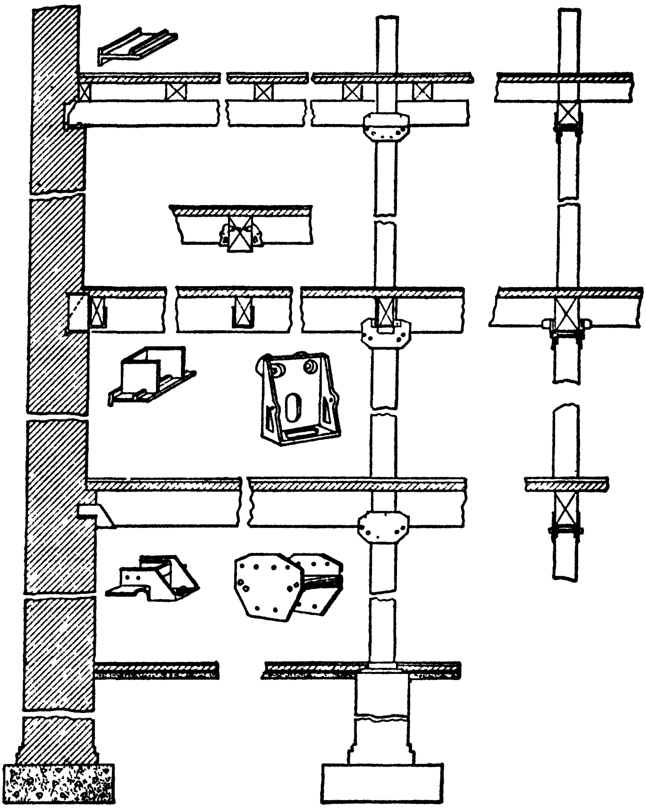


Fig. 26. Mill-construction. Column, Girder and Joist-framing.

carried clear of the wall by means of approved WALL-HANGERS and the beams are carried by the girders in malleable-iron JOIST-HANGERS.

Cast-Iron Post-Caps and Bases. Fig. 28 illustrates other details of construction. The lowest post rests on a heavy, cast-iron, ribbed POST-BASE. The first-story floor-girders are carried at the post by means of heavy, cast-iron POST-CAPS and are built into the wall in cast-iron WALL-BOXES. When cast-iron is used for POST-CAPS it is essential that it be made extra-heavy, as cast-iron is very uncertain on account of the uneven shrinkage when cooling, which often

causes internal stresses and weakens the caps. Flaws, also, may develop during the manufacture which weaken the caps and greatly impair the safety of the building. An objection to cast-iron is its tendency to crack and break during a fire when cold water is thrown on it. The POST-CAPS shown in Fig. 28

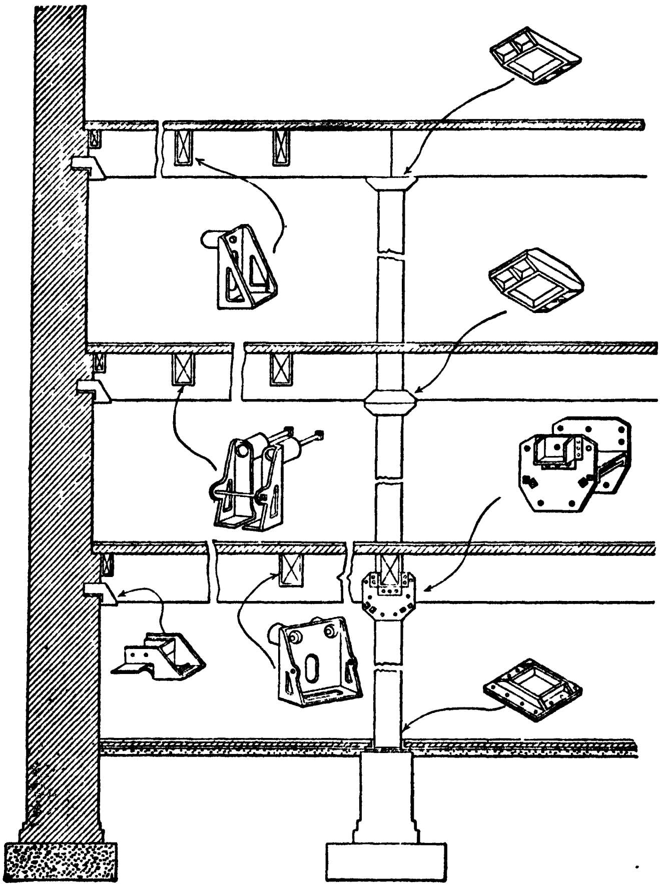


FIG. 27. Mill-construction. Malleable-iron Post-caps and Bases

are of cast-iron for the first and second floors, Duplex steel for the third floor, and malleable iron on the top post.

Duplex Combination Post-Cap. Fig. 29 illustrates the use of the DUPLEX COMBINATION POST-CAP on the bottom post. This cap is made with a mal-

leable-iron lower part and a steel upper part. The POST-CAP shown on the second post is called the IDEAL POST-CAP and consists of a steel upper part with

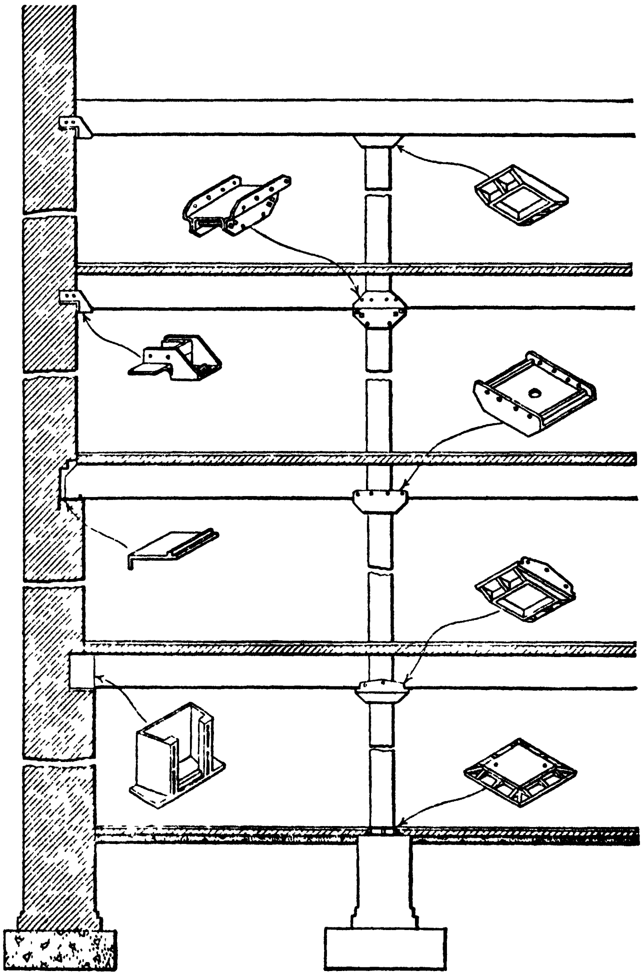


Fig. 28. Mill-construction. Cast-iron Post-caps and Bases

steel angles riveted underneath to fit the post. The cap shown on the top post is the old-style, cast-iron cap. The WALL-HANGER, WALL-BOX, WALL-PLATE and JOIST-HANGER shown are used in STANDARD CONSTRUCTION.

Steel Post-Caps. Fig. 30 illustrates various forms of steel POST-CAPS. The IDEAL POST-CAP is shown on the bottom post and the VAN DORN POST-CAP on the post next above. On the top post the STAR POST-CAP is shown. This has a fin for which the top of the post must be slotted to receive it. Steel JOIST-

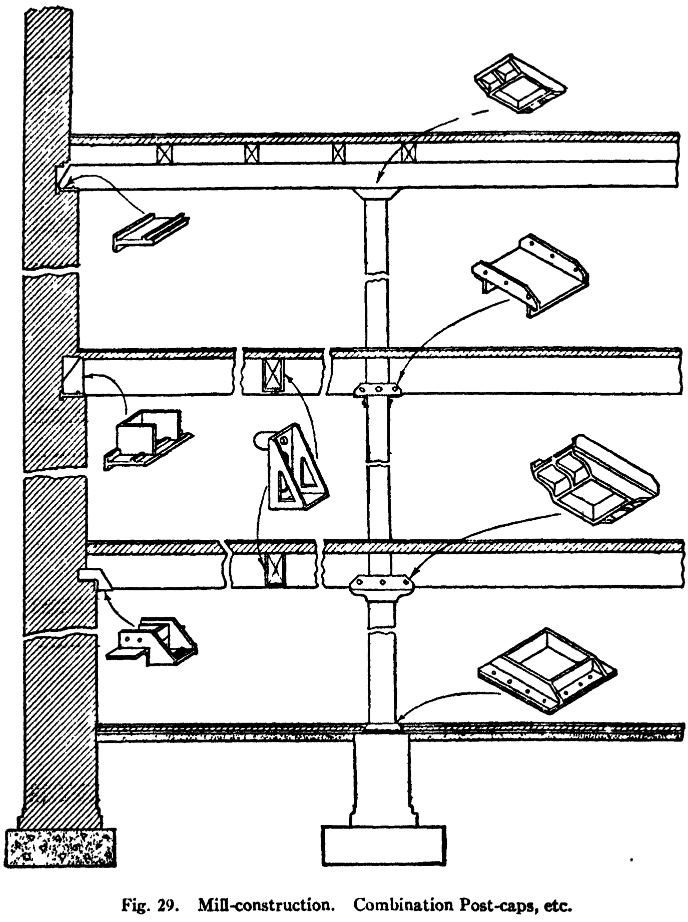


Fig. 29. Mill-construction. Combination Post-caps, etc.

HANGERS are shown for the two lower floors. The IDEAL JOIST-HANGER is illustrated in the lower floor. It is spiked to the sides and top of the girder. The VAN DORN JOIST-HANGER is shown in the second floor, while the old-style STIRRUP is shown in the top floor. The WALL HANGERS illustrated are of the approved type.

Framing Steel Beams and Girders. Fig. 31 illustrates the use of I-BEAM GIRDERS in place of WOODEN GIRDERS and their connections with wooden beams. In this kind of construction it is necessary to fireproof the steel beams, as they are more readily affected by heat in case of fire than large wooden timbers Intense heat often causes them to collapse and ruin a building.

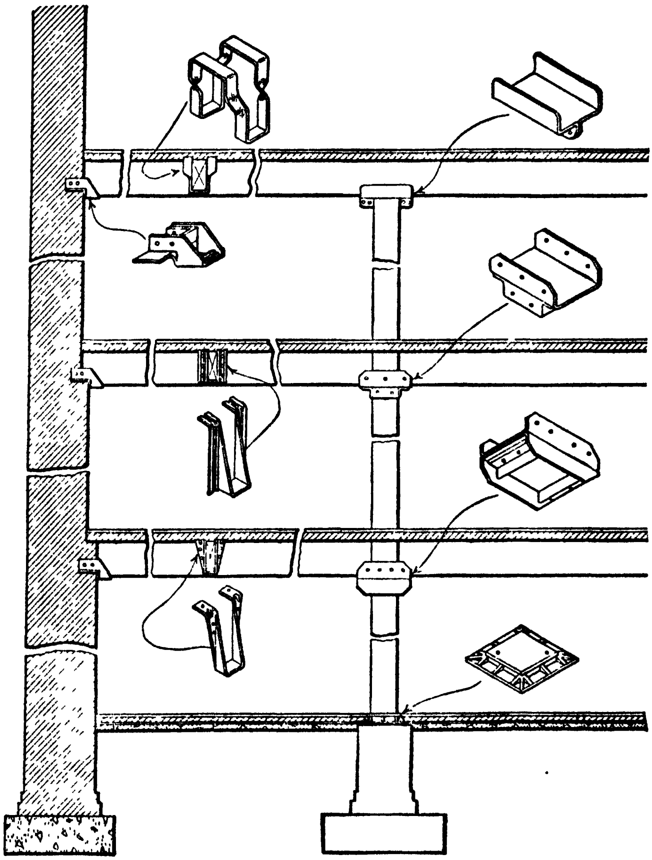


Fig. 30. Mill-construction. Steel Post-caps, etc.

The HANGER shown in the first floor is used where the I beams and wooden beams are of the same height. This HANGER provides an extra bearing for the timber and has proved very satisfactory. The HANGER shown in the second floor is used when it is necessary to raise the wooden beam above the lower flange of the steel beam. This HANGER brings all the load on the lower flange of the I beam and provides an anchorage for the wooden beam. It is used singly or in pairs on the I beam as required, and is bolted through

the web of the I beam. This has been found to be a very economical and efficient construction. In the third floor the wooden beam is shown framed

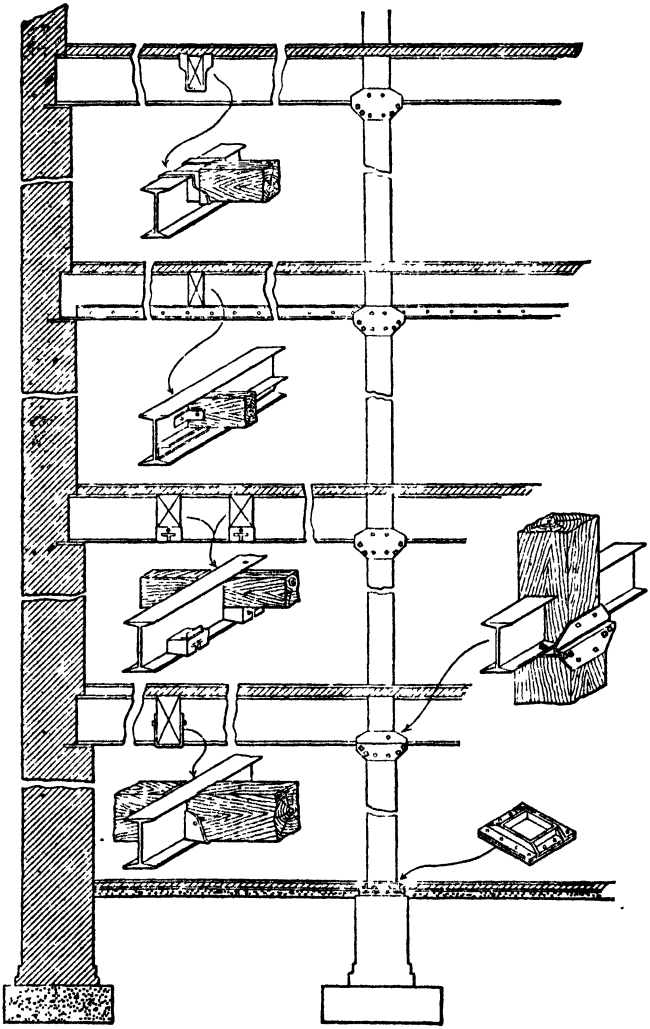


FIG. 31. Mill-construction. Framing Steel Beams and Girders

to the I beam by means of a SHELF-ANGLE. With this form of construction it is necessary to rivet the SHELF-ANGLE to the web of the I beam. The upper detail shows the old-fashioned STIRRUP passing over the top flange of

the I beam and carrying the wooden beam. The POST-CAPS shown are the DUPLEX STEEL POST-CAPS which are approved by the National Board of Fire Underwriters.

12. Connections of Floor-Beams and Girders

Girder-Hangers and Joist-Hangers. To render the construction, and particularly the girders, SLOW-BURNING, it is important to have no hollow spaces between the top of the girders and the flooring, that is, to have the top surface of the floor-beams flush with that of the girders. This, of course, necessitates framing the floor-beams into the girders. For HEAVY CONSTRUCTION the only kind of framing that is permissible is one in which some kind of JOIST-HANGER is used. When the floor-beams are 6 by 12 in or larger in cross-section, and the girders are of wood, the author would give the preference to the DUPLEX HANGER shown in Fig. 32.

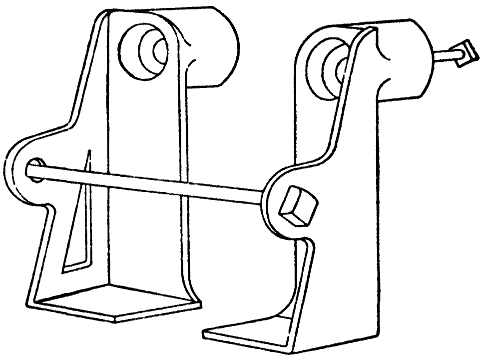


Fig. 32. Duplex Hanger for Heavy Floor-beams

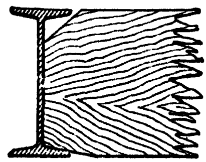


Fig. 33. Framing I Beam and Wooden Beam of Same Depth

If STEEL-BEAM GIRDERS are used in place of WOODEN GIRDERS, there are several methods in use for framing the wooden beams. Fig. 33 shows a steel I beam, and a wooden beam of the same depth framed into it and resting on its lower flange. In most cases, however, this does not afford a sufficient bearing for the wooden beam. Fig. 34 shows a SHELF-ANGLE riveted to the web of the I beam. Whenever this method of supporting the beams is used, enough bolts or rivets should be used to support the load carried by the SHELF-ANGLES. Each ¾ in bolt may be considered to support 3 000 lb on each side of the girder, and each ⅞-in bolt, 4 000 lb. The methods shown in Figs. 35 and 36 are sometimes used, but are open to objection on account of the weakening of the wooden beams when loaded

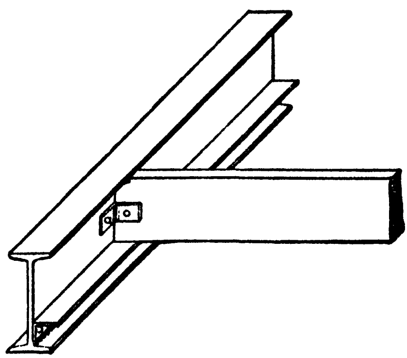


Fig. 34. Wooden Beam Framed to I Beam with Shelf-angle

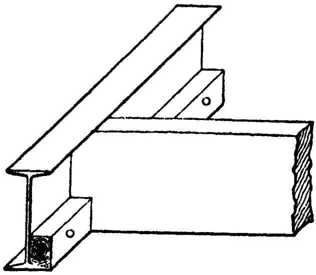


Fig. 35. Wooden Beam Framed to I Beam with Wooden Cleat

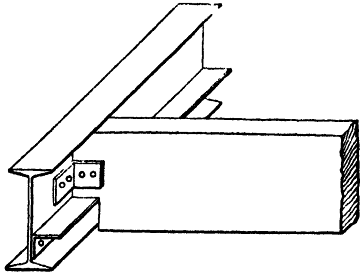


Fig. 36. Wooden Beam Framed to I Beam with Shelf-angle

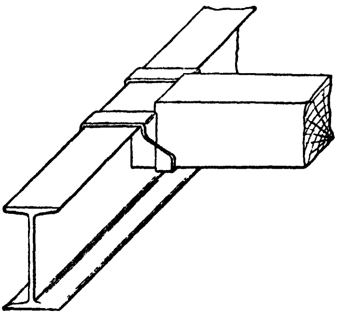


Fig. 37. Wooden Beam Framed to I Beam with Stirrup-hanger

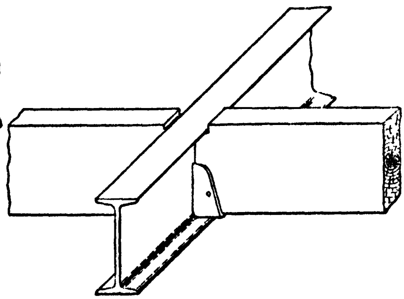


Fig. 38. Wooden Beam Framed to I Beam with Duplex Hanger

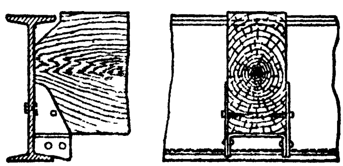


Fig. 39. Wooden Beam Framed to I Beam with Duplex Shelf-hanger

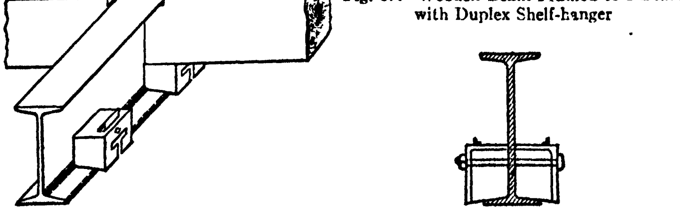


Fig. 40. Wooden Beam Framed to I Beam with Duplex Box-hanger

Fig. 37 shows a STIRRUP-TYPE of hanger. This construction permits the framing of the wooden beam at any desired height, and has proved satisfactory. These hangers can be used with any depth of beam or girder, and are furnished by all manufacturers of steel JOIST-HANGERS of the various types, as well as by blacksmiths who can make WROUGHT-IRON STIRRUPS. Fig. 38

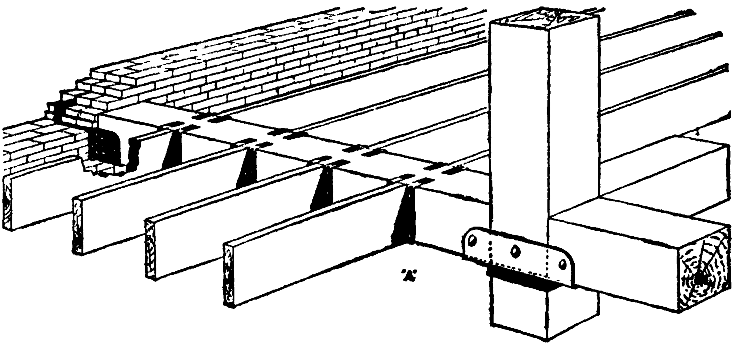


Fig. 41. Floor-framing with Van Dorn Hangers and Post-caps

shows the DUPLEX-TYPE OF HANGER for framing a wooden beam flush with the lower flange of the I beam. This hanger is attached by means of bolts. Fig. 39 shows the same design of HANGER, with the SHELF-CONSTRUCTION used to carry the wooden beams up to 4 in above the lower flange of the I beam.

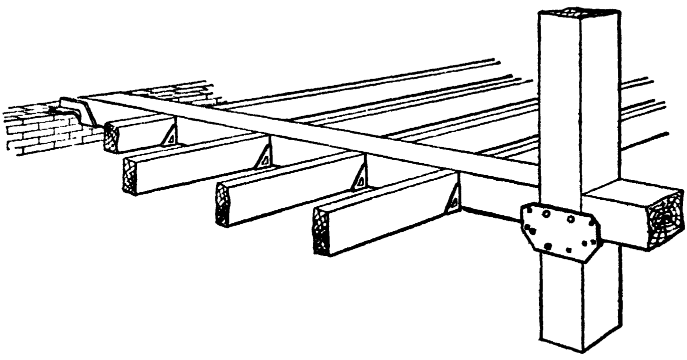


Fig. 42. Floor-framing with Duplex Hangers and Post-caps

Fig. 40 shows a HANGER for carrying the wooden beams 4 in or more above the lower flange of the I beam.

The HANGERS described in Figs. 38, 39 and 40 are all of the DUPLEX TYPE, and are so constructed that all the load is carried on the lower flange of the I beam, which is a very satisfactory and ideal construction whenever it is necessary to frame wooden beams into and not rest them on the I beams. The design is a very economical one for framing wooden beams to I beams, as the

holes for attaching these HANGERS can be punched while the steel is being fabricated, and the HANGERS are attached to the steel beams by means of bolts when the wooden beams are put in place. These HANGERS are provided with lugs or lag-screws for anchoring the wooden beams securely to the steel girder. Fig. 41 shows a floor-framing with the VAN DORN STEEL HANGERS. Fig. 42 shows the floor framed with the DUPLEX TYPE OF HANGER and POST-CAP. The same principle of construction is applicable to larger wooden beams spaced farther apart.

13. Wall-Supports and Anchors for Joists and Girders

Box Anchors, Wall-Hangers, etc. Anchoring. In a warehouse intended to be constructed on the SLOW-BURNING PRINCIPLE, the floor-beams and girders should be anchored to, and supported by the walls in such a way that in case the beams are burned through, the ends may fall without injuring the walls; and where large timbers are used, provision should be made against the possibility of dry rot.

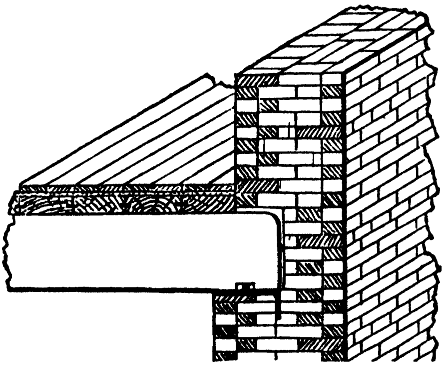


Fig. 43. Early Form of Beam-support in Mill-construction

Box Anchors. The method of supporting the beams in MILL-CONSTRUCTION as originally developed in the New England mills is shown in Fig. 43. This fulfilled the requirements above mentioned, but it weakened the walls to some extent. The

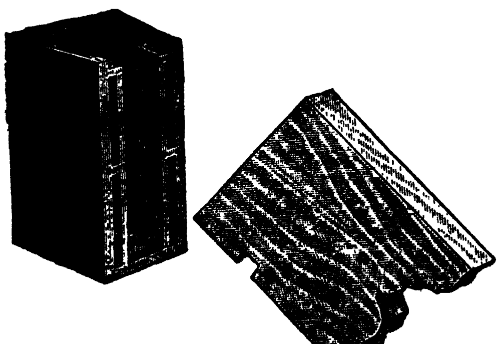


Fig. 44. Goetz Box Anchor for Wooden Beams

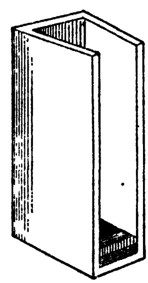


Fig. 45. Goetz Box Anchor for Wooden Beams

GOETZ CAST-IRON BOX ANCHORS shown in Figs. 44, 45 and 46 and the DUPLEX WALL-BOX shown in Fig. 47 are decided improvements on the anchor shown in

Fig. 43, as they afford all the advantages of the latter without weakening the walls, unless the floor-beams are very wide. The WALL-BOX as shown in Fig. 47 is made with a malleable-iron bottom plate and a steel box above. It has a rib on the plate at the back, which extends up and down, and acts as a secure anchorage in the brickwork. These WALL-BOXES are made wedge-shape, and it is therefore impossible to pull them out of the wall. The more weight there is on the beam, the stronger will be the bond that holds the beam to the box and the box to the wall.

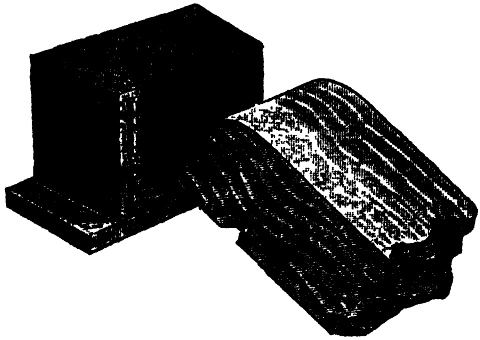


Fig. 46. Goetz Box Anchor for Wooden Girders

In case of fire or accident, the joists can burn through or break; and in falling they can free themselves from the anchorage and leave the wall standing The wall is not even weakened by the space left in it, because the box remains, and the crushing strength of this CAST-IRON BOX is much greater than that of the wall. No break or breach is made in the wall, and the box that remains, securely held, forms a space for the easy replacement of the wooden beam. The box provides a perfect and secure foundation for each beam. Fire from a defective flue cannot ignite a beam-end, because it is protected by a ventilated, CAST-IRON BOX. The WALL-BOXES have air-spaces, also, in the sides, ½ in wide, which permit a circulation of air around the ends of the beams, effectually preventing dry rot. If timber is wet or unseasoned these wall-boxes allow it to dry out after it is put in the building. The average weight of a box like that shown in Fig. 45, for 2 by 12 in joists, is 10 lb.

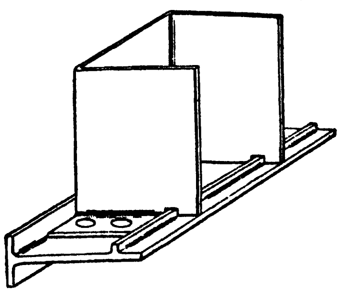


Fig. 47. Duplex Wall-box with Ribbed Plate

Wall-Hangers. Another device for obtaining the same results in a different way is the WALL-HANGER. Figs. 48 and 49 show DUPLEX WALL-HANGERS for large timbers. The hanger shown in Fig. 49 is made of open-hearth steel and is extra-heavy. Each of these hangers is provided with a plate which has an 8-in bearing on the wall, and the bearing of the timbers on the hanger is also 8 in. For beams not exceeding 10 in in breadth there is probably little choice between the BOX ANCHOR, Fig. 46, and the WALL-HANGERS, Figs. 48 and 49, except perhaps in the price and appearance. When the WALL-HANGER is used, no hole is left in the wall, and a saving of 6 in in the length of the beams is effected, which in some cases would be a consideration. For girders 12 by 14 in and upwards in cross-section, the author believes that the hanger shown in Fig. 49 is preferable to the BOX ANCHOR. WALL-HANGERS

made from STIRRUPS should not be used for heavy beams. The use of any one of the hangers or boxes is obviously greatly superior to the ordinary method of anchoring beams or girders to walls, and the use of such hangers will undoubtedly save much loss which would be caused by the falling of the walls. These are almost invariably pulled down by the ORDINARY IRON ANCHORS when the beams fall. Fig. 50 shows the application of a WALL-HANGER.

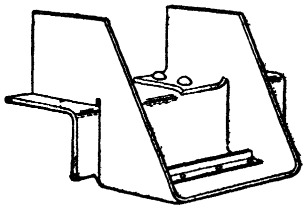


Fig. 48. Duplex Wall-hanger for Large Wooden Girder

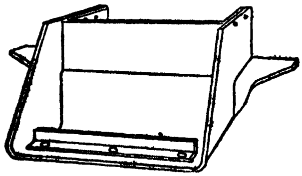


Fig. 49. Duplex Extra-heavy Wall-hanger for Large Wooden Girder

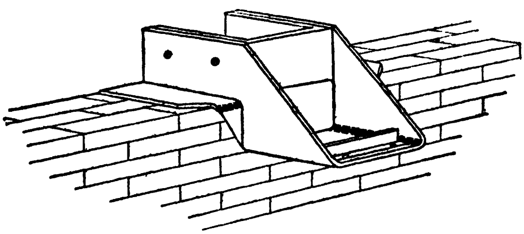


Fig. 50. Application of Wall-hanger to Brick Wall

14. Weakness of Wrought-Iron Stirrups when Exposed to Fire

Stirrups and Fire-Tests. Referring to this subject, Professor J. B. Johnson, of Washington University, said: "The recent fire tests of STEEL STIRRUPS and brick walls which were made under my supervision in this city (St. Louis), show very conclusively that unprotected stirrups are extremely dangerous. These stirrups become red-hot in a few minutes and then rapidly char and burn away the ends of the beams; and they also bend down, so that in from twenty to thirty minutes after the fire reaches the stirrups, the beam is dropped right out of the twisted steel by the straightening out of this bend or twist."

The Duplex Hangers possess an advantage over STEEL STIRRUPS because, being of malleable iron, they are not as quickly affected by heat, there are no twists or bends to straighten, and the bearing in the trimmer or header is to a great degree protected by the form of construction. During a severe fire at Paterson, N. J., some DUPLEX WALL-HANGERS were subjected to a most severe test without apparent injury. It is undoubtedly desirable that all structural iron should be protected from fire, but it is almost impracticable to effectively protect the STIRRUPS used in connection with wooden beams without going to a greater expense than the character of the construction warrants.

15. Post and Girder-Connections

Iron Cap-Plates, Wooden Bolsters, etc. Whenever a building is constructed with wooden posts extending through several stories, each upper post should rest on an IRON CAP-PLATE, fitted over the post below, and never on a girder or even on a WOODEN BOLSTER. A BOLSTER would not be objectionable were it not for the fact that the pressure under the post is generally sufficient to crush the fibers of any kind of wood. Then, too, there is always some settlement due from shrinkage. As posts are used expressly for the support of beams or girders, the IRON CAPS must, of course, extend sufficiently beyond the upper post to afford ample bearing for the end of the girder. This bearing in square inches should be equal to at least one-half the load on the girder divided by the safe resistance of the wood to crushing across the grain

Example. A 12 by 14 in yellow pine girder is designated to support a possible load of 38 000 lb. What bearing should it have at the ends?

Solution. The safe resistance given for long-leaf yellow pine to crushing across the grain is 350 lb per sq in. One-half the load on the girder is 19 000 lb, and hence the bearing area should be 19 000 divided by 350 or about 54 sq in. As the breadth of the beam is 12 in this would require a bearing lengthwise of the girder of 4½ in. In no case should the bearing be less than that required by the above rule

16. Form and Material of Post-Caps

Cast-Iron versus Steel Post-Caps. Formerly CAST-IRON POST-CAPS were used for the framing of the girders at the columns and posts. But the uncertainty attached to the use of cast-iron, and the necessity of extremely heavy caps to assure safe construction have led most engineers to specify STEEL POST-CAPS, as they are unquestionably the strongest form of construction for framing posts and girders. The use of STEEL POST-CAPS is to be recommended, there being no uncertainty regarding the strength of steel as there is concerning the strength of cast iron used for post-cap construction. Internal stresses due to uneven cooling may seriously affect the strength of a CAST-IRON CAP, while a honeycombed casting may be used, undetected, and affect the safe carrying capacity; so that failure of the cap may occur even from the vibration due to the machinery in the building.

Cast-Iron Post-Caps are still used in some localities and a few of the common forms as well as those of STEEL POST-CAPS are shown. Fig. 51 shows a form which is frequently used for light construction. Fig. 52 shows a similar cap for a cylindrical post. These caps permit the use of girders wider than the post. When the girders and floor-beams are in place, and especially when the building is occupied, there is no danger of the girders or posts slipping on the plate; in fact it would require a greater force to move them. The girders should be tied together longitudinally by IRON STRAPS spiked to their sides. Many persons, however, consider it important in a building of SLOW-BURNING CONSTRUCTION, to have the posts tied together in vertical lines, and the girders secured in such a way that they will be self-releasing without pulling down the posts. Figs. 53 and 54 show TWO POST-CAPS which fulfill these requirements. With these caps the ends of the girders are not fastened by bolts or spikes, but are held in place and tied longitudinally by means of the LUG L on the GOETZ CAP, and by PINS on the DUVINAGE CAP; so that in case the girder is burned to the breaking point, it can fall without pulling on the post. Pro-

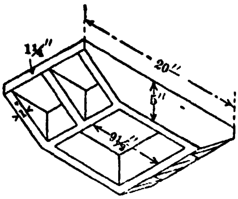


Fig. 51. Cast-iron Post-cap for Square-section Wooden Post

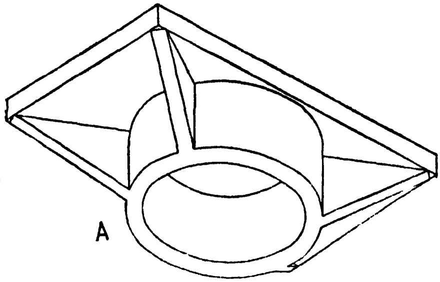


Fig. 52. Cast-iron Post-cap for Cylindrical Wooden Post

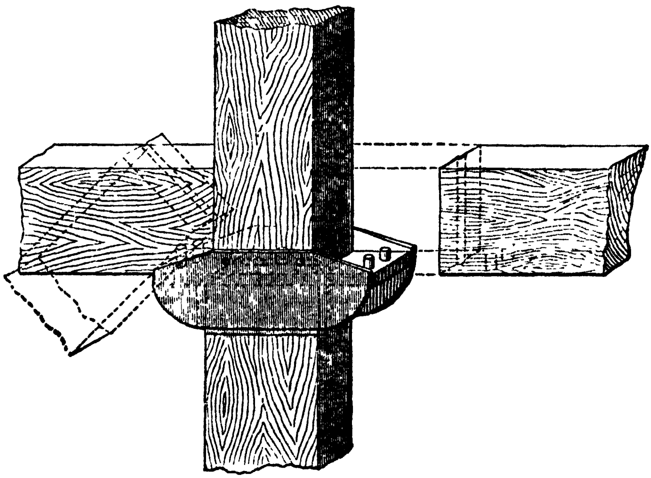


Fig. 53. Cast-iron Duvinage Post-cap with Beam-pins

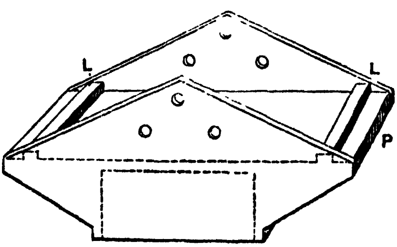


Fig. 54. Cast-iron Goetz Post-cap with Beam-lugs

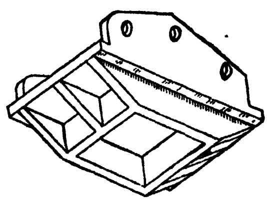


Fig. 55. Cast-iron Post-cap with High Sides

vision is also made for bolting the cap to the upper post. The author doubts very much, however, if posts bolted together in this way will stand after the girders have fallen, as the planking will be likely to pull the posts over, even if they do not burn as quickly as the beams. Fig. 55 shows another form of

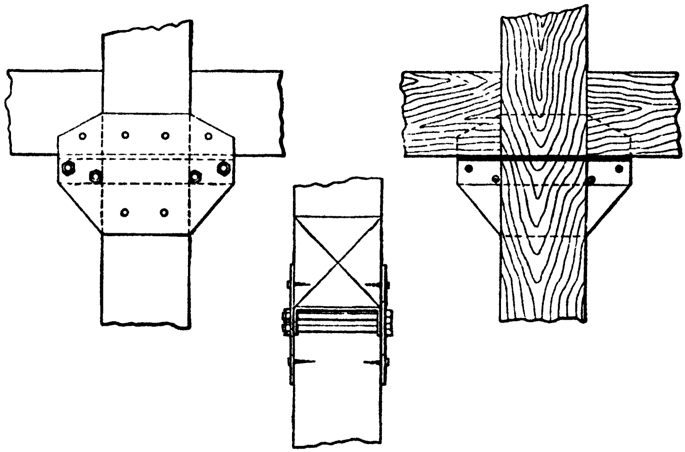


Fig. 56. Steel Post-cap with Side-plates and Brackets

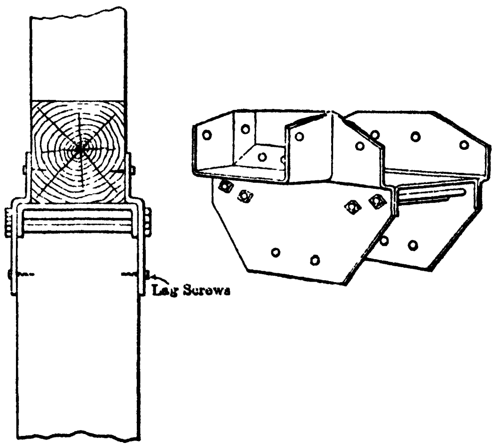


Fig. 57. Steel Post-caps for Posts Varying in Section. Second Figure Shows Four-way Beam-construction

CAST CAP with high sides, allowing lag-screws to be driven in the holes to tie the girders.

A Steel Post-Cap, which is approved by the National Board of Fire Underwriters and bears their label, is shown in Fig. 56. This POST-CAP is made up of

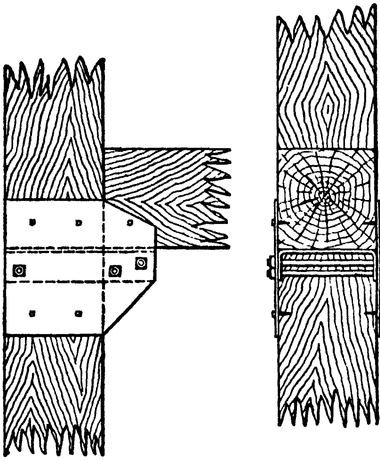


Fig. 58. Steel Post-cap. One-way Beam-construction

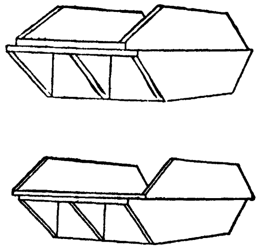


Fig 59. Malleable-iron Post-caps

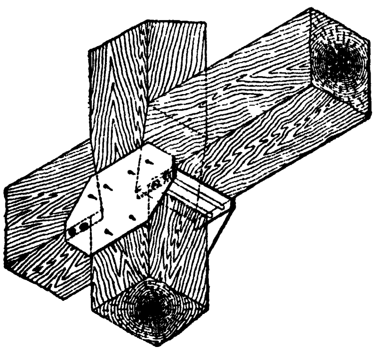


Fig. 60. Steel Post-cap for Continuous Post

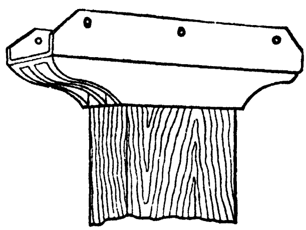


Fig. 61. Malleable-iron Post-cap with Steel Top-plate

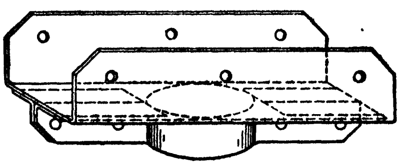


Fig. 62. Steel Post-cap for Cylindrical Wooden Post. Perspective.

steel side-plates and heavy steel brackets, all held rigidly together by means of four heavy bolts. The posts and girders are fastened to the cap by means of lag-screws, permitting the girders to release themselves in case of fire. By this method the entire construction is tied together vertically and longitudinally. This cap, on account of its simple design, lends itself readily to every form of construction desired.

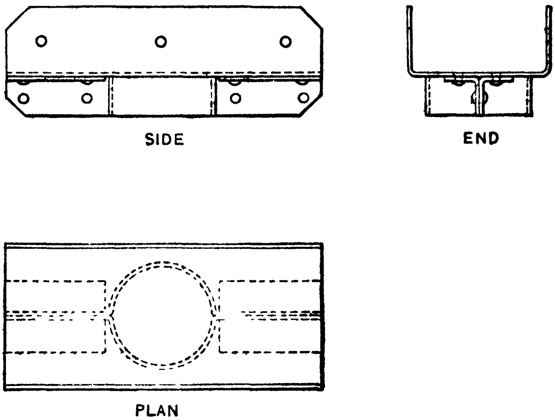


Fig. 63. Steel Post-cap for Cylindrical Wooden Post. Elevations and Plan

Various Types of Post-Caps. Fig. 57 illustrates one POST-CAP in which the width of the girder is less than that of the post below, and also another POST-CAP in which the width of the girder is greater than that of the post below. In the latter FOUR-WAY BRACKETS are riveted to the side-plates to provide for the FOUR-WAY CONSTRUCTION. Fig. 58 shows a ONE-WAY CONSTRUCTION. Fig. 60 shows a POST-CAP which is used when it is required to run a post through two stories. This is what is known as a CONTINUOUS POST-CAP. The bracket instead of being made clear across the cap is made short on both sides and fitted into shoulders notched into the post, so as to make a more rigid construction. Fig. 59 shows two POST-CAPS made of malleable iron which are preferable to cast-iron caps as they will not break off in case of a fire when cold water comes in contact with them. This danger is present when CAST-IRON POST-CAPS are used. The cap shown is made in two parts so that it will fit posts and girders of different sizes. This cap, also, is approved by the Board of Fire Underwriters. Fig. 61 shows a COMBINATION POST-CAP, the upper part of which is made of steel plate, and the lower part of malleable iron. Figs. 62 and 63 show STEEL POST-CAPS FOR ROUND POSTS. They are also frequently used for pipe-columns and concrete-filled columns. Fig. 64 shows a STEEL POST-CAP intended for lighter

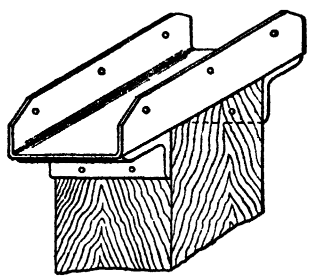


Fig. 64. Steel Post-cap for Light Construction

construction. Fig. 65 shows VAN DORN POST-CAPS. Fig. 66 illustrates the STAR POST-CAP which is made of a bent steel plate with a fin projecting below into a slot in the post. Both are approved by the Underwriters. It is necessary

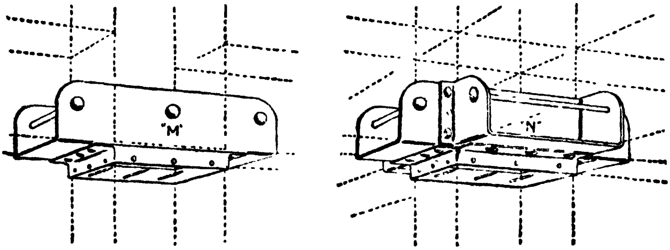


Fig 65. Van Dorn Steel Post-caps

to slot out the post in order to insert this fin. POST-CAPS which completely encircle the top of the post in a socket, to a great measure tend to prevent the twisting effect of the post, which is so noticeable when the posts are of wood. There is an objection to the use of the FOUR-WAY POST-CAP when the girders are of wood, because the floor-beams that are hung from a girder drop a distance equal to the shrinkage in the girder, if the beams are hung in stirrups, or by one-half this amount if they are hung in DUPLEX HANGERS. The beams supported on the POST-CAP cannot drop at all, and consequently the floor will be higher over the beam supported by the posts, than over the intermediate beams. In one building where deep beams were used, the unevenness in the floor amounted to nearly an inch and was very noticeable. Wherever wooden girders are used it is, therefore, a much better construction to support all of the floor-beams from the girders, in which case the shrinkage will be uniform. With steel girders there is no shrinkage, and a beam may be placed opposite the posts with advantage.

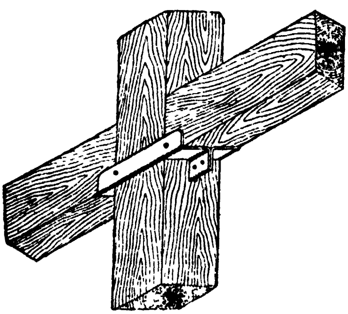


Fig. 66. Star Steel Post-cap with Fin

17. Roofing-Materials

Warehouse-Roofs are almost always flat and, like floors, should be continuous from wall to wall, without openings. The occupancy of such buildings calls for little light, and hence skylights and other roof-structures are not required.

Dampness and Leaks. Stored goods may be very easily damaged by water, and roofs, therefore, should be of such construction that they will prevent dampness, either through leakage or condensation. While roofs are usually built as flat as possible, the incline should be sufficient to drain readily, and the

outlets should be of sufficient capacity to quickly drain the roof of a maximum amount of water.

Slag or Tin are almost exclusively used on buildings of this type, although asphalt or other mastics are sometimes used with good results.

Slag Roofs should generally be not less than 5-ply, with the maximum amount of coating. The flashings and counterflashings should be of copper or heavily-coated best terne-plates.

Tin Roofs should be laid with the best open-hearth, palm-oil process terne-plates, laid on felt or other suitable materials which will avoid condensation and act as a fire-retardant.

Canvas Roofing will stand hard usage, as is shown by its continued use on decks of vessels and steamers; but it is not adapted to large buildings.

Provisions for Flooding Roofs. When warehouses are located in congested districts, surrounded by higher buildings, or by buildings of light construction or hazardous occupancy, their roofs should be so constructed that they may be flooded during severe fires in such surrounding buildings. This can be accomplished by using good roofing-materials, making high flashings, water-proofing the walls above the roof-line, and providing roof-outlets of types that will allow the placing of stoppers at the scuppers. (See Fig. 11)

18. Partitions

Non-Bearing Partitions. This refers only to those light walls or enclosures which separate rooms, etc., and not to those walls which divide the building into sections. PARTITIONS, as here defined, bear no floor-loads. Buildings of the SLOW-BURNING TYPE, for occupancies described above, need but few partitions, and these should be built of non-inflammable materials, preferably metal lath and plaster on light, metal studding. All cupboards, closets, lockers, etc., in a building of this type should be of metal, or other equally non-inflammable material.

19. Doors and Shutters

Fire-Underwriters' Specifications. Doors and shutters should be built as outlined in the Rules and Requirements of the National Board of Fire Underwriters for the construction and installation of fire-doors and fire-shutters, as these specifications are accepted by architects and builders as the standard.

Door-Openings should be limited to 80 sq ft, or less, each, and all communication between buildings or sections of a building protected with double, automatic, sliding doors.

20. Fire-Protection

Automatic Sprinklers, supplied with an ample quantity of water at a good pressure, are needed in mills, storehouses, factories, warehouses, etc., where combustible goods are made or stored, or where large values are at stake. They may, in fact, be installed in buildings of any type of construction and occupancy, but are most effective in buildings of FIRE-PROOF or MILL-CONSTRUCTION.

Inside Standpipes, with outlets in each story, in the basement and on the roof, should be installed at points readily accessible in case of fire, and should have a sufficient quantity of good hose attached at each outlet.

Roof Nozzles. If a building is badly exposed to other buildings of inferior construction or hazardous occupancy, a Monitor-nozzle of large size, located on the roof, is advisable.

Public Water-Supplies. If these are not available, a private fire-service may be advisable.

Competent Supervision. All of the above FIRE-PROTECTION EQUIPMENTS should be installed by men familiar with their operation, and supervised by competent FIRE-PROTECTION ENGINEERS, under plans approved by underwriters having jurisdiction.

CHAPTER XXII

FIRE-PROOFING OF BUILDINGS

By
GEO. E. STREHAN
CONSULTING ENGINEER

1. Fire-Resistive Construction

Definitions and Scope. FIRE-PROOF as applied to building-construction cannot be defined in terms of precise requirements; all building materials resist fire to some degree, which has led to the quite general adoption of the substitute term FIRE-RESISTIVE; the degree to which any material or combination of materials in a structural assembly resists the action of fire is called its FIRE-RESISTIVE or FIRE-RETARDANT CLASSIFICATION. Within the range of temperatures that may occur in a building either normally or under fire conditions, materials are also classified as COMBUSTIBLE or INCOMBUSTIBLE. The term INCOMBUSTIBLE is generally applied to materials of construction which will neither ignite nor support combustion at temperatures variously assumed at 1 200° to 1 700° F. Under the standard requirements for FIRE-PROOF construction of the last decade, it was possible to design a structure which would permit a complete burnout of its contents without invalidating any important structural element; nevertheless the actual experience of a fire in such a building frequently resulted in serious property damage to contents and occasionally in loss of life. The tendency of modern regulation of building-construction is to fix the degree of fire-resistance built into the structure in accordance with the potential fire hazard involved in its proposed occupancy, to require more careful design of the subdivision of areas to control fire and the provision of adequate exit facilities to minimize life hazard, and to protect the exterior of the building against the spread or communication of fire. In addition, the structure must be equipped and maintained in a state of preparedness to cope with the ever-present fire threat. The purpose of this chapter is to furnish the architect with the required information of the fire-resistive values of materials and assemblies, the limitations of their use and application, and the general precautionary measures that should be taken to control the fire hazard. He must choose the most economic materials and type of construction consistent with the desired degree of fire-protection, apply them to the best advantage in the structure, but always, however, within the limitations imposed by the building laws having jurisdiction. One illustration will be quoted of fire conditions in a modern type of fire-resistive construction.* On Feb. 16, 1926, a fire occurred in the file-room of a suite of offices occupying a cut-off section or fire-area on the thirty-fifth floor of the 40-story Equitable Building in New York City. The building (whose predecessor on the site, one of the earliest types of fire-proof construction, was totally destroyed by fire in 1912) is subdivided into four fire-areas or sections by fire-resistive walls of brick with

* Quarterly, National Fire Protection Association, April, 1926.

fire-doors at the openings, and all vertical communications are enclosed in fire-resistive partitions with protected openings. Workmen engaged in repairing piping in the building entered a pipe-shaft through a protected opening on the thirty-fifth floor, leaving the door held open by a mechanical check. From an unknown cause, fire originated at the third-story level in the shaft, fed on the insulation of the piping, traveled up through the shaft, caused the expansion and final rupture of a 5-inch unprotected gas-main, and entered the suite of offices at the unprotected opening on the thirty-fifth floor. After 12 minutes of delay and unsuccessful combat of the flames, the fire department was notified and the fire was extinguished after one hour of fighting by the aid of the standpipe system in the building. The fire though severe was confined to the cut-off section of the thirty-fifth floor; it completely destroyed all the contents of office-furniture, files, books and paper stock, and caused a very small amount of water and smoke damage on other floors above and below. The damage to structural floors and ceilings was such as to be readily repaired. The rapid travel of the fire was in part due to escaping gas, which burned in the shaft, until the broken pipe was automatically sealed by the water poured on the flames. The meter records indicated a probable consumption of 8 000 to 9 000 cu ft of illuminating-gas above the normal. The building was not equipped with automatic fire-extinguishing devices except in especially hazardous locations, the exterior openings were not generally glazed with wire-glass, and several other minor principles of modern-day practices were not followed in the design.

Fire-Resistive Classification of Materials refers to the actual time-period classification of assemblies of building materials in structural units of the building (such as walls, columns, floors and partitions), as determined by laboratory fire-tests conducted and controlled in accordance with the test procedure established by the Standard Fire Test Specifications of the American Standards Association.* Under this procedure, the performance of the construction is expressed as "2-hour," "½-hour," "6-hour," etc. Materials which involve combustibles, or which continue to burn after the desired test-period, are designated by the term COMBUSTIBLE after the assigned period. The intensity of the test-fire is controlled by a time-temperature curve passing through the following points: 1 550° F. at 30 minutes; 1 700° F. at 1 hour; 1 850° F. at 2 hours; 1 925° F. at 3 hours; 2 000° F. at 4 hours; and 2 300° F. at 8 hours. The area under the curve is a measure of the total amount of heat to which the test specimen has been subjected. For economical design, the building should incorporate only that degree of fire-resistance which is commensurate with the fire hazard involved in its occupancy and location, in place of the old general standard of 4-hour construction as now required by the New York Code for all fire-proof buildings. The application of this graduated scale of fire-resistance is further aided by the principles of zoning and restricting specific urban areas to particular occupancy uses. Although fire-testing under controlled conditions has been developing for the last quarter century, methods of testing have only recently been reasonably stabilized. The available data on fire-resistive classifications of all representative structural assemblies are not yet adequate to establish a comprehensive structural classification of buildings. More full-size tests on various assemblies are required. Several partial classifications have been issued by the U. S. Bureau of Standards, the Underwriters' Laboratories of Chicago, the Columbia University Testing Laboratories, and

* A S.A. Tentative Standard A_2—1926.

to a limited extent in several municipal building regulations.* The present tendency is to establish an intermediate class of building for occupancies of lighter hazard and lower permissible heights, between the fully protected class and the non-fire-proof or ordinary construction. This is called PARTIALLY PROTECTED in the tentative draft of the proposed code of the Department of Commerce, and FIRE PROTECTED for the intermediate class as distinguished from FIRE-RESISTIVE for the first class in the Philadelphia Code. Modern building codes for the cities of New York, Detroit, Minneapolis, Chicago, and Milwaukee, and the Recommended Practices of the National Board of Fire Underwriters, are now in the course of preparation. Among the fire-resistive time-period classifications already promulgated are those contained in the Tentative Draft of Fire-Resistive Construction of the Building Code Committee of the Department of Commerce, January, 1929; on Hollow Load-Bearing Wall Tile, Research Paper No. 37, Vol. 2, No. 1 Journal of Research; on Brick Walls of Clay, Shale, Concrete and Sand Lime Brick in Letter Circulars Nos. 228 and 229, Bureau of Standards; and on Masonry Wall Assemblies, National Board of Fire Underwriters, August, 1929. Other types of construction besides the fully and partially protected buildings possess some degree of fire-resistance, among which should be especially noted the MILL TYPE OR SLOW-BURNING CONSTRUCTION.†

Severity of Building Fires. Another factor that should be determined for proper design of fire-resistive buildings, is the equivalent time-temperature period of the inherent fire hazard in buildings of different occupancies. The temperatures and durations of fire in a building depend upon the occupancy, contents and character of construction. Observation of actual fires and conflagrations indicates average temperatures of 1 500° F. with a maximum of 2 500° F, the most intense heat lasting probably not over 60 minutes, in the general case ‡ On June 17, 1928, a full-scale fire-test was conducted under the direction of Mr. S. H. Ingberg, U. S. Bureau of Standards, on wood-joisted, brick-wall buildings, separated by a fire-wall, one two stories and the other five stories high, with the floors loaded in different sections with waste lumber to the extent of 7½, 15 and 30 lb per sq ft.§ These buildings comprised part of a group of structures being demolished to make way for a government building-operation in Washington, D. C. More than a hundred thermocouples were distributed throughout the structures and fires were started simultaneously in both buildings. In twenty minutes the temperatures reached 2 000° F., and in forty minutes a 2 400° F. maximum. The fire extended to the top story of the five-story building in five minutes through an open elevator-shaft, and in twenty minutes the wooden joists burned through, allowing steel safes which had been planted in the building to fall through. In two hours the buildings were burned down except for smouldering refuse. Mr. Ingberg reached the conclusion that the fire was not more severe than a two-hour demonstration of the Standard Fire Test, and probably less. In the numerous floor and partition tests conducted by the Columbia University Testing Laboratory for the Bureau of Buildings of New York City,|| which are performed in fire-resistive test-chambers of reinforced concrete, involving to date perhaps some hundred floor and a like number of partition assemblies, the combustibles used

* Building Codes of Pittsburgh, 1925; Philadelphia, 1929; proposed Detroit, 1930; West Coast Joint Code, 1927.

† See Chapter XXI.

‡ Freitag, Fire Prevention and Fire Protection.

§ Quarterly, N.F.P.A., July, 1928.

|| ee fire-resistive floors, Article 7; partitions, Article 12.

for firing the furnace are kindling wood, and various amounts of oak and pine cord-wood, with kerosene as an accelerator. Records of the tests show that in a one-hour standard partition or a one-hour standard floor-test, approximately 20 to 30 lb of combustibles are consumed per square foot of floor area enclosed, generally more in the partition than in the floor-test, and in a four-hour floor test approximately 60 lb of combustibles per square foot are consumed. Much heat is lost in these tests by temperature transmission through the partitions and walls, through the calcining effects of the materials entering into the construction and especially through the evaporation of the free water in the construction and the chemically combined water in the materials. The severity of the fire does not vary in direct proportion to the amount of combustibles, but the general conclusion would be that a one-hour period of the Standard Time-Temperature curve would equal the fire-consumption of at least 20 lb of fuel per square foot, and a four-hour period, approximately 60 lb per square foot.* To study the probable fire intensity corresponding to different-use occupancies of buildings, Mr. S. H. Ingberg conducted a series of burn-out tests on one-story fire-resistive test-structures erected at the U. S. Bureau of Standards, Washington, D. C.† These buildings were fitted with furnishings and records to represent light office, commercial and record-storage occupancies and were fired in several ways to represent slow and rapid ignition and spread of fire, with combustible and incombustible floor finishes. The conclusions reached by Mr. Ingberg as to the equivalent time-temperature periods for light office occupancy (represented by a storage of 10 to 12 lb of combustibles) and heavy record-room occupancy (represented by 50 lb of combustibles) appear ultra-conservative. The fire areas under test were comparatively small and the fire conditions were different from those obtaining in an actual building. The objective of these studies is of exceeding importance in the design of a fire-resistive building, but the data are not yet comprehensive enough to establish definite conclusions. The uniform building code requirements for a "fire-proof" building undoubtedly provided for more than the needed protection for buildings to house the lighter occupancies and uses, and in a like degree failed to provide for those occupancies with a maximum fire hazard due to the quantity and character of the combustible contents.

From experience in actual fires and from the fire-test investigations of The U. S. Bureau of Standards and the Underwriters' Laboratories at Chicago, it appears probable that buildings having office, residential and habitational occupancies generally do not suffer fires exceeding in effect and severity the one-hour test-exposure conditions contemplated by the Standard Fire Test. Exceptional fires in such buildings would probably never exceed the equivalent of the 1½-hour standard exposure. Mercantile, manufacturing and warehouse buildings of more hazardous contents may develop fires which would equal in severity two to four-hour periods of the Standard Test Specifications.

Municipal Code Specifications. From the preceding discussion, it will be seen that the art of fire-resistive construction is not stabilized and the

* Since the area under the Time-Temperature curve is a measure of the heat supplied to the constructions during a fire-test, accurate planimeter measurements were made of both the New York Fire Test and the Standard Test curves. The area of the New York Code curve for one hour is 49.92 sq in, and for the American Standards curve is 43.61 sq in. Reduced to average temperatures for the one-hour period, the values are respectively 1664° F and 1454° F.

† Quarterly, N.F.P.A., July, 1928.

requirements vary in important details in all municipal codes. A few typical examples will be cited, but the detailed requirements of the law having jurisdiction must be consulted for each individual case.

Philadelphia.* "FIRST CLASS TYPE A defined. FIRST CLASS TYPE A buildings shall be those having frames of reinforced concrete or structural steel wherein the fireproofing of the skeleton-frame and the floor-systems is in intimate contact with all parts of the steel, and the floor-systems are rigidly connected to the skeleton-frame.

"FIRST CLASS TYPE B defined. FIRST CLASS TYPE B buildings shall be those having frames of reinforced concrete or structural steel wherein the fireproofing of the skeleton-frame is in intimate contact with all parts of the steel, and the floor-systems are of steel, rigidly connected to the skeleton-frame, and fireproofed on top and bottom flanges, but need not be in contact.

"FIRST CLASS TYPE C defined. FIRST CLASS TYPE C buildings shall be those having frames of reinforced concrete or structural steel wherein the fireproofing of the skeleton-frame entirely envelops each member, but need not be in intimate contact with the steel, and the floor-systems are of steel which need not be rigidly connected to the skeleton-frame and which is fireproofed on top and bottom flanges, but need not be in contact. INTERMEDIATE CLASS, FIRE-PROTECTED, construction is the same as FIRST CLASS TYPE C above except that structural frames and floor-systems may be fire-protected instead of fireproofed and each individual member of the skeleton-frame need not be entirely developed.

"FIREPROOFING MATERIAL. A material of the thickness specified in this code for fireproofing, providing it is capable of passing the standard four-hour test in conformity with requirements as promulgated by the Chief.

"FIRE-PROTECTION. A protection of the materials acceptable for fireproofing, but of thickness not less than one-half those required for fireproofing. Cement or gypsum plaster properly held may be considered as fire-protection, providing that the constructions so protected will pass the standard one-hour fire test."

Pittsburgh. "FIRE-RESISTIVE CONSTRUCTION. Fire-resistive building materials, systems, units and forms of construction shall be classified in accordance with the degree of resistance they afford when measured by a fire-test conducted in conformity with the standard time-temperature requirements herein provided as follows:

FOR FLOORS, ROOFS AND COLUMNS

Four-hour fire-resistive construction, or full fire-protection.
Three-hour fire-resistive construction.
Two-hour resistive construction.

FOR WALLS AND PARTITIONS

Four-hour fire-resistive construction, or full fire-protection.
Three-hour fire-resistive construction.
Two-hour fire-resistive construction.
One-hour fire-resistive construction.

"For all fire-resistive buildings, the following time-period classifications are required:

"For all columns and for all girders and trusses which support columns or outside enclosing or bearing masonry walls; three-hour fire-resistive construction or protection.

* The material in these codes has been edited to conform in spelling, punctuation etc.. with the style of the handbook.

"For all floor-construction and for all trusses, girders and beams not otherwise regulated by this section; two-hour fire-resistive construction or protection."

West Coast Joint Code. (Adopted by about 80 cities.) "TYPE 1 or TYPE 1 BUILDINGS. The structural frame of TYPE 1 buildings shall be of structural steel or iron which shall be fireproofed, or shall be of reinforced concrete. The foundation, exterior walls, inner-court walls, and walls enclosing vent openings shall be of masonry or reinforced concrete. The roof-construction and floors shall be of fire-resistive materials. Exterior doors and windows, except as specified in Section 1813, shall be of fire-resistive construction. (NOTE: Fire-resistive materials and fire-resistive construction have a specific meaning in this code, as specified in Chapters 42 and 43.)

"All structural steel or iron members, not including forms or structural members for elevators and elevator enclosures, shall be thoroughly fireproofed with not less than four-hour fire-resistive protection for columns, beams and girders, and three-hour fire-resistive protection for floors, for all buildings more than eight stories or 85 ft in height; and with three-hour fire-resistive protection for columns, beams and girders, and two-hour fire-resistive protection for floors for all buildings which are eight stories or 85 ft or less in height; and all such fire-resistive protection shall be as specified in Chapter 43.

"All exterior walls, fire-walls and fire division walls shall be of masonry or reinforced concrete, as specified in Chapter 29, and shall be of not less than four-hour fire-resistive construction, as specified in Section 4302.

"Inner-court walls shall be of masonry or reinforced concrete of not less than three-hour fire-resistive construction, as specified in Section 4302.

"Interior partitions shall be constructed of incombustible materials and shall be of not less than one-hour fire-resistive construction as specified in Section 4302.

"Enclosures for elevator-shafts, vent shafts, stair-wells and other vertical openings when required because of occupancy in Part III shall be of two-hour fire-resistive construction and all openings therein shall be protected by fire-resistive doors or windows, as specified in Chapters 30 and 43."

New Orleans. "CLASS NO. I. FIRE-RESISTIVE CONSTRUCTION. A building is of fire-resistive construction, if all the walls, partitions, piers, columns, floors, ceilings, roofs, stairs, stair and elevator enclosures are built of incombustible materials, except hand-rails for stairs, minor interior finish and interior doors, also window trim, and if all metallic structural members except trusses supporting roof with fire-resistive ceiling beneath, are protected by an incombustible fire-resisting covering of low-heat-conductivity. Plastering shall not be applied to wood lath or wood furring strips.

"CLASS NO. III. ORDINARY CONSTRUCTION PROTECTED. This is Class III Construction (Ordinary Construction) * except that protection is afforded by requiring all combustible walls, ceilings and partitions to be covered with incombustible lath and cement or gypsum plaster or equivalent protection which will withstand a one-hour fire-test, and further protection is to be afforded by a complete standard sprinkler equipment. All openings in floors are to be enclosed in an approved manner to prevent passage of fire from one floor to another; all openings in side or rear walls shall be provided with approved fire-windows, doors or shutters, and all openings on streets less than 50 ft wide shall be provided with such protection above the first floor. in the event that buildings opposite are not of fire-resistive construction."

* Matter in parenthesis not contained in original.

2. Limitations of Non-Fire-Proof Construction

Limiting Areas. Because of the difficulty of controlling fires, the area which a non-fire-proof building may cover is generally limited in municipal building-regulations unless such areas are subdivided by fire-walls. The difficulty of fighting a fire depends also upon its accessibility from the outside and the relative depth and width of the building. Maximum areas permitted in some of the cities are found in the following classification of representative cities.

Limiting areas	Unsprinklered	Sprinklered
Baltimore, Md. (1927) (Public buildings, department stores, factories)	3 000 sq ft if one stairway is provided	3 600 sq ft
	9 000 sq ft if two stairways are provided	10 800 sq ft
	15 000 sq ft if three stairways are provided	18 000 sq ft
	24 000 sq ft if four stairways are provided	28 800 sq ft
Boston, Mass. (1924)	10 000 sq ft	
New York, N. Y. (1915)	7 500 sq ft if facing one street	15 000 sq ft
	12 000 sq ft if facing two streets	24 000 sq ft
	15 000 sq ft if facing three streets	30 000 sq ft
Philadelphia, Pa. (1929) (Ordinary construction)	5 000 sq ft if facing one street	10 000 sq ft
	7 500 sq ft if facing two streets	15 000 sq ft
	10 000 sq ft if facing three streets	20 000 sq ft
(Mill-construction)	10 000 sq ft if facing one street	20 000 sq ft
	15 000 sq ft if facing two streets	22 500 sq ft
	17 500 sq ft if facing three streets	25 000 sq ft
Chicago, Ill. (1924) (All ordinary construction)	9 000 sq ft if facing two streets	12 000 sq ft
(All mill-construction)	12 000 sq ft if facing two streets	16 000 sq ft
Cleveland, Ohio (1920) (Ordinary construction)	5 000 sq ft if facing one street	7 500 sq ft
	7 500 sq ft if facing two streets	11 250 sq ft
	10 000 sq ft if facing three streets	15 000 sq ft
(Mill-construction)	9 000 sq ft if facing one street	13 500 sq ft
	12 000 sq ft if facing two streets	18 000 sq ft
	15 000 sq ft if facing three streets	22 500 sq ft
Pittsburgh, Pa. (1924) Zone 1	5 500 sq ft if facing one street	
	8 800 sq ft if facing two streets	
	11 000 sq ft if facing three streets	
Zone 2	10 000 sq ft if facing one street	
	12 000 sq ft if facing two streets	
	15 000 sq ft if facing three streets	
Los Angeles (1929) (Protected steel, non-fire-proof floors)	10 000 sq ft, all locations	25 000 sq ft
(All others)	7 500 sq ft, all locations	17 000 sq ft
St. Louis, Mo. (1927)	7 500 sq ft, all locations	
West Coast Joint Code (Ordinary and mill-construction)	12 000 sq ft if facing one street	24 000 sq ft
	15 000 sq ft if facing two streets	30 000 sq ft
	18 000 sq ft if facing three streets	36 000 sq ft

Limiting Heights. The height of a building is an important factor in determining the requirements for fire-resistive construction. The rate of increase in the difficulty of coping with fire is greater than the direct increase in height, and the extreme limit will vary to some extent in different cities. With modern fire-fighting equipment, municipal fire departments cannot generally fight a fire effectively from the street to a height of more than six stories, or at the most 75 ft, without the aid of auxiliary devices in the building itself or from adjoining buildings. Some of the municipal height limits placed upon non-fire-proof construction are given in Table I.

Table I. Limiting Heights for Non-Fire-Proof Buildings

City	Factories	Hotels	Schools
Baltimore, Md. (1927)	50 ft, 4 stories†¶ 75 ft‡	3 stories and basement¶	2 stories and basement
Boston, Mass. (1924	75 ft	65 ft, 5 stories ¶	*
New York City (1915)	4 stories	20 ft	20 ft
Philadelphia, Pa (1929)	3 stories, 40 ft † 85 ft‡	3 stories and basement 50 ft	2 stories
Washington, D. C. (1925)	3 stories† 60 ft‡	3 stories¶	3 stories¶
Chicago, Ill. (1924)	50 ft, 4 stories† 100 ft, 7 stories‡	3 stories and basement	200 persons 2 stories and basement
Cleveland, Ohio (1920)	2 stories† 100 ft‡	4 stories	1 story and basement
Detroit, Mich. (Proposed 1930)	40 ft, 3 stories† 60 ft, 5 stories‡	3 stories 40 ft	2 stories 40 ft
New Orleans, La (1927)	30 ft, 2 stories† 57 ft, 4 stories, 72 ft, 5 stories ‡	3 stories 40 ft	3 stories
Pittsburgh, Pa (1924)	3 stories† 80 ft, 6 stories‡§	3 stories	1 story
Fort Worth, Tex. (1928)	3 stories† 4 stories‡	2 stories	1 story
Kansas City, Mo. (1927)	40 ft, 3 stories† 85 ft‡	2 stories 25 ft	1 story
Los Angeles, Cal. (1929)	3 stories† 100 ft‡	3 stories	*
San Francisco, Cal. (1924)	55 ft† 84 ft‡	55 ft	1 000 persons
St. Louis, Mo. (1927)	2 stories† 90 ft‡	3 stories and basement 50 ft	2 stories and basement
West Coast (1927)	55 ft, 5 stories† 75 ft, 7 stories‡	5 stories 55 ft	2 stories

* Fire-proof construction. † Ordinary construction. ‡ Mill-construction. § Sprinklered. ¶ First floor must be of fire-proof construction.

Table I (Continued). Limiting Heights for Non-Fire-Proof Buildings

City	Hospitals and asylums	Residence buildings	Other buildings
Baltimore, Md. (1927)	2 stories and basement	3 stories and basement¶	4 stories and basement¶
Boston, Mass. (1924)	3 stories 40 ft	5 stories 65 ft ¶	75 ft
New York City (1915)	20 ft	6 stories¶	75 ft
Philadelphia, Pa. (1929)	Hosp., 2 stories Asyl. *	3 stories and basement 50 ft	3 stories and basement 50 ft
Washington, D. C. (1925)	3 stories¶	3 stories¶	60 ft
Chicago, Ill. (1924)	2 stories and basement	3 stories and basement	Less than 50 ft
Cleveland, Ohio (1920)	*	60 ft	60 ft
Detroit, Mich. (Proposed 1930)	Hosp., 2 st., 30 ft Asyl. *	3 stories 40 ft	3 stories 40 ft
New Orleans, La. (1927)	2 stories 30 ft	3 stories 40 ft	4 stories 57 ft
Pittsburgh, Pa (1924)	1 story	3 stories	3 stories
Fort Worth, Tex. (1928)	*	3 stories	Over 5 stories
Kansas City, Mo. (1927)	Hosp., 1 story Asy. *	3 stories, 44 ft 2 stories if 2 apts. are on 1 floor	3 stories 44 ft
Los Angeles, Cal (1929)	*	3 stories	50 ft
San Francisco, Cal (1924)	1 000 persons	55 ft	55 ft
St. Louis, Mo. (1927)	Hosp., 3 stories and basement Asyl., 2 stories and basement	3 stories and basement 50 ft	75 ft
West Coast (1927)	Hosp., 1 story Asyl. *	5 stories 55 ft	5 stories 55 ft

* Fire-proof construction. † Ordinary construction. ‡ Mill-construction. §Sprinklered. ¶ First floor must be of fire-proof construction.

3. Comparative Costs

Type of Construction. The cost of construction is only a secondary measure of economy, and the designer must make the most careful analysis to select the best and most adaptable type of construction for the particular use to which the structure will be put, bearing in mind the importance of its maintenance in service without interruption and with minimum probability of life or property loss. The problem of fire-proofing, and whether the additional cost is warranted, is probably the most important for the architect to consider. In addition to the two major types of fire-resistive construction, the protected-steel frame and the all-reinforced-concrete frame, the first class is subdivided into many degrees of fire-resistance depending on the thickness and character of protective coverings used, and the second class may be built of four or more radically different types of construction all of which are subject to competitive cost advantages and disadvantages, and have varying fire-resistive properties.

Height and Area. Not only do the relative costs of the two main types of fire-resistive construction vary in different localities, but the unit costs of buildings of the same type of construction are never alike, owing to differences in plan and other physical characteristics, such as spacing of columns, floor-loads and ceiling heights. Especially does the unit cost vary with height and area of the project. Generally speaking, beyond a certain minimum limit, the taller the structure the greater the unit costs, whereas the larger the area with greater duplication of units, the lower the unit costs.

Use Requirements. A building must be planned either to return an income proportional to the capital investment or to provide an economical plant for the particular use operation for which it is designed. The requirements for no two industrial or commercial occupancies are exactly alike, nor are buildings for similar occupancies in different localities rarely planned to be comparative in cost. Because of these many variables, the fallacy of quoting differential percentage costs for all classes of buildings in various localities is evident. In an analysis of costs of types of fire-proof construction* Mr. Arthur F. Klein, Vice-President of R. C. Wieboldt Co., Chicago, compares four types of construction for a sixteen-story hotel, three types for a six-story light-manufacturing building and three types for an eight-story warehouse. The general analysis indicates that there is one type of construction that proves most economical for each of these buildings.

In selecting the most economical design for any building, the architect must resort to a test analysis of all available types of construction for a typical panel. Such a cost analysis was prepared for a typical panel of a four-story light-manufacturing building, comparing four types of construction: TYPE A, Brick Wall, Wood-Joisted; TYPE B, Mill Construction; TYPE C, Reinforced Concrete Girder and Slab Type; and TYPE D, Protected Steel Frame with Concrete Slabs. The estimates were based on current cost of building materials and wage-scales obtaining in New York City in 1930. In this analysis, interior completion, builder's profits, and financing costs were not included. On a net estimated cost of $0.20 per cu ft for the wood-joist construction, Type B was estimated to cost 3% more, Type C 20% more and Type D 30% more than Type A.

In a check analysis of the net field construction costs in the Metropolitan District of New York City, including all classes of buildings on the basis of

* Journal of the Western Society of Engineers, Vol. XXIX, No. 7, July, 1924.

July, 1930, delivered-on-the-job costs of basic building materials and current wage-scales, Mr. Allen E. Beals * gives the computed cubic-foot costs of some 125 buildings of various classes and heights. This analysis indicates that the estimated cubic-foot costs of these buildings average as follows:

Residential Buildings	
Frame, 2½ stories	$0.35
Brick, ordinary, 6 stories	0.45
Semi-fire-proof, 6 stories, steel joists	0.55
Fire-resistive, 15 stories, protected-steel frame	0.75
Hotel Buildings	
Brick, ordinary, 5 stories	0.65
Fire-resistive, 7 stories, reinforced concrete	0.67
Fire-resistive, 20 stories, protected-steel frame	0 90
Fire-resistive, 40 stories, protected-steel frame	1 25
Public Buildings	
Brick, ordinary, 5 stories	0 68
Fire-resistive, 12 stories, protected-steel frame	0.84
Office-Buildings	
Fire-resistive, 15 stories, protected steel	0 78
Fire-resistive, 40 stories, protected steel	0.92
Loft-Buildings (Tenant Factories)	
Fire-resistive, 15 stories, protected steel	0.53
Fire-resistive, 30 stories, protected steel	0.70
Factory-Buildings	
Mill construction, 2 stories	0.18
Fire-resistive, 4 stories, reinforced concrete	0.35
Fire-resistive, 4 stories, protected-steel frame	0.45
Warehouse-Buildings	
Fire-resistive, 7 stories, reinforced concrete	0.30
Fire-resistive, 14 stories, protected-steel frame	0.38

Under average conditions, where the several types of construction are of practical application, the data seem to indicate that mill construction will add 5% or less to the cost of ordinary construction, reinforced concrete 0 to 15% to mill construction, and protected-steel construction of the highest fire-resistive type from 5 to 10% to the cost of reinforced-concrete construction.

4. Fire-Resistance of Building Materials

Fire Effects on Durability of Materials. All building materials will ignite, flame, disintegrate or fuse if the temperature is high enough. Materials used in fire-resistive construction can be classified according to their uses as·

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(*a*) Those which serve as STRUCTURAL-LOAD-BEARING MEMBERS; (*b*) those which serve as a PROTECTIVE against the destructive action of fire on the structural element proper; (*c*) those which serve in assemblies as BARRIERS against the spread of fire; and (*d*) those which serve as ARCHITECTURAL TRIM and have no structural value at all. Materials of the first class, such as those used in floor and wall-constructions, must be incombustible, must not disintegrate or disrupt under sudden changes of temperature, and must retain their structural integrity throughout fire-exposure. Materials of the second class, such as column fireproofing, may or may not possess structural qualifications, but must be low conductors of heat and maintain their integrity as a protection to the enclosed load-bearing section. Materials of the third class, such as those used in fire-resistive partitions, must withstand the action of high temperatures and flames without transmitting sufficient heat to ignite combustible materials in the protected area or without breaking down to permit passage of flame, smoke or water. Materials of the fourth class such as flooring and trim, must be incombustible and not support or transmit flames or add fuel to the fire.

Effects of High Temperatures on Strength. Incombustible fire-resistive materials will lose their structural qualities to some degree under exposure to high temperatures or sudden changes of temperatures. Fire-resistive clays will soften, fuse or disintegrate from unequal expansions; Portland cement concretes will spall and disintegrate, or dehydrate and lose their strength, gypsum or plaster composition will calcine and break down under the action of heat; and steel follows certain definite structural property changes, depending upon its chemical analysis and manufacture.

Effects on Structural Assemblies. When materials are built or joined together in structural units to make up a building member, the fire-resistive rating of the assembly * is dependent upon the relative action of the component materials. Unequal expansions will cause the unit to warp, deflect or induce stresses in itself and adjoining members in the structure. The relative position of one part of the assembly with respect to another, as for example, the distance between a ceiling-protective and the steel floor-beam, vitally affects the fire-resistance of the whole unit. The American Standard Fire Test Specification sets up the following criteria. BEARING-WALLS and PARTITIONS, FLOORS and ROOFS must sustain "the applied load during the controlled fire and fire stream test, without passage of flame, stream or gases hot enough to ignite cotton waste, and after cooling, but within 72 hours after its completion, shall sustain a total load equal to the dead load plus twice the superimposed load. . . . Transmission of heat through the wall, partition (or floor) during the fire endurance test shall not have been such as to raise the temperature on its unexposed surface more than 250° F. above its initial temperature." FINISH FOR WALLS, PARTITIONS and CEILINGS must withstand the fire and fire-stream test "without developing openings capable of passing flame, hot gases or stream," and "transmission of heat through the finish during the fire endurance test shall not have been such as to raise the temperatures at its contact with the structural members of the test panel or elsewhere on its unexposed surface more than 250° F. above the initial temperature at these points."

* See Fire-Resistive Classification, Article 1.

5. Fire-Resistive Specifications and Properties of Materials*

Brick. COMMON CLAY BRICK, when of good quality, will stand exposure to severe fire for a considerable length of time.† Experience has shown that thick walls are less affected by heat than thin walls, and that hard-burned bricks stand better than soft or underburned bricks. "In the Baltimore and San Francisco fires, it was demonstrated that for outside walls, brick is superior to any other material used in wall-construction as a fire-resistive material." This resistance is due in large measure to the form and size of the unit and method of erection. SAND LIME BRICK ‡ though not equal to clay brick, is an excellent fire-resistive material. CONCRETE BRICK possesses the fire-resistive qualities of the concrete mixtures and aggregate used in its manufacture.

CLAY or SHALE building brick are manufactured in accordance with the Standard Specifications of the American Society for Testing Materials, C62–30, in three grades based on strength requirements. (See Table II.) Sizes of brick are fixed by these standards with a permissible variation of plus or minus $\frac{1}{16}$ in in depth, $\frac{1}{8}$ in in width and $\frac{1}{4}$ in in length. (See Table III.)

Table II. Grades of Clay Building Brick

Name of grade	Compressive strength (brick flatwise), lb per sq in		Modulus of rupture (brick flatwise), lb per sq in	
	Mean of 5 tests	Individual min.	Mean of 5 tests	Individual min.
Grade A.....	4 500	3 500	600	400
Grade B.....	2 500–4 500	2 000	450–600	300
Grade C.....	1 250–2 500	1 000	300–450	200

NOTE. The specifications for physical strength properties and tests of sand lime brick, A.S.T.M. Standards C73–30, are exactly similar to those for clay or shale brick. These specifications do not determine weather-resistance.

Table III. Sizes of Clay Building Brick

Type	Depth, inches	Width, inches	Length, inches
Common brick	$2\frac{1}{4}$	$3\frac{3}{4}$	8
Rough-face brick . .	$2\frac{1}{4}$	$3\frac{3}{4}$	8
Smooth-face brick....	$2\frac{1}{4}$	$3\frac{7}{8}$	8

Natural Stones.§ Very few natural stones successfully withstand the action of high or sudden changes in temperature, and stone in general should be used with discretion in fire-resistive building-construction, and cer-

* References to the Simplified Practice Recommendations (S.P.R.) are given in all cases where these have been formula ed and approved by the indu tries and endorsed by the U. S. Department of Commerce.

† See Wall Assemblies, Bureau of Standards, Article 12.

‡ S.P.R. No. 38.

§ Bulletin No. 370 U. S. Geological Survey Tests.

tain stones not at all. GRANITE will explode and disintegrate into sand when exposed to severe flames. LIMESTONES and MARBLES are usually ruined if not totally destroyed by an ordinary fire. They are the least desirable of all stones from the fire-resistive point of view, and the granites come next. SANDSTONES when fine-grained and compact sometimes stand fire without serious injury, but in very serious fires may be affected so badly as to require replacement.

Terra Cotta. Clay shaped into the desired building forms and baked at high temperatures in special kilns is of two general types: ARCHITECTURAL OR ORNAMENTAL TILE and STRUCTURAL CLAY TILE.

Architectural or Ornamental Terra-Cotta Tile is generally formed by hand to the desired units, and when burned with a glazed or monolithic colored enamel finish is well adapted to the architectural treatment of a fire-resistive building. The shells and webs must however be proportioned properly and made heavy enough to resist expansion and contraction stresses, and when incorporated in a wall the tile must be of sufficient strength to support its share of the wall-load as well as its own weight.

Structural Clay Tile.* Hollow clay masonry units were formerly variously known as structural terra cotta, terra-cotta building-blocks and hollow tile. In the trade, hollow clay masonry units having cells or voids greater than 25% of the gross area are now called STRUCTURAL CLAY TILE. It is formed into the various commercial shapes by mechanically extruding the plastic clay mass through special dies and is generally wire-cut into units of desired length before burning. Three classes of structural clay tile are manufactured from surface clay, shale, fire clay, or admixtures thereof,† conforming to Standard Specifications and Tests of the American Society for Testing Materials.

(*a*) LOAD-BEARING WALL TILE ‡ is manufactured and classified according to the structural requirements based on strength and absorption given in Table IV. For use in load-bearing walls or in walls partly or wholly exposed

Table IV. Physical Properties of Hollow-Clay, Load-Bearing Wall Tile

Class	Absorption, per cent			Compressive strength based on gross area, lb per sq in			
				End construction		Side construction	
	Mean of 5 tests	Indiv. max.	Indiv. min.	Mean of 5 tests	Indiv. min.	Mean of 5 tests	Indiv. min.
Hard....	6 to 12	15	5	2 000 or more	1 400	1 000 or more	700
Medium.	12 to 16	19	5	1 400 or more	1 000	700 or more	500
Soft.....	16 to 25	28	5	1 000 or more	700	500 or more	350

* S P R. 12.
† Definitions, A. S.T.M. Spec. C-43-24.
‡ A.S.T M. Spec. C-34-30.

to the weather, tile of at least MEDIUM GRADE should be used. The tile are manufactured to the following standard sizes and weights:

Size of unit, inches	Number of cells in thickness	Weight, lb
3¾ × 12 × 12	3	20
6 × 12 × 12	6	30
8 × 12 × 12	6	36
10 × 12 × 12	6	42
12 × 12 × 12	6	48
12 × 12 × 12	9	52
3¾ × 5 × 12	1	9
8 × 5 × 12	2	16
8 × 5 × 12	3	16
8 × 5 × 12 (L-shaped)	.	16
8 × 6¼ × 12 (T-shaped)	4	16
8 × 7¾ × 12 (Square)	6	24
8 × 10¼ × 1 (H-shaped)	7	32
8 × 8 × 8 (Cube)	9	18

(*b*) FLOOR TILE is manufactured and classified in accordance with the structural requirements * given in Table V.

Table V. Physical Properties of Hollow-Clay Floor Tile

Class	Absorption, per cent			Compressive strength based on net area, lb per sq in			
				End construction		Side construction	
	Mean of 5 tests	Indiv. max.	Indiv. min.	Mean of 5 tests	Indiv. min.	Mean of 5 tests	Indiv. min.
Hard	6 to 12	15	5	4 600 or more	3 000	2 400 or more	1 700
Medium	12 to 16	19	5	3 200 or more	2 250	1 600 or more	1 100
Soft	16 to 25	25	5	2 000 or more	1 400	1 200 or more	850

The standard weights of floor tile per square foot of flat and segmental arches are as follows:

Depth of arch, inches	Pounds per sq ft floor	
	Flat arch	Segmental arch
6	26	30
7	29	....
8	32	36
9	35	....
10	38	40
11	42	....
12	50	....

* A. S.T.M. Spec. C-57-30

For use in COMBINATION FLOORS * structural clay tiles are manufactured in the following sizes and weights. A tolerance of 3% is allowed in dimensions and 5% in weights.

Size of units, inches	Minimum number of cells in thickness	Standard weight, lb
4 ×12×12	3	16
6 ×12×12	3	22
6 ×12×12	4	25
8 ×12×12	4	30
10 ×12×12	4	35
12 ×12×12	4	40
4 ×16×16	4	32
4½×16×16	4	34
5 ×16×16	8	42
6 ×16×16	8	46
7 ×16×16	8	48
8 ×16×16	8	50
9 ×16×16	12	62

(c) FIRE-PROOFING, PARTITION AND FURRING TILE † are classified in accordance with absorption requirements based on standard methods of test as given in Table VI.

Table VI. Hollow Clay Fire-Proofing, Partition and Furring Tile

Class	Absorption, per cent		
	Mean of 5 tests	Individual maximum	Individual minimum
Hard	6 to 12	15	5
Medium	12 to 16	19	5
Soft	16 to 25	25	5

No strength requirements are specified for this class of materials, but the FIRE-RESISTIVE RATING should be specified and the manufacturer be required to supply information of the fire-test performance of the material for each particular use.‡ Standard PARTITION TILE is furnished in the following sizes and weights:

Dimensions, inches	Minimum number of cells in thickness	Standard weight, lb
3×12×12	3	15
4×12×12	3	16
6×12×12	3	22
6×12×12	4	25
8×12×12	4	30
10×12×12	4	35
12×12×12	4	40

* See Gypsum Tile Fillers, Article 7.
† A.S.T.M. Spec. C-56-30.
‡ See Tables IX and XXVI for test performance of assemblies of hollow clay tile.

Monolithic Concrete. Stone Concrete. Under the action of fire, stone concrete shows a general typical behavior, but the degree of destructive effects varies with the nature of the aggregate and the density of the concrete. Owing to low conductivity, the heated surface expands while the unexposed side remains cool, and the result is warping, cracking or even local spalling or popping of the surface. The heat also dehydrates the cement mortar, causing disintegration for a depth of 1 in or more, while very high temperatures will cause fusion and flowing of the concrete (temperatures of 2 000 to 2 500° F.) depending on the aggregate. This occurred in the Edison, East Orange, fire in 1912. CALCAREOUS aggregates such as limestones show the lowest heat-transmission and least destructive effect from fire. After fire, the calcined surfaces are apt to slake from mositure in the air, fall off and require surface repair of structural members. FELDSPATHIC aggregates comprising igneous rocks such as the feldspars and trap rock give slightly higher heat-transmission but do not cause cracking or spalling of the concrete to as great an extent. Artificial aggregates of burnt clay and furnace cinders are generally grouped in this class so far as fire-resistance is concerned. GRANITE and SANDSTONES and other igneous rocks, containing larger quantities of free silica and usually with a binder of siliceous or calcareous cements, under the action of fire show a tendency to disrupt, crack or spall when used in concrete. The temperature of transmission is also likely to be higher. SILICEOUS rocks consisting mostly of uncombined silica when used as aggregate generally result in the most disruptive effects under fire conditions. The temperature of fusion, however, is beyond the range of normal fires in buildings. For the ordinary range of fire conditions concrete construction can utilize any of the commonly accepted aggregates. For the most severe fire exposures, it may be necessary to select aggregates or even resort to special reinforcement of the protective layer of concrete on the outside of the structural member.

Slag Concrete. Blast-furnace slag has been used as aggregate with highly satisfactory results as to both fire-resistance and strength. Floor systems of slag concrete have withstood the four-hour fire-tests conducted for the New York Bureau of Buildings, and buildings using this system have been in existence for twenty to thirty years. Care must be exercised, however, in the selection of the slag. Sanford Thompson states that the slag must be "air-cooled, crushed, screened from dust, and free from foreign material," and that "exceptional care must be used in proportioning, mixing and placing." * Air-cooled slag is now extensively used in localities adjacent to steel mills and where the item of freight does not enter into the cost of the material, and has apparently resulted in very satisfactory concrete construction. Attempts have been made in recent years to apply commercial treatments to slag aggregates for purposes of easy handling of the material and lightening the weight of the concrete. The H. H. Potts Co. of Chicago, Ill., make a granulated slag aggregate under the trade name "POTTSCO," by a process of cold-water treatment of the hot slag. The resulting material is graded and screened to definite proportions and used in the manufacture of concrete products. "CALICEL" is produced by the Calicel Company, Inc., of Hammond, Indiana, and Chicago, Ill., as a processed slag aggregate for concrete products and for acoustic tiles. It is also used in the manufacture of high-temperature insulation for lining furnaces. The aggregate for concrete production is a highly expanded stone

* For a series of tests and description of materials, see pamphlet issued by the Carnegie Steel Company, Furnace Slags in Concrete. See also Proc. Am. Soc. for Test. Mats., 1914, and a full discussion of slag concrete published in the Iron and Coal Trade Review (London) for November 22 and 29, 1918.

consisting of silicates of lime and aluminia, expanded to approximately forty times its original volume. The loose weight of the expanded stone varies from 7 to 10 lb. per cu ft and produces concrete of low strength but high insulation value.

Cinder Concrete. Concrete made from well-burned cinders resulting from forced-draft combustion of coal, free from foreign matter and containing unburned coal not to exceed 35%, makes an excellent fire-resistive material Owing to the porous and cellular nature of the aggregate, it is difficult to secure a workable consistency without the use of sand as a fine aggregate and within practical limits of cement content. The variable quality of the commercial cinders procured in practice results in a wide range of physical strength properties.* For FIREPROOFING PURPOSES, cinder concrete should be used in a viscous, wet mixture and should not be deposited dry. "Anthracite cinder concrete, well mixed, and deposited in such manner as to coat the reinforcement with mortar, will not cause corrosion of embedded steel." † Owing to the variability of the available commercial cinders, the Corlite Corporation of New York City developed a controlled manufactured aggregate for light-weight concrete, by submitting a mixture of ordinary house ash and silica sand to a smelting process, whereby all the combustible constituents of the ash are consumed and the non-combustible components of the mass are fused into a hard, vitreous, clinker-like iron-aluminum silicate. This clinker is then submitted to a crushing and screening process to size the material in a definite predetermined gradation of particles. This aggregate, known as "CORLITE," possesses all the superior characteristics of a good cinder in concrete plus the advantages of a controlled material of definite composition and quality. In the smelting process, practically all of the unburned coal in the ash is consumed. Commercial cinders as secured in New York City contain anywhere from 10 to 35% or more of unburned coal, which does not seem to vitiate its effectiveness as fire-proofing within the scope of practical requirements. In 1930, the Underwriters' Laboratories of Chicago undertook an investigation of the fire-resistance of cinder-concrete units containing 20, 35 and 45% of combustibles in the aggregate. As a result of these tests, the following specification was issued in connection with the use of cinders for concrete-block construction: "When cinders are used, the average combustible content of the mixed fine and coarse aggregate shall not exceed 35% by weight of the dried mixed aggregates." ‡ The weight of cinder concrete can be controlled from 84 to 120 lb per cu ft, depending upon the strength required. The fire-proofing mixture used in New York City averages 108 lb per cu ft and consists of 1 part of cement, 2 parts of sand and 5 parts of cinders.

Processed Concretes. "AEROCRETE," § a light-weight, expanded structural concrete is produced by adding a small amount of metallic aluminum powder to the mixture of Portland cement and sand or cinders. On the addition of water, a gas is generated which expands the wet mix and forms small air-cells throughout the material. (See Fig. 1.) "Aerocrete" can be precast or poured in place in the field; it requires no tamping in the forms, merely being brought to an approximate level when pouring. "Aerocrete" is made in three grades, depending upon the uses for which the material is designed and the strength required: HEAVY-WEIGHT STRUCTURAL, weighing 60 to 80 lb per cu ft and having a compressive strength of 600 lb per sq in; INTERMEDIATE

* See Cinder-Concrete Floor-Construction, Article 7.

† Cinder-Concrete Floor-Construction, Trans. A.S.C.E., Vol. LXXIX, p. 523.

‡ Research 22, Retardant No. 2226, Underwriters' Laboratories, April 30, 1930.

§ Aerocrete Corporation of America, 51 East 42d St., New York City.

for use in partitions, furring, floor-construction filler tile and fireproofing generally, weighing 50 to 60 lb per cu ft, with a compressive strength of 300 lb per sq in; and INSULATING, weighing 30 to 50 lb per cu ft, for floor fill and insulation uses generally. It is used for structural floor and roof slabs, partition blocks for soundproofing, wall insulation in rooms of refrigeration plants and light-weight fill on top of structural floor and roof slabs. In addition to its light weight, it has excellent fire-resistive qualities and has been approved for use in fire-proof floor-construction in New York City, after satisfactory four-hour fire, water and load tests.

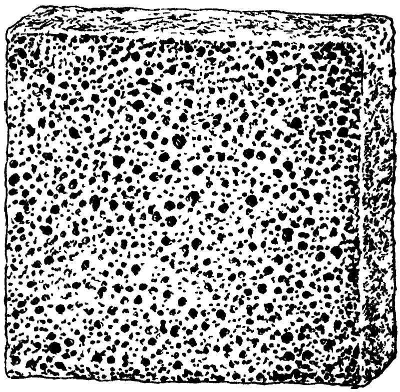


Fig. 1. "Aerocrete" Block

GUNITE.* The mixture of sand and cement deposited under high pneumatic pressure with a machine manufactured under the trade name "CEMENT-GUN," to which the required supply of water is added just before the dry constituents emerge from the nozzle, has in recent years been extensively applied in the field of fire-resistive building-construction Tests have shown that "GUNITE" is superior in tensile and compressive strength, and adhesion, with less permeability, absorption and porosity than good hand-made or deposited concretes or cement mortars.

(a) For encasing structural steel, the method of application insures direct contact between the steel surface and protecting coat, results in absolute protection against corrosion and eliminates possibilities of air-pockets and porous conditions. It is possible to apply the protective coatings in successive layers to the required thickness, which do not break down under exposure to high temperatures and fire-streams. The first use of the CEMENT-GUN in steel encasement was at the Grand Central Station, New York City, where over five million square feet of gunite was placed as a protective against fire and corrosion. The great saving in dead load on the structure of the Bronx Terminal Market in New York City, by the use of "Gunite" encasement, was stated to result in "a lighter structural-steel floor-system; lighter column sections and smaller foundations; decrease in the cost of beam encasement and rapidity of construction." †

(b) Reinforced Gunite for floor and roof slabs in a mixture of 1 part of cement to 3 parts of sand may be reduced to a thickness of 1½ to 3½ in. Because of its impervious qualities, "Gunite" roofs can be built without other roof-covering. Special provision however must be made for expansion-joints along lines of trusses and walls at from 20 to 40-ft centers. These joints must be protected with a weather shield and apron in the usual manner of expansion-joints.

(c) Curtain walls of "Gunite" from 1½ to 2 in thick reinforced with steel mesh of not less than 0.5 of 1% in each direction are constructed on structural

* Cement-Gun Company, Inc., Allentown, Pa.
† The American Architect, June 3, 1925

steel frames for industrial buildings and for enclosure walls of stair and elevator shafts. Supports for the slabs should be spaced not less than 4 nor more than 8 ft on centers. Double-wall construction with concrete, steel or wood studs has also been used in building-construction. When composed of two slabs, each 2 in thick with an enclosed 8-in air-space, "Gunite" wall-construction has been accepted as the equivalent of a 12-in brick wall in skeleton wall construction.

(*d*) "Gunite" has also been used for the restoration and strengthening of both concrete and steel structures, as well as for reinforcement of otherwise weak structural members of buildings, bridges, etc. The strength of columns, beams, girders and slabs, is commonly increased by adding rods or other reinforcement and encasing the whole with a 2-in layer of "Gunite."

(*e*) Several municipal building codes have approved the "Gunite" method of increasing the thickness of deficient masonry walls when additional stories are added to the building *

(*f*) "Gunite" is also applied as a stucco over hollow tile, wood or metal lath, resulting in positive adhesion of the dense protective coat with low absorptive properties, insuring protection and weather resistance.

(*g*) As a result of a series of tests conducted at the Underwriters' Laboratories, the following time-temperature ratings were secured on various walls and partitions of "Gunite": Hollow 12-in walls on "Gunite" studs, 3-hour; solid 2-in non-bearing "Gunite" partitions, 1-hour.†

PORETE ‡ (see Fig. 2) is a Portland-cement concrete to which a chemical foam is added to generate gases in the process of deposition, resulting in a light-weight precast or shop-made unit in both hollow and solid forms. It is

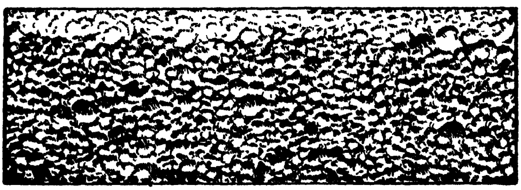


Fig. 2. "Porete"

manufactured in solid slabs for short-span roofs, and siding of industrial buildings, the slabs being reinforced for use in both flat and sloped roofs, and also in field mixes for a fill and nailing base.

HAYDITE, a concrete aggregate controlled by the American Aggregate Company of Kansas City, Mo., is manufactured by crushing a special clay, shale or shale rock containing a high percentage of carbonaceous matter and burning it in a rotary kiln until clinkers of a porous nature are formed. After passing through the kiln, it is allowed to cool, after which it is crushed, screened and graded to predetermined sizes and proportions, in accordance with definite screen analyses. "HAYDITE" concrete averages 30 to 40% lighter than sand-and-gravel or stone concretes and is chemically pure and free of silt or other deleterious matters. It is claimed to have superior fire-resistance. Its uses include fire-proofing for structural-steel frames, general reinforced-concrete construction, and the manufacture of light-weight pre-

* Los Angeles Building Code.

† Report on Gunite Walls, Retardant No. 1327, May 12, 1922.

‡ Porete Manufacturing Co., Newark, N. J.

cast concrete products, artificial building stones and incombustible nailing bases. "Haydite" concrete has a coefficient of heat conductivity of $\frac{1}{62}$ Btu per hour as compared to approximately 6 Btu per hour for stone concrete and 5 Btu per hour for brickwork of the same thickness. The modulus of elasticity of concretes made with coarse "Haydite" and natural sand is approximately 75%, and for all "Haydite" concrete about 55% of that of sand-and-gravel concretes of the same strength.

Concrete Units.* Numerous brick, block and tile forms of concrete building units are manufactured by the WET or DRY PROCESS or various modifications of these methods,† for use as substitutes for brick, stone or terra-cotta. They have their greatest application in the walls of residential or industrial buildings of relatively low height, in general fire-resistive partition-construction and fireproofing of steel members, and in combination floors of concrete ribs and tile or block fillers. The various manufacturing processes employed show no marked differences upon the fire-resistive properties developed in tests of assemblies of these units. The test data do not seem to justify preference as to AIR VS. STEAM CURING or as between dry, damp or wet consistencies used in the manufacture, provided the products are thoroughly CURED when incorporated in the construction.

(a) **Concrete Building Brick** ‡ are manufactured in accordance with standard specifications for strength requirements as given in Table VII. The standard size of brick is the same as clay brick, $2\frac{1}{4}$ by $3\frac{3}{4}$ by 8 in, with a

Table VII. Concrete Building Brick

Classification	Compressive strength (brick flatwise), lb per sq in		Modulus of rupture (brick flatwise), lb per sq in	
	Mean of 5 tests	Individual minimum	Mean of 5 tests	Individual minimum
Grade B	2 250	1 500	450	300
Grade C	1 250	1 000	300	200

permissible variation of plus or minus $\frac{1}{16}$ in in depth, $\frac{1}{8}$ in in width and $\frac{1}{4}$ in in length.

(b) **Hollow Concrete Blocks and Tiles.** Hollow blocks or tiles of Portland cement and sand or gravel mixtures when subjected to fire show the same effects of expansion and temperature stresses that develop in brick or hollow-tile walls. The web walls of concrete blocks develop general cracking and the exterior walls may spall and explode, depending upon the nature of the aggregate used. The heat-transmission through a hollow-block wall is generally greater than through a solid brick wall, provided the latter does not open along mortar joints owing to expansion under heat. On the other hand, the greater size of the concrete block, being equivalent to 12 or more brick units in the wall, results in a considerable reduction in the number of joints, and the likelihood of fewer direct means of heat-communication through a wall. Fire-resistive tests indicate that for a time-performance of four hours as a fire- or exterior wall the shells and webs of 12-in hollow-concrete block

* S.P.R. 32.
† See Proceedings American Concrete Institute.
‡ A.S.T.M. Spec. C 55–28T

should have a minimum thickness of 1½ in and have at least 2 cells in the thickness of the wall; and for a three-hour performance of 8-in blocks, the shells and webs should be at least 1¾ in thick.

Practice in the manufacture of CONCRETE BUILDING BLOCK AND TILE is based on the STANDARD SPECIFICATIONS of the American Concrete Institute.* The requirements for strength are based on the gross cross-sectional area of the block as laid in the wall with a minimum compressive strength at 28 days of 700 lb per sq in. CINDER-CONCRETE units are similarly manufactured in tamp or pressure machines of a relatively dry mix, under careful control of the aggregate and process, in plants licensed by the patentees of the product.† Strength and absorption requirements are based on the same specifications as sand-concrete products. The cinder aggregate is prepared by crushing and screening to predetermined sizes to produce a graded mixture of fine and coarse material which is combined with the cement without the addition of sand. In fire-tests, the material shows less tendency to spall or crack from unequal expansion than the sand or stone units; the weight is approximately 50% less than sand-concrete mixtures; and porous mixtures for interior non-bearing partitions, because of their cellular nature, possess good sound and heat insulation and allow easy driving of nails and cutting of blocks without breakage or waste. Tests at the Underwriters' Laboratories show that coal combustibles in the aggregate up to a maximum of 35% do not affect the fire-resistance of the product, and the same fire-resistive ratings are accorded approved blocks as the sand-cement mixtures. However, since the blocks are manufactured of a dry mixture, special precautions should be taken to insure their thorough curing before they are incorporated in the wall; otherwise the changes of moisture content in the wall will result in continued working, with expansion and contraction of the units.

Cinder-concrete blocks are also manufactured, composed of cinders, cement and asbestos fiber binders,‡ in both solid and hollow units with compressive strength of 600 to 1 200 lb per sq in. Greater ease in nailing and nail-holding power is claimed for this unit, as well as lighter weight of block, than for the usual cinder-concrete unit.

Gypsum is composed of one part of calcium sulphate and two parts of chemically combined water of crystallization, its chemical formula being $CaSO_4 \cdot 2H_2O$. The fire-resisting properties of gypsum are due to the water of crystallization, which, when exposed to the action of fire, is slowly liberated. The practical significance of this action is that the temperature of materials protected by gypsum cannot exceed 212° F., until all of the chemically combined water has been driven off, which means that a thinner protection is required than for most other materials for the same fire-resistive ratings. Two processes of manufacture are used for calcining the gypsum rock, the KETTLE PROCESS and the ROTARY-KILN PROCESS. Because of this and the variation in the chemical constituents of the raw material, the resulting product from different plants and locations varies considerably in consistency and physical properties. By a recalcination process, a SECOND-SETTLE gypsum is produced which possesses greater strength properties. First-settle gypsum has been referred to in the trade as STUCCO and second-settle gypsum as STRUCTOLITE. By the addition of accelerators or retarders, the

* Standard Specifications P-1A-29S, published by Portland Cement Association, Chicago, Ill.

† National Building Units Corporation, Philadelphia, Pa.

‡ Nailcrete Corporation, New York City, N. Y.

time of set of calcined gypsum is controlled for the purposes intended as provided in Standard Specifications of the American Society for Testing Materials. Among the various commercial forms for building-construction are plasters, wall boards, plaster boards, partition tile, floor filler-block, and fireproofing for structural-steel framing. When combined with an aggregate, it is employed as a structural material for precast and poured-in-place floor and roof-construction. These compositions are light in weight, good non-conductors of heat and possess fire-resisting properties of considerable importance. The common aggregates include wood shavings or chips, cocoa, asbestos fiber, coke breeze, cinders, and sand. With cinder aggregate, the product weighs 70 to 80 lb per cu ft, and with wood fiber, from 48 to 60 lb per cu ft. The superior fire-resistance is partly offset by the low structural strength. Under high temperatures, the material calcines and softens and is readily attacked and washed away on the surface by water streams.

KEENE CEMENT is an anhydrous calcined gypsum with an accelerated set, used for the base and finish coats of plastering and in Scagliola. After first set, it can be retempered and will reset quite as hard and strong as the original plaster. It is hard and dense and is particularly adaptable to highly decorative plastering work and for sanitary purposes where subject to moisture.

GYPSUM WALL-BOARDS for use without plastering are manufactured under Standard Specifications C 36–25 of the American Society for Testing Materials with a nominal thickness of 3/8 in, widths of 32, 36 and 48 in, and lengths of 4 to 12 ft. For laying with filled joints, the above widths are reduced 1/4 in. They weigh from 1 1/2 to 2 lb per sq ft.

GYPSUM PLASTER BOARDS are manufactured to conform to A.S.T.M. Standards C 37–30 to comply with definite strength requirements in a width of 32 in and lengths of 24, 36 or 48 in. The thickness and weights of the boards are as follows:

Thickness, inches	Weights, lb per sq ft	
	Minimum	Maximum
1/4	1.2	1.5
5/16	1.25	1.65
3/8	1.4	2.0
1/2	2.0	3.0

GYPSUM PARTITION TILE OR BLOCK are manufactured under A.S.T.M. Standards C 52–27, generally in lengths of 30 in and width of 12 in. They must have a compressive strength of 75 lb per sq in when tested dry, and 25 lb per sq in when tested wet. The thickness of blocks and shells are given in Table VIII.

Mortars and Plasters. The relatively fire-proof qualities of mortars and plasters have led to much controversy in the past and are still the subject of much discussion. LIME MORTAR for masonry was formerly considered the most satisfactory in fire-resistance for use in dry locations, and in recent fire-tests, clay-brick walls built of it have developed the least amount of deflection and distortion. Under load, however, it is deficient in strength, compared to PORTLAND-CEMENT mortar, and for this reason the latter is generally preferred for all brick and tile masonry work. Admixtures of

Table VIII. Thickness of Gypsum Tiles and Shells

Nominal size of tile, inches	Actual thickness, inches		Shell th ckness, inches	
	Minimum	Maximum	Circul ır core	Elliptical or rectangular core
1½ (Furring).....	1⅜	1⅝		. ...
2 (Furring)....	1⅞	2⅛	...	... . . .
2	2	2¼	Solid	Solid
2½........... .	2⅜	2¾	½	9/16
3.	2⅞	3¼	½	11/16
4.......... .	3⅞	4¼	¾	13/16
5	4⅞	5¼	⅞	1⅛
6........ . .	5¾	6¼	½	11/16
8.......... .	7¾	8¼	¾	13/16

NOTE. Tile for furring purposes are secured by splitting 3 or 4-inch cored tiles.

lime and cement mortar are highly desirable because of their greater workability and appear to give the best results in fire-resistance. GYPSUM mortars composed of one part of unfibered gypsum neat plaster and not more than three parts of clean, sharp, well-graded sand by weight, possess superior fire-resistive properties and should always be used for setting precast gypsum units and in gypsum construction generally.

LIME PLASTER applied on wire lath will withstand a high degree of heat without injury, but under the action of water will disintegrate and wash away.

GYPSUM PLASTER in which sand is used should be mixed in the proportion of one part of gypsum plaster to not more than two parts of sand by weight for the first or scratch coat on wood lath, gypsum lath and metal lath. For the scratch or first coat on clay tile, gypsum tile and brick, and for the second or browning coat on all the aforementioned bases or backgrounds, the proportion should be one part of plaster to not more than three parts of sand by weight. Although sand float finishes and colored finishes for float and texture work are quite frequently employed, the finish coat of plaster, regardless of the scratch and browning coats, in the majority of cases, is a lime-putty finish consisting of one part of gypsum gauging plaster and three parts of lime putty by volume, or the white coat consists of one part of lime putty and one part of Keene's cement by volume. Keene's cement is also used in connection with lime plaster for scratch and browning coats. For plastering concrete, specially prepared gypsum plaster for concrete surfaces, to which water only is added, should be used. These plasters should not be sanded. In tests conducted at the Bureau of Standards in Washington on masonry construction, it was found that ¾ in of sanded gypsum plaster is the equivalent of ½ in of neat gypsum plaster and adds from ½ hour to ¾ hour to the fire-resistive rating of a masonry assembly for each plastered surface.

PORTLAND-CEMENT plaster is the equivalent of sanded gypsum plaster in fire-resistance. Whenever plaster coats exceed ¾ in in thickness they should be reinforced with wire mesh or lath.

Asbestos Products. Asbestos fiber and Portland cement united under heavy hydraulic pressure is manufactured in the form of plain and corrugated sheathing; siding and roofing; roof coatings and shingles; wall-boards; building lumber; heat-insulation packing and blocks; theater-curtains;

and various fire-resistive compounds, and plasters and stucco for use in covering heating equipment. Because of incombustibility, low coefficient of expansion, and low heat-conductivity, asbestos products will withstand high temperatures without disintegration or loss of strength. In tests for the New York Bureaus of Buildings, a single layer of ¼-in asbestos boards attached to wood joist resulted in a ½-hour fire-resistive rating. The principal companies manufacturing asbestos building products are: Ambler Asbestos Shingle and Sheathing Company, Ambler, Pa., Johns-Manville, Inc., New York, N. Y.; Philip Carey Co., Cincinnati, Ohio; Ruberoid Company, New York, N. Y.; and Asbestos Mfg. Co., Ltd., LaChine, Canada. ASBESTOS WALL-BOARD is successfully used as a fire-resistive protection for covering walls and ceilings, in single or double layers, for lining garages, and generally where the use of a fire-retardant is desirable. It is manufactured in sheets 48 by 48 in to 48 by 96 in, and from 3⁄16 to ⅜ in thick.

ASBESTOS BUILDING LUMBER or "Transite Asbestos Wood" is made in standard sheets 36 by 48 in, 42 by 48 in, and 42 by 96 in, varying in thickness from ⅛ to 2 in, and on special order to 4-in or greater thicknesses. It is considerably harder than ordinary wood, but can be manipulated with heavier metal-working tools. It can be punched, drilled, or sawed, and is secured in place with bolts, nails, or screws. It is sufficiently elastic to withstand ordinary vibration, expansion, and contraction due to normal temperature changes, and can be formed to easy curvatures without splitting. Sudden extreme changes of temperature such as produced by hose-streams on heated surfaces may result in cracking

ASBESTOS CORRUGATED sheathing is used for both roofing and siding of industrial buildings. This is made by the same pressure process as the flat boards into a dense, unlaminated, monolithic sheet of great strength and rigidity. The corrugations of "TRANSITE" roofing and siding are 1½ in in over-all height and 4.2 in in pitch. It is manufactured in standard sheets 42 in wide and 4 to 8½ ft long. It is generally 7⁄16 in thick at ridges and valleys and 5⁄16 in thick on the slope of the corrugations. It is designed for application directly over roof-purlins with a maximum spacing of 5 ft, and directly on the girts for siding with a maximum spacing of 6 ft. It is generally secured with bolts or clips of special design for the specific use. In addition to fire-resistive qualities, corrugated asbestos sheathing offers high atmospheric resistance without the need for paint preservatives.

ASBESTOS ROOFING-SHINGLES, are approved by the Underwriters' Laboratories as Class A roof-coverings when laid over a layer of asphalt-saturated asbestos felt. The advantages claimed in addition to fire-resisting qualities are light weight; strength; elasticity; ease of manipulation, cutting, sawing and shaping to fit dormer windows, chimneys, etc.; immunity from corrosive action of salt air; and general weather-resistance.

Steel and Iron.* Structural steel and cast iron used in building-construction conforming with the Standard Specifications of the American Society for Testing Materials † will not sustain its design loads at the usual working stresses under fire exposure.‡ In the temperature range of 480 to 500° F., structural steel increases in compressive strength about 25% above normal, and in similar degree in tensile strength. Beyond this region, the strength decreases until the 800° F. temperature range, when the strength is again normal. With further increase of temperature, the strength falls off rapidly.

* Proceedings, A.S.T.M., Vol. 26, 1926, p. 33.

† A.S.T.M. Standards A9–29.

‡ See Tests of Columns and Column Protectives, Article 6.

At a temperature region of 932° F. to 1319° F., the ultimate compressive strength is approximately the same as the permissible working stress, and the column or structural member is at the point of failure under working loads. Cast-iron is subject to approximately the same temperature-strength laws, but the increased strength at early temperature rises cannot be utilized in design in either material, for it is accompanied by a decrease in elastic limit or yield-point. With temperature increases also, there is a falling off in coefficient of elasticity of the metal, and all structural-steel and cast-iron members must be insulated with a protective coat of the required fire-resistive rating as specified under Column and Girder Fireproofing to prevent the yield-point of the steel from being exceeded.

Cold-Drawn Steel.* Because of the effects of the cold working of the metal in the process of manufacture, cold-drawn steels develop higher ultimate strengths without any definite yield-point in the elastic behavior of the metal. This material is frequently used in the form of wire mesh in floor-construction and hooping or ties in columns at higher permissible working stresses than ordinary structural steel. However, the exposure of such material to fire at temperatures beyond 1 000° F. removes all the effects of cold working and reduces the steel to the condition of normal structural-grade material. The use of increased working stresses should therefore be applied with discretion and only when proper insulating coverings are provided.

Robertson Protected Metal is primarily a roofing and siding material for industrial buildings, although fabricated in a number of other metal products, such as ventilators, hoods and ducts, and wherever protection from rust and corrosion in addition to some fire-retardant value is required. The ROBERTSON PROCESS is designed to take the place of periodic paint coatings and will preserve the steel-sheet core for a life of 10 to 15 years. The process of manufacture consists in dipping plain or corrugated steel sheets of Nos. 18 to 26 gauge in a chemically inert molten asphalt to secure a first or steel coating. The coated plate is then covered on both sides with a layer of asbestos felt saturated with asphalt under pressure and folded around the edges. The FELTED sheet is then passed through coating rolls which apply an outside bitumen waterproofing cover at a temperature of approximately 425° F. The sheets are made either flat, corrugated, or V-shaped, and colored with mineral pigments during manufacture to give a permanent red color if desired. The manufacture of the material is controlled by the H. H. Robertson Company of Pittsburgh, Pa. The Underwriters' Laboratories have approved asphalt asbestos protected sheet steel as a Class A roof-covering on either combustible or incombustible roof-decks. When fastened to structural members the sheets are secured with galvanized iron clips and screws, with the two sheets bolted together at laps.

Full-size fire-tests on this material were conducted by the Factory Mutual Laboratories in 1926, as a result of which, approval was granted for the use of the STANDARD SIDING and STANDARD ROOFING material in structural-steel frames for industrial buildings, when no other combustible material occurs in the construction, without automatic sprinkler protection.

Glass and Wire-Glass. Glass conducts heat about one-hundredth as fast as metal, and the change in volume from thermal expansion or contraction is very small. Sudden and extreme changes of temperature, however, result in fracture of glass. Recent developments in glass manufacture have

* A.S.T.M. Standards A82-27.

demonstrated the possibility of manufacturing annealed glass which will absorb heat with low thermal expansion and is capable of withstanding considerable heat shock. Such glass is now in use in optical projecting apparatus. The Labor Law of the State of New York requires ¼-in plate-glass or wired-glass in window-sash of incombustible construction on the fronts of buildings. Steel-wire netting incorporated in glass of ¼-in thickness or greater increases resistance to the passage of flames and to the cooling and impact of hose-streams by holding the glass in the sash after it has been cracked from the heat, or heated to the temperature of fusion. WIRE-GLASS windows afford an effective fire-stop under ordinary conditions of fire exposure. Wire-glass is furnished in smooth polished plate, roughened, hammered, or obscured surfaces, and in various forms of figured, prism, pyramid and pressed-lens effects for the purpose of eliminating direct transmission, reducing glare and substituting diffusion or redirection of light. For fire-resistance, the glass should be not less than ¼ in thick, reinforced with wire mesh of No. 24 B and S gauge wire, with mesh not larger than ⅞ in square. The size of pane allowed is governed by underwriters' requirements * based on the severity of the probable fire exposure, but in no case should it be more than 48 in in either dimension or exceed 720 sq in in area. Only non-inflammable material is allowed in setting the glass in the sash. It is manufactured in thicknesses up to ½ in, depending on the type of surface; in widths up to 48 in, and lengths up to 12 ft, and weighs approximately 2 lb per ⅛ in in thickness. For deck, vault or floor lights, it is furnished in thicknesses of ¾ in, widths up to 30 in, and lengths up to 6 ft, in hammered, ribbed, ground, or polished plate. The twist of the wire in all styles of wire-glass runs with the length of the sheet; in ordering wire-glass, the width should always be specified first, as the twist should be set vertically in the sash.

CORRUGATED WIRE-GLASS for skylights, canopies, marquises or side-wall panels is manufactured in sheets 27¾ in wide and not exceeding 96 in in length. DEEP-ANGLE corrugated glass is 2½ in center to center and SHALLOW-ANGLE 2$^{11}/_{16}$ in center to center of corrugations. In skylight construction, the sheets of glass are laid edge to edge, with ½-in open joints, and exterior and interior metal caps or aprons shaped to conform to the corrugations of the glass, and secured together with bolts. The chief manufacturers of wire-glass are the Mississippi Wire Glass Co. of New York, N. Y.; the Pennsylvania Wire Glass Co. of Philadelphia; the Highland-Western Glass Co. of Washington, Pa.; and the Manufacturers' Glass Co. of Chicago, Ill.

Prism and Structural Glass. Glass prisms or tiles installed for the purposes of diffusing and distributing light in exterior openings, office-partitions and skylights are frequently not contained in frames which are designed to withstand severe heat. The prism-glass units are usually made 4 in or 6 in square or in rectangles of 5 in by 7 in, and ¼ or $^{5}/_{16}$ in thick. They should preferably be set in elastic compounds in rigid cast-iron frames embedded in a rigid rib construction of reinforced concrete to complete the panel. In Report No. 11 of the Insurance Engineering Experiment Station, C. L. Norton describes a series of comparative fire-tests of electroglazed Luxfer prisms, 0.35 in thick and 4 in square; electroglazed plate, ¼ in thick and 4 in square; and ¼-in wire-glass. The results of these tests indicate that the three materials, in sheets up to 24 by 30 in, are of equal value in FIRE-RESISTANT PROPERTIES and remain in effective operation up to the time

* See Opening Protectives, Fire-Doors and Fire-Windows, Article 20.

when the temperature of melting glass is reached. PRISM TILES are furnished by the American Three-Way Luxfer Prism Co. of New York and Cicero, Ill.

During the last decade, the use of prism glass in reinforced-concrete frames has been extensively applied in Europe for exterior wall enclosures as well as for interior partitions.* Glass prisms of various types and designs manufactured by the Siemens Glass Works, Ltd., of Dresden, Germany, have recently been introduced in the United States under the trade name STRUCTURAL GLASS.† (See Fig. 3.) These products include prisms for floor and roof lights, windows, walls and partitions, of solid-section, plain and wired glass, from 1 in

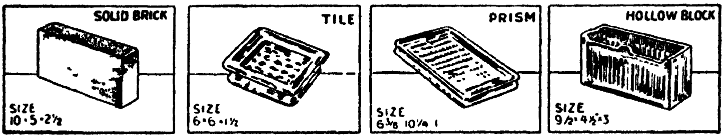


Fig. 3. "Structural Glass" for Windows, Walls and Partitions

to 3 in in thickness; hollow blocks 3 in in thickness; vacuum blocks 2½ and 3 in in thickness; and solid brick 2½ in in thickness. In a series of tests conducted for the New York Bureaus of Buildings in September, 1930, four different kinds of structural-glass units were erected in the form of panels and subjected to the American Standards Fire Test. Each of these panels was approximately 6 ft by 6 ft in area, except Panel No. 4, which was approximately 6 ft by 8 ft. Panel No. 1 was built of solid "Structural Glass" units (No. 400), 10 in by 5 in by 2½ in in size, laid up in mortar of 1 part of cement to 2 parts of sand with ⅜ in horizontal and vertical joints, with a ½ in expansion joint on all sides. Panel No. 2 was built of "Structural Glass" tile (No. 10), 6 in by 6 in by 1½ in in size in a grid frame of reinforced-concrete members and expansion-joints on all sides. Panel No. 3 was built of "Structural Glass" prism tile (No. 13), 6⅜ in by 10¼ in by 1 in in size, set in individual metal forms in a reinforced-concrete grid. The spaces between the tiles were filled with a cement mortar producing joints 1 in wide on one face of the panel. Panel No. 4 was built of "Structural Glass" hollow block (No. 100 W), 9¼ in by 4½ in by 3 in in size, with ⅜-in walls reinforced with chicken-wire mesh. The horizontal joints were of the mechanical interlocking type and were laid up ¼ in thick in cement mortar. The vertical joints averaged ⅜ in in thickness, and an expansion-joint was provided on all sides. (See Fig. 4.) After 15 minutes of firing, all the glass units were generally cracked in panels Nos. 2, 3 and 4, and approximately 14 of the solid brick were filled with minute interior cracks in Panel No. 1. At the end of half an hour, the temperatures on the outside of panels Nos. 2, 3 and 4 had all reached approximately 1 000° F., and on the solid brick panel No. 1 approximately 300° F., and one-half of the brick were still in good condition in this panel. The furnace temperature at this time was 1589° F. In one hour's time, all of the glass panels were still intact except panel No. 1, which had warped inwardly toward the fire, leaving a 2-in wide opening at the top through which smoke and fire passed freely. The fire was continued for a total period of 1 hour and 23½ minutes, when the upper third of Panel No. 1 fell inward toward the fire.

* A comprehensive article on the physical properties of glass, both optical and fire-resistive, together with illustrations of its application and use in Europe, is contained in Architectural Record for October, 1930.

† Structural Glass Corporation, 101 Park Avenue, New York City.

Application of water at 30-lb pressure punched a hole through Panel No. 3 after 1 minute of application, but Panels Nos. 2 and 4 withstood the water application for the full test requirement of 2½ minutes. After half an hour of firing, the heat transmitted through the panels was sufficient to cause the charring of a wood structure located approximately 4 ft away from the side of the test house containing Panels Nos. 3 and 4, and thereafter this frame structure had to be wet down to prevent actual ignition. Panel No. 4 of the wired-glass hollow-brick type was apparently in the best condition at the end of the test. The edges of several of the blocks in this panel had begun to fuse at the end of the test. The temperature of the furnace had reached approximately 1 750° F. at this time. The test demonstrated at least equivalent fire-resistance to the standard wire-glass construction, normally assumed as a one-hour fire-retardant.

Fire-Retardant Wood. To meet the requirements of the New York Building Code for elimination of combustibles in the interior trim and finish of buildings, several processes have been developed to render wood fire-resistive or flame-resistive. This material is generally referred to as fire-proofed wood.

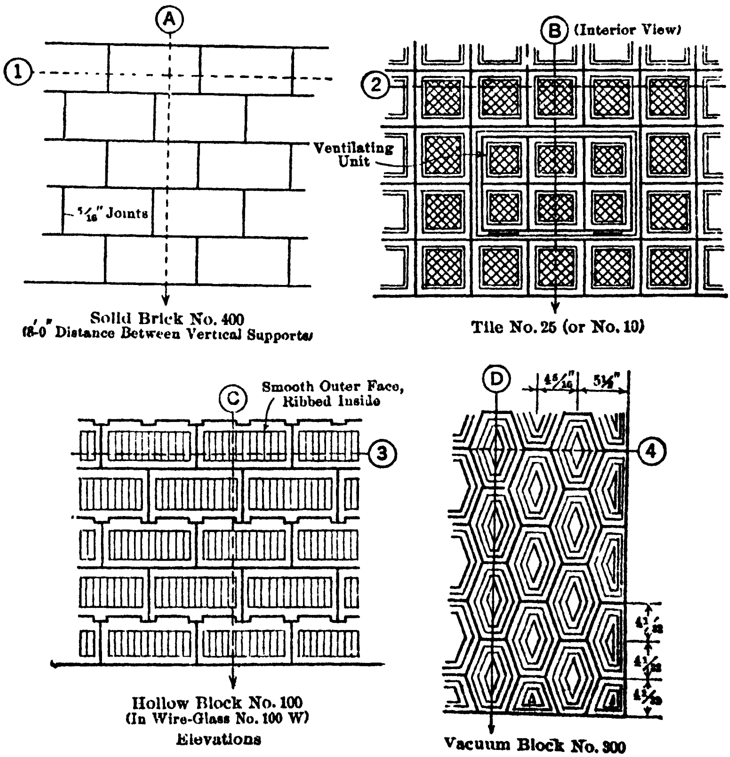


Fig. 4. Structural Glass for Walls, Partitions and Windows

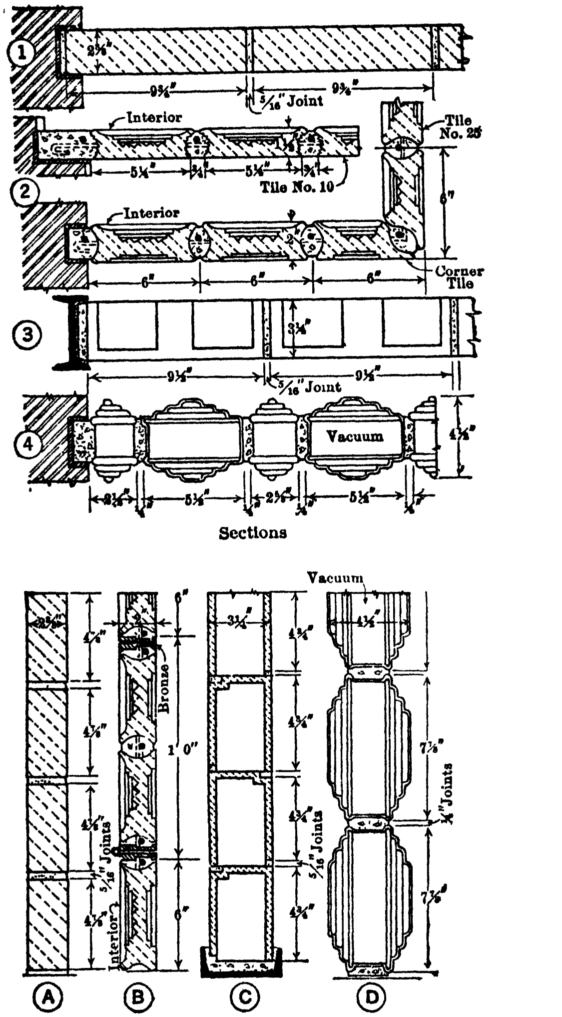


Fig. 4 (Continued). Structural Glass for Walls, Partitions and Windows

Experience has demonstrated that these processed woods have a definite application in building-construction for reducing the fire hazard. The effectiveness of so-called fire-proofed wood can be expressed in general terms of the percentage of wood consumed under laboratory test conditions of fixed temperatures and time of fire application as 90 per cent for untreated soft

woods, 50 per cent for hard woods and 20 per cent or less for efficient commercially treated woods. The processing of wood to make it fire-resistive raises the ignition-point and so renders the material less likely to ignite when subjected to flame or intense heat; the wood substance carbonizes without sustaining flame and tends to prevent the transmission of flame and fire to adjoining units of the structure and its contents.

Among the disadvantages cited against fire-retardant wood is the increased difficulty in working the wood because of a tendency of the included salts to dull wood-working tools. This necessitates the use of special heat-treated steels in milling and increased cost in manufacturing trim. Some of the processes involve the use of salts which are hydroscopic in character and which tend to keep the woodwork damp and destroy varnish finishes as well as corrode hardware and other metal work in contact with the wood. Improper processing of the wood also has a tendency to affect the cell structure and make the wood brittle. By proper preparatory drying of the wood and selection of the material best adapted to treatment, and with the use of the proper admixture of chemical salts, these defects can be largely overcome. As the wood is impregnated under pressure far in excess of 250 lb per sq in, and even as high as 1 000 lb per sq in in dense wood structures, a thorough control of the process is essential at all stages. Moreover, the wood must be heated uniformly without distortion and without destroying the cell structure or discoloring the wood. The perfection with which these ends are attained accounts for a wide range in the cost of available commercial processes, but only the most carefully controlled treatments can be recommended for effective and dependable construction material.

The fire-retardance of wood is effected by injecting chemicals of various kinds into the cellular structure with equipment similar to that used for vacuum pressure preservative treatments of timber. The wood must be thoroughly dried as required for cabinet work and each charge of wood selected for similarity of specific weight and density, general wood structure and especially uniform moisture content, to be efficiently treated in any one impregnation. There are in general three classes of chemicals employed in the treatment: those which give off incombustible gases when decomposed by heat; those which fuse under heat and cover the wood fibers with a protective coating, and those which reduce flammability by releasing sufficient water of crystallization to saturate the wood with moisture. The treatment of the wood increases the weight from 15 to 20%, depending on the amount of salts deposited in the cellular structures. Permanence of the treatment for rendering the wood fire-resistant has been attested by investigations of the U. S. War Department on processed wood removed from the torpedo boat "Winslow" and by a series of tests conducted on samples selected from buildings in New York City after 20 to 30 years' use by Mr. Ernest F. Hartman* in cooperation with the officials of the New York Bureau of Buildings.

In New York City, the use of fire-retardant wood for interior-trim purposes is controlled by check tests made on samples of the material selected by a representative of the Bureau of Buildings after delivery at the job. A representative set of samples is taken from each shipment and subjected to Shavings, Timber and Crib Tests as defined in rules of the Board of Standards and Appeals.† If the tests are not satisfactory, the entire shipment is condemned. During late years, with improvements in the details of commercial processes, the use of fire-retarded wood has been extended to the

* Protexol Corporation, Kenilworth, N J.

† Bulletin Board of Standards and Appeals, New York City, July 12, 1927

safeguarding of temporary structures and staging such as construction-scaffolding and hoist-towers on high buildings and more recently to assemblies of wood for partitions and doors in building-construction. To meet this commercial expansion a new method of test has been developed at the Forest Products Laboratory of the U. S. Department of Agriculture, known as the Fire Tube Test. The purpose of this test is to provide an accurate measure of the loss in weight and increase in temperature resulting from combustion of a specimen of uniform dimensions. With improvements in details of the method of procedure and construction of test-apparatus, the Fire Tube Test promises to provide an accurate method of measuring the effectiveness of the treatment of the wood for fire-resistance.*

Flaimpruf Wood. In the application of fire-retardant wood to a more extended field, a product known as "FLAIMPRUF WOOD" has been developed in the last five years for use in partitions, wainscoting, bank and hospital equipment, fire-safe furniture and filing cabinets and especially for doors and opening protectives where low heat-conductivity and barriers to flame and smoke are desirable from the viewpoint of safety to life.† This treatment has been successfully applied to all the common woods used in decorative treatment of buildings, including walnut, birch, red oak, maple, whitewood, basswood, pine, spruce and fir, in stock up to 3 in in thickness. In this process, the natural moisture content of the wood is uniformly reduced before impregnation to about 7% by combined air and kiln drying. After treatment "Flaimpruf Wood" is again very slowly dried in especially equipped dry-kilns to reduce its moisture content to 7% and the pores are sealed against further absorption of moisture. Careful check weights are maintained throughout the process so that the degree of treatment is thoroughly controlled and the treatment does not interfere with grain, structure or color of the wood. The chemicals used are inactive to moisture, the wood can be glued as effectively as untreated material, the applied finishes last as long as those on untreated wood, and the treatment for interior uses unexposed to the weather is claimed to be as permanent as unprocessed wood. "Flaimpruf" can also be manufactured for exterior use by properly sealing the pores with a water-repellent after the fire-retarding process.

Fire-Retardant Paint. Numerous so-called FIRE-PROOF PAINTS have been introduced in recent years. When applied to woodwork they provide a more or less effective protection against fire, and may, for this reason, prevent the spread of local sources of fire, such as burning cigarettes or matches. Ordinary calcimines or whitewash have proved in tests to be as effective as any of the trade paints tried. The following regulations regarding fire-proof paint were given in the annual report of the Manhattan Bureau of Buildings, New York, for 1904.

"(1) The term FIRE-PROOF PAINT shall be understood to mean any preparation used to cover the surfaces of wood or other materials for the purpose of protecting the same against ignition.

"(2) No fire-proof paint will be considered satisfactory unless it so protects the wood or other material to which it is applied that the same will not flame or glow after having been subjected to the flame of a gasoline torch for two minutes.

"(3) Before applying fire-proof paint to any material the surfaces must be cleaned.

* Report Committee C-5, A.S.T.M., 1930.
† Henry Klein & Co., 40 West 23d Street, New York City.

"(4) Application of fire-proof paint must be repeated whenever it is found that the material to which it is applied is no longer protected to fulfil Specification No. 2."

6. Column, Truss and Girder Assemblies

General Requirements. As the columns, trusses and girders form the backbone of the structure, it is vitally important that they be so constructed that in a complete burnout of the contents of the building there will be no structural collapse. Iron and steel as well as reinforced-concrete columns must be protected to maintain the temperature of the structural or load-bearing core within safe limits.* Unprotected cast-iron columns and steel columns were tested in 1896 by a committee of the American Society of Mechanical Engineers, in cooperation with other organizations, in full-size specimens loaded to their rated safe-load capacities. These tests showed that unprotected steel columns failed at an average temperature of 1 150° F., and the cast-iron columns at an average temperature of 1 300° F. Failure occurred after an exposure of from 23 minutes to 1 hour and 20 minutes of fire. For some years thereafter practice in the protection of structural columns was based primarily on small-size tests of the insulating properties of the common protective materials, basing the requirement for thickness of protection on the ability of the covering material to keep the temperature of the steel within 550° F.

Fire-Tests of Building Columns. Investigation of the ultimate fire-resistance of representative types of building columns under their full rated loads, both protected and unprotected, was undertaken in 1917 by three cooperating organizations, the Associated Factory Mutual Fire Insurance Companies, the National Board of Fire Underwriters and the U. S. Bureau of Standards.† The testing authorities assigned a fire-resistive rating to the various types of columns and protective coatings equal to approximately two-thirds of the ultimate resistance periods, thus allowing for a factor of safety of 1½. The conditions under which controlled time-temperature fire-tests are conducted are unusually severe as compared to the average severity of building fires. Under the procedure established by the American Standards A_2–1926,‡ the test period during which the assembly will still perform its function is reported as the fire-resistive rating of the material or assembly, and it is this period upon which building codes are generally based. It is felt that either the required performance periods for different units of building-construction should be reduced, or a smaller factor than 1½ should be applied to the ultimate test-periods. In Table IX, the average time to failure of the various types of columns and protectives has been deduced from a study of the available test data of ultimate periods. The time periods indicated in the tables are the rated periods or fire-resistive ratings generally assumed in building codes. These 1917–1919 tests included a total of 106 column sections covering practically all the types employed in general building practice. Since the above tests, a series of 60 typical reinforced-concrete column fire-tests conducted at the U. S. Bureau of Standards' Pittsburgh Laboratory

* See Steel and Iron, Article 5.

† Fire Tests of Building Columns. Copies of the complete report can be secured from the Associated Factory Mutual Fire Insurance Companies, 31 Milk St., Boston, Mass.; or the Underwriters' Laboratories, 207 East Ohio Street, Chicago, Ill. Also Technical Paper 184, U. S. Bureau of Standards.

‡ See Fire-Resistive Classification of Material, Article 1.

Table IX. Minimum Protective Column Coverings

Type of column	Description of protection	Fire-resistive rating, hours
Structural steel	Unprotected	1/4
Cast iron	Unprotected	1/2
Cast iron, conc. fill	Unprotected outside	3/4
Steel pipe, conc. fill	Unprotected outside	1/2
Steel pipe, conc. fill	Interior reinforcement	1
Struct. steel, conc. fill	Unprotected outside	1
Structural steel	1 layer Portland cement or gypsum mortar on metal lath	1
Structural steel	2 layers Portland cement or gypsum mortar on metal lath with 3/4-in air-space	2
Structural steel	2-in poured gypsum reinforced	4
Structural steel	3-in poured gypsum reinforced	5
Structural steel	2-in siliceous aggregate concrete, reinforce	2
Structural steel	2-in trap or cinder aggregate concrete	4
Structural steel	2-in limestone aggregate concrete	6
Structural steel	3-in siliceous aggregate concrete, reinforce	3
Structural steel	3-in trap or cinder aggregate concrete	5
Structural steel	3-in limestone aggregate	7
Structural steel	4-in siliceous gravel concrete, reinforced	4
Structural steel	4-in trap or cinder aggregate concrete	7 1/2
Structural steel	4-in limestone aggregate concrete	8
Structural steel	2 1/4-in clay or concrete brick	2
Structural steel	3 3/4-in clay or concrete brick	4
Structural steel	2-in solid or 3-in hollow structural clay tile outside metal ties and mortar joints between tile and steel (shells 1 in thick)	4
Structural steel	4-in hollow clay tile with 1-in walls and webs outside metal ties; minimum of 2 1/2 in solid material	4
Structural steel	Double covering of 2 in solid or 3 in hollow clay tile metal ties or mesh in joints	5
Structural steel	2-in solid or 3-in hollow gypsum blocks	3
Structural steel	3-in solid or 3-in hollow gypsum blocks, plastered	4
Structural steel	4-in solid or hollow gypsum block	5
Reinforced concrete	2-in integral siliceous gravel concrete	4
Reinforced concrete	2-in integral trap-rock concrete	6
Reinforced concrete	2-in integral limestone concrete	8

NOTE. Poured gypsum, block or tile coverings, and the lower grades of concrete applied to structural-steel columns require the use of metal ties or reinforcement. Such reinforcement must consist of at least No. 10 gauge wire with a spacing or pitch of 6 in, or wire mesh, weighing not less than 1 1/2 lb per sq yd, with a mesh opening not exceeding 4 in by 4 in.

were reported in a publication,* covering various types of aggregates and thickness of protection. Several other programs have been carried out at the Bureau of Standards on special problems of protection of structural-steel sections, including the use of various types of solid and hollow gypsum blocks; some of these investigations are still in progress and have not yet been reported. In

* Technological Paper No. 272.

both the Chicago and Pittsburgh tests, temperature observations were made at various distances beneath the surface of the column protection, to determine the rate of heat-penetration.

Application to Trusses and Girders. The required protective coatings for truss members and girders are of almost equal importance with those for columns. Since these members are mainly in horizontal positions, they may be affected in larger degree by fire conditions and be more subject to rupture by expansion with resulting cracking and spalling. Unless properly protected, the expansion of a steel truss or built-up girder supporting several

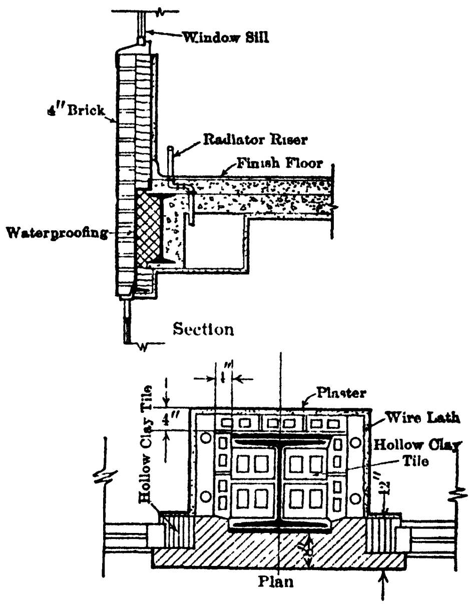


Fig. 5. 40 Wall Street, New York City

stories of a building may cause secondary failure of the columns which it supports or by which it is supported. In general, however, the quality and dimensions required for column protections are usually applied to these members.

The projecting flanges of beams and girders are subject if anything to more serious fire effects than the columns, particularly when projecting below the general ceiling-line. For this reason, the fireproofing of girders and trusses is considered in conjunction with column protections. The close relation between these assemblies is especially of importance in preserving the integrity of any one. Unless the column-protection is continuous and interlocked with the girder and floor fireproofing, any one or all of these members may fail under fire attack. This is particularly true at connections of girders to columns, where the efficiency of the protective coat may be lost by failure to provide the necessary covering. The edges of lugs, brackets

and ends of flanges, however, are not required to be covered with the full thickness of material required on the body of the section, and are usually permitted to project to within one inch of the outside of the fireproofing coat.

Brick Protection. Four-inch brick protection on INTERIOR COLUMNS will provide all the necessary fire-resistance for most occupancies and results in a rigid construction of great strength and stability that will not easily be damaged in warehouse spaces devoted to trucking or heavy manufacturing uses. This construction, however, is not common on interior columns in modern buildings, owing to its weight and cost. On EXTERIOR COLUMNS, brick protection is quite common, and most building codes require a minimum covering of 8 in on the outside and 4 in on the inside faces, which is ample to meet the fire-resistive requirements for all occupancy hazards as well as exterior fire exposure. As a protection against corrosion, however, the steel columns

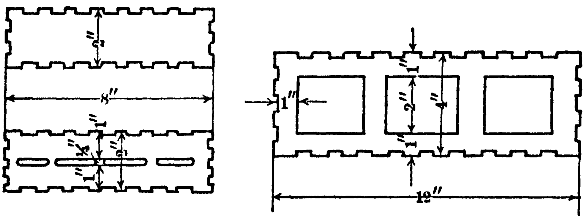
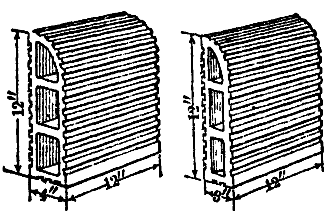


Fig. 6. Structural Clay Tile

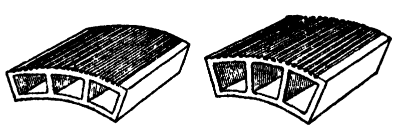
should be parged with cement mortar or "Gunited" with a $\frac{1}{4}$-in protective coat. Corrosion of the steel Z-bar and plate exterior columns of a structural-steel frame, after 15 years of service, caused the formation of a $\frac{1}{4}$- to $\frac{3}{8}$-in scale of rust to form and resulted in the splitting of the 4-in brick encasement. This defect appeared, however, only on the walls subject to the



4''×12''×12'' Round Corner Column Covering 3 cell Wt. Approx. 16 lb

3'×12'×12'' Round Corner Column Covering 3 cell Wt. Approx. 15 lb

Fig. 7. Column Covering



3'' Round Column Covering 3 cell Wt. Approx. 15 lb

4'' Round Column Covering 3 cell Wt. Approx. 16 lb

Fig. 8. Round Column Covering

driving wind and rain-storms from the Northeast, which is the prevailing direction of storms where the building is located.

Structural Clay-Tile Protection. By reason of light weight combined with structural strength and ease of handling, STRUCTURAL CLAY-TILE is

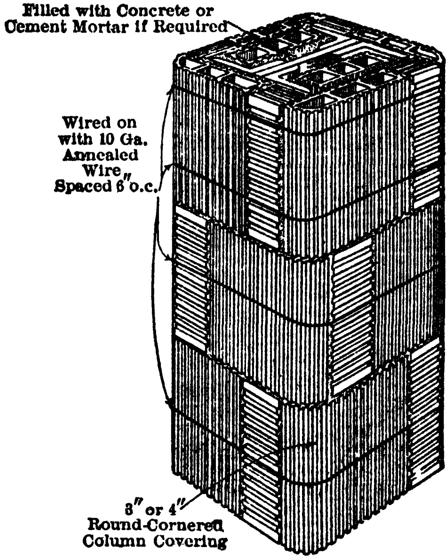


Fig 9. Application of Round Column Covering to Built-up I Column, Showing Use of Partition-tile "Pieces" as Fillers and Method of Wiring Tile in Place

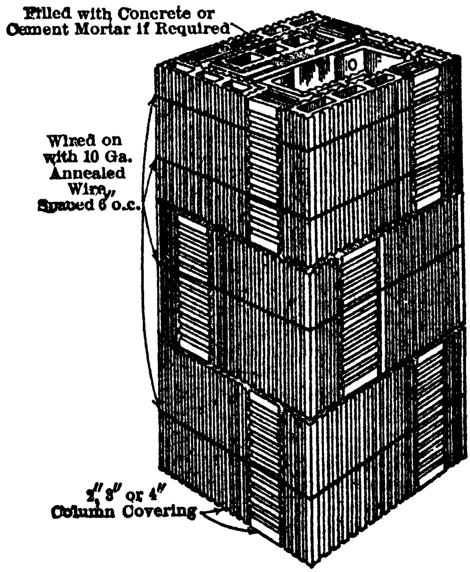


Fig. 10. Built-up I Column Protected with Square Column Covering Partition-tile

well adapted to this use. The units for fireproofing of columns and girders are manufactured in various shapes and sizes. In special cases, structural drawings should be submitted to the manufacturers for estimates and prices. The simplest form of structural clay-tile column-protection is the partition-tile, rectangular in form. Sometimes these tiles are rounded on one corner

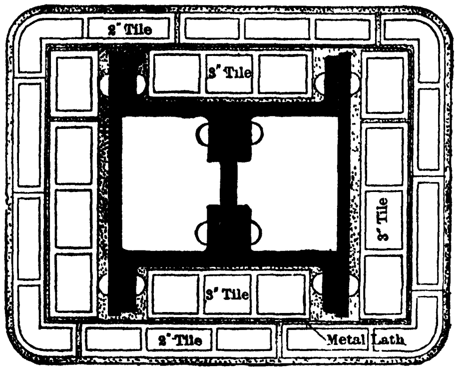


Fig 11. Double-tile and Metal-lath Column-Protection

as shown in Fig. 7. For round columns, a special form of radial tile is used, Fig. 8. Under the rigid requirements of the New York Code, and on the information developed in column tests, the standard partition-tile does not contain a sufficient thickness of solid clay for fireproofing,* and Fig. 6 shows three types of protecting tile widely used in New York City and the East generally. For a four-hour protection, at least $2\frac{1}{2}$ in of solid material are required outside of the steel face. The entire height of column or length of girder must be solidly encased in the protecting coat. The column-covering must start on the floor-construction proper and not on the top fill or the finished wood flooring, and should be carried to the underside of the floor-construction of the upper story with a full mortar joint at top and bottom. Where there is a difference in level between tops of beams and top of floor-construction, a level base should be provided for the base course with slabs of broken tile laid in cement mortar. Block or tile protection should be secured with No. 10 gauge wire spaced not over 6 in on centers, or U-shaped clips of No. 16 gauge strap iron slipped over the shells of abutting blocks in each course, or with wire mesh not more than 4 in by 4 in in size and not less than No. 14 gauge wires. Figs. 9 and 10 show common methods of protecting rectangular sections. Similar coverings are provided for cylindrical sections. Clay-tile fireproofing should generally be erected with the cells vertical, although horizontal fillers may be employed where necessary to

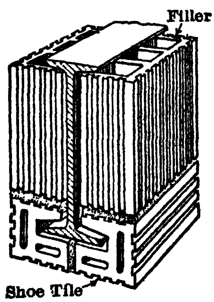


Fig. 12. Girder Protected by Use of Shoes and One-piece Filler

* See Table IX.

complete the course of blocks. To increase the efficiency of the protection, two superimposed layers of tile can be secured around the columns, each being

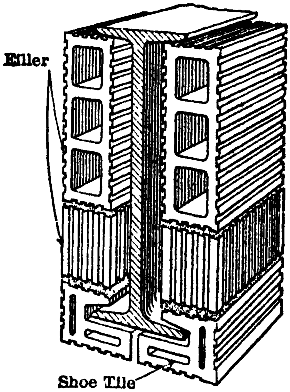


Fig. 13. Showing application to Larger Girders Using More than One Piece for Filler

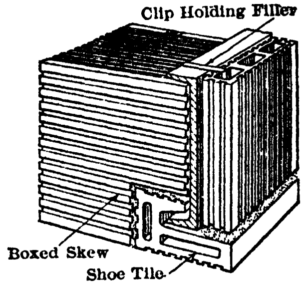


Fig. 14. Facia Covering at Stairwells, etc., with Arch on Opposite Side

wrapped independently of the other. Not only is the fire-resistive rating * thus increased, but the method insures the integrity of at least part of the protection against the destructive action of hose-streams. Fig. 11 shows

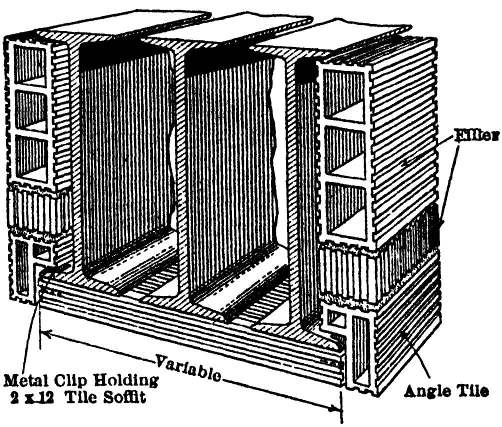


Fig. 15. Angle and Soffit Method of Protecting Triple Girders, Showing Use of Metal Clips to Retain Soffit

a column protected by this method. The inner layer as well as the outer is wrapped with wire ties or mesh, and all joints between the layers of tile and

* See Table IX.

the metal faces of the column are filled with cement mortar. Steel guards of ¼- or ⅜-in metal are sometimes provided, to a height of 5 or 6 ft from the floor, in garage spaces, mercantile and factory buildings to protect the fireproofing from injury. This covering can be welded into a tight-fitting jacket flush with the face of the finished plastering or exterior surface of the

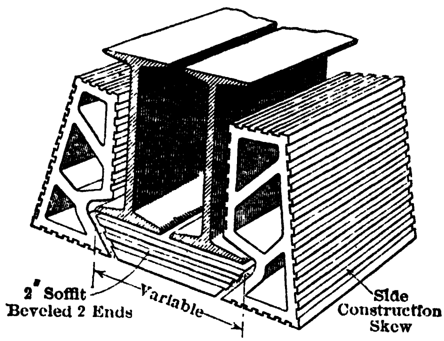


Fig. 16. Illustrating Method of Protecting Double Beams Supporting End-construction Flat-arch Floor with Side-construction Skews

column assembly. Figs. 31 and 36 show the general arrangement of typical exterior columns and the relation to outside wall face.

Standard shapes of BEAM and GIRDER structural clay-tile protection are carried in stock and schedules are furnished by the manufacturers for all

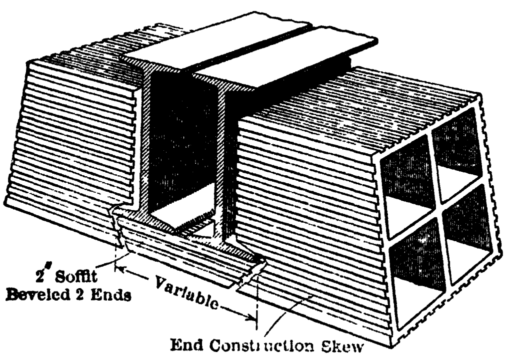
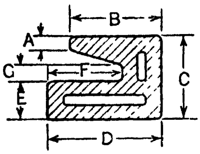


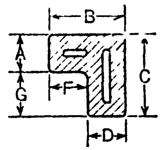
Fig. 17. Illustrating Method of Protecting Double Beams Supporting End-construction Flat-arch Floor with End-construction Skews

sizes and shapes of tile clips and fillers required by the structural-steel design. Figs. 12, 13, 14, and 15 show several common methods employed with clay-tile protections. In these applications, the SHOE TILE encases the lower flange and is held in place by the web fillers. On very wide flanges, the tile soffit is held by metal clips, as shown in Fig. 20.

Table X. Standard Clip and Angle Tile



Standard Clip and Angle Tile



NOTE Clips and angles are 12 in long. A percentage of halves is shipped with each order.

Die No.	Approx. sq ft outside measure	Approx. weight per piece	A	B	C	D	E	F	G
G20	.65	13	¾	3⅜	3¾	3¾	2	2	¾
G25	.70	14	¾	3⅞	4	4½	2	2½	13/16
G30	.75	15	⅞	4¼	4¼	4¾	2	3	⅞
G35	.80	16	¾	4¾	4¼	5½	2	3½	1
G40	.85	17	¾	4¾	4¼	6	2	4	1
G45	.90	18	¾	4¾	4¼	6½	2	4½	1
G46	.95	19	¾	4¾	4¾	6⅝	2	4⅝	1½
G50	.95	19	¾	4¾	4¼	7	2	5	1
G55	1.00	20	¾	4¾	4½	7½	2	5½	1¼
G60	1.05	21	1	5½	4⅝	8	2	6	1⅜
G61	1.05	21	1	4½	4⅝	8	2	6	1⅜
G70	1.16	23	1	6	4⅞	9	2	7	1½
G71	1.16	23	1	4½	4⅞	9	2	7	1½
G75	1.35	27	2	7½	6¾	9½	2	7½	2⅜
G80	1.50	30	2	8	6⅞	10	2	8	2⅞
L23	.60	12	2	4	5	2	.	2	3
L26	.90	18	2	4	8½	2		2	6½
L43	.65	13	2	6	5	2	.	4	3
L46	.95	19	2	6	8½	2		4	6½

Other typical clay-tile beam-protections used in conjuction with structural clay-tile floors are shown in Figs. 16 and 17. If a girder, a strut, or truss member is exposed on all sides, it is protected as shown in Fig. 18. All clay-tile fire-proofing is sold on the basis of square feet of superficial area. Table X gives the dimensions of FLANGE CLIPS and ANGLE TILES required for varying widths of flanges.

The protection of TRUSSES is accomplished in the same manner as that of columns and girders, each member being completely encased in unit tiles, each tile being secured in place by clamps or ties or wire wrapping. Fig. 19 shows some of the units employed for this purpose in addition to the standard partition, angle and clip tiles and soffit pieces.

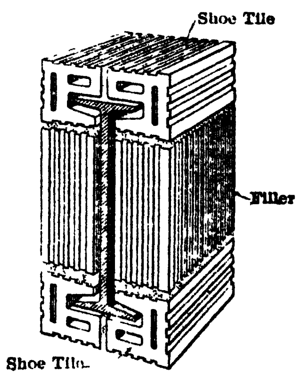


Fig. 18. Method of Fire-proofing Girders Exposed on All Sides

Gypsum Column-Protection. The prime function of the fire-protective coat applied to structural steel is to insulate the steel against an increase in temperature above the 800° F. temperature range, and the material used must not expand or contract unduly. It should also be light in weight, readily adaptable to building conditions and be reasonable in cost. GYPSUM FIREPROOFING for steel beams, girders, columns and trusses is either POURED-

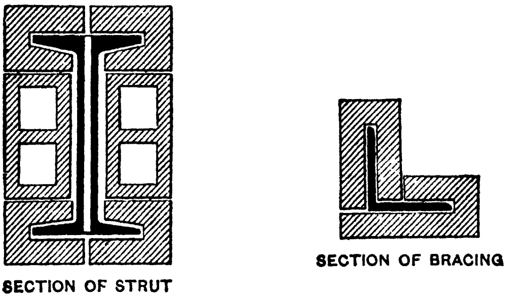


Fig. 19. Tile Protection for Members of Steel Trusses

IN-PLACE or PRECAST. When POURED-IN-PLACE, the structural member is wrapped with wire ties or mesh and fireproofed with gypsum fibered concrete. The common mix employed is 87½% of gypsum and 12½% of wood chips. The wrapping should consist of wire fabric of No. 14 gauge wires and 4-in by 4-in mesh. PRECAST fireproofing for columns consists of solid or hollow-gypsum partition-tile, Fig. 21. It is furnished in the standard

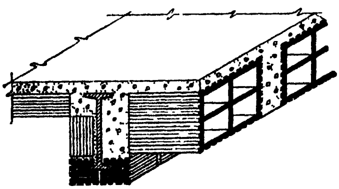


Fig. 20. Method of Using Tile Fire-proofing with Long-span Concrete-joist Floor-construction. Note that the covering is put in place before the floor is poured and that the concrete of the joist fills the cells of the tile and bears directly on the flange of the beam.

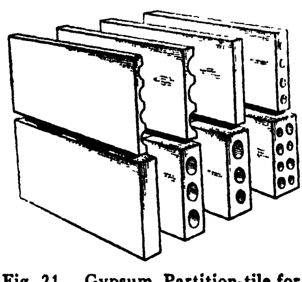


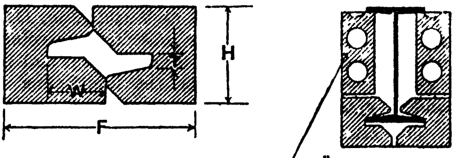
Fig. 21. Gypsum Partition-tile for Fire-proofing

12-in by 30-in unit,* which can be quickly and easily cut or sawed to fit difficult corners or angles and slight curves.

Beam, girder and truss PRECAST fireproofing consists of 2-in gypsum SHOE TILE or ANGLE AND SOFFIT tile for lower flanges. Gypsum partition-tile are used as side covers for webs. The size of SHOE TILES for various flange widths are shown in Table XI, and are mounted in a double unit, 18 in long. SOFFIT TILE is specially molded at the mill of the proper width for each size

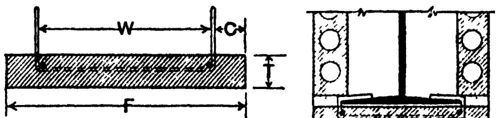
* See Gypsum Partition Blocks, Article 5.

Table XI. Gypsum Shoe and Soffit Tile



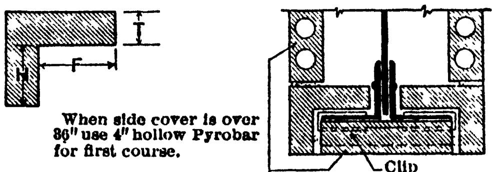
Pyrobar Shoe Tile

Type	F	H	T	W	Weight
B - 40	7⅝"	4¼"	½"	2"	12.5 Lb
B - 50	8¼"	4¼"	⅝"	2½"	13
B - 65	9¾"	4¼"	13/16"	3¼"	15
B - 80	10¼"	4¼"	15/16"	4"	16



Pyrobar Soffit Tile

Type	F	C	T	W	Weight
S - 10	15¼"	2⅛"	2"	10"	13 Lb
S - 11	15¼"	2"	2"	11"	13
S - 12	17¼"	2⅛"	2"	12"	14.5
S - 13	17¼"	2"	2"	13"	14.5
S - 14	19¼"	2½"	2"	14"	16
S - 15	19¼"	2"	2"	15"	16



When side cover is over 36" use 4" hollow Pyrobar for first course.

Pyrobar Angle Tile

Type	F	H	T	Weight
L - 34	3"	4"	2"	15 Lb
L - 45	4"	5"	2"	18.5
L - 46	4"	6"	2"	20

Type Shoe
B - 40 Max. Flange Width = 5"
B - 50 Max. Flange Width = 6"
B - 65 Max. Flange Width = 7¼"
B - 80 Max. Flange Width = 9"

flange, with the reinforcement straps embedded. For deep girders, these are used in combination with angle tile. Soffit protection can also be cut from ordinary hollow partition-tile on the job and secured in place with pipe straps.

Tests were conducted at the Bureau of Standards at Washington, D. C, in 1930, of both 2-in POURED and 3-in PRECAST gypsum protections on columns under their rated loads, which developed approximately seven-hour and five-hour resistance periods, respectively. In both tests, the fireproofing was plastered with ½ in of sanded gypsum plaster.*

Concrete Protection. The relative values of various concretes have already been discussed. (See Table IX.) In general it is advisable in all cases to reinforce the protecting layer by means of wire ties or wire mesh as herein discussed for all poured-in-place plastic coats. Effective wrapping of reinforce-

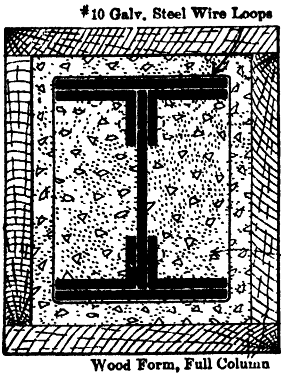


Fig. 22. Concrete Column-protection and Wooden Form

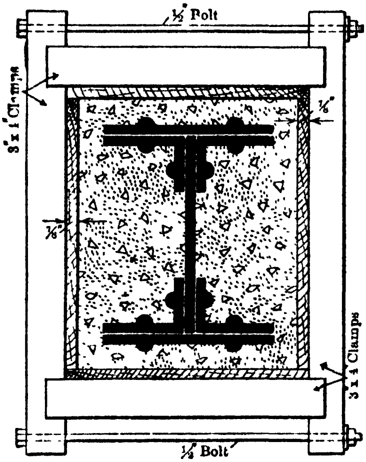


Fig. 23. Concrete Column-protection and Wooden Form

ment can be secured either with a No. 14 gauge wire mesh, size of mesh not to exceed 4 in by 4 in, or No. 10 gauge spirals with a pitch of not over 6 in. In the Midwestern States, effective results are also secured with concrete made from burned-clay aggregates such as "Haydite." † Figs. 22 and 23 show the effective encasement of the steel section secured with poured concrete. The lighter weight and more insulating concretes are preferable for this purpose, and very efficient results have been secured with 1 : 2 : 5 CINDER CONCRETE in New York City. Wood forms should be placed the full length of the column and the concrete be poured through a hole in the floor-construction to insure the continuity of the protection.

CONCRETE-BLOCK UNITS for fireproofing structural steel must incorporate, in the cross-section, the same amount of concrete as required in Table IX for solid concrete brick fireproofing, that is not less than 3¾ in for three- and

* See Mortars and Plasters, Article 5.

† See Haydite, Article 5

four-hour protection. In a single-cell block, the webs may be $1\frac{3}{4}$ in thick

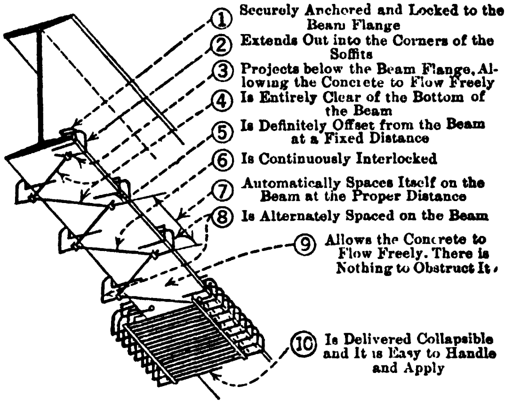


Fig. 24. Beam Wrapping

and in a double-cell block $1\frac{1}{2}$ in thick. The same methods of installation must be followed as for clay or gypsum unit construction.

BEAM and GIRDER-protection can be more thoroughly accomplished with the use of concrete, particularly at inaccessible places such as connections of girders to columns, than with any precast unit construction. Around the lower flanges, reinforcement in the form of mesh or especially designed expanded clips (see Figs. 24 and 25) should be used to protect the projecting edges from injury and destruction. Precast concrete units in the form of soffit pieces are also manufactured with the hangers or clamp precast in a shop-made unit and have proved advantageous in the protection of steel girders. By allowing an insulated air-space of $\frac{3}{4}$ to 1 in between steel and fireproofing, the temperature of the entire protected section remains uniform and no unequal expansion-stresses are produced in the section. Provided sufficient protection is installed to maintain the temperature below 800° F., no deflection will result due to

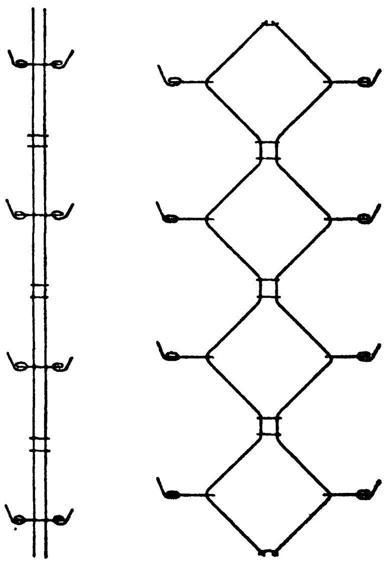


Fig. 25. Beam Wrapping

heat effects. A fire-test of this form of protection in the Butterick Building, New York City, thoroughly established its efficiency.

Metal Lath and Plaster. This protection serves for FIRE-RESISTIVE RATINGS of not more than one-hour to two-hour time-temperature performance.

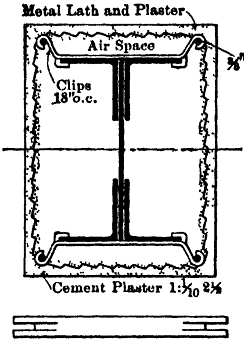


Fig. 26. One-hour Column Protection

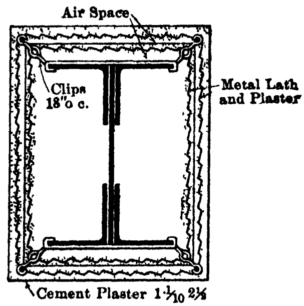


Fig. 27. Two-hour Column Protection

For lighter occupancy hazards or for the INTERMEDIATE CLASS of construction, in which one-hour floor-construction may be acceptable for multistory dwellings and similar uses up to a limiting height of 80 to 90 ft, the single layer of No. 24 gauge metal lath weighing 3.4 lb per sq yd or No. 18 gauge ⅜-in mesh wire lath, supported at a distance of ¾ in from the structural

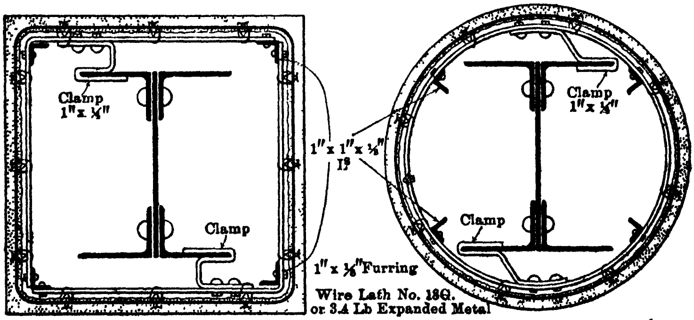


Fig. 28. Metal-lath and Plaster Column Protection

section on steel furring (see Fig. 26), protected with one ¾-in layer of sanded gypsum * or Portland-cement mortar, will fulfil the requirements for all normal fire exposure. A double layer of reinforcement separated by a ¾-in air-space and two plaster casts as shown in Fig. 27, reduces to a minimum the danger of the entire protection being demolished by hose-streams, and

* See Mortars and Plasters, Article 5

under time-temperature performance results in a two-hour fire-resistive rating. Similar protections are also applicable to beams, girders and members of trusses, as shown in Fig 29.

SUSPENDED METAL-LATH and PLASTER CEILINGS are not however generally recommended for the protection of MAIN GIRDERS. On SECONDARY-FLOOR MEMBERS * this practice is permissible when the area thus protected is limited in extent and is enclosed on all limiting sides by girders thoroughly encased in protective materials or by other bulkheads to reduce the possibility of heat and flames being transmitted through to the main structural members of the building. In severe fires, the experience has been that such ceilings are likely to collapse or be broken down at some one point, thereby destroying any advantage that the protection may otherwise have afforded. Individual members of TRUSSES can also be protected with metal lath furred away from the steel sections with ¼-in pencil rods or ¾-in steel channels wired to the members with No. 18 gauge wire and covered with 3.0-lb metal lath and ¾ in of neat gypsum plaster or 1 in of sanded plaster or Portland-cement mortar.† This protection affords a one-hour fire-resistive rating and is satisfactory for secondary trusses or those not supporting columns or other important members of the structure. For ROOF-TRUSSES a metal-lath and plaster ceiling continuously applied and suspended from the lower chord of the truss is suitable for a fire-retarded construction of one-hour rating, particularly if the truss is at a sufficient height to preserve the ceiling from mechanical injury. Roof-trusses located more than 20 ft above the floor and thus protected are generally installed without fireproofing on the roof-purlins and with a sub-standard roof-deck.‡

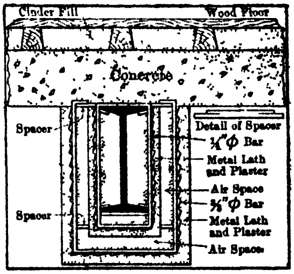


Fig. 29. Two-hour Beam Protection

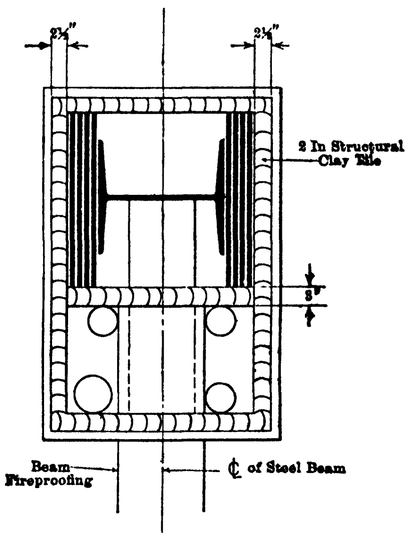


Fig. 30. Typical Pipe-Shaft, Interior Column, 40 Wall Street, New York City

Provision for Recesses and Pipe Chaises. A practice in construction methods that is subject to the most serious condemnation still persists,

* See Metal Joist Floors, Structural Platform Slabs, Article 7, also Table XVI.
† See Plaster and Mortar, Article 5.
‡ See Roofs, Article 10.

although attention has been directed to it by failures in many fires and in the literature on the subject. To effect economy in first cost and space, builders frequently locate pipe and vent-spaces alongside both interior and exterior columns within the protective encasement. Correct methods of installing such piping and vent-ducts are shown in Figs. 30 and 31. Fig. 5 also indicates how heating risers and returns for the radiators should be installed with expansion-elbows so located as not to injure the girder-protection.

Columns should always be fireproofed independently of ducts or shafts. The piping, conduits and duct risers should be installed outside the column-

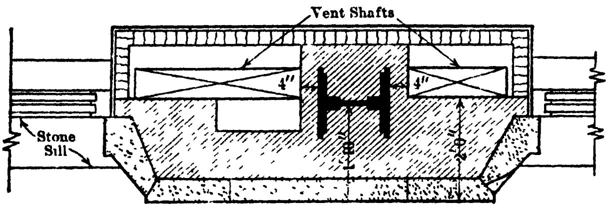


Fig. 31. Plan at Column

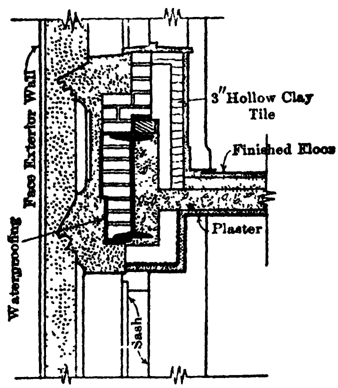


Fig. 32. Spandrel Section

Figs. 31 and 32. Bank of Manhattan, N. Y. C.

covering with a 3- or 4-in approved fire-resistive partition between the column and pipe space.

The spandrel sections shown in Figs. 32 and 33 incidentally indicate methods of overcoming a defect that is more or less prevalent in modern skeleton-frame construction. Because of the gradual reduction in thickness of exterior walls and the use of hollow-tile construction, wind-driven rain and moisture enter the structure through the face brick and mortar joints. The result is the formation of water-pockets which eventually make contact with ceiling and wall plaster. This condition is especially aggravated at all set-backs and where horizontal planes and breaks in the structure serve to collect the water. The most vulnerable point of entry is where spandrel beams occur

in the exterior walls, and provision must be made at such points to provide an impenetrable water-proofing membrane. (See also Fig. 34.) A still more

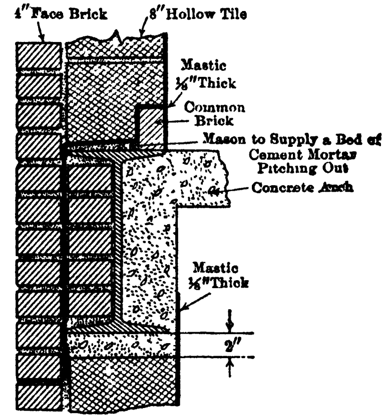


Fig. 33. Method of Spandrel Waterproofing

recent development is the return of metal panels over the spandrel walls and in some cases over the piers between windows for the entire height of the

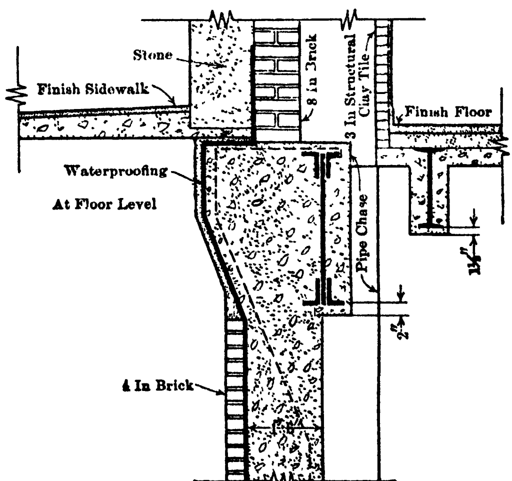


Fig. 34. Section through Outside Wall

building. (See Fig. 35.) The metals now used in place of the former cast-iron veneers are aluminum and non-corroding steel alloys. This type of covering properly installed with expansion diaphragms and joints eliminates

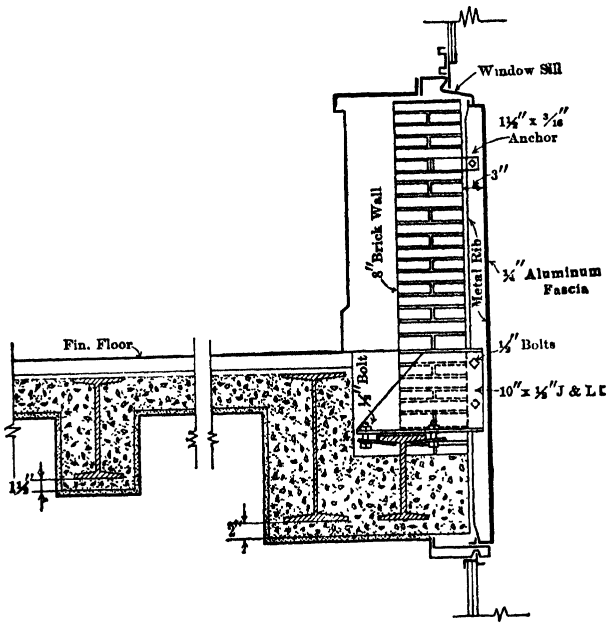


Fig. 35. Section through Spandrel, Empire State Building

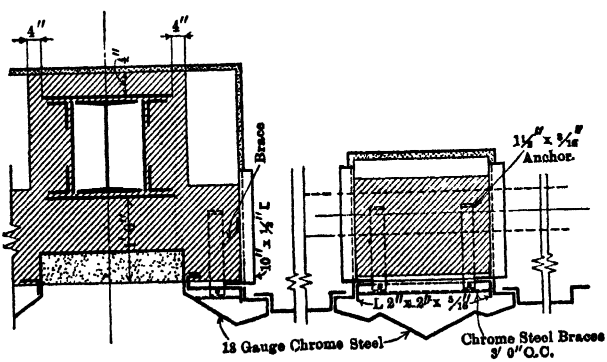


Fig. 36. Pilaster Section, Empire State Building

the necessity for membrane waterproofing around the spandrel beams, and when backed up with 8 in of approved masonry results in an effective treatment both for appearance and efficiency.

Isolation of Column-Covering. To safeguard the column-covering, subdividing room partitions of a temporary nature should never be incorporated in the protective enclosure, but should be separately erected and anchored to the covering by metal ties, hooks or metal bands, cut into the mortar-joints or anchored as required. PERMANENT partition or division walls such as stair or elevator-enclosures may be directly incorporated or interlocked with the column-protection, as removal or disturbance of such assemblies is not likely.

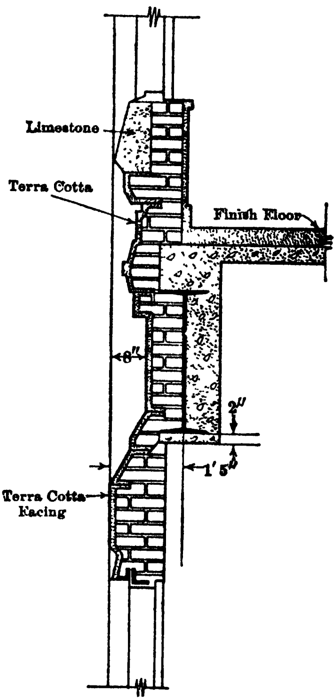


Fig. 37. Section through Spandrel, 40 Wall St., New York City

7. Fire-Resistive Floor-Construction

Fire-Resistive Floors. In the study of fireproofing-materials by far the greatest attention has been given to FLOOR-CONSTRUCTION; and of the very large number of types which have been developed, the characteristic and leading ones are here considered.

Requirements for a Fire-Resistive Floor. It goes without saying that a fire-resistive floor must be made of incombustible materials. It seems unnecessary, also, to mention that it must resist as much as possible the transmission of heat, so as to affo·d thorough protection to the metal incased by it or forming an essential part of it. The materials used should not disintegrate or otherwise fail when exposed to heat or flame. They should also resist the action of water that may be used to extinguish a fire. The floor-construction should be essentially water-tight, so as to prevent damage by water in stories below. It should be designed to safely carry its load at all times. The New York City Building Code describes certain acceptable forms of fire-proof floors, but also provides for the acceptance of other forms which successfully meet the prescribed fire and strength tests. Fully 100 tests have been made under the auspices of the New York City authorities and these, together with a few made by the authorities of other cities, comprise practically all that have been made in this country. The British Fire-Prevention Committee of London has also made a number of such tests.* Of the New York tests, approximately

* For a list of these tests made in the United States and in London, see Proc. A.S.T.M., Vol VI, page 128.

one-half the number have been conducted at the Columbia University Fire-Testing Station, Greenpoint, N. Y., in a test house constructed of 12-in 1 : 2 : 5 cinder-concrete walls. This construction has withstood fully 200 hours of 1 700° fire tests and is still intact.

Fire-Tests for Floors. The AMERICAN STANDARDS FIRE-TEST* is less severe than that required by the New York City Building Code, and the one used by the British Fire-Prevention Committee. Briefly, the New York test consists in subjecting the floor in question, carrying a load of 150 lb per sq ft, to a fire maintained at 1 700° F. for four hours; and then in applying a stream of water, at 60-lb nozzle-pressure, for ten minutes, the floor being considered satisfactory if there has been no appreciable deterioration due to the test and if it has resisted the passage of flames during the test, and will subsequently support four times the design live load.

Types of Floor-Constructions. In considering the several systems of floor-construction, they are for convenience divided into the following types or groups:

(1) Brick arches,
(2) Structural clay-tile floors:
  a. Segmental,
  b. Flat-end construction,
  c. Flat combination construction,
  d. Reinforced-tile arches,
  e. Guastavino,
(3) Concrete floors:
  a. Segmental,
  b. Flat reinforced floors,
  c. Sectional systems,
(4) Gypsum floors,
(5) Steel joist-construction,
(6) Special metal floors.

Brick Floor-Arches. The first attempt at fire-proof floor-construction between wrought-iron beams was made by using BRICK ARCHES sprung between

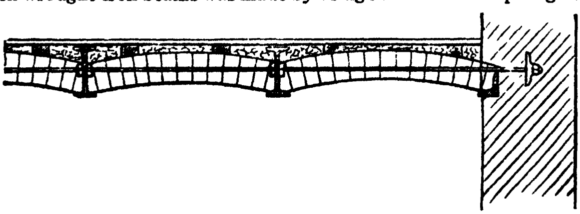


Fig. 38. Brick Floor-arch

the beams and resting on the bottom flanges, as illustrated by Fig. 38. When this form of construction is used the bricks should be hard, well-burned bricks, or hollow bricks of good shape, laid to a line on centers without mortar, with their lower edges touching; and all the joints should be filled in with cement grout. The bricks of one line should break joints with those of the next

* See Year Book, A S.T.M.

adjoining, and in case there is more than one row, the joints of one row should also break joints with those of the row above or below. The arches need not be over 4 in thick for spans between 6 and 8 ft, provided the haunches are filled with a good cement and gravel concrete, put in rather wet. The RISE of the arch should be about one-eighth the span, or $1\frac{1}{2}$ in to the foot; and the most desirable span is between 4 and 6 ft. The building laws of many cities provide that when the spans exceed 5 ft the arches must be increased in thickness, generally to 8 in. The HAUNCHES should be filled with concrete, level with the top of the arch. In first-class fire-proof construction the bottom flanges of the beams should be protected by structural clay-tile fireproofing. as in Fig. 39 which shows the construction used for the floors of the principal stories of the Government Printing Office at Washington, D. C. A 4-in brick arch of 6-ft span, well grouted and leveled off with Portland-cement

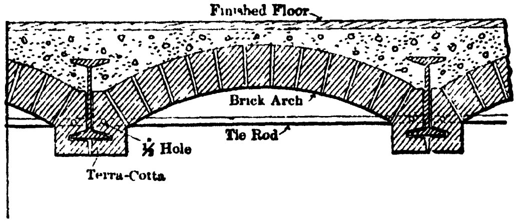


Fig 39 Brick Floor-arch. Government Printing Office, Washington, D. C.

concrete, should safely carry 300 or 400 lb to the square foot. Experiments have shown that brick arches will stand very severe pounding and a great amount of DEFLECTION without failure. The WEIGHT of a floor, such as is shown in Fig. 38, is about 40 lb per sq ft, without the concrete fill or finish. TIE-RODS should always be provided. The brick arch is the strongest type of arch for the span it occupies, with the exception, perhaps, of the stone-concrete arch. It is perhaps, also, the most expensive. Its weight necessitates a heavier framework than is required for other types; and, on account of its appearance, it is adapted only to buildings of the warehouse type, and finds very little or no application in the modern building.

Structural Clay-Tile Floor-Arches. STRUCTURAL CLAY-TILE as a fire resistive material and the standard specifications and strength requirements of the THREE GRADES OF FLOOR-TILE are discussed in Article 5. For floor-construction, the MEDIUM-GRADE tile is the best as a compromise between strength and fire-resistance. For the large number of systems in use, a great variety of shapes and sizes of blocks are manufactured in this country. Representative companies devoted to the manufacture and erection of structural clay-tile fireproofing are the National Fireproofing Corporation, Pittsburgh, Pa.; Henry Maurer & Son, New York; Whitacre-Greer Fireproofing Co., Waynesburg, Ohio; Fraser Brick Co., Dallas, Texas. Any one of the many large companies can make any form of block desired, except such as are covered by letters patent.

Advantages of Clay-Tile Floor-Arches. Many architects prefer the use of STRUCTURAL CLAY-TILE ARCHES in buildings because the setting of them causes less disturbance to the mechanics of other branches of the construction. The work of installing tile arches is generally more rapid than for other types and it is not necessary to wait for them to dry out. The quality of the tile can be

readily judged from its appearance, not only before it is put in place but also after it is set. Thus it does not require the constant supervision necessary for materials that are mixed as they are put in place.

Disadvantages of Tile Floor-Arches. The principal DISADVANTAGE OF TILE ARCHES for floor-construction is the difficulty of adapting any system to the filling of irregular-shaped spaces. The arches must be set between I beams or channels, and to get the best effect the supporting beams must be parallel or nearly so. Tile arches, especially of END-CONSTRUCTION, are weakened more by holes for pipes than are monolithic floors. As there is no bond between the rows of tiles in the END-CONSTRUCTION arch, if a single tile in a row is cut out or omitted, there is nothing to hold up the remaining tiles in the row except the adhesion of the mortar in the side joints. In this respect SIDE-CONSTRUCTION arches have an advantage over the END-CONSTRUCTION. Where it is necessary to use considerable concrete filling over the arch the weight of the floor-construction and its thickness will usually greatly exceed that of the concrete systems, and this additional weight means, also, additional expense. The floor-tiles are liable to breakage and chipped blocks in the floor are not unusual.

Inspection of Floor-Arches. Flat arches of hollow tile require close INSPECTION during erection to see that broken or imperfect tiles are not used; that

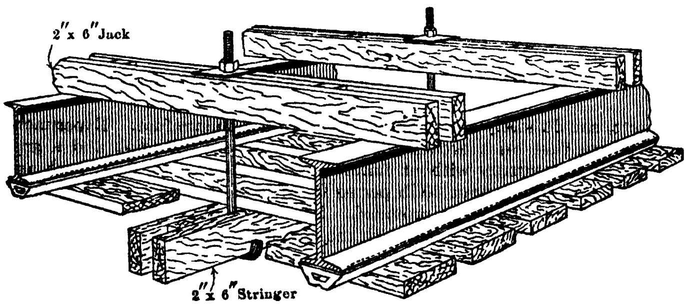
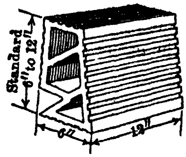


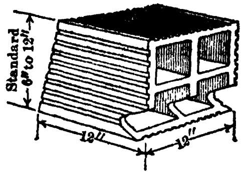
Fig. 40. Wood Form for Flat-arch Construction, Showing Method of Applying Soffits before Skews are Placed in Position

the ribs in END-CONSTRUCTION tiles abut opposite each other; that all joints are properly mortared and that all of the steelwork is properly protected. Much poor workmanship has been allowed to pass in order to avoid delay, and also because it cannot be discovered until the centering is removed. A tile arch generally looks better on the top surface than it does on the bottom.

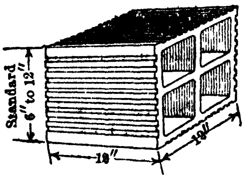
Setting of Tile Floor-Arches. Tile arches are always SET on wooden CENTERS suspended by bolts supported by stringers over the tops of the I beams. For all spans of 5 ft and over, the centers should be slightly CAMBERED. The usual type of floor centering used is shown in Fig. 40. Before any floor-arches are set, all girders projecting below floor-beams should be completely covered on the bottom and sides, independently of the floor-construction. To protect the steel from rust it should have a good coat of Portland-cement mortar before the tiles are set. The various types of floor tile used in FLAT-ARCH construction are shown in Fig. 41. After the centers are in place the soffit-tiles should be placed under the bottom of the



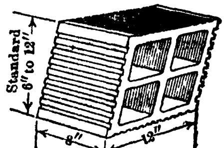
Side-construction Skew
3 cell



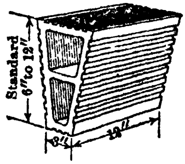
End-construction Skew
4 cell



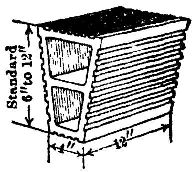
12" Inter
4 cell



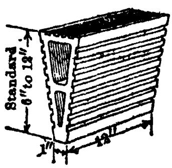
8" Inter
4 cell



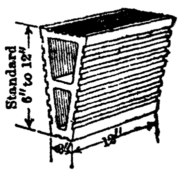
3" Key
2 cell



4" Key
2 cell



1" Key
2 cell



2" Key
2 cell

Fig. 41. Flat-arch Tiles

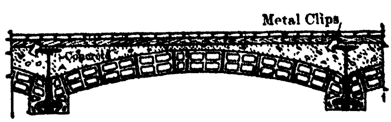


Fig. 42. Typical Segmental-arch Construction

beams and mortar slushed on the sides. These serve to bridge across gaps between the tile under beams in flat-arch construction and under girders generally, for an all tile fireproof covering. The entire sides of the SKEWBACKS

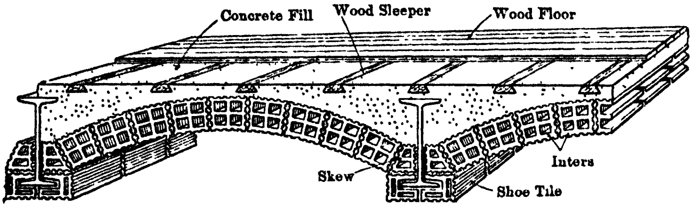


Fig. 43. Segmental Floor-arch, Illustrating Method of Protecting Exposed Portions of Girder

which rest against the floor-beams should then be covered with just enough mortar to give them a perfect bearing, and shoved up against the beams After this, the INTERMEDIATE BLOCKS, with their ribs on one end and one side

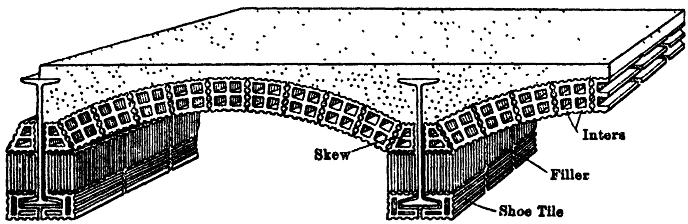
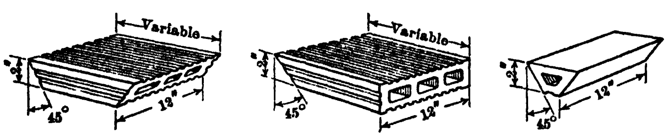


Fig. 44. Segmental Floor-arch, Illustrating Method of Protecting Deep Girders, Concrete Fill and Concrete Floor

covered with a full bed of mortar, should be shoved into place. The KEYS should have mortar on both sides and one end, if SIDE-METHOD KEYS are used, and they should fit snugly, but not tight. Under no conditions should a key be rammed in place. It is better to use a smaller key and fill out the space



2″ End Construction Soffit	2″ End Construction Soffit	2″ Side Construction Soffit
3 cell	3 cell	1 cell
Beveled both ends	Beveled one end	Width variable to suit
Length variable to suit	Length variable to suit	width of beam
width of beam	width of beam	

Fig. 45. Hollow Clay Soffit Tiles

left with either a solid slab of tile, or, if the opening is too small, with a piece of slate. In setting tile arches it is very common to build the arches in STRING-COURSES, first fitting all the skews, then all the intermediates, and finally all the keys. This is bad practice, as it loads the center, both planks and string

ers, to excess, causing too great a deflection. In the END-CONSTRUCTION the arches should be built one by one, each being complete before the next is started. In SIDE-CONSTRUCTION, where joints are broken longitudinally, the arches should be keyed up or completed at the first point where the intermediates meet the lines of the key, thus completing the successive arches as rapidly as possible. All JOINTS in the arches should be filled with mortar, especially at the top.

Wetting the Floor-Tiles. In warm weather all hollow tiles, whether hard or soft, should be well wet or water-soaked before laying. In freezing weather they must be kept dry.

Mortar for Setting Floor-Tiles. Mortar for setting porous hollow tile should never be made of cement and sand alone, as such mortar is too SHORT, rolls off the tile, and does not insure a full joint. A good mortar is made by mixing the cement and sand in the proportion of 1 : 3, and adding cold lime putty or hydrated lime to the extent of 15% of the cement-content. The mortar should be thoroughly worked. Hot lime mortar should never be used. In dry weather the centers can be removed in 36 hours after the tiles are in place, but it is much better to allow 48 hours and even longer in cold or wet weather.

Filling above Tile Floor-Arches. The strength of all tile arches is greatly increased by wetting their top surface and covering it with a rich cinder concrete, mixed with Portland cement, well tamped and brought level with the tops of the steel beams. If the floors are to be finished in wood, NAILING-STRIPS are required to secure the flooring. These nailing-strips are usually dovetail-shape in cross-section, about 2½ in wide at the top, 3½ in at the bottom and from 1¾ to 2 in thick. It is preferable to lay them at right-angles to the steel beams, so that they may be secured to the top flanges by metal clips. Before the nailing-strips are laid, all piping and wiring which must go above or through the tile arches should be put in place. After the nailing-strips are in place the tops of the steel beams should be covered with a thin coat of Portland-cement-and-sand grout, applied with a brush. The spaces between the nailing-strips should be filled with a 1 : 8 or 1 : 10 cinder concrete, finished about ¼ in below the tops of the strips. Some architects claim better results with strips of rectangular section, with nails driven horizontally into the vertical sides to form the grip in the concrete. This method avoids the loosening of the strips and flooring from any shrinkage of the strips.

Cement Floors. If the floors are to be finished with cement, the cement and concrete should be at least 2½ in and preferably 3 in thick above the steel beams, and should be blocked out in sections of not over 6 ft square, with joints extending through the concrete. When practicable the joints in one direction should be over the beams. See Fig. 46.

Weather-Protection. Terra-cotta arches should always be protected against rain or snow, especially in freezing weather, as both the blocks and the mortar in the joints are injured by freezing. Soft floor-tile, especially, may be utterly ruined by freezing when soaked with water.

Segmental Tile Floor-Arches. "This form of arch is the strongest and cheapest. It is particularly adapted to warehouses, lofts, factories, sidewalks, or wherever great strength is required and a flat ceiling is not necessary. When a light, strong arch is required in deep beams and a flat ceiling is also demanded, this result can be obtained by using a metal-lath ceiling suspended below the beams." * These arches are usually formed by either 6- or 8-in hol

* Bevier. National Fire Proofing Company, New York City

low tiles, set on the SIDE-CONSTRUCTION principle and bonded endwise like a brick vault. END-CONSTRUCTION blocks may be used, but they are unsatisfactory, unless the arches are of uniform span and rise throughout. The rise of the SIDE-CONSTRUCTION arch can be varied by increasing the thickness of the upper or lower part of the mortar joint, but this cannot be done with the END-CONSTRUCTION method." Segmental arch construction may be used for spans up to 12 ft and on wider spans where the arch is especially designed in accordance with accepted engineering formulas. The supporting beams in arch floor-construction are tied together with steel tie-rods at least ¾ in in diameter, spaced as required to resist the thrust of arch and not to exceed eight times the depth of the beam or fifteen times the width of the beam flange. Tie rods should be placed as near the point of thrust of the arch as practicable, but should be completely incased within the construction to a depth of at least 2 in unless the tie-rods are otherwise protected with fire-

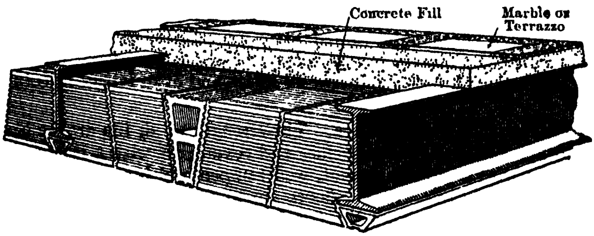


Fig. 46. Section of End-construction Flat Arch with End-construction Skews and with Marble or Terrazo Finish, Illustrating Placing of Units and Methods of Making Concrete Fill

proof covering below the soffit of the arch. All arches should have at least two cellular spaces in depth unless reinforced by steel. The total depth of structural-clay hollow-tile segmental arches should not be less than 6 in, having two cellular spaces in depth, and no segmental arch should have a rise of less than ¾ in per ft of span, and preferably from ⅛ to 1⁄12 the span above the spring line, depending on the live loads. Figs. 43 and 44 show typical forms of SEGMENTAL ARCHES. The WEIGHT of segmental floor-arches is given in the discussion in Article 5. To these weights should be added the weight of concrete filling, flooring, plaster, etc.

Thickness of Webs. For general use the WEBS of segment-tile should be not less than ½ in thick and the SKEWBACK should be at least ¾ in thick. For printing-establishments or any other building where a large amount of vibration occurs the webs of all tiles must be designed in proportionate thickness to the load they are required to carry. These thicknesses apply to Chicago practice more particularly, where a stronger tile is produced than in the East. In New York City webs are generally ⅝ in thick.

Filling the Haunches. The HAUNCHES of SEGMENTAL ARCHES should be filled with good cement concrete, leveled up to a point not less than 1 in above the CROWN of the arch.

Strength of the Segmental Structural Clay-Tile Floor-Arches. The SAFE LOADS per square foot on 6-and 8-in segmental arches, with side-construction, medium tile, webs and shells ⅝ in thick, and with a factor of safety of 7, as obtained from the tables of the National Fire Proofing Company are given in Table XII.

Table XII. Safe Superimposed Loads for Segmental Medium Clay-Tile Floor-Arches

Spans, ft and in	Rise in	6-in arch, lb	8-in arch, lb	Spans, ft and in	Rise, in	6-in arch, lb	8-in arch, lb	Spans, ft and in	Rise, in	6-in arch, lb	8-in arch, lb
	¾	902	1 078		¾	411	491		¾	261	312
	1	1 184	1 414		1	551	658		1	351	419
4	1¼	1 485	1 774	8–6	1¼	678	810	13	1¼	437	522
	1½	1 740	2 079		1½	806	963		1½	519	620
	1¾	1 986	2 373		1¾	926	1 106		1¾	596	712
	2	2 233	2 667		2	1 037	1 239		2	670	801
	¾	792	946		¾	386	461		¾	240	287
	1	1 044	1 247		1	518	619		1	326	390
4–6	1¼	1 313	1 568	9	1¼	645	770	14	1¼	406	485
	1½	1 539	1 838		1½	758	906		1½	482	575
	1¾	1 775	2 121		1¾	871	1 041		1¾	553	661
	2	1 975	2 359		2	977	1 167		2	619	740
	¾	709	847		¾	364	435		¾	225	268
	1	957	1 143		1	489	584		1	302	361
5	1¼	1 172	1 400	9–6	1¼	608	726	15	1¼	377	450
	1½	1 379	1 647		1½	721	862		1½	447	534
	1¾	1 592	1 902		1¾	823	983		1¾	515	616
	2	1 773	2 118		2	923	1 102		2	577	690
	¾	641	766		¾	344	411		¾	209	249
	1	864	1 032		1	462	552		1	281	336
5–6	1¼	1 062	1 269	10	1¼	576	688	16	1¼	353	421
	1½	1 266	1 512		1½	683	816		1½	419	500
	1¾	1 439	1 719		1¾	784	937		1¾	481	575
	2	1 619	1 933		2	879	1 050		2	540	645
	¾	585	699		¾	331	396		¾	194	232
	1	788	941		1	438	523		1	265	316
6	1¼	969	1 157	10–6	1¼	546	652	17	1¼	330	394
	1½	1 154	1 379		1½	648	774		1½	392	468
	1¾	1 315	1 570		1¾	744	889		1¾	452	540
	2	1 476	1 763		2	832	994		2	506	605
	¾	551	658		¾	315	376		¾	182	218
	1	724	864		1	421	503		1	248	296
6–6	1¼	902	1 077	11	1¼	519	621	18	1¼	310	370
	1½	1 058	1 264		1½	617	737		1½	370	442
	1¾	1 218	1 455		1¾	709	847		1¾	425	507
	2	1 358	1 622		2	794	948		2	477	570
	¾	508	606		¾	299	358		¾	173	206
	1	669	799		1	401	480		1	233	279
7	1¼	834	996	11–6	1¼	499	596	19	1¼	293	350
	1½	981	1 171		1½	592	707		1½	348	416
	1¾	1 127	1 346		1¾	680	812		1¾	402	480
	2	1 264	1 510		2	761	909		2	451	539
	¾	471	563		¾	285	341		¾	163	194
	1	621	741		1	383	458		1	221	265
7–6	1¼	774	925	12	1¼	477	569	20	1¼	277	331
	1½	920	1 099		1½	566	676		1½	330	395
	1¾	1 049	1 253		1¾	649	776		1¾	381	455
	2	1 176	1 405		2	727	869		2	427	510
	¾	439	525		¾	273	326		¾	155	183
	1	588	703		1	366	437		1	209	250
8	1¼	724	864	12–6	1¼	456	545	21	1¼	263	315
	1½	859	1 026		1½	541	646		1½	314	375
	1¾	987	1 179		1¾	621	742		1¾	361	432
	2	1 099	1 312		2	696	832		2	406	485

Safe loads for tile with the following sectional areas (per foot of arch parallel with beams): 4-in arch, 28 sq in; 6-in, 36 sq in; 8-in, 43 sq in. Factor of safety, 7.

Rise in inches per foot of span. Example: Rise 1½ for 12-ft span = 18 in.

NOTE. The weight of the arch tile has been deducted in table so that only the dead load of concrete fill, plastering, etc., must be deducted to obtain net live load.

Flat End-Construction Floor Arches. In this construction the sides and voids of the individual tile run at right-angles to the beams, so that the pressure on the webs is endwise of the tile. It has been conclusively demonstrated that hollow tiles are much stronger in END COMPRESSION than SIDE-COMPRESSION. The objection urged against this construction is that it is wasteful of mortar and difficult to get the edges of the blocks properly bedded. They do require slightly more mortar, but the second objection is not serious, for, if the blocks are cut to a proper bevel, the tighter they are set the stronger the arch. The individual tiles in END-CONSTRUCTION are commonly made rectangular in shape, advancing by 1 in from 6 to 15 in in depth.

The total depth of flat-arch construction should not be less than 6 in and never less than $1\frac{1}{2}$ in per foot of span. The length and width, also, of the blocks may be varied, but the standard size is 12 in for both dimensions. The number of partitions or webs in the blocks varies with the size of the tile and also with the strength desired. The 6-in, 7-in, and 8-in blocks usually have two vertical partitions and one horizontal partition, or one vertical and one horizontal, for blocks 8 in wide. The 10-in and 12-in arches may have either one or two horizontal partitions. Floor-tile over 12 in deep should always have at least two horizontal partitions. In the strongest

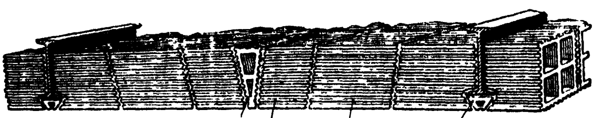


Fig. 47. End-construction Flat Tile Arch

tile the voids are about 3 in square. "The arch-blocks must be set end to end in straight courses from beam to beam, and cannot be set breaking joints, as in the SIDE-CONSTRUCTION method."*

Flat tile arches are now generally made either of end-construction or combination side-and-end construction, as shown in accompanying illustrations, Figs. 47 and 48. End-construction is made with end-construction skews and inters, and side-construction keys. Combination side-and-end construction is made with side-construction skews and keys, and end-construction inters.

Form of Skewback. An end-construction arch may have SKEWBACKS formed of the same blocks, with notches in the ends of the blocks to fit over the bottom flanges of the beams, as in Fig. 46. It is generally considered that the end-construction skewback is much stronger than the side-construction skewback, but on account of the large amount of mortar lost in the voids and the difficulty of obtaining an even bearing with end-construction skewbacks, and, also, because of the greater facility with which the side-construction skewbacks can be used contractors generally prefer to use the latter; and this has given rise to the form of arch shown in Fig. 48. But a more important reason for using side-construction skewbacks with end-construction arches is the better protection against fire that they afford to the beam or girder. To develop the necessary strength, side-construction skewbacks should have a large sectional area and a sufficient number of partitions, following, approximately, the lines of thrust. With any form of skewback the recess for the beam-flange should be of ample width, so that when the tiles are set the protecting flanges on the skewbacks will not touch the bottom of the beams, but

* Bevier

will be at least $\frac{1}{4}$ in below them. Many varieties of side-construction skew-backs are made to meet all possible conditions.

Keys. Both end-construction and side-construction KEYS are used with end-construction arches, the choice of the key depending principally upon its length. If the span of the arch is such that the standard intermediate blocks require a key 6 in or more in width, the END-METHOD KEY is used, but if the space for the key is small, a SIDE-METHOD KEY is used. As the key is almost entirely in compression, a side-construction key 6 in or less in width will usually give all the strength required, provided that the horizontal webs are in the same line with those in the intermediate blocks. Side-construction keys are now generally employed in all flat arches.

Depth, Span, and Weight. A deep tile makes a much stronger floor than a shallower one, and for the same depth of beams a lighter and cheaper floor. A 12-in arch weighs less per square foot than a 10-in arch with 2 in of concrete filling; and it costs less.

The DEPTH OF ARCH most frequently used is 10 in, the girders being spaced to use 10-in I beams for joists spaced from 5 to 6 ft apart. As a rule the depth of the arch should be about equal to the depth of the beam, as it is just about

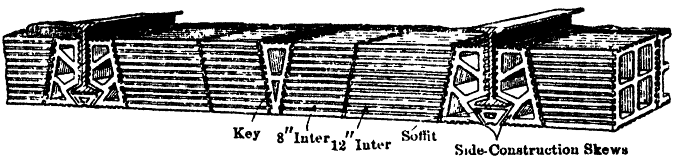


Fig. 48. Combination Flat Tile Arch

as cheap and much better construction to use deeper tiles and less concrete filling

Structural clay-tile flat-arch floors are customarily used for spans up to about 8 ft 6 in. The total depth of flat-arch construction should not be less than 6 in, and never less than $1\frac{1}{2}$ in per foot of span. The weights per square foot of standard flat-arch construction are given in Table V.

Safe Loads for Flat Structural Clay-Tile Arches. The SAFE LOADS for the COMBINATION-CONSTRUCTION which are applicable to all shapes of tile of the net sectional-areas noted are shown in Table XIII. These loads have been figured for the National Fireproofing Company's tile of SIDE-CONSTRUCTION skews and key with END-CONSTRUCTION inters. The strength of arches of different tile are directly proportional to the NET SECTIONAL-AREA of the key tile on a section parallel to the end supports of the arch.

Reinforced-Tile Floor-Arches. In order to obtain a wide-span flat arch or to obtain a reduced depth of arch-block for the shorter spans, the manufacturers of terra-cotta have applied to their floor-construction the principle of REINFORCEMENT WITH METAL, which is the basis of reinforced-concrete construction. Compared with reinforced concrete, even when cinders are used for the aggregate, the greater depth and hollow construction of these REINFORCED-TILE ARCHES secure for them greater strength per square foot for the same weight of construction. On the other hand, they are undoubtedly more expensive than cinder-concrete floor-construction, because of the material used and the increased height of the building due to thicker floors.

Table XIII. Safe Gross Loads per Square Feet (Dead and Live)

Factor of safety of 7

Arches	6 in	7 in	8 in	9 in	10 in	12 in	15 in
Net sectional-areas	27 sq in	27 sq in	27 sq in	27 sq in	36 sq in	36 sq in	36 sq in
Average weight per sq ft	26 lb	30 lb	32 lb	36 lb	40 lb	48 lb	56 lb
Span, ft and in	lb	lb	lb	lb	lb	lb	lb
3 0	420	490	560	630	933	1 120	1 400
3 3	357	417	477	537	795	954	1 193
3 6	308	360	411	462	685	823	1 028
3 9	268	313	358	403	597	716	895
4 0	236	276	315	354	525	630	786
4 3	...	244	279	314	465	558	697
4 6	...	218	249	279	415	497	622
4 9	...	...	223	251	372	447	558
5 0	...	...	201	227	336	402	504
5 3	...	...	182	205	305	365	457
5 6	...	...	...	187	277	333	417
5 9	...	...	...	171	254	305	381
6 0	...	...	...	157	233	280	350
6 3	...	...	...	...	214	258	322
6 6	...	...	...	...	198	238	298
6 9	...	...	...	...	...	221	276
7 0	...	...	...	...	...	206	257
7 6	...	...	...	...	...	178	223
8 0	...	...	...	...	...	157	197
8 6	...	...	...	...	...	...	174
9 0	...	...	...	...	...	...	155
9 6	...	...	...	...	...	...	140
10 0	...	...	...	...	...	...	126

NOTE. Weight of arch, fill and finish must be deducted from above loads for the safe live load. For weight of arches, see Standards, Table V

Natco "New York" Reinforced-Tile Floor-Arch. This arch (Fig. 49) was designed by P. H. Bevier, of the National Fire-Proofing Company, for use where a light and cheap, but strong, sound-proof, fire-proof floor-construction with a flat ceiling is required, and is particularly adapted to wide spans in shallow beams. Where light floor-construction with deep beams is necessary, it can be secured by setting the tile level with the tops of the beams and using a flat metal-lath ceiling, or by omitting the ceiling, a paneled effect is obtained. Where shallow beams are used, the tile are set level on the bottom and 1½ or 2 in below the bottom of the beams, as may be required by local building codes. Light cinder concrete should be used to level up to the top of the beams.

This arch is formed of self-centering units which are set dry, except the skewbacks, permitting installation by unskilled labor. The "lip" tile inters when set tight form channels for the mortar ribs 1 in wide in the direction of the arch, approximately 12 in on centers, and these inters are provided in such lengths as to allow for grouting a mortar key at or near the center of the arch. In order to prevent the mortar from flowing into the cells of the end con-

struction inters, 1-in tile slabs are used to close the cells. The feature of the mortar key is important in that it eliminates the tile key of rigid dimension, thus avoiding much loss of time in laying out an arch to allow proper space for the final insertion of the key.

Reinforcement for the arch consists of $\frac{1}{4}$- or $\frac{3}{8}$-in round rods, placed in the mortar ribs, one rod to each rib, the size of the steel depending upon the span of the arch, and the load to be carried. Finally, the tile being set, the ribs and keys are grouted with a mortar of one part Portland cement to three parts sand The skewbacks should be carefully bedded against the beams with mortar, and clip tile used with raised arches should also be set with

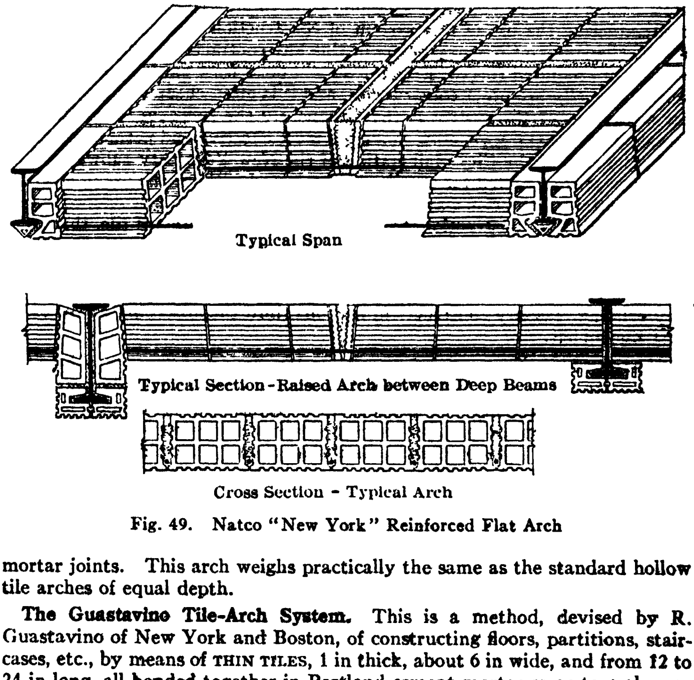


Fig. 49. Natco "New York" Reinforced Flat Arch

mortar joints. This arch weighs practically the same as the standard hollow tile arches of equal depth.

The Guastavino Tile-Arch System. This is a method, devised by R. Guastavino of New York and Boston, of constructing floors, partitions, staircases, etc., by means of THIN TILES, 1 in thick, about 6 in wide, and from 12 to 24 in long, all bonded together in Portland-cement mortar so as to make one solid mass. The floors are built by spanning the spaces between the girders with single arches, vaults, or domes, constructed of two, three, or more thicknesses of 1-in tiles, the number of thicknesses depending upon the dimensions of the arches or vaults. In its best application, steel is used in tension only in tie-members; and in place of steel girders, tile girders are constructed of the same material. Wherever steel is used it is embedded in the masonry construction. One of the earliest notable buildings in which this system was used is the Boston Public Library Building, completed in 1895. Some of the later important constructions are the Cathedral of St. John the Divine, New York City; the Minnesota State Capitol Building, St. Paul, Minn.; the Girard Trust Company's Building, Philadelphia; the Chicago and Northwestern

Railway terminal station, Chicago; the Pennsylvania and the New York Central Railroad terminal stations, New York City; and the Hall of Fame, University of New York, New York City.

An illustration of the wide spans that can be safely used with this system of construction is seen in the Cathedral of St. John the Divine in New York City. The floor above the crypt, measuring 56 by 60 ft, with no interior supports, and designed to carry a safe load of 400 lb per sq ft, was constructed on this principle. Wherever a VAULTED CEILING is desired this form of construction seems to be well adapted for use. Floors built in this way have been tested under the supervision of the New York City Building Department up to 3 700 lb per sq ft, on spans of 10 ft. When used between I beams the only steel beams required are those spanning from column to column. Architects contemplating the use of this system of construction are advised to consult the R. Guastavino Company before letting any contracts. Wherever vaulted ceilings are required this construction should be at least as cheap as any other form of equally fire-proof construction, and it is often cheaper. One particular advantage of the system is that frequently the soffit-course of tile is of PRESSED OR GLAZED MATERIAL, making a most effective and permanent finish, as in the case of the City Hall station of the New York City subway. This station was constructed for very heavy loads and without the use of steel.

Incidentally, attention may be called to the RUMFORD TILE developed in connection with this construction, to be used as the first course of tile, that is, on exposed surfaces on the interior of auditoriums, on account of its sound-absorbing character. Professor Sabine of Harvard University concluded from his investigations that this tile has over sixfold the absorbing power of any existing masonry construction, and one third the absorbing power of the best-known felt.

Concrete Floor Systems. Concrete used in fire-proof floors may be either PLAIN OF REINFORCED. Without reinforcement its use is generally practicable for very short spans only, on account of its weight. In this chapter it is considered only as a FLOOR-FILLING between steel beams in structural-steel frames Chapter XXIII is devoted to a discussion of the principles governing the design and use of reinforced concrete.

Advantages of Reinforced Concrete for Floor-Construction. Although many ADVANTAGES are claimed for reinforced concrete over other systems the principal advantage is that of economy, taking into account the cost of both the steel framework and the filling between. The other important advantages are less weight per square foot of floor (usually but not always the case), adaptability to irregular framing, and rapidity of construction. Except in the immediate locality of the clay tile-factories, fire-resistive floors of concrete can usually be placed at less expense than is incurred in setting floors of hollow tile; and when the spans permit the use of cinder and other light-weight concretes, the concrete floors are lighter than those of the tile, when both floors have the same strength. The combination of concrete with hollow tile of both clay and other materials has resulted in the most economical floor system in respect to WEIGHT and THICKNESS of construction. The materials entering into the construction of reinforced-concrete floors are readily obtained in almost any locality; no specially prepared material is required, except perhaps in a few special forms of construction, and the work can be done almost entirely by unskilled labor. Less capital is required for concrete work than for the tile-constructions, and no material need be carried in stock during an idle period, except tools, mixing-machines, old centering, etc. That the above advantages are real is sufficiently proved by

the immense amount of reinforced concrete now under construction throughout the world. Wherever a floor is to have a finished, cement surface, reinforced-concrete constructions are considerably cheaper than any tile system, because in the former, the entire concrete can be used to give strength, while with the flat-tile arches it merely increases the dead weight.

Disadvantages of Reinforced Concrete for Floor-Construction. One decided DISADVANTAGE connected with concrete floor-construction is the interference in a large measure with the progress of other parts of the work. During its installation, there is a constant dripping from the floor, making it sometimes impossible to continue other lines of work. After the completion of the floors a long time is required, depending upon the weather, for the drying out, before interior finishing can proceed.

Composition of the Concrete. The materials used for concrete are discussed in Article 5 and also in Chapter XXIII. Portland cement, only, should be used in any floor-construction. For most reinforced-concrete floors, having a span between the steel beams of 8 ft or less, CINDER CONCRETE is generally used for the reason that cement mixed with cinders is much lighter than that mixed with broken stone or gravel. The usual PROPORTIONS OF CINDER CONCRETE are one of cement, to two of sand, and five or six of cinders. Concrete mixed with common ashes, a mixture occasionally used for rough fills, has little strength and is totally unreliable. Cinders should be selected to be free from all fine, powdery ash, secured from combustion of coal under forced-draft burning, with all large clinkers broken up to pass the ¾-in mesh screen. The source of the material should be investigated to eliminate possible contamination of cinder by oil or other deleterious substances. For a discussion of CINDER CONCRETE, see Article 5. The WEIGHT OF CINDER CONCRETE will vary from 80 to 110 lb per cu ft, depending upon the coarseness of the material, the quantity of sand, and the amount of tamping. For ordinary purposes a 1 : 2 : 5 cinder concrete should be used, weighing 108 lb per cu ft, or 9 lb per sq ft per inch of thickness. For LONG-SPAN floor-construction, concrete should be made from stone or gravel or aggregates producing concrete of equivalent strength and durability.

Segmental Concrete Floor-Arches. For warehouse-floors the SEGMENTAL ARCH SYSTEMS are adaptable to short spans and heavy loads. For spans between floor-beams of 5 ft or less, a 1 : 6 gravel-concrete arch, 3 in thick at crown and without any reinforcement, should sustain, without cracking, a distributed load of 1 500 lb per sq ft.

The concrete arch, considered as a monolithic construction, if built of stone concrete, is superior to the brick arch. The cinder-concrete arch is inferior only in point of strength. Such an arch should be at least 4 in deep at the crown, and the rise should be not less than one-eighth the span. Cinder concrete should not be used for spans exceeding 8 ft. The strength of such an arch for ordinary cinder concrete is about the same as that of a 6-in segmental-tile arch of the same span, as given in Table XII. All arch systems, whether of concrete or tile, require tie-rods between the beams to take up the thrust of the arches. With the development of modern methods of design and construction of the two-way, long-span reinforced-concrete flat slab, the use of the segmental arch in building frames has become obsolete.

Weight of Segmental Concrete Arches. The weight of solid segmental arches may be found by the following formula, which gives results approximately correct when the rise of the arch is not more one-sixth of the span:

$$W = (w/12)\,(c + 4\,S/p)$$

in which W = weight of arch, in pounds per square foot;
w = weight of material, in pounds per cubic foot;
c = thickness of arch at crown, in inches;
S = span of arch, in feet;
p = ratio of span to rise of arch.

Flat Reinforced Floors. These floors consist of slabs of concrete, varying in thickness according to the span and load, constructed between the steel floor-beams and reinforced near the lower surface with steel in one of the shapes. For LONG SPANS and ordinary loads the thickness of the slab should be at least 3/8 in for each foot of span when reinforced in both directions and constructed of approved concrete of stone, gravel, or burnt clay aggregates, with a minimum thickness of 3½ in. Thinner slabs have been used, but the thickness should be carefully considered for each particular case. For SHORT-SPAN slabs with structural reinforcement in one direction, the thickness will depend upon the character of concrete employed and type of reinforcement, but for a FIRE-RESISTIVE building should never be less than 3½ in. The floor-slabs are not usually of the same depth as the beams supporting them. The position of the slabs, therefore, determines the character of the ceiling. When the bottom of the slabs is placed at or below the lower flanges, a flat ceiling results, and the space over the slabs must be filled to the under side of the flooring with some incombustible material, thus often increasing the weight. When the slabs are set at the top flanges, a paneled ceiling results, unless a hung ceiling is provided.

Strength of Short-Span Flat Floor-Construction. The following empirical formula, representing the practice established by the New York Building Code, is based on an investigation of cinder-concrete floor-construction made under the joint auspices of Columbia University and the Bureau of Buildings, Manhattan, New York.* This formula is in use in New York City for all short-span concrete floor-fillings between steel beams. (See also Figs. 50 and 51.)

$$w = Kda/S^2$$

in which w = safe load, in pounds per square foot, including the weight of slab;
d = distance, in inches, from top of slab to center of reinforcement;
a = cross-sectional area, in square inches, of the reinforcement, for each foot of width of slab;
S = span, in feet, of slab;
K = a coefficient with values as follows: when cinder concrete is used, 26 000 if the reinforcement consists of steel fabric continuous over supports; 18 000 if the reinforcement consists of steel rods or other shapes securely hooked over or attached to the supports; and 14 000 if the reinforcement is not continuous over the supports; and when stone or gravel concrete is used, 30 000, 20 000, and 16 000, respectively, for the corresponding conditions.

The material contemplated by this formula is a concrete consisting of one part of Portland cement, and not more than two parts of sand, and five parts of stone, gravel, or cinders with a compressive strength of not less than 800 lb per sq in. The reinforcement consists either of steel rods or other suitable shapes, or steel fabric. Hot-rolled bar reinforcement should be of structural

* Trans. Am. Soc. C. E., Vol. LXXIX, 1915, page 523.

or intermediate grades,* with an ultimate strength of 60 000 lb per sq in. Cold-drawn steel fabric should have an ultimate strength of not less than 80 000 lb per sq in. In case cold drawn steel fabric is used, the tensional reinforcement should not be less than $^{12}/_{100}\%$, and in case other forms of reinforcement are used, not less than $^{25}/_{100}\%$, the percentage being based on the sectional area of the slab above the center of the reinforcement. For proper protection against fire and corrosion the center of the reinforcement should be at least 1 in above the bottom of the slab, but there should always be at least ¾ in of concrete outside of any part of the reinforcement. The formula should not be applied to spans exceeding 8 ft. Cinder-concrete floors should be limited to that span in any case. For reinforcement of less ultimate strength than noted above, the coefficient K should be decreased proportionately. The cold-drawn fabrics being delivered into the market at the present time develop safe loads 20% less than the above recommended values of w.

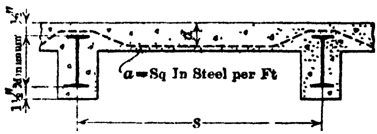


Fig. 50. Short-span Concrete Top Arch

For SHORT-SPAN FLAT ARCHES, any of the types of reinforcement described in Article 9 may be used. For FIRE-RESISTIVE CONSTRUCTION, the steel bars or fabric must be protected with at least ¾ in of concrete at the bottom of the slab; and where continuous over the top flanges of the supporting beams, the reinforcement should be incased in at least ½ in of protection. See Fig. 50. When the slab is installed on the bottom flanges of the structural beams, Fig. 51, special care must be taken to insure the correct placement of continuous mesh or fabric to eliminate excess length and possibilities of slack in the fabric. In addition, in such cases, a straight piece of reinforcing mesh should be installed under each floor-beam extending for a distance of 1 to 2 ft on each side of the flange, to avoid cracking of the arch from vibration or impact loads. When sheet fabric is installed under the safe-load requirements for continuous mesh, it must be lapped at the ends sufficiently to develop the steel area by bond with the concrete. In placing expanded metal in the concrete, it is necessary to lap the sheets on the ends up to and including 3–9–20, one diamond (8 in); from 3–9–25 to 3–6–60, one and a half diamonds (12 in); and heavier than 3–6–60, two diamonds (16 in).

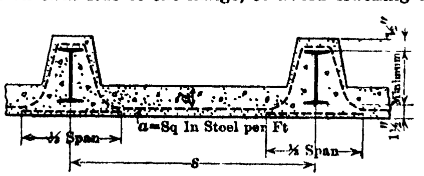


Fig. 51. Short-Span Concrete Bottom Arch

Aerocrete Arches. "AEROCRETE" concrete has been approved for use in SHORT-SPAN floor-arches in the same manner as CINDER OR STONE CONCRETE. For a discussion of the properties of this processed concrete see Article 5. Welded wire cloth is used for the reinforcement in arches of a minimum thickness of 4 in; and the beams and girders are solidly encased with the same material, as shown in Fig. 52. The weight of the mixture can be controlled within a range of 30 to 80 lb per cu ft. The compressive strength is directly proportional to its weight; that of the LIGHT-WEIGHT grade being 250 lb per

* A.S.T.M. Spec. A15–30.

sq in; the MEDIUM grade, 650 lb per sq in; and the HEAVY STRUCTURAL grade 1 300 to 1 500 lb per sq in.

A LONG-SPAN form of construction, consisting of light-weight structural beams or open-web bar-joists spaced from 3 to 4 ft on centers rigidly connected by rivets or welding to the main floor girders, with a uniform thickness of fill of light-weight "AEROCRETE," has been approved for use in New York City. See Fig. 53. The test construction consisting of 6-in standard I beam on a 24 ft 0 in span filled solidly with 7½ in. of "Aerocrete" supported a load of 300 lb per sq ft with a deflection of 1/360 of the span. The weight of

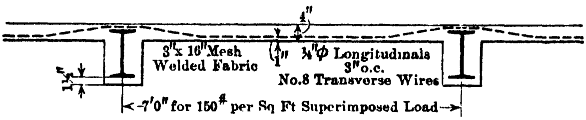
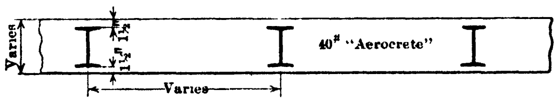


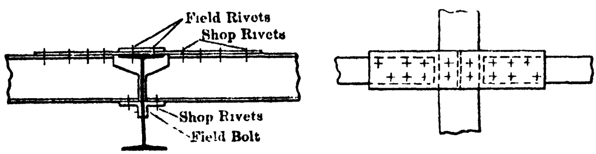
Fig. 52. "Aerocrete" Short Span Arch Construction

the floor including steel and fireproofing of girders is only 30 lb per sq ft. The particular advantage of this type of construction, besides its light weight, is the speed of erection. Forms can be removed in from one to two days The cast-in-place construction is controlled by the Aerocrete Corporation of America, New York, who also furnish a precast shop-made unit for installation between steel or reinforced-concrete floor-beams.

Gypsum Floors. Gypsum has been extensively used for floors and roofs in fire-resistive buildings because of its excellent fire-resistive properties * Under a 1 700° F. fire, the temperature on the unexposed side of a floor or



Section of Floor



Details of Beam Connections

Fig. 53. "Aerocrete" Long Span Construction

partition of gypsum will remain below 212° F. for a considerable period of time, while the water of crystallization is being evaporated from the gypsum. The duration of this period depends on the thickness and quantity of material involved, and which must all be calcined before the transmitted temperature can exceed the temperature of boiling water. It furnishes a light construction, which, with the additional advantage of the rapidity with which it can be put in place, is economical not only with respect to the floor itself but also on account of a saving in the amount of the structural steel supporting it. Another favorable feature is the great heat-insulating property of gypsum,

* See Discussion of Gypsum. Article 5.

resulting in absence of condensation and a reduction in the cost of heating the building. Its disadvantage lies in the variation of physical properties of gypsum from different sources and the necessity for special methods of design and installation because of its comparatively low strength properties.

The Suspension System. This construction, originally known as the METROPOLITAN SYSTEM, consists of a series of steel cables suspended from the supporting steel beams and encased in a slab of pure calcined gypsum containing about 12½% of wood chips. The cables are generally composed of two No. 12 galvanized steel wires, twisted. They are made continuous over the supports, being securely fastened over the flanges of the end-beams or channels by heavy S hooks or other suitable means. The cables are spaced from 1 to 3 in apart, depending on the carrying capacity desired. They are held taut by a ⅝-in round steel rod, laid at the middle of the span at right-angles to their direction. The mixture of gypsum and chips is sent to the work in bags and placed on wooden centers, as in the case of concrete floors, wet, and allowed to set. The sides and flanges of the supporting steel beams are encased in the same material, all as shown in Fig 54. The minimum

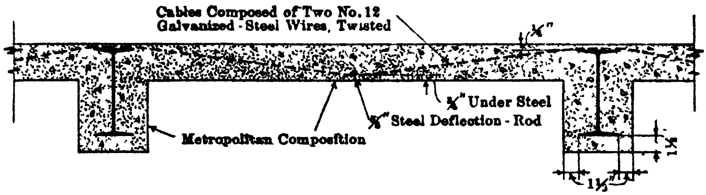


Fig. 54. Metropolitan Fire-resistive Floor

thickness of floor-slabs is 4 in; the usual thicknesses for roofs are 3 and 3½ in. The finished slabs weigh about 4 lb per sq ft per in in thickness. Spans of from 6 ft 6 in to 7 ft are said to make the most economical arrangement, all things considered. Spans should not exceed 10 ft. This system is designed to secure all the advantages of the fire-resisting values of gypsum with as little dependence on its structural value as possible. The safe gross strength of the construction in practice is determined by the formula

$$w = \frac{24Td}{bL\sqrt{9L^2 + d^2}},$$

in which w = the safe gross load per sq ft of floor or roof-surface, in pounds;
T = the safe tensile strength of the twisted cables in pounds, which for the ordinary case of two No. 12 cold-drawn steel wires, may be taken at 365 lb;
d = the deflection of the cable, in inches, and equals the slab-thickness less the sum of the protection of the cables at the center of the slab and over the supports, that is, ordinarily, the slab-thickness less 1 in;
b = the spacing of the cables, in inches;
L = the span, or distance, between centers of supports of the slab, in feet.

The above formula is in error in that it does not consider the inclination of the REINFORCEMENT. It would be more correct if two deflection-rods were provided at the third or fourth points of the span. The stress in the cable as

installed should be increased by a component of the reaction of the gypsum fill on the deflection rod at the center. With one deflection-rod, the maximum stress under working loads occurs at the points of support where the reinforcement circles the structural beams, and the correct formula for SAFE LOAD should read

$$w = \frac{24Td}{bL\sqrt{36L^2 + d^2}}.$$

By this latter formula, the actual safe load is considerably reduced. No tie-rods are necessary in this floor-construction; but in the end-bays, when the lateral stiffness of the beams together with the compressive strength of the floor-slab is not sufficient, struts must be provided of such size and spacing as are necessary to resist the pull of the cables. This system is installed by all the large companies which install gypsum floor and roof constructions. It has been approved for a full 4-hour fire-resistive rating.

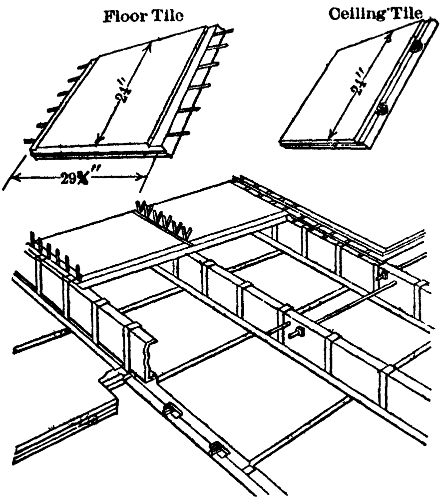


Fig 55. "Gypsteel" Pre-cast Floor-construction, Showing Floor- and Ceiling-slabs and Hangers

Gypsteel Floor-Construction. The "GYPSTEEL" FLOOR and CEILING-CONSTRUCTION consists of a 2½-in precast gypsum floor-slab molded at the factory and reinforced with 3/16 in cold-drawn wire rods spaced 4 in on centers, projecting about 2½ in at both ends of the tile. The floor-beams or steel joists are erected in the structural-steel frame at intervals of 2 ft 6 in, the slabs are set in the top flanges, and the projecting rods are drawn up taut and twisted together at their connections The ends of the slabs are rabbeted, and these depressions are filled solidly with gypsum grout and leveled off to form an unbroken surface with the tops of the slabs. Floor slabs are 24 in wide by 29¾ in long. Before the structural floor-slabs are installed, precast gypsum ceiling slabs, 2 in thick, reinforced with two flat steel bars, ¼ in by ⅜ in, projecting 1 in from the ends of the slabs, are suspended from the floor-beams or joists by inserting the projecting reinforcement in slots of ⅛-in by 1-in hangers clamped around the top flanges of the floor-beams. Beveled edges on the top surface of the ceiling-slabs form a V-shaped joint ½ in wide which is filled with gypsum grout before the floor-slabs are set. The steel girders supporting the secondary floor-beams are fireproofed by precast soffit slabs clamped to the lower flanges, with the open spaces between soffit pieces and ceiling-slabs filled with plastic gypsum to provide a minimum protection of 1½ in outside of the edges of the steel flanges; or the sides of

the steel girders can be enclosed with gypsum blocks resting on the soffit slabs. The SAFE LOADS are based on the SUSPENSION design, the gypsum serving merely as a fire-resistive filler.

"GYPSTEEL" CONSTRUCTION is controlled by the Structural Gypsum Corporation of Linden, N. J. The system with 24-in span slabs, in place of the 30-in now commonly installed, plastered with ¾-in gypsum plaster tested by the Columbia University Testing Laboratories for the City of New York, developed a four-hour fire-resistive rating.

Clip-On Fire-Resistive Construction. The CLIP-ON SYSTEM, installed by the American Clip-On Corporation of Philadelphia, Pa., possesses a distinct advantage in that the fire-resistive ceiling can be placed under any existing

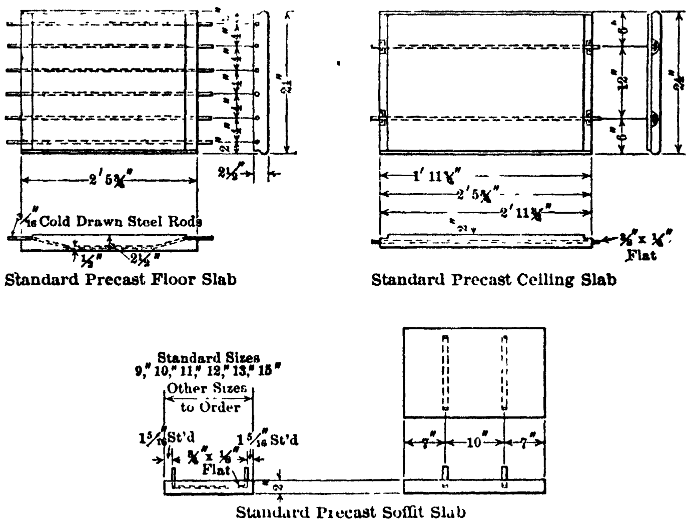


Fig. 56. "Gypsteel and Ceiling-slabs

floor-construction, or after the top structural platform has been installed; or in the event of fire damage requiring the removal of a part of the ceiling, the injured portion can be replaced without interference with the structural parts of the floor. The system proper consists of a precast ceiling-block or tile of gypsum-fiber concrete, 18 in wide by 36 in long, tongue-and-grooved on the long sides, reinforced with two longitudinal No. 5 gauge wires and two transverse yokes to which S hooks are attached at both ends of the block, as shown in Fig. 57. The under surface of the blocks is corrugated to insure plaster bond. The blocks are suspended from 1-in furring channels spaced 3 ft on centers and secured to the lower flanges of the steel floor-beams or steel joists in the usual manner. The hooks can be readily manipulated and the blocks hung in place from the floor below. U-shaped sockets left in the block to accommodate the S hooks are filled with gypsum plaster. The ceiling surface is then finished with brown coat and hard finish, making the total thickness of fireproofing 2 or 2½ in.

These blocks can be secured in thicknesses of 1½ and 2 in. In the Strawbridge and Clothier 14-story building erected in Philadelphia, Pa., in 1930, the 1½-in block, plastered to a total thickness of 2 in, was used as a ceiling throughout the structure under a top slab of 2½ in of reinforced concrete on 4-ft spans. During construction, a fire occurred in the temporary wood partitions and false-work which communicated through the entire 14 stories and burned for over one hour. The "Clip-On Ceiling" which had already been installed and plastered was not damaged beyond smoke discoloration of the ceiling surfaces.* The "Clip-On" system of plastering includes the

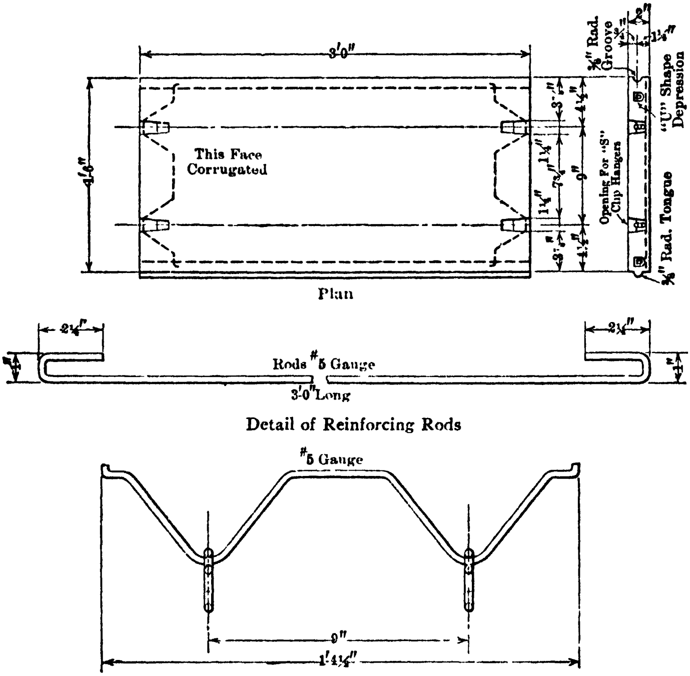


Fig. 57. Detail of "Clip-on" Block

use of special dry-mixed sanded-gypsum bond-coats and lime and molding-plaster finish-coats, and is installed only under the supervision of the company. In the fire-test for the City of New York, the plastering remained practically intact during the entire four-hour test, whereas the usual plastering is destroyed or falls off after ½ hour to 1 hour of fire. The temperature of transmission through the ceiling was uniform throughout the hollow space surrounding the steel floor beams and never exceeded 242° F. on the bottom flanges of the beams during the four-hour test period. The 2-in "Clip-On" block ceiling, plastered to 2½ in. developed a fire-resistive rating of four

* See Philadelphia Evening Public Ledger of July 17, 1930.

hours. From analysis of the test details and results, the 1½-in block plastered to a total thickness of 2 in would develop at least a three-hour rating.

Pyrofill Monolithic Floor-Construction. The design of the "Pyrofill" floor-construction controlled by the U. S. Gypsum Co., Chicago, Ill., is based on the "Suspension" system and is installed either as a SOLID slab with suspension cables anchored to the top flanges of the floor-beams with the bottom flanges of the beams solidly encased for fireproofing similar to Fig. 54, or as a COMPOSITE top slab and separate ceiling-slab. This construction differs from the "Gypsteel" or "Clip-On" systems in that it is poured-in-place. It suffers from the same disadvantage of the "Gypsteel" floor, in that the ceiling-slab must be installed before the top floor-slab. The "Pyrofill" mixture is a GYPSUM-FIBERED CONCRETE consisting of 12½ parts of wood chips to 87½ parts of gypsum, mixed with water varying from 60 to 70 per cent of the combined weights of gypsum and wood chips, depending upon the consistency of the gypsum and the porosity of the chips.

The ceiling, which is poured first, consists of a 2½-in thick slab reinforced with galvanized wire fabric of No. 12 gauge wires, 2-in by 4-in mesh, sup-

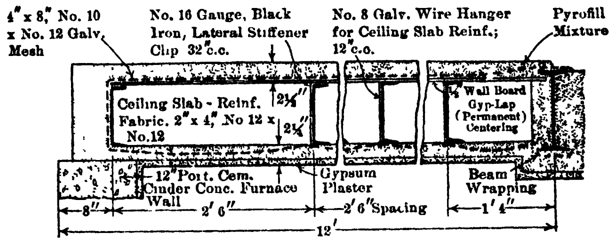


Fig. 58. Gypsum "Pyrofill" Floor-construction

ported on ⅝-in round rods suspended from the floor-beams with No. 8 gauge wires. The mesh is installed on a wood form so as to provide a sag of 1⅛ in on the 30-in span. The floor-slab is constructed on a permanent centering of ½-in gypsum wall-board and the "Pyrofill" slab poured to a total thickness of 2½ in, reinforced with galvanized welded wire fabric, 4-in by 8-in mesh, No. 10 longitudinals and No. 12 transverse wires, laid directly on the plaster board. In practice, the platform slab is made 2½, 3, or 3 in thick, depending upon the desired load-bearing capacity. Fig. 58 shows the construction as installed in the test house of Columbia University. In the test construction, the ceiling-slab was plastered with three-coat gypsum plaster to a thickness of ¾ in, making a total ceiling thickness of 3 in. The construction developed a four-hour fire-resistive rating.

Application to Steel-Joist Floors. In common with all GYPSUM CONSTRUCTIONS, the application of water through a 1⅛-in nozzle hose-stream, as required by the STANDARD FIRE-TEST, entirely demolishes a ceiling construction 2 or 2½ in thick, whether precast or poured-in-place. A reinforced slab, 3 in or more in thickness, when exposed to the fire-stream is generally eroded to the depth of the calcined material exposing the reinforcement. The latter can be replastered with gypsum to practically restore the original fire-resistive qualities; but unless a suspended fire-resistive ceiling is designed so that it can be rehung from underneath, it is not practicable to replace it with ordinary methods of construction.

As described under Steel Joist and Junior Beam Floors, precast or poured-in-place gypsum slabs of the types here described are extensively used with joist-spacings of 16 to 30 in, where comparatively light live loads are required, as in HOTELS and APARTMENT-HOUSES.

Open-Web Steel-Joist Floors. Steel-joist floors are designed to meet the requirements of the INTERMEDIATE CLASS of fire-resistive construction, which is both economical and safe for buildings of lighter occupancy and heights not exceeding 80 to 90 ft. The type of floor-construction generally used with OPEN-WEB STEEL-JOIST or LIGHT-WEIGHT STRUCTURAL BEAMS * consists of a top plate of incombustible material designed as a structural platform to support the loads and a bottom plate or ceiling-construction to provide a protective ceiling of fire-resistive construction. These light-weight floor-constructions are of various types and afford different fire-resistive

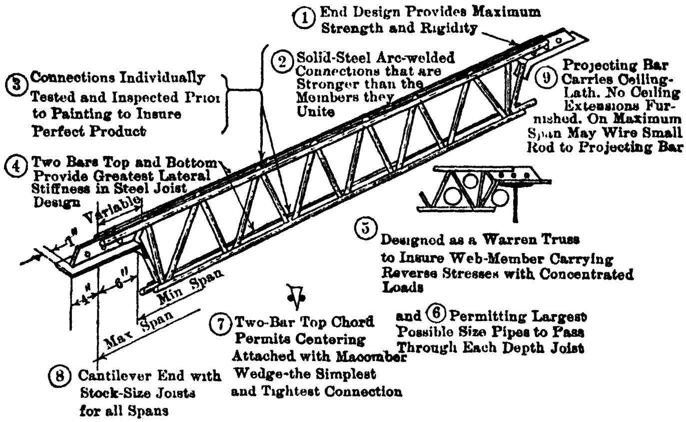


Fig. 59. Welded Bar Joist

ratings, dependent upon the character and construction of the details. STRUCTURAL PLATFORM SLABS are described later in Article 7, and FIRE-RESISTIVE CEILINGS in Article 8.

The STEEL JOIST in its present form is the result of progressive steps starting with the steel channel forms of METAL LUMBER pressed from blue annealed sheets which were either cold-riveted back to back or spot-welded. This form was followed by a 5-piece section-joist of typical PLATE-GIRDER design formed of a web-member of strip steel and flanges of four light-weight structural angles, spot-welded in place.

The modern forms of STEEL JOIST are made in numerous sections by various manufacturers but can be classified in three general types: (*a*) WELDED BAR (see Fig. 59), formed by welding bars into the form of trusses; with top and bottom chords consisting each of two round bars and web-members of smaller round rods bent to suit and welded between the chord bars at junctions. The bottom chords are curved upward at the ends, and seats for bearings consist of pressed-steel angles welded to the bars. (*b*) WELDED STRUCTURAL, with top and bottom chords of wide special tee or other structural-shaped mem-

* See Junior Beams in Article 7.

bers continuous to and including the seats or bearings, and the web-members consisting of round rods bent to proper shape and welded between the chords. The top flange is straight and the bottom flange generally curved upward at the ends to the supports. At the supports the flange-members are connected by a web-plate forming a solid beam section. High-pressure electric welding is used at all joints. (See Fig. 60.) (*c*) EXPANDED JOIST, made by slitting, heating and expanding light beam sections to form one-piece trusses, including inclined web-members and having a depth considerably greater than the original beam. Both top and bottom chords may be straight from end to end with the bottom chord shortened to set between supports, and a plate welded to the top chord for the bearing, or the original section of the unexpanded beam may be left at both ends. (See Fig. 61.) At the supports all joists are generally $2\frac{1}{2}$ in deep, regardless of the depth at the center. By supporting these under-slung joists $2\frac{1}{2}$ in below the top chord, they are held in proper position by gravity, and erection problems are greatly

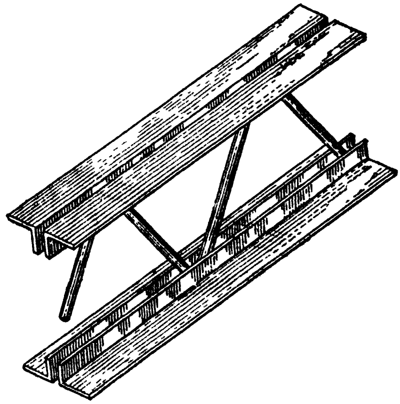


Fig. 60. Welded Structural Joist

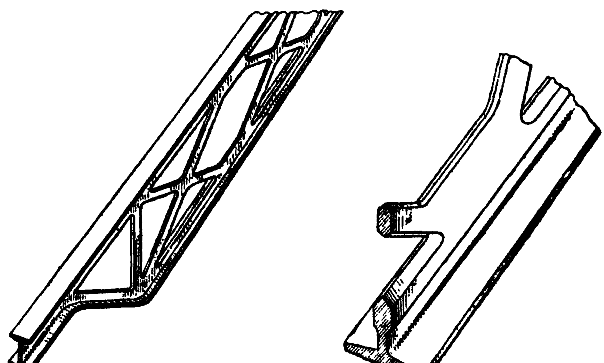


Fig. 61. Expanded Joist

simplified. "EXTENSION BOTTOM" chords are provided for use at the ends of joists where bottom chords are bent up to accommodate the flat ceiling. Bridging of the joists is accomplished by the use of 2-in by 1-in channels U bolted to the top flanges of the joists supplemented by heavy wire bridging, to stay the bottom chords, or by various other types of diagonal strut channel, rod or wire bridging. Many other special forms of STEEL JOISTS are

manufactured of which the following examples indicate some of the special devices employed. The NAILER STEEL JOIST manufactured by the Truscon Steel Co. of Youngstown, Ohio, provides nailing strips in the top flange (see Fig. 62) clamped or secured in position for attaching wood flooring or wood roof-decks. The NAILER JOIST incorporates the same basic features of the typical STEEL JOISTS previously described, allowing for the passage of pipes and conduits in any direction through the floor-construction without

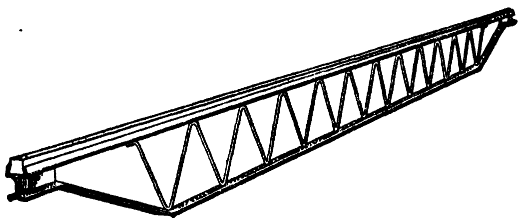


Fig. 62. Nailer Joist

requiring the cutting of joists or the installation of a special suspended ceiling. With a metal lath-plastered ceiling, and double wood flooring with insulating felt between, this construction will provide a fire-resistance of at least ½ hour and fire safety to a substantial degree. The "RIVET GRIP" steel floor-joist is designed for use with an interlocking ribbed steel decking, by the Rivet-grip Steel Co. of Cleveland, Ohio. (See Fig. 63.) The upper chord consists of an inverted channel section to which the steel decking is inter-

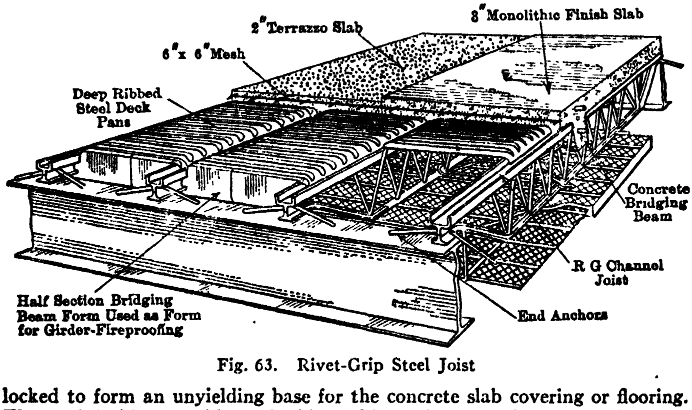


Fig. 63. Rivet-Grip Steel Joist

locked to form an unyielding base for the concrete slab covering or flooring. The steel decking provides a rigid lateral brace between the top chords of the joist.

LOAD-BEARING CAPACITIES.* In cooperation with the Department of Commerce a Simplified Practice Recommendation was promulgated and became effective on October 1, 1930, and was adopted by all manufacturers of open-web steel joists for safe loads of spans not exceeding 24 times the depth of the member. (See Table XIV.) In order to develop these safe

* S.P.R.94-30.

Table XIV. Properties and Allowable Total Loads in Pounds per Linear Foot of Open-Web Steel Joists

American Institute of Steel Construction Stresses

Steel joist institute designation	SJ81	SJ82	SJ102	SJ103	SJ104	SJ123	SJ124	SJ125	SJ126	SJ145	SJ146	SJ147	SJ166	SJ167
Depth in inches.	8	8	10	10	10	12	12	12	12	14	14	14	16	16
Resisting moment in in-lb..........	29 500	52 500	63 000	82 000	100 000	92 000	115 000	142 000	175 000	156 000	205 000	246 000	232 000	281 000
Maximum end reaction in lb.........	1 600	1 900	1 900	1 950	2 200	2 200	2 300	2 500	2 700	2 900	3 100	3 400	3 200	3 600
Span 4 Feet	800													
5	640													
6	530													
7	402													
8	308													
9	243													
10	197	350												
11	162	289												
12	137	243	292											
13	116	207	248											
14	100	178	214											
15	87	155	187	243		272								
16	77	137	164	213	260	240								
17			145	189	230	212								
18			130	169	205	189	236							
19			116	151	184	170	212			288				
20			105	137	167	153	192	237		260				
21						139	174	215		236				
22						127	158	196	241	215				
23						116	145	179	221	197	258			
24						106	133	164	202	180	237			
25										166	218	262	247	
26										154	202	243	229	
27										143	187	225	212	257
28										133	174	209	197	239
29													184	223
30													172	208
31													161	195
32													151	183

loads with the same factor of safety customary in standard structural-steel framing, the joists must be rigidly supported in a lateral direction either by one of the methods mentioned above or by WELDED-STEEL horizontal top and bottom braces and diagonal members. With present practices, it can be stated that generally the open-type joist, owing to lack of stiffness and continuity of web-construction, does not equal the computed equivalent rolled-beam section.

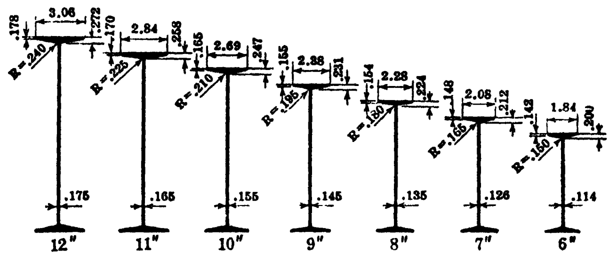
Advantages of Open-Web Joist Floors. For residential occupancies this type of construction results in a light-weight structure with a dead load of construction of not more than 35 to 40 lb per sq ft, and in GENERAL ECONOMY of structural frame, bearing walls and foundations. It insures RAPID CONSTRUCTION without any special equipment and can be installed at all seasons of the year with ordinary labor and in any kind of weather. As compared to WOOD-JOIST construction, the additional cost is small, and the steel joist eliminates one cause of settlement, namely, SHRINKAGE of beams, with less tendency toward separation of baseboard and floor and PLASTER CRACKING. The UNDERSLUNG DESIGN of the bearings permits of maximum head-room under supporting girders, and the open webs allow the installation of pipes and conduits without INCREASED THICKNESS of floor-construction. The addition of a fire-resistive ceiling provides FIRE SAFETY to a substantial degree. With the ceiling-construction shown in Table XVI, a 1½-hour resistance can be secured.

Disadvantages of Open-Joist Floors. Destruction of the ceiling at any one place destroys the FIRE-RESISTANCE of a large part of the construction. The joists do not provide a rigid attachment to supporting girders and are of little value in resisting lateral pressures due to wind. For these reasons, their application is limited to buildings of medium height, and occupancies not involving severe fire hazard or heavy loads.

Junior Beams. Light-weight structural-steel sections rolled from the billet to a full I section in a continuous mill from structural-grade copper-bearing steel are manufactured by the Jones and Laughlin Steel Corporation of Pittsburgh, Pa., for use as SECONDARY MEMBERS in structural-steel floor-framing. Tests at Columbia University, Massachusetts Institute of Technology, and the University of Illinois demonstrated that this product develops the same safe stresses as those used in the design of the STANDARD-WEIGHT STRUCTURAL-STEEL section, and approvals have been granted accordingly throughout the United States. The shape and dimensions of these beams are shown in Fig. 64.

JUNIOR BEAM end-support is provided through riveted STANDARD END-CLIP ANGLES or through STANDARD EXTENSION ANGLES, as shown in Fig. 65. The beam can be provided with a 45° end-cut to facilitate piping and conduit installation. The beams are braced laterally either by continuous diagonal wire bridging of No. 12½ galvanized steel wire, by tie-rods or by welded horizontal braces and diagonal ties. JUNIOR BEAMS can be used in structural design with any of the types of fire-proof floor-construction, employing standard engineering formulas, and should be included in Specifications under STRUCTURAL STEEL.

Structural Platform Slabs. Floor-systems for light-weight secondary structural members adaptable to use with JUNIOR BEAMS or with STEEL JOIST can be classified in three groups: (*a*) CONCRETE SLABS ON PERMANENT CENTERING; (*b*) CONCRETE SLABS ON REMOVABLE FORMS, and (*c*) PRECAST or POURED-IN-PLACE GYPSUM SLABS. In addition, the junior beam can be



Depth, in	Weight, lb per ft	Flange width, in	Web thickness, in	Area, sq in	Radius of gyration		Moment of inertia		Section-modulus	
					Hor. axis	Vert. axis	Hor. axis	Vert. axis	Hor. axis	Vert. axis
12	11.74	3.06	0.175	3.45	4.573	0.532	72.21	0.978	12.01	0.638
11	10.23	2.84	.165	3.01	4.200	.498	53.08	.746	9.63	.525
10	8.96	2.69	.155	2.64	3.847	.480	39.01	.608	7.78	.452
9	7.48	2.38	.145	2.20	3.450	.423	26.20	.394	5.81	.332
8	6.54	2.28	.135	1.92	3.116	.423	18.67	.343	4.65	.301
7	5.48	2.08	.126	1.61	2.744	.393	12.13	.248	3.45	.239
6	4.41	1.84	.114	1.30	2.372	.357	7.30	.165	2.42	.179

Fig. 64. Sizes, weights and properties of Junior beams

utilized with standard types of reinforced slabs or clay-tile floor-arches, including solid encasement of the beams with protective materials. (See Fig. 66.)

STRUCTURAL PLATFORM SLABS of GYPSUM are described under GYPSUM FLOOR-CONSTRUCTIONS.* For CONCRETE platform slabs of TYPE A, Fig. 66, Expanded Metal, Sheet Metal or Composite Laths of the weights given in Table XV are recommended. The lath is applied with sheets running at right-angles to the structural beams or joists. The sheets must be secured, lapped and wired together at sides and ends and clipped to the joists. When flooring other than wood is used, temperature rods should be installed in both directions at least 24 in on centers.

OPEN-JOIST floor-construction has been approved for use in fire-resistive construction in many of the large cities throughout the country, among which may be mentioned the following:

PHILADELPHIA, PA., permits the construction with a 2-in top plate and a ¾-in plastered ceiling in RESIDENTIAL BUILDINGS not over 85 ft high; and in COMMERCIAL BUILDINGS with a 2-in gypsum ceiling, either poured-in-place or precast for full fire-resistive buildings of unlimited height, when the joists are rigidly connected to the steel girders.

WASHINGTON, D. C., allows similar construction with a 1-in cement or gypsum plaster ceiling in RESIDENTIAL BUILDINGS up to 85 ft in height; and a 1½-in ceiling of gypsum or other approved materials for full fire-resistive construction of unlimited height.

PITTSBURGH, PA., requires a three-hour ceiling on metal joists for full fire protection in OFFICE and MERCANTILE buildings, but accepts the ¾-in plaster protection for two-hour construction in RESIDENCE buildings.

* See Gypsum Floors, Article 7.

Table XV. Steel-Joist Floor—Type A Construction

Slab—thickness, in	1 : 2 : 4 stone or cinder—2	
Spacing of joists or beams—in	19	24
Expanded metal lath	⅜-in rib—3 4 lb sq yd	⅜-in rib—4 0 lb sq yd
Sheet metal lath perforated	4.5 lb sq yd	4.5 lb sq yd
V-beam or rib metal steel sheathing	1 6 lb sq ft	1.6 lb sq ft
Composite lath or mesh or reinforcing bars	Long. No. 12 gauge—3-in o.c. Trans No. 12 gauge—4-in o c.	Long. No. 12 gauge—3-in o.c. Trans. No. 12 gauge—4-in o.c
Ceiling furring and lath	⅜-in rib lath—3 0 lb	⅜-in rib lath—3 4 lb
	No. 18 gauge wire lath or No. 20 gauge V-stiffened	No. 18 gauge V-stiffened wire lath
	Flat lath—3.4 lb with ¼-in ϕ 12-in o c.	Flat lath—3 4 lb with ¼-in ϕ 12-in o c.
Dead load exclusive of joist and floor finish; lb per sq ft	25-cinder concrete	25-cinder concrete
	35-stone concrete	35-stone concrete

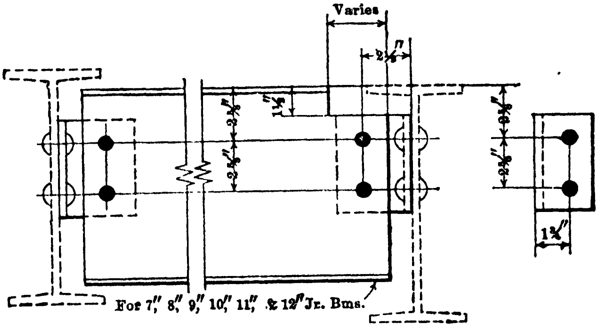
Slab—thickness, in	1 : 2 : 4 stone or cinder—2½	
Spacing of joists or beams—in	30	36
Expanded metal lath	¾-in rib—0.56 lb sq ft	Light loads—¾-in rib—0 56 lb sq ft Heavy loads—¾-in rib—0.65 lb sq ft
Sheet metal lath perforated	6 75 lb sq yd	Light loads—6.75 lb sq yd Heavy loads—7 50 lb sq yd
V-beam or rib metal steel sheathing	1.6 lb sq ft	Light loads—1.6 lb sq ft Heavy loads—2 1 lb sq ft
Composite lath or mesh or reinforcing bars	Long. No. 12 gauge—3-in o c. Trans. No. 12 gauge—4 in o.c.	Light loads—No. 12 long. and No. 12 trans. Heavy loads—No. 10 long. and No. 12 trans.
Ceiling furring and lath	Flat lath 3.4 lb on ¾-in channels 16-in o c.	Flat lath 3.4 lb on ¾-in channels 16-in o c.
	No. 18 gauge wire lath or No. 20 gauge V-stiffened wire lath, all on furring channels—16-in o.c.	No. 18 gauge wire lath or No. 20 gauge V-stiffened wire lath, all on furring channels—16-in o.c.
	⅜-in rib lath 3.0 lb on ¾-in channels 16-in o c.	⅜-in rib lath 3 0 lb on ¾-in channels 16-in o.c.
Dead load exclusive of joist and floor finish; lb per sq ft	30-cinder concrete	30-cinder concrete
	40-stone concrete	40-stone concrete

Note: With steel deck centering, spacing of joists can be increased depending on weight of metal and live loads (consult manufacturers). Reinforcing bars not less than ¼-in diameter, spaced 12-in on centers longitudinally, and 24-in on centers transversely, can be used with removable metal or wood centering, or mesh of equivalent areas for types B or D, Fig. 66.

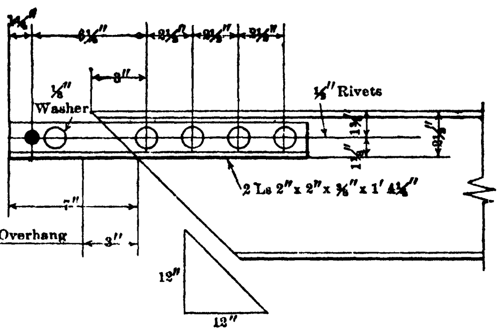
Los Angeles, Cal., permits the metal joist with plaster ceiling in buildings up to 100 ft in height.

Detroit, Mich. (Proposed Code), requires a 2-in precast or poured-in-place ceiling of gypsum or other approved materials in a first-class fire-resistive building of unlimited height.

In a paper presented before the Building Officials Conference at Madison. Wis., on April 21, 1925, Mr. J. J. Calvin describes a test conducted at the Underwriters' Laboratories under Standard Time-Temperature procedure on a floor-construction consisting of 8-in built-up strip-steel joists manufactured by the Berger Manufacturing Co. spaced 15¼ in on centers on a 16-ft span, with a top slab consisting of ⅜-in rib lath weighing 4 lb per sq yd, covered with 2 in of 1 : 2 : 4 concrete and a ceiling-protection of ⅜-in rib lath weighing 3 lb per sq yd, plastered ⅞ in thick with three coats of fibered gypsum plas-



Clip-angle Connections



Standard End Connection for 11" and 12" Junior Beams

Fig 65. Junior Beam Connections

ter. The floor-construction was loaded with a superimposed load of 90 lb per sq ft, producing a fiber-stress of 16 000 lb per sq in in the joists under the total load of 130 lb per sq ft. The fire was continued for a period of 2 hr 2½ min. At the end of the test, the temperature of the fire was 1 835° F.; the temperature of transmission on the top slab was 300° F.; the temperature of the air-space surrounding the joists was 835° F. After termination of the fire, the floor-assembly continued to absorb heat, and the deflection increased from 2.7 in at the end of the test to a maximum of 3.1 in. After removal of the load the construction recovered approximately 1 in.

With the usual three-coat plastered ceiling, $\frac{3}{4}$ in thick, the fire-resistive rating would be at least $1\frac{1}{2}$ hours.*

Sectional Systems. Many systems of precast units have been devised during the development of REINFORCED-CONCRETE CONSTRUCTION, consisting

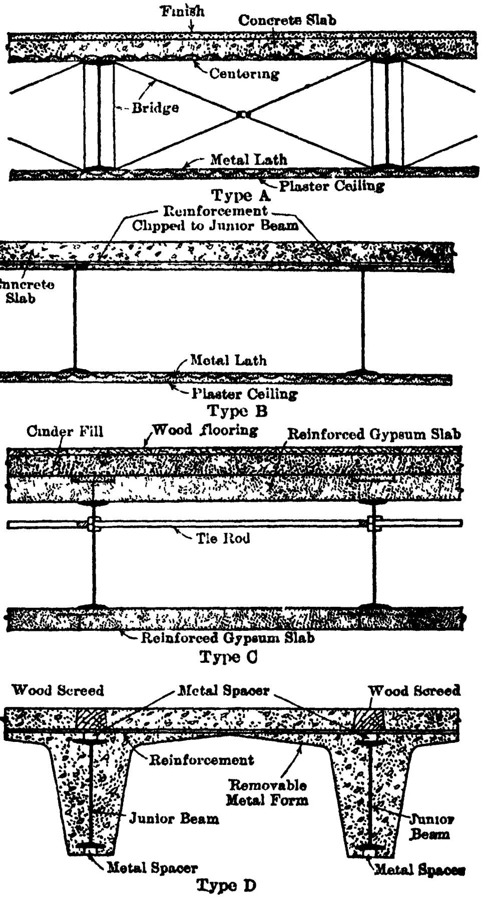


Fig. 66. Structural Platform Slabs

of shop-made reinforced-concrete members, such as girders, lintels, floor-slabs, and wall-panels, made at a factory and shipped to the sites of building operations, both in this country and abroad. Such systems are more completely discussed in Chapter XXIII. Precast shop-made members have

* See Fire-resistive Ceilings, Article 8.

been used between the steel beams of fire-resistive floor-construction as a substitute floor-filling for the usual clay-tile or concrete floor-arches. Such members are usually made as large as can be conveniently handled and of comparatively long span.

Disadvantages of Sectional Systems. The reason that the SECTIONAL SYSTEMS have not found favor is because they necessitate a fairly uniform spacing of beams throughout a structure, and this is generally impracticable. The casting of the parts has hitherto not been commercially successful, as the forms, although used repeatedly, have been more expensive than the usual centering at the building; and it is also generally necessary to use a concrete that is richer and more carefully prepared in order that it may stand the additional handling. Even with all possible care, the breakages in transportation are considerable. As the methods of manufacture of factory-made members are constantly being perfected, chiefly in mechanical contrivances for cheapening the forms and reducing the handling during the process of manufacture, the economy of this method of construction may be substantiated, particularly when used in combination with a light structural-steel fire-proofed frame.

Waite's Concrete Beam. In Fig. 67 is shown a type of SECTIONAL FLOOR-CONSTRUCTION that has been used in a number of buildings by the Standard

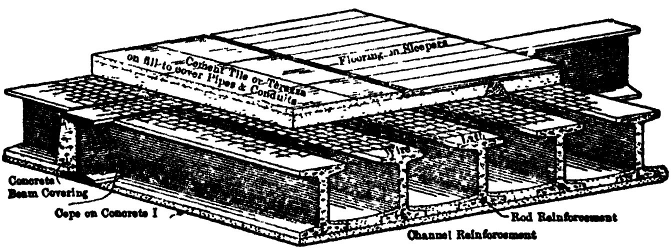


Fig 67. Waite's Concrete I Beams

Concrete Steel Company of New York City. The floor-construction consists of a series of concrete I beams 10 or 12 in in depth, supported on the lower flanges of the steel beams, which are spaced from 5 to 7 ft apart. The concrete beams are set about 18 in apart and the spaces between the lower flanges are filled in with a cinder concrete of the same composition as the I beams. On the tops of the concrete beams is placed a metal fabric of small mesh on which a lean-concrete slab is laid. This makes a comparatively light floor-construction, because of the large spaces between the concrete beams. The concrete I beams are cast at the shop and allowed to harden before they are sent to the building. In the lower flange is inserted, as shown, a steel reinforcement, of small circular or other cross-section, to furnish the necessary tensile strength. The beams are cast with the proper lengths, in accordance with the drawings; and any slight variations at the building are made up by filling the spaces between the ends of the concrete beams and the webs of the steel beams, and covering the webs of the latter with concrete. A similar construction, consisting of a series of T beams, with lower flanges $1\frac{1}{2}$ in thick and 12 in wide and stems 2 in thick and 12 in deep, of 1 : 4 cinder concrete, reinforced with $\frac{3}{16}$-in rods near the flanges, and without floor-finish of any kind, successfully withstood the fire, water, and load tests of the New York City Bureau of Buildings after having been constructed 28 days.

This system has proved to be practical in cases in which a flat or level ceiling is required and the steel floor-beams are 10 in or more in depth. The cost of construction compares favorably with that of other flush-ceiling types, but the system does not meet the present-day demand for shallow depth floors.

The Watson Floor System. Two types of sectional floor systems for fire-proof floor-fillings between steel beams are shown in Figs. 68 and 69. For long

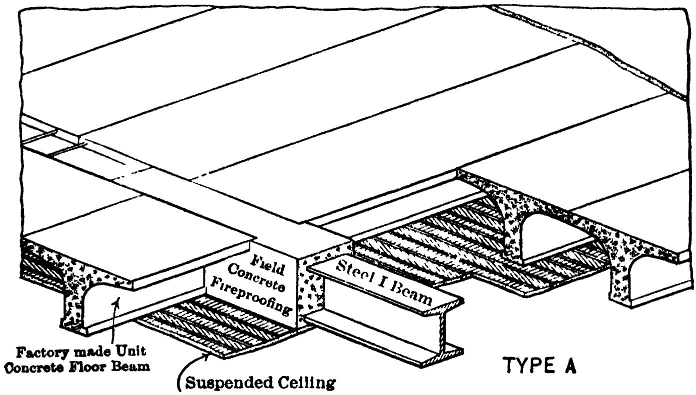


Fig 68. Watson Reinforced-Concrete Floor-Construction. Without Slabs

spans and heavy loads, the ⊤ sections are used, laid side by side; and for spans less than 20 ft and loads of 200 lb per sq ft or less, the beams are spaced 5 ft on centers with flat slabs between. This system is controlled and installed by the Unit Construction Company of St. Louis, Mo. Beams and girders are

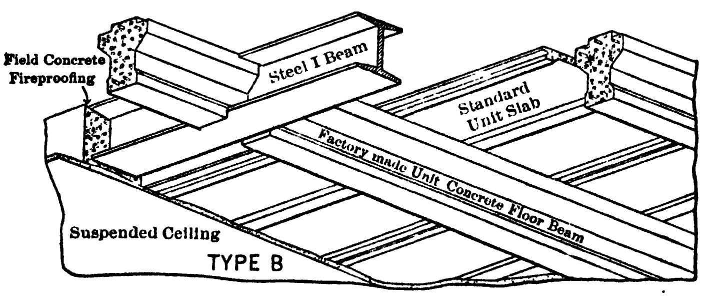


Fig. 69. Watson Reinforced-Concrete Floor-Construction. With Slabs

cast with unit frames in horizontal molds, and slabs are made on edge in steel forms. In the American School Board Journal for August, 1912, Theodore H. Skinner describes the construction and erection of a story-and-basement schoolhouse with a structural-steel frame and shop-made reinforced-concrete joists, with unit-ribbed reinforced-concrete slabs.

Miller Precast Unit. A system that is claimed to solve the problem of producing, handling and transporting large-size floor units to fit any column

spacing or irregularity of bays has recently been devised by Mr. Max Miller, Chief Engineer, PRECAST FLOORS CORPORATION, NEW YORK.* The floor units instead of being cast in one piece are shipped to the building-site in three separate lengths, with projecting reinforcement which loops into the gap between the precast units. A comparatively dry and stiff mixture of concrete is poured into these gaps, to secure the continuity of the reinforcement and to transmit the shear and compressive stresses from one section to the other. (See Fig. 70.) The upper edges of the units are beveled to form a trough in which the negative reinforcement and the anchorage of the topping reinforcement are grouted with cement mortar. The steel girders are protected with precast soffit and web-plates of cinder concrete.

The joint-sections or gaps in each span-length to be concreted are 9 to $10\frac{1}{2}$ in wide, and 6, 8 and 10 in deep. The STANDARD UNIT is 12 in wide and 6, 8, 10, 12 in deep. The CENTER UNITS are of fixed length, variations in span being taken care of in the end units which are stocked in lengths

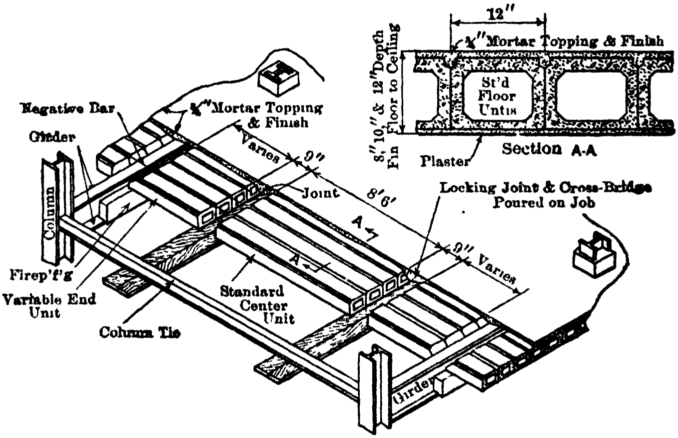


Fig. 70. Miller System

varying by 3- or 6-in intervals. Closer adjustments for length are made in the width of the concreted field-joints. The "MILLER SYSTEM" has been used in several apartment-buildings and in the erection of a building over the subway tracks of the New York transit system. Precast beams were installed over the tracks for the first floor of the building to minimize formwork interferences and the possibility of dripping concrete. It is claimed for this system that it is soundproof and will prevent condensation because of the hollow core; it produces a smooth, beamless ceiling; it can be installed in any weather; and it is economical because of light weight. The system has been approved in the City of New York for use on spans up to 26 ft and loads as computed by accepted engineering formulas.

Lith-I-Bar System. The Lith-I-Bar Company, Kalamazoo, Mich., controls a system of shop-made concrete joists manufactured by introducing two layers of dry-mixed concrete in a long metal mold with a special electric-

* Engineering News Record, March 28, 1929.

welded truss-reinforcement between the layers. The mold is then passed under a set of heavy cast-iron rollers which compact the concrete sufficiently to develop the required density and strength. The JOIST UNITS are used on a spacing of 18 or 24 in, and a 2-in top slab is cast in place either on removable metal pans or self-centering metal lath as shown in Fig. 71, to make a monolithic floor-system. Tests by the Engineering Department of the University of Michigan indicate that the bond between the poured-in-place slab and the shop-made unit is sufficient to develop T beam action. "Haydite" * lightweight concrete is used in the construction of this unit. The beam ceiling is readily adaptable to residence-construction, but where desired, a suspended plaster ceiling can be installed in the usual manner. It is claimed for this construction that its cost for small residence-buildings exceeds wood-frame construction by approximately 9 to 10 cents per sq ft of floor-area.

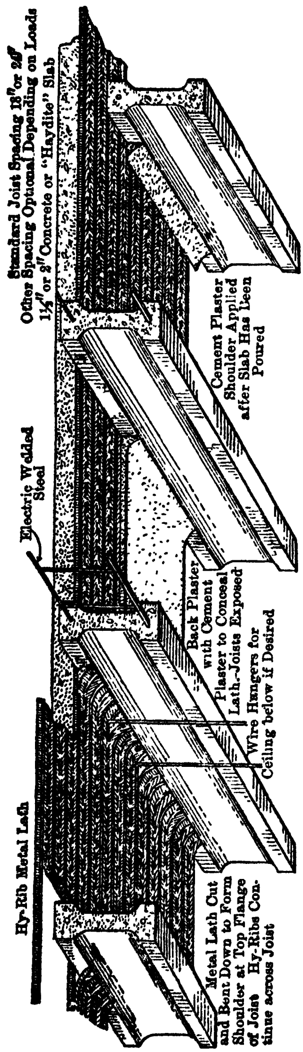


Fig. 71. "Lith-I-Bar" System

"Porete" Floor System. The Porete Manufacturing Co., Newark N. J., manufactures a precast hollow concrete-form in STANDARD lengths of 4 to 6 ft. These units are set up in rows on formwork with pockets between the ends of abutting sections designed to be filled with cement-mortar grout in the field and to envelop the reinforcing bars at these points with not less then 1 in of grout on all sides, at the same time that the ribs are poured between the units. Lugs are cast in the bottom flange of the shop-made units which act as chair supports for the reinforcing bars. Variations in span-length are provided for by any combination of the standard units required. Negative reinforcing steel is laid in the rib spaces over the steel beam-supports. The concrete units are closed at the ends to prevent concrete flowing into the enclosed air-spaces. It is claimed for this system that it is simple and adapted to spans of 10 to 25

* For discussion of "Haydite," see Article 5.

ft and to live loads, based on the steel reinforcement designed according to standard reinforced-concrete practice.

"Tee-Stone" System. The "TEE-STONE" unit is a precast reinforced-concrete tee section designed for use as a floor-beam, wall-stud or roof-rafter.

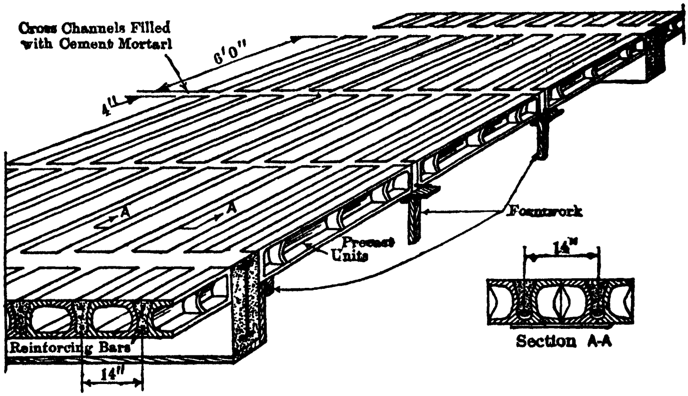


Fig. 72. "Porete" Long Span Floor System

It is manufactured in the dimensions and form shown in Fig. 73, and in STANDARD lengths of 8 ft to 16 ft. For floor-construction, it can be installed with the flange up or down, and as a wall-section the unit is usually erected with the stem inside. For flat ceilings and interior-wall construction, lath and plaster is attached directly to the stems, producing a hollow, enclosed space. The floor-section when placed with stem up has a wood nailing-strip attached to the top of the stem for nailing wood flooring, the flange produc-

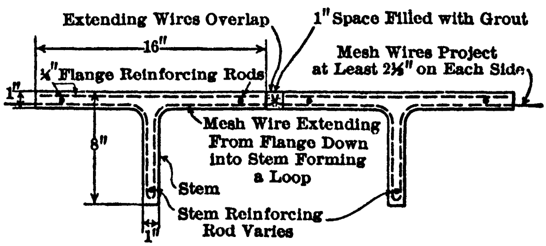


Fig. 73. Tee Stone System

ing a flat, level ceiling. The flange reinforcement, which projects 2½ in, is looped to that of the adjoining unit, and the 1-in space between the units is grouted or pointed up to produce a monolithically reinforced wall or floor-slab. Longitudinal and transverse FLANGE RODS are usually ¼ in in diameter, and longitudinal STEM RODS are varied to meet the design requirements in accordance with accepted practice. The FLOOR UNITS installed weigh 20 lb per sq ft, and WALL UNITS from 12 to 20 lb per sq ft. They have been

used in the construction of small residence-buildings and as a fire-resistive roof-construction over steel-frame buildings. It is claimed that the construction is not only fire-resistive, but damp-proof and indestructible, and can be installed in residence-construction at a cost very little in excess of frame buildings. The system is controlled by the Tee Stone Corporation, New York. No fire-tests are available on the completed units of this system and it is questionable whether sufficient protection has been provided for the main steel-reinforcement.

Long-Span Concrete Floors. Little or no exact data on the fire resistance of LONG-SPAN reinforced-concrete construction are available. For obvious reasons, no large-size controlled fire-tests have been made on this form of construction. In actual fires and conflagrations, the behavior of such floors as have been observed has resulted in conflicting conclusions, owing primarily to the varied quality of the concrete, and particularly, the character of the aggregates. For a discussion of these properties see Stone Concrete, Article 5. The structural design of this type of floor is treated in Chapter XXIII; only a brief discussion will be given here of the principles controlling the fire-resistance. Flat ceilings and the absence of sharp corners generally result in minimizing the destructive effects of fire. On the other hand, the expansion of solid masses of concrete in large areas, restrained in a rigid frame, will frequently result in spalling and "popping" of the concrete surfaces, exposing the steel reinforcement. The details of construction of the larger units must be based on the knowledge developed in tests of small units as regards protection, character, and distribution of reinforcement. The more uniformly the steel is distributed in the concrete, the better the fire-resistance; a close arrangement of small areas of steel results in better fire protection than larger reinforcing units widely spaced; and the TWO-WAY construction with reinforcement in both directions is superior to the ONE-WAY type. The type of floor-slab constructed of hollow units of light-weight materials properly incorporated with solid structural elements of reinforced concrete produces a more elastic assembly with better fire-resistance, and is discussed in the following paragraphs.

Ribbed and Combination Floors. Ribbed floors with concrete joists extending in one or both directions and a top slab of reinforced concrete should have a minimum thickness of at least 2½ in of concrete above the ribs. When, however, a ceiling of ¾ in of gypsum plaster on metal lath is suspended below the ribs, the floor-plate thickness may be reduced to 2 in. When incombustible floor-fillers are used, as in the COMBINATION FLOOR, and become part of the structural floor-slab, the topping of concrete may be entirely omitted, provided a protection or floor-fill of cinders or other light, porous, incombustible material continuously covers the top of the slab to a minimum thickness of 2 in, and provided further that the ceiling is protected with a ½-in coat of fibered gypsum plaster or a ¾-in coat of sanded gypsum or Portland-cement plaster. In all cases, the ribs should be of sufficient width and depth to provide minimum protection of 1 in outside of the reinforcement on all sides.

Gypsum Tile Fillers. A form of COMBINATION FLOOR has been used to considerable extent in recent years, in which the space between the reinforced concrete ribs is filled with GYPSUM PARTITION-TILE. To secure a uniform plastering base, soffit blocks are used for facing the joists, 1 in thick and of the required width of joist. This floor-construction requires a topping of not less than 1½ in of concrete on a system reinforced in two directions, and a ceiling of ½ in of fibered gypsum. Such fillers are not of sufficient strength

to be included in the computations of the resistive section for load-bearing capacity.

Clay Tile-Combination Floors. STRUCTURAL CLAY-TILE for use in COMBINATION FLOORS should at least comply with the MEDIUM grade * of the American Society for Testing Materials. The shells of the tile in contact with the concrete and at least 50% of top and bottom shells not in contact with concrete can be included in the resisting section† when the tile is selected to be of equal strength with the concrete. When the top slab is omitted from the combination section, a plastered ceiling should be provided and a top fill of incombustible materials should be placed to form an unbroken blanket over the top of the system at least 2 in thick. To provide for a uniform plaster base, soffit slab tiles can be installed under the concrete ribs, scored on one side, as are the tile filler-blocks.

"Natcoflor" System. The STRUCTURAL CLAY-TILE blocks for this system are manufactured by the National Fireproofing Company, with special bottom flanges to provide a continuous base for plastering, as shown in Fig. 74. The design of the tile provides for the largest section of clay at the top of the block to resist compression stresses, and the tile are made of high-strength clays. The 2-in joints are generally poured of a rich cement-mortar grout which thoroughly protects the steel reinforcement and completely fills the joint. The claims made for this system are: greater speed of erection, lighter-weight construction, economical floor-depth, and a saving in plastering and other labor costs.

SPECIAL TILE are also furnished with this system for the ends of continuous spans at supports, the relative top and bottom areas of the tile walls being reversed. (See Fig. 75) Shallow tile are also provided to take care of conduit runs when cement or terrazo floor finishes are used, as shown in Fig. 76. Where wood floors with sleepers and cinder fill are installed over the slab, the conduit can be run between the sleepers.

"Schuster" Floor System. "SCHUSTER" TWO-WAY COMBINATION FLOORS are composed of structural clay floor-tile 12 in by 12 in or 16 in by 16 in in size, of medium or hard grades.‡ The steel reinforcement is installed 16 or 20 in on centers in both directions, as illustrated in Fig. 77, producing ribs 4 in wide. When the concrete mixture is poured into the ribs, it flows into the cells of the filler tiles, bonding tile and concrete together, and the cross-section of the tile is included in calculations for bending-moment and shear. As previously noted, when the topping is omitted for fire-resistance, the slab should be finished with a ¾-in plaster ceiling below and a 2-in incombustible fill above. The advantages of this construction, properly designed, include: flat ceilings; minimum floor thickness; elimination of intermediate beams and girders; rapid and economical construction. FLOOR-TILE are furnished 4, 6, 8, 10 and 12 in in thickness. The thickness of the floors should not be less than 1/36 of the average span and in no case less than 4 in, but is dependent on the required design strength for heavy loads and long spans. The weight of the floor averages approximately 8½ lb per sq ft per in of slab thickness.

"Republic Slagblok" System. The "SLAGBLOK" COMBINATION FLOOR controlled by the Republic Fireproofing Company, Inc., New York, is installed as a ONE-WAY or a TWO-WAY system, using a special concrete floor-block made of a slag-aggregate concrete. The "SLAGBLOK" unit is 16 in by 16 in by 3,

* See Floor-Tile Specifications, Table V, Article 5.

† Technologic Paper No. 291 and Research Paper No. 181, U. S. Bureau of Standards.

‡ See discussion of Structural Clay-Tile, Table V, Article 5.

4½, 6, 7 or 8 in. The concrete ribs are 4 in wide, and reinforcement is spaced 20 in on centers, as shown in Fig. 78. The "SLAGBLOK" is manufactured exclusively by this company of a slag-concrete mix in steel molds under pressure. Each unit is made up of two blocks 8 in by 16 in in area with one open end and a tapering core, so that the two halves, when set in position on the

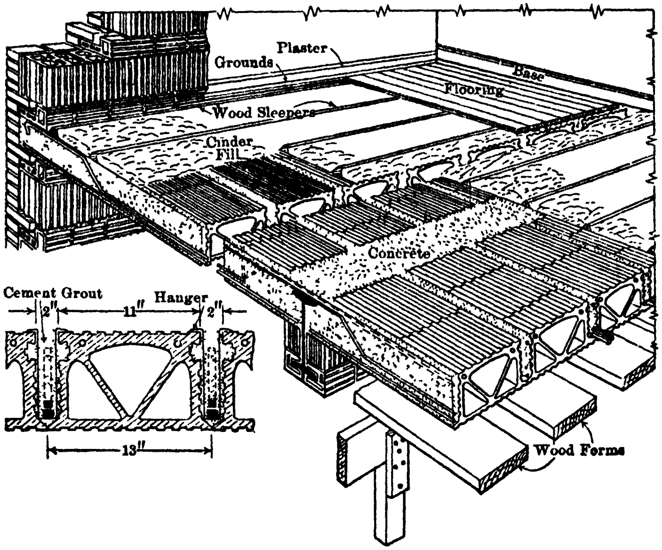
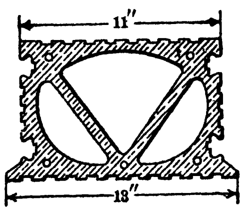


Fig. 74



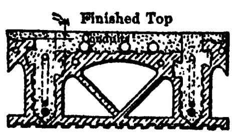
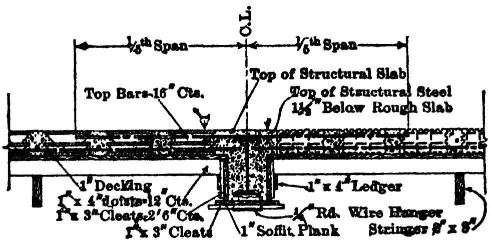


Fig. 75 Fig. 76

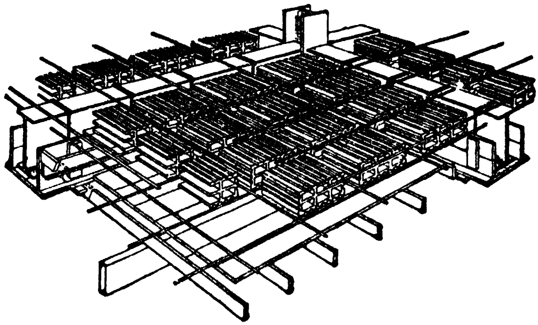
Figs. 74, 75, 76. "Natcoflor" Long Span System

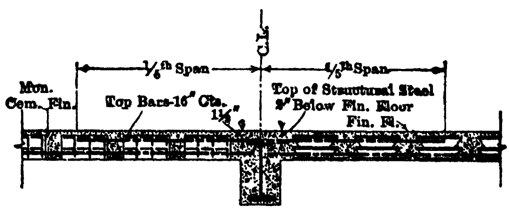
forms, result in a rectangular unit with a hollow core tapering from the center toward the ends, and alternating in direction in successive units. Direct tests on the net area of the block develop strengths of approximately 2 000 lb per sq in. The most economical design with this system is secured on spans of 15 ft to 25 ft, but it can be used on any span based on accepted principles of design. The advantages claimed for this system besides its fire-resistance are: economy of materials, low dead weight, minimum thickness of floors,

and sound-proofness. The same limitation as to thickness of this system and to fire-resistive protection applies as in other COMBINATION FLOORS. To be used as a composite member of the system, with omission of the top plate,



Typical Centering Details
Steel Beam Construction





Typical Details
Showing Use of Monolithic Cement Finish and Tile Soffit where an All Tile Ceiling is Desired

Fig. 77. Schuster Floor System

the "SLAGBLOK" should develop strength properties equal to those of the concrete which composes the ribs.

"Battledeck" Floor-Construction. The "BATTLEDECK" FLOOR-CONSTRUCTION developed by the American Institute of Steel Construction consists of a structural floor of $\frac{3}{16}$ or $\frac{1}{4}$-in steel plates welded to each other on

all edges and to the top flanges of the supporting floor-beams, with a fire-resistive ceiling of metal lath and plaster or any of the forms described in Article 8, suspended from the lower flanges of the floor-beams as shown in Fig. 79. The claims made for this construction are: extreme lightness in weight, averaging 10 to 15 lb per sq ft exclusive of weight of floor-finish and fire-resistive ceiling; ease and rapidity of erection; economical design in that the parts are assembled into a structural unit, with a statically determinate resisting section; flexibility in adapting the systems to irregular panels; and low labor costs. As a fire-resistive flooring, cork or rubber tile, composition

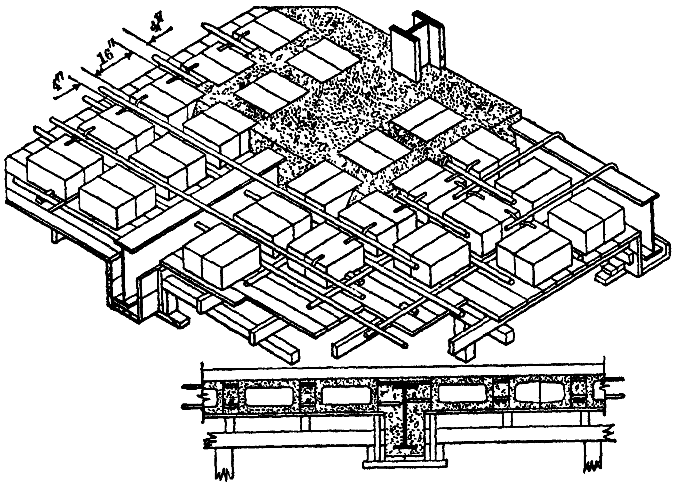


Fig. 78. "Slagblok" Floor System

floors, linoleums or plastic asphalt can be laid directly on the steel floor-plates. Among its probable disadvantages may be included the inadequacy of a continuous fire-resistive ceiling as a protection for structural steel in first-class fire-resistive construction; insufficient mass and stability in the floor-construction to transmit horizontal shears and bending-stresses from wind and other lateral forces for tall building frames; and probability of transmission of sound and vibrations throughout the frame through continuous steel contact without insulation. Tests are now in progress at the U. S. Bureau of Standards on methods and materials for fireproofing the members of the floor. The system is quite new, and extended experience with the construction is not yet available.

Tie-Rods. In all segmental arches and other types in which a thrust is exerted against the beams, TIE-RODS must be provided to prevent the beams from being pushed apart, and especially to prevent the outer bays from spreading. They should run from beam to beam from one end of the floor to the other. If the outer arches spring from an angle, as in Fig. 38, the tie-rods in this bay should be anchored into the walls with large plate-washers. The tie-rods should be located in the LINES OF THRUST of the arches, which are ordinarily below the half-depth of the beams, and in some cases near the bot-

tom flanges. If their appearance is objectionable, they should be hidden by a hung ceiling. As a rule tie-rods are proportioned and spaced according to some RULE OF THUMB rather than by actual calculations of the thrust. For the interior arches this practice is probably safe enough, but for outside spans, and particularly for segmental arches, the thrusts of the arches should be computed and the rods proportioned accordingly. The spacing of the rods is generally eight times the depth of the supporting beams, but never more than 8 ft. For interior flat-tile arches, the following rule can usually be safely followed: for spans of 6 ft or less, use ¾-in rods spaced about 5 ft apart; for 7-ft spans, ⅞-in rods, 5 ft apart; and for 9-ft spans, ⅞-in rods, 4 ft apart.

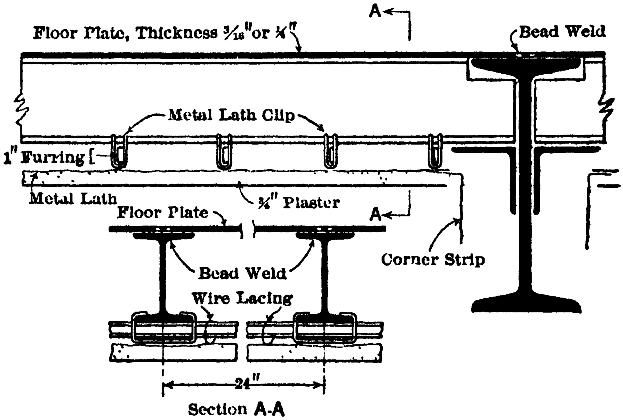


Fig. 79. Battledeck Floor

The HORIZONTAL THRUST of an arch may be found by the following formula:

$$T = \frac{3\,wL^2}{2R}$$

in which T = pressure or thrust in pounds per linear foot of arch;
w = load on arch in pounds per square foot, uniformly distributed;
L = span of arch, in feet;
R = rise of segmental arch, or effective rise of flat arch, in inches.

Load-Tests. It may be desirable at times to test fire-resistive floors after they have been installed. The same precautions should be taken as for tests on reinforced-concrete construction. If it is desired to determine from such tests the ULTIMATE STRENGTH, a section of the floor of a width equal to the span should be cut loose from the rest and loaded to destruction, the supporting steel beams being shored up during the test. The SAFE WORKING LOAD is found by dividing the BREAKING-LOAD by the proper FACTOR OF SAFETY.

8. Fire-Resistive Ceilings

Flat Versus Paneled Ceilings. In BEAM and ARCH floor-construction without HUNG CEILINGS, the projection of the beams below the ceiling-line forms pockets for the retention of heat and flame, increases the exposed area,

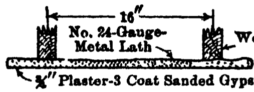
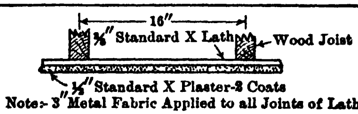
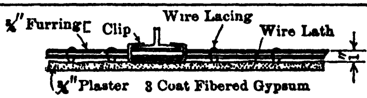
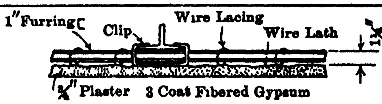
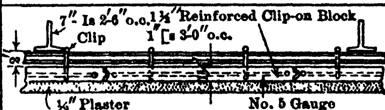
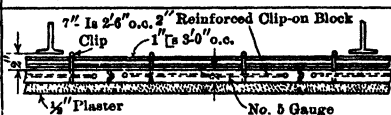
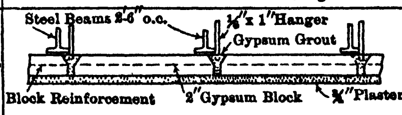
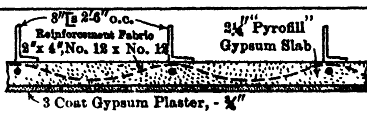
and, when attacked by fire, results in greater structural damage. A HUNG CEILING below such floor systems provides additional fire protection and minimizes the destructive effects of flames and high temperatures. In addition, a flat ceiling can be more readily and economically protected with automatically operated sprinklers than a paneled ceiling. Moreover, a perfectly flat ceiling reflects more light, makes a better-lighted room, and dissipates the heat. For these reasons, a FLAT ceiling is generally to be preferred to a PANELED ceiling. In STEEL-FRAME construction, it is frequently possible to lower the concrete slab or clay tile arch to the bottom flanges of the floor-beams, thus accomplishing the same results. A discussion of the fire-resistance of a plaster ceiling will be found under Mortars and Plasters, Article 5, and in the description of floor systems which depend for their resistance on precast or field-constructed ceilings suspended below the floor-beams.

Types of Ceilings. Combustible Construction. Modern building codes generally require the protection of combustible floor and partition construction of wood joists or studs with coverings of ½-hour or one-hour fire-retardant value in special locations such as cellar ceilings, soffits of stairways, and stair-enclosures. In 1914, a test to determine the relative fire-resisting merits of six different types of CEILING-PROTECTION was made for the City of New York in the STANDARD FIRE-TEST CHAMBER on a WOOD-JOIST roof-construction divided into six panels, protected as follows: No. 1, No. 25 gauge expanded metal lath plastered with ¾ in of lime plaster; No. 2, ½-in plaster boards with ⅞-in furring strips and 29 gauge stamped ceiling metal; No. 3, ⅞-in furring strips and 29 gauge stamped ceiling metal (same as No. 2 with plaster boards omitted); No. 4, wood lath plastered with lime plaster similar to No. 1; No. 5, ⅞-in furring strips covered with 2 layers of 29 gauge metal; No. 6, ⅞-in tongue-and-groove boards covered with 29 gauge stamped ceiling metal. The top of the floor-joists was covered with a 2-in slab of reinforced-cinder concrete. The construction was subjected to the standard fire-test for a period of one hour, the temperature being gradually raised to reach 1 700° F. at the end of the period. Approximately 12% of the wood beams was charred in Panel No. 1; 5% in Panel No. 2; practically all charred in Panel No. 3; almost all burned away in Panel No. 4; 50% charred in Panel No. 5; and 24% charred in Panel No. 6. The temperature on the top surface of Panels 3 and 4 reached critical end-points in 10 minutes, of Panel 5 and 6 in 15 minutes, of Panel 1 in approximately 20 minutes, and of Panel 2 in ½ hour. The general observations led to the rating of these protections in the following order of merit: Nos. 2, 1, 6, 5, 3, and 4, and demonstrated a full one-hour rating for the PLASTER BOARD AND METAL protection, approximately ½ hour for the METAL LATH AND LIME PLASTER, and not more than ⅙ hour for the WOOD LATH AND PLASTER. Although this test was not conclusive proof of the definite ratings of these several protections, the above conclusions seem to be warranted, and are further confirmed in the light of more recent data developed in fire-tests of full-size constructions in accordance with the American Engineering Standards procedure.

Table XVI shows the fire-resistance rating developed in several investigations of ceiling-constructions tested under carefully controlled TIME-TEMPERATURE procedure. A description of these several types of construction is given in the following paragraphs.

Metal Lath and Plaster (Combustible Construction). Type 1. Ceiling constructed of ⅜-in standard diamond mesh metal lath, No. 25 U. S. gauge weighing 3 lb per sq yd, attached to wood joists with 2-in, 6-penny

Table XVI. Fire-Resistive Ceilings

Type	Rating	Description	Remarks
1 Combustible	¾ - Hr.*		Underwriters Tests 1922
2 Combustible U.S. Gypsum Co. Standard "X"	1 - Hr.		Columbia Univ.-N.Y.City Test 1929
3 Incombustible	1 - Hr.		Underwriters Tests
4 Incombustible	1½-Hr.		Underwriters Tests and Columbia Univ.-N.Y.City Test
5 Precast ("American Clip-on")	3 - Hrs.		(Interpolated)
6 Precast ("American Clip-on")	4 - Hrs.		Columbia Univ.-N.Y.City Test
7 Structural Gypsum Precast ("Gypsteel")	4 - Hrs.		Columbia Univ.-N.Y.City Test
8 U.S. Gypsum Co. Poured-in-Place ("Pyrofill")	4 - Hrs.		Columbia Univ.-N.Y.City Test

* If finished on top of joists with rough flooring, insulating felt, furring strips and finished flooring, a 1-hour FIRE-RESISTIVE rating is given to the floor-construction complete.

nails or staples spaced 6 in; lath lapped at junctions with walls and partitions; plastered with 3 coats of sanded gypsum plaster or Portland-cement mortar, results in a protection on one side of wood studs or floor-joists with a ½-hour fire-resistive rating. When applied on both sides of wood studs, such construction is rated with one-hour fire-resistance.

Type 2. Standard X Ceiling. Constructed of ⅜-in plaster board, 16 in by 48 in, finished on the back with a moisture-resisting fiber felt, erected with

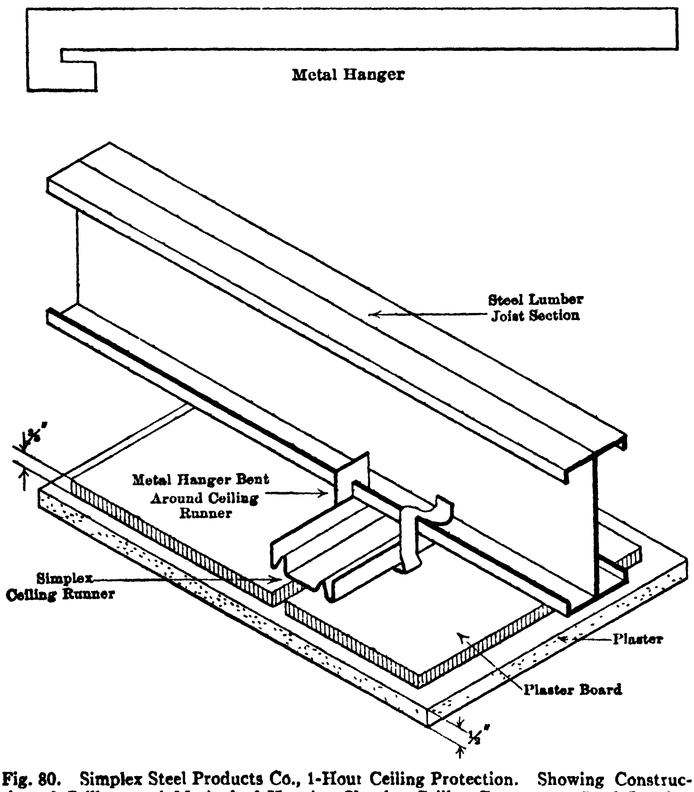


Fig. 80. Simplex Steel Products Co., 1-Hour Ceiling Protection. Showing Construction of Ceilings and Method of Hanging Simplex Ceiling Runners to Steel Lumber

broken joints, each lath nailed to the joists with ten 1¼ in by No. 11 gauge nails with flat thin heads, placed around the edges of the boards; all lath joints, nail-heads, and corner angles covered with 3-lb STANDARD X diamond mesh fabric, 3 in wide by 48 in long, secured with 2-in finishing nails, driven and bent over; plastered with 2 coats of fibered gypsum STANDARD X plaster and hard finish, results in a one-hour fire-resistive rating.

Plaster Board ceilings can also be applied to steel floor-beams or joists, as shown in Fig. 80. The Simplex metal hanger here shown is readily

attached to the runner channels, and the board is held in place by steel pins or nails in punched prongs.

Type 3. Light Metal Lath and Plaster (Incombustible Construction). All the available data secured from fire-tests and actual fires in buildings indicate that METAL LATH AND PLASTER construction on steel column, girder or ceiling construction will develop a one-hour fire-resistance. The value of this protection depends first on the details followed in the construction, and secondly, on the actual insulation afforded between the protecting coat and the structural member. Following are the specifications for FURRING, LATHING and PLASTERING which are the minimum requirements for a properly executed ceiling-protection of metal lath and plaster. For details of construction see Figs. 81 to 88.

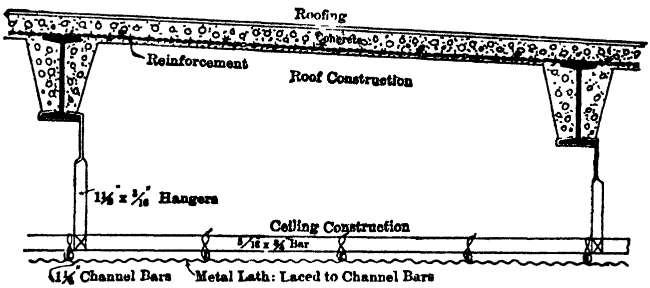


Fig. 81. Suspended-Ceiling Construction

(*a*) RUNNING BARS. When furring channels are not directly clipped to floor-beams, RUNNER BARS to consist of 1½-in channels or 1½-in by ¼-in flats suspended by No. 8 galvanized steel hangers or 1-in or 1½-in steel flats.

(*b*) FURRING CHANNELS. To be ¾ in cold or hot-rolled channels for spans not exceeding 4 ft, and 1-in channels for spans not exceeding 5 ft.

(*c*) METAL LATH. For spacing of furring channels not exceeding 12-in centers, 3 lb No 25 gauge flat expanded lath or No. 19 gauge wire lath; not

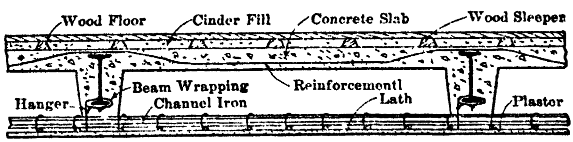


Fig. 82. Directly Attached Typical Fire-proof Floor and Ceiling-construction

exceeding 16-in centers, 3.4 lb No. 24 gauge flat expanded lath or No. 18 gauge wire lath, or 3 lb ⅜ in rib lath, or No. 20 gauge V-stiffened wire lath; not exceeding 24-in centers, 3.4 lb ⅜ in rib lath, or No. 18 gauge V-stiffened wire lath.

(*d*) PLASTERING. For one-hour fire-resistive rating, plastering should not be less than ¾ in thick, of gypsum fibered or sanded plaster, or Portland-cement mortar.

Type 4. Heavy Metal Lath and Plaster. By minimizing the transfer of heat from plaster ceiling to structural beam through increasing the insulation

between the two, the retardant rating of Type 3 protection can be increased. Fire-tests conducted at the laboratories of Columbia University indicate that this object can be accomplished by suspending the ceiling so as to break the contact as completely as possible between the beam and its covering, or by increasing the thickness of the ceiling coat. For a $1\frac{1}{2}$-hour rating, the minimum thickness required is a poured ceiling of $1\frac{1}{4}$ in of reinforced fibered gypsum or Portland-cement concrete, or a $\frac{3}{4}$-in plastered ceiling suspended $1\frac{1}{4}$ in from the supporting beam. Types 5, 6, 7, and 8 are described under Gypsum floors, Article 7.

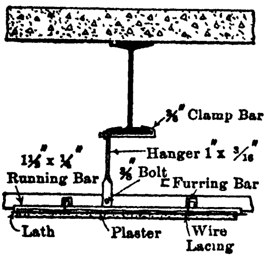


Fig. 83. Suspended Ceiling

Suspended Ceiling Details. Office-buildings, apartment-houses, etc., having flat roofs, require ceilings below the roofs in order to make a proper finish in the rooms and also for heat-insulation; also with beam-and-girder type floors, suspended ceilings may be required. In office-buildings the ceilings of the top story are often framed and constructed like the floors, but with a lighter construction. More often the ceilings are suspended from the roof, as this requires much less steel and is consequently much cheaper. It answers

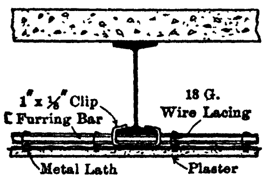


Fig. 84 Clipped Ceiling on Unprotected Beam

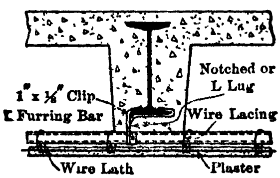
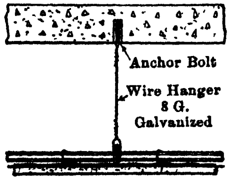
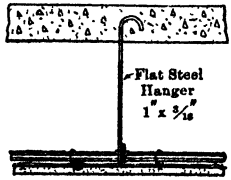


Fig. 85. Clipped Ceiling on Fire-proofed Beam

the purpose fully as well, that is, if the roof-beams are efficiently protected. Fig. 81 shows a common construction for such ceilings. Flat steel hangers. about $1\frac{1}{2}$ by $\frac{3}{16}$ in or 1 by $\frac{3}{16}$ in, split at one end to hook over the lower



Wire Hanger with Anchor Bolt Connection



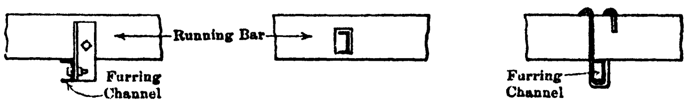
Flat Hanger Anchored in Concrete Slab

Fig. 86. Suspended Ceiling. Intermediate Hangers

flanges of the roof-beams, are used to support $\frac{5}{16}$ by $\frac{3}{4}$-in or $1\frac{1}{2}$ by $\frac{1}{4}$-in flat steel bars, spaced from 4 to 5 ft on centers; and to the under-side of these are laced $\frac{3}{4}$-in, $\frac{7}{8}$-in, or $1\frac{1}{4}$-in channels, 12 to 24 in, to receive the metal

lathing. The bottom of each hanger is bent at right-angles to form a seat for the bar, and the bar is laced to the hangers. No bolting or riveting is required, all connections being made by lacing wire, or by bending the iron. If ordinary lime mortar is used for plastering a 12-in spacing is the maximum permissible.

Figs. 83 and 88 show very satisfactory details for the construction of these systems. Where the ceiling is suspended below clay-tile arches, toggle-bolts



Bolted
¾ × ¾ × ⅛-in angle lug with ¼-in bolts

Passed through
Running bar punched to receive furring bar with snug fit

Clipped
Hair pin Clip of No. 9 annealed galvanized wire.

Fig. 87. Methods of Connecting Furring Bar to Running Bar

are used for the support of the hangers. The ends of the small bars supporting the lathing are usually spliced by means of sheet-iron clamps, about 6 in long, wrapped closely around the bars and hammered tight. For suspended ceilings under segmental or paneled floor-construction, the same methods are employed, except that the hangers are replaced by clips holding the ceiling-bars close to the soffits of the beams, as shown in Fig. 82.

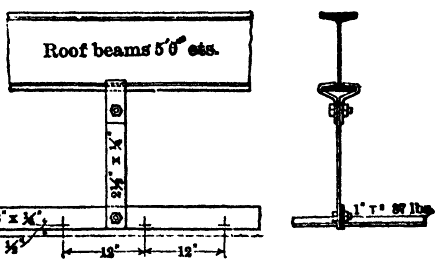


Fig. 88. Suspended Ceiling. Details of Two-Bar System

9. Types of Reinforcement

The Types of Reinforcement used in monolithic concrete construction are described and discussed in Chapter XXIII. All of these forms may be used in FIRE-RESISTIVE FLOOR-CONSTRUCTION. In addition, however, there are other types of reinforcement that are not generally adaptable to reinforced-concrete members, but are particularly useful in short-span floor arches in skeleton steel frames. They are generally manufactured in units of small cross-section in order to give a uniform distribution of steel throughout the floor-area to resist stresses produced by HIGH TEMPERATURES as well as the tension due to FLEXURE. The reinforcement discussed in this Chapter may be classified in the following types: (*a*) WIRE FABRIC; (*b*) EXPANDED MESH; (*c*) SELF-CENTERING; (*d*) DEFORMED SHEET STEEL.

(a) **Wire Fabrics** are especially advantageous for use in floor systems of structural steel or reinforced-concrete frames, as they are less likely to be displaced in erection and the distribution of the reinforcement provides more effectively against unequal temperature-stresses than larger units more widely spaced. The wire is manufactured under Standard Specifications * for cold-drawn wire. The basic difference between HOT-ROLLED BARS and COLD-DRAWN WIRE is an increase in ultimate strength of the latter, using the material of the same low carbon-content and ductility as that employed in the hot-rolled bar, through the action of cold-working of the metal. The cold-drawn wire has no definite yield-point, the wire continuing to stretch gradually with increased stress even after the true elastic limit of the material has been passed, compared to the sudden large elongation of the hot-rolled product at its yield-point. Slabs reinforced with wire fabric will develop a large number of small and well-distributed cracks with large deformation and deflection before failure. With steel of the same ultimate strength, cold-drawn wire can be stressed with the same degree of safety about 25% beyond the hot-rolled product.

On the other hand, the increased strength due to cold working may be offset by the high temperatures transmitted through the protective covering outside of the reinforcement when attacked by fire. In a 2 000° F. fire, the temperature of the reinforcement may reach as high as 1 700°, sufficient to anneal the wire when cooled.† Another disadvantage of the cold-drawn product is the lack of flexibility of the material and the difficulty of unrolling the wire and holding it in place in the concrete floor-forms. Because of this difficulty, low-carbon mild steel is frequently substituted in practice, still using the higher working stresses permitted with cold-drawn wire. Furthermore, electric welding of the fabric at mesh intersections is more effectively accomplished on mild steel hot-drawn products, and samples of welded fabric selected in the field are apt to show the properties of ordinary hot-rolled mild steel in physical tests.

Triangle Mesh Wire Reinforcement. Under this name the American Steel and Wire Company is manufacturing a wire fabric of cold-drawn steel wire for the reinforcement of fire-proof floors. A detail of the standard material is shown in Fig. 89. The triangular mesh is built up of either single or stranded longitudinals with the cross-wires or bond-wires running diagonally across the width of the fabric. It is claimed that the triangular mesh affords an even distribution of the steel, reinforcing in every possible direction, and that the strength is increased by reason of the truss-construction. The longitudinal wires in Triangular Mesh are invariably spaced 4 in on centers, but the diagonal wires may be spaced either 4 or 8 in apart. The manufacturers can furnish different styles, giving variations in the cross-sectional area as shown in Table XVII. The material is furnished either galvanized or plain. This type of fabric results in greater load-bearing capacity than electrically welded rectangular mesh of equal areas and weights.

American Electrical Welded Fabric, shown in Fig. 90, is also manufactured by the American Steel and Wire Company in sizes of mesh and wire given in Table XVIII. The cross-wires are rigidly secured to the longitudinals and prevent the latter from slipping in place. It is furnished either plain or galvanized.

Clinton Welded-Wire Fabric is manufactured by the Wickwire Spencer Steel Co., New York, of sizes and mesh similar to American fabric. Fig. 91

* A.S.T.M. Spec. A 82-27.

† Cinder Concrete Floor Construction, Trans. Am. Soc. C. E., Vol. LXXIX, page 523.

Table XVII. Triangle Mesh Woven Wire Concrete Reinforcement

Style No.	Number and gauge of wires each, longitudinal, A. S. & W. Co.'s steel wire gauge	Sectional area longitudinals, sq in per ft width	Effective sectional areas, sq in per ft of fabric: Transverse	Effective sectional areas, sq in per ft of fabric: Longitudinal	Approximate weight, lb per 100 sq ft
Longitudinals Spaced 4 in	Cross Wires No 14 Gauge, Spaced 4 in				
032	1—No 12 gauge	026	.022	.032	22
040	1—No. 11 gauge	.034	.022	.040	25
049	1—No. 10 gauge	.043	.022	.049	28
058	1—No. 9 gauge	.052	.022	.058	32
068	1—No. 8 gauge	.062	.022	.068	35
080	1—No. 7 gauge	.074	.022	.080	40
093	1—No. 6 gauge	.087	.022	.093	45
107	1—No. 5 gauge	.101	022	.107	50
126	1—No 4 gauge	.120	022	.126	57
146	1—No. 3 gauge	.140	022	.146	65
153	1—¼ in	.147	.022	.153	68
168	1—No. 2 gauge	.162	.022	.168	74
180	2—No 6 gauge	.174	.022	.180	78
208	2—No. 5 gauge	.202	.022	208	89
245	2—No 4 gauge	.239	.022	245	103
267	3—No 6 gauge	.261	022	267	111
287	3—No. 5½ gauge	.281	022	287	119
309	3—No. 5 gauge	303	022	.309	128
336	3—No. 4½ gauge	.330	.022	336	138
365	3—No 4 gauge	.359	.022	.365	149
395	3—No 3½ gauge	389	022	395	160
Longitudinals Spaced 4 in.	Cross Wires No. 14 Gauge, Spaced 8 in				
036P	1—No. 12 gauge	.026	.009	036	17
044P	1—No 11 gauge	.034	009	044	20
053P	1—No. 10 gauge	.043	.009	.053	24
062P	1—No 9 gauge	052	.009	.062	27
072P	1—No. 8 gauge	.062	.009	.072	31
084P	1—No 7 gauge	.074	.009	.084	35
097P	1—No. 6 gauge	087	009	097	40
Longitudinals Spaced 4 in	Cross Wires No 12½ Gauge, Spaced 8 in				
041R	1—No 12 gauge	.026	.014	.041	21
049R	1—No. 11 gauge	034	.014	.049	24
058R	1—No. 10 gauge	.043	.014	.058	28
067R	1—No 9 gauge	.052	.014	.067	31
077R	1—No. 8 gauge	.062	.014	.077	35
089R	1—No. 7 gauge	.074	.014	.089	40
102R	1—No. 6 gauge	.087	.014	.102	44

Longitudinals Spaced 4 in Cross Wires Nos. 14 and 12½ Gauge, Spaced 2 in
(This material is used principally for cement gun work)

Style No.	Number of wires each, longitudinal	Gauge of wires each, longitudinal	Gauge of cross wires	Approximate weight, lb per 100 sq ft
7A	1	12	14	31
6A	1	10	14	37
5A	1	8	14	44
4A	1	6	14	53
29A	1	12	12½	42
28A	1	10	12½	48
27A	1	8	12½	55
26A	1	6	12½	64

Rolls: Standard lengths 150, 200 and 300 ft.
Widths: Approximately 16, 20, 24, 28, 32, 36, 40, 44, 48, 52 and 56 in.
Finish: Plain or galvanized.
Tensile strength: Standard, 70,000 to 85,000 lb per sq in.

Table XVIII. American Electrically Welded Fabric

Spacing of wires, in		Gauge number		Sect. area, sq in per ft		Weight per 100 sq ft lb
Longit.	Trans.	Longit.	Trans.	Longit.	Trans.	
2	16	1	7	.377	.018	140
2	16	2	8	.325	.015	119
2	16	3	8	.280	.015	104
2	16	4	9	.239	.013	89
2	16	5	10	.202	.011	75
2	16	6	10	.174	.011	65
2	16	7	11	.148	.009	55
3	16	1	7	.252	.018	96
3	16	2	8	.216	.015	83
3	16	3	8	.187	.015	72
3	16	4	9	.159	.013	61
3	16	5	10	.135	.011	52
3	16	6	10	.116	.011	45
3	16	7	11	.098	.009	38
3	16	8	12	.082	.007	32
4	16	3	8	.140	.015	56
4	16	4	9	.120	.013	48
4	16	5	10	.101	.011	40
4	16	6	10	.087	.011	35
4	16	7	11	.074	.009	30
4	12	8	12	.062	.009	26
4	12	9	12	.052	.009	22
4	12	10	12	.043	.009	19
4	12	12	12	.026	.009	13
4	12	5	5	.101	.034	48
4	12	6	6	.087	.029	42
4	12	7	7	.074	.025	35
4	12	8	8	.062	.021	30
6	12	0	6	.148	.029	65
6	12	2	2	.108	.054	59
6	12	3	3	.093	.047	51
6	12	4	4	.080	.040	44
6	12	5	5	.067	.034	37
6	12	6	6	.058	.029	32
6	12	7	7	.049	.025	27
6	8	12	12	.017	.013	11
6	6	0	0	.148	.148	107
6	6	1	1	.126	.126	91
6	6	2	2	.108	.108	78
6	6	3	3	.093	.093	68
6	6	4	4	.080	.080	58
6	6	5	5	.067	.067	49
6	6	6	6	.058	.058	42
6	6	7	7	.049	.049	36
6	6	8	8	.041	.041	30
6	6	9	9	.035	.035	25
6	6	10	10	.029	.029	21
6	6	12	12	.017	.017	13

Table XVIII (Continued). American Electrically Welded Fabric

Spacing of wires, in		Gauge number		Sect. area, sq in per ft		Weight per 100 sq ft lb
Longit.	Trans.	Longit.	Trans.	Longit.	Trans.	
4	4	4	4	.120	.120	85
4	4	6	6	087	.087	62
4	4	8	8	.062	.062	44
4	4	10	10	043	.043	31
4	4	12	12	.026	026	19
4	4	14	14	015	.015	11
3	3	10	10	057	.057	41
3	3	12	12	.035	.035	25
2	2	10	10	086	.086	60
2	2	12	12	.052	.052	37
2	2	14	14	030	.030	21
2	4	12	12	052	026	28
2	4	14	14	.030	.015	16

Rolls: Standard lengths 150, 200 and 300 ft. For No. 2 gauge wires and larger, flat sheets only.

Widths: Multiples of the spacing of longitudinal wires up to a maximum width which varies with the size and spacing of the longitudinals Approximate maximums 56 to 72 in for 2-in spacing, 84 to 96 in for 3 or 4-in spacing, and 96 to 120 in for 6-in spacing.

Tensile Strength: Standard, 60,000 to 70,000 lb per sq in, 70,000 to 80,000 lb furnished when ordered.

Weights: All above weights are based on a width of 60 in measured from center to center of the outside or selvage longitudinal wires.

shows the monolithic character of the weld, and in tension tests strands of wire always fail outside of the junction. The claim is made of perfect adhesion between reinforcement and concrete and no obstructions to

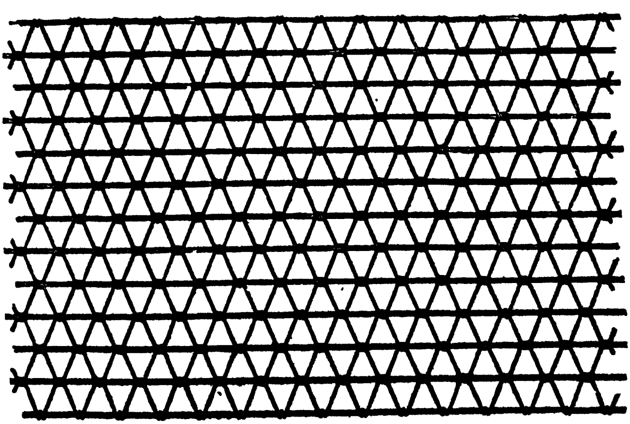


Fig. 89. Wire-Fabric Reinforcement, Triangular Mesh

pouring of the slab mixture. The material is galvanized thoroughly before being welded, or is furnished plain. ELECTRICALLY WELDED FABRIC is also manufactured by the National Steel Fabric Co. of Pittsburgh, Pa., of No. 16 gauge to 3/8-in longitudinal wires and 16 gauge to 15/32-in transverse wires. Longitudinal wires are spaced 2, 3, 4, 6, 8, 9 or 12 in on centers and transverse wires 2, 3, 4, 6, 8, 9, 10, 12, 16 or 18 in on centers. It is furnished in rolls or sheets of any desired lengths and in widths of 2 ft to 11 ft 6 in. Stock lengths of rolls are 100 to 300 ft.

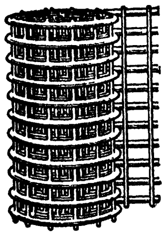


Fig. 90. Electrically Welded Wire Fabric Reinforcement

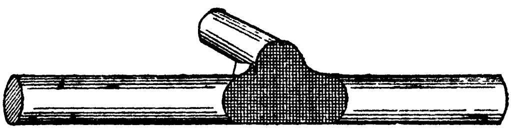


Fig. 91. Cross-section of Weld, Illustrating the Fusing Together of Longitudinal and Transverse Members

(b) Expanded Mesh. Steelcrete. Steelcrete reinforcing fabric is a diamond mesh, cold-drawn from a solid steel plate by the Consolidated Expanded Metal Companies, Wheeling, W. Va. It possesses many of the properties of cold-drawn wire with a guaranteed theoretic elastic limit 60 per cent higher than commercial medium steel.* STEELCRETE MESH is free from mechanical and welded joints, and provides distribution of reinforcement in all directions. It is furnished in large, flat sheets and is much more

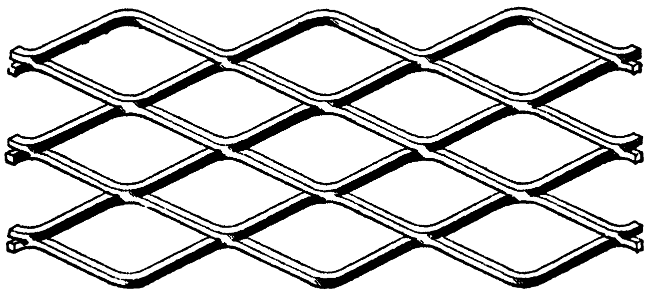


Fig. 92. Expanded Metal, Diamond Mesh

readily handled on the job than rolled wire fabric. Table XIX gives the sizes of sheets and the effective reinforcing area per foot of width of fabric. Expanded metal reinforcement is also furnished by the Genfire Steel Company, Youngstown, Ohio.

(c) Self-Centering. In addition to the laths noted in Article 13 as possessing the necessary characteristics and rigidity to serve as combined centering and reinforcement for concrete floor or roof slabs generally supported on top flanges of steel floor joists or light-weight structural beams, permanent centering or SELF-CENTERING is produced in special forms by many of the

* A. S. T. M. Spec. A15-30.

Table XIX. Standards for "Steelcrete" Expanded Metal

Designation of mesh	Effective sectional area sq in per ft width	Approx. weight per sq ft lb	Standard width of sheets, ft-in	Sq ft in a standard sheet of various lengths 8 ft	10 ft	12 ft	16 ft	Sheets in a standard bundle
3-13-075	.075	25	6-4	50.67	63.33	76.00	101.33	10
3-13- 10	.10	34	7-0	56 00	70.00	84 00	112 00	7
3-13-125	.125	.42	5-8	45 33	56.67	68.00	90.67	7
3- 9- 15	.15	.51	7-0	56 00	70.00	84.00	112.00	5
3- 9-175	.175	.60	6-0	48.00	60.00	72.00	96.00	5
3- 9- 20	.20	.68	5-3	42.00	52.50	63.00	84.00	5
3- 9- 25	25	85	4-2½	33.67	42.08	50.50	67.33	5
3- 9- 30	.30	1.02	7-0	56.00	70.00	84.00	112.00	2
3- 9- 35	.35	1 19	6-0	48.00	60.00	72.00	96.00	2
3- 4- 40	.40	1 36	4-7	36 67	45.83	55.00	73 33	2
3- 4- 45	45	1 53	4-1	32 67	40 83	49 00	65 33	2
3- 4- 50	50	1 70	7-4	58 67	73 33	88.00	117 33	2
3- 4- 55	55	1.87	6-8	53 33	66.67	80.00	106.67	2
3- 4- 60	.60	2.04	6-1½	49 00	61.25	73.50	98.00	2

NOTE. All the above sizes are furnished in a standard diamond 8-in length and approximately 3-in width. All sizes are furnished in stock lengths 8, 12 and 16 ft. In addition, all sizes from 3-13-075 to 3-9-35, inclusive, are furnished in stock lengths of 10 ft.

lath manufacturers. These laths are furnished in either FLAT or SEGMENTAL sheets, pressed into a series of solid ribs, between which the metal is stamped, perforated, or deformed into an open mesh-work. The size of the mesh must not be large enough to permit fairly dry mixes of concrete to leak through. These laths are furnished painted or galvanized, and in open-hearth mild steel or in special copper-bearing or alloy steels.

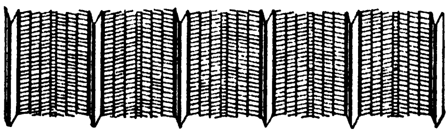


Fig. 93. Section of ¾-in "Ribplex" Sheet

Ribplex, shown in Fig. 93, is manufactured by the Berger Manufacturing Co. of Canton, Ohio, with ¾-in ribs spaced 4.8 in apart, with herringbone-mesh panels between the ribs, all formed cold from the same sheet of metal. The sizes of available sheets and the cross-section area of metal per foot of width of sheet are given in the following table. Under the weight of wet con-

Ribs are ¾ in high spaced 4.8 on centers

Painted Steel ¾-In Ribplex

Gauge No.	Weight per sq ft, lb	Total sect. area per ft, sq ft	Width of sheets, in	Length of sheets for all gauges, ft
28	.50	.1406	24	4, 5, 6, 7, 8, 9 10, 11, and 12
26	.60	.1688	24	
24	.75	.2250	24	

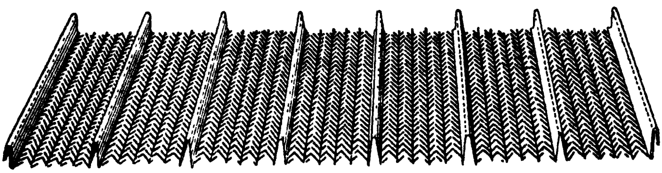
crete, the SELF-CENTERING must be selected of sufficient strength to support the construction loads, or temporary shores or reinforcement must be provided to prevent deflection of the forms. Maximum permissible spans for RIBPLEX under a wet concrete are given in the following table. When the

Maximum Spans with Wet Concrete

Slab thickness......in	2	2½	3	3½	4
Gauge of ¾-in Ribplex	28 26 24	28 26 24	28 26 24	28 26 24	28 26 24
Maximum span....ft	3¼ 3½ 4	3 3¼ 3½	2¾ 3 3	2½ 2¾ 3	2¼ 2½ 2¾

span exceeds these limits, temporary supports must be installed until the concrete sets.

Three-quarter-in Hy-rib is manufactured by the Truscon Steel Co. of Youngstown, Ohio, in the weights and sizes shown in the following table.



¾-in. Hy. Rib

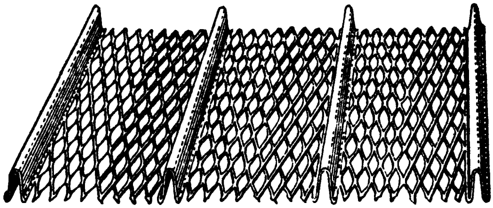
Weight per sq ft, lb	Height of ribs, in	Spacing of ribs, in	Number ribs per sheet	Width of sheet, in	Cross-sectional area per foot width, sq in
.58	¾	4 c. to c.	8	28	.1172
.68	¾	4 c. to c.	8	28	.1407
.78	¾	4 c to c.	8	28	.1642

Lengths of 6, 8, 10 and 12 ft. Packed 8 sheets per bundle.

Fig. 94. Properties of ¾-in Hy-Rib

Self-Sentering is manufactured by the Genfire Company, Youngstown, Ohio, in three sizes as shown in the attached sketch and table, Fig. 95. The ribs are approximately ¾ in high, spaced 3⅝ in on centers.

(d) Deformed Sheet Steel, rolled with ribs or corrugations, as described in Article 10, is applicable as a self-centering reinforcement in short-span concrete slabs built between or on the top flanges of structural steel or light-weight open-joists. The sheets generally should be installed with the flat faces in contact with the bearings and the ribs projected upwards into the slab. The span of the sheet should be limited so as not to produce deflection under the weight of the wet concrete exceeding 1/250 of the span. Safe load tables are furnished by the manufacturers of these deformed plates from which the required gauge of sheet can be selected. The metal, whether of steel or alloy metals, should either be painted on the under-side, galvanized, or otherwise protected for this use. When the corrugations or ribs are of the open type and do not furnish sufficient tensional steel area for reinforcement



Painted steel weight per sq ft, lb	Effective sectional area per ft of width, sq in	Standard lengths, ft
.56	167	8, 10 and 12
.65	.193	8, 10 and 12
.75	.223	8, 10 and 12

Fig. 95. "Self-Sentering'

of the concrete, auxiliary bars must be installed in the corrugations to act as reinforcement, particularly on spans exceeding 30 in. This is true of plates deformed as shown in Figs. 112, C-113, 114, and 115. Practically all of the sheets can be used for self-centering reinforcement in top platform slabs on light-weight open-web steel joists. (See Table XV.)

10. Fire-Resistive Roof-Construction

Flat Roofs. Flat roofs are constructed in the same way as the floors, except that the beams and girders may be set so as to give a slight pitch to the roof to drain the water. As the ROOF-LOADS are usually less than the FLOOR-LOADS and as there are no partitions to be supported, the arches or roof-panels are usually considerably lighter than the floor-panels, but the general construction is practically the same for both. When the roof is formed of reinforced concrete, the beams may be set so that the concrete will give the desired inclination to the roof and will have a nearly uniform thickness, as this reduces the amount of concrete required, and also the weight. In cases where level ceilings are desired, however, it would be cheaper to set the roof-beams level and to grade the roof with dry cinders, as the cost of the hung ceiling would more than offset the cost of the extra construction necessary to take the added weight of cinder fill. If the roof is to be covered with tin or copper, nailing-strips should be embedded in the concrete, as for wooden floors, preferably impregnated with preservative chemicals. Gravel or tile roofs may be built without woodwork of any kind. Whether terra-cotta, gypsum tile, or concrete is used for the roof-panels, the sides and bottoms of the steel beams and girders should be efficiently protected, as well as all columns and all other structural metal in the roof-space. In an ordinary building, in which there are stair-wells or elevator-wells, the roof and upper ceiling are likely to be more severely tested by heat, in case of fire, than any of the floors below, and experience has shown that this part of the building often has the poorest protection.

Pitched Roofs. Pitched roofs may be constructed in various ways, according to the material that is to be used and the kind of roofing that is to be employed. When structural clay or gypsum tile is to be used for the fire protection, the most common method of construction is that which involves the framing of the roof with I-beam rafters and T-iron purlins, set horizontally and spaced 1 in farther apart than the lengths of the tile. Between the tees, book tiles, or roofing-tiles are placed as in Fig. 96, and the roofing is applied

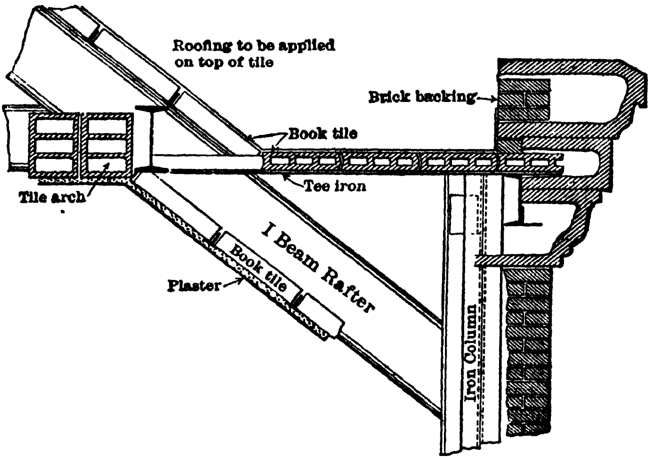


Fig. 96. Tile Fire-proofing for Roof-Construction.

directly to the surface of the tiles. If the roofing is to be of slate or of clay tiles, one of the nailing compounds or a fill with sleepers must be installed above the structural roof. The same construction may be used for flat roofs; but on account of the expense of the tees it will usually be more expensive than the construction above described, and not as strong or desirable. With the construction shown in Fig. 96, it is impossible, by any economical method, to efficiently protect the bottom of the T irons from the effects of heat. Reinforced-cinder concrete, precast gypsum tile, Porete and substitutes of like economy, afford excellent and also economical construction for fire-resistive pitched roofs. Three-inch plates of reinforced concrete have been successfully used in spans of from 6 to 7 ft and in some cases even in 8-ft spans, eliminating the intermediate purlins. The concrete is deposited on wooden centerings, as in the floor-construction, and the upper side is smoothed off during the setting and floated smooth and straight to receive the roof-covering. The roof-covering, usually slate, or clay tiles, may be nailed directly to the concrete, as nails are held nearly as well by cinder concrete as by wood. This applies to cinder concrete or other admixtures of light-weight porous aggregates, as it is quite impossible to nail into rock concrete or gravel concrete. In concrete roofs the rafters, also, should be surrounded with concrete held in place by metal lath. With structural clay roofs, the beams should be incased with structural clay blocks. Fig. 97 shows the standard shape of book tiles. These are made 2, 2½, and 3 in thick, and from 16 to 24 in long. Three-inch book tiles weigh about 13 lb per sq ft, and 2½-

in solid tiles about 16 lb per sq ft. Book tiles are also used for bulkhead and ceiling-construction in store display windows, raised floors for toilets, and in suspended ceilings.

Federal Reinforced-Cement Tiles. Cement tiles of interlocking types, made in the factory and reinforced with metal fabric or mesh, may be laid without sheathing directly on steel purlins. This type of construction, however, is suitable only as a semifire-resisting roof-covering, as it is usually made with plates of insufficient thickness and does not contemplate the thorough incasing of the steel understructure with concrete or other fire-resisting materials.

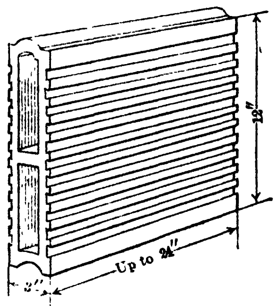


Fig. 97. Book Tile, Two Cell

Federal Interlocking precast concrete roof tile is primarily designed for use on unprotected steel purlins of sloping roofs. This unit is manufactured by the Federal-American Cement Tile Co., Chicago, Ill. The tile are laid with the shoulder hooked over the steel purlins, each tile overlapping the one below it. (See Fig. 98.) The next course is staggered to overlap, the center roll of each tile fitting over the rib of the tile below. A squeeze joint of waterproof elastic cement hermetically seals this overlap. The longitudinal joint is similarly made water-proof. The tiles are colored red with a permanent integral pigment. The STANDARD tile is 52 in long by 24 in wide and $1\frac{1}{8}$ in thick, covering a roof area of 4 ft by 2 ft. Using a purlin spacing of 4 ft, 100 sq ft of roof requires $12\frac{1}{2}$ tiles and weighs 17 lb per sq ft. FEDERAL GLASS TILE are also manufactured incorporating a wire-glass light, 21 by 35 in in area, with an asphalt cushion joint between the glass and the concrete. Each GLASS TILE contains 5 sq ft of glass and is interlocking with the STANDARD RED TILE.

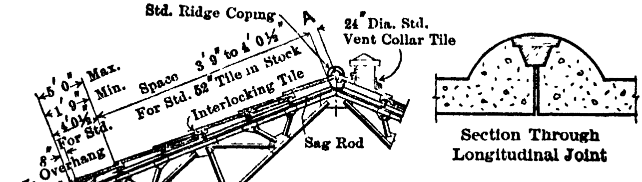


Fig. 98. Federal Interlocking Tile

FEDERAL "LIGHT-SIX" roof slab is manufactured by the same company for use on flat roofs. It is a reinforced channel-shaped slab 24 in wide by 6 ft long in stock size, weighing 16 lb per sq ft. It is designed for a safe live load of 60 lb per sq ft for use on 6-ft spans, but can be secured with a maximum length of 6 ft 6 in. The under-side of the tile is poured with a smooth finish, eliminating the necessity for a suspended ceiling in industrial and manufacturing plants. The top of the slab is covered with any of the waterproof composition roofings.

Federal Nailing Concrete roof-slabs are manufactured for use on roof decks of any slope, and are designed for a live load of 60 lb per sq ft on a beam or purlin spacing of 6 ft. The top surface of the slab presents a smooth surface for application of ornamental tile, slate, copper, or roofing felt. (See Figs. 99 and 100.) The channel section is reinforced with steel rods in the legs and welded wire mesh in the webs. These tile are made of Haydite concrete

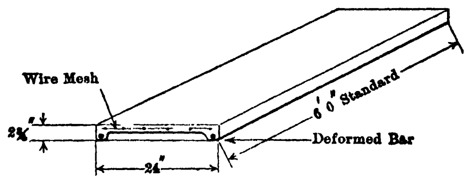


Fig. 99. Federal Flat Roof Tile

with the top section of a porous mixture which takes and holds nails. The tile weigh 20 lb per sq ft. STANDARD LIGHT-WEIGHT TILE, Fig. 101, are 2¾ in thick and weigh 10 lb per sq ft for use on spans up to 6 ft 8 in. All joints are cemented on the upper side with an asphaltic cement and the tile are finished with composition roof-coverings. For longer spans, thicker tile are furnished and weigh correspondingly more. The claims made for FEDERAL PRECAST ROOF-SLABS are: permanence; no maintenance costs; economy

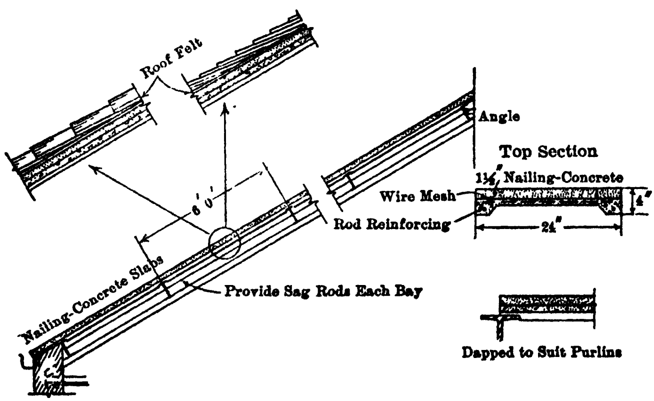


Fig. 100. Details of Featherweight Nailing-Concrete Roof-Slabs

because of light weight; insulating value of porous Haydite concrete; rapidity of erection; and perfect base for finished roofing. Slabs of greater fire-resisting value can be secured with thicker webs. When erected on roof-spans of industrial plants at elevations of 20 ft or more above the floor-level, such that the fire-protection of the truss members can be safely omitted, the STANDARD precast unit will give sufficient fire-resistance for all normal fire hazards. No full-sized standard TIME-TEMPERATURE controlled fire-tests have been made on these constructions.

Porete Roof Tile.* The Porete Manufacturing Co., Newark, N. J., makes a precast roof slab 24 in by 32 in by $1\frac{1}{4}$ in in size for installation on structural ⊥ purlins spaced 33 in on centers. These slabs are reinforced for a live load of 50 lb per sq ft and weigh 7 lb per sq ft. They are secured to the subpurlins with nails or steel wedges driven through holes in the stem of the tees on 18-in centers. After all the slabs are laid, a cement grout is worked into the rough surface and joints, and the entire surface is finished with a thin cement coat. The total weight of the roof including subpurlins and water-proof roofing is 15 lb per sq ft. The advantages claimed for PORETE ROOF DECKS are: light weight; rapidity of erection; high heat-insulation value; and clean, smooth, stone finish on the under-side. A light aerated Portland-cement mortar insulating fill can be poured in place in the

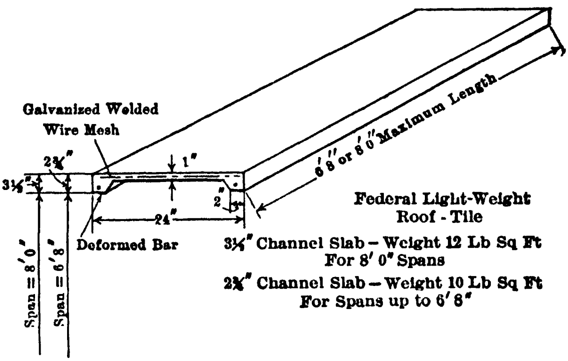


Fig. 101. Federal Channel Slabs

field on top of the roof-slab. This material, weighing 36 lb per cu ft, has a compressive strength of 200 lb per sq in. The weight and porosity of the fill can be predetermined and carefully controlled in the field. It is claimed that approximately $2\frac{1}{2}$ in of this fill equal a 1-in thickness of cork insulation. Where slate, tile or metal roofing is to be applied to the roof deck, PORETE NAIL FINISH can be applied in the field in thicknesses of $\frac{1}{2}$ in or more. The finish forms a tough, resilient sheet which bonds integrally with the Porete slab and adds to its strength. (See Fig. 114.)

Long-Span Porete Slabs are also furnished precast for spans of 4 to 6 ft., varying in thickness from $2\frac{1}{8}$ to 3 in and weighing from 11 to 16 lb per sq ft. These roof tile must be provided with a finish weather-proof coat in the field.

Sheetrock-Pyrofill Roof. This system of roof-construction is sold completely erected by the United States Gypsum Co. It consists of a fibered-gypsum concrete slab reinforced with electric-welded galvanized-steel fabric on a system of permanent SHEETROCK † forms, as shown in Fig. 102. The plaster board forms are supported on subpurlins of standard structural steel ⊥'s or light rail sections spaced $32\frac{1}{2}$ in on centers and clipped to the main purlins. The gypsum-fibered concrete consists of PYROFILL composed of $12\frac{1}{2}$ parts of wood shavings and $87\frac{1}{2}$ parts of calcined gypsum. The welded

* See Properties of Porete, Article 5.

† For a description of Sheetrock wall board, see Article 13.

fabric is composed of No. 12 gauge longitudinal wires 4 in on centers and No. 14 gauge transverse wires 8 in on centers laid on top of the plaster board. The construction is poured in place to a total thickness of 2½ or 3 in, and weighs with the water-proofing 15 lb per sq ft, including the subpurlins. On roofs with a pitch of 45° or over, it is necessary to back form from the top,

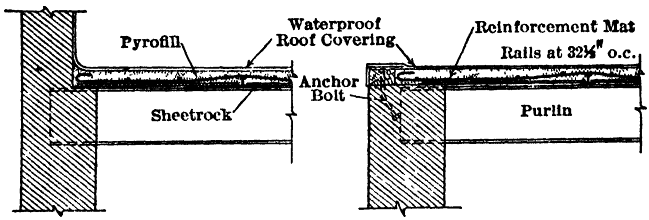


Fig. 102. Sheetrock-Pyrofill Roof-Slab

pouring the slab in sections. With subpurlin spacing of 32½ in, the weight of slab and rail sizes required for varying spans between main purlins are given in the following tabulation. No provision is made for protecting the

Span of main purl ns	Size of rail, lb	Weight of rail per sq ft of roof, lb	* Minimum thickness of slab, in	Weight of slab including rail, lb
l p to 6′-4″ . ..	8	1 00	2½	11 50
6′-4″ to 8′-11″	12	1.50	2½	12.00
8′-11″ to 11′-3″... ...	16	2.00	2½	12 50
11′-3″ to 13′-2″.....	20	2.50	3	15.00

* Includes thickness of Sheetrock.

lower flanges of the purlins with this system. It is intended primarily for use in manufacturing plants and industrial buildings generally, with roof-trusses erected at some distances above the floor-level. The system is designed on the theory of a reinforced composite slab similar to the PYROFILL FLOOR-SLAB.* A ¾-in furred gypsum-plaster ceiling on metal lath suspended from the lower chord of the roof-truss will furnish a one-hour fire-resistance to the construction. It is also used for roofs of school-buildings, auditoriums, gymnasiums, theaters, hospitals and hotels. The advantages claimed for Sheetrock-Pyrofill roofs are: light weight; high insulation; reduction of heat-losses in winter and maintenance of low temperatures in summer; low construction costs; and minimum upkeep charges. In buildings where high humidity conditions combined with high temperatures prevail, instances of deterioration of gypsum roofs have been noted, resulting in a gradual decomposition of the gypsum mixture. Where such deterioration has occurred, there are indications that too much water was used in the mixture and the slab was poured with too thin consistency, resulting in a porous construction. A load test was made on a 3-in SHEETROCK-PYROFILL slab one hour after it was poured on a 32-in span for the New York Bureau of Buildings. The

* See Pyrofill Monolithic Floor Construction, Article 7.

slab was loaded to 148 lb per sq ft, or three times the design load, without any deflection or cracking. A duplicate slab thoroughly cured developed an ultimate strength of 499 lb per sq ft of load. The test demonstrated that saturation of the slab by rain or snow or from a leaky roof would not result in a dangerous reduction of strength.

To secure greater fire-resistance, the PYROFILL ROOF-SLAB can be poured on temporary wood forms suspended ¾ in below the bottom of the subpur-

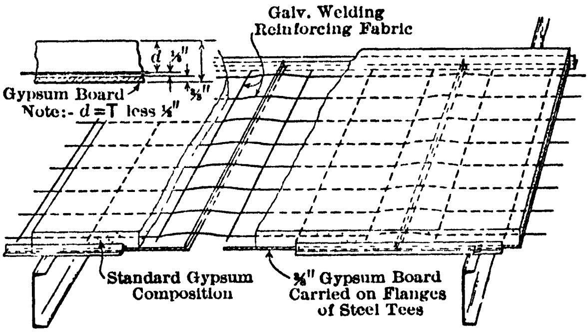
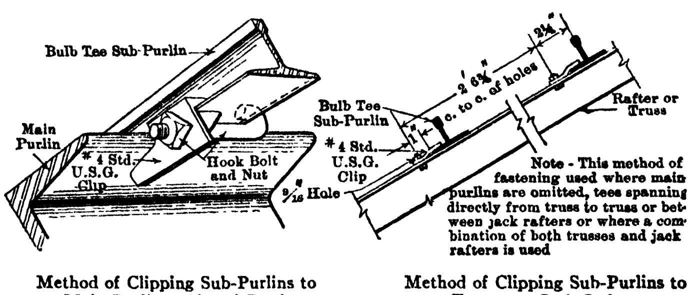


Fig. 103. Marks-T-System Poured-in-Place Roof-Construction

lins, and the main purlins can be encased with Pyrofill at the same time. The construction has then the same appearance and virtually the same fire-resistive rating as the MONOLITHIC SUSPENSION SLAB.*

Marks-T-System, Fig. 103, consists of a permanent centering of ⅜-in gypsum boards set on ⊥ irons spaced 2 ft 8 in on centers with cross tees 2 ft 8 in long resting on the main longitudinal tees at intervals of 3 ft to support



Method of Clipping Sub-Purlins to Main Purlins—Sloped Roof

Method of Clipping Sub-Purlins to Trusses or Jack Rafters

Fig. 104 Details of Tees and Short-Span, 30" Type, Pyrobar Roof-Tile

the gypsum boards on all sides. After a row of the panel centering has been installed, rectangular welded-wire reinforcement, 4 in by 12 in mesh is unrolled over the cross-tees and the gypsum-fibered concrete is poured in place. The MARKS-T-SYSTEM is designed on the same principles of flexure as the composite beam of concrete reinforced with steel assuming a fiber-stress in com-

* See The Suspension System, Article 7.

pression for the concrete and the usual steel stresses. No allowance is made for the structural tees in computing the safe loads. This system is controlled by the U. S. Gypsum Co., Chicago. Among the advantages claimed for this ROOF-CONSTRUCTION are: a clear-paneled ceiling insuring good light reflection; high heat-insulation value due to uniform thickness of slab above the purlins; low first cost; and rapidity of construction. The MARKS-T-SYSTEM possesses the same characteristics and adaptability as the SHEETROCK-PYROFILL construction.*

Pyrobar Precast Roof. Shop-made roof tile of quick-setting gypsum are manufactured by the U. S. Gypsum Company, Chicago, Ill. Both SHORT-SPAN and LONG-SPAN tile are furnished, as shown in Figs. 105 and 106.

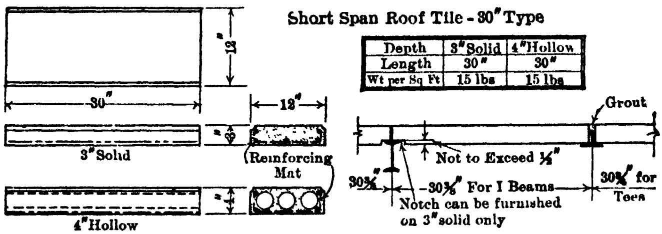


Fig. 105. Details of Short Span Pyrobar Roof-Tile. All Tile Reinforced with Electrically Welded Galvanized Steel Mat

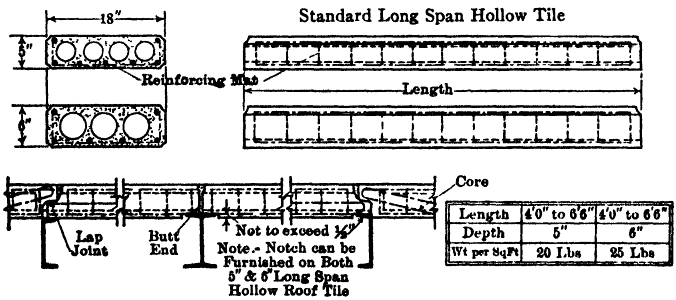


Fig. 106. Details of Long Span Pyrobar Precast Roof-Tile

SHORT-SPAN PYROBAR TILE are made either SOLID or HOLLOW. The SOLID TILE, 3 in thick, and the HOLLOW TILE, 4 in thick, each weigh 17-lb per sq ft. For pitched roofs, the solid tile can be notched at the ends to take the thrust. They are also furnished with HOOKED ENDS for use with 16 or 20-lb rail purlins.

LONG-SPAN PYROBAR TILE are made hollow in two thicknesses, 18 in in width, and for spans of 4 ft to 6 ft 6 in long. The 5-in thick tile weighs 20 lb per sq ft, and the 6-in thick tile weighs 25 lb per sq ft. Both tile are made BUTT END for I-beam purlins, and LAPP END for [purlins, and can also be notched for thrust on pitched roofs.

* See Sheetrock-Pyrofill Roof, Article 10.

ALL PYROBAR ROOF-TILES are designed for a safe superimposed load of 50 lb per sq ft. The 3-in SOLID tile is especially adaptable for nailing purposes; the HOLLOW tile is recommended for use in connection with a built-up roof-covering. The reinforcement consists of welded, galvanized fabric of 4 by 4 in mesh with No. 12 longitudinal and No. 14 cross-wires. The subpurlins for the SHORT-SPAN tile are usually bulb tees of special wide flanges. The tile are laid directly on the subpurlins without mortar and with tightly butted sides. The grouting joints are then filled with gypsum mortar and the roof is immediately ready for the application of the water-proof covering.

Gypsteel Roof-Construction. The Structural Gypsum Corporation, Linden, N. J., furnishes both POURED-IN-PLACE and PRECAST gypsum roof-constructions. The POURED-IN-PLACE system is of the same general character and design as the SUSPENSION TYPE floor-system. The PRECAST slabs are furnished in LONG-SPAN construction for spans from 4 to 7 ft, varying in length by 3 in, and in SHORT-SPAN construction for fixed spans between subpurlins of 30½ in.

The LONG-SPAN roof-slab is furnished in widths of 18, 21 and 24 in, and in thicknesses of 3 and 3½ in, weighing respectively 14 and 17 lb per sq ft. The reinforcement consists of $\frac{3}{16}$ in cold-drawn suspension wires, projecting 2½ in at both ends of the tile, which are rabbeted at the ends on the top surface where the reinforcing cables emerge. The supporting purlins are placed parallel with the top flanges in the same plane. When the top flanges are less than 2½ in wide, bearing-plates must be provided for temporary support of the slabs until the ends of the suspension cables are tied up and grouted. These plates are furnished with the GYPSTEEL slabs. Angle or tie-rod bracing must be installed in end or anchor spans to resist the cable tension. For pitched roofs of slopes greater than 1 in 3, stop angles must be framed on the lowest purlin to take the side thrust, with a minimum bearing of 3½ in on the slab.

The SHORT-SPAN roof-tile are made either SOLID in thicknesses of 2½, 3 or 3½ in, or HOLLOW, 3 in thick. The 2½-in slabs are furnished 29¾ in long by 24 in wide, and the 3 and 3½-in solid slabs are furnished 30 in long by 24 in wide. The HOLLOW SLABS are 30 in long by 12 in wide. The tile are set directly upon the flanges of the subpurlins, and the joints formed by the beveled edges are flushed solid with gypsum grout. The weights of these tile are as follows:

3-in hollow	11 lb per sq ft
2½-in solid	12 lb per sq ft
3-in solid	14 lb per sq ft
3½-in solid	17 lb per sq ft

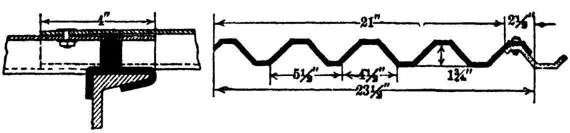
Nailcrete Roof. A fire-resistive roof-construction made of a poured-in-place mixture of Portland cement, sand and mineral fiber binder is furnished by the Nailcrete Corporation, New York, for use as a light-weight structural roof-slab on spans not exceeding 6 ft and a slab thickness of 4 in. It is installed between structural steel beams similarly to the types described under Strength of Short-Span Flat Floor-Construction, Article 7, reinforced with wire mesh consisting of No. 6 gauge longitudinals 2 in on centers, and No. 10 gauge transverse wires 16 in on centers, for a superimposed load of 60 lb per sq ft. It has the compressive strength of 1 : 2 : 6 cinder concrete and weighs 96 lb per cu ft. In addition to its fire-resistance, NAILCRETE provides a nailing base to which tile, slate or metal roofing can be directly attached. It is poured like any other Portland-cement concrete and is subject to the same

precautionary measures. From 2 to 3 days are required for the mixture to set hard enough to walk on, depending upon temperature and weather conditions. It can also be used as a structural platform slab over light floor-beams or open-web steel joists.*

A similar material is manufactured by the National Naylegrip Company, Inc., Erie, Pa., under the trade name "NAYLEGRIP." These materials come packed in bags ready for mixing in the field with Portland cement and sand or cinders. They are also used for floor-fill and nailing bases on structural slabs and on masonry walls.

Steel Roof-Decks are coming into wide use on one-story manufacturing plants and industrial buildings generally, because of their adaptability, ease of erection, and low cost. These steel-roof-plates form a rigid and permanent base to which insulating and water-proofing coats can be readily applied. They are fabricated from protected, copper-bearing, or galvanized-steel sheets reinforced by various shaped ribs, rolled into the sheet, and interlocking and dovetailing together both on sides and ends. The deformed sheets are readily and quickly erected on the steel roof-trusses and are either anchored by special clips or welded to the roof-purlins. ROOF-DECK PLATES can be classified into three general groups: CORRUGATED, RIB, and V-BEAM TYPE. They are also used to some extent for the siding of isolated industrial plants,† but must be specially processed or insulated to minimize heat losses, or keep out cold, and to deaden noise. The construction is incombustible and when protected with heat-resisting materials possesses some degree of fire-resistance. No full-size time-temperature controlled fire-tests are available; but as this type of construction is generally installed on unprotected steel frames, its strength and incombustibility are sufficient qualifications.

ROBERTSON PROTECTED METAL,‡ fabricated by the H. H. Robertson Company, Pittsburgh, Pa., is formed with STANDARD CORRUGATIONS characteristic of other corrugated metal sheets, and also in deeper corrugations known as the V-BEAM shape. It is fastened to the steel framework of buildings by metal straps or rivet clips, or to wood framework by nails and screws. The straps are also made of the special patented protected metal, and are as durable as the roofing itself. The V-beam sheet is also largely used as a permanent form for built-up roofs. Insulating materials of the desired thick-



Method of Fastening Robertson V-Beam Sheets to Roof-Purlins

Fig. 107. Robertson V-Beam Roof Deck

ness are applied to the top of the sheets and covered with the weather-proof coatings. The corrosion-resisting and non-maintenance characteristics of this metal offer great advantages to its use for this purpose. When a high degree of insulating value is required, the PROTECTED METAL can be used in a double layer, with a core of insulating materials. By this construction, the roof can be given the heat-insulating value of masonry floors or walls at

* See Structural Platform Slabs, Article 7

† See Metal Enclosure Walls, Article 12.

‡ See Robertson Protected Metal, Article 5.

considerably less cost. The V-beam sheets are available for a 30-lb per sq ft superimposed load with a limiting deflection of 1/100 of the span-length in the following gauges:

Length of span, ft	in	V-beam sheet Gauge	Length, ft
4	9	24	10
5	3	24	11
5	9	24	12
6	6	24	12
7	6	22	12
8	6	20	12
9	6	18	12

In a series of Flame Exposure, Burning Brand, Spread and Radiation tests at the Underwriters' Laboratories,* this material used as a covering over wood-decking demonstrated that it "provides a substantial barrier to the passage of flame, affords a fairly high degree of heat-insulation to the roof-deck, and will effectively resist large burning brands and pieces of red-hot iron . . . "

Genfire Steeldeck is manufactured by the Genfire Steel Co., Youngstown, Ohio, in three standard sections of the RIB TYPE, as shown in Fig. 108.

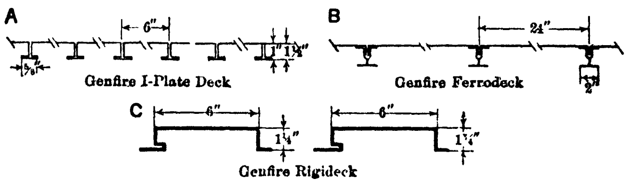


Fig. 108. Genfire Steel Deck

"I-PLATE" deck is manufactured of Nos. 18 and 20 gauge copper-bearing steel in standard lengths of 8 and 10 ft and 24 in wide for the 1-in depth, and 18 in wide for the 1½-in depth. The sheets interlock and are secured together by sleeve splice clips that telescope over the ends of the I-section and wrap around the roof-purlins.

"**Ferrodeck**" is an assembly of 18-gauge subpurlins of channel sections spot-welded back to back and roof sheets of No. 18 or 20 gauge metal with flanged edges that fit into the open slot of the subpurlins. The subpurlins are secured to the steel frame with spring clips and the roof sheets are locked to the subpurlins with reinforcing members attached to the sheets at intervals of 2 ft, and bent down under the top flanges of the subpurlins. The roof sheets are 2 ft wide by 4 ft long, and special shorter sheets are furnished where roof lengths are not multiples of 4 ft. The flat and smooth top surface of the deck receives the layer of insulation which is installed in thicknesses of ½ to 2 in, and a covering of bituminous waterproofing is applied to furnish the necessary weather-resistance. The thickness of insulation

* Fire Retardant Report No. 1727.

used is determined by the required air conditions in the building, by the amount necessary to prevent condensation under these conditions, and by the capacity of the heating equipment installed.

"**Rigideck**" is rolled in one interlocking piece of No. 18 or 20 gauge Armco Ingot Iron with a finished width of 6 in and depths of 1¼ and 1¾ in. It is fastened to the structural purlins with clips at 6-in centers that also lock the adjoining sections together. Rigideck, insulated and waterproofed, is adaptable to flat or pitched as well as curved roofs with a minimum radius of 40 ft.

When the pitch of roofs exceeds 1 in 4, the insulation and waterproofing must be secured to the steel decks with metal cleats in the ribs and copper nails through the roofing to prevent movement of the coverings. Soft copper nails are generally used to fasten the built-up roofing to the insulation, which, when driven, turn and clinch on the steel deck. The dead load of these roof-decks including 1 in of insulation and waterproofing is 5 to 6 lb per sq ft. The safe superimposed loads and spans shown in the following tabulation are based on a steel stress of 18 000 lb per sq in, and an allowable deflection of 1/250 of the span. Similar roof-deck sections are furnished by the Truscon Steel Company of Youngstown, Ohio.

Spans for Roof-Deck Sections

Maximum Spans

Type	Live load—lb per sq ft	
	30	40
I Plate 20 G × 1 in	6 ft 0 in	
I Plate 20 G × 1½ in. . . .	7 ft 0 in	6 ft 6 in
I Plate 18 G × 1½ in	7 ft 6 in	7 ft 0 in
Ferrodeck.	6 ft 5 in	5 ft 9 in
Rigideck 20 G × 1¼ in.	6 ft 6 in	6 ft 0 in
Rigideck 20 G × 1¾ in..	8 ft 6 in	8 ft 0 in
Rigideck 18 G × 1¾ in	9 ft 0 in	8 ft 6 in

Blawsteel Deck, manufactured by the Blaw-Knox Company, Pittsburgh, Pa., is of a combination RIB and V-BEAM TYPE, furnished for spans of 3 ft

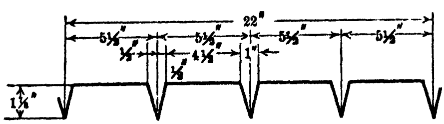


Fig. 109. Blawsteel Long-Span Roof-Sheathing

10½ in to 4 ft 10½ in, with a depth of 1¼ in; and for spans up to 8 ft 6 in with a depth of 1⅞ in. It is made of Nos. 18, 20 and 24 gauge galvanized

steel sheets in widths of 22 in as shown in Fig. 109. The maximum spans for live loads of 30 and 40 lb per sq ft are shown in the following tabulation:

Maximum Spans

Gauge No.	Depth, inches	Live load, lb per sq ft	
		30	40
24 .	1¼	3 ft 10½ in	3 ft 10½ in
20	1⅞	7 ft 6 in	7 ft 0 in
18	1⅞	8 ft 6 in	7 ft 6 in

Mahon Steel Deck, manufactured by the R. C. Mahon Company, Detroit, Mich., is provided in Nos. 18 and 20 gauge copper-bearing steel sheets of the RIB TYPE as shown in Fig. 110. The sheets lay 12 in wide with 1½-in

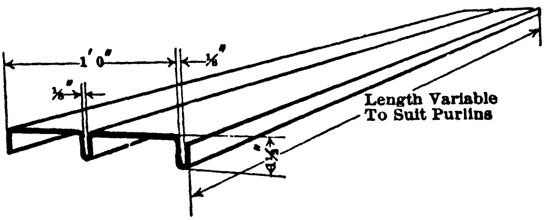


Fig. 110. Standard Mahon Roof-Deck Plate

ribs spaced 6 in on centers, and are furnished in any length up to 12 ft 4 in. Allowance for lateral expansion and contraction is made in the center rib of each sheet. The 18 gauge sheets weigh 3.25 lb, and the 20 gauge, 2.5 lb per sq ft. STANDARD-LENGTHS in stock are 8 ft 2 in for the No 16 gauge sheets; and 6 ft 2 in, 7 ft 2 in, 8 ft 4 in, 10 ft 4 in, 11 ft 6 in, and 12 ft 4 in, for the No. 20 gauge sheets. The gauge of metal deck, span, and deflection for superimposed loads of 30 and 40 lb per sq ft are shown in the following tabulation. A similar roof-deck of the RIB TYPE is manufactured by the St. Paul Corrugating Co., St. Paul, Minn.

Maximum Spans

Gauge No.	Live load, lb per sq ft		
	Depth, inches	30	40
20.	1½	6 ft 6 in	6 ft 0 in
18.	1½	7 ft 0 in	6 ft 6 in

Ribsteel Deck is furnished by the Porete Manufacturing Co., Newark, N. J., of 20 to 24 gauge copper-bearing steel, and is designed for use with

a light-weight concrete fill. The ribs are $1\frac{1}{2}$ and 2 in deep, spaced 6 in on centers as shown in Fig. 111. If built-up roofing is used, the concrete fill is 1 in thick; if to be used as a nailing base for tile, slate or metal roofing,

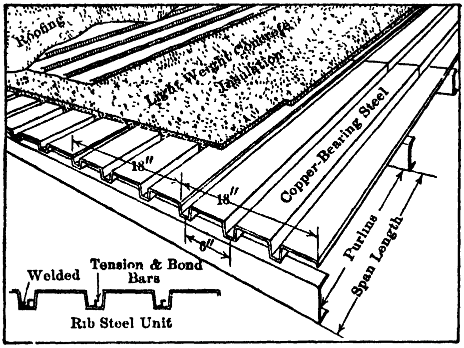


Fig. 111. Rib-Steel Roof-Slab Construction

the concrete fill is $1\frac{1}{2}$ in deep. Tension steel is provided in the bottom of the ribs of ⅃⎾ bar shape, welded to the steel plate. The advantages claimed for this construction are: rapid erection; light weight; economy; ease of application on steep roofs; and nailable base if used for floor-construction. Table XX shows the safe superimposed loads for different gauges of metal and depths of corrugations.

Table XX. Recommended Construction Specifications for Given Span of Rib Steel Roofs and Floors

Span, in feet	4	6	8	10	12
Gauge Corrugations, in deep	24 $1\frac{1}{2}$	24 $1\frac{1}{2}$	22 $1\frac{1}{2}$	22 2	20 2
Safe live load, lb per sq ft with 1-in solid cement finish (dead load 15–20 lb per sq ft)	120	90	65	55	45
Safe live load, lb per sq ft with $1\frac{1}{2}$-in light-weight nailable-concrete finish (dead load 9–15 lb per sq ft)	120	75	50	...	....

Holorib, manufactured by the Detroit Steel Products Company, Detroit, Mich., is a V-BEAM TYPE roof-deck of copper-bearing steel sheets with closed triangular ribs forming a complete girder. It is applicable to flat, pitched or curved roofs. It is made of Nos. 24 and 26 gauge sheets with ribs spaced

3¼ in on centers and ¾ in deep for spans up to 5 ft; and of Nos. 20 and 22 gauge sheets with ribs spaced 6 in on centers, and 1½ in deep for spans up to 8 ft, as shown in Fig. 112. All HOLORIB sheets are coated with an oven-baked gray paint. The sheets can be cut to special lengths at the factory,

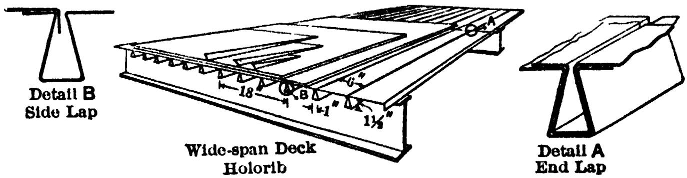


Fig. 112. Holorib Roof Deck

but the standard width is 18 in. The spans and gauges of roof-deck for live loads of 30 and 40 lb per sq ft are given in the following tabulation:

Maximum Spans

Gauge No.	Rib size, inches	Limiting spans, live load, lb sq ft	
		30	40
26	¾	3 ft 0 in	3 ft 0 in
24	¾	4 ft 6 in	4 ft 6 in
22 .	1½	7 ft 0 in	6 ft 3 in
20. . . .	1½	8 ft 0 in	7 ft 0 in

Insulation Data. The purpose of the roof-insulation is to prevent heat loss and condensation, to effect economy in fuel and heating equipment, and to limit excessive expansion and contraction. The available commercial materials for insulation purposes are of three general types: SEMI-FLEXIBLE materials, including flax and rye fibers; SEMI-RIGID materials, including cork, rock wool and straw pulp; and STIFF FIBROUS materials, including wood pulp, root and sugar-cane fiber. They are all available in sheet or board form in thicknesses varying from ½ to 1½ in. With the thin, light-weight, steel roof-deck, some form of insulation is necessary to minimize the transfer of heat and to prolong the life of the construction.

Mansard Roofs are usually framed with rafters, riveted or bolted to wall-plates. The space between the rafters may be filled with cinder concrete, fire-resistive nailing compounds, or hollow partition-tiles, or blocks extending from rafter to rafter. Slates or tiles may be nailed directly to cinder concrete or to such nailing bases. Probably the best way to attach slates or tiles is to nail 1¼ by 2-in wooden strips to the outer face of the concrete or structural clay tile, set them at the proper distances apart to receive the slates or tiles, and then plaster between the strips with cement mortar. The

wooden strips are not affected by fire until the slate is practically destroyed. Figs. 113 and 114 show mansard and tower roof-constructions with clay

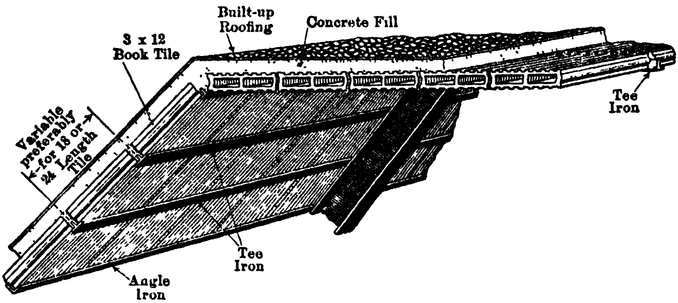


Fig. 113. Standard Book Tile in the Construction of Light-weight Fire-proof Roofs

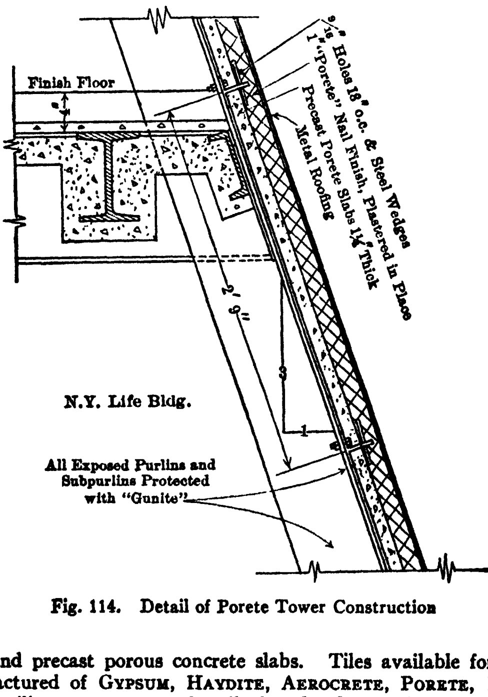


Fig. 114. Detail of Porete Tower Construction

book tile and precast porous concrete slabs. Tiles available for such uses are manufactured of Gypsum, Haydite, Aerocrete, Porete, Nailcrete and other nailing concretes, as described under floor and roof-constructions.

11. Roof-Coverings

Roof-Coverings. The materials ordinarily used for the roof-covering of fire-resistive buildings are: (1) tar and gravel; (2) asphalt and gravel or sand; (3) vitrified tiles, bricks, or slate tiles over tarred felt. Tar and gravel, or asphalt felting and gravel or sand, offer the cheapest roof suitable for a fire-resistive building; and when a good quality of felt and distilled pitch or the best grades of asphalt are used, make a very satisfactory covering. Such roofs, however, require to be renewed every ten to twenty years. The roofing is put on in the same manner as over wooden construction, the felt being laid directly on the concrete. Probably the best flat roof that can be put on a building is one of vitrified or slate tiles, laid over five plys of tarred felt. The felt is laid and mopped as for a gravel roof, and the tiles are bedded on the felt in cement mortar, preferably reinforced with light wire mesh. Vitrified tiles, 6 or 8 in square and 1 to 1½ in thick, are made for this purpose, and slate tiles, 12 in square by 1 in thick, have been used. Flat, vitrified-brick tiles, also, are used. Gravel roofing should not be used on roofs which have an inclination exceeding ¾ in in 1 ft. For pitched or inclined roofs, slates, clay, asbestos, concrete, or metal tiles may be used. Clay tiles are superior to slate when exposed to fire and are generally to be preferred to slate; this is especially true of some of the patent interlocking tiles.

Tests. The Bureau of Standards has been conducting an extensive investigation of the relative merits of different ROOF COVERINGS for several years. The tests commonly applied include FLAME and BRAND tests to determine the resistance of the materials to heat, flame penetration, spread of fire, and absence of brand production. All approved fire-resistive roof coverings have been classified by the Underwriters' Laboratories as Class *A*, *B*, or *C* as a result of similar tests; and building codes in general approve the two first classes for use in fire-resistive construction.

12. Fire-Resistive Wall Assemblies

Classification. The spread of fire to or through a building is prevented in part by building a predetermined degree of fire-resistance into the enclosure walls and interior division walls of the structure. The ultimate object of the division and enclosure-wall-construction is to restrict the building to areas in which a fire can be confined or controlled or even permitted to completely burn out the contents without endangering the structural integrity of the building or without being communicated to other areas or other buildings. The tendency in modern building regulation is to define such assemblies of this character on the basis of actual performance standards rather than in terms of minimum dimensions and materials. A FIRE-WALL is one which subdivides a building into restricted areas, extending continuously from foundation through all stories to the roof, and generally required to have a fire-resistive rating of four hour. Underwriters' and building code requirements restrict the number and size of openings in fire-walls and require approved FIRE-DOORS * at both sides of the opening. The ENCLOSURE WALL is generally required to have a fire-resistant rating of 4 or 3 hours and must be constructed of approved masonry. Underwriters' rating schedules are usually based on the use of the solid brick wall construction as a standard to which all other types of walls are compared and rated in accordance with the relative fire-resistance. The New York Insurance Exchange within recent years

* See Opening Protectives, Article 20.

has accepted the "Combination Wall" * as the equivalent of the solid brick wall in skeleton frame buildings of unlimited height. The FIRE-DIVISION WALL is a subdividing assembly used in any one or more stories of a building, usually to provide an area of refuge in connection with exit facilities and to restrict the spread of fire in that story. The fire-resistant rating for such walls is variously specified as two-hour or three-hour construction in building codes, including in this classification vertical shaft and frequently stairway enclosures. The FIRE-PROOF PARTITION sometimes designated "FIRE-DIVISION PARTITION" is the partition used for subdividing the interior space in any story, such as individual office spaces and communicating corridors. The general requirement for this assembly is a fire-resistant rating of one hour.

Metal Enclosure Walls. For isolated industrial buildings in outlying undeveloped districts, or even within urban areas for miscellaneous one-story buildings of limited dimensions, enclosures of metal or other incombustible substitutes are commonly acceptable.†

The Philadelphia Building Code (1929) specifies that "Buildings with structural steel columns and floor and roof systems not fireproofed or fire-protected from the inside, shall be considered FIRST-CLASS BUILDINGS. Steel roof-decking, properly insulated and weatherproofed, may be used in such buildings or in one-story buildings of commercial occupancy [except manufacture of explosives or storage of highly inflammable materials] without fire-proofing or fire-protection on the outside, provided the building is separated on all sides from other structures by not less than twenty-five feet."

Masonry Fire-Wall Requirements. The fire-resistive requirements for fire-walls and partitions are essentially the same as for other structural units. They must withstand the attack of fire without disintegration or failure, prevent the ignition of combustible contents in the building or protected area by transmitted heat, and they must retain their structural integrity without dangerous distortion or failure in whole or in part. In addition the assembly must withstand the possible blow of falling bodies and the destructive effects of fire hose-streams applied while the walls are hot or still under fire. For exterior walls, the structural and weather-resistive requirements are frequently the controlling factors determining the thickness and character of the materials, rather than the fire-resistance. Available fire-tests of walls cover a wide range of materials and types of construction. Numerous tests have been conducted by the Underwriters' Laboratories, covered in the reports of the various fire-retardants, and by the Fire Resistive Section of the U. S. Bureau of Standards. Many of these fire-tests were made under rated load-bearing capacity, and include walls of brick, structural clay tile, concrete block and tile and numerous proprietary forms of construction. The time performance period of the various assemblies has been determined either by failure under load or by the time period required to reach the CRITICAL TEMPERATURE end-point as established by the Standard Fire Test of not more than 250° F. temperature rise above its initial temperature on the unexposed surface of the wall or partition.

Underwriters' Fire-Retardant Classification. In the classifications issued by the Underwriters' Laboratories, the performance periods are modified by a factor of safety to allow for prevention of damage to structural members by fire and the consequent expense of repair or replacement. Classifications based on this principle using a factor of 1¼ have been issued for Clay Brick

* See Clay Tile Combination Walls, Article 12.
† See Robertson Protected Metal, Article 5.

Walls under date of Aug. 3, 1927, Hollow Concrete Building Units under date of June 12, 1928, and Hollow Clay Tile Units under date of June 14, 1929. Table XXI gives a digest of these fire-retardant ratings. Merely from a consideration of safe construction, these ratings might be considered too severe.*

Table XXI. Underwriters' Classification.* Wall Assemblies

Material	Thickness, in	Description	Fire-Resistive Rating, hrs. Combustible floor members	Fire-Resistive Rating, hrs. Incombustible floor members
Clay brick	4	Non-bearing	1	.
Clay brick	8	Bearing or non-bearing	2	5
Clay brick	12	Bearing or non-bearing	9	9
Hollow concrete units	8	Bearing or non-bearing	1	3
Hollow concrete units	12	Bearing or non-bearing	1–3	3–6
Hollow clay tile A.S.T.M. spec C34–27	8	2-cell	1	1¾
		3-cell	1¼	2¼
Hollow clay tile A.S.T.M. spec. C34–27	12	3-cell, single unit	2¾	3
		3-cell, 2 units	3½	4
		4-cell, single unit	4	6

NOTE. Three-quarters inch Portland cement or gypsum plaster or Portland-cement stucco on one side will increase classifications one-half hour.

* Underwriters' Fire Retardant Classifications of Wall Assemblies, August, 1929.

U. S. Bureau of Standards' Classifications. Common Clay Brick. As a result of a series of tests of 26 large and 22 small wall panel fire-tests and 7 fire and water tests, including BRICK from SHALE and SURFACE CLAY, solid and "Rolok" hollow design, with and without load,† the U. S. Bureau of Standards issued a summary of fire-resistive ratings for brick walls given in Table XXII. Some of the 4-in walls failed from excessive deflection or under applied working loads, but not until the critical temperature transmission had been exceeded. Most of the thicker walls failed in temperature transmission in the time periods stated. One 12-in brick wall failed under the working load after 10 hours of fire due to the fusion-point of the clay resulting in a fluxing of the brick. (Temperature of fire approximately 2 500° F.) FIRE damage to the brick consisted of cracks from $\frac{1}{16}$ to $\frac{3}{8}$ in wide on the unexposed side, but not extending through the wall; headers were seldom found cracked and the structural integrity of the wall was maintained. Shale brick were more susceptible to cracking than the clay. "Walls laid up with lime mortar had lower deflections and fewer and smaller cracks than those laid up in cement or cement-lime mortar."

Sand-Lime and Concrete Brick. Tests on large and small panels of sand-lime and concrete brick, with and without load, developed results shown in

* See discussion on Column Tests, Article 6.

† Letter Circular 228 U. S. Bureau of Standards, June 8, 1927.

Table XXIII.* For restrained walls, deflection toward the fire averaged from 1¼ in to 2 in at the center. For unrestrained walls, the maximum deflection occurred at the top of the walls away from the fire, exceeding 9 in for the 8-in walls at the end of 6 hours. The behavior of the walls and the fire effects were similar to the clay brick walls.

Table XXII. Bureau of Standards' Classification.* Wall Assemblies of Common Clay Brick

Type	Thickness, in	Description	Ultimate resistance periods, hrs: Combustible floor members	Ultimate resistance periods, hrs: Incombustible floor members
Solid	4	Unplastered	1	Non-bearing
		Plastered both sides	2½	Non-bearing
Solid	8	Unplastered	2	5
		Plastered both sides	4	9
Solid	12	Unplastered	9	10
Hollow rolok	8	Unplastered, hollow space filled at floor-line	2	2½
Hollow rolok	8	Plastered both sides, filled at floor-line	4	5
		Plastered both sides, unfilled at floor-line	2½	5
Hollow rolok	12	Unplastered, filled at floor-line	5	5
		Unplastered, unfilled	3½	5
Hollow rolok	12	Plastered both sides, filled at floor-line	9	9
		Plastered both sides, unfilled	6	9
Brick-faced	8	Plastered both sides, filled at floor-line	4	5
		Unplastered both sides, unfilled	2½	5
Brick-faced	12	Unplastered, filled at floor-line	9	10
		Unplastered, unfilled	6	10

Note. All plaster coats ½-in thick of neat gypsum or Portland-cement mortar, 1 part cement to 3 parts sand.

* Bureau of Standards, Letter Circular 228.

Structural Clay-Tile. From 1921 to 1927 the U. S. Bureau of Standards in cooperation with the Hollow Building Tile Association conducted an exhaustive series of fire-tests on bearing and non-bearing structural clay-tile

* Letter Circular 229 U. S. Bureau of Standards, June 8, 1927.

Table XXIII. Bureau of Standards' Classification.* Wall Assemblies of Concrete and Sand-Lime Brick

Type	Thickness, in	Description	Ultimate resistance periods, hrs	
			Combustible members	Incombustible members
Concrete	8	Unplastered.	3	6½
Concrete	12	Unplastered.	12½	15
Sand-lime	8	Unplastered.	3½	8
Sand-lime	12	Unplastered.	10	10

NOTE. For periods not exceeding 6 hours, Portland cement or lime mortar can be used. For longer fire-resistance periods, only Portland cement or cement-lime mortar should be permitted from the fire-resistant point of view, and not as regards structural strength.

walls and partitions.† The test specimens for the large-size tests represented tile made from shales, fire-clays, mixtures of shale and fire-clay, and surface clays, secured from 8 manufacturing plants throughout the country, as representative of 40 different natural sources investigated. In the Load-Bearing Series, 167 fire-endurance tests and 4 fire and water tests were conducted on test panels representing end and side construction with varying number of cells through the thickness of the walls, special types of tiles with double-shell walls, varying thicknesses of walls and various special types of construction. Of these tests, 113 were made on 8-in walls, 36 on 12-in walls and 18 on 16-in walls, both plastered and unplastered. Shale and the hard-burning fire-clays produced walls of higher strength than those of less dense tile, but suffered greater fire damage. Tile made from open-burning fire-clays and most surface clays gave walls of less high initial strength but which suffered less from fire damage; and the shells of such tile showed a tendency to remain in place even after cracking. The fire-resistive rating for the wall assemblies was generally determined by the time required to meet the CRITICAL-TEMPERATURE rise of 250° F. on the unexposed side of the wall specimen, and the tests indicated the suitability of the range of design of tile and source and character of the raw material to meet the requirements of both the fire-endurance and hose-stream tests. The periods given in Table XXIV are based on the assumption that walls are laid up with Portland cement or cement-lime mortar. Although lime-mortar walls may show equal or better fire-resistance, strength tests indicate a lower bearing capacity for lime-mortar walls.

FIRE EFFECTS on structural clay-tile walls are generally caused by unequal expansion of integral parts of the block units, resulting in flaking and spalling of laminated tiles and rupture of the transverse webs generally. In the double-exterior shell tile,‡ the damage is confined to the exposed shell and the attached short webs. The shape of the unit appeared to have only a minor effect on the damage. Individual tiles in restrained or in loaded walls are damaged more than in unrestrained walls. Practically all of these walls

* Bureau of Standards, letter circular 229.

† Research Paper No. 37, U. S. Bureau of Standards, January, 1929.

‡ Manufactured by the National Fireproofing Company.

Table XXIV. Bureau of Standards' Classification. Bearing-Wall Assemblies of Hollow Clay-Tile

Type	Thickness, inches	Description	Plaster coats	Ultimate resistance periods, hours: Combustible floor-members*	Ultimate resistance periods, hours: Incombustible floor-members*
3-cell	8	End-construction tile, or T-shaped tile, or double-shell tile	None	1¼	2¾
			P 1 S	1¼	3¾
			P.2 S.	2¼	5
4-cell (2 units)	12	End-construction tile, or T-shaped tile, or double-shell tile	None	4	6
			P.2.S	5½	9
5 or 6-cell (2 or 3 units)	16	End-construction tile, or T-shaped tile, or double-shell tile	None	8	11
			P 2 S	10	15
3-cell	8	Side-construction tile, or double-shell with 1 or 2 interior cells	None	1	2¼
			P 1 S	1	3
			P 2 S	2	4
4-cell (2 units)	12	Side-construction tile, or double-shell with 1 or 2 interior cells	None	3	5
			P 2.S.	4½	7½
5 or 6-cell (2 or 3 units)	16	Side-construction tile, or double-shell with 1 or 2 interior cells	None	6½	8½
			P 2 S	8	12
2-cell	8	End or side-construction	None	1	1¾
			P 1 S	1	2¾
			P 2 S	2	4
3-cell (1 unit)	12	End or side-construction	None	2¾	3
			P 2 S	4	6
3-cell (2 units)	12	End or side-construction	None	3½	4
			P 2 S	4½	6½
4-cell (2 or 3 units)	16	End or side-construction	None	5½	6
			P.2 S.	7	9
Double tile 2-in. air sp. 2-cell	10	End or side-construction Two units	P.2 S.	1	4
2-cell Furred one side	8	End or side-construction Furring	None	1	3¾
			P 2.S.	2	5½

* Note. When the hollow space is filled at floor-lines and between combustible floor-members, the periods are the same as for incombustible floor-members, except that the period for 8-in walls with combustible floor-members should not exceed 2 hours (P.1.S.) or 4 hours (P.2.S.).

P.1.S. denotes plastered one side.

P.2.S. denotes plastered two sides.

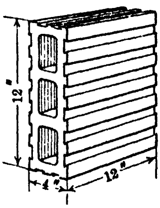
were repairable after fire damage, by one or more of the following methods: Replastering; removing outer shells and filling with cement mortar; removing exposed shells of single-wall tiles and filling cells on the exposed side with cement mortar; or repairing with "Gunite."* Recurrent fire-tests showed that repaired and replastered walls developed an efficiency in withstanding fire equal to that of the original wall.

Table XXV. Bureau of Standards' Classification. Combination Wall Assemblies—Hollow Clay-Tile

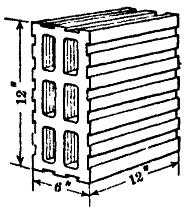
(Bearing or Non-Bearing)

Type	Thickness, inches	Description	Plaster coats	Ultimate resistance periods, hours	
				Combustible floor-member *	Incombustible floor-member *
Brick-faced (1-cell tile)	8	End or side-construction	P 1 S	2	4½
Brick-faced (2 or 3-cell tile)	12	End or side-construction	None	3½	6
			P 1 S	3½	7

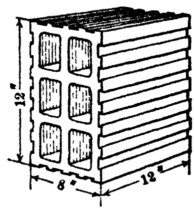
* See Note, Table XXIV



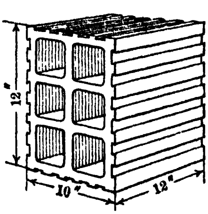
4"×12"×12"
3-cell
Wt. approx. 20 lb



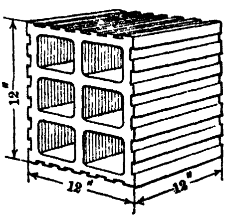
6"×12"×12"
6-cell
Wt. approx. 30 lb



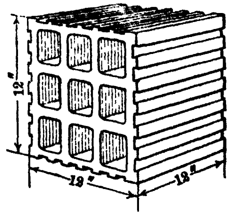
8"×12"×12"
6-cell
Wt. approx. 36 lb



10"×12"×12"
6-cell
Wt. approx. 42 lb



12"×12"×12"
6-cell
Wt. Approx. 48 lb



12"×12"×12"
9-cell
Wt. approx. 52 lb

Fig. 115. Load-Bearing Structural Clay Tile

* See Gunite, Article 5.

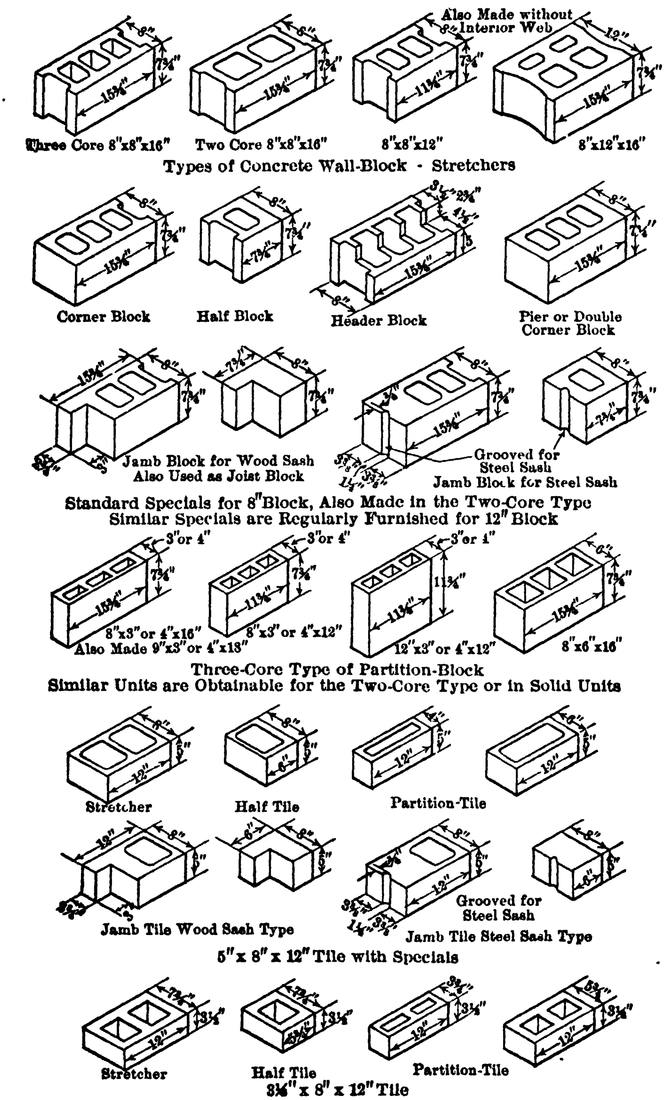


Fig. 116. Sand-Concrete Wall Units

Load-Bearing Clay-Tile. Load-bearing structural clay-tile are scored for the mechanical bonding of plaster or stucco on four sides and may be laid with their cells either horizontal or vertical. They have the same outside dimensions as partition tile and are similar in general appearance, except that the shells and webs are thicker. The New York City Building Code requirements call for webs and shells 1 in thick.

Standard load-bearing tile are used in all types of load-bearing walls. When used for exterior walls they are generally finished outside with brick veneer or stucco and plastered inside if a finished inside wall is desired. They are also used for interior load-bearing walls, being plastered on one or both sides or left unplastered.

Load-bearing concrete units are shown in Figs 116 and 117. They are built in sizes fixed by Simplified Practice Recommendations of the Department of Commerce,* both in the sand and cinder-concrete units. For a discussion of the properties of HOLLOW CONCRETE UNITS, see Hollow Concrete Blocks and Tiles, Article 5.

Clay-Tile. Combination Walls. Building Code requirements for the protection of STRUCTURAL CLAY TILE exterior walls vary considerably from permitting 8-in walls with exposed tile to a requirement for stucco or stone finishes, and in the more recent developments for panel walls in skeleton frames, to not less than 4 in of clay brick facing.

Originally the purpose of back-up tile was to provide a heat-insulating membrane in residence-construction which would serve the purpose of furring and also provide a load-bearing element. Types of these back-up tile or block, used for the last 15 years in buildings of limited height, are indicated in Fig. 118.

For the backing-up of masonry walls, and some face brick walls, where the added insulation is wished, a two-cell HOLLOW BRICK is also used. These units are more flexible than the larger back-up units, the small size of the hollow brick being adaptable to irregular stonework. (See Fig. 119.) Fig. 120 indicates such back-up tile used in FACED WALLS or brick-veneered walls in buildings of limited height.

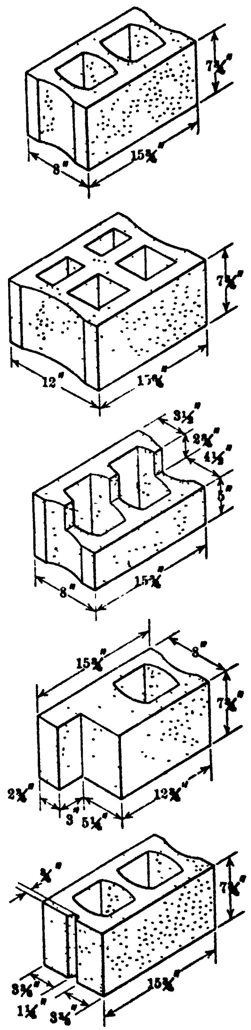
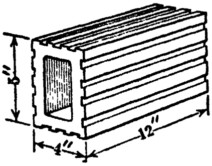


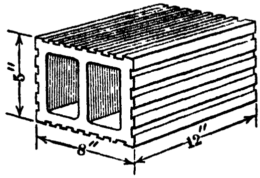
Fig. 117. Cinder-Concrete Wall Units

In recent years the advent of the multistory building with structural-steel

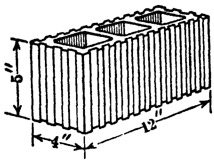
* S.P.R. 32.



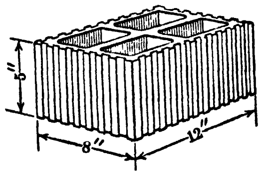
5"×4"×12"
1-cell
Wt. approx. 9 lb.



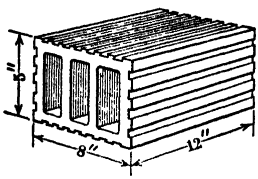
5"×8"×12"
2-cell
Wt. approx. 16 lb



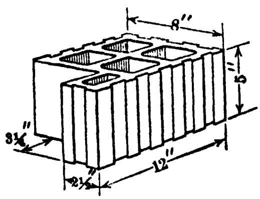
5"×4"×12" Corner Tile
3-cell
Wt. approx. 9 lb



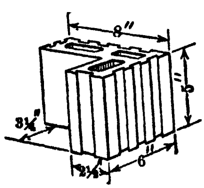
5"×8"×12" Corner Tile
4-cell
Wt. approx. 16 lb



5"×8"×12"
3-cell
Wt. approx. 16 lb



5"×8"×12" Jamb Tile
5-cell



5"×8"×6" Half Jamb
3-cell

Unless otherwise specified, Back-up Tile are furnished plain on one 5"×12" side.

Fig. 118. Back-up Tile

or reinforced-concrete frame has added another reason for the wider use of back-up material. The great saving in the weight of supporting steel experienced by the lighter weight of the exterior walls of important buildings has made the Combination Wall exceedingly popular, especially in New York City and the East. The better types of back-up tile for this use are those which do not materially retard but preferably speed erection of the exterior face brick. As in the case of any extruded structural-clay product, back-up tile must of necessity be either SIDE or END-CONSTRUCTION, depending upon whether the cells are laid horizontally or vertically. The selection of the method of construction depends upon the comparative efficiencies of the two types. It is claimed by some that the HORIZONTAL cell or SIDE-CONSTRUCTION will insure dry walls, owing to the fact that the moisture when condensed will not run down through the vertical cells and accumulate at the floor-line, and better and easier construction

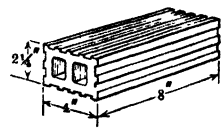
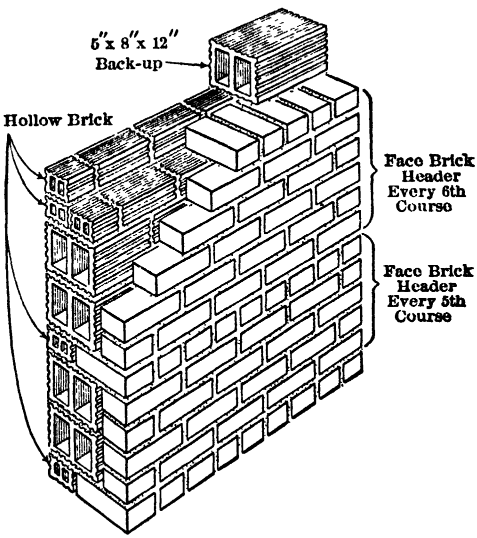


Fig. 119. Back-up Hollow Brick, Two Cell

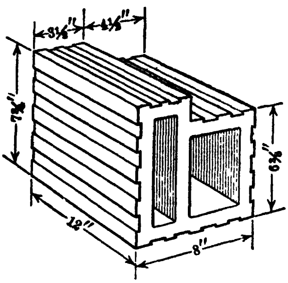


Twelve-inch brick veener load bearing wall, using 5"×8"×12" back-up material as backing and hollow brick to provide header bond

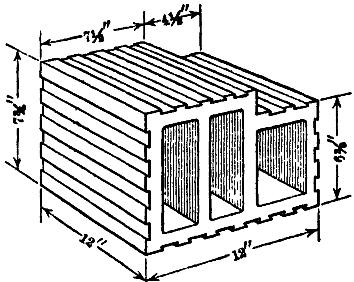
Fig. 120. Back-up Hollow Brick Used in Faced Walls

results from the more positive bed for horizontal joints. Others do not desire to sacrifice the strength differential afforded by end-construction in favor of the more or less doubtful advantage above stated.

The New York City Building Code requires the shells and webs of back-up clay tile used in skeleton wall construction to be 1 in thick, faced with 4 in of brickwork bonded every sixth course with headers extended into the back-

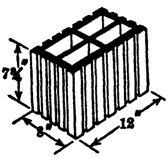


8″ Perfection Tile
8″×7¾″×12″ 2-cell
Wt. approx. 26 lb
Full inch web and shell

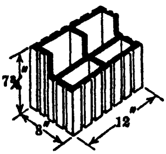


12″ Perfection Tile
12″×7¾″×12″ 3-cell
Wt. approx. 39 lb
Full inch web and shell

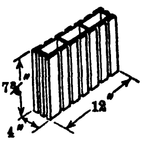
Fig. 121. Side-Construction Back-up Tile



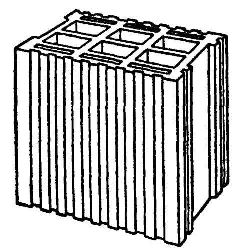
Stretcher Block

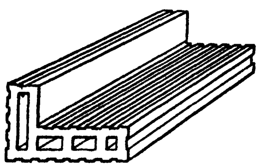


Header Block



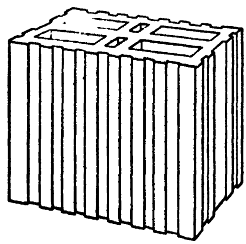
Column Covering

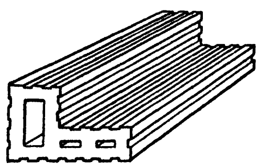




Western Header Backer

Note that the backer tile is similar to Natco Double Shell Load Bearing Tile except in height which is 10½″ instead of 12″. Header tile matches the backer tile in web construction





Eastern Header Backer

Note that the backer tile is similar to Natco Triple X Load Bearing Tile except in height which is 10½″ instead of 12″. Header tile matches the backer tile in web construction

Fig. 122. End-Construction Back-up Tile

ing. In place of one solid web, two adjacent ⅝-in thick longitudinal webs are accepted. Structural clay-tile used in exterior walls should be low in absorption. With absorption higher than 12 to 16%,* there is likelihood of gradual disintegration caused by freezing and thawing. Examples of SIDE and END-CONSTRUCTION tile are shown in Figs. 121 and 122. A standard weight of 8-in tile for use in exterior walls has been established by the U. S. Department of Commerce at 36 lb per sq ft, both for load and non-load bearing constructions.

All three types of tile insure the laying of a brick-faced wall with five stretcher courses and one header course every sixth course. The usual method of laying back-up tile is to lay five stretcher courses, then two back-up block courses, followed by the stretcher course. This method insures regularity without affecting the usual procedure understood by the average bricklayer. However, it is recommended that one or two courses of tile backing be laid ahead of the face brick in order that the space between brick and backing may be filled with mortar as the brick are laid up. This has been found by experience to be the best method of insuring water- and storm-proof walls. All exterior doors and window frames should be properly caulked with elastic caulking cement. Mr. P. H. Bevier of the National Fireproofing Company, in an article "Brick and Hollow Tile Walls" † describes the precautions necessary in laying up COMBINATION WALLS, as follows: "The joints of the face work should be thoroughly filled with mortar and carefully struck smooth with the trowel or convex tool to compress the mortar to make it more dense and impervious. Raked-out joints or rough-cut joints for artistic effect are common causes for leaking walls. Some architects specify an integral waterproofing to be mixed with the cement. There are several brands on the market and their use will lessen the absorption of the mortar, but it will not close shrinkage cracks or cure the effects of bad workmanship or poor mortar. Mortar made of well-graded sand, not too coarse, mixed 3 parts to 1 part Portland cement and 15% by volume of the cement of hydrated lime, mixed in a machine mixer, will give good results, if care is used by the bricklayer to fill all cross joints as well as the bed joints. Mortar is more porous than tile and a tight wall cannot be expected with lean mortar. In a tile-backed wall the back of the face brick should be entirely covered by parging with mortar before the tile are set. The space between the brick and tile should be filled with mortar and this can be better done by parging than by trying to fill with mortar after the tile are in place. For some years many advocated leaving this space open on the theory that the moisture was carried through the wall by capillary action of the mortar, which is greater than that of the tile; but this was found to be a mistake. The open space admits the water freely and it runs down the back of the face brick until it hits the brick headers and then finds its way to the plastering. Unfortunately, some contractors persist in this practice because it costs something to fill joints with mortar. The place to keep out water is the outside of the wall. When the tile are being set, care must be taken to fill and strike smooth all the inside joints."

To gain the space lost by furring exterior masonry walls, either solid brick or back-up construction, DAMPPROOFING is generally applied to the brick or tile. Either a ⅛-in thick mastic coat is applied with a trowel, or two coats of a semi-mastic are applied with a brush, and in both cases the plas-

* A. S. T. M. Standards, see Table IV, Article 5.

† Building Age and National Builder, April, 1928.

tering must be done while the water-proofing is still tacky. A third method is to apply a scratch or parging coat of water-proofed mortar to the masonry surface. Spandrel sections * are especially subject to infiltration of dampness

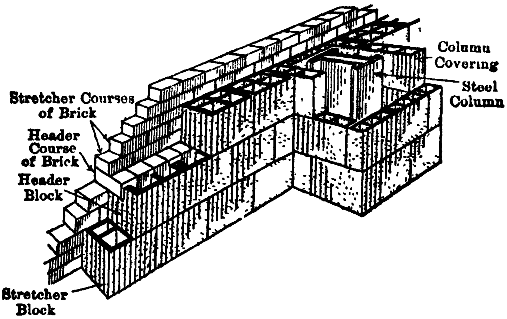


Fig. 123. "Raritile" Back-up Blocks

and moisture, which show as leaks between and over window-heads. Frequently trouble has been experienced from wet inner surfaces in the erection of brick-faced clay-tile walls, making it necessary to repoint the mortar joints. This experience has been traced to the use of brick of a non-absorptive character. When hard-burned or dense face brick are laid in the usual mortar mixture a hair crack appears at the top and bottom of the mortar joint, owing to the poor bond between the brick and the mortar and general lack of SUCTION. Moisture percolates through these cracks and stains the plaster on unfurred walls; and subsequent freezing and weathering magnify the defects. Care should be exercised to select brick of sufficient suction to secure a permanent bond between the mortar and the brick, thus eliminating the hair cracks as a source of trouble.

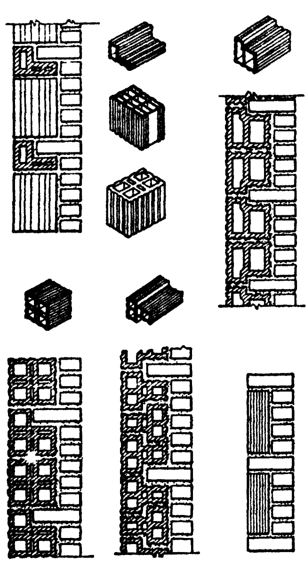


Fig. 124. Combination Clay-tile Wall Sections

In Figs. 123, 124, 125, and 126, examples are shown of the various types of back-up structural clay-tile in combination with brick face for walls 8, 10, 12, 14, and 16 in in thickness.

Concrete-Tile Combination Walls. Brick facing in COMBINATION WALLS of concrete block or tile is usually bonded with one header course in every six courses. Header and back-up blocks used for this purpose are shown in Figs. 117 and 127, in the form of cinder-concrete units. In the Integrity

* See Girder Protection, Article 6.

Trust Building, Philadelphia, Pa., cinder-concrete units were used for the

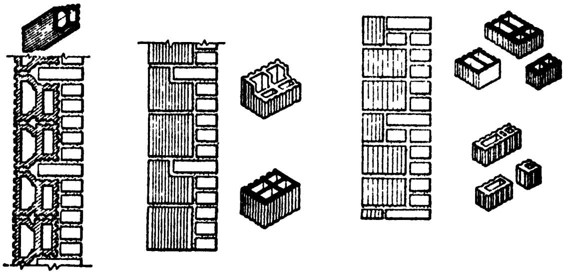


Fig. 125. Combination Clay-tile Wall Sections

backing, with stone, brick, and terra-cotta facings, as well as for fire-proofing of columns, shafts, stair-enclosures and partitions of this twenty-five-story skeleton-frame structure. As with other concrete block or tile products, special precautions must be used to insure that the material is thoroughly cured and dry before placing in the wall construction.

Weldcrete Masonry. Within recent years a form of hollow concrete-block masonry construction, either alone or combined with a brick facing, welded together by "Gunite,"* has been developed by the Covell Corporation of 1600 Walnut Street, Philadelphia, Pa. Special cinder-concrete masonry units, Fig. 128, designed to provide open joints directly accessible from one or both faces, are laid up dry in the walls and bonded together with "Gunite" shot into the open joints by means of the cement gun. Walls, 6 in or more

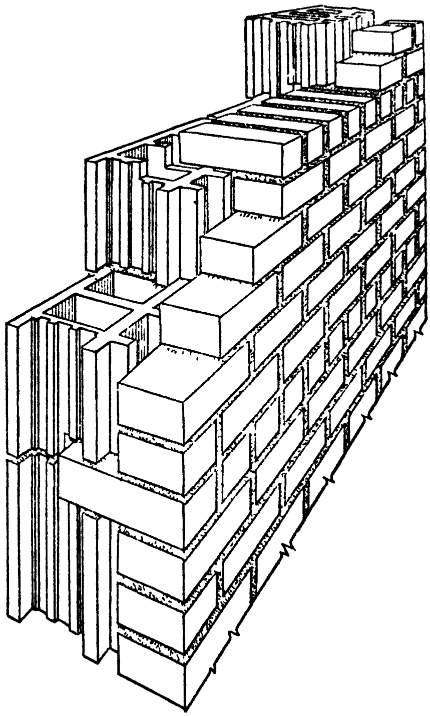


Fig. 126. "Kubak" Back-up Tile

* See Gunite, Article 5.

in thickness, are erected dry and plumb to line without auxiliary support; partition walls less than 6 in in thickness are temporarily braced with studding as a means of support until the joints are filled with "Gunite." In COMBINATION WALLS, faced or veneered with brick, the process consists in erecting the face brick with hand-placed mortar to scaffold height or sill height as determined by field conditions, which is then parged on the inside with a ½-in coat of "Gunite," and the dry WELDCRETE units are then laid up against the parging coat and the joints shot and bonded to the facing with "Gunite" webs. The inside face of the wall is either plastered directly on the blocks, or a base coat of "Gunite" is deposited after the joints have been filled. The surface is then screeded, straight-edged or otherwise made ready for the finish plaster. A wall thus erected with two membranes of "Gunite" is practically proof against weather and infiltration of air and moisture.

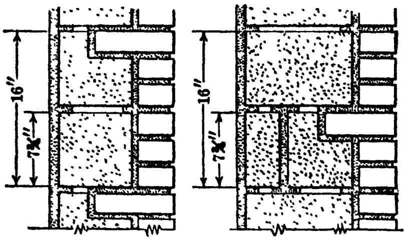


Fig. 127. Cinder-concrete Block Combination Wall

Three principal types of wall constructions are built by the "Weldcrete" method; CONCRETE-BLOCK wall with interior and exterior coats of "Gunite"; COMBINATION wall with exterior facing of brick or stone; and the FURRED wall, consisting of the face wall of brick or stone laid up in the usual manner by masons with trowel and mortar, and furring blocks of a minimum thickness of 3 in are Gunited together and to the parge coat on the masonry wall. The "Gunite" usually employed is the standard 1 : 3 mixture of Portland cement and sand. For lighter loads and skeleton walls, a mixture of 1 part of Portland cement, 1 part sand, 3 parts of ⅜-in crushed cinders and ⅓ sack of hydrated lime to a bag of cement has been substituted. This parging provides a better base coat for plastering, and is deposited with greater ease and exactness than the regular "Gunite" mixture. An example of "Weldcrete" construction is the ten-story Riviera Apartment at Atlantic City, New Jersey,* erected in 1930. At Asbury Park, N. J., the new Fitkin Memorial Hospital is a four-story skeleton reinforced-concrete frame in which the walls consist of 4-in face brick backed up with the 8 by 8 by 16-in "Weldcrete" units with ½-in parge coat of "Gunite" and finish plaster. In the Masonic Home at Elizabethtown, Pa., 70 000 sq ft of 3-in partition blocks were used, grouted both sides with "Gunite" to a total thickness of 4 in. The outside walls of the structure were water-proofed with a cinder "Gunite" coat in which an integral water-proofing was incorporated. In addition to eliminating scratch and brown coats of plaster by the use of the "Gunite" parge coat, the "Weldcrete" masonry is insured for a period of ten years against penetration of moisture or injury to the wall from such cause. Strength tests conducted at the University of Pennsylvania on wall specimens, approximately 3 ft high by 3 ft wide and 8 and 12 in thick with and without brick facings, developed ultimate loads in excess of the strength of the individual units owing to the monolithic nature of the completed construction, and the continuous grid of vertical and horizontal webs. Exterior or interior "Gunite" parge coats

* September, 1930, issue of Concrete, Chicago, Ill.

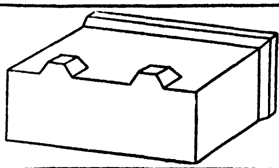
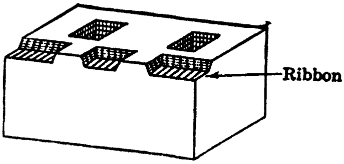
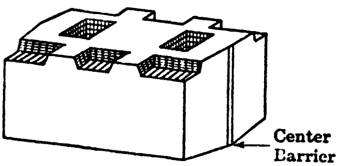
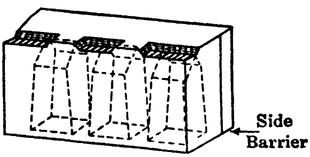
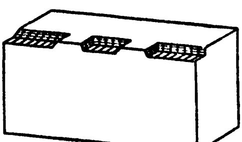
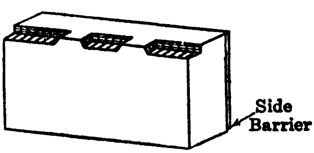
	Uses: Foundations and heavy load bearing walls Sizes: 6 in × 8 in × 16 in 8 in × 8 in × 16 in 12 in × 8 in × 16 in
	Single Ribbon.—No barrier (square end) Uses: Back-up Sizes: 6 × 8 × 16 hollow 8 × 8 × 16 hollow 12 × 8 × 16 hollow Halves of above Sizes
	Double Ribbon.—Center barrier Uses: Fire walls, fire division walls, party walls, elevator enclosures fire and stair towers, exterior walls for stucco finish Sizes: 6 × 8 × 16 hollow 8 × 8 × 16 hollow 12 × 8 × 16 hollow Halves of above Sizes
	Single Ribbon.—Side barrier Uses: Non-load bearing partitions Sizes: 3 × 9 × 16 hollow with solid top 4 × 9 × 16 hollow with solid top Halves of above Sizes
	Single Ribbon.—No barrier Uses: Back-up Sizes: 3 × 8 × 16 solid (furring) 4 × 8 × 16 solid Halves of above Sizes
	Single Ribbon.—Side barrier one end only Uses: Fireproofing columns Sizes: 3 × 8 × 12, 14, 16, 18 or 20 solid

Fig. 128. "Weldcrete" Masonry Units

can be deposited of natural or white Portland cement colored with mineral pigments and the surface finish left with natural "Gun" finish, screeded, troweled, sacked, or with other tooled effects. The advantages claimed for "Weldcrete" masonry are: Economy and lower cost than ordinary masonry walls of equal strength, water-proofness and fire-resistance; greater strength, efficiency, rigidity, and impact resistance; and ease and rapidity of erection.

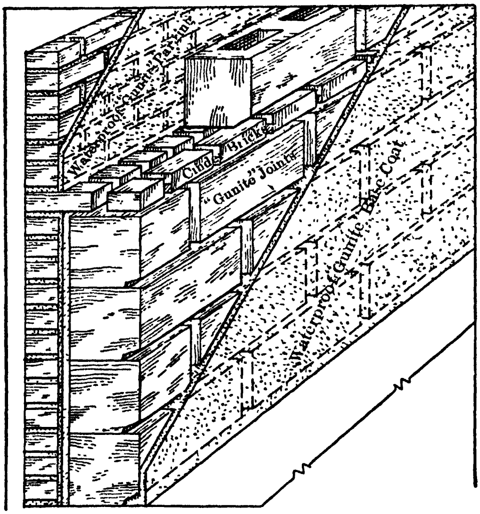


Fig. 129. Weldcrete Wall of Cinder Units and "Gunite"

No full-size fire-tests and very few strength-tests are yet available on this type of construction.

13. Fire-Resistive Partitions

Requirement of Fire-Resistive Partitions.* As a rule the partitions in fire-resistive buildings are not required to support any weight, but merely to serve the purpose of dividing the spaces into rooms, and to confine a fire to the compartment in which it originates. No greater strength, therefore, is required in a partition than is necessary to carry its own weight. Rigidity, however, is required, and a rigidity in proportion to its height and unsupported length. When partitions separate apartments or sections of a story, that is, when they are practically without window-openings or door-openings, they should be rigid enough to prevent the passage of water from a hose-stream as well as the passage of flame. In other cases this may be unnecessary; in fact, at times it may seem desirable to construct partitions which can be easily removed to get at a fire spreading through doors or windows. The materials of partitions should be incombustible. They should be poor conductors of heat. It is desirable, also, to have them unaffected by water. Lightness is

* See Fire-Resistive Wall Assemblies, Article 12.

a desirable property, as any increase in the dead weight of the construction adds to the cost of the structure. Partitions should be as sound-proof as possible. Window-openings should be avoided, when possible, in fire-resistive partitions, and even door-openings should be reduced in number to a minimum. In many buildings, however, in which halls have no openings into streets or courts, such windows are necessary for lighting the halls. When this is the case the frames should be made fire-resistive, wire-glass should be used, and, if possible, the sash made stationary.

Fire-Tests on Partitions. In New York City no materials or types of construction are permitted for interior permanent partitions in fire-proof buildings that have not met the required fire tests. Briefly, these tests require that the partition shall resist for one hour the destructive action of a wood fire, the heat of which has been gradually increased to 1 700° F. during the first half-hour and maintained at that temperature for the balance of the time; and that it shall resist, also, for two and a half minutes at the conclusion of the fire test, the application of a hose-stream at 30 lb pressure.

In addition to the WALL ASSEMBLY tests previously described and classified in this article, fire-testing of partitions has been actively carried on in the testing laboratories throughout the country, and has been especially prosecuted at the Columbia University Testing Station for the New York Bureau of Buildings. In this laboratory, at least one hundred tests have been made involving hollow-clay tile, concrete and concrete block, gypsum and gypsum block, metal lath and plaster on both combustible and incombustible furring, plaster and wall boards of various materials and reinforced concrete in various forms, as well as patented assemblies of these and other materials.

In the New York Tests there has not been generally any specified requirement for limitation of HEAT-TRANSMISSION, although this factor has been considered in limiting the approvals. No data have been published giving the wealth of information contained in these tests, except for a brief résumé on the behavior of GYPSUM BLOCKS * by S. H. Ingberg. Analyzing the merits of the various materials used in the constructions, and correlating all the other available test data, the FIRE-RESISTIVE RATINGS given in Table XXVI are believed to represent an equitable and just classification of the types of construction commonly found in buildings.

Types of Partitions. Fire-resistive partitions that are in common use may be grouped, according to the materials or the method of construction used, as follows:

(1) Brick;
(2) Hollow clay-tile;
(3) Concrete (stone or cinder);
(4) Gypsum block;
(5) Plaster or concrete, with metal lath or reinforcement.

Some of the properties of these constructions are discussed in the following paragraphs; the choice of the materials and the type of construction will be determined by the character of the building and the purposes for which it is used.

Partition-Walls. For bearing-partitions, that is, those which support floor-beams, there are probably no materials more satisfactory than brick and concrete. The latter may be used either in the form of blocks, or may be poured into forms. MEDIUM or HARD grades of tile are also being used with satisfactory results for bearing-walls.

* Proceedings A. S. T. M., Vol. 25, 1925, page 299.

Table XXVI. Minimum Fire-Resistive Partitions for Buildings

(Exclusive of Plaster Finish, Except Where Noted)

Thickness, inches	Description of wall or partition	Fire-resistive rating, hours
2	Solid fibered gypsum plaster on ⅝-in gypsum bd.; steel studs 26 in o.c., maximum spacing	1
2	Solid sanded gypsum plaster on metal studs and metal lath	1
2	Solid Portland-cement plaster on metal studs and metal lath	1
2	"Gunite" shot on metal lath consisting of No. 13 gauge 1¾ in exp. metal	1
2	Solid gypsum block	1
3	Hollow gypsum block	1
3	Hollow structural clay-tile, 1-cell, plastered ½ in	1
3	Hollow cinder-concrete tile-plastered both sides ½ in	1
3	Hollow, metal studs, metal lath or ⅜ in gypsum bds. both sides plastered	1
4	Hollow structural clay-tile, 1-cell, plastered 1 side ½ in	1
4	Hollow cinder-concrete tile	1½
4	Hollow clay-tile, 1-cell, plastered both sides ½ in	1½
4½	Hollow, metal studs, metal lath both sides, ¾ in sanded gypsum plaster	1½
6	Hollow clay-tile, 2-cells	1½
2	Solid fibered gypsum plaster on metal studs and lath	2
2½	Solid Portland-cement plaster on metal studs and lath	2
2½	Solid sanded gypsum plaster on metal studs and lath	2
3	Hollow gypsum block, plastered both sides ½ in	2
6	Hollow structural clay-tile, 2-cell; plastered one side	2
8	Hollow structural clay-tile; 3-cell	2
2½	Solid fibered gypsum plaster, on metal studs and lath	3
4	Hollow gypsum block	3
5	Reinforced stone or cinder concrete	3
6	Hollow structural clay-tile, 2-cell, plastered ½ in both sides	3
6	Solid cinder-concrete block	3
8	Hollow brick plastered ½ in both sides	3
8	Hollow concrete block	3
8	Hollow structural clay-tile, 3-cells, plastered one side	3
12	Hollow structural clay-tile, 3-cell	3
12	"Gunite" on both sides of "Gunite" studs, 2 in thick; 6 × 8 in studs	3
6	Reinforced concrete	4
6	Solid cinder-concrete block, plastered both sides ½ in	4
8	Solid brick, solid cinder or solid stone concrete	4
8	Solid cinder concrete block	4
8	Hollow concrete block, plastered both sides ½ in	4
8	Hollow brick, plastered both sides ½ in	4
8	Hollow structural clay-tile, 2-cell, plastered both sides	4
12	Structural clay-tile, 2 units, 3-cell	4
12	Structural clay-tile, 1-unit, 3-cell, plastered 1 side	4
12	Hollow Rolok wall, plastered one side	4
8	Reinforced concrete	6
12	Hollow Rolok wall, plastered both sides	6
12	Combination wall, 8-in clay-tile, 4-in brick, 2-cell, plastered one side	6
12	Structural clay-tile, 2-units, 4-cell	7
12	Reinforced concrete	9
12	Solid brick	9

NOTE 1. Rating increased ½ hour for each coat of ½ in filtered gypsum or ¾ in 1 : 2 Portland-cement mortar on assemblies shown above unplastered.

NOTE 2. Number of cells in clay-tile refers to number through thickness of partition.

Hollow Clay-Tile Partitions. These are usually built of tile that are 12 by 12 in on the face, and of varying thicknesses, as shown in Fig. 130. The 2-in partition-tile available in the Eastern market in 2 by 8 by 12 in and 2 by 12 by 12 in dimensions are impracticable and are not recommended for partition-work. The 3-in, 4-in, and 6-in tile are commonly used, the 4-in tile being the most popular for ordinary work. For the more important partitions, such as stair and elevator-enclosures, nothing less than the 6-in tile with the double row of cells should be used. The blocks are commonly set with the voids vertical.

Strength and Fire-Resistance. Under Fireproofing, Partition and Furring Tile, Article 5, the specifications of the American Society for Testing Mate-

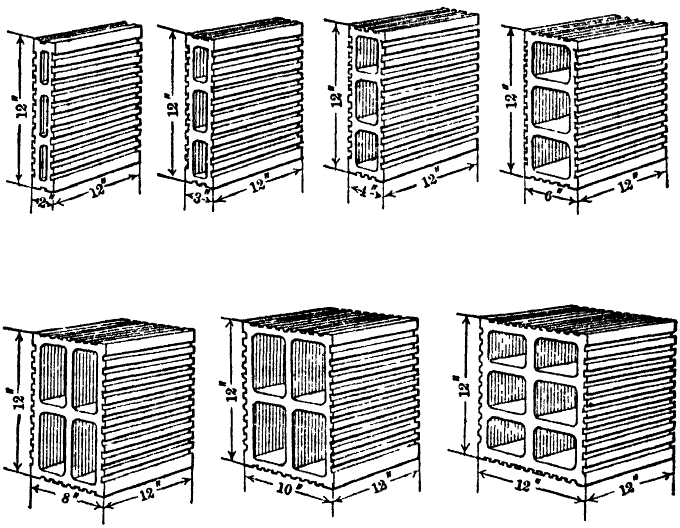


Fig. 130. Structural-Clay Partition-Tile

rials are given for the CLASSIFICATION and the STANDARD WEIGHTS and DIMENSIONS of STRUCTURAL CLAY PARTITION-TILE.* Where the FIRE-RESISTANCE is an essential property, the purchaser must specify the performance period required, and the manufacturer will supply the information on the fire-test performance of the specified product. For inside non-bearing partitions the SOFT GRADE is preferable to the HARD, while for outside walls the MEDIUM or HARD GRADES should be used. With DENSE TILING it is necessary to insert either wooden nailing-strips, which are very objectionable, or blocks of porous tile to take their place. Wood and other trim is usually attached to structural clay-tile partitions or walls by means of wedges or plugs, set in the mortar joints. Several attempts have been made to eliminate this troublesome procedure as it interrupts the speed of the tile-setters. Probably the most effective solution of the problem is the use of the WEDGE feature on tile manufactured by the Whitacre Greer Fire Proofing Company. (See Fig. 131.) The tile is deeply scored at one point in either or both faces of the tile, and

* A. S. T. M. C56-30.

definitely dovetailed. After the tile are erected in the finished partition, wooden wedges are inserted in these deep scorings in pairs. The trim is nailed to these wedges. The advantage of this method is that the wedges may be inserted after the tile are erected, the trim can be applied at any point without previous manipulation, and the tile-setters are not interrupted in their work. Special wood wedges are provided for nailing bases, waterproofed with a stainless solution so as not to expand or contract under varying temperatures and humidity. Strength tests made at the Case School of Applied Science show that one pair of wooden wedges with two No. 4 Clout Nails developed a vertical pull of 573 lb and a horizontal pull of 232 lb. Fig. 131 shows how the nail when driven clinches in the wedge block.

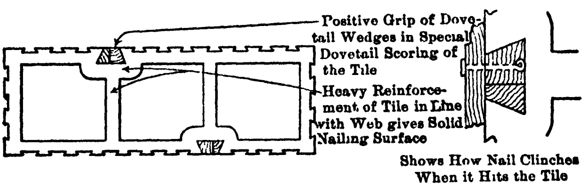


Fig. 131. Wood Wedges in Tile for Attaching Trim

Mortar. Tile partition-blocks should be set in mortar made of one part lime-putty, two parts cement, and from two to three parts sand. The blocks should be well wet before setting and the partition wet down before the plastering is applied.

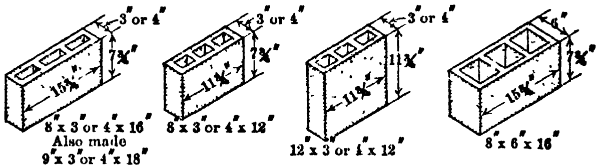
Heights and Lengths of Structural Clay-Tile Partitions. The safe HEIGHT of terra-cotta partitions in inches may be approximated by multiplying the thickness in inches by 40. Common practice allows a safe height of 12 ft for 3-in, 16 ft for 4-in, and 20 ft for 6-in partitions. For partitions without side-supports the LENGTH should not materially exceed the safe height. Doors and high windows may be considered as side supports, provided the studs run from floor to ceiling.

Scored vs. Smooth-Surfaced Tile. Structural clay-tile are generally plastered on one or both faces. To insure an effective mechanical bond, the tile are scored. That is, the dies through which they are extruded are so contrived that the finished outer surfaces are all scored to a depth of about $\frac{1}{4}$ to $\frac{5}{16}$-in width, with under-beveled edges. This device provides a definite, positive bond for plaster. Some manufacturers extrude the clay in a consistency which is sufficiently dry to cause the side of the beveled scoring to DRAG in the dies, thereby producing a feathered edge, which is claimed to enhance still further the value of the scoring. Tile are made with plane surfaces, without scoring, when plaster is not to be applied. The uses of smooth tile are confined to industrial plants, power-houses, etc., where a reasonably smooth, unplastered wall or partition is required, at a minimum of expense, which can be painted with standard oil paints; but precaution must be taken to prevent the mortar joints when fresh from burning through the paint.

Clay Tile-Partition Weights. The weights of structural clay tile-partitions should be assumed not less than the following, to which plastering will add an additional 5 lb for each surface plastered.

2-in partition	(3-cell tile)	14 lb per sq ft
3-in partition	(3-cell tile)	16 lb per sq ft
4-in partition	(3-cell tile)	18 lb per sq ft
6-in partition	(3-cell tile)	22 lb per sq ft
6-in partition	(4-cell tile)	26 lb per sq ft
8-in partition	(4-cell tile)	32 lb per sq ft
10-in partition	(4-cell tile)	38 lb per sq ft
12-in partition	(4-cell tile)	42 lb per sq ft

Concrete Partitions. Partitions of monolithic stone concrete are seldom used because of the forms necessary for their erection, and their weight, which make them comparatively expensive. Unless reinforced they take up too much room. They are the heaviest of all partitions. Even in buildings that are entirely of reinforced concrete they are not generally used. Cinder-concrete monolithic partitions are somewhat lighter and considerably cheaper than those of stone concrete. Yet even these are too heavy and too troublesome to construct to be satisfactory. Among the partitions tested and approved by the New York City Building Bureau is one that consists of cinder-concrete blocks, 2½ and 3 in thick, the thicker ones being hollow, 12 in high, and 18 in



Three-core Type of Partition Block—Sand-Concrete
Similar Units are Obtainable for the Two-core Type or in Solid Units

Fig. 132. Sand-Concrete Partition-Blocks

long. They have their edges cast with tongues and grooves that furnish more or less of a bond between the blocks when they are set. Hollow sand-cement concrete building-blocks make fairly good partitions, but are frequently objectionable on account of their thickness and weight.

The sizes of concrete block and tile are fixed by Simplified Practice Recommendations of the Department of Commerce.* Typical tile are shown in Fig. 132.

Cinder-Concrete Tile-Partitions and Walls are made of units of the size and dimensions shown in Figs. 117 and 133 in STANDARD THICKNESSES of 3, 4, 6, 8, 10, and 12 in. The manufacture is controlled by the National Building Units Corporation, Philadelphia, Pa. The weight of unplastered partitions should be assumed not less than the following:

3-in partition	(3-cell tile)	18 lb per sq ft
4-in partition	(3-cell tile)	20 lb per sq ft
6-in partition	(2-cell tile)	28 lb per sq ft
8-in partition	(2-cell block)	36 lb per sq ft
10-in partition	(2-cell block)	48 lb per sq ft
12-in partition	(4-cell block)	56 lb per sq ft

In a fire test conducted on a 4-in cinder tile-partition, unplastered, for the Labor Department of the State of New York in 1928, the construction was

* S. P. R. 32.

subjected to the standard test for a period of three hours without any visible cracking of the partition. The blocks on the fire side were discolored for a depth of ½ to 1 in; no appreciable surface pitting occurred, but the mortar joints were eroded to a depth of approximately ½ in. The 250° F. temperature rise through the partition was reached in 1 hr 40 min. The average surface temperature on the side unexposed to fire exceeded 800° F. at the end of the 3-hour test. The partition was not plastered.

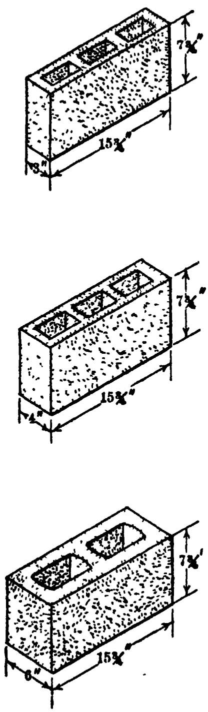


Fig. 133. Cinder-Concrete Partition-Blocks

Light-Weight Concretes made with artificial aggregates such as HAYDITE are extensively used through the West in the manufacture of HOLLOW PARTITION and WALL BLOCKS in all standard sizes and thicknesses. (See Fig. 116.) No large-size fire-tests are available of these partitions; but equal thicknesses should develop the same resistance as CINDER-CONCRETE units and weigh approximately 30% more. FIRE-TESTS* on the 8-in load-bearing Haydite block manufactured by the Western Brick Company, Danville, Ill., indicate that blocks of this thickness successfully meet the 3-hour test requirements and that failure generally occurs in temperature rise beyond the critical permissible limits on the unexposed side of the partitions.

Another LIGHT-WEIGHT concrete partition-unit is made of AEROCRETE, an expanded concrete, in blocks 3 and 4 in thick by 12 by 24 in. It can also be cast in special sizes on order. The AEROCRETE used for this purpose weighs 50 to 60 lb per cu ft and has a compressive strength of 300 lb per sq in. It is claimed for these blocks that the cellular structure of the AEROCRETE absorbs sound to a very high degree and that they are desirable for use in partitions of hospitals, apartment-houses, and office-buildings. The blocks are manufactured by the Aerocrete Corporation of America, New York, N. Y., H. W. Bell Company, New York, N. Y., and the Aerocrete Western Corporation, Chicago, Ill. These blocks can be plastered with two coats of GYPSUM or painted with plastic paint. The height of partitions should not exceed 50 times the thickness of block.

Gypsum-Blocks. The terms GYPSUM-BLOCKS and TILES are now more generally employed than the term PLASTER-BLOCKS, as they are more accurately descriptive. GYPSUM PARTITION-TILE are manufactured 12 in wide and 30 in long; and 2 in solid, 3 in solid, and 3, 4, 5, and 6 in hollow. The most common size is the 3-in HOLLOW BLOCK. Among the manufacturers are the U. S. Gypsum Co., Chicago; Structural Gypsum Corporation, Linden, N. J.; Atlantic Gypsum Products Co., Boston; Certain-teed Products Corporation, New York; Universal Gypsum and Lime Co., Chicago The FIRE-RESISTANCE, STANDARD THICKNESS OF SHELLS and other properties of GYPSUM and GYPSUM TILE have been previously discussed. (See Article 5.)

* Retardants Nos. 2051 and 2390, Underwriters' Laboratories, Chicago.

Gypsum-Block Partitions. Blocks made of gypsum combined with various substances, such as cinders, wood chips, cocoanut fiber, asbestos, etc., have been largely used for partitions in fire-resistive buildings. They are generally made of unretarded first-settle gypsum with 1 to 2½% by weight of wood fiber. The principal advantages claimed for these partitions are their great lightness and reduced cost compared with other forms of partitions. Gypsum blocks can be readily cut with a saw, and have a considerable holding power for nails. In the fire tests, made for the Bureau of Buildings, New York City, they have generally shown considerable resistance to the flame and have transmitted less heat than partitions of any other form. They did not, however, always stand the hose-stream, some of them being easily pierced, and all of them being more or less washed away by the water. The combination of large units with lightness in weight permits more rapid erection of gypsum tile-partitions than any other type of fire-resistive partition. An objectionable characteristic of these blocks is their tendency to absorb moisture while being stored and to draw water from the plastering when it is applied. This moisture works down to the bottom of the partition where it is likely to injure the wooden base. The height should not exceed from 50 to 60 times the thickness of the blocks, unplastered. Hollow blocks should always be set with the cells horizontal. In some forms of these partitions the blocks are BONDED together by means of metal dowels,* running across the horizontal and vertical joints from one block into the adjoining one, as shown in Fig. 134. The cut illustrates the use of the block in the construction of dumb-waiter shafts and shows how the blocks are anchored at the corners by iron dowel-angles. Gypsum-sanded mortar in the proportions of 1 part of gypsum to 2 or 3 parts of sand is used in laying plaster-blocks; and occasionally fibered-gypsum plaster, tempered with sand, may be employed. All the partitions in the Chrysler Building, New York, and in prominent buildings of Chicago and New York City, are of gypsum blocks. Gypsum blocks make the lightest practical partition known. The weight of these partitions per square foot may be taken as follows:

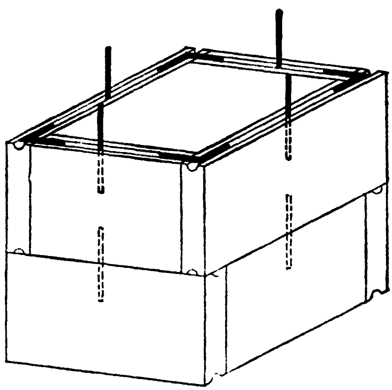


Fig. 134. Gypsum-Blocks. Doweled Construction.

Thickness of block, inches	Weight of partition, lb sq ft
2 solid	10
3 solid	13
3 hollow	10
4 hollow	13
5 hollow	16
6 hollow	17

* Patented by the Sanitary Fireproofing and Contracting Co., New York.

About 6 lb per sq ft should be added to obtain the weight of the partition when plastered on both sides.

Metal-Lath and Plaster Partitions. Partitions constructed of metal lath and plaster are of three general types: SOLID, HOLLOW, and DOUBLE. "SOLID partitions are preferred where cost and space saving are the important factors and no piping exceeding 1 in in outside diameter is to be concealed. Hollow metal-lath partitions on metal studs are recommended for fire-resistive separations to conceal service piping, rolling doors, etc.; those on wood studs are preferred for bearing partitions in non-fireproof buildings and for positions where such studs are required by the building code to have metal-lath and plaster protection to provide one-hour fire retardance, and in general for better plastering in non-fireproof buildings. Soundproof Double Metal Lath partitions are particularly useful where special consideration is given to sound insulation."*

The SOLID PARTITIONS are remarkably rigid owing to the adhesion of the plaster to the steel making a two-way reinforced plate, and they are lighter and occupy less space than any other practical fire-resisting partition of equal strength and stability. In the FIRE-TESTS, these partitions thoroughly resist the passage of flames very much like the plaster block partitions. When the hose-stream is applied, the calcined plaster on the fire side washes off and exposes the metal lath. The rigidity of the metal fabric on the metal studding has been considered by firemen a disadvantage, because of the difficulty of cutting through it to get at a fire. Any of the reinforcing fabrics, described under Types and Properties of Metal Lath, Article 14, can be used in the construction. The lath or fabric has been in some instances subject to the CORROSIVE EFFECTS of the gypsum plaster.

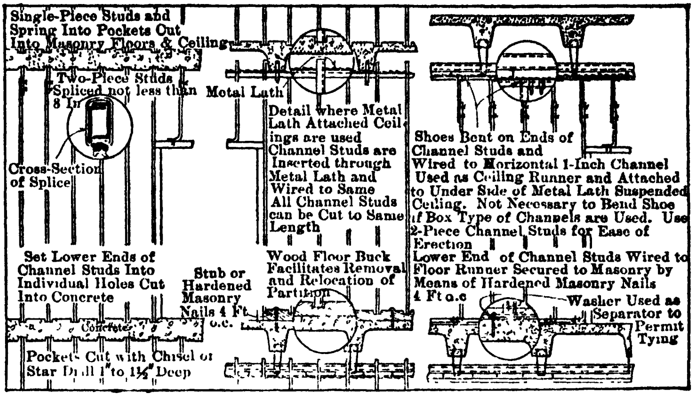
In the demolition of the Pabst Building, New York City, the metal lath used throughout in the partitions was found to be considerably corroded, after about four years, even though the lath had been painted. On the other hand, other cases are cited by the manufacturers, such as the Chess residence at Pittsburgh, Pa., the Sturtevant residence at Springfield, Mass., and the West End Trust Building at Philadelphia, Pa , in which after twenty years no corrosion of the metal lath in plaster partitions was observed. The investigations of the United States Bureau of Standards of various forms of stucco-construction seem to bear out the manufacturers' contention. The lath should in all cases be protected against initial or incipient corrosion by painting or galvanizing before being embedded in the cementitious material.

Weights of Plaster-and-Metal Partitions. The WEIGHT of a 2-in solid partition, when dry, is about 20 lb per sq ft. The weight of partitions of greater thickness may be estimated on a basis of 120 lb per cu ft of gypsum or cement-mortar plaster.

Construction of Solid Partitions.† The usual construction of these partitions consists of ¾, 1 or 1½-in standard or hot-or-cold-rolled box channels spaced from 12 to 30 in on centers, depending upon the rigidity of the lath used and the height of the partition. Figs. 135 and 136 show the various methods used for attaching the studs, either in one or two pieces as may be required, to the fire-resistive floor and ceiling-construction. For time-saving erection, it is a good practice to install track channels at floor and ceiling secured to the masonry by hardened nails. In 2-in solid partitions, the channel studs are placed slightly off center to provide ¾ in of plaster on the lath

* See Partition Handbook of Associated Metal Lath Manufacturers, Chicago, Ill.

† Some of the details are taken from the Associated Metal Lath Manufacturers' Handbook, to which the reader is referred for further information.



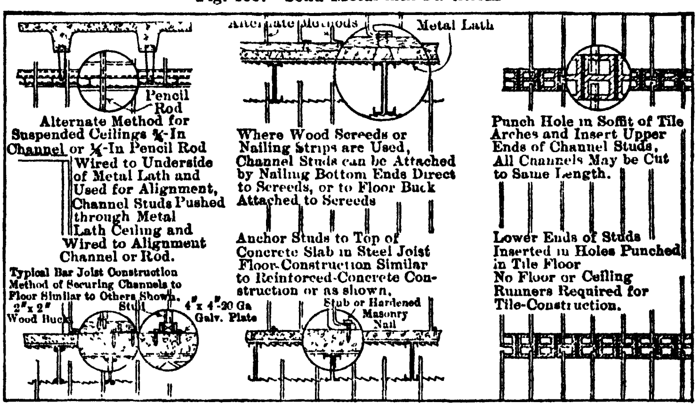
Anchorage in Flat Slab Concrete Floors and Ceilings

(Above) Upper Ends of Studs Secured to Contact Metal Lath Ceiling Attached to Concrete Joists (Below) Lower Stud Ends Secured to Wood Floor Buck, for Removability (Office-Buildings)

(Above) Upper Ends of Studs Secured to Suspended Metal Lath

(Below) Lower Ends Secured to Metal Channel Floor Runner on Masonry Floor

Fig. 135. Solid Metal-lath Partitions



(Above) Upper Ends of Studs Pushed through Suspended Ceiling

(Below) Another Method for Attaching Wood Buck to Concrete Floors —Steel Joist Construction

(Above) Studs Secured to Wood Floor Buck and to Metal Lath Ceiling—Steel Joist Construction (Below) Lower Ends of Studs Attached Direct to Concrete with Masonry Nails — Steel Joist Construction

Top and Bottom Ends of Studs Secured to Structural Tile Floor-Construction without Use of Runners or Floor Bucks

Fig. 136. Solid Metal-lath Partitions

side of the partition. The anchorage of the studs is desirable for stability and maximum fire-resistance. Table XXVII gives the size of studs required

Table XXVII. Size of Studs and Thickness of Partition for Various Heights of Solid Partitions

Height not to exceed	Thickness of partition	Size and weight per ft of channels (cold-rolled)
9 ft	$1\frac{3}{4}$ in	$\frac{3}{4}$ in .276 lb
12 ft	2 in	$\frac{3}{4}$ in .276 lb
14 ft	2 in	1 in .332 lb
14 ft	$2\frac{1}{4}$ in	$\frac{3}{4}$ in .276 lb
16 ft	$2\frac{1}{4}$ in	1 in .332 lb
18 ft	$2\frac{1}{2}$ in	1 in 332 lb
20 ft	$2\frac{3}{4}$ in	1 in .332 lb
24 ft	3 in	$1\frac{1}{2}$ in 442 lb
30 ft	$3\frac{1}{2}$ in	$1\frac{1}{2}$ in .442 lb

NOTE. For partitions over 20 ft high, horizontal channels or rods spaced 6 ft on centers are required on the stud side of the partition.

for the necessary stiffness in solid partitions of different thicknesses and heights. The LATH is tied to the studs with No. 18 gauge galvanized annealed ties spaced 6 in on centers, with the long dimension of the sheet across the studs. Flat laths are lapped not less than $\frac{1}{2}$ in on sides and 1 in on ends. Rib and sheet laths are lapped at sides by nesting the outside ribs or selvage, and on the ends, the lap is at least 1 in; or on sheet lath at least one series of loops are nested. Wire lath should be lapped at least 1 in on the ends of the sheet, and sufficiently at the sides to bind the mesh together. All sheet lath should be installed so that the splices are staggered. The SPACING OF STUDS as required for different weights of lath are given in the Table XXVIII. All

Table XXVIII. Spacing of Studs in Solid or Hollow Metal-Lath Partitions

Kind of lath	Spacing of channel studs, inches					
	12	14	16	19	24	32
Sheet metal, lb per sq yd .				4.5	4.5*	.
Expanded flat, lb per sq yd .	2 5 2.2*	3.0	3.4*		.	
$\frac{3}{16}$ in flat rib, lb per sq yd			3 0	3 4	.	.
$\frac{3}{8}$ in flat rib, lb per sq yd. . .		.	2 5	3 0 2 5*	4 0 3 4*	4.0*
Unstiffened wire-gauge. . .	No 20	No 19	No. 18	. . .		.
Stiffened wire-gauge...	No. 21	No. 21	No. 20	No 18 No. 20*	No. 18*	. . . .

* Solid.

types of metal-lath partitions are temporarily braced with horizontal channels usually wired midway in the height of the studs to insure a straight and plumb wall for the correct application of lath and grounds.

After the lathing is in place the carpenter should attach wooden grounds to secure the base, and pegs or spot-grounds for chair-rail, picture-molding, etc. These grounds are secured by staples or wire ties or clips, and when the partition is plastered, become very rigid. "When wainscots or bases of Portland cement, terazzo or similar materials are used, it is recommended that metal base screeds or grounds be employed." * At every door-buck and

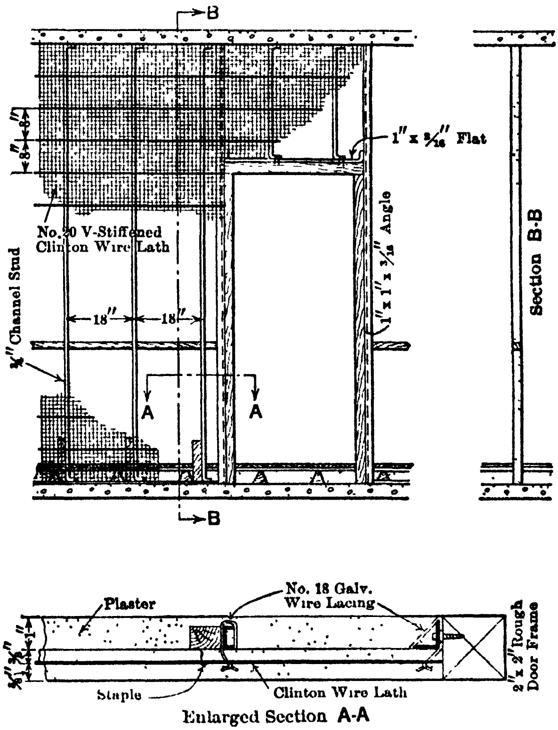


Fig. 137. Solid Metal-lath Partition

similar opening, a channel stud, either single or double, or of heavier sections, should extend from floor to ceiling and across the top of the opening. These are drilled and tapped for attaching the bucks or frames, or separated from and unattached to the buck to avoid plaster cracking from impact blows. The manufacturers of metal doors and trim also provide metal door-bucks and frames adaptable to these partitions.† In plastering these partitions, five or six coats of plaster are required to make a good job; a scratch coat on one

* Associated Metal Lath Manufacturers.

† See Interior Finish and Trim, Article 18.

side, a brown coat on each side, and the usual white coat on each side for finishing. It is essential for all thin partitions that a HARD-SETTING mortar be used of either fibered or sanded gypsum plaster or Portland-cement mortar. The partitions acquire their stiffness largely from the solidity of the plastering, hence the firmer and harder the plastering the more substantial the walls. Fig. 137 shows a SOLID partition with wood door-bucks.

Construction of Hollow Partition. HOLLOW, DOUBLE metal-lath and plaster partitions are constructed of $1\frac{1}{2}$ or 2-in channel studs, or a double

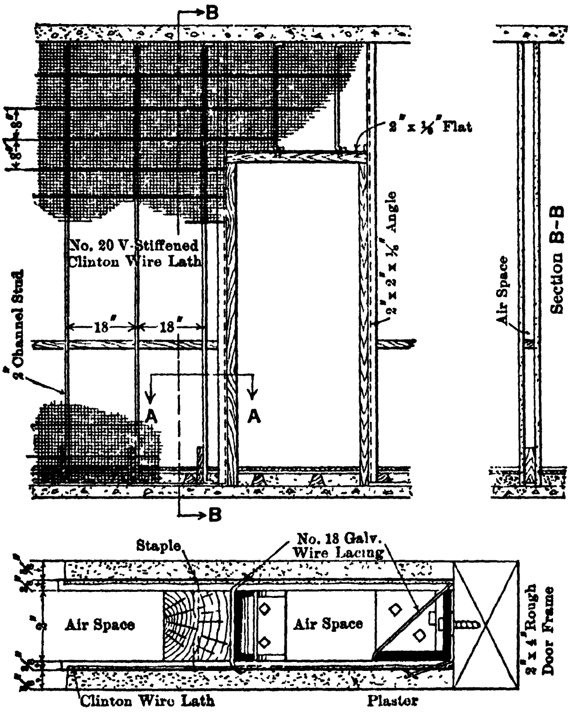


Fig. 138. Hollow Metal-lath Partition

row of channel studs held together by spacers, the thickness or depth being determined by the space required to conceal the pipes and ducts to be installed in the partition. For fire-retarding WOOD-STUD partitions, the metal lath is applied on both sides of the 2 in by 4 in wood studs. In each case, the outside surface of the lath on both sides of the partition is plastered with ¾ in of plaster. The sizes of studs for varying heights and thicknesses of HOLLOW PARTITIONS are given in Table XXIX. The spacing of the studs is governed by the same consideration as in the SOLID PARTITION, and the values given in Table XXVIII also apply. The DOUBLE PARTITION can also be

made solid by filling the hollow space with light-weight concretes, either of cinder aggregates or of any one of the expanded concrete fillers, "Aerocrete" and "Haydite" (Article 5) and "Nailcrete" and "Naylegrip" (Article 10). The use of nailing-concrete fillers eliminates the necessity for furring or nailing-strips for the attachment of base-boards, chair-rails or other interior trim. The DOUBLE PARTITION can also be SOUNDPROOFED by erecting the double row of studs independently of each other without cross-braces or attached separators.* Each line of studs is separately reinforced with horizontal channels placed in the hollow space with the web horizontal and wired to the studs, erected on 4-ft centers.

Table XXIX. Size of Studs and Thicknesses of Hollow Partitions

Type	Face-to-face plaster thickness	Maximum height
Double row of ¾-in channels	3 in	18 ft
Double row of ¾-in channels	4 in	24 ft
Double row of ¾-in channels	5 in	30 ft
Single row of 1½-in channels	3 in	18 ft
Single row of 2-in channels	3½ in	20 ft
Single row of 3-in channels	4½ in	26 ft
Single row of 4-in channels	5½ in	32 ft

NOTE. A horizontal stiffener should be provided on both sides of all hollow partitions more than 10 ft long or more than 9 ft high.

Berloy Standard Studs. A patent pressed-steel stud is manufactured by the Berger Manufacturing Company, Canton, Ohio, for use in the construction of HOLLOW LATH and PLASTER partitions. It is furnished in [and H sections in depths of 4, 5, 6, 7 and 8 in, the partition finishing about 1½ in thicker than the depth of the stud. The properties of the sections are given in the following table. As shown in Fig. 139, the feature of these studs is the positive provision made for securing the lath through the prongs punched out of the sheet metal. The lath is thus fastened more firmly and securely than by the usual wiring or clips.

Dickson Studs and Lath. Another patented form of stud is shown in Fig. 140, which incorporates a novel form of horizontal plaster lath. This device is manufactured by the Dickson Patent Partition Co., New York, for both SOLID and HOLLOW partitions. The advantages claimed for this construction are rapidity of erection and economy in cost. The studs are spaced 3 ft on centers, and secured to floor and ceiling with ¼ by 1¼ hard-steel nails or expansion bolts. The partition is plastered with a scratch coat containing excelsior or hair-fiber binder, and the usual brown and finish coats. For DOUBLE PARTITIONS, a paper felt backing COMBINATION LATH is fastened to the inside of each line of studs and lath.

Wall- and Plaster-Board Partitions. There are various forms of WALL-BOARD of an incombustible or flame-resisting nature, made of GYPSUM reinforced with pulp, news or felt stock, or a mixture of hydrated calcined gypsum and fibrous binding materials, and boards of treated wood fiber compressed into rigid sheets. They are used either as LATH for plastering, or as insu-

* See Sound Insulation, Article 15.

lating and wall coverings without plaster finish. They are very light in weight, require minimum plaster quantities for finishing, and cost less than METAL LATH and but little more than WOOD LATH with three coats of plaster. Approved types of these boards, properly erected, possess considerable fire-

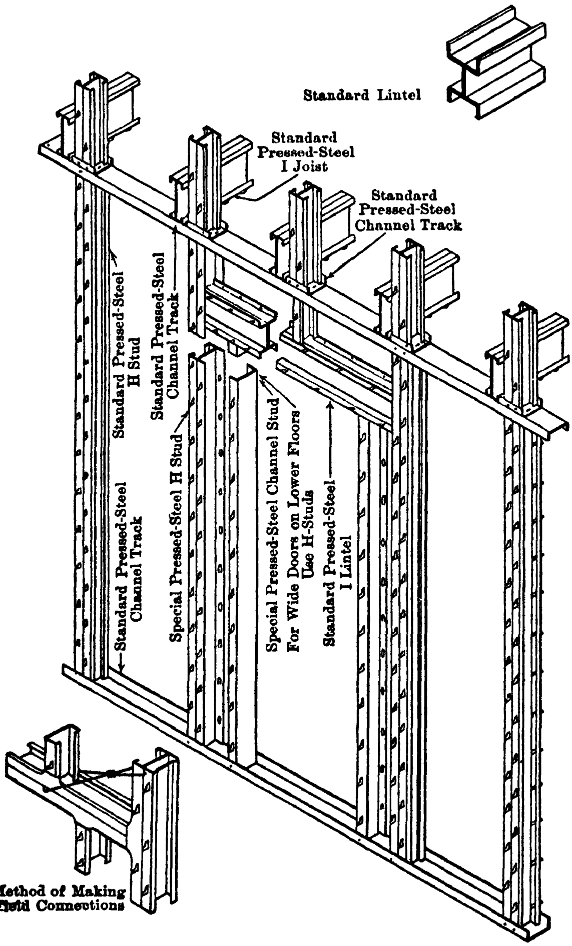


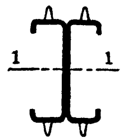
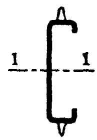
Fig. 139. Berloy Standard Steel Studs

resistance. Wall boards of ASBESTOS are described under Asbestos Products, Article 5.

GYPSUM WALL and PLASTER BOARDS are manufactured in conformity with STANDARDS of the American Society for Testing Materials.* WALL-BOARDS,

* A. S. T. M. Specs. C36–25 and C37–30. See Gypsum Plaster Boards, Article 5.

Table XXX. Physical Properties—Standard Supporting Studs

Depth, in	Weight per ft lb	Flange, width, in	Thickness, metal, in	Thickness web, in	Net area of section, sq in	Axis 1–1 r	
4	3.70	3.00	.072	.144	.972	1.4988	
4	6.00	3.00	.120	.240	1.620	1.4896	
5	4.20	3.00	.072	.144	1.116	1.8358	
5	6.80	3.00	.120	.240	1.860	1.8276	
6	4.70	3 00	.072	.144	1.260	2.1669	H-studs
6	7.60	3.00	.120	.240	2.100	2.1598	
7	5 50	3 50	.072	.144	1 476	2.5406	
7	8 80	3 50	.120	.240	2.460	2.5326	
8	6.10	4 00	.072	.144	1.692	2.9166	
8	10 00	4 00	120	240	2.820	2 9083	
Channel studs							
4	1 85	1½	072	.	.486	1 4988	
4	3 00	1½	.120		.810	1.4896	
5	2 10	1½	.072	.	.558	1 8358	
5	3.40	1½	120	...	.930	1.8276	
6	2 35	1½	.072	. .	.630	2.1669	
6	3 80	1½	.120	. .	1.050	2.1598	
7	2.75	1¾	072	.	.738	2 5406	
7	4 40	1¾	.120	. . .	1.230	2 5326	Channel Studs
8	3 05	2	072		.846	2 9166	
8	5 00	2	.120	. .	1.410	2 9083	

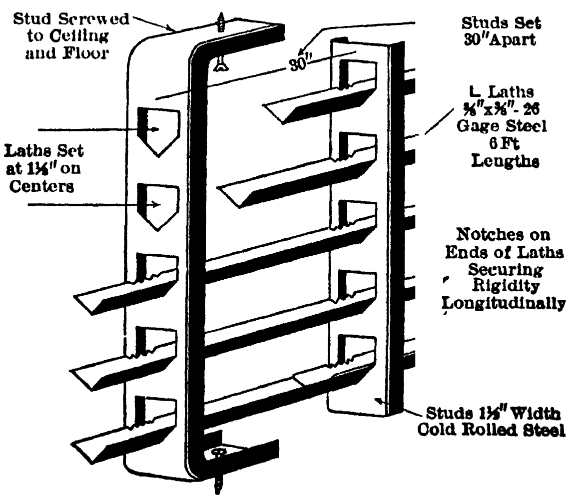


Fig. 140. Dickson Stud

designed for use without plastering, are manufactured in nominal widths of 32, 36 and 48 in and lengths of 4 to 12 ft. PLASTER BOARDS are manufactured 16, 24 and 32 in wide in nominal lengths of 24, 36 and 48 in. In addition to the boards described in the following paragraphs, plaster and wall-boards are also manufactured by the Kelly Plaster Board Co., Delawanna, N. J.; Atlantic Gypsum Products Co., Boston, Mass.; Universal Gypsum and Lime Co., Chicago; Ebsary Gypsum Co., Rochester, N. Y.; National Gypsum Co., Buffalo, N. Y.; and the American Gypsum Co., Port Clinton, Ohio.

Gyplap. This is a WALL-BOARD consisting of a solid sheet of gypsum encased in a moisture-proof fibrous covering, with one edge V-shaped and the opposite edge cupped to nest in the notch and form a wind-tight joint. It is manufactured by the U. S. Gypsum Company ½ in thick, 24 in wide and 6 ft 8 in and 8 ft long. It cuts and nails as easily as lumber and possesses considerable rigidity as sheating on studs.

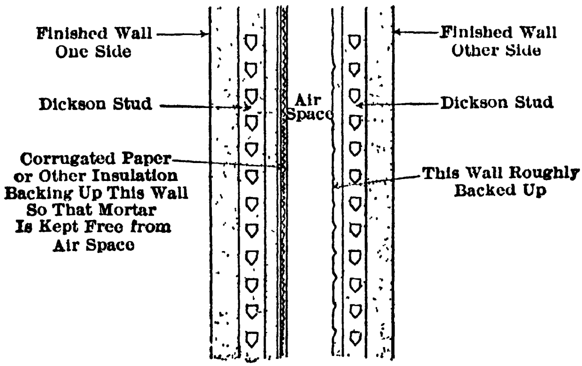


Fig. 141. Dickson Double Partition

Sheetrock. This WALL-BOARD is manufactured by the U. S. Gypsum Company, with a close-grained calendered surface of ivory color adaptable to enamel and paint finishes to represent tile work. It is furnished ¼ and ⅜ in thick in sheets 32 and 48 in wide and 4 to 10 ft long. A special system of joint construction developed by the manufacturer consists in applying either a perforated metal tape or a processed-cloth fabric embedded in a special cement over all joints.

Sackett's Wall-Board. This is a composite board of three layers of pure gypsum and four thin layers of wool-felt. The boards can be nailed to wooden studding or set flat against solid beams or planks, and can be cut with a saw. For plastering, the best results are obtained by applying first a brown coat of hard wall-plaster, ¼ to ⅜ in thick, and when this is thoroughly set, finishing it with a thin coat of regular hard finish of lime-putty and plaster. Tests and investigations at the Underwriters' Laboratories "have shown Sackett Board, Perfection Brand, to be suitable as a base for fibered-gypsum plasters; and when attached to walls, ceilings, and partitions and coated with ½ in of plaster, possess fire-retarding properties considerably higher than those of wooden lath with gypsum or lime-and-cement plaster." The Perfection Brand, Sackett's Wall-Board, is ⅜ in thick, and is attached with No. 10½, ½-in,

flat-headed, $\frac{3}{4}$-barbed wire nails, $1\frac{1}{4}$ in long, and spaced not more than 6 in at each support. SACKETT'S BOARD is made by the United States Gypsum Company, Chicago, Ill., and the Grand Rapids Plaster Company, Grand Rapids, Mich.

Bestwall. BESTWALL is primarily intended for use as an interior finish on side walls and ceilings in buildings of all classes. It may be used in all situations where finishes of lath and plaster may be used, and in many situations where the latter finish is not adaptable. It consists of a single layer of fibered calcined gypsum, surfaced on each side with specially prepared water-proofed paper securely bonded to the surface. Bestwall is $\frac{3}{8}$ in in thickness, and is furnished in stock sizes $47\frac{3}{4}$ in wide, and in lengths of 5, 6, 7, 8, 9, and 10 ft. The finished product presents a smooth, true surface, which is light cream in color on the face-side and gray on the reverse side. The edges are slightly beveled to provide for the filling of the joints, and are doubly reinforced. Its weight is approximately 1 850 lb per 1 000 sq ft. Interior finish composed of Bestwall is applied by nailing the Bestwall directly to the joists, studs, and furring, and filling the joints between the pieces of Bestwall with a specially prepared filler of the same composition as the core of the board. For the nailing, threepenny fine wire nails, spaced from 2 to 3 in at the edges and from 8 to 12 in at the intermediate supports, are used. The filling consists of two operations: first, ROUGHING IN and then, TROWELING OUT, to a smooth, true finish, flush with the surface. Bestwall is cut and fitted either with a saw or by scoring and breaking over a straight edge. The completed finish presents a smooth, true, continuous surface, without showing joints or nail-heads, and ready to receive, if desired, a decoration of paint, paper, tint, etc. This board is manufactured by the Bestwall Manufacturing Co., Chicago, Ill., and the Certain-teed Products Corporation, New York, N. Y.

Cork-Board. CORK-BOARD plaster base is furnished by the Armstrong Cork and Insulation Company, Lancaster, Pa., 1 in thick, 12 in wide and 32 to 36 in long, for attaching to studs or wide faces of walls and ceilings as both an insulation and a lath for plastering. Tests of plaster bond on cork-board show that it can be satisfactorily finished with either gypsum or Portland-cement sanded plasters. The use of CORK-BOARD as a plaster base will add approximately 5 per cent to the cost of construction with ordinary plaster bases.

Burson Partition. Fire-resistive partitions of plaster-board, on metal studs are constructed either of SOLID or HOLLOW types. The BURSON PARTITIONS are shown in Figs. 142 and 143. Both of these constructions were tested for the New York Bureau of Buildings in 1919 and are acceptable for use in fire-proof buildings. The maximum temperature of transmission through the HOLLOW PARTITION was 175° F. and the temperature in the hollow space was 869° F. at the end of the 1-hour test. These constructions are controlled by the Dennos Products Co., Chicago, Ill.

Simplex Partition. The Simplex Steel Products Co., Chicago, Ill., manufactures hollow partition studs of pressed steel] section, with punched prongs for securing the plaster boards, of the following sizes: $1\frac{1}{4}$ in for partitions to finish 3 in thick; $2\frac{1}{4}$ in for 4-in partitions; 3 in for $4\frac{3}{4}$-in partitions; $4\frac{1}{4}$ in for 6-in partitions; and 6 in for 8-in partitions. The boards are secured by steel pins or nails as shown in Fig. 144, and the construction is rigid and yet readily salvaged or rebuilt. As shown in Table XXVI, the 3-in HOLLOW or 2-in SOLID PLASTER-BOARD PARTITION has a 1-hour fire-resistive rating

The weight of the HOLLOW PLASTER-BOARD PARTITION varies from 10 to 12 lb per sq ft of partition.

American Clip-on System. The American Clip-On Corporation, Philadelphia, Pa., has developed a system of LATHING and PLASTERING for FIRE-RESISTIVE FLOORS * and PARTITIONS, using precast gypsum-block and plaster-

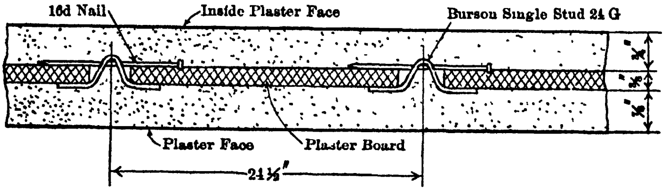


Fig. 142. Two-inch Burson Solid Plaster-board Partition

board units, standard furring channels and special clips and hangers. Fig. 145 shows the CLIP-ON devices used for attaching the plastering bases to the furring. These constructions are all installed by licensed contractors under the supervision of the company.

WOOD STUD PARTITION. The ⅜-in plaster boards are attached to both sides of the studs spaced 16 in on centers by CLIP 301, shown in Fig. 145, and are secured to each other at all corner junctions with CLIP 103. The plaster

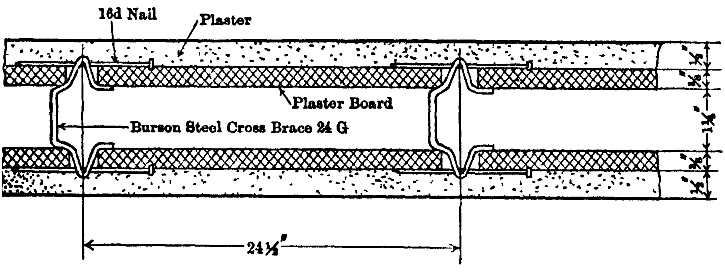


Fig. 143. Three-inch Burson Hollow Partition

boards are not nailed to wood studs or joists at any point to minimize any tendency to crack from unequal shrinkage or expansion. Standard CLIP-ON plastering is applied ¾ in thick on both sides of the partition.

SOLID PARTITIONS. Standard ¾-in furring channels rigidly secured to floor and ceiling are spaced 16 in on centers and the ⅜-in plaster board, 16 in by 48 in in area, is attached to one side of the studs with CLIP 101, Fig. 145, and to each other with CLIP 103. Before plastering, the carpenter secures the necessary wood nailing grounds directly to the metal studs with wires or clips. A bond coat of sanded gypsum plaster is applied on the channel side to cover the clips, and followed by scratch and brown coats of sanded gypsum on both sides with the usual hard-finish coat to a total thickness of 2 in. SOLID PARTITIONS of greater thicknesses are made with precast blocks between 1-in

* See Clip-on Fire-Resistive Construction, Article 7. See also Table XVI.

channels spaced 36½ in on centers, using CLIP 408 on the studs and CLIP 407 between the blocks. These partition-blocks are 18 in by 36 in in size, finished with square edges. The plaster is applied ½ in thick on both faces of the block.

HOLLOW PARTITIONS. Hollow partitions, 3½ to 5½ in in thickness, are erected on two rows of ¾-in hot-rolled channels spaced 16 in on centers. The 16 in by 48 in plaster boards are secured to the outside of both rows of chan-

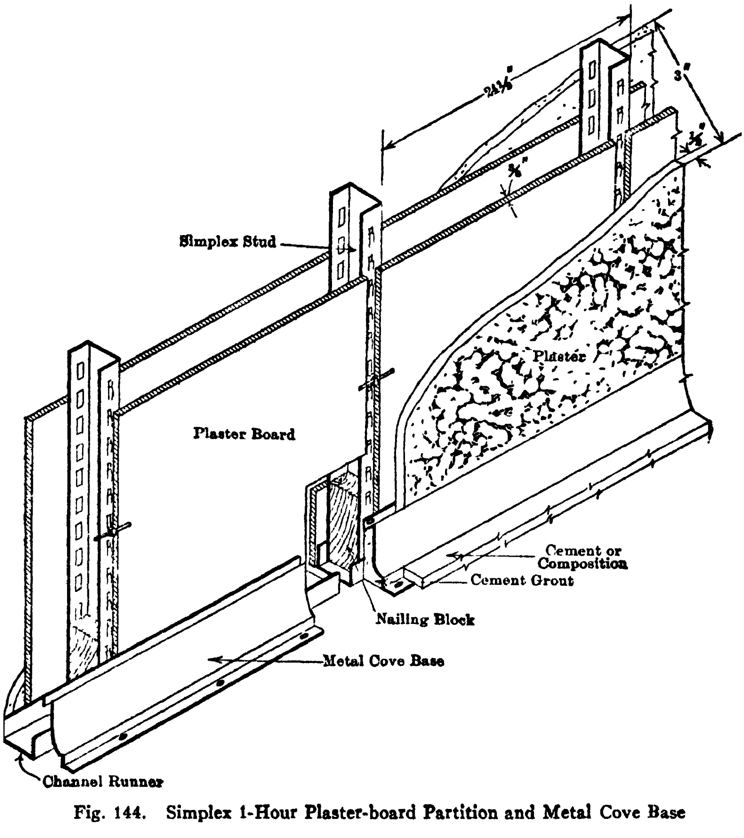


Fig. 144. Simplex 1-Hour Plaster-board Partition and Metal Cove Base

nel studs with CLIP 103, Fig. 145, with adjacent boards held in alignment with CLIP 101. At every sixth channel stud, a toggle tie CLIP 105 is introduced at every junction of three plaster boards. The boards are finished with a bond and finish coat ½ in thick on both outside faces.

Shaft-Construction. The most important partitions in a building are those inclosing interior shafts. Vertical openings through buildings form flues and cause up-drafts. In all buildings they should be inclosed for two reasons: first, to prevent a fire that would find a natural outlet in such openings from

spreading to other floors; and secondly, to prevent, as far as possible a fire from getting into these openings where the draft would greatly increase its fury. To be thoroughly effective the enclosure walls should be constructed of materials of not less than 3-hour fire-resistance. In the walls inclosing elevator-shafts no openings except those necessary for entrance-doors should be permitted. The doors should be of fire-resistive construction.* Glass lights are sometimes provided in such doors, although this is not good practice; if they are used, wire-glass, only, should be used, in accordance with the limitations noted under fire-door construction. The architectural features of open grilles may still be retained for the fronts and doors of such elevators by using them in conjunction with approved wire-glass construction. In interior light-shafts and vent-shafts, openings must necessarily be provided, but here again the construction of the window-frames, sashes, and glazing should be as far as possible as described in Article 20. Whenever the occupancy of a building admits, the stairs, also, should be inclosed in fire-resistive partitions, with fire doors at the openings. Unless so inclosed, the stair-

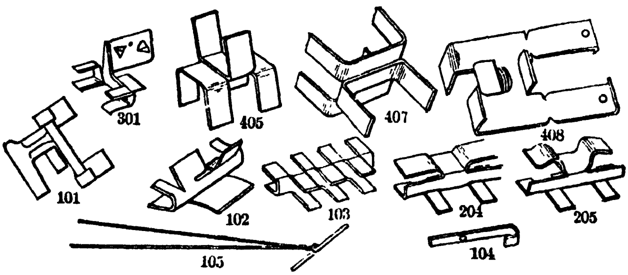


Fig. 145. "Clip-on" Anchors and Ties

ways form flues for the flames, and the stairs themselves, consequently, are exposed to intense heat. Open shaft-walls should in all cases be carried 3 ft or more above the roof.

14. Types and Properties of Metal Lath †

General Classification of Lath. Numerous styles of METAL LATH have been placed on the market in recent years under various trade names, for which special advantages are claimed either in economy of plastering materials, efficiency of use, or durability. In addition to those laths which are used primarily for plastering, there are other forms known as SELF-CENTERING, which are applicable both as reinforcement and centering in concrete construction.‡ PLASTERING LATH proper may be classified in the following types: (*a*) SHEET; (*b*) EXPANDED; (*c*) RIB; (*d*) WIRE; and (*e*) COMBINATION. STUCCO lath, § for use with Portland-cement mortars on exterior walls or for "GUNITE," are either of expanded metal or wire fabric of heavier weight and larger mesh than ordinary plastering lath. All types of lath are furnished in various non-corrosive metals as well as plain or galvanized steel, such as COPPER-BEARING, ARMCO INGOT IRON, and TONCAN METAL. When made of

* See Fire-Doors, Article 20.
† S. P. R. R3-28.
‡ See Self-Centering, Article 9.
§ See Stucco Lath, Article 14.

ordinary steel or non-corrosive alloys, it should be at least painted with an asphaltum or other preservative paint, or be galvanized, to prevent initial corrosion until the protective coat of Portland or gypsum-cement mortar has been applied. No unprotected lath is manufactured for PLASTERING use. For STUCCO work, galvanized lath with Portland-cement plastering should be used.

The general function of metal lath is to serve as a plaster base, but in addition it reinforces the plaster, distributes stresses and precludes the possibilities of cracking. The large size of the sheets with lapped joints between abutting units, when properly wired together, insures a monolithic construction. It also tends to eliminate streaking and staining when condensation of moisture occurs, because of the uniform distribution of the metal reinforcement. Owing to the large number of plaster keys resulting with a good metal lath, the damage of broken keys from vibration is more or less eliminated, and there is little or no pressure from swelling or shrinking of the base. In conjunction with gypsum plaster or Portland cement, metal-lath constructions have demonstrated their fire-resistive qualities both in tests and actual fires. Its superiority from the fire-protection point of view depends in large

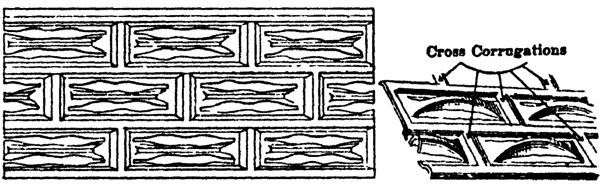


Fig. 146. Bostwick Truss-Loop Lath

measure on its ability to hold the softened and calcined plasters and mortars in place by mechanical bond, after the adhesive and strength properties of the plaster have been destroyed by fire.

Typical laths and their characteristics are illustrated in the following paragraphs.

(a) Sheet Metal Laths. SHEET LATHS in general are economical in plaster, which covers one side of the lath, latches through the perforations, but does not embed the metal completely. Their rigidity makes for ease and rapidity of erection.

Bostwick Loop Lath, shown in Fig. 146, has a cross-ridged corrugation at right-angles to the rib structure and possesses considerable rigidity. As a plaster base it permits wider spacing of supports, and because of the multiple keys and bearing surfaces, the claim is made that the brown coat of plaster can be applied immediately over a moist base coat without removing the scaffold, resulting in economy of construction. It is manufactured by the Bostwick Steel Company of Niles, Ohio, in sheets 24 in by 96 in, and weighs 4.5 lb per sq yd, painted, and 5.5 lb per sq yd, galvanized. It is also used in SELF-CENTERING * in TOP PLATFORM SLABS on steel joists.

Wheeling Arch Lath, shown in Fig. 147, is furnished painted or galvanized in plain steel or copper-alloyed steel by the Wheeling Corrugated Company of Wheeling, W. Va., in sheets 27 in by 96 in, weighing 4.5 lb per sq yd, and galvanized, 5.2 lb per sq yd. This lath can also be applied as SELF-CENTERING.

* See Self-Centering, Article 9.

Troff Kalmanlath, shown in Fig. 148, manufactured by the Kalman Steel Co. of Chicago, Ill., comes in the sizes and weights shown in the table. The shape of the ribs and rigidity of the lath are claimed to produce speed and ease in plastering.

	Weight per sq yd lb	Size of sheet, in
Steel painted black	4 5	13½×96 24 ×96
Galvanized	5 5	13½×96 24 ×96

Also furnished in copper bearing and commercial pure iron.

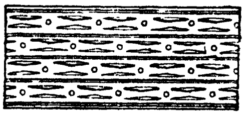


Fig. 147. Wheeling Arch Lath

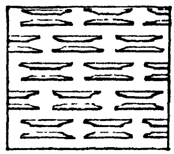


Fig. 148. Troff Kalmanlath

Shurebond Lath, shown in Fig. 149, is manufactured by the Goldsmith Metal Lath Co. of Cincinnati, Ohio, in ½-ft lengths up to 6 ft and foot lengths up to 12 ft, in widths of 2 ft exclusive of laps and in four different weights 0.625, 0.80, 0.95 and 1.25 lb per sq ft. The dovetail ribs, a distinctive feature of this lath, are spaced 4 in on centers, with a flange on all sides providing for the lap of sheets. It can be formed into arches or cylinders with a minimum radius of bend of 3 in. In common with other sheet metal laths it can also be used in SELF-CENTERING.

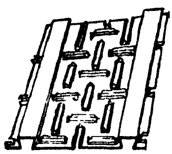


Fig. 149. Shurebond Metal Lath

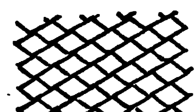
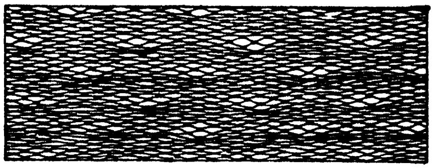


Fig. 150. "Key" Expanded Metal Lath

(b) Expanded Metal Lath is economical in weight and generally considered superior for plastering when the mesh is formed to take the plaster freely, key it thoroughly, and provide a uniform distribution of metal reinforcement throughout the area, wholly embedded in the plaster coat. Different methods are followed in the manufacture of the expanded sheets for which various advantages are claimed for the shape of the key produced and the cold-working effects resulting in increased strength of the metal.

Key Expanded Lath, shown in Fig. 150, manufactured by the Genfire Steel Company, Youngstown, Ohio, is a general-purpose lath uniformly pliable in all directions, and is furnished in sheets 27 in by 96 in, in four weights, 2.2,

2 5, 3.0, and 3.4 lb per sq yd. It is furnished in two sizes of mesh, Standard Size KEY and in a smaller mesh KEYLOK. The weights of the two meshes are the same, but the latter provides slightly more rigidity and cannot be CUPPED for self-furring purposes as the Standard Key lath. Similar DIAMOND MESH LATHS are furnished by the Truscon Steel Co., Berger Mfg. Co., Youngstown Pressed Steel Co., Penn Metal Co., Northwestern Expanded Metal Co., and others.



Self-Furring Key Lath is furnished by the manufacturers of diamond mesh lath, to meet the demand for a flexible self-furring lath. The sheets are deformed in various ways at regular intervals to form a cup or projection beyond the plane of the lath, as shown in Fig. 151.

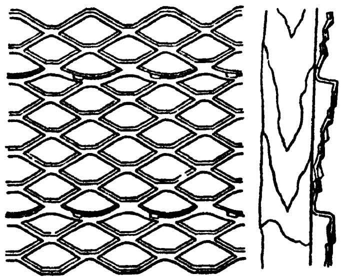


Fig. 151. Self-furring Metal Lath

Cup Kalmanlath, shown in Fig. 152, manufactured by the Kalman Steel Company of Chicago, Ill., is a specially deformed self-furring lath furnished in sheets 18 in by 96 in and weights of 2.5 and 3.4 lb per sq yd. It insures a heavy coat of plastering with an excellent mechanical bond and is commonly used in the Middle West for stucco work.

Besides the KEY or diamond mesh lath, other flat expanded laths are manufactured with small mesh and either flat or very shallow ribs not more than ⅛ in in height, for which is made the claim of plaster saving without invalidating the efficiency of the protective coat.

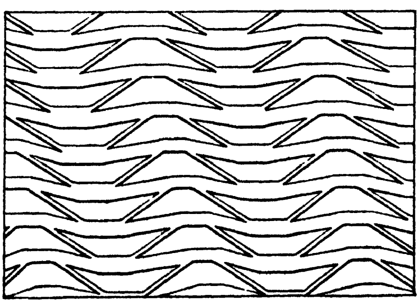


Fig. 152. Cup Kalmanlath

Herringbone Double-mesh Expanded Lath, manufactured by the Genfire Steel Co. of Youngstown, Ohio, is shown in Fig. 153. It is furnished in sheets 24 in by 96 in and standard weights of 2.75, 3.0 and 3.4 lb per sq yd. The longitudinal ribs lie at an angle to the plane of the sheet and the ribs are formed to make the lath self-furring.

"Plasta-Saver" Expanded Lath, shown in Fig. 154, manufactured by the Northwestern Expanded Metal Company of Chicago, Ill., has a flat rib ½ in wide with a ⅛-in bead at the center connected by sections of small diamond

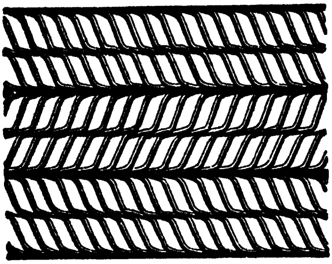


Fig. 153. Herringbone Double Mesh

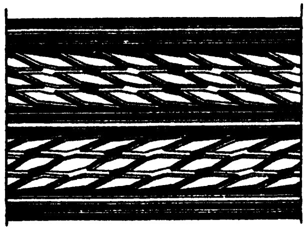
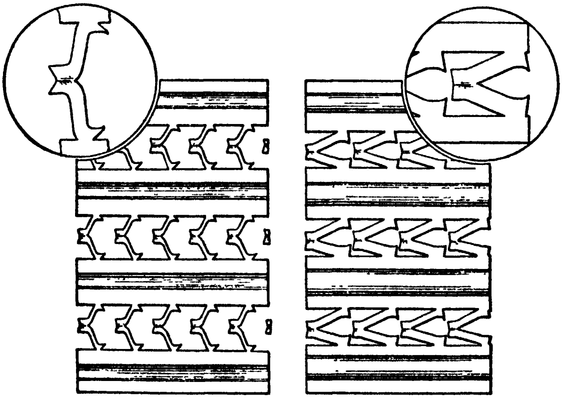


Fig. 154. "Plasta-Saver" Expanded Lath



Truscon 1-A Lath Truscon 2-A Lath

Fig. 155. Truscon "A" Expanded Lath

type mesh and is designed to facilitate coverage. It is furnished in sheets 24 in by 96 in and weighs 3.4 lb per sq yd.

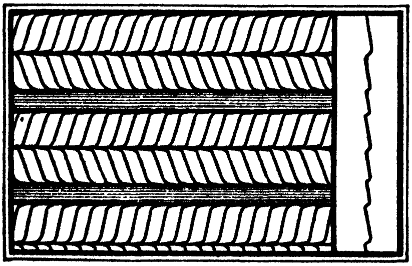


Fig. 156. Y. P. S. Z-rib Lath

Truscon "A" Expanded Lath, shown in Fig. 155, is manufactured by the Truscon Steel Co. of Youngstown, Ohio, in 3 0 lb and 3.4 lb per sq yd sheets, 18 and 16⅞ in wide, respectively, and 96 in long.

Z-Rib Lath, shown in Fig. 156, manufactured by the Youngstown Pressed Steel Co. of Warren, Ohio, is a lath of universal application for one-side interior plaster work and is

furnished in sheets 24 in wide by 96 in long and in weights of 2.75, 3.0 and 3.4 lb per sq yd.

(c) **Rib Lath** is usually formed with U-shaped ribs 3/8 in in height designed for use where added strength and rigidity are required, as on suspended ceilings. These laths are particularly adaptable for use as a base on ceilings under STEEL JOIST FLOORS for spacing of joists up to 30 in.

Diamond Rib Lath, shown in Fig. 157, manufactured by the Genfire Steel Co. in sheets 24 in by 96 in, has the ribs 4.8 in on centers but can be furnished in special lengths on mill order. It weighs 3.0, 3.4 and 4.0 lb per sq yd. A similar lath is furnished by the Truscon Steel Co., Northwestern Expanded Metal Co., U. S. Gypsum Co., and others. The Youngstown Pressed Steel Company's Rib Lath employs sections or panels of herring-bone design between the ribs. The Berger Mfg Co. furnishes 3/8-in V-shaped Rib Lath in lengths of 102 in in plain steel and Toncan Iron.

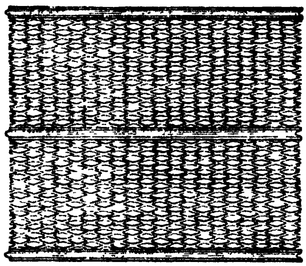


Fig. 157. "Diamond Rib" Lath

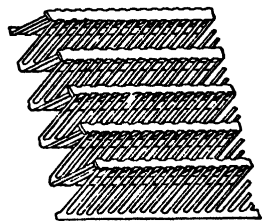


Fig. 158. "Trussit" Expanded Lath

Trussit Expanded Lath, manufactured by the Genfire Steel Co., is a special form of lath adapted to fire-resistive walls and partitions of solid construction. It is uniformly expanded (see Fig. 158) in both directions so that it can be plastered equally on both sides. No studding is necessary. It is furnished in sheets 19 in wide and 8 to 12 ft long painted, and 8 ft long in the galvanized and Armco Iron sheets. The galvanized sheets weigh 0.68 and 0.88 lb per sq ft, and the painted steel 0.57, 0.62 and 0.83 lb per sq ft.

(d) **Wire Lath** for plastering is furnished in WELDED mesh, made of cold-drawn wire of No. 18 to 21 Washburn Moen gauge wire, either 2 or 2½ meshes to the inch in STOCK GRADES and SIZES. Other combinations of gauges and meshes can be secured on mill order. Where special rigidity is required for wide spacing of furring or studs, V-shaped stiffening ribs of No. 24 gauge steel are attached at intervals of 8 in to the wires by flat clips or are interwoven in the mesh. V-stiffeners make the lath SELF-FURRING, enabling the lath to be applied directly to flat surfaces without furring bars. Rod stiffeners ¼ or 5/16 in in diameter may also be obtained on special order, bound to the lath by wire clips. (See Figs. 159 and 160.) For average construction with 12-in furring, Nos. 18 or 19 gauge wire plain lath, or Nos. 20 or 21 gauge stiffened-lath should be used. With supports on 16-in centers or more, stiffened wire lath should always be used. All end joints should be lapped at least 1 in, and side laps not less than ½ in, and all transverse joints alternated so as not to be in one line. The lath should be ordered in the proper width to provide for laps, preferably located at furring or supports, and to eliminate waste and bulky laps.

Clinton Wire Lath is furnished either painted or galvanized-after-woven, in widths of 36 in, by the Wickwire Spencer Steel Co. of New York, and in Nos. 18, 19, 20, and 21 gauge wires, in rolls 150 ft long.

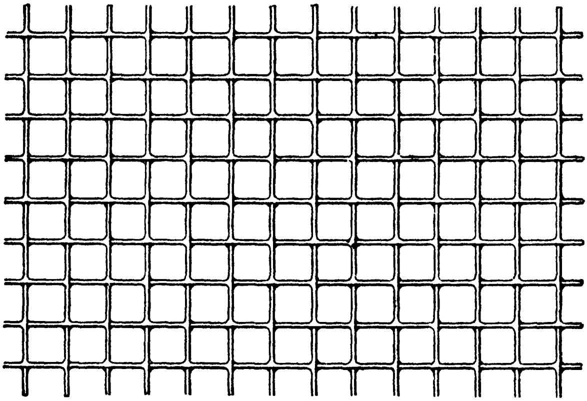


Fig. 159. Plain Wire Lath, Galvanized after Woven

Roebling Wire Lath is furnished by the New Jersey Wire Cloth Co., New York, N. Y., in widths up to 10 ft and in rolls 150 ft long.

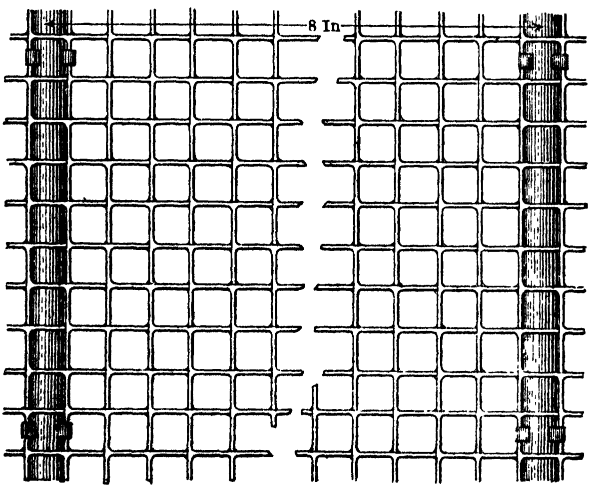
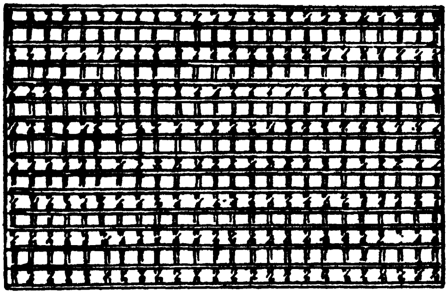


Fig. 160. Stiffened Wire Lath, Galvanized after Woven

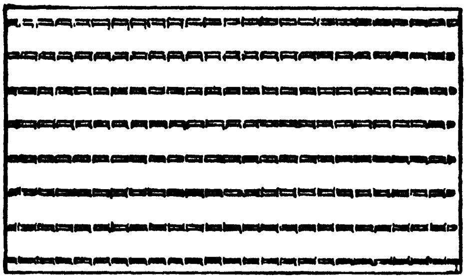
(e) Combination Lath is composed of either EXPANDED METAL or WIRE LATH combined with an integral backing of water-proof felt, heavy reinforced

paper or fibrous boards. The backing excludes moisture in the case of exterior stucco work, provides a back-stop for the plaster to assist in complete embedment of the lath, and with the proper selection of backing materials serves as both a heat and sound insulation. The backing can also be furred out from the wall to provide for a fixed dead-air space between outer walls and their weather-proof covering. COMBINATION lath can also be used as a SELF-CENTERING and reinforcement for concrete and gypsum platform floor-construction.*

Steeltex, shown in Fig. 161, is manufactured by the National Steel Fabric Company, Pittsburgh, Pa. RIBBED STEELTEX for plastering consists of



Front of Steeltex Sheet



Back of Steeltex Sheet

Fig. 161. Steeltex Sheet Lath

sheets $28\frac{1}{2}$ in wide by 50 in long, with 2 in by 2 in No. 16 gauge mesh, cold-drawn, electric-welded, galvanized wire fabric, which is secured to a fibrous felt backing with 26 gauge V-shaped ribs on the reverse side.

For STUCCO work, 14 gauge mesh is attached to a double layer of the fibrous backing, with a mastic filler between the felt sheets. It is furnished in rolls, 50 in wide and 108 ft long, or in sheets 50 in by 52 in. A similar material is furnished for WATER-PROOFING brick-faced walls. STEELTEX for PLATFORM FLOOR SLABS, on spans up to 36 in, is supplied in rolls 4 ft wide, consisting of No. 12 gauge galvanized steel wires with a 3-in by 4-in mesh

* See Structural Platform Slabs, Article 7.

secured to the water-proof backing in the usual manner. In pouring the concrete fill, the lath sags sufficiently to embed the steel reinforcement to a depth of ½ in.

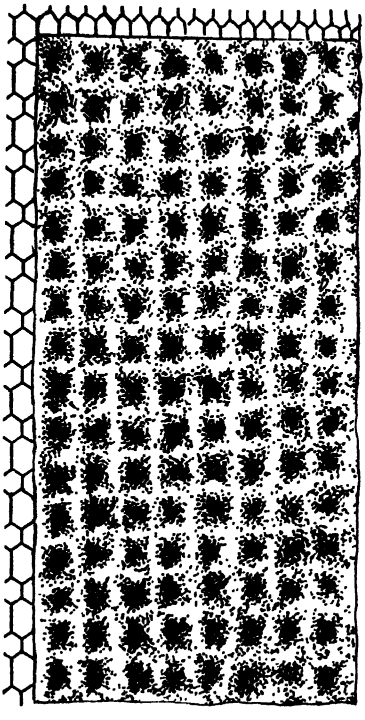


Fig. 162. "Biflax" Lath

Biflax, shown in Fig. 162, is manufactured by the Flax-Li-Num Insulating Co of St. Paul, Minn., in sheets 24 in by 48 in, as an insulated base for general interior plastering. It consists of a base of extra heavy FLAX-LI-NUM, ½ or 1 in thick, applied to expanded metal lath with No. 18 gauge wire ties. A layer of water-proofing felt is attached between the lath and the FLAX-LI-NUM base. The metal lath extends 1 in beyond the insulation at one side and one end of the sheet, to allow the metal lath to overlap the tightly butted joints of the FLAX-LI-NUM insulation.

Foster Floor Form. This is a form of COMBINATION LATH used for permanent centering and reinforcement of concrete slabs on top flanges of light structural beams spaced 4 ft on centers. The form is a shop-made unit consisting of a ⅜-in plaster board with 2-in square wood nailing-strips cemented to the board with minwax. Welded wire fabric of Nos. 10 and 12 wires with 4 in by 12 in mesh is secured to the nailing-strips and form as shown in Fig. 163. The unit comes to the job ready for installation, and the

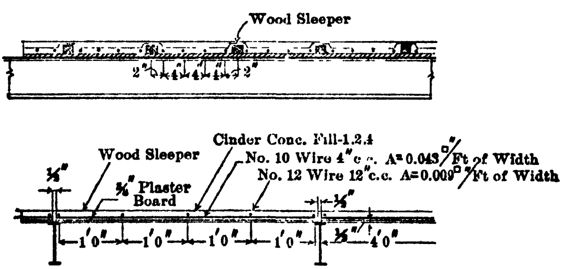


Fig. 163. Foster Floor Form

concrete mixture is deposited in place. It is adaptable for light loads for residence and office occupancies. The plaster board serves also as a sound

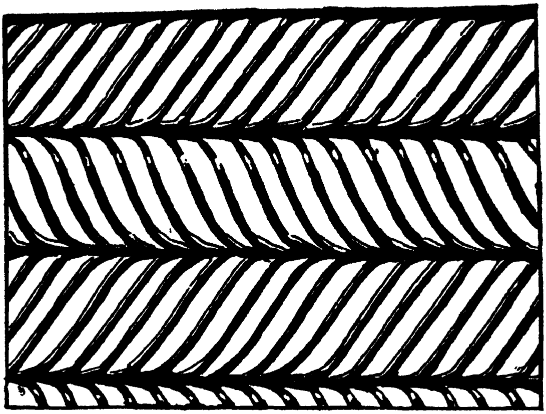


Fig. 164. Herringbone Rigid Lath

deadener and insulator between the floor-construction and the steel beams. It is manufactured by the National Bridge Works, New York, N. Y.

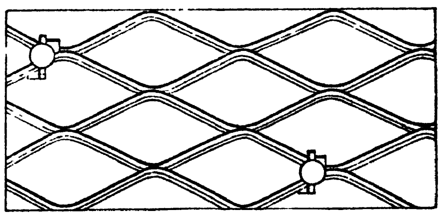


Fig. 165. Expanded Metal Lath for Stucco

Stucco Laths. The larger mesh Expanded Metal laths, such as HERRINGBONE RIGID METAL LATH manufactured by the Genfire Steel Co. in galvanized sheets 20¼ wide by 96 in long, is a rigid lath adaptable for stucco work (see Fig. 164), weighing 3.4 lb per yd.

RED TOP 1½-in-mesh EXPANDED METAL LATH, controlled by the U. S.

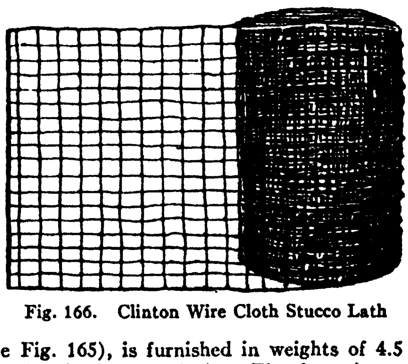


Fig. 166. Clinton Wire Cloth Stucco Lath

Gypsum Co. (see Fig. 165), is furnished in weights of 4.5 and 6.18 lb per sq yd, black and galvanized, respectively. The sheet sizes are 24 in by 96 in.

CLINTON WIRE CLOTH, galvanized in 1⅜ by 2 in mesh of No. 14 gauge wire in rolls of 24¾-in width and 219 ft long (Fig. 166) and GALVANIZED TRI-

ANGULAR MESH in 2 in by 4 in and 4 in by 4 in mesh, widths of sheet 36 and 48 in and 150-ft rolls (Fig. 167) are also furnished for stucco plastering or "GUNITE" work.

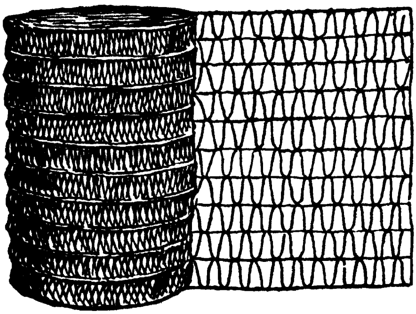


Fig. 167. Galvanized Triangular Mesh Stucco Lath

15. Sound Insulation * and Fire-Resistance

Acoustical Treatments. QUIETING or NOISE REDUCTION is an important consideration in modern multiple-dwellings, office-buildings, hotels, hospitals, music and broad-casting studios, and in buildings for many other occupancies. Many materials and forms of treatment are available for application to walls, partitions, floors and ceilings which possess to varying degrees the property of SOUND ABSORPTION and reduction of REVERBERATION and RESONANCE, but the problem of FIRE-PROOFING is concerned only with the fire-resistive values of such methods of construction. ACOUSTICAL TREATMENTS involve the use of soft and porous materials such as organic felts and fibers (hair felt, wood fiber, asbestos, cane fiber, flax, etc., bonded together by cements or other means); or inorganic plasters and tile of gypsum and cement plasters of a porous nature. The purpose of these treatments is to absorb sound at the source, and eliminate reflected sounds. Discretion must be exercised in the selection of such materials for use in fire-resistive construction to avoid the introduction of highly combustible materials which when improperly installed will increase the fire hazard.

Sound Insulation. The sound-insulating factors or efficiency of all types of partitions and floor-constructions to prevent the transmission of sound from one room to another have been investigated in various laboratories,† but the results from different sources are not entirely consistent, owing to the difference of equipment used, the sound-producing medium, the variations in the size of sound room, and other attendant conditions. The factors that have the greatest bearing on efficiency of sound insulation of the usual types of construction are: (1) The avoidance of cracks and crevices resulting from defective construction, or connecting air-passages such as ventilating ducts or other openings; (2) snug fitting of windows and doors and introduction of baffles or sound-absorbent materials in ducts, around pipes, etc.; (3) selection of the most rigid type of construction to eliminate vibration of the structure as a whole; and (4) wherever possible, the insulation of one face of the construction from the opposite face, and of partitions from floors, to reduce direct conductance of sound. In the many investigations made in sound laboratories throughout the country, the general laws controlling sound insulation have been fairly well determined; sound transmission can be reduced by using as massive construction as possible, or double construction with the two parts isolated; increased separation of the parts generally

* See Architectural Acoustics, Chapter XXXV.

† American Architect, July 4, 1923, Aug. 5, 1926, Jan. 30, 1928; Armour Engineer, May, 1926; Bulletin No. 127, Univ. of Illinois; F. R. Watson, Acoustics of Buildings; Research Bulletin, Bell Telephone Laboratories, N. Y. C.; Paul E. Sabine, Measurement of Sound Absorbent Coefficients, etc.

increases the insulation; a layer of absorbent material between the parts is effective if it does not bridge the gap; coarse cinder fills on floor-slabs make an effective deadening pad; hung ceilings on insulated hangers and flooring of wood, cork, rubber, asphaltic combinations, or similar cushioning materials, effectively deaden sound. The best sound-insulating construction, according to tests of Mr. V. L. Chrysler at the U. S. Bureau of Standards, consists of a structural center core, either hollow or solid, with a light facing unit on each side, preferably furred or so attached as to give as little direct connection between the two faces of the construction as possible.

Comparison of Construction Types. At the average pitch of sounds produced by speech or music, materials used in common building practice absorb sound to the degree shown in Table XXXI. The balance of the sound is reflected and therefore cannot be transmitted through the construction. It

Table XXXI. Percentage of Total Sound Absorbed by Building Materials

Material	Per cent
Hard plaster on metal lath	2½
Sand and lime plaster on wood lath	3
Plaster on hollow tile ..	2
Wood sheathing, unfinished...	6
Varnished wood .. .	3
Common brick wall.	3
Common brick wall, painted . .	1¾
Glass, single thickness	2¾
Linoleum... .	3
Marble .	1
Glazed tile....	1¾
Concrete and stone . . .	1½

is this small percentage of the total sound which is involved in the problem of insulation. The tests on insulating values of assemblies must include a wide range of sound frequencies, and the particular assembly must be selected for the noise level corresponding to the predominating sounds in the particular room. Table XXXII gives a measure of the amount of sound insulation

Table XXXII. Loudness of Sounds*

Kind of sound	Sensation units	
High-pitch whistle........ . . .	120	
Gunfire................	110–100	
Noise in airplane........	90	
Noise in New York subway.....	80	
Noise in stenographic room.... . .	70	
Noise 34th St. and 6th Ave., New York City....	60	↑ Range of
Interior, railroad train.........	50	conversa-
Noisy office....	40	tional
Soft radio in apartment	40–30	↓ speech
Quiet office.........................	30	

To determine the intensity of sound transmitted through a building assembly, deduct from these values, the values of the construction given in Tables XXXIII and XXXIV.

* Harvey Fletcher, Speech and Hearing.

Table XXXIII. Sound Reduction Factors—Partition Construction

Ascending order of efficiency	Description of partition	Reduction factor in sensation units: Residence noises	Average office	Noisy office	Extreme noises
1	¼-in standard Celotex board	14	18	24	24
2	¼-in 3-ply plywood	21	26	26	22
3	½-in standard Celotex board	22	23	27	25
4	⅛-in double-strength glass (edges thoroughly sealed)	26	31	33	29
5	¼-in plate glass	33	34	34	32
6	3-in hollow gypsum block; one side plastered; 2 coat gypsum	34	37	41	44
7	Hollow partition; ¾-in [studs; metal lath and plaster both sides; 3 coat gypsum	36	38	45	47
8	4-in Haydite tile; both sides plastered; 2 coat gypsum		38		*
9	3-in hollow gypsum block; both sides plastered; 2 coat gypsum	43	39	42	44
10	2-in solid partition; ¾-in [stud; ⅜-in gypsum boards plastered both sides, 3 coat gypsum	37	41	47	47
11	4-in hollow clay tile; both sides plastered; 2 coat gypsum	41	42	50	47
12	Wood studs; metal lath and plaster both sides; 3 coat gypsum	47	42	55	60
13	Wood studs; wood lath and plaster both sides; 3 coat gypsum	31	43	42	55
14	3-in hollow clay tile; both sides plastered; 2 coat gypsum	41	43	51	51
15	2-in solid partition; ¾-in [studs, metal lath plastered both sides; 3 coat gypsum	42	45	49	49
16	Wood studs; wire lath and plaster both sides; 3 coat gypsum.	49	48	58	60
17	4-in brick; both sides plastered; 2 coat gypsum	46	49	58	61
18	Two 2-in solid, back plastered metal lath and ¾-in [stud partitions; 2-in air space; 3 coat gypsum	36	50	62	68
19	Same as No. 18 except ⅜-in layer of cabots quilt in air space	49	49	75	70
20	Wood studs; ⅜-in gypsum boards and plaster both sides; 3 coat gypsum	53	51	55	58
21	Same as No. 18 except 1-in hair felt in air space	49	51	72	69
22	Two 3-in hollow clay tile partitions, unplastered, spaced 1¾-in apart, with 1-in Flax-Li-Num in space. One partition insulated all edges.	55	51	66	73
23	Same as No. 18 except ½-in Celotex in air space	54	55	76	72
24	Two 3-in hollow gypsum blocks, plastered on outsides; with 1½-in air space, 2 coat gypsum	53	56	65	74
25	4-in hollow clay tile partition; wood furring strips; ½-in masonite board and plaster both sides; 2 coat gypsum	55	57	69	70
26	4-in hollow clay tile partition; wood furring strips, paper, metal lath and plaster both sides; 3 coat gypsum	56	57	58	64
27	4-in hollow clay tile partition; wood furring strips, ½-in insulite board and plaster both sides; 2 coat gypsum	52	61	61	62

* From other sources and not directly comparable

Note. Where insulating sheathings and fillers are used, they should preferably be incombustible in fire-resistive construction.

Table XXXIV. Sound Reduction Factors—Floor Construction

(Relative values for sounds with frequency of 500–550 cycles)

Ascending order of efficiency	Description of floor	Reduction factor in sensation units
1	Wood joists; 3 coats of gypsum plaster on wood lath, double wood flooring	41
2	8-in flat hollow tile arch; plastered 2 coat gypsum ceiling; 2-in cinder fill and wood flooring	47
3	Same as No. 2, except 2-in cinder concrete fill and 1-in cement floor finish.	47
4	6-in combination structural clay tile and concrete; 2-in cinder concrete fill, and 1-in cement finish; 2 coat gypsum plastered ceiling	48
5	Wood joists; 3 coats of gypsum plaster on wood lath; double wood flooring with ½-in insulite between layers	54
6	4-in combination structural clay tile and concrete; ½-in insulite furred ceiling; 2 coats of gypsum plaster.	56
7	4-in reinforced concrete slab; ½-in insulite furred ceiling plastered; ½-in insulite between slab and double wood flooring	56
8	Wood joists; insulated suspended ceiling of ½-in insulite with 2 coats of gypsum plaster, double wood flooring with ½-in insulite between layers.	57
9	4-in combination structural clay tile and concrete; ½-in insulite furred ceiling, 2 coats of gypsum plaster, cinder fill and double wood floor with ½-in insulite between layers	61
10	Same as No. 9 except insulite between slab and double wood floor.	63
11	Same as No. 9 except suspended ceiling of ½-in insulite on insulated suspenders	66

Note. Many of these constructions are partly composed of combustible materials. In Fire-Resistive assemblies incombustible materials should be substituted in insulating sheathings and ceilings.

required in terms of SENSATION UNITS for different degrees and kinds of sound.

Most constructions show a greater reduction factor for the higher frequencies of sound than for the lower. To make speech unintelligible through a partition, it must have a reduction factor of at least 40 sensation units, and to make it entirely inaudible, between 50 and 60 sensation units. The intensity of any sound as heard through a partition or floor-assembly would be measured by the difference between the original intensity of the sound and the reduction factor of the construction. The REDUCTION FACTORS in terms of sensation units given in Tables XXXIII and XXXIV have been assembled from tests of the U. S. Bureau of Standards.* In the solution of sound-absorption and insulation problems, it is generally customary to assume sounds with a frequency of 512 cycles as the average for SPEECH and MUSIC, and 1 000 to 2 000 cycles as the range for OFFICE USES. Under conditions of average building use, however, very little actual difference can be distinguished between the SOUND-INSULATING properties of approved types of fire-resistive partitions.

16. Wall Furring

Furring for Outside Walls. The outside walls of fire-resistive buildings are generally finished on the inside by plastering applied directly to the masonry. When the walls are of brick, it is often desirable to fur them so that there will be an air-space between the plaster and the masonry to prevent the passage of moisture. This furring should be of incombustible materials, such as hollow brick, clay tile, or metal. FURRING BRICK are made of brick-clay and of the same size as common bricks; but they are hollow. They are built up with the rest of the wall, on the inside face, and bonded into the wall by the usual header-courses. STRUCTURAL CLAY FURRING TILE is used for insulation against the weather and to produce architectural effects such as deep reveals.† It consists either of STANDARD PARTITION-TILE or SPLIT-FURRING TILE. SPLIT FURRING consists of 3- or 4-in tile with 12 by 12 in face extended with deep scorings along the center line of webs and shells that lie normal to the wall face. When the tile reach the field they can be readily split into two separate units, $1\frac{1}{2}$ or 2 in thick, as indicated in Fig. 168. The furring is supported or tied to the wall with metal ties. The face is grooved to give proper bond for the plastering. At recesses in the walls partition-blocks are substituted across the openings, making a continuous wall-surface. When using furring-tiles, the mason should be careful not to drop mortar into the hollow spaces. When walls are furred or lined with tile, "soft grade" tile should be built in wherever nailings are required for bases, picture-moldings, etc.

Split furring cannot be built free-standing and must be anchored to the masonry wall by the use of spikes or heavy wire ties built into the mortar joints. A butter joint is used, and mortar should be omitted between ribs and wall. When furring more than 2 in in thickness is desired, it is more practical to use 3- or 4-in partition-tile. When the furring stands free of the main wall, it should be anchored at intervals by cross-walls of similar tiles. Funing around PIPES or DUCTS is usually accomplished with 3-in partition-tile.

* Scientific Paper Nos. 526 and 552, and Research Paper No. 48.
† See Combination Walls, Article 12.

Metal Furring. To produce architectural forms in the interior decoration of fire-resistive buildings, METAL FURRING and METAL LATH are now almost universally used. The furring is always of a false nature, and never employed to carry loads other than its own weight; so that the only requirement is that it shall be incombustible and furnish a satisfactory ground for attaching the metal lath. For coves, cornices, false beams, etc., the furring-members are made of light bars, angles, tees, or channels, attached to the walls by means of nails, staples, or toggle-bolts, and to the steel beams by means of bolts, hangers, clips, etc. The furring-pieces are bent or shaped to the approximate outlines of the finished plaster-work, so that when the lathing is applied it will require not more than $1\frac{1}{2}$ or 2 in of plaster to give the desired outline. For plane surfaces, the furring should be brought to within $\frac{3}{4}$ in of the plaster line. Deep beams, etc., should be braced by diagonal rods, to prevent distortion. All structural-steel members should always be fire-proofed back of the furring. The lathing is secured to the furring by means of No. 18 galvanized lacing-wire. The spacing of the furring should be either 12 or 16 in, according to the kind of lath that is to be used. When chases in walls are covered over, the covering should be done with metal furring and lath. The casings for vertical pipe-lines, also, should be of this construction and the space about the pipes at the floor-level should be filled solidly with fire-resistive material, to cut off all connection between stories.

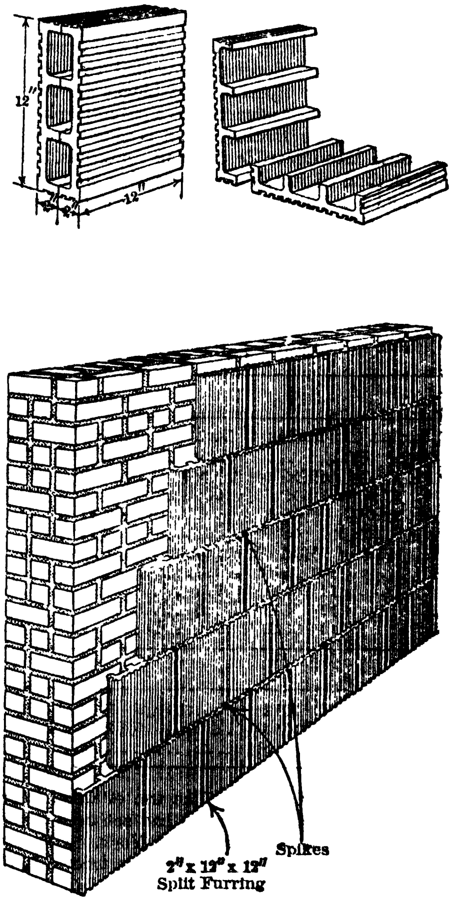


Fig. 168. Split Clay Furring Tile

17. Fire-Resistive Floorings

Untreated Wood. The floor surfaces of fire-resistive buildings are commonly made of hardwood, but this should always be backed up solidly with incombustible materials. In the building code of New York City, the use of untreated wood flooring is limited to buildings not exceeding 150 ft in height. Special precautions must be taken to prevent free combustion and transmission of flame. This object is accomplished in the so-called "CELLIZED" hardwood flooring controlled by the Cellized Oak Flooring, Inc., Memphis, Tenn. Special wood blocks are laid directly in contact with the fire-resistive floor-construction in a patented plastic cement, "EVERBOND." It is claimed for this flooring that it is sound-deadening, and more resilient than the usual incombustible flooring materials, as well as being fire-resistive. This method of installing the flooring eliminates air-flues between the concrete and wood and thus minimizes the possibility of combustion and transmission of fire. "CELLIZED" flooring is installed by licensed contractors in blocks of the following woods and dimensions:

Oak, 13/16 in thick	Maple, 13/16 in thick	Maple, 11/16 in thick	Beech, 13/16 in thick	Walnut, 13/16 in thick	Philippine hardwood, 13/16 in thick	Yellow pine, 13/16 in thick
Squares						
Inches	Inches	Inches	Inches	Inches	Inches	Inches
6¾ × 6¾		6¾ × 6¾	6¾ × 6¾	6¾ × 6¾	6¾ × 6¾	9¾ × 9¾
7½ × 7½	6¾ × 6¾	9 × 9	7½ × 7½	9 × 9	9 × 9	
9 × 9	7½ × 7½		9 × 9	11¼ × 11¼	11¼ × 11¼	
10½ × 10½	8 × 8		11¼ × 11¼			
11¼ × 11¼	9 × 9		12 × 12			
12 × 12	9¾ × 9¾					
	10 × 10					
	11¼ × 11¼					
	12 × 12					
	13 × 13					
Rectangles						
6 × 12	6 × 12		6 × 12	6¾ × 13½	6¾ × 13	13 × 13
6¾ × 13½	6½ × 13		6¾ × 13½			
	6¾ × 13½					

Fire-Retardant Wood. The use of wood impregnated with fire-resisting chemicals for flooring surfaces has not been generally applied in the United States outside of the City of New York. The characteristics of this material and its use in buildings are elsewhere treated in detail in this chapter.*

Cork Tile. CORK FLOORING is generally laid in tiles of several shades of brown and in various sizes, 5/16 and 1/2 in in thickness. This material is not incombustible but burns very slowly, and when laid in close contact with the under-flooring does not freely support combustion. It is made from a select grade of cleaned and screened cork-wood shavings, compressed in molds of

* For properties, see Fire-Retardant Wood, Article 5; also Fire-Retardant Wood Trim, Article 18.

the desired size and baked at a temperature of 600° F. By this process the natural wood resins are liquefied and serve to coat and bind the cork particles together. The tile are laid in elastic cement directly on the smooth cement finish of concrete floors or on any thoroughly dry base of either wood or metal. The flooring is resilient, noiseless, non-slip, a low conductor of heat, and will not dust or crumble.

Rubber Tile. RUBBER FLOORING is manufactured in two styles, Interlocking and Block Tiles. Interlocking Tile is 2⅜ in square and ⅜ in thick, and Block Tile is 6 in and 9 in square and ⅜ in thick. The INTERLOCKING TILE introduced by the New York Belting and Packing Co., is laid in alternate male and female tiles, in a range of colors adaptable to designs in harmony with the interior decoration. Rubber tiling should be laid on a base course of cement mortar, troweled smooth and level and absolutely dry before the tile are laid. The interlocking tile are frequently installed without cementing to the base course, and it is claimed for this type that the interlocking feature assures permanence of installation without buckling or blistering of the flooring. The resilience of rubber minimizes wear from abrasion or friction and cracking of the material; and expansion of the tile is taken up in the many interlocking joints. It is adaptable as flooring in office-buildings, hospitals, public buildings, schools and residences, and wherever a sanitary, durable and safe floor is required. Interlocking tile is especially useful where subject to vibration, or where exposed to dampness and to the weather. The first cost is considerably higher than cork and other resilient flooring-materials, but it is one of the longest wearing floor-surfaces. Though combustible at high temperatures, the material does not readily ignite or transmit fire and is generally accepted as a fire-resistive finish flooring.

Linoleum. The chief ingredient of linoleum is linseed oil, which is oxidized by exposing it to the air until it hardens into a tough, rubber-like substance and is then thoroughly mixed with powdered cork, wood flour, various gums and suitable color pigments. The resultant plastic mass is pressed on burlap. The "green" linoleum then passes into drying buildings, where it is cured and seasoned. Linoleum should be laid on dry, smooth floors. When cemented in direct contact with incombustible underflooring, ½-in linoleum will not freely support combustion.

A PLASTIC LINOLEUM manufactured and laid in place in plastic form consists of an asphaltum composition combined with asbestos fiber, silica sand, and mineral pigments. The materials are received in containers on the job and mixed with naphtha or gasoline as a solvent and troweled in place on the floor in several coats. Owing to the volatility of the solvents used, the surrounding air becomes charged with an explosive vapor during construction and serious fires have resulted from the ignition of these vapors.* After the wearing surface has dried and cured thoroughly, the flooring, although combustible, does not offer a serious fire hazard if laid directly on the fire-resistive floor-assembly. Extreme care, however, must be taken during installation to eliminate the hazard of lighted matches and smoking.

Cement Mortars and Granolithic. Cement mortar and granolithic wearing surfaces are applied to concrete floors either in a MONOLITHIC finish integral with the structural slab or as a BONDED or separately applied finish after the structural slab has cured. The former method results in absolute assurance that the wearing surface will adhere to the base, but may result in damage to the finish before it has hardened sufficiently to withstand wear and injury

* Quarterly N. F. P. A., January, 1926.

Table XXXV. Comparison of Types of Flooring in Factory and Mercantile Buildings

Type of floor surface	Average life in years		Safety (non-slip quality)	First Cost	Annual Cost (foot traffic)	Cost of renewal	Annual Cost after renewal	Advantages (1) and disadvantages (2)
	Trucking	Foot wear		Dollars per square foot				
Integral granolithic	10 to 20	25	C	10	.004	.25	01	(1) Most economical of all floor surfaces. Longer life. Clean. Easily renewed. (2) Objection of operatives on account of hardness. Possibility of dusting. More difficult to attach machinery Slippery when wet.
Bonded or "laid after" granolithic	10 to 15	25	C	14	0056	20	008	Integral granolithic, as compared with bonded, has the advantage of slightly lower first cost and gives absolute assurance of bonding; but has the disadvantage, particularly in multi-story construction of delaying progress to an extent that much more than offsets the advantage of lower first cost.
Hardwood top, paper, screeds and cinder-concrete fill	Not suitable	15	B	30	02	18	012	(1) Cheapest of wood floors. (2) Takes 60 to 90 days longer to finish building than in case of granolithic floor. Moisture due to cinder fill takes 10 to 20 days longer than when tar base is used.
Hardwood top, paper, intermediate floor, screeds and cinder concrete fill	Suitable for light trucking	20	B	36	018	.18	09	(1) A low-cost wood surface adaptable to some purposes (2) Not suitable for attaching machinery or for heavy trucking.

Table XXXV (Continued). Comparison of Types of Flooring in Factory and Mercantile Buildings

Type of floor surface	Average life in years		Safety (non-slip quality)	First Cost*	Annual Cost (foot traffic)	Cost of renewal	Annual Cost after renewal	Advantages (1) and disadvantages (2)
	Trucking	Foot wear		Dollars per square foot				
Hardwood top, plank, screeds and cinder-concrete fill	6 to 12	20	B	47	0235	18	009	(1) Adaptable for trucking Light machinery can be secured to floor without drilling into concrete. (2) Greater weight and higher cost.
Hardwood top, pank and tar base	6 to 12	20	B	40	02	18	009	(1) Omiss on of cinder concrete saves 10 to 20 days. Saves 2 in of depth due to screeds. (2) Twisting and warping of plank apt to occur
Hardwood top, intermediate floor, plank and tar base	6 to 12	20	B	.46	023	18	009	(1) Twisting and warping of planks prevented. Better foundation for securing machinery. (2) Greater weight and higher cost.
Wood block floors	10 to 20	25	A	34	0136	34	0136	(1) Withstands extra hard service, Dustless, resilient, noiseless. (2) Not adaptable for some manufacturing purposes. Shrinkage or swelling of blocks sometimes causes trouble.

from other causes. The bonded method requires careful and special preparation of the base to insure a bond between structural slab and wearing surface. This latter method, properly executed, results in a better floor.

Cement and granolithic finishes are incombustible and add to the fire-resistance of the structure. Owing to hardness and greater heat-conductivity, however, they have an unfavorable effect on the efficiency, comfort, and health of operatives in factory and mercantile buildings. They are also more noisy and less resilient than the wood flooring. CEMENT floors have been used for rooms and corridors in hotels when covered with carpets attached to inlaid fastenings. GRANOLITHIC floors have much more durable and wearing qualities than the ordinary CEMENT-MORTAR finish troweled smooth. The latter are apt to dust badly unless treated with surface or integral chemical hardeners.

Composition Floors. These are usually admixtures of magnesite, asbestos, fine sand and sawdust, mixed with linseed oil and binders of magnesium chloride. They are spread over the entire floor in a plastic state in ¼-in layers to a total thickness of ½ to ¾ in. The flooring hardens in 12 to 24 hours. The base coat contains coarse fibrous fillers which take up the inequalities in the foundation, while the top wearing coat is of a fine-grained texture giving a smooth finish. Properly laid, this type of flooring is as durable as cement, without its hardness; it is elastic, wears well, withstands water and acids, and is incombustible. The dry materials are shipped to the job with pigments incorporated in the mix, in red, white, yellow, brown, gray, black, blue, and green colors. The flooring can be laid on bases of wood, stone, concrete, asphalt, cement, or metals. It cannot be installed directly over gypsum plasters, gypsum concretes, or over bricks, terra-cotta blocks, or concrete containing lime or lime compounds in the materials or joints. In this case, a water-proof membrane or felt must be first laid with a wire mesh reinforcement secured thereto as a foundation for the MAGNESITE. In any case, provision must be made for expansion and contraction by marking off the finish coat in squares, rectangles or sections of suitable size and at junction with walls and partitions.

Zenitherm,* precast magnesite composition blocks consisting of a combination of wood fiber and calcined magnesium oxide, are made by the Zenitherm Company, Kearney, N. J., to be laid over concrete or wood under-flooring. The blocks or tiles are furnished in STANDARD SIZES and make a fire-resistive and sanitary floor-surface which is not affected by climatic conditions.

Asphalt Mastic. This flooring is composed of selected asphalts, rubbers and gums, inert mineral fillers, and non-fading mineral and chemical pigments. It is resilient, non-absorbent, non-slip and fire-resistive, and can be installed over wood or cement sub-flooring. When used over wood a special cushion coat with expanded metal lath is necessary. It is laid in a plastic form, and after setting can be brought to a polish by waxing; it is usually about ½ in thick.

Asphalt mastic floor-tiles are similar to asphalt mastic flooring, except that they are precast. They are laid on a dry base of either wood, cement or composition. The advantage of the tiles is that any number can easily be replaced in damaged floors without unsightly patching.

Cement, Clay and Slate Tile.† For public corridors, banks, lobbies, and toilet rooms, encaustic, vitreous, ceramic and quarry tiles and slates are

* See Zenitherm, Article 18.
† See Condensed List of Clay Tiles, Table XXXVI.

frequently used. CEMENT tiles have also been introduced to a limited extent. These materials are all incombustible and fire-resisting but are cold and trying to the comfort of occupants and operatives.

Mr. A. B. MacMillan, Chief Engineer of the Aberthaw Company, Boston, Mass., makes the following comparison of the life and cost of various common types of flooring found in factory and mercantile buildings.* (See Table XXXV.)

18. Interior Finish and Trim

Fire Hazard. That combustible trim in an otherwise FIRE-RESISTIVE structure adds fuel to the flames and therefore increases the fire hazard is not generally recognized in the design of buildings. The New York Building Code stands alone in this requirement that above the 150-ft height limit, the flooring is required to be of incombustible material, the interior finish including doors, door-jambs, window-frames, sashes, bases, and all other interior trim must be constructed of "metal or wood covered with metal, or of fire-proofed wood, or of any incombustible materials or any combination of materials that will show a fire-resistance not less than that of a fire-proofed wood." The same materials that are accepted for the top flooring can be used for the interior finish also.† This requirement of the building code of New York City is supplemented by the provisions of the New York State Labor Law that in all fire-proof factory buildings (which include all new factory buildings over four stories in height) all trim, interior finish and partitions, if not constructed "fire-proof," must be of incombustible materials. Certain exceptions to these requirements of the New York Code have gradually developed in practice. Wearing surfaces of cork, linoleum and similar materials, ½ in or less in thickness, when firmly cemented to the fire-resistive floor-construction are generally permitted in places other than public halls and stair enclosures. It is also generally permitted to install free standing moldings with a cross-sectional area of 2 sq in or less, and ⅛ in thick veneers secured to incombustible cores of ordinary wood-construction. In fire-proof partitions, door-bucks of wood are also allowed when backed up solidly and covered on all exposed sides with metal or other incombustible materials. In the burn-out tests conducted at the U. S. Bureau of Standards, Washington, D. C.,‡ the amount of combustible finish entering into the construction proved a factor in the intensity of fire measured by the time-temperature period.

The New York Fire Insurance Exchange allows a reduction of 5% on the insurance building rate when all interior trim is incombustible.

Fire-Retardant Wood Trim.§ There are several companies in the United States engaged in the commercial processing of wood to render it fire-resistive, employing different chemicals in the treatment but following the same general procedure for impregnating the wood structure. Among these companies are: Queens Lumber Fireproofing and Transforming Co., 1350 Broadway, New York City; Batavia and New York Wood Working Co., Batavia, N. Y.; Hardwood Products Corporation, Neenah, Wis.; Firesafe Process Co., 3821 Sherman Street, L.I.C., New York City; Tennessee Firesafe Lumber Co., Knoxville, Tenn.; Henry Klein & Co., 40 West 23rd Street, New York City;

* Factory Floor Surfaces, Aberthaw Company, Boston, Mass.

† See Fire-Resistive Floorings, Article 17.

‡ See Severity of Building Fires, Article 1.

§ See Fire Retardant Wood, Article 5.

Protexol Corporation, Kenilworth, N. J.; Sloane and Moller, Inc., 316 East 65th Street, New York City; Geo. H. Storm & Co., Park Avenue and 135th Street, New York City. As the material is not subject to check by external visual examination and no field tests have yet been developed to determine the uniformity and efficiency of the treatment, it is necessary to resort to field sampling and laboratory check tests. This involves considerable effort, time and expense in controlling the proper use of this material in practice, and has sometimes resulted in the use of unreliable material. As now controlled in New York City, every shipment to the job is subject to test on field samples before it is incorporated in the building structure. Among the recent buildings in which fire-retardant wood has been used for interior trim are the following: Albany Main Telephone Building, Albany, N. Y.; Fogg Art Museum, Cambridge, Mass.; Museum of Natural History, New York City; New Jersey Public Service Offices, Newark, N. J.; Orpheum Theatre, Boston, Mass.; and Holstein-Friesian Association Building, Brattleboro, Vt.

Metal-Covered Trim.* Among the first attempts in the United States to fire-retard the interior trim of buildings were those made in New York City, about the year 1880, in the form of metal-covered woodwork by the firm of Campbell & Bantossell of that city. The metal-covered wood industry has now developed a product for the interior trim of buildings which possesses all of the requirements necessary for architectural and decorative treatment. By this process the metal envelope is drawn on to a thoroughly kiln-dried, sound core of white pine, chestnut or oak, through steel dies, producing a profile of sharp angles and arrises, free from waves and buckles. All manufacturers maintain a representative line of standard moldings, but dies can be secured for special details. It is more economical, however, to use moldings from standard dies. The metals generally used are long terne plate; galvanized iron; furniture steel; Armco ingot iron or kalamein iron of 18 to 26 gauge; copper of 16 to 32 oz weight; or bronze of 16 to 24 gauge. The required weight of metal is dependent upon the severity of usage to which the doors or other trim will be subjected. Where the duty is likely to be heavy, 24 gauge steel or wrought-iron sheets should be used. The term KALAMEIN is commonly applied in the trade to all metal-covered woodwork to distinguish it from hollow metal constructions, whereas the true application is to a special form of galvanized iron. Door opening sizes and dimensions of stiles and rails are established by the Simplified Practice Recommendations of the Bureau of Standards, of Washington, D. C.† Panels are usually made of three-ply laminated pine or of incombustible materials such as asbestos sheets. For bronze doors, the jambs and trim are either bronze-covered or extruded bronze, stiles and rails are covered with No. 16 gauge bronze sheets, and panel and glass moldings are usually extruded sheets; where edge pieces and moldings are used they should lap the door on both sides. Standard thicknesses of doors are 1¾ and 2¼ in. Doors must be designed not to warp or twist under temperature stresses, by gluing the strips in stiles and rails with reversed grain, and introducing mortise, tenon and box-wedged joints where necessary. Metal-covered doors are sometimes made with continuous sheets of metal on both sides, without seams or joints and with the recessed panels formed by presses. Panel and moldings can be made an integral part of stiles and rails. In one type of

* See Kidder's Building Construction and Superintendence, Part II by the late Professor Thomas Nolan.

† S. P. R., No. 83-28.

door, the metal sheets of the two sides are made to overlap in a depression on the edges of the door * and are secured in place by screws. Metal-covered trim can be painted in any color or grained to match natural hardwoods. Copper and bronze-covered trim is furnished in antique, statuary bronze or any other suitable finish. Among examples of recent installations are: Bridgeport City Trust Co., Bridgeport, Conn.; Y.M.C.A., Springfield, Mass.; and N. Y. Central Office Building, New York City. Metal-covered trim is accepted as an incombustible material under most building code specifications, it cannot be considered as possessing a high degree of fire resistance especially in members of small dimensions.

Hollow Metal Trim. This type of construction when expertly carried out results in details for interior work which are efficient in fire-resistance and attractive in appearance. The details of construction lend themselves to economical installation in the building, with a minimum of labor costs, and possibilities of elaborate design in a variety of attractive finishes. Many of the outstanding buildings of recent times are equipped with hollow metal doors, frames, sashes, trim of corridors, hallways, stair and elevator enclosures, and interior office-partitions. Because of the non-absorbent character of the baked-enamel finish, it is readily cleaned and sanitary, especially if elaborate moldings are omitted and panels are made simply as smooth depressions. Among modern buildings fitted with hollow metal trim are the following: Philadelphia Free Public Library, Philadelphia, Pa.; Federal Reserve Bank, New York City, David Stott Building, Detroit, Mich.; Board of Trade Building, Chicago, Ill.: Empire State Building, New York City; and Fidelity Trust Building, Philadelphia, Pa

The COLD-DRAWN process by which the cold metal is drawn through special dies to give it the desired shape, retaining the bright smooth finish, is now generally employed in manufacturing frames, trim and moldings. A large number of metal dies are maintained for stock designs of panel and stop moldings, door and window-casings, wainscoting and chair-rails, and miscellaneous trim of all kinds. The corners and angles are sharp and true, and the pieces possess greater strength and rigidity than HOT-ROLLED metal of greater thickness. All joints are accurately fitted, assembled by either electric or gas welding, and are carefully dressed and finished. The steel doors are generally finished with baked enamel, but they frequently are shipped to the job with a priming coat of red lead or other metal paint, to be finished by the painting contractor. Besides hot and cold-rolled strip steel, galvanized steel and ingot iron, brass, bronze, copper and aluminum, monel metal and chrome stainless steels are employed in the manufacture of hollow metal trim, window-sash, frames and doors. Hollow metal doors of stainless steel with etched panels in any desired pattern and colored backgrounds of plastic enamel have recently been introduced for highly decorative treatments.† The finish is claimed to be fire-proof, weather-proof and water-proof. Hollow metal window-frames and sash of steel, wrought iron, cast and drawn bronze are also used in those parts of buildings where exposure to fire may not be serious enough to warrant labeled devices or where a more pleasing appearance is required than that resulting from protective devices such as hinged or rolling fire-shutters. They are glazed with plate glass, wire glass, prism glass, etc.

The following weights of metal are commonly used in the manufacture of hollow steel trim. The work for bronze doors is executed similarly to steel doors, except that the material is generally two gauges heavier.

* Thorp Fire Proof Door Co., Minneapolis, Minn.
† United Metal Products Co., Canton, Ohio.

Doors. Stiles and rails of No. 18 gauge; panels and cold-drawn panel mouldings of No. 20 gauge, interlocking with concealed metal fasteners, screwed on with special oval-headed screws, or riveted in place. Panels are usually formed of two sheets of No. 18 gauge with an incombustible filler of asbestos board to reduce heat-transmission, with an over-all thickness of $\frac{5}{16}$ or $\frac{3}{8}$ in. The rails and stiles are filled with cork or asbestos filler to eliminate metallic sound. Doors are furnished in standard size and designs in accordance with Simplified Practice Recommendations of the U. S. Department of Commerce.*

Frames. Nos. 10, 12, 14, 16 or 18 gauge. Frames lighter than No. 14 gauge should be installed with structural-steel bucks or pressed hollow-steel bucks of Nos. 10, 12 or 14 gauge steel, welded at the corners, and provided with anchors and clip angles for securing to walls and fastening to floor. Frames are mitered or coped and welded at the corners. Doors and frames are prepared and reinforced for all hardware selected in advance of the trim manufacture.

Trim Members. Either pressed into the frame, or made of No. 20 gauge cold-drawn shapes applied to the casings by concealed fastenings.

Hardware. Templates must be furnished to the door and trim manufacture for the proper provision of reinforcing plates and diaphragms.

Transom Bars. Nos. 10, 12, 14, 16 or 18 gauge, with plain, paneled or molded faces, welded or attached together by concealed screws.

Window Trim. Jamb lining, mullions, casing and aprons of No. 18 gauge; and stools of No. 16 gauge.

Trim Moldings. Chair-rails, staff, scribe, picture and wire moldings, of cold-drawn shapes of No. 20 gauge, accurately fitted and secured by concealed screws, or welded and dressed.

Partitions. Of adjustable, knock-down sectional type of No. 18 gauge, with filled or plain panels.

Cement Trim. Precast concrete stone set in Portland-cement mortar is used for finish and trim on both interiors and exteriors. The surface finish can either be plain, surfaced or cut in a variety of ways, depending upon the nature of the building and the type of architecture. The appearance of the cast stone depends largely upon the selection of the aggregates and the mineral colors added, and is manufactured to imitate any natural stone. Many reliable manufacturers are engaged in producing this product, and the results secured in a properly manufactured stone are both attractive and lasting. Where subjected to the weather, the joints should be calked and filled with cement-mortar grout. Finishes of Portland cement and marble or other stone chips, furnished in the precast form for floors, wainscot and molding details require a minimum of 2 in for the finish from the rough on concrete or other masonry bases, and 3 in from the rough on frame construction.

Colored Portland-Cement Mortars are furnished in ready dry-mixtures, requiring only the addition of water, to be applied as stucco-work directly to masonry surfaces or on metal lath, and are used to some extent as interior finishes, manufactured in place. Interior finishes, to reproduce artificially marble, travertine, or any other synthetic stone, are also accomplished by skilled craftsmen with admixtures of cement and lime or gypsum.

Keene's Cement has been used for many years for running base moldings, door and window trim, and in many European buildings practically all of the

* S. P R., No. 82-28.

interior finish is of this material. Moldings can be run with sharp angles, and the surface is sufficiently hard to withstand ordinary usage. The cement finish can be painted as desired.

Clay Tile. (a) **Architectural Tile** for interior finish of walls as well as flooring,* is a burned-clay product made in various plain and decorated forms and sizes, of natural burned colors and enamel finishes. Table XXXVI † gives the range of shapes, colors and sizes and their uses. All these products are manufactured to minimum grade specifications in accordance with the Simplified Practice Recommendations of the Department of Commerce.‡ Fifty-six standard forms of complementary angles, corners and stops have been adopted by the trade to care for every normal trimmer requirement, and the use of these standards will insure economy and prompt delivery. ARCHITECTURAL-TILE interiors are probably the most expensive interior finishes available, but are also adapted to the most elaborate as well as the most original treatments.

(b) **Structural or Hollow Clay Tile** as a combination structural tile with a decorative finish, is also furnished by several manufacturers for use where the economy of a combined structural wall with an attractive finish is desired. They are adapted to interior bearing and non-bearing walls and partitions as well as exterior walls, and are furnished with the finish on one or both faces. The Clay Products Co., Inc., of Brazil, Indiana, make several varieties of these tile under trade names of "AR-KE-TEX," "INSUL-GLAZE," "CAEN-TILE." They are manufactured of a common base of vitrified fire-clay with textured, salt-glazed, stippled or mottled finish. They are furnished in cream, buffs, grays, cream whites and mottled brown. Besides the sanitary finish, with permanence of color, the tiles possess structural strength and are adaptable to use in public or commercial buildings, in corridors, laboratories, etc.

"VITRITILE" are made by the National Fireproofing Corporation, Pittsburgh, Pa., by applying successive coats of glaze material to the fire-clay base, which gives a silica glaze to the tile when burned. A variety of colors is secured by adding powdered calcined colors to the glaze before burning. Glazed "TEX-TILE" is also produced by the same company with the exterior scratched or combed-face and a smooth glazed sanitary interior face. "NATCO FACE TILE" with either smooth or TEX finish is especially adapted to panel or curtain walls between piers or columns for factories, warehouses, and farm-buildings. The tile are also furnished in special forms for jambs and lintels. This company also burns a DOUBLE-SHELL FACE TILE, adding to the strength and stability of the wall. In this construction all mortar joints are non-continuous, and the multiple air-cells reduce heat-transmission through the walls. The construction is economical in that each unit in the wall is equivalent to six ordinary bricks. The exterior surface is finished to resemble tapestry brick.

The Kraftile Company, Niles, Alameda County, California, manufacture two forms of structural tile with a glazed face. "KRAFT-ENAMEL" hollow tile is manufactured true to size and free from warps, cracks and checks, with a colored enamel finish produced by a one-fire monolithic method. The body of the tile and the enamel face are fired in one continuous burning, fusing both together. Either plain or decorative effects are produced. "KRAFTILE FAIENCE" consists of a body of plastic high-temperature fire-clay molded when wet, with special enamel glaze of a wide range of colors and shades

* See Cement, Clay and Slate Tile, Article 17.
† Associated Tile Manufacturers.
‡ S. P. R. R61–30.

Table XXXVI. Condensed List of Clay Tiles, Their Characteristics and Uses

Kind of tiles	Thickness	Shapes and sizes	Colors	Surfaces and glazes	General uses
CERAMIC MOSAIC	$\frac{1}{4}$	Square—$2\frac{3}{16}$, $1\frac{9}{16}$, $1\frac{1}{16}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{11}{32}$ Oblong—$2\frac{3}{16}\times1\frac{1}{16}$, $1\frac{9}{16}\times\frac{3}{4}$, $1\frac{1}{16}\times\frac{1}{2}$ Hexagon—$1\frac{1}{4}$, 1 Pentagon—$2\frac{3}{16}$ Trapezoid—$2\frac{3}{16}\times1\frac{1}{16}$	See Vitreous and semivitreous tiles, also flat and variegated colors such as textures and flashings	Unglazed	Floors walls swimming pools plunge-baths kitchen-sinks interior exterior
Enameled mosaic Glazed mosaic			See Dull-glazed tiles	Bright	
Dull-glazed mosaic Matt-glazed mosaic				Dull Matt	
Faience mosaic	Variable	Any shape or size less than $2\frac{1}{4}$ sq in in area	See Faience tiles	Plain or embossed Bright, dull or matt	
Plastic mosaic			See Plastic tiles	Plain or embossed Unglazed	
VITREOUS TILES	$\frac{1}{2}$	Square—3, $2\frac{1}{8}$, $1\frac{1}{2}$, $1\frac{1}{16}$ Oblong—$3\times1\frac{1}{2}$, 3×1, $3\times\frac{1}{2}$, $2\frac{1}{8}\times1\frac{1}{16}$, $1\frac{1}{16}\times\frac{1}{2}$ Hexagon—3, 2 Octagon—3 Triangle—3, $1\frac{47}{64}$, $1\frac{5}{32}$	White, celadon, silver-gray, green, blue-green, light blue, dark-blue, pink, cream, and granites of these colors	Unglazed	Floors walls fireplaces interior exterior
Glazed				Bright	
Dull-glazed Matt-glazed			See Dull-glazed tiles	Dull Matt	
SEMIVITREOUS TILES		Same as vitreous tiles, also Square—6, $4\frac{1}{4}$ Oblong—9×3, 6×4, 6×3, 6×2, $6\times1\frac{1}{2}$, $6\times\frac{3}{4}$, $6\times\frac{1}{2}$, $4\frac{1}{4}\times2\frac{1}{8}$, $4\frac{1}{4}\times1\frac{1}{16}$ Hexagon—6, $4\frac{1}{4}$, 6×3, $4\frac{1}{4}\times2\frac{1}{8}$ Octagon—6, $4\frac{1}{4}$ Pentagon—$5\frac{5}{16}$, $2\frac{1}{8}$	Buff, salmon, light-gray, dark-gray, red, black, chocolate, and granites of these colors	Unglazed	
Glazed				Bright	
Dull-glazed Matt-glazed			See Dull-glazed tiles	Dull Matt	

Table XXXVI (Continued). Condensed List of Clay Tiles, Their Characteristics and Uses.

Kind of tiles	Thickness	Shapes and sizes	Colors	Surfaces and glazes	General uses
Paving-Tiles Flint, Hydraulic	¾	Square—6, 4¼ Oblong—6×4, 6×3, 6×½ Hexagon—6, 4¼ Octagon—6	White, light gray, dark gray, celadon, sage, light blue, dark blue, green, cream	Unglazed	Floors, walls interior-exterior
	¾ and 21/32	Square—6, 4¼ Oblong—10×5, 9×3, 6×3, 6×½ Hexagon—6, 4¼	See Semivitreous tiles		Floors interior-exterior
White-Glazed Tiles	½ and ⅜	Basic sizes for fields: Square 6, 4¼; Oblong 6×3; Hexagon 3, 2; Octagon 3 Other sizes: 3, 2⅛, 1½, 1 1/16, ¾, ½ 6×2, 6×1½, 6×1 6×¾, 6×½, 4¼×2¼ 4¼×1 1/16, 3×1½, 3×1 3×¾, 3×¼, 2⅛×1 1/16	White	Plain or embossed bright	Walls interior
Dull-Glazed Tiles Matt-Glazed Tiles	⅜ and ½		Unlimited color range Selection must be made from samples and colors specified by number	Plain or embossed Dull Matt	Walls fireplaces interior
Enamels				Bright	
Plastic Tiles	½ and over	Obtainable in all of the above, and special shapes and sizes	Colors that result from firing of natural clays	Plain or embossed Smooth or rough unglazed	Floors Walls fireplaces interior exterior
Faience			See Dull-glazed tiles	Plain or embossed bright, dull or matt	
Quarry Tiles	½, ¾, ⅞, and 1¼	Square—12, 9, 6, 4¼, 3 Oblong—12×6, 9×6, 9×4¼, 9×3, 6×3 Hexagon—8×4	Red, gray, buff, brown, and flashed	Unglazed	Floors interior-exterior

applied and burned by the monolithic method. It is available in a large number of stock designs for wainscoting, bases, borders, panels and inserts. "MOSAIC KRAFTILE" is a special decorative type with a groove outlined around the design· the groove may be painted after erection to produce a mosaic effect.

Glass Tile. Structural glass tiles are furnished in a variety of colors and textures for trim and wainscot in lobbies, hallways and places of assembly. They consist of opaque and highly polished vitreous slabs made by fusing feldspar, china clay, sand and other minerals at high temperatures, subsequently tempering them in annealing ovens to relieve internal stresses. GLASS TILE is non-porous, non-absorbent, impervious to moisture, stainproof, sanitary and easy to keep clean. It has a high compressive value, fairly high tensile strength and high resistance to abrasion; but it will not stand sudden changes of temperature. To offset this defect, it can be obtained with wire reinforcement. It is produced in a variety of pure and combination-flashed colors for base, door, window and border trim and large panel wainscotting.

Proprietary Forms. The "Pyrono Process" is a mechanical method of fireproofing wood cores used in the manufacture of interior trim. Cores of non-resinous wood are covered with long-fiber asbestos sheathing, mechanically bonded and indented into the wood under pressure. Doors and panels are made flush with or without inlays, and recessed.* The process is controlled by the Compound and Pyrono Door Company of St. Joseph, Mich. Standard designs of casings and trim are manufactured, conforming generally with standard millwork products, but these can also be adapted to special architectural details. The special features of "PYRONO" trim are natural wood facings, weight not more than unprocessed wood, and the installation handled as regular millwork by carpenters. Typical installations of this material have been made in the Metropolitan Life Insurance Annex, New York City; Massachusetts General Hospital, Boston, Mass.; Cadillac Motor Co., Engineering Building, Detroit, Michigan; Civic Opera House, Chicago, Ill.

"Heirloom" Panel, manufactured by the Housing Company of Boston Mass., was developed to meet the requirements for a fire-resisting trim and still retain the desired features of antique wood paneling. It consists of a mineral core of 3/8-in thick plaster board with a wood veneer bonded on both sides. The back surface is protected with a special water-proof paint. It does not warp, shrink, swell or crack. To determine its incombustibility, tests conducted for the New York Bureau of Buildings showed a very low transmission of heat and very little tendency to support combustion. The panels are delivered ready for installation, either in large sections or batten-type construction, and either finished or unfinished in any kind of wood veneer, smooth and antique hand-planed surface, plain or knotty wood. The outer surface of the panel is thick enough to be molded.

"Magnesite,"† an admixture of wood and asbestos fiber and other aggregates with calcined magnesium oxide and magnesium chloride, has been employed for both plastic laid-in-place flooring and trim and in precast board or tile form. The material is colored with mineral pigments and generally retains its surface and appearance under severe conditions of service. It is both incombustible and fire-resistive in that it will not crack or disin-

* See Fire-Doors, Article 20.
† See Composition Floors, Article 17.

tegrate under a temperature of 1 700° F. It is also low in cost. Special care, however, must be taken in the installation of the plastic forms to provide proper foundation and to guard against expansion and contraction. It can be laid on a base of wood, cement, concrete or steel, but will not bond to cinder or wet concrete, lime compositions or any form of gypsum. If laid on such foundations, plastic magnesite compositions must be installed with a membrane waterproofing and lath reinforcement properly secured to the subfloor.

"**Zenitherm,**" a precast tile or board made by the Zenitherm Company, Inc., Kearny, N. J, is a combination of selected wood fiber with the magnesium oxide treated with a weather-proof binder and subjected to hydraulic pressure in steel molds. It is furnished in fifteen standard sizes of sheets, varying in widths from 5½ to 17½ in and lengths of 5½ to 47½ in, and in standard moldings and solid corner-pieces to eliminate edge-mitering and joining. It is made in twenty-one colors and shades produced by pigments, pressed in perfect detail so that no surfacing or finishing are required. Possessing many of the physical characteristics of wood, it can be sawed, drilled, planed, nailed, screwed, or glued, and veneered with hardwoods; or in natural finish it can be made to imitate stone and marble without having the coldness of stone. Tests made for the New York Bureau of Buildings in 1920 developed a compressive strength of over 1 000 lb per sq in, a tensile strength of 325 lb per sq in, and a bending strength of 772 lb per sq in. It has low absorption, and withstands frost, moisture and other climatic conditions without warping, shrinking, swelling or checking. Specimens subjected to twenty repetitions of the "Standard" freezing tests for clay-brick showed a 13½% loss in weight. Two partitions were subjected to the New York Standard two-hour fire-test, including the application of water through a 1⅛-in nozzle at 30 lb pressure. One partition consisted of a single plate of 1-in Zenitherm board nailed to wood studs 24 in on centers; the second partition, of double construction with a plate on each side of the studs. The temperature on the exterior of the single plate partition increased from 80° F. to 300° F. at the end of one hour, and no flame or fire was transmitted through either partition throughout the test. In heat-insulation, 1-in boards of this material nailed on wood studs compare favorably with solid or hollow fire-proof block construction. The material is highly fire-resistive, possesses a low coefficient of thermal conductivity, and is, furthermore, a good sound-insulator. It can be used for both interior and exterior walls and is furnished in thicknesses from ⅝ in for wall material and stair treads up to 1¼ in for flooring-material. It is of moderate cost, being comparable to wood-construction, and it can be erected by any competent carpenter. On masonry-construction, the material can be erected on wood furring strips secured with expansion or toggle bolts about 18 in on centers; or toe-nailed with fourpenny nails to a ¾-in minimum brown coat of plaster floated to an even finish. Typical installations of Zenitherm are found in the following buildings: Michaelangelo School, Boston, Mass.; Fordham University Chapel, New York City; Capitol City Bank, Madison, Wis.; Philadelphia Hospital, Philadelphia, Pa.; Shoreham Hotel, Washington, D. C.

19. Stair Construction

Stairs. In a majority of fire-resistive buildings the architects have contented themselves with putting in INCOMBUSTIBLE STAIRS of iron, with perhaps slate or marble treads. As pointed out in the first pages of this

chapter, unprotected iron cannot be considered fire-resisting, but it is difficult to protect the ironwork of a stairway, as it is usually built, and at the same time preserve an ornamental effect. If exposed metal construction is to be used, slate and marble treads and platforms should be supported underneath. When subjected to heat, marble and slate crack and fall away, leaving the stairs impassable. A fire-department captain in New York City lost his life through the collapse of a marble platform. Such materials should always have a subtread of iron or concrete beneath them. It is possible and practicable to build stairs of clay tiles, bricks, or reinforced concrete, that are fire-resisting to a high degree. The stairs in the Pension Building at Washington, D. C., are built of brick, with the exception of the treads, which are slate; and in many of the earlier government buildings the stairs are of stone. Stones suitable for stairs, however, are not as resistant as cast iron to heat. Part I of Building Construction and Superintendence * contains descriptions and illustrations of brick stairs. The Guastavino Company has built several staircases according to its system of construction, using flat clay tile embedded in cement. No iron-work whatever is used in this construction; hence it is eminently fire-resistive. Reinforced concrete, with slate or marble treads, is a good material for the construction of stairs, and permits of very elaborate and complicated construction. The details and design of reinforced concrete stairs are covered in Chapter XXIII.

20. Opening Protectives

Basic Requirements. Openings in interior partitions and division walls of a building and in the exterior enclosure walls must be protected as efficiently as practicable to prevent the spread of fire into and through the structure. The desired objectives are the protection of property and the safeguarding of life. Owing to practical structural difficulties, standard opening protectives now generally accepted as approved devices for these purposes offer less fire-resistance than the wall or partition-assemblies of which they form a part. Regulations and standards for the construction of these devices are the development of experience secured under actual fire conditions and in performance tests of such assemblies under controlled fire conditions, and are based on the ability of the device to prevent the transmission of flame, to reduce the transmission of heat by conduction and radiation below temperatures of ignition for combustible contents, to stop the passage of smoke from the fire to the protected area, and to remain in position as an effective barrier against fire for the desired period of protection. The device also must be of sufficient structural strength to withstand the impact of blows from falling bodies and to resist the distortion caused by settlement, expansion and other temperature changes in the wall-assembly. Where protection of life is an important function, the ability of the protective to retard the passage of smoke and heat for periods sufficient to permit the escape of occupants through enclosed stairways and other egress facilities should be one of the criteria for acceptance.

Fire-Doors. The Underwriters' Laboratories and the Factory Mutual Laboratories classify and label doors and shutters for the situation or location of the opening to be protected as: Class A for openings in fire-walls, installed on both sides of the wall and generally regarded as together providing a 3-hour rating. Class B for openings in continuous vertical enclosures such

* By Frank E. Kidder, rewritten by the late Professor Thomas Nolan.

as elevator and vent-shafts, with a single door at each opening, but requiring the failure of two such protectives before the fire can spread to adjoining areas or fire sections, and generally regarded as providing a 1½-hour rating. CLASS C for openings in subdividing rooms and corridor partitions and interior subdivisions, and generally regarded as possessing a ¾- to 1-hour rating. CLASS D and CLASS E protectives for openings in exterior walls, capable of resisting fires of moderate and light exposure and generally regarded as possessing a fire-retardant rating of not more than ½ hour.*

In this connection, it should be noted that municipal building codes in general specify a 1-hour rating for opening protectives in fire-partitions and vertical enclosures and a double 1-hour door, one on each side of the opening, in fire-walls. The building code of New York City † specifies a 1-hour fire-test for doors and fire-windows which has never been applied in practice, nor would the devices generally accepted for protectives in such openings meet the requirements of this test. In fact the New York test specified for fire-doors is more severe than the test for fire-proof partitions.

Detail specifications for the construction of fire-doors, shutters and other opening protectives are contained in the Regulations of the National Board of Fire Underwriters, effective Oct. 15, 1930. In CLASS A and CLASS B protectives, wire-glass panels are prohibited; CLASS C devices may be furnished with panels of standard wire glass, with an exposed area of no individual glass light in excess of 1296 sq in. Wire-glass panels are prohibited in CLASS D protectives, but are permitted up to an area of 720 sq in in CLASS E devices with a maximum height of pane of 54 in. The door-frames, mounting, hardware and other accessories should be selected for suitability with the particular opening protective to be used. These accessories are also subject to approval of the insurance authorities having jurisdiction and are generally furnished with separate label service by the Underwriters' Laboratories; except that for sliding doors on passenger-elevator enclosures, the label applies to both door and frame, and on CLASS A swinging, hollow-metal doors for fire-wall situations, the label on a door applies to both door and hardware.

General Types of Doors. Fire-doors and shutters are either of the horizontal or vertical sliding, swinging or rolling type. SLIDING DOORS can be mounted most effectively against the wall-assembly to prevent the spread of fire but are not adapted to service in exit facilities. SWINGING DOORS are best adapted to the protection of openings in exit enclosures and can readily be made either self-closing or automatic and should generally be hung to open in the direction of travel. The SELF-CLOSING DOOR, normally held in the closed position, is generally preferable as an exit protective. ROLLING DOORS either of metal or processed metal are less likely to be rendered inoperative by obstructions but are not adapted to serve as exit doors or to furnish much insulation against the transmission of heat. Automatic devices actuated by rise of temperature are available to provide quick operation for closing doors normally held open where rapid spread of fire is likely or where serious danger to life may exist.‡ Where the spread of fire through openings protected by

* The fire-retardant ratings here quoted are not definite determinations under usual standard test procedure, but the ability of the device to serve efficiently in the respective situations is a matter of experience and judgment. No standard test procedure has been adopted for the control of fire-tests of opening protectives, and many of the devices now in general use are either deficient in heat-insulating properties or are structurally defective, or pass flame or smoke at early periods of fire exposure, when measured by the criteria applied in the fire-testing of materials.

† 1915 draft.

‡ See Automatic Releasing Devices and Systems, Article 22.

automatic doors is not likely to be very rapid or to seriously endanger life, automatic doors are generally operated by fusible link closing-devices.

Fire-doors are also classified by construction as: TIN-CLAD, PLATE STEEL, SHEET METAL, ROLLING STEEL, METAL-COVERED and HOLLOW METAL types.

Tin-Clad Doors. Standard tin-clad fire-doors and shutters are made with either 3-ply CLASS A or 2-ply CLASS B wood cores and depend for their effectiveness on their construction. The core, being entirely combustible, when exposed to fire will char and flame and generate sufficient gas frequently to bulge the tin envelope and even rupture the joints, causing the transmission of flame and smoke at early periods to the protected areas. To vent the generated gases in case of fire, 3-in holes should be provided in doors up to 50 sq ft in area, and 4-in holes for larger doors. The doors must be completely tinned before mounting, with lock-jointed seams and hardware bolted through the door. They are substantial of construction, and fairly resistive to the action of hose-streams, but they are likely to generate and transmit considerable smoke and a moderate amount of flame. They are also likely to deteriorate rapidly and are difficult to maintain. Normally equipped with a heavy type of hardware and somewhat difficult to operate, they are designed primarily for use in factory buildings and warehouses. Fire-shutters for exterior openings are also manufactured in this way.* The detail construction and accessory hardware should conform with the specifications of the National Board of Fire Underwriters.

Plate-Steel Doors. For a satisfactory fire-door, the construction should be of not less than No. 12 gauge plate reinforced with a structural steel angle frame not less than 1½ by 1½ by ¼ in. They are made and installed either to swing on hinges or slide on tracks, or are counterbalanced to slide vertically in two panels one above the other for elevator enclosures, or in a flexible form to roll on a shaft. Plate-steel doors are durable if protected against corrosion, but are mounted with heavy hardware and are difficult to operate and are not intended for use in exit enclosures. They are substantial in construction, resist the action of hose-streams very well but are subject to a high degree of heat-transmission and to warping and distortion under heat.

STANDARD ROLLING STEEL fire-doors are labeled for use in the protection of openings not more than 80 sq ft in area, with neither width nor height exceeding 12 ft, for interior fire-walls and elevator-shafts; nor more than 100 sq ft for exposure openings in exterior walls. Their operation is either: (*a*) Self-closing, push-up and pull-down type; (*b*) by chain and sprocket; (*c*) by hand-crank and gear; (*d*) by electric-motor lifting and lowering mechanisms. Labeled doors are furnished with an automatic closing mechanism actuated by temperature rise in case of fire. When motor-operated, they are also designed to close automatically under the action of heat, generally at a temperature of 150° F.

Rolling steel fire-doors are constructed of a series of interlocking steel plates or slats forming a flexible curtain, retained in flame-tight guides at the sides and flame-stopped hoods at the top enclosing a rolling shaft, controlled by springs to counterbalance the door at any point in its travel. They are especially adapted to large exterior openings in warehouses and mill buildings and for loading-platforms, railroad entrances, garages and ramps; they occupy little room and provide a full 100 per cent opening, and are "normally open" or "normally closed" as demanded by the conditions of operation. Doors for larger openings than the labeling-size limits can be constructed and secured with a certificate of inspection and approval from the Underwriters'

* See Tin-Clad Shutters, Article 20.

Laboratories. Rolling steel fire-doors or shutters normally held open in service, and closed only in case of necessity, are counterweighted and held open by fusible links or other approved devices, so that they will close automatically in case of fire. Large outside doors are equipped with wind-locks to prevent the curtain from blowing out of the guides. The installation involves special details for correct operation, and the manufacturer should be consulted in the design of all new openings in a building for which this type of protection is desired. Cold-rolled open-hearth or copper-bearing steel is usually employed in rolling-door construction of No. 16 gauge for fire-walls, No. 20 gauge for elevator-enclosures, and 18 to 22 gauge for exterior openings, and is also finished electro-plated with a coat of pure zinc. Where subjected to severe weather exposure, they can be obtained with special patented, non-corrodible curtain bottoms, as on the Cornell Rolling Doors. Some other distinctive features of the Cornell Door are: Wide slats to give rigidity; oil-less graphite bushings in the shaft bearings; and floating or self-alignment bearings on the main shaft-bracket. Rolling Steel Doors are manufactured by the following companies: Cornell Iron Works, Long Island City, N. Y.; Geo. W. Johnson Mfg. Co., Kansas City, Mo.; Kinnear Mfg. Co. Columbus, Ohio; R. C. Mahon Co., Detroit, Mich.; J. G. Wilson Corp., Norfolk, Va.

Sheet-Metal Doors. These doors may be secured of two thicknesses of No. 24 gauge steel, plain or corrugated, interlined with a fire-resistive core of asbestos roll board or similar fire-retardants, with a continuous rigid steel frame, and one usually provided with reinforcing angles and members similar to hollow steel doors for the attachment of hardware. In this type of door, provision should be made for expansion and contraction to prevent distortion of the frame; when properly constructed, they are less liable to deterioration under general conditions of service in mill and factory buildings than the tin-clad wood door, to which they are comparable in cost.*

Metal-Covered Doors.† Wooden core doors covered with kalamein iron or other metals described under interior trim ‡ are used as opening protectives where general appearance is a consideration. Hardware can be of the mortised, concealed type, but cores and envelope must be of substantial construction to withstand heavy usage. Latch-bolts or unit locks must have throw of not less than ¾ in on doors to exit facilities. They are furnished for CLASSES B, C, D and E situations, with framed wooden cores, covered with sheet metal of 16 to 26 gauge thickness, with stiles and rails at least 1½ in in thickness and with depressed or flush panels. Recessed panels of this type of door are frequently filled with an incombustible core of asbestos board or similar material, and the moldings and trim are secured by either welded, nailed or screw joints and fastenings. Metal-covered doors resist fire-streams very well, with a fairly high degree of resistance to transmission of heat. The combustible cores, however, generate gases at low temperatures, and at early stages of fire are likely to emit both smoke and flames to the protected area. Careful consideration therefore should be given to the selection of these doors where human occupancy is high or the life hazard is a serious factor. The doors are subject to rapid deterioration under adverse conditions of service.

Hollow Metal Doors.§ Fire-doors of hollow metal construction are the highest type of labeled opening protective available where appearance is a

* Merchant and Evans Co., Philadelphia, Pa.
† S.P.R. 83-28.
‡ See Metal-Covered Trim, Article 18.
§ S.P.R. 82-28.

factor. They are substantial in construction, practical under all conditions of service and permit the use of concealed hardware. Both frames and doors are usually fabricated by welding, and must be reinforced when manufactured with interior plates and diaphragms for the hardware and accessories with which the door is to be mounted. CLASS A doors are provided with insulated stiles and rails at least 1¾ in thick, and insulated panels at least 5/16 in thick; and CLASS B doors, 1½ in and ¼ in, respectively. The weight of metal should be selected for the severity of duty to which the doors will be subjected in service; it generally consists of No. 18 gauge for stiles, rails, panels and casings, Nos. 10 to 16 gauge for frames and No. 20 gauge for moldings.

Proprietary Forms. In addition to the standard types of fire-doors and trim heretofore covered, several types of patented door-constructions involving the use of processed wood and gypsum have been developed which adapt themselves to individual designs and details for specific architectural effects to harmonize with the features of interior finishes and treatments.

Flaimpruf Door.* The fire-door patented and manufactured by the Henry Klein & Co. of New York City is constructed of a "Flaimpruf" hardwood core, 1¾ in thick, with treated plywood cross-bands on each side approximately 1/7 in thick and a 1/20-in untreated veneer of any desired hardwood. The hardwood core is made of strips secured together and to the rails with special glue and joints. By actual time-temperature test-performance, conducted for the New York Bureaus of Buildings, the 2-in "Flaimpruf" door has demonstrated that it will perform the functions of a complete barrier against communication of fire from one area to another and a safeguard for the protection of human life for a period of one hour without the passage of smoke or flame during that period and with a maximum temperature rise at the end of one hour of less than 200° F. on the side of the door opposite the fire exposure. Doors of greater thicknesses can be made to withstand corresponding longer periods of exposure. The metal-covered or "kalamine door" with which the "Flaimpruf" door is comparative in cost under actual time-temperature performance-tests will evict smoke in less than 12 minutes and gas flames in less than 15 minutes, and the temperature on the unexposed side of the door will reach 500° F. in 20 minutes and 1 000° F. in one-half hour.† The veneer surface on the "Flaimpruf" door can be treated with inlaid designs of various textured and colored woods and finished to harmonize with any decorative scheme. The doors are handled as any other millwork, the hardware being applied and the doors hung by carpenters. Doors of this type have recently been installed in the Williamsburgh Savings Bank Building, New York City.

Pyrono Door. (Fig. 169.) Another processed wood treatment used for interior trim has been successfully applied to the manufacture of "PYRONO" fire-resistive doors. The wood core is covered with a fire-resistive sheathing of asbestos which is pressed and indented into the wood by heavy pressure. It is

* See Fire-Retardant Wood, Article 5.

† Although the labeled door may be considered as having failed in test after less than fifteen minutes as a protection against life hazard and safeguard against communication of fire to adjoining areas, it should be noted that after the combustible core has burned out completely, the door subsequently behaves better and eventually may withstand the action of a hose-stream which is usually included in standard test-procedure. The 2-in "Flaimpruf" door after 1 hour 5 minutes to 1 hour 10 minutes fire exposure is practically burned out. Throughout the 1-hour period, however, it is a perfect safeguard for the protection of life in buildings of human occupancy and a barrier against the spread of flames.

claimed that the sheathing excludes oxygen to prevent support of flame and at the same time permits the escape of gases generated by high temperature under fire exposure. The processed core is covered with veneers of untreated cabinet woods, which in gluing are brought into contact with the core through the indents. "Pyrono doors" are made in flush or recessed panels in accordance with any design or details of architectural treatment, with moldings and stops to conform. Under extreme temperatures, the doors do not warp or twist, and time-temperature performance tests at the Underwriters' Laboratory for a period of 42 minutes indicated that the $1\frac{3}{4}$-in door for this period possesses a fire-resistive rating equal to a two-ply tin-clad door. Doors of this type have been installed in the Cunard Building in New York City and the Federal Reserve Bank Building in Chicago. This door is also manufactured with a thin sheathing of sheet metal under the veneers for which construction greater fire-resistance is claimed.

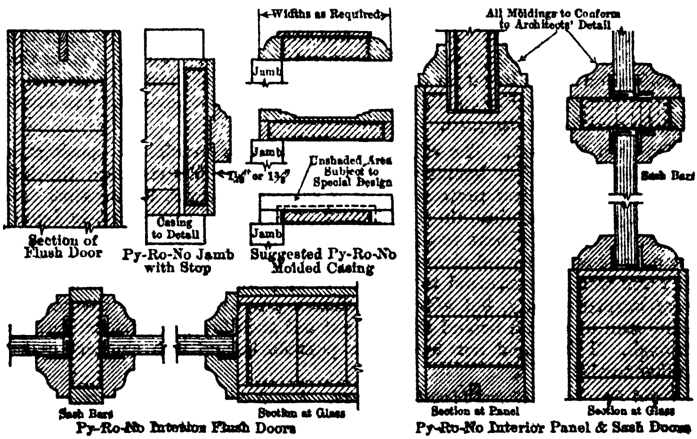


Fig. 169. Py-Ro-No Doors

Openings in Exterior Walls. The efficient protection of exterior openings in buildings is an important factor in preventing the spread of fire and in safeguarding life at times of fire and is largely determined by the proximity and use of surrounding structures. Statistics indicate that approximately 15% of fires have their origin in communication from exposure, but frequently such fires are of extreme severity resulting in large property and life losses. An excellent illustration of the communication of fire through exterior exposure is found in the Chicago Fire of March 15, 1922, involving the 15-story fire-resistive Burlington Building.* This fire swept through a group of nine buildings, in a congested part of a manufacturing district, mostly of the ordinary or mill constructed type, except for the Burlington Building, spreading from one structure to the other by the aid of a high wind prevailing at the time. After the fire had been in progress for approximately one-half hour, the wood window-frames on an unprotected side of the Burlington Building were ignited by sparks blowing across an

* Quarterly N F.P. A., July, 1922.

80-ft street, and eventually the wood flooring, doors, trim and combustible contents were in flames on all floors above the eighth. The fire was apparently confined to each floor area and burned there without vertical communication until the wire glass in the metal frames on the opposite side of the building facing a narrow court fused and melted out of the sash from the intense heat, permitting the flames to communicate upward through openings from floor to floor. The upper seven floors of the building were completely burned out, but no serious damage was done to structural supports.

The factors controlling the proper design of these protective devices are: (*a*) horizontal distances between buildings; (*b*) height above roof of adjoining buildings; (*c*) character of construction of surrounding buildings; (*d*) occupancy and combustible hazard of contents. Five general types of protective devices are in use for exterior opening protectives: tin-clad shutters; solid steel and sheet-metal shutters; rolling steel shutters; fire-windows glazed with wire glass; and water curtains. The construction of these devices and their mounting and hardware should comply with the Regulations of the National Board of Fire Underwriters, and the Inspection Department of the Associated Factory Mutual Fire Insurance Companies.* In these rules the classification is not based on a definite time-temperature performance under fire-test conditions, but on the general suitability of the device for the situation for which it is designed.

Tin-Clad Shutters. The effectiveness of shutters on window openings is dependent on fallible human agency; when closed the standard tin-clad shutter gives excellent protection. It is subject, however, to the same disadvantages cited against tin-clad doors.† "In a very severe fire, in Lynn. Mass., in which the heat was intense enough to melt most of the tin from the outside of the tinned plates covering the shutters, it was found afterward that the wood was charred to a depth of only about 3/8 in. The shutters were warped slightly, but afforded sufficient protection against the heat to allow men to remain behind them to put out such fire as occasionally crept through. This would not have been possible behind iron shutters under similar conditions."‡

Solid Steel and Sheet-Metal Shutters. STEEL-PLATE shutters are usually made of No. 14 gauge sheet iron or steel reinforced with frames of 1½ in by ¼ in angle iron with cross-bars and rails, and should either lap the opening 1½ to 2 in or fit closely within the masonry reveal. They are of the earliest types of window-protectives, but under the action of fire, they warp and distort and transmit a high degree of heat. They are not dependable under severe fire conditions and cost more than the tin-clad shutter. SHEET-METAL shutters are also made of corrugated No. 22 gauge galvanized steel corrugations running horizontally on the outside, an inner plate with vertical corrugation and a sheet of 12-lb asbestos filler.§ A pin projects through the shutter to permit opening from the outside. This device is of general local application in the vicinity where manufactured, and furnishes a high degree of protection. All metal shutters should be kept thoroughly protected against corrosion by a good quality of red lead or other metal paint. Shutters adjoining fire-escapes and above adjoining buildings should be operated from both inside and outside, as should also at least one shutter

* Regulations for the Protection of Openings, Oct. 15, 1930, National Board of Fire Underwriters.

† See Tin-Clad Doors, Article 20.

‡ Insurance Engineering, December, 1902.

§ Saino Mfg. Co., Memphis, Tenn.

in every three from the second to the seventh story, for means of access by the Fire Department.

Rolling Steel Shutters. This type of protective is better adapted to installation on the inside of an opening than the swinging type of shutter. For this reason, it is not subject to as much corrosion or deterioration and is generally more accessible and more likely to be closed. The rolling shutter is also better adapted to automatic operation. It should be installed to allow a 100% opening, overlapping the brick at sides and top; and the hood should be placed above the top of the window so that the bottom of the curtain will be entirely above or flush with the top of the masonry opening. For exterior exposure openings, rolling steel shutters are limited to an area of 100 sq ft and are usually constructed of 18 to 22 gauge metal.

Fire-Windows. The general modern acceptance of exterior opening protective is the ¼-in wire-glass window in metal sash and frames, arranged either for manual or automatic operation. The wire-glass may be either clear polished, ribbed, figured or rough type, and the assembly lends itself to architectural effects and pleasing appearance, which generally preclude the use of shutters entirely. The LABELED FIRE-WINDOW as now constructed in SOLID STEEL, COMBINATION, or HOLLOW METAL sash gives greater assurance of fire protection at the opening because it is more likely to be closed at night, more readily closed when required, does not conceal an interior fire, and when necessary is more readily opened than the fire-shutter. Up to the melting-point of glass, it forms a satisfactory barrier against the spread of flames, hot gases and smoke. The disadvantages of the wire-glass fire-window consist in its high transmission of radiant heat to combustible contents and its susceptibility to injury from falling walls and other objects. The performance of wire-glass windows is especially effective when supplemented by OPEN SPRINKLERS.* The area of wire glass between supports is limited by Underwriters' requirements to 720 sq in in area, 54 in maximum vertical to 48 in maximum horizontal dimensions for Class E openings of moderate fire exposure; and to 2916 sq in exposed area and 54 in maximum dimension in either direction for Class F openings of light fire exposure, except where noted for specific types of sash. The local underwriters' inspection bureaus have jurisdiction over the classification as to light or moderate fire exposure, and their ruling must be secured before manufacture of the window. The rules of the National Board of Fire Underwriters give complete regulations for the glazing and installation of fire windows.

(a) Metal-Covered Sash.† Metal-covered windows are manufactured to conform to special requirements for all classes of buildings. They are incombustible rather than fire-resistive, and are used where the exposure hazard does not require full protection and where there is a prohibition against the use of wood trim in buildings over a limiting height. Their efficiency as a substandard protection was demonstrated in the Kohl Building of the San Francisco Conflagration, 1906.‡ They are usually made of No. 26 gauge galvanized steel or copper-bearing steel, 16 to 32 oz. copper, or 16 to 24 gauge bronze or kalamein iron drawn on to the wood core through steel dies for the sash and trim.

(b) Solid-Steel Sash. Solid-steel sash is manufactured in standard sizes under simplified practice recommendations,§ and all architectural designs

* See Water Curtains, Article 20.

† See Metal-Covered Trim, Article 18.

‡ Fire Prevention and Fire Protection, Freitag.

§ S. P. R. No. 72-30.

should incorporate standardized materials only of the recommended opening sizes and ventilator requirements. It is furnished in Standard Weight and Heavy Type. This sash is desirable in factories, lofts and wherever large window surfaces giving maximum light are required. The Underwriters' Laboratories approve and provide labels for Horizontally Pivoted, Commercial Projected, and Architectural Projected as well as Heavy Type Casements, except inwardly projecting and bottom-pivoted units. Glass sizes must not exceeed in general 350 sq in in area, and window units are limited to 7 ft in width by 12 ft in height. There is no limitation on over-all width of opening provided the mullions and window units conform to Underwriters' requirements. The operators can be either manually or automatically controlled. Not more than two pivoted ventilators are permitted in any window unit, and the closing chains must contain fusible links or a special approval secured for stay-bar operation from the local inspection bureau. The National Board of Fire Underwriters require all glass to be held in place with glazing angles or molding on INSIDE GLAZED windows, and by wire or angle clips or wedges on OUTSIDE GLAZED windows. The Factory Mutuals approve clip and putty glazing, and this method passes practically all city and state industrial codes. Special Underwriters' Hardware, such as stop lugs on the pivoted ventilator, limit the opening so that gravity will insure automatic closing, and interlocks are provided to hold the ventilator closed. The operating hardware must be malleable iron or steel; bronze hardware is not permissible. In addition, solid-steel sash is furnished in BALANCED WINDOW, AUSTRAL TYPE, and CONTINUOUS SASH. All these types of window can also be secured in bronze. Continuous windows, however, cannot be labeled, and labeled bronze windows are limited in size below the maximum for steel. A solid section metal window-sash has been placed on the market made of genuine wrought iron conforming to the specifications A84–30, Grade C, of the American Society for Testing Materials.*

HEAVY-TYPE CASEMENT WINDOWS, as manufactured by the David Lupton's Sons Co., with exposed area of individual glass lights of 565 sq in, are also labeled by the Underwriters, with a maximum height of pane of 54 in and width of 48 in. Single side-hinge casements are limited to 3 ft in width and 7 ft in height; stationary windows to 5 ft by 9 ft in either dimension; and double casements to 5 ft in width by 7 ft in height. Assemblies of various types of windows in this sash are limited to 9 ft in width and 9 ft in height. This company also furnishes automatic-closing projected out-at-bottom ventilators in HEAVY CASEMENT SECTIONS. The Detroit Steel Products Co. make a light-section casement, limited to 5 ft in width and 6 ft 6 in in height, which can be labeled.

The principal companies furnishing Solid-Steel Window Sash are: William Bagley Co., Springfield, Ohio; Detroit Steel Products Co., Detroit, Mich.; Federal Steel Sash Co., Waukesha, Wis.; Lupton's Sons' Co., Philadelphia, Pa.; J. S. Thorn Co., Philadelphia, Pa.; Truscon Steel Co., Youngstown, Ohio; United States Metal Products Co., San Francisco, Calif.

HOLLOW METAL-PLATE STEEL. (Combination) sash is furnished in Double-Hung and Counterbalanced Windows and is limited to a maximum size of 6 ft in width and 10 ft in height for the Underwriters' label with a maximum area of any one light of 720 sq in. Sash are manufactured by Campbell Metal Window Corp., Baltimore, Md.; Kawneer Co., Niles, Mich.; Lupton's Sons' Co., Philadelphia, Pa.; S. H. Pomeroy Co., New York, N. Y.; Truscon Steel

* Mesker Bros. Iron Co, St. Louis, Mo.

Co., Youngstown, Ohio; Voightmann Metal Window Corp., Kalamazoo, Mich.; Voightmann & Co., Chicago, Ill.

(c) **Hollow Metal Sash.*** Tests of HOLLOW METAL SASH show that the device, when properly installed and equipped, results in details which are very efficient in resisting fire, and it makes an exceptionally good appearance. This sash is commonly used in the best examples of fire-resistive construction. Hollow Metal Sash is generally manufactured of sheet steel, hot galvanized steel, copper-bearing steel, ingot or genuine wrought iron of No. 18 gauge for jamb linings, mullions, casings and aprons, and No. 16 gauge for stools. It is now also furnished in aluminum and special rust-resisting steels. Bronze work is similarly executed but usually of two gauges heavier than steel. All seams and joints are accurately fitted, mitered, welded and dressed with stock designs of stop moldings and casings; and the sash are made to be weathertight and practicable in all respects. The sash is especially prepared for glazing with wire glass, the glass moldings being fitted and secured with invisible fastenings or oval-headed screws. Two thicknesses of wire glass have been used with a ventilated air-space of approximately one inch between the lights. The heat-transfer through a double pane is less than one-half that through a single pane. Labeled windows are provided in Stationary, Double-hung, Pivoted Sash, and combinations of all these types, and are subject to the following size limitations. In Single and Double Sash, other than casement, up to 5 ft wide by 5 ft high and 5 ft by 10 ft, respectively; Multiple Sliding Sash when specially reinforced is standard for openings up to 6 ft by 10 ft; Single Casements are limited to 3½ ft by 10 ft. Individual glass lights are controlled as to maximum permissible size by the character of the groove sections used by the individual manufacturer but never to exceed 720 sq in in area. Non-bearing sheet-metal mullions are also furnished with LABEL SERVICE for use with HOLLOW METAL and COMBINATION HOLLOW METAL and PLATE-STEEL and SOLID-SECTION sash for unlimited widths and wall openings not exceeding 12 ft in height.

Among manufacturers furnishing Hollow Metal Window Sash are: Chapman Metal Products Co., Inc., New York City; Mesker Bros. Iron Co., St. Louis, Mo.; S. H. Pomeroy Co., New York City; J. S. Thorn Co , Philadelphia, Pa.; United States Metal Products Co. of the Pacific Coast, San Francisco, Cal.; Voightmann & Co., Kalamazoo, Mich.; Western Steel Products, Ltd., Winnipeg, Can.

Water Curtains. The OUTSIDE OPEN-SPRINKLER EXPOSURE PROTECTION or water-curtain system properly installed with ample water-supply and suitable arrangements for periodic testing has demonstrated its value in many exposure fires such as the Fall River, Mass., fire of Feb. 2, 1928, in which the old Herald Building, Globe and Telephone Buildings were involved.† The outside window sprinklers saved these structures. This device has been universally successful either alone or in combination with other protectives. The development of open sprinkler systems for the protection of buildings against exposure fires was practically concurrent with that of automatic sprinkler systems for protection against fires inside of buildings.‡ The original form consisted of lines of pipes with open heads (ordinary sprinklers with the automatic struts removed) extended along cornices or eaves of buildings or peaks of sloping roofs, especially of frame buildings. Opening of the system would spread a thin sheet of water over the roofs and down the

* See Hollow Metal Trim, Article 18.

† Report, National Board of Fire Underwriters, 60 Batterymarch St., Boston, Mass.

‡ See Automatic Sprinkler Systems, Article 22.

walls. Subsequently special types of open sprinklers were designed to protect all window openings. In the MODERN SYSTEM, an open sprinkler at the top of each window, or window unit in continuous sash, covers the entire area of the opening with a copious sheet of water. Until recently, these systems have been controlled manually by inside valves in the building and have been independent of the inside automatic sprinkler-system and sources of water-supply. In 1930, the National Fire Protection Association recommended acceptance of outside open-sprinkler systems connected to the inside system of automatic sprinklers when water-supplies are adequate. Thermostatic devices are now in process of development to actuate these systems automatically with exterior temperature rises.

21. Fire-Resistive Furniture and Equipment

Reduction of Fire Hazard. In a building of fire-resistive construction, the fuel that a fire can feed on consists of the trim, furnishings, fixtures and combustible contents. In office-buildings, banks, libraries and public buildings, the fixtures, books and file stocks are frequently the only combustibles. If the furniture and fixtures are made of materials that will not support combustion, fire cannot spread and damage will be reduced.

Metal Furniture and Fittings. Almost anything in the way of furniture and fittings, including desks, cabinets and files, can now be obtained in metal, and many libraries, banks and court-houses have been furnished entirely with steel cabinet-work. Catalogues can be obtained from the leading manufacturers of metal furniture, such as the Art Metal Construction Co., Jamestown, N. Y.; Berger Manufacturing Co., Canton, Ohio; General Fireproofing Co., Youngstown, Ohio; David Lupton's Son's Co., Philadelphia, Pa.; Metal Office Furniture Co., Grand Rapids, Mich.; Yawman & Erbe Mfg. Co., Rochester, N. Y. Not all of these metal products are of equal effectiveness in resisting fire or safeguarding contents of safes, cabinets, etc. In the burnout tests conducted at the Bureau of Standards,* Mr. S. H. Ingberg found that with approximately 49 lb of combustibles per square foot, steel shelving of the skeleton type with open backs resulted in an equivalent fire duration of 4 hours 40 minutes, as compared to a period of 3 hours 41 minutes, in a test of a room equipped with partition steel shelving with closed backs; whereas the room with wood shelving and the same intensity of combustible loading resulted in an equivalent fire period of over 6 hours. Moreover, various methods of insulating metal cabinets with linings and incombustible cores result in different degrees of protection to their contents.

Fire-Retardant Wood Furniture. Recent attempts to construct cabinets, safes and other cabinet work of "Flaimpruf" fire-retardant wood cores with beautiful veneers of hardwoods to harmonize with the fittings, either plain or inlaid in decorative designs, have resulted in equipment which is highly resistive to the conduction of heat, which suffers no distortion or breakdown from the application of high temperatures and flames and adds no combustible fuel to a fire.†

22. Fire-Preventive and Fire-Fighting Devices

General Precautionary Measures. Although the building structure can be erected of such fire-resistive construction as to permit a complete burning out of the combustible contents without collapse of vital structural members,

* See Severity of Building Fires, Article 1.

† See Flaimpruf Wood, Article 5.

certain precautionary and preventive measures must be taken to limit the spread of fire in the building and its communication to adjacent structures. Such precautionary measures involving only a small initial investment may result in large saving by a reduction of the possible fire loss and a decrease in the municipal cost of fighting fires. In addition, by such measures, savings are affected in the insurance rate on building and contents. Furthermore, the installation of fire-preventive and fighting devices is an effective means of increasing the safety of occupants and is reflected in lower building code requirements for means of egress and greater leeway in permissible areas and heights of buildings. In warehouses, stores or factories, containing large quantities of combustible contents or even highly inflammable materials, there is always the possibility of a fire which if not checked in its incipient stage, may develop to such proportions as to cause a great loss to the contents and even damage to the structure. There are now available many devices of value for detecting and checking fires, which should be considered in the planning of every building in connection with the potential fire hazard of the proposed occupancy and use of the building. The automatic control of fire has been so highly developed that these available devices can cope with all kinds of fire hazard whether in a dwelling or in a powder-mill. Chief among these devices are (1) Automatic fire-alarms; (2) automatic sprinkler systems; (3) standpipe and hose systems; (4) first-aid fire appliances.

Automatic Fire-Alarm Systems. The prompt discovery of fire frequently assures its speedy control. It is not practically possible to have some one on duty in all parts of a building at all times, and unless some method of automatic detection and notification is used, fires may gain dangerous headway before being discovered. This can be accomplished by the installation of an automatic fire-alarm system or by an automatic sprinkler system equipped with automatic alarm apparatus. There are two general classes of automatic alarm systems; one in which the detecting devices function at a FIXED or PREDETERMINED TEMPERATURE; the other on a RISE OF TEMPERATURE. The systems in the former class employ thermostats and a fusible-core wire as detecting devices, actuating an electrical circuit. The thermostats are designed to operate at a predetermined temperature, and open or close an electric circuit actuating electrical signaling apparatus or alarm gongs. The fusible solder type of thermostat, once widely used, is now obsolete. Differential expansion of dissimilar metals is the principle upon which these detecting devices are now constructed. In the RISE OF TEMPERATURE class most of the systems utilize the principle of expansion under heat of air contained either in tubes or small air-chambers, known as heat-actuated devices. One system uses specially designed thermopiles as heat detectors which generate electrical currents on rise of temperature. RISE OF TEMPERATURE systems function whenever the temperature rises at a certain minimum rate under fire conditions, but are not affected by gradual temperature increases under normal conditions.

The "Aero" System utilizes the principle of heat expansion of air in a small copper tube attached to ceilings or walls and arranged in loops of limited lengths having at each end a diaphragm in a detector unit. A quick rise in temperature expands the air in the tube, causing one or both diaphragms to close an electrical contact actuating a transmitter arranged to send coded signals to a central station and also to operate local electrical gongs. This system, installed by the Aero Alarm Co., Inc., New York City, is approved for central station, proprietary or local service by the under writers' laboratories.

The "A.D.T." System, also approved for both central station and proprietary service, installed by the American District Telegraph Company and Controlled Companies, New York City, utilizes the principle of expansion under heat of air in small copper tubes connected to diaphragms in detector units which close an electric circuit actuating a transmitter arranged to send coded signals to a central station. If desired, local signals may be produced on vibrating electrical gongs in the building.

The "Atmo" System, installed by the Atmo Automatic Fire Alarm Co. of America, Nutley, N. J., utilizes the same principle as the systems explained above and functions similarly. It is approved for local fire-alarm service.

The "Automatic" System, installed by the Automatic Sprinkler Corporation of America, Cleveland, Ohio, and licensee companies, utilizes the principle of expansion under heat of air contained in small chambers, heat-actuated pressure-producing units, connected through small-bore bronze tubing with a diaphragm-operated releasing mechanism which functions to open a valve allowing water under pressure to flow to a water motor-gong or a circuit closer controlling an electric gong. This system operates under a predetermined rate-of-rise temperature. It is approved for local fire-alarm service.

The "Derby" System, installed by the American Fire Prevention Bureau, Inc., New York City, utilizes the "Derby Fire Sentinel" thermostat, a fixed-temperature device, set to operate at different degrees of temperature. The devices are wired on a closed circuit, and the system is under constant electrical supervision. This system may be arranged to utilize electric current from either primary or storage batteries or from alternating or direct current light and power service up to 125 volts, and to operate local vibrating bells, a fire-alarm box, or a transmitter to produce coded signals on single-stroke gongs; it is approved for local fire-alarm service.

The "Garrison" System, installed by the Garrison Fire Detecting System, Inc., New York City, is a fixed-temperature system employing as the detecting device a special form of thermostatic wire with a fusible core which melts when heated by a fire, thus causing contact between a slotted core sheath and a brass sleeve, closing the electric circuit and actuating the system. The actuating current is supplied from a 6- or 24-volt storage battery. The system can be arranged to provide either coded or non-coded signals on single stroke or vibrating electric gongs.

The "Reichel" System, installed by the Fire Detection Service, San Francisco, Calif., employs thermopiles of special design as heat detectors, connected in a series circuit with special supervising and operating batteries, relays, local transmitter and sounding devices. The transmitter is arranged to send coded signals to a central station. Under a sudden rise of temperature the thermopile generates sufficient electric current to actuate a relay causing the transmitter to operate. Slow temperature changes do not affect the system.

The "Watkins" System, installed by the Automatic Fire Alarm Co., of New York City, employs a fixed-temperature thermostat in which the expansion of thermostatic strips makes a scraping contact, closing an electrical circuit and actuating a transmitter arranged to send coded signals to a central station.

Automatic Sprinkler Systems. The automatic sprinkler system is considered an important device for controlling fire, by limiting its severity to the point of origin, by reducing the life hazard as well as the property damage and indirectly the amount of water damage, through the extinguishing of

the fire in its incipiency by the sprinkler heads locally affected. The statement has been made that "it is as impossible to prevent the start of a fire as it is impractical to manually put it out in every instance and in its early stages. The true solution is the automatic sprinkler, for unless fire is stopped promptly, no amount of foresight and no excellence of building design can prevent its ravages."*

Standard Class A Automatic Sprinkler System. The Standard Automatic Sprinkler System, as approved by the National Board of Fire Underwriters, consists of a number of sprinkler devices screwed into pipe fittings precisely spaced and joined together by sections of pipe of specified sizes which convey water from the source of supply to the automatic releasing and distributing devices. The automatic distributing device consists of an automatic sprinkler head held in the closed position by a metal strut composed of separable parts secured together with fusible solder. When heated by fire, the solder melts, the head opens, and the water is discharged through a ½-in nozzle. The released stream of water is broken up by a deflector which diffuses a coarse spray of water over an area of 100 sq ft, more or less; and the heads are spaced as required by the underwriters' rules for the kind of building and conditions of occupancy. In one type of automatic-sprinkler device, the holding strut is a frangible quartz bulb containing a liquid which expands when heated, bursting the bulb and actuating the sprinkler. Automatic sprinkler heads are made to open at various temperatures, and are customarily rated as follows: Ordinary Heads, 155° F. to 165° F. for the solder type and 135° F. for the uncolored quartz bulb type; Intermediate Heads, 212° F. for the solder type and 175° F. for the white quartz bulb type; Hard Heads, 286° F. for the solder type and 250° F. for the blue quartz bulb type; Extra Hard Heads, 360° F. for the solder type and 325° F. for the red quartz bulb type. High temperature heads are designed for locations where the normal temperatures are above those that generally obtain throughout a building, as boiler-rooms, drying-rooms, locations close to heat or light sources, etc.

The Standard Class A Automatic Sprinkler System may be of two general types: WET PIPE and DRY PIPE. In the WET-PIPE SYSTEMS, which are required in all buildings normally maintained heated, the piping contains water under pressure. In the dry-pipe systems, for use in unheated buildings in localities where water might freeze, the piping system is maintained filled with air under moderate pressure. The opening of one or more sprinkler heads in case of fire allows the air to escape, and the drop in pressure releases the dry-pipe valve, admitting the water into the system. In modern practice, it is customary to equip dry-pipe valves with auxiliary quick-opening devices which accelerate the action of the dry-pipe valve and open the valve a few seconds after the sprinkler head is released.

A complete automatic sprinkler system includes an alarm apparatus so arranged that a flow of water in the system will operate a water-motor gong, or an electric gong or both. In wet-pipe systems this apparatus consists of an alarm check-valve and appurtenances; in dry-pipe systems an alarm attachment to the dry-pipe valve. Automatic sprinkler-system alarm-apparatus may be so arranged that notification of its operation can be communicated to the municipal fire department or other central stations, thus adding an automatic fire-alarm feature to the sprinkler system.

Approvals of the various types of automatic sprinklers and other related devices and apparatus for standard sprinkler systems are accredited by the laboratories of the stock and mutual fire insurance companies in Chicago and

* National Fire Prevention Association.

Boston to the following companies: "Automatic" Sprinkler Corp. of America, Cleveland, Ohio; C. S. B. Sprinkler Co., Boston, Mass.; Central Automatic Sprinkler Co., Philadelphia, Pa.; Crowder Bros., St. Louis, Mo.; Esty Sprinkler Co., H. G. Vogel Co., sole agents, New York City; Fire Protection Co., Chicago, Ill.; Globe Automatic Sprinkler Co., Philadelphia, Pa.; Grinnell Co., Inc., Providence, R. I.; Hodgman Mfg. Co., Taunton, Mass.; P. Nacey Co., Chicago, Ill.; Raisler Sprinkler Co., New York City; Reliable Automatic Sprinkler Co., Inc., New York City; Rockwood Sprinkler Co., Worcester, Mass.; Star Sprinkler Corp., Philadelphia, Pa.; and the Viking Corp., Hastings, Mich.

Standard Class B Sprinkler Systems. In 1930, the National Board of Fire Underwriters established a Class B Standard of automatic sprinkler equipment for light hazard occupancies such as apartment-houses, asylums, clubhouses, colleges, churches, dormitories, dwellings, hospitals, hotels, libraries, museums, office-buildings, school-houses and tenements, in which some modifications are permissible from standard Class A requirements. The Class B standard provides for wider spacing of sprinklers, smaller pipe-sizes and more moderate water supplies. In other respects Class A requirements apply to WET or DRY-PIPE Class B systems.

Grinnell "Simplex" Dry-Pipe System, Grinnell Co., Inc., Providence, R. I , is a special type of complete dry-pipe automatic sprinkler system constructed according to the Class B requirements as to spacing of sprinklers and pipe-sizes and having a limited water supply from a pressure tank of about 1 500-gal capacity. The air-pressure in both the tank and sprinkler system is constantly maintained by an automatic, electrically operated air-compressor, and controlled between the tank and the distribution system by a special discharge device. When a sprinkler head opens releasing air from the system, the discharge device functions and the water in the tank is forced into the sprinkler system by the air-pressure in the tank. This system is intended for use where not more than ten sprinklers are expected to operate at one time.

Thermostatically Operated Sprinkler Systems. Automatic sprinkler systems have recently been developed, depending for operation upon auxiliary thermostatic devices, functioning on a certain rate-of-rise of temperature, instead of the fusing of individual fixed-temperature automatic sprinkler heads. In case of fire the operation of the thermostatic devices releases one or more automatic valves, permitting the entry of water into the sprinkler system. There are two general classes of these systems: one which utilizes the principle of expansion of air when subjected to an increase of temperature in the actuating devices, which are independent of the sprinkler system except for connection with the automatic water-release valves; the other which employs the principle of unequal expansion of metals in the actuating devices which are attached direct to the sprinkler system. Typical of the former class is the "Automatic" system, and of the latter, the "Tyden" system.

The "Automatic" System, "Automatic" Sprinkler Co. of America, Cleveland, O., utilizes the principle of expansion of air when subjected to a predetermined rate of increase of temperature. The heat-actuated device consists of an air-chamber with a special compensating vent, located in the area covered by the sprinkler system, and connected through copper tubing run in rigid conduit to the water-supply valve-releasing mechanism. The heat of a fire causes expansion of air in the air-chambers, and the resulting pressure trips the release mechanism and opens the automatic valve controlling the water-supply, permitting the water to flow into the sprinkler system

and to the alarm devices. The thermostat system may actuate an independent alarm. There are two types of this system: one in which open, non-automatic sprinklers are used, and the other in which regular automatic sprinklers are used. In the former, called the DELUGE SYSTEM, the installation is divided into sections, each having one automatic water-release valve with not more than 75 sprinklers controlled by one valve. The other type, called a PREACTION SYSTEM, comprises an automatic sprinkler system having individual automatic sprinklers, normally dry-pipe, with the water-supply automatically controlled by one release valve thermostatically actuated. The operation of the thermostat system sounds an alarm, releases the automatic control valve and admits water into the sprinkler heads in case of fire. The THERMOSTATIC and the DELUGE type may be combined in one system, arranged to function in different parts of the structure. The Automatic Sprinkler Corp. of America licenses the following companies to use its thermostatic system for operating sprinkler systems or other kinds of apparatus for the extinguishing and other purposes incidental to the control of fire: Aero Alarm Co., New York City; American-LaFrance and Foamite Industries, Inc., New York City; Fyrout Carbon Dioxide Extinguishing Co., Inc., New York City; Globe Automatic Sprinkler Co., Inc., Philadelphia, Pa.; Grinnell Company, Inc., Providence, R. I.; Independent Aetna Sprinkler Corporation, New York City; Rhode Island Supply and Sprinkler Company, Providence, R. I.; Rockwood Sprinkler Company, Worcester, Mass.; H. G. Vogel Company, New York City.

The " Tyden " System, The Viking Corp., Hastings, Mich., uses a thermostatic device comprising two metal rods of different diameters the dissimilar expansion of which under heat actuates the device and causes the opening of the automatic valves controlling the entry of water into a sprinkler system, in which the sprinklers may be open, non-automatic, or individually automatic. These devices may be attached directly to a regular dry-pipe automatic sprinkler system and be made an integral part of it, appreciably increasing the speed of its operation.

Thermostatically Operated Fire-Control Systems. There are a number of systems, in addition to sprinkler systems using water, which employ thermostatic devices for their actuation to extinguish fire, such as systems diffusing carbon dioxide (CO_2) gas, or foam fire-extinguishing blankets. All of the approved devices are listed by the Underwriters' Laboratories, Inc., Chicago, and the Inspection Department, Associated Factory Mutual Fire Insurance Companies, Boston.

Sypho Chemical Sprinkler System, installed by " Automatic " Sprinkler Corporation of America, is an automatic stationary chemical fire-extinguisher system, especially designed for moderate hazards and particularly well adapted where suitable water-supply is not available. This system involves the use of a distribution system of modified pipe-sizes, filled with a non-freezing solution; a chemical engine containing bicarbonate of soda solution and sulphuric acid, and means by which, when a sprinkler opens, this acid is syphoned into the soda solution. The automatic sprinkler heads used have been tested and found to resist the chemicals contained in the system which is designed for use in cases where a limited number of sprinklers are expected to control the fire. This system is maintained under a periodic inspection service by the installing company, and has had 15 years of satisfactory fire record.

Sprinkler System Supervisory Service. Experience reveals that the majority of fires not successfully controlled by sprinkler systems have been due to faulty maintenance of the system and to the unlicensed shutting off

of the water-supply control-valves. The most reliable method of assuring the proper operating condition of all parts of a sprinkler system is the so-called Sprinkler Supervisory Service. This comprises apparatus for giving a constant check on the normal condition of essential parts of an automatic sprinkler system and of transmitting signals to a central station whenever a water-supply valve is closed, when the water-level or temperature in a gravity tank falls below the proper point, when the air-pressure weakens or the water-level rises or falls in a pressure tank, whenever a water flow occurs in any part of the system, and in all respects constantly supervising the entire sprinkler system. The complete supervisory service comprehends a standard central station system, such as the "Aero" system, Aero Alarm Co., Inc., New York City, and the "A.D.T." system, American District Telegraph Co., New York City. Another kind of this service is the PROPRIETARY system in which the signals are transmitted to a local station in the property protected. Types of these systems are the "A.D T.," also the "Autocall," The Autocall Co., Shelby, Ohio. Partial supervisory service and water-flow signals are given locally by isolated systems, such as the Derby, American Fire Prevention Bureau, Inc., New York City; by a system of the Automatic Fire Alarm Co., New York City; and the "U.S.E.M. Waterwatch," U.S.E.M. Co., New York City. In addition to the customary local alarm service in automatic-sprinkler systems, supervisory service attachments can be made to any standard water-flow alarm-valve to transmit characteristic signals to central or proprietary stations or to actuate special local alarms.

Automatic Releasing Devices and Systems. For many years the fusible link was the sole means available for automatically releasing self-closing fire-doors in case of fire. Where conditions favor rapid spread of fire through wall openings, the need for more rapid closing has led to the development of devices and systems for releasing fire-doors more speedily through thermostatic fire-detectors that cause the actuation of electrical and mechanical devices. The fire-detectors used in these automatic release systems are either of the fixed-temperature or rise-of-temperature types. Such sensitive devices are also adapted to the closing of dip-tank covers, automatic skylights and windows, dropping theater curtains, stopping blowers, engines and electric motors, opening or closing valves for various purposes, operating different kinds of chemical extinguishers, controlling of fires in flammable liquids, and generally controlling mechanical movements and arresting power energy in event of fire.

The "Derby" System. American Fire Prevention Bureau, Inc., New York City, employs the "Derby Electric Release," an electromechanical device, designed to hold a rope or chain controlling the position of device to which it is attached. The actuating mechanism is an electromagnet, arranged in a series circuit of 110 volts, alternating or direct current, and taking from 120 to 350 milliamperes to operate the releasing levers. This electric circuit and the "Derby Fire-Sentinel" fixed-temperature thermostat located as many be required for transmitting and controlling the current to actuate the automatic release constitute the complete automatic release system.

The "Grinnell" System, Grinnell Co., Inc., Providence, R. I., employs a mechanical release automatically actuated by expansion of air in copper tubing. The principle of automatic operation and apparatus used is the same as in the "Aero" and "Atmo" automatic fire-alarm systems. The release mechanism consists of a series of levers and a set of diaphragms with associated air leaks and compensating chambers with which the fire-detector tubing is connected.

The "Lowe" System, "Automatic" Sprinkler Corp. of America, Cleveland, Ohio, utilizes the "Lowe Release," a diaphragm-operated lever mechanism enclosed in a cast-iron case, which is operated by heat-actuated pressure-producing units, affected by rate-of-rise of temperature, installed in the fire area, and connected through small-bore bronze tubing with the diaphragms of the releasing devices. For releasing fire-doors, the heat-actuated device and the releasing mechanism may be combined in one self-contained unit.

The "Saveall" System, Globe Automatic Sprinkler Co., Inc., Philadelphia, Pa., employs the "Saveall" automatic release, a diaphragm-operated lever mechanism operated by fixed-temperature heat-actuated vacuum-producing units installed in a fire area with small-bore bronze tubing for connecting with the diaphragm of the release mechanism.

Stand-Pipe and Hose Systems. In buildings not protected by inside automatic sprinkler systems, stand-pipe risers with hose reels attached in each story and on the roof constitute a ready means for fire control. In some cities such a system is required by law in buildings exceeding a certain height, or maximum area, whether or not the building is sprinklered, particularly for fire-department use in the upper stories. For ordinary purposes stand-pipes are 3 in or 4 in in diameter with connections for 1½-in hose to enable all points to be reached with at least one stream from a line of hose not over 100 ft long. Where designed for fire-department connection the stand-pipe size is 6 in with 2½-in hose-connections in each story, to which 1½-in hose may be attached for ordinary fire service by reducer valves. Water-supply may be from the public source or from a gravity or pressure tank, or preferably by automatically controlled fire-pumps in the building. The fire-pump is probably the best control for a stand-pipe system, as it makes available an inexhaustible water-supply with any desired practical pressure at the hose-nozzle. Siamese connections at the street-level for fire-department use should be included in any stand-pipe and hose system regardless of size or kind. A properly designed stand-pipe system includes the correct layout of pipes and check-valves to permit the use of one or more sources of water-supply without interference with others. The number and location of stand-pipes should be such as to protect all parts of the structure, and they are generally located within stair-enclosures or other readily accessible parts of the building. Such cross-connected stand-pipe systems become very extensive and elaborate in a modern fire-resistive structure. In the 73-story Chrysler Building in New York City, the stand-pipe system was installed at a cost of $150,000. The stand-pipe is primarily intended for fire-department use. With modern high-pressure pumping apparatus and ordinary deck or street hose, effective fire-streams cannot be thrown above the second story; and with the 65-ft fire-department water-tower, the limit of effectiveness for properly placed streams is not much above the sixth story of a building. With the aid of a properly equipped stand-pipe system, the height, area as well as the exposure hazard is considerably reduced in high building-structures.

In tall stand-pipes, to overcome excessive pressures at lower hose-outlets due to the height, reducer valves must be introduced in the riser. A pressure-reducer valve manufactured by William F. Conran of 26 Ferry Street, New York City, is designed to be attached on the down-stream side of the hose-outlet valve, each device being marked for the specific floor for which it is adjusted to give a safe nozzle pressure of 25 to 35 lb per sq in at the base of the hose-nozzle. The device is marked with thirteen notches to allow of thirteen pressure-variations, and is locked in the required position by a removable

dog fitting into the notch. For fire-department use, the dog can be readily broken by the hose-spanner, and the device can be adjusted to give any desired pressure up to the full static head. After such use, the valve can be set back to the proper floor adjustment as indicated by the stamped number on the casing, and a new locking dog installed.

Conran Stand-pipe System. This system consists of one or more stand-pipes located on the inside of the structure, generally near outer walls, with a nozzle and hydraulically-operated valve on each riser in each story. The riser can be raised or lowered hydraulically from a control box at the street-level, and rotated manually from the same point. From the control box, hydraulically operated valves control a nozzle at each story, by means of which a stream of water can be thrown on every one or all stories to reach all parts of the floor-area within the radius of action of the apparatus. It is primarily intended for fire-department use, and with the rigidly mounted nozzles at each story and pump water-supply, pressures as high as 100 lb per sq in can be effectively handled at each outlet. The stand-pipe must be so located that the control mechanism is accessible in the first story.

Conran Hydraulic Distant Control Valve. This dry-pipe valve is designed to operate stand-pipe systems where maintained dry, in locations subject to freezing. In case of fire, the valve located in the main stand-pipe line is operated hydraulically either by a manual-controlled actuating device or thermostatically through a detector actuating an electric circuit accompanied by a fire alarm on local gongs or coded signal transmitted to a central station or to the fire department. The Conran Valve can also be used on a cross-connected stand-pipe system to eliminate any one riser in the event of its being broken, or several of such devices can be installed throughout each stand-pipe riser to eliminate sections of the pipe if they should become broken, without affecting the balance of the system. The Conran stand-pipe systems and control valves are approved by the Underwriters' Laboratories.

The "Automatic" Hose Valve, manufactured by "Automatic" Sprinkler Corporation of America, Cleveland, Ohio, consists of an approved angle hose-valve so equipped that, when the hand wheel is opened part of a turn, a diaphragm is collapsed suddenly. The air compressed by the collapsing of the diaphragm is transmitted through small copper tubing to a remote location, resulting in the tripping of a pneumatically controlled "Automatic" Deluge valve, permitting water to flow into the normally dry hydrant or stand-pipe system. The use of these hose valves permits the intallation of hydrant or stand-pipe systems in unheated risks, or where the system would be subject to freezing. The delivery of water is accomplished by the same operation as on a wet system, by opening the hose valve.

First-Aid Fire-Appliances. Readily available auxiliary fire-extinguishing apparatus for controlling incipient fires are generally desirable. Of more or less value, according to various types of such apparatus, and dependent upon the structural conditions and occupancy of the building, may be mentioned the following: Soda-acid hand extinguishers, soda-acid engines on wheels, soda acid stand-pipe systems, automatic foam cans, foam hand-extinguishers, foam engines on wheels, foam stand-pipe systems, carbon tetrachloride extinguishers, calcium chloride extinguishers, carbon dioxide (CO_2) gas extinguishers, and dry-chemical fire-extinguishers. All of these devices can be secured for manual operation and some for automatic control, and approved types are labeled and listed by the underwriters' testing laboratories, both stock and mutual.

CHAPTER XXIII

REINFORCED-CONCRETE CONSTRUCTION

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1. Introduction

Definition. Concrete is a mixture of Portland cement, water and inert materials such as sand and crushed stone or gravel.

Reinforced concrete is concrete in which metal is embedded in such a manner that the two materials act together in resisting forces; the principal function of the concrete is to resist compressive stresses, and that of the reinforcement to resist tensile stresses.

Historical Notes. Concrete has been used as a structural material for many centuries. The Romans made an excellent cement by combining volcanic dust with slacked lime. This product was used in both masonry and concrete employed for the construction of buildings, viaducts and other engineering structures. Apparently, the art of mixing and placing concrete was much the same as in our own times, the plastic material being cast in wooden forms. During the medieval days there is no evidence that concrete was used to any extent in western or southern Europe. In 1756, while constructing the Eddy Stone Lighthouse in the English Channel, John Smeaton discovered that by burning an impure limestone containing considerable clayey matter, a cement could be produced that would set under water. In following years natural cements came to be widely accepted for use in masonry and concrete construction.

In 1842 Joseph Aspden, an English bricklayer, took out a patent applying to the manufacture of cement made from a definite amount of clay and limestone amalgamated and calcined. Because the resulting product when hardened resembled a limestone quarried near Portland on the southern coast of England, the inventor named his product Portland cement. It was nearly 30 years later that the material began to be used extensively in England. Portland cement was first manufactured in the United States at Coplay, Pennsylvania, in 1875.

Reinforcement was probably first used in concrete by the Romans but in a very elementary way. Even after the invention of Portland cement, there is no further record of reinforcement being used until about the middle of the last century. The first experiments made by François Coignet of Paris are recorded by a patent taken out by him in 1869 applying to a combination of iron and concrete. Two years before this, P. A. J. Monier, a gardener living in the neighborhood of Paris, applied the principle of reinforcement in the making of ornamental flower-pots.

Reinforced concrete began to be applied to engineering structures and buildings toward the end of the nineteenth century. Following its use for bridges, the art of design and construction rapidly developed during the next twenty years. The discussion of the subject in this chapter is confined to

the use of concrete as applied to building construction. Since the first recorded example of a reinforced-concrete building in this country was erected by W. E. Ward near Port Chester in 1875, about 700 buildings have been constructed in the United States over ten stories in height. In 1928, the Master Printers' Building, twenty stories in height, illustrated the economy of concrete for city structures designed for manufacturing purposes. During the last decade, the use of concrete for apartment houses and institutional buildings has supplemented its established use for factories and warehouses.

Economic Requirements. The planning of a concrete building requires both a knowledge of structural design and a familiarity with field methods. Because concrete construction is a manufacturing process carried on in the field, the engineer or architect must be sufficiently familiar with construction methods to design the type of work that can be economically erected. The ground plan of an industrial building, of the factory or warehouse type, may be governed by the size and shape of the available property, or by the type of occupancy for which the building is intended. The height of a building is also largely dependent upon land value and operating conditions, but there are a few general principles, the application of which results in more economical construction.

When the location of the columns is not controlled by architectural requirements, it is desirable to make the bays approximately square with column spacings about 20 ft on centers. In factories, it is often undesirable to place columns on the center line of the building, and when possible the width should be divided into at least three bays in order that continuity may be assumed in the design of the floor construction. The exterior and interior columns should be placed on the same center lines in order to simplify the framing plans. Columns that are to remain exposed, such as those in the interior of an industrial building, are generally made round in section. When columns are to be concealed in partitions, square or oblong sections are used. In skeleton construction, the exterior columns are generally of rectangular cross-section with a constant face width for the entire height of the building, the thickness being changed to give the necessary cross-sectional area. It is customary to fill the entire wall spaces, above panel walls from column to column, with steel sash.

There are at present three broad classifications or types of concrete floor construction besides the various uses of concrete as a floor material in steel-framed buildings: the girderless or flat slab, the beam-and-slab, and the so-called joist construction. For the typical industrial building of three bays or more in width, when the column locations can be chosen to give approximately square bays, of equal or nearly equal size, the girderless type is usually more satisfactory for superimposed loads of 100 lb per sq ft or more. It should be remembered that the saving in head room with this type of construction may, on a fifteen-story building, permit an extra floor within the same total height. There is also the added advantage of a comparatively flat ceiling, which favors the installation of sprinkler outlets and overhead piping, as well as the lighting of interior bays. In buildings with an irregular column spacing, for floors sustaining extremely heavy loads, or where bays are broken up by large openings, a beam-and-slab construction may be a desirable choice.

Joist construction is particularly suitable for long spans and light live loads. The ordinary conditions encountered in the construction of school buildings, hospitals, club-houses, apartment-houses and fire-resistant residences lend themselves to this type of construction. As the dead load of the floor itself

is much less than that of a solid concrete slab, there is usually a distinct saving except for heavy loads such as encountered upon industrial work. It should be remembered, however, that for short spans such as the corridors of schoolhouses, a solid concrete slab is usually cheaper. The various types of joist-floor construction comprise systems employing wooden or tin pans, fillers of terra cotta, gypsum and slag block, which may be supported on bearing walls or a structural steel or reinforced-concrete frame.

Design Procedure. The first step in the design of the structural portion of a concrete building should be a comparative study of the relative costs of the various designs which can be satisfactorily used to meet the architectural requirements. Typical portions of the building should be designed, prices obtained for labor and material and comparative estimates prepared in order that the designer may choose a system that will not only satisfy the requirements of good practice but one that will also be economical.

Having determined upon the system to be employed, the chief structural members such as the columns, girders and beams should be located on a preliminary framing plan showing all walls, bearing partitions and openings requiring framing. The first floor to be designed should be the one which represents typical conditions, that is, the system that will repeat the greatest number of times in the structural work of the building. Slabs are designed first, followed in order by beams and girders or, for girderless floors, as described later. Columns are designed from the roof down for total loads computed from the live and dead loads per square foot on the area of the bay supported, to which are added the weights of the columns themselves, superimposed walls, etc. The footings come next. Curtain walls, parapets, stairs, cornices, etc., are the last structural elements of a building to be designed but their sizes must be closely approximated in the early stages of the work in order that their proper dead loads may be included in the computations for supporting members.

2. Materials Used in Reinforced-Concrete Construction

Cement. PORTLAND CEMENT is the product obtained by finely pulverizing the clinker made by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials with no additions subsequent to calcination except water and calcined or uncalcined gypsum. Only Portland cement should be used in reinforced-concrete work and every shipment or carload should be tested as prescribed by "Standard Specifications and Tests for Portland Cement" (Serial Designation: C9–26 of the American Society for Testing Materials).

NATURAL CEMENTS, used quite extensively in bricklaying mortars, are not desirable for concrete. NON-STAINING PORTLAND CEMENTS are used only for decorative work and for applied finishes.

HIGH EARLY STRENGTH CEMENTS, some of which are true Portlands and others of the alumina variety, are of particular value where it is necessary to obtain concrete that will gain strength more rapidly than that utilizing standard Portland cement. The cost of most special cements designed to give a high early strength is considerably above that of the standard material. Especially is this true of the alumina cements which produce a concrete having a 28-day strength at an age of 24 hours. Many designers, in fact, prefer to use larger proportions of ordinary Portland cement to obtain a high early strength at the 3 to 7 day periods. Increasing the cement content permits

drier placement with equal workability, thereby resulting at all ages in a stronger and more impermeable concrete.

Reinforcement. In the design of reinforced concrete, steel is employed to resist tensile stresses and in some cases to assist the concrete in carrying compressive stresses. The various grades of steel used in the past for the purpose of reinforcement have now largely given place to the so-called "intermediate grade," conforming to the requirements of the "Standard Specifications for Billet-Steel Concrete Reinforcement Bars of Intermediate Grade" (Serial Designation: A 15–14) of the American Society for Testing Materials. This specification was also established by the United States Department of Commerce as the single standard for billet-steel reinforcement (Commercial Standard No. 1). Supplementing this specification is the standard for "Rail-Steel Concrete Reinforcement Bars" (Serial Designation: A16–14) of the American Society for Testing Materials and the "Standard Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement" (Serial Designation: A82–27) of the American Society for Testing Materials. Structural steel should conform to the requirements of the "Standard Specifications for Structural Steel for Buildings" (Serial Designation: A9–24) of the American Society for Testing Materials.

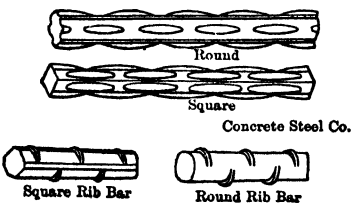


Fig. 1. Typical Types of Deformed Bars

The INTERMEDIATE GRADE of billet-steel can be generally used for all purposes where reinforcement is required. RE-ROLLED RAIL-STEEL, owing to its somewhat brittle character, should be used with caution on work where it is necessary to bend rods or bars larger than ¾ in.

DEFORMED BARS (Fig. 1) are those made in various shapes other than plain rounds and plain squares, for the purpose of developing a mechanical bond between steel and concrete. Such are usually rolled with lugs or projections. Their use permits an increase of 25% in the unit bond stress allowed for plain rods or bars.

All reinforcing steel should be purchased under the appropriate specification of the American Society for Testing Materials which controls the chemical and physical properties.

The WORKING TENSILE STRESS on steel reinforcement should never exceed 50% of that developed at the yield-point of the material. It may, however,

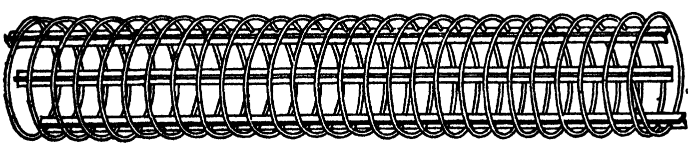


Fig. 2. Spiral Reinforcement for Columns

safely approach this value as a limit. On this basis it would seem almost as logical to allow a working stress of 20 000 lb per sq in on intermediate grade reinforcing bars (yield-point 40 000 lb) as to permit 16 000 lb per sq in on structural grade steel (yield-point 33 000 lb). The higher stress is proposed

by the Joint Code (1928) and is used in some localities but has not been generally accepted by the profession. It is recommended that designs be based on a working-stress in tension of 18 000 lb per sq in and that the specifications call for INTERMEDIATE GRADE steel. This matter should, however, be checked with the local building department as some ordinances still limit the tensile stress to 16 000 lb per sq in in all reinforcement, irrespective of its character.

The following are the STANDARD SIZES of reinforcing bars:

Table I. Areas, Weights and Perimeters of Rods and Bars

ROUND RODS

Size in in	¼	⅜	½	⅝	¾	⅞	1
Area in sq in	0 049	0 11	0 19	0 30	0.44	0.60	0.78
Weight per ft in lb	0.167	0 376	0 668	1 04	1.50	2.04	2.67
Perimeter in in	0 78	1.18	1.57	1.96	2.35	2.75	3.14

SQUARE BARS

Size in in	½	1	1⅛	1¼
Area in sq in	0 25	1.00	1.26	1.56
Weight per ft in lb	0 85	3 40	4.30	5.31
Perimeter in in	2.00	4.00	4.50	5.00

WIRE FABRIC (Fig. 3) made of cold-drawn steel wire is widely used for the reinforcement of concrete slabs and floors, as well as for stuccoed work. The so-called "metal-lath" does not ordinarily make a suitable reinforcement for either concrete or stucco as it is much more difficult to properly embed in the plastic concrete or mortar. Wire fabrics of different types may be obtained to meet various conditions.

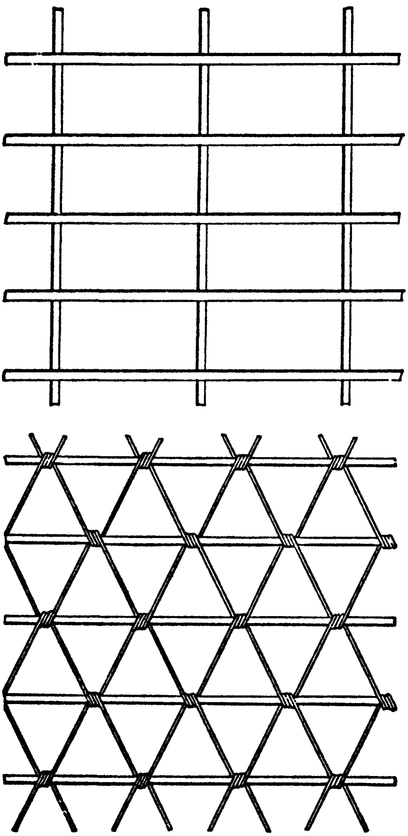
Water. Water used in mixing concrete should be clean and free from acids, alkalis or organic materials. All water suitable for drinking may be used for concrete. Even serious pollution due to sewage, or manufacturing waste, produces hardly any effect upon the strength of concrete unless resulting in an appreciable concentration of some harmful substance. In an exhaustive series of tests performed at the Lewis Institute, the only waters giving concrete strengths below 80% of that obtained with clear water, were the following: acid waters, lime-soak from a tannery, refuse from a paint factory, mineral water from Canada, and water containing 5% or more of common salt.

Concrete Aggregates. Aggregates for plain or reinforced concrete should consist of natural sands and gravels, crushed rock, crushed air-cooled blast-furnace slag, or other inert materials having clean, uncoated grains of strong and durable minerals.

The particles should be well graded, varying in size from fine to coarse. The combined mixture of the aggregates should have a low void content and be graded up to as large a size as can be conveniently placed under the conditions existing on any particular job. Standard Tyler sieves (Fig. 4) are ordinarily used for the purpose of sieve analysis, from the results of which the proportions of fine and coarse materials may be more accurately determined

than by the crude approximations usually employed. Particles passing a sieve having four meshes to the linear inch are ordinarily referred to as FINE AGGREGATES while those retained on a sieve of this size are classed as COARSE AGGREGATES.

The following tests are applied to determine the fitness of an aggregate for concrete work; the designations refer to the Proceedings of the American Society for Testing Materials:



American Steel and Wire Co.

Fig. 3. Welded Wire Fabric for Slabs or Walls

Standard Method of Test for Organic Impurities in Sands for Concrete (Serial Designation: 040–27). See Fig. 5.

Standard Method of Decantation Test for Sand and Other Fine Aggregates (Serial Designation: B136–28).

Standard Method of Test for Sieve Analysis of Aggregates for Concrete (Serial Designation: C.E.41–24).

Tension Tests on Mortar Briquettes as Described in Specification for Concrete Aggregates (Serial Designation: C33–28T).

Standard Methods of Making Compression Tests of Concrete (Serial Designation: C39–27).

SAMPLES OF AGGREGATES for test purposes should be representative specimens. The "Standard Methods of Sampling Stone, Slag, Gravel, Sand and Stone Block for Use as Highway Materials, Including Some Material Survey Methods" (Serial Designation: D75–22) should be referred to.

The sequence given above might well be applied following a visual inspection which should first establish the physical character of the material. If a sand, or other type of fine aggregate, is dirty, lacking gradation in size of particle, or excessively fine, there is seldom any use in performing the more expensive tests, as good concrete cannot be economically produced from such material.

The test for ORGANIC IMPURITIES may be very easily made without the assistance of a laboratory. It consists in immersing the sand for a period of 24 hours in a 3% solution of sodium hydroxide (NaOH) and comparing the

color of the resulting liquid with a standard color card, or glass. Injurious amounts of organic compounds are indicated by the fact that the color of the solution is darker than the standard. The purpose of the DECANTATION TEST is to determine the amount of silt, loam or clay which the aggregate contains. Good practice and many building ordinances specify limiting amounts for "fines" of this kind. The requirement recommended by the Joint Code (1928) is that silt and crusher dust, finer than the No. 100 standard sieve,

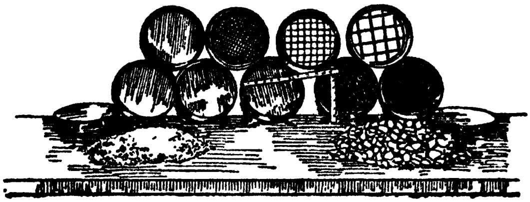


Fig. 4. Set of Standard Tyler Sieves

shall not exceed 2% of the weight of the combined aggregate as used in the concrete.

On important work these preliminary tests should be supplemented by TENSION TESTS upon mortar briquettes (Fig. 6) in which it is customary to require the sand under test to develop at least the strength of standard Ottawa sand and, in fact, a well-graded building sand should develop 15% more than the standard sand.

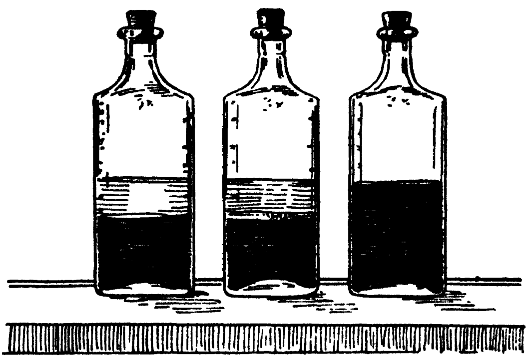


Fig. 5. Colorimetric Test Used to Detect the Presence of Harmful Amounts of Organic Matter in Aggregates

The value of a SIEVE-ANALYSIS is to assist the designer in choosing an aggregate, or a combination of aggregates, that gives the strongest concrete for any fixed amount of cement and, at the same time, produces a workable mixture which can be easily placed in the forms and around the reinforcement.

COMPRESSION TESTS on concrete cylinders (Fig. 7), using the materials and proportions that it is intended to employ on the job, furnish a very fair indication of the strength that can be obtained for any given set of conditions. When making laboratory experiments it is extremely important to bear in mind the situation on the work in order to avoid designing a mixture that would be too dry, or too coarse, to be placed under job conditions.

Compression tests also give a very valuable check on the execution of the work. The "Standard Method of Making and Storing Specimens of Concrete in the Field" (Serial Designation: D31-27) describes the way in which

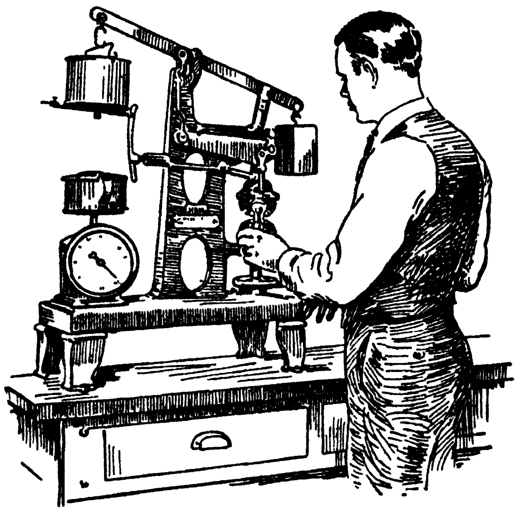


Fig. 6. Briquette Tensile Testing Machine

samples of concrete should be obtained. "Standard Methods of Securing Specimens of Hardened Concrete from the Structure" (Serial Designation: C42-27) describes a method for obtaining specimens from completed work.

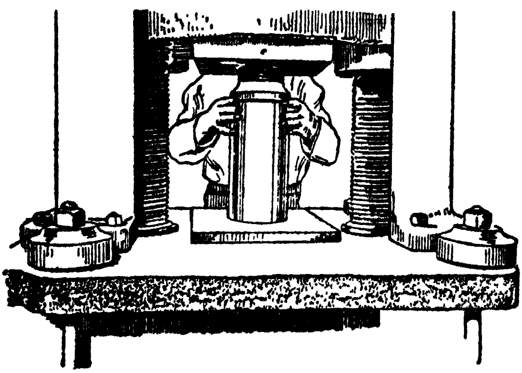


Fig. 7. Cylinder Compression Testing Machine

The JOINT CODE * promulgated by Committee 501 of the American Concrete

* Tentative Building Regulations for Reinforced Concrete: Proc. American Concrete Inst., Vol. 24, page 791.

Institute and the Concrete Reinforcing Steel Institute's Committee on Engineering Practice, gives the following requirements for concrete aggregates:

"Concrete aggregates shall consist of natural sands and gravels, crushed rock, crushed air-cooled blast-furnace slag, or other inert materials having clean, uncoated grains of strong and durable minerals. Aggregates containing soft, friable, thin, flaky, elongated, or laminated particles totaling more than 3%, or containing shale in excess of 1½%, or silt and crusher dust finer than the No. 100 standard sieve in excess of 2% shall not be used. These percentages shall be based on the weight of the combined aggregate as used in the concrete. When all three groups of these deleterious materials are present in the aggregates, the combined amounts shall not exceed 5% by weight of the combined aggregate.

"Aggregates shall not contain strong alkali or organic material which gives a color darker than the standard color when tested in accordance with the 'Standard Method of Test for Organic Impurities in Sands for Concrete' (Serial Designation: C40–27) of the American Society for Testing Materials.

"The maximum size of the aggregate shall be not larger than one-fifth of the narrowest dimension between forms of the member for which the concrete is to be used nor larger than three-fourths of the minimum clear spacing between reinforcing bars. By maximum size of aggregate is meant the clear span between the sides of the smallest square opening through which 95% by weight of the material can be passed."

Dangerous Aggregates. Although most types or rock which have a good physical appearance and conform with the specification given above are satisfactory for concrete work, when crushed and graded to the proper sizes, there have been instances where apparently satisfactory material, having the appearance of limestone, has gone to pieces after incorporation in the concrete. The result has been complete disintegration. In the light of such experiences, it would seem desirable to investigate the character of any stone used for the first time as a concrete aggregate.

Cinders and Light-Weight Aggregates. ANTHRACITE CINDERS are widely used in concrete intended for fireproofing purposes and also in structural work where the design does not call for a material of greater strength. Bituminous cinders from bituminous coal, or those containing unburned particles, or ashes from domestic plants, should not be used as such do not produce satisfactory concrete. BLAST-FURNACE SLAG makes a every good aggregate, developing adequate strength in properly designed mixtures and giving a lighter dead load than stone concrete.

Many other LIGHT-WEIGHT AGGREGATES are now upon the market, some of which may be used to produce concrete of strength sufficient for structural use and giving a dead load ½ to ⅓ that of stone concrete. For example, light-weight aggregate made from burnt clay produces a concrete weighing about 100 lb per cu ft, and by a process of aeration another manufacturer has developed a mixture weighing less than 50 lb per cu ft which is well suited to the lighter structural uses.

Admixtures. This term is applied to substances other than cement, aggregates and water which are added to concrete mixtures for the purpose of imparting to the latter certain improved qualities. Hydrated lime, calcium chloride and many similar proprietary materials fall in this classification. Where such substances are recommended for use in structural work it is either for the purpose of gaining a more impervious concrete, increasing the workability of the plastic mass, or as insurance against injury from frost. Admixtures are used in floor finishes as accelerators and hardeners.

Materials such as HYDRATED LIME may be added in percentages up to 8% by weight of cement. Such amounts generally benefit lean mixtures of concrete but tend rather to reduce than increase the strength of rich mixtures. Apparently, hydrated lime does not act to fill the voids in concrete except in so far as its addition may reduce the amount of water required in order to obtain a given workability. Hydrated lime and similar powdered admixtures are added to the cement at the time of mixing.

CALCIUM CHLORIDE alone and in combination with other substances is one of the most widely used of the various admixtures. This material, when applied in the form of flakes, also acts as a curing agent as it has the power of drawing moisture from the air. As an INTEGRAL COMPOUND it is used in solutions up to 2% chlorine by weight of cement, and its principal function is to hasten the hardening of the concrete. The following statement, the result of a large number of tests, is abstracted from Bulletin 13 of the Lewis Institute:

"In the use of calcium-chloride no advantage was gained for percentages of the commercial product greater than 2 or 3% of the weight of cement (chlorine content 1 to 1½%). This amount when used in mixes of about 1:5 and in consistencies suitable for building construction, showed an increase in compressive strength of from 100 to 200 lb per sq in, which increase was practically constant at ages of two days to three years. For richer mixes and drier consistencies the strength increase was greater and for leaner mixes and wetter concrete it was less."

When calcium chloride or proprietary compounds are used in reinforced-concrete work, it is important that the reinforcement be thoroughly embedded. It is not believed that the use of calcium chloride causes efflorescence. As certain brands of Portland cement harden four or five times as fast as others when incorporated with a solution of calcium chloride, tests should be made with the particular brand of cement intended for use before determining upon the specification.

CAL, which is a product obtained by pulverizing a mixture of either quicklime or hydrated lime, calcium chloride and water, is similar to calcium chloride in its application as an admixture. It gives considerably greater strength at the 2 and 7-day periods and practically the same strength at the 28-day period. In using Cal the same precautions should be taken as in the case of calcium chloride. Cal is usually somewhat more expensive and produces a slightly greater workability of the plastic concrete.

At one time COMMON SALT was quite widely used as an admixture for the purpose of lowering the freezing-point of water, when concrete was to be placed under cold weather conditions. This practice is not to be recommended, as appreciable amounts of salt (over 5% by weight of cement) affect the strength of the concrete, and may cause the reinforcement to rust, and very probably will result in efflorescence.

In general, it seems to be well established that calcium chloride, cal and similar admixtures manufactured by reputable dealers have a distinct value as ACCELERATORS. They are widely used in floor finishes for the purpose of permitting the finishing process to proceed more rapidly and in certain types of structural work to allow the earlier removal of forms.

The value of admixtures for the purpose of HARDENING floor surfaces has also been widely accepted; there are excellent materials available for this purpose which have been demonstrated to give good results.

HYDRATED LIME, KAOLINE, and CELITE represent a class of powdered admixtures which are generally beneficial when used with concrete of the leaner mixtures, increasing the workability and impermeability.

The use of admixtures to prevent concrete from FREEZING is usually a

futile procedure except in weather which is merely frosty. Real cold weather operations should be organized to provide for the heating of the materials and the protection of the concrete after depositing, as described later.

3. Design of Reinforced-Concrete Construction

Loads and Stresses

Design Loads. The DEAD LOAD is the weight of the permanent parts of the structure including all floors, roofs, walls, columns, stairs, partitions, etc., together with any specific surface finishes such as plaster, granolithic or wooden flooring or roofing. Table I gives the weights of materials ordinarily used in design to determine the dead load.

Table I. Weights of Building Materials

Materials	Lb per cu ft	Lb per sq ft
Ashlar masonry, granite, gneiss	165	......
Ashlar masonry, limestone, marble	160	......
Ashlar masonry, sandstone, bluestone	140	......
Cinder-concrete fill	60	......
Cinder-concrete floor-arches	105	......
Cinder fill, rammed in place	50 to 55	......
Plain-stone or gravel concrete	144	......
Reinforced-stone or gravel concrete	150	......
Common brickwork	120	......
Pressed brickwork	140	......
Brickwork, 4-in with 4-in tile backing	......	60
Brickwork, 4-in with 8-in tile backing	......	75
Cement-mortar finish, 1 in thick	......	12
Clay-tile partitions, 3-in	......	18
Clay-tile partitions, 4-in	......	19
Clay-tile partitions, 6-in	......	25
Clay-tile partitions, 8-in	......	31
Creosoted wood-block flooring, 3 in thick	......	15
Gypsum partitions, 3-in, hollow	......	11½
Gypsum partitions, 4-in, hollow	......	14½
Hardwood flooring, ⅞ in thick	......	4
Lime or gypsum plasters, ¾ in thick	......	5
Pine, spruce, or hemlock sheathing	......	2½
Roofing-felt, 3-ply, and gravel	......	5½
Roofing-felt, 4-ply, and gravel	......	6
Roofing-felt, 5-ply, and gravel	......	6½
Roofing-ply, 3-ply, and slag	......	4½
Roofing-felt, 4-ply, and slag	......	5
Roofing-felt, 5-ply, and slag	......	5½
Roofing-tile, laid in place, book-tile 2-in	......	12
Roofing-tile, laid in place, book-tile 3-in	......	20
Roofing-tile, laid in place, flat, cement	......	15 to 20
Roofing-tile, laid in place, shingle-type, clay	......	12 to 14
Roofing-tile, laid in place, Spanish	......	8 to 10
Skylights with ⅜-in wire-glass and frame	......	7½
Slate, laid in place, ¼-in	......	9½
Slate, laid in place, ⅜-in	......	14½
Slate, laid in place, ½-in	......	19½
Suspended ceilings, metal lath and cement plaster	......	10
Wall-tiles, 8-in	......	33
Wall-tiles, 12-in	......	45

The LIVE LOAD is the load in excess of its own weight which the structure is designed to carry, and includes all loads other than the wind and dead load. It is usually expressed as so many pounds per square foot of floor area and unless the specification calls for special concentrations, the design is made upon an assumption of a uniform distribution of load. On important work it is then desirable to check the design for the effect that would be produced by loading certain bays to the maximum and leaving others entirely unloaded. The present American practice is to combine live load with dead load in the design of concrete buildings except where it is necessary to provide for moving loads such as on crane ways. Tables II and III give the minimum requirements for uniformly distributed live loads.

Table II. Minimum Live-Load Requirements for Floors for Industrial or Commercial Occupancy

From the Report of the Building Code Committee, Department of Commerce

Buildings, structures, etc.	Lb per sq ft
Storage-purposes, general	250
Storage-purposes, special	100
Manufacturing, light	75
Printing-plants	100
Wholesale stores, light merchandise	100
Retail salesrooms, light merchandise	75
Stables	75
Garages, all types of vehicles	100
Garages, passenger-cars only	80
Sidewalks, 250 lb per sq ft uniformly distributed or 8 000 lb concentrated, whichever gives the larger bending moment or shear	....

Table III. Minimum Live-Load Requirements

From the New York City Building Code

Buildings, structures, etc.	Lb per sq ft
Residences	40
Places for assembly or for public purposes	100
Classrooms of schools or other places of instruction	75
Offices	50
Floors of any other class not included above	120
Roofs with a pitch of 20° or less	40
Roofs with a pitch of more than 20°, measured on the projection on the horizontal plane	30
Sidewalks betweeen curb and building	300
Yards and courts inside the building-line	120

The WIND-LOAD is the vertical component of stresses resulting from wind-pressure. In the design of industrial buildings wind-pressure is not usually an important consideration, but wind-pressure should be investigated for high buildings and for those of moderate height and extremely narrow width.

IMPACT LOADS, such as those sustained by beams supporting elevator mechanism, are provided for by adding 100% to the actual known loads in the design of the supporting beams; this increment is either entirely dropped

Table IV. Unit Working Stresses for Static Loads

From various building codes

Classification of stresses	Working stresses, in lb per sq in			
	1928 * San Francisco	1929, Philadelphia: Proportionment by arbitrary volumes †	1929, Philadelphia: Proportionment by water/cement ratio ‡	1930 Boston
Extreme fiber-stress in concrete in compression:				
In general.	715	650	40% f'_c	715
Adjacent to supports of continuous beams	825	747	45% f'_c	825
Concentric compression in concrete	495	500	25% f'_c	495
Shearing-stress in concrete when no steel is provided to resist diagonal-tension. . . .		40	2% f'_c	44
Vertical shearing-stress when the diagonal-tension requirements are satisfied.	132	120	9% § f'_c	132
Bond-stress:				
Between concrete and plain bars	88	80	4% f'_c	88
Between concrete and deformed bars	110	100	5% f'_c	110
Maximum tensile stress in steel reinforcement . . .	18 000	18 000	18 000	18 000
Maximum tensile stress in cold-drawn steel wire. . . .	20 000	. . .	. . . . .	22 500

* Values based on a 28-day compressive strength of 2 200 lb per sq in, and corresponding to a mixture of one part cement to six parts of combined aggregate, where the coarse aggregate is of granite or trap-rock. The proportions are by volume of cement to the combined aggregates, measured separately. For example, a 1 : 2 : 4 mixture might also be referred to as a 1 : 6 mixture.

† Values based on a 28-day compressive strength of 2 000 lb per sq in; proportions, one part of cement to six parts of combined aggregate. The amount of water, including the moisture content of the aggregate, is limited to 7¼ gallons per bag of cement.

‡ The value of $f'c$ represents the 28-day compressive strength as determined by actual tests. The concrete is proportioned by the inspector. The following table gives the approximate quantities of combined aggregates, water ratios, and corresponding minimum 28-day strengths for the various mixtures. Water or moisture contained in the aggregates and ascertained by daily tests, is to be included in determining the amount of water corresponding to a required water-cement ratio.

§ Beams with web-reinforcement and longitudinal bars having special anchorage.

Approximate volume of Portland cement to sum of separate volumes of dry and rodded fine and coarse aggregate	Water-cement ratio, United States gal per 94-lb sack of cement	Assumed ultimate strength at 28 days
1 : 8½	8¼	1 500
1 : 6½	7½	2 000
1 : 5½	6¾	2 500
1 : 5	6	3 000

Table V. Unit Working Stresses from the Joint Code

Classification of stresses	Allowable unit working stresses in lb per sq in			
	For any strength of concrete as fixed by test in accordance with code requirements $n = \frac{30\,000}{f'_c}$	When strength of concrete is fixed by the water-cement ratio in accordance with code requirements		
		$f'_c =$ 2 000 lb $n = 15$	$f'_c =$ 2 500 lb $n = 12$	$f'_c =$ 3 000 lb $n = 10$
Flexure:				
Extreme fiber stress in compression	$0.40\,f'_c$	800	1 000	1 200
Extreme fiber stress in compression adjacent to supports of continuous or fixed beams or of rigid frames	$0.45\,f'_c$	900	1 125	1 350
Shear:				
Beams with no web reinforcement and without special anchorage of longitudinal steel	$0.02\,f'_c$	40	50	60
Beams with no web reinforcement, but with special anchorage of longitudinal steel	$0.03\,f'_c$	60	75	90
Beams with properly designed web reinforcement, but without special anchorage of longitudinal steel	$0.06\,f'_c$	120	150	180
Beams with properly designed web reinforcement and with special anchorage of longitudinal steel	$0.09\,f'_c$	180	225	270
Flat slabs at distance d from edge of column cap or drop panel	$0.03\,f'_c$	60	75	90
Footings where longitudinal bars have no special anchorage	$0.02\,f'_c$	40	50	60
Footings where longitudinal bars have special anchorage	$0.03\,f'_c$	60	75	90
Bond:				
In beams and slabs and one-way footings:				
Plain bars	$0.04\,f'_c$	80	100	120
Deformed bars	$0.05\,f'_c$	100	125	150
In two-way footings:				
Plain bars	$0.03\,f'_c$	60	75	90
Deformed bars	$0.0375\,f'_c$	75	94	112
(Where special anchorage is provided double these values in bond may be used.)				
Bearing:				
Where a concrete member has an area at least twice the area in bearing	$0.25\,f'_c$	500	625	750

Table V (Continued). Unit Working Stresses from the Joint Code

The following unit stresses in reinforcing steel shall not be exceeded:

Tension:	
Intermediate grade billet-steel	$(f_s) = 20\,000$ lb per sq in
Rail-steel bars	$(f_s) = 20\,000$ lb per sq in
Web reinforcement	$(f_v) = 16\,000$ lb per sq in
Structural steel shapes	$(f_s) = 18\,000$ lb per sq in
Other steel reinforcement 50% of the yield-point, but not to exceed	$(f_s) = 20\,000$ lb per sq in
Compression:	
Bars	nf_c *
Structural steel section in composite columns . .	15 000 lb per sq in
Cast iron section in composite columns	9 000 lb per sq in

Special values are given by this code for f_c in column design.

Under special conditions of design and supervision a value of $0.12f'_c$ is permitted for the unit shearing stress.

* Values of n are taken from the preceding table.

or reduced by ½ for the secondary members such as the girders, or supporting columns.

Soil loads are treated under the design of footings.

Reduction of live loads in column design is treated under columns.

Working Stresses. The present practice in this country is to design reinforced-concrete members on a basis of the working stresses in the materials rather than on the ultimate stresses. Although many building ordinances still assume fixed values for concrete mixed in certain specified proportions, it is more logical, when preliminary tests can be made, to compute the working stresses as percentages of the ultimate compressive strength actually developed at an age of 28 days. Our engineering societies have suggested this practice and recommended that the tests be made in accordance with the requirements of the American Society for Testing Materials (Serial Designation: C39–25). Table IV gives unit working stresses for static loads taken from the codes of representative cities. Table V gives unit working stresses for static loads taken from the Joint Code.*

Flexure Formulas for Reinforced-Concrete Members

Concrete and Steel in Combination. As concrete is very much stronger in compression than it is in tension, it has become the generally accepted practice to provide steel reinforcement in the concrete to resist all of the principal tensile stresses developed by any system of loading.

Steel is also used to resist compressive stresses in columns, in rectangular beams of limited cross-section, and in the webs of T beams to increase the resistance to negative bending moments over supports. Steel, however, is not as economical as concrete for this purpose and should be used to resist compressive stresses only when it is impracticable to increase the sectional area of the concrete.

In a typical reinforced-concrete beam, all compression through the middle part of the span is resisted by the concrete and all tension is assumed to be resisted by the steel. The compressive stress diminishes from a maximum f_c,

* Tentative Building Regulations for Reinforced Concrete recommended by the American Concrete Institute and the Concrete Reinforcing Steel Institute: Proc. American Concrete Institute, Vol. 24, page 791.

in the concrete at the upper surface of the beam, to zero at the neutral surface, which lies a little above the center of the section. The tensile stress is considered to act with a uniform intensity f_s, at the centroid of the longitudinal reinforcement. The resultant of the compressive stresses in the concrete and the tensile stresses in the steel form the forces of a resisting mechanical couple. As the forces of a couple must be equal, the most economical beam is one in which the respective areas of steel and concrete are so proportioned that each will develop, simultaneously with the other, its full unit working stress. RECTANGULAR BEAMS and slabs are often designed according to this principle, the sectional area of the reinforcement being taken as a percentage of the area of the concrete. In the case of T BEAMS, however, there is usually an excess of concrete and it would be wasteful to furnish an equivalent, and therefore excessive, area of steel. Consequently, the reinforcement of T beams is usually designed independently of the concrete section.

Fundamental Assumptions. The design of reinforced-concrete members is based on the following assumptions:

(1) A PLANE CROSS-SECTION of a beam, before bending, remains a plane section after bending.

(2) The MODULUS OF ELASTICITY of the concrete in compression remains constant within the assumed working stresses and the distribution of compressive stress in beams is rectilinear.

(3) PERFECT ADHESION exists between concrete and steel within the range of working stresses.

(4) In calculating the moment of resistance of reinforced-concrete beams and slabs, the TENSILE RESISTANCE of the concrete is negligible.

(5) The INITIAL STRESS in the steel caused by contraction or expansion of the concrete is negligible, except in some column formulas.

(6) The MODULUS OF ELASTICITY of concrete in computations for the position of the neutral axis, for the resisting moment of beams, and for compression of concrete in columns, is expressed by the formula:

$$n = \frac{E_s}{1\,000 f'_c} = \frac{30\,000}{f'_c}$$

Notation Used in Reinforced-Concrete Design. The following notation is used in this discussion. (See Fig. 1.)

d = effective depth of beam, distance from extreme fibers in compression to center of gravity of tensile reinforcement in inches;

b = width of beam or section of slab in inches;

k = ratio of distance of neutral axis of cross-section from extreme fibers in compression to effective depth of beam;

kd = distance of neutral axis from extreme fibers in compression in inches;

j = ratio of distance between the center of compression of concrete and center of tension of steel to effective depth of beam or ratio of arm of resisting couple to d;

jd = distance between center of compression in concrete and center of tension in steel or arm of resisting couple in inches;

f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch;

f''_c = tensile unit stress in extreme fibers of concrete in pounds per square inch;

f_s = tensile unit stress in steel in pounds per square inch;

f'_s = compressive unit stress in steel in pounds per square inch;
A_s = area of cross-section of main tensile reinforcement in square inches;
E_s = modulus of elasticity of steel, in pounds per square inch;
E_c = modulus of elasticity of concrete, in pounds per square inch;
n = modulus of elasticity of steel divided by modulus of elasticity of concrete, or E_s/E_c;
M_s = resisting moment of reinforcement in inch-pounds; when computed on the basis of the full area of steel, A_s, times an assumed stress, f_s;
M_c = resisting moment of concrete in inch-pounds when computed on the basis of an assumed extreme fiber stress in the concrete, f_c;
M = bending moment in inch-pounds;
p = percentage of reinforcement, or A_s/bd.

Working Formulas for Rectangular Beams and Slabs. The following formulas are based on the above assumptions.

By assumption (1) the deformation in any fiber of a beam is proportional to its distance from the neutral surface, or from the neutral axis of the cross-section of the beam. Since, by assumption (2), the modulus of elasticity

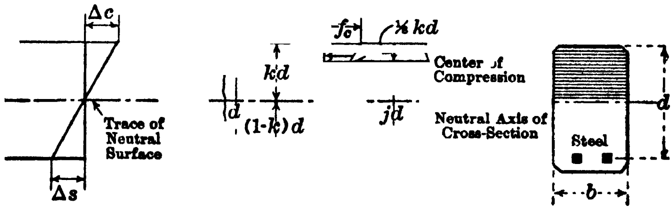


Fig. 1. Diagrams of Deformations and Stresses

remains constant, the stresses are proportional to their deformations and hence these stresses also vary in proportion to their distances from the neutral axis. At the neutral surface of the beam, the stress is zero and on the compression side increases to a maximum value, f_c, in the extreme compressive fibers.

A triangle of compressive stresses is thus formed, with the center of compression at one-third the altitude of the triangle measured from its base. The center of tension is at the center of gravity of the cross-section of the steel.

In Fig. 1, Δ_c and Δ_s represent the maximum deformations in the concrete and steel, respectively, when the beam is in a state of flexure under any system of loading. The resultant compressive stress in the concrete is the sum of the varying compressive stresses, and its line of action is through the center of gravity of the triangle of stresses. Since the modulus of elasticity of each material equals the unit stress in the material divided by the corresponding unit deformation,

$$\text{Modulus of elasticity} = \frac{\text{unit stress}}{\text{unit deformation}}$$

Substituting,

$$E_c = \frac{f_c}{\Delta_c} \quad \text{and} \quad E_s = \frac{f_s}{\Delta_s}$$

Transposing,

$$\Delta_c = \frac{f_c}{E_c} \quad \text{and} \quad \Delta_s = \frac{f_s}{E_s}$$

By similar triangles,

$$\frac{\Delta_c}{\Delta_s} = \frac{kd}{(d - kd)}$$

Substituting the values found for Δ_c and Δ_s,

$$\frac{f_c/E_c}{f_s/E_s} = \frac{kd}{(d - kd)} \quad \text{or} \quad \frac{f_c}{f_s} \times \frac{E_s}{E_c} = \frac{k}{1 - k}$$

Substituting the value of $n = \dfrac{E_s}{E_c}$

$$\frac{nf_c}{f_s} = \frac{k}{1 - k} \quad \text{and} \quad \frac{f_c}{f_s} = \frac{k}{n(1 - k)} \tag{1}$$

Since the LONGITUDINAL REINFORCEMENT can develop a resisting moment equal to its sectional area A_s, or pbd, multiplied by the unit working tensile stress, f_s, times its lever arm, jd, it follows that

$$M_s = A_s \times f_s \times jd \quad \text{or} \quad M_s = pbd \times f_s \times jd = pf_sjbd^2 \tag{2}$$

Since the CONCRETE can develop a resisting moment equal to the area of the cross-section in compression, kdb, multiplied by the average unit compressive stress, $\frac{1}{2}f_c$, times its lever arm, jd, it follows that

$$M_c = kbd \times \tfrac{1}{2}f_c \times jd \quad \text{or} \quad M_c = \tfrac{1}{2}f_c\, jkbd^2 \tag{3}$$

Since the TENSILE STRESS IN THE REINFORCEMENT must be balanced by COMPRESSIVE STRESSES IN THE CONCRETE, these two expressions may be placed equal to each other, hence

$$pf_sjbd^2 = \tfrac{1}{2} f_cjkbd^2$$

Canceling common factors

$$2f_sp = f_ck \quad \text{or} \quad \frac{f_c}{f_s} = \frac{2p}{k} \tag{4}$$

Equating the values of $\dfrac{f_c}{f_s}$ from equations (1) and (4)

$$\frac{k}{n(1 - k)} = \frac{2p}{k} \quad \text{or} \quad k^2 = 2pn(1 - k)$$

And

$$k^2 + 2pnk = 2pn$$

Completing the square of the first number of this equation by adding $(pn)^2$ to both numbers,

$$k^2 + 2pnk + (pn)^2 = 2pn + (pn)^2$$

Extracting the square root of the first number of the equation, and indicating the operation for the second number,

$$k + pn = \sqrt{2pn + (pn)^2}$$

and

$$k = \sqrt{2pn + (pn)^2} - pn \tag{5}$$

By this formula may be obtained the VALUE OF k, DETERMINING the LOCATION OF THE NEUTRAL AXIS, when p and n are known.

From formula (2) by transposition, and by substituting M for M_s

$$f_s = \frac{M}{pjbd^2} \tag{6}$$

Similarly from formula (3)

$$f_c = \frac{2M}{jkbd^2} \tag{7}$$

These formulas are used to CHECK THE STRESSES IN THE STEEL AND CONCRETE, respectively, for members already designed.

From formula (6), substituting $A_s = pbd$, and transposing,

$$A_s = \frac{M}{f_s jd} \tag{8}$$

By definition,

$$A_s = pbd \tag{9}$$

These last are the two formulas for DETERMINING THE CROSS-SECTIONAL AREA OF THE MAIN TENSILE REINFORCEMENT to resist a given bending moment. Formula (8) is for general use. The value of j, corresponding to any given stresses and value of n, may be taken from Tables VI and VII. Although accurate only in the case that the amount of reinforcement is such that both steel and concrete develop simultaneously their full working stresses, the error, on the side of safety, is usually negligible.

Formula (9) furnishes a means for determining the sectional-area of the reinforcement as a percentage, p, of the concrete section, bd. If the value of p is taken from Table VI or VII, the result will be to supply the exact amount of steel necessary to "balance" the concrete. This procedure is satis-

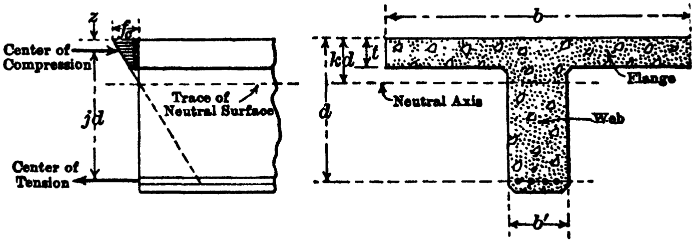


Fig. 2. Stress Diagram and Cross-Section of a Reinforced-Concrete T Beam

factory in the case of slabs and rectangular beams, or other members for which the concrete section has been determined by moment considerations. It is obviously inappropriate for the typical design of T beams or others where the entire concrete section is not needed for moment requirements.

From formula (7), transposing,

$$M = \tfrac{1}{2} f_c jkbd^2$$

from which

$$d = \sqrt{\frac{M}{\tfrac{1}{2} f_c jkb}} = \sqrt{\frac{M}{Kb}} \tag{10}$$

This is the formula for DETERMINING THE DEPTH OF A SLAB OR BEAM TO RESIST A GIVEN BENDING MOMENT without exceeding a specified unit stress in the concrete. If this formula is used, either formula (8) or (9) may be employed to determine the steel area.

Working Formulas for T Beams. The design of T beams and T girders is divided into two classifications: first, that in which the neutral surface falls either within the flange or at the bottom of it; and second, that in which the neutral surface falls within the web. See Fig. 2.

Table VI. Formula-Factors for Rectangular Beams and Slabs

To be used in the following formulas:

Depth of slab or beam, $d = \sqrt{\dfrac{M}{\frac{1}{2} f_c jkb}} = \sqrt{\dfrac{M}{Kb}}$

Area of reinforcement, $A_s = pbd$

and $A_s = \dfrac{M}{f_s jd}$

Tensile stress in longitudinal reinforcement, $f_s = \dfrac{M}{pjbd^2}$

Compressive stress in concrete, $f_c = \dfrac{2M}{jkbd^2}$

Ratio of moduli, $n = 15$

f_s Lb per sq in	f_c Lb per sq in	k*	j	p†	K
16 000	500	0.319	0.894	0.0050	71.3
	550	0.339	0.887	0.0058	82.9
	600	0.359	0.881	0.0067	94.5
	650	0.378	0.874	0.0077	107.7
	700	0.397	0.868	0.0087	120.4
	750	0.414	0.862	0.0097	133.5
	800	0.429	0.857	0.0107	146.9
	850	0.443	0.852	0.0118	160.6
	900	0.458	0.847	0.0129	174.5
18 000	500	0.294	0.902	0.0041	66.3
	550	0.314	0.895	0.0048	77.4
	600	0.333	0.889	0.0056	88.9
	650	0.351	0.883	0.0063	100.8
	700	0.368	0.877	0.0072	113.1
	750	0.385	0.872	0.0080	125.7
	800	0.400	0.867	0.0089	138.7
	850	0.415	0.862	0.0098	151.9
	900	0.429	0.857	0.0107	165.3
20 000	500	0.272	0.909	0.0034	62.0
	550	0.292	0.903	0.0040	72.5
	600	0.311	0.896	0.0047	83.5
	650	0.328	0.891	0.0053	95.0
	700	0.344	0.885	0.0060	106.5
	750	0.359	0.880	0.0067	118.5
	800	0.374	0.875	0.0075	131.0
	850	0.389	0.870	0.0083	144.0
	900	0.403	0.866	0.0091	157.0

* The value of k applies to all types of beams, including T beams.

† In members with compression-reinforcement, p becomes p_1.

In the first case, the design of concrete and reinforcement to resist the stresses due to bending moments is the same as for rectangular beams, and the preceding formulas apply. The width of the flange, however, is used for the value of b, and the steel ratio p is based upon the total area bd.

Table VII. Formula-Factors for Rectangular Beams and Slabs

To be used in the following formulas:

Depth of slab or beam, $d = \sqrt{\dfrac{M}{\frac{1}{2} f_c jkb}} = \sqrt{\dfrac{M}{Kb}}$

Area of reinforcement, $A_s = pbd$

and $A_s = \dfrac{M}{f_s jd}$

Tensile stress in longitudinal reinforcement, $f_s = \dfrac{M}{pjbd^2}$

Compressive stress in concrete, $f_c = \dfrac{2M}{jkbd^2}$

Ratio of moduli, $n = 12$

f_s Lb per sq in	f_c Lb per sq in	k*	j	p†	K
16 000	700	0 344	0 885	0 0075	106.7
	750	0 360	0 880	0 0084	118.8
	800	0 375	0 875	0 0094	131.3
	850	0.389	0 870	0.0103	144 0
	900	0 402	0 866	0 0113	157.0
	950	0 415	0 862	0.0123	170.0
	1 000	0.428	0.857	0.0134	183.5
18 000	700	0.318	0 894	0 0062	99 5
	750	0 333	0.889	0 0070	111.0
	800	0 348	0.884	0.0077	123.0
	850	0.362	0.879	0.0085	135.0
	900	0.375	0 875	0 0094	147 0
	950	0 388	0 871	0 0102	159 5
	1 000	0.400	0 867	0.0111	172.5
20 000	700	0 296	0 901	0.0052	93.3
	750	0.310	0 897	0.0058	104 5
	800	0.324	0.892	0.0065	115.6
	850	0.338	0.887	0.0072	127.5
	900	0.351	0.883	0 0079	139.5
	1 000	0.375	0.875	0.0094	164.0

* The value of k applies to all types of beams, including T beams.
† In members with compression-reinforcement, p becomes p_1.

In the second case, which more often occurs, the following formulas apply. These are developed from the same fundamental assumptions as those used in the design of rectangular beams.

Neglecting the amount of compression resisted by the web,

Let t = total slab-thickness on either side of the web, in inches;
z = distance from the compression-surface of the beam to the center of compression, in inches;
b' = width of web in inches;
b = allowable width of flange, in inches;
$p = A_s/bd$.

The remaining notation is the same as for rectangular beams and as previously given.

Then

$$kd = \frac{2\,ndA_s + bt^2}{2\,nA_s + 2\,bt}$$

And

$$k = \frac{pn + (\tfrac{1}{2})(t/d)^2}{pn + t/d}$$

$$z = \frac{3\,kd - 2\,t}{2\,kd - t} \times \frac{t}{3}$$

$$jd = d - z$$

$$k = \frac{1}{1 + f_s/nf_c}$$

$$M_c = f_c \times \left(1 - \frac{t}{2\,kd}\right) \times bt \times jd$$

$$f_c = f_s k/n\,(1 - k)$$

$$A_s = \frac{M}{f_s jd}$$

Formulas giving approximate results, but erring slightly on the safe side, are deduced by substituting $\left(d - \frac{t}{2}\right)$ for jd, and $(\tfrac{1}{2}f_c)$ for $f_c\left(1 - \frac{t}{2\,kd}\right)$ in the more exact formulas,* as follows:

$$M_c = (\tfrac{1}{2})\,f_c bt\left(d - \frac{t}{2}\right)$$

Or, if the bending moment is substituted for the resisting moment,

$$f_c = \frac{2\,M}{bt\left(d - \frac{t}{2}\right)}$$

And

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)}$$

Standard textbooks give more complicated formulas which take into consideration the compression resisted by the small portion of the web lying between the neutral axis and the flange, but the two last equations are the ones ordinarily employed in practical design for the purpose of determining the sectional-area of reinforcement required to resist the tensile stress devel-

* See page 78, Concrete Building Construction, by Theodore Crane, published by John Wiley & Sons, Inc.

oped by the bending moment and to check the value of the extreme fiber stress in the concrete due to the same cause.

Bending Moments

Bending Moments on Beams and Girders. In concrete construction, owing to the monolithic character of the material, the bending moments both positive and negative depend upon the degree of restraint at the supports. For instance, a simply supported beam, one merely resting on two supports, has a maximum positive moment at the middle equal to $\frac{WL}{8}$ in-lb and no moment at either support. In this formula W is the total uniformly distributed load in pounds and L the span in inches. If a beam is fixed at one end and simply supported at the other, the maximum positive bending moment is $\frac{3}{8}L$ from the free end and its value is approximately $\frac{WL}{14.2}$ in-lb. A negative moment equal to $\frac{WL}{8}$ in-lb is developed at the fixed end. Likewise, a beam fixed at both ends has a positive moment at the middle equal to $\frac{WL}{24}$ in-lb and a negative moment at each support which is twice as great, or $\frac{WL}{12}$ in-lb.

A reinforced-concrete beam which is continuous over a support always develops a negative bending moment over the support, and this is usually greater than the maximum positive moment which normally occurs near the mid-span. The following charts give the THEORETICAL BENDING MOMENTS for the conditions indicated. The MOMENT COEFFICIENTS USED IN PRACTICE should never be less than the theoretical moments and, in the case of positive moments, are usually taken considerably greater in order to provide for variations in live-load distribution except where fixed concentrations occur, as in girder design. For example, a series of fully continuous beams of equal and uniformly loaded spans has negative moments at supports equal to $\frac{WL}{12}$ and positive moments at mid-spans equal to $\frac{WL}{24}$ (see diagram 1, Fig. 3); the general practice, however, is to use the same value, that is, $\frac{WL}{12}$ for both positive and negative moments, as appears in the following recommendation.

Design-Moment Coefficients. For special cases, design-moment coefficients must be determined from a study of the theoretical moment diagrams, or worked out by the Theorem of Three-Moments, or other method for computing indeterminate stresses. For uniform loads and equal spans, however, the following design moments are recommended and represent present practice except where an exception is noted.

These recommendations are taken from the Joint Code published by the American Concrete Institute, Volume 24, page 807, proceedings of 1928.

Let M = bending moment in inch-pounds;
W = total uniformly distributed load on the member in pounds;
L = span, in inches;
I = moment of inertia of a cross-section of a beam, girder, or column about the neutral axis for bending in biquadratic inches;
h = unsupported length of a column in inches.

The span length of freely supported beams and slabs shall be the clear span plus the depth of beam or slab, but shall not exceed the distance between centers of the supports. The span length for continuous or restrained beams built to act integrally with supports shall be the clear distance between faces of supports.

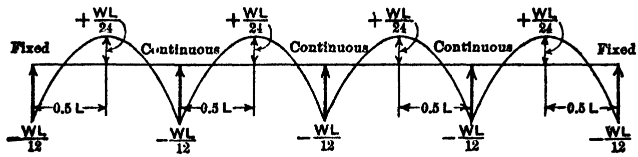


Diagram 1. Four-Span Continuous Beam with Fixed Ends.

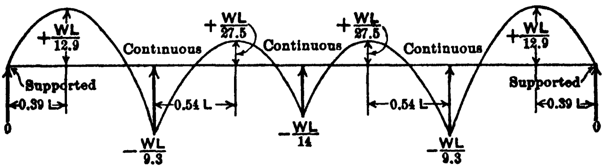


Diagram 2. Four-Span Continuous Beam with Simply-Supported Ends.

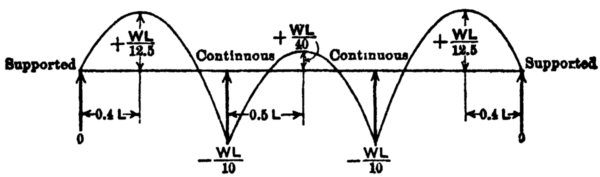


Diagram 3. Three-Span Continuous Beam with Simply-Supported Ends.

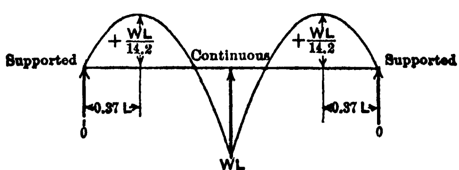


Diagram 4. Two-Span Beam, Each Span Semi-Continuous.

Fig. 3. Theoretical Bending-Moment Diagrams, All for Uniform Loads and Equal Spans

MOMENT COEFFICIENTS FOR FREELY SUPPORTED OR SLIGHTLY RESTRAINED CONTINUOUS BEAMS OR SLABS OF APPROXIMATELY EQUAL SPAN: UNIFORM LOAD. Beams and slabs of approximately equal spans, freely supported or built to act integrally with beams, girders, or other SLIGHTLY RESTRAINING

SUPPORTS, or beams and slabs built into brick or masonry walls in a manner which develops only PARTIAL END RESTRAINT, and carrying uniformly dis-

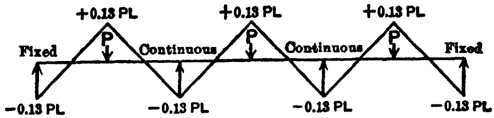


Diagram 5. Three-Span Continuous Beam with Fixed Ends. Single Concentrated Load P at the Middle of Each Span

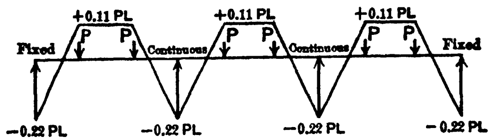


Diagram 6. Three-Span Continuous Beam with Fixed Ends. Concentrated Loads P at the Third-Points of Each Span

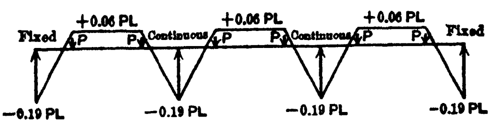


Diagram 7. Three-Span Continuous Beam with Fixed Ends. Concentrated Loads P at the Quarter-Points of Each Span

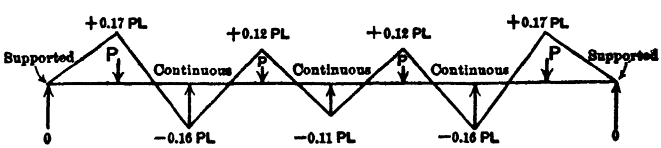


Diagram 8. Four-Span Continuous Beam with Simply-Supported Ends. Single Concentrated Load P at the Middle of Each Span

Fig. 4. Theoretical Bending-Moment Diagrams, All for Concentrated Loads and Equal Spans

tributed loads, shall be designed for the following bending moments at critical sections:

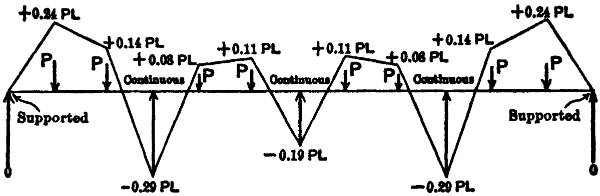


Diagram 9. Four-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Third-Points of Each Span.

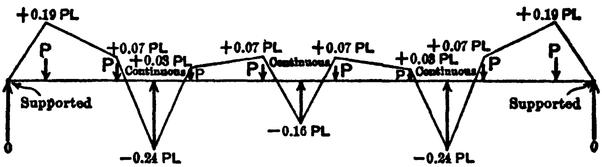


Diagram 10. Four-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Quarter-Points of Each Span.

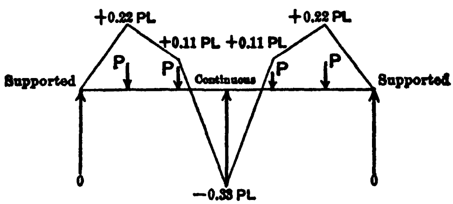


Diagram 11. Two-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Third-Points of Each Span.

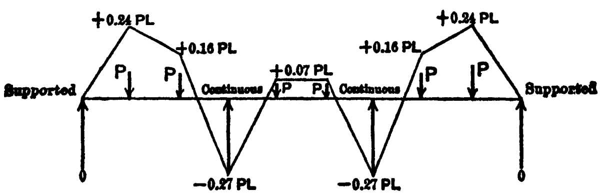


Diagram 12. Three-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Third-Points of Each Span.

Fig. 5. Theoretical Bending-Moment Diagrams, All for Concentrated Loads and Equal Spans

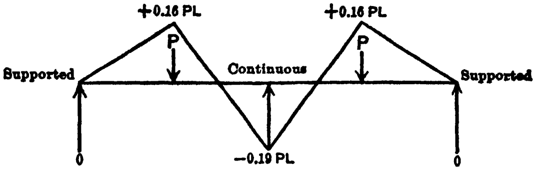


Diagram 13. Two-Span Continuous Beam with Simply-Supported Ends. Concentrated Load P at the Middle of Each Span

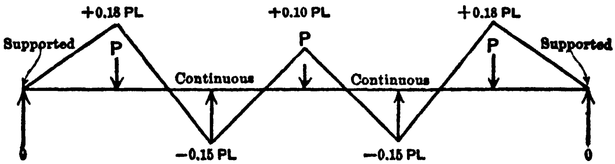


Diagram 14. Three-Span Continuous Beam with Simply-Supported Ends. Concentrated Load P at the Middle of Each Span

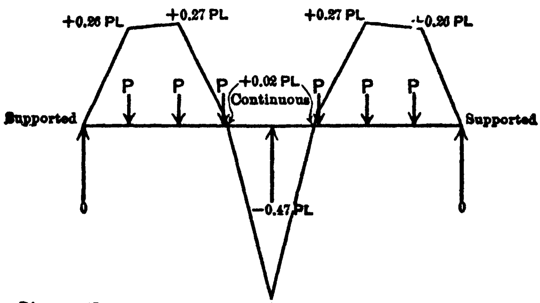


Diagram 15. Two-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Middle and at the Quarter-Points of Each Span

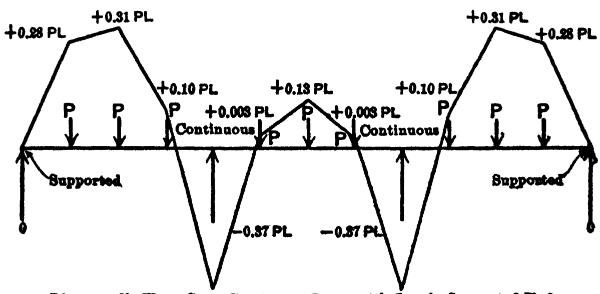


Diagram 16. Three-Span Continuous Beam with Simply-Supported Ends. Concentrated Loads P at the Middle and at the Quarter-Points of Each Span

Fig. 6. Theoretical Bending-Moment Diagrams, All for Concentrated Loads and Equal Spans

(1) Beams and slabs of one span,

Positive moment near center,

$$M = \frac{WL}{8}$$

(2) Beams and slabs continuous for two spans only,

Positive moment near center,

$$M = \frac{WL}{10}$$

Negative moment over interior support,

$$M = \frac{WL}{8}$$

(3) Beams and slabs continuous for more than two spans,

Positive moment near center and negative moment at support of interior spans,

$$M = \frac{WL}{12}$$

Positive moment near centers of end spans and negative moment at first interior support,

$$M = \frac{WL}{10}$$

(4) Negative moment at end supports for cases (1), (2), and (3) of this section,

$$M = \text{not less than } \frac{WL}{24}$$

MOMENT COEFFICIENTS FOR FULLY RESTRAINED CONTINUOUS BEAMS OF SLABS OF APPROXIMATELY EQUAL SPAN: UNIFORM LOAD. Beams and slabs of approximately equal spans built to act integrally with columns, walls, or other RESTRAINING SUPPORTS and assumed to carry uniformly distributed loads, shall, except as provided above, be designed for the following bending moments at critical sections:

(1) Interior spans;

Negative moment at interior supports except the first,

$$M = \frac{WL}{12}$$

Positive moment near centers of interior spans,

$$M = \frac{WL}{16}$$

(2) End spans of continuous beams or slabs, and beams or slabs of one span;

Where I/L is less than twice the sum of the values of I/h for the exterior columns above and below which are built into the beams:

Positive moment near center of span and negative moment at first interior supports,

$$M = \frac{WL}{12}$$

Negative moment at exterior supports,

$$M = \frac{WL}{12}$$

Where I/L is equal to or greater than twice the sum of the values of I/h for the exterior column above and below which are built into the beams:

Positive moment near center of span and negative moment at first interior support,

$$M = \frac{WL}{10}$$

Negative moment at exterior support,

$$M = \frac{WL}{16}$$

In this section, I represents the moment of inertia which, for these calculations, shall be computed on the assumption that the member is homogeneous, neglecting the reinforcement, but including that portion of the concrete section outside of the reinforcement which is ordinarily considered as fireproofing.

MOMENT COEFFICIENTS FOR CONTINUOUS BEAMS OF SLABS OF UNEQUAL SPANS OR WITH NON-UNIFORM LOADS. Continuous beams with substantially unequal spans, or with other than uniformly distributed loads, whether freely supported or restrained, shall be designed for the maximum moments

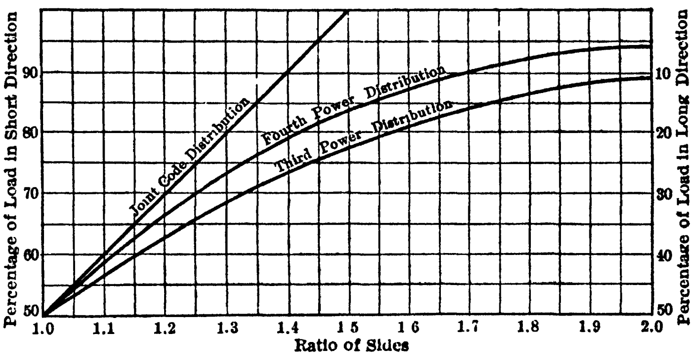


Fig. 7. Load-Distribution in Slabs with Two-Way Reinforcement

resulting from the most severe PROBABLE COMBINATION of loading and restraint. Provision shall be made where necessary for negative moment near the center of short spans which are adjacent to long spans, and for the negative moment at the end supports, if restrained.

In order to conform with the notation of the text, W has been substituted for wl in the equations.

Bending Moments on Slabs. The bending moments used for uniformly loaded beams apply also to uniformly loaded slabs having one-way reinforcement. If two-way reinforcement is employed, the amount of load carried in each direction is determined by the chart in Fig. 7. Using the correct proportion of load, the two systems of reinforcement are independently designed. Even in the case of TWO-WAY DESIGNS, most building codes still require that the bending moments for simply supported spans be computed as equal to $\frac{WL}{8}$, for semi-continuous spans as $\frac{WL}{10}$ and for fully continuous spans as $\frac{WL}{12}$.

in which W = total load on the slab (including the weight of the slab) carried in the direction considered;

L = span length as defined for beams expressed in inches, as the result is to be in inch-pounds.

It has been amply demonstrated, however, that where two-way reinforcement is employed on panels which are square, or nearly square (see the limitations shown in the chart in Fig. 7), the bending moments are lessened by reason of flat-plate action and the values given above may be safely reduced by approximately 30%. It is true that the building ordinances of many of our larger cities permit such a reduction for proprietary systems which have gained their acceptance through submitting empirical data, and there is no reason why this same reduction should not also apply to solid concrete slabs.

Shear and Diagonal Tension

Working Formula for Shearing-Stresses. Diagonal tension is a combination of horizontal and vertical shear For practical purposes the latter is taken as a measure of the diagonal tension which a beam is designed to resist. The general formula used for the design of slabs, beams, and girders, as controlled by shearing-stress, is derived as follows:

V = total vertical shear at the section considered, in pounds;
v = unit shearing-stress at the section in pounds per square inch;
j = 0.875 (an approximation used for shear-computations);
b = width of beam for rectangular beams, and width of web for T beams, in inches;
d = effective depth of beam in inches.

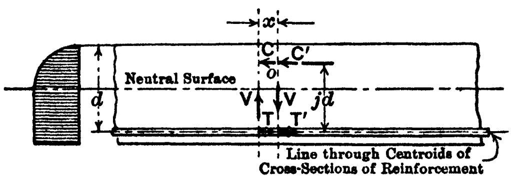


Fig. 8. Diagram of Shearing Stresses in a Reinforced-Concrete Beam

The intensity of the unit shearing-stress on any vertical section taken through a beam varies from a value of zero at the extreme fiber on the compression-face, to a maximum value at the neutral surface. As the tensile value of the concrete is neglected in computation, the intensity of the shearing-stress is considered as if it were constant from the neutral surface of the beam to the centroid of the reinforcement. Fig. 8 illustrates this condition.

The portion of the beam shown in the illustration is considered to be cut by two vertical planes normal to the side of the beam. These planes are separated by only a short distance, x, which is so small that the value of the external shear, V, is practically the same for each section. As the reinforcement is assumed to resist all tensile stress, the resultants of all tension on the two adjacent planes may be represented by T and T', acting in the plane of the steel. Similarly, C and C' may represent the resultants of all compressive stresses acting above the neutral surface. On any horizontal section between the neutral surface and the reinforcement, the increase in stress between the two vertical planes, x distance apart, is $T - T'$. This horizontal shear is

resisted by the concrete over an area equal to xb, where b is the breadth of the beam.

The unit shearing-stress is then, $v = T - T'/bx$. Since the section must be in equilibrium, the sum of the moments about the point o must equal zero and $(T - T')\,jd = Vx$, or $T - T' = Vx/jd$. Substituting in the equation, above,

$$v = Vx/bxjd$$

Or, canceling the x,

$$v = V/jbd = V/.875\,bd$$

The use of this equation is TO CHECK THE UNIT SHEARING-STRESSES on slabs and rectangular beams or girders, the sizes of which are normally determined by bending moment considerations. In the form

$$d = \frac{V}{jvb} = V/.875\,vb$$

the equation is used in the DIRECT DESIGN OF T BEAMS, the maximum allowable value being substituted for v. It should be noted that the depths of T beams are usually determined by shear, owing to the fact that the amount of concrete provided by the floor construction is normally more than sufficient to resist the compressive stresses due to moment.

Working Formulas for Web-Reinforcement. The DESIGN OF THE WEB-REINFORCEMENT consists in determining the size and spacing of stirrups in connection with the inclined portions of the main tensile steel. The point at which the longitudinal reinforcement is raised should be determined by moment considerations, but the bars usually cross the beam at sections where web-reinforcement is required and are, consequently, serviceable as such, thereby reducing the number of stirrups.

The design of web-reinforcement is not standardized. Some building ordinances demand that STEEL BE PROVIDED TO TAKE TWO-THIRDS OF THE TOTAL SHEARING-STRESS at all sections where such exceeds the value permitted for plain concrete; others allow the concrete to carry its permitted stress at all sections and require that STEEL BE USED ONLY FOR THE EXCESS. If the latter method is used,

Let v' = allowable unit shearing-stress resisted by the concrete alone, in pounds per square inch;

V = total vertical shear at the section considered in pounds;

A_s = cross-sectional area of stirrups, or of inclined portion of double-bend rods, in square inches. In a U-shaped or W-shaped stirrup, this is the total cross-sectional area of all legs;

b = width of beam in inches for rectangular beams, and width of web for T beams;

d = effective depth of beam in inches;

j = 0.875 (an approximation used for shear-computations);

f_s = allowable unit tensile stress in stirrups in pounds per square inch;

s = spacing of stirrups in inches.

Then, if $v'bjd$ represents the total shearing stress carried by the concrete,

$$s = \frac{f_s jdA_s}{(V - v'bjd)}$$

If it is required that the reinforcement be designed to take two-thirds of the total shearing stress, the formula becomes

$$s = \frac{3}{2} \times \frac{f_s jdA_s}{V}$$

These formulas apply to vertical stirrups; INCLINED STIRRUPS OR INCLINED BARS may be designed or their effective areas may be checked by the same formulas, merely introducing the sine of the angle of inclination to the horizontal as a multiple of the shear.

When designing stirrups, it is customary to choose either a 3/8 or 1/2-in round rod, and to determine upon either a U or W type: A_s is then known and the spacing is computed by the above formulas. The maximum spacing for stirrups should be limited to 3/4 times the depth of the beam. This is also the maximum distance through which the inclined portion of a longitudinal rod, or several rods, bent at the same section, may be considered effective as web-reinforcement. Stirrups should encircle the main longitudi-

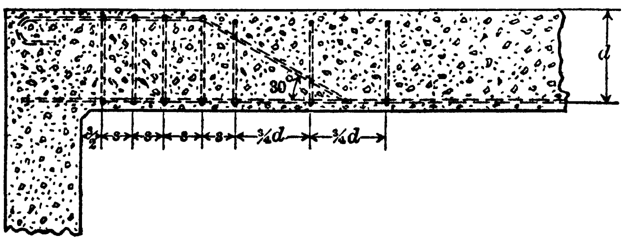


Fig. 9. Diagram of Web-Reinforcement in a Rectangular or T Beam

nal reinforcement and all free ends should be anchored by hooks. (See Fig. 9.)

Anchorage of Reinforcement. In the design of concrete structures it is often necessary to anchor reinforcing bars. The LENGTH OF EMBEDMENT required to develop the strength of a rod is based upon pull-out tests and naturally depends upon the tensile strength of the steel, the character of its surface and the quality of the concrete in which it is embedded. The following formula is generally accepted as giving the length of embedment in inches required to develop the full working stress in reinforcing bars:

$$l = (1/4)\,(f_s/u)i$$

in which f_s is either the tensile or compressive stress in the steel in pounds per square inch, i the diameter of a round rod or the side of a square bar in inches, and u the permissible bond stress, usually 80 lb or 100 lb per sq in, depending upon whether plain steel or deformed steel is used. Although the formula contains no factor representing the character of the concrete, it is assumed to be such as would give an ultimate compressive strength of 2 000 lb per sq in at an age of 28 days.

Substituting values in the above formula shows that an embedment of 50 diameters (or side of a square bar) is required to develop a working stress of 16 000 lb per sq in in a steel rod or bar; if the steel stress is 18 000 lb per sq in, an embedment of 56 diameters is required for plain steel and 45 diameters for deformed rods or bars.

It is customary to use this same formula to determine the LENGTH OF LAPS WHEN SPLICING REINFORCEMENT. Although square bends are generally used, as such are easier to make, heavy reinforcing bars should be bent to a radius of 4 diameters as illustrated. In general, BENDS ARE DESIGNED AS A MATTER OF INSURANCE where there is a possibility of unexpected load condi-

tions developing a greater stress in a reinforcing bar than the embedment up to that section would permit it to carry.

Working Formulas for Bond. As the tensile stresses in the longitudinal reinforcement of a beam increase toward the sections of maximum bending moment, the increment must be transmitted to the concrete, and bond stresses are developed along the surface of the rod or bar. To resist the tendency of the steel to pull loose from the surrounding concrete there are both the grip due to the initial shrinkage of the concrete in hardening and the frictional resistance of the rod against slipping, which latter is considerably higher for the deformed types

The BOND STRESS is a function of the EXTERNAL VERTICAL SHEAR at the section considered, and general practice accepts the following formula:

$$u = \frac{V}{\Sigma ojd}$$

in which u = unit bond stress in pounds per square inch;
V = total shear at the section, in pounds;
Σo = the sum of the perimeters of all horizontal rods under tensile stress at the section considered, in inches;
j = 0.875 (an approximation used for shear-computations);
d = distance from extreme fibers in compression to center of gravity of tensile reinforcement, in inches.

LIMITING VALUES for the bond stress, usually 80 lb per sq in for plain rounds or squares, and 100 lb per sq in for deformed rods or bars, are given by all building ordinances and design standards. The bond stress should be tested at CRITICAL SECTIONS which, in continuous or restrained beams, are usually considered to be at the faces of the supports for the tensile steel near THE TOP OF THE BEAM. In simply supported beams, or freely supported end spans of continuous beams, the CRITICAL SECTIONS are considered to be at the faces of the supports, and the Σo is the sum of the perimeters of the rods carried straight through into a support in the BOTTOM OF THE BEAM.

Some standards, such as the Joint Code (1928), recommend that, for continuous and restrained beams, bond stresses be tested at the POINT OF INFLECTION, ordinarily about the fifth point of the clear span, as well as at the sections described above. When this is done, however, a somewhat greater value is allowed for u provided that the reinforcement is securely anchored.

External Shears. The external vertical shear at any cross-section of a slab, beam, or girder, is determined by considering all the loads and reactions on either the right or left of the section under design and disregarding those on the other side. The loads are considered negative and the reactions positive and their ALGEBRAIC SUM is the shear at the section. The maximum vertical shears, V, occur immediately at the right or left of the supports and the NUMERICAL SUM of the positive and negative shears at any support is the reaction at that support.

For all uniformly or symmetrically loaded members, these maximum positive or negative shears are equal to one-half the span load, W, provided the condition of support is similar at both ends of the member under design. If one end of a beam or girder is continuous and the other end simply supported, which condition often exists in the end spans of a series of continuous beams, the shears at the ends of the exterior beam will not be equal, as may be seen by referring to Fig. 10.

The vertical shears for simply supported members UNSYMMETRICALLY LOADED are found by taking moments about either support and computing the reactions. The vertical shear at any section is then equal to the sum of the reactions minus the sum of the loads to the right or left of the section.

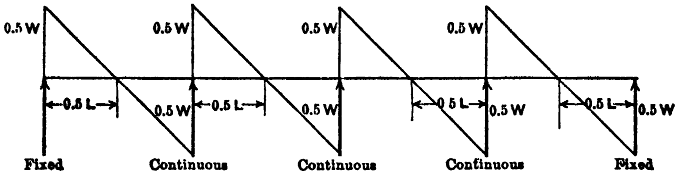


Diagram 1. Shear-Diagram for a Four-Span, Uniformly Loaded Continuous Beam, with Fixed Ends.

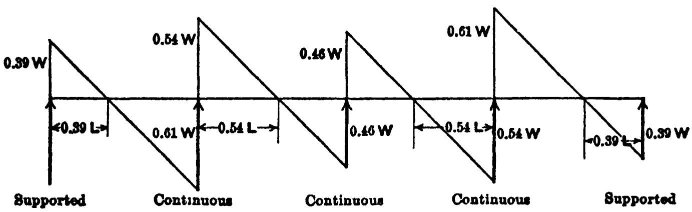


Diagram 2. Shear-Diagram for a Four-Span, Uniformly Loaded Continuous Beam, with Simply-Supported Ends

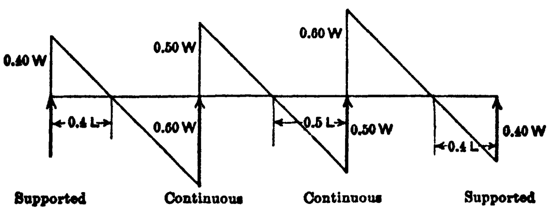


Diagram 3. Shear-Diagram for a Three-Span, Uniformly Loaded Continuous Beam, with Simply-Supported Ends.

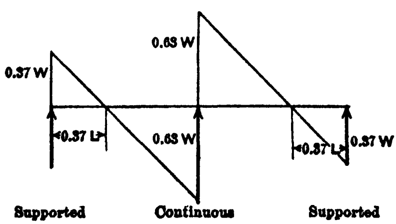


Diagram 4. Shear-Diagram for a Two-Span. Uniformly Loaded Semi-Continuous Beam, with Simply-Supported Ends.

Fig. 10. Theoretical Shear Diagrams, All for Uniform Loads and Equal Spans

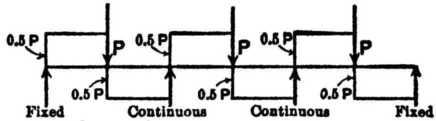


Diagram 5 Shear-Diagram for a Three-Span, Continuous Beam with Fixed Ends, and with a Single Concentrated Load P at the Middle of Each Span

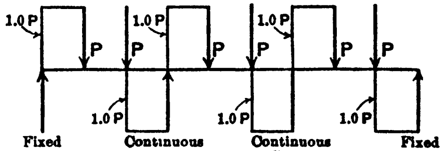


Diagram 6. Shear-Diagram for a Three-Span, Continuous Beam with Fixed Ends, and with a Concentrated Load P at Each Third Point of Each Span

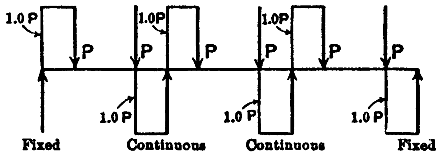


Diagram 7. Shear-Diagram for a Three-Span, Continuous Beam with Fixed Ends, and with a Concentrated Load P at Each Quarter Point of Each Span

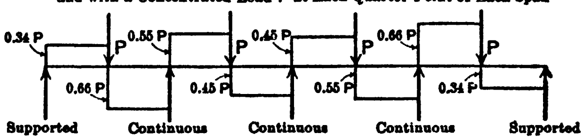


Diagram 8. Shear-Diagram for a Four-Span, Continuous Beam with Simply-Supported Ends, and with a Single Concentrated Load P at the Middle of Each Span

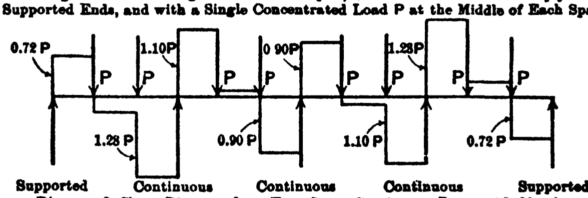


Diagram 9. Shear-Diagram for a Four-Span, Continuous Beam with Simply-Supported Ends, and with a Concentrated Load P at Each Third-Point of Each Span

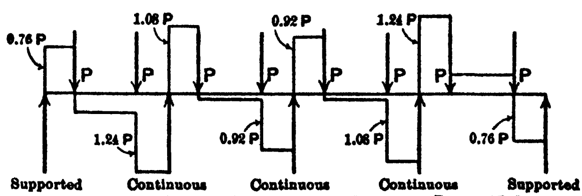


Diagram 10. Shear-Diagram for a Four-Span, Continuous Beam with Simply-Supported Ends, and with a Concentrated Load P at Each Quarter-Point of Each Span

Fig. 11. Theoretical Shear Diagrams, All for Concentrated Loads and Equal Spans

The VERTICAL SHEARS on CONTINUOUS and RESTRAINED MEMBERS supporting heavy, unsymmetrical loads are determined after the negative bending moments at the supports are found by the Theorem of Three Moments.

The Design of Rectangular Beams

Choice and Distribution of Reinforcement in Beams and Girders. The NUMBER and SIZE of rods or bars are selected to give the required sectional area as closely as possible. There is no particular choice between round and square steel, but the design is somewhat neater if the two are not mixed on the same portion of the work. The choice between PLAIN ROUNDS and SQUARES, or DEFORMED RODS or BARS, depends upon the bond stresses used in the design. If the higher values are desired, deformed bars should be used, otherwise plain steel is just as satisfactory. When designing a series of uni-

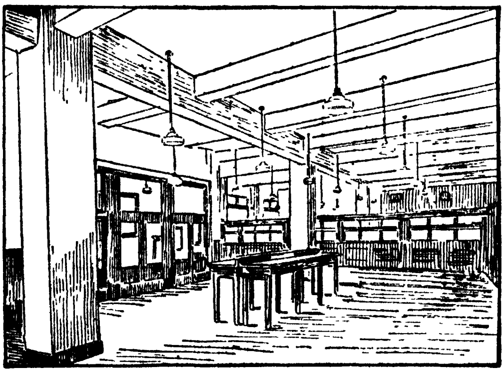


Fig. 12. Typical Beam-and-Slab Construction

formly loaded, continuous beams of equal, or nearly equal spans, it is desirable to choose an EVEN NUMBER OF RODS for the reason that present-day practice generally requires, under such conditions, equal positive and negative design moments. This choice permits raising 50% of the reinforcement at the FIFTH POINT OF THE CLEAR SPAN on all series of uniformly loaded beams where adjacent members have approximately equal spans and equal loads.

The MAIN LONGITUDINAL REINFORCEMENT should be raised at an angle varying between 30° and 45° to the horizontal (Fig. 13), carried over supports on continuous work a distance equal to $\frac{1}{5}$ of the adjacent span, or properly anchored into the supporting member on non-continuous spans.

When designing RECTANGULAR BEAMS and slabs it is usually desirable to employ the exact amount of steel that will "balance" the required concrete; this usually results in a sectional area of about $\frac{7}{10}$ to $\frac{8}{10}$% computed on the breadth of the beam times its effective depth. In the design of T BEAMS, however, there is usually an excess of concrete, and the percentage represented by the area of the longitudinal steel, divided by the sectional area of the concrete, has no particular significance.

The same general principles apply when selecting the reinforcement for GIRDERS SUPPORTING SYMMETRICAL LOADS, or others subjected to unsymmetrical concentrations. In order to locate the point at which a portion of

the longitudinal reinforcement may be raised, as the steel approaches a support, it is necessary to construct the bending-moment curve and to determine the area of the steel required at the various sections. In general, it is desirable to carry reinforcing bars about 6 in beyond the section at which they are needed. For instance, in a girder supporting beams framing in at the third points of the girder span, 50% of the main longitudinal steel is raised

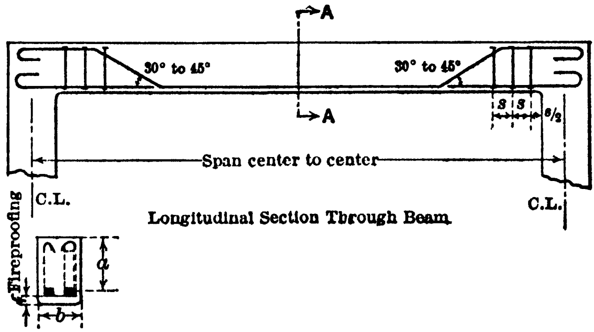


Fig 13. Diagram of a Single Span, Rectangular Beam

about 6 in from the face of the beam, on the side toward the adjacent support. (See Fig. 14)

Spacing and Protection of Reinforcement in Beams and Girders. There should be a CLEAR DISTANCE BETWEEN RODS lying in the bottoms of beams, in the same horizontal plane, equal to $1\frac{1}{2}$ times the diameter of a round rod or $1\frac{1}{2}$ times the diagonal of a square bar, unless the steel is thoroughly anchored. Satisfactory anchorage may be obtained by means of hooking

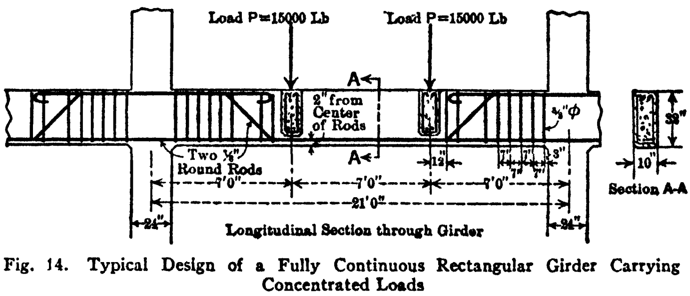


Fig. 14. Typical Design of a Fully Continuous Rectangular Girder Carrying Concentrated Loads

the bars into supporting members at beam terminations, or, for continuous beams, by carrying the negative reinforcement over the tops of supports and into the adjacent beams a distance of at least ⅕ of the span. If such provision is made, the clear spacing between rods may be taken as the diameter of a round rod or as the diagonal of a square bar, but never less than 1 in or $1\frac{1}{4}$ times the maximum size of the coarse aggregate. (See Fig. 15.)

Although it is occasionally necessary to put in two or more layers of steel,

particularly in large girders carrying heavy loads, it is usually more economical to slightly widen a beam, thereby permitting all of the main tensile reinforcement to lie in the same plane. If more than one layer is used the CLEAR VERTICAL DISTANCE between two layers of rods, or bars, should be taken at a minimum of 1 in or 1¼ times the maximum size of the coarse aggregate.

The THICKNESS OF THE PROTECTIVE CONCRETE required around the reinforcement depends upon the type of exposure. The fire risk normally demands 1½ in between the face of the concrete and the face of the steel

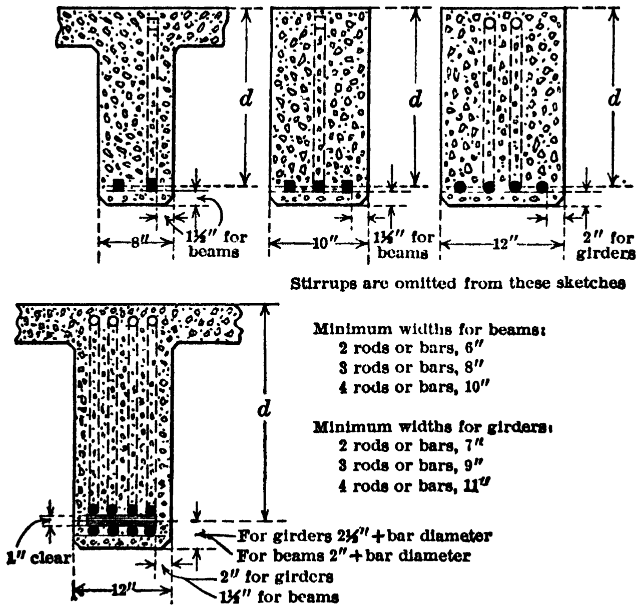


Fig. 15. Typical Beam and Girder Sections

on beams and walls; this distance may be reduced to 1 in, or even ¾ in, for slabs and should be increased to 2 in for columns. Under ground, all reinforcement should be surrounded by at least 3 in and preferably 4 in of concrete. On EXTERIOR WORK it is recommended that a 2-in minimum be employed wherever the size of the unit or member permits such a thickness of protective covering. One in of good concrete should be adequate, but there is always the danger of inferior workmanship resulting in permeability. The slightest moisture entering as far as the reinforcement will cause rusting and rapid disruption of the surface.

Design Procedure for Rectangular Beams and Girders. (1) Determine the width, b, of the beam or girder large enough to accommodate the probable reinforcement, including the necessary fire protection, as stated in the preceding paragraph. Compute the weight of the beam on a basis of the assumed width and a total depth estimated as equal to 1¼ in for each foot

of clear span and 2 in for fireproofing. The clear distance between supports should not exceed 32 times the width of the compression flange.

(2) Determine the maximum bending moments at the supports for continuous or restrained beams and near the mid-spans for those simply supported, as described under Bending Moments. Compute the depth, d, by the formula,

$$d = \sqrt{\frac{M}{Kb}}$$

in which d = effective depth of beam; distance from compression face to center of tensile reinforcement, in inches;
M = the moment in inch pounds;
K = a factor corresponding to the stresses employed and obtained from Tables VI and VII;
b = the width of the beam in inches as assumed in (1).

(3) Compute the area of the main tensile reinforcement by the formulas,

$$A_s = pbd \quad \text{or} \quad A_s = \frac{M}{f_s jd}$$

in which A_s = steel area in square inches;
p = the balancing percentage of steel obtained from Tables VI and VII;
M, b, and d are as noted above;
f_s = steel stress in pounds per square inch;
j = ratio of arm of resisting couple to d, obtained from Tables VI and VII.

(4) Determine the maximum shear, V, as described under Shear and Diagonal Tension. This is equal to one-half the load, W, for uniformly or symmetrically loaded members, except as influenced by the conditions of end support. In the case of unsymmetrical loads, the shears are determined by finding the reactions. Compute the value of the maximum unit shearing-stress by the formula,

$$v = \frac{V}{jbd}$$

in which v = unit shearing-stress in pounds at the section where the shear (V) is computed;
V = total vertical shear at the section considered, in pounds;
j = 0.875 (an approximation used for shear-computations);
b and d are as noted above.

If this value is more than that ordinarily allowed for concrete without web reinforcement (40 to 60 lb per sq in), stirrups or bent bars are required.

(5) Design the web-reinforcement, determining, first, the distance from the support through which either stirrups or bent bars are required. For uniformly loaded beams this distance, x, is given by the formula,

$$x = \frac{L}{2} \times \left(1 - \frac{v'}{v}\right)$$

in which L = span of beam in feet;
v' = allowable unit shearing-stress on plain concrete, usually 40 lb per sq in;
v = actual unit shearing-stress as found under (4).

For girders, or beams with concentrated loads, v, the unit shearing-stress, should be computed at various sections and web-reinforcement used where its value exceeds that permitted for plain concrete. Assume a diameter for the stirrup steel which should not exceed $\frac{1}{50}$ of the beam depth. This usually results in a choice of $\frac{3}{8}$-in round rods for ordinary work and $\frac{1}{2}$ in round rods for very heavy girders and footing beams. The stirrup spacing is then found by the formula,

$$s = \frac{f_s j d A_s}{(V - v'bjd)}$$

in which s = spacing of stirrups in inches;

A_s = sectional area of the stirrup steel; for instance, if a $\frac{3}{8}$-in round rod is used, the area of which is equal to 0.11 sq in, A_s for a U-shaped stirrup would be 0.22, and for a W-shaped stirrup 0.44.

The other factors are as noted above.

Having determined the minimum spacing, which occurs adjacent to the support, it is customary to place the first stirrup at a distance of $\frac{s}{2}$ from the face of the support. If 50% of the main longitudinal reinforcement is raised at the fifth point of the clear span, the bent rod or rods may be considered to act as web-reinforcement through a distance along the beam equal to $\frac{3}{4}$ the depth d. Inside of this section, toward the support, the stirrups should be spaced approximately s distance apart. If the formula indicates that additional stirrups are needed beyond the section covered by the bent rods, one or two stirrups may be placed at a spacing not to exceed $\frac{3}{4}$ the depth of the beam. It is not customary to place stirrups through the central portions of uniformly loaded beams nor elsewhere in sections where the unit shearing-stress is under the limit allowed for v (usually 40 or 60 lb per sq in).

(6) Check the bond stress by the formula,

$$u = \frac{V}{\Sigma o j d}$$

in which u = unit bond stress in pounds per square inch;

V = total vertical shear in pounds at the support or other section under investigation;

Σo = sum of the perimeters of all rods in the tensile side of the beam at the section considered;

j and d are is noted above.

The value of u is generally limited to 80 lb per sq in for plain steel and 100 lb per sq in for deformed rods or bars. If the bond stress is over the allowable limit, reduce the size of the rods, thereby increasing their number, or deepen the beam.

(7) Specify the length of laps, location of hooks for anchorage and the position of bends.

Typical Design of a Rectangular Beam, Fig. 16

Specification data: f_s = 16 000 lb per sq in

f_c = 650 lb per sq in (over supports increased 15%)

v' = limited to 40 lb per sq in

v = limited to 150 lb per sq in

u = limited to 80 lb per sq in (plain rounds or squares)

n = 15

Uniformly loaded beam with ends fully continuous.

Span = 20 ft between faces of supports
Load = 25 000 lb (exclusive of the weight of the beam)

(1) Assume $b = 10$ in. Estimating the effective depth, d, as $1\frac{1}{4}$ in for each foot of span, and adding 2 in for fireproofing below the center of the reinforcement ($1\frac{1}{2}$ in protection) the total depth is 27 in and the total uniformly distributed load is

$$W = 25\,000 + \frac{27 \times 10}{144} \times 20 \times 150 = 30\,625 \text{ lb}$$

in which 150 is the weight, in pounds, of 1 cu ft of concrete.

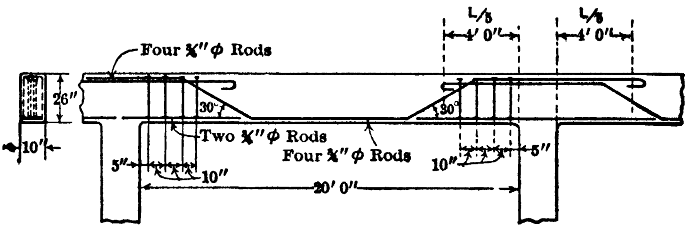


Fig. 16. Typical Design of a Fully-Continuous Rectangular Beam

(2) By formula

$$M = WL/12 = (30\,625 \times 20 \times 12)/12 = 612\,500 \text{ in-lb}$$

By formula

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{612\,500}{107.6 \times 10}} = 23.8 \text{ in}$$

$23.8 + 0.5 + 1.5 = 25.8$ Accept a value of 26 in for the total depth.

(3) By formula

$$A_s = pbd = 0.0077 \times 10 \times 23.8 = 1.83 \text{ sq in}$$

Accept 4 ¾-in rounds ($A_s = 1.76$ sq in). Two of these rods are raised at the fifth-points of the clear span at an angle of 30° to the horizontal.

(4) The maximum shear

$$V = 30\,625/2 = 15\,312 \text{ lb}$$

By formula

$$v = V/jbd = 15\,312/0.875 \times 10 \times 24 = 73 \text{ lb per sq in}$$

(5) By formula

$$x = \frac{L}{2} \times \left(1 - \frac{v'}{v}\right) = 10 \times \left(1 - \frac{40}{73}\right) = 4.52 \text{ ft} \quad \text{or} \quad 4 \text{ ft } 6 \text{ in}$$

as the distance out from the face of the supports to the section where the shear can be carried by the concrete alone and web-reinforcement is unnecessary.

By formula

$$s = \frac{f_s jdA_s}{V - v'bjd} = \frac{16\,000 \times 0.875 \times 24 \times 0.22}{15\,312 - (40 \times 10 \times 0.875 \times 24)} = 10.7 \text{ in}$$

The first stirrup is placed at a distance of 5 in ($\frac{1}{2}s$) from the face of the support. Three more stirrups, placed 10 in apart, provide web-reinforcement out to the section where the bent rods are effective. Beyond this section, as shown by the equation above, none is needed.

(6) By formula, the bond on the negative reinforcement at the face of the support,

$$u = \frac{V}{\Sigma o j d} = \frac{15\,312}{(4 \times 2.36) \times 0.875 \times 24} = 77 \text{ lb per sq in}$$

in which 2.36 is the perimeter of the $\frac{3}{4}$-in rods (see Table VIII), there being two raised from the beam under design, and two others from the adjacent beam with which it is continuous. It is not necessary to use deformed rods.

(7) The straight bars are carried to the center lines of columns and the double-bend rods are lapped to the fifth-points of the adjacent spans. It is generally desirable to hook the terminations of the bent bars. Hooks are not usually required at the terminations of the straight bars of continuous beams.

The Design of Slabs

Load Distribution on Slabs. Concrete slabs, forming a structural floor between beams or bearing partitions, are usually comparatively narrow. Their length, depending upon the arrangement of the supporting members and the building dimensions, is usually two or three times their width. Under such conditions, the reinforcement running across the shorter dimension of the slab should be designed to carry the entire load, and so-called TEMPERATURE STEEL, comprising $\frac{3}{8}$-in rounds spaced at 1 ft 6 in on centers, is considered adequate in the longer direction. In some types of buildings, however, it is practicable to use panels that are square, or nearly so, and then it is necessary to determine, by reference to Fig. 7, the PROPORTION OF THE LOAD per sq ft that may be assumed to be distributed IN EACH DIRECTION. Building ordinances vary in their requirements, but all demand definite proportionment for panels where the longer dimension is equal to, or less than, $1\frac{1}{2}$ times the short dimension. Where this condition occurs, computations are made for each direction: the larger slab thickness is used, and the reinforcements designed to resist the bending moments in each direction.

On SQUARE PANELS, with supports of equal rigidity, the live and dead loads are equally distributed in both directions and the reinforcements are the same each way except as affected by the fact that one set of rods must of necessity lie on top of the other, thereby requiring a slightly greater sectional area to give the same resisting moment, owing to the reduced depth. This matter is practically negligible except in very thin slabs where the thickness of a rod, assumed as $\frac{1}{2}$ in, makes an appreciable difference.

The design procedure is substantially the same for slabs with either ONE-WAY OR TWO-WAY REINFORCEMENT except that there is no proportionment of load in the former case and computations are made for only one direction. Knowing the load per sq ft, the total load on a strip of slab 1 ft wide, extending from support to support, is obtained by multiplying the unit load by the span. This strip is then treated as a rectangular beam; the breadth, b, being 12 in. The bending moments in one or two directions, depth of slab, and steel areas are found as explained in the following procedure.

Design Procedure for Concrete Slabs. (1) Assume a slab thickness equal to approximately $\frac{1}{2}$ in for each foot of clear span. Using this thickness to determine the dead load, add the weight of floor finish and live load, obtaining a total load per sq ft of slab area.

(2) Determine the proportion of the load carried in each direction by reference to the chart, Fig. 7. Using the percentages taken from the chart, determine the total load on a strip 1 ft wide, for both the long and short spans.

(3) Compute the maximum bending moments, on a strip 1 ft wide, for each span by using the same formulas as applied to beams.

(4) Using the larger of the two bending moments, or the single moment found in the case of one-way reinforcement, determine the depth d by the formula

$$d = \sqrt{\frac{M}{Kb}}$$

in which d = effective depth of slab, distance from compression face to center of tensile reinforcement, in inches;
M = bending moment in each direction in inch-pounds;
K = factor obtained from Tables VI and VII;
b = the width of the strip of slab taken for purposes of design; 12 in.

The total thickness of the slab will be this depth plus half the diameter of the reinforcing rods, plus the allowance for fireproofing.

(5) Apply the same formula, using the smaller moment for the longer span. The thickness required will be less than in the more heavily loaded span but, owing to the fact that the rods of the longer span must be on top of those in the short direction, it is necessary to add an extra half inch to the depth; this additional amount may, in the case of lightly loaded spans, give a total depth greater than that required by the heavier moment on the shorter span.

(6) Compute the area of the reinforcement in both directions by the formulas

$$A_s = \frac{M}{f_s j d} \quad \text{or} \quad A_s = pbd$$

in which A_s = sectional-area of tensile reinforcement for a slab width of 1 ft;
f_s = steel stress in pounds per square inch;
j = ratio of arm of resisting couple to d, obtained from Tables VI and VII;
d = depth from the top of the slab to the center of the steel as determined for each direction in (4) and (5);
p = the balancing percentage of steel obtained from Tables VI and VII;
M and b are as noted above.

(7) Determine the maximum vertical shear adjacent to the support which, in the case of uniformly loaded floors, is equal to one-half of the load carried in each direction as found in (2). Compute the value of the unit shearing-stress at the support by the formula

$$v = \frac{V}{jbd}$$

in which v = unit shearing-stress in pounds at the section where the shear (V) is computed;
V = total vertical shear at the section considered, in pounds;
j = 0.875 (an approximation used for shear-computations);
b and d are as noted above.

As it is not practicable to use web-reinforcement in slabs, the value of v should not exceed 40 or 60 lb per sq in. A higher shearing-stress, which, however, seldom occurs except in slabs of short span very heavily loaded, should be reduced by deepening the slab.

(8) Check the bond stresses by the formula

$$u = \frac{V}{\Sigma o j d}$$

in which u = unit bond stress;

V = total vertical shear in pounds at the support or other section under investigation;

Σ_o = sum of the perimeters of all rods in the tensile side of the slab for a 1-ft width at the section considered;

j and d are as noted above.

The value of u is generally limited to 80 lb per sq in for plain steel and 100 lb per sq in for deformed rods or bars. If the bond stress is over the allowable limit, reduce the size of the bars, thereby increasing their number

Table VIII. Sizes, Spacing, and Sectional Areas of Bars and Rods in Reinforced-Concrete Slabs

Sectional area of steel bars per lin ft of slab

Size of bars, in	Perimeter of bars, in	Weight per lin ft, lb	Spacing of SQUARE bars in slabs, in in											
			3 in	3½ in	4 in	4½ in	5 in	5½ in	6 in	7 in	8 in	9 in	10 in	12 in
½	2 00	0 850	1.00	0 86	0 75	0 67	0 60	0 55	0 50	0 43	0 37	0 33	0 30	0 25
1	4 00	3 400	4 00	3 43	3 00	2 67	2 40	2 18	2 00	1 71	1.50	1 33	1 20	1 00
1⅛	4 50	4 303	5 06	4.34	3.80	3 37	3 04	2.76	2 53	2.17	1 89	1.69	1 52	1 27
1¼	5.00	5 313	6 25	5 36	4.69	4 17	3 75	3 41	3 12	2 68	2.34	2 08	1 87	1.56

Sectional area of steel rods per lin ft of slab

Size of rods, in	Perimeter of bars, in	Weight per lin ft, lb	Spacing of ROUND rods in slabs, in in											
			3 in	3½ in	4 in	4½ in	5 in	5½ in	6 in	7 in	8 in	9 in	11 in	12 in
¼	0.785	0 167	0 20	0 17	0 15	0.13	0.12	0 11	0 10	0.08	0.07	0.07	0 06	0 05
⅜	1.18	0.376	0 44	0 38	0 33	0 30	0 26	0 24	0 22	0 19	0 17	0 15	0 13	0 11
½	1.57	0.668	0.78	0.67	0.59	0.52	0 47	0.43	0 39	0 34	0 29	0 26	0 24	0 20
⅝	1.96	1.043	1.23	1 05	0.92	0.82	0.74	0.67	0 61	0 53	0 46	0.41	0.37	0.31
¾	2.36	1 502	1 77	1.51	1.32	1.18	1 06	0.96	0.88	0 76	0 66	0.59	0 53	0.44
⅞	2.75	2.044	2 40	2.06	1 80	1 60	1.44	1 31	1.20	1 03	0.90	0 80	0.72	0 60
1	3.14	2.670	3 14	2.69	2 36	2 09	1 88	1 71	1.57	1.35	1.18	1.05	0.94	0 78

Typical Design of a Concrete Slab: One-Way Reinforcement, Fig. 17

Specification data: f_s = 18 000 lb per sq in

f_c = 650 lb per sq in

v' = limited to 40 lb per sq in

n = 15

Panel width between faces of supports, 8 ft 0 in

Live lo'.d, 100 lb per sq ft; allow 25 lb per sq ft for floor fill, finish-flooring and plaster

End-conditions: Semi-continuous

(1) Assume a slab thickness of $4\frac{1}{2}$ in

Then,	Live load	= 100 lb
	Floor finish, etc....	= 25 lb
	Weight of slab.......... .	= 54 lb
	Total load...............	= 179 lb per sq ft

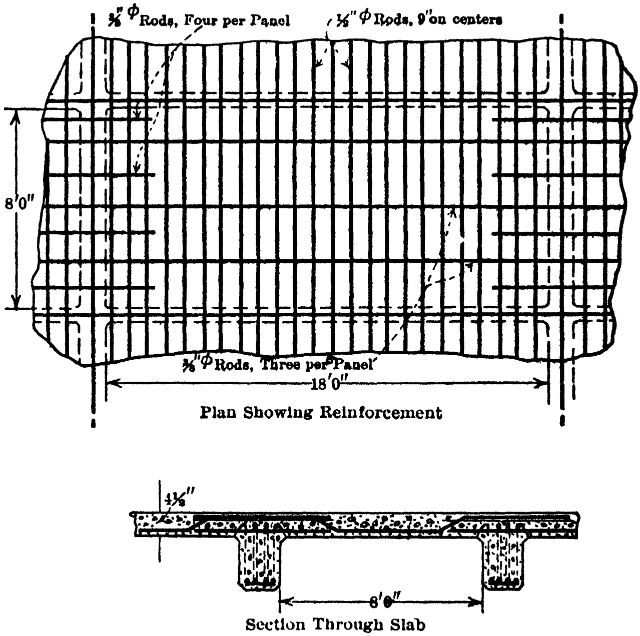


Fig. 17. Concrete Slab with One-Way Reinforcement

(2) On a strip of slab 1 ft wide the total load is

$$179 \times 8 = 1\,432 \text{ lb}$$

(3) By formula

$$M = \frac{WL}{10} = \frac{1\,432 \times 8 \times 12}{10} = 13\,472 \text{ in-lb}$$

(4) By formula

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{13\,472}{100.8 \times 12}} = 3.34 \text{ in}$$

Total thickness of slab = 3.34 + 0,25 + 1.00 = 4.59 in, in which 0.25

is the approximate half diameter of the reinforcing rods, and 1 in the allowance for fireproofing. Accept a total slab-thickness of 4½ in.

(5) This step does not apply to one-way slabs.

(6) By formula

$$A_s = pbd = .0063 \times 12 \times 3.34 = 0.25 \text{ sq in}$$

Accept ½-in rounds 9 in on centers.

(7) The maximum shear

$$V = 1\,432/2 = 716$$

By formula

$$v = \frac{V}{jbd} = \frac{716}{0.875 \times 12 \times 3.25} = 20 \text{ lb per sq in}$$

The shearing-stress is seldom critical and this step may be omitted except for short, heavily loaded spans.

(8) By formula

$$u = \frac{V}{\Sigma ojd} = \frac{716}{(1.33 \times 1.57) \times 0.875 \times 3.25} = 120 \text{ lb per sq in}$$

in which 716 is one-half the total load on the 1-ft strip under design; 1.33 the number of rods in the bottom of the slab, per ft of width (9 in on centers); 1.57 the perimeter of the cross-section of a ½-in round rod in inches; 0.875 the value of j used in shear and bond computations; and 3.25 the actual depth, d, of the slab in inches, as designed. This bond stress is high but can be accepted for deformed rods if the reinforcement is anchored at terminations and lapped over supports. The use of ⅜-in rounds at 5-in centers gives the same sectional area (Table VIII) and reduces the bond stress to 88 lb per sq in.

Typical Design of a Concrete Slab: Two-Way Reinforcement, Fig. 18

Specification data: f_s = 18 000 lb per sq in
f_c = 650 lb per sq in
v' = limited to 40 lb per sq in
n = 15

Panel dimensions between faces of supports, 16 ft 0 in × 18 ft 0 in

Live load, 60 lb per sq ft; allow 25 lb per sq ft for floor fill, finish-flooring and plaster

End-conditions: Short span fully continuous; long span semi-continuous

(1) Assume a slab thickness of 6½ in, then

Live load..............................	= 75 lb
Floor finish, etc.........................	= 25 lb
Weight of slab.........................	= 78 lb
Total load.............................	= 178 lb per sq ft

(2) From chart (Fig. 7), using the straight line distribution, 62% is carried in the short direction and 38% in the long direction.

On a strip of slab 1 ft wide the total load is

Short span, 178 × 0.62 × 16 = 1 765 lb
Long span, 178 × 0.38 × 18 = 1 224 lb

(3) Maximum bending moment on short span

$$M = \frac{WL}{12} = \frac{1\,765 \times 16 \times 12}{12} = 28\,240 \text{ in-lb}$$

Maximum bending moment on long span

$$M = \frac{WL}{10} = \frac{1\,224 \times 18 \times 12}{10} = 26\,438 \text{ in-lb}$$

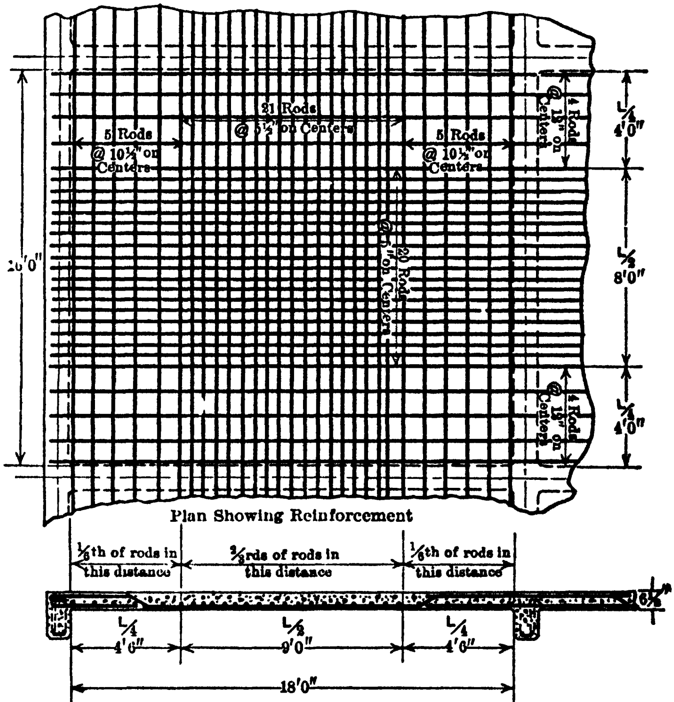


Fig. 18. Concrete Slab with Two-Way Reinforcement

(4) Depth by short-span moment

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{28\,240}{100.8 \times 12}} = 4.84 \text{ in}$$

Total thickness of slab = 4.84 + 0.25 + 1.00 = 6.09 in, in which 0.25 is the approximate half diameter of the reinforcing rods and 1 in the allowance for fireproofing.

(5) Depth by long-span moment

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{26\,438}{100.8 \times 12}} = 4.67 \text{ in}$$

Total thickness of slab = 4.67 + 0.25 + 1.50 = 6.42 in, in which the distance below the rods is increased from 1 in, to 1½ in, as the two layers of rods cannot lie in the same plane.

Accept the larger of these values, giving a slab-thickness of 6½ in.

(6) By formula

$$\text{Short direction}\quad A_s = \frac{M}{f_s jd} = \frac{28\,240}{18\,000 \times 0.883 \times 5.25} = 0\ 34 \text{ sq in}$$

$$\text{Long direction}\quad A_s = \frac{M}{f_s jd} = \frac{26\,438}{18\,000 \times 0.883 \times 4.75} = 0.35 \text{ sq in}$$

Accept ½-in rounds 7 in on centers both ways.

Note that the formula $A_s = pbd$ could be used to determine the steel area, but in this case the depth required by moment (see (4) and (5)) has been slightly increased to give an even half inch (6½ in) and there is no object in supplying steel to balance the additional concrete.

(7) The computation for shear is omitted as unnecessary.

(8) By formula

Bond stress in short direction

$$u = \frac{V}{\Sigma ojd} = \frac{1\,765/2}{(1.7 \times 1.57) \times 0.875 \times 5.25} = 72 \text{ lb per sq in}$$

Bond stress in long direction

$$u = \frac{V}{\Sigma ojd} = \frac{1\,224/2}{(1.7 \times 1.57) \times 0.875 \times 4.75} = 55 \text{ lb per sq in}$$

in which V is one-half the load carried in each direction on a 1-ft strip; 1.7 the number of rods in the slab, in each direction, per ft of width (7 in on centers); 1 57 the perimeter of the cross-section of a ½-in round rod in inches; 0.875 the value of j used in shear and bond computations; 5.25 and 4.75 are the effective depths, d, of the slab in each direction, in inches.

This bond stress is acceptable for plain rods. All reinforcement should be anchored at terminations and lapped over supports.

This design is made in accordance with the requirements of the more severe building ordinances, employing the full computed amount of reinforcement without any reductions by reason of flat-plate action. The distribution of the reinforcement is shown in Fig. 18. Under Bending Moments in Slabs will be found the moment reductions recommended by the author for two-way slabs.

Distribution of Reinforcement in Slabs. In order to provide for the negative bending moments over supports in continuous slabs, a portion of the main reinforcement is raised at about the fifth point of the clear span and carried over the supporting beam, or wall, at a height of one inch below the top of the slab. In the case of SLIGHTLY RESTRAINED ENDS, it is good practice to raise from one-third to one-half of the steel adjacent to the supports; this occurs where concrete slabs are carried by spandrel beams. Where the slabs are FULLY CONTINUOUS, however, it is desirable to raise alternate rods from each span, thereby giving the same sectional area over supports as at mid-spans.

Three times the effective depth of the slab is usually taken as the MAXIMUM SPACING FOR CONCRETE REINFORCEMENT. Where a small unframed opening occurs in a slab, the rods should be spread past the sides and in no case terminate against the face of an opening.

Fig. 19 shows various methods of arranging slab reinforcement. Some building codes permit a reduction in the steel placed in the outer quarters of panels reinforced in two directions. The Joint Code recommends, on two-way slabs, that the bending moments and resulting steel areas be reduced by 50% in the outer quarters of the panel.

When designing one-way slabs it is necessary to provide shrinkage and temperature reinforcement at right-angles to the principal reinforcement. The Joint Code recommends that:

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement shall be provided in floor and roof slabs where the principal reinforcement extends in one direction only. Such reinforcement shall provide for the following minimum ratios of reinforcement area to

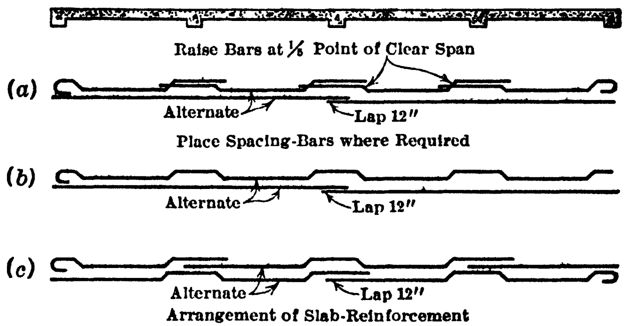


Fig. 19. Arrangement and Bending of Slab Steel

concrete area, but in no case shall such reinforcing bars be placed farther apart than five times the slab thickness nor more than 18 in:

Floor slabs where plain bars are used.	0.0025
Floor slabs where deformed bars are used	0.002
Floor slabs where wire fabric is used, having welded intersections not farther apart in the direction of stress than 12 in	0.0018
Roof slabs where plain bars are used	0.003
Roof slabs where deformed bars are used	0.0025
Roof slabs where wire fabric is used, having welded intersections not farther apart in the direction of stress than 12 in	0.0022

The thickness of PROTECTIVE COVERING of concrete required below the reinforcement of slabs is normally 1 in; this should be taken as a clear distance measured from the face of the rod or bar.

Compression Reinforcement in Beams and Girders

Method Used in Design. The use of compression reinforcement is occasionally necessary through the MID-SPANS OF RECTANGULAR BEAMS when architectual considerations require a limited cross-section of concrete. A critical condition also exists OVER THE SUPPORTS OF CONTINUOUS BEAMS OF T SECTION where the lower portion of the member is subjected to compressive stresses due to the action of the negative moment. Since most codes permit

the use of a higher unit compressive stress over the supports than at the mid-span, it is usually possible to obtain sufficient compressive strength by carrying the straight rods of each beam far enough beyond the face of the support to develop by bond the compression value required of the steel, as an aid to the concrete. When, however, there is any probability of the allowable stress in the concrete being exceeded, the percentage of the main tension steel should be computed and compared with the limiting percentages which the plain concrete will carry for the given values of f_s and f_c and n. (See Table V.) If the actual amount of tensile-steel, as required for the bending moment, exceeds that required for a balanced beam, the sectional area of the concrete must be increased, or compression-steel utilized to resist the excess compression. If it is impracticable to increase the size of the beam, the following method is employed to determine the amounts of tension and compression reinforcement. When designing T beams at supports it should be remembered that the width, b, is that of the web and not the flange.

Let f_s = tensile unit stress in steel in pounds per square inch;
f'_s = compressive unit stress in the steel in pounds per square inch;
f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch;
b = width of the web of the beam in inches;
d = effective depth of beam, distance from compressive face to center of tensile reinforcement, in inches;
j and k are factors taken from Tables VI and VII corresponding to the stresses used;
d' = distance from compression-face of beam to center of compression-steel in inches;
A_s = cross-sectional area of main tension-steel in square inches;
$p = A_s/bd$ = ratio of cross-section of tension-steel to cross-section of beam.
A'_s = cross-sectional area of compression-steel in square inches;
$p' = A'_s/bd$ = ratio of cross-section of compression-steel to cross-section of beam;
p_1 = steel-ratio for a balanced beam;
p_2 = steel-ratio of the added tension-steel;
$p = p_1 + p_2$;
M_1 = resisting moment of the beam without compression steel in inch-pounds;
M_2 = resisting moment of the compression-steel and added tension-steel in inch-pounds;
$M = M_1 + M_2$.

Design Procedure for Computing Compression Reinforcement in Beams and Girders, Fig. 20. (1) Compute the beam dimensions and the reinforcement as outlined in the procedures given for rectangular or T beams, according to the type of the member under design. Compute the maximum bending moment that the beam can resist without compression reinforcement, by formula

$$M_1 = \tfrac{1}{2} f_c jkbd^2$$

(2) Subtract from the actual bending moment the moment which the beam can support without compression reinforcement. Let M_2 designate the difference between these two moments expressed in inch-pounds. This is the moment which the compression-reinforcement must be designed to resist.

(3) Determine the steel ratio of the additional tension-steel, which it is

necessary to supply in order to balance the additional compression-steel, by the formula

$$p_2 = \frac{M_2}{f_s b d^2 \left(1 - \frac{d'}{d}\right)}$$

The total ratio of tension-steel is then

$$p = p_1 + p_2$$

and the total sectional-area of tension-reinforcement is

$$A_s = pbd.$$

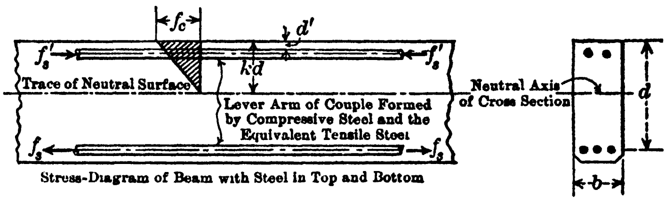


Fig. 20. Stress Diagram of a Concrete Beam Reinforced for Both Tension and Compression

(4) Since the concrete in compression can only balance in tension an amount of steel represented by p_1, compression-reinforcement must be added to take care of the additional tension-steel represented by the percentage p_2. The areas and consequently the percentages are inversely proportional to their distances from the neutral surface of the beam, and the amount of compression-reinforcement is determined by the formulas

$$p' = p_2 \times \frac{1 - k}{k - \frac{d'}{d}}$$

and

$$A'_s = p'bd$$

The values of k, j and p are taken in all cases from Tables VI and VII. It should be noted that, as these depend upon the values of n, f_c and f_s, they will be different over supports than at mid-span as it is customary to allow a 15% increase in the value of f_c at the former sections. For example, if an extreme fiber stress of 650 pounds was used with $n = 15$ and $f_s = 16\,000$, the values of these coefficients, as taken from Table VI, would be: $k = 0.378$, $j = 0.874$, $p_1 = 0.0077$; over supports where $f_c = 750$, $k = 0.414$, $j = 0.862$ and $p_1 = 0.0097$. Provide ties or stirrups, not less than ¼-in diameter, spaced not more than 8 in apart throughout the distance where the compression-steel is required.

Typical Design of Compression Reinforcement for a Rectangular Beam, Fig. 21

Specification data: $f_s = 16\,000$ lb per sq in
$f_c = 650$ lb per sq in
$n = 15$

Uniformly loaded beam with ends fully continuous
Span = 18 ft between faces of supports
Load = 25 000 (exclusive of the weight of the beam)

(1) It is assumed that the size of the beam is controlled by architectural requirements and that b and d are limited to 10 in and 18 in, respectively. The maximum bending moment that this beam can resist without compression-reinforcement is given by formula

$$M_1 = \tfrac{1}{2} f_c jkbd^2 = \tfrac{1}{2} \times 650 \times 0.874 \times 0.378 \times 10 \times (18)^2 = 348\,870 \text{ in-lb}$$

(2) But the actual bending moment is

$$M = WL/12 = \frac{(25\,000 + 3\,750) \times 18 \times 12}{12} = 517\,500 \text{ in-lb}$$

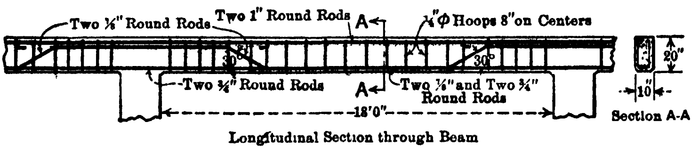


Fig. 21. Typical Design of a Fully Continuous Rectangular Beam, Under a Uniformly Distributed Load and with Compression Reinforcement

in which the weight of the beam is added to the superimposed load. Compression-reinforcement must be provided for

$$M_2 = M - M_1 = 517\,500 - 348\,870 = 168\,630 \text{ in-lb}$$

(3) Placing the compression-steel 2 in below the compression face of the beam, $d'/d = 2/18 = 0.11$, and the steel ratio of the added tension-steel is found by formula

$$p_2 = \frac{M_2}{f_s bd^2 \left(1 - \frac{d'}{d}\right)} = \frac{168\,630}{16\,000 \times 10 \times (18)^2 \times (1 - 0.11)} = 0.0036$$

The total ratio of tension-steel is equal to

$$p = p_1 + p_2 = 0.0077 + 0.0036 = 0.0113$$

and the sectional area of the tension-steel is, by formula,

$$A_s = pbd = 0.0113 \times 10 \times 18 = 2.03 \text{ sq in}$$

Accept 2 ⅞-in and 2 ¾-in round rods (A_s = 2 08 sq in).

(4) The total ratio of compression-steel is found by formula

$$p' = p_2 \times \frac{1 - k}{k - \frac{d'}{d}} = 0.0036 \times \frac{1 - 0.378}{0.378 - 0.11} = 0.0084$$

and the sectional area of the compression-steel is

$$A'_s = p'bd = 0.0084 \times 10 \times 18 = 1.51 \text{ sq in}$$

Accept 2 1-in round rods (A'_s = 1.56 sq in)

Place ¼-in round hoops, 8 in on centers through the central portion of the beam as shown in Fig. 21.

The Design of T Beams and T Girders

Load Distribution on T Beams and T Girders. When designing a floor system comprising slabs, beams and girders, the slabs are first considered. Knowing the unit live and dead load per sq ft, the load carried by a T beam or T girder can then be found by multiplying this unit load by the floor-area which the beam supports and adding to the product any other incidental loads together with the weight of the stem of the beam.

If the slabs have ONE-WAY REINFORCEMENT, their loads being carried directly to the marginal beams on two sides, the floor-area supported by a beam is equal to the product of the length of the beam by the sum of the two half slab spans on either side.

If the slabs have TWO-WAY REINFORCEMENT, the loads carried to the supporting beams are not uniformly distributed, the loading being a maximum at the middle of the span and diminishing toward the ends. The following formulas give the bending moments in foot-pounds for marginal beams supporting concrete panels reinforced in two directions and assumed to be simply supported. Bending moment for the longer beam

$$M = \frac{w_1 b l^2}{12}$$

Bending moment for the shorter beam

$$M = \frac{w_2 l b^2}{12}$$

in which M = bending moment in foot-pounds;
w_1 = load per square foot of slab carried to longer beam;
w_2 = load per square foot of slab carried to shorter beam;
l = longer dimension of slab in feet (length of longer beam);
b = shorter dimension of slab in feet (length of shorter beam).

For SEMI-CONTINUOUS or FULLY CONTINUOUS beams, these results are multiplied by ⅘ and ⅔, respectively.

The LOADS CARRIED BY GIRDERS are usually a combination of concentrated loads due to beam reactions and the uniformly distributed weight of the girder itself, plus a small portion of the floor-load the weight of which is transmitted directly to the girder. This last, however, is usually disregarded, all of the slab weight being considered as supported by the beams and the girder carrying merely its own weight and the concentrated loads from the beams or other sources.

For computing the DEAD LOADS needed in design, the weights of materials given in Table I may be used provided that they are not less than those given in the particular building ordinances governing the design of the work.

If granolithic floor-finish is placed monolithically with the structural slab, its thickness may be included, for design purposes, as part of the slab itself.

Width of Flange for T Beams and T Girders. For purposes of design in computing the sectional-area of concrete available to resist compressive stresses, the WIDTH OF SLAB which may be considered as acting as the flange of a T beam should not exceed ¼ of the span-length of the beam. Neither

should the overhanging width on either side of the web exceed eight times the thickness of the slab, nor ½ the clear distance to the next beam. As the floor slab, forming the flange in T-beam construction, usually supplies an excess of concrete, such provisions seldom govern except in the case of angle beams. For these latter, the overhanging width of flange should be limited to 1/12 the span or ½ the clear distance to the next beam. (See Fig. 22.)

It is always necessary to pour the floor forming the flange of the beam monolithically with the web, and there must be steel in the form of stirrups, or bent rods, to properly bond the flange to the web. This condition is ordinarily met in all beams that are designed for the usual shearing-stresses.

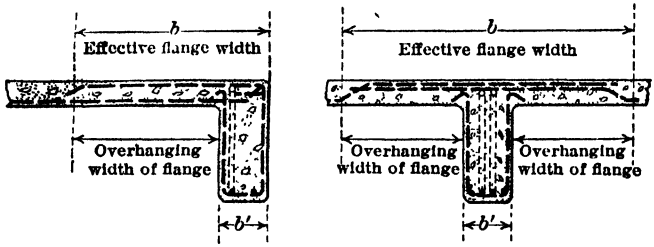


Fig. 22. Diagrams of Typical Beams

Design Procedure for T Beams and T Girders. (1) Determine the width b', of the web of beam or girder large enough to accommodate the probable reinforcement, including the necessary fire protection. (See Thickness of Protective Concrete.) Compute the weight of the beam on a basis of the assumed width and a total depth estimated as equal to 1 in for each foot of clear span and 2 in for fireproofing.

(2) Determine the maximum shear, V, as described under Shear and Diagonal Tension. This is equal to one-half the load, W, for uniformly or symmetrically loaded members, except as influenced by the conditions of end support. In the case of unsymmetrical loads, the shears are determined by finding the reactions.

(3) Compute the depth of the beam or girder by a derivative of the formula

$$d = \frac{V}{vjb'}$$

in which d = effective depth of beam, distance from compression face to center of steel, in inches;
v = allowable unit shearing-stress where web-reinforcement is employed, in pounds per square inch;
V = total shear at the critical section, in pounds;
j = 0.875 (an approximation used for shear-computations);
b' = width of web in inches;

If v = 120 lb per sq in, the formula becomes

$$d = \frac{V}{105\,b'}$$

If v = 150 lb per sq in,

$$d = \frac{V}{131\,b'}$$

(4) Determine the maximum bending moments at the supports for continuous or restrained beams and near the mid-spans for those simply supported, as described under Bending Moments.

(5) Compute the area of the main tensile reinforcement by the formula

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)}$$

in which A_s = steel area in square inches;
M = bending moment in inch-pounds;
f_s = tensile unit stress in steel in pounds per square inch;
d = effective depth of beam; distance from compression face to center of tensile reinforcement in inches;
t = thickness of slab forming flange of T beam, in inches.

(6) For simply supported members, the unit compressive stress in the concrete near the mid-span is checked by formula

$$f_c = \frac{2M}{bt\left(d - \frac{t}{2}\right)}$$

in which f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch, and should not exceed the limit set in the specification;
b = effective flange width;
M, d and t are as noted above.

For continuous and semi-continuous members, the unit compressive stress in the concrete over supports is checked by the method explained under Compression Reinforcement in Beams and Girders.

(7) Design the web-reinforcement, determining, first, the distance from the support through which either stirrups or bent bars are required. For uniformly loaded beams this distance, x, is given by the formula

$$x = \frac{L}{2} \times \left(1 - \frac{v'}{v}\right)$$

in which L = span of beam as defined under Design Moment Coefficients;
v' = allowable unit shearing-stress on plain concrete, usually 40 lb per sq in;
v actual unit shearing-stress as used under (3).

Assume a diameter for the stirrup steel which should not exceed 1/50 of the beam depth. This usually results in a choice of 3/8-in round rods for ordinary work and 1/2-in round rods for very heavy girders and footing beams. The stirrup spacing is then found by the formula

$$s = \frac{f_s j d A_s}{(V - v'b'jd)}$$

in which s = spacing of stirrups in inches;
A_s = sectional area of the stirrup steel; for instance, if a 3/8-in round rod is used, the area of which is equal to 0.11 sq in, A_s for a U-shaped stirrup would be 0.22 and for a W-shaped stirrup 0.44.
The other factors are as noted above.

Having determined the minimum spacing, which occurs adjacent to the support, it is customary to place the first stirrup at a distance of $\frac{s}{2}$ from the face of the support. If 50% of the main longitudinal reinforcement is raised at the fifth point of the clear span, the bent rod or rods may be considered to act as web-reinforcement through a distance along the beam equal to the depth $\frac{3}{4}d$. Inside of this section toward the support, the stirrups should be spaced approximately s distance apart. If the use of the formula indicates that additional stirrups are needed beyond the section covered by the bent rods, one or two stirrups may be placed at a spacing not to exceed the depth of the beam. It is not customary to place stirrups through the central portions of uniformly loaded beams nor elsewhere in sections where the unit shearing-stress is under the limit allowed for v (usually 40 or 60 lb per sq in).

(8) Check the bond stress by the formula

$$u = \frac{V}{\Sigma ojd}$$

in which u = unit bond stress; see under Working Formulas for Bond for limiting values;

V = total vertical shear in pounds at the support or other section under investigation;

Σo = sum of the perimeters of all rods in the tensile side of the beam at the section considered;

j and d are as noted above.

The value of u is generally limited to 80 lb per sq in for plain steel and 100 lb per sq in for deformed rods or bars. If the bond stress is over the allowable limit, reduce the size of the rods, thereby increasing their number, or deepen the beam.

(9) Specify the length of laps, location of hooks for anchorage and the position of bonds. (See Anchorage of Reinforcement)

Typical Design of a T Beam, Fig. 23

Specification data: f_s = 18 000 lb per sq in
f_c = 650 lb per sq in
v' = limited to 40 lb per sq in
v = limited to 120 lb per sq in
u = limited to 100 lb per sq in (deformed bars)
n = 15

Uniformly loaded beam with ends semi-continuous
Span = 20 ft between faces of supports
Load = 40 000 lb (exclusive of the weight of the beam)
Slab-thickness = 4 in

(1) Assume b' = 10 in. Estimating the effective depth, d, as 1 in for each foot of span, and adding 2 in for fireproofing below the center of the reinforcement ($1\frac{1}{2}$-in protection) the total depth is 22 in and the total uniformly distributed load is

$$W = 40\,000 + \frac{22 \times 10}{144} \times 20 \times 150 = 44\,600 \text{ lb}$$

in which 150 is the weight, in pounds, of 1 cu ft of concrete.

(2) The maximum shear

$$V = 44\,600/2 = 22\,300 \text{ lb}$$

(3) By formula, making $v = 120$ lb

$$d = \frac{V}{vjb'} = \frac{22\,300}{105 \times 10} = 21.2 \text{ in}$$

$21.2 + 0.5 + 1.5 = 23.2$. Accept a value of 23 in for the total depth. The estimated weight is sufficiently accurate.

(4) By formula, $M = \dfrac{WL}{10} = \dfrac{44\,600 \times 20 \times 12}{10} = 1\,070\,400$ in-lb

(5) By formula

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} = \frac{1\,070\,400}{18\,000\left(21 - \frac{4}{2}\right)} = 3.13 \text{ sq in}$$

Accept 4 1-in rounds ($A_s = 3.12$ sq in). Raise two rods over supports as shown in Fig. 23.

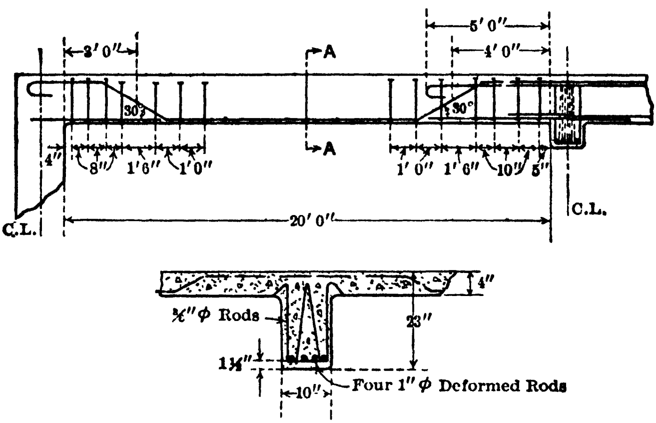


Fig. 23. Typical Design of a Reinforced-Concrete T Beam

(6) As the beam is continuous at one end, the compressive stress in the concrete is checked at the face of the continuous support by the method given under Compression Reinforcement in Beams and Girders. This step is only occasionally required, as the practice of carrying the straight rods to the center lines of supports usually provides enough steel to assist the concrete in resisting the negative moment that causes compression on the lower half of the beam. Without compression-reinforcement the beam, as designed, can resist a negative bending moment at the face of the support of

$$M = \tfrac{1}{2} f_c jkbd^2 = \tfrac{1}{2} \times 750 \times 0.872 \times 0.385 \times 10 \times (21)^2 = 555\,220 \text{ in-lb}$$

in which 0.872 is the value of j and 0.385 the value of k corresponding to a unit compressive-stress of 750 lb per sq in on the concrete (650 + 15% allowed over supports) and 18 000 lb per sq in on the reinforcement. (See Table VI.) Then, the moment for which compression-steel must be provided is

$$M_2 = M - M_1 = 1\,070\,400 - 555\,220 = 515\,180 \text{ in-lb}$$

The additional tension-steel is then found from

$$p_2 = \frac{M_2}{f_s b' d^2 \left(1 - \frac{d'}{d}\right)} = \frac{515\,180}{18\,000 \times 10 \times (21)^2 \left(1 - \frac{2}{21}\right)} = 0.0071$$

The total tension-reinforcement is then checked as

$$p = p_1 + p_2 = 0.0080 + 0.0071 = 0.0151$$

and

$$A_s = pb'd = 0.0151 \times 10 \times 21 = 3.17 \text{ sq in}$$

which value is approximately that already computed in (5).

The total compression-reinforcement is then computed as

$$p' = p_2 \times \frac{1 - k}{k - \frac{d'}{d}} = 0.0071 \times \frac{1 - 0.385}{0.385 - \frac{2}{21}} = 0.014$$

and

$$A_s = p'b'd = 0.014 \times 10 \times 21 = 2.94 \text{ sq in}$$

As this area is practically equivalent to that of the main tension-reinforcement, the straight rods in the bottom of the beam should be lapped past the further face of the support a distance of 20 diameters, so that all four may act as compression-reinforcement at both faces of the supporting column or girder. This necessity seldom occurs except at the first interior support in a series of continuous beams.

(7) By formula

$$x = \frac{L}{2} \times \left(1 - \frac{v'}{v}\right) = 10 \times \left(1 - \frac{40}{120}\right) = 6.66 \quad \text{or} \quad 6 \text{ ft } 8 \text{ in}$$

in which v has the value used in (3). Beyond this distance from the face of each support no web-reinforcement is required.

$$s = \frac{f_s j d A_s}{V - v'b'jd} \; \frac{18\,000 \times 0.875 \times 21 \times 0.44}{22\,300 - (40 \times 10 \times 0.875 \times 21)} = 9.7 \text{ in}$$

in which $A_s = 0.44$ represents the sectional area, in square inches, of a ⅜-in diameter stirrup of the W-type. The first stirrup is placed at a distance of 5 in (½ s) from the face of the support. Three more stirrups, placed 10 in apart, provide web-reinforcement out to the section where the bent rods are effective. Beyond this section, as shown by the illustration, three stirrups are placed at a distance of 12 in apart.

(8) By formula, the bond on the negative reinforcement at the face of the support,

$$u = \frac{V}{\Sigma o j d} = \frac{22\,300}{(4 \times 3.14) \times 0.875 \times 21} = 97 \text{ lb per sq in}$$

in which 3.14 is the perimeter of the 1-in rods (see Table VIII), there being two raised from the beam under design and equivalent reinforcement from the adjacent beam with which it is continuous. Deformed rods are desirable.

(9) At the continuous end, the double-bend rods are lapped to the quarter-point of the adjacent span and hooked. (Lapping to the fifth point is usually adequate, except at first interior supports.) The straight rods are treated as described in (6). At the non-continuous end, the bent rods are anchored

into the supporting member by means of hooks and the straight rods are cut off at the center line of the support.

Typical Design of a T Girder (Fig. 24)

Specification data same as for T beam
Equal, concentrated loads at third-points
Ends fully continuous

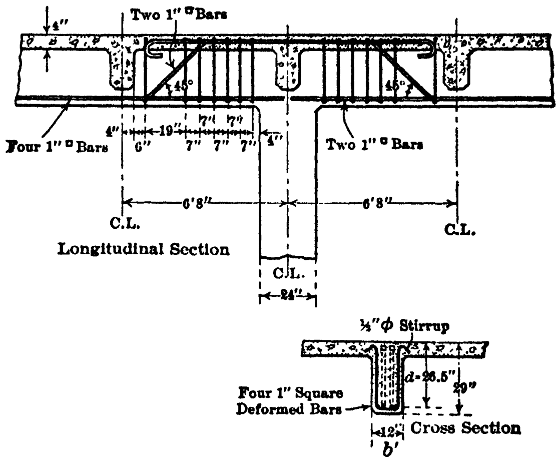


Fig. 24. Typical Design of a Reinforced-Concrete T Girder

Span = 20 ft (center to center); clear span 18 ft
Load at each concentration 30 000 lb (exclusive of the weight of the girder)
Slab-thickness = 4 in
(1) Assume b' = 12 in. Estimating the total depth as 30 in,

$$W = 30\,000 + 30\,000 + \frac{30 \times 12}{144} \times 18 \times 150 = 66\,750 \text{ lb}$$

(2) The maximum shear, $V = 66\,750/2 = 33\,375$ lb
(3) The depth as determined by shear

$$d = \frac{V}{vjb'} = \frac{33\,375}{105 \times 12} = 26.4 \text{ in}$$

26.4 + 2.5 = 28.9. Accept a value of 29 in for the total depth. The estimated weight is sufficiently accurate.

(4) From Diagram 6, Fig. 4, the value of the maximum bending moment (negative) caused by the concentrated loads, is taken as 0.22 PL. The uniform load, due to the weight of the girder, causes an additional bending moment that is equal to $WL/12$, therefore,

$$M = 0.22 \times 30\,000 \times 18 \times 12 = 1\,425\,600 \text{ in-lb}$$
$$M = 6\,750 \times 18 \times 12/12 = 121\,500$$

Total design moment = 1 547 100 in-lb

If this member were semi-continuous, the moment over the first interior support, 0.29 $P\ L$, would be used in place of 0.22 $P\ L$ (see Diagram 9, Fig. 5) and the moment due to the uniform load would be given by the equation $M = WL/10$.

(5) By formula

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} = \frac{1\ 547\ 100}{18\ 000\left(26.5 - \frac{4}{2}\right)} = 3.50$$

Accept 4 1-in square bars ($As = 4.00$ sq in).

Raise two bars over supports as shown in Fig. 24.

(6) Omit this step except for critical sections such as the first interior support in a series of continuous girders; the procedure, in this case, is identical with that given for T beams.

(7) Inspection of Diagram 9, Fig. 11, shows that stirrups are needed only between the ends of the girder and the sections where the concentrations (beams) occur. The spacing is

$$s = \frac{f_s jdA_s}{V - v'b'jd} = \frac{18\ 000 \times 0.875 \times 26.5 \times 0.38}{33\ 375 - (40 \times 12 \times 0.875 \times 26.5)} = 7.1 \text{ in}$$

in which $A_s = 0.38$ represents the sectional area, in square inches, of a ½-in diameter stirrup of U-type The first stirrup is placed at a distance of 4 in from the face of the support and 6 additional stirrups are located as shown in Fig. 24.

(8) By formula, the bond on the negative reinforcement at the face of the support,

$$u = \frac{V}{\Sigma ojd} = \frac{33\ 375}{(4 \times 4) \times 0.875 \times 26.5} = 89 \text{ lb per sq in}$$

in which 4 is the perimeter of the 1-in bars, there being two raised from the girder under design, and equivalent reinforcement from the adjacent girder with which it is continuous. Deformed bars are desirable.

(9) The double-bend bars are raised at an angle of 45° and lapped to the fifth points of the adjacent spans and hooked. The straight bars are cut off at the center lines of the supports.

Contraction and Expansion Joints

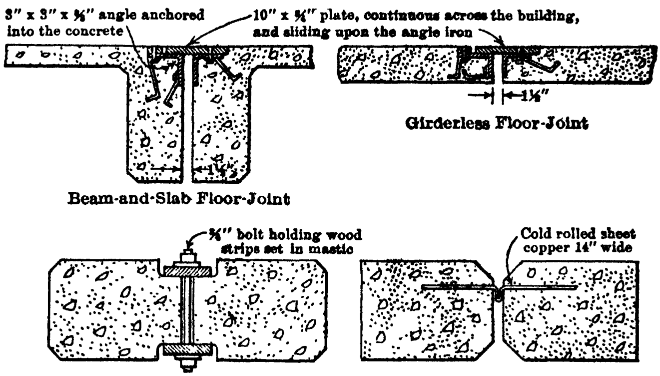
The Contraction and Expansion of Concrete.* The hardening of concrete, due to the hydration of the cement, is normally accompanied by VOLUMETRIC CHANGES. Although when cured in water, there may be an appreciable increase in volume, under job conditions concrete usually contracts while hardening. Changes in the temperature of the surrounding air and variations in atmospheric moisture also cause changes in volume even after the concrete has hardened.

Contraction-Joints in Concrete Buildings. In order to meet all these conditions, it is necessary to provide joints at sufficiently frequent intervals so that the strength of the concrete will not be exceeded, or to employ an amount of reinforcement sufficient to distribute the resulting deformations in such a way that they will not seriously affect the utility or appearance of the building.

* See under Shrinkage and Expansion of Concrete, Part IV, for a full discussion of this subject.

For buildings not exceeding 300 ft in length, the amount of reinforcement ordinarily required by the principal stresses is usually sufficient to properly distribute the secondary stresses due to shrinkage and temperature variation.

Buildings over 300 ft in length should be separated by a TRANSVERSE JOINT providing a plane of separation so that free movement of the two adjacent parts may take place. In beam-and-slab construction this can be done by constructing two sets of columns, resting, however, upon a common footing, and supporting parallel floor beams on either side of the joint. (Fig. 25.) Reinforcement should never extend across an expansion joint; the break between the two sections should be complete and exposed joints should be filled with an elastic joint-filler. In girderless floor-construction contraction joints are usually placed in the middle of a bay, the two sections of which are designed as cantilevers.



Joints between Columns

Fig. 25. Typical Designs for Contraction Joints

Expansion-Joints in Concrete Buildings. Although most of the injury caused through temperature and moisture changes is traceable to contraction, it is occasionally necessary to provide EXPANSION JOINTS where sections of concrete are poured between rigid terminations. Wherever expansion might cause buckling, joints should be provided, particularly around the edges of concrete roof-fills adjacent to parapet and pent-house walls. Besides the actual joints required to provide for contraction and expansion, it is customary to use what is often referred to as TEMPERATURE REINFORCEMENT in the form of small rods, usually ⅜-in round rods, in all members exposed to extreme temperature changes. The amount of such reinforcement is ordinarily based upon the sectional area of the member, and should represent about $\frac{3}{10}\%$ of the concrete area. It is not, however, desirable to build monolithically long sections of parapet; it is better practice to divide work of this nature into a number of short sections, each one separated from contiguous work by a mastic joint.

Combination Floor Systems

Ribbed Floor Construction. Reinforced concrete is largely used in the floor systems of all fire-resistant buildings whether framed of concrete or

structural steel. Various types of floor construction are described in Chapter XXII, some of which employ reinforced concrete as a structural material. As the RIBBED FLOORS, however, are directly developed from the concrete floor slab and treated as a series of parallel T beams, their design is included in this chapter. Although the girderless type of floor construction, as described in this chapter, is usually the more economical for factories and warehouses it is seldom suitable for other types of occupancy owing to the size of the column capitals and the fact that arbitrary limitations placed upon slab thickness make the construction over-heavy.

For BUILDINGS SUPPORTING LIGHT AND MEDIUM LIVE LOADS, such as hotels, hospitals, and apartment-houses, some system comprising reinforced-

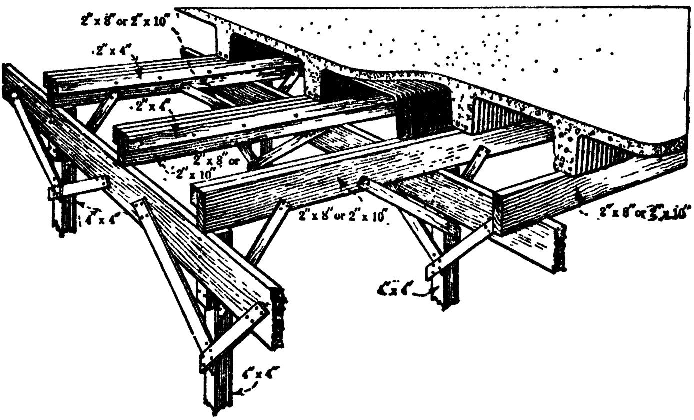


Fig. 26. Metal Pans Alternating with Concrete Ribs

concrete ribs cast between either steel or wooden forms, or filler blocks, usually proves economical. Ribbed floor construction includes floor systems of ribs and slabs placed monolithically, in which the ribs are not farther apart than 3 ft face to face. The ribs should have a minimum width of 4 in and a depth not over three times the width.

STEEL FORMS, generally referred to as "tin pans" (Fig. 26), can be easily supported upon open form work; the pans are ordinarily about 20 in wide and are used with a 5-in rib. A continuous slab cast over the tops of the pans, varying in thickness from 2½ * to 3 in, and cast integrally with the concrete forming the ribs, furnishes a T section which is designed in the same manner as an ordinary T beam.

The steel forms may be of either the PERMANENT or REMOVABLE TYPE; the former are of lighter weight, usually corrugated to give additional strength, and are left in place at the completion of the work. The removable forms (Fig. 27), of somewhat heavier design, are taken down with the temporary supports and re-used upon successive floors.

STEEL DOMES (Fig. 28), forming voids between concrete ribs extending in two directions, at right-angles to each other, have been placed upon the market but are not extensively used.

* Some building ordinances permit a 2-in thickness.

WOODEN PANS are employed in the same way as tin pans and both the design and construction are in every way similar.

GYPSUM BLOCKS (Fig. 29), approximately 19 in wide, make an excellent filler when used between reinforced-concrete ribs normally of 5-in width;

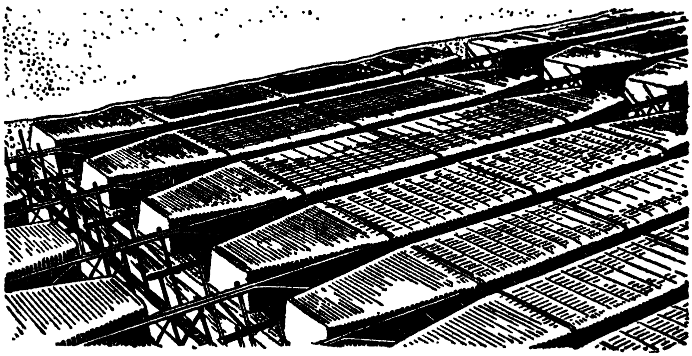


Fig. 27. Metal Pans Alternating with Concrete Ribs—Tapered Type

in this case, also, a continuous concrete slab is cast over the top of both blocks and ribs, the slab varying in thickness between 2½ and 3 in. Soffit blocks may be used at the bottoms of the ribs in order to obtain a uniform gypsum surface for the application of plaster.

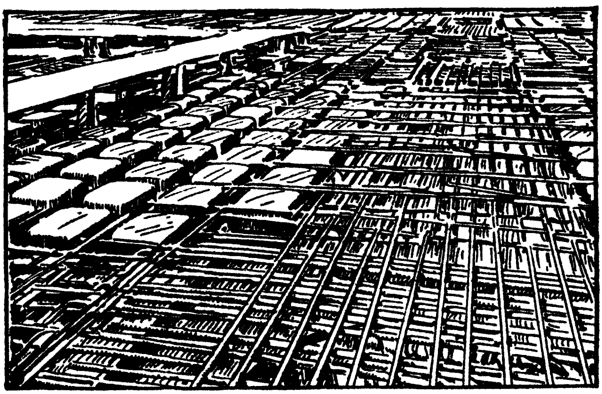


Fig. 28. Two-way System—Steel Domes Alternating with Concrete Ribs

TERRA-COTTA BLOCKS (Fig. 30) also make excellent fillers; as the blocks are 12 in square and the ribs normally 5 in wide, their use results in a center-to-center spacing of 17 in.

Units made of slag-concrete and employed as fillers between reinforced-concrete ribs under the name of the SLAGBLOK SYSTEM, have been used for the same purpose of providing a fire-resistant floor for light and medium

loads on comparatively long spans. This system has the particular feature of being adaptable to two-way design, thereby obtaining a two-way distribu-

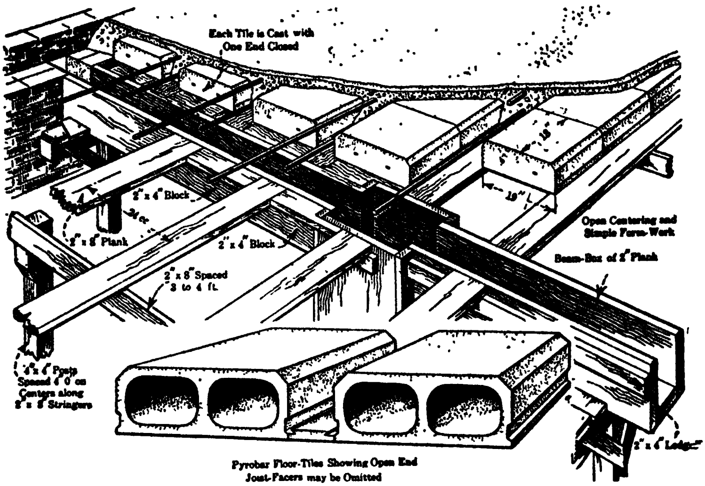


Fig. 29. Gypsum Blocks Alternating with Concrete Ribs

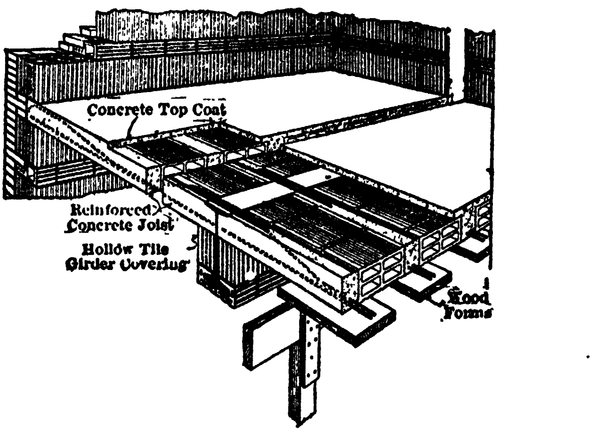


Fig. 30. Terra Cotta Blocks Alternating with Concrete Ribs

tion of the load which results in a thinner floor construction wherever the framing can be arranged to give support on all four sides of a panel.

In systems of this kind employing block fillers (Fig. 31), it is possible to

plaster directly upon the soffit of the structural floor. The thickness of the concrete slab on tile or cement block can also be reduced to a 2-in minimum, whereas a 2½-in thickness is necessary to give a substantial job over wood or metal forms. Where pan-construction is used, metal lath is ordinarily attached to the soffits of the ribs in order to provide a base for the plaster of the ceiling below. All of these systems have the advantage of being CONSIDERABLY LIGHTER than a solid reinforced-concrete slab of similar depth. The reinforcement in the ribs normally consists of one or two rods, or bars, together with a light reinforcement in the slab above the pans or fillers, which is placed at right-angles to the main reinforcement in the ribs.

The sectional area of the ribs is usually determined by shearing considerations, and the amount of the tensile reinforcement by the bending moment. As the width of the flange cannot exceed the center-to-center spacing of the ribs, it is also necessary to check the extreme fiber stress in the concrete. When terra-cotta or cement block are used, the webs of the tile, or block, in

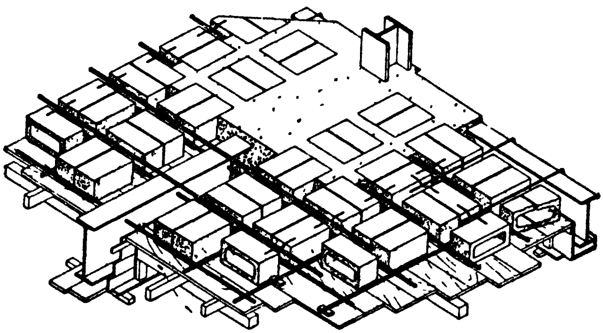


Fig. 31. Two-way System—Concrete Blocks Alternating with Concrete Ribs

contact with the ribs, may be included in the calculations for shear and negative bending moment, provided that the joints between the tile in alternate rows are staggered.

Most building codes permit a unit shearing-stress of 60 lb per sq in on the concrete of the ribs provided that the reinforcement is properly anchored. This would seem to be conservative practice. If the unit shearing-stress, however, is more than that which may be allowed, it is possible to widen the rib by the use of specially TAPERED PANS, in the case of the tin-pan construction, or by using HALF BLOCKS adjacent to supports, in the case of block construction. If it does not seem desirable to do this, or if the value of the shearing-stress is still too high, CONTINUOUS STIRRUPS may be used without excessive expense.

The depth of the tile or filler block is chosen to give the depth of rib required by the design. Terra-cotta block are sold in thickness varying from 4 to 12 in. Gypsum block may be obtained from 6 to 12 in and metal tile from 4 to 14 in in depth. The Slagblok units are manufactured in 4½-in, 6-in, and 8-in depths. Open form work with centers lying beneath the ribs is used except where a two-way arrangement of ribs necessitates a complete decking.

When planning any of these systems for use with a STRUCTURAL STEEL FRAME, the floor system should be placed at a height in relation to the sup-

porting beams, to permit the continuity of the reinforcement; this can be arranged if the tile or block fillers are placed so that their upper surfaces are flush with the tops of the steel members. When used in a building with a REINFORCED-CONCRETE FRAME, this same principle should be observed, and in order to leave sufficient room for the reinforcement of the ribs to cross that of the supporting girders, the tile or block fillers should be kept two inches below the upper surface of the structural slab.

Design Procedure for Ribbed Floor Construction. Substantially the same procedure is followed in the design of all ribbed floor systems. The value of the dead load is taken from the manufacturer's tables, or computed by the designer, for the particular system employed. The center-to-center spacing will also depend upon the width of rib and filler block.

(1) Assume a depth for the tile or block and a thickness for the concrete slab. Obtain the weight per square foot from the manufacturer's catalogue.

(2) Add the live load to the weight of the construction as determined in (1) and compute the total load on each rib.

(3) Compute the bending moment by the appropriate formula

$$\frac{WL}{8}, \quad \frac{WL}{10} \quad \text{or} \quad \frac{WL}{12}$$

(4) Compute the sectional area of the main tensile-reinforcement required in each rib by the formula

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)}$$

in which A_s = steel area in square inches;
M = bending moment in inch pounds;
f_s = tensile unit stress in steel in pounds per square inch;
d = the depth of pan or filler block plus the thickness of the concrete slab, minus $1\frac{1}{2}$ in;
t = total slab thickness.

Choose one, or preferably two, rods or bars, giving the required sectional area.

(5) Check the compressive stress in the concrete by the formula

$$f_c = \frac{2M}{bt\left(d - \frac{t}{2}\right)}$$

in which f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch;
b = center-to-center spacing of the ribs. in inches;
M, d and t are as noted above.

(6) Determine the maximum vertical shear, V, adjacent to the support. In the case of uniformly loaded floors this is equal to one-half of the load on the rib as found in (2). Compute the value of the maximum unit shearing-stress by the formula

$$v = \frac{V}{b'jd}$$

in which v = unit shearing-stress in pounds at the section where the shear, V, is computed;
V = total vertical shear at the support in pounds, on a width of slab equal to the center to center spacing of the ribs;
b' = width of rib in inches;
j and d are as noted above.

(7) Check the bond stresses by the formula

$$u = \frac{V}{\Sigma ojd}$$

in which u = unit bond stress in pounds per square inch;
Σo = sum of the perimeters of all rods (in the width of one rib) comprising the reinforcement in tension over the supports;
V, j and d are as noted above.

On continuous spans it is desirable to raise one-half of the main longitudinal steel at an angle of 30° to the horizontal at approximately the fifth point of the clear span. If only one rod is used in each rib, it is customary to raise the rods in alternate ribs. This practice, however, is not to be recommended, the better design being to use two smaller rods in each rib instead of one large rod.

Rods raised over supports should be carried to the fifth points of adjacent spans if the rib is continuous across the girder, and should be hooked into the supporting member at non-continuous termination, such as at a spandrel beam. Rods lying in the bottoms of the ribs should be cut off at the center lines of the girders if the rib is continuous across the girder and should be hooked into the supporting member at non-continuous terminations. All ribs should have a bearing of eight inches on brickwork or stone masonry.

Typical Design of Ribbed-Floor Construction

This example applies to the tin-pan system; the same method is used for all ribbed designs.

Specification data: f_s = 18 000 lb per sq in
f_c = 650 lb per sq in
v' = limited to 60 lb per sq in
n = 15
Minimum thickness of concrete over metal = 2½ in

Panel width between faces of supports 22 ft

Live load 60 lb per sq ft; allow 25 lb per sq ft for floor fill, finish-flooring and plaster.

End-conditions: semi-continuous

(1) Assume a 10-in depth for the pan with a 2½-in topping: the weight per sq ft is found from the manufacturer's catalogue, or Table I, to be 63 lb.

(2) Live load	=	60 lb
Floor finish, etc.	=	25 lb
Weight of construction	=	63 lb
Total load	=	148 lb per sq ft

Load per linear foot of rib = 148 × 25/12 = 308 lb
Total load on each rib = 308 × 22 = 6 776 lb

(3) Bending moment, $M = WL/10 = \dfrac{6\,776 \times 22 \times 12}{10} = 178\,886$ in-lb

(4) Steel area, $$A_s = \frac{M}{f_s\left(d - \dfrac{t}{2}\right)} = \frac{178\,886}{18\,000 \times 9.75} = 1.02 \text{ sq in}$$

in which $d = (10 + 2\frac{1}{2}) - 1.50 = 11$ in

$t/2 = 2.5/2 = 1.25$ in

Accept $1\frac{3}{4}$-in and $1\frac{7}{8}$-in round rods

($A_s = 1.04$ sq in) in each 5-in rib.

(5) The stress in the concrete is checked by determining the value of

$$f_c = \frac{Mkd}{bt\left(kd - \dfrac{t}{2}\right)jd}, \quad \frac{178\,886 \times 0.351 \times 11}{(25 \times 2.5)(0.351 \times 11 - 1.25)(0.883 \times 11)}$$

$= 435$ lb per sq in

(6) The maximum shear, $V = 6\,776/2 = 3\,388$ lb

By formula $$v = \frac{V}{b'jd} = \frac{3\,388}{6 \times 0.875 \times 11} = 59 \text{ lb per sq in}$$

in which 6 is the average width of the web in inches. This stress is permissible with anchored reinforcement.

(7) By formula,

$$u = \frac{V}{\Sigma ojd} = \frac{3\,388}{(2.35 + 2.75) \times 0.875 \times 11} = 69 \text{ lb per sq in}$$

Deformed rods are not necessary.

The $\frac{7}{8}$-in rods should be raised at the fifth points of the clear spans adjacent to all continuous terminations, and lapped across supports to the fifth points of the adjoining spans. At non-continuous terminations, the $\frac{7}{8}$-in rods should be raised at a section about half-way between the fifth and tenth points of the clear spans, and anchored by hooks into the supporting spandrel beam or column. It is not necessary to anchor into supporting masonry walls, but a 4-in bearing should be considered an absolute minimum.

The $\frac{3}{4}$-in rods are straight and lie in the bottoms of the ribs. When the ribs are continuous across supports, these are cut off at the center lines. Where the ribs terminate at spandrel beams or walls, the rods should have at least a 4-in bearing and be hooked at the ends.

Design of Girderless Floor Construction, Fig. 32

Characteristics of the Various Systems. The so-called "flat slab" or girderless type of floor construction has largely superseded beam-and-girder design for industrial work. If a building is three or more bays in width, and the column locations such as to give approximately square bays, of equal or nearly equal size, this type is usually more satisfactory for superimposed loads of 100 lb per sq ft or more. The advantages are a saving in story height due to the elimination of beams and girders, better lighting facilities

and structural economy. A girderless floor system may be designed either with or without DROPS OVER THE COLUMNS; if drops are used over the interior columns, HALF DROPS should be used at the exterior columns. The FLARED HEADS of columns are characteristic of this system; beams are used to frame openings around stair wells and elevators as well as to form the lintels connecting exterior columns.

These general features are common to practically all designs of the girderless variety. The variations, often referred to as "systems," apply, in most cases, only to the disposition of the reinforcement. There are at present

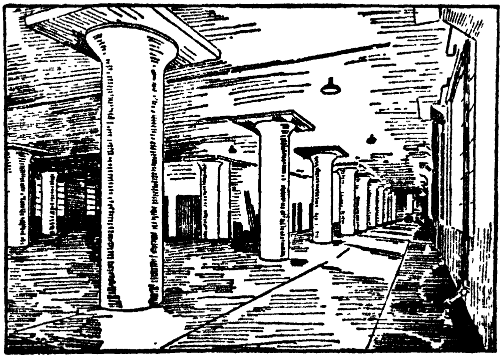


Fig. 32. Typical Girderless Floor Construction

but two widely used SYSTEMS: the two-way (Fig. 33), and the four-way (Fig. 34). In the former, bands of steel rods are carried from column to column and a TWO-WAY REINFORCEMENT placed in the central, rectangular portion of the panel between the bands and running parallel to them.

In the FOUR-WAY SYSTEM (Fig. 34), reinforcement is placed in four directions and comprises two bands of steel rods carried directly from column to column and two other bands placed diagonally across the panels from column to column. Supplementary reinforcement is added in the form of short rods lying through the mid-spans perpendicular to the rectangular band steel.

There have, from time to time, been many patents awarded for various systems either proposing some new distribution of the reinforcement, or advocating a variation in structural arrangement. Among the PROPRIETARY SYSTEMS, that known as the S.M.I. flat-slab system, and controlled by the S.M.I. Engineering Company, has probably been the most widely used. In this system the reinforcement is in the form of rings and radial bars.

Codes and Standards of Design. At present practically every large city HAS ITS OWN PARTICULAR CODE governing the design of girderless floors, and the Joint Code published by the American Concrete Institute offers an excellent and carefully studied recommendation. For work done under any particular jurisdiction, the designer has little option and must follow the regulations of his locality; he is free to choose, however, either a two-way or four-way system, or one of the proprietary methods, provided that he meets the demands of the building ordinances in regard to the structural requirements. When designing work outside of any particular jurisdiction,

it is generally customary for designers to select one of the better city codes as a guide; those of New York and Chicago have both been very widely used for this purpose and represent well-tried designs.

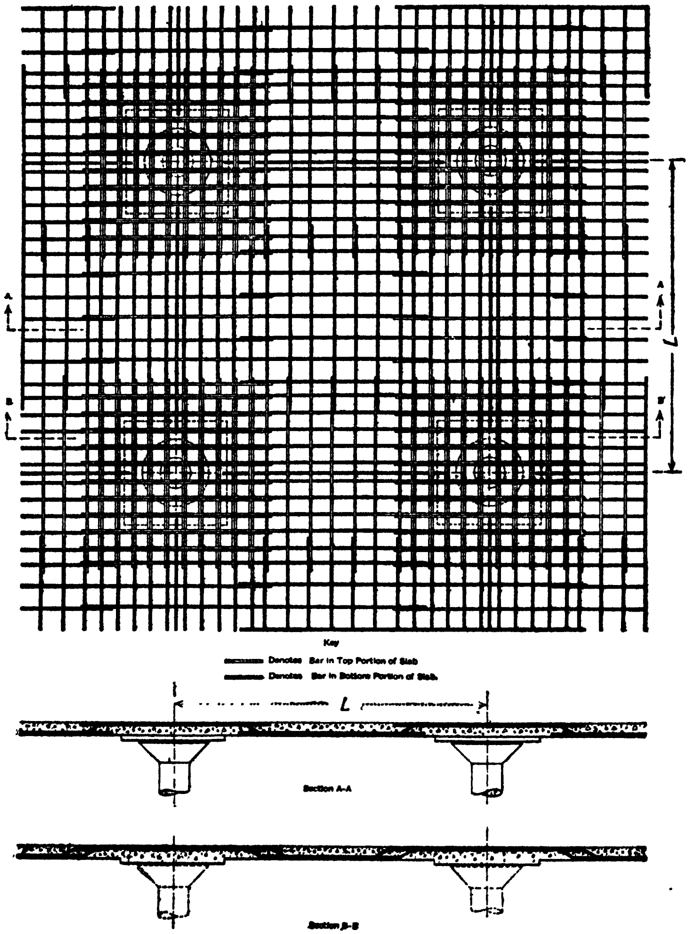


Fig. 33. Two-way Flat Slab Construction. Typical Interior Panel

Design Procedure for Girderless Floors. Having at hand a sketch showing the over-all dimensions of the building and the column locations, the steps in the design of a girderless floor system can be planned as follows:

(1) Determine the average center to center span, L, for any particular

panel; span lengths are taken between center lines of columns, or supporting members. Spans are defined in building ordinances, which permit, within certain limits, the use of average spans for oblong panels, but if the ratio of

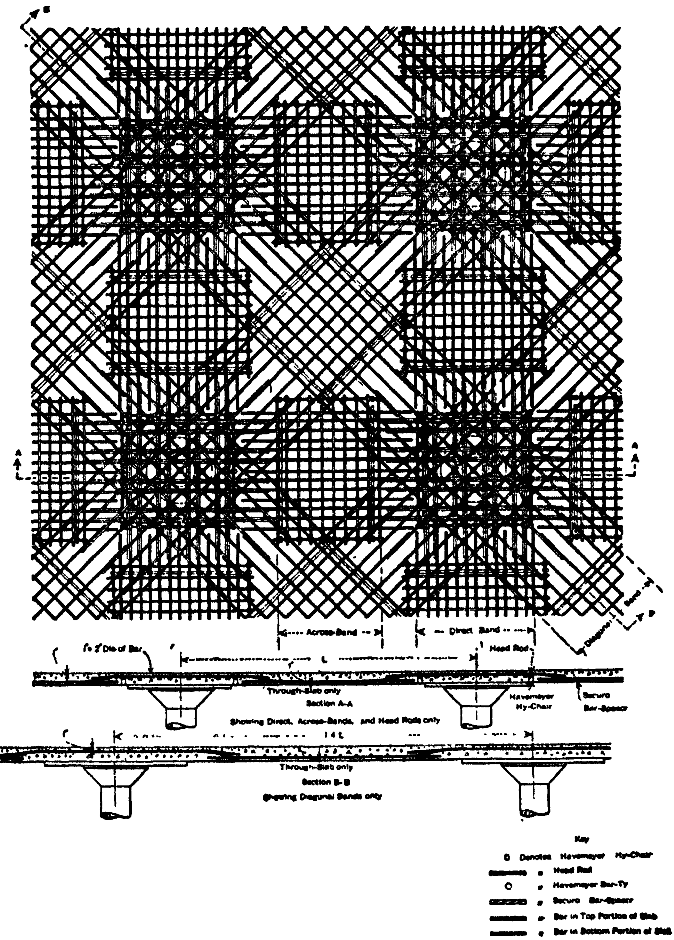


Fig. 34. Four-way Flat Slab Construction. Typical Interior Panel

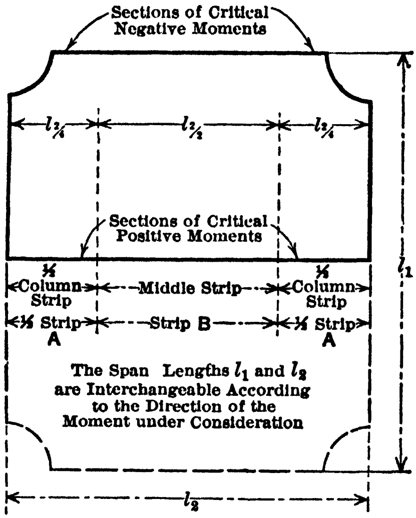
panel length to width exceeds 1.33, a special analysis should always be made, as the standard method of design is inapplicable.

(2) Determine the slab thickness; most codes give a formula for this purpose which makes the total thickness a function of the span and load. Typ-

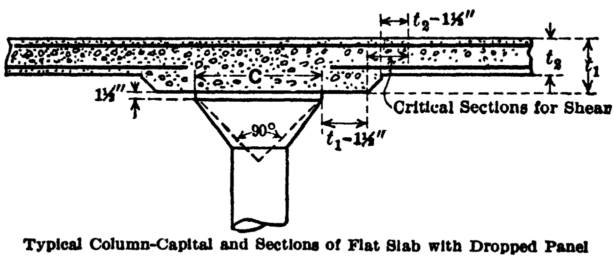
ical of the general requirements is that of New York City, based on an ultimate 28-day compressive strength of 2 000 lb per sq in in the concrete:

$$t = 0.02\ L\sqrt{w} + 1 \quad \text{(for slabs with drops)}$$

$$t = 0.024\ L\sqrt{w} + 1\tfrac{1}{2} \quad \text{(for slabs without drops)}$$



Principal Design-Sections of a Flat Slab



Typical Column-Capital and Sections of Flat Slab with Dropped Panel

Fig. 35. Flat Slab Construction. Details of Drop. Designations Used in the Joint Code

in which t = total thickness of slab, in inches;
L = average span of the slab, in feet;
w = total live and dead load in pounds per square foot of floor area.

Slab thicknesses for floors are also often limited to $L/32$ or 6 in as a minimum. The corresponding minimum thicknesses for roof slabs are $L/40$ and 5 in. These limiting dimensions are based on a 28-day compressive strength of 2 000 lb per sq in in the concrete; where higher stresses are justified, the Joint Code permits the use of a reduction formula.

(3) Determine the size of the drop-panel if used; the size is usually limited to a proportion of the span. For example, the Joint Code specifies that a side of the drop shall not be less than 0.35 L, and many city codes require at least 0.33 L. The size of the drop is also governed by the vertical shearing-stress around its sides. If the unit shear is higher than the limit

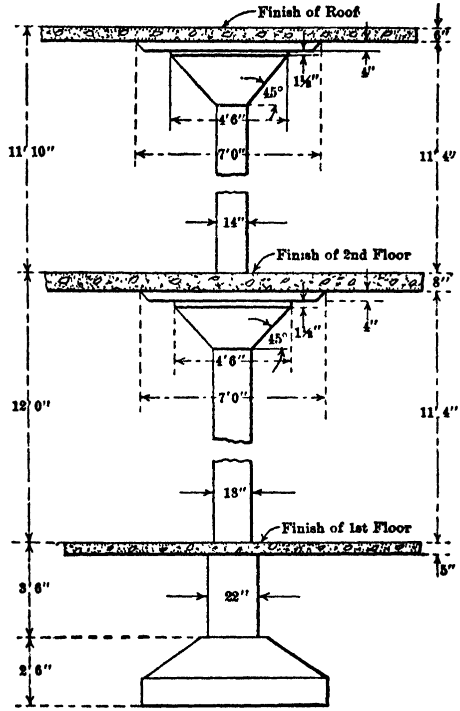


Fig. 36. Flat Slab Construction. Vertical Section Showing Column Caps and Drops

permitted by the code, it could be reduced by increasing the slab thickness, but this is generally an uneconomical procedure, it being better to increase the size of the drop itself. The shear around the perimeter of the drop is found by the formula

$$V = w\,(L^2 - b^2)$$

in which V = total vertical shear acting on all four sides of the drop, in pounds;

w = total live and dead load in pounds per square foot of floor area;

L^2 = area of the panel in square feet;

b^2 = area of the drop in square feet.*

* The Joint Code computes the unit shearing-stress on a vertical section which lies at a distance of t_2 − 1½ in from the edge of the drop-panel and parallel with it; t_2 = thickness of slab.

Having found the value of V, the unit shearing stress, usually limited by code to 60 lb per sq in is found by the formula

$$v = \frac{V}{bjd}$$

in which v = unit shearing-stress around the perimeter of the drop, in pounds per square inch;
b = perimeter of the drop in inches;
j = 0.875 (an approximation used in shear computations);
d = effective depth of the slab; distance from the compression face to the center of tension-steel, in inches.

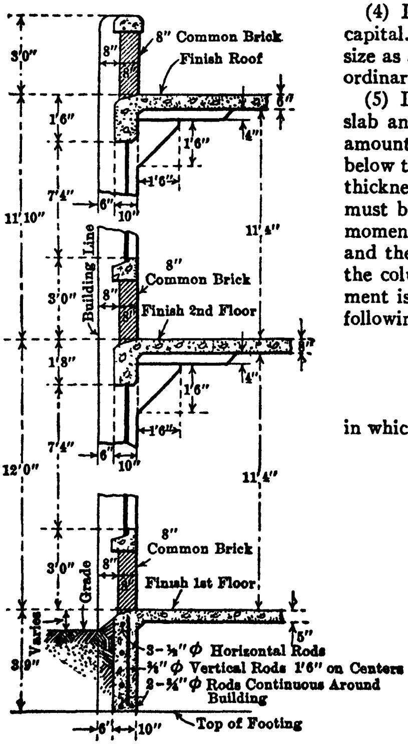


Fig. 37. Flat Slab Construction. Vertical Section Showing Lintel Beams and Panel Walls

(4) Determine the size of the column capital. Most codes give a limiting size as a function of the span; this is ordinarily 0.225 L.

(5) Determine the thickness of the slab and drop, thereby obtaining the amount which the drop should project below the soffit of the slab. The total thickness of concrete at this section must be such as to resist the negative moment on the column-head section and the shear around the perimeter of the column capital. The first requirement is met by the application of the following formula,

$$d = \sqrt{\frac{2M}{f_c jkb}}$$

in which d = effective depth of combined slab and drop in inches, from lower surface of slab to center of reinforcement;
M = bending moment in inch-pounds at the column head section;
f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch;
b = width of drop in inches;
j and k are factors obtained from Tables VI and VII, corresponding to the stresses employed.

The total vertical shear is computed and the unit shearing-stress is checked by the formulas

$$V = w\left(L^2 - \frac{\pi}{4} \times c^2\right) \text{ and } v_2 = \frac{V}{bjd}$$

in which V = total vertical shear, in pounds, acting on the section around the perimeter of the column capital;
w = total live and dead load in pounds per square foot of floor area;
L^2 = area of the panel in square feet;
c = diameter of the column capital, in feet;
v_2 = unit punching shear around the perimeter of the column capital, in pounds per square inch;
b = perimeter of the column capital, in inches;*
j = 0.875 (an approximation used in shear computations);
d, as noted above.

The value of v_2 is usually limited in this computation to 120 lb per sq in.

The Joint Code limits the total thickness of slab and drop to between $1\frac{1}{4}\,t$ and $1\frac{1}{2}\,t$, where t represents the slab thickness.

(6) Compute the moments on the various design-sections as required by the code selected. This is a simple matter requiring first the computation for W, which is merely the area of the panel multiplied by the total live and dead load per sq ft on the panel, including the weight of the drop; and a determination of the value to be used for L as defined in the code.

All codes and rulings divide the panel, for purposes of design, into two strips, each of which, in each direction, is reinforced to resist positive bending moments near the centers of spans, and negative bending moments along lines connecting the columns. These strips are defined by the Chicago Building Code as follows:

"For the purpose of establishing the bending moments and the resisting moments of a square panel, the panel shall be divided into strips known as strip A and strip B. Strip A shall include the reinforcement and slab in a width extending from the center line of the columns for a distance each side of this center line equal to one-quarter (¼) of the panel length. Strip B shall include the reinforcement and slab in the half width remaining in the center of the panel. At right-angles to these strips, the panel shall be divided into similar strips A and B, having the same widths and relations to the center line of the columns as the above strips. These strips shall be for designing purposes only, and are not intended as the boundary lines of any bands of steel used."

With drops and capitals conforming to the previous requirements, the following moment coefficients are used in Chicago and New York; the third column gives those recommended by the Joint Code, for the special case

Table IX. Comparison of Design Moments for Girderless Floors

Two-way reinforcement with drops

Section	Moment coefficients		
	Chicago	New York	Joint code
Strip A over columns....... ..	$-WL/30$	$-WL/32$	$-WL/30.5$
Strip A midway between columns	$+WL/60$	$+WL/80$	$+WL/77$
Strip B over columns........	$-WL/120$	$-WL/133$	$-WL/102.5$
Strip B midway between columns	$+WL/120$	$+WL/133$	$+WL/102.5$

* The Joint Code computes the unit shearing-stress on a vertical section which lies at a distance of $t_1 - 1\frac{1}{2}$ in from the edge of the column capital and parallel with it; t_1 = combined thickness of slab and drop.

when the diameter of the column capital $c = 0.225$ times the average length of span, L.

(7) Compute the steel areas required by the bending moments at the various design sections.

(8) Select the rods, or bars, to give the desired areas for the sections of positive moment (at center spans); ½-in or ⅝-in rounds are usually the most suitable, although ⅜-in and ¾-in rounds are sometimes used. For the sake of economy, the designer should avoid a close spacing and good practice does not permit a distance between rods more than 1½ times the slab thickness. These considerations usually result in spacings between 6 and 10 in.

(9) Determine the arrangement of rods to give the areas required for the sections of negative moment (over supports), by means of lapping. As the area of the reinforcement for the positive moments is not often exactly one-half that for the negative moments, short rods are used to make up the required sectional areas.

(10) Note the points of bend, length of laps and anchorage at terminations. Make a final check on the detailed requirements of the code, or ruling governing the design.

Typical Design of Girderless Floor Construction

(Chicago Code, 2-way reinforcement with drops)

Specification data: $f_s = 18\,000$ lb per sq in
$f_c = 700$ lb per sq in
$v = 60$ lb per sq in (unit shearing-stress around perimeter of drop)
$v_2 = 120$ lb per sq in (unit shearing-stress around perimeter of column capital).

Panel data: Dimensions center-to-center of columns, 20 ft by 20 ft
Superimposed load, 200 lb per sq ft
Drops are to be used

Under this code the panel is divided into two strips known as strip A and strip B. Strip A corresponds to the column-strip of the Joint Code designation (Fig. 35) and strip B corresponds to the middle strip.

(1) Length of span, $L = 20$ ft

(2) Slab thickness not less than

(a) $t = 6$ in
(b) $t = (1/32)\,L = 20/32 = 0.625$ ft $= 7\tfrac{1}{2}$ in
(c) $t = \sqrt{W}/44$

A slab thickness of 8 in is assumed. Then, allowing 4 lb per sq ft of bay for the weight of the drop

$$W = (200 + 100) \times 400 = 120\,000 \text{ lb}$$

and

$$t = 7.88 \text{ in}$$

Accept a slab-thickness equal to 8 in

(3) Width of drop not less than

(a) $0.33\,L = 0.33 \times 20 = 6$ ft 8 in
A width of $0.35\,L = 7$ ft is assumed

(b) Vertical shear around perimeter of drop:

$$V = w(L^2 - b^2) = 300\,(20)^2 - (7)^2 = 105\,300 \text{ lb}$$

$$v = \frac{V}{bjd} = \frac{105\,300}{4 \times 84 \times 0.875 \times 6.5} = 55 \text{ lb per sq in}$$

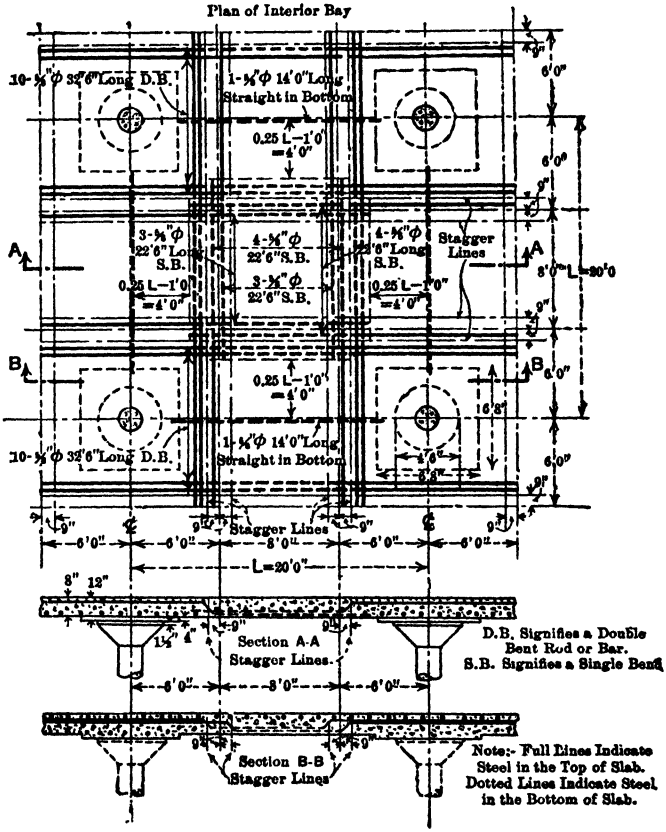


Fig. 38. Flat Slab Construction. Details of Reinforcement for a Typical Interior Panel

in which (4 × 84) is the perimeter of the drop and 6.5 the effective depth of the slab, both in inches. As 60 lb per sq in is permitted, accept a square drop with side equal to 7 ft 0 in.

(4) Size of column capital

A circular capital with a diameter equal to 0.225 L is chosen from code requirements. Then,

$$0.225\,L = 0.225 \times 20 = 4 \text{ ft } 6 \text{ in}$$

(5) Depth of drop not less than

(*a*) By formula

$$d = \sqrt{\frac{2M}{f_ckjb}} = \sqrt{\frac{2 \times 963\,700}{700 \times 0.368 \times 0.877 \times 84}} = 10.05 \text{ in}$$

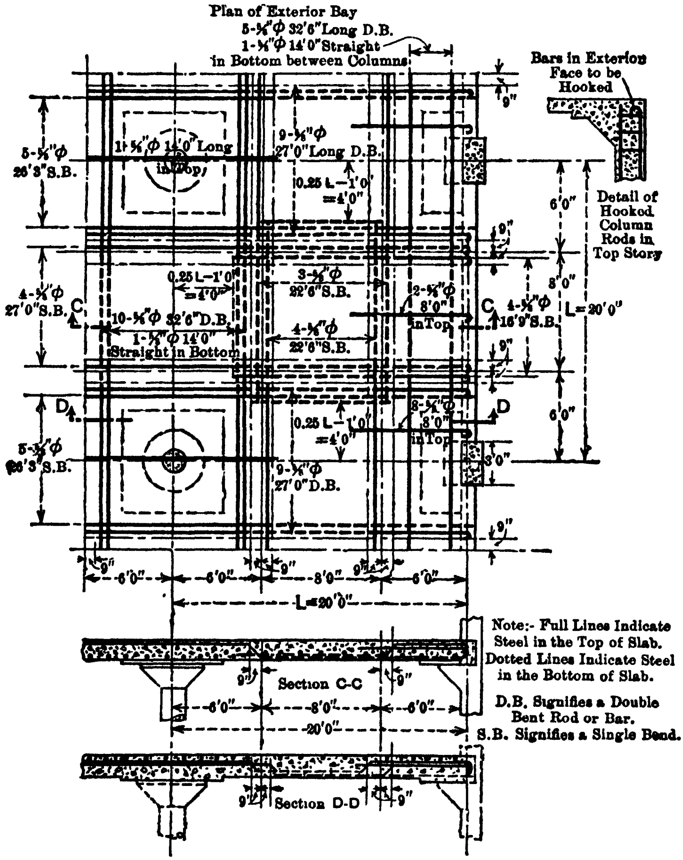


Fig. 39. Flat Slab Construction. Details of Reinforcement for a Typical Exterior Panel

in which M = the bending moment, in inch-pounds, in band A over the column-head computed as in (6), following, and based upon an assumed depth of $11\frac{1}{2}$-in. The values of k and j are taken from Tables VI and VII and correspond to $f_c = 700$ and $f_s = 18\,000$ lb per sq in

Total thickness is 10.05 + 1.50, or $11\frac{1}{2}$-in

(b) Punching-shear around perimeter of column capital.

$$V = w\left(L^2 - \frac{\pi}{4}c^2\right) = 300\,(20)^2 - \frac{3.14 \times (4.5)^2}{4} = 115\,200 \text{ lb}$$

By formula

$$v_2 = \frac{V}{bjd} = \frac{115\,200}{3.14 \times 54 \times 0.875 \times 10.25} = 76 \text{ lb per sq in}$$

in which (3.14 × 54) is the perimeter of the column capital, and 10.25 the effective depth of the combined slab and drop, both in inches. As 120 lb per sq in is permitted on this section, the depth of the drop is controlled by the compressive strength of the concrete (see paragraph (a) above) and the offset may be $3\frac{1}{2}$ in, making the total depth $11\frac{1}{2}$ in.

(6) Moments:

$$W = (20 \times 20 \times 296) + (7 \times 7 \times 42) = 120\,460 \text{ lb}$$

Band A, from formulas in code,

$$-M = WL/30 = 120\,460 \times 20 \times 12/30 = 963\,700 \text{ in-lb}$$
$$+M = WL/60 = 120\,460 \times 20 \times 12/60 = 481\,840 \text{ in-lb}$$

Band B, from formulas in code,

$$-M = +M = WL/120 = 120\,460 \times 20 \times 12/120 = \pm 240\,900 \text{ in-lb}$$

(7) Steel areas:

Band A:

Negative reinforcement $A_s = \frac{M}{f_sjd} = \frac{963\,700}{18\,000 \times 0.877 \times 10.25} = 5.95$ sq in

Positive reinforcement $A_s = \frac{M}{f_sjd} = \frac{481\,840}{18\,000 \times 0.877 \times 7} = 4.36$ sq in

Band B:

Both positive and negative $A_s = \frac{240\,900}{18\,000 \times 0.877 \times 7} = 2.18$ sq in

(8) Band A: Accept 15 $\frac{5}{8}$-in round rods ($A_s = 4.60$ sq in) each way, which give the sectional area required for the positive moment.

Band B: Accept 12 $\frac{1}{2}$-in round rods ($A_s = 2.28$ sq in) each way, which give the sectional area required for the positive moment.

(9) Band A: As the negative moment requires a sectional area of 5.95 sq in over supports (along center lines of columns), 10 of the $\frac{5}{8}$-in rods are raised from each bay along the line of inflection, making 20 rods in all and supplying 20 × 0.307 = 6.14 sq in to resist the negative bending moment.

Band B: As the negative moment is equal to the positive moment, 6 of the 12 $\frac{1}{2}$-in rods are raised from each bay along the line of inflection, making 12 rods in all and supplying 12 × 0.19 = 2.28 sq in to resist the negative bending moment.

(10) The bent rods of both bands, in both directions, should extend entirely across the panel and to the quarter points of adjacent bays.

The straight rods should extend twenty rod-diameters each side of the lines of inflection. The point of inflection is considered as being one-quarter the distance center to center of columns, both cross-wise and diagonally, from the center of the column.

Design of Reinforced Concrete Columns

Loads on Columns. The weight carried by a column is usually computed by multiplying the area of the supported bay by the live load and dead load per sq ft of floor area. If floor loads are not uniformly distributed, the load on a column may be found by adding together the reactions of the members framing into the column. In any case, the live load should be kept separate from the dead load in order that any allowable reductions may be easily made.

The DEAD LOAD on a column is the weight of the permanent structure which the column supports including partitions, floor slabs, flooring, the webs of T beams and the weight of the column itself. The LIVE LOAD is the load per square foot on the floor, or roof area for which the building is designed in excess of its own weight and may be reduced under certain conditions. For example, when the character of the occupancy is such that the full load is never placed simultaneously on all floor areas, most city ordinances permit a 5% REDUCTION OF LIVE LOAD on each floor below the top floor, for buildings more than five stories in height, provided that the total reduction shall be not more than 50% of the live load. This means that the live load on the floor next below the top floor may be assumed to be 95% of the live load for which the building is designed, on the next lower floor 90%, and each succeeding lower floor correspondingly decreasing percentages, down to 50% of the required load; this percentage is used for the remaining lower floors.

Although the end reactions on beams and girders, and consequently the loads carried by supporting columns, vary within narrow limits, following the conditions of end-support, it is customary to assume that uniformly distributed loads are equally divided between the supporting members.

Bending Moments on Columns. The two general cases that occur in building design are, first, an interior column supporting beams of unequal spans, or beams of equal spans with unbalanced live loads and, secondly, an exterior column supporting one end of a non-continuous beam. The procedure, when bending must be considered, is to make a somewhat liberal design and then to check the fiber-stresses by combining the effect of axial load and moment. As the computations are somewhat involved, they are not given in this text.

Types of Concrete Columns. (See Fig. 40.) There are three styles of columns now extensively used for buildings constructed of reinforced concrete. These are characterized by the type of reinforcement employed.

The RODDED OF HOOPED COLUMN, of round, square, or oblong section, contains vertical reinforcement in the form of rods or bars tied together by steel hoops or ties, placed from 8 to 12 in apart and formed of ¼-in or ⅜-in round steel.

The SPIRALLED OF LATERALLY REINFORCED COLUMN contains vertical reinforcement in the form of rods or bars and lateral reinforcement in the form of spirally wound wire held in place by 3 or 4 spiral spaces and varying in diameter from ¼-in round to ⅝-in round.

The third type of column which has recently come into general use, for the purpose of reducing the size of the cross-section required through the lower stories of comparatively high buildings, is known as the CORED COLUMN. (See Fig. 41.) Such are composed of structural steel H sections, often weighing over 200 lb per lin ft, surrounded by 4 in of stone concrete reinforced by vertical rods, or bars, enclosed within a medium-weight spiral. The column

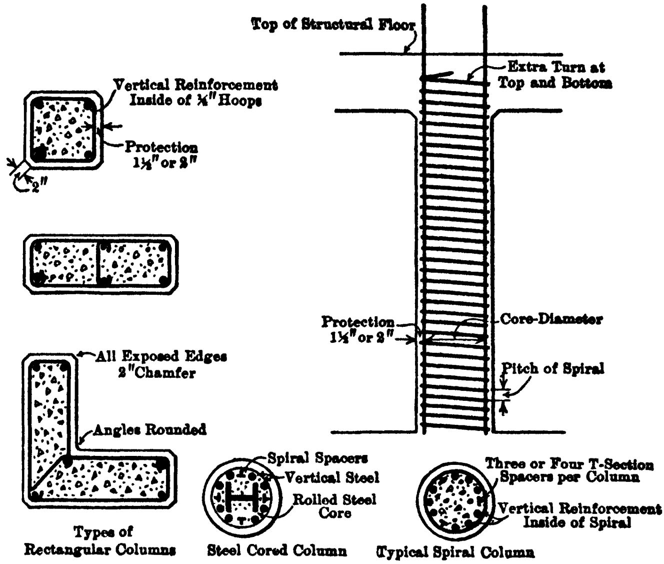


Fig. 40. Types of Reinforced-Concrete Columns and Arrangement of Reinforcement

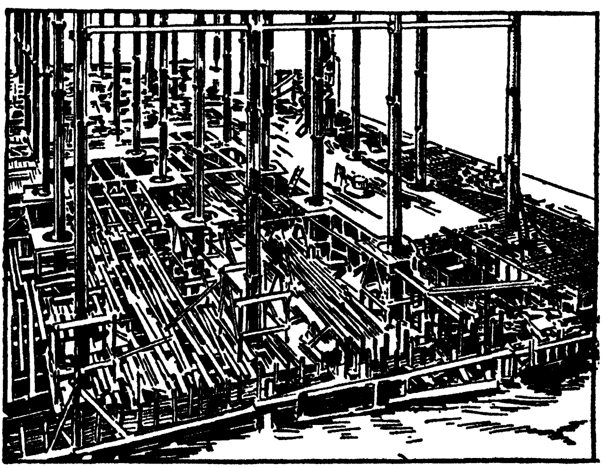


Fig. 41. Structural Steel H Sections Used as Cores for Reinforced-Concrete Columns Supporting Girderless Floor Construction

cores, often extending through five or six stories, are equipped with angle brackets at the floor levels and are used for the interior supports of buildings otherwise framed throughout in reinforced concrete.

Column Dimensions. A PIER is defined as a compression member, the height of which does not exceed three times its least lateral dimension or diameter; when concentrically loaded reinforcement is not required and the cross-sectional area is determined by dividing the load to be carried by the allowable compressive unit stress in the concrete, assumed to be ¼ of the ultimate crushing strength. The height of a column is usually limited by a so-called SLENDERNESS RATIO as well as a minimum or least width. If l denotes the unsupported height, and r the least width or diameter, measured from the outside surface, it is desirable to limit the ratio of l/r to 15.

If the height of a SPIRALLED or CORED COLUMN, divided by the least radius of gyration of the column core, exceeds 50, the following formula may be used to determine the safe load:

$$P' = P\left(1.50 - \frac{h}{100\,R}\right)$$

If the height of a RODDED COLUMN divided by the least radius of gyration of the column section exceeds 40, the following formula applies:

$$P' = P\left(1.33 - \frac{h}{120\,R}\right)$$

In both formulas

P' = total safe axial-load on long column, in pounds;
P = total safe axial-load on column the length of which does not exceed 11 times its least cross-sectional dimension, in pounds;
h = unsupported length of column, in feet;
R = least radius of gyration of column core, in feet.

The radius of gyration for any section is determined by the formula

$$R = \sqrt{\frac{I}{A}}$$

in which I = moment of inertia of the section;
A = the section-area.

Although SMALL COLUMNS of a secondary nature such as those supporting stair-landings may be built 6 in square, the principal columns of a building should never be less than 12 in in diameter, or least width, measured from the outside surface. Good practice requires a minimum diameter of 16 in for round columns supporting girderless floor construction, and a minimum 14 in thickness for the exterior columns of industrial buildings, even if the sectional area of concrete is in excess of that required to carry the imposed loads. Neither should the diameter or least width of a column be less than $1/15$ of the average center-to-center span of the supported bay.

General Principles Governing the Economic Design of Reinforced-Concrete Columns. Comparatively light loads, such as those carried by the interior columns in the upper stories of buildings, are most economically supported by RODDED COLUMNS. The amount of vertical steel that may be used as reinforcement is expressed as a percentage of either the gross area of the column, or of the net area, the latter being determined by deducting the

fire-proofing allowance from the over-all dimensions. Both the maximum and minimum percentages of steel are controlled by building ordinances and the values range from ½% to 4% of the effective concrete-section.* From the viewpoint of economy, it is usually desirable to increase the column size or use a richer mixture than to add vertical steel for the purpose of carrying load. It is extremely important, however, to have enough vertical steel to resist BENDING which may be induced by the unsymmetrical loading of interior bays and which is always present in exterior columns. There is no choice between the use of square or round steel; rods are ordinarily lapped a distance of 20 diameters or a minimum of 2 ft at floor levels. Vertical steel in the lowest tier of columns is either anchored into the footings or bonded by dowels. The vertical steel in top-story columns is normally cut off 3 in under the surface of the rough slab forming the roof. Exposed bonds, intended to join present work with future extensions, should be protected from rusting.

The HOOPS or TILES serve the purpose of preventing the vertical reinforcement from buckling under load. Most building codes require a vertical spacing of hoops not over 15 times the diameter of the vertical steel with a maximum of 12 in. Good practice demands ¼-in round hoops at least 12 in on centers. The vertical reinforcement in typical columns of this class usually consists of from 4 to 10 round rods varying in diameter from ⅝ in to 1¼ in; four ⅝-in rounds should be considered a minimum for square columns and six ⅝-in rounds a minimum for round columns. At floor levels where column sizes are reduced, the reinforcement of the larger column below is bent toward the center so as to lap the rods above. The lengths of laps should be at least 30 bar diameters for plain steel and 24 diameters for deformed rods or bars.

SPIRALLED COLUMNS are generally used throughout the interiors of buildings to carry comparatively heavy loads. The amount of vertical reinforcement is expressed as a percentage of the area of the concrete core enclosed within the spiral; maximum and minimum limits are established by all building ordinances, varying between 1 and 4%. The spiral is expressed as a percentage of the volume of the enclosed concrete and varies from ½ to 2%.

Although it is important to employ enough vertical reinforcement to resist any possible BENDING STRESSES, vertical steel is an expensive material for carrying load in reinforced-concrete columns. ECONOMY is obtained by using a rich mixture of concrete and as high a percentage of spiral as the code, or specification, will permit.

The PITCH OF SPIRALS varies between 1⅝-in minimum and 3-in maximum; spirals are usually cut off 9 in below the surface of the rough floor slab in order that they may not interfere with the reinforcement of the floor system.

Columns of this type require three or four SPIRAL SPACERS which retain the spiral in its desired position. The rods comprising the VERTICAL REINFORCEMENT should always be sufficiently separated to permit the concrete to thoroughly surround the steel. Six ½-in round rods should be considered a minimum in columns of this type.

The Effective Area of Columns. The EFFECTIVE AREA of a column is that area which is considered as capable of carrying load. In the case of rodded columns some city ordinances and many designers permit the GROSS CROSS-SECTION to be used. Many others demand that the two outer inches be considered only as fireproofing. This makes a considerable difference in small

* The minimum allowed by the Joint Code is ½%, and the maximum to be considered in the computations is 2%; both percentages are based on the total area of the column.

columns; for example, a 12-in × 12-in column under the first method would have 144 sq in of effective sectional-area and only 64 sq in under the second method. Whatever method is applied, the percentages of reinforcement are always based upon the area that is considered effective.

In the case of spiralled columns there is no difference in treatment; the SECTION ENCLOSED WITHIN THE SPIRAL IS ALWAYS CONSIDERED AS THE EFFECTIVE AREA and the concrete, usually 2 in thick outside of the spiral, is disregarded.

When computing the load-carrying capacity of CORED COLUMNS the value of the concrete casing is also disregarded, the structural steel core being designed to carry the entire load.

Design Formulas for Rodded Columns. The design of rodded columns is practically the same in all parts of the country. The total safe AXIAL-LOAD is computed as the sum of the loads which can be carried by the concrete and the steel. Each is found by multiplying the effective area of the material by its working stress in compression.

Let A = total effective cross-sectional area of column, in square inches;
A_O = section-area of longitudinal steel, in square inches;
A_C = effective section-area of concrete, in square inches;
p = steel-ratio, A_O/A;
P = total safe axial-load on column the length of which does not exceed 11 times its least cross-sectional dimension, in pounds;
n = ratio of modulus of elasticity of steel to modulus of elasticity of concrete;
f = average compressive unit stress, P/A, in the column, in pounds per square inch;
f_c = average compressive unit stress in concrete, in pounds per square inch;
f'_c = ultimate crushing unit strength of concrete at age of 28 days; in pounds per square inch;
f'_s = compressive unit stress in vertical steel, in pounds per square inch.

Then

$$P = f_c A_c + f'_s A_o$$

Since the unit compressive stress in the steel can be considered equal to the unit compressive stress in the concrete, multiplied by the ratio of the modulus of elasticity of concrete, f'_s is replaced by nf_c and

$$P = f_c A + nf_c A_o$$

But

$$A_c = A - A_o$$

Substituting,

$$P = f_c [(A - A_o) + nA_o] = f_c [A + (n - 1) A_o]$$

Replacing A_o by pA and transposing

$$P = f_c A [1 + (n - 1) p]$$

Let

f = the average compressive unit stress $\frac{P}{A}$, in the column, in lb per sq in,

Then

$$f = \frac{P}{A} = f_c [1 + (n - 1) p]$$

Tables X, XI, XII, XIII, and XIV have been developed to assist in the design of hooped and spiralled columns.

Design Procedure for Rodded Columns. (1) Determine the value of f equal to $f_c\,[1 + (n - 1)\,p]$ for the required concrete stress, value of n and minimum percentage of vertical steel which the code requirements and good judgment permit. It should be remembered that as the value of n is less for the richer mixtures of concrete, there is a reduction in the load-bearing value of the reinforcement as that of the concrete is increased. For example, if $f_c = 500$, n is assumed as 15 and $f'_s = 7\,500$ lb per sq in. If $f_c = 600$, n is assumed as 12 and the unit compressive stress on the reinforcement f'_s would be 7 200 lb per sq in.

(2) Compute the effective area of the column by the formula $A = \dfrac{P}{f}$.

(3) From Tables X and XI pick out a column having an effective cross-section equal to or greater than this value.

(4) From Tables X and XI choose the vertical reinforcement to give a sectional area corresponding to the value of p assumed in (1).

(5) Note the size and spacing of hoops, laps and the thickness of insulation for fire or weather resistance. Knowing the size and percentage of reinforcement, the load-carrying capacity of a column may be checked by the formula

$$P = f_cA\,[1 + (n - 1)\,p]$$

Typical Design for Rodded Columns

These examples apply to columns of any section (round, square, oblong, etc.) provided that the least lateral dimension is not less than one-fifteenth of the unstayed height.

Specification data: $f_c = 600$ lb per sq in

$n = 12$

Minimum ratio of vertical steel; 0.5% based on net area of cross-section: fireproofing 2 in thick.

Example 1. Design a round column to carry an axial load of 160 000 lb, including the weight of the column. For the sake of economy use the minimum amount of vertical reinforcement, *i.e.*, 0.5% of the net sectional area.

(1) The value of the average unit compressive stress, $\dfrac{P}{A}$, is then,

$$f = f_c\,[1 + (n - 1)\,p] = 633 \text{ lb per sq in.}$$

(2) $A = P/f = 160\,000/633 = 252$ sq in.

(3) From Table XI it is seen that this area is given by an 18-in round column, gross diameter 22 in.

(4) Area of vertical steel $A_0 = A \times p = 254.4 \times 0.005 = 1.28$ sq in.

(5) Accept 5⅝-in round rods with ¼-in round hoops spaced at ⅝ in × 15 = 9 in on centers. The steel rods should be placed 2 in inside of the surface. Splices at floor levels should be lapped 24 in.

Example 2. If it is desired to compute the maximum load that a round column of 22-in gross diameter can carry under any specified stresses, the procedure is to use the maximum allowable percentage of vertical reinforcement and to apply formula

$$P = f_cA\,[1 + (n - 1)\,p]$$

If 4% of steel is used with the same stresses as employed in the first example,

$$P = 600 \times 254.4\,[1 + (12 - 1) \times 0.04] = 219\,800 \text{ lb}$$

The reinforcement is then $A_0 = A \times p = 254.4 \times 0.04 = 10.18$ sq in. Accept 13 1-in round rods with ¼-in round hoops spaced at 12 in on centers. The steel should be placed 2 in inside of the surface. Splices at floor levels should be lapped 24 in. In this case it would be more economical to use either a spiral reinforcement (Fig. 40), or a somewhat larger section instead of so much vertical steel (about 35 lb per lin ft).

Design Procedure for Columns with Spiral Reinforcement. Owing to the lack of uniformity in the design of spiralled columns, it is impossible to offer any method which will be generally applicable. If the designer is governed by the requirements of a local building ordinance, there is very little choice in method.

Where the designer is free to select his own formulas, those proposed by the Joint Code offer a satisfactory solution. This recommendation specifies that:

"The permissible axial load on columns reinforced with longitudinal bars and closely spaced spirals enclosing a circular core, shall not be greater than that determined by formula

$$P = A_c\,[1 + (n - 1)p]\,f_c$$

in which A_c is the area of the concrete core within the outer circumference of the spiral hooping, and the value of f_c is given by the formula,

$$f_c = [300 + (0.10 + 4p)\,f'_c]$$

"The longitudinal reinforcement shall consist of at least six bars of minimum diameter of ½ in and of an effective cross-sectional area not less than 1%, nor more than 6% of that of the core. The number of longitudinal bars concentrated in the ring at the periphery of the core shall be governed by the spacing requirements previously cited. If all the bars cannot be placed at the periphery of the core, the bars within shall be stayed at intervals of 24 in, and shall not be nearer to the outer ring than two-tenths times the core diameter. When the ratio of reinforcement in a spirally reinforced column is greater than 4%, special placing drawings illustrating the proper distribution of steel shall be submitted with the detail plans. Splices in longitudinal reinforcement shall provide a lap of at least 24-bar diameters for deformed bars, and 30 diameters for plain bars.

"The ratio of the spiral reinforcement shall be not less than one-fourth the ratio of the longitudinal reinforcement. Spiral reinforcement shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical spacer bars. At the ends of all spirals and at points of splice, the outside diameter shall be maintained. The spacing of the spirals shall not be greater than one-sixth of the diameter of the core and in no case more than 3 in.

"Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core which shall have a minimum thickness of 1½ in."

It is generally economical, however, to use as low a percentage of vertical steel and as high a percentage of spiral steel as the code or specification permits. Having determined upon these percentages, the value of $f = \dfrac{P}{A}$ may be computed for the most economical combination of concrete mixture,

vertical and spiral steel. Individual columns can then be designed by dividing this value into the column load, giving an effective area which will be satisfactory provided the designer then chooses vertical steel and spiralled reinforcement to meet the assumed percentages. As it is not always possible to obtain the exact values desired, some adjustment may be necessary and, within the limits previously given, vertical steel can always be added when it is necessary to reduce the diameter of a column. Tables XII, XIII and XIV give spiral percentages which are of general application.

Design Procedure for Cored Columns. Cored columns are customarily used through heights of several stories; the section is designed to carry the load in the uppermost story, and plates added through the lower stories to take the increments due to floor loads.

The STRUCTURAL STEEL CORE is designed to carry the entire load employing whatever fiber stress may be allowed in compression for the grade of steel used. The spiral, of diameter equal to the maximum horizontal dimension of the core, plus 4 in, is designed to supply a steel volume equal to ½% of the enclosed core. Four in of stone-concrete fireproofing is then designed to surround the structural-steel core and this shell is reinforced by vertical rods, their sectional area totaling ½% of the area within the spiral. The limiting unstayed height to which this method of design should be applied is given by the ratio of l/r which should not exceed the value of 15.

Typical Design for Spiralled Columns

Spiralled columns are usually round in section, but this type of reinforcement may be used in columns of any shape provided that the least lateral dimension of the core, enclosed by the spiral, is not less than one-fifteenth of the unstayed height.

Specification data: $f_c = 600$ or $f'_c = 2\,500$ lb per sq in

$$n = 12$$

Design a round column to carry an axial load of 220 000 lb, including the weight of the column:

Example 1, in accordance with the New York City Building Code. This ordinance places limits of 1% and 4% on the amount of vertical reinforcement, and confines the spiral within the range from ½% to 2%. Assuming that the minimum allowable percentage of vertical steel is satisfactory, a value of $f = \dfrac{P}{A}$ is found for 1% vertical and 2% spiral. From the building ordinance,

$$f = \frac{P}{A} = f_c\,[1 + (n - 1)\,p] + 2p' f_{s_1}$$

in which p' = percentage of spiral, based on the volume of the enclosed core;
f_{s1} = unit working tensile stress in spiral reinforcement (20 000 lb per sq in for cold drawn steel wire)

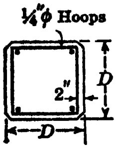
(1) $f = 600\,[1 + (12 - 1)0.01] + 2 \times 0.02 \times 20\,000 = 1\,466$ lb per sq in

(2) $A = P/f = 220\,000/1\,466 = 150$ sq in

(3) This area is given by a 14-in (154 sq in) round column. Allowing the minimum thickness of fireproofing permitted by this code, 2 in, the gross diameter is 18 in.

(4) Vertical steel $A_0 = A \times p = 154 \times 0.01 = 1.54$ sq in. Accept 8 ½-in round rods.

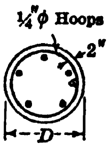
Table X. Safe Loads on Concrete in Units of 1 000 Lb for Square Concrete Columns, Hooped



D = width in inches
A = D^2 = gross section-area in square inches
P = safe loads on gross area of concrete in units of 1 000 lb

1	2	3	4	5	6
D in	A sq in	P at 500 lb per sq in	P at 600 lb per sq in	A ×0.005	A ×0.04
12	144	72.0	86.4	0 72	5.76
13	169	84 5	101 4	0 85	6.76
14	196	98.0	117.6	0.98	7.84
15	225	112.5	135 0	1.13	9.00
16	256	128.0	153 6	1.28	10.24
17	289	144 5	173 4	1.45	11.56
18	324	162 0	194 4	1 62	12.96
19	361	180 5	216 6	1 81	14.44
20	400	200 0	240 0	2.00	16.00
21	441	220 5	264 6	2.21	17.64
22	484	242.0	290.4	2.42	19.36
23	529	264 5	317 4	2.65	21 16
24	576	288.0	345.6	2.88	23.04
25	625	312 5	375 0	3.13	25.00
26	676	338 0	405.6	3 38	27 04
27	729	364 5	437.4	3 65	29.16
28	784	392 0	470.4	3.92	31.36
29	841	420 5	504 6	4 21	33.64
30	900	450 0	540 0	4.50	36.00
31	961	480 5	576.6	4 81	38.44
32	1 024	512.0	614.4	5.12	40.96
33	1 089	544.5	653 4	5.45	43.56
34	1 156	578.0	693.6	5.78	46.24
35	1 225	612.5	735.0	6.13	49.00
36	1 296	648 0	777.6	6 48	51.84
37	1 369	684 5	821 4	6.85	54.76
38	1 444	722.0	866 4	7 22	57.76
39	1 521	760.5	912 6	7 61	60.84
40	1 600	800 0	960 0	8.00	64.00
41	1 681	840 5	1 008 6	8.41	67.24
42	1 764	882.0	1 058.4	8 82	70.56
43	1 849	924.5	1 109.4	9 25	73.96
44	1 936	968 0	1 161.6	9 68	77 44
45	2 025	1 012.5	1 215.0	10 13	81.00
46	2 116	1 058.0	1 269.6	10.58	84.64
47	2 209	1 104 5	1 325.4	11.05	88.36
48	2 304	1 152 0	1 382.4	11.52	92.16
49	2 401	1 200 5	1 440.6	12.01	96.04
50	2 500	1 250 0	1 500.0	12 50	100.00
51	2 601	1 300 5	1 560.6	13.01	104.04
52	2 704	1 352.0	1 622.4	13.52	108.16
53	2 809	1 404.5	1 685.4	14 05	112.36
54	2 916	1 458.0	1 749.6	14.58	116.64
55	3 025	1 512.5	1 815 0	15.13	121.00
56	3 136	1 568.0	1 881.6	15.68	125.44
57	3 249	1 624.5	1 949.4	16.25	129.96
58	3 364	1 682.0	2 018.4	16.82	134.56
59	3 481	1 740.5	2 088.6	17.41	139.24
60	3 600	1 800.0	2 160.0	18.00	144.00
61	3 721	1 860.5	2 232.6	18 61	148.84

Table XI. Safe Loads on Concrete in Units of 1 000 Lb for Round Concrete Columns, Hooped



D = diameter in inches
A = 0.7854 D^2 = gross section-area, in square inches
P = safe loads on gross area of concrete, in units of 1 000 lb

1	2	3	4	5*	6
D in	A sq in	P at 500 lb per sq in	P at 600 lb per sq in	A ×0.005	A ×0.04
12	113.1	56.55	67.86	0 56	4.52
13	132.7	66.35	79.62	0.67	5.30
14	153.9	76.95	92.34	0.77	6.15
15	176.7	88.35	106.02	0.89	7.06
16	201.0	100.50	120.60	1.01	8.04
17	227.0	113.50	136.20	1.14	9.08
18	254.4	127.20	152.64	1.28	10.17
19	283.5	141.75	170.10	1.42	11.34
20	314.1	157.05	188.46	1.58	12.56
21	346.3	173.15	207.78	1.74	13.85
22	380.1	190.05	228.06	1.91	15.20
23	415.4	207.70	249.24	2.08	16.61
24	452.3	226.15	271.38	2.27	18.09
25	490.8	245.40	294.48	2.46	19.63
26	530.9	265.45	318.54	2.66	21.23
27	572.5	286.25	343.50	2.87	22.90
28	615.7	307.85	369.42	3.08	24.62
29	660.5	330.25	396.30	3.31	26.42
30	706.8	353.40	424.08	3.54	28.27
31	754.7	377.35	452.82	3.78	30.18
32	804.7	402.10	482.52	4.03	32.16
33	855.3	427.65	513.18	4.28	34.21
34	907.9	453.95	544.74	4.54	36.31
35	962.1	481.05	577.26	4 82	38.48
36	1 017.8	508.90	610.68	5.09	40.71
37	1 075.2	537.60	645.12	5.38	43.00
38	1 134.1	567.05	680.46	5.68	45.36
39	1 194.5	597.25	716.70	5.98	47.78
40	1 256.6	628.30	753.96	6.29	50.26
41	1 320.2	660.10	792.12	6.61	52.80
42	1 385.4	692.70	831.24	6.93	55.41
43	1 452.2	726.10	871.32	7.27	58.08
44	1 520.5	760.25	912.30	7.61	60.82
45	1 590.4	795.20	954.24	7.96	63.61
46	1 661.9	830.95	997.14	8.31	66.47
47	1 734.9	867.45	1 040.94	8.68	69.39
48	1 809.5	904.75	1 085.70	9.05	72.38
49	1 885.7	942.85	1 131.42	9.43	75.42
50	1 963.5	981.75	1 178.10	9.82	78.54
51	2 042 8	1 021.40	1 225.68	10.22	81.71
52	2 123.7	1 061.85	1 274.22	10.62	84.94
53	2 206.1	1 103.05	1 323.66	11.04	88.24
54	2 290.2	1 145.10	1 374.12	11.46	91.60
55	2 375.8	1 187.90	1 425.48	11.88	95.03
56	2 463.0	1 231.50	1 477.80	12.32	98.52
57	2 551.7	1 275.85	1 531.02	12.76	102.06
58	2 642.0	1 321.00	1 585.20	13.21	105.68
59	2 733.9	1 366.95	1 640.34	13.67	109.35
60	2 827.4	1 413.70	1 696.44	14.14	113.09
61	2 922.4	1 461.20	1 753.44	14.62	116.89

Table XII. Percentages of Spiral Reinforcement in Round Concrete Columns

Diameter of spiral reinforcement, ⅜ in

D = diameter of spiral in inches
p = pitch of spiral reinforcement in in
Asp = section-area of spiral reinforcement
% = percentage of spiral reinforcement $= 4 \times \frac{Asp}{Dp}$

Limits of percentage, %
Minimum = 0.5, maximum = 2.0
Limits of pitch, p
Maximum = $D/6$, and 3 in

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	Pitch of spiral wire in inches = p													
	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3
12					1.96	1.84								
13				1.94	1.81	1.70	1.60							
14			1.94	1.80	1.68	1.58	1.48	1.40						
15		1.96	1.81	1.68	1.57	1.47	1.38	1.31	1.24	1.17				
16		1.84	1.69	1.57	1.47	1.38	1.30	1.22	1.16	1.10	1.05			
17	1.89	1.73	1.59	1.48	1.39	1.30	1.22	1.15	1.09	1.04	0.99	0.94		
18	1.78	1.63	1.50	1.40	1.31	1.22	1.15	1.09	1.03	0.98	0.93	0.89	0.85	0 81
19	1.69	1.55	1.43	1.33	1.24	1.16	1.09	1.03	0.98	0.93	0.88	0.84	0.80	0 77
20	1.60	1.47	1.35	1.26	1.18	1.10	1.04	0.98	0.93	0.88	0.84	0.80	0.76	0.73
21	1.53	1.40	1.29	1.20	1.12	1.05	0.99	0.93	0.88	0.84	0.80	0.76	0.73	0 70
22	1.46	1.34	1.23	1.15	1.07	1.00	0.94	0.89	0.84	0.80	0.76	0.73	0.69	0.66
23	1.39	1.28	1.18	1.09	1.02	0.96	0.90	0.85	0.80	0.76	0.73	0.69	0.66	0 63
24	1.34	1.22	1.13	1.05	0.98	0.92	0.86	0.81	0.77	0.73	0.70	0.67	0 64	0.61
25	1.28	1.17	1.08	1.01	0.94	0.88	0 83	0.78	0.74	0.70	0.67	0.64	0.61	0 58
26	1.23	1.13	1.04	0.97	0.90	0.85	0.79	0.75	0.71	0.67	0.64	0.61	0.59	0.56
27	1.19	1.09	1.00	0.93	0.87	0.81	0.77	0.72	0.68	0.65	0.62	0.59	0.56	0.54
28	1.14	1.05	0.97	0.90	0.84	0.78	0.74	0.70	0.66	0.63	0.60	0.57	0.54	0.52
29	1.10	1.01	0.93	0.87	0.81	0.76	0.71	0.67	0.64	0.60	0.58	0.55	0 52	0.50
30	1.07	0.98	0.90	0.84	0.78	0.73	0.69	0.65	0.62	0.58	0.56	0.53	0.51	
31	1.03	0.95	0.87	0.81	0.76	0.71	0.67	0.63	0.60	0.56	0.54	0.51		
32	1.00	0.92	0.84	0.78	0.73	0.69	0.65	0.61	0.58	0.55	0.52	0.50		
33	0.97	0.89	0.82	0.76	0.71	0.67	0.63	0.59	0.56	0.53	0.51			
34	0.94	0.86	0.79	0.74	0.68	0.65	0.61	0.57	0.54	0.51				
35	0.91	0.84	0.77	0.72	0.67	0.63	0.59	0.56	0.53	0.50				
36	0.89	0.81	0.75	0.70	0.65	0.61	0 57	0.54	0.51					
37	0.86	0.79	0.73	0.68	0.63	0.59	0.56	0.53	0.50					
38	0.84	0.77	0.71	0.66	0.62	0.58	0.54	0.51						
39	0.82	0.75	0.69	0.64	0.60	0.56	0.53	0.50						
40	0.80	0.73	0.67	0.63	0.58	0.55	0.52							
41	0.78	0.71	0.66	0.61	0.57	0.53	0.50							
42	0.76	0.70	0.64	0.60	0.56	0.52								
43	0.74	0.68	0.63	0.58	0.54	0.51								
44	0.73	0.67	0.61	0.57	0.53	0.50								
45	0.71	0.65	0.60	0.56	0.52									
46	0.69	0.64	0.59	0.54	0.51									
47	0.68	0.62	0.57	0.53	0.50									
48	0.66	0.61	0.56	0.52										
49	0.65	0.60	0.55	0.51										
50	0.64	0.58	0.54	0.50										

Table XIII. Percentages of Spiral Reinforcement in Round Concrete Columns

Diameter of spiral reinforcement, ½ in

D = diameter of spiral in inches
p = pitch of spiral reinforcement in in
$A_s p$ = section-area of spiral reinforcement
% = percentage of spiral reinforcement $= 4 \times \frac{A_s p}{Dp}$

Limits of percentage, %
Minimum = 0.5, maximum = 2.0
Limits of pitch, p
Maximum = $D/6$, and 3 in

1	2	3	4	5	6	7	8	9	10	11	12	13	14
D	Pitch of spiral wire in inches = p												
	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3
12	...	...	...	...		...			...	...	...	...	...
13	...	...	...	...	...	...			...			...	...
14		...	...	...				...	...			...	...
15	...	...	...	...	...							...	...
16	...	...	...	...		...			1.96	1.86	...	...	...
17	...	...	...	...	...	...		1.94	1 85	1.76	1.68	...	...
18	...	...	...	...	...	...	1.94	1.83	1.74	1.66	1.58	1.51	1.45
19	...	...	...	...	...	1.94	1.83	1.74	1.65	1.57	1.50	1.44	1.37
20	...	...	...	...	1.96	1.85	1.74	1.65	1.57	1.49	1.43	1.36	1.30
21		...	...	1 99	1.87	1.76	1.66	1.57	1.49	1.42	1.36	1.30	1.24
22	...	...	...	1.90	1.78	1.68	1.58	1.50	1.43	1.36	1.30	1.24	1.19
23	...	...	1.95	1.82	1.70	1.60	1.51	1.43	1.36	1.30	1.24	1.18	1.13
24	...	...	1.87	1.74	1.63	1.54	1.45	1.37	1.31	1.24	1.19	1.14	1.09
25	...	1 93	1.79	1.67	1.57	1.47	1.39	1.32	1.25	1.19	1.14	1.09	1.04
26	...	1.86	1.72	1.61	1.51	1.42	1.34	1.27	1.20	1.15	1.10	1.05	1.00
27	1.94	1.79	1.66	1.55	1.45	1.37	1.29	1.22	1.16	1.10	1.06	1.01	0.96
28	1.87	1.72	1.60	1.49	1.40	1.32	1.24	1.18	1.12	1.06	1.02	0.97	0.93
29	1.80	1.66	1.54	1.44	1.35	1.27	1.20	1.14	1.08	1.03	0.98	0.94	0.90
30	1.74	1.61	1.49	1.39	1.31	1.23	1.16	1.10	1.04	0.99	0.95	0.91	0.87
31	1.69	1.56	1.44	1.35	1.26	1.19	1.12	1.06	1.01	0.96	0.92	0.88	0.84
32	1.63	1.51	1.40	1.31	1.22	1.15	1.09	1.03	0.98	0.93	0.89	0.85	0.81
33	1.58	1.46	1.36	1.27	1.19	1.12	1.05	1.00	0.95	0.90	0.86	0.82	0.79
34	1.54	1.42	1.32	1.23	1.15	1.08	1.02	0.97	0.92	0.87	0.84	0.80	0.76
35	1.49	1.38	1.28	1.19	1.12	1.05	0.99	0.94	0.89	0.85	0.81	0.78	0.74
36	1.45	1.34	1.24	1.16	1.09	1.02	0.96	0.91	0.87	0.83	0.79	0.75	0.72
37	1.41	1.30	1.21	1.13	1.06	0.99	0.94	0.89	0.84	0.80	0.77	0.73	0.70
38	1.37	1.27	1.18	1.10	1.03	0.97	0.91	0.87	0.82	0.78	0.75	0.71	0.68
39	1.34	1.24	1.15	1.07	1.00	0.94	0.89	0.84	0.80	0.76	0.73	0.70	0.67
40	1.31	1.21	1.12	1.04	0 98	0.92	0.87	0.82	0.78	0.74	0.71	0.68	0.65
41	1.27	1.17	1.09	1.02	0.95	0.90	0.85	0.80	0.76	0.72	0.69	0.66	0.63
42	1.24	1.15	1.06	0.99	0.93	0.88	0.83	0.78	0.74	0.71	0.68	0.65	0.62
43	1.21	1.12	1.04	0.97	0.91	0.85	0.81	0.76	0.73	0.69	0.66	0.63	0.60
44	1.19	1.09	1.02	0.95	0.89	0.84	0.79	0.75	0.71	0.68	0.65	0.62	0.59
45	1.16	1.07	0.99	0.93	0.87	0.82	0.77	0.73	0.69	0.66	0.63	0.60	0.58
46	1.13	1.05	0.97	0.91	0.85	0.80	0.75	0.71	0.68	0.65	0.62	0.59	0.56
47	1.11	1.03	0.95	0.89	0.83	0.78	0.74	0.70	0.66	0.63	0.60	0.58	0.55
48	1.09	1.00	0.93	0.87	0.81	0.77	0.72	0.68	0.65	0.62	0.59	0.56	0.54
49	1.07	0.98	0.91	0.85	0.80	0.75	0.71	0.67	0.64	0.61	0.58	0.55	0.53
50	1.04	0.96	0.89	0.83	0.78	0.73	0.69	0.66	0.62	0.59	0.57	0.54	0.52

Table XIV. Percentages of Spiral Reinforcement in Round Concrete Columns

Diameter of spiral reinforcement, ⅝ in

D = diameter of spiral in inches
p = pitch of spiral reinforcement in in
A_sp = section-area of spiral reinforcement
% = percentage of spiral reinforcement $= 4 \times \frac{A_sp}{Dp}$

Limits of percentage, %
Minimum = 0 5, maximum = 2.0
Limits of pitch, p
Maximum = $D/6$, and 3 in

1	2	3	4	5	6	7	8	9	10	11	12	13
	Pitch of spiral wire in inches = p											
D	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3
12	....	. .	. . .	..	....	.	. .	.	. .			
13	....		. . .	. .		.	. . .	.	.	.		
14	.....	.	. .	.	..	..	.	. .	.	.		
15	. .	. . .	.	.	...			.	.	.	.	
16	.....	.	. .	. .		.		.	.	.	.	
17	.. ..	.. .	. .	.		.	.			.		
18	.. .	.. .	.	.	. .	.			. .			
19	.....	. . .	.	....	.	.			.		. .	
20	.. .	....	.	..	..		.	.	. .		.	
21	....	. ..	.	.	.	.			.	.		1.94
22	....	....	.		.			.		.	1.94	1.86
23	. ..	....	.	...	.				.	1.94	1.86	1 77
24	.. .	. .	. .	.	.	. .	.		1 94	1.86	1.78	1.70
25	.....	. .	.			.	.	1 96	1.87	1.78	1.71	1.63
26	.....	. .	.	.	.		1.98	1.89	1.80	1.71	1.64	1.57
27	. . . .	. ..	.	.			1.91	1.82	1.73	1.65	1.58	1.51
28	....	.. .	.. .	. .		1.95	1.84	1.75	1.67	1.59	1.52	1.46
29	... .	. .		.	1.99	1 88	1.78	1.69	1.61	1.54	1.47	1.41
30	.....	. ..	.	.	1.92	1.82	1.72	1.64	1.56	1.49	1.42	1.36
31	....	. .	. .	1.98	1.86	1.76	1.66	1.58	1.50	1.44	1 38	1.32
32	.. .	. .	.	1.92	1.80	1.70	1 61	1.53	1.46	1.39	1.33	1.27
33	.....	...	1 98	1.86	1.75	1.65	1.56	1.49	1 41	1 35	1 29	1 24
34	.....	. .	1.92	1.80	1.70	1.60	1.52	1.44	1.37	1.31	1.25	1.20
35	.. ..	.. ..	1.87	1.75	1 65	1.55	1.47	1.40	1.33	1.27	1.22	1.16
36	.....	1.95	1.82	1.70	1.60	1.51	1.43	1.36	1.29	1.24	1.18	1.13
37	.....	1.89	1.77	1.66	1 56	1.47	1.39	1.33	1.26	1.20	1.15	1.10
38	1.99	1.84	1.72	1.61	1.52	1.43	1.36	1.29	1.23	1.17	1.12	1.07
39	1.94	1.80	1.68	1.57	1.48	1.40	1.32	1.26	1.19	1.14	1 09	1 05
40	1.89	1.75	1.64	1.53	1.44	1.36	1.29	1.23	1.17	1.11	1.06	1 02
41	1.84	1.71	1.59	1.49	1.41	1.33	1.26	1.19	1.14	1.09	1.04	0.99
42	1.80	1.67	1.56	1.46	1.37	1.30	1.23	1.17	1.11	1.06	1.01	0.97
43	1.76	1.63	1.52	1.42	1.34	1.27	1.20	1.14	1.08	1.03	0.99	0.95
44	1.72	1.59	1.49	1.39	1.31	1.24	1.17	1.11	1.06	1.01	0.97	0.92
45	1.68	1.56	1.45	1.36	1.28	1.21	1.14	1.09	1.04	0.99	0.95	0.90
46	1.64	1.52	1.42	1.33	1.25	1.18	1.12	1.06	1.01	0 97	0.92	0.88
47	1.61	1.49	1.39	1.30	1.23	1.16	1.10	1.04	0.99	0.95	0.91	0.87
48	1.57	1.46	1.36	1.28	1.20	1.13	1.07	1.02	0.97	0.93	0.89	0.85
49	1.54	1.43	1.33	1.25	1.18	1.11	1.05	1.00	0.95	0.91	0.87	0.83
50	1.51	1.40	1.31	1.22	1.15	1.09	1.03	0.98	0.93	0.89	0.85	0.81

(5) Spiral steel, from Table XII, $\frac{3}{8}$-in reinforcement, $1\frac{5}{8}$-in pitch (1.94%).

(6) The vertical rods are placed inside the spiral which requires, under this code, 4 spiral spaces. The vertical steel should be lapped 18 in at floor levels and the spiral stopped sufficiently below the level of the floor to avoid interference with the reinforcement of the floor slabs or girders.

Example 2, in accordance with the Joint Standard Building Code. This specification places limits of 1% and 6% on the amount of vertical steel, and requires that the ratio of the spiral reinforcement be not less than one-fourth the ratio of the longitudinal reinforcement. Assuming a ratio of 4% vertical and 1% spiral, a value of $f = \frac{P}{A}$ is determined. From the code,

(1) $f_c = [300 + (0.10 + 4 \times 0.04) \times 2\,500] = 950$ lb per sq in.

$$f = \frac{P}{A} = [1 + (n - 1)\,p]\,f_c = [1 + (11 \times 0.04)] \times 950 = 1\,370 \text{ lb}$$

per sq in.

(2) $A = P/f = 220\,000/1\,370 = 161$ sq in.

(3) This area is given by a 15-in (177 sq in) round column (Table XI) Allowing the minimum thickness of fireproofing permitted by this code $1\frac{1}{2}$ in, the gross diameter is 18 in.

(4) Vertical steel $A_0 = A \times p = 177 \times 0.04 = 7.06$ sq in. Accept 12 $\frac{7}{8}$-in round rods.

(5) Spiral steel, from Table XII, $\frac{3}{8}$-in reinforcement, $2\frac{7}{8}$-in pitch (1.02%).

(6) The vertical rods are placed inside the spiral which requires, under this code, 3 spiral spaces. The vertical steel should be lapped 24 bar-diameters for deformed bars and 30 diameters for plain rods or bars.

Design of Reinforced-Concrete Footings

Loads on Footings. The load used in determining the soil-bearing area of a footing is the load used in the design of the column in the story immediately above the footing considered (which load has already been subjected to any allowable live load reductions, see under Loads on Columns) plus any live load or dead load in that story, plus the weight of the pedestal if there is one, plus the estimated weight of the footing itself. Basement floors resting on the material of the foundation bed are not ordinarily considered as contributing to the loads on footings.

The design load used in determining the thickness and reinforcement of a footing slab, is the load as defined above, less the weight of the footing. In the design of SOIL FOOTINGS this load is assumed to be uniformly distributed over the area of the footing; when designing footings over PILE GROUPS, the pile reactions are treated as concentrated loads, each acting at the center line of the pile and equal to the safe-bearing capacity of the pile.

Proportionment of Footing Areas. The proportionment of footing areas follows the principles outlined in Chapter II.

Pile Foundations under Concrete Footings. The supporting power of wood and concrete piles is computed as described in Chapter II. The concrete composing the footing slab is designed of sufficient size and thickness to cover the piles and to provide for the various stresses. The design of footing slabs over either WOOD OR CONCRETE PILES is entirely similar to the design of footings upon soil except that it is customary to compute the punching shears around the perimeters of piles located near the edges of the footing slab and to base the computations for diagonal tension and bending moment upon the

individual concentrations represented by the pile reactions, instead of the uniform load per sq ft represented by the soil reaction.

The butts of WOODEN PILES should extend from 4 to 6 in into the concrete of the footing-slab, and there should be from 3 to 4 in of concrete between the tops of the piles and the reinforcement. (See Fig. 42.)

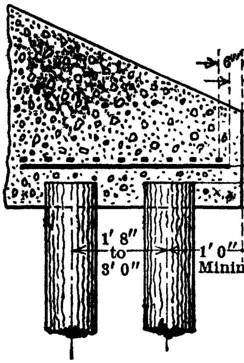


Fig. 42. Detail of Reinforced-Concrete Footing on Wood Piles

CONCRETE PILES are extended at least 4 in into the concrete of the footing-slab, and where a steel casing surrounds the pile, 3 to 4 in of concrete is required between the tops of the piles and the slab reinforcement, unless the casing is trimmed back this distance, in which case the reinforcement is allowed to lie directly upon the butts of the piles. (See Fig. 43.)

Types of Concrete Footings. PLAIN CONCRETE FOOTINGS are used only for low, wall-bearing structures and for columns resting upon solid rock. The width of a wall footing for a concrete, brick, or masonry bearing wall is determined by dividing the load per lin ft, including an assumed weight of the footing, by the allowable soil pressure. The footing may then be designed without reinforcement with a sufficient number of offsets, or steps, to obtain the required width, each step projecting beyond the face of the one above, a distance equal to one-half the thickness of the step.

PLAIN CONCRETE COLUMN FOOTINGS may be designed by determining the footing area in the usual way and employing a stepped, or pyramidal form of such shape that all re-entrant angles lie outside of a line drawn from the base of the footing, at an angle of 60° to the horizontal.

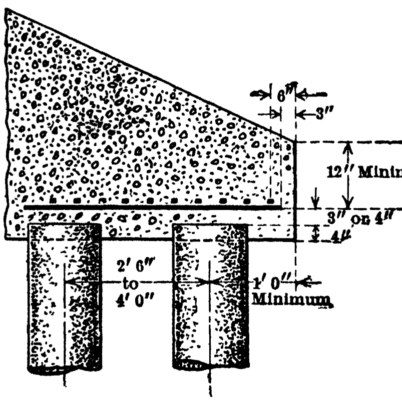


Fig. 43. Detail of Reinforced-Concrete Footing on Concrete Piles

REINFORCED-CONCRETE FOOTINGS (Fig. 44) may be roughly divided into five groups; WALL FOOTINGS, INDEPENDENT COLUMN FOOTINGS, COMBINED FOOTINGS, carrying usually two columns and constructed as inverted beams, CANTILEVER FOOTINGS, in which the eccentricity of an exterior footing is

resisted by a strap connected with an adjacent interior footing, and CONTINUOUS FOOTINGS constructed as inverted beams and carrying a number of columns. All of these types of footings may be used either with or without piles.

General Principles Governing the Design of Reinforced-Concrete Footings. The design of reinforced-concrete footings employes the same formulas as were developed for concrete beams and slabs. The footing must be sufficiently strong to resist stresses produced by bending moments and diagonal tension. Most building ordinances also require a computation for a stress known as

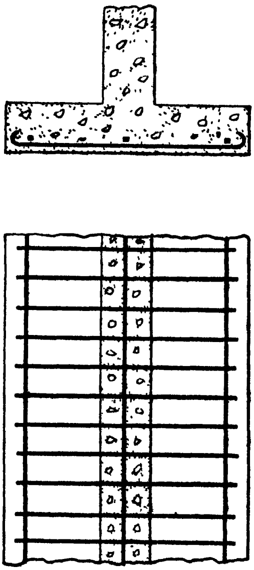


Fig. 44. Reinforced-Concrete Wall Footing

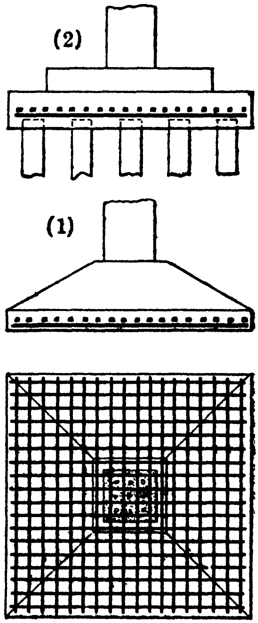


Fig. 45. Footing for One Column: (1) Sloped, (2) Stepped

"punching shear" which is supposed to insure the security of the footing against being punched through by the superimposed column. In the case of pile footings, as mentioned above, it is also necessary to consider this matter of punching shear around the heads of the piles.

Footings for INDEPENDENT COLUMNS are designed in both STEPPED and PYRAMIDAL FORMS (Fig. 45); the former is probably preferable owing to the simplicity of the form work, although the pyramidal shape uses less concrete. Present practice tends towards the use of a two-way reinforcement without diagonal steel in footing slabs of this type. The plan is normally square, but an oblong shape is satisfactory where space is limited. The use of a pedestal between the footing-slab and the base of the superimposed column is of value, not only in reducing the unit compression on the top sur-

face of the footing, but also in furnishing an easy means of leveling up the footings, when they are of different heights, and thus obtaining greater uniformity in column-construction.

Except where pedestals are unusually high, the reinforcement consists of vertical rods, or dowels, connecting the footing with the basement column. The dowels should extend into the column or pedestal and into the footing a distance of 30 diameters for plain steel and 24 diameters for deformed rods or bars. Their sectional area and number should be equivalent to the vertical reinforcement in the supported column. The SIZE OF A PEDESTAL is determined by the allowable compressive stress in the concrete, normally 1/4 of the ultimate compressive strength at an age of 28 days.

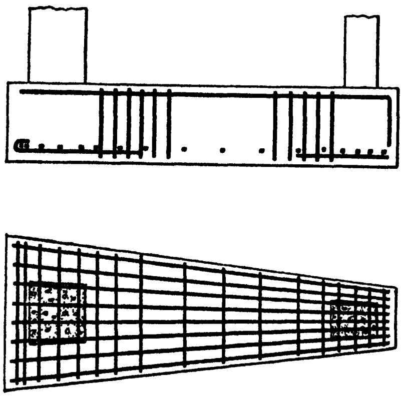


Fig. 46. Combined Footing for Two Columns

Good practice limits the COMPRESSIVE STRESS on top of a pedestal, or footing, directly under the columns to the value given by the following formula:

$$r_a = 0.25 f'_c \sqrt[3]{\frac{A}{A'}}\,*$$

in which r_a = permissible working stress over the loaded area, in pounds per square inch;

A = total area at the top of the pedestal, or footing, in square inches;

A' = loaded area at the column-base, in square inches;

f'_c = ultimate compressive strength of the concrete, in pounds per square inch.

In sloped or stepped footings A may be taken as the area of the top horizontal surface of the footing, or as the area of the lower base of the largest frustum of a pyramid, or cone, contained wholly within the footing, and having for its upper base the loaded area A', and having side slopes of 1 vertical to 2 horizontal.

The COMBINED FOOTING, carrying two columns, is used where it is impossible, owing to the proximity of a property line, or other obstruction, to center an exterior footing beneath its supported column. A footing-slab is constructed of sufficient size to carry the combined loads of the eccentric column and an adjacent, interior column, and placed so that the center of gravity of the combined loads passes through the center of gravity of the footing area. The plan is normally rectangular as this shape is simpler to build and easier to design. Where obstructions prevent sufficient elongation to enable a pro-

* If the pedestal is reinforced by a spiral, the Joint Code allows a somewhat increased value.

portionment of length to breadth which permits the center of gravity of the area to coincide with the center of gravity of the superimposed loads, a trapezoidal plan is used. (See Figs. 46 and 47.)

Fig. 47. Cantilever Footing for Two Columns

The CANTILEVER FOOTING is resorted to under the same conditions as the combined footing, namely to overcome the eccentricity of an exterior footing which cannot be built concentric with the superimposed column. This type is cheaper to build than the combined footing and is ordinarily used where the loads do not require the greater bearing area supplied by the latter.

The CONTINUOUS FOOTING (Fig. 48) is merely an inverted beam distributing the concentrated loads from the columns over the soil beneath. This type can be advantageously employed when the loads and the column spacing are such as to supply sufficient footing-area without making the offset on the interior of the building wider than that which is permitted on the exterior.

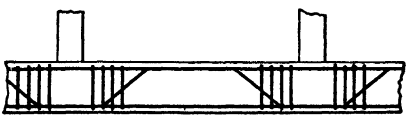


Fig. 48. Continuous Footing for More than Two Columns

Notation Applying to Footing Formulas.

a = side of square column, or pedestal, in feet;
A_s = area of cross-section of main tensile reinforcement in square inches;
b = width of footing or of the section considered, parallel to the wall or face of column in inches;
d = effective depth of section in inches;
D = diameter of round column in feet;
e = the width of the horizontal top of the footing, in feet;
f_c = compressive unit stress in extreme fibers of concrete in pounds per square inch;
f_s = tensile unit stress in the steel in pounds per square inch;
j = ratio of distance between center of compression of concrete and center of tension of steel to effective depth of section; ratio of arm of resisting couple to d;
jd = distance between center of compression in concrete and center of tension in steel; arm of resisting couple in inches;
k = ratio of distance of neutral axis of cross-section from extreme fibers in compression to effective depth of section;
kd = distance of neutral axis from extreme fibers in compression in inches;
l = length of the projection of the footing-slab, or distance between center lines of columns in combined footings, in feet;
L = length of footing in feet;
M = bending moment in inch-pounds;
n = modulus of elasticity of steel divided by modulus of elasticity of concrete, E_s/E_c;
p = percentage of reinforcement = A_s/bd;
V = total vertical shear at section considered, in pounds;
v = unit shearing-stress in pounds per square inch;

v_2 = unit punching shear in pounds per square inch;
w = unit soil-pressure = design-load divided by footing-area in pounds per square foot.

Design Procedure for Concrete Wall Footings. (1) Determine the projection of the footing slab on each side of the wall by dividing the weight to be supported, in lb per lin ft, including the weight of the wall, and the estimated weight of the footing itself by the unit soil pressure in lb per sq ft. Then w = net soil pressure in lb per sq ft = design load in lb divided by footing area in sq ft

(2) Compute the maximum bending moment at the face of the wall by the formula

$$M = 6\, wl^2$$

(3) Compute the depth as governed by moment

$$d = \sqrt{\frac{M}{\frac{1}{2} f_c jkb}}$$

(4) Check the depth as governed by diagonal tension

$$v = \frac{[l - (d/12)] \times w}{10\,5 \times d}$$

(5) Compute the steel area per lin ft of footing by the formula

$$A_s = \frac{M}{f_s jd}$$

(6) Compute the bond stress and anchorage by the formulas

$$u = \frac{V}{\Sigma o j d}$$

$$l = (\tfrac{1}{4})\,\frac{(f_s)}{u}\,i$$

in which l = length in inches required to develop the full working stress in the reinforcement;
i = diameter of round rod or side of square bar.

(7) For pile-footings, check the depth as governed by the punching shear over the exterior row of piles

$$d = \frac{V}{v_2 b}$$

in which V = pile reaction in pounds;
b = perimeter of pile in inches.

Typical Design of a Reinforced-Concrete Wall Footing

Specification data: f_s = 18 000 lb per sq in
f_c = 650 lb per sq in
n = 15
v, limited to 60 lb per sq in (reinforcement anchored)
u, limited to 150 lb per sq in (deformed bars, anchored)
Soil load = 4 000 lb per sq ft

It is required to design a footing for a 24-in concrete bearing wall carrying a load, including its own weight, of 23 000 lb per lin ft.

(1) Estimating the weight of the footing at 1 000 lb per lin ft, the footing width is

$$24\,000/4\,000 = 6 \text{ ft}$$

The net soil pressure, $w = 23\,000/6 = 3\,835$ lb per sq ft

(2) Maximum moment at face of wall,

$$M = 6\,wl^2 = 6 \times 3\,835 \times 4 = 92\,040 \text{ in-lb}$$

(3) The depth as governed by moment is found by formula

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{92\,040}{100.8 \times 12}} = 8\ 7 \text{ in}$$

Accept 9 in.

(4) The depth is checked by formula

$$v = \frac{[l - (d/12)] \times w}{10\ 5 \times d} = \frac{(2\ 0 - 0\ 75) \times 3\,835}{10.5 \times 9} = 51 \text{ lb per sq in}$$

Accept a depth of $9 + 3 = 12$ in.

(5) $A_s = \dfrac{M}{f_s jd} = \dfrac{92\,040}{18\,000 \times 0\ 883 \times 9} = 0.64$ sq in

Try ½-in rounds, 3½ in on centers.

(6) $u = V/\Sigma ojd = \dfrac{2 \times 3\,835}{1.57 \times 3.4 \times 0.875 \times 9} = 180$ lb per sq in

in which 1.57 is the perimeter of the cross-section of a ½-in round rod and 3.4 the number per linear foot at the required spacing. As this stress is too high, the number of rods is recalculated on the basis of $u = 150$ lb per sq in, for anchored reinforcement

$$\text{No of rods} = \frac{2 \times 3\,835}{1.57 \times 0.875 \times 9 \times 150} = 4 \text{ per lin ft}$$

Accept ½-in round deformed rods spaced 3 in on centers: hook both ends.*

Design Procedure for Independent Column Footings (Fig. 50). (1) Determine the area of the footing-slab by dividing the load used in the design of the column in the story immediately above the footing, plus any live load or dead load in that story, plus the weight of the pedestal if there is one, plus the estimated weight of the footing itself, by the unit soil-pressure, in lb per sq ft. Then

w = net soil-pressure in pounds per square foot
= design-load divided by footing-area

Note that if the footing-areas are to be proportioned for equal settlement, the procedure given in Chapter II, Foundations. should be followed in determining the areas of the footing-slabs.

* The amount of steel required by such a design results in a very expensive footing; as the use of bearing walls is usually limited to low buildings, a plain concrete section is to be preferred.

(2) Compute the depth as governed by punching shear at the face of the column or pedestal by the formulas

$$d = \frac{V}{v_2 b}$$

$$d = \frac{(L^2 - a^2) \times w}{48\, a v_2} \text{ (for square footings with square columns)}$$

$$d = \frac{(L^2 - \pi D^2/4) \times w}{37.7\, D v_2} \text{ (for square footings with round columns)}$$

in which V = total vertical shear or upward reaction on the portion of the footing-slab tributary to the side of the column, or pedestal, of length b, in pounds per square foot;
b = the side of the column or pedestal in inches;
d = effective depth of footing at face of column, or pedestal, in inches

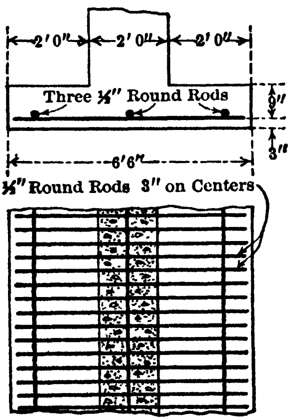


Fig. 49. Reinforced-Concrete Wall Footing

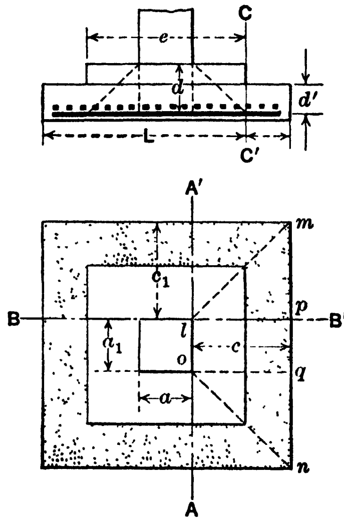


Fig. 50. Design Diagram for a Single Column Footing

For general notation see Notation Applying to Footing Formulas.

For pile-footings, check the depth as governed by the punching shear over the exterior row of piles

$$d = \frac{V}{v_2 b}$$

in which V = pile reaction in pounds;
b = perimeter of pile in inches.

(3) Compute the maximum bending moment at the face of the column or pedestal by the formulas

$$M = 6\,w\,(a_1 + 1.33\,c_1)\,c^2 \text{ (for oblong footings)}$$

$$M = 6\,w\,(a + 1.33\,c)\,c_1^2 \text{ (for oblong footings)}$$

$$M = 6\,w\,(a + 1.20\,c)\,c^2 \text{ (for square footings. with square or round columns)}$$

in which a = side of column, or pedestal or diameter of round column × 0.886, either in feet;

a_1 = the corresponding dimension, where two are required in the case of oblong footings;

c = projection, in feet, of the footing slab beyond the face of the column, or pedestal (or the equivalent square, for round columns);

c_1 = the corresponding dimension, where two are required in the case of oblong footings

For general notation see Notation Applying to Footing Formulas.

(4) The depth may be governed by the bending moment at the face of the column or pedestal. Check the compressive stress on the concrete at this section by the formula

$$f_c = \frac{2\,M}{jkbd^2}$$

in which b = the width of the horizontal top of the footing in in = 12 e.

(5) Determine the depth of the footing slab as governed by diagonal tension, at a distance out from the face of the column, or pedestal, equal to the effective depth of the footing. For pile footings, the shear is computed along a section distant $d/2$ from the face of the column or pedestal; the loads from piles with centers within this section are excluded when computing V.

$$d' = V/bjv$$

$$d' = \frac{[L^2 - \{a + (d/6)\}^2] \times w}{42 \times v \times [a + (d/6)]} \text{ (for square footings with square columns)}$$

in which d' = effective depth of footing in inches at a distance out from the face of the column, or pedestal, equal to the depth, d, of the footing;

V = total vertical shear, or upward reaction, on the portion of the footing outside of the section d', in pounds;

b = perimeter of the section d' (on all four sides of the column) in inches.

(6) The section-area of the steel, in each direction, is computed by formula

$$A_s = M/f_sjd$$

(7) The bond-stresses and anchorage are computed by formulas

$$u = V/\Sigma ojd$$

$$l = (1/4)(f_s/u)i$$

in which l = length in inches required to develop the full working stress in the reinforcement;

i = diameter of round rod, or side of square bar

For general notation see Notation Applying to Footing Formulas.

Typical Design of an Independent Reinforced-Concrete Column Footing

Specification data: f_s = 16 000 lb per sq in
f_c = 650 lb per sq in
n = 15
v, limited to 40 lb per sq in
v_2, (punching shear) limited to 120 lb per sq in
u, limited to 100 lb per sq in (deformed bars)
Soil load 4 000 lb per sq ft

It is required to design a footing for a column 30 in square sustaining a load of 475 000 lb, including the weight of the column.

The following example applies to structural steel columns as well as to concrete columns, the only difference being that, in the former case, the dimensions of the billet, or grillage, are used instead of the column dimensions.

(1) Estimating the weight of the footing at 50 000 lb,

$$\text{Area of footing slab} = \frac{525\,000}{4\,000} = 132 \text{ sq ft}$$

As a square footing is more economical than one of oblong shape, the dimensions are made 11′ − 6″ × 11′ − 6″

$$w = 475\,000/132 = 3\,600 \text{ lb per sq ft}$$

(2) By formula, the depth as governed by punching shear is

$$d = \frac{(L^2 - a^2) \times w}{48\, a\, v_2} = \frac{(11\ 5^2 - 2.5^2)\ 3\,600}{48 \times 2.5 \times 120} = 31.4 \text{ in}$$

As a trial make d = 32 in.

(3) By formula, the maximum moment (at face of column) is

$$M = 6\, w\, (a + 1.20\, c)\, c^2 = 6 \times 3\,600\, [2.5 + (1.20 \times 4.5)]\, 20.25$$
$$= 3\,465\,000 \text{ in-lb}$$

(4) By formula, the unit compressive concrete stress as governed by bending moment is

$$f_c = \frac{M}{1.99\ ed^2} = \frac{3\,465\,000}{1.99 \times 3 \times 32^2} = 566 \text{ lb per sq in}$$

in which 3 is the length of the flat top of the footing in feet. If type "b" or type "c" is used, the value of e will be 7 ft or 10 ft 6 in, respectively, and the stress f_c will be lower: it is seldom critical.

Accept a depth of 32 in, total depth 36 in.

(5) By formula, the depth as governed by diagonal tension is

$$d' = \frac{[L^2 - \{a + (d/6)\}^2] \times w}{1\,680 \times [a + (d/6)]} = \frac{11.5^2 - \{2.5 + (32/6)\}^2 \times 3\,600}{1\,680 \times [2.5 + (32/6)]} = 19.5 \text{ in}$$

Whatever type of footing is selected must have this depth at section $C—C'$ (Fig. 50), d distance from the face of the column base or pedestal.

(6) The sectional area of the reinforcement is

$$A_s = \frac{M}{f_s jd} = \frac{3\,465\,000}{16\,000 \times 0.875 \times 32} = 7.75 \text{ sq in}$$

As a trial, 18¾-in round rods are selected.

(7) By formula, the bond stress at the face of the column is

$$u = V/\Sigma ojd$$

Since the shear at each face of the square column is equal to

$$V = \frac{(11.5^2 - 2.5^2) \times 3\,600}{4} = 113\,200 \text{ lb}$$

$$u = \frac{113\,200}{18 \times 2.35 \times 0.875 \times 32} = 96 \text{ lb per sq in}$$

In the upper equation, 11.5^2 is the area of the footing slab, and 2.5^2 the area of the column base, both in square feet. In the lower equation, 2.35 is the perimeter of each of the 18¾-in round rods.

Accept the reinforcement as selected.

Design Procedure for Cantilever Footings. Where the proximity of another building, or property line, makes a concentric footing impossible, it is necessary to overcome the eccentricity of an exterior footing by means of combining the footing-slab with that of an adjacent interior column or by the use of a connecting strap, the function of which is to resist the bending moment caused by the eccentricity of the exterior footing. This strap is considered to act as a balanced cantilever and assumed to rest on the center of gravity of the exterior footing. Although the stresses in the strap are highly indeterminate, an approximate solution may be obtained by assuming the ends of the cantilevers free to rotate which results in a theoretical zero bending moment beneath each column. If the pressure under soil footings is assumed to be uniformly distributed over the entire footing area, the distance from the exterior edge of the footing to the section of zero shear, and of maximum moment, is found by dividing the column load P, by the soil reaction per lin ft of footing. For footings on piles, the section of maximum moment and zero shear is found by considering the column load in relation to the pile reactions. This section is usually along the center line of the last row of piles on the side toward the interior footing.

The following steps apply to the design of cantilever footings:

(1) Design the interior footing as an independent unit, following the procedure for individual column-footings, as given in this chapter, and neglecting any effect of the uplift.

(2) Compute the approximate area of the exterior footing for the load used in the design of the column, plus any additional load in the basement story, and an allowance of 25% of the column load to cover the weight of the footing, the weight of the strap, and the increment due to uplift.

(3) Knowing the distance e (Fig. 51), or eccentricity of the column in relation to the footing, calculate the amount of uplift equal to P_1e/l.

(4) Compute $R_1 = P_1 + P_1e/l$, and p, the reaction upon the footing per lin ft, in a direction parallel to the strap. Then, $a_2 = P_1/p$ and a_1 is also known.

(5) Compute the maximum bending moment in the strap by formulas:

For soil footings:

$$M = -P_1e \quad M = -P_1(a_1 - a_2/2)$$

For pile footings:

$$M = -P_1a_1 + (q_1a_2 + q_2a_3 + \ . \ . \ .)$$

(6) Assume a width, b, for the strap, usually equal to the column width

and compute the depth and steel area at the interior face of the exterior column by formulas

$$d = \sqrt{\frac{M}{Kb}}$$

$$A_s = M/14\,000\,d$$

Notation is given under Notation Applying to Footing Formulas.

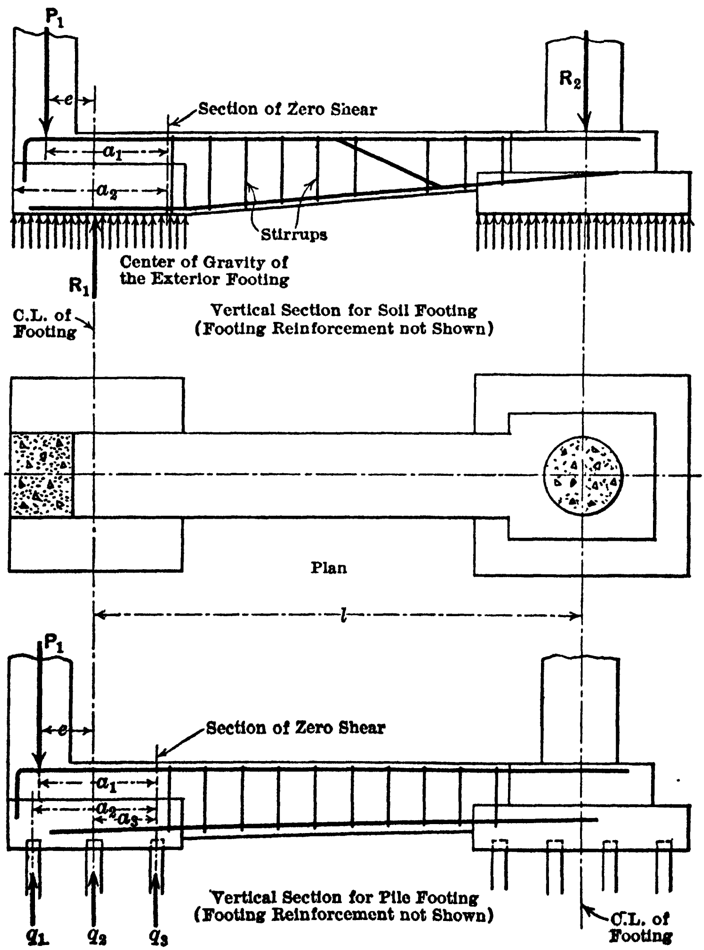


Fig. 51. Design Diagram for a Cantilever Footing

(7) Compute the vertical shear at the exterior face of the interior footing assuming that

$$V = P_1e/l$$

(8) Determine the section of the strap, as governed by shear at the exterior face of the interior footing by formula

$$d = V/vjb$$

(9) Check the concrete section and steel-area of the strap adjacent to the interior footing and provide for a positive bending moment in the strap, varying from one-quarter to one-half of the maximum negative moment. Add reinforcement, if necessary, along the bottom of the strap, to obtain an amount varying from one-fifth to one-third of that required at the top.

(10) Knowing the value of the up-lift and the size of the strap, determined by the computations in steps (5) to (9), check the load assumption in (2), and redesign if necessary.

(11) Provide proper anchorage for all the main longitudinal reinforcement at the top of the strap, particularly beneath the exterior column.

Typical Design of a Cantilever Footing

Specification data: $f_s = 16\,000$ lb per sq in
$f_c = 650$ lb per sq in
$n = 15$
v, limited to 40 lb per sq in
v_2 (punching shear) limited to 120 lb per sq in
u limited to 100 lb per sq in (deformed bars)

(1) Assume that the interior footing has been designed as previously described, and that the area of the base is 8 ft $\times$ 8 ft.

(2) The size of the exterior footing, similarly designed, but with an allowance of 25% of the column load to cover the weight of the footing, the weight of the strap, and the increment due to uplift, is found to be 5 ft $\times$ 8 ft.

(3) From the drawings and column schedule the following data are taken:

Center-to-center spacing between columns,	$l = 20$ ft;
Design load for interior column,	$P_2 = 425\,000$ lb;
Design load for exterior column,	$P_1 = 250\,000$ lb

Eccentricity of exterior footing in relation to the exterior column,

$$e = 1 \text{ ft } 6 \text{ in.}$$

Then, uplift $= P_1e/l = 250\,000 \times 1.5/20 = 18\,750$ lb.

(4) Taking moments about R_2,

$$R_1l - P_1(l + e) = 0$$

$$R_1 = P_1 + \frac{P_1e}{l} = 250\,000 + 250\,000 \times 1.5/20 = 268\,800 \text{ lb}$$

If this represents the total reaction upon the exterior footing, the reaction per linear foot in a direction parallel to the strap is,

$$p = 268\,800/5 = 53\,760 \text{ lb}$$

The distance from the outside edge of the footing to the section of zero shear is, then,

$$a_3 = P_1/p = 250\,000/53\,760 = 4.65 \text{ ft}$$

Assuming that the exterior column is 2 ft wide, in the direction of the strap,

$$a_1 = a_2 - 1.0 = 3.65 \text{ ft}$$

since the difference between a_1 and a_2 is one-half the column width. (See Fig. 51.)

(5) Maximum moment for soil footings by formula,

$$M = -P_1(a_1 - a_2/2) = -250\,000\,(3\,65 - 4.65/2) = -332\,500 \text{ ft-lb}$$

(6) As a trial, assume a width of 30 in for the strap, then,

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{332\,500 \times 12}{107.6 \times 30}} = 35 \text{ in}$$

$$A_s = \frac{M}{14\,000\,d} = \frac{332\,500 \times 12}{14\,000 \times 35} = 8.15 \text{ sq in}$$

Accept a total depth of 39 in and 10 1-in round rods at the interior face of exterior footing.

(7) The vertical shear at the exterior face of the interior footing is equal to the uplift

$$V = P_1e/l = 250\,000 \times 1.5/20 = 18\,750 \text{ lb}$$

(8) By formula

$$d = V/vjb = 18\,750/40 \times 0.875 \times 30 = 18 \text{ in}$$

(9) Checking the same section as governed by a positive bending moment equal to one-third of the maximum moment computed above,

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{332\,500 \times 12/3}{107.6 \times 30}} = 20 \text{ in}$$

$$A_s = \frac{M}{14\,000\,d} = \frac{332\,500 \times 12/3}{14\,000 \times 20} = 4.75 \text{ sq in}$$

Accept a total depth of 24 in at the exterior face of the interior column and 6 1-in round rods placed in the bottom of the strap.

(10) The approximate weights are as follows:

Exterior footing........................	15 000 lb
Connecting strap....	13 500 lb
Uplift..........	18 750 lb
	47 250 lb

The approximation made in step 2 is satisfactory.

(11) Ten 1-in round rods are placed along the top of the strap; of this number 8 are straight, except for the bend beneath the exterior column, and 2 are bent down where indicated. These 2 rods, together with the 4 straight 1-in rods in the bottom of the strap, give the 6 rods required at the exterior face of the interior footing. The rods lying in the top of the strap should be hooked or bent down beneath the exterior column.

Design Procedure for Continuous Footings. (See Fig. 52.) The design of a continuous footing supporting a line of columns is exactly parallel to the design of a continuous beam carrying a uniform floor load. In this type of work the proportion of dead load to live load is higher than for the beams comprising a floor system but there is also more danger to be feared from

unequal settlement. These two facts favor the use of positive moment coefficients somewhat nearer the theoretical values than those used for ordinary beam design and a more general distribution of the reinforcement to provide for unexpected stresses.

It is recommended that the concrete section be determined as in the case of beams by the formulas for shear and bending moment, and that the steel-area computed from the requirements of the bending moments be increased one-third. One-third of the originally computed amount would then be in the form of straight bars at the top of the footing, one-third in the form of straight bars at the bottom of the footing, and two-thirds bent at an angle of forty-five degrees, the center of the bend being located at the fifth point of the clear span between columns. This arrangement is suitable when it is desired to provide for equal positive and negative bending moments and to give some security against unequal settlement. If conditions are such as to warrant the use of a positive moment factor less than that of the negative moment, the sectional area required for the latter may be obtained by the usual method of lapping the reinforcement, the proportion of bent steel being varied to give the relative areas required.

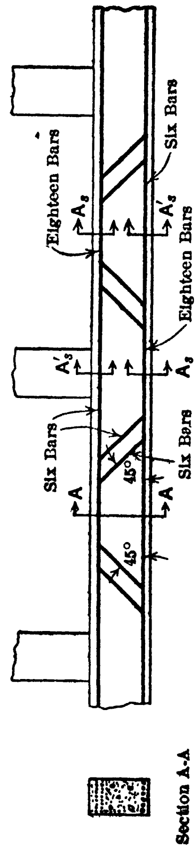


Fig. 52. Typical Design for a Continuous Footing, Showing Arrangement of Bars

Stairs, Walls, and Parapets

Stairs. (See Figs. 53 and 54) Steel has largely replaced concrete for the construction of stairs in reinforced-concrete buildings. The former has the advantage of quicker installation and earlier use during the erection of the building. It is often desirable, however, to build both exterior steps and interior stairs of reinforced concrete, and such are usually constructed as inclined slabs, poured monolithically with treads and risers and spanning from floor to floor, or from floor to intermediate landing. The thickness of the stair-slab varies from 4 to 8 in according to the spans and loads. The proportion of tread width to riser height is generally specified by building codes. A customary requirement is to make the sum of the two approximately 17 in and their product between 70 and 75 in. SAFETY TREADS, with or without a projecting nosing, are almost invariably used, and it is customary to apply a monolithic finish to the concrete surface. In order not to delay the work upon the main features of a building, recesses and bonds are usually left at the junctions of floors with stair-slabs, and the stairs are constructed later.

The THICKNESS and REINFORCEMENT of a stair-slab are determined by considering it as simply supported and of a length equal to the *horizontal* distance between supports. The main longitudinal reinforcement, preferably of

small bars, is carried up the stair-run and across the landings, the bars being bent so that they lie near the lower surface of the landing. The design is

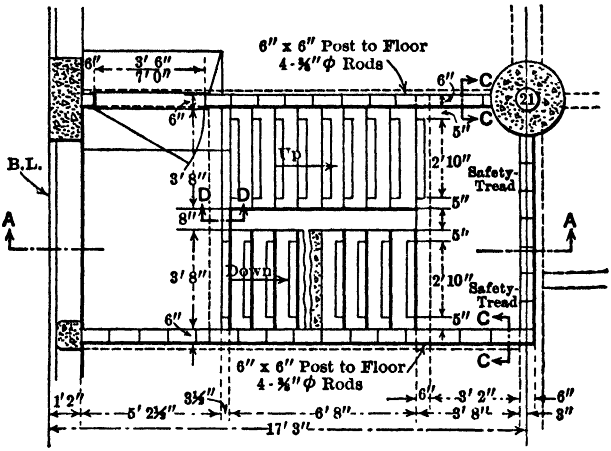


Fig. 53. Structural Design of Reinforced-Concrete Stairs. Plan

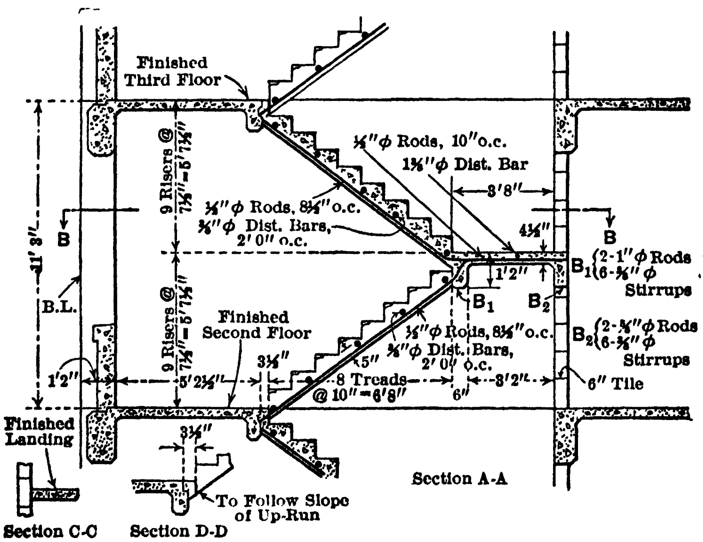


Fig. 54. Structural Design of Reinforced-Concrete Stairs. Section

then entirely similar to that of ordinary reinforced-concrete slabs, with one-way reinforcement.

Concrete Walls Subjected to Earth Pressure. The design of retaining walls is covered in Chapter IV but it is often necessary to compute the reinforcement and check the thickness of a BASEMENT WALL subjected to earth pressure under conditions which are slightly different from those encountered in the design of ordinary retaining walls.

When the basement of a building is located below ground, the exterior walls must resist earth pressure. When the height of the wall is not appreciably over one-half the clear span between columns, it is desirable to carry the thrust of the earth on the vertical reinforcement spanning from the basement floor to the first structural floor. If there is no footing beneath the wall, as sometimes happens when the wall spans from column to column,

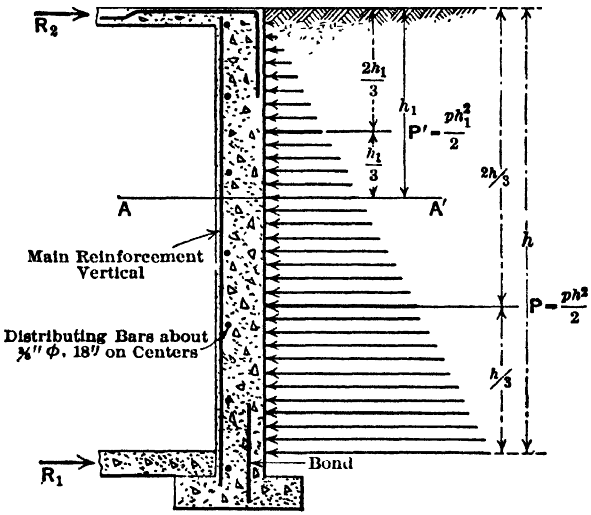


Fig. 55. Diagram of Basement Wall Subjected to Earth Pressure but Without Surcharge

it must also be designed to carry its own weight. The MINIMUM THICKNESS for basement walls should be 10 in, and some building codes require a minimum of 12 in.

For the purpose of computing the sectional area of the concrete and reinforcement required for walls subjected to earth pressure, the weight of the soil may be assumed to be 100 lb per cu ft and to exert a pressure equal to that of a fluid weighing 30 lb per cu ft.

Design Procedure for Basement Walls Subjected to Earth Pressure but without Surcharge. (See Fig. 55.) (1) Let

Let P = the total pressure per linear foot against the back of the wall in pounds;

h = the height of the wall, that is, the span length, in feet;

p = equivalent fluid pressure of the soil in pounds per square foot assumed at 30 pounds.

The total pressure is

$$P = \frac{ph^2}{2}$$

and the resultant acts at a height of $\frac{h}{3}$ above the bottom of the wall; compute the reactions $R_1 = \frac{2P}{3}$ and $R_2 = \frac{P}{3}$.

(2) Determine the section of maximum bending moment at the distance h_1 in feet below the top of the wall;

$$h_1 = 0.58\ h$$

(3) Compute the value of the maximum bending moment at this section,

$$M = 0.064\ ph^3 \text{ ft-lb}$$

(4) Design the concrete section and the steel-area in the same manner as for ordinary slabs.

Place the reinforcement vertically near the inside face of the wall, adding ⅜-in rounds 1 ft 6 in on centers horizontally for the purpose of supporting the vertical reinforcement and to act as temperature steel.

If the wall spans between column footings it should be reinforced as a simple beam with sufficient steel to resist the bending moment due to its own weight.

If the height of the basement wall is much more than one-half of the clear span between columns, or if openings interfere with the vertical reinforcement, the walls should be designed to span horizontally between columns with only enough vertical steel to support the horizontal reinforcement and serve as temperature reinforcement.

Typical Design of a Basement Wall Subjected to Earth Pressure but Without Surcharge

Specification data: $f_s = 18\,000$ lb
$f_c = 650$ lb
v' limited to 40 lb per sq in

It is required to design a basement wall 12 ft high and subjected to earth pressure for the full height. Then $p = 30$ and $h = 12$

(1) $P = ph^2/2 = 30 \times 12^2/2 = 2\,160$ lb per lin ft

$R_1 = 2\,P/3 = 1\,440$ lb and $R_2 = P/3 = 720$ lb

(2) Section of maximum moment is at

$h_1 = 0.58\,h = 0.58 \times 12 = 6.96$ ft below the top of the wall.

(3) Maximum bending moment,

$M = 0.064\,ph^3 = 0.064 \times 30 \times 12^3 = 3\,317$ ft-lb $= 39\,800$ in lb

(4) Thickness of wall, by formula,

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{39\,800}{100.8 \times 12}} = 5.75 \text{ in}$$

in which the value of b is the width of a vertical strip 1 ft wide.

Under these conditions the wall could be made 5.75 + 2.00 = 7.75 in thick but a minimum thickness of 10 in is usually imposed for this type of work.

Assuming that the wall spans vertically, from basement to first floor, the steel area is found by formula

$$A_s = \frac{M}{f_s jd} = \frac{39\ 800}{18\ 000 \times 0.883 \times 8} = 0.314 \text{ sq in}$$

Accept ½-in rounds spaced at 7 in on centers for vertical reinforcement, and ⅜-in rounds spaced at 1 ft 6 in on centers horizontally.

Checking the maximum shearing-stress at the base of the wall by the formula

$$v = \frac{V}{jbd} = \frac{1\ 440}{0.875 \times 12 \times 8} = 17 \text{ lb per sq in}$$

in which the external shear per lin ft, V, is equal to the reaction, R_1. As this value is less than 40 lb the design is accepted.

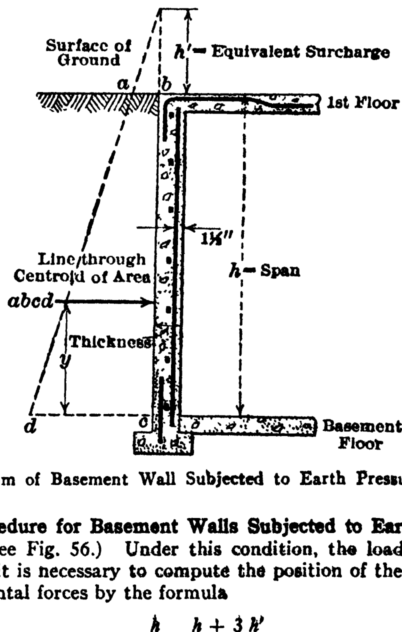


Fig. 56. Diagram of Basement Wall Subjected to Earth Pressure and Surcharge

Design Procedure for Basement Walls Subjected to Earth Pressure and Surcharge. (See Fig. 56.) Under this condition, the load diagram is not a triangle and it is necessary to compute the position of the resultant of the external horizontal forces by the formula

$$y = \frac{h}{3} \times \frac{h + 3\,h'}{h + 2\,h'}$$

in which y = the distance from the bottom of the span to the center of gravity of the external thrust, in feet;

h = the height of the wall; that is, the span-length, in feet;

h' = the height of the surcharge, in feet.

Assuming the weight of earth as 100 lb per cu ft, it is customary to represent a SURCHARGE diagrammatically as an equivalent height of earth. For exam-

ple, a surcharge of 300 lb per cu ft, which would be customary for sidewalk construction, is considered to add 3 ft in height to the load diagram.

The actual steps in the design of a wall of this type are arranged as follows:

(1) Compute the position of the center of gravity of the external horizontal forces by the formula given above.

(2) Compute the pressure per lin ft at both top and bottom of wall; using the average of these two values, multiply by the height of the wall obtaining the total load on a vertical strip 1 ft wide.

(3) Determine the reactions, R_1 and R_2 for 1 ft of wall, by taking moments about the top and bottom of the wall.

(4) Find the value of the maximum moment under the assumption that the total load found in (2) is uniformly distributed. This will give a result sufficiently accurate for ordinary purposes.

(5) Design the concrete section and the steel area in the same manner as for ordinary slabs. The reinforcement is placed as described for basement walls without surcharge.

Typical Design of a Basement Wall Subjected to Earth Pressure and Surcharge, (Fig. 57)

Specification data: f_s = 18 000 lb
f_c = 650 lb
v' limited to 40 lb per sq in

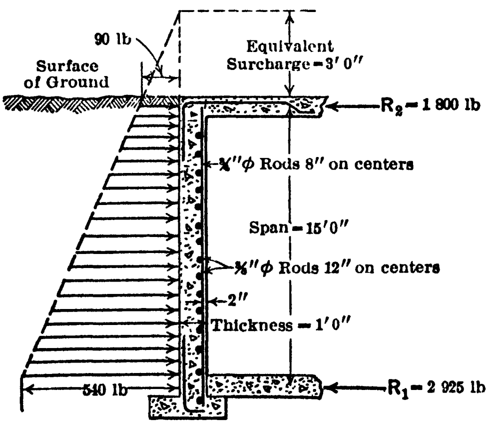


Fig. 57. Typical Design of a Basement Wall Subjected to Earth Pressure and Surcharge

It is required to design a basement wall 15 ft high and subjected to a surcharge, due to an adjacent sidewalk, which requires a live load of 300 lb per sq ft.

(1) By the formula given above,

$$y = \frac{h}{3} \times \frac{h + 3\,h'}{h + 2\,h'} = \frac{15}{3} \times \frac{15 + (3 \times 3)}{15 + (2 \times 3)} = 5.71 \text{ ft}$$

(2) Pressure at top of wall = 30×3 = 90 lb per lin ft.
Pressure at bottom of wall = 30×18 = 540 lb per lin ft.
Average = $(90 + 540)/2$ = 315 lb per lin ft.
Total load on a strip of wall 15 ft high and 1 ft wide is, then, $315 \times 15 =$ 4 725 lb.

(3) Taking moments about the top of the wall, the reaction at the bottom, $R_1 = 4\,725 \times \frac{(15 - 5.71)}{15} = 2\,925$ lb.

Reaction at the top, $R_2 = 4\,725 - 2\,925 = 1\,800$ lb.

(4) Assuming that the total load is uniformly distributed,

$$\text{Maximum moment, } M = \frac{WL}{8} = \frac{4\,725 \times 15 \times 12}{8} = 106\,310 \text{ in-lb}$$

in which L is the height of the wall between supporting floors or other structural members.

(5) Thickness of wall, by formula,

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{106\,310}{100.8 \times 12}} = 9.4 \text{ in}$$

in which the value of b is the width of a vertical strip 1 ft wide. Allowing 2 in from the inside face of the wall to the center of the vertical reinforcement, accept a total thickness of 12 in

$$A_s = \frac{M}{f_s jd} = \frac{106\,310}{18\,000 \times 0.883 \times 10} = 0.66 \text{ sq in}$$

Accept ¾-in rounds spaced at 8 in on centers for vertical reinforcement and ⅜-in rounds spaced at 1 ft on centers horizontally.

Checking the maximum shearing-stress at the base of the wall by the formula

$$v = \frac{V}{jbd} = \frac{2\,925}{0.875 \times 12 \times 10} = 28 \text{ lb per sq in}$$

in which the external shear per lin ft, V, is equal to the reaction R_1. As this value is less than 40 lb the design is accepted.

Parapet Walls. Parapet walls, built of reinforced concrete, are normally 8 or 9 in thick, but there is no structural reason why a 5-in thickness could not be used if permitted by local ordinances. It is desirable to reinforce walls of this type by means of comparatively small rods, usually ⅜-in rounds, and the amount of reinforcement depends upon the length of the wall and its exposure. For units about 20 ft between joints, it is good practice to use $\frac{3}{10}$ of 1% of REINFORCEMENT, based on the cross-sectional area of the wall. This amount of steel should be run horizontally with ⅜-in round verticals placed about 1 ft 6 in on centers for walls of only a few feet in height. High walls usually require more steel and should be especially designed to meet the conditions of load and exposure. Parapets should be bonded to the spandrel beams of the roof construction; for this purpose ½-in, or ⅜-in, round rods, 3 ft long, are spaced 1 ft 6 in on centers.

When designing parapet walls it is desirable to LEAVE JOINTS BETWEEN THE PANELS AND THE PILASTERS OR COLUMNS.

If the roof surfaces are graded by means of cinder, or cinder-concrete fill, vertical EXPANSION JOINTS should be left around the inside faces of all parapet walls, as otherwise the expansion of the roof-fill, when subjected to the heat

of mid-summer, may be sufficient to crack even strongly reinforced-concrete parapets.

Panel and Curtain Walls. When built of reinforced concrete, curtain and panel walls are usually made from 6 to 12 in in thickness, depending upon the height and unstayed length. The minimum reinforcement should comprise $\frac{3}{8}$-in round rods spaced about 1 ft on centers horizontally and about 1 ft 6 in on centers vertically. Additional steel should be added to resist any tensile stress that may be present and, for exterior walls, the total should not be less than about $\frac{3}{10}$ of 1% of reinforcement, based on the cross-sectional area of the wall. As the function of this steel is to distribute stresses due to volumetric changes, caused by the temperature and moisture content of the air, the rods should be placed near the exterior of the wall. Because of the danger of corrosion, however, there should always be a 2-in insulation except where particularly impervious concrete is used.

In some cases, panel walls are poured monolithically with the spandrel beams, and reinforced to form upturned spandrels extending the full height of the wall. In this case it is necessary to use a diagonal girder in the corner panels to provide proper support for the floor beams.

If floor areas are protected by a sprinkler system, the panel walls should be equipped with scuppers to carry off the water.

The following recommendation is abstracted from the Joint Code.

Reinforced-concrete bearing walls shall have a thickness of at least one twenty-fifth ($\frac{1}{25}$) of the unsupported height or width, provided, however, that approved buttresses, built-in columns, or piers designed to carry all the vertical loads, may be used in lieu of greater thicknesses. Working compressive stresses in such walls shall not exceed $0.0625 f'_c$ when the wall is 25 times the thickness in height, increasing proportionately to $0.125 f'_c$ when the wall is 15 times the thickness or less in height.

The lateral support for such walls shall consist of a fire-resistive floor when the framing is on one side of the wall only, but may be of a fire-resistive or of a non fire-resistive type where framing is on both sides of the wall, provided that for residences, wood-frame construction properly tied may be used as support.

In fire-resistive buildings, bearing walls shall be reinforced with an area of steel in each direction, vertical and horizontal, at least equal to 0.0025 times the cross-sectional area. Walls 8 in or more in thickness shall have half the steel at each face of the wall. The bars shall not be farther apart in either direction than 18 in, regardless of whether the steel is disposed in one or two layers, nor shall less than the equivalent of $\frac{3}{8}$-in round rods be so used. The vertical steel shall not be relied on to carry load unless tied and arranged as in reinforced-concrete columns.

All bearing walls shall be designed for any lateral pressure to which they are subjected. Eccentric loads and wind stresses shall be fully provided for.

In buildings of skeleton construction, panel or other walls supported on the structural frame shall not be less than 5 in thick, nor less than one-thirtieth ($\frac{1}{30}$) of the horizontal distance between columns, cross walls, or equivalent anchorage. Such walls shall be reinforced in the same manner as bearing walls in fireproof buildings.

Stairway and elevator enclosures in all classes of buildings may be built of reinforced concrete, when the wall thicknesses and reinforcements are in accordance with the requirements given above.

Concrete Sills. Reinforced-concrete sills may be either precast or poured-in-place. If brick panel walls are used, it is usually more desirable to employ

PRECAST SILLS. When the entire panel wall is of concrete, the sills can easily be cast monolithically with the wall itself.

Table XV. Minimum Wall Thickness, in Inches, in Story Indicated

No. of stories	Basement	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1	6	6	...	...	...	...	...	...	...	...	...
2	7	6	6	...	...	...	...	...	...	...	...
3	8	7	7	6	...	...	...	...	...	...	...
4	8	8	7	7	6	...	...	...	...	...	...
5	9	8	8	7	7	6	...	...	...	...	...
6	9	9	8	8	7	7	6	...	...	...	...
7	10	9	9	8	8	7	7	6	...	...	...
8	10	10	9	9	8	8	7	7	6	...	...
9	12	10	10	9	9	8	8	7	7	6	...
10	12	12	10	10	9	9	8	8	7	7	6

Sills should be reinforced with small rods laid longitudinally and retained in place by lateral ties. In placing reinforcement of this nature it is very important that it be PROTECTED by an adequate thickness of concrete to prevent corrosion.

Pits and Areas. Elevator pits should provide sufficient depth to accommodate the buffer installation and meet the requirements of the local building code. Walls of elevator pits are usually constructed 8 or 12 in in thickness. The design must consider the thrust of earth pressure, surcharge from basement floors and any hydrostatic head that may exist. The MAIN REINFORCEMENT is normally run horizontally around the pit with $\frac{3}{8}$-in round rods 1 ft 6 in on centers vertically.

4. Erection of Reinforced-Concrete Construction

Forms for Concrete

General Requirements. Forms for concrete construction are generally built of wood except for round columns. Many excellent designs have been made for metal floor-forms, and steel forms are used, to a certain extent, for wall-construction, but wood is more universally employed. An exception might be mentioned in the case of METAL TILE or the so-called "tin-pan," which have become very popular for use in ribbed construction, as illustrated in Figs. 26 and 27, pp. 1168 and 1169.

Whatever material is employed for the forms, it is essential that they be built true to line and grade and sufficiently strong to support, without sagging or bulging, the weight of the plastic concrete and the additional loads incident to construction. Forms should also be sufficiently tight to avoid any appreciable leakage of mortar. Some designers bear in mind the STOCK SIZES OF LUMBER when computing the size of concrete beams or other structural members. But as the builders have various methods of forming the concrete work, refinements of this nature are of questionable value. It is desirable, however, to allow a reasonable TOLERANCE in the sizes of structural members in order to permit the use of stock sizes without excessive waste.

Any good sound lumber which is free from knots, shakes, or decay can be used for forms. Spruce and fir have been widely employed for this purpose and short-leaf pine is often substituted, although not quite as satisfactory.

Hemlock is not to be recommended. For floor sheathing and wall panels tongue-and-groove roofers are generally used. In the East, these are of North Carolina pine; any tight boarding is satisfactory.

Although forms are naturally in the class with temporary structures, it is not desirable to use green lumber, and where careful work is required, air-dried, rather than kiln-dried, material is preferable. All faces of lumber coming in direct contact with concrete should be planed.

Design and Construction of Forms. These matters are usually left to the builder, who should design the forms to withstand the weight, or pressure, of the plastic concrete, figured at 150 lb per cu ft, together with a construction load usually assumed at 75 lb per sq ft of floor area. The SUPPORTING MEMBERS should be able to sustain such weights without deflecting over $\frac{1}{360}$ of the span. These computations are usually made with safe-load tables. In the case of columns and walls, the strength of the forms should be adequate to resist the hydrostatic pressure of the liquid concrete, which may be taken at 150 lb per sq ft for each vertical foot of height. The entire frames of concrete buildings, with the exception of certain types of unit construction now seldom employed, are ordinarily poured monolithically, except for any necessary contraction joints. As it is not expedient to delay the progress of a job by the construction of curtain walls, stairs and concrete partitions, such portions of the work are usually left until the skeleton of the building is completed. Consequently, it is often necessary to leave RECESSES in the faces of columns and in the edges of floor-slabs to receive walls and stairs which are poured at a later date. (See under Stairs, Part 3.)

Wooden column forms and beam and girder troughs (Figs. 1, 2, 3, and 4) are usually handled as units. Similarly, sections of slab and wall forms are constructed as large as can be conveniently moved about with the means available. Girder forms for bays of normal size are built continuous from column to column, except in cases where the beams are of the same or greater depth, when it may be more practicable to build the girder sides in sections between beam openings.

Forms should be constructed in such a way as to STRIP EASILY without breaking, as they are often used many times over. In fact, many builders construct only one set of floor forms for a building ten or twelve stories in height; provided that there is no requirement to the contrary, it is perfectly practicable to pour ONE STORY A WEEK on typical skeleton construction, using the same set of forms as many times as there may be stories in the building. City ordinances, however, often demand that the forms be left in place for ten days or two weeks after the placement of the concrete. Such provisions naturally add to the expense of construction as the contractor is forced either to delay his work or to build several sets of floor forms. In order to prevent the breaking off of square corners or exposed edges, it is desirable to insert either a COVE MOULDING or a small TRIANGULAR FILLET in the forms along the edges of columns, beams, and girder soffits. This practice not only facilitates stripping, but gives a good appearance to the finished work. A like effect is obtained at the re-entrant angle, where the sides of beams or girders join the slabs, by BEVELING THE EDGES of the floor sheathing.

It is important to provide openings at the bases of all columns for the purpose of cleaning out shavings or other rubbish. It should be the duty of an inspector to require that all sections of form-work be thoroughly cleaned out immediately before the placement of concrete.

Erection of Forms. Forms are usually handled by a small derrick or gin-pole operated by a hand-winch; on large operations the form units are

raised from story to story by power. The LOCATIONS of footings and columns should be laid off with a transit and the levels of the former checked by an instrument from an established bench mark. Forms for low walls and soil footings may be held in place by means of stakes driven into the ground. When shoring upon SOFT SOIL, care should be exercised to distribute the load from vertical supports over a sufficient area, by means of planks bedded on the ground, to obviate any appreciable settlement when the wet concrete is

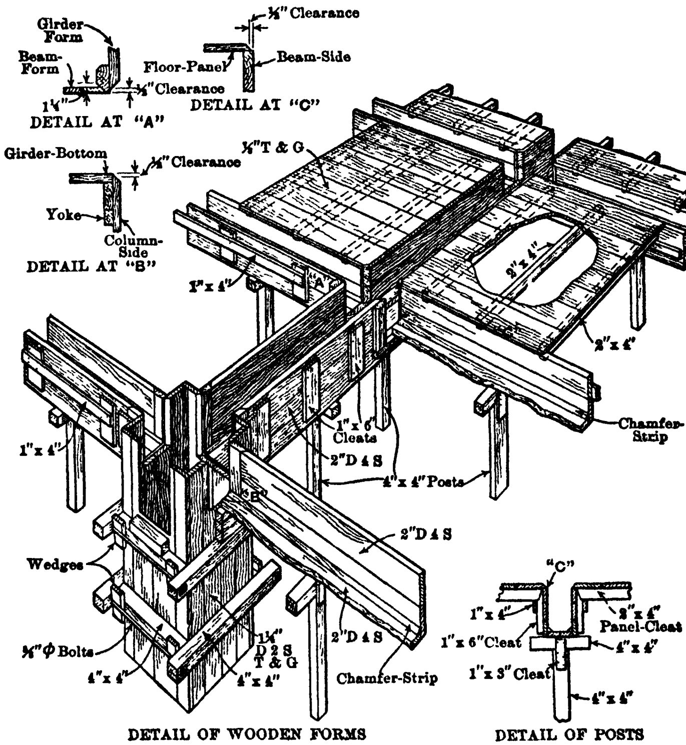


Fig. 1. Wood Forms for Beam-and-Slab Construction

placed upon the forms. If it is necessary to erect shores on FROZEN GROUND, the area beneath the floor should be enclosed and heated by means of salamanders, or any other effective means, for a sufficient time before pouring the concrete to insure the removal of frost and to provide a stable foundation.

When concreting large floors, particularly those with thick slabs such as are encountered in industrial buildings subjected to heavy live loads, a man should be constantly on watch beneath the floor that is being poured to see that the forms take their load without sag or appreciable leakage.

Where practicable, beam-and-girder forms may be given a slight CAMBER

of ¼ in for every 10 ft of length; this same practice may be applied to slabs of long span. It is very important to place long horizontal members, such as window-sills, with particular care. They should be formed from planed lumber, accurately leveled and held rigidly in position.

On unimportant work it is customary to WET THE FORMS immediately before placing the concrete. On large jobs where the forms are to be re-used a number of times, it is customary to apply a brush coat of FORM OIL. This operation should be done before the reinforcement is placed in order to pre-

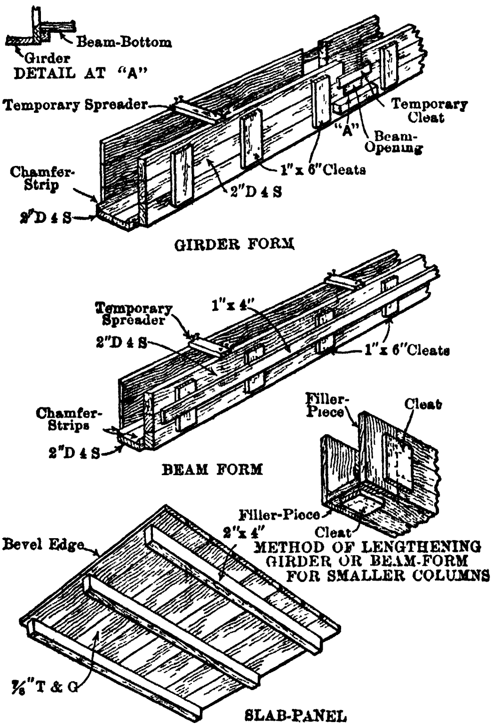


Fig. 2. Units of Form Work before Assembling

vent greasing the steel, as a coat of oil lessens the adhesion of the concrete. In case it is intended to plaster the concrete surfaces, oil or grease should not be used and the forms may be merely wet down.

Removal of Forms. There is no uniform practice in regard to the stripping of concrete. Walls and columns, carrying only their own weight in vertical compression, can naturally be stripped much earlier than beams, girders, or slabs in which the bending stresses due to the dead load require bond resistance between concrete and steel. Similarly, members in which the dead load is proportionally large as compared with the live load must be supported

for a longer period. Various cements set more rapidly than others, a rich concrete gains strength much more quickly than a lean mixture, and both the amount of water and the temperature during the curing period greatly influence the strength at an early age.

On account of all these varying conditions it is extremely difficult to lay down any DEFINITE RULE in regard to the time at which it is safe to remove forms from structural concrete. In fact, it is most inadvisable to make any

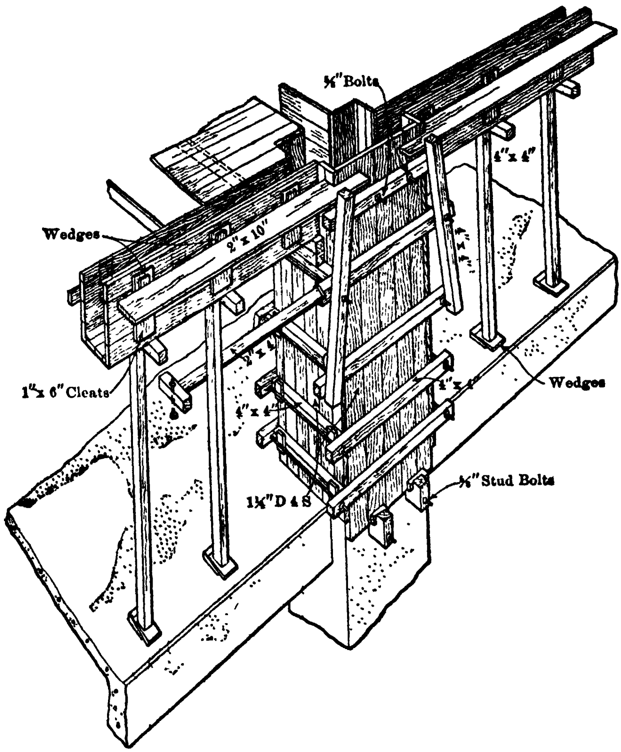


Fig. 3. Forms for Exterior Column and Spandrel Beam

hard and fast rules, as MANY PARTIAL FAILURES IN CONCRETE-BUILDING CONSTRUCTION HAVE BEEN DUE TO PREMATURE REMOVAL OF THE FORMS where the concrete, owing either to cold weather or inferior quality, had not attained sufficient strength to carry the imposed loads.

As a guide, however, the following periods may be given, which approximate the MINIMUM TIME that the forms for the different members should remain in place upon buildings conservatively designed and built of concrete which, under ACTUAL CONDITIONS EXISTING AT THE JOB, would give a compressive strength of 2 000 lb per sq in at an age of 28 days. If higher values are

obtained by the use of a high early strength cement, or by employing a richer mixture, the removal of forms may be hastened, but the actual strengths should, under such conditions, be carefully checked by test cylinders taken at the job and cured under field conditions.

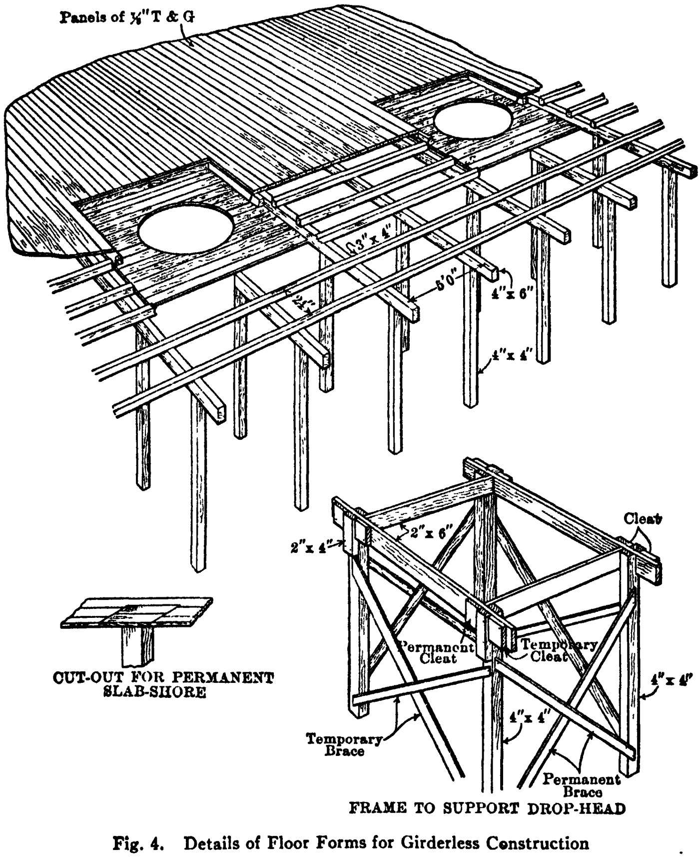


Fig. 4. Details of Floor Forms for Girderless Construction

Beam-and-girder floor construction. Columns, two days, provided that the girders are shored, or reposted, to prevent any appreciable load being transferred to the columns.

Girders, four days, provided that each girder is immediately reposted so that only one member at a time is left unsupported.

Beams, five days, provided that the same procedure is followed as in the case of girders.

Panels, up to 7 ft; any time after the beam forms are removed.

GIRDERLESS FLOOR CONSTRUCTION. Columns, as noted above for beam-and-girder construction.

DROPS, four days, provided that the slabs are shored or reposted to prevent any appreciable load being transferred to the unsupported sections.

SLABS, up to 20 ft, five days, provided that each slab is immediately reposted, and permanent 6 × 6-in shores left beneath the center of each bay and not disturbed at the time of stripping.

Reposting. In following a schedule such as that given above it is necessary to repost the concrete after the removal of the forms. When the shores of consecutive stories are upheld upon the floor construction beneath, the latter should be adequately supported either by the original shores or by reposting, until the concrete has gained the strength ordinarily resulting from an AGE OF 28 DAYS UNDER GOOD CURING CONDITIONS. It is consequently often necessary to have three stories reposted besides the floor which is actually formed. In reposting successive floors it is desirable that the shores be placed as near as possible directly above those in the lower stories. REPOSTING is usually done with 4 × 4-in or 6 × 6-in timbers brought to a firm bearing by the use of wedges, stayed against bending and spaced according to the load which they are assumed to carry. Where there is danger of punching into the unhardened concrete, the load should be distributed by the use of a section of plank placed as a cap.

Reinforcement

The Care of Reinforcement. The size and length of reinforcing bars should be CHECKED by the general contractor's representative as soon as the material is delivered to the job. In order to facilitate the selection of the proper steel it is desirable to construct WOODEN FRAMES with different compartments for the various lengths and sizes. The use of such a framework is of further advantage in keeping the steel clean. If a large quantity of reinforcement is stored for an extended period, it is well to build sheds over the storage racks. A BRIGHT-RED RUST, such as forms in a few days on reinforcement exposed to rain, is not in any way detrimental. Actual rust scales, however, may indicate a reduction in the effective cross-section of the bar. DEEP SCALING should be considered a sufficient reason for condemning the use of reinforcement unless it can be used as the equivalent of a smaller size. After the steel is embedded in the concrete, there is no reason to believe that the rusting is progressive; in fact, slightly rusted bars have been found, when broken out after several years' embedment, to be freer of rust than at the time when the concrete was poured. All reinforcing steel should be kept FREE FROM OIL as such tends to reduce the bond between concrete and steel.

Fabricating and Placing Reinforcement. Steel is often cut to length and bent in the shop, but many firms prefer to do their own fabrication upon the job. In the latter case, LIGHT-WEIGHT CUTTERS are installed and a special bench constructed for bending the rods composing the reinforcement of beams and girders. The smaller bars, used for the reinforcement of floor slabs, are usually bent on the floor. Bars or rods of structural grade should not be bent to a radius of less than FOUR TIMES THE LEAST DIMENSION OF THE ROD OR BAR and this should be increased to eight times the least dimension in the case of intermediate or hard grade steel. When RAIL-STEEL is used, it is difficult to bend the rods without heating, and usually it is more practicable to avoid the use of bent rods as far as may be consistent with the design.

The reinforcement of beams and girders is usually assembled in the beam or girder troughs. It is essential that stirrups be wired to the horizontal rods which they encircle. The steel assembly is ordinarily held up from the bottom of the troughs by means of CHAIRS or by SMALL BLOCKS of mortar, previously cast to the required thickness. When more than one layer of bars

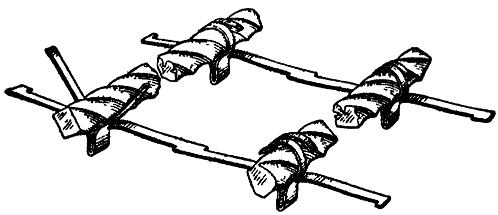


Fig. 5. Slab Bar Supporter and Spacer

lies at the bottom of a beam or girder the layers are separated by SPACERS, providing a clear vertical distance of at least one inch between the bars. (See Figs. 5 and 6.)

It is particularly important that all LOOSE BARS, such as those sometimes required to furnish additional negative reinforcement over supports, be securely held in place. Templets for the correct location of DOWELS are often required where steel columns are superimposed upon concrete footings. SLAB REINFORCEMENT should be adequately secured against lateral displacement and held at the correct height above the forms by means of suitable supports. There are many satisfactory devices upon the market to serve these purposes, and cement mortar blocks are also employed as supports for the steel, using, ordinarily, reinforcing rods as spacers placed at right-angles to the main bands of reinforcement. In GIRDERLESS FLOOR-CONSTRUCTION (Fig. 8), it is of particular importance to keep the bars comprising the reinforcement of the various bands at the desired height. The reinforcement for HOOPED or TIED COLUMNS is made up as a frame, by wiring the hoops to the vertical rods, and placed in the forms as a unit. SPIRAL reinforcement is bent in the shop and shipped collapsed, being assembled with the vertical spacers upon the job; three or four spacers are required for every spiral. (See Fig. 9.)

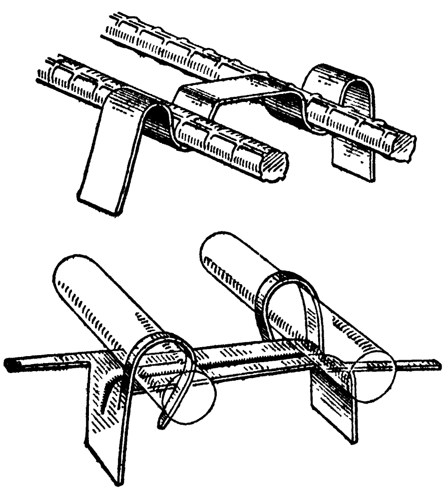


Fig. 6. Beam Bar Supporters and Separators

Splicing Reinforcement. Reinforcing bars may be spliced except at sections of maximum stress. It is customary to make LAPPED SPLICES of sufficient length to transfer the stress between the bars by bond and shear. The lengths of such splices are the same as given for anchorage. (See under Anchorage of Reinforcement, Part 3.) Wire mesh or fabric is ordinarily spliced by overlapping a distance of from 4 to 8 in and wired merely to hold it in proper position.

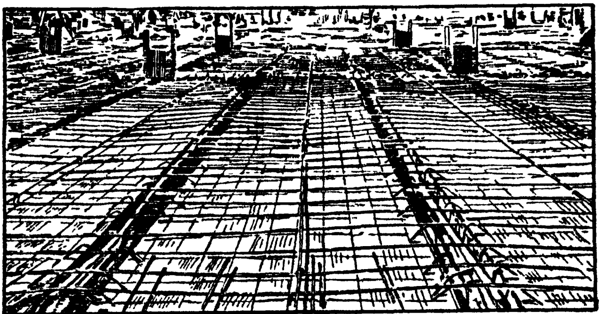


Fig. 7. Reinforcement in Place for Beam and Slab Construction

SPLICES IN VERTICAL REINFORCEMENT are made by lapping the rods a sufficient distance to develop the compressive stress by bond. Column steel is normally lapped an arbitrary length of 2 ft. Bonds should be generally used wherever it is necessary to join new work with that previously cast. These are usually of $3/8$-in or $1/2$-in diameter rods from 3 to 4 ft in total length.

Protection of Reinforcement. It is extremely important that all reinforcing steel be properly PROTECTED FROM FIRE OR MOISTURE. Building codes

Fig. 8. Reinforcement in Place for Girderless Construction

specify a minimum covering for the various structural members of buildings which, in general, is 2 in for COLUMNS and GIRDERS, $1\frac{1}{2}$ in for BEAMS and WALLS and 1 in or $\frac{3}{4}$ in for SLABS. These figures are generally assumed to apply to the major elements of the reinforcement, and hoops or stirrups are permitted to encroach upon these dimensions. In the case of spiralled columns there should be $1\frac{1}{2}$ in in the clear from the outside face of the spiral wire.

In order to provide this insulation some effective means should be employed, such as the use of cement-mortar doughnuts, which are cast of such thickness

as to give, when slipped over the vertical rods, the design distance between the reinforcement and the face of the forms.

Attention should be paid to RECESSES and ORNAMENTAL MARKINGS to insure a sufficient protection at these sections to provide against corrosion. The amount of protection required on the exteriors of buildings depends upon the quality of the concrete and the degree of exposure. In general, it would seem to be good practice to allow at least 2 in of concrete as ordinarily designed for structural purposes, outside of all reinforcement, to insure against moisture penetrating to the steel.

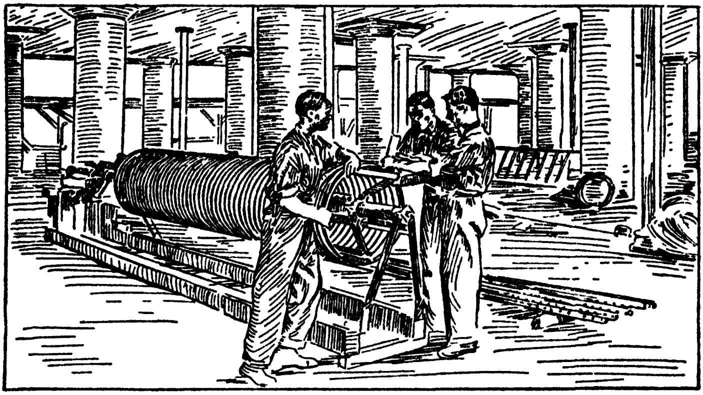


Fig. 9. Assembling Spiral Reinforcement

Proportioning the Ingredients of Concrete *

Methods of Proportioning Concrete. The erection of a concrete building is a manufacturing process carried on in the field and consequently demands efficient supervision. Over a period of many years, different experimenters suggested various methods for proportioning the ingredients of concrete. Although it is a comparatively simple matter to make concrete of sufficient strength to satisfy the ordinary requirements of a structural frame, it is a much harder problem to determine the most ECONOMICAL PROPORTIONS for a concrete required to meet given CONDITIONS OF USE AND EXPOSURE.

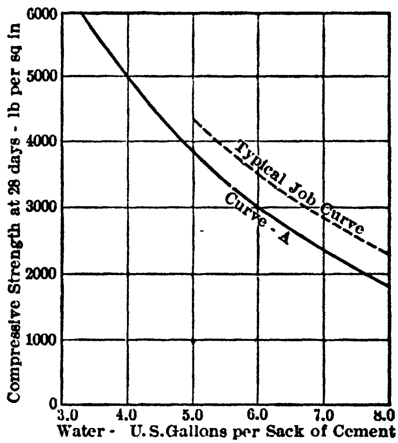
Various methods have been devised for obtaining this result, several of which are undoubtedly preferable to the old system of proportionment by arbitrary measure or weight. FULLER'S METHOD, starting from a carefully made sieve-analysis of the dry materials, was based upon a reproportionment for maximum density. EDWARDS developed the theory of surface-areas which helped to clarify our conception of concrete mixtures. DUFF ABRAMS, besides his contribution, of inestimable value, concerning the QUANTITATIVE RELATIONSHIP OF THE WATER-CEMENT RATIO to the physical properties of the hardened concrete, also proposed a scientific proportionment by means of the fineness modulus, another function of the sieve-analysis.

* Tables, charts, and photographs in this portion of the text have been supplied by the Portland Cement Association.

In spite of all this valuable research work, much concrete, particularly on small operations, is still proportioned by arbitrary volumes and without any specification in regard to the water content. As the strength of concrete, within the range of workable mixtures, is a direct function of the WATER-CEMENT RATIO (see Fig. 10), such a specification may very well result in obtaining concrete of ¾ or ½ the strength assumed in the design of the structure.

If it is desired to use a set formula in the specification, such as one part of cement, two parts of sand, and four parts of crushed stone or gravel, it is essential that a definite stipulation be made in regard to the WATER CONTENT, otherwise there is no assurance that the CONCRETE STRENGTH will meet the assumptions made in the design. Such an ARBITRARY PROPORTIONMENT, although not scientific and seldom economical, has proven to be fairly satisfactory provided that the water is strictly controlled. This can be done by any one of the automatic control-devices furnished with the concrete-mixers.

A much more desirable method for proportioning concrete is that developed by Frank R. MacMillan on the basis of the water-cement ratio and known as PROPORTIONING BY TRIAL. By this method, fully described under Procedure in Proportioning Concrete Mixtures by the Water-Cement Ratio, the designer controls the strength by the water-cement ratio, and determines the most economical mixture that can be properly placed under job conditions by making trial batches with varying amounts of aggregates.



Effect of quantity of mixing water on the strength of concrete. Curve "A" is based on average values from nine series of tests made over a period of four years. In the absence of field tests, it may be used for design where the water-cement ratio is carefully controlled by accurate measurement of quantities of water, cement and aggregate with proper correction for water carried by the aggregate. The job curve is a representative curve obtained from tests of the materials to be used on a specific project.

Fig. 10. Effect of Quantity of Mixing Water on Strength of Concrete

Basic Principles of Proportionment. By referring to the chart, Fig. 10, which is the result of exhaustive experimentation carried on in the Lewis Institute under the auspices of the Portland Cement Association, the designer may determine the number of gallons of water for each sack of cement corresponding to the strength which he desires in the finished concrete. When choosing the water-cement ratio, it is necessary to bear in mind the type of work for which the concrete is intended. Table I gives an approximate idea of the quantities of water suitable for the different degrees of exposure, as well as for the different types of concrete used in building construction. Equally important with strength is the workability of the plastic mass. In other

Table I. Classes of Concrete for Different Degrees of Exposure

Type of structure	Degree of exposure	U. S. gal of water * per sack of cement
Walls, dams, piers and other structures exposed to sea or alkali waters....	Extreme	5½
Walls, dams, piers, reservoir linings, etc, exposed to alternate wetting and drying in fresh water in northern climate. Watertight structures. Sewers, pressure pipe, tanks, piles, athletic stadia, pavements, all thin structural members exposed to severe weather and frost action	Severe	6
Walls, dams, piers, reservoir linings exposed to fresh water in southern climate. Exterior columns and beams of reinforced concrete buildings. Basement walls. Thin structural members of all types exposed to moderate weather and frost action .	Moderate	6¾
Ordinary enclosed structural members. Heavy piers and retaining walls in moderate exposure. Mass concrete, footings, etc, protected from alternate wetting and drying and from severe weather conditions..........	Protected	7½

* These quantities should not be exceeded even when resultant strength is higher than required for structural stability. Free water or moisture carried by the aggregate must be included as part of mixing water.

words, the freshly mixed concrete must have a consistency sufficiently plastic to flow sluggishly without segregation, being neither crumbly nor watery.

In CHOOSING THE AGGREGATES and in combining the proportions of sand and stone, it is necessary to remember that stiff, plastic mixtures with large aggregates, which might be suitable for large units, would not be workable in structural members such as thin floor-slabs. Furthermore, in determining the proportions of the sand and coarse aggregate, economy should be considered. Generally speaking, a coarse aggregate presents a smaller total SURFACE-AREA to be coated with cement-paste than an aggregate composed of finer particles. It is consequently more economical up to the point where a too great preponderance of large particles makes the mixture too hard to place or, as we say, too harsh. Consequently, it is desirable to use the lowest PROPORTION OF FINE AGGREGATE that will completely fill the void spaces in the coarse aggregate. It is generally true that aggregates that are evenly graded from fine to coarse are more economical than those in which one or two sizes predominate, for the simple reason that the former have less VOID-SPACE to fill with a more costly material. An intelligent proportionment of commercial aggregates, however, usually results in a satisfactory combination. The mixture should contain enough FINE MATERIAL, such as sand, to work smoothly, but no more than is required. An excess of fine materials results in a greater surface area to be covered and more voids to be filled with cement paste, reducing the total AMOUNT OF AGGREGATE that can be used with any given amount of cement.

Bearing in mind the principles outlined in the foregoing paragraphs, Table II has been made to indicate the usual LIMITS IN PROPORTIONING fine and coarse aggregates to produce plastic and workable mixtures based on a water-cement ratio of 7½ gal or less, per sack of cement.

Having determined upon the proportions of fine and coarse aggregate, it will be found that the total amount of the combined aggregates that can be used with any given amount of cement and mortar depends upon the consistency required, in order that the concrete may be properly placed under the existing conditions. ECONOMY is gained by using a STIFFER MIXTURE, which is another way of expressing the fact that more aggregate is crowded into the same cement paste. But although a stiff mixture saves material, it costs more to handle, and when EXCESSIVELY DRY, invariably results in HONEYCOMBED WORK. On the other hand, OVER-WET MIXTURES USE MORE CEMENT in order to obtain the same strength and cannot be placed without separation of the materials.

Table II. Recommended Proportions of Aggregates

Maximum size of coarse aggregate, inches	Ratio of fine * to total aggregate on basis of dry, compact volumes, measured separately	
	Minimum	Maximum
3/8	0 55	0.70
3/4	0.40	0.60
1 and over	0.30	0.50

* The finer the sand, the lower will be the percentage required.

Proportioning Concrete by the Water-Cement Ratio *

Concrete Quality. The working stresses used in DESIGNING reinforced-concrete buildings should be based upon the minimum ultimate 28-DAY STRENGTH of the concrete in the structure and determined in accordance with the values given in Table III. For specification purposes, the strength of the concrete may be fixed in terms of the water-cement ratio by either (1) using results established for AVERAGE MATERIALS, as described in the following article, or (2) making actual tests of concrete composed of the MATERIALS TO BE USED in the structure, as described in the second article following.

The water-cement ratio means the proportion of water to cement entering the mixture, including the surface-water carried by the aggregates; this quantity is expressed in terms of the quantity of cement, that is, U. S. gallons per sack (94 lb) of cement.

Water-Cement Ratio for Average Materials. When NO PRELIMINARY TESTS are made, the water-cement ratios should not exceed the values in Table III. The mixes shown in the table are approximate only, and may require adjustment to give proper workability.

Throughout the progress of the work, at least one specimen should be tested for each 100 cu yd of concrete, and the architect or engineer may require a reasonable number of additional tests. All tests should be made in accordance with the provisions given under the paragraph entitled Field Tests of Concrete. If the average 28-day strength falls below the minimum ultimate strength used in the design, the architect would have reason to require a LOAD TEST.

* Based on the requirements of the Joint Code-Building Regulations for Reinforced Concrete as recommended by the American Concrete Institute and the Reinforcing Steel Institute.

Table III. Assumed Strength of Concrete Mixtures

Water-cement ratio U. S. gal per 94-lb sack of cement	Approximate mix volume of Portland cement to sum of separate volumes of fine and coarse aggregate as measured dry	Assumed compressive strength at 28 days in pounds per square inch
	Plastic Concrete	
8¼	1 : 7	1 500
7½	1 : 6	2 000
6¾	1 : 5¼	2 500
6	1 : 4½	3 000
	Moderately Wet Concrete	
8¼	1 : 6½	1 500
7½	1 : 5½	2 000
6¾	1 : 4¾	2 500
6	1 : 4	3 000

NOTE: In interpreting this table, surface water contained in the aggregate must be included as part of the mixing water in computing the water-cement ratio.

Water-Cement Ratio for Special Materials. When the water-cement ratios for the various concrete strengths are ESTABLISHED BY TEST, such should be made in advance of the operations. The specimen cylinders should be composed of the same materials and mixed to the same consistencies as it is intended to use on the work. The COMPRESSIVE STRENGTH of the concrete is then determined in accordance with the "Standard Method of Making Compression Tests of Concrete" (Serial Designation C39–27) of the American Society for Testing Materials, including the provision for curing in a moist room at 70° F. and testing wet. The next step is to draw a curve representing the relation between the average 28-day strength of the concrete and the water-cement ratio, for a range of values including all the strengths called for by the design. Tests should include at least four different water-cement ratios and at least four specimens for each. The water-cement ratio to be used in the structure should be that corresponding to a point on the curve established by these tests representing a concrete strength 15% higher than the minimum ultimate strength called for by the design, and satisfactory evidence should be submitted to show that these water-cement ratios are not exceeded during the construction of the building. No substitution of materials should be permitted without additional tests establishing new water-cement ratios.

During the progress of the work, a reasonable number of additional 28-day compression tests may be required by the architect, but at least one specimen should be tested for each 50 cu yd of concrete of any one strength, and not less than two specimens of each strength of concrete for any one day's operation. Such tests should be made in accordance with the recommendations given under the following article. Should the AVERAGE STRENGTH of the control cylinders as shown by these tests, for any portion of the structure,

fall below the minimum ultimate 28-day strength used in the design, the architect may order a change in the mixture, or in the water-cement ratios, for the remaining portion of the structure and require load tests on the portions of the building affected. Should the average strengths shown by the cylinders cured on the job, and tested subsequent to 28 days, fall below the required strength, the architect may require special conditions of temperature and moisture as may be necessary to obtain the required strength.

Field Tests of Concrete. When field tests are required, specimens should be obtained in accordance with the "Standard Method of Making and Storing Compression Test Specimens of Concrete in the Field," (Serial Designation C 31-27) of the American Society for Testing Materials. Compression tests should be made in accordance with the "Standard Method of Making Compression Tests of Concrete" (Serial Designation C39–27) of the American Society for Testing Materials with the following exceptions:

(1) Two sets of samples for test specimens should be taken as the concrete is being delivered at the point of deposit, care being exercised to obtain a sample representative of the entire batch.

(2) One set designated as "control cylinders" should be placed under moist curing conditions at approximately 70° F. within 24 hours after molding and maintained therein until tested.

(3) The second set, designated as "job cylinders" should be stored as near as possible to the point of sampling and receive the same protection from the elements as is given to like portions of the structure. Specimens should be protected while on the work and sent to the LABORATORY within seven days of the date of test; while in the laboratory they should be kept in air at a temperature of approximately 70° F.

Concrete Proportions and Consistency. The proportions of aggregates to cement, for any chosen water-cement ratio, should be such as to produce concrete that works readily into the corners and angles of the form and around the reinforcement, without excessive puddling or spading, and without permitting the materials to segregate, or free water to collect on the surface. The COMBINED AGGREGATE should be of such a size that when separated by a No. 4 standard sieve, the weight retained on the sieve is not less than one-third nor more than two-thirds of the total; neither should the amount of coarse material be such as to produce HARSHNESS IN PLACING OF HONEYCOMB in the structure. When forms are removed, the faces and corners of the members should appear smooth and sound throughout.

The METHODS OF MEASURING concrete materials should be such that the proportion of water to cement can be accurately controlled during the progress of the work and easily checked at any time by the architect or his representative.

Although the acceptance of this specification may seem a rather radical departure from the old method of specifying an arbitrary mixture, it must be acknowledged that the basic idea is sound. IT IS LOGICAL and PERFECTLY PRACTICABLE TO SPECIFY A STRENGTH INSTEAD OF ARBITRARY MIXTURES. The economy of this practice has also been established by many of our foremost builders. Although it might appear that the results of strength tests would arrive too late for practical use, it has been found entirely feasible to develop a STANDARD QUALITY of concrete during the early stages of the operation and to accurately predicate the 28-day strength from results obtained by crushing cylinders at an age of 7 and 14 days.

The formula proposed by Mr. W. A. Slater of the Bureau of Standards:

$$f'_c = f + 30\sqrt{f}$$

in which f is the ultimate compressive strength of the concrete at an age of 7 days in lb per sq in, and f'_c the probable strength at 28 days, enables the architect or his representative to closely approximate the 28-day strength of concrete on a basis of 7-day tests. Charts such as shown in Fig. 15 also enable those in control of the work to closely approximate the ultimate strength of concrete from tests on newly poured cylinders.

Correction for Bulking of Aggregates. The proportions of a concrete mixture are determined for materials in a dry, compact condition. On the job both the sand and coarse aggregate will be measured in loose volumes (unless weighing devices are employed, in which case this part of the correction is eliminated), and any water that may be present will not only have to be deducted from the quantity added to the batch, but may also cause a very appreciable bulking of the fine aggregate. This correction is made by adding proportionately larger amounts of the bulked aggregate to secure the desired volume of dry, compact aggregate. Thus, if the computations call for a dry, compact mixture of 1 : 2¼ : 3½ and the bulking factors have been found, as in the following article, to be 1.21 and 1.06, the proportion of sand, based on loose volume, will be $2\frac{1}{4} \times 1.21 = 2.7$, and of the coarse aggregate $3\frac{1}{2} \times 1.06 = 3.7$, giving a field mix of 1 : 2.7 : 3.7. To maintain uniform amounts of aggregate at the mixer, allowance for changes in bulking due to changes in MOISTURE-CONTENT should be made by corresponding changes in the measured proportions. Thus, if the bulking factor of the sand increases to 1.25, the relative volume of sand in the same mix should be changed to $\frac{1.25}{1.21} \times 2.7 = 2.8$.

Absorption and Moisture. The amount of water absorbed by most aggregates is negligible. It is important, however, to determine the amount of FREE WATER that may be carried by the aggregates, particularly the fine aggregate, and to deduct this quantity from the total water required for the mixture as shown on the chart giving the water-cement ratio.

The amount of water in sand can be easily determined by drying tests but may be closely approximated by the figures given in the following table. Assuming that the sand carries free water to the amount of ½ gal per cu ft, and the coarse aggregate ¼ gal per cu ft, using the proportions of the previous article, the corrections would be

$$\begin{aligned} 2.7 \times 0.5 &= 1.35 \\ 3.7 \times 0.25 &= \underline{.92} \\ &\ 2.27 \quad \text{or} \quad 2\tfrac{1}{4} \text{ gal} \end{aligned}$$

The amount of water to be added at the mixer for each sack of cement is then the quantity shown by Table III corresponding to the desired strength, less 2¼ gal.

Procedure in Proportioning Concrete Mixtures by the Water-Cement Ratio. Although several methods of proportionment have been recommended, based on the law that the strength of concrete is a function of the water-cement ratio (provided that suitable materials are used and the resulting mixture is sufficiently plastic to be workable), the simplest method is that known as "Proportioning Concrete Mixtures by Trial" proposed by Frank R. MacMillan. The procedure is substantially as follows:

(1) Select from Table III, or the diagram shown in Fig. 10, a water-cement ratio that will produce concrete of the strength required by the design: this ratio is expressed in gallons per bag of cement.

Table IV. Approximate Quantity of Surface Water Carried by Average Aggregates *

Very wet sand	$\frac{3}{4}$ to 1 gal per cu ft
Moderately wet sand	about $\frac{1}{2}$ gal per cu ft
Moist sand	about $\frac{1}{4}$ gal per cu ft
Moist gravel or crushed rock	about $\frac{1}{4}$ gal per cu ft

Approximate Absorption of Aggregates

Average sand	1.0 per cent by weight
Pebbles and crushed limestone	1.0 per cent by weight
Trap-rock and granite	0.5 per cent by weight
Porous sandstone	7.0 per cent by weight
Very light and porous aggregate may be as high as	25 per cent by weight

* The coarser the aggregate, the less free water it will carry.

(2) Choose materials that meet the requirements given under Basic Principles of Proportionment, and dry the aggregates sufficiently to remove all surface moisture.

(3) Prepare a cement-water paste of the chosen proportions. Record the proportions, noting the volume of the water and the weight of the cement.

(4) Make up several mixtures combining the fine and coarse aggregates in some reasonable proportion within the limits set by Table II, such as $2\frac{1}{4}$ parts sand and $3\frac{3}{4}$ parts stone, sift the mixture into a container, such as a pail, and determine the weight. Proportions can best be taken by weight and later, if desired, transferred into dry rodded volumes by determining the weight per cubic foot of each aggregate. Well-compacted dry sand usually weighs about 105 lb per cu ft and crushed stone about 100 lb. Changing slightly the first proportions, for example, using 2.4 parts sand to 3.6 parts stone, determine the weight of the same volume. Making small changes in each direction, a proportion is eventually found that gives the maximum weight for the same volume. Record the proportions (the materials being in a dry, compact condition) noting separate weights (or volumes) and combined weights (or volumes).

(5) Add the cement-paste to that mixture of aggregates which shows the maximum weight for any given volume until the consistency of the concrete is as stiff as conditions will permit for proper placement.

(6) Compute the quantities of cement, sand, coarse aggregate, and water by deducting the amounts unused from the quantities originally measured and write the proportions by volume or by weight.

(7) Find the "field mix" by correcting the proportions to allow for moisture, bulking and loose measurement, as described in the preceding paragraphs.

The AMOUNT OF COARSE AGGREGATE that any given volume of cement paste can carry naturally depends upon the character of the work involved; the designer should put in as much coarse aggregate as possible without losing the degree of workability, or plasticity, which the type of work demands.

During the early stages of the work full-sized batches may be made using the proportions determined by trial, and corrections made if the concrete is not of a suitable consistency. As a guide in selecting trial mixtures Table V may be used: these mixtures are based on dry, compact volumes and proper allowance should be made for LOOSE MEASUREMENT and the effect of BULKING or increase in volume due to moisture.

Slump Test and Flow-Table. The two best-known methods employed for measuring the consistency and workability of plastic concrete are the slump

Table V. Trial Mixtures for Various Water-Cement Ratios

Water-cement ratios indicated include moisture contained in the aggregate.

Proportions are given by *volume, aggregate dry* and *compact.* Thus 1–2–3½ indicates 1 volume of cement, 2 volumes of sand and 3½ volumes of coarse aggregate.

If the aggregates are to be measured in the *damp* and *loose* condition they will occupy greater volume than when dry and compact. Amount should be determined by test. Approximate average value for sand 20%, for coarse aggregate 6%.

For approximate proportions *by weight* add 15% to proportions of aggregate shown in the table.

The mixes are given as a *guide only.* The first batch should be made with measured water content and the proportions thereafter adjusted to give the desired workability, maintaining the specified water-cement ratio.

Slump inches	Trial mix, dry compact volumes for maximum size of aggregate indicated	
	1 inch	2 inches
Water-Cement Ratio 5½ Gallons per Sack		
½–1	1–2–3	1–2–3½
3–4	1–1¾–2½	1–1¾–3
5–7	1–1½–2	1–1½–2½
Water-Cement Ratio 6 Gallons per Sack		
½–1	1–2¼–3¼	1–2¼–4
3–4	1–2–3	1–2–3½
5–7	1–1¾–2½	1–1¾–3
Water-Cement Ratio 6¾ Gallons per Sack		
½–1	1–2½–3½	1–2½–4
3–4	1–2 –3¼	1–2¼–3¾
5–7	1–2–3	1–2–3½
Water-Cement Ratio 7½ Gallons per Sack		
½–1	1–3–4	1–3–4¾
3–4	1–2½–3¾	1–2½–4¼
5–7	1–2¼–3½	1–2¼–3¾

test and the flow-table. The former is described in the Proceedings of the American Society for Testing Materials under Serial Designation: D 138–26 T. The sample to be tested is molded in a steel form, shaped as a truncated cone without top or bottom, 12 in in height, and with upper and lower bases 4 and 8 in in diameter, respectively. The plastic concrete is deposited in the mold in four successive layers with a specified amount of tamping. Three minutes after filling the mold is removed by raising vertically. The supported mass then either sinks down, tumbles over, or remains standing. (See Fig. 11.) The so-called SLUMP is determined as the difference in inches

between 12 in and the upper surface of the specimen. Figs. 11 and 12 illustrate the procedure. In FIELD PRACTICE, as a measure of the workability of various batches in which the proportion of cement and the character of aggregate is constant, the SLUMP TEST is a useful device. However, it is not sufficiently reliable to furnish a standard for consistency in laboratory work, and very erratic results may be obtained when the test is applied to the comparison of variously graded aggregates, particularly to the coarser sizes

Fig. 11. Slumps of Stiff, Medium and Wet Mixtures

and leaner mixtures. As it is a fair index of workability, and such depends upon the grading of aggregates and richness of mixture as well as quantity of water, it is apparent that the test should only be used for comparing the relative water-content between similar mixtures employing the same proportions and the same aggregate.

The FLOW-TABLE is more appropriate for laboratory purposes than for use upon the job, as it indicates the relative water-content, or consistency rather than the workability, which latter is usually of prime importance in the field. As designed by the Bureau of Standards the apparatus consists of a

Fig. 12. Measuring the Slump

metal-covered table with the top so arranged that it can be raised and dropped freely through a distance of ½ in by one revolution of a cam. The plastic concrete is molded in a truncated cone, the mold withdrawn, and the table raised and dropped 15 times in 10 seconds. The resulting diameter of the mass, divided by its diameter as molded, and multiplied by 100, is considered an index of the consistency or flow. A slump of 6 to 7 in might be obtained from a concrete showing a flow of 210 to 230.

Quantities of Materials for Concrete Mixtures. The quantities of materials in a concrete mixture may be accurately determined from the fact that the volume of concrete produced by any combination of materials, so

LONG AS THE CONCRETE IS PLASTIC, is equal to the absolute volume of the cement plus the absolute volume of the aggregate plus the volume of water. The absolute volume of a loose material is the actual total volume of solid matter in all the particles. This can be computed from the weight per unit volume and the specific gravity as follows:

$$\text{Absolute volume} = \frac{\text{unit weight}}{\text{specific gravity} \times \text{unit weight of water}}$$

The method is best illustrated by an example. Assume that the concrete consists of one sack of cement (94 lb), 2.2 cu ft of fine aggregate weighing 110 lb per cu ft, and 3.6 cu ft of coarse aggregate weighing 100 lb per cu ft, and is to be mixed with a water-cement ratio of 7 gal per sack. The specific gravity of the cement is usually about 3.1 and of the more common aggregates about 2.65. The volume of concrete produced by the above mix is calculated as follows:

$$\text{Cement} = \frac{94}{3.1 \times 62.5} = 0.49 \text{ ft}$$

$$\text{Fine aggregate} = \frac{110 \times 2.2}{2.65 \times 62.5} = 1.46 \text{ cu ft}$$

$$\text{Coarse aggregate} = \frac{100 \times 3.6}{2.65 \times 62.5} = 2.18 \text{ cu ft}$$

$$\text{Volume of water} = \frac{7.0}{7.5} = 0.93 \text{ cu ft}$$

$$\text{Total volume of concrete produced} = 5.06 \text{ cu ft}$$

Thus, one sack of cement produces 5.06 cu ft, neglecting absorption or loss of water in manipulation. The cement required for one cubic yard of concrete is, therefore, $\frac{27}{5.06} = 5.34$ sacks of 1.33 barrels. The quantities of sand and stone required can be found by a simple computation based on the number of cubic feet used with each sack of cement; thus, for the sand, $\frac{5.34 \times 2.2}{27} = 0.43$ cu yd required, and for the stone, $\frac{5.34 \times 3.6}{27} = 0.71$ cu yd of stone.

For unusual materials such as blast-furnace slag and similar light-weight aggregates, the exact specific gravities should be used. It will be found that, for the purpose of estimating quantities, the average value of 2.65, given previously, will be sufficiently accurate for sand and gravel and the common varieties of crushed rock.

Measuring Equipment. On most building operations the CEMENT is measured by counting the sacks, and the AGGREGATES are measured by volume. Some builders measure aggregates by weight, which is more desirable as this method eliminates any necessity for correcting the volumes because of the bulking of the sand due to moisture. It should be noted that this may be a very important item as some sands increase in volume as much as 40% through the addition of 5% by weight of water. VOLUME MEASUREMENT, however, is generally employed and is entirely satisfactory provided that it is accurately done.

The AMOUNT OF WATER should be carefully controlled by a measuring device which is calibrated to give accurate quantities and enable adjustments

to be quickly made. It should be provided with a means of locking so that the proportion of water cannot be changed except by some one in authority. Various appliances may also be purchased for measuring the sand in a submerged condition in order to eliminate differences in volume caused by variations in moisture.

Mixing, Conveying and Depositing Concrete

Mixing. Even for small operations concrete is ordinarily machine mixed in batch mixers. CONTINUOUS MIXERS are not to be recommended as it is difficult to control the proportions of the materials. HAND-MIXING can be done satisfactorily on a water-tight platform, but is so laborious that the cost is usually prohibitive.

Small mixers designed for a one-bag batch are very practical for small operations and for the mixing of plaster or mortar. For reinforced-concrete buildings a mixer of ¾-yd capacity is extensively used although the 1-yd capacity is occasionally seen on large operations.

Good practice requires that the mixing of each batch of concrete should continue not less than ONE MINUTE after all the materials are in the mixer, during which time the mixer should rotate at a peripheral speed of about 200 ft per minute. This requirement should be taken as an absolute minimum as tests show that the strength of concrete is appreciably increased by longer periods of mixing.

By referring to Fig. 16, it appears that the strength of concrete, on a 28-day basis, is increased nearly 15% by extending the PERIOD OF MIXING from one to two minutes. It is also true that longer mixing tends to develop a more homogeneous mixture and one which can be placed with less difficulty. The effective time of mixing is the actual duration of the mixing period after all the materials, including the water, have been placed in the mixer. Many mixers are now equipped with TIME-LOCKING DEVICES which prevent the mixer being discharged until the allotted time has elapsed.

Retempering Concrete. Although retempering (remixing with additional water) has been found of value in lessening the initial shrinkage of the concrete, the practice should not generally be allowed, as the retempered material may be quite unsatisfactory for structural purposes. Therefore the rule against using concrete that is partially set should be strictly followed.

Conveying Concrete. The principal agencies at present used for conveying concrete from the mixer to the forms are the TOWER AND HOIST as a means of elevation, and the BUGGY or two-wheeled carry-all for the purpose of distribution. CHUTES can be satisfactorily employed on large jobs which lend themselves to this type of distribution, but the tendency during the last few years has rather been in favor of buggy distribution. If chutes are used, the slope and length of the chute should be adjusted to permit the movement of the mixture in such a way as to avoid separation of the materials. In general, the SLOPE should not be flatter than 1 : 3 nor steeper than 1 : 2. The mixture should be sufficiently plastic to enable the concrete to travel at a speed that will keep the chutes clean, but not so fast that the materials segregate.

Where long chutes are used the concrete should be delivered into a HOPPER before it is deposited in the forms. By this means, a certain amount of remixing occurs to correct any segregation which may have taken place. If concrete is placed directly in the forms, it must not be allowed to fall any great distance from the end of the chute. Both before and after each day's run, the troughs or chutes should be thoroughly flushed with water which

should be discharged outside of the forms. The use of WHEELBARROWS in place of buggies for conveying concrete from hoppers to forms is usually confined to small jobs, or types of work where the shape of a wheelbarrow permits more easy placement of the material.

Within recent years CENTRAL MIXING-PLANTS have come into general use in all of our larger cities. Concrete mixed to a definite specification is conveyed to the job in trucks equipped with rotating drums. Under proper supervision the results are excellent.

Depositing Concrete. Before depositing concrete all debris should be removed from the forms. In COLD WEATHER it may be desirable to use steam for the purpose of removing ice and snow. Except in freezing weather wooden forms are often wet down immediately ahead of the concrete. It is somewhat better practice, however, to oil the forms as mentioned under Erection of Forms. This operation can be carried on at all seasons of the year, whereas it is not safe to permit the wetting of forms in freezing weather.

The TEMPORARY OPENINGS mentioned under Design and Construction of Forms should be used for the purpose of cleaning out the bottoms of column and wall forms and for the drainage of excessive water. A final check should be made immediately prior to placing concrete to make certain that all openings are properly closed, the forms thoroughly cleaned and true to the lines and dimensions given upon the drawings.

The depositing of concrete has only recently received the attention that it merits. In the past many inspectors have been satisfied to see the forms filled with a properly proportioned mixture and have given little attention to the method used in placing the material, other than to enforce the general requirement of spading. This has resulted in segregation, laitance, and lines of weakness which have been the cause of serious weathering on many types of work.

In the first place, the mixture must be of such a consistency that it will FLOW into the corners and angles of the forms and thoroughly embed the reinforcement. The custom of pouring a few buggies of "soup" into the bottoms of beam forms before depositing the concrete forming these structural members is not to be recommended, as the mortar surrounding the reinforcement is of such weak consistency that it has little strength or fire resistance. A careful proportionment of the materials, transportation that will avoid segregation, and a reasonable degree of CARE IN DEPOSITING the concrete will permit the proper embedment of reinforcement in any beam correctly designed without resorting to "soupy" mixtures.

In filling a length of form-work, concrete should not be deposited continuously at one point and allowed to flow to distant points, as this practice causes SEGREGATION of the water and "fines" from the rest of the mixture. Contrary to opinions once held, excessive amounts of tamping have been shown to cause separation of materials. Spading of the concrete mixture against the forms is another operation which is usually overdone by the conscientious workman. A little spading is sufficient to prevent honeycomb, and excessive manipulation results in drawing a layer of fines against the forms which produce an inferior wearing surface more subject to disintegration.

When concrete is placed in deep layers water is forced to the surface. The EXCESS WATER should be worked to a low point and removed without actually causing a flow.

When water exists in an excavation it should be removed before depositing concrete. A flow of water is particularly to be avoided as it tends to wash the cement from the newly deposited mass.

Laitance. This term is applied to the whitish, chalk-like substance of very little strength forming on the upper surfaces of concrete which is placed in too wet a condition. Specifications have, in the past, required the removal of laitance before continuing the successive stages of an operation; the mixture should be of such a consistency that laitance will not form—its presence indicates that the concrete is not properly mixed, or that it has been allowed to segregate. Lines of laitance between the work of successive days leave COURSES OF WEAKNESS through walls, or other structural work, which not only offer easy passage for water in foundations and retaining walls, but also hasten the SURFACE DISINTEGRATION of work above grade.

Construction Joints. In building construction, beams and girders are normally poured at the same time as the floor slabs. Except when cold weather makes a different procedure imperative, it is desirable to pour the columns and walls up to the bottoms of beams, or, in girderless construction, up to the bottoms of drops, or floor slabs, at least a few hours before the remainder of the floor-system is cast. This is done to permit the concrete in the columns to settle before placing the supporting members; otherwise there is a probability of cracks occurring around the necks of the columns.

Planes separating work done on different days, or at periods sufficiently apart to permit the partial hardening of the concrete, should be either horizontal or vertical. If the concrete is deposited in horizontal layers, the construction joints will be level. This matter is of course negligible in floor construction but of great importance in pouring walls. When applied finishes are not employed, a satisfactory appearance depends upon keeping the board marks and lines of deposit either horizontal or vertical.

LONG WELLS can be broken up by vertical construction joints, spaced at equal intervals, which may also serve as contraction joints, but, in any case, should be formed by a stopper which makes a straight vertical plane between the faces of old and new concrete. As joints in structural floors should conform to planes of minimum shearing-stress, work is stopped along the center lines of SLABS and at the mid-spans of BEAMS and GIRDERS. If a beam intersects a girder at this point, the joint in the girder is offset a distance equal to twice the width of the beam.

Before commencing the continuation of work, the surfaces previously cast are cleaned, roughened, and thoroughly saturated with water. It is also good practice to coat the old surfaces with a neat cement-grout, depositing the new concrete before the grout has attained its initial set.

As construction joints have no permanent function and are merely convenient divisions between work done on different days, or periods, it is often customary to place BONDS in the concrete for the purpose of strengthening the joint and, in the case of walls, to leave recesses in the concrete first poured to make a tight joint with the new work. The bonds employed are usually $\frac{3}{8}$-in or $\frac{1}{2}$-in round rods, extending 30 or 40 diameters each side of the joint.

When forming construction joints that are required to be water-tight, good practice recommends that horizontal joints be constructed by forming a CONTINUOUS KEYWAY in the lower portion of the concrete before it has hardened. Vertical joints may be made by the use of a METAL WATER STOP of non-corrodible material.

Curing Concrete

Effect of Temperature. The hardening of concrete is a chemical process which requires warmth as well as moisture. When newly mixed concrete is subjected to a FREEZING TEMPERATURE the water crystallizes and is not

available for action with the cement. If the concrete freezes immediately after depositing, it may be permanently injured and when eventually thawed out, even several months later, will probably have only a small proportion of the strength which would normally be attained at that age. Owing to this fact, a number of serious accidents have occurred through stripping FROZEN CONCRETE which had become hard by reason of freezing and not through the process of normal hydration. It is also true that if newly placed concrete is frozen for only a short period it may never attain its normal strength and ALTERNATE FREEZING AND THAWING IS EXTREMELY HARMFUL, causing structural weakness and the crumbling and spawling of exposed surfaces.

Most men in charge of concrete work realize the injury that may be caused by actual freezing, but fail to appreciate the extent to which the hardening

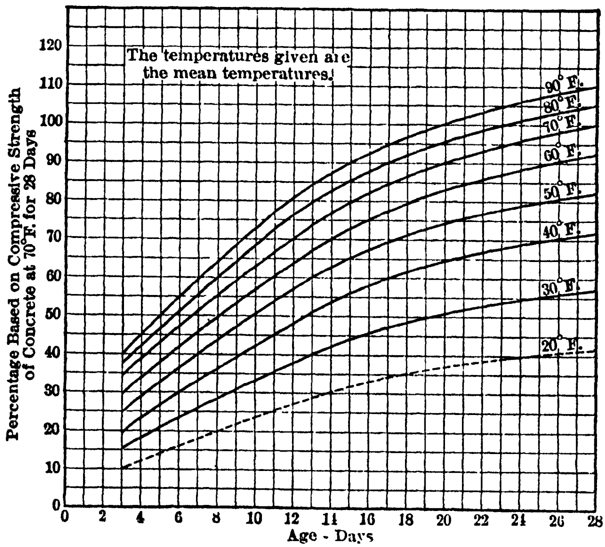


Fig. 13. Percentages of Strength for Different Temperatures

process may be delayed by LOW TEMPERATURES. Fig. 13 shows the relative strength of concrete cured at different temperatures. From these data it is apparent that a mixture which would normally acquire 52% of its 28-day strength when cured at a temperature of 70° F., for a period of 7 days, would only develop about 33% of the same strength when cured for a like period at a temperature of 40° F.

Effect of Moisture. As concrete requires water for the hydration of cement, which is a slow process extending over a long period, it is necessary to supply moisture during the time that the concrete is curing. Fig. 14 illustrates the value of MOIST CURING. All specimens were tested at an age of four months, but they were cured differently, being placed in damp sand for the various periods indicated and then allowed to cure in dry air until tested

The chart shows that the same concrete which developed only 1 400 lb in compression when cured for 120 days in dry air, gave a compressive strength of 2 500 lb when cured for 10 days in damp sand and for 110 days in dry air. It may also be noted that a very appreciable increase in strength is obtained by moist curing for approximately the first 30 days.

FLOORS and SIDEWALKS should be covered with a layer of burlap, sand, earth, or sawdust as soon as the finish is sufficiently hard to prevent injury, and kept wet for at least 10 days after the concrete is deposited. This practice not only gives a stronger concrete, but also greatly lessens the wear and dusting of the surfaces, which qualities are generally considered to depend upon the strength. Structural concrete such as WALLS, COLUMNS, and the undersides of FLOOR-SLABS, should be supplied with moisture by frequent sprinkling. In cold weather, when such a procedure is of course impossible.

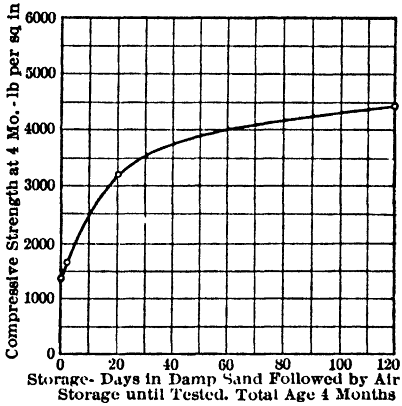


Fig. 14. Relation Between Curing Conditions and the Compressive Strength of Concrete

moisture may be supplied to the enclosed portion of a building by the use of steam or by water containers placed over salamanders.

Applied WALL FINISHES such as stucco should be protected from both sun and hot winds for a period of a week to 10 days and thoroughly sprinkled at least twice a day during this time. Such work should never be carried on in freezing weather except when the entire area is enclosed.

Effect of Age. Although, for design purposes, the strength of concrete at an age of 28 days is usually taken as a basis for determining working stresses, Fig. 15 shows a CONSTANT INCREASE through a period of 5 years. These data represent a large number of specimens made with various water-cement ratios and of different mixtures. It is interesting to note that concrete which would normally give a strength of about 2 700 lb per sq in at an age of 28 days attains a strength approximately 5 200 lb per sq in at an age of 5 years.

Winter-Weather Operations. Concrete work may be carried on successfully in weather with FROSTY NIGHTS, without enclosing the work, provided the water used in mixing is heated to a temperature of about 130° F., and

both the fine and coarse aggregates are sufficiently warmed to insure the removal of frost. When stock piles are placed upon the ground this can be easily accomplished by piling the materials over old sewer pipes, or on a sheet of corrugated metal, under which a wood fire is kept burning. A few

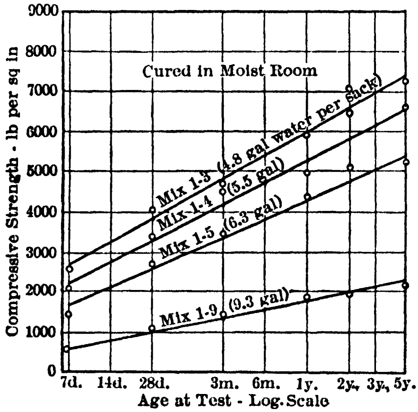


Fig. 15. Relation Between Age and the Compressive Strength of Concrete

pails of boiling water are usually sufficient to temper that used for mixing purposes. If there is steam available on the job, this can be used to heat the water as well as to remove all frost and ice from the faces of forms and reinforcement. Each night after finishing a day's work the newly placed con-

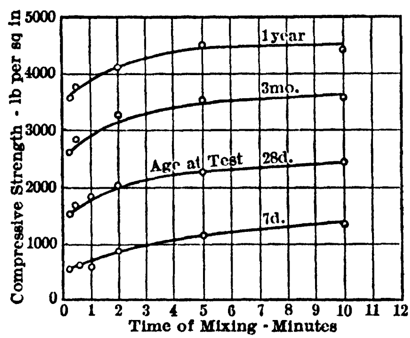


Fig. 16. Relation Between Time of Mixing and the Compressive Strength of Concrete

crete should be protected with canvas or tar paper, or a layer of salt hay or similar material. Protection of this kind should be replaced each night or when frost is imminent for at least one week after the concrete has been poured.

Although the foregoing precautions are satisfactory for frosty weather and

particularly suited to small operations, it is necessary to provide a much more elaborate plant for large buildings constructed in WINTER WEATHER.

Most builders engaged in the erection of reinforced-concrete structures consider it necessary during really cold weather to enclose several stories with canvas and to provide artificial heat through the interior either by means of salamanders or steam. The water and aggregates are heated by steam supplied from a boiler provided for the purpose and every precaution is used to insure that the concrete is placed at a temperature not less than 50° F. nor over 120° F. To obtain a desirable temperature, under average conditions, the water should be heated to about 150° F. and the aggregates to a temperature of at least 40° F., the exact temperature depending upon that of the outside air, the character of the plant, and the type of work under construction. On all cold-weather operations, temperatures should be taken at various critical points at regular intervals throughout the day and night, in order that the rapidity of curing may be closely estimated before removing the forms.

Concrete in Alkali Soils. If concrete is subjected to ground water, alkaline in nature, disintegration may result. Concrete sewers are particularly vulnerable, but trouble has occurred with building foundations which should be protected by a system of subsoil drainage where high concentraions of soluble sulphates occur.

When DESIGNING concrete mixtures to resist destructive agents of this nature, it should be remembered that impermeability is a most important quality. Care should be exercised to obtain a well-graded aggregate, neither porous nor weak in structure, to use a consistency as dry as can be properly placed, and to insure proper mixing and careful curing under moist conditions. It is also desirable to increase the quantity of cement in the concrete, some designers requiring a minimum of 1¾ barrels per cu yd.

When PLACING concrete subject to the action of alkali, the number of horizontal construction joints should be reduced to a minimum, as such are planes of weakness along which surface disintegration is most likely to start. Reinforcement should also be more thoroughly protected for serious exposure of this nature, 2 in of concrete being a minimum for this purpose, and in foundations and footings a 4-in protection is generally required.

Construction under Water. Only as a last resort should concrete be deposited under water, as the results are always subject to a degree of uncertainty. When, however, such a procedure is necessary, a somewhat richer mixture should be used in order to provide for any loss of cement during the process of depositing. One and three-quarter barrels of cement per cu yd is a desirable minimum. A satisfactory method of placing concrete under water is with a TREMIE, or steel pipe of sufficient length to permit the lower end reaching the bottom of the space within which the concrete is placed, while the upper end is above water-level. When first lowered in place, the tremie is plugged at the lower end to prevent the entrance of water, or the escape of the concrete; when in position, the plug is removed and the concrete allowed to flow out slowly. A hopper at the upper end provides a continuous stream of concrete and, as the lower end is kept submerged in the plastic mass already deposited, the loss of cement is minimized.

Salts in Sea-Water. The salts present in sea-water are of the sulphate-chloride type which produce disintegration on concrete surfaces by forming soluble compounds with the elements of the cement. The action of salt water has little or no effect upon impervious concrete. Permanent results may be accomplished if the same precautions are followed as recommended

in the case of alkali soils. Impermeability demands sound aggregates of low porosity. The AMOUNT OF MIXING WATER should not exceed six gallons per sack of cement, which should include the free moisture in the aggregates. (See Table IV.) The aggregates should be combined and proportioned with the cement as described, mixed until thoroughly homogeneous, and transported without segregation. During the process of depositing, the plastic mass should be of such a consistency as to flow easily into place without excessive tamping and free from any accumulation of water upon freshly poured surfaces. Where it is impossible to pour a unit of work in one continuous operation, CONSTRUCTION JOINTS should be left, and a good bond obtained between old and new work, as described under Construction Joints. Favorable curing conditions are extremely important, and where the highest quality of concrete is necessary, the surfaces should be kept damp for a period of two weeks after placement.

Water-Tight Concrete. It is perfectly possible to construct water-tight concrete walls; the fact that most concrete work is extremely porous is not due to any inherent failure in the materials but to a lack of knowledge in construction methods. Providing that a structure is properly designed, walls can easily be constructed to resist HIGH WATER-PRESSURES without the use of water-proofing compounds. Although it is essential to apply a flexible membrane where movement due to unequal settlement or other cause may be expected, and surface applications of cement mortar are of great service in certain types of work, the use of an integral compound for water-proofing purposes would seem to be of very little value. It is extremely difficult, if not impossible, to make poor concrete water-proof by the use of admixtures and, if the recommendations given in the previous paragraphs be scrupulously followed, water-tight concrete can be built without the use of any proprietary compounds.

Shrinkage and Expansion of Concrete. VOLUMETRIC CHANGES normally occur during the hardening process of concrete. Specimens that are cured in dry air commence to SHRINK from the time that they are formed and continue to decrease in volume for a period of several months. If the same specimens are placed in water after their initial hardening, a slight INCREASE IN VOLUME occurs over a like period. If subsequently dried, they will shrink to about the same volume as those cured in dry air.

The PRINCIPAL CAUSE OF SHRINKAGE is the loss of water due to evaporation, or absorption by the aggregate or by the forms. The preliminary shrinkage which takes place while the concrete is still plastic is fairly rapid, depending upon the rate of the evaporation of the mixing water, as affected by the relative humidity of the air and the exposure of the mass. For example, concrete placed in a thin wall exposed to hot sun and wind shrinks to an extent that may result in surface checking.

SECONDARY SHRINKAGE, occurring after the member has assumed a definite form, is also due to loss of water from the concrete, but is very much slower than that occurring immediately after placement. RICHER MIXTURES shrink more than those containing less cement, as the cement paste is the material responsible for the volume changes. It is also true that the degree of shrinkage increases as the mixture becomes wetter, that is, as a function of the water-cement ratio.

TEMPERATURE CHANGES in the surrounding air result in volumetric changes throughout the life of a structure. The theoretical coefficient of expansion of concrete for normal temperatures is approximately 0.000005, which means that for a change in temperature of 100° F. a concrete member 100 ft long

would increase in length approximately $\frac{5}{8}$ in; the actual change in length of concrete structures, however, does not usually approach this value.

Changes in ATMOSPHERIC MOISTURE also cause volumetric changes in concrete structures which may be as great as that due to a change of 100° F. in temperature. These various factors make it necessary to provide for the contraction and expansion of concrete members as described under Contraction Joints in Concrete Buildings, Part 3.)

Finishing Concrete Surfaces

Exterior Finishes of Monolithic Buildings. The first essential for an attractive exterior is that the structure be built true to line and level. Any amount of surface treatment cannot overcome the ill effect of bulging walls, drooping beams, or irregular arrises.

Within the last few years some very attractive work has been done in concrete leaving the SURFACES PRACTICALLY AS STRIPPED. Unless covered by an applied finish of appreciable depth, board marks will show on surfaces formed in wood, no matter how carefully the work may be done. These lines, however, if kept absolutely horizontal, are unobjectionable for certain types of work. When constructing columns, it is customary to run the boards comprising the forms vertically. The method used foı securing the forms should also be considered in relation to the surface, as it is impossible to cut back tie-wires without patching the holes. There are at present some very satisfactory devices which secure the forms in place without leaving unsightly holes or patches that must later be filled.

The consistency of concrete is another matter which affects the APPEARANCE OF THE FINISHED SURFACE where such is not covered by an applied finish. If it is intended to remove the film of cement by either proprietary compounds, or by scrubbing, or tooling the surface, the consistency of the mixture should be very carefully controlled in order to avoid separation of the materials, which produces a spotty surface. Laitance due to excessive moisture, although always objectionable from the structural viewpoint, is particularly disastrous where concrete surfaces are to be given an exposed aggregate treatment, as this necessitates patching with its accompanying discoloration of the surfaces.

Although excessive spading along the faces of forms results in carrying too great a proportion of "fines" to the exterior surfaces of the concrete, particular care should be exercised to avoid honeycomb, which can be remedied from a structural viewpoint, but which is extremely hard to make presentable in appearance.

FORMS should be carefully designed for all detail work cast in concrete, in order to permit stripping without danger of breaking moldings and other small members. If it is necessary to repair exterior work, this should be CAST IN PLACE in wood or metal forms. The practice of "running" concrete moldings with a templet, as might be done on interior plaster work, is seldom successful and should not be permitted, as the movement of the templet over the partially hardened concrete almost invariably causes structural weakness resulting in rapid deterioration under the action of rain and frost. If it is necessary to patch work previously cast, such as broken moldings or the corners of posts, the old work should be drilled and the new portions thoroughly bonded by the use of dowels and reinforced with small rods.

The first step in finishing the exteriors of monolithic buildings is the removal of all nails, wires, or bolts. If tie-wires are cut back they should

be protected by at least one inch of mortar. The next step, which is also essential for even structural reasons, is the filling of stone-pockets or honeycomb with a cement mortar of approximately the same proportions as used in the concrete mixture. If it is necessary to remove defective work, care should be exercised to cut sharp shoulders, at least ½ in deep, around the edges of all cavities: if a patch of mortar is spread out thinly over the surrounding concrete, the edges invariably crack off. The best work of an ARCHITECTURAL CHARACTER now avoids the use of ordinary bolts, thereby eliminating the holes, but where such are used, the holes can be stopped with corks driven to a sufficient depth to permit a 1-in protective coat of mortar to be placed over the top. All patches of whatever nature should be kept wet for a week after their application. If this is not done, the shrinkage of the mortar in hardening tends to develop cracks between old and new work.

Types of Exterior Finishes. One of the most satisfactory treatments for the exterior surfaces of industrial buildings is a finish applied by means of smoothing and slightly GRINDING THE SURFACE with carborundum stones applied by hand. The first step in the process is to thoroughly rub the surfaces with a No. 20 carborundum stone, using plenty of clean water, which develops a thin paste later on removed by washing and brushing. This operation is more effective if carried on while the concrete is still green, preferably about 24 hours after placement. If the work is delayed until the concrete has thoroughly hardened, a cement wash is used instead of plain water, which assists in the formation of a thin paste. After the structural work is finished, and when the building is ready for final cleaning, the surfaces are again rubbed with a No. 24 carborundum stone, using plenty of clean water. This operation is followed by brushing and washing without the application of any other surfacing material.

For a number of years various means have been employed to remove the thin layer of cement forming on the exterior of concrete surfaces, in order that the aggregate may be exposed. Although satisfactory results are obtained over limited areas by scrubbing partially hardened concrete surfaces with wire brushes, this and similar treatments produce very uneven results when applied to the exteriors of entire buildings. One of the principal difficulties is the impracticability of stripping all portions of the work at exactly the same age, which makes it impossible to obtain uniformity in the appearance of treated surfaces. Where an EXPOSED AGGREGATE FINISH is desired it is recommended that the surface treatment be at least supplemented by the use of a proprietary compound, similar in appearance to medium-weight oil, with which the forms are coated, and which has a property of killing the action of the cement on the surface against which it is applied. The manufacturers claim that by painting the surfaces of the forms with this material, the aggregate may be easily exposed after stripping by merely using a wire brush with water.

TOOLING can be used for the finishing of monolithic concrete as readily as for the various types of artificial stone. It is of course essential that the concrete be of a homogeneous nature and sufficiently hard to avoid pitting or spalling of the surfaces. As this is a rather expensive operation, its application is usually limited to panels and used more in a decorative way than as a general exterior finish.

Excellent CEMENT PAINTS may be obtained which are prepared for exterior use where it is desirable to paint concrete surfaces. Such coatings are prepared to resist the action of the lime in the concrete, and they cover about the same range of colors as found in ordinary oil paints.

Although not of particular value as a permanent finish, a CEMENT WASH composed of 1 part of white or gray cement to 1 part of finely screened sand with 5% of hydrated lime, measured by volume of cement, may be applied with a whitewash brush and is satisfactory for some types of work. The sand should be screened through a No. 18 sieve. This application may be later rubbed into the concrete surface by means of a CORK FLOAT or carborundum stone, or the surface may be brushed; in any case, all excess material should be removed and the work left eventually with only the thinnest possible coating. In order to avoid dusting, washes of this nature should be sprinkled with a fine spray, two or three times a day, for at least three days following the operation.

Although it is extremely difficult to make TROWEL APPLICATIONS adhere to hardened concrete walls, unless the latter are thoroughly roughened, it is possible to successfully apply a thin finish thrown on by means of a brush or paddle. Applications of this nature should not be attempted upon surfaces cast in metal forms, and they require, in any case, careful workmanship. The surfaces should be partially saturated with water shortly in advance of the work, but not by any means drenched, the idea being to avoid an excessive absorption of moisture from the mortar but to retain enough suction so that the applied material will adhere to the concrete surface. Only a thin coating can be used, and if the surface is not sufficiently covered, two thin applications should be made instead of one of greater thickness. The success of such an operation may well depend upon the method of curing, which should insure protection from both sun and hot winds, and sprinkling two or three times a day for a week after application.

Granolithic Floor Finishes. Concrete floor surfaces may be laid either MONOLITHICALLY with the slab forming the structural floor, bonded to the slab if the latter has hardened, or laid quite independently over a fill such as cinders or cinder-concrete. For industrial buildings where there are comparatively few pipes or conduit, and these can be laid out in advance of the work and run in the thickness of the structural slab, it is more economical to use a monolithic finish. If so laid, the wearing surface, ¾ in or 1 in in thickness, can be considered, for design purposes, a part of the structural thickness of the floor, which makes a very appreciable saving in the cost of a large building.

On WINTER WORK, however, it is not practicable to finish and protect large floor-areas so that it often becomes necessary, even for industrial operations to place the finish later. For office-buildings and particularly for apartments, where granolithic finishes are employed in halls and service portions, it is usually necessary to use a fill between the structural slab and the finished floor, for the purpose of supplying space for pipes and conduit. Under these conditions, the granolithic finish, not having the support of the structural slab, is usually made somewhat thicker, varying in different specifications from 1 to 2 in.

The principles involved in securing a good floor finish are the same in any case. Research work has proven that the ABRASIVE RESISTANCE of concrete increases with its compressive strength and that only a poor concrete is subject to DUSTING. The problem is then to design a mixture for the granolithic surface on exactly the same principles as previously outlined for concrete mixtures used in structural work.

MATERIALS FOR FLOOR FINISH should conform to the standard requirements except that the size of the aggregate will of course be much smaller. Proportions are varied for different classes of work, a popular mixture being 1 part cement to 1 part sand to 1 part of crushed stone, the latter passing a screen

having $\frac{3}{8}$-in openings, and not more than 10% passing a screen having $\frac{1}{4}$-in openings. One part of cement combined with $\frac{1}{2}$ part of sand, and $1\frac{1}{2}$ or 2 parts of coarser aggregate, may sometimes make a better mixture, or the specification might be written to accept the entire run of a crushed stone or gravel passing a $\frac{1}{2}$-in or $\frac{3}{8}$-in screen with the dust only removed, depending upon the relative grading of the aggregates. It is a general principle that better work will be obtained if the aggregate is chosen as coarse as practicable.

The limitation on the amount of water is one of the most vital considerations in the specification of granolithic floors. Owing to the variation in materials, it is impossible to make any definite statement in regard to the quantity that should be permitted, but this should be limited to that which is necessary to produce a consistency as dry as can be smoothed upon the floor without tearing under the strikeboard. In general, this corresponds to a slump of between 2 and 3 in.

Where COLORING MATERIALS are used, it is desirable to choose a white cement as the ordinary grade of Portland cement does not produce clear colors. Although many builders are accustomed to use calcium chloride, or various proprietary compounds, for the purpose of accelerating the HARDENING PROCESS, this practice is not by any means universal. The use of such admixtures is, however, often economical owing to the saving in overtime labor on the part of the finishers. Hardeners are sometimes recommended as being absolutely necessary in order that a good wearing surface may be produced. Although these compounds probably assist in gaining this result, they are not essential, and excellent work is done without them.

Granolithic floor surfaces are normally troweled to a smooth, hard finish by the use of both wooden float and steel trowel. Two separate trowelings are required, the first being done as soon as the surface is hardened sufficiently to support the knee-boards upon which the workmen kneel, and the second or last troweling just before the initial set takes place. A wooden float has the advantage of not exerting as much suction as a steel trowel but, on the other hand, the steel is necessary to form a hard metallic surface. Excessive troweling should be avoided, and the practice of sprinkling dry cement upon the surface of the concrete in order to hasten the drying process tends to form a skin coat which will later be subject to dusting and crazing.

Exterior work, such as sidewalks and the surfaces of stadiums, should not be troweled to the same extent as the floor surfaces. In fact, it is recommended that such be finished by means of a wooden float and untouched by the steel trowel. Most defective work in this field is due to two causes: excessive water in the mixture, and excessive troweling. Failure usually occurs through surface crazing due to the fact that all of the "fines" in excessively wet mixtures have been drawn to the surface by the suction of the trowel. This surface layer of fine sand and cement is very quick to succumb to the action of rain and frost.

CHAPTER XXIV

TYPES OF ROOF-TRUSSES

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1. Definitions

Truss. A TRUSS is a structural framework composed of a series of straight members so arranged and fastened together that external loads applied at the joints will cause only direct stress in the members.

The only geometrical figure which is incapable of a change in shape without a change in length of its sides is the triangle, which is the basic arrangement of the members composing a truss.

A SIMPLE TRUSS is one in which (*a*) the combination of members forms a complete series of triangles; (*b*) the axes of all members at each joint meet in a common point; (*c*) the reactions are not restrained horizontally but are vertical under vertical loading; and (*d*) the reactions due to inclined loading can be determined by the fundamental equations of static equilibrium.

Simple trusses are designed to act as beams, when spans and superimposed loads are too great to permit the economical use of sections commonly referred to as beams. A ROOF-TRUSS, being composed of a series of joined members, can be adjusted to conform to the shape of the roof outline and also to provide transverse support for the ceiling construction.

Chord Members. The upper and lower flange members of a truss are called the UPPER and LOWER CHORD, respectively. When the external loads act downward and the truss is supported at its ends, the upper chord is always in compression and the lower chord always in tension similarly to the upper and lower flanges of a simple beam.

Web-Members. Those members of the truss which are framed between, and join the upper and lower chord, are called WEB-MEMBERS. They are subjected to direct stresses of either tension or compression of amounts such that the vertical components of their stress on a given vertical section will neutralize the unbalanced vertical summation of all external loads, reactions and vertical components of chord-members on that section.

Web-members which are subject to tensile stress are called TENSION WEB-MEMBERS, and those which are subject to compression are called COMPRESSION WEB-MEMBERS.

Counterbrace. A web-member which is designed to resist either tension or compression is called a COUNTERBRACE. In certain positions a web-member may be subjected to tension through the action of a load applied at one point, and when applied at another point that load may produce compression in the member, hence at one time it will be subjected to tension and at another time to compression.

When the stress in any member of a truss is the result of several external forces, some of which cause tension and others of which cause compression, the maximum combination of the algebraic sum is the MAXIMUM DESIGN

STRESS for the member. Such a member is not a counterbrace unless under two different possible combinations of loads the net stress is tension in one case and compression in the other.

Counter. A COUNTER is a member of a truss system which acts only for a particular partial loading, and which has zero stress when the truss is completely loaded.

Panel. A PANEL, or PANEL LENGTH, is the distance between two adjacent joints along either the upper or lower chords. The quadrangular space, crossed by an inclined web-member, is also referred to as a panel of the truss.

Bay. The portion of the roof between two adjacent trusses is called a BAY.

Bent. When a truss is supported at its ends by columns, the truss together with its columns, considered as a unit, is called a BENT.

Panel-Point. The intersection of two or more members of the truss is called a JOINT, or PANEL-POINT.

Purlin. Beams supported by the upper chords, spanning from truss to truss and supporting the roof construction, are called PURLINS. Whenever possible purlins should be supported at the panel-points.

Ceiling-Beams. Beams supported by the lower chords, spanning between trusses and supporting the ceiling-construction, are called CEILING-BEAMS. Ceiling-beams should, wherever possible, be framed to transfer the ceiling loads to the panel-points of the supporting truss.

Rafters. An inclined beam resting on and supported by the purlins, usually about 16 in to 24 in on centers, and which supports the sheathing directly, or may support sub-purlins, is called a RAFTER.

Sub-purlin. A secondary system of beams parallel to the purlins and supported by the rafters is sometimes employed to support tile or slate weathering surfaces. These beams are called SUB-PURLINS.

Weathering Surface. The finished upper surface of the roof construction such as tile, slate, wooden shingles, copper, etc., is called the WEATHERING SURFACE.

Structural Covering. The construction above the purlins, such as rafters and sheathing or a concrete slab designed to support the weathering surface, is called the STRUCTURAL COVERING.

Span. The SPAN of a roof-truss is the distance between the centers of the supports, which in a simple truss is the distance between end-joints.

Rise. The RISE is the distance between the apex, or the highest point, of the truss and the line joining the points of support.

Pitch. The PITCH of a roof-truss is the ratio of the rise to the span for a truss symmetrical about its center line.

Slope. The SLOPE of an inclined chord-member is the tangent of the angle of inclination with the horizontal, usually specified in inches rise per 12-in horizontal run.

2. Classification of Trusses

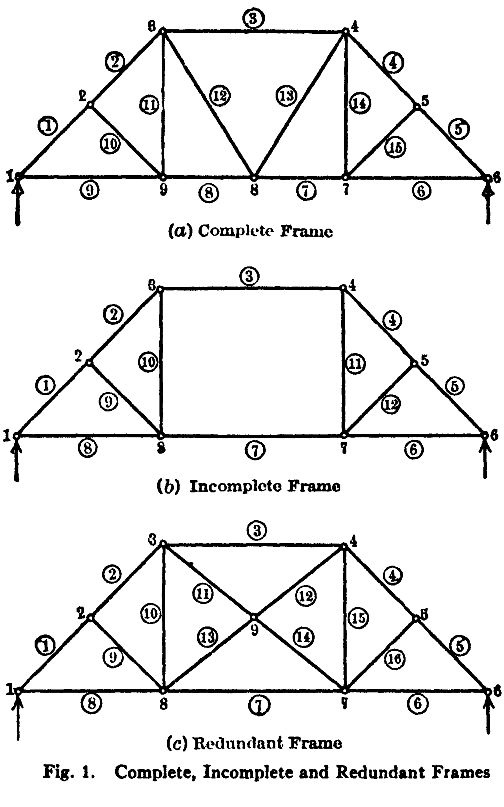
Fundamentally, trusses may be divided into three general classes, relative to the number and arrangement of the members composing the truss:

(a) Complete Frame. A complete structural frame, or truss, is one which is made up of the minimum number of members required to provide a com-

plete system of triangles fixing the relative positions of a given number of panel-points. Beginning with the first triangle of any series, the three sides locate three panel-points, and each additional panel-point requires two additional sides for its location, hence, if p = the number of panel-points in the entire structure and n = the minimum number of necessary members:

$$n = 2p - 3 \tag{1}$$

A simple form of a complete structural frame is shown at (a), Fig. 1. In



(a) Complete Frame

(b) Incomplete Frame

(c) Redundant Frame

Fig. 1. Complete, Incomplete and Redundant Frames

this frame there are fifteen members (1) to (15), and nine panel-points as shown, whence, from Equation (1), the frame is complete:

$$n = (2 \times 9) - 3 = 15$$

(b) Incomplete Frame. An incomplete frame is one in which the number of members is less than that required by Equation (1). Such a frame is

shown at (*b*), Fig. 1, where the total number of panel-points is 8, hence the minimum number of necessary members is:

$$n = (2 \times 8) - 3 = 13$$

The number of members is 12 or one less than the required minimum, and the frame is, therefore, incomplete.

A structure such as illustrated at (*b*) is unstable except under symmetrically arranged loads, and from the general definition given in Article 1 this frame, or any other incomplete frame, is not a true truss.

(c) Redundant Frame. A redundant frame is one which contains a greater number of members than required by Equation (1).

The frame shown at (*c*), Fig. 1, is a redundant frame since there are 9 panel-points and 16 members, as shown. The number of members should be:

$$n = (2 \times 9) - 3 = 15$$

or one less than the number shown.

If a second diagonal is added to any quadrilateral panel, the added diagonal is always a redundant member; however, if such an added member is capable of resisting only one kind of stress, the redundancy is only apparent.

If in Fig. 1 (*c*) the diagonals in the center panel are made of rods with no joint at 9, the number of members is 14, but for eight joints the limiting number is $(2 \times 8) - 3 = 13$.

Since the rods cannot take compressive stress, only one of the diagonals will act at a time, depending upon the direction of the shear in the center panel as the applied load is shifted from one side of the center line of the structure to the other; hence, in reality the frame is complete and the redundancy only apparent.

3. Types of Roof-Trusses

Roof-trusses may be classified as to type with respect to form, method of support, or arrangement of the web-bracing system.

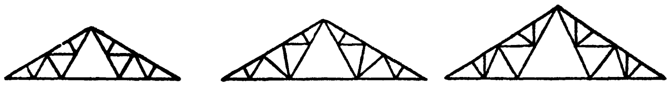
(*a*) The general form or outline of a truss is often determined by architectural considerations, and it may be TRIANGULAR, QUADRANGULAR, CRESCENT, SCISSORS, ARCHED, etc.

(*b*) A truss may be supported at each end-joint, known as a SIMPLE TRUSS SPAN, or at one end-joint and one other joint not an end-joint, known as an OVERHANGING END SPAN, or the entire support may be supplied at one end only, known as a CANTILEVER SPAN.

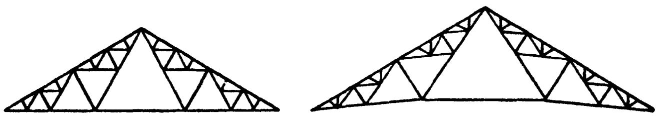
(*c*) The subdivision of a truss into triangular elements may be accomplished by various arrangements of the web-members. The systems of web-bracing in general use bear the names of the men who introduced them, such as Howe, Fink, Pratt and Warren.

Figs. 2 and 3 illustrate some of the more common types of roof-trusses used in building-construction.

All the types of trusses illustrated are well adapted to steel construction, but on account of practical difficulties in making satisfactory joints, the more suitable types for timber construction are the HOWE, SCISSORS, and HAMMER-BEAM types.

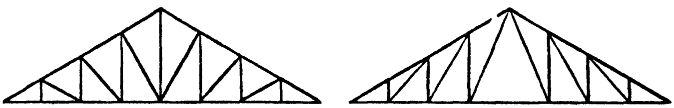


8 Panels 10 Panels 12 Panels

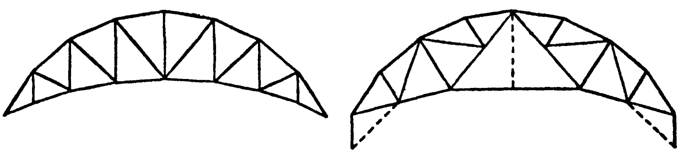


16 Panels 20 Panels (Cambered)

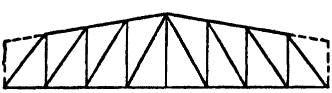
Types of Fink Trusses



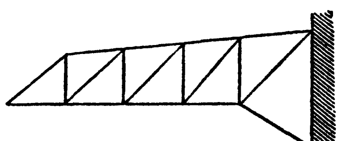
Howe Pratt



Crescent Trusses



Quadrangular Truss
Howe Type



Cantilever

Fig. 2. Common Types of Roof-trusses

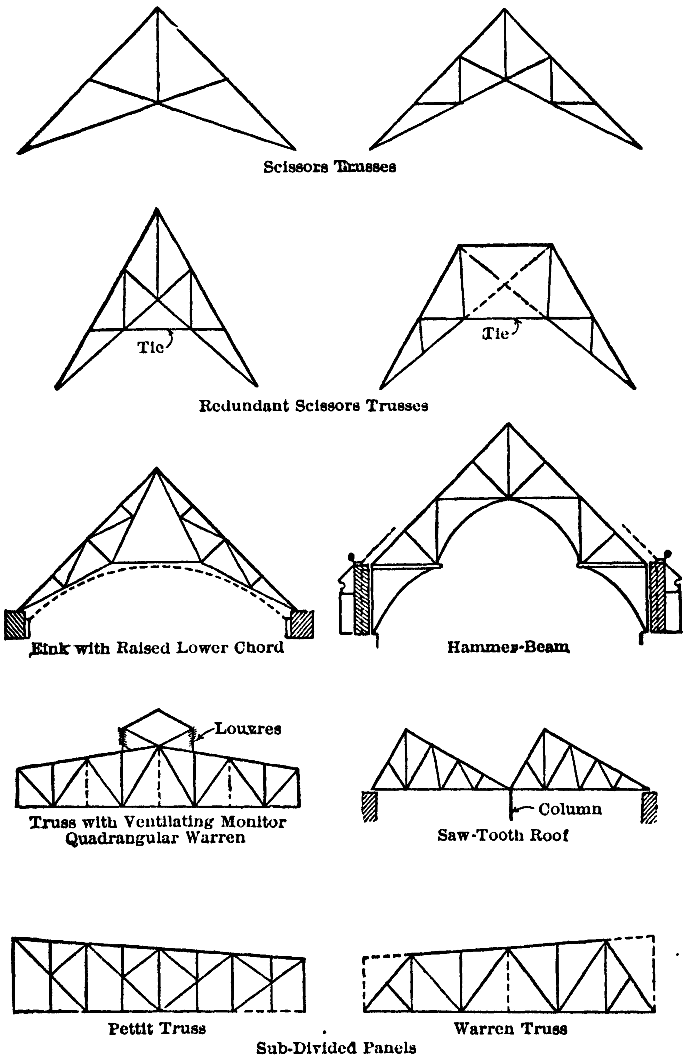


Fig. 3. Common Types of Roof-trusses

4. Composite Trusses of Timber and Steel Rods

The term TIMBER ROOF-TRUSS is commonly used to designate a truss constructed principally of wood, but with the tension web-members composed of steel rods. It is possible to construct many types of roof-trusses entirely of timber; however, it is practically impossible to design economical joints

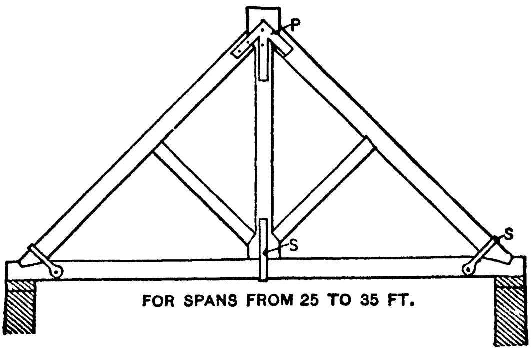


Fig. 4 Modern King-post Truss

which will effectively transfer tensile stress in an all-timber truss, except for the most simple types of short-span trusses.

The HOWE TRUSS, Fig 2, is the type most extensively used for composite trusses of timber and steel rods. The number of panels depends upon the span of the truss and the purlin spacing and when there are only two panels

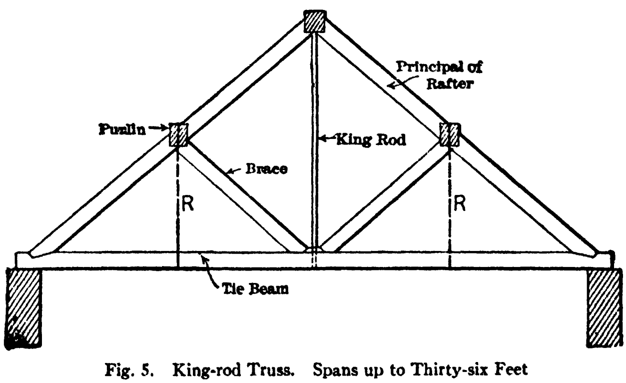


Fig. 5. King-rod Truss. Spans up to Thirty-six Feet

for each one-half of the truss span, as shown in Fig. 4, the truss may be built entirely of timber, excepting the steel strap fastenings for the vertical tension member. Such a truss is called a KING-POST TRUSS. A more practical construction of such a truss is shown in Fig. 5, where the center tension-member is a steel rod passing through the chord members, and secured at each end

by a nut which takes bearing on a washer. When an attic-floor or a ceiling is to be supported by the lower chord additional panel-points are provided by additional tension-members, as *R*, *R*, shown by dotted lines.

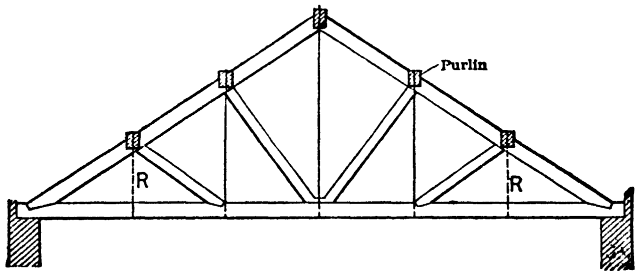


Fig. 6. Six-panel Triangular Howe Truss Spans from Thirty-six to Fifty Feet

Figs. 6 and 7 illustrate the extension of the Howe type of composite timber and steel-rod truss for spans up to 48 and 60 ft, respectively.

Trusses with Parallel Chords. Trusses with parallel chords are frequently

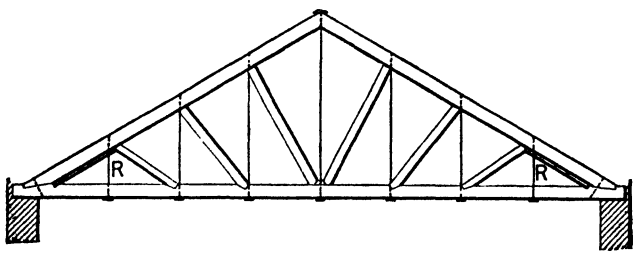


Fig. 7. Eight-panel Triangular Howe Truss Spans from Forty-eight to Sixty Feet

used in deck-roof construction, for intermediate supports of long hips in hipped-roof construction, and for flat-roof construction

When, as is generally the case, there is a possibility of unsymmetrical load-

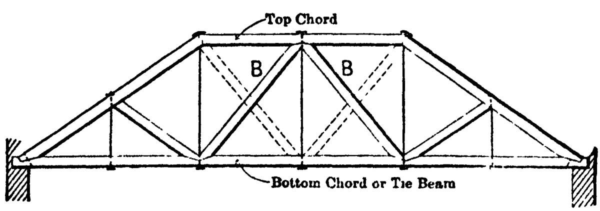


Fig. 8. Queen-rod Truss. Spans from Forty to Fifty-two Feet

ing, due to wind, snow, or unequal distribution of dead loads, counters must be supplied in those panels where the direction of the shear under partial

load differs from that for full loading, as illustrated in Figs. 8 and 9. Trusses of this type, as in Fig. 10, may be employed for spans up to 80 ft; however, excepting in localities where timber is relatively cheap and easily obtainable,

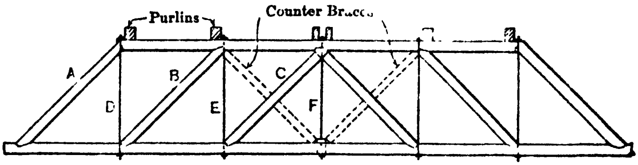


Fig. 9. Six-panel Howe Truss

steel trusses are more economical than composite timber trusses for spans in excess of 60 ft.

When the truss is placed in the longitudinal direction of a flat roof, as shown in Fig. 11, the upper chord may be made to conform to the inclination of the roof so as to support the rafters directly without blocking. It should

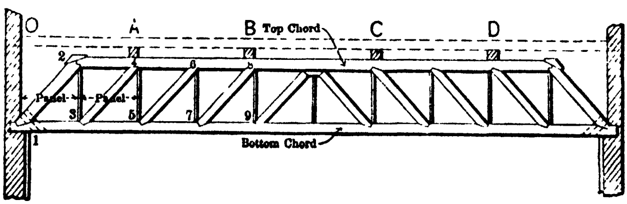


Fig. 10. Ten-panel Howe Truss

be noted that in this, as in any construction, where the loads are applied between panel-points of a chord member, the supporting member must function as a beam in addition to its function as a member of the truss. The design stress for such a member is the direct sum of the stress due to flexure and the stress due to truss action as discussed more fully in Chapter XXVI.

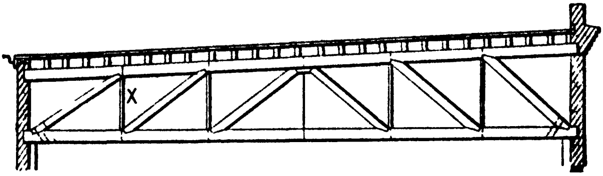


Fig. 11. Six-panel Howe Truss with Top Chord Inclined

For deck or mansard roofs the upper chord may be made to conform to the outline of the roof as shown in Fig. 12, and those panels in which the character of stress in the inclined web-member is subject to reversal must be supplied with counters, unless connections capable of transferring tensile stress are provided at the ends of the single diagonal.

Scissors Trusses. The SCISSORS TRUSS, so named from its resemblance to a pair of scissors, is, in its various forms, adaptable to composite timber and steel-rod construction.

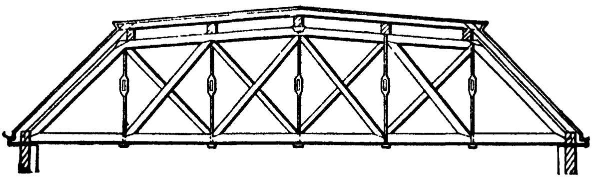


Fig. 12. Howe Truss for Deck Roofs

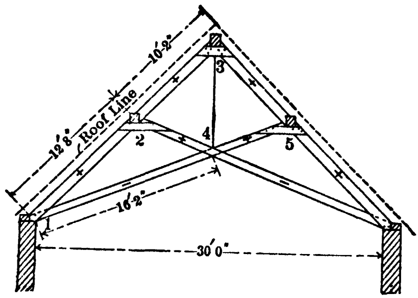


Fig. 13. Simple Scissors Truss. Spans up to Thirty Feet

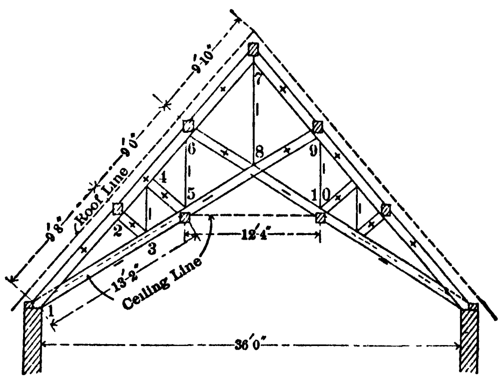


Fig. 14. Scissors Truss. Spans Exceeding Thirty Feet

The scissors truss is generally employed in open timber roof-construction or where raised ceilings are desired over halls and auditoriums. The types of scissors trusses illustrated in Figs. 13 and 14 are the most frequently used. Figs. 15 and 16 illustrate forms of modified scissors trusses and both are stat-

ically indeterminate, since, by Equation (1), each has one redundant member. The elastic deformation of the scissors truss results in a considerable hori-

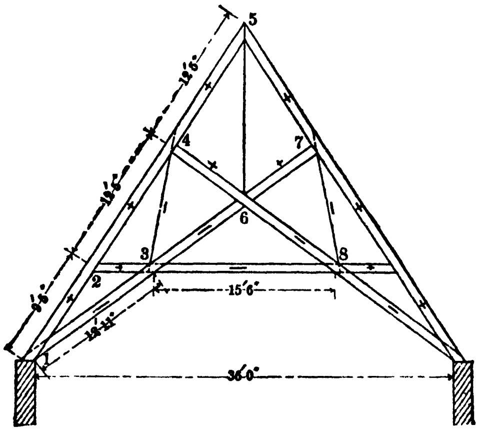


Fig. 15. Scissors Truss. For Steep Roofs

zontal thrust at the points of support, as shown in Chapter XXV. The amount of horizontal thrust can be reduced through the use of excess area in the members, especially the upper and lower chords, which in reality

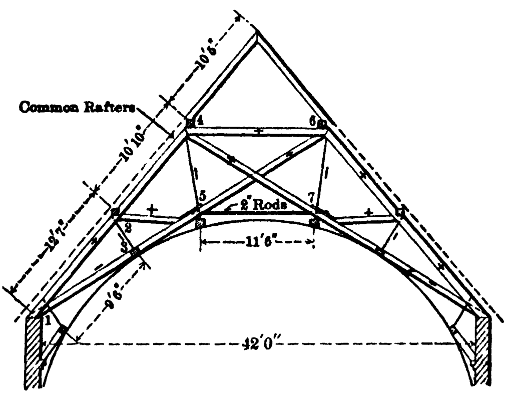


Fig. 16. Modified Scissors Truss. Spans Exceeding Thirty-six Feet

amounts to the adoption of a decreased value of the working unit stresses used in design.

Fig. 17 illustrates a CAMBERED FAN TYPE of truss, which is somewhat similar in appearance to the scissors type and may be used for roofs of moderate slope.

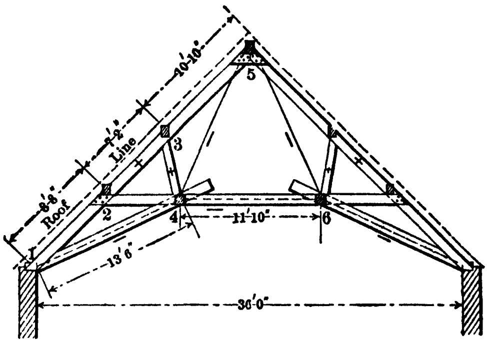


Fig. 17. Modified Scissors Truss. For Medium Pitch

5. Lattice Trusses of Wood

Lattice trusses are those which have multiple systems of web bracing and may have either parallel chords or curved upper chords. Such trusses are statically indeterminate, since the distribution of stress to the various systems of web-members is complicated by the fact that any assumed distribu-

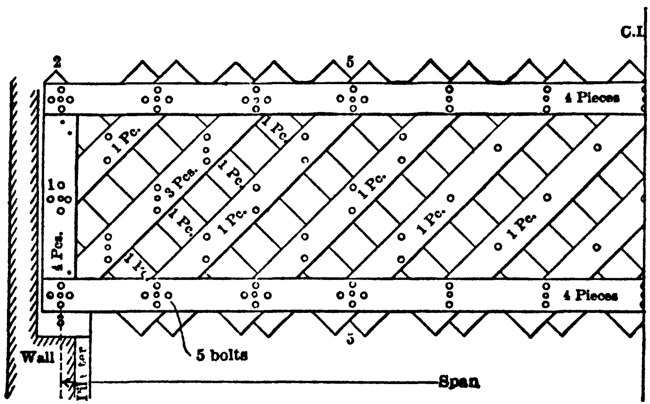


Fig. 18. Lattice Truss

tion of load on any one system will affect, to some extent, the distribution of stress in all other systems. Moreover, the web-members are usually fastened at their points of intersection and independent action of any one system is impossible. For usual span lengths it is customary and satisfactory to analyze a lattice truss, of the types shown in Figs. 18, 19 and 20, as beams. The moments and shears are calculated at various points throughout the span; the chord stresses are then determined by assuming the entire resistance to moment to be offered by the chord sections, or $S_c = M/h$, where S_c

is the total stress in each chord at the section, where M = the bending moment, and h = the depth of the truss, center to center of chords. The stress in the web-members is obtained by assuming the shear to be equally divided among the web-members cut by a vertical plane. Any web-member, sloping in such a manner that the axial internal force equilibrating the shear

Table I. Dimensions for Lattice Trusses Uniformly Loaded. Timber—Norway Pine, Douglas Fir or Yellow Pine

Dead Load = 15 lb per sq ft of Horizontal Projection
Live Load = 30 lb per sq ft of Horizontal Projection
Ceiling Load = 10 lb per sq ft of Horizontal Projection

Trusses as shown in Fig 18

Span	Spacing of trusses	Height out to out of chords		No. of spaces	No. and size of pcs of bottom chord		No. and size of pcs of top chord		Size of braces	No. and diameter of bolts, joints 1–5, Fig. 18	
ft	ft	ft	in			in in		in in	in in		in
40	12	5	6	16	4	2×6	4	2×6	2×6	4	1
		7	2	12	4	2×6	4	2×6	2×6	4	1
	14	5	7	16	4	2×6	4	2×8	2×6	4	1
		7	3	12	4	2×6	4	2×8	2×6	4	1
	16	5	8	16	4	2×8	4	2×8	2×8	4	1⅛
		7	4	12	4	2×8	4	2×8	2×8	4	1⅛
50	12	6	8	16	4	2×8	4	2×8	2×10	5	1⅛
		8	8	12	4	2×8	3	2×8	2×10	5	1⅛
	14	6	8	16	4	2×8	4	2×8	2×10	5	1⅛
		8	8	12	4	2×8	4	2×8	2×10	5	1⅛
	16	6	9	16	4	2×8	4	2×10	2×10	5	1⅛
		8	8	12	4	2×8	4	2×8	2×10	5	1⅛
60	12	8	4	16	4	2×10	4	2×10	2×10	5	1¼
		10	10	12	4	2×10	4	2×10	2×10	5	1¼
	14	8	4	16	4	2×10	4	2×10	2×10	5	1¼
		10	10	12	4	2×10	4	2×10	2×10	5	1¼
	16	8	4	16	4	2×10	4	2×10	2×10	5	1¼
		10	10	12	4	2×10	4	2×10	2×10	5	1¼
70	14	9	5	16	4	2×10	4	2×12	2×10	5	1¼
		12	4	12	4	2×10	4	2×10	2×10	5	1¼
	16	9	5	16	4	2×10	4	2×12	2×10	5	1¼
		12	4	12	4	2×10	4	2×10	2×10	5	1¼
	18	9	6	16	4	2×12	4	2×12	2×10	7	1¼
		12	6	12	4	2×12	4	2×12	2×10	7	1¼
80	14	11	0	16	4	2×12	4	2×12	2×12	7	1¼
		14	0	12	4	2×12	4	2×12	2×12	7	1¼
	16	11	2	16	4	2×14	4	2×14	2×12	7	1¼
		14	0	12	4	2×12	4	2×12	2×12	7	1¼
	18	11	2	16	4	2×14	4	2×14	2×12	7	1¼
		14	1	12	4	2×12	4	2×14	2×12	7	1¼

NOTE. All joints should be thoroughly spiked and packing blocks used where necessary.

assigned to that member necessitates a thrust against the cut section, is a COMPRESSION WEB-MEMBER. When the equilibrating force is directed away from the cut section of a web-member, that member is a TENSION WEB-MEMBER.

The parallel chord type of lattice truss, Fig. 18, was invented by Ithiel Towne in 1820. Each chord member is made up of four pieces arranged in pairs on each side of the web. The lower chord pieces should be as long as can conveniently be obtained and arranged so that no two splices occur at the same point.

Each member of the web-system consists of a single piece, inclined at an angle of about 45°. There should be at least three bolts at each web and chord connection, and this number must be increased at the joints near each support where the larger shears assigned to the web-members exist. All web-members are spiked or bolted at their intersections. Since one-half of the web-members are in tension, these members should project beyond the bolted connection a sufficient distance to provide the necessary longitudinal

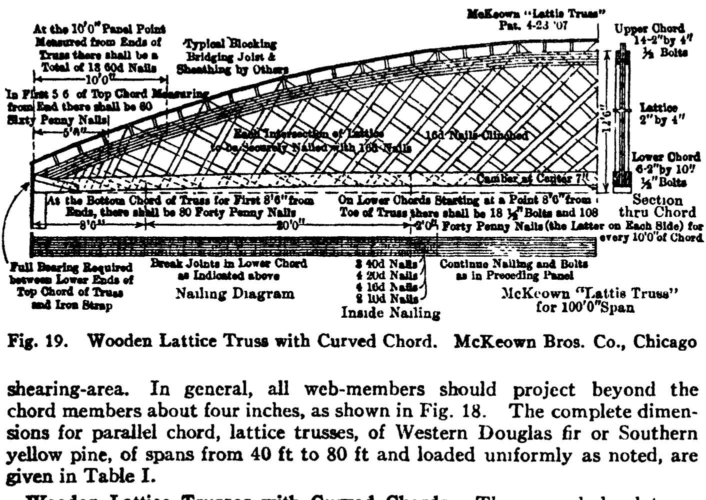


Fig. 19. Wooden Lattice Truss with Curved Chord. McKeown Bros. Co., Chicago

shearing-area. In general, all web-members should project beyond the chord members about four inches, as shown in Fig. 18. The complete dimensions for parallel chord, lattice trusses, of Western Douglas fir or Southern yellow pine, of spans from 40 ft to 80 ft and loaded uniformly as noted, are given in Table I.

Wooden Lattice Trusses with Curved Chords. The curved-chord types of wooden lattice trusses illustrated in Figs. 19 and 20 are patented, the former by the McKeown Bros. Co. of Chicago, and the latter by the Double Strength Truss Co. of Chicago.

Curved-chord lattice trusses have been used extensively in the construction of armories, assembly-halls, car shops, coliseums, dance-halls, factories, foundries, garages, gymnasiums, hangars, riding academies, skating-rinks, etc.

The curve of the upper chord approximates a parabola, thus providing approximately uniform chord stress under uniform loads. The curved upper chord acts somewhat as an arch, and the lower chord as a tie, and under uniform load, therefore, the stresses in the web-members are relatively small.

The web-members are rigidly connected at each intersection in order to provide lateral stiffness against the buckling tendency of the compression web-members. It is obvious that the buckling tendency of the compression

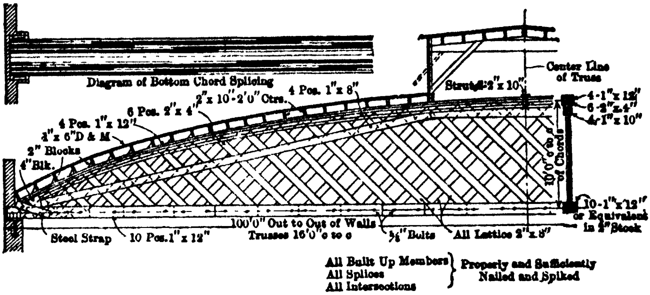


Fig. 20. Wooden Lattice Truss with Curved Chord. Double Strength Truss Co., Chicago

members can be largely reduced by rigid connections to the tension members which it crosses, thereby promoting increased stiffness throughout the entire structure.

6. Ornamental Timber Roof-Trusses

The open timber roof, architecturally treated, is almost exclusively an English feature. The roofs of the basilican churches of Italy which were framed with timber were of simple construction and the only decoration was paint. The great oak forests of medieval England and the expertness of the early English wood-worker probably account for the unsurpassed excellence of the English open timber roof.

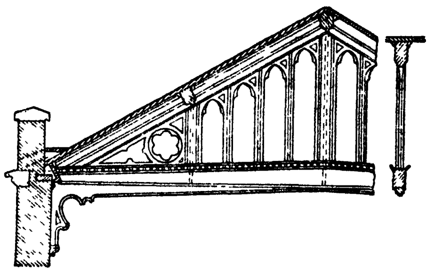


Fig. 21. Tie-beam Roof Construction.

In these roofs all the timber framing is visible from wall to ridge and in most cases all timbers are carved or molded and were originally richly painted and gilded.

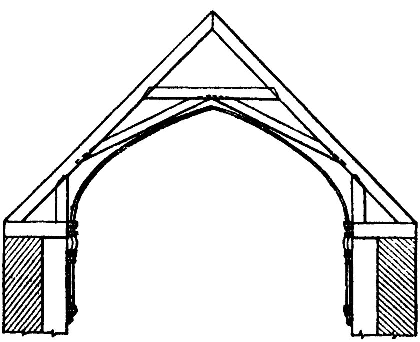


Fig. 22. Braced-rafter Type of Roof Construction

Medieval open timber roofs may be classified as follows:

(1) Tie-Beam Roof
(2) Braced-Rafter Roof
(3) Arch-Braced Roof
(4) Hammer-beam Truss

Tie-Beam roof-construction consists of the simple triangular frame composed of two rafters

and a horizontal cross-beam joining their lower ends. The tie-beams are seldom straight, but made with a slight rise at the center, and the space between the tie-beam and rafters was frequently filled with pierced or carved paneling, as illustrated in Fig. 21.

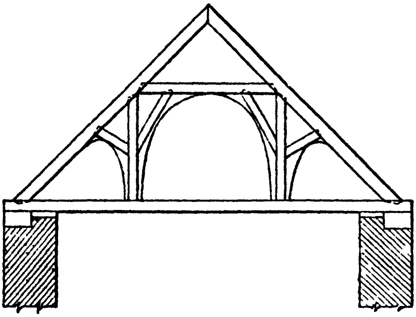


Fig. 23. Queen-post Truss

Braced-Rafter type of open timber roof-construction consists of various forms of intermediate support for the rafter member of the frame. The construction illustrated in Fig. 21 might be classified as braced rafter; however, the term is more commonly used to designate an incomplete frame such as shown in Fig. 22. The type of framing illustrated in Fig. 23, and a modern adaptation of the same, shown in Fig. 24, is called by early writers the QUEEN-POST TRUSS. The frame is not a truss, but a braced-rafter type of frame in which the tie-beam serves as a support for the vertical braces.

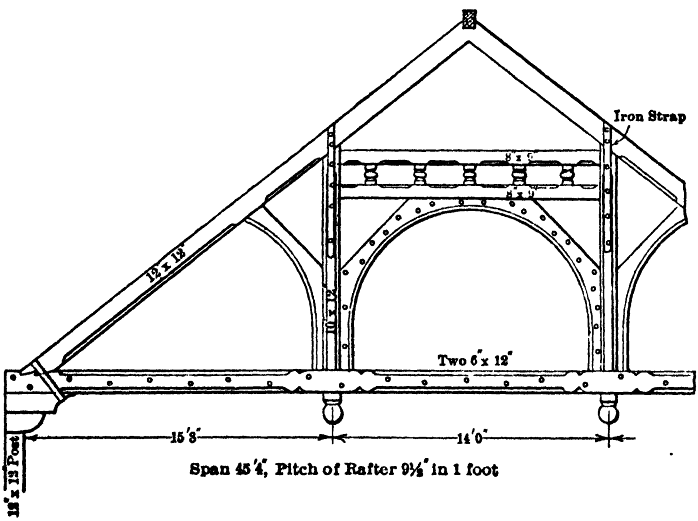


Fig. 24. Queen-post Truss. Massachusetts Charitable Mechanics' Association Building, Boston, Mass.

The tie-beam in frames of the type shown in Fig. 23 is subjected to undesirable deflections due to the intermediate supports of the rafter members. Two distinct methods were employed to obviate this unsightly depression.

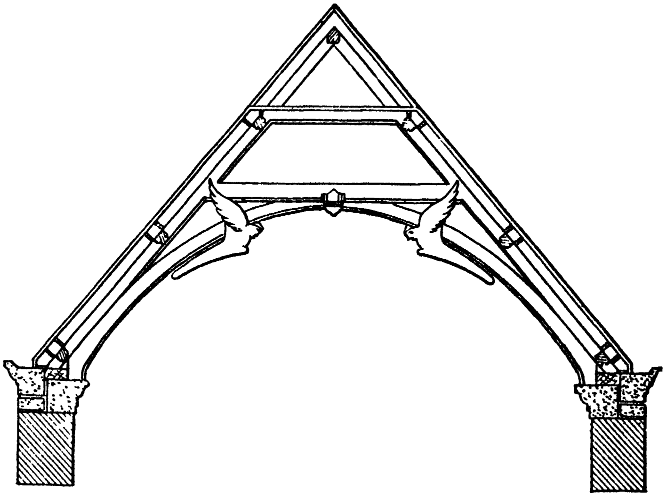


Fig. 25. Arch-braced Roof Construction

One method was to CAMBER the tie-beam, as in Fig. 21, and the other was to replace the tie-beam by a collar-beam and some system of bracing such as shown in Fig. 22. The second method is known as ARCH-BRACED CONSTRUCTION.

Arch-Braced Roofs. In this type of construction the tie-beam is omitted and an arch-brace placed in the angle formed by the rafter and collar-beam, as in Fig 25, or both the tie and collar-beams are omitted and a continuous arch rib is placed beneath the rafters from wall to peak, as in Fig. 26. The arch rib is securely fastened to a vertical member at, and built into, the wall, known as the wall-post. The rafter thrust, where the collar-beam was omitted, is resisted by a height of wall equal to the height of the wall-post, as illustrated in Fig. 26.

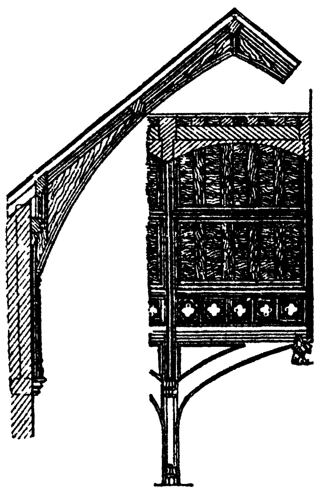


Fig. 26. Arch-braced roof over nave of Briton Church, Norfolk, Eng.*

Hammer-Beam Roofs. Arch-braced roofs are often assumed to be a modification of the HAMMER-BEAM type of roof-construction. Chronologically this is not the case, since arch-braced roofs were used as early as 1320, while the first hammer-beam roof of record is that

* From Brandon.

of Westminster Hall, completed in 1399. Early hammer-beam roofs were similar in general appearance to, and their action under load was probably not unlike that of, the arch-braced roof. The only structural difference between the arch-braced roof and the early hammer-beam types, such as illustrated in Figs. 27 and 28, is the framing at the supports. In the former, the sole-piece does not project beyond the inner face of the wall, while in the

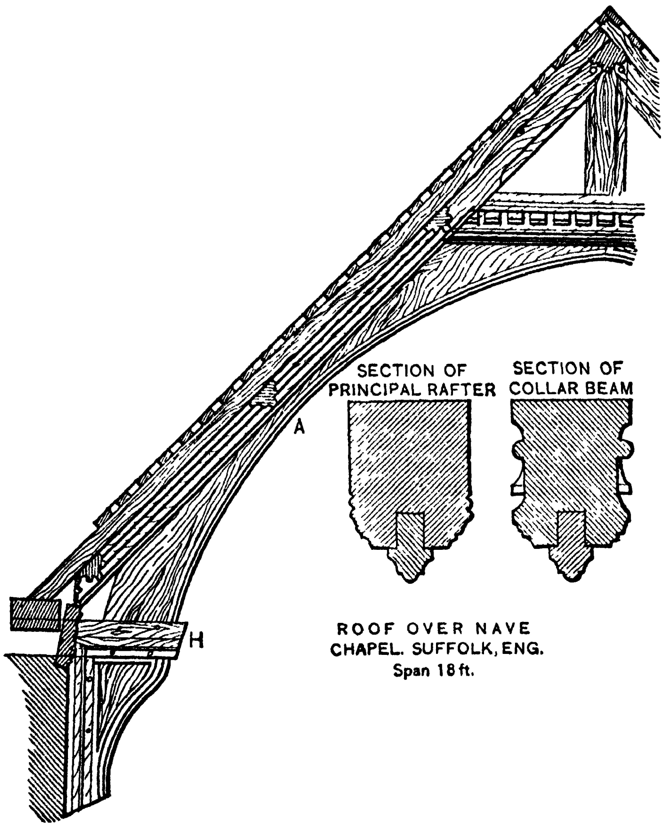


Fig. 27. Early English Type of Hammer-beam Construction

latter it is elongated, may have a considerable projection and is called the HAMMER-BEAM. This distinguishing member is noted at *H* in Figs. 27 and 28.

A modern adaptation of the arch-braced hammer-beam type of construction is shown in Fig. 29. The hammer-beam truss, Figs. 30 and 31, is a modern development of the early English arch-braced hammer-beam type.

Ornamental Scissors Trusses. In many examples of modern open timber roof-construction various forms of architecturally treated scissors trusses have been employed instead of the more expensive forms just described.

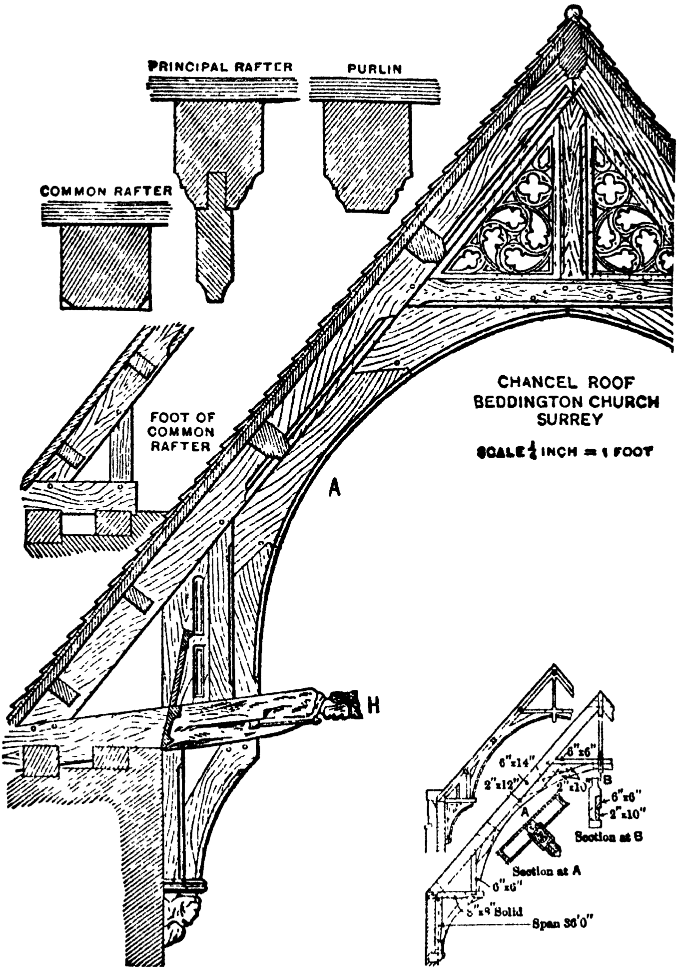


Fig. 28. Early English Type of Hammer-beam Construction

Fig. 29. Modern Adaptation of Hammer-beam Roof Construction

Structurally there is as much merit in the scissors type of truss as in any of the earlier types of framing employed in open timber roof-construction. Somewhat the same spirit of loftiness, characteristic of the earlier types, is

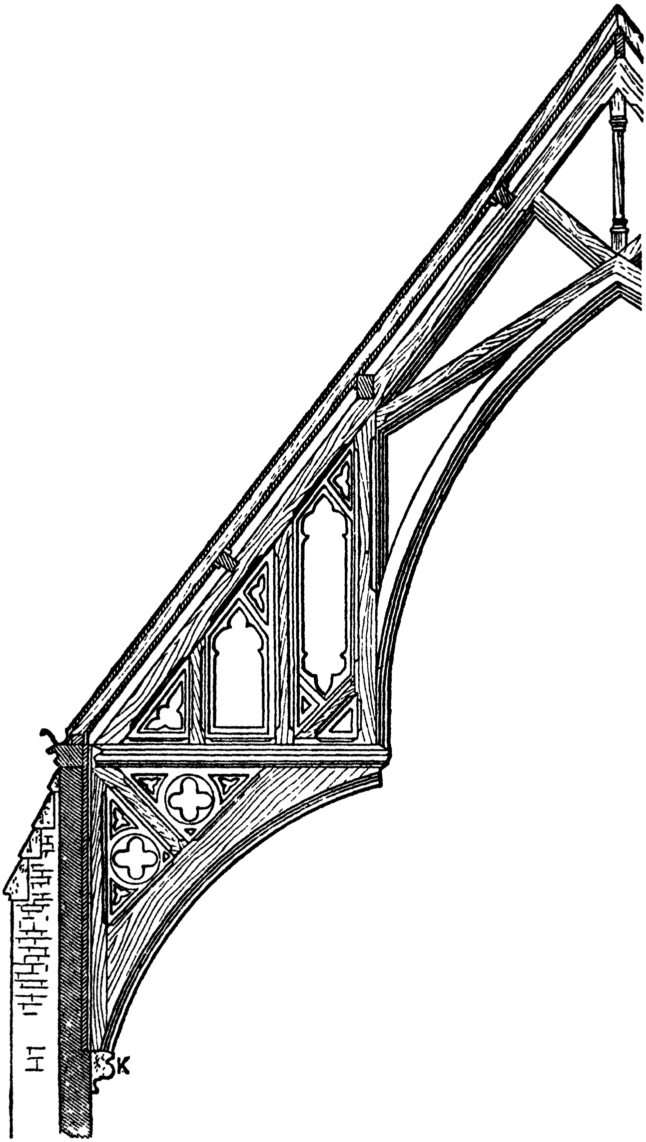


Fig. 30. Modern Adaptation of Hammer-beam Roof-construction

obtained through the use of the scissors truss, and when properly designed, with members amply large to provide for relatively low working unit stresses, wooden scissors trusses are satisfactory for roofs over halls and churches up to spans of 40 to 48 ft. A simple form of the scissors truss used in open

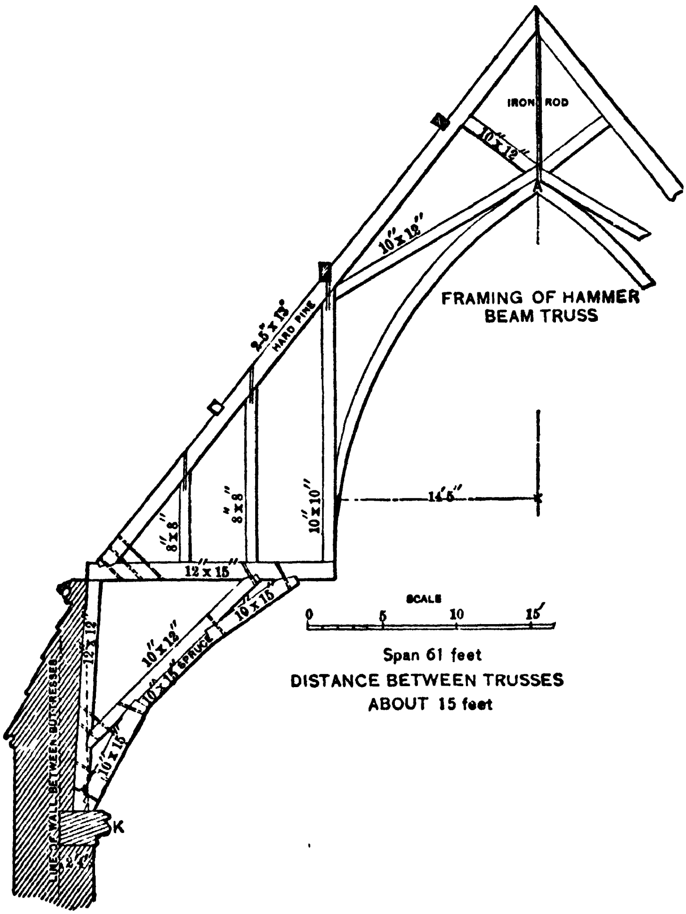


Fig. 31. Framing of Truss Shown in Fig. 30

timber roof-construction is shown in Fig. 32, wherein the inclined members give a pleasing arched effect.

When the span is more than 48 ft and an open timber roof of the scissors type is desired, the structural truss may be constructed of steel, covered or boxed with wooden paneling. The result is not a truthful expression and the

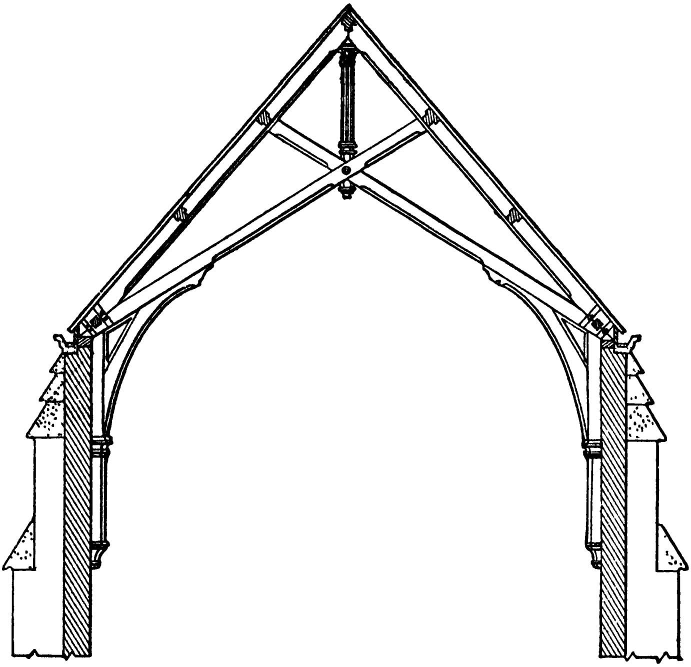


Fig. 32. Simple Form of Scissors Truss

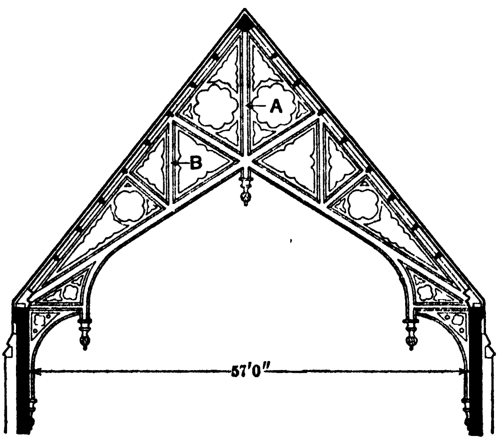


Fig. 33. "Boxed-in" Scissors Truss

general effect is usually poor as compared with actual timber-construction. An example of a BOXED steel truss of the scissors type is illustrated in Fig. 33.

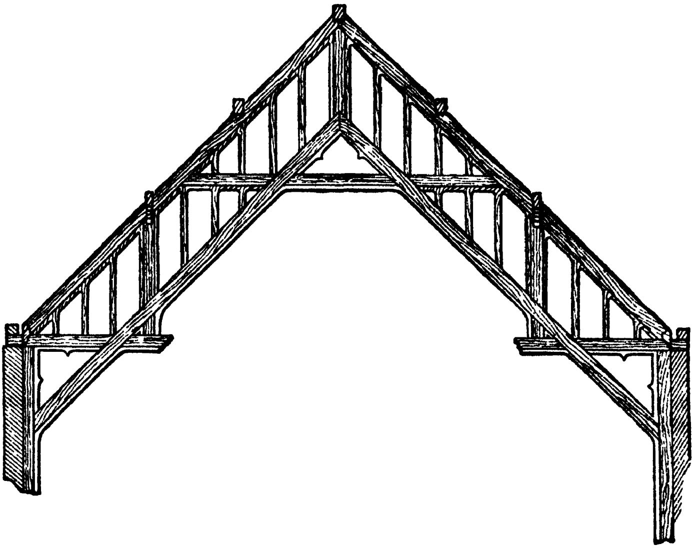


Fig. 34. Truss for Emmanuel Church, Shelburne Falls, Mass.

Miscellaneous Types of Ornamental Trusses. Many variations, combinations and hybridizations of the various forms of timber roof-construction have been fabricated in modern work.

The laminated, SEMI-HAMMER-BEAM TRUSS, Fig. 34, from Emmanuel Church, Shelburne Falls, Mass., Van Brunt and Howe, Architects, is an interesting architectural adaptation of an indeterminate timber frame which depends for its strength upon the rigidity of the joints. A frame of this type should be constructed of well-seasoned timber, and the connection bolts should be drawn up from time to time after the building has been put into service. The arched-braced hammer-beam with a tie-rod used to eliminate excessively heavy wall construction is shown in Fig. 35.

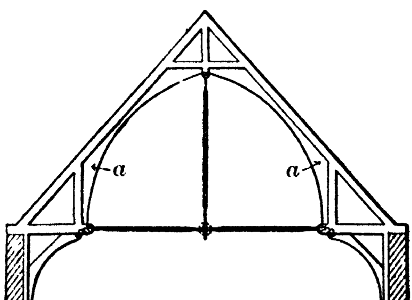


Fig. 35. Hammer-beam Truss for Grace Chapel, New York City

A similar construction, spanning about 54 ft, in which the wooden arch rib is ornamented with sawed scroll work, Fig. 36, was used in the Metropolitan Concert Hall, New York

City, Mr. George B. Post, Architect In both of the above examples the tie-rods are prevented from sagging by a vertical rod suspended from the crown of the arch.

A simple form of the FINK TRUSS, in which the timbers are chamfered to

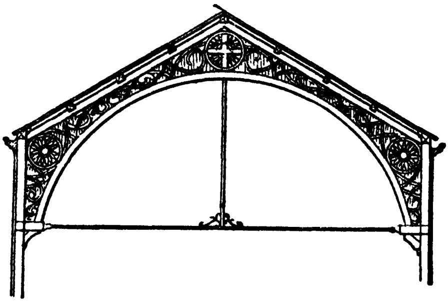


Fig. 36. Truss for Metropolitan Concert Hall, New York City

give a more pleasing finish, is shown in Fig. 37. The general effect obtained is similar to that obtained by the use of the simple scissors truss, Fig. 32.

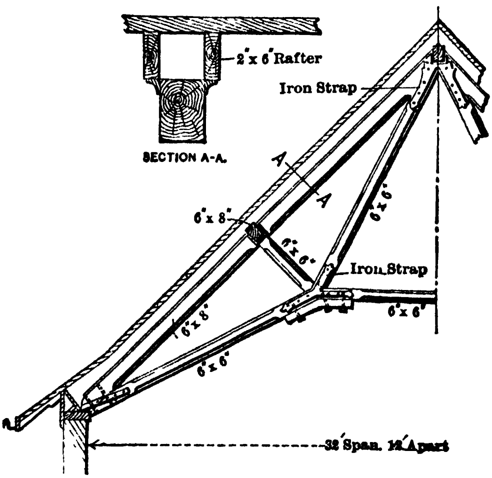


Fig. 37. Simple Fink Truss with Raised Lower Chord

7. Arched Roofs of Timber-Construction

Difference between the Arch and the Truss. An ARCH is a structure employed to support a system of loads over a given span, usually supported at its extremities by pins, which are known as hinges, and has inclined reactions for both vertical and inclined loading.

The lower chords of arches are generally cambered to follow the arch form; however, a SIMPLE TRUSS becomes an arch if its extremities are restrained against horizontal movement. The distinguishing structural characteristic between an arch and a truss is the manner of support In an arch the length of the structure between supports is always constant or practically so, while in a truss the elongation of the tension chord, due to stress or a rise in temperature, increases the distance between the points of support.

It is evident that the restrained tendency of an arch to spread produces horizontal thrusts at the supports, necessitating horizontal reactions, which may be supplied either by a rigid abutment, or by a tie-member connecting the two supports, thus relieving them of the duty of supplying the necessary horizontal resistance.

Classification of Arches Arches used in roof-construction may be classified as follows:

(1) **Arched Trusses,** or a simple truss having an arched form, as in Fig. 38 (*a*). The structure is not an arch since one end is free to adjust itself to horizontal displacements.

(2) **Two-Hinged Arches.** When the arched structure is supported and restrained horizontally as shown at (*b*), Fig. 38, it is known as a TWO-HINGED ARCH. The vertical reactions are the same as for an arched truss having the same span and loading and can be determined by simple statics. The horizontal reactions, indicated in this case as tie-rod stress, depend upon the deformation of the framework and cannot be determined by the principles of statics alone.

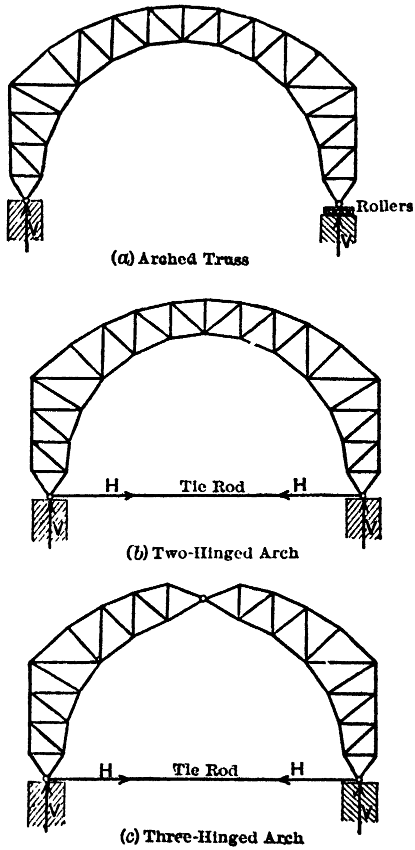


Fig. 38. Types of Arched Trusses

(3) **Three-Hinged Arches.** When a third hinge is provided at the crown of the arch, as shown at (*c*), Fig. 38, the structure is known as a THREE-HINGED ARCH. The vertical reactions are the same as for a simple arched truss and since the structure may be considered as two simple trusses with

one common and interdependent support (the intermediate hinge), the horizontal reactions may be determined by the principles of statics. The three-hinged arch is, therefore, the only type of arch which is statically determinate. Arches may also be constructed without hinges, being fixed rigidly to the abutments and continuous throughout the span; such arches are seldom used in building-construction.

Arches are also classified according to the manner of framing. When the structure consists of a trussed framework as in (*b*) or (*c*), Fig. 38, it is called a BRACED ARCH. When the arch consists of a curved beam with solid web

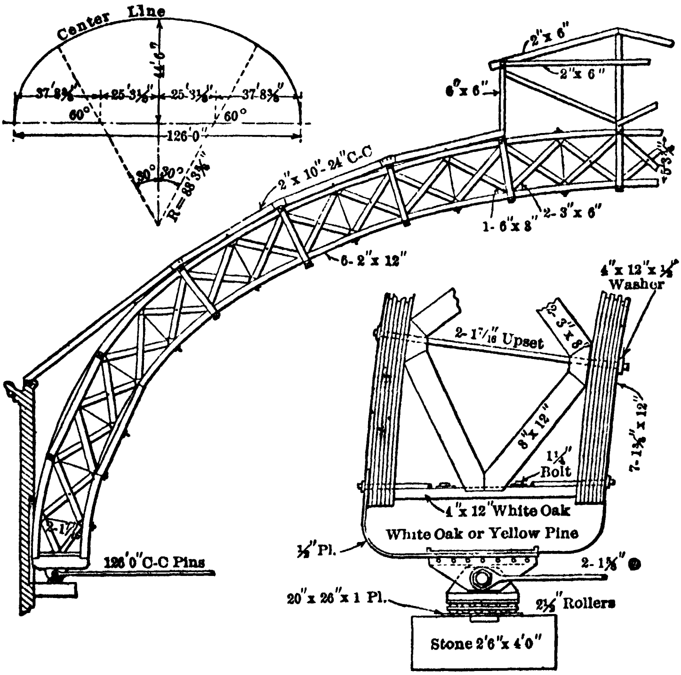


Fig. 39. Two-hinged Timber Arch

and flanges the structure is known as a RIBBED ARCH. The latter type is seldom used in building-construction.

Examples of Arched Timber Roofs. The various forms of open timber roof-construction, where the horizontal tie-beam member is omitted, are the best examples of timber arched truss forms.

The two-hinged timber arch shown in Fig. 39 consists of a multiple-web braced timber arch, the thrust of which is resisted by a steel tie-rod member placed beneath the floor.

The objectionable feature in this particular structure is the use of a multiple web-system with the consequent uncertainty of stress distribution as deter-

mined by the usual methods of assuming independent systems; however, since the web stresses are the only ones affected, the bracing used can be justified when the second web-system has been considered as wholly redundant throughout the analysis.

A three-hinged timber arch of approximately 80-ft span is illustrated in

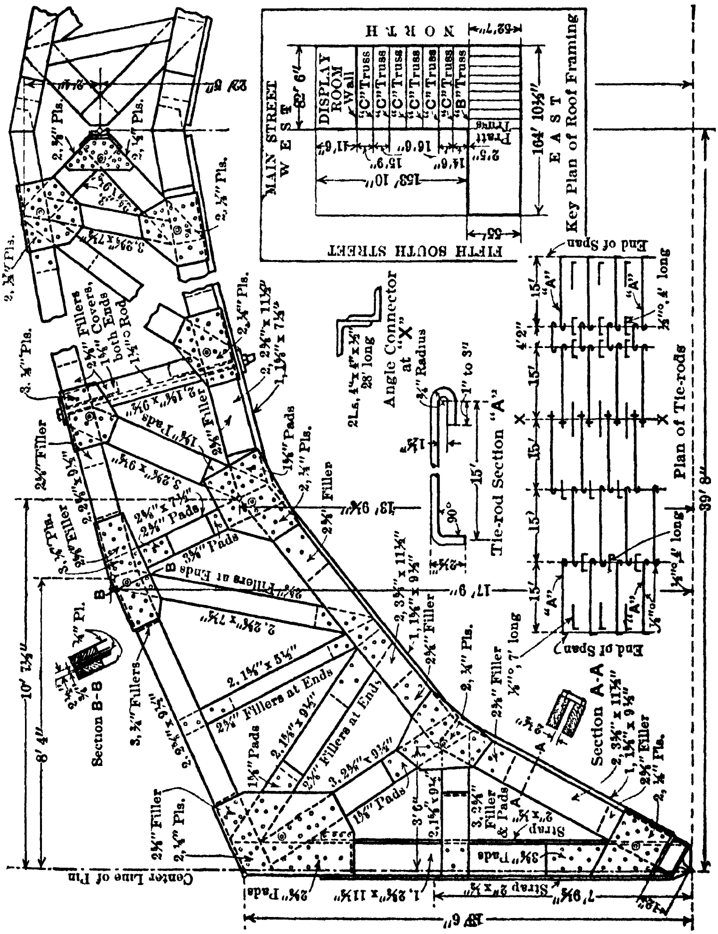


Fig. 40. Three-hinged Timber Arch * (* From E. N.-R. Vol. 80, page 595.)

Fig. 40. All members of the arch are wood; the tension members, however, are reinforced with steel rods, and all joints throughout are bolted connections with steel gusset-plates.

The thrust of the arch is taken by a tie member composed of four parallel lines of $\frac{7}{8}$-ϕ rods, hooked together in 15-ft lengths, and the whole assemblage

embedded in a concrete-filled trench beneath the floor. At mid-span the rods pass through a pair of angles and are there fitted with nuts to permit tightening up before being concreted in. The arches, including all hardware, weigh about 6 lb per sq ft of horizontal projection.

The design loads were as follows:

Dead load	16.5 lb per sq ft
Snow-load	25.0 lb per sq ft
Wind-load (equivalent vertical)	10.0 lb per sq ft

The arches were used in the roof construction of a Salt Lake City garage.*

Lamella Roof Construction A recent solution for the problem of spanning large floor-areas, where it is undesirable to use columns, is found in the

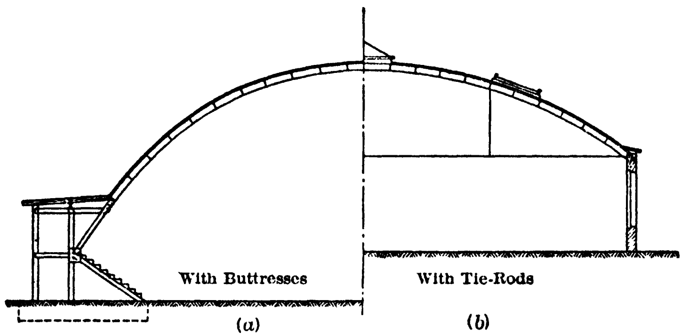


Fig. 41. Lamella Roof Construction

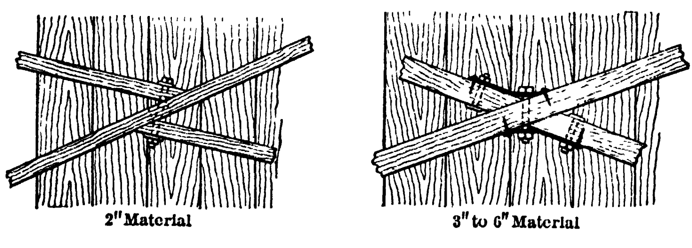


Fig. 42. Lamella Joints

LAMELLA ROOF. This type of roof-construction was developed in Germany in 1923 and has, within the past few years, been extensively used in this country.

The construction is essentially a wooden arch, either two- or three-hinged, the horizontal thrust of which is taken by buttresses, Fig. 41 (*a*), or by tie-rods, Fig. 41 (*b*), depending upon the type and arrangement of supports. The arch is made of short timbers, called LAMELLAS, which are bolted together as shown in Fig. 42, to form a network of diamond-shaped panels, Fig. 43 (*a*).

* Eng. News-Record, Vol. 80, page 595.

The lamellas are curved on one side and beveled on both ends, as shown in Fig. 43 (*b*), and for a given roof they are all the same size.

A notable example of lamella roof construction is the Convention Hall at Houston, Texas, built to house the Democratic National Convention in 1928.* This structure consists of three bays roofed with lamella arches. The central bay has a span of 120 ft and a rise at the crown of 58 ft 4 in above the floor. Each side bay has a span of 75 ft 8 in and a rise at the crown of 41 ft 10 in.

There were 2 000 lamellas 3 in × 14 in and 12 ft long required for the central bay and 1 864 lamellas 2 in × 10 in and 9 ft long required for each side bay.

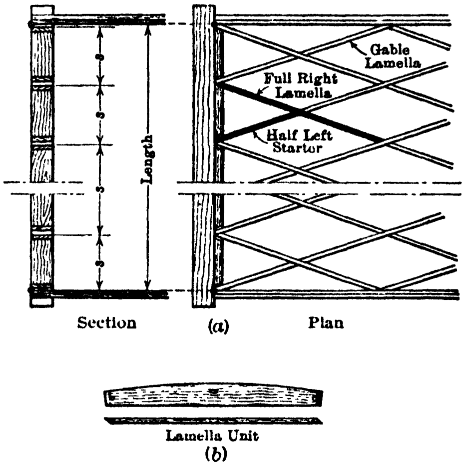


Fig. 43. Details of Lamella Roof Construction

Another notable example of lamella construction is the arena of the National Exhibition Company of St. Louis, Mo.,† described in Article 13.

8. Types of Steel Roof-Trusses

General Considerations. With the exception of a few types, such as the hammer-beam and its various modifications, all forms of roof-trusses may be readily and economically constructed of standard structural-steel shapes. Steel roof-trusses may be supported directly on masonry walls in wall-bearing construction, or they may be supported by steel columns in steel-frame construction.

Standard structural-steel angles are the most practical and economical sections, both for chord and web-members of steel roof-trusses of usual spans and loading. For long-span construction roof-truss members may be composed of rolled-beam sections or built-up sections of various types.

Joints in steel roof-truss framing are generally riveted connections, although within the past few years great progress has been made in the use of welded joints for steel-frame structures. Arc-welded curved-chord trusses of relatively light construction have been successfully fabricated up to span lengths of 80 ft, and when welding becomes less dependent upon skilled labor, when values can be predetermined for the safe working allowance of the various types of welding, when inspection methods are perfected, and economical procedure established, welding may replace riveting, to a large extent, in ordinary steel-frame roof-construction.

* Eng. News-Record, Vol. 100, page 815.

† Eng. News-Record, Vol. 104, page 935.

9. Special Forms of Steel Roof-Trusses

In addition to the more or less standard types of roof-trusses illustrated in Figs. 1 and 2, steel roof-trusses may assume almost an endless variety of forms to comply with esthetic or utilitarian requirements of the structure.

The division of a given span and general outline of roof-construction into panel lengths depends largely upon the type of roof-covering and method of roof-construction to be employed. If purlin and rafter construction is to be employed the panel length may be as great as 12 ft; if sheathing is to be placed directly on the purlins, panel lengths of 6 ft to 8 ft are required.

The truss shown in Fig. 44 was designed for a roof-construction composed of 2 × 8 in rafters supported by steel purlins placed at points *A*, *B*, *C*, *D*, *E*, and *F*. The roof covering is slate laid over slaters felt on 1-in matched sheathing. There are no loads at the panel-points of the lower chord, excepting the gallery load at *X*, and this joint is located with reference to the position of the gallery support, otherwise the slope *AC* might have been more economically divided into equal lengths.

In order to relieve the feeling of depression and the optical illusion of sag-

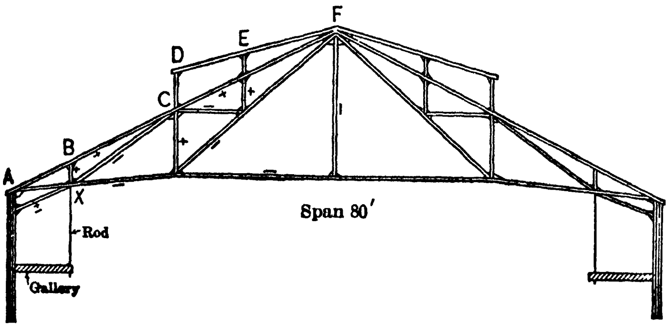


Fig. 44. Fink Truss with Vertical Struts, for Drill-hall. Span Eighty Feet

ging resulting from a horizontal lower chord, this member was raised, or cambered, as shown.

The members *CD*, *DE* and *EF* are not structurally a part of the truss proper, but serve merely as a framework to elevate the central portion of the roof in order to provide a series of glazed panels in the rise *CD*, for the purpose of central light and ventilation.

The truss shown in Fig. 44 is similar in outline and function to the truss of the same span illustrated in Fig. 45. In the latter truss, the type of roof-construction necessitated a smaller panel length and the Howe type of bracing was adopted for the arrangement of the web-members.

The Fink type of web-system, illustrated in Fig. 46, might have been used with equal economy. The reactions of the trusses in both Fig. 45 and Fig. 46 are delivered to the tops of the supporting columns at points *A* and *C*. The members *AD* and *CE* are the end-panel members of the lower chords. The continuations of *DE* to meet the columns at some distance below *D* and *E*, respectively, serve merely as stiffening braces for the connection joints of the truss to the columns.

In each of these trusses the framework, above chords AB and BC, which carries fixed or movable sash or LOUVERS for the purpose of ventilation and light, is known as a MONITOR.

Quadrangular trusses of the general types illustrated in Figs. 45 and 46

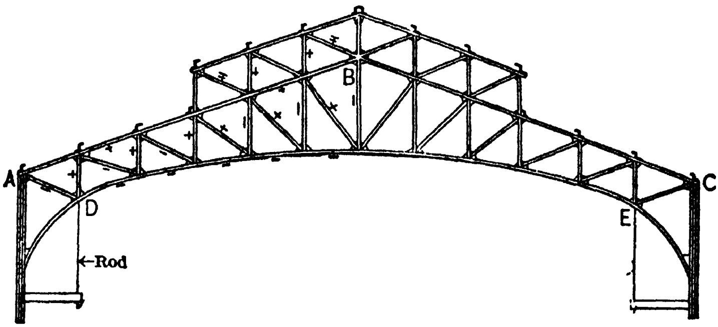


Fig. 45. Quadrangular Truss. Span Eighty Feet

may be used for spans up to 200 ft or more when the horizontal displacement due to the deflection of the truss under load and due to temperature change is properly provided for. Spans greater than 80 to 100 ft, supported on steel columns, should have an expansion bearing at one support which will permit

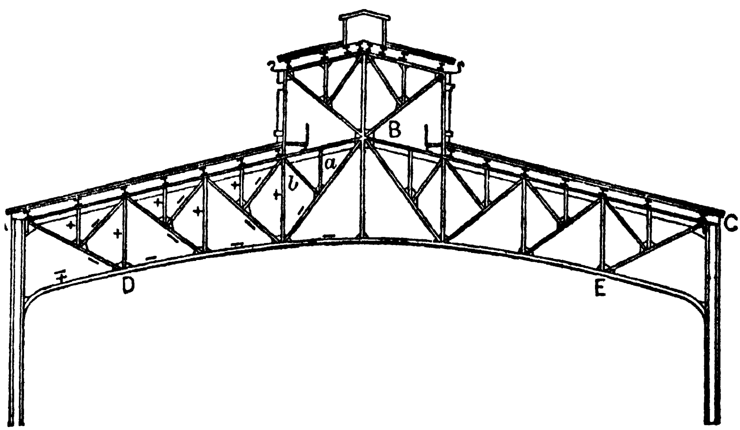


Fig. 46. Quadrangular Truss. Span One Hundred Feet

of a lateral movement of at least $\frac{1}{8}$ in for every 10 ft of span, or each bent, composed of a truss and its supporting columns, should be designed as a rigid unit.

10. Long-Span Roof-Truss Construction

Long-span Roof-Construction is a relative term; however, in general practice it is applied to truss spans greater than 120 ft.

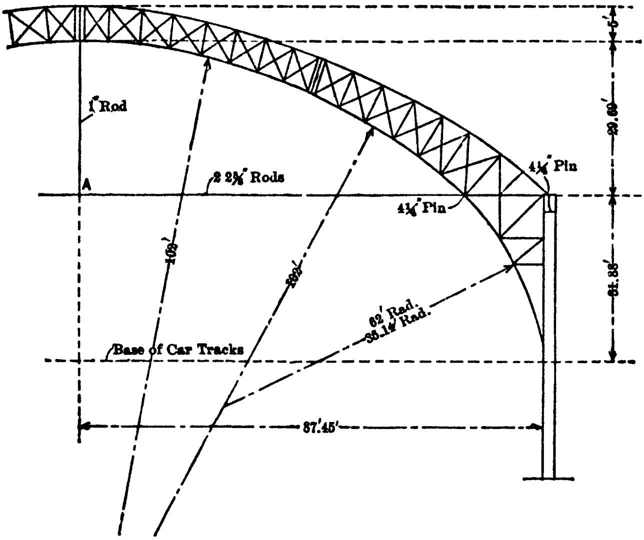


Fig. 47. Arched Truss for Sullivan Square Station, Elevated Railway, Boston, Mass.

Long-span roof-truss construction may be conveniently divided into three general classes:

(1) ARCHED TRUSSES with horizontal ties.
(2) SIMPLE TRUSSES with one end free horizontally.
(3) RIGID UNIT BENT CONSTRUCTION.

(1) Arched Trusses with Horizontal Ties have been used rather extensively in the past. This type of construction, an example of which is shown in Fig. 47, consists of an arched truss with a horizontal tension member connecting the end supports. The structure as a whole, including the horizontal member, may be considered as a simple truss. It should be observed, however, that in the construction illustrated the ends of the span are not free to move horizontally and that the resistance to horizontal displacement of the arched portion is taken partially by the tie-member and partially by the supporting column, the amounts of each depending upon the relation between the elongation of the tie-member and the

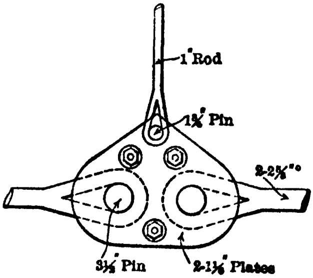


Fig. 47A. Detail at *A*, Fig 47

stiffness of the column. In reality this type of construction probably more nearly approaches a two-hinged arch with a tie-rod than it does a simple truss. Because of its indeterminate nature involving rather complex analysis on the one hand or purely arbitrary assumptions on the other, this type of construction has been largely replaced in present-day practice by one of the other two methods mentioned above.

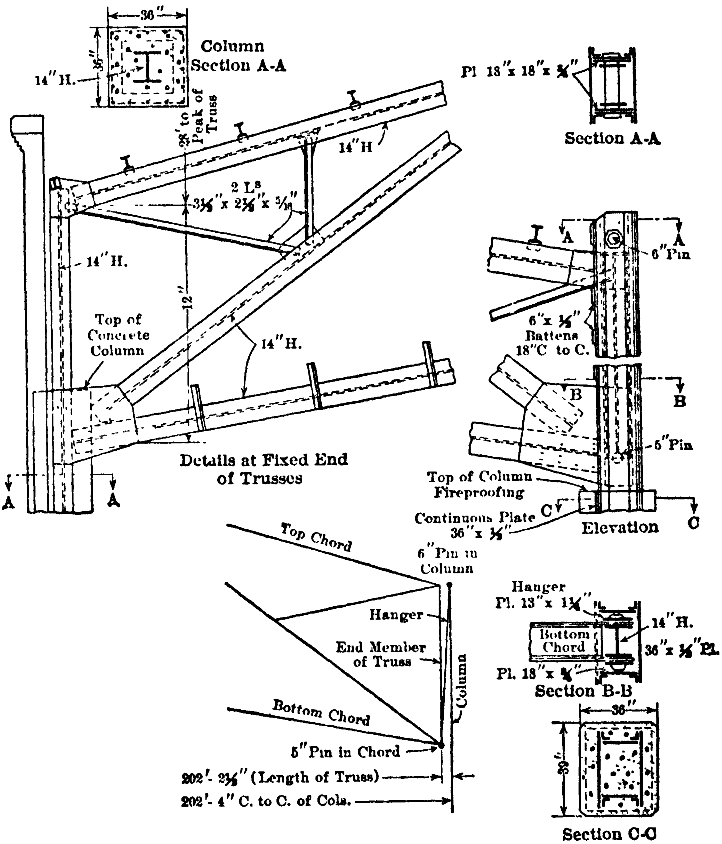


Fig. 48. Details of Pendulum Hanger for Long Spans *

* From E. N.-R., Vol. 103, page 801.

(2) Simple Trusses with One End Free Horizontally. When a simple truss is subjected to its load, the lower chord, being in tension, will elongate and the truss span will deflect downward. Both of these actions will combine to produce an increase in the length of the truss between points of support.

If the truss is supported on masonry walls, or piers, one end may be placed on an expansion bearing of the ROLLER, ROCKER, or SLIDING-PLATE TYPE. When the trusses are supported by steel columns an expansion bearing such as used for a masonry support is not practical and some type of an expansion link detail must be employed.

The roof-trusses for the Industrial Mutual Association Auditorium at Flint, Michigan,* Fig. 48, provide an excellent example of long-span construction wherein the expansion link or pendulum detail is employed.

The truss span is approximately 202 ft, with panel lengths of about 25 ft. The upper and lower chords are composed of 14-in H-sections, with their flanges in the plane of the truss. The principal web-members are 14-in I-beam sections connected to the chord members by gusset-plates on both flanges.

The horizontal displacement due to deformation and the variation in the length due to temperature changes were both considered in the design of the movable end detail.

This detail as illustrated consists of pin-connected plate-hangers about

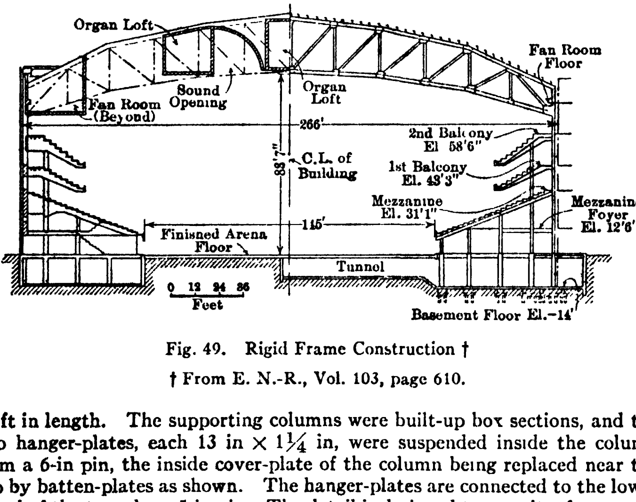


Fig. 49. Rigid Frame Construction †

† From E. N.-R., Vol. 103, page 610.

12 ft in length. The supporting columns were built-up box sections, and the two hanger-plates, each 13 in × 1¼ in, were suspended inside the column from a 6-in pin, the inside cover-plate of the column being replaced near the top by batten-plates as shown. The hanger-plates are connected to the lower chord of the truss by a 5-in pin. The detail is designed to permit a free movement of 4 in, which was the maximum total horizontal movement estimated for a combination of maximum conditions.

(3) Rigid Unit Bent Construction. The roof-construction of the Chicago Stadium,‡ Fig. 49, provides a striking example of that type of framing in which each truss and its supporting columns are designed as a rigid unit frame. This method of construction was adopted rather than the two-or three-hinged arch in order to avoid the deep arch sections which would have interfered with sight lines.

The trusses are 28 ft deep at the center and 15 ft deep at their ends, with a

* Eng. News-Record, Vol. 103, page 801.
‡ Engineering and Contracting, Vol. 68, p. 147.
‡ Eng. News-Record, Vol. 103, p. 610.

bottom chord rise of 21 ft 6 in. The chord members are built-up box-type sections composed of plates and angles, the typical bottom chord having 2 plates 22 in × 5/8 in and 4 angles 4 in × 4 in × 1/2 in; the typical top chord is made up of two plates 22 in × 1/2 in, 2 angles 4 in × 4 in × 1/2 in, 2 angles 6 in × 4 in × 5/8 in, and one cover-plate 26 in × 5/8 in. The web-members are all 12-in H-sections. The roof design-load used was 67 lb per sq ft, proportioned as follows:

Live load	25 lb per sq ft
Dead load, Trusses and Bracing	29 lb per sq ft
Purlins	3 lb per sq ft
Roof-deck	5 lb per sq ft
Ceiling	5 lb per sq ft
	67 lb per sq ft

In addition to these loads two of the central trusses support an exceptionally large theater organ and its equipment, while four other trusses support the main portion of the ventilating equipment.

11. Trussed Arch Roof-Construction

Types of Arches. There are two types of steel arches in general use for long-span roof-construction, classified with respect to the number of hinges as (1) THREE-HINGED ARCHES, (2) TWO-HINGED ARCHES, as described in Article 7.

The THREE-HINGED ARCH has been more widely used because it is the only type of true arch in which the stresses are statically determinate. TWO-HINGED ARCHES are rigid and economical and the rather limited use of this type of framing in roof-construction has been largely due to the difficulties encountered in analysis and design, which involves the theory of elastic deformations.

The arch ribs are usually built of angles, plates and angles, or channels, with riveted connections, forming an open framework. Solid web arch ribs are not common in roof-construction; however, a good example of solid web rib construction is that of the Central Armory, Cleveland, Ohio, illustrated in Engineering Record, Vol. 35, page 76.

Advantages and Disadvantages of Arched Roofs. For a roof of very long span, such as used for armories, train-sheds, assembly and exhibition halls, large garages, hangars, etc., an arch both with respect to form and method of construction provides an economical structure of pleasing appearance. A large part of the economy results from the omission of wall columns, which are replaced by the haunches of the arch. Another advantage lies in the fact that the stresses due to elastic displacements under load and to temperature change may be provided for by a comparatively simple expansion bearing detail. The base of the arch being usually near the ground or rigid foundation, the arch structure is an excellent type of framing for resisting to wind-loads.

The one serious objection to the use of the arch as ordinarily designed is the diminution of the clear floor-space at each cross-section of framing, due to the width of the arch haunch. These projecting end portions of the arch also interfere with the continuity of balconies along the side-walls and obstruct the view of spectators who might be seated along the side-walls between arches. This objection may be overcome by supporting the arch at the

desired elevation on cantilever arms, as illustrated in Fig. 62, or by a framed abutment where the working space will permit, as illustrated in Fig. 56. In the latter example the vertical and inclined struts are designed to provide the necessary vertical and horizontal reactions for the end-pins.

Two Three-Hinged Arches Used in Exposition Buildings, Chicago World's Fair (1893). The roof-construction of the Manufactures and Liberal Arts Building, illustrated in Fig. 50, consisted of three-hinged arches 368 ft center

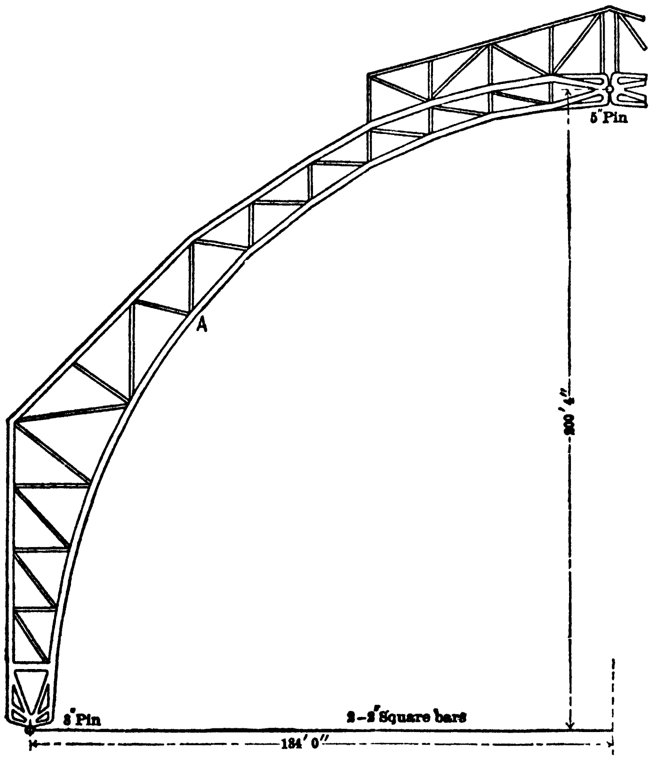


Fig. 50. Three-hinged Arch, Manufactures and Liberal Arts Building, Chicago Exposition

to center of end-pins and with a vertical dimension of 200 ft between the centers of the end-pins and the crown-pin.* The roof of the Machinery Hall was also supported by three-hinged arch construction as shown in Fig. 51. The general form of the arch was a stilted semicircle so designed to minimize the horizontal thrust due to dead load.†

* Eng. Record, Vol. 26, page 299.
† Eng. Record, Vol. 27, page 77.

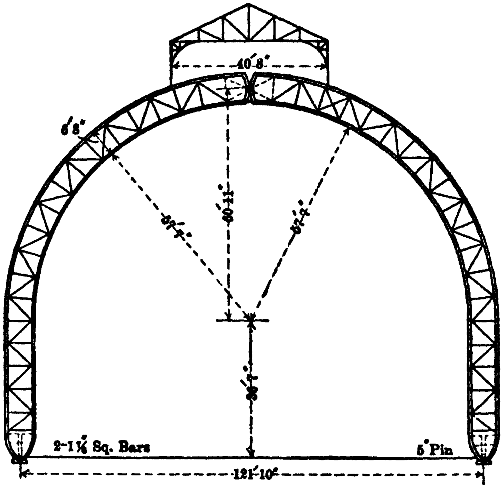


Fig. 51. Three-hinged Arch, Machinery Hall, Chicago Exposition

Roof-Construction for Drill Hall, Brooklyn, N. Y. The Engineering Record of December 23, 1899, describes the roof-construction illustrated in Fig. 52. The arch ribs are constructed with a double system of web-bracing, which is somewhat objectionable in view of the assumptions of stress distribution thereby necessitated.

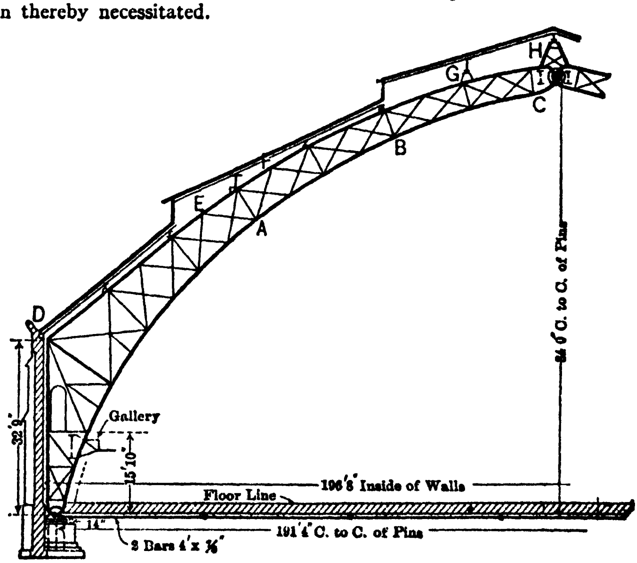


Fig. 52. Three-hinged Arch, Drill-hall, Brooklyn, N. Y.

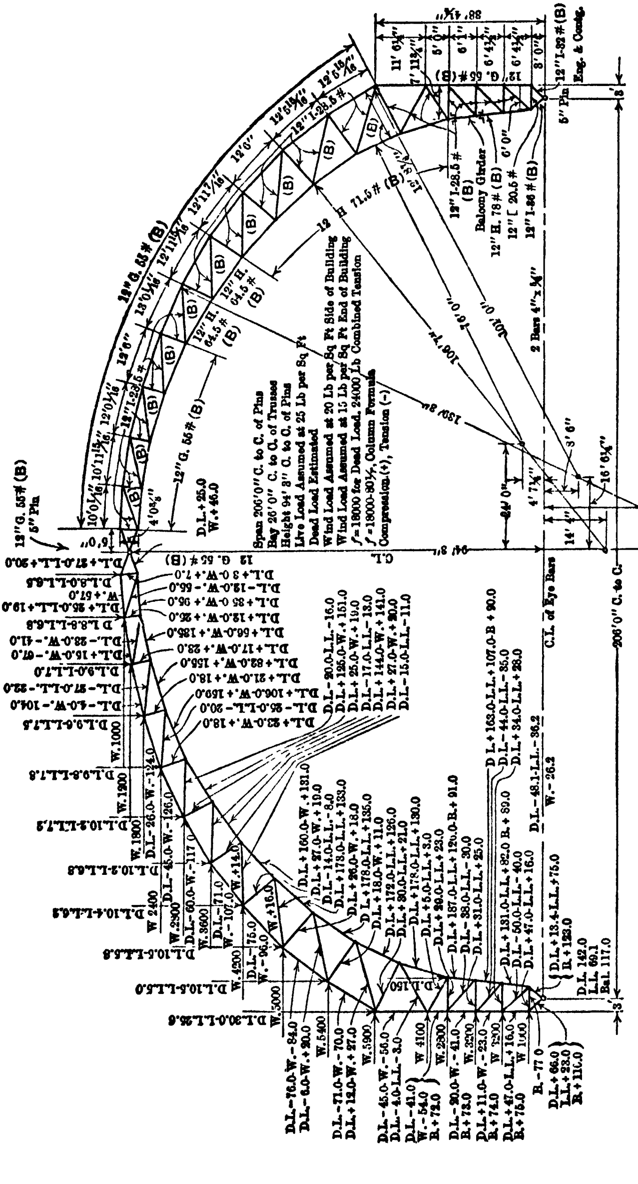


Fig 53. Three-hinged Arch, University of Illinois Armory *

* From Engineering and Contracting, Vol. 40, p. 160, given also in E. N.-R., Vol. 70, page 1182.

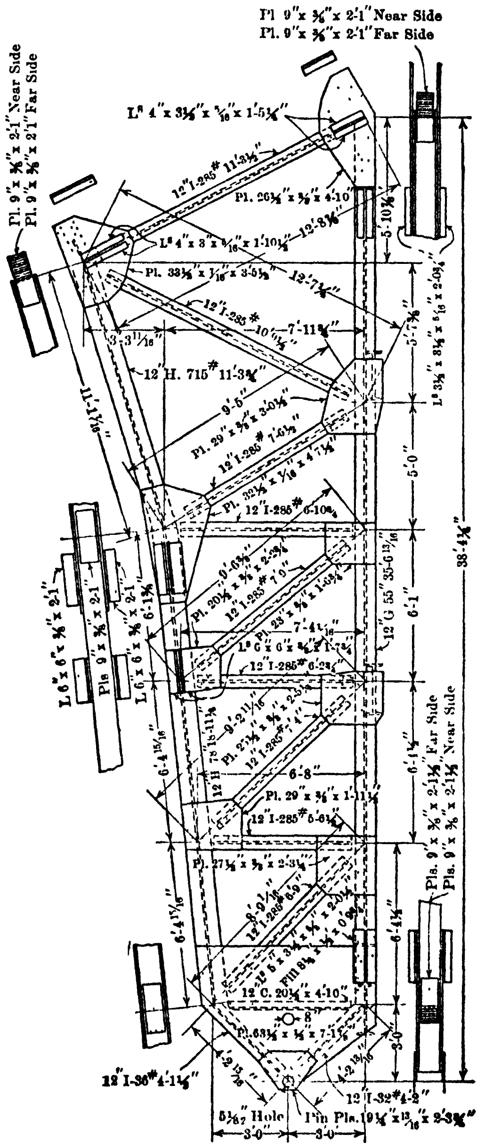


Fig. 54. Detail of Haunch, University of Illinois Armory *

* From Engineering and Contracting, Vol. 40, p. 161

University of Illinois Armory. The central unit or drill-hall has a clear span of 206 ft center to center of the end-pins of the three-hinged arches which support the roof and side-wall partitions. The outline of a typical arch rib

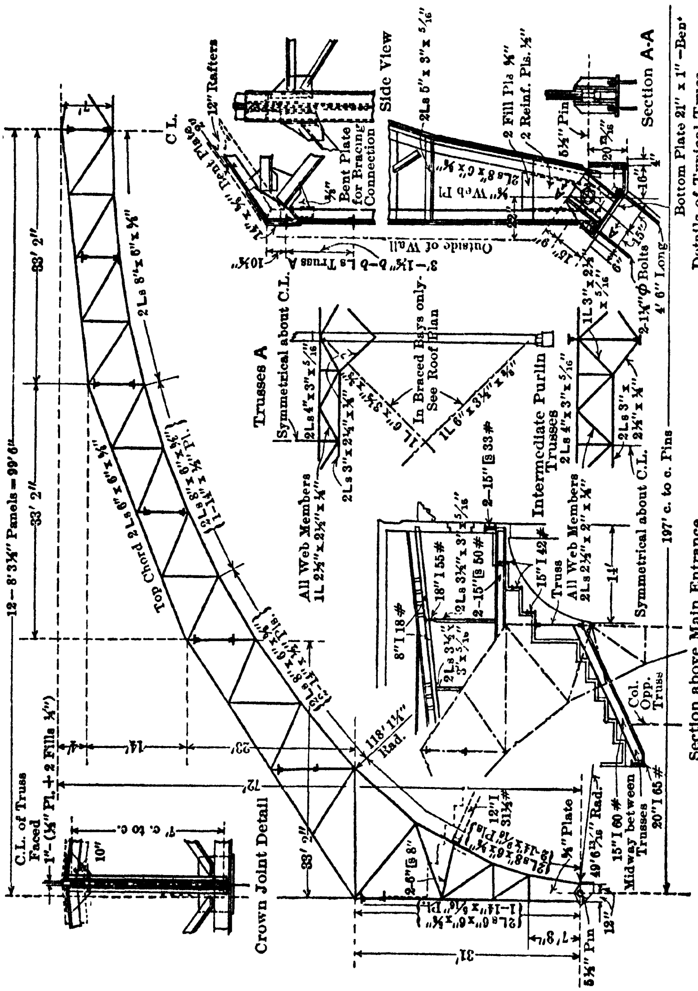


Fig. 55. Three-hinged Arch, Coliseum, Springfield, Mass.*

* From E N R, Vol 74, p 442

is shown in Fig. 53 and a detail of the same is illustrated in Fig. 54. The chord members of the braced ribs are composed of 12-in Bethlehem sections throughout and all web-members are of uniform section, being 12-in–I–28.5 lb. These sections are excessive for many of the web-members; however, the

simplicity of fabrication and the additional stiffness supplied led to the adoption of the uniform section. The arches rest on masonry piers, and the horizontal thrust is resisted by eye-bar ties connecting the end-pins. The tie-bars are encased in a concrete covering below the drill floor and the ends of the arches are also encased in concrete coverings so designed as to provide for lateral expansion and contraction.

Exposition Coliseum, Springfield, Mass. The coliseum erected at Springfield, Mass., for the Eastern States Agricultural and Industrial Exposition (Eng. Record, Vol. 74, page 442) provided an unobstructed floor area of about 200 ft × 305 ft. The roof was supported by ten three-hinged arches as illustrated in Fig. 55. These arches were designed as three-hinged arches, with the usual pin-ends, but instead of a pin detail at the crown an abutting joint was employed composed of bearing-plates with faced surfaces at the top chord joint, and an expansion detail with bolts and slotted holes was provided at the bottom chord joint. This crown joint detail is shown in Fig. 55. The arches were constructed without tie-rods, the end bearings being supported by concrete piers with inclined upper surfaces perpendicular to the thrust of the arch, designed to offer full resistance to the horizontal component of the end reactions.

Passenger Station, Rochester, N. Y. The general design of the three-hinged arch roof-construction for the passenger station of the N. Y. C. and H. R. Railroad at Rochester, N. Y., is illustrated in Fig. 56. In order to eliminate the projections of the arch ribs at the mezzanine floor the end-pins were supported 10 ft above this floor, or 21 ft 4 in above the waiting-room floor, by a structural frame composed of vertical and inclined posts. The supporting frames were erected, bolted to their footings, and braced in pairs by horizontal and diagonal bracing, prior to the erection of the arches.

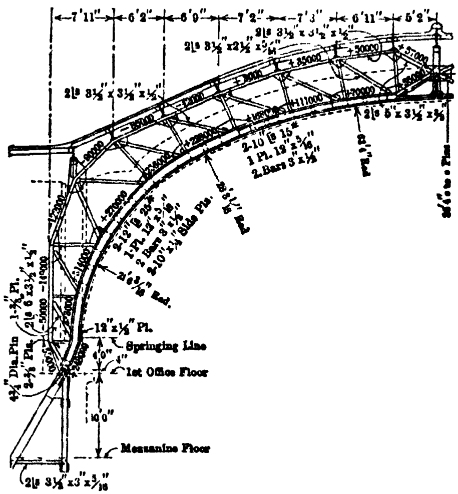


Fig. 56. Three-hinged Arch, Passenger Station, Rochester, N. Y.*

* From E. R., Vol. 68, p. 666

A similar method was employed in the design of the roof-construction for the Government hangar at Scott Field, Illinois (Eng. News, Vol. 90, page 234), where the end-pins were supported approximately 48 ft above the hangar floor.

12. Two-Hinged Arch Roof-Construction

Advantages of Two-Hinged Type of Roof Arch. Although the three-hinged type of arch has been most commonly used for long-span arched roof-construction, a properly designed two-hinged arch is more economical and has certain other advantages. The two-hinged type of roof arch provides a more rigid system of framing than the three-hinged arch, and where the arch is employed to support floor-construction above as well as to span floor-space below, rigidity of construction is an important consideration. Two-hinged arches require rigid supports; however, the horizontal components of the

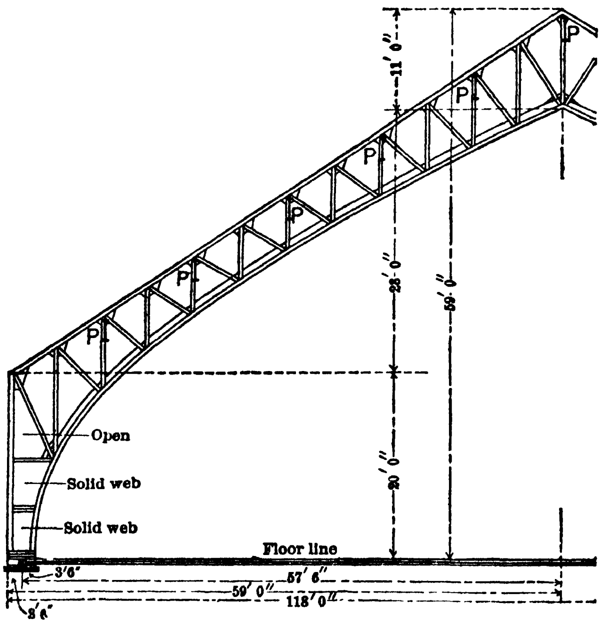


Fig. 57. Two-hinged Arch, Exposition Hall, Providence, R. I.

reactions on the supporting pins can be taken by tie-rods as in the three-hinged arch.

Where tie-rods are used, one end of the arch is anchored to one abutment and the other end is placed on an expansion detail such as sliding-plates or rollers. The abutments can, therefore, be designed for the vertical components, and the tie-rod for the horizontal components, of the reactions.

Braced Arches without definite hinge details at the supports, as usually constructed, are essentially two-hinged arches and should be analyzed as such.

Providence Exposition Hall.* This structure, 118 ft in width and 196 ft in length, has an arched roof supported by 7 main trusses, as illustrated in Fig. 57. The span center to center of end-bearings is 115 ft 0 in and the height from the floor-line to the mid-point at the center of the arch is 64 ft 6 in.

* Architects' and Builders' Magazine, Vol. 3, page 9.

The arch has a flat bearing at each end, the two ends being connected by a tie-rod with a turnbuckle adjustment, and one end being provided with an expansion detail. The total weight of steel, per square foot of horizontal projection, including arches, purlins, details, bracing, and tie-rods, is 11.5 lb.

Chicago Live Stock Pavilion. The roof of the amphitheater of the exhibition pavilion at the Chicago Union Stock Yards is supported by two-hinged arches, spanning 198 ft 0 in center to center of end-pins.

A line drawing of the arch, as illustrated in Engineering News, Vol. 55, page 716, is shown in Fig. 58.

The arches are 42 to 0 in on centers and the design loading was as follows:

(1) Dead load * = 25 lb per sq ft of horizontal projection
(2) Wind-load = 30 lb per sq ft of vertical projection
(3) Snow-load = 10 lb per sq ft of horizontal projection
(4) Arch = 9.5 lb per sq ft of horizontal projection

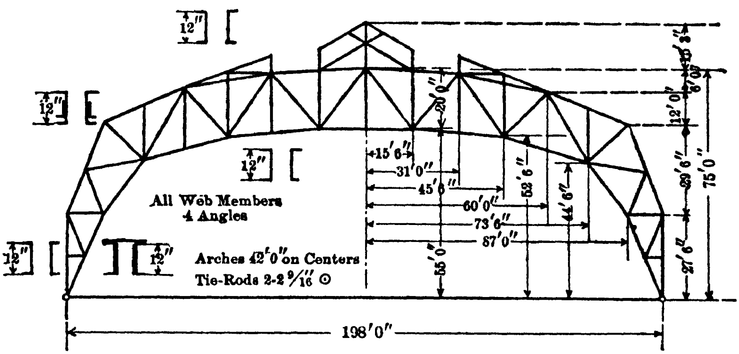


Fig. 58. Two-hinged Arch, Chicago Live Stock Pavilion

The chord members are composed of two 12-in channels of varying weights, reinforced by 12-in side-plates, where necessary, and connected by lacing bars over the flanges. The web-members are all composed of four angles, either connected by lacing or plates. The tie-rods for the main arches consist of two $2\frac{9}{16}$-in ϕ, designed to take the maximum thrust of 112 000 lb. The top and bottom chords break joints on alternate panel-points and at the breaks along the lower chord; longitudinal trusses of a depth equal to the depth of the arch are framed between arches.

Top lateral bracing occurs in all panels of the top chord and the lower flanges of the roof-rafters. Bottom-chord bracing occurs in all panels of the end bays and in all intermediate panels where the bottom chord is in compression as a result of arch action.

13. Cantilever Roof Construction

Cantilever roof construction may be divided into three general classes:

(1) One anchor span and one cantilever span
(2) Two anchor spans, two cantilever spans, and a simple span
(3) A single cantilever span anchored against rotation at its support, as in sidewalk canopies and similar structures.

* Superimposed.

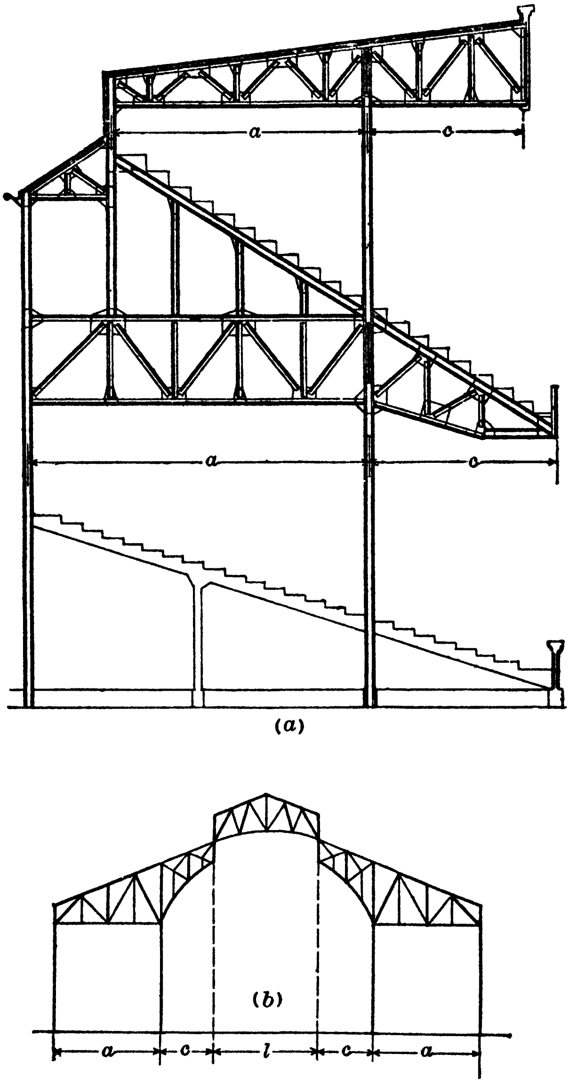


Fig. 59. Cantilever Roof Construction

The first type, illustrated in Fig. 59A, is limited, in general, to shelters for grandstands built for out-of-door events, such as ball-games, track-meets, racing events, and pageants.

The second type, Fig. 59B, may be employed in buildings where the interior is to be divided into a wide central aisle, unobstructed by columns, and flanked on each side by arcades. The central simple truss span is supported at the free ends of the cantilever arms, *c*, which are extensions of the anchor arms, *a*. The outside columns are subject to negative reactions from loads on the central span and the cantilever arms, and if the total resultant reactions are negative, due provision must be made for anchorage.

Advantages and Disadvantages of Cantilever Roof Construction. The advantages or points of merit possessed by the type of cantilever construction shown in Fig. 59B are: (1) greater clear height in the center than is obtainable by any other type of framing excepting the arch; (2) the construction presents a light and graceful appearance; (3) there is no horizontal thrust, and consequently no tie-rods are necessary; (4) on long spans the entire structure can be assembled in place, piece by piece, without falsework.

The relative economy of two- and three-hinged arches as compared to cantilever construction has never been thoroughly investigated. The arch type

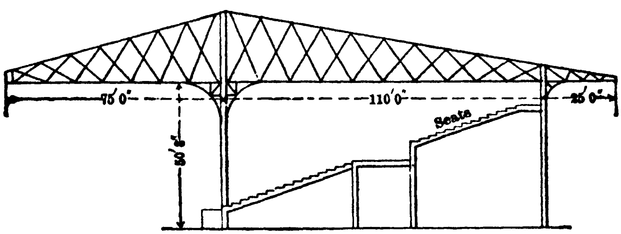


Fig. 60. Cantilever Truss, Grandstand, Monmouth Park, N. J.

of construction is probably more economical for spans greater than 120 ft. The choice of type, however, is more apt to be determined by architectural requirements than by relative economy, since the cantilever type of construction requires suitable anchor spans and intermediate columns or very heavy brackets, which are often undesirable if not impossible.

Grandstand, Monmouth Park, N. J An example of type (*a*) Fig. 59, is illustrated in Fig. 60, which shows the cantilever roof truss construction for the grandstand at Monmouth Park, N. J.* The forward supporting column, as is usually the practice in this type of cantilever construction, is continuous through to the top chord of the truss, and acts as a vertical compression member of the truss system as well as a supporting member of the frame. The upper and lower chords are composed of two angles and a plate forming a T-section. The web-members consist of two angles back to back and are riveted at their ends to the continuous web-plate between chord angles.

The Mining Building, World's Columbian Exposition. Examples of the type of cantilever construction illustrated in Fig. 59(*b*) are not common in general practice; however, the Mining Building at the Chicago World's

* Eng. News, Vol. 23, page 57.

Fair * and the Machinery Hall at the Exposition in Geneva, Switzerland,† are excellent examples of this type.

The general scheme of framing for the Mining Building at Chicago is shown in Fig. 61; the heavy lines represent compression members, the lighter lines,

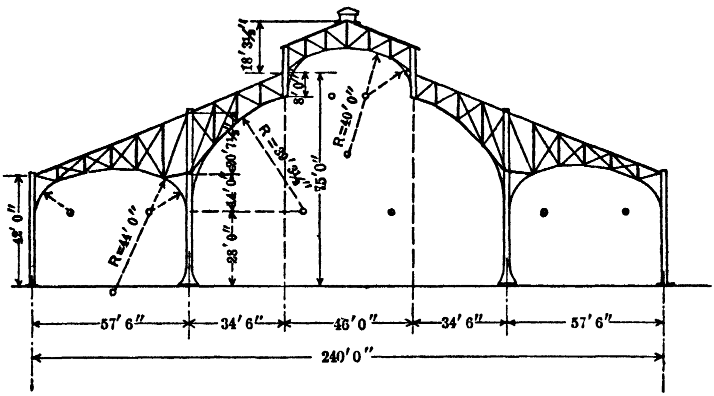


Fig. 61. Mining Building, World's Columbian Exposition

tension members, and counters. This arrangement of framing was possible because of the wide central aisle and the continuous side aisles. The cantilever method was probably adopted for the novel effect and graceful outlines rather than from any necessity or relative economy, since the trusses were noticeably heavy in section and detail.

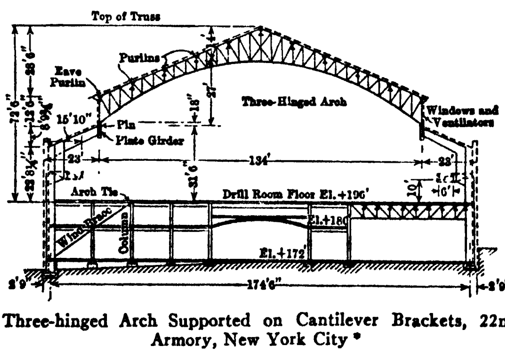


Fig. 62. Three-hinged Arch Supported on Cantilever Brackets, 22nd Regiment Armory, New York City *

* From E. N., Vol. 63, p. 520

Armory for the 22nd Regiment, New York City. In order to avoid reduction in width of the main floor by wide arch haunches, and to eliminate the interruptions in the balcony seating, the roof-construction of this building

* Engineering Record, Vol. 29, page 9.
† Engineering News, Vol. 37, page 38.

was designed as shown in Fig. 62. The roof arch is of the usual three-hinged type but rests on cantilever side brackets instead of abutments at or near the floor-level.

The thrust of the arch is taken up by a tie-member consisting of a 12-in, 35-lb channel, laid over a 15-in, 38-lb I-beam in the plane of the bent. The I-beam serves as a floor-beam and as a support for the channel-tie.

The maximum vertical design-reaction on the supporting end-pins was

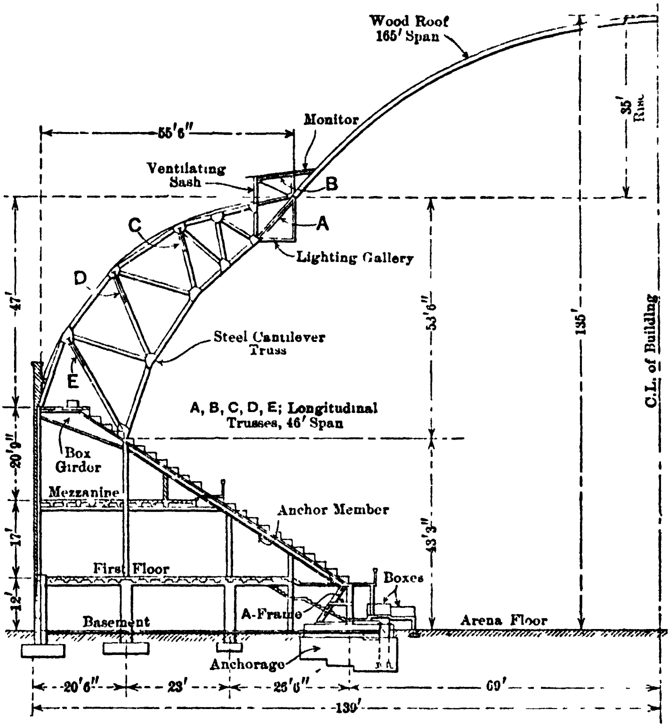


Fig. 63. Steel Cantilever Trusses Supporting Lamella Arch Span. Arena of National Exhibition Company's Coliseum, St. Louis, Mo.*

* From E. N.-R., Vol. 104, page 935

taken at 133 300 lb with a simultaneous outward thrust on these pins of 113 600 lb. These reaction components produce moments of opposite character in the supporting columns.

The width of the supporting columns at the floor-line is relatively small (3 ft 6 in back to back of angles); this is made possible by the fact that for the design loads the bending in the column is relatively small. The end-pins of the arch are located in such positions that the clockwise and counter-clockwise moments at the floor-line are approximately equal. The locations

of the supporting pins were determined by trial and calculations, taking into account the proper combinations of dead, live, and wind-loads.

An account of the problems involved in the design, together with details of the construction of this roof, are given in Engineering News, Vol. 63, page 520.

National Exhibition Co. Arena, St. Louis, Mo. The building, 278 ft in width out to out of walls, has an arched roof consisting of a row of tall steel cantilever trusses on each side, the upper ends of which carry a central arch of "lamella" type timber roof-construction, the latter having a span of 165 ft.* The steel cantilever trusses are supported at the level of the top row of seats, as shown in Fig. 63. The base of each cantilever is a heavy inclined box girder carrying the terraced seats. Below this girder, and forming a continuation of it, is an inclined girder composed of two 15-in channels with cover-plates top and bottom. The lower end of the inclined girder is attached to the apex of an A-frame built up of 12-in H-sections and mounted on a pair of 30-in plate girders, which are anchored to a massive concrete base. The uplift at the inner flange of the cantilever truss is transmitted through the inclined girder and resisted by the anchorage.

The lamella construction has a rise of 35 ft above its spring line, and is 135 ft 0 in above the arena floor at its crown. It is composed of Douglas fir units as illustrated in Fig. 43(*b*), each unit being 3¾ in × 17½ in and 15 ft long.

Expansion is provided for in the structure by means of slotted hole connections in the steelwork and by permissible BREATHING of the wooden arched roof.

14. References to Engineering Literature Relative to Steel Roof-Construction

Steel roof-construction may be divided for general reference into four classifications:

(1) Simple span trusses
(2) Three-hinged arches
(3) Two-hinged arches
(4) Cantilever construction

The general characteristics of these four classes of roof-construction, together with typical examples of each, have been briefly discussed in the preceding paragraphs.

There are many variations in form, detail and dimensions of the construction in any one classification and accordingly a number of selected references to engineering periodicals are given in the following tabulations, wherein the designer or the student may find illustrations and descriptions of the various details of design. No attempt has been made to refer to all engineering periodicals, nor to include every reference in the periodicals selected.

By simple span trusses is meant a trussed span designed with one end free to move horizontally and wherein the reactions are assumed to be vertical under vertical loading.

Spans. All span lengths of simple trusses and arches are given center to center of bearing.

Cantilever Spans. The span lengths *a*, *c* and *l* noted in the table of cantilever construction refer to anchor span, cantilever span and supported span, as noted in Figs. 59(*a*) and 59(*b*).

* Eng. News-Record, Vol. 104, page 935.

Rise. The RISE in the case of SIMPLE TRUSSES is given as the vertical distance between chord centers at the center line of span. For THREE-HINGED ARCHES the rise is given as the vertical dimension between the horizontal line connecting end-pins, and the crown-pin. The rise as given for TWO-HINGED ARCHES is the vertical dimension taken from the horizontal line between supports to the mid-point of the arch frame at the center line of span.

Abbreviations. The titles of the engineering periodicals are abbreviated as follows:

E.R.	Engineering Record
E.N.	Engineering News and Engineering News-Record
A. & B.M.	Architects' and Builders' Magazine

Simple Span Steel Roof-Trusses

Location	Span		Rise		Reference
	Ft	In	Ft	In	Vol. Page
New York City, Pier Shed...	96	0	10	0	E R. 33: 115
Chicago, Assembly Hall	104	3	10	0	E.N. 79: 1 105
Baltimore, Pier Shed	107	0	15	0	E R. 52: 6
New York City, Hippodrome	108	4	12	6	E R 51: 354
Peoria, Illinois, Train Shed..	109	4	18	0	E R. 42: 536
New York City, Pier Shed.	118	3	15	0	E N 61: 30
Cleveland, Ohio, Theater.	120	7	15	3	E N. 88: 1 041
New York City, Railway Station	123	8	25	0	E R. 67: 211
East Orange, N. J, Armory. ...	126	0	20	0	E R. 65: 711
Camden, N. J, Train Shed.	126	0	14	9	E R. 44: 242
Chicago, Riding Club	158	7	15	0	E N. 97: 250
New York City, Madison Sq. Garden	166	10	17	0	E R 23: 124
Boston, Mass, Coliseum....	174	0	24	0	E N. 102: 324
Brooklyn, N. Y, Armory	179	2	14	0	E N. 58: 221
Jacksonville, Fla, Pier Shed..	180	0	19	5	E N. 74: 494
Buffalo, N Y, Arsenal	181	2	27	0	E R. 67: 302
Kansas City, Mo., Auditorium	187	4	32	6	E.R. 40: 170
New York City, Armory	189	8	24	0	E R. 50: 7
Quincy, Mass, Shipbuilding Plant *	195	0	35	0	E R. 46: 85
Flint, Mich, Auditorium.	202	4	22	0	E N. 103: 801
Cleveland, Ohio, Auditorium...	209	0	28	6	E.N. 84: 414
Chicago, Train Shed	212	0	25	6	E R. 48: 302
Boston, Mass, Train Shed ..	228	6	18	0	E R. 39: 135
Harrison, N. J, Steel Mill ...	235	9	42	9	E.N. 82: 898
Chicago, Stadium ..	261	9	25	0	E N. 103: 610
Birmingham, Eng, Train Shed..	275	0	8	0	E.N. 72: 1 112

* With 60 ft 0 in cantilever overhang, each end.

Three-Hinged Steel Arch Roof-Construction

Location	Span Ft	Span In	Rise Ft	Rise In	Reference Vol.	Page
Rochester, N. Y., Railway Station	90	8	36	4	E.R. 68:	666
Dallas, Texas, Stock Pavilion *	100	0	35	0	E.N. 65:	728
Syracuse, N.Y., University Gymnasium	101	4	27	0	E.R. 58:	216
Cleveland, Ohio, Armory *	120	0	52	6	E.R 35:	76
Chicago, World's Fair Machinery Hall	121	10	97	6	E R 27:	77
New York City, Armory †	134	0	31	6	E.R. 63:	520
Chicago, Recreation Pier	136	8	74	4	E N 74:	197
Chicago, Coliseum	149	9	66	6	E R. 43:	627
Chicago, Armory	155	6	78	0	E N 32:	176
Scott Field, Illinois, Hangar	160	0	114	0	E N. 90:	234
Newark, N. J, Armory	163	6	73	5	E R 41:	500
Providence, R. I, Drill-Hall	166	8	60	0	E R. 55:	474
St. Louis, Exposition, Govt. Bldg	172	0	69	9	E N 52:	282
St. Louis, Mo, Coliseum	178	6	80	0	E R 37:	383
Hartford, Conn, Armory	181	0	90	2	E R. 58:	291
New York City, Armory	189	8	103	4	E R 51:	620
Baltimore, Md, Armory	190	0	74	6	E.R 49:	604
Brooklyn, N. Y, Armory	196	8	84	0	E R 40:	704
Springfield, Mass., Coliseum	197	0	68	6	E R. 74:	442
Chicago, Armory	198	0	90	0	E.N 75:	152
Urbana, Illinois, University Armory	206	0	94	3	E N 70:	1 182
Minneapolis, University Field House	220	0	100	0	E.N. 100:	578
Buffalo, N. Y., Armory	227	0	94	0	E R. 41:	549
Jersey City, N. J., Train Shed	252	8	92	3	E R. 40:	216
Pittsburgh, Penna., Train Shed	255	0	96	0	E R. 46:	173
Lakehurst, N. J, Hangar	258	0	183	0	E.N. 84:	892
Philadelphia, Pa., Train Shed	259	0	88	3	E R. 27:	22
New York City, Armory	288	10	102	7	E N 71:	1 339
Philadelphia, Penna, Train Shed	300	8	108	5	E N. 29:	512
Akron, Ohio, Airship Dock	325	0	197	6	E N. 105:	135
Paris, France, Exposition, Mach Hall	362	9	149	0	E R. 20:	318
Chicago, Exposition Manuf Bldg	368	0	206	4	E.R. 26:	299

* Solid web ribs.
† Cantilevered from column brackets

Two-Hinged Steel Arch Roof-Construction

Location	Span Ft	Span In	Rise Ft	Rise In	Reference Vol.	Page
New York City, Church	73	0	75	0	E.R 44:	370
New York City, Express Co. Bldg	75	0	27	0	E R 50:	495
Portland, Maine, Armory	92	0	45	0	A & BM 3:	10
Brockton, Mass., Exposition Hall	94	0	60	0	A & BM 3:	11
Providence, R. I., Exposition Hall	115	0	53	6	A & BM 3:	9
Scranton, Penna., Armory	156	0	49	6	E R. 44:	180
New York City, Armory	176	0	67	0	E N 21:	332
New York City, Armory	179	2	73	0	E N 58:	220
Frankfort, Germany, Train Shed	184	0	93	6	E R 25:	230
West Baden, Indiana, Hotel	195	0	43	0	E N. 48:	158
Chicago, Stock Pavilion	198	0	64	0	E.N. 55:	716

Cantilever Roof Construction

a = spans of anchor arms
c = spans of cantilevers
l = supported spans

Location	Spans						Reference
	a		c		l		
	Ft	In	Ft	In	Ft	In	Vol. Page
New York City, Ball Park.......	40	0	22	0	.	.	E R. 64: 126
New York City, Armory		.	23	0	134	0	E N. 63: 520
Chicago, Ball Park...	64	0	24	0	.	.	E R. 62: 261
Brooklyn, N. Y., Ball Park.......	64	3	25	0	...		E.R. 69: 647
Chicago, Ball Park..........	64	0	26	0	.		E.N. 91: 173
Chicago, Speedway Grandstand	47	0	27	8	.	.	E.N. 74, 1 167
Brooklyn, N. Y., Ball Park.. .	29	0	28	0	..		E.R. 67: 274
Boston, Mass., Ball Park..	80	0	30	0	....		E.N. 74: 376
Salt Lake City, Utah, Ball Park...	18	0	30	0	...		E.N. 76: 707
Cleveland. Ohio, Ball Park... ..	44	6	32	0	.	.	E.R. 62: 705
Chicago, Exposition, Mining Bldg..	57	6	34	6	46	0	E.R. 29: 8
Geneva, Switzerland, Exp. Bldg...	80	0	50	0	26	0	E.N. 37: 38
Balboa, Canal Zone, Pier Shed	23	3	52	0	.		E N. 80: 996
St Louis, Mo, Arena *....		.	55	6	165	0	E N. 104: 936
Quincy, Mass, Shipbuilding Plant	195	0	60	0	.		E.R. 46: 85
Monmouth Park, N. J, Grandstand	110	0	75	0	.	.	E.N. 23: 57
Prince Rupert, B C., Dock Shed..	80	0	80	0	.	.	E.N. 67: 2
Ely, Minn, Mine Head House..	48	0	88	6	.	.	E.N. 80: 1 002

* Steel cantilevers, lamella arch span.

CHAPTER XXV

STRESSES IN ROOF-TRUSSES

By

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1. General Considerations and Definitions

External Forces. The EXTERNAL FORCES, OR LOADS to which a roof-truss is subjected, consist of the weights of materials of construction, snow, ice, and wind-pressure, together with the reactions developed at the supports as a result of these loads.

Internal Forces. The INTERNAL FORCES, or STRESSES, are those developed within the different parts of the structure under the action of the external forces. In a prismatic member the imaginary line connecting the centers of gravity of every cross-section of the piece is called its LONGITUDINAL AXIS, or simply its AXIS. The stress in the member is said to be AXIAL when the resultant of the stresses at every cross-section passes through the center of gravity of that section.

Axial stresses in the members of a roof-truss are usually considered to be uniformly distributed over the entire cross-section, at each section of any member. The UNIT STRESS, or intensity of stress, is the total stress at a given section divided by the area of that cross-section. Unit stresses are generally expressed in pounds per square inch.

Strain, or Deformation. When any material is subjected to the action of a force, the piece so acted upon undergoes a change in shape. This change in shape is called STRAIN OF DEFORMATION.

The cohesion of the particles of which the material is composed tends to resist deformation and as a result INTERNAL STRESS is produced. Internal stress is, therefore, a result of strain and is of an amount just sufficient to hold the external forces in equilibrium.

Statics is that branch of mechanics which, in its application to structures, treats of the conditions under which every part of a structure, and therefore the structure as a whole, is in equilibrium under the action of external forces and the resulting internal stresses.

Elements of a Force. A force, acting upon a structure, is completely defined when its LINE OF ACTION, DIRECTION, POINT OF APPLICATION, and MAGNITUDE are given. The LINE OF ACTION of a force is the path along which the force tends to produce motion. The DIRECTION of a force is the specification relative to which of the two ways along the line of action the force tends to produce motion. Direction is indicated, graphically, by an arrow placed on the line of action showing the direction in which the force tends to produce motion. The POINT OF APPLICATION is the place (considered as a point) where the force is brought to bear upon the structure. The LINE OF ACTION of the force is located by the point of application and the direction.

The MAGNITUDE of a force is the ratio of the effectiveness of that force in producing motion to that of a unit force. The magnitude of a force may be expressed, graphically, by the length of the action line.

Equilibrium of Forces. When a system of given forces acts upon a structure without producing a change with respect to the state of rest or motion, the system of forces is said to be in EQUILIBRIUM. Any one of the forces may be considered as the EQUILIBRANT (or balancing force) of all the other forces.

Resultant. The RESULTANT of a system of two or more forces is a single force which would produce the same effect as the given system of forces. The resultant of a given force system and the equilibrant of that system, are, therefore, equal in magnitude, have the same line of action, and are opposite in direction.

Components of a Force. Any number of forces whose combined effect is the same in all respects to a single force are called the COMPONENTS of that force. It is evident, therefore, that a force may have any number of components.

Composition of Forces is the process of finding, for a given system of forces, an equivalent system having a smaller number of forces than the original system. The finding of a single force to replace the given system is the most important case of composition.

Resolution of Forces. Resolution is the process of finding, for a given system of forces, an equivalent system having a greater number of forces than the original system. The process of finding two (reactions) forces which are equivalent to a given (resultant) force is the most important case of resolution.

Space and Force Diagrams. In the graphical solution of problems involving statics (graphic-statics), it is usually desirable to draw two separate figures, one of which shows the lines of action and points of application of the given force system with respect to the given structure, and the other, drawn independent of the structure, showing the magnitude and direction of the various forces. The former is called the SPACE-DIAGRAM and the latter the FORCE-DIAGRAM.

Loads. The loads to be considered preliminary to the problems of stress determinations in roof trusses may be classified as follows:

(*a*) DEAD LOADS
(*b*) SNOW-LOADS
(*c*) WIND-LOADS

The DEAD LOAD consists of the weights of the following items: roof-covering; (2) sheathing, rafters, purlins; (3) roof-trusses and bracing; (4) ceiling-loads; (5) any other permanent or fixed loads supported by the roof-construction.

The SNOW-LOAD varies greatly for different localities, depending upon the latitude and relative humidity of the locality.

The WIND-LOAD on a roof surface depends upon the velocity and direction of the wind and upon the inclination and exposure of the roof. The wind is assumed to move horizontally and the component normal to an inclined surface is considered in calculating wind-loads on roof-trusses.

The methods used in the determination of the magnitudes of external loads for the design of roof-trusses are discussed in detail in Chapter XXVI.

2. Reactions

General Considerations. The REACTIONS of a roof-truss are the external forces, which if applied at the center of the truss supports will hold in equilibrium the weight of the truss and the loads supported by it. The reactions are, therefore, equal, numerically, to the pressures exerted by the truss against the supports. It is evident that before the reactions can be determined, the

loads that are supported by the truss, together with the weight of the truss itself, must be calculated.

The data relative to the calculation of loads, which is a part of the problem of design, are given in detail in Chapter XXVI.

Since the two reactions of a given truss must equilibrate the system of applied loads, their resultant must be numerically equal and opposed in direction to, and have the same action line as, the resultant of the system of applied loads.

It is highly important, therefore, that the calculation of the reactions be exact since the subsequent calculations of stresses throughout the truss depend upon the established equilibrium of the structure as a whole.

Since the methods of finding the reactions for vertical load systems differ somewhat from those for inclined load systems, the procedure for the two cases will be considered in separate articles, the first dealing with the problems relative to reactions for dead and snow-loads, and the second dealing with the problems of finding reactions for wind-loads.

Reactions for Dead, Snow, and Ceiling-Loads. Dead, snow, and ceiling-loads are gravity loads, and their action lines are, therefore, taken normal to the horizontal plane. The reactions and stresses due to snow-loads are usually considered separately from those due to dead and ceiling-loads in order to permit separate combinations of dead and wind-load stresses since maximum snow and wind-loads are assumed never to act at the same time.

Example 1. Let it be required to find the reactions for the six-panel Howe truss loaded as shown in Fig. 1. The vertical panel loads comprise a simple

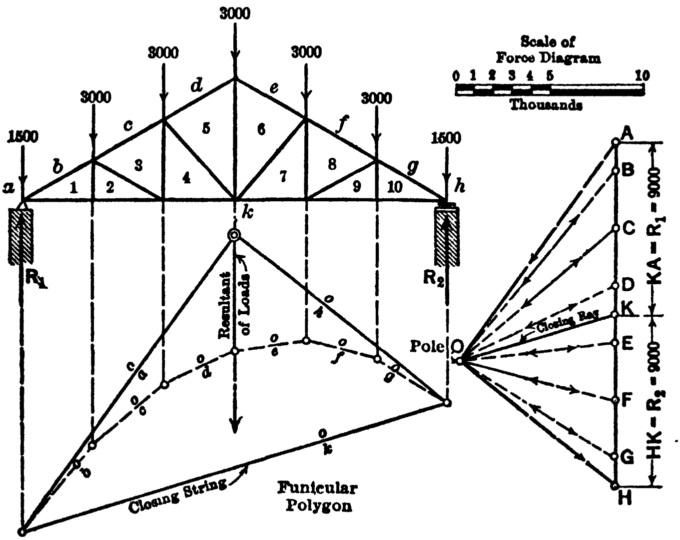


Fig. 1. Stress-diagram for Howe Truss

system of parallel loads and since the points of application of the reactions are known only two unknowns remain for each reaction, namely, magnitude and direction. Construct the force-polygon $A—H$ for the given loads

taken in order reading clockwise around the truss from the left support. In this case the force-polygon is a straight line, and is called the LOAD LINE. Assume any pole O, draw the rays OA, OB—OH and construct the funicular polygon ob, oc—og, noting that each string of the funicular polygon, such as od, crosses the interval in the space diagram denoted by d and is parallel to the ray OD.

The closing string ok, which is a component of $ka = R_1 = KA$ and $hk = R_2 = HK$, defines the direction of ray OK dividing the load line into the two required reactions, thus determining their magnitudes and directions.

It should be noted that strings oa and oh are of zero length in the funicular polygan since the loads ab and gh are coincident with the action lines of the reactions R_1 and R_2, respectively.

These half-panel loads are transferred to the supports directly without affecting the truss and could be subtracted from the reactions directly and thus eliminated. The resulting reactions are called the NET or EFFECTIVE REACTIONS. The effective reactions for the truss in Fig. 1 are 9 000 − 1 500 = 7 500 lb.

The reactions might have been found algebraically, by taking moments about one of the supports, or in the case of symmetrical loading, by merely dividing the total load into two equal reactions.

Example 2. In this case the loading on the truss is unsymmetrical on account of the added load of 5 000 lb located at the second panel-point of the lower chord, as shown in Fig. 2.

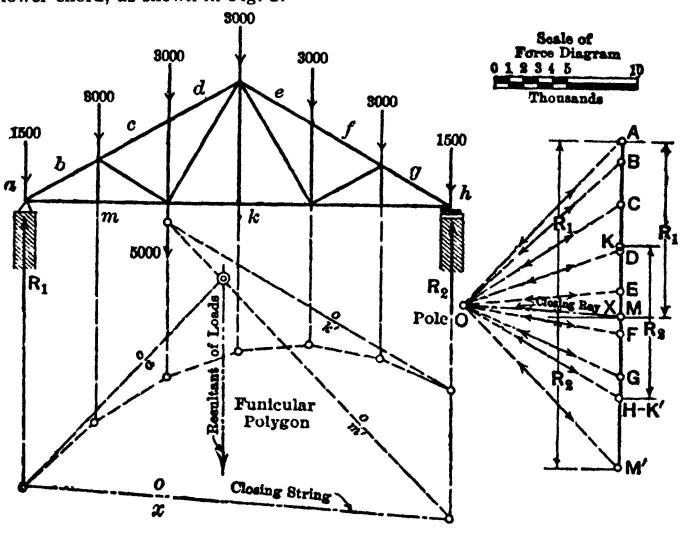


Fig. 2. Truss with Lower Chord Load

Starting at a, and reading clockwise around the truss, lay off the load line AM'. Since the values of $R_2 = HK$ and $R_1 = MA$, are unknown, their designations must be temporarily omitted from the load line, as shown. Select a convenient pole, construct the ray-diagram, and draw the funicular polygon as before.

It should be noted that the ray-diagram is in reality a force-polygon in which each force is resolved into two components; thus force AB is resolved into AO and OB as indicated by the arrows and BC is resolved into BO and OC. It is evident that, since all intermediate rays are used twice, and in reverse order, the entire force-polygon, with the exception of the first and last rays, is neutralized. The first and last rays are, therefore, the components of the resultant of the two reactions. This resultant may be divided into the two required reactions by the closing string transposed to the force polygon as the necessary components of the reactions.

In Fig. 2, ray OK' is a component of load gh and load km, while ray OM' is the remaining component of load km and also one component of the unknown reaction R_2. The string om' must therefore be drawn from the intersection of string ok' and the action line of load km, to intersect the action line of R_2, thus locating the required closing string ox. Then, in the force-polygon, $M'X = R_2$, and $XA = R_1$. Since the reactions are now determined the load line may be laid off in consecutive order as $A \downarrow H$, $H \uparrow K = R_2$, $K \downarrow M$ and $M \uparrow A = R_1$, as indicated in Fig. 2.

Example 3. When a series of ceiling-loads are applied to the lower chord, as in Fig. 3, the same procedure as outlined in Example 2 may be employed, provided the ceiling-loads are replaced by their resultant.

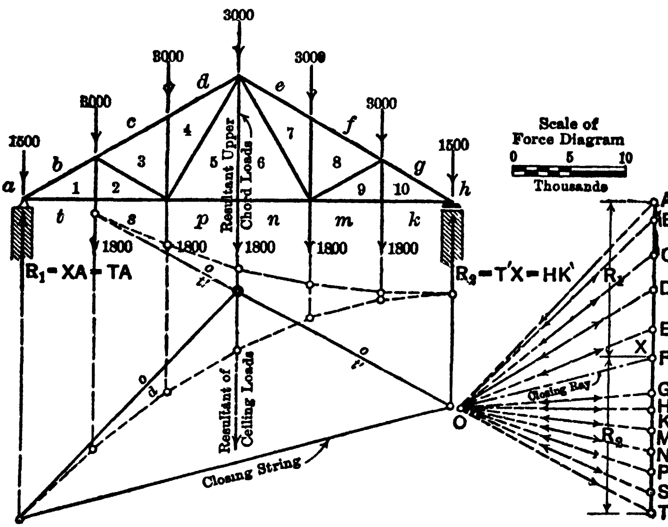


Fig. 3. Truss with Suspended Loads

Example 4. Fig. 4 shows an unsymmetrical truss with the one-half panel loads at each end deducted directly from the load system. The effective reactions are found as before by considering the ceiling-loads to be replaced by their resultant of 6 000 lb which may be subdivided, after it is properly located between the reactions on the load line, into the five panel loads of 1 200 lb each.

Reactions for Wind-Loads. The magnitudes and directions of the wind-load reactions for a given truss depend upon the manner in which the truss is supported at its ends. When the span is small, or when the truss is made of timber, the ends are usually fixed or restrained against horizontal movement at the supports by means of anchor bolts, or by being built into the support with no provision for horizontal movement.

For spans of 40 to 50 ft and more, the change in length of the truss due to deformations caused by the stresses in the members, together with changes in length due to changes in temperature, necessitate an end bearing detail, at one support, which will permit of a horizontal movement.

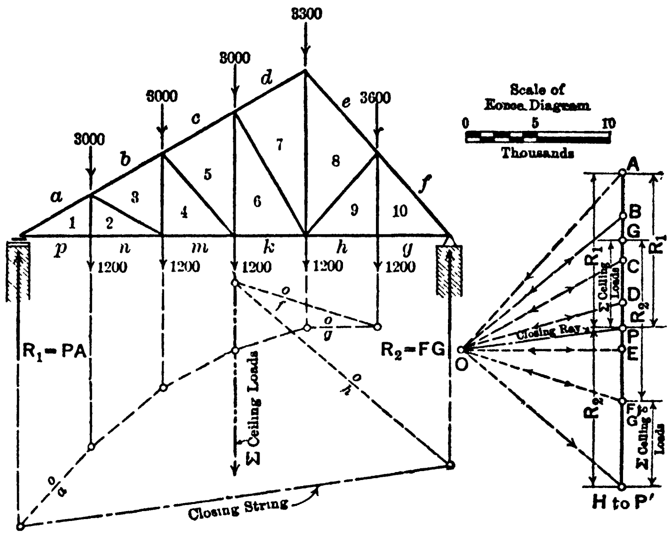


Fig. 4. Unsymmetrical Truss

If the ends of a truss are both fixed, deformation under load and temperature change may produce excessive secondary stresses in the truss as well as horizontal thrusts at the supports. The usual methods of providing for horizontal movement is to place one end of the truss upon a planed base plate and to provide a slotted connection of the truss to this plate, or in the case of heavier trusses, to place one end upon a roller or rocker bearing. Wind-load reactions will be determined for the three following assumptions:

(*a*) that both ends of the truss are fixed horizontally;
(*b*) that one end of the truss is supported upon rollers;
(*c*) that one end of the truss is supported upon a planed base plate with a coefficient of friction of 0.30.

(a) Truss Fixed at Both Supports. The problem of finding the lines of action of the wind-load reactions for this case is indeterminate since the amount of horizontal resistance offered by each support is unknown. The vertical components of the reactions are independent of the horizontal com-

ponents and of any assumption which may be made relative to the horizontal components.

When the roof is comparatively flat, so that the resultant of the normal components of the wind-loads approaches a vertical direction, the reactions can be assumed PARALLEL to each other and parallel to the wind-load resultant.

When the roof is steep and the wind-load is large as compared to the dead load, the assumption of parallel reactions may lead to absurd results.

The assumption which probably approximates the truth more nearly for the average conditions is that the HORIZONTAL COMPONENTS OF THE REACTIONS ARE EQUAL. If the truss is assumed to be rigid, the supports equally elastic and no horizontal tendency other than the wind-pressure existing, this assumption would be correct, since the horizontal forces causing equal yielding of the rigid frame, on equally rigid supports, must be equal.

The usual assumptions employed in determining the wind-load reactions for a truss with fixed ends may be briefly stated as follows:

(1) that the wind-load reactions are parallel;
(2) that the horizontal components of the wind-load reactions are equal.

Case 1. Reactions Parallel. Let it be required to find the wind-load reactions for the truss loaded as shown in Fig. 5. To determine the reactions

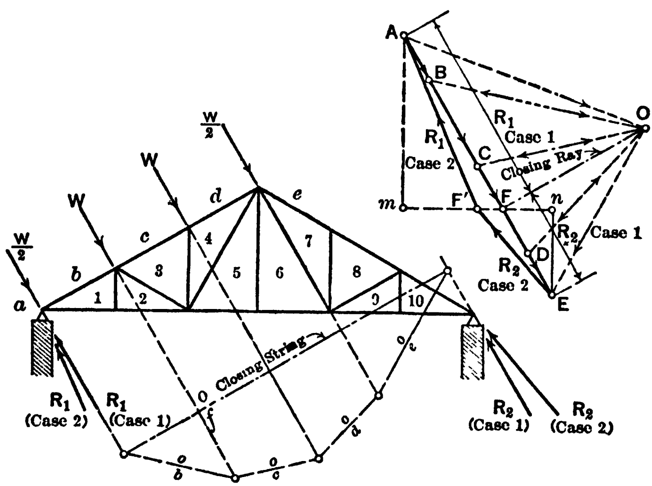


Fig. 5. Wind-load Reactions

construct the load line A—E, assume a convenient pole, O, and draw the funicular polygon. The string oa is zero length since the action line of ab is coincident with the action line of R_1 and the closing string of determining the direction of ray OF divides the load line into $EF = R_2$ and $FA = R_1$.

Case 2. Horizontal Components of Reactions Equal. The reactions will be found for the same truss and loading as in Case 1. First, find the reactions as in Case 1, assuming that their action lines are parallel. The vertical components of the reactions thus found, being independent of the end conditions,

are the true vertical components for any and all assumptions relative to the distribution of horizontal components.

Therefore, draw a horizontal line mn, Fig. 5, through F, and project A and E vertically to determine m and n respectively. The line mn is then equal to the horizontal component of the total wind-load. Bisect mn, making $mF' = F'n$, and draw $EF' = R_2$ and $F'A = R_1$, as shown in Fig. 5.

(b) One End of Truss Supported on Rollers. If one end of the truss is supported on rollers, then the reaction at that end cannot have a horizontal component (neglecting the friction of the rollers) since the rollers are so designed and placed as to permit an unrestrained horizontal movement of that end of the truss. The reaction at the end supported on rollers is, therefore, vertical, and the entire horizontal resistance must be supplied by the fixed end.

Since the wind-pressure may be applied on either side of the truss, depending upon the direction of the wind, the rollers may, at one time, be under the end to the windward side and at another time under the end to the leeward side. There are, therefore, two cases to be considered, as follows:

(1) Rollers under the windward end of the truss;

(2) Rollers under the leeward end of the truss.

Case 1. Rollers Under the Windward End of the Truss. The truss and loading is shown in Fig. 6, and since the left or windward end is on rollers, the

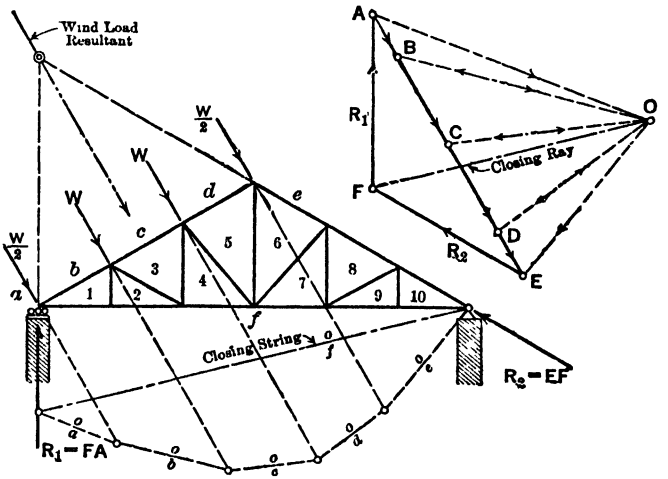


Fig. 6. Wind-load Reactions

reaction at this end, or R_1, is vertical. The right reaction, R_2, is unknown in direction, one point only on its line of action being known.

The reactions may be determined by one of two methods:

(a) Lay off the load line A—E, locate the resultant of the total wind-load, and since the direction and point of application of R_1 are known, extend the action line of R_1 to intersect the action line of the resultant wind-load.

Draw the action line of R_2 through the known point (right support) and the intersection of the action lines of R_1 and the wind resultant, thus determining

the direction of R_2. (Three forces in equilibrium meet in a common point.)

Complete the force-polygon to equilibrate the load line A-E (= wind resultant) by drawing through E parallel to the direction of R_2 and through A parallel to R_1 to intersect at F. Then $EF = R_2$ and $FA = R_1$ to the scale of the load line.

(*b*) Choose a convenient pole, such as O, and from the ray-diagram constructed upon the load line, draw a funicular polygon. Since the only point

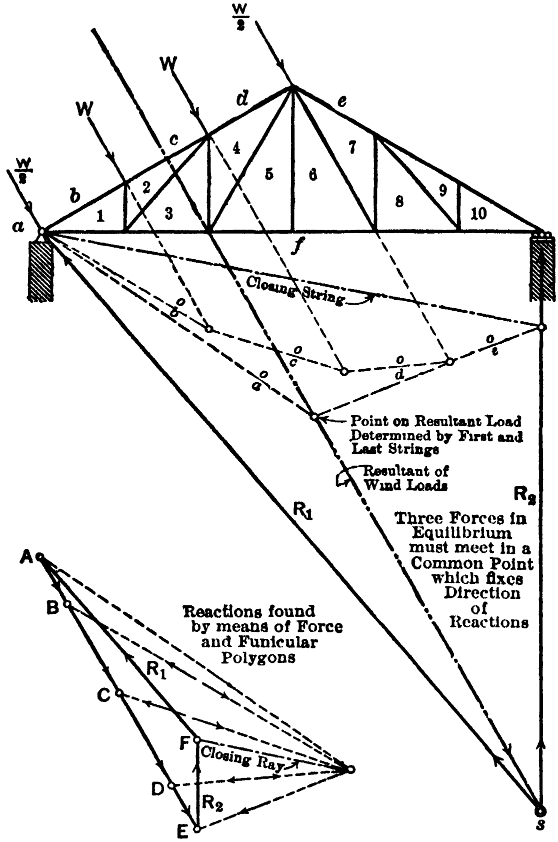


Fig. 7. Wind-load Reactions

on the line of action of R_2 is that point at the right support, string *oe* must pass through this point. Start the funicular polygon with string *oe* drawn through the point of support at the leeward end and let the last string *oa* intersect the known vertical line of action of R_1 where it will. Draw the closing string *of*, which when transposed as a ray to the ray-diagram will

locate point F, the lower end of the vertical line through A, designating R_1. Then, $EF = R_2$ in magnitude and direction.

Case 2. Rollers Under the Leeward End of the Truss. The reactions for this case are determined by the general methods outlined in Case 1.

(a) Draw the load line $A—E$, Fig. 7, and by symmetry locate the resultant of wind-loads. Prolong the action line of the resultant of wind-loads to intersect the known vertical action line of R_2 in s. Connect s and the other known point (the left support) on the action line of R_1, thus determining the direction of R_1. Through point E on the load line draw parallel to R_2 (vertical) and through A draw parallel to R_1, to intersect in point F. Then $EF = R_2$ and $FA = R_1$ in direction and in magnitude to the scale of the load line.

(c) One End of the Truss Supported upon a Planed Base-Plate with a Coefficient of Friction of 0.30. For truss spans of 40 ft to 120 ft supported on

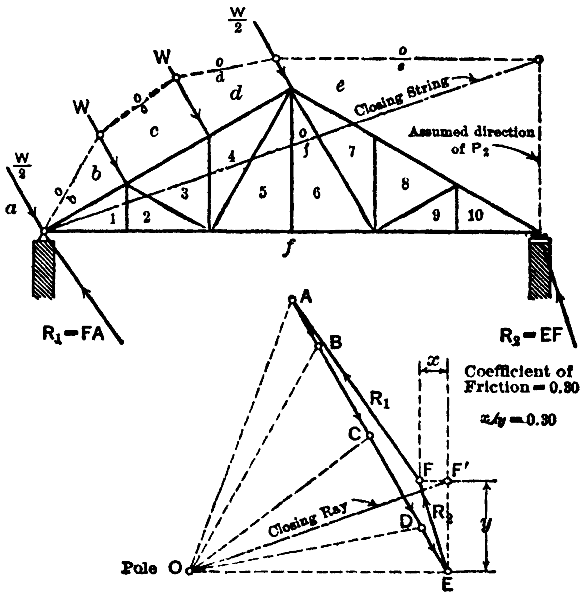


Fig. 8. Wind-load Reactions

masonry walls or piers, the usual detail at the free end of the truss consists of a surfaced base-plate upon which the outstanding legs of the chord or shoe angles rest and over which they are free to move horizontally as the length of the truss changes with deformation and temperature. Such a detail is not without friction on the bearing surfaces, and an amount of horizontal resistance will be offered equal to the frictional resistance of the detail. The coefficient of friction of steel on steel (at rest) is from 0.30 to 0.35.

Lay off the load line $A—E$, Fig. 8, and tentatively assume the free end of the truss to be on frictionless rollers. Determine the reactions as in Case 2 of the preceding article. The first string of the funicular polygon oa must be

drawn through the only known point on the windward reaction, its point of application at the left support. The closing string *of* determines the direction of ray *OF* and locates *F'*, assuming $EF' = R_2$ to be vertical. The reaction R_2, however, is not vertical, unless the expansion detail at the right support is without friction.

If the coefficient of friction for the sliding detail, as specified, be taken at 0.3, lay off $F'F = 0.30\ EF'$, then the reactions are $EF = R_2$ and $FA = R_1$ in the directions shown, and of magnitudes as determined by the scale of the load line.

3. Stresses in Roof-Trusses

The dead loads, snow-loads and wind-loads acting upon a truss are transferred, throughout the members of the truss, to the supports. The stresses developed in the truss through this action tend to shorten some of the members, and to elongate others; and since the materials of which trusses are constructed are not inelastic, the lengths of the members are changed and the form of the truss is distorted. The actual deformations are very small but none the less significant since they indicate the character, and are a direct measure of, the stresses in the members of the truss. There are three kinds of stress which may be developed by the action of the external forces: (*a*) TENSION; (*b*) COMPRESSION; (*c*) SHEAR.

(a) Tension. When the deformation of a member is such that there is a tendency for the particles of the member to be pulled apart or elongated in a direction normal to the surface of cleavage, the member is said to be subjected to TENSILE STRESS. The forces external to and producing tension in a member act in a direction from the center toward the ends of the member; hence, the internal forces, or stresses, act from the ends toward the center.

(b) Compression. When the deformation of a member is such that there is a tendency for the particles of the member to move toward each other, or to be shortened in a direction normal to the surface of cleavage, the member is said to be subjected to COMPRESSIVE STRESS. The forces external to and producing compression in a member act in a direction from the ends toward the center; hence, the internal stresses, or balancing forces, act from the center toward the ends.

(c) Shear. A member is said to be subjected to a SHEARING-STRESS if there is a tendency for the particles of the member to slide past each other. The external loads supported by a truss are generally applied at the panel-points, subjecting the members to longitudinal stresses of tension or compression without producing shear. However, shearing-stresses are developed in the joint details and as a result of bending due to the rigidity of the joints.

While the truss as a whole acts like a beam, the individual members are not subjected to bending stresses from external forces applied at the joints. Unless a member is vertical it is subjected to bending due to its own weight; however, such stresses are relatively small and may, in most cases, be entirely neglected.

General Methods of Determining Stresses in Trusses. The determination of the stresses in the members of any framed structure is based upon the principles of static equilibrium. The structure as a whole must be in equilibrium under the action of the applied loads and the reactions. Any portion of the structure taken separately as a free body must also be in equilibrium under the action of the external forces, or stresses, acting upon the ends of the members cut in separating the portion under consideration from the original

structure. The three fundamental equations of equilibrium which must be satisfied are:

(1) Σ horizontal components of all forces = 0.
(2) Σ vertical components of all forces = 0.
(3) Σ moments of all forces about any point = 0.

The first two equations of static equilibrium involve the RESOLUTION OF FORCES, and they may be solved either algebraically or graphically. The third equation involves the MOMENTS OF FORCES, both external and internal, and it may also be solved either algebraically or graphically. When the reactions have been found, the stresses in the various members may be determined either by Equations (1) and (2) or by Equation (3). There are, therefore, four methods of determining the stresses in the members of a truss:

(*A*) Resolution of forces
  (*a*) Algebraic method
  (*b*) Graphical method
(*B*) Moments of forces
  (*a*) Algebraic method
  (*b*) Graphical method

The choice of method best adapted to the determination of the stresses in any given truss is governed partly by the conditions of the problem and

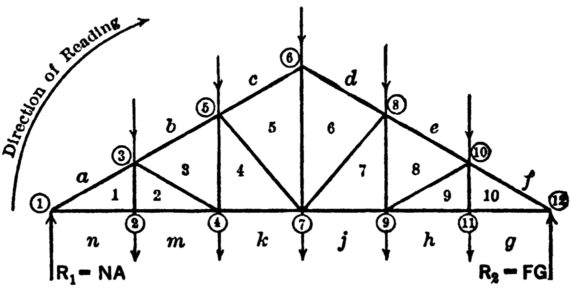


Fig. 9

partly by individual preference. In general, the graphical methods are preferable in the analysis of the stresses in roof-trusses for the following reasons: (1) the inclinations of the members are usually irregular; (2) the external loads are generally assumed to act along fixed lines and at fixed points; (3) graphical methods are self-checking; (4) graphical methods present a concise summary of relative magnitudes. The application of algebraic methods demands, generally, a more skilful command of the principles of statics and a more judicious choice of procedure, than is required in the use of usual graphical methods. By the algebraic method, however, stresses may be determined to any degree of accuracy, whereas, in the graphical method, accuracy depends upon the care used in the execution of the drawing, and the scale adopted.

Notation. It is highly important that some orderly form of notation be adopted whereby reference to the various members of the truss can be easily and clearly made.

The notation that will be used throughout this chapter is illustrated in Fig. 9. External loads are designated by the lower-case letter on each side

of that load, as for example, the load at joint ③ is a–b. Each section of the upper and lower chord is designated by a lower-case letter and a figure, as for example, the upper chord between joints ③ and ⑤ is b–3. Each web-member is designated by two figures, as for example, the member joining panel-points ③ and ④ is member 2–3 as read about joint ④, and 3–2 as read about joint ③. A tensile stress will be recorded by prefixing a plus (+) sign, and a compressive stress by prefixing a minus (−) sign before the quantity representing the magnitude of the stress.

Stresses by Algebraic Resolution. The term ALGEBRAIC, applied to the methods of stress analysis, is used to differentiate it from graphical methods, although the process may be numeric rather than algebraic.

The stresses may be computed algebraically either by (a) the METHOD OF SECTIONS, or by (b) the METHOD OF JOINTS. It is often desirable to compute the stresses in some of the members by method (a) and the stresses in other members by method (b).

In either method the procedure requires the separation of the given truss into two parts, by an imaginary section, either plane or curved. One portion is considered to be removed together with all external forces on that portion, and the members that are cut by the section are loaded at their cut ends by forces equal to the stresses acting in those members. The first two fundamental equations of static equilibrium are then applied to the removed portion of the truss and the stresses found by supplying the necessary numerical values in those equations:

$$\Sigma \text{ horizontal components} = 0 \tag{1}$$

$$\Sigma \text{ vertical components} = 0 \tag{2}$$

In writing the numerical values for the equations, forces acting upward and to the right will be considered positive, and those acting downward and to the left, negative. The unknowns in each case will be assumed to act away from the section. If then the final sign stands plus (+) the assumption is correct, or away from the plane, the stress being tension and the plus (+) sign properly indicating the same. If the final sign is minus (−) the assumption that the stress is acting away from the plane is incorrect; the stress is compression and the minus (−) sign properly indicates the character of stress.

Example 5. Method of Sections. Let it be required to find the stresses in the members of the truss loaded as shown in Fig. 10.

To find the stresses in members a–1 and 1–g:

Assume any plane, such as mm, cutting the members in question, as at (I), then:

$$\Sigma H = 0; \quad +\overline{a\text{–}1}\,(\cos 30°) + \overline{1\text{–}g} = 0 \tag{a}$$

$$\Sigma V = 0; \quad +\overline{a\text{–}1}\,(\sin 30°) + 7\,500 = 0 \tag{b}$$

from (b) $\quad +0.50\,\overline{a\text{–}1} + 7\,500 = 0$

or $\quad \overline{a\text{–}1} = -\,15\,000 \text{ lb}$

from (a) $\quad -15\,000\,(0.866) + \overline{1\text{–}g} = 0$

or $\quad \overline{1\text{–}g} = +\,13\,000 \text{ lb}$

To find the stresses in members 1–2 and 2–g:

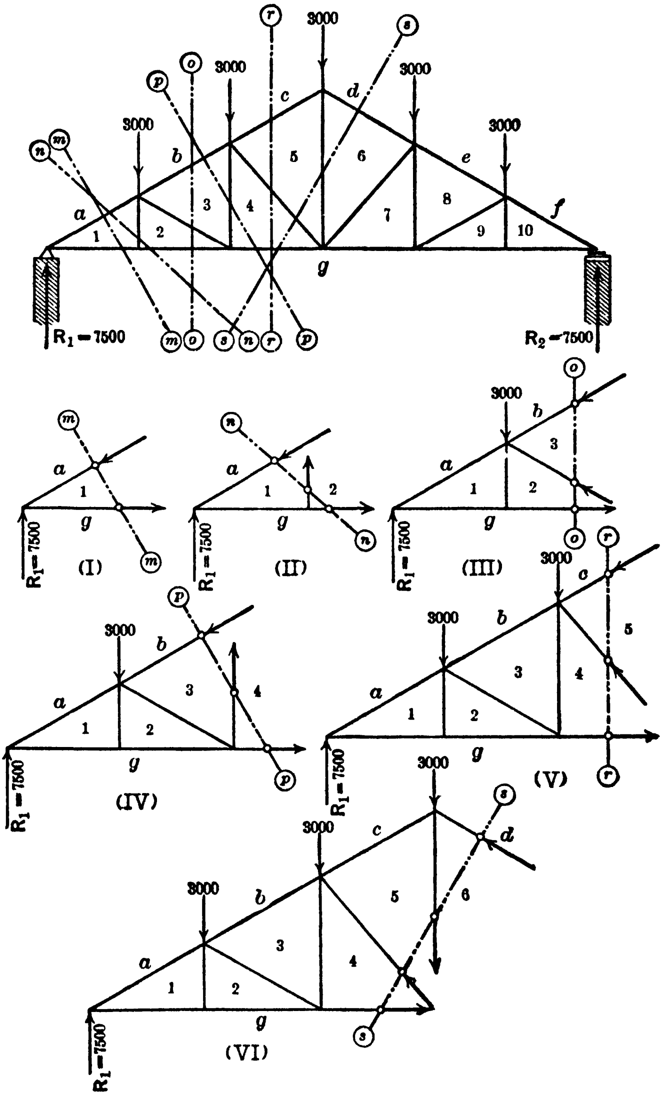


Fig. 10. Method of Sections

Assume any plane, such as $\overline{nn}$, cutting the members in question, as at (II). then:

$$\Sigma H = 0; \quad +\overline{a\text{–}1}\,(\cos 30°) + \overline{2\text{–}g} = 0 \qquad (a)$$

$$\Sigma V = 0; \quad +\overline{a\text{–}1}\,(\sin 30°) + 7\,500 - \overline{1\text{–}2} = 0 \qquad (b)$$

from (*a*) $\quad -15\,000\,(0.866) + \overline{2\text{–}g} = 0$

or $\quad \overline{2\text{–}g} = +\,13\,000$ lb

from (*b*) $\quad -15\,000\,(0.50) + 7\,500 - \overline{1\text{–}2} = 0$

or $\quad \overline{1\text{–}2} = 0$

To find the stresses in members *b*–3 and 3–2:

Assume any plane, such as $\overline{oo}$, cutting the members in question, as at (III), then:

$$\Sigma H = 0; \quad +\overline{b\text{–}3}\,(\cos 30°) + \overline{3\text{–}2}\,(\cos 30°) + \overline{2\text{–}g} = 0 \qquad (a)$$

$$\Sigma V = 0; \quad +\overline{b\text{–}3}\,(\sin 30°) - \overline{3\text{–}2}\,(\sin 30°) + 7\,500 - 3\,000 = 0 \qquad (b)$$

from (*a*) $\quad +\overline{b\text{–}3}\,(0\,866) + \overline{3\text{–}2}\,(0.866) + 13\,000 = 0$

from (*b*) $\quad +\overline{b\text{–}3}\,(0\,50) - \overline{3\text{–}2}\,(0.50) + 4\,500 = 0$

whence $\quad +1.732\,\overline{b\text{–}3} = -\,20\,800$

or $\quad \overline{b\text{–}3} = -\,12\,000$ lb

from (*b*) $\quad +\,6\,000 - \overline{3\text{–}2}\,(0.50) + 4\,500 = 0$

or $\quad \overline{3\text{–}2} = -\,3\,000$ lb

To find the stresses in members 3–4 and 4–*g*:

Assume any plane, such as $\overline{pp}$, cutting the members in question, as at (IV), then:

$$\Sigma H = 0; \quad +\overline{b\text{–}3}\,(\cos 30°) + \overline{4\text{–}g} = 0 \qquad (a)$$

$$\Sigma V = 0; \quad +\overline{b\text{–}3}\,(\sin 30°) + \overline{3\text{–}4} + 4\,500 = 0 \qquad (b)$$

from (*a*) $\quad -12\,000\,(0.866) + \overline{4\text{–}g} = 0$

or $\quad \overline{4\text{–}g} = +\,10\,400$ lb

from (*b*) $\quad -12\,000\,(0.50) + \overline{3\text{–}4} + 4\,500 = 0$

or $\quad \overline{3\text{–}4} = +\,1\,500$ lb

To find the stresses in the members *c*–5 and 5–4·

Assume any plane, such as $\overline{rr}$, cutting the members in question, as at (*V*), then:

$$\Sigma H = 0; \quad +\overline{c\text{–}5}\,(\cos 30°) + \overline{5\text{–}4}\,(\cos 49°) + \overline{4\text{–}g} = 0 \qquad (a)$$

$$\Sigma V = 0; \quad +\overline{c\text{–}5}\,(\sin 30°) - \overline{5\text{–}4}\,(\sin 49°) + 7\,500 - 6\,000 = 0 \qquad (b)$$

from (*a*) $\quad +\overline{c\text{–}5}\,(0.866) + \overline{5\text{–}4}\,(0.655) + 10\,400 = 0$

from (*b*) $\quad +\overline{c\text{–}5}\,(0.50) - \overline{5\text{–}4}\,(0.755) + 1\,500 = 0$

whence $\quad +(1.965)\,\overline{5\text{–}4} + 7\,800 = 0$

or $\quad \overline{5\text{–}4} = -\,3\,960$ lb

from (*b*) $\quad +\overline{c\text{–}5}\,(0.50) + 3\,900\,(0.755) = -\,1\,500$

or $\quad \overline{c\text{–}5} = -\,9\,000$ lb

To find the stress in member 5–6:

Assume any plane such as $\overline{ss}$, cutting the member, as at (VI), then:

$$\Sigma H = 0; \quad +\overline{d\text{–}6}\,(\cos 30°) + \overline{4\text{–}g} + \overline{4\text{–}5}\,(\cos 49°) = 0 \qquad (a)$$

$$\Sigma V = 0; \quad -\overline{d\text{–}6}\,(\sin 30°) - \overline{5\text{–}6} - \overline{4\text{–}5}\,(\sin 49°) + 7\,500 - 9\,000 = 0 \qquad (b)$$

from (a) $\quad +\overline{d\text{–}6}\,(0.866) + 10\,400 - 3\,900\,(0.655) = 0$

from (b) $\quad -\overline{d\text{–}6}\,(0.50) - 5\text{–}6 + 3\,900(0.755) = 1\,500$

whence $\quad \overline{d\text{–}6} = -\,9\,000$ lb

and $\quad \overline{5\text{–}6} = +\,6\,000$ lb

Example 6. Method of Joints. This method differs from the preceding one only in that the portion of the truss which is assumed to be removed by the imaginary section is that portion immediately surrounding a joint.

The two fundamental equations, $\Sigma H = 0$ and $\Sigma V = 0$, are again employed, assuming that the unknown forces replacing the stresses on the cut ends of the members act away from the joint.

Let it be required to find the stresses in the members of the truss loaded as shown in Fig. 11.

To find the stresses in members a–1 and 1–g:

Take the section around joint (1) as at I, and assume the coordinate axes X and Y, then:

$$\Sigma X = 0; \quad +\overline{a\text{–}1}\,(\cos 30°) + \overline{1\text{–}g} = 0 \qquad (a)$$

$$\Sigma Y = 0; \quad +\overline{a\text{–}1}\,(\sin 30°) + 7\,500 = 0 \qquad (b)$$

whence $\quad \overline{a\text{–}1} = -\,15\,000$ lb

and $\quad \overline{1\text{–}g} = +\,13\,000$ lb

To find the stresses in members 1–2 and 2–g:

Take section around joint (2) as at II, and assume the coordinate axes X and Y, then:

$$\Sigma X = 0; \quad -\overline{g\text{–}1} + \overline{2\text{–}g} = 0$$

$$\Sigma Y = 0; \quad -\overline{1\text{–}2} = 0$$

whence $\quad \overline{2\text{–}g} = \overline{g\text{–}1} = +\,13\,000$ lb

and $\quad \overline{1\text{–}2} = 0$

To find the stresses in members b–3 and 3–2:

Take the section around joint (3), as at III, and assume the coordinate axes X and Y, then:

$$\Sigma X = 0; \quad -\overline{a\text{–}1}\,(\cos 30°) + \overline{b\text{–}3}\,(\cos 30°) + \overline{3\text{–}2}\,(\cos 30°) = 0 \qquad (a)$$

$$\Sigma Y = 0, \quad -\overline{a\text{–}1}\,(\sin 30°) + \overline{b\text{–}3}\,(\sin 30°) - \overline{3\text{–}2}\,(\sin 30°) - 3\,000 = 0 \qquad (b)$$

from (a) $\quad -(-15\,000)\cdot(0.866) + \overline{b\text{–}3}\,(0.866) + \overline{3\text{–}2}\,(0.866) = 0$

from (b) $\quad -(-15\,000)\cdot(0.50) + \overline{b\text{–}3}\,(0.50) - \overline{3\text{–}2}\,(0.50) - 3\,000 = 0$

or $\quad +\overline{b\text{–}3}\,(0.866) + \overline{3\text{–}2}\,(0.866) = -\,13\,000$

and $\quad +\overline{b\text{–}3}\,(0.50) - \overline{3\text{–}2}\,(0.50) = -\,4\,500$

whence $\quad \overline{b\text{–}3} = -\,12\,000$ lb

and $\quad \overline{3\text{–}2} = -\,3\,000$ lb

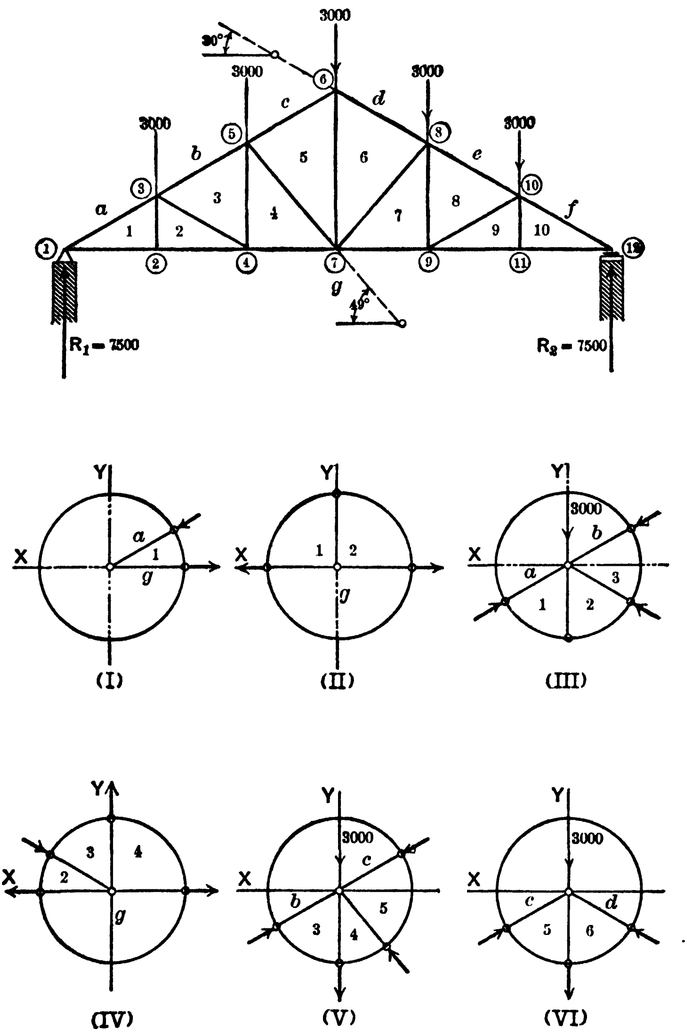


Fig. 11. Method of Joints

To find the stresses in members 3–4 and 4–g:

Take the section around joint ④, as at IV, and assume the coordinate axes X and Y, then:

$$\Sigma X = 0; \quad -\overline{2\text{–}g} - \overline{2\text{–}3}\,(\cos 30°) + \overline{4\text{–}g} = 0 \qquad (a)$$

$$\Sigma Y = 0; \quad +\overline{2\text{–}3}\,(\sin 30°) + \overline{3\text{–}4} = 0 \qquad (b)$$

from (a) $\quad -13\,000 - (-3\,000)\,(0.866) + \overline{4\text{–}g} = 0$

from (b) $\quad -3\,000\,(\sin 30°) + \overline{3\text{–}4} = 0$

whence $\quad \overline{4\text{–}g} = +\ 10\,400$ lb

and $\quad \overline{3\text{–}4} = +\ 1\,500$ lb

To find the stresses in members c–5 and 5–4:

Take the section around joint ⑤, as at V, and assume the coordinate axes X and Y, then:

$$\Sigma X = 0; \quad -\overline{b\text{–}3}\,(\cos 30°) + \overline{c\text{–}5}\,(\cos 30°) + \overline{4\text{–}5}\,(\sin 49°) = 0 \qquad (a)$$

$$\Sigma Y = 0; \quad -\overline{b\text{–}3}\,(\sin 30°) + \overline{c\text{–}5}\,(\sin 30°) - \overline{4\text{–}5}\,(\sin 49°) - \overline{3\text{–}4} - 3\,000 = 0 \qquad (b)$$

from (a) $\quad -(-12\,000)\cdot(0.866) + \overline{c\text{–}5}\,(0.866) + \overline{4\text{–}5}\,(0.655) = 0$

from (b) $\quad -(-12\,000)\cdot(0.50) + \overline{c\text{–}5}\,(0.50) - \overline{4\text{–}5}\,(0.755) - 1\,500 - 3\,000 = 0$

whence $\quad +\overline{c\text{–}5}\,(0.866) + \overline{4\text{–}5}\,(0.655) = -\ 10\,400$

$\quad +\overline{c\text{–}5}\,(0.50) - \overline{4\text{–}5}\,(0.755) = -\ 1\,500$

or $\quad +(1.965)\,\overline{4\text{–}5} = -\ 7\,800$

then $\quad \overline{4\text{–}5} = -\ 3\,960$ lb

and $\quad \overline{c\text{–}5} = -\ 9\,000$ lb

To find the stress in member 5–6:

Take the section around joint ⑥, as at VI, and assume the coordinate axes X and Y, then,

$$\Sigma X = 0; \quad -\overline{c\text{–}5}\,(\cos 30°) + \overline{d\text{–}6}\,(\cos 30°) = 0 \qquad (a)$$

$$\Sigma Y = 0; \quad -\overline{c\text{–}5}\,(\sin 30°) - \overline{d\text{–}6}\,(\sin 30°) - \overline{5\text{–}6} - 3\,000 = 0 \qquad (b)$$

from (a) $\quad \overline{d\text{–}6} = \overline{c\text{–}5} = -\ 9\,000$ lb

from (b) $\quad -(-9\,000)\cdot(0.50) - (-9\,000)\cdot(0.50) - \overline{5\text{–}6} - 3\,000 = 0$

or $\quad \overline{5\text{–}6} = +\ 6\,000$ lb

Stresses by Graphical Resolution. The method of graphical resolution, which is the most convenient for determining the stresses in the members of roof-trusses, consists of the application of the principles of the force-polygon to the external forces and stresses acting at each panel-point of the truss. Since the external forces and the stresses in the members at each joint are in equilibrium, the force-polygon must close and the forces must form a continuous system around the polygon. Since the action lines of both external forces and stresses are known, it is evident that, if a sufficient number of forces at a given panel-point are completely known to permit a closed polygon

to be drawn, the magnitudes and directions of the unknown stresses can be determined.

Let it be required to determine the stresses in the members of the truss, loaded as shown in Fig. 12.

Joint ①. There are three forces acting at joint ①, shown at (I), of which one, the reaction R_1, is shown in magnitude as well as direction. The force-polygon at (I) is constructed by laying off $GA = R_1$ at some convenient

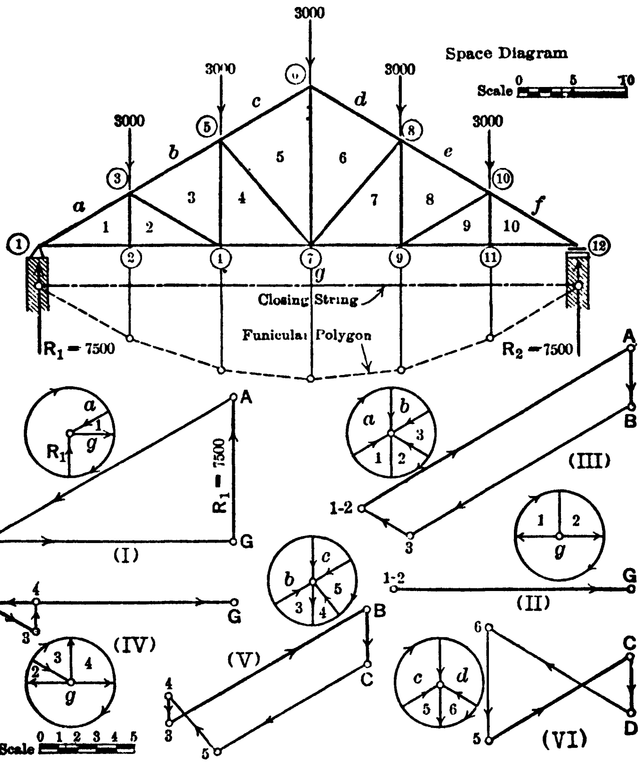


Fig. 12. Method of Graphical Resolution

scale, acting upward and parallel to the known action line of the reaction. From the extremities G and A of the vector GA, draw G–1 parallel to g–1 and A–1 parallel to a–1 to intersect in point 1. Then G–1 and A–1 represent, to the scale used to lay off GA, the magnitudes of the stresses in members g–1 and a–1, respectively. These forces, to be in equilibrium, must act around the polygon in consecutive order, thus determining the directions of the various stresses with respect to the joint. Since the direction of GA is up, as indicated by the arrow, A–1 must act in the direction from A toward 1 and 1–G must act from 1 toward G to close the polygon. Applying these direc-

tions to joint ① it is seen that a–1 acts toward the joint and is, therefore, compression, while 1–g acts away from the joint and is, therefore, tension.

The forces at joint ② are considered next, since at that joint there are but two unknowns, while at all other joints there are three or more unknowns. At joint ②, shown at (II), the force g–1 is equal and opposite to 1–g as found at the preceding joint, and as there are no external forces, it is evident by inspection that the stress in member 1–2 must be zero, since $\Sigma V = 0$. The same conclusion is evident by considering $\Sigma H = 0$, since g–1 has the same action line as 2–g and there is no other force which has a possible horizontal component at this joint, therefore, the force-polygon consists of two equal and opposite forces having the same action line. The force-polygon is read, G–1, $\overline{1-2} = 0$ and 2–G = G–1.

The forces acting at joint ③ are next considered. At this joint 1–A, equal and opposite to A–1, is known and $\overline{2-1} = 0$ is known from the solutions of joints ① and ②, respectively, and the panel load A–B is also known

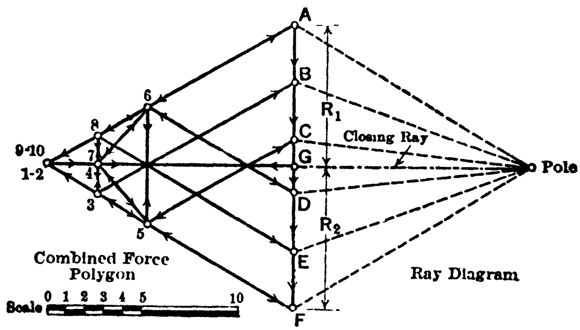


Fig. 13. Stress-Diagram

in magnitude and direction. Lay off 1–A in its proper magnitude and direction, draw A–B in magnitude and direction and since $\overline{2-1} = 0$ points 1 and 2 are coincident. Through point B draw B–3 parallel to the direction of the action line of member b–3 and at point 2 draw 2–3 parallel to the action line of member 3–2 to intersect B–3 in point 3. The directions of the stresses are found from the known directions of 1–A and A–B by following consecutively around the polygon as shown by the arrows. It is apparent, then, that the stresses 1–a, b–3, and 3–2 all act toward the joint and are, therefore, all compressive stresses.

There are four forces acting at joint ④, two of which ($\overline{g-2}$ and $\overline{2-3}$) are already known. The force-polygon is shown at IV. Lay off G–2 and $\overline{2-3}$ in magnitude and direction; draw $\overline{3-4}$ and $\overline{G-4}$ parallel to members 3–4 and 4–g, respectively, to complete the polygon. Applying the forces of this polygon to joint ④ it is seen that $\overline{g-2}$, $\overline{3-4}$, and $\overline{4-g}$ act away from the joint, indicating tension in those members, while $\overline{2-3}$ acts toward the joint, and therefore the stress in this member is compression.

Of the five forces acting at joint ⑤, three are known. The force-polygon, shown at (V), may be started at point 3, drawing 3–B in its proper direction

and magnitude, then B–C, and from the polygon for joint (IV) the magnitude of $\overline{4-3}$ is obtained. The stress in member 4–3 being tension, as determined for joint ④, will act away from joint ⑤ or from 4 toward 3 as shown in the force-polygon at (V). Through point 4 draw 4–5 parallel to member $\overline{4-5}$ and through C draw C–5 parallel to member $\overline{c-5}$ to intersect 4–5 in point 5, thus closing the polygon. The stresses $\overline{3-b}$, $\overline{c-5}$, and $\overline{5-4}$ act toward the joint, indicating compression, and stress 4–3 acts away from the joint indicating tension.

The forces at joint ⑥ are shown at VI. The unknown forces $\overline{d-6}$ and $\overline{6-5}$ are found by constructing the force-polygon beginning with point 5 and drawing the known forces 5–C and C–D. The stress in member $\overline{d-6}$ is compression since D–6 in the force-polygon acts upward or toward the joint, while the stress in member $\overline{6-5}$ is tension since the force 6–5 acts downward, or away from the joint.

The Stress-Diagram. In constructing the force-polygons for the separate joints, certain forces in one polygon are necessarily repeated in subsequent polygons. The separate polygons may be grouped together to avoid repetition, as shown in Fig. 13, and the result of the combined series of joint polygons is known as a stress-diagram. Considering the separate polygons, it is seen that the stress in any member acts in one direction in one polygon and in the opposite direction in the next polygon. When the polygons are combined, the line representing the stress in a given member will, therefore, have two arrows pointing in opposite directions.

If the external forces are laid off on the load line in the order determined by reading around the structure in a clockwise direction, the true direction of the arrows for each stress, in the stress-diagram, will be given by following the stress-diagram in the order fixed by reading around the joint in question, also in a clockwise direction. Thus for joint ⑤, Fig. 12, the character of stress from the diagram, Fig. 13, is:

b–c, downward
c–5, downward to the left, toward joint, compression (–)
5–4, upward to the left, toward joint, compression (–)
4–3, downward, away from joint, tension (+)
3–b, upward to the right, toward joint, compression (–)

Graphical Resolution, Loads on Upper and Lower Chords. In many buildings, ceiling-loads and other miscellaneous loads are supported at the lower chord panel-points, while the upper chord panel-points support the roof-construction in the usual manner. The stresses resulting from upper and lower chord loading may be found by:

(1) Drawing separate stress-diagrams for each system of loads.
(2) Drawing a combined-load stress-diagram.

Example 1. Separate Stress-Diagrams. Let it be required to determine the stresses in the truss, loaded with both upper and lower chord loads, as shown in Fig. 14. The stress-diagram for the upper chord loads is shown at (b). The load line A to F is laid off to a convenient scale, reading around the truss in a clockwise direction. The reactions for the upper chord loads are equal and each equal to one-half of the load line, $R_2 = FG$ and $R_1 = SA$, since the truss and loading are symmetrical. Draw A–1 and S–1 parallel to members a–1 and 1–s, respectively, and locate, at their intersection, point 1; draw 1–2 and P–2 to locate point 2. Since S and P are coincident P–2 = S–1 and $\overline{1-2} = 0$; draw 2–3 and B–3 parallel to their respective members to locate point 3, and continue this process until the stress-diagram is completed.

The stress-diagram for the lower chord loads is shown at (*c*). The load line *G–S* is laid off beginning at *g* in the space-diagram, and reading around

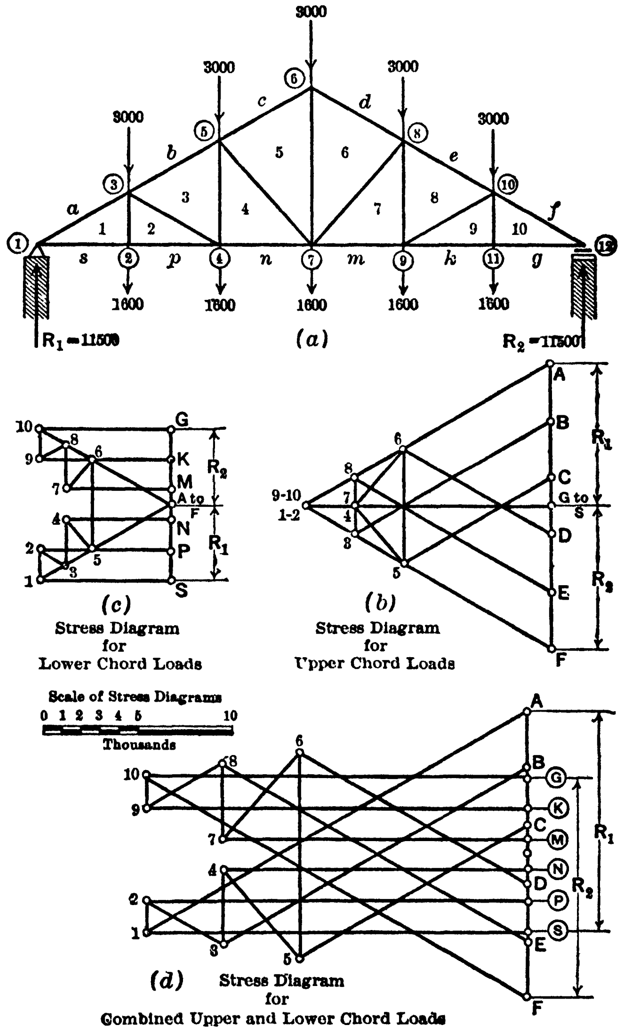


Fig. 14. Separate Stress-diagrams

the truss clockwise, and $SA = R_1 = FG = R_2$. Draw A–1 and S–1 parallel to their respective members, in the truss, to locate point 1; draw 1–2 and

P–2 parallel to members 1–2 and 2–p to locate point 2, noting that for lower chord loads member $\overline{1-2}$ is stressed in tension an amount equal to the panel-load applied at its lower end. The usual process of constructing the stress-diagram is continued until the diagram is completed, or closed.

The stress in any member of the truss for combined loading may be found by the direct addition of the value of the stress due to upper chord loads and the stress due to lower chord loads.

Example 2. Combined Stress-Diagram. The combined stress-diagram for both upper and lower chord loads is shown at (d), Fig 14. In laying off the combined load line AF, FG, GS, SA, it is necessary to find first the reactions FG and SA, since $FG = R_2$ and $SA = R_1$ are external loads and FG must be introduced in order, between loads EF and GK. In this case, where the truss and the loading are symmetrical, the reactions are equal, each being equal to one-half of the total load, or ½ of 23 000 lb = 11 500 lb. The load line is then laid off, from A to F, then $FG = R_2$ is laid off acting upward; G to S represents the ceiling-loads acting down, and $SA = R_1$, acting upward to close the load-line polygon. The stress-diagram is then drawn upon the combined load line in the usual manner. The magnitude of each stress vector in the stress-diagram at (d) is equal to the sum of the stress vectors, for the same member, as given in diagrams (b) and (c).

Method of Computing Stresses by Algebraic Moments. The stresses in the members of trusses may be conveniently found by algebraic moments, although this method is not generally as simple as the method of sections or the method of joints. To obtain the stress in any given member, a section is taken such that it will cut the member whose stress is desired; and if possible, the section is so taken that the other members cut intersect in a common point, which point is taken as the center of moments. The sign of the moment and also of the resulting stress is determined by assuming the unknown force to act away from the section, as in the method of sections.

For any section in equilibrium $\Sigma M = 0$, or the sum of the moments of all forces external to, and including the forces placed on the ends of the members cut by the section must be equal to zero with respect to any center of moments. Clockwise moments are to be considered positive (+) and counter-clockwise moments negative (−). Let it be required to find the stresses in the members of the truss loaded as shown in Fig. 15.

To determine the stress in member a–1;

Assume plane mm cutting members a–1 and 1–g, and take the center of moments at joint ②:

$$\Sigma M = 0; \quad +R_1 \times 8 + \overline{a-1} \times 4 = 0$$

or

$$+7\,500 \times 8 + \overline{a-1} \times 4 = 0$$

whence

$$a-1 = -\ 15\,000 \text{ lb}$$

To determine the stress in member 1–g;

Assume plane mm as before, and take the center of moments at joint ③:

$$\Sigma M = 0; \quad +R_1 \times 8 - \overline{1-g} \times 4.62 = 0$$

or

$$+7\,500 \times 8 - \overline{1-g} \times 4.62 = 0$$

whence

$$\overline{1-g} = +\ 13\,000 \text{ lb}$$

To determine the stress in member 2–g:

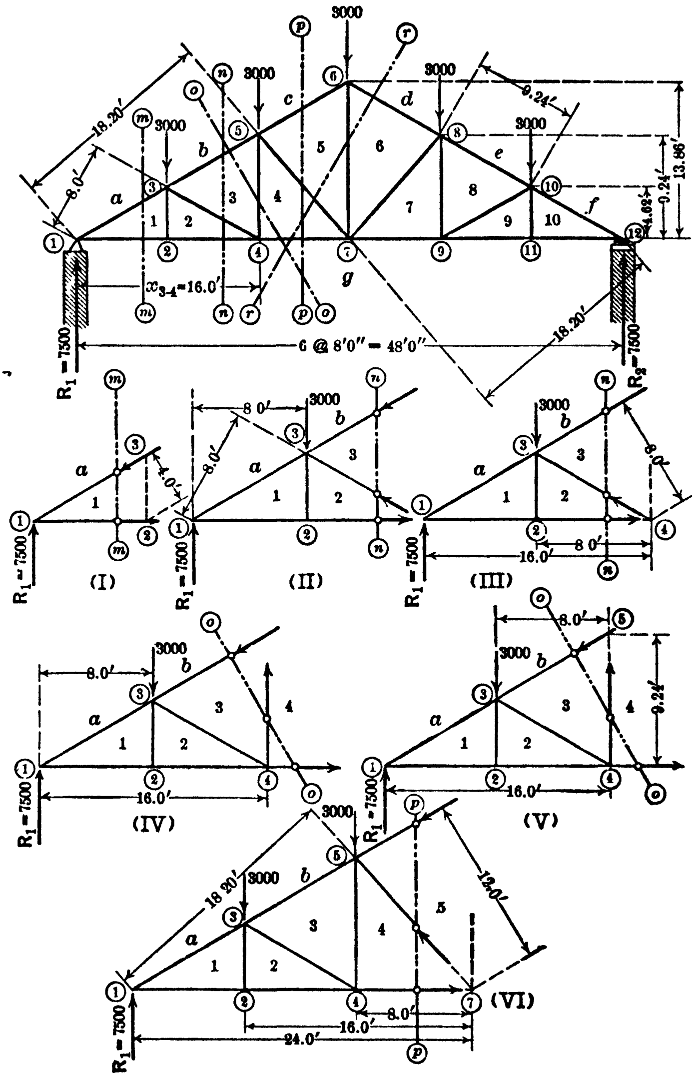


Fig. 15. Method of Algebraic Moments

Assume plane $\overline{nn}$ cutting members $\overline{b\text{–}3}$, $\overline{3\text{–}2}$, and $\overline{2\text{–}g}$, and take the center of moments at joint ③:

$$\Sigma M = 0; \quad +R_1 \times 8 - \overline{2\text{–}g} \times 4.62 = 0$$

or $$+7\,500 \times 8 - \overline{2\text{–}g} \times 4.62 = 0$$

Whence $$\overline{2\text{–}g} = +\ 13\,000 \text{ lb}$$

To determine the stress in member $\overline{2\text{–}3}$:

Assume plane $\overline{nn}$, as before, and take the center of moments at joint ①;

$$\Sigma M = 0; \quad +3\,000 \times 8 + \overline{2\text{–}3} \times 8 = 0$$

or $$\overline{2\text{–}3} = -\ 3\,000 \text{ lb}$$

To determine the stress in member b–3:

Assume plane $\overline{nn}$ as before, and take the center of moments at joint ④;

$$\Sigma M = 0; \quad +R_1 \times 16 - 3\,000 \times 8 + \overline{b\text{–}3} \times 8 = 0$$

or $$+7\,500 \times 16 - 3\,000 \times 8 + \overline{b\text{–}3} \times 8 = 0$$

whence $$\overline{b\text{–}3} = -\ 12\,000 \text{ lb}$$

To determine the stress in member 3–4:

Assume plane $\overline{oo}$, cutting members $\overline{b\text{–}3}$, $\overline{3\text{–}4}$, and $\overline{4\text{–}g}$, and take the center of moments at joint ①:

$$\Sigma M = 0; \quad +3\,000 \times 8 - \overline{3\text{–}4} \times 16 = 0$$

whence $$\overline{3\text{–}4} = +\ 1\,500 \text{ lb}$$

To determine the stress in member 4–g:

Assume plane $\overline{oo}$, as before, and take the center of moments at joint ⑥:

$$\Sigma M = 0; \quad +R_1 \times 16 - 3\,000 \times 8 - \overline{4\text{–}g} \times 9.24 = 0$$

or $$+7\,500 \times 16 - 3\,000 \times 8 - \overline{4\text{–}g} \times 9.24 = 0$$

whence $$\overline{4\text{–}g} = +\ 10\,400 \text{ lb}$$

To determine the stress in member c–5:

Assume plane $\overline{pp}$ cutting member $\overline{c\text{–}5}$, $\overline{4\text{–}5}$ and $\overline{4\text{–}g}$, and take the center of moments at joint ⑦:

$$\Sigma M = 0; \quad +R_1 \times 24 - 3\,000 \times 16 - 3\,000 \times 8 + \overline{c\text{–}5} \times 12 = 0$$

or $$\overline{c\text{–}5} = -9\,000 \text{ lb}$$

To determine the stress in member 4–5:

Assume plane $\overline{pp}$, as before, and take the center of moments at joint ①:

$$\Sigma M = 0; \quad +3\,000 \times 8 + 3\,000 \times 16 + \overline{4\text{–}5} \times 18.2 = 0$$

$$\overline{4\text{–}5} = -\ 3\,960 \text{ lb}$$

To determine the stress in member 5–6:

Assume plane $\overline{rr}$, cutting members $\overline{d\text{–}6}$, $\overline{5\text{–}6}$, $\overline{5\text{–}4}$ and $\overline{4\text{–}g}$ and take the center of moments at the right support:

$$\Sigma M = 0; \quad +R_1 \times 48 - 3\,000\,(40 + 32 + 24) - \overline{4\text{–}5} \times 18.2 - \overline{5\text{–}6} \times 24 = 0$$

or $$\overline{5\text{–}6} = +\ 6\,000 \text{ lb}$$

Stresses by Graphical Moments. This method is similar to the method of algebraic moments, the difference being that the external moments are

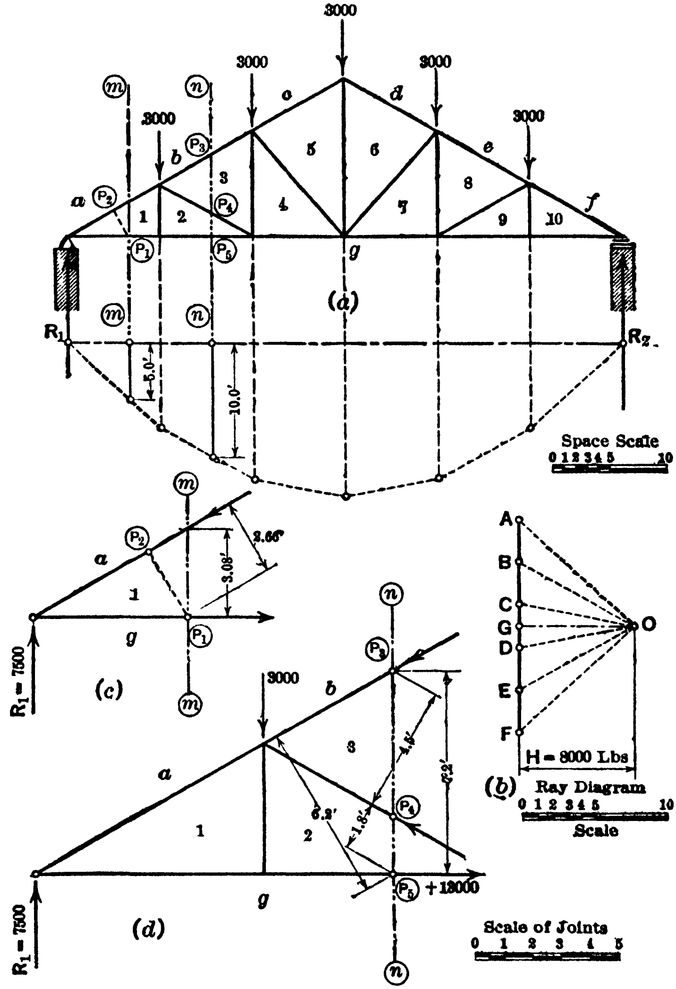


Fig. 16. Method of Graphical Moments

determined graphically. The method of graphical resolution by joints is a more desirable method, hence the following explanation will be brief:

Example. Let it be required to find the stresses in the members of the truss, loaded as shown in Fig. 16, by the method of graphical moments.

The load line is laid off at (*b*) and with a selected pole distance H, the ray-diagram and funicular polygons are drawn. The funicular polygon, together with the closing string, is a moment-diagram, such that its vertical intercept, in feet to the space scale, multiplied by the pole distance H, in pounds, to the load-line scale, gives the moment at that section in pounds-feet.

To find the stress in member a–1:

Assume plane $\overline{mm}$. The moment of the external forces about this section is $M_m = 5 \times 8\,000 = +\,40\,000$ lb-ft, and this moment must be balanced by the moments of the internal forces cut by the section.

Take the center of moments at p_1, and scale the perpendicular distance, or moment arm of a–1, which is $p_1p_2 = 2.66$ ft, then,

$$\Sigma M = 0; \quad + \overline{a\text{–}1}\,(2.66) + 40\,000 = 0$$

or

$$\overline{a\text{–}1} = -\ 15\,000 \text{ lb}$$

To find the stress in member 1–g:

Assume plane $\overline{mm}$, as before, and take the center of moments at the intersection of $\overline{mm}$ and $\overline{a\text{–}1}$;

$$\Sigma M = 0; \quad - \overline{1\text{–}g}\,(3.08) + 40\,000 = 0$$

or

$$\overline{1\text{–}g} = +\ 13\,000 \text{ lb}$$

To find the stresses in members b–3 and 3–2:

Assume plane $\overline{nn}$ cutting members $\overline{b\text{–}3}$, $\overline{3\text{–}2}$, and $\overline{2\text{–}g}$. The stress in $\overline{2\text{–}g}$ = stress in $\overline{1\text{–}g} = +\ 13\,000$ lb.

The moment of the external forces about this section is

$$M_n = 10 \times 8\,000 = +\ 80\,000 \text{ lb-ft}$$

Take the center of moments at p_3:

$$\Sigma M = 0; \quad -13\,000 \times (7.2) - \overline{3\text{–}2} \times (4.5) + 80\,000 = 0$$

or

$$\overline{3\text{–}2} = -\ 3\,000 \text{ lb}$$

Take the center of moments at p_6:

$$\Sigma M = 0; \quad + \overline{b\text{–}3}\,(8.0) + \overline{3\text{–}2}\,(1.8) + 80\,000 = 0$$

or

$$+ \overline{b\text{–}3}\,(8.0) + (-5\,400) + 80\,000 = 0$$

whence

$$\overline{b\text{–}3} = -\ 12\,000 \text{ lb}$$

The remainder of the stresses can be found in a similar manner. The accuracy of this method depends entirely upon careful drawing and exact reading of scaled values.

Wind-Load Stresses. Graphical Resolution. The following examples will illustrate the procedure, in the determination of the wind-load and combined wind- and dead-load stresses, in trusses, by the method of graphical resolution. The reactions will be found in each case in the manner which has been explained in Article 2. The stresses throughout a given truss, which is subjected to any load or system of loads, having a horizontal component, will depend upon the assumed distribution of the horizontal components of the truss reactions. The four following cases will be considered:

(1) Both ends of the truss fixed—reactions parallel.
(2) Leeward end of the truss on rollers.
(3) Both ends of the truss fixed—horizontal components of reactions equal.
(4) Windward end of the truss on rollers.

Case 1. Both Ends of Truss Fixed—Reactions Parallel. Let it be required to find the stresses, due to wind-loads only, in the members of the truss loaded as shown in Fig. 17. The reactions are found by means of the force and funicular polygons as shown at (*b*) and (*a*) respectively.

The stress-diagram (*c*) is then constructed upon the load-reaction polygon *AE–EF–FA*, beginning at joint ① and continuing in the order indicated by the joint numbering.

The numerical values of the stress in any member can be found by scaling the vector in the stress-diagram corresponding to that member of the truss, using the given load-line scale.

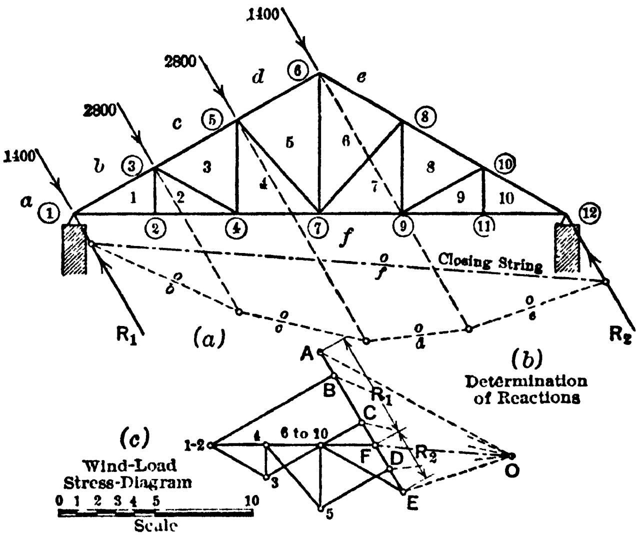


Fig. 17. Wind-load Stress-diagram. Reactions Parallel

The character of the stress in any member can be found by reading clockwise around the joint at either end of the member and tracing the same sequence of notation on the stress-diagram. As an example, consider joint ⑤ and beginning at *c*, read clockwise: *c–d*, *d*–5, 5–4, 4–3, and 3–*c*; tracing this sequence on the stress-diagram the directions of the action lines of the balanced force system with respect to joint ⑤ indicate the following character of stress:

c–d, toward the joint = external load
d–5, toward the joint = compression (−)
5–4, toward the joint = compression (−)
4–3, away from joint = tension (+)
3–*c*, toward the joint = compression (−)

Case 2. Leeward End of Truss on Rollers. Consider the same truss and loading as in the preceding case, but instead of assuming the truss to be fixed at both supports, let the leeward reaction be a roller-bearing, without friction, as shown in Fig. 18.

The right reaction is known in direction, line of action, and point of application, the magnitude being the only unknown element. The left reaction is known only in point of application. Lay off the load line AE and through E draw a vertical line of indefinite length, to represent the direction of the line of action of R_2, the magnitude of which is at present unknown. Select a convenient pole and construct the ray (or force) and funicular polygons.

Since the point of support is the only point known on the action line of the left reaction, the funicular polygon must be started at this point with the string parallel to OA, which is a component of load ab and $R_1 = fa$, drawn from the action line of ab to the action line of fa. The string oa is, therefore, of zero length, and string ob is the first string of significant length, drawn from the point of left support, crossing the space b and intersecting the action line

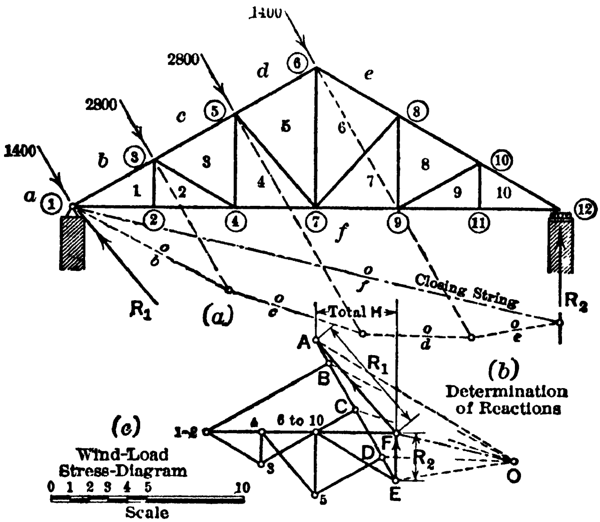


Fig. 18. Wind-load Stress-diagram. Leeward Reaction on Rollers

of load bc. The closing string of, being a component of both $ef = R_2$ and $fa = R_1$, is drawn from the beginning point on R_1 to the intersection of oe and the known action line of R_2, closing the funicular polygon. Ray OF drawn parallel to of will locate point F on the vertical, or known direction EF, drawn through E. Then $EF = R_2$ in magnitude and direction, and FA, closing the force-polygon, gives the magnitude and direction of R_1. These reactions might have been found by tentatively assuming them parallel and parallel to the wind resultant, as in Fig. 17 (c), thereby determining the unalterable values of the vertical components. Then, since the leeward support is on rollers, the total horizontal component must be resisted by the windward support.

The stress-diagram is constructed upon the load-reaction force-polygon as before to determine the stresses in all the members of the truss. It should be noted that the stress-diagram is a composite force-polygon and that when accurately and correctly drawn it will close. Thus, in Fig. 18 (c) the stress-diagram which is started at joint ① in the order a–b, b–1, 1–f, f–a, is contin-

ued joint by joint in the order of joint numbering. Since there are no loads on the right half of the truss e–6, e–8 and e–10 will be identical in the stress-diagram, whence 6–7, 7–8, 8–9 and 9–10 are zero stress and 7–f, 9–f and 10–f will also be identical. In order for the stress-diagram to close, therefore, point 6, in the stress-diagram as determined by the position of stress 5–6, must also lie at the intersection of E–10 and F–10.

Case 3. Both Ends of Truss Fixed—Horizontal Components of Reactions Equal. When the horizontal component of the wind resultant is large and the supports equally elastic, the assumption of an equal division of horizontal reaction, to each support, closely approximates actual conditions. The truss in Fig. 19 is loaded with uniform dead load at the upper chord panel-point, a miscellaneous load P = 5 000 at joint 4 on the lower chord, and with wind-load on the left slope. Let it be required to find the reactions, and stresses in the members, of the truss for the assumed condition of fixed reactions with equal horizontal components. Replace each system of loads by its resultant, reducing the external loads to a system of three forces, ΣW = resultant of wind-loads, ΣU = resultant of upper chord loads, and P = load at joint 4. Draw the load line at (b) for this system of forces, and with any pole O, construct the ray and funicular polygons, assuming tentatively any convenient distribution of horizontal reaction in order to proceed with the determination of the true vertical components of the reactions. In this example it is tentatively assumed that the leeward support is on rollers (R_2 vertical).

The three loads are taken in the order W, U, and P, which may be designated A_1–E_1, E_1–M_1, and M_1N_1, as shown at (b). The closing string of the funicular polygon ox_1 transferred to the ray polygon locates X_1 on a vertical through N_1; then N_1X_1 = the vertical component of R_2. The total horizontal component, H, is divided into equal parts: $X_1X = H/2$; then $N_1X = R_2$ and $XA_1 = R_1$ in magnitudes and directions.

The reactions having been found for the assumed conditions, the load-reaction force-polygon may be drawn as at (d). At each upper chord panel-point the dead and wind-load may be combined as shown at (c) to facilitate the drawing of the load line. All external forces are laid off in regular order starting at joint 1 and reading clockwise around the truss: AE = total load on left half; EH = total load on right half; $HM = R_2$ is drawn parallel and equal to N_1X as found at (b); KN = the load of joint 4; and NA is drawn parallel and equal to $XA_1 = R_1$ as found at (b), to close the load-reaction polygon.

Beginning at joint 1 the stress-diagram is drawn in the usual manner, closing at point 10 and checked by the known directions of G–10 and K–10.

Case 4. Windward End of Truss on Rollers. Since the wind may act from either direction, it is necessary to find the stresses in the members of a symmetrical truss when the rollers are at the leeward support and also when the rollers are at the windward support. When the truss is unsymmetrical with rollers at one support, it is necessary to consider (a) wind-load on the right, and (b) wind-load on the left.

Let it be required to determine the reactions for, and stresses in the members of, the truss loaded as shown in Fig. 20.

The external loads are reduced, by considering the resultant of each system, to three forces: ΣV = resultant of upper and lower chord loads, acting along the center line of the truss, since both the truss and loading are symmetrical about the center line; ΣW_N = resultant of the wind-loads normal to roof slope; and ΣW_H = resultant of the wind-loads on the vertical end of the structure. In this example the vertical components of the reactions

are found by tentatively assuming the reactions to be parallel and parallel to the total resultant of all external loads; then ZT (vertical) $= R_2$ and $TX = R_1$.

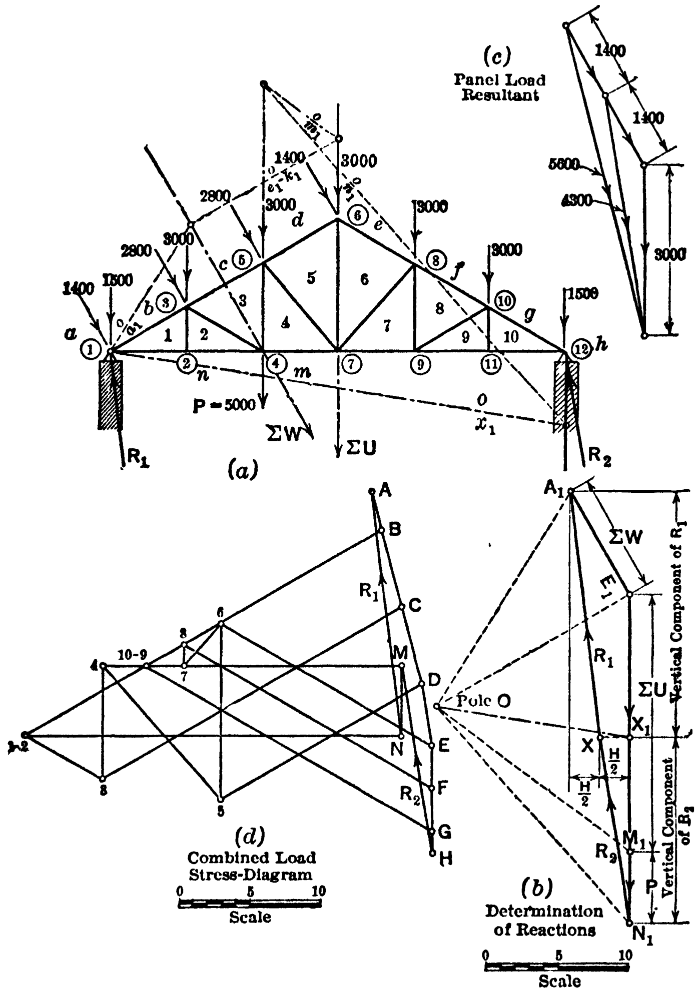


Fig. 19. Wind-load Stress-diagram. Horizontal Components of Reactions Equal

The load-reaction force-polygon may be drawn, as shown at (*d*), after the reactions have been determined. The diagram at (*c*) gives the direction and magnitude of combined wind and dead loads for the upper chord panel-points

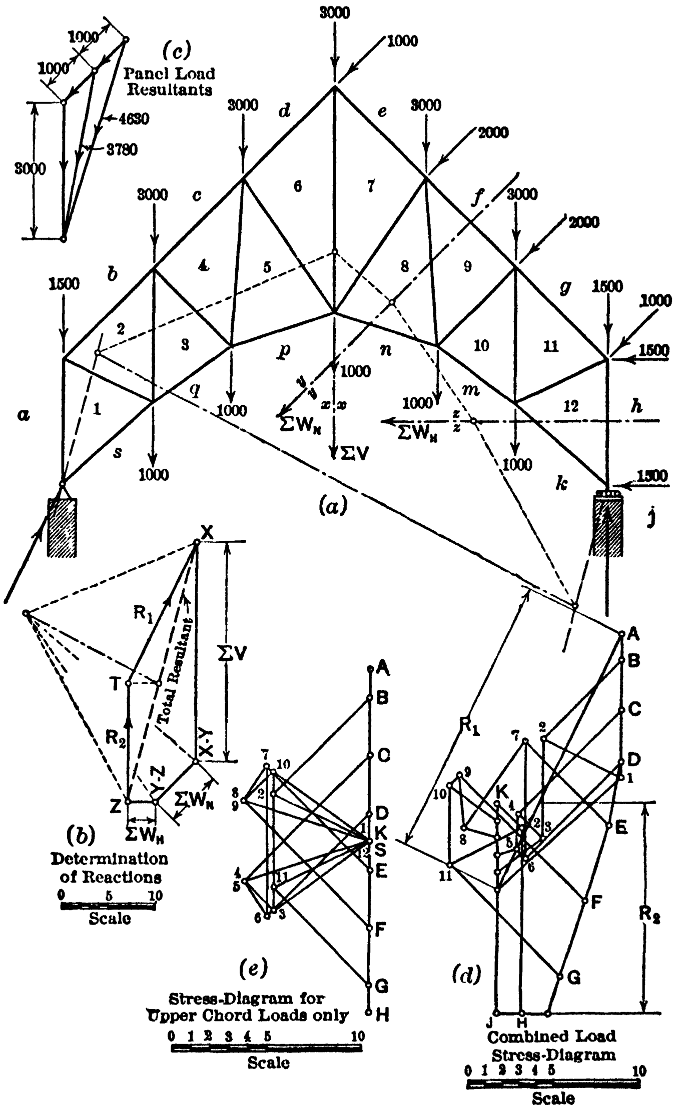


Fig. 20. Stress-diagrams

on the right half of the truss. The load-reaction force-polygon is drawn in the usual manner reading clockwise around the truss, starting at a; R_2 being included in its proper sequence as JK, then the ceiling-loads KM, MN,—RS, and finally SA equal and parallel to $TX = R_1$ closes the force-diagram.

The stress-diagram may then be drawn, beginning at the left support and drawing A–1 (vertical) parallel to a–1, then S–1 parallel to s–1 to locate, at their intersection, point 1. The process is continued, as explained in preceding examples, until the stress-diagram closes at point 12 checking the known directions of the stresses in members h–12, 11–12 and k–12.

In all cases the character of the stress in any member is determined by the direction of the action of the stress with respect to the joint, and this direction is given by reading in sequence the closed force-polygon for the joint, as given in the stress-diagram, and taken in the order of the notation around the joint in question, reading in the same direction as was used in laying off the load line (generally clockwise).

4. Stresses in Cantilever and Unsymmetrical Trusses

A cantilever truss is one which is supported at one end, the other end being entirely free. Cantilever trusses are often used to support the roof-construction over receiving and shipping platforms, over driveways and entrances to

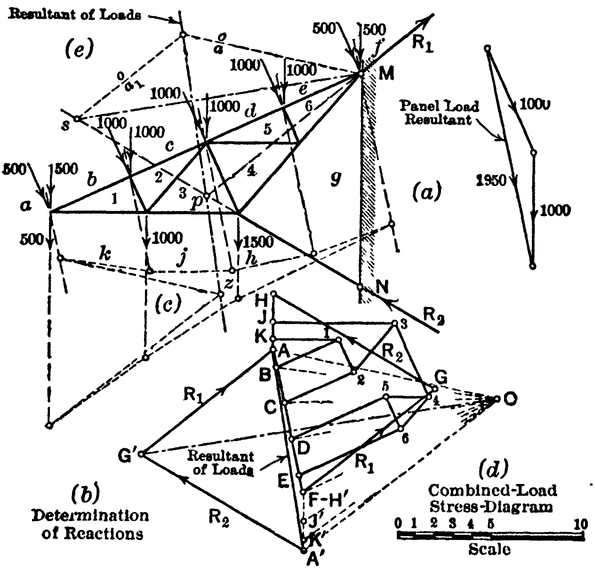


Fig. 21. Cantilever Truss

buildings. Such trusses may be supported by being fastened to the walls of the buildings or directly to the building columns.

Example 1. Let it be required to find the reactions for, and the stresses in the members of, the cantilever truss loaded as shown in Fig. 21.

The points of application of the reactions M and N are known, and the direction of R_2 at point N must be along member g–h since any stress in g–h must be counteracted by an external force parallel to g–h, there being no other member to neutralize another component.

The line of action of R_2 is therefore fixed, and the line of action of R_1 can be found by applying the principle that three forces in equilibrium must intersect in a common point. The three forces in this case are R_1, R_2, and the resultant of all loads.

The resultant of all loads may be found by drawing the force and funicular polygons, as shown at (*b*) and (*c*) respectively. Beginning at A, Fig. 21 (*b*), lay off the load line; A to F represents the combined dead and wind-loads at the upper chord panel-points; then H to A_1 (shown as a broken line) represents the ceiling-loads, not in their proper position, since the reactions FG and GH (to be found) must be introduced between F and H.

With a convenient pole-point, such as O, construct the ray-diagram and then the funicular polygon at (*c*).

The first and last rays AO and OA_1 are the components of A–A_1, the total resultant load. The strings a–o and o–a_1, corresponding to these rays, intersect at z, a point on the resultant, the direction of which is given by AA_1 in the force-polygon. The action line of the resultant, drawn parallel to A–A' through z intersects the known action line of R_2 in point P; then PM is the direction of the action line of R_1. The magnitudes of R_1 and R_2 can be found from the force-polygon by drawing A_1G_1 parallel to the action line of R_2 and G_1A parallel to the action line of R_1 forming the closed polygon AA_1G_1A. At point F lay off FG equal and parallel to $G_1A = R_1$; from G, lay off GH equal and parallel to $A_1G_1 = R_2$, then, $HA = H_1A_1$ (the ceiling-loads) must close the true load line, upon which the stress-diagram may be drawn as shown at (*d*).

The action lines of the reactions might have been determined, after the position and direction of the resultant load was found, by means of the funicular polygon at (*e*) constructed as follows: Since M is the only known point on R_1 draw string oa, parallel to OA, through M to intersect the resultant load, and from this intersection draw string oa_1 parallel to OA_1 to intersect the known direction of the action line of R_2 in point s. The closing string SM establishes the direction of the closing ray OG_1 intersecting the line through A_1, parallel to R_2, in point G_1. The reactions thus determined are, as before, $A_1G_1 = R_2$ and $G_1A = R_1$.

Example 2. The cantilever truss shown in Fig. 22 is supported at N in such a manner that reaction R_2 can have both a vertical and horizontal component. The support at M is a link, or tie, which offers only horizontal resistance.

The resultant wind-load, normal to the upper chord, is 6 400 lb, and the resultant vertical load is 8 000 lb, both acting at the mid-point of the upper chord. Lay off, at (*b*), the known magnitudes and directions of ΣW and ΣV, select any pole-point, and draw the ray or force-polygon (*b*) and the funicular polygon (*a*). Since N is the only known point on the action line of R_2, draw string o–I, a component of R_2 and ΣW, through N to intersect the action line of W at point w. Through w, draw string O–II, a component of ΣW and ΣV, to intersect ΣV at point v. String o–III, a component of ΣV and R_1, is drawn from v to intersect the known action line of R_1 at point r, then rN, the common component of R_2 and R_1, is the closing string. Through O, in the force-polygon, at (*b*), draw ray rN parallel to the closing string, locating G_1 on the line III–G_1, the known direction of the action line of R_1, then

III–G_1 = R_1 and G_1–I = R_2. The reactions having been thus completely determined, the true load-reaction force-polygon, with the external forces in proper sequence, can be drawn, as at (*c*), and the stress-diagram constructed thereon in the usual manner.

Example 3. Let it be required to determine the reactions for, and the stresses in the members of, the grandstand truss, loaded as shown in Fig. 23. In this structure, the knee-brace and seat-beam meet at point *M*, and the left-hand column is, therefore, braced laterally at this point. The total

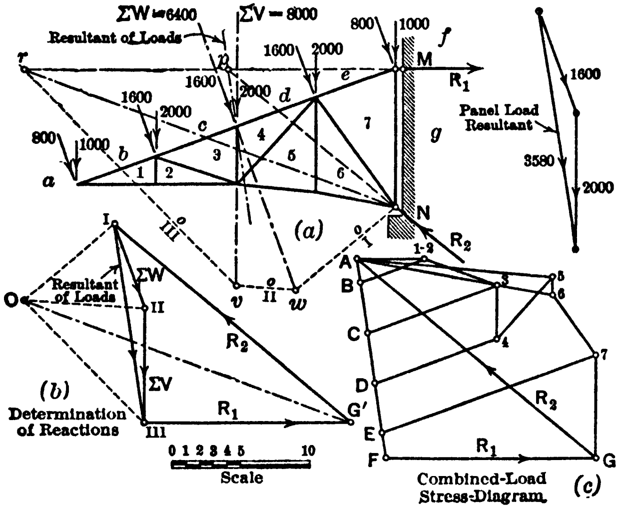


Fig. 22. Cantilever Truss

horizontal resistance will act at point *M* and be supplied by the seat-beam. The right-hand column, being unsupported laterally, can offer little horizontal resistance, and the reaction R_2 is, therefore, logically assumed to be vertical. The vertical loads are replaced by their resultant ΣV and the wind-loads by their resultant ΣW. The load line is drawn at (*b*), pole-point *O* taken in any convenient location, and the funicular polygon drawn as shown at (*a*). The first string *o*–*a* is drawn through *M*, the only known point on the action line of R_1; *ok′* is drawn parallel to *OK′*, to include only the vertical load at the outermost joint of the upper chord. The ray *OP* drawn parallel to the closing string *o*–*p* to intersect the known direction of R_2 at point *P*, gives the reactions *NP* = R_2 and *PA* = R_1, in magnitude and direction. Since there is no external load at the joint of member 1–1′ and the vertical end member through *M*, the stress in member 1–1′ is zero and the member is shown by a broken line in the space-diagram at (*a*).

The reactions having been determined, to complete the force-polygon at (*b*), the stress-diagram (*c*) may be constructed by beginning at the left reaction and proceeding in the order *P*–*A*, *A*–1, 1–*P*. This procedure is continued joint by joint, as shown at (*c*). The stress-diagram closes on point 15, where the vertical *M*–15 checks point 15 as determined by the directions of members *k*–15 and 14–15.

Example 4. Truss to Support an Electric Sign. Let it be required to determine the reactions for, and the stresses in the members of, the vertical

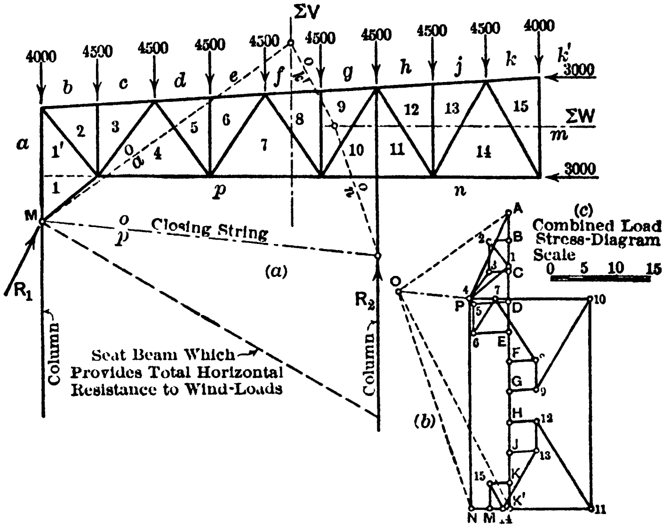


Fig. 23. Grandstand Truss

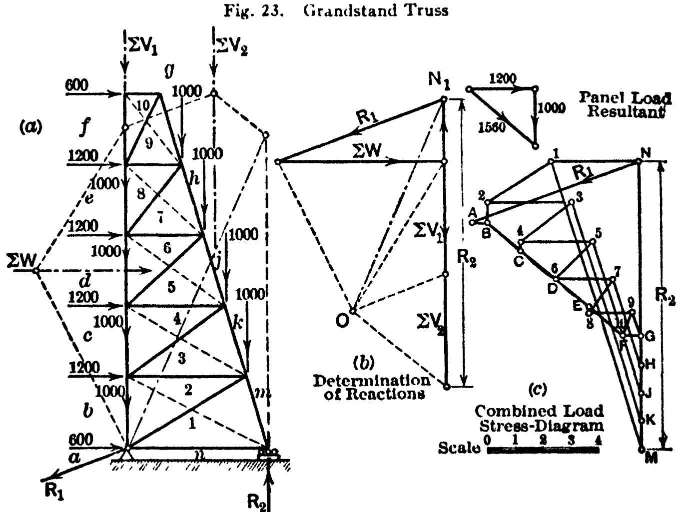


Fig. 24. Truss to Support Electric Sign

cantilever truss loaded as shown in Fig. 24. The left reaction is fixed horizontally by a more rigid detail than is possible at the right reaction. The

former will, therefore, be assumed to offer all the horizontal resistance, the latter being vertical, as shown. The dead loads consist of the weight of the structure, which is distributed in this example to the four intermediate

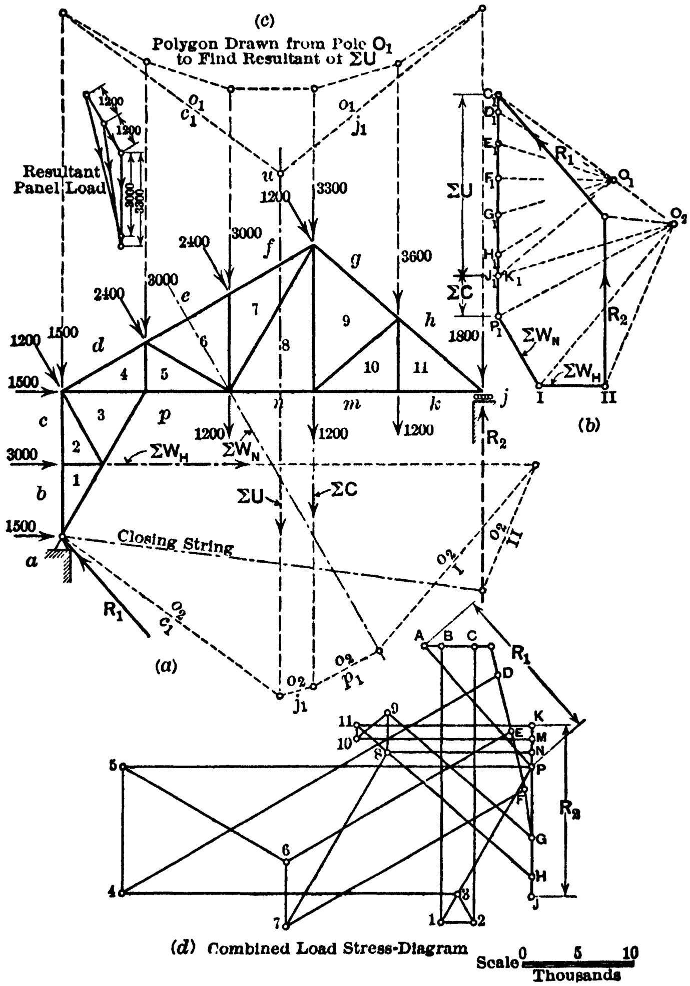


Fig. 25. Stress-diagram for Unsymmetrical Truss

joints along each chord. Resolve the external loads into three resultants ΣW, ΣV_1 and ΣV_2, and lay off the load line as at (b). Construct the funicular polygon as shown at (c), starting at the left support, which is the only known

point on the action line of R_1. The closing string determines the direction of the closing ray and locates point N_1, which gives the magnitudes and directions of R_1 and R_2 as shown. It should be noted that the vertical reaction R_2 is greater than the total vertical load as a result of the overturning tendency of the structure. The direction of R_1 is, therefore, downward to the left, indicating that for the given loading this reaction must be anchored to the support.

The reactions having been determined, the stress-diagram may be drawn in the usual manner, as shown at (*z*). All diagonal web-members in this truss are to be designed to take tensile stress only. It is necessary, therefore, to supply two sets of diagonals as indicated by the full and broken lines. The members represented by the full lines are acting when the wind-load is from the left, as in the example. The members represented by the broken lines comprise the active diagonal web-system when the direction of the wind is reversed.

Example 5. Unsymmetrical Truss. Combined Load Stress Diagram. Let it be required to find the reactions for, and the stresses in the members of, the unsymmetrical truss loaded as shown in Fig. 25. The given external loads are replaced by their four resultant forces:

(1) ΣW_H = the total horizontal wind-load
(2) ΣW_N = total normal wind-load
(3) ΣU = total vertical loading on upper chord
(4) ΣC = total ceiling-load

The resultant loads are laid off in any convenient order as shown at (*b*). In order to find the position of the action line of ΣU, the individual components are laid off to scale, and the ray-diagram with pole O_1 constructed, as shown at (*b*), from which the funicular polygon at (*c*) is drawn. The first and last strings of the funicular polygon determine, at their intersection, point *u*, a point on the action line of ΣU. With O_2 as a pole-point draw the ray-diagram for the four resultant forces of the external loads and, beginning at the point of left support, construct the funicular polygon as shown at (*a*). The closing string transposed to the force-polygon through pole O_2 determines the magnitudes of R_1 and R_2 and the hitherto unknown direction of R_1.

At (*d*) the load-reaction polygon is drawn with R_1 and R_2 as found in (*b*) neutralizing the load line. The stresses in the members of the truss are found by drawing the stress-diagram as shown at (*d*).

5. Maximum and Minimum Stresses

In the preceding examples the stresses in various trusses have been found for different systems of loading and for certain combinations of loading. For the purposes of design it is necessary to know the MAXIMUM STRESS which may be produced in each member, and in some cases it is important to know the MINIMUM STRESS. By MAXIMUM and MINIMUM STRESSES is meant the range of stress that may occur due to the different combinations of loading.

If the stress in a given member is of the same character throughout its range of stress, there is no reversal, but if under certain conditions of loading the character of stress is changed, that member must be designed for the possible REVERSAL OF STRESS. In designing, the minimum stresses are not important unless they are of opposite sign to the maximum stress, since if no reversals occur the members are designed for the maximum stress. If there are reversals of stress the members involved must be designed for both maximum and minimum stress.

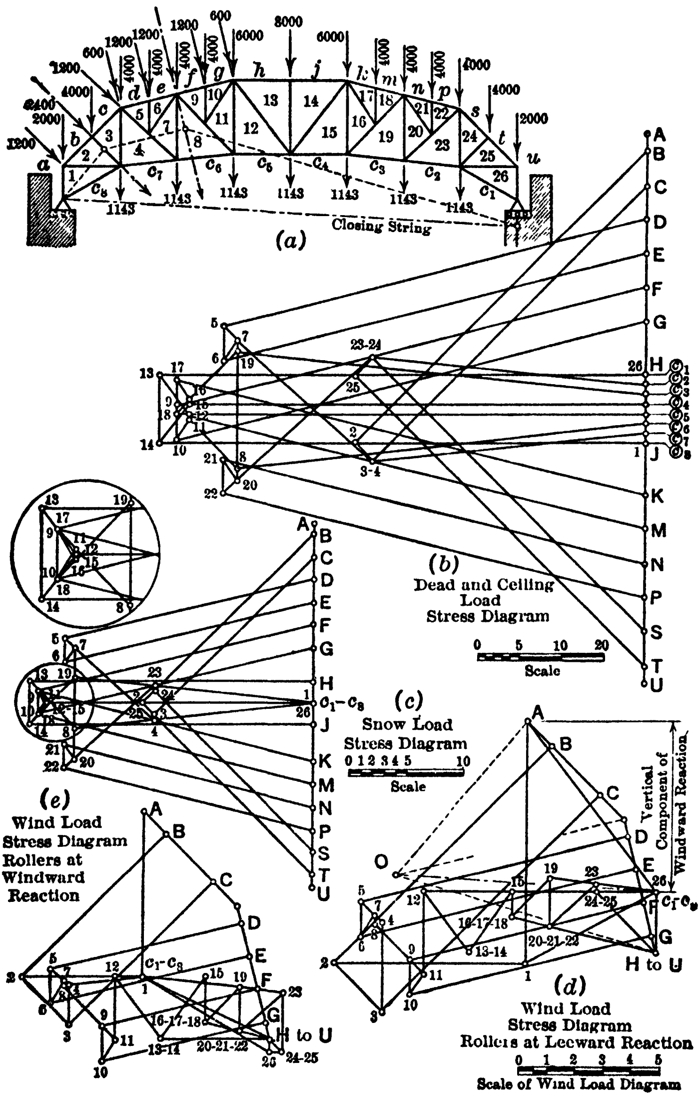


Fig. 26. Camel-back Truss

In general, compression-members are able to take tensile stress up to and somewhat beyond their compression values; however, most tension members are able to take but a relatively small amount of compression. COMPRESSION REVERSALS are, therefore, the important minimum stresses to be considered in design. In finding minimum stresses it should be noted that the dead load is always present and that there is no reversal of stress in any member unless the wind, or some other loading, produces, when combined with dead load, a stress of opposite kind to that caused by dead load alone. Since the dead load is always acting upon the truss, dead-load stresses must be included in all combinations for maximum and minimum stress.

When the maximum snow-load is included, the wind-load is not considered, since it is highly improbable that the maximum wind-load and maximum snow-load will ever occur at the same time. The minimum snow- or ice-and-sleet load may occur simultaneously with the maximum wind-load.

The following combinations of loading must, therefore, be considered in calculating the maximum and minimum stresses in a truss subjected to the usual roof-loads:

(1) DEAD LOAD (including CEILING-LOAD) alone.
(2) DEAD LOAD plus maximum SNOW-LOAD.
(3) DEAD LOAD plus WIND-LOAD (rollers leeward).
(4) DEAD LOAD plus WIND-LOAD (rollers windward).
(5) DEAD LOAD plus MINIMUM SNOW plus WIND-LOAD (rollers leeward).
(6) DEAD LOAD plus MINIMUM SNOW plus WIND-LOAD (rollers windward).

Since the snow-loads are generally given in pounds per square foot of horizontal projection, maximum and minimum snow-loads can be determined from a single stress-diagram. The stress in any member is directly proportional to the unit loads for all systems of loads having the same lines of action and points of application.

Example. Let it be required to determine the maximum and minimum stresses in the members of the camel-back truss, loaded as shown in Fig. 26.

The combined dead and ceiling-load stress-diagram is shown at (*b*). The truss and the dead plus ceiling-loads are symmetrical about the center line; therefore, each reaction is equal to one-half of the total load, or:

$$R_2 = UC_1 = \tfrac{1}{2}(64\,000 + 8\,000) = 36\,000$$

and

$$R_1 = C_8A = 36\,000$$

The maximum snow-load stress-diagram is shown at (*c*). The snow panel load in this case is equal to one-half the dead panel load on the upper chord. The minimum snow-load is one-half the maximum snow-load, and the stress-diagram at (*c*), read to twice the scale at which it is drawn for maximum snow-load, gives the values of minimum snow-load stresses. The stress-diagram for wind-load on the left with rollers under the right reaction is shown at (*d*). The reactions are found in the usual manner by means of the funicular polygon (*a*) drawn from the ray diagram with pole O.

At (*e*) the rollers are assumed to be at the left reaction for the wind-load, as shown, on the left half of the truss. When the truss is symmetrical, the reaction conditions can be changed instead of changing the direction of the wind for the purpose of determining the stresses under the two conditions of loads and reactions.

The vertical component of R_1 as found at (*d*) then becomes the vertical reaction, assuming rollers at the left support. With this value determined, R_2 is drawn to close the load line-reaction force-polygon, and the stress-diagram is constructed in the usual manner.

In drawing the stress-diagrams for the truss in this example it is impossible to proceed beyond point 15 by the usual methods when the diagram is started at the left support. After reaching point 15, the diagrams in Fig. 26 were completed by starting at the right support and working back to point 15.

Truss Member	Dead and Ceiling Load Stress	Snow Load Stress		Wind Load Stress Windward Members		Combination for Maximum	Maximum Stress	Wind Load Stress Leeward Members		Truss Member
		Max.	Min.	Rollers Leeward	Rollers Windward			Rollers Leeward	Rollers Windward	
	①	②	③	④	⑤			⑥	⑦	
$a-1$	−28.0	−16.0	−8.0	−8.5	−5.9	①③④	−44.5	−2.1	+0.5	$u-26$
$b-2$	−48.3	−21.2	−10.6	−10.8	−7.2	①③④	−69.7	−3.0	+0.7	$t-25$
$c-3$	−45.4	−19.8	−9.9	−10.8	−7.2	①③④	−66.1	−3.0	+0.7	$s-24$
$d-5$	−51.3	−22.5	−11.3	−9.7	−6.9	①②	−13.8	−3.8	−1.1	$p-22$
$e-6$	−51.3	−22.5	−11.3	−10.0	−7.2	①②	−13.8	−3.8	−1.1	$n-21$
$f-9$	−56.8	−25.0	−12.5	−8.5	−5.7	①②	−81.8	−5.2	−2.3	$m-18$
$g-10$	−56.8	−25.0	−12.5	−8.8	−5.9	①②	−81.8	−5.2	−2.3	$k-17$
$h-13$	−57.1	−25.0	−12.5	−6.6	−3.8	①②	−82.1	−6.6	−3.8	$j-14$
c_8-1	0	0	0	+5.2	0	①⑦	−5.2	0	−5.2	c_1-26
c_7-4	+32.3	+14.0	+7.0	+9.7	+2.6	①③④	+49.0	+2.1	−5.0	c_2-23
c_6-8	−48.4	+21.2	+10.6	+10.0	+2.8	①②	+69.6	+3.8	−3.5	c_3-19
c_5-12	+53.6	+23.5	+11.8	+8.2	+1.0	①②	+77.1	+5.1	−2.2	c_4-15
1−2	−34.3	+15.0	+7.5	+6.8	+4.3	①②	+49.3	+2.2	−0.5	26-25
2−3	−3.0	−1.5	−0.8	−2.4	−2.4	①③④	−6.2	0	0	25-24
3−4	0	−0.5	−0.3	+3.2	+1.4	①④	+3.2	−0.3	−2.1	24-23
4−5	+24.0	+10.7	+5.4	+1.1	+0.9	①②	+34.7	+2.2	+2.0	23-22
5−6	−4.0	−2.0	−1.0	−1.2	−1.2	①③④	−6.2	0	0	22-21
6−7	+3.0	+1.5	+0.8	+0.9	+0.9	①③④	+4.7	0	0	21-20
7−4	+21.5	+9.5	+4.8	+0.4	+0.2	①②	+31.0	+2.2	+2.0	23-20
7−8	−15.0	−7.0	−3.5	−0.3	−0.2	①②	−12.0	−1.7	−1.5	20-19
8−9	+10.3	+4.6	+2.3	−1.8	−1.9	①②	+11.9	+1.9	+1.7	19-18
9−10	−4.0	−2.0	−1.0	−1.2	−1.2	①③④	−6.2	0	0	18-17
10−11	+3.0	+1.5	+0.8	+0.9	+0.9	①③④	+4.7	0	0	17-16
8−11	+8.0	+3.6	+1.8	−2.5	−2.6	①③⑥	+11.7	+1.9	+1.7	19-16
11−12	+0.7	−0.5	−0.3	+2.9	+2.2	①④	+3.6	−0.9	−1.6	16-15
12−13	+5.6	+2.5	+1.3	−2.7	−2.7	①③⑥	+9.6	+2.7	+2.7	15-14
13−14	−8.0	−4.0	−2.0	0	0	①②	−12.0			

Fig. 27. Summary of Stresses

Various methods of overcoming the difficulty arising from the presence of three unknowns at an intermediate joint are discussed in Article 7.

Summary of Stresses. Preliminary to the design of the members of the truss, the value of the stresses due to the various load conditions are scaled from the stress-diagrams in Fig. 26 and recorded in tabular form as shown in Fig. 27. For symmetrical trusses it is generally unnecessary to employ the

last three columns of the table in Fig. 27 unless it is evident from the stress-diagrams that reversals may be present in some members of the leeward half of truss under the action of dead and wind-loads.

The notation in the column headed "Combination for Maximum" refers to the designation of the columns from which the stress values, to be added in arriving at the maximum stress, are taken.

As an example of the manner in which the table is compiled and used, consider the member C_7–4, which occupies a position in the left half of the truss similar to C_2–23 in the right half. The recorded data show:

(*a*)	Dead + ceiling-load stress	= 32 300 lb (tension)
	Maximum snow-load stress	= 14 000 lb (tension)
	① + ②	= 46 300 lb (tension)
Windward Member		
(*b*)	Dead + ceiling-load stress	= 32 300 lb (tension)
	Minimum snow-load stress	= 7 000 lb (tension)
	Wind-load stress (rollers leeward)	= 9 700 lb (tension)
	① + ③ + ④	= 49 000 lb (tension)
(*c*)	Dead + ceiling-load stress	= 32 300 lb (tension)
	Minimum snow-load stress	= 7 000 lb (tension)
	Wind-load stress (rollers windward)	= 2 600 lb (tension)
	① + ③ + ⑤	= 41 900 lb (tension)
Leeward Member		
(*d*)	Dead + ceiling-load stress	= 32 300 lb (tension)
	Minimum snow-load stress	= 7 000 lb (tension)
	Wind-load stress (rollers leeward)	= 2 100 lb (tension)
	① + ③ + ⑥	= 41 400 lb (tension)
(*e*)	Dead + ceiling-load stress	= 32 300 lb (tension)
	Minimum snow-load stress	= 7 000 lb (tension)
	Wind-load stress (rollers windward)	= − 5 000 lb (compression)
	① + ③ + ⑦	= 34 300 lb (tension)

The maximum design stress for members C_7–4 and C_2–23 is therefore, 49 000-lb tension, which occurs in the windward member with the roller reaction leeward and under the combination of loading shown in (*b*). The simultaneous stress in the homologous member is 41 400-lb tension, as given by the combination of loading shown in (*d*).

The stress resulting from all loadings, excepting wind-load stress in the leeward member with rollers windward, is tension. The condition of loading which produces a compression of 5 000 lb exists in combination with the dead and ceiling-load stress of 32 300-lb tension. The minimum stress in members C_7–4 and C_2–23 is, therefore, 32 300 − 5 000 = 27 300 (tension). There is no reversal of stress in this or in any other member of the truss.

6. Examples of Stress-Diagrams for Various Types of Trusses

The principles outlined in the preceding articles can be applied to any type or form of roof-truss which complies fully with the definition of a truss as given in Article 1 of Chapter XXIV.

The stress-diagrams for each type of truss possess certain distinct characteristics peculiar to the form and type of bracing which are the distinguishing features of that particular type of truss. It is advisable, therefore, to become more or less familiar with the general characteristics of the stress-diagrams for the more usual types of bracing and forms of trusses which have not been employed as examples in the preceding articles of this chapter.

Example 1. Simple Fan Truss. This type of truss, shown in Fig. 28, is a simple modification of the Fink system of web-bracing. The reactions for

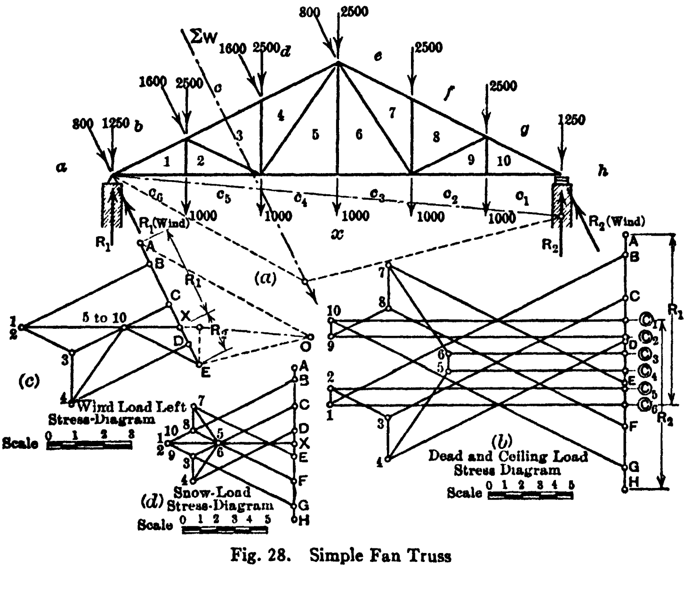


Fig. 28. Simple Fan Truss

dead and ceiling-loads are equal and each equal to one-half the total load, since the truss and loading are symmetrical about the center line. The reactions for wind-load are most readily determined by means of the force and funicular polygon. Since the roof slope is relatively flat it will be assumed that the wind-load reactions are parallel and that the frictional resistance in the bearing detail at R_2 is sufficient to provide the necessary horizontal component. The action line of both reactions having been established by assumption, the funicular polygon might have been started at any convenient point on the action line of R_1, without employing the tentative assumption that R_2 was vertical.

The combined dead and ceiling-load stress-diagram is shown at (*b*), the

wind-load stress-diagram is shown at (*c*), and the snow-load stress-diagram is shown at (*d*).

Example 2. Scissors Truss. Fig. 29 (*a*) illustrates a simple scissors truss with both upper and lower chord loading. The dead plus ceiling-load stress-diagram is shown at (*b*), in which the reactions are equal since the truss and loading are both symmetrical.

When the roof slope is relatively steep, as in this example, an assumption of equal horizontal components of wind-load reactions is more logical, provided the expansion detail offers sufficient horizontal resistance. When the slope of the upper chord is 45°, the resultant wind-load intersects the line

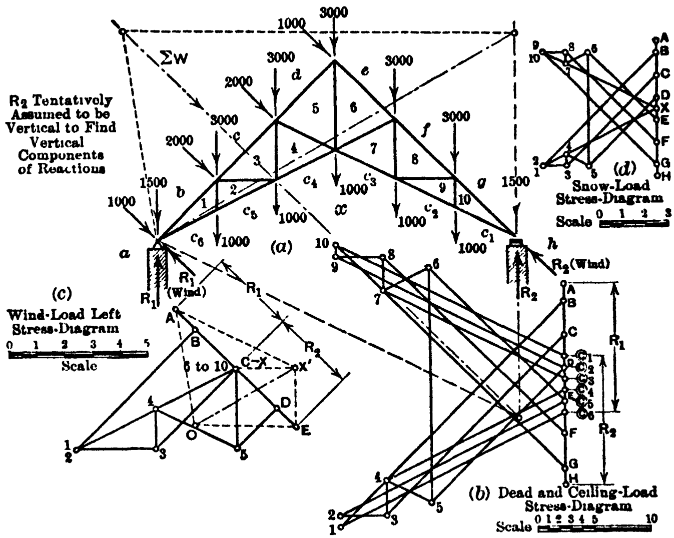


Fig. 29. Scissors Truss

joining the two supports at the mid-point, hence, for the assumption of equal horizontal components, the reactions are parallel. The vertical components of the wind-load reactions are found by the tentative assumption that R_2 is vertical. In this example it is convenient to determine the direction of R_1 for the tentative assumption by the principle: "Three forces in equilibrium meet in a common point." The reactions are also found for the tentative assumption by means of a funicular polygon, and are shown at (*c*) by broken lines. It should be noted that the chord stresses in the scissors-type truss are excessively large as compared to the reactions, on account of the steep inclination of the lower chord which is also responsible for the large stress in the center vertical.

For these reasons the scissors type of truss is not economical for spans much in excess of 36 ft. The high chord stresses are conducive to excessive deformation which in turn results in large horizontal thrusts. (See Article 9.)

Example 3. The Howe Truss, which is one of the most satisfactory types for timber construction, is shown in Fig. 30.

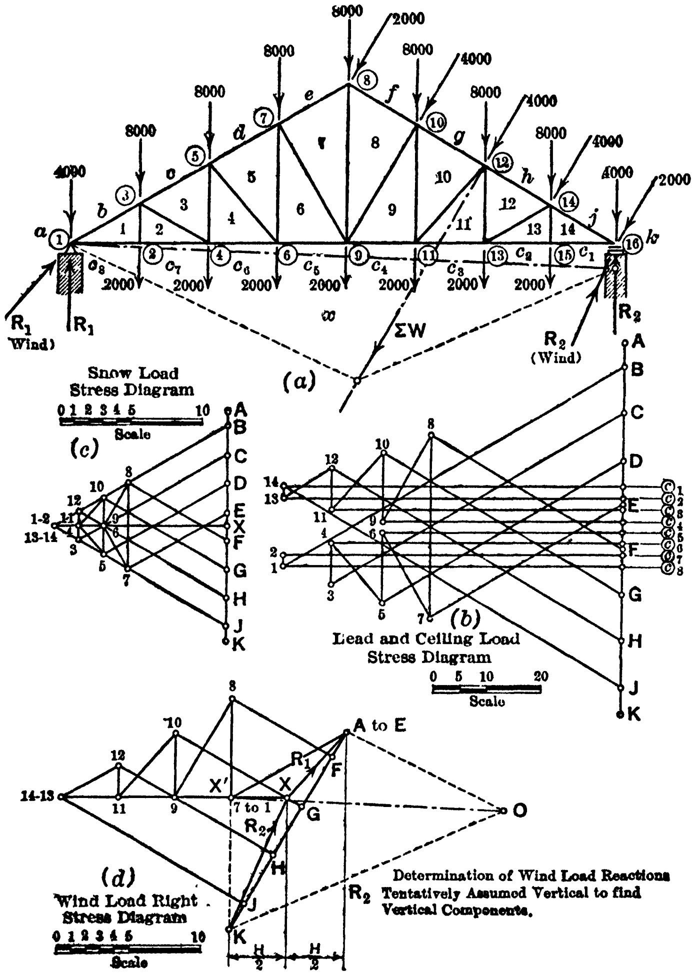


Fig. 30. Howe Truss

The combined dead and ceiling-load stress-diagram is constructed in the usual manner, as shown at (*b*). The order in which the joints are considered in constructing the diagram is indicated by the joint notation at (*a*). The leeward reaction is assumed to be a sliding-plate detail having a coefficient

of friction of one-quarter. The total horizontal resistance of the sliding detail under dead, ceiling and wind-loads is:

$$\tfrac{1}{4}\,(8 \times 8\,000 + 7 \times 2\,000 + 0.866 \times 16\,000) = 22\,900 \text{ lb}$$

The total horizontal component of the wind-load is only 8 000 lb; there-

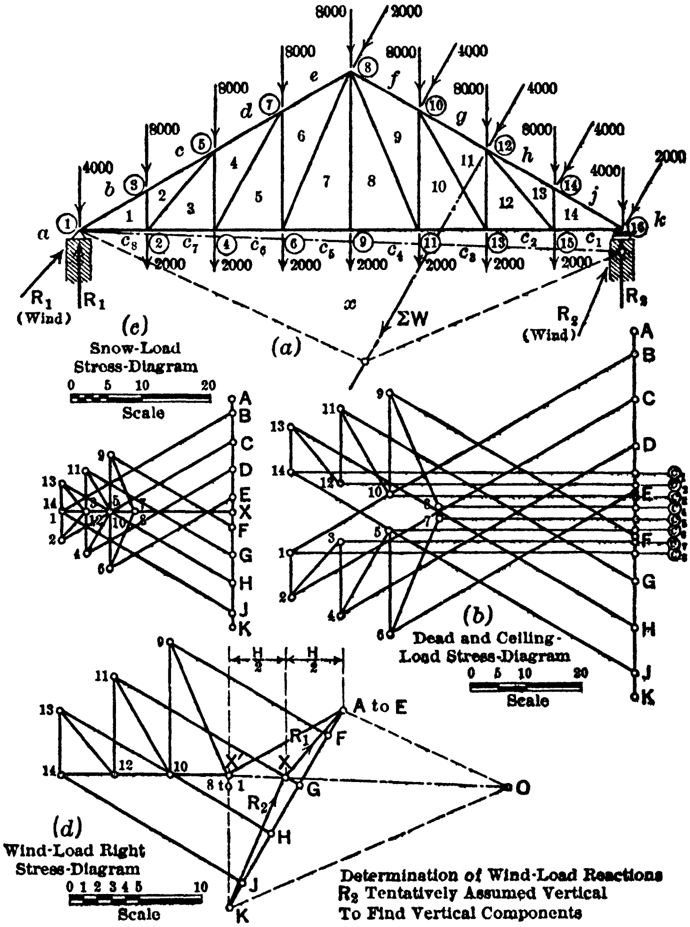


Fig. 31. Pratt Truss

fore, even with the sliding detail the reactions due to wind may be assumed to have equal horizontal components or may be assumed parallel. In this example the former assumption will be made. The vertical components of the wind reactions are found by first assuming R_2 to be vertical (frictionless

sliding detail), after which the horizontal component of the wind-load is divided equally to the two reactions as shown at (*c*) and the stress-diagram drawn upon the closed force-polygon thus determined.

Example 4. The Pratt Truss, shown in Fig. 31, is identical in outline to the Howe truss of the preceding example, the only difference being in the inclination of the diagonal web-members. In the Howe truss the diagonal web-members are in compression and the vertical web-members are in tension, while in the Pratt truss these conditions are reversed with the exception of the center vertical, which is a tension-member in both types of bracing.

As a rule, long tension-members are more economical than long compression-members, and in steel construction the Pratt truss is generally more economical than the Howe, while in timber construction the latter is more economical on account of the less complicated details at the intersections of chord and web-members.

The stress-diagrams for the Pratt truss are constructed in the same general manner as those in the preceding example.

Example 5. Parallel Chord Howe Truss, Loaded at Alternate Joints. In some types of roof-construction, especially where sub-purlins are used, it is economical to place the main purlins at relatively large intervals. It may also be economical to reduce the lengths of the compression-members of the truss by the adoption of intermediate joints, as shown in Fig. 32 (*a*). The stress-diagram is constructed in the usual manner, as shown for dead and ceiling-loads at (*b*), and for snow-load at (*c*).

Example 6. The Warren Truss, shown in Fig. 33, is of the same general dimensions and supports the same system loads as the Howe truss of the preceding example. The stress-diagrams are drawn to the same scale as the stress-diagram in Fig. 32 (*b*) and (*c*). A direct comparison of the stress conditions in the two trusses under the same loading may be made; however, the question of final economy cannot be settled until the design of the members is completed, since the unsupported lengths of the compression-members of the Warren truss are greater than those of the Howe truss.

Example 7. Quadrangular Truss. Fig. 34 illustrates a quadrangular truss used for a relatively flat roof, and in which all diagonal web-members are in tension. The inclination of the two diagonals in the panels on each side of the center line must be of opposite inclination to that of the outer panels as shown by the stress-diagram. When it is desired to have the diagonal web-members in tension (or compression) and doubt exists as to their inclination, they may be assumed tentatively at either inclination and the character of stress determined as the stress-diagram is constructed. If the tentative assumption results in opposite character of stress desired, the inclination of the member should be reversed in the truss and the result in the stress-diagram will be the character of stress desired.

In constructing the stress-diagram shown at (*b*), it should be noted that the lower chord stresses in the end panels is zero and that members 1–2 and 19–20 are in reality the lower chord members in these panels.

Example 8. Quadrangular Truss. The slope of the roof supported by the quadrangular truss shown in Fig. 35 is less than that of the preceding example. The spans and the loading for the two examples are identical and the stress-diagrams for each are drawn to the same scale. A direct comparison of the stresses in both chord and web-members for homologous panels provides an interesting study.

In the truss illustrated in Fig. 35 the inclination of the diagonal web-members, which are to be in compression, changes only in the panels adjacent

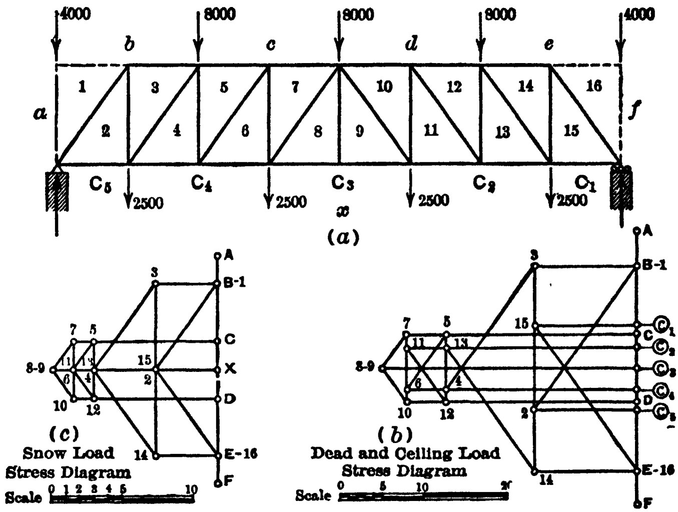


Fig. 32. Parallel Chord Howe Truss

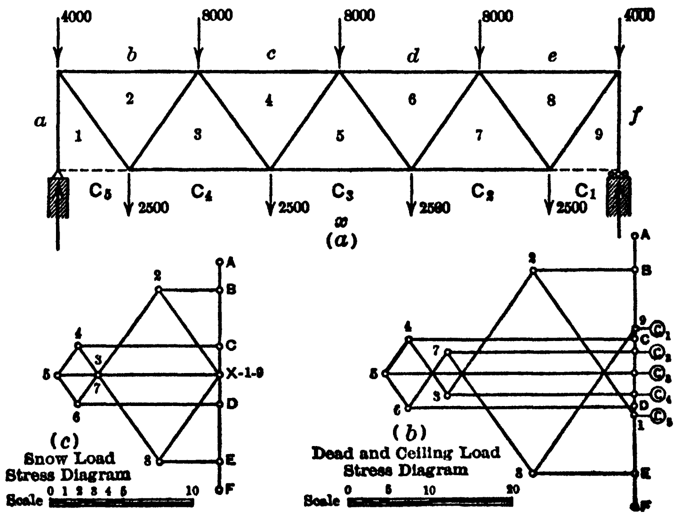


Fig. 33. Warren Truss

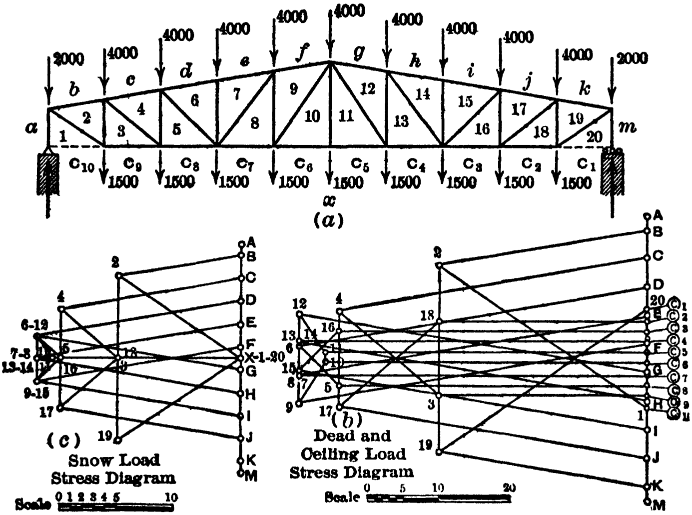


Fig. 34. Quadrangular Truss

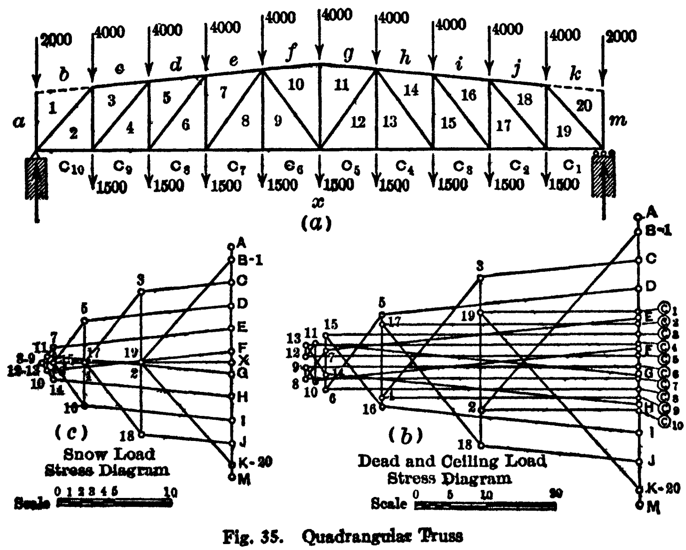


Fig. 35. Quadrangular Truss

to the center line, and the stress in these diagonals is very small. With a still less inclination of the upper chord the stress in members 9–10 and 11–12 becomes zero; and when the upper chord becomes horizontal the stress in

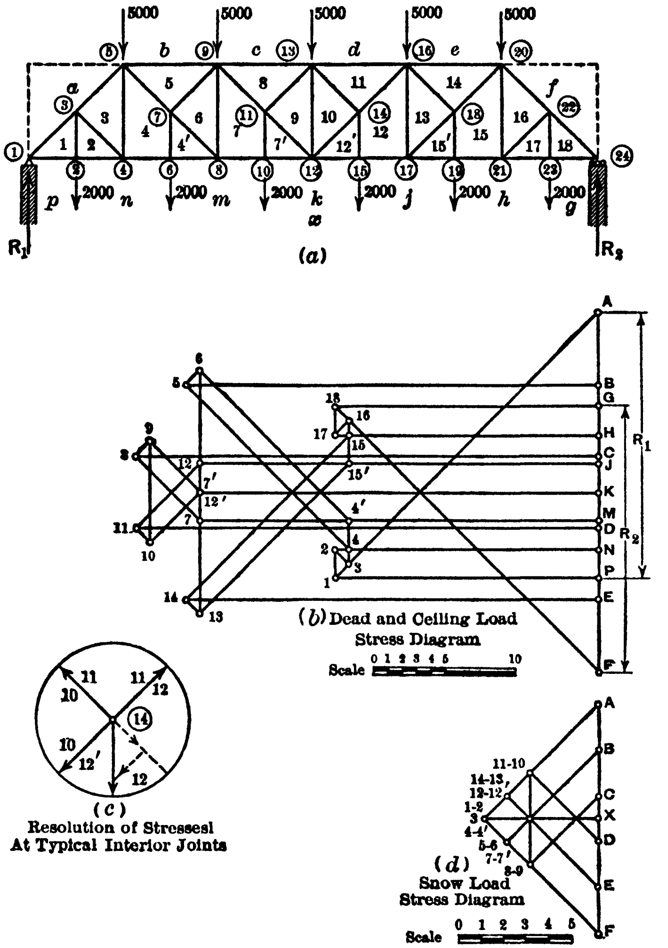


Fig. 36. Petit or Baltimore Truss

these members changes from compression to tension, as can be determined by considering the sum of vertical shears on a vertical plane cutting either of these diagonals and the two horizontal chord sections in their respective panels. The direction of the stress in the web-member must act away from

the section in order to provide the vertical equilibrating component, and must, therefore, be tension.

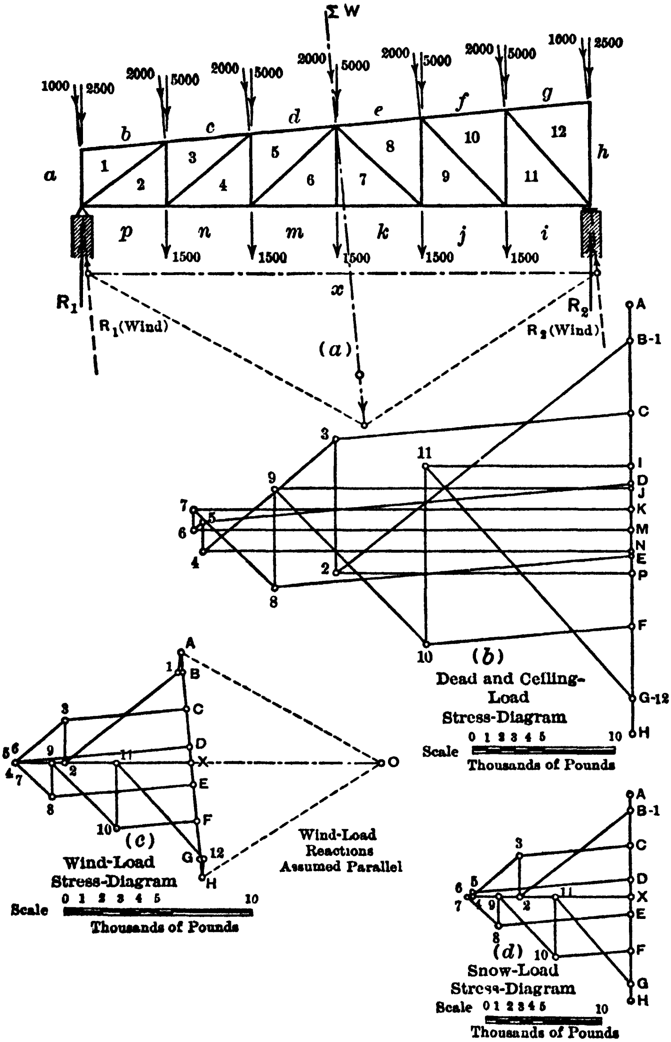


Fig. 37. Howe Truss with Sloping Upper Chord

Example 9. Petit Truss. The parallel chord truss shown in Fig. 36 is sometimes called the BALTIMORE TRUSS, but is more commonly known as

the PETIT TRUSS. In reality the truss in this example is a Howe truss with subdivided panels to permit intermediate loading on the lower chord. The

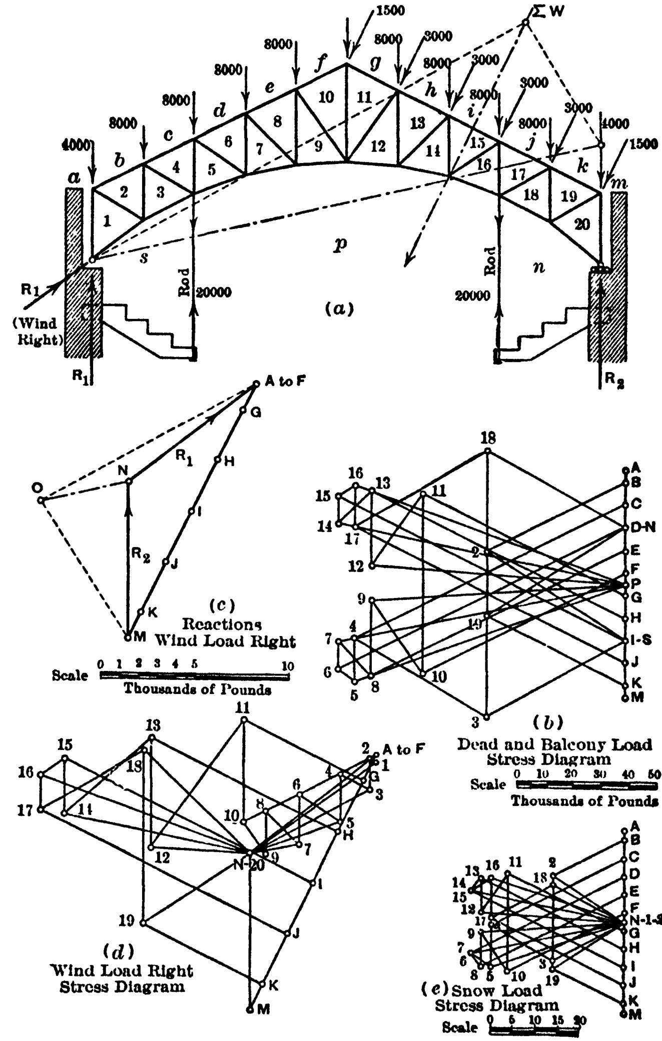


Fig. 38. Truss Supporting a Balcony

web-system of the Petit truss may be employed to subdivide the panel lengths of the upper chord, as illustrated in Fig. 26.

In the present example, on first inspection it might appear that the truss is an incomplete frame, since the members 4–4′, 7–7′, 12–12′, and 15–15′ divide a triangular figure into a smaller triangle and a quadrilateral figure. These dividing members, however, act merely as hangers, and the effect on the other members of the truss is exactly the same as though the lower chord loads were applied at the mid-points of the main diagonals.

The stress-diagram for the combined upper and lower chord loads is shown at (*b*). In beginning the stress-diagram at the left support and following the order of procedure indicated by the numbering of the joints at (*a*) it is evident upon reaching point 9 in the stress-diagram as determined by the solution of joint (11) that further progress is impossible since at joint (12) and joint (13) there are three unknowns. The diagram for the right-hand half of the truss can be started at the right support, joint (24), and carried through to point 10, then point 9 as previously found will lie on a vertical through point 10, thus closing the stress-diagram.

Example 10. Howe Truss with Sloping Upper Chord. When the roof has a single slope, a truss of the general form shown in Fig. 37 is employed. In this particular truss all the diagonal web-members are in compression. The dead plus ceiling-load stress, diagram is shown at (*b*) and the wind-load stress-diagram is shown at (*c*). Since the roof slope is comparatively flat and the normal wind-loads nearly vertical, the wind-load reactions are assumed to be parallel.

Example 11. Truss Supporting a Balcony. The quadrangular truss shown in Fig. 38 supports the roof and balcony of a small assembly-hall.

The truss is supported on masonry walls with a roller-bearing provided at one end, assumed in the example to be the right or windward end. The combined upper chord and balcony-load stress-diagram is shown at (*b*) and the wind-load stress-diagram is shown at (*c*).

Example 12. Crescent Truss. The truss shown in Fig. 39 is sometimes referred to as a CAMEL-BACK TRUSS, but is more commonly known as a CRESCENT TRUSS. This type of truss is used where an arched effect is desired both on the exterior and interior.

The dead plus ceiling-load stress-diagram is shown at (*c*). The stress-diagram for wind on the left is shown at (*d*). The wind-load on each panel is taken normal to the upper chord in that panel and the total resultant wind-load is determined by means of the force-polygon at (*b*) and the funicular polygon at (*a*). The known vertical direction of R_2 prolonged through its point of application to intersect the wind-load reaction at Z establishes a point on the action line of R_1, since R_1, R_2, and ΣW, being in equilibrium, must intersect in a common point. The direction of R_1 is then fixed by a line drawn through the left support and point Z.

The reactions for the wind-load on the right may be determined as follows: The direction of R_2 is known, and since the wind-load on the right will produce vertical components at R_1 and R_2 of the same magnitude as produced by wind on the left at R_2 and R_1, respectively, these values may be taken from the force-diagram at (*b*). Lay off the load line for wind right at (*e*); through K draw the vertical KM equal to the vertical component of R_1 in (*b*); then $KM = R_2$ and $MA = R_1$. The striking differences in the two wind-load stress-diagrams should be noted, especially as to the magnitudes of the chord stresses in the panels near the supports and the changes in character of stress in both chord and web-members.

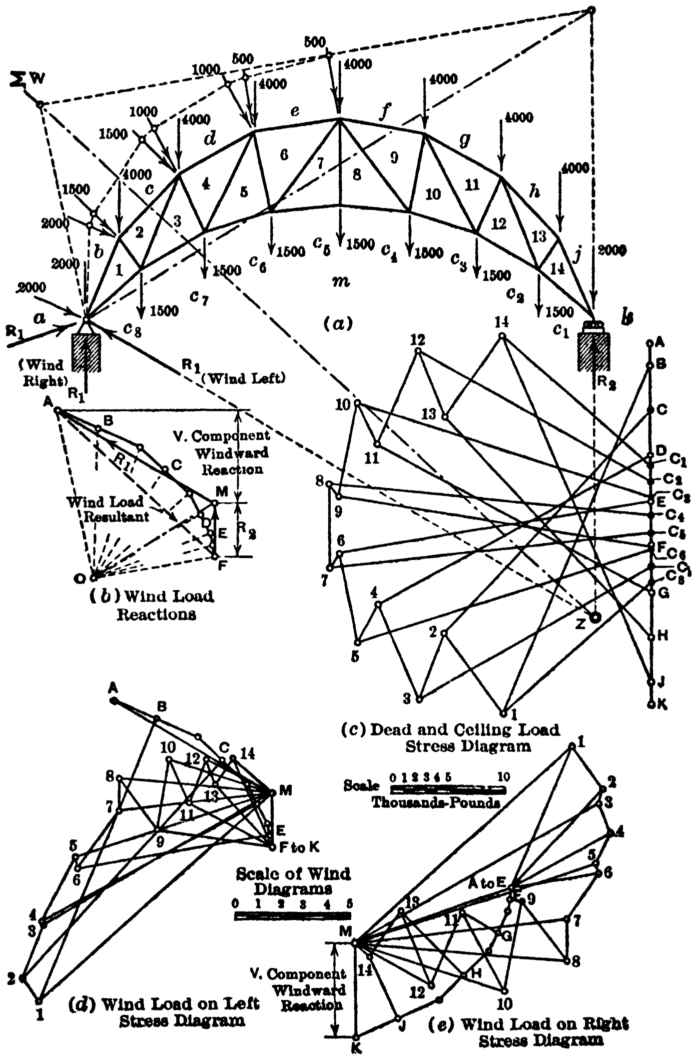


Fig. 39. Crescent Truss

7. Types of Trusses Requiring Special Consideration

The stress-diagrams for the various types of trusses illustrated in the preceding article are all constructed in the same manner, with the single exception of the Petit truss discussed in Example 9. In general, stress-diagrams are started by drawing the closed force-polygon for the joint at the left support and proceeding to the next adjacent joint where there are but two unknown magnitudes, progressing always in a clockwise direction.

It was seen in Example 9, Fig. 36, that when point 9, determined by the force-polygon for joint (11), was reached, further progress was impossible by following the clockwise sequence of joints, since at both joint (12) and joint (13) there were three unknowns, or one more than the number of equations represented by a polygon of forces, that is, $\Sigma V = 0$ and $\Sigma H = 0$. Continuing to joint (24), however, only two unknowns were encountered, and it was therefore possible to construct the force-polygon for that joint and thence in a counter-clockwise direction the successive polygons for the joints up to and including joint (14), thus eliminating one of the previous unknown magnitudes at both joint (12) and joint (13). This example might also have been solved as illustrated by the free body diagram at (*c*), Fig. 36. After the force-polygon for joint (11) has been completed, consider the forces acting on the free ends of the members at joint (14) removed from the structure. Assume coördinate axes Y and X along members 10–11 and 11–12. If these members are not at right-angles to each other, as in the example, take the X axis coincident with 11–12. Resolve the two unknowns not on the X axis into components along the Y axis. Since the magnitude of 12–12′ is known and since $\Sigma Y = 0$, the magnitude of 10–11 can be determined, and at joint (13) there remain only two unknowns.

There are, however, certain types of trusses in which neither of these two methods are possible, and in which certain special methods of procedure are necessary.

The Fink Truss. The Fink type of web-bracing is the most common example in which three unknowns are encountered at interior panel-points and in which neither of the general methods outlined above is applicable. Four methods of procedure, applicable to Fink trusses supported on masonry walls or piers, will be given:

Method 1. Let it be required to construct the dead and wind-load stress-diagrams for the Fink truss, loaded as shown in Fig. 40.

The dead load stress-diagram is shown at (*b*). The load line A to K is laid off and the reactions $R_2 = KM$ and $R_1 = MA$, are each equal to one-half the load, as determined by symmetry. The stress-diagram is started, as usual, at joint (1) and continued to joint (4), where the condition of three unknowns is encountered. There are also three unknowns at joint (5). If the stress-diagram is started at joint (15) it may be continued to joint (12), where again three unknowns are encountered and the same condition exists at joint (11).

From an inspection of the truss it is seen that the portion bounded by joints (1), (3), and (5) is similar to the portion bounded by joints (5), (6), and (8) and that under uniformly distributed chord loads the stress in member 2–3 is equal to, and of the same character as, that in member 4–5. The members make the same angle with member 3–4 and therefore the difference in stress in the two upper chord members at joint (6) is equal to the component of the panel load parallel to the chord. It is also evident that this same difference exists in the two chord members at joints (2) and (7). The following

general law for Fink trusses, of the type shown in Fig. 40, and for any number of uniform subdivisions of the upper chord, may be stated: "The rate of decrease in stress in each successive upper length of top chord panels is constant, and equal to the component of the uniform panel load normal to the struts."

In Fig. 40 (*b*) the slope of the line 1–2 represents the rate of decrease of stress for the first two panel lengths of upper chord, therefore line 1–2 produced will locate point 5 on the line through *D* and point 6 on the line through *E* drawn parallel to the members *d*–5 and *e*–6, respectively.

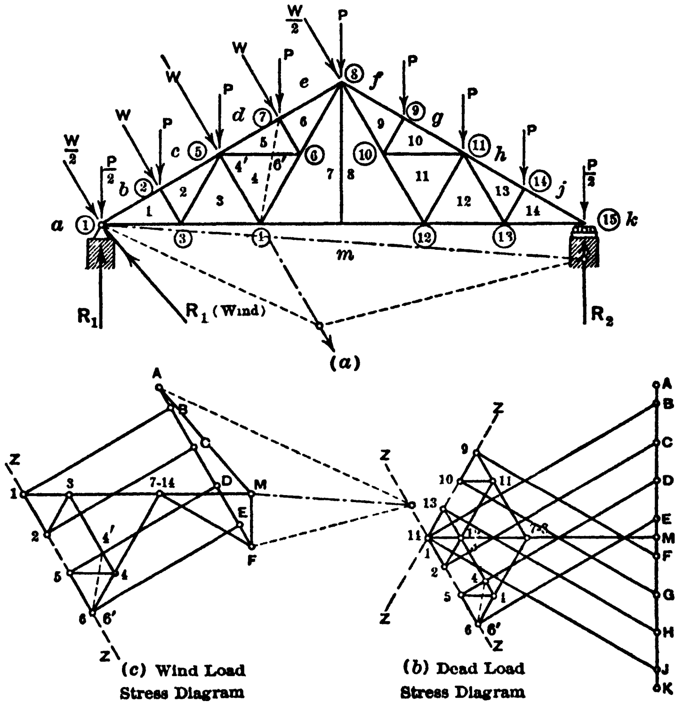


Fig. 40. Fink Truss

The same law applies to the wind-load stress-diagram shown at (*c*). The struts of the Fink truss may be placed perpendicular to the lower chord, as in Fig. 41. The law of the rate of decrease in stress for successive upper chord panel lengths as stated before is applicable also to this arrangement of web-members, as illustrated in the dead load stress-diagram at (*b*) and the wind-load stress-diagram at (*c*).

Method 2. The method of the solution of the Fink truss shown in Fig. 42 is known as the METHOD OF THE SUBSTITUTE MEMBER. The stress-diagram is drawn in the usual manner up to point 3. Members 4–5 and 5–6 are

replaced tentatively by member 4′–6′, shown by the broken line at (*a*). If a plane, such as ZZ, be passed to cut members *e*–6, 6–7 and 7–*m*, and the portion of the truss to either side of the plane be considered as a free body in equilibrium, it is evident that the stresses in members *e*–6, 6–7 and 7–*m* are unaffected by the assumed rearrangement of the web-members.

With the substituted member replacing 4–5 and 5–6, there remain only two unknowns at joint (5), namely, *d*–4′ and 3–4′. Through point 3, in the stress-diagrams (*b*) and (*c*), Fig. 42, draw 3–4′ and *d*–4′ parallel to the

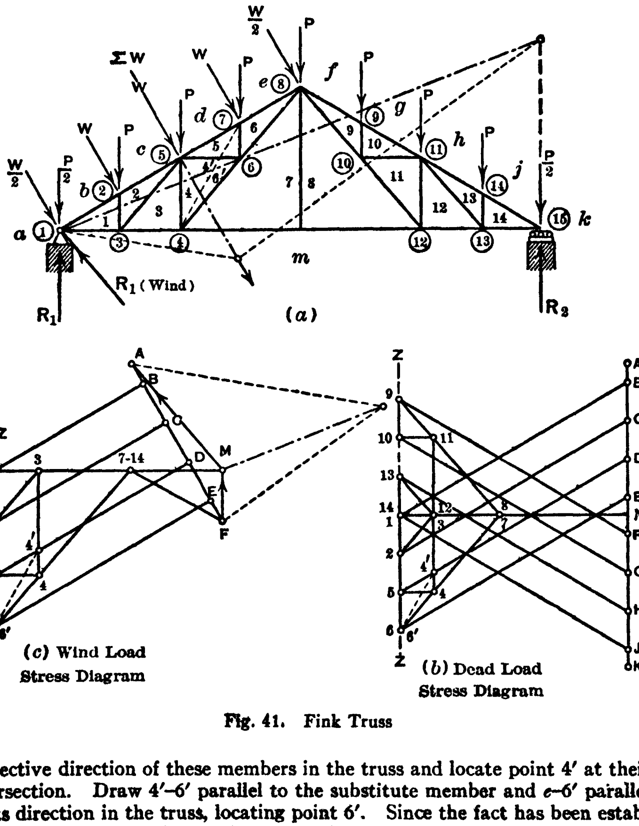


Fig. 41. Fink Truss

respective direction of these members in the truss and locate point 4′ at their intersection. Draw 4′–6′ parallel to the substitute member and *e*–6′ parallel to its direction in the truss, locating point 6′. Since the fact has been established that the stress in member *e*–6 is independent of the arrangement of web-members, below joints (6) and (7), the stress *e*–6′ = *e*–6 or the point 6′ as found is coincident with the true position of point 6. Draw 6–5 to intersect the line through *D* parallel to *d*–5, locating point 5, and through 5, parallel to 5–4, draw a line to intersect 3–4 in the true position of point 4. Points 6, 4 and 7 are on the same straight line parallel to members 4–7 and 6–7. No further difficulty occurs in the completion of the stress-diagrams.

Method 3. In so far as the web-members of the Fink truss are concerned, each half of the truss may be considered independently. The right half of the truss together with member 7–*m* may be assumed to be removed and an external force, Q_2, supplied at joint (8) such that it together with an external force, Q_1, applied at joint (1) will serve tentatively as the equilibrating reactions for the load system on the left half of the truss.

A solution of the stresses in the members of the half truss for the given load system as equilibrated for that half will determine the true stresses in the web-

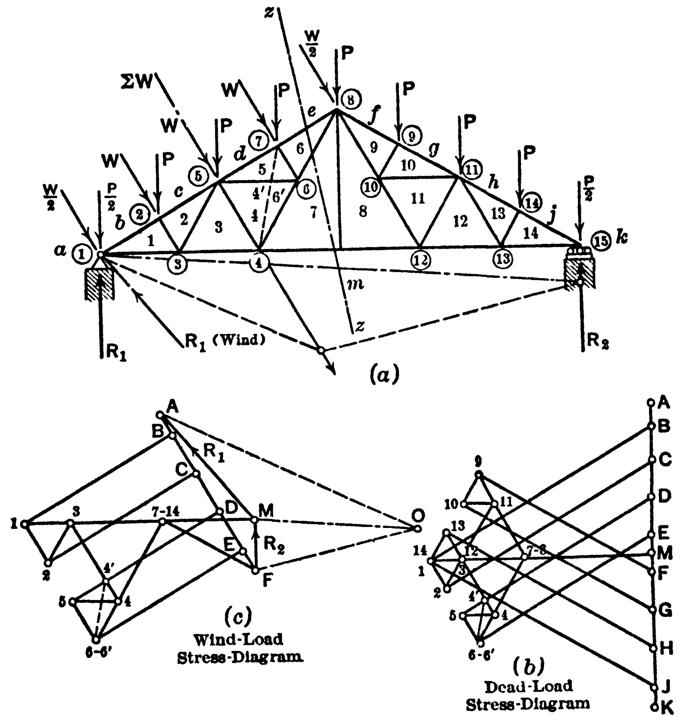


Fig. 42. Fink Truss. Method of the Substitute Member

members. To construct the dead load stress-diagram: Lay off the total load line *A* to *K* as at (*b*), Fig. 43, and proceed in the usual manner up to point 3. Considering only the loads on the left half of the truss, *A* to *M*, let these loads be equilibrated by Q_1 at joint (1) and Q_2 at joint (8). On the load line *AM* lay off $MM' = Q_2$ and $M'A = Q_1$, each being equal to one-half of the load on the half truss. Construct the stress-diagram for the half truss as shown by the broken lines at (*b*). The stresses thus found for the web-members are correct in magnitude for the structure as a whole; the positions of points 1′, 2′, 3′, etc., are incorrect with respect to the load line. Since the true position of point 3 has been previously determined, the stress-diagram for the web-

members may be transposed to its proper position by making $3'$ coincident with point 3 and projecting all other points of the web-member stress-diagram parallel and equal to the transposition of point $3'$.

The wind-load stress-diagram as constructed by this method is shown at (*c*). The equilibrating forces Q_1 and Q_2 are assumed to be parallel and each equal to one-half of the wind-load.

Transverse Bents. The roof-trusses in the preceding examples have been assumed to be supported on masonry walls or on piers. In many cases of

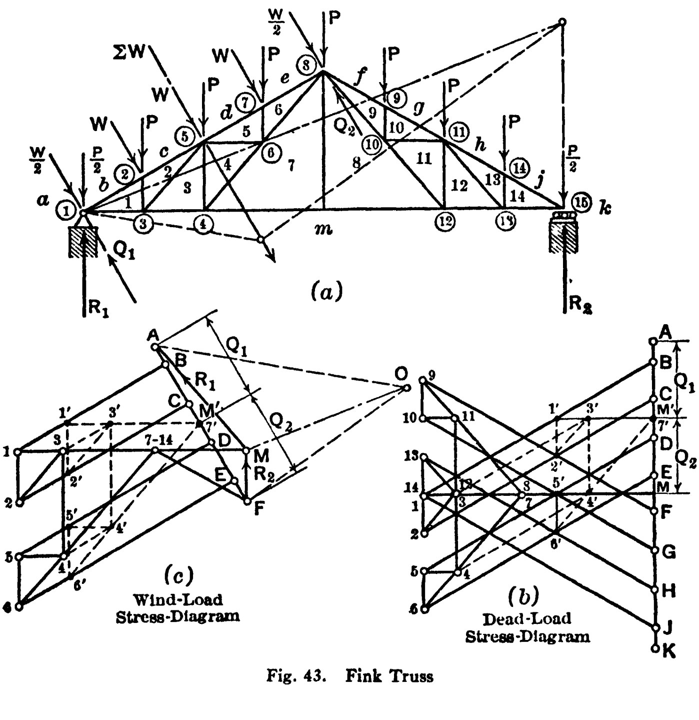


Fig. 43. Fink Truss

building-construction, however, the roof-trusses are supported by columns to which they are rigidly connected by means of riveted joints and braced by inclined members called KNEE-BRACES. The truss together with the supporting columns and the knee-braces forms a transverse unit known as a BENT. The dead and snow-load stresses in the members of the truss of a transverse bent are the same as for a truss supported upon masonry walls. Wind-loads or other horizontal forces produce stresses throughout the truss which depend to a large extent upon the action of the supporting columns. The columns may be assumed as:

(1) hinged at the top and the base;
(2) hinged at the top and fixed at the base; or
(3) rigidly fixed at the top and at the base.

The assumption to be made will depend upon the judgment of the designer and the knowledge he may have of the probable conditions. It is difficult, if not impossible, exactly to realize any one of the conditions which may be assumed, as there are so many variable factors entering into the problem.

If the columns are merely anchored in the usual manner to the foundation with no effectual attempt to fix them against rotation by a deep embedment in concrete or by some other means, they should be considered as hinged at the base and the top.

When the columns are fixed at the base so that the tangent to the elastic curve, under flexure, remains parallel to the original position of the neutral axis of the unloaded column, a point of CONTRA-FLEXURE will occur about midway between the base and the foot of the knee-brace. In this case the column may be taken as hinged at the point of contra-flexure, which is equivalent to a decrease in the effective length of the column by an amount equal to the distance from the base to the point of contra-flexure. The maximum positive bending moment in the columns is at the foot of the knee-brace of the leeward column and is equal to the product of the horizontal component of the wind reaction into the distance from the point of contra-flexure to the foot of the knee-brace. The maximum negative moment occurs at the base of the leeward column and is numerically equal to the positive moment when the point of contra-flexure is assumed at the mid-point between the base and foot of the knee-brace. It is rarely possible to fix the supporting columns at both the top and the base. When a positive attempt is made to fix the columns, the resulting condition is probably more nearly that of a column fixed at the base and hinged at the top.

In the case of any one of the above end conditions, when the columns are similar, the most reasonable assumption relative to wind-load reactions is that their horizontal components are equal.

Example 1. Fig. 44 illustrates an eight-panel Fink truss supported on steel columns and knee-braced to them by the members 3–x and 17–x. Taken as a whole, the bent is an incomplete frame, and in order to permit the construction of the stress-diagrams the columns must be braced temporarily by the auxiliary members b–1, c–2, and 1–2 at the left column and by a similar set of members at the right column. The addition of these members provides a trussed support eliminating the bending moment in the columns. The stresses in the members of the truss are not affected, as may be shown by considering the portion of the structure to the left of plane ZZ as a free body in equilibrium. The stress in member e–5 can be determined by taking the summation of moments of the external forces, to the left of ZZ about joint 6 and dividing the result by the moment arm of e–5 about joint 6. The external loads and the reaction considered are not changed in magnitude, direction or action line by the addition of the auxiliary members, and hence the stress in member e–5 is not affected. Since, at joint 5, the stress in e–5 remains unaltered and the external loads remain unchanged, the stresses in members d–4 and 4–5 are not affected by the auxiliary members. In a similar manner it can be shown that the stress in each member of the truss is independent of the temporary members added to the supporting columns.

Since the dead and ceiling-loads are symmetrical about the center line of the structure, $R_1 = R_2 = \frac{1}{2}\Sigma V$. The dead plus ceiling-load stress-diagram is shown at (b). The reactions are vertical for this loading, and since the deformation of the truss is neglected, there are no stresses in the knee-braces and the stress-diagram is the same as for a truss supported on masonry walls.

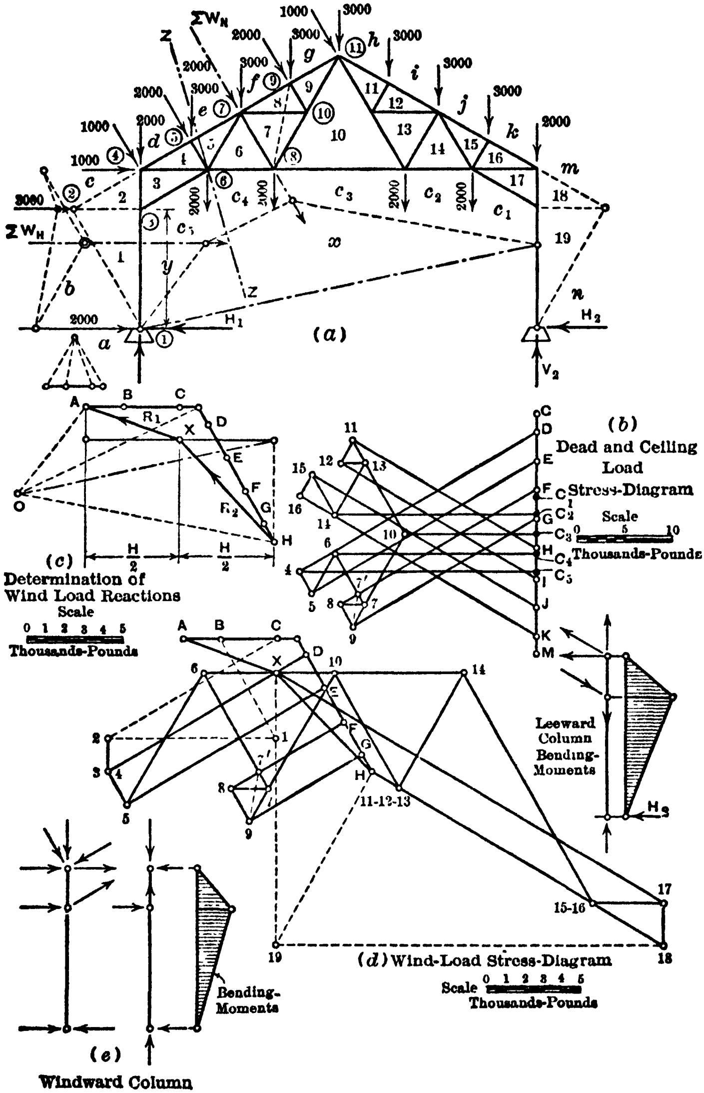


Fig. 44. Transverse Bent. Columns Hinged at Base

In this example the columns are assumed to be hinged at the top and the base and the wind-load reactions, therefore, are at the bases of the columns. The horizontal components of these reactions will be assumed to be equal. These horizontal components will produce moments in the columns of a maximum value, at the foot of the knee-braces, equal to $\left(+H_1 \cdot y - w\frac{y_2}{2}\right)$ in the windward column and $+H_2 \cdot y$ in the leeward column, as indicated at (*a*), Fig. 44. With the temporary bracing in place the moments are eliminated and the true stresses in the columns due to the combination of direct stress and bending will be determined as a subsequent problem. The wind-loads are laid off in order at (*c*); the pole O selected and the funicular polygon constructed as shown at (*a*). The right reaction was tentatively assumed to be vertical in order to determine the true magnitudes of V_1 and V_2. The total horizontal component of the wind-loads is assumed equally divided to each reaction, thus determining R_1 and R_2, as shown at (*c*). The wind force-polygon, loads and reactions, is redrawn at (*d*) and the stress-diagram constructed in the usual manner. The stress-diagram gives the true values and characters of the stresses in all the members of the bent, except the columns.

The true direct stresses in the windward and leeward columns are respectively equal to V_1 and V_2. The maximum bending moment occurs at the foot of leeward knee-brace and is equal to $H_2 \cdot y$. The columns may be considered as beams, in so far as the moments are concerned The leeward column is supported at the top by the horizontal resultant of the stresses in the chord members, supported at the base by H_2, and loaded at the foot of the knee-brace by the horizontal component of the knee-brace stress. The maximum combined unit stress in the leeward column is:

$$f_{max} = \frac{V_2}{A} + \frac{(H_2 \cdot y)c^*}{I} \tag{1}$$

The combined stress in the windward column may be found in the same manner. The moment in this column includes the moment of the external wind-load in the height of the column, which in constructing the stress-diagram was concentrated at joints ①, ③ and ④.

Example 2. The columns of the transverse bent shown in Fig. 45 are assumed to be fixed at the base, and the point of contra-flexure is assumed to be midway between the base and the foot of the knee-brace. The columns may, therefore, be considered as hinged at the points of contra-flexure, and in so far as the upper portion of the structure is concerned the horizontal and components at this hinge supply the horizontal reactions.

The general procedure in finding the reactions and constructing the stress-diagrams is the same as in the preceding example.

The Hammer-Beam Truss. A typical hammer-beam truss such as illustrated in Fig. 46 may be considered to be composed of three interdependent parts:

(*a*) A simple truss supported at joints ⑤ and ⑩ with the apex at joint ⑧

(*b*) A supporting truss on the left bounded by joints ①, ②, ③, and ⑤.

(*c*) A supporting truss on the right bounded by joints ⑭, ⑬, ⑫ and ⑩.

Members 1–2 and 9–10 are known as the hammer-beams. The entire framework is supported at joints ① and ⑭ by masonry walls which continue

* The value determined by this expression is approximate, but near enough for practical design. For an exact treatment of combined stresses see Modern Framed Structures, by Johnson Bryan and Turneaure, Part II, page 516, Ninth Edition.

upward to the level of joints 2 and 13, and members *a*–1 and *k*–10 are usually secured rigidly to the masonry walls. Hammer-beam trusses of the general

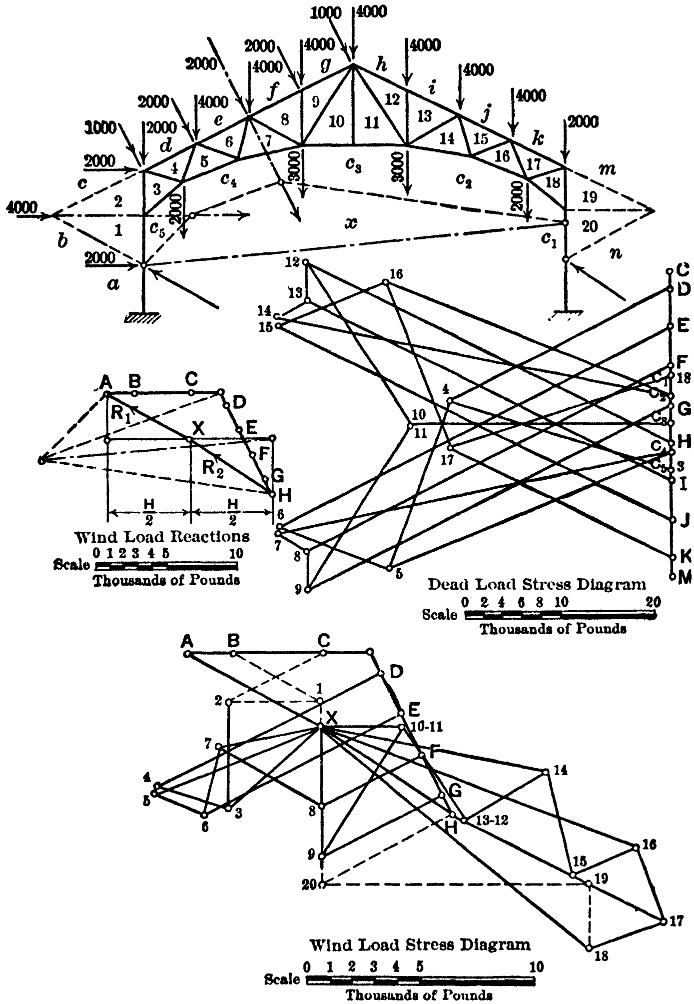


Fig. 45. Transverse Bent. Columns Fixed at Base

type illustrated in Fig. 46 are, strictly speaking, statically indeterminate structures, since joints 1, 2 and 6 as well as the corresponding joints of

the opposite half of the truss are rigid as actually constructed and are, therefore, subjected to moments under all conditions of loading.

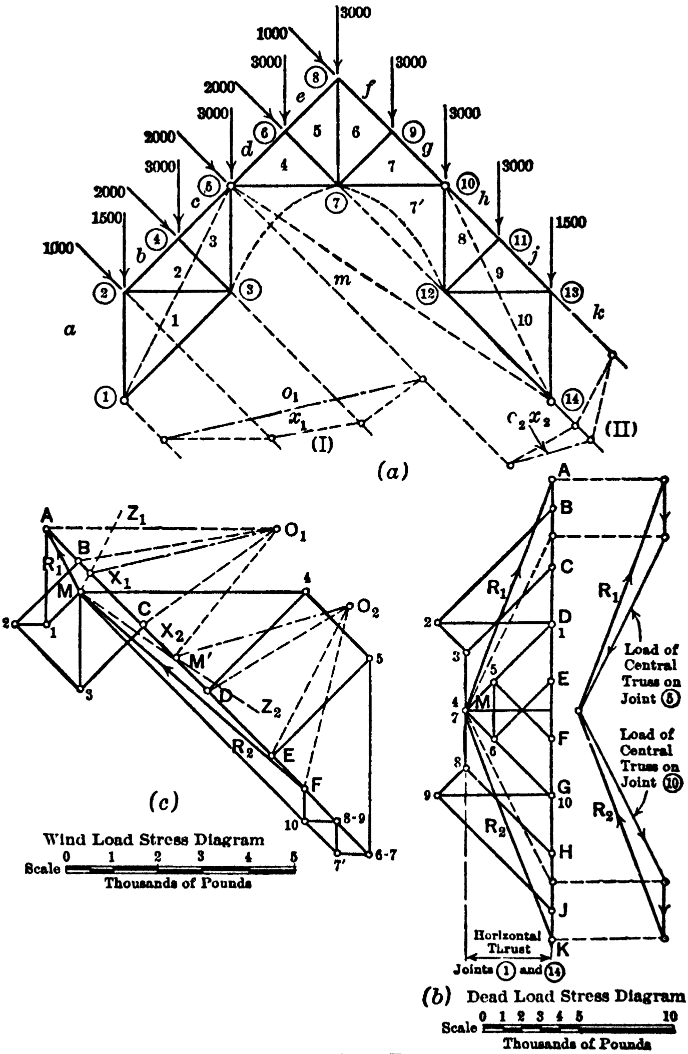


Fig. 46. Hammer-beam Truss

However, by assuming that the two side portions, containing the hammer-beams, are supported at joints (1) and (14), respectively, by hinge-like details,

and also assuming hinges at joints ⑤ and ⑩, the stresses become statically determinate for symmetrical vertical loads. Under unsymmetrical loads either the hinge at ⑤ or at ⑩ must be assumed to be locked by the ornamental curved members shown in broken lines at (a).

The reactions at joints ① and ⑭ are inclined for all conditions of loading, and therefore a horizontal thrust is always present at these points of support. This thrust must be provided for by heavy masonry wall construction and buttresses. This is an important consideration, since the following analysis is based upon the assumption that lateral rigidity of the supports is available.

The stress-diagram for uniform vertical loading over the entire structure is shown at (b), Fig. 46. Since the truss is assumed to be supported by hinges at joints ① and ⑭ and since hinge-joints are also assumed at ⑤ and ⑩, the reaction at joint ① must have a direction fixed by the right line connecting the hinge-joints at ① and ⑤, otherwise a moment would be introduced at the hinge not on the line of action of the equilibrating force. For the same reason R_2 must have the fixed direction as determined by the line connecting joints ⑭ and ⑩. The direction of the reactions being fixed, the force-polygon may be drawn and the stress-diagram constructed as shown at (b). The stress-diagram for wind-load on the left-hand slope of the roof is shown at (d), Fig. 46.

As stated above, to provide stability under the action of unsymmetrical loading one of the curved members ③–⑦ or ⑦–⑫ must be assumed to act. Although the member thus provided is curved, the stress can be determined as for a straight member connecting joints ③ and ⑦ or ⑫ and ⑦. Since it is more practical to provide the proper end details for compression-members in timber framing than to provide suitable tension connections, the leeward member ⑦–⑫ will be assumed as connecting joints ⑦ and ⑫ along a right line. When the stress in the straight member has been found, the stress in the curved member may be determined by adding the moment stress due to the eccentricity to the direct stress found in the straight member as given by Equation (1) of the preceding example.

Since the member ⑦–⑫ eliminates the hinge at ⑩, the structure may be considered as divided into two parts by the intermediate hinge at ⑤. Each part of the structure may then be considered as supported by one wall reaction and by the other part at the common hinge. There are four reactions then to be determined, two of which, the right reaction of the left portion and the left reaction of the right portion, both at the intermediate hinge ⑤, must be equal and opposite.

Lay off the wind-load line at (c), Fig. 46. One-half of the total wind-load is on each portion of the structure, the left portion supporting the amount shown on the load line as AM'. With any pole, as O_1, construct the force and funicular polygons for the left portion, assuming for convenience that the reactions at ① and ⑤ are parallel. The closing string o_1–x_1 from (I) determines the magnitudes of the reactions as assumed. In like manner construct the force and funicular polygons, using pole O_2 for the wind-loads on the right portion. The closing string o_2–x_2 from (II) locates X_2 dividing $M'F$ into two reactions, each assumed parallel to the wind-load. It will be noted that x_2 in this example coincides with M', showing, for the assumption made relative to the direction of the reactions, that the left reaction of the right portion on hinge ⑤ is zero, for the assumption of parallel reactions. This would be expected, since the resultant of the wind-loads on the right portion passes through the right support.

Point X_1 was determined by the closing string o_1x_1 and the tentative assumption of parallel reactions; but the closing string of the true funicular

polygon for the left-hand portion must pass through hinges 1 and 5 since no other points on the reactions at these hinges are known. The correct pole must therefore lie on line X_1Z_1, drawn through X_1 parallel to 1–5, and for similar reasons the true pole for the right-hand portion must be on X_2–Z_2. Therefore, a single pole satisfying these conditions for the load line AF is at the intersection of X_1–Z_1 and X_2–X_2, which is point M, Fig. 46(*c*). Then F–M and M–M' are the true reaction components of M'–F, while M'–M and M–A are the reaction components of A–M'. It is evident, therefore, that F–$M = R_2$ and $MA = R_1$, the desired reactions at joints 14 and 1, respectively. The wind-load stress-diagram is constructed in the usual manner, as shown at (*c*).

Cantilever Trusses with Central Supported Span. The structure illustrated in Fig. 47 consists of two cantilever trusses supporting a central simple truss, the three so membered as to form a single unit.

It will be assumed that the intermediate supports R_2 and R_3 are slender columns which are unable to supply any horizontal reaction. The horizontal component of the wind-load may then be equally divided between the supports at the outer walls.

Let it be required to find the reactions for, and the stresses in the members of, the structure loaded with dead and wind-loads as shown at (*a*).

Lay off the load line for the resultant wind and dead loads on the central truss at (*b*), and with any convenient pole O_1 construct the funicular polygon for these loads, assuming tentatively that the right reaction, R'', of the central truss, is vertical. The closing string locates X_1, following the assumption that R'' is vertical. As one-half of the horizontal component of the wind-load has been assigned to R_4, and since R_3 has been assumed vertical, R'' must have a horizontal component equal to the horizontal component of R_4, since these are the only horizontal forces acting on the right cantilever.

At (*c*) lay off the wind-loads to the same scale as the load line at (*b*) and divide H into two equal parts x_1x and xx_2. The horizontal component of R'' must be equal to x_1x; therefore, at (*b*) make $x_1x = \frac{H}{2}$, then, $Px = R''$ and $xG = R'$.

The loads on the left-hand cantilever, including the reaction R', are laid off at (*d*):

1 = resultant of upper chord loads;
2 = resultant of lower chord loads;
3 = resultant of horizontal wind-load and R' at the inner end of cantilever.

With any convenient pole O_2 construct the ray-diagram at (*d*) and the funicular polygon for the loads on the left cantilever as shown in (*a*).

The ray through O_2 parallel to the closing string locates Y on a vertical through F, then, $FY = R_2$ and $YA = R_1$. It will be noted that the direction of R_1 is downward to the left, indicating a negative reaction at the windward wall. In this case the vertical reaction R_2 must equal the total vertical component of the applied loads plus the vertical component of R_1. The load line for the right cantilever is shown at (*e*), where:

4 = the resultant of upper chord loads;
5 = the resultant of lower chord loads;
6 = the central truss reaction R'', at the inner end of the cantilever.

With any pole such as O_3 the funicular polygon for the loads on the right cantilever is drawn, and the closing string paralleled through O_3 locates Z on a vertical through W, then, $WZ = R_3$ and $ZQ = R_4$. The four reactions

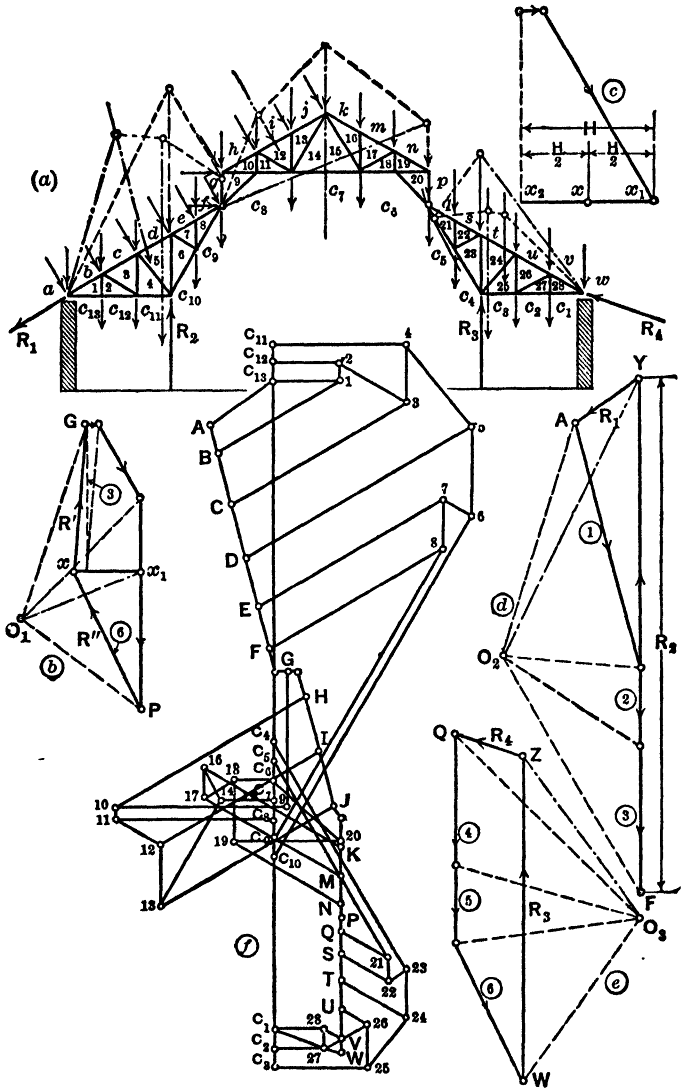


Fig. 47. Cantilever Trusses with Central Supported Span

having been determined the complete force-polygon is laid off in clockwise order to include all loads and reactions, as shown at (*f*). The stress-diagram is drawn in the usual manner beginning at the left support, and closing points are encountered at points 8, 20, and 28, indicating that the three portions of the structure might be considered separately after the correct reactions are determined.

8. The Three-Hinged Arch

The hammer-beam truss under wind-loads with assumed hinges at ①, ⑤ and ⑭, as analyzed in the preceding article, is in reality a THREE-HINGED ARCH.

The more common form of the three-hinged arch as employed in modern building-construction is illustrated in Fig. 48. The principal problem involved in the solution of a three-hinged arch is the determination of the reactions. Reactions occur at the three hinges and are assumed to act through the centers of the hinges. The reactions of one segment of the arch upon the other at the intermediate hinge must necessarily be equal and opposite.

The reactions at the end hinges are inclined for all types of loading and may be supplied by an abutment or base which is capable of resisting both the horizontal and vertical component. In many cases, however, it is desirable to relieve the footing of a part of the horizontal component by the introduction of a tie-member connecting the end hinges.

When the loads on a three-hinged arch are vertical, the tie provides the total horizontal resistance resulting from arch action, and the reaction of the footings is vertical.

In the case of wind-loads, the horizontal component must be divided in some manner between the two footings.

In general, the most logical assumption is that the horizontal component of the wind-load is equally divided between the two reactions. In any case where the external loads have a horizontal component the tie-rod stress is equal to the difference between the horizontal reaction due to arch action and the amount of horizontal component assigned to the leeward support.

Example 1. Reactions. Case 1. Fig. 48 (*a*). The general method of finding the reactions of a three-hinged arch may best be explained by considering a single load on one segment.

Let it be required to find the reactions of the three-hinged arch loaded as shown at (*a*), Fig. 48.

The load *a–b*, supported by the left-hand segment, causes reactions at hinges Ⓐ, Ⓑ and Ⓒ. The reactions at Ⓒ and Ⓑ are the only forces acting upon the right-hand segment. Since this segment is held in equilibrium by these two forces, they must have the same line of action and be of opposite direction. The common action line of the reactions at Ⓑ and Ⓒ is, therefore, determined by the right line joining the centers of the hinges at Ⓑ and Ⓒ The left-hand segment is held in equilibrium by three forces, namely, the hinge reactions at Ⓐ and Ⓒ and the applied load *a–b*.

Three forces in equilibrium must meet in a common point; hence, the direction of the reaction at hinge Ⓐ may be determined since the direction of the other two forces are known. Prolong the line of action of the reaction at Ⓒ, direction Ⓑ–Ⓒ, to intersect the line of action of *a–b* in *m*; then Ⓐ–*m* must be the direction of the reaction at hinge Ⓐ.

The directions of the lines of action of R_1 and R_2 being known, their magnitudes may be determined by drawing the force-polygon as shown at (*b*).

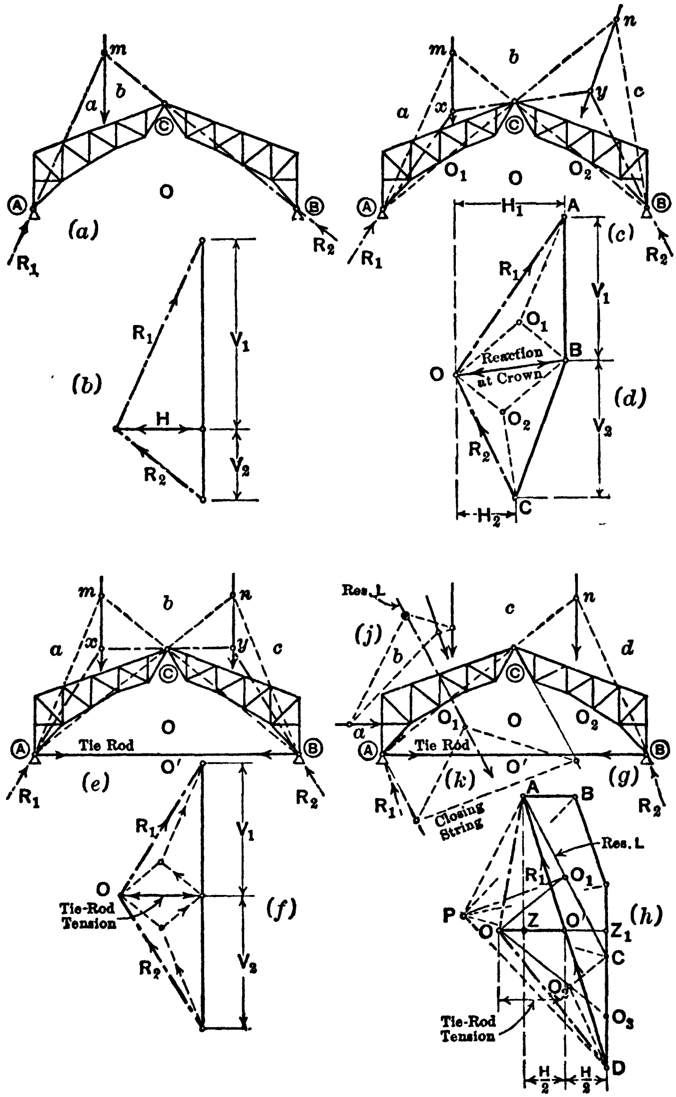


Fig. 48. Three-hinged Arch

The reaction of each segment of the arch against the other at hinge Ⓒ must be equal and opposite. From the force-polygon at (*b*), it is evident that the left segment is in equilibrium when R_2 is applied either at Ⓒ or Ⓑ and the reaction at Ⓒ due to the left-hand segment is R_2 acting downward to the right, which is neutralized by R_2 at Ⓑ acting upward to the left along the same action line. The three-hinged arch taken as a whole is, therefore, in equilibrium under the action of the three forces R_1, $a-b$, and R_2. In this example the horizontal components of R_1 and $R_2 = H$ are assumed to be provided by the foundation.

Case 2. Both Segments Loaded. Let it be required to find the reactions for the three-hinged arch loaded as shown in Fig. 48 (*c*).

When both segments of the arch are loaded, the reactions at any hinge are equal to the resultant of the reactions due to the loads taken separately as in Case 1.

Consider the left-hand segment and let the resultant of external loads on that segment be $a-b$, and assume temporarily that there is no load on the right-hand segment. As in Case 1, prolong Ⓑ–Ⓒ to intersect the action line of $a-b$ in m and draw Ⓐ–m.

Lay off $A-B$ (equal to load $a-b$) in the force-polygon at (*d*). Draw BO_1 parallel to the action line of R_2 (Ⓑ–Ⓒ), and O_1A parallel to the action line of R_1 (Ⓐ–m), closing the force-polygon for the load on the left-hand segment only. Next, consider the right-hand segment, and let the resultant of external loads on that segment be $b-c$, and assume temporarily that there is no load on the left-hand segment. Prolong Ⓐ–Ⓒ to intersect the action line of load $b-c$ in n, and draw Ⓑ–n, thus determining the direction lines of the reactions for the load on the right-hand segment only. Lay off $B-C$ (equal to load $b-c$) in the force-polygon at (*d*), and draw $C-O_2$ and O_2-B parallel, respectively, to the action lines of R_2 and R_1 for the single load $b-c$. When both segments are loaded simultaneously

$$R_2 = \text{vector sum } BO_1 \text{ and } CO_2$$
$$R_1 = \text{vector sum } O_1A \text{ and } O_2B$$

The separate reactions for each loading condition are readily determined by force-triangles as shown at (*d*), from which the reactions for the combined loading are given by $C-O = R_2$ and $O-A = R_1$ in magnitude and direction. The reactions at hinge Ⓒ are $O-B$ and $B-O$ upon the left and right segments respectively. The action lines of R_2 and the hinge reaction at Ⓒ must intersect on the line of action of $b-c$, as shown, at point y, since the three forces are in equilibrium. Likewise, the action lines of R_1 and the hinge reaction at Ⓒ must intersect on the line of action of $a-b$, as shown, at x, Fig. 48 (*c*).

The horizontal components of R_1 and R_2 in this case are supplied by the foundations.

Case 3. Both Segments Symmetrically Loaded. End Hinges Connected by a Tie-Rod. In the case of a symmetrical arch, symmetrically loaded, Fig. 48 (*e*), it is evident from the preceding case that the reactions at hinge Ⓒ will be horizontal. The reactions of the end hinges can be found directly for either segment. The point x or y is determined by the intersection of a horizontal line through Ⓒ and the action line of the resultant load on one segment, as shown in (*e*), Fig. 48.

The construction in this and the preceding examples has an important interpretation, as follows: The point O can be considered as a pole-point, and the lines Ⓐ–x, $x-y$, and y–Ⓑ the strings of a funicular polygon for the given loads. It is evident, then, that the reactions at the hinges can be determined

by constructing a funicular polygon for the given load system such that the three strings, representing the three-hinge reactions, will pass through their respective hinge points. The lengths of the corresponding rays, in the force-polygon, give the magnitudes of these reactions.

Case 4. Combined Wind and Dead Load Reactions for Arches with Tie-Rods. Let it be required to find the reactions for the three-hinged arch, loaded as shown in Fig. 48 (*g*). Lay off the load, line A to D, as shown at (*h*), and with any convenient pole, such as P, construct the force-polygon, and at (*j*) the funicular polygon, to determine the position of the resultant of the loads on the left-hand segment.

The reactions may now be determined for the two loads A–C, on the left segment, and C–D, on the right, by locating the proper pole such that a funicular polygon for these loads will pass through points Ⓐ, Ⓒ and Ⓑ.

Consider the left-hand segment as an independent structure, assume tentatively that the reactions at Ⓐ and Ⓒ are parallel to the resultant load A–C, and with the pole P draw the funicular polygon at (*k*). The closing string locates, in the force-polygon, point O_1 dividing A–C into two reactions having the proper magnitude and direction for the tentative assumption. The funicular polygon must, however, have a pole O, such that strings C–O and O–A will pass through hinges Ⓒ and Ⓐ, respectively. The true closing line of such a funicular polygon is, therefore, parallel to Ⓐ–Ⓒ. Through point O_1 draw O_1–O parallel to the known direction of the true closing line; then O_1–O is the locus of all pole-points which will satisfy the required condition.

Consider the right segment independently and prolong Ⓐ–Ⓒ to intersect the action line of load C–D in n. Draw D–O_2 and O_2–C parallel to Ⓑ–n and n–Ⓒ, respectively, to determine, at their intersection, the point O_2, which is the pole-point of the funicular polygon, for load C–D, which will pass through points Ⓑ and Ⓒ. The closing string of such a polygon is O_2–O_3, parallel to line Ⓑ–Ⓒ. The locus of all points satisfying the reaction requirements for the right-hand segment is line O_2–O_3 prolonged. Prolong O_2–O_3 to intersect O_1–O at point O, then point O is the true pole-point for which the reaction conditions for all three hinges are satisfied.

The end reactions are, therefore, D–$O = R_2$ and O–$A = R_1$ in magnitude and direction as shown at (*h*).

Let it be assumed that the horizontal component of the external load system is equally divided to the two footings. Project A vertically to the horizontal line through O, and project D vertically to this line, locating points Z and Z_1, respectively. Divide Z–$Z_1 = H$ into two equal parts locating point O', then DO' and $O'A$ are the reactions of the arch upon the footings and O'–O is the tension in the tie-rod.

Stresses in Three-Hinged Arches. After the reactions for the arch, under the given system of loads, have been determined, the stress diagram may be constructed in the usual manner.

Example 1. Dead Load Stress-Diagram. Let it be required to construct the stress-diagram for the three-hinged arch loaded as shown in Fig. 49 (*a*). Lay off the total load line as shown at (*b*), and with any convenient pole P construct the ray-diagram (force-polygon) for the loads on the left-hand segment and at (*c*) draw the funicular polygon to determine the position of the resultant load on the left segment. Since the loading is symmetrical, the resultant load on the right segment has the same relative position to that segment as the resultant load on the left has to the left segment.

The reactions at hinge Ⓒ are horizontal, and the stress-diagram for one seg-

ment is identical, but of opposite hand, to the other segment. In this example the complete stress-diagram is shown to illustrate the symmetry. The reactions are found by the method described under Case 3 of the preceding article and illustrated in Fig. 47 (*f*).

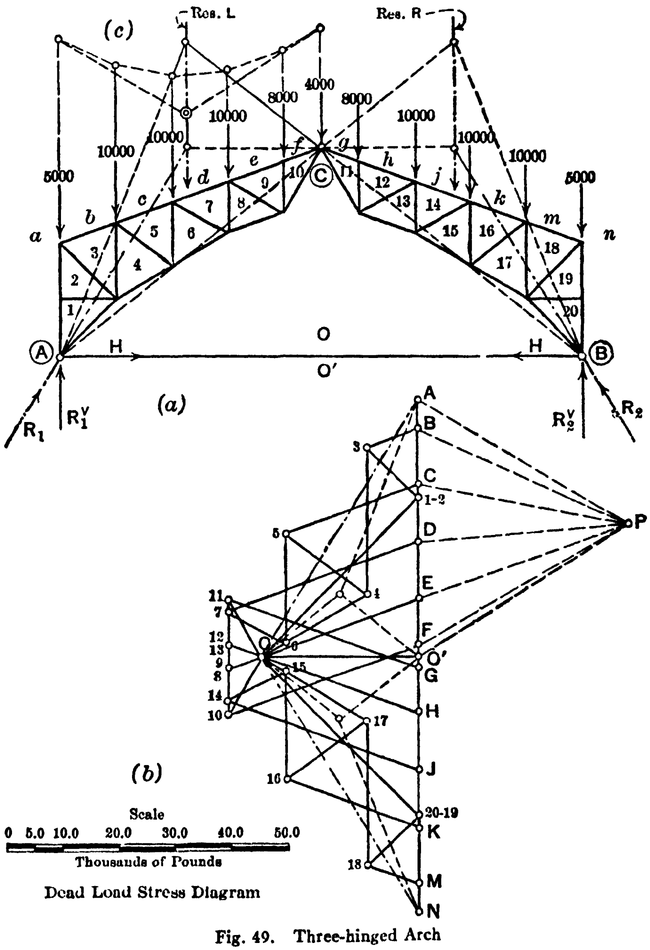


Fig. 49. Three-hinged Arch

The stress-diagram started at hinge Ⓐ and will close with members *O*–10 and 10–*f* at hinge Ⓒ. The stress-diagram is then continued for the right segment beginning at hinge Ⓒ and closing with members *n*–20 and 20–*O* at hinge Ⓑ.

If the tie-rod is removed from the arch in Fig. 49 (*a*) the stress-diagram at

(*b*) is unaltered. The only difference exists in the reactions upon the foundation, which, instead of being $N\text{-}O' = R_2$ and $O'A = R_1$ for the arch with the tie-rod in place, become $NO = R_2$ and $O\text{-}A = R_1$, and the horizontal thrust $O\text{-}O'$ must be supplied by the footings.

Example 2. Wind-Load Stress-Diagram. Let it be required to construct the wind-load stress-diagram for the three-hinged arch, loaded as shown in Fig. 50 (*a*).

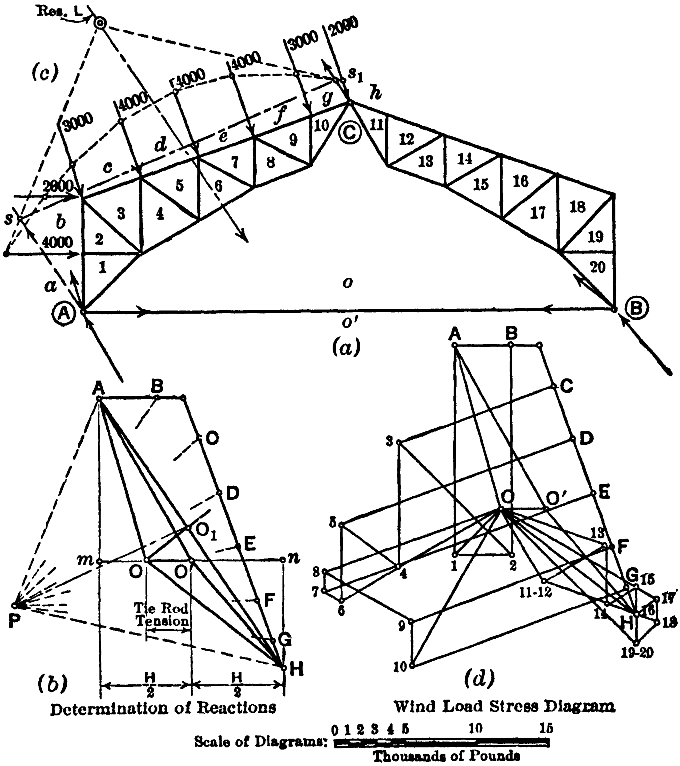


Fig. 50. Three-hinged Arch

Lay off the load line A to H as shown at (*b*) and with pole P construct the funicular polygon as shown at (*c*) to determine the position of the resultant $A\text{-}H$ of the given load system. Assume the reactions at Ⓐ and Ⓒ to be parallel to the wind-load resultant and extend the first and last string of the funicular polygon at (*c*) to intersect the assumed action lines of these reactions in points s and s_1. Line $s\text{-}s_1$ is the closing string for the reactions as assumed and locates point O_1 in the force-polygon at (*b*).

Through O_1, draw $O_1\text{-}O$ parallel to Ⓐ-Ⓒ, the known direction of the closing line for all funicular polygons passing through hinges Ⓐ and Ⓒ. Since there

is no load on the right segment draw H–O through H parallel to Ⓑ–Ⓒ, the known direction of the reactions passing through hinges Ⓒ and Ⓑ.

The intersection of O_1–O and H–O locates point O, the true pole-point for the one funicular polygon which will pass through the three hinges. The reactions at the end hinges are, therefore, H–$O = R_2$ and O–$A = R_1$. Assuming the horizontal component of the wind resultant to be equally divided between the foundations at Ⓐ and Ⓑ, as in Case 4 above, the reactions on the foundations are H–$O' = R_2$ and O'–$A = R_1$, and the tension in the tie-rod is O'–O, as shown at (*b*).

In order to avoid confusion the load-line reaction-polygon is redrawn at (*d*) and the stress-diagram for the entire arch constructed thereupon. It should be noted that this diagram also closes for each arch segment. If the tie-rod were omitted in this case the horizontal thrust upon the footings would be o–m at Ⓐ and o–n at Ⓑ, and the total reactions would be

$$R_1 = OA \quad \text{and} \quad R_2 = HO.$$

9. Stresses in Redundant Members

The addition of members beyond the minimum number required to provide a complete frame, as given by Equation (1), Chapter XXV, results in a statically indeterminate condition. Every redundant member introduces one unknown quantity in excess of the number which can be determined by the methods of statics.

The calculation of the stresses in the members of a truss, when redundant members are present, is made possible by determining the relation between the distortion of the necessary members and the redundant members.

Consider the scissors truss with a tie-rod connecting the two end-joints A and E, as shown in Fig. 51. The minimum number of members necessary is $n = 2p - 3$, or $n = 2 \times 6 - 3 = 9$.

The structure, therefore, has one redundant member. In this case any member may be taken as the redundant member, and the tie-rod, member x–x', will be so taken. Assume, temporarily, that the tie-rod is replaced by a horizontal stress at A and E equal to S_x (the stress in the tie-rod). Under the action of the given loads, the right end of the truss, which is on a sliding detail, will move to the right some distance, Δ. Let S_x be the horizontal force necessary to bring the right reaction back to its original position. The various members of the truss will be affected by the action of S_x, and the stress in any member due to S_x will vary directly with the magnitude of S_x.

Let U_1, U_2, U_3, etc., represent the stresses in members 1, 2, 3, etc , due to S_x = unity, and let the deformations of these members due to their stresses from the given loading be $\delta 1$, $\delta 2$, $\delta 3$, etc.

The total internal work done on the members of the truss by the application of S_x = unity, is equal to the sum of the average stress produced in each member by S_x multiplied by the distance through which this stress acts, or:

$$\text{Total internal work} = S_x \cdot \Sigma_1^9 \tfrac{1}{2} U \cdot \delta \qquad (1)$$

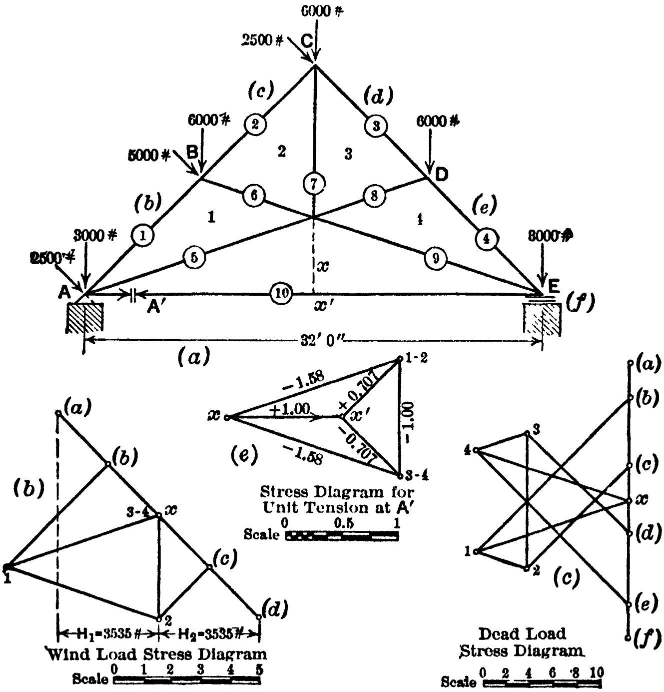
$$\text{Total external work} = \tfrac{1}{2} S_x \cdot \Delta \qquad (2)$$

Since for any elastic structure in equilibrium the external work of applied forces must equal the internal work of the stresses due to those forces:

$$\Delta = \Sigma_1^9 U \cdot \delta \qquad (3)$$

The deformation of any member, δ, is given by the fundamental equation of elasticity as:

$$\delta = \frac{S \cdot L}{A \cdot E} \tag{4}$$



Member	Stresses in Kips for				Section	Area Sq Ins
	Dead Load	Wind Left	Wind Right	Maximum Combination		
1	−19.0	−5.0	−5.0	−24.00	2Ls 3"x 3"x 3/8"	4.22
2	−12.7	−2.5	−5.0	−17.70	do.	do.
5	+14.2	+5.6	0	+19.80	2Ls 2½"x2"x¼"	2.12
6	−4.8	−5.6	0	−10.40	do.	do.
7	+12.0	+3.5	+3.5	+15.50	1L 2½"x 2"x 5/16"	1.31

(d)

Fig. 51. Scissors Truss with Tie-rod

in which S = the TOTAL STRESS in the member in pounds per square inch;
L = the length of the member in inches;
A = the cross-sectional area of member in square inches;
E = the modulus of elasticity of material in pounds per square inch.

Substituting the value δ as given by (4) in (3):

$$\Delta = \sum_1^9 \frac{SUL}{AE} \tag{5}$$

The total stress S in any member may be considered as composed of two parts: (1) a stress S', due to the given load system with the redundant member removed; (2) a stress $U \times S_x$, due to the stress in the redundant member, where U is the stress due to S_x = unity.

Then from Equation (5):

$$\Delta = \sum_1^9 \frac{S'UL}{AE} + \sum_1^9 \frac{(US_x)UL}{AE} \tag{6}$$

which is the expression for the horizontal displacement of the right end of the truss and must be equal and opposed to the deformation of the tie-rod, the latter being

$$\delta_x = (S_xL_x)/A_x \cdot E$$

then

$$\sum_1^9 \frac{S'UL}{A \cdot E} + \sum_1^9 \frac{(US_x)UL}{A \cdot E} = -\frac{S_xL_x}{A_x \cdot E} \tag{7}$$

whence

$$S_x = -\frac{\sum_1^9 \frac{S'UL}{A \cdot E}}{\sum_1^9 \frac{U^2L}{A \cdot E} + \frac{L_x}{A_xE}} \tag{8}$$

Careful attention in the above procedure must be paid to sign, tension being considered plus and compression minus. The stress U is the stress in any member due to a one-pound TENSION in the redundant member.

Example 1. Let it be required to find the stresses in the scissors truss with a horizontal tie-rod, illustrated in Fig. 51.

The simple stress-diagrams for the given loading, assuming the tie-rod to be removed, are shown at (*b*) and (*c*). It is assumed in finding the wind-load reactions that the friction in the sliding detail, at reaction E, is sufficient to permit the assumption that the horizontal component of the wind-load is equally divided to the two reactions.

The stresses as found in diagrams (*b*) and (*c*) are tabulated at (*d*) and the combination for maximum stress determined. Suitable members are then selected and their sections together with the section-areas tabulated in the last two columns of table (*d*). A one-inch round tie-rod (area = 0.785 sq in) will be assumed for member 10.

Assume the tie-rod to be in place, and subjected to a one-pound tension. The tie-rod may be imagined as cut at A' and forces of one pound introduced on each of the cut ends.

The stress-diagram for this load of unit tension in member 10 is shown at (*e*), from which the values of U are determined. The data necessary for the solution of Equation (8) are given in convenient form in Fig. 52, in which the common term E is omitted. The stress in the one-inch round tie-rod is found to be +7 540.0 lb, or a unit stress of 7 540/0.785 = 9 600 lb per sq in, tension. If the rod is not upset, the net area at the root of the threads is 0.55 sq in and the unit tension is 13 700 lb per sq in.

Column 9, Fig. 52, gives the combined stress $S = S' + S_{10} \cdot U$, which is the true stress in each member when the tie-rod is acting. It is evident from a comparison of the values of S' and the final true stresses that a considerable saving could be made by redesigning the members of the truss, which, however, would necessitate a new set of stress calculations.

Example 2. Stresses in a Scissors Truss without a Tie-Rod. If the tie-rod is omitted from the truss shown in Fig. 51 and the ends of the truss BUILT IN or anchored to the supporting walls, as shown in Fig. 53, the horizontal thrust resulting from the elastic deformation of the truss must be resisted by the walls.

①	②	③	④	⑤	⑥	⑦	⑧	⑨
Member	L Ins	A Sq Ins	S′ Kips.	U Lbs	$\frac{S'UL}{A}$ (Thousands)	$\frac{U^2 L}{A}$	$S_{10}U$ Kips.	True Stress S Kips.
1	136	4 22	−24.0	+0 707	−546.840	+16 106	+5 330	−18 670
2	136	4 22	−15.2	+0.707	−346.330	+16 106	+5 330	−9.870
3	136	4 22	−17.7	+0 707	−403 290	+16 106	+5 330	−12 370
4	136	4.22	−24 0	+0 707	−546 840	+16 106	+5 330	−18 670
5	204	2 12	+19 8	−1.580	−3010 350	+240 220	−11 913	+7 887
6	102	2.12	−10.4	0	0	0	0	−10 400
7	128	1 31	+15.5	−1.000	−1514.500	+97 710	−7.540	+7 960
8	102	2.12	−4.8	0	0	0	0	−4 800
9	204	2.12	+14.2	−1.580	−2158 940	+240.220	−11.913	+2.287
				Σ =	−8527 090	+642.574		

$$\frac{L_{10}}{A_{10}} = +490\ 000$$

$$+1132.574$$

$$S_{10} = -\frac{-8527090.000}{+1132.574} = +7540.0^{\#}$$

or 9600 Lbs per Sq Inch

Fig. 52. Data for Solution of Equation (8)

If one end, as at E, is placed on an expansion detail, that end of the truss will move outward an amount determined by the horizontal displacement of point E.

From the preceding analysis it may be seen that the horizontal displacement of the end free to move can be determined by introducing a unit tensile force acting along the line of the desired displacement at that end, then:

$$\Delta = \sum_{1}^{9} \frac{S'UL}{A \cdot E}$$

and the numerical value of this expression is the total of column 6, Fig. 52 divided by the modulus of elasticity of steel, or

$$\Delta = -\frac{8\ 527\ 000}{29\ 000\ 000} = -0.294$$

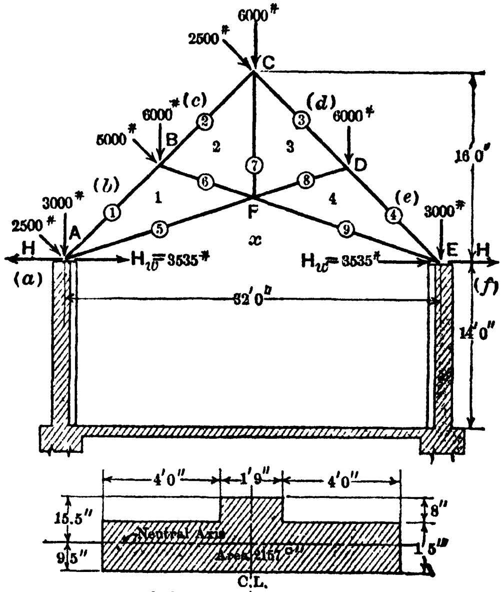
indicating that point E, at which point the horizontal unit tension is applied, moves in an opposite (minus sign) direction to the unit load, 0.294 in.

Thus, if one end of the truss is allowed a free horizontal movement the supporting walls will be relieved of a large part of the horizontal thrust due to elastic displacement. For relatively short-span trusses of the type illustrated in Fig. 53, both ends are often anchored, or built into the masonry walls.

Unless the supporting walls are absolutely rigid, which is practically impossible, they will move outward at their upper ends, following in general the laws governing the deflection of cantilever beams loaded at the free end, or:

$$\Delta = \frac{PL^3}{3\,EI} \tag{9}$$

As in the preceding example, one-half of the horizontal component of the



(a) Section through Pier

Fig. 53. Scissors Truss without Tie-rod

wind-load will be assumed to be resisted by each support. The primary stresses (S') are the same as before and are shown in column 4 of Fig. 52.

Let the horizontal thrust at each support, due to elastic deformation, be denoted by H, and let the equal horizontal wind-reactions be denoted by H_W. The outward deflections of the upper ends of the supporting walls are given by Equation (9) as follows:

$$\text{Left-hand wall:}\quad \Delta_L = (H - H_W)\,\frac{L^3}{3\,EI} \tag{10}$$

$$\text{Right-hand wall:}\quad \Delta_R = (H + H_W)\,\frac{L^3}{3\,EI} \tag{11}$$

The net change in length between points A and E is:

$$\Delta = \Delta_L + \Delta_R = \frac{2\,HL^3}{3\,EI} \tag{12}$$

The elastic change in length of the truss from A to E may be found as in the preceding example, noting, however, that the absence of the tie-rod gives rise to the horizontal thrusts H, acting in a direction opposite to the tension in the tie-rod of the preceding example.

The sense of H must therefore be taken as minus (thrust instead of tension).

Since the elastic change in length A to E must be equal to the net change in that length due to the deflections of the supporting walls, and as in Equation (7):

$$\sum_1^9 \frac{S'UL}{AE} + \sum_1^9 \frac{(-H)U^2L}{A\cdot E} = -\frac{2(-H)L_W{}^3}{3\,E_WI_W} \tag{13}$$

whence

$$H = \frac{\sum_1^9 \frac{S'UL}{A\cdot E}}{\sum_1^9 \frac{U_2L}{AE} + \frac{2\,L_W{}^3}{3\,E_WI_W}} \tag{14}$$

where L_W, E_W and I_W refer to the height, modulus of elasticity, and moment of inertia of the supporting walls.

The cross-section of the pilastered supporting walls is shown at (a), Fig. 53. The extent of wall available to offer horizontal resistance is limited by openings at 4 ft on each side of the pilaster. The moment of inertia of the wall section about its neutral axis is 73,089 in^4; its height above the point where the two walls are tied together is 14 ft (= 168 in), and the modulus of elasticity of the masonry will be taken at 2 000 000 lb per sq in; then

$$\frac{2\,L_W{}^3}{3\,E_WI_W} = \frac{2 \times \overline{168}^3}{3\,(2\,000\,000)\,73\,089} = 0.0000216$$

From column 6, Fig. 52:

$$\sum_1^9 \frac{S'UL}{AE} = \frac{8\,527\,000}{29\,000\,000} = 0.294$$

From column 7, Fig. 52:

$$\sum_1^9 \frac{U^2L}{AE} = \frac{642.574}{29\,000\,000} = 0.0000222$$

The horizontal thrust at the tops of the supporting walls is then given by Equation (14) as:

$$H = \frac{0.294}{0.0000222 + 0.0000216} = 6\,700\text{ lb}$$

The critical wall stress will occur on the inside face of the leeward pilaster at the top of the base. The bending moment at this section is:

$$M = (H_W + H)\,168 = (3\,535 + 6\,700)\,168 = 1\,720\,000\text{ lb-in}$$

The direction compression on the cross-section of the wall at the top of the base is equal to the sum of the vertical truss reaction and the weight of the wall, or:

$$12\,000 + 3\,500 + \frac{2\,157}{144} \times 14 \times 125 = 41\,700\text{ lb}$$

The unit compression is:

$$f_c = \frac{41\ 700}{2\ 157} = 19.4 \text{ lb per sq in}$$

The unit tension at the inner face of the pilaster is:

$$f_t = \frac{Mc}{I} = \frac{1\ 720\ 000 \times 15.5}{73\ 089} = 364 \text{ lb per sq in}$$

The resultant net stress on the masonry at this point is:

$$f = f_t - f_c = 364 - 19.4 = 344.6 \text{ lb per sq in, tension}$$

Since brick and stone masonry are never permitted to be subjected to tensile stress, a wall of these materials and of the dimensions shown is inadequate.

The design might be remedied by one or the other of the following methods:

(1) Place one end of the truss on an expansion bearing, and if anchorage of both ends is advisable or desirable in the completed structure, BUILD IN the expansion detail after the dead loads have been applied.

(2) Design a buttress for the outside of the wall of suitable proportions to reduce the tensile unit stress on the leeward pilaster to within allowable values.

10. Stresses in a Two-Hinged Arch

A two-hinged arch is a trussed frame, with hinged end supports, which has inclined reactions for vertical loads. The horizontal components of the reactions may be supplied either by the foundations or by a tie-member connecting the hinges. Two-hinged arches are statically indeterminate structures since the reactions cannot be determined by the usual methods of statics.

The VERTICAL REACTIONS are identical with the vertical reactions of a three-hinged arch or a simple truss having the same span and loading. The HORIZONTAL REACTIONS depend upon the deformation of the arch, and before the deformation can be calculated the sizes of the members must be known.

Any method of procedure for the calculation of the stresses in a two-hinged arch, therefore must consist of a series of successive approximations.

After the reactions have been completely determined, the stresses in the members of the arch may be found by either the algebraic or graphical methods, as in the case of simple trusses.

Calculation of Horizontal Reactions. The horizontal reactions of a two-hinged arch must be the forces which are necessary to prevent a change in length of the distance between the hinges if the ends of the arch were free to move horizontally.

In the preceding article it was shown by Equation (5) that the horizontal displacement of the free end of a simple truss is:

$$\Delta = \sum \frac{SUL}{A \cdot E} \tag{15}$$

If one end of the arch is temporarily assumed to be on rollers, that end must be prevented from moving by the application of a horizontal force, H. The negative displacement of the free end due to H will be:

$$\Delta_1 = \sum \frac{(-H)U^2L}{A \cdot E} \tag{16}$$

where U, as before, is the stress in any member due to H = unity. In this case H is applied as a thrust and the sign is, therefore, minus.

Since the values of Δ and Δ_1 given by Equations (15) and (16), respectively, must be equal:

$$H = -\frac{\sum \frac{SUL}{A \cdot E}}{\sum \frac{U^2L}{A \cdot E}} \tag{17}$$

When a tie-member is employed to provide the resistance to the horizontal displacement, the movement of the free end of the arch will be equal to the elongation of the tie-member.

The deformation of the tie is:

$$\delta = \frac{S_T L_T}{A_T \cdot E} \tag{18}$$

The final horizontal displacement of a two-hinged arch, with a tie-member and with one end free to move horizontally, is equal to the algebraic sum of: (*a*) the displacement of the arch as a simple truss; (*b*) the distance the stress in the tie will bring the free end back; and (*c*) the elongation of the tie, whence the stress in the tie-rod is (as in Equation 8):

$$S_T = -\frac{\sum \frac{S'UL}{A \cdot E}}{\sum \frac{U^2L}{A \cdot E} + \frac{L_T}{A_T E}} \tag{19}$$

Temperature Stresses in Two-Hinged Arches. When a horizontal tie is used, together with an expansion detail at one support, and all parts of the structure are equally exposed, the arch together with the tie-rod will expand and contract equally and the range of temperature will not affect the stresses in the arch. When the tie-member is placed in a protected trench beneath the floor or otherwise insulated from temperature variation, or where the arch is supported by rigid abutments, the stresses due to temperature differences must be considered.

The horizontal deformation due to a uniform change of temperature is:

$$\Delta_t = c \cdot t \cdot L \tag{20}$$

in which c = the coefficient of expansion of the material (for structural steel, $c = 0.0000065$ per degree F.);

t = the change in temperature in degrees Fahrenheit (or difference between temperature of arch and tie-rod in degrees Fahrenheit);

L = the length of the span in inches.

The horizontal reaction due to a change in temperature, $(-)$ = rise, $(+)$ = fall, is:

$$H_t = -\frac{\pm(c \cdot t \cdot L)}{\sum \frac{U^2L}{A \cdot E}} \tag{21}$$

or

$$S_t = -\frac{-(c \cdot t \cdot L)}{\sum \frac{U^2L}{A \cdot E} + \frac{L_T}{A_T E}} \tag{22}$$

The resulting stresses in the various members of the arch will, then, be equal to the algebraic sum of the stresses due to the given loads and the stresses due to temperature change, or

$$S = S' + (H_t \text{ or } S_t)U \qquad (23)$$

Procedure for Determination of Stresses in a Two-Hinged Arch. The following outline of procedure will facilitate the work of finding the stresses in a two-hinged arch:

(1) With one end of the arch free to move horizontally, find the stresses, as in a simple truss, for the given loads.

(2) Assume a unit horizontal reaction at each support and find the stresses in the members of the arch due to unit load, considering the arch to be a simple truss.

(3) Tabulate the values $\Sigma S'UL$ and ΣU^2L and assume tentatively that the areas of all members are unity and that E is constant. Then

$$H = -\frac{\Sigma S'UL}{\Sigma\, U^2L} \qquad (24)$$

in which S' = the stress in any member, considering the arch as a simple truss,

U = the stress in each member due to a horizontal reaction at each support of one pound.

(4) With the approximate value of H as found in (3), acting with the given loads, find the stresses in the structure by the usual methods of statics.

(5) Design the members for the stresses as found in (4).

(6) Tabulate the value of $\sum \frac{S'UL}{A \cdot E}$ and $\sum \frac{U^2L}{A \cdot E}$, using the areas as found in (5).

(7) Find a more accurate value of H, using the values found in (6) in Equation (17).

(8) Recalculate the stresses, using the more accurate value of H, redesign the members and procedure as in (6) and (7) until satisfactory results have been obtained.

The second approximation is usually sufficient. Gross areas may be used in the calculations; however, when it is evident that a tension-member will be considerably reduced, by punching, the average of gross and probable net areas should be used.

Example of the Determination of the Stresses in a Two-Hinged Arch. Let it be required to find the reactions for, and the stresses in the members of, the two-hinged arch loaded as shown in Fig. 54.

(1) Assuming the right support to be on rollers, the combined dead and wind-load stress-diagram is constructed at (*a*), Fig. 54, and the stresses as scaled from this diagram (values of S') are recorded in Table I, column 3.

(2) Assume a horizontal reaction of one pound (H = unity) at each support, and construct the stress-diagram for this loading as at (*b*), Fig. 54.

Record the stresses (values of U) in column 4 of Table I.

(3) From the tabular values, calculate $S'UL$ and U^2L for each member and record these values in columns 5 and 6 of Table I. Add, algebraically, the values in columns 5 and 6 and find the first approximate value of H by Equation (24), as shown at the bottom of Table I.

(4) With approximate value of H = 50 200 lb as found, lay off the load line for the arch as shown at (*c*), Fig. 54. The approximate reactions are

$px = R_2$ and $xa = R_1$, found by adding $H = 50\,200$ lb to the reactions as determined at (*a*).

(5) Using the approximate stresses as given by the stress-diagram at (*c*), design the members of the structure and record the sections in column 4, Table II. Record the areas of the members thus designed in column 5, Table II.

(6) From the data given in columns 2, 6, and 7, Table II, calculate the values of $\frac{S'UL}{A \cdot E}$ and $\frac{U^2L}{A \cdot E}$ for each member and record these values in columns 8 and 9 of Table II.

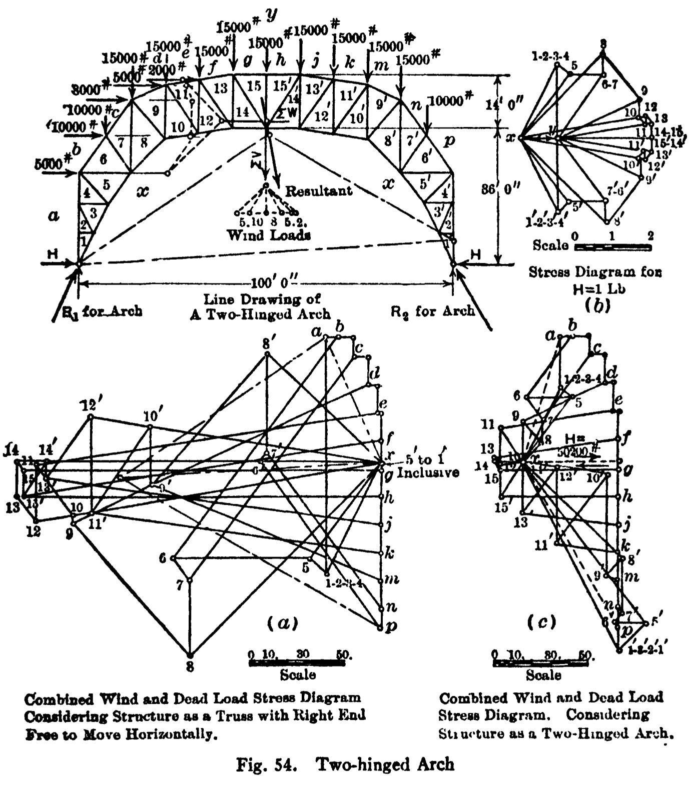


Fig. 54. Two-hinged Arch

(7) From the algebraic summation of columns 8 and 9, a more accurate value of $H = 52\,900$ lb is found, as shown at the bottom of Table II.

(8) The resultant stress values for the given loads and the value of H found in (7) are given in column 10, Table II.

No further recalculation of value of H will be made in this example since the members as designed are fairly satisfactory, as indicated by column 11, Table II. With the correct value of H the horizontal displacement of the

TABLE I

(1) Member	(2) Length Inches	(3) Stress as a Free End Truss S' Lb	(4) Stress for H = 1 Lb U Lb	(5) S' UL Kips	(6) U^2 L Lb
$a-1$	96	− 126 000	+ 1.95	− 23 600.	+ 365
$a-2$	96	− 126 000	+ 1.95	− 23 600.	+ 365
$a-4$	96	− 126 000	+ 1.95	− 23 600.	+ 365
$b-6$	144	− 146 000	+ 2.07	− 43 600.	+ 618
$c-7$	144	− 148 500	+ 2.07	− 44 300.	+ 618
$d-9$	120	− 175 000	+ 2.47	− 51 900.	+ 733
$e-11$	108	− 184 000	+ 2.40	− 47 600.	+ 622
$f-13$	108	− 196 000	+ 2.55	− 54 000.	+ 703
$g-15$	108	− 191 000	+ 2.50	− 51 500.	+ 675
$h-15'$	108	− 191 000	+ 2.50	− 51 500.	− 675
$j-13'$	108	− 180 500	+ 2.55	− 49 600.	+ 703
$k-11'$	108	− 156 500	+ 2.40	− 40 600.	+ 622
$m-9'$	120	− 132 000	+ 2.47	− 39 400.	+ 733
$n-7'$	144	− 103 000	+ 2.07	− 30 800.	+ 618
$p-6'$	144	− 110 000	+ 2.07	− 32 800.	+ 618
$p-4'$	96	− 88 000	+ 1.95	− 16 400.	+ 365
$p-2'$	96	− 88 000	+ 1.95	− 16 400.	+ 365
$p-1'$	96	− 88 000	+ 1.95	− 16 400.	+ 365
$1-x$	108	+ 67 000	− 2.20	− 15 900	+ 522
$3-x$	108	+ 67 000	− 2.20	− 15 900.	+ 522
$5-x$	120	+ 64 000	− 2.10	− 16 200.	+ 530
$8-x$	156	+ 145 000	− 3.10	− 70 100	+ 1 500
$10-x$	108	+ 167 000	− 3.25	− 58 600.	+ 1 140
$12-x$	108	+ 187 000	− 3.40	− 68 600.	+ 1 250
$14-x$	108	+ 194 000	− 3.50	− 73 500.	+ 1 325
$14'-x$	108	+ 179 000	− 3.50	− 67 700.	+ 1 325
$12'-x$	108	+ 157 000	− 3.40	− 57 700.	+ 1 250
$10'-x$	108	+ 124 500	− 3.25	− 43 500.	+ 1 140
$8'-x$	156	+ 84 000	− 3.10	− 40 600.	+ 1 500
$5'-x$	120	0	− 2.10	0	+ 530
$3'-x$	108	0	− 2.20	0	+ 522
$1'-x$	108	0	− 2.20	0	+ 522
$1-2$	48	0	0	0	0
$2-3$	108	0	0	0	0
$3-4$	96	0	0	0	0
$4-5$	135	+ 12 000	− 0.40	− 650.	+ 22
$5-6$	156	+ 72 500	− 0.95	− 10 700.	+ 141
$6-7$	144	− 14 500	0	0	0
$7-8$	234	− 40 000	+ 0.55	− 5 150.	+ 71
$8-9$	165	+ 95 000	− 1.55	− 24 300.	+ 396
$9-10$	172	+ 3 500	− 0.50	− 300.	+ 43
$10-11$	192	+ 33 000	− 0.22	− 1 390.	+ 9
$11-12$	168	− 30 000	+ 0.20	− 1 010.	+ 7
$12-13$	192	+ 17 000	− 0.25	− 820.	+ 120
$13-14$	168	+ 18 000	− 0.40	− 1 210.	+ 27
$14-15$	204	− 6 000	0	0	0
$15-15'$	168	− 15 000	0	0	0
$15'-14'$	204	+ 22 500	0	0	0
$14'-13'$	168	− 9 000	− 0.40	+ 605.	+ 27
$13'-12'$	192	+ 40 000	− 0.25	− 1 920.	+ 120
$12'-11'$	168	− 51 000	+ 0.20	− 1 720.	+ 7
$11'-10'$	192	+ 57 000	− 0.22	− 2 420.	+ 9
$10'-9'$	172	− 31 000	− 0.50	+ 2 670.	+ 43
$9'-8'$	165	+ 94 000	− 1.55	− 24 000.	+ 396
$8'-7'$	234	− 53 000	+ 0.55	− 6 800.	+ 71
$7'-6'$	144	6 500	0	0	0
$6'-5'$	156	+ 65 000	− 0.95	− 9 650.	+ 141
$5'-4'$	135	0	− 0.40	0	+ 22
$4'-3'$	96	0	0	0	0
$3'-2'$	108	0	0	0	0
$2'-1'$	48	0	0	0	0
Approximate $H = -\frac{-1\,274\,665}{+25\,378} \times 1000 = +50{,}200$ #				− 1 274 665.	+ 25 378

TABLE II

(1) Member	(2) Length Inches	(3) Arch Stress for H=50900 S_1 Lbs	(4) Section Designed for S_1 Stress	(5) Area Sq Ins	(6) Stress as a Free End Truss S' Lbs	(7) Stress for H=1 Lb U Lbs	(8) $\frac{S' U L}{A E}$	(9) $\frac{U^2 L}{A E}$ x 10 000	(10) Corrected Values Arch Stress S=S′+52 900U	(11) $\frac{S U L}{A E}$
a-1	96	− 26 500	2 Ls 3″x 3″x 5/16″	3 56	− 126 000	+ 1.95	−0 221	+ 0 0342	− 22 500	−0 0394
a-2	96	− 26 500	do		− 126 000	+ 1.95	−0.221	+ 0 0342	− 22 500	− 0 0394
a-4	96	− 26 500	do		− 126 000	+ 1 95	−0 221	+ 0.0342	− 22 500	− 0 0394
b-6	144	− 41 000	2 Ls 5″x 3″x 3/8″	5 72	− 146 000	+ 2 07	−0 254	+ 0.0360	− 36 000	− 0 0625
c-7	144	− 43 500	do		− 148 500	+ 2 07	−0 258	+ 0.0360	− 38 500	− 0 0670
d-9	120	− 48 000	do		− 175 000	+ 2 47	−0 302	+ 0.0425	− 44 000	− 0 0758
e-11	108	− 61 500	2 Ls 5″x 3″x 7/16″	6 62	− 184 000	+ 2.40	−0 240	+ 0 0314	− 56 700	− 0 0740
f-13	108	− 66 500	do		− 196 000	+ 2.55	−0 272	+ 0 0352	− 60 700	− 0 0842
g-15	108	− 62 500	do		− 191 000	+ 2 50	−0 259	+ 0 0339	− 58 400	− 0 0793
h-15′	108	− 62 500	do		− 191 000	+ 2 50	−0 259	+ 0 0339	− 58 400	− 0 0793
j-13′	108	− 50 500	do		− 180 500	+ 2 55	−0 250	+ 0 0352	− 45 200	− 0 0625
k-11′	108	− 32 500	do		− 166 500	+ 2 40	−0 240	+ 0 0314	− 29 300	− 0 0382
m-9′	120	− 6 500	2 Ls 5″x 3″x 3/8″	5 72	− 133 000	+ 2 47	−0 230	+ 0 0425	− 2 000	−0 0034
n-7′	144	+ 4 000	do		− 103 000	+ 2 07	−0 179	+ 0 0360	+ 7 000	+ 0 0122
p-6′	144	− 2 000	do		− 110 000	+ 2 07	−0 191	+ 0.0360	0	0
p-4′	96	+ 11 500	2 Ls 3″x 3″x 5/16″	3 56	− 88 000	+ 1 95	−0 154	+ 0 0342	+ 15 500	+ 0 0273
p-2′	96	+ 11 500	do		− 88 000	+ 1 95	0 154	+ 0.0342	+ 15 500	+ 0 0273
p-1′	96	+ 11 500	do		− 88 000	+ 1 95	−0 154	+ 0 0342	+ 15 500	+ 0 0273
1-x	108	− 45 000	2 Ls 5″x 3½″x 3/8″	9 84	+ 67 000	− 2 20	−0 054	+ 0 0177	− 49 700	+ 0 0402
3-x	108	− 45 000	do		+ 67 000	− 2 20	−0 054	+ 0 0177	− 49 700	+ 0 0402
5-x	120	− 43 500	do		+ 64 000	− 2 10	0 055	+ 0 0180	− 47 200	+ 0 0405
8-x	156	− 14 000	2 Ls 5″x 3½″x 5/16″	8 94	+ 145 000	− 3 10	−0 262	+ 0 0560	− 19 000	+ 0 0344
10-x	108	− 0	2 Ls 5″x 3″x 5/16″	4 80	+ 167 000	− 3 25	−0 406	+ 0 0792	− 5 200	+ 0 0127
12-x	108	+ 13 000	do		+ 187 000	− 3 40	−0 476	+ 0 0870	+ 7 000	− 0 0178
14-x	108	+ 15 000	2 Ls 2½″x 2″x 1/4″	2 12	+ 194 000	− 3 50	−1.155	+ 0 2080	+ 9 000	− 0 0532
14′-x	108	− 0	do		+ 179 000	− 3 50	−1 061	+ 0 2080	− 6 500	+ 0 0388
12′-x	108	− 18 000	2 Ls 5″x 3″x 5/16″	4 80	+ 157 000	3 40	−0 400	+ 0.0870	− 23 000	+ 0 0589
10′-x	108	− 44 500	do		+ 124 500	− 3.25	−0 302	+ 0.0792	+ 48 000	+ 0 1175
8′-x	156	− 73 000	2 Ls 5″x 3½″x 5/16″	8.94	+ 84 000	− 3 10	−0 152	+ 0.0560	− 80 300	+ 0 1450
5′-x	120	−107 000	2 Ls 5″x 3½″x 3/8″	9 84	0	− 2 10	0	+ 0 0180	− 111 300	+ 0 0956
3-x	108	−111 500	do		0	− 2 20	0	+ 0 0177	− 116 700	+ 0 0945
1′-x	108	−111 500	do		0	− 2 20	0	+ 0 0177	− 116 700	+ 0 0945
1-2	48	0	2 Ls 2½″x 2″x 1/4″	2 12	0	0	0	0	0	0
2-3	108	0	do		0	0	0	0	0	0
3-4	96	0	do		0	0	0	0	0	0
4-5	135	− 8 500	2 Ls 4″x 3″x 5/16″	4 18	+ 12 000	− 0 40	−0 005	+ 0 0017	− 9 200	+ 0 0040
5-6	156	+ 25 000	2 Ls 2½″x 2″x 1/4″	2 12	+ 72 000	+ 0 95	−0 168	+ 0 0222	− 21 700	+ 0 0507
6-7	144	− 14 000	2 Ls 4″x 3″x 5/16″	4 18	− 14 500	0	0	0	− 14 500	0
7-8	234	− 13 000	2 Ls 6″x 6″x 3/8″	8 72	− 40 000	+ 0 55	−0 019	+ 0 0027	− 10 800	− 0 0053
8-9	165	+ 16 000	2 Ls 2½″x 2″x 1/4″	2 12	+ 95 000	− 1 55	−0 882	+ 0 0620	+ 12 800	− 0 0514
9-10	172	− 21 500	2 Ls 5″x 3½″x 7/16″	7 06	+ 8 500	− 0 50	−0 001	+ 0 0020	− 28 000	+ 0 0094
10-11	192	+ 21 500	2 Ls 3″x 3″x 5/16″	3 56	+ 83 000	− 0.22	−0 013	+ 0 0008	+ 21 300	− 0 0081
11-12	168	− 19 500	2 Ls 5″x 3½″x 3/8″	6 10	− 80 000	+ 0 20	−0 005	+ 0 0004	− 19 400	− 0 0035
12-13	192	+ 4 500	2 Ls 2½″x 2½″x 1/4″	2 38	+ 17 000	− 0 25	−0 012	+ 0 0168	− 8 700	− 0 0025
13-14	168	− 1 500	2 Ls 5″x 3½″x 5/16″	5 12	+ 18 000	− 0 40	−0 008	+ 0 0018	− 8 200	+ 0 0014
14-15	204	− 5 500	2 Ls 6″x 4″x 3/8″	7 22	− 6 000	0	0	0	− 6 000	0
15-15′	168	− 15 000	2 Ls 5″x 3½″x 5/16″	5 12	− 15 000	0	0	0	− 15 000	0
15′-14′	204	+ 22 500	2 Ls 6″x 4″x 3/8″	7 22	+ 22 500	0	0	0	+ 22 500	0
14′-13′	168	− 27 500	2 Ls 5″x 3½″x 5/16″	5 12	− 9 000	− 0 40	+ 0 004	+ 0 0018	− 30 200	+ 0 0132
13′-12′	192	− 30 500	2 Ls 2½″x 2½″x 1/4″	2 38	+ 40 000	− 0 25	−0 027	+ 0 0168	+ 26 700	− 0 0179
12′-11′	168	− 42 000	2 Ls 5″x 3½″x 3/8″	6 10	− 51 000	+ 0 20	−0 009	+ 0 0004	− 40 400	− 0 0074
11′-10′	192	+ 46 000	2 Ls 3″x 3″x 5/16″	3 56	+ 57 000	− 0 22	−0 023	+ 0 0008	+ 45 800	− 0 0176
10′-9′	172	− 54 000	2 Ls 5″x 3½″x 7/16″	7,06	− 31 000	− 0 50	+ 0 012	+ 0.0020	− 57 500	+ 0 0231
9′-8′	165	+ 13 000	2 Ls 2½″x 2″x 1/4″	2,12	+ 94 000	− 1 55	−0 378	+ 0 0620	− 11 700	− 0.0469
8′-7′	234	− 30 500	2 Ls 6″x 6″x 3/8″	8 72	− 53 000	+ 0 55	−0 026	+ 0 0027	− 23 800	− 0.0117
7′-6′	144	− 7 000	2 Ls 4″x 3″x 5/16″	4 18	− 6 500	0	0	0	− 6 500	− 0
6′-5′	156	+ 15 000	2 Ls 2½″x 2″x 1/4″	2 12	+ 65 000	− 0 95	−0 152	+ 0 0222	− 14 600	− 0 0340
5′-4′	135	− 19 000	2 Ls 4″x 3″x 5/16″	4 18	− 0	− 0 40	0	+ 0 0017	− 21 200	+ 0.0092
4′-3′	96	0	2 Ls 2½″x 2″x 1/4″	2 12	0	0	0	0	0	0
3′-2′	108	0	do		0	0	0	0	0	0
2′ 1′	48	0	do		0	0	0	0	0	0
							Σ = −9.648	+1.8232		−0.0040

$$H = -\frac{-9\ 648}{+1.8232} \times 10\ 000 = +52{,}900\#$$

right support should equal zero. The value of Δ as given by Equation (15) should, therefore, equal zero, or:

$$\Delta = \sum \frac{SUL}{A \cdot E} = 0 \tag{25}$$

A Two-Hinged Arch with a Tie-Member. In the preceding example the horizontal reactions were supplied by the foundations. If the two supporting hinges of the arch in Fig. 54 are connected by a horizontal tie, the stress in the tie-member will be somewhat less than the horizontal reactions of the abutments as found in the preceding example.

Let it be assumed that the two supports are connected by a tie-member composed of two eye-bars 3 in $\times$ $\frac{5}{8}$ in with an area of 3.75 sq in. The value of $\frac{L}{AE}$ for the tie-member is:

$$\frac{L}{A \cdot E} = \frac{1\ 200}{3.75 \times 29\ 000\ 000} = 0.00001105$$

The stress in the tie-member is given by Equation (19) in which the values $\sum \frac{S'UL}{A \cdot E}$ and $\sum \frac{U^2L}{A \cdot E}$ are the totals of columns 8 and 9, respectively, of Table II, whence:

$$S_r = -\frac{-9\ 648}{\frac{1.8232}{10\ 000} + 0.00001105}$$

$$= +\ 49\ 800 \text{ lb (Tension)}$$

or

$$\frac{49\ 800}{3.75} = 13\ 300 \text{ lb per sq in}$$

A Check of Temperature Stresses. If the arch as originally designed, without a tie-member, is subjected to a temperature increase of 100° F., the horizontal thrust due to rise in temperature is given by Equation (21) as:

$$H_t = -\frac{-(0\ 0000065) \times 100 \times 1\ 200}{\frac{1\ 8232}{10\ 000}}$$

$$= +\ 4\ 350 \text{ lb (Acting with H)}$$

The combined stress in any member is given by Equation (23). As an example, consider the combined stress in member l–x; the value of the stress with no temperature-change is given in column 10, Table II, as $-49\ 700$, then for a rise in temperature:

$$S_{1-x} = -\ 49\ 700 + 4\ 350\,(-2.20)$$

$$= -\ 59\ 300 \text{ lb (Compression)}$$

If the arch is not erected at a time of approximately average temperature, the difference should be taken into account in setting the foundations, provided the horizontal thrust is to be resisted by them without the aid of a tie-member.

11. The Fixed Arch

The Fixed Arch has no hinges and is a type which is seldom employed by architects in the truss-form. The rigid analysis of a TRUSSED FIXED ARCH is very long and tedious, so a few formulas will be given, necessary for the solution of ARCHES WITH SOLID WEBS, such as PLATE-GIRDER ARCHES. These formulas may be applied to truss-forms, where the chords are approximately parallel, without serious error. Midway between the top and bottom chords draw a smooth curve, called the ARCH-AXIS, and designate the distance between

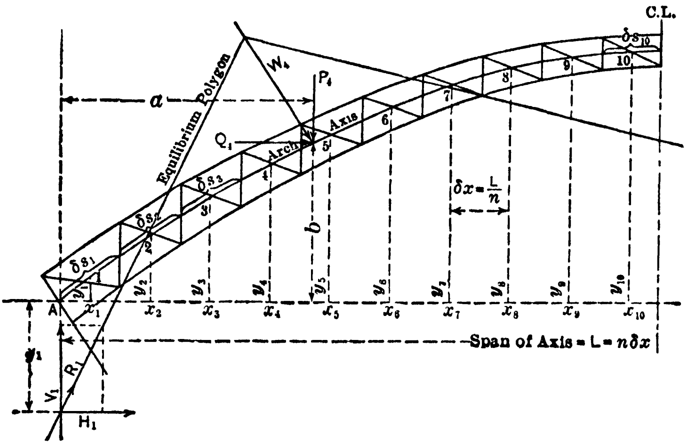


Fig. 55. Fixed Arch. Truss-diagram

its ends as L or the SPAN OF THE AXIS. Divide the span into n equal parts and at the centers of these divisions draw perpendiculars until they cut the arch-axis. Number the points 1, 2, 3, etc., as shown by Fig. 55, which also indicates the nomenclature employed.

Determination of H_1, V_1 and H_1y_1. The equilibrium-polygon for a single inclined load is shown in Fig. 55, in its true position with reference to the arch-axis. This locates the point of application of H_1. The following formulas are very close approximations for arches having a rise greater than one-eighth the span.

$$H_1 = \Sigma m_{xy}A'' \div \Sigma yA''$$

$$A'' = K\left\{y - \frac{\Sigma yK}{\Sigma K}\right\} \qquad K = \frac{\delta s}{EI}$$

$$\left.\begin{matrix} H_1y_1 \\ H_2y_2 \end{matrix}\right\} = H_1\frac{\Sigma yK}{\Sigma K} - \left\{\frac{\Sigma m_{xy}K}{\Sigma K} \pm \frac{\Sigma m_{xy}K(z-n)}{n^2\Sigma K - \Sigma_z{}^2K}n\right\}$$

$$V_1 = \frac{H_1y_2 - H_2y_2}{L} + r_1 \qquad y_1 = \frac{H_1y_1}{H_1}$$

y_1 is measured down from A when H_1y_1 is negative. Σ is the sum of quantities it governs for each point on the arch-axis numbered 1, 2, 3, . . . n. For example

$$\Sigma K = \left(\frac{\delta s}{EI}\right)_1 + \left(\frac{\delta s}{EI}\right)_2 + \left(\frac{\delta s}{EI}\right)_3 + \text{etc.}$$

I is the moment of inertia of the chords about an axis midway between them. The sections of the chords are to be taken on radial lines passing through points 1, 2, 3, etc. x and y are the coördinates of the points 1, 2, 3, etc., in Fig. 55.

$$x = z\frac{\delta x}{2} = z\frac{L}{2n}$$

m_{xy} is the moment at the point on the arch-axis having the coordinates xy, assuming that the given loading is supported by the axis hinged at the right end and on rollers at the left end. r_1 is the reaction at the left support under the conditions specified for m_{xy}. In the above formulas the only terms which depend upon the loading are those containing m_{xy} and r_1, the others being constant for any given arch. While but one load has been used, any number may be used by considering m_{xy} and r_1 as the sum of the respective quantities for each load.

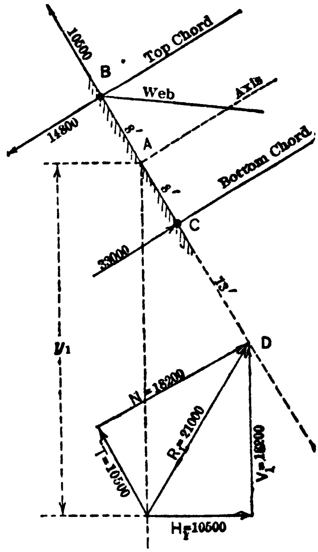


Fig. 56 Fixed Arch. Reactions

Stresses. The STRESSES in the truss-members can be found by the ordinary graphical methods when H_1, V_1 and Hy_1 are known. For example, assume the numerical values shown in Fig. 56. The resultant of V_1 and H_1 is resolved into two components parallel and perpendicular to the bottom-chord member at the support. Then T must act at the upper-chord joint as shown. The two reactions parallel to the bottom chord are found by moments. The stress-diagram can now be drawn beginning with these forces and proceeding until the right support is reached.

Symmetrical Loading. When the loading is symmetrical, $H_1y_1 = H_2y_2$ and hence $V_1 = r_1$. Also

$$H_1y_1 = H_1\frac{\Sigma yK}{\Sigma K} - \frac{\Sigma m_{xy}K}{\Sigma K}$$

Changes of Temperature. For temperature-changes,

$$H_t = et^\circ L \div \Sigma yA''$$

$$H_1y_1 = H_2y_2 = H_t\frac{\Sigma yK}{\Sigma K}$$

$$y_1 = \frac{\Sigma yK}{\Sigma K} \qquad V_1 = 0$$

12. Arches with Solid Ribs

Arches with Solid Ribs. While this chapter considers TRUSSES only, it may not be out of place to briefly consider ARCHES HAVING SOLID RIBS. The computations for V_1, H_1 and H_1y_1 remain unchanged, excepting that I now is the moment of inertia of the radial section of the rib at points 1, 2, 3, etc.

Fiber-Stresses. If x and y are the coördinates of any point on the gravity-axis of the rib, which should coincide with the arch-axis, the bending moment at this point is, for each load,

$$M_x = H_1y_1 + V_1x - H_1y - \overset{x>a}{P\,(x-a)} - \overset{x>a}{Q\,(y-b)}$$

H_1y_1 is negative when y_1 is measured below A in Fig. 56.

$$S = \frac{M_x y c}{I} + \frac{Nxy}{A}$$

where c is the distance from the gravity-axis to the outermost fiber. For the TWO- and THREE-HINGED ARCHES, $H_1y_1 = 0$.

Radial Shear. Let H_x be the algebraic sum of all the horizontal components on the left of the section, V_x the algebraic sum of all the vertical components on the left of the section and θ the angle which the radial section, upon which the shear is wanted, makes with the vertical. Then $T_x = V_x \cos\theta - H_x \sin\theta$.

Two-hinged Parabolic Arch. If the center line of the SOLID RIB is a PARABOLA, when $EI\cos\theta$ is a constant, the following simple formulas give the values of V_1 and H_1:

$$V_1 = P\,(1-k) - Q\frac{4f}{L}k\,(1-k)$$

$$H_1 = \frac{5}{8}\frac{L}{f}P\,[k\,(1-2k^2+k^3)] - Q\left\{1 - \frac{k}{2}\,[5\,(1-k-2k^2+4k^3)) - 8k^4]\right\}$$

and

$$H_t = \frac{15}{8f^2}EI_0et^\circ$$

in which $k = a \div L$ (Fig. 55), f is the rise of the axis, P is the vertical load acting down, Q is the horizontal load acting from left to right and I_0 is the moment of inertia of the section of the rib at the crown.

Fixed Parabolic Arch. In like manner the following formulas apply for the arch without hinges:

$$V = P\,(1-k)^2(1+2k) - \frac{12f}{L}Q\,(k-k^2)^2$$

$$H_1 = \frac{15\,L}{4f}Pk^2(1-k)^2 - Q\,\{1 + k^2(-15 + 50k - 60k^2 + 24k^3)\}$$

$$H_1y_1 = \frac{L}{2}Pk\,(1-k)^2(5k-2) - fQ\{2k\,(1-k)^2(2-7k+8k^2)\}$$

$$H_t = \frac{45}{4f}EI_0et^\circ \qquad H_ty_1 = \frac{15}{2f}EI_0et^\circ$$

The values of the factors containing k in the above formulas are given in tabular form in A Treatise on Arches.*

Circular Arches, with solid ribs of constant cross-section and the center line an arc of a circle, may be considered by using formulas somewhat similar to those given for PARABOLIC ARCHES but very much longer and more complex. Formulas and tables for their solution are given in the treatise on arches referred to above.

* A Treatise on Arches, by Malverd A. Howe, John Wiley & Sons, Inc., New York.

CHAPTER XXVI

DESIGN AND CONSTRUCTION OF ROOF-TRUSSES

BY

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1. Data for the Design of Roof-Trusses

General Procedure. Certain items, preliminary to the design of a roof-truss, must be decided upon before the loads to be supported and the stresses to be provided for can be determined. The character of the roof-covering, the span, and the geometrical shape of the truss are generally fixed by architectural requirements. The spacing of trusses is also, quite often, established by the location, in plan, of supporting columns or piers.

The location of the building and its exposure will fix the amount of snow- and wind-loads which must be considered in the design.

Roof-Loads. The loads which must be considered in the design of a roof-truss consist of:

(*a*) The weight of the ROOF-COVERINGS
(*b*) The weights of PURLINS, BRACING and RAFTERS
(*c*) The weight of the TRUSS itself
(*d*) CEILING-LOADS
(*e*) MISCELLANEOUS LOADS, such as suspended balconies and floors, heating and ventilating equipment, etc.
(*f*) Maximum and minimum SNOW-LOADS
(*g*) WIND-LOADS

These loads, and the methods of determining their magnitudes, will be outlined in the following paragraphs:

Weight of Roof-Coverings. The actual weight of ROOF-COVERINGS should, if possible, be carefully calculated from the manufacturer's data for a given material placed in accordance with a given specification. Table I gives the approximate weight of the more usual materials employed for roof-coverings, in pounds per square foot of roof-surface.

Weight of Purlins, Bracing and Rafters. PURLINS, whether of steel or of timber, will weigh from 1.5 to 4.0 lb per sq ft of roof-surface, depending upon the span and spacing of purlins and the weight of the roof-covering.

BRACING is a variable quantity, depending upon the type of construction. When a rigid structural roof-covering is used, no bracing is required in the plane of the upper chord of the trusses, excepting temporary bracing during erection. When the ceiling-construction provides sufficient lateral support in the plane of the lower chord, no additional bracing is necessary. For short spans up to 50 or 60 ft, and with a reasonably rigid structural covering, it is rarely necessary to provide bracing in vertical planes between trusses. For spans exceeding 60 ft, and especially where the structural covering is anything other than nailing concrete or an equally rigid medium, bracing to the extent of from 0.5 to 1.0 lb per sq ft of horizontal projection should be provided.

Table I. Weights of Roof-Coverings in Pounds per Square Foot of Roof

Material	Weight in lbs per sq ft
Shingles:	
Common wood	2 5 to 3.0
Asphalt, slate surfaced	2 0 to 3.0
Asbestos, plain and tapered	4.5 to 6.5
Slate, Plus one layer of felt:	
3/16 in thick	7.0
1/4 in thick	9 0
3/8 in thick	10.0 to 12
1/2 in thick	14.0 to 16
Tile, Clay:	
Flat or plain	10.0 to 17.0
Mission	12 0 to 15 0
Spanish	10.0 to 14 0
French	10.0 to 16.0
Tile, Cement:	
Flat or plain	17.0 to 20.0
Interlocking	15.0 to 18.0
Gypsum:	
Pre-cast roof tile, 1 in thick	5.0 to 6.0
Poured-in-place, 1 in thick	4.0 to 5.0
Nailing concrete:	
Per inch of thickness	7.5 to 8.0
Tin, plus one layer of felt:	
Sheets or shingles, painted	1.0 to 1.5
Corrugated steel:	
24 to 26 gauge	1.0 to 1.5
20 to 22 gauge	1.5 to 2.0
16 to 18 gauge	2.5 to 3.0
Copper, plus one layer of felt:	
Sheets	1.5
Shingles or tile	2.0 to 2.5
Felt and gravel:	
5 ply	5.0 to 7.0
4 ply	4.0 to 5.0
3 ply (ready roofing)	1.5 to 2.0
Skylights, including frames:	
1/4 in glass	4.5 to 5.0
5/16 in glass	5.0 to 6.0
3/8 in glass	6.0 to 7.0
Sheathing:	
White pine, spruce and hemlock, 1 in thick	3.0
Southern yellow pine, 1 in thick	4.0

The weights of RAFTERS per square foot of roof-surface are given in Table II.

Table II. Weights of Rafters in Pounds per Square Foot of Roof-Surface

Kind of timber		White pine, spruce and hemlock				Yellow pine			
Spacing		12 in	16 in	20 in	24 in	12 in	16 in	20 in	24 in
Nominal Size	2× 4 in	1.3	0.9	0.8	0.6	1.7	1.2	1 0	0.8
	2× 6 in	1 9	1.4	1.1	1.0	2.5	1 9	1.5	1.3
	2× 8 in	2.5	1.9	1.5	1.3	3.4	2.5	2.0	1.7
	2×10 in	3.2	2 4	2.0	1.7	4.3	3.2	2.6	2.2
	2×12 in	3.9	2.9	2.3	2.0	5.2	3.9	3.1	2.6

Weight of Roof-Trusses. The weight of a ROOF-TRUSS varies with the span and spacing of trusses, the magnitude of the total load carried by the truss, the pitch of the roof, the type of truss, the working unit stresses and specifications, and the personal equation of the designer.

Various emperical formulas have been proposed for determining an approximate weight of the truss for the purpose of determining a value to be added to the design loads.

The following empirical formulas are perhaps the least complex of those which have been proposed and at the same time are sufficiently accurate for an estimate of the probable weight of a proposed truss:

(*a*) Weights of timber roof-trusses:

$$W = 0.5\,SL + 0.075\,SL^2 \qquad \text{H. S. Jacoby}$$

(*b*) Weights of steel roof-trusses:

$$W = 0.4\,SL + 0.04\,SL^2 \qquad \text{C. E. Fowler}$$

In these formulas, W = total weight of truss in pounds, S = the spacing of trusses in feet, and L = the span of the truss in feet. The approximate weights of roof trusses expressed in terms of the total load supported by the truss and for given working unit stresses, may be found from the following empirical equations:

Weights of timber roof-trusses for an allowable tension = allowable extreme fiber-stress in flexure = 1 500 lb per sq in:

$$W = \frac{P}{65}\left(1 + \frac{L}{30} + \frac{L}{5\sqrt{S}}\right) \tag{1}$$

Weights of steel roof-trusses for an allowable working unit stress in tension of 18 000 lb per sq in:

$$W = \frac{P}{110}\left(1 + \frac{L}{30} + \frac{L}{5\sqrt{S}}\right) \tag{2}$$

In Equations (1) and (2) W = the total weight of truss in pounds, L = the span of truss in feet, S = the spacing of trusses in feet, and P = the total vertical or equivalent vertical load supported by the truss.

In general, the weight of the truss for all usual spans and loadings is only about 10% to 15% of the total load supported by the truss, consequently the

values given by Equations (1) and (2) may be adopted without subsequent correction.

Weight of Ceiling-Construction. Suspended metal lath and plaster including steel ceiling-beams and pressed-steel furring-channels weighs about 10 lb per sq ft of ceiling. Ceilings constructed with wooden joists, fiber or gypsum plaster bases and with steel or timber ceiling-beams will also weigh about 10 lb per sq ft of ceiling. In designing the joists and beams for ceiling-supports, care must be taken to provide against a deflection greater than $\frac{1}{360}$ of the span of the supporting member.

Miscellaneous Loads. In addition to the roof and ceiling-loads, roof-trusses must often be designed to support special loads such as motors, fans, pipe-lines, and coils, storage tanks, etc. In some cases upper floors, balconies or mezzanine floors are hung from roof-trusses. The magnitudes of MISCELLANEOUS LOADS which may consist of a combination of DEAD, LIVE, and in some cases, IMPACT LOADS, should be accurately calculated and the supporting members should be so arranged that the loads are transferred to the truss as concentrations at panel-points.

Snow-Loads. The maximum SNOW-LOAD to be assumed in the design of a given roof depends upon the slope of the roof, and the latitude and humidity of the locality. Dry and freshly fallen snow weighs about 8 lb per cu ft, while packed or wet snow may weigh as much as 10 to 12 lb per cu ft, and snow mixed with hail or sleet may weigh as much as 25 to 30 lb per cu ft. The amount of snow on a roof at any one time is also a variable quantity. Table III gives the probable range of snow-load intensities for various localities and roof-slopes.

Table III. Snow-Loads for Roof-Truss Design in Pounds per Square Foot of Roof-Surface

Locality	Slope of roof				
	45°	30°	25°	20°	Flat
Northwestern and New England States	10–15	15–20	25–30	35	40
Western and Central States..	5–10	10–15	20–25	25–30	35
Southern and Pacific States.......	0–5	5–10	5–10	5–10	10

A high wind may follow a sleet or ice-storm, hence there is a possibility of combination of WIND and SLEET (minimum snow) LOADS. A value for minimum snow-load of 10 lb per sq ft of roof should be used for all roof-slopes and for all localities with the possible exception of Southern and Pacific Coast states, where half the tabular values of maximum snow may be used.

Wind-Loads. The WIND-PRESSURE against a roof surface depends upon the velocity and direction of the wind, and upon the exposure and inclination of the roof-surface. The wind is assumed to move horizontally, and the pressure against a flat surface, normal to the direction of the wind, is given quite accurately by the equation:

$$p = 0.004\ V^2 \tag{3}$$

where p = the pressure in pounds per square foot on a flat, normal surface, and V = the velocity of the wind in miles per hour.

When the wind blows upon a surface other than a plane, the pressure on the projected area depends upon the form of the surface. Table IV gives the values of wind-pressure on various geometrical surfaces in a ratio to that upon a flat surface, as given by Equation (3).

Table IV. Ratios of Wind-Pressures on Various Surfaces to That Upon a Flat Plate *

Form of surface	Ratio
Sphere—Convex surface	0.35 to 0.40
Cylinder—Convex surface	0.55 to 0.65
Cone—90° base angle, apex to wind	0.65 to 0.75
Sphere—Concave surface	1.30 to 1.70
Cylinder—Concave surface	1.15 to 1.30
Rectangular side of building	0.75 to 0.85

* Mechanics Applied to Engineering, Goodman (Longmans, Green & Company, 1904).

In the design of roof-trusses it is usual to assume that the resultant wind-pressure is normal to the roof-surface. Experiments made by Colonel Duchemin, a French army officer, in 1829, resulted in the widely accepted FORMULA FOR WIND-PRESSURES ON INCLINED SURFACES:

$$P_n = P\frac{2 \sin A}{1 + \sin^2 A} \tag{4}$$

where P_n = the normal wind-pressure in pounds per square foot of surface, P = wind-pressure in pounds per square foot on a vertical surface, and A = the angle of inclination of the roof surface with the horizontal.

The Duchemin formula was verified, without knowledge of its existence, by Mr. S. P. Langley in 1888.

The Duchemin formula, and its verification by Langley, are based upon experiments on small thin plates for which the conditions are quite dissimilar from those encountered in roof-surfaces. In view of the uncertainty as to the intensity and distribution of wind-loads, refinements in the computations are not wholly warranted. For the purpose of calculating the value of normal wind-pressures to be used in roof-truss design, some designers prefer the simple, straight line formula:

$$P_n = \frac{A \times P}{45} \tag{5}$$

where P_n and P are the same as in Equation (4), and A = the angle of inclination of the roof with the horizontal, in degrees; which tacitly assumes that for a slope of 45° the normal pressure is equal to the pressure on a vertical plane.

The intensities of normal wind-pressure for roof-surfaces of various slopes as given by Equations (4) and (5) are recorded in Table V.

Table V. Wind-Load in Pounds Per Square Foot of Roof-Surface

Slope of roof	Normal pressure P_n			
	$P = 20$ lb		$P = 30$ lb	
	Duchemin	Straight line	Duchemin	Straight line
10°	6.7	4.4	10.0	6 7
15°	9.7	6.7	14 6	10 0
20°	12 3	8 9	18.4	13 3
25°	14.4	11.0	21.5	16.7
30°	16 0	13 3	24.0	20 0
35°	17 3	15 5	26 0	23 3
40°	18 2	17 8	27.3	26.6
45°	18 9	20.0	28.3	30.0

Experiments by J. O. V. Irminger (Copenhagen) in 1894,* and by T E. Stanton (London) in 1903,† indicate that the action of the wind produces a suction on the leeward side of the roof. Summarizing these experiments, Professor Marburg ‡ concludes: "The experiments . . . were made on much too small a scale to admit of quantitative deductions applicable to conditions in practice. They are valuably suggestive, however, in calling attention to conditions which were previously not generally or adequately recognized."

This condition should be provided for to the extent that individual members of the truss be designed so as to be capable of developing compressive as well as tensile stress in order to avoid possible buckling, and the truss itself should be well anchored to its supports.

Combined Loads for Maximum Stresses. The combination of dead, snow, and wind-loads which are to be considered in calculating the maximum stresses in the individual members of a truss is largely a matter of the individual judgment of the designer.

It is highly improbable that snow will remain undisturbed on a roof with a wind velocity of 61 miles per hour, which is equivalent to a pressure of 15 lb per sq ft.

Wet snow or sleet may remain on the roof-surface in the presence of a relatively high wind velocity, but the probable maximum for such a combination could not be reasonably taken at a greater value than full wind-load plus one-half of maximum snow-load, or wind plus minimum snow.

The combinations of loading for maximum stresses which appear the most reasonable are those given in Chapter XXV:

(1) Dead load plus maximum snow.
(2) Dead load plus wind right plus minimum snow.
(3) Dead load plus wind left plus minimum snow.

* Eng. News, Feb. 14, 1895.
† Proc. Inst. C. E., Vol. CLVI, page 78, 1904.
‡ Framed Structures and Girders, McGraw-Hill, 1911.

Equivalent Vertical Loads. Instead of considering separately the dead, snow-, and wind-loads and the various combinations of snow- and wind-load which may act with the dead load to produce maximum stresses in the members of a truss, an EQUIVALENT VERTICAL LOAD is sometimes assumed.

For simple roof-trusses of the usual types, supported on masonry walls, the combined stresses in the various members due to wind and minimum snow-loads are practically the same as the stresses resulting from a properly assumed vertical load, uniformly distributed over the truss.

This method is efficient since only one set of calculations, or one stress-diagram, is required for the determination of maximum stresses. The selection of the proper equivalent uniform vertical load, however, requires considerable judgment on the part of the designer. A study of the particular case under consideration together with preliminary trials will enable an experienced designer to select a satisfactory equivalent load for the particular type of truss which is to be used and the existing local conditions which are to be provided for.

The average values of uniform vertical loads equivalent to combined wind and minimum snow-loads, or to maximum snow-load, in producing stresses in the members of roof-trusses of the usual types, supported by masonry walls, are given in Table VI.

Table VI. Equivalent Uniform Vertical Loads to Replace Combined Snow- and Wind-Loads for Calculating Maximum Stresses in Roof-Trusses. Values in Pounds per Square Foot of Roof-Surface *

Locality	Slope of roof				
	60°	45°	30°	20°	Flat
Northwestern and New England States	28	26	24	35	40
Western and Central States . .	28	26	24	30	35
Southern and Pacific States.......	28	26	24	20	30

* Values corrected for an allowable increase in working unit stress for wind-stresses of $33\frac{1}{3}\%$.

In calculating the values of equivalent loads given in Table VI account has been taken of the usual specification relative to the working unit stress for combined stresses due to wind and other loads.

The dead load is always present and must be added directly to the values given in Table VI. The maximum snow-load may be present at intervals, and when present can be expected to exist for a considerable time. The maximum wind-load occurs at infrequent intervals and its effect lasts but a short time. The working unit stress for wind-loads may, therefore, be greater than the value specified for dead and snow-loads. The specification of the American Institute of Steel Construction recommends an increase of $33\frac{1}{3}\%$ for combined stresses due to wind and other loads. With this factor provided for in the equivalent values, the tabulated values may be used directly with the basic stress as specified for fixed or permanent loads.

Roof and Ceiling-Loads Supported at Any Joint. Calculations for the direct stresses in the members of a roof-truss are based upon the assumptions that the loads are transferred to the panel-points, and that the members at any point are united by a frictionless hinge, although in practice the joints may be riveted or otherwise rigidly connected.

The upper chord panel-loads are the PURLIN REACTIONS, and the lower chord panel-loads are the REACTIONS OF THE CEILING-BEAMS.

When the roof and ceiling-loads are uniformly distributed, as is usually the case, the simplest method of computing the panel-loads is to determine the roof or ceiling-area tributary to the joint, and to multiply this area by the unit load.

2. Stress Coefficients

Stress coefficients or the algebraic or graphical methods given in Chapter XXV may be used to determine the stresses in the individual members after the loads at each panel-point of a truss have been determined.

When a truss is uniformly loaded the stresses in the members throughout the truss are each a direct function of the form of the truss and the magnitude of the uniform panel-load.

If the panel-load is taken as unity the resulting stress in any member is known as the stress coefficient for that member for the given system of loading.

The stress in any member of a truss, for which the stress coefficients are known, may be determined by multiplying the actual panel-load by the stress coefficient for the member in question.

The stress coefficients for some of the more usual forms, and pitch ratios, of trusses in general use are given in Tables VII to XIII, inclusive.

In these tables, the members of the truss are designated on the truss-diagrams by letters, and the character of stress is noted as tension or compression in the table, or is indicated by the weights of the lines in the truss-diagram. Heavy lines indicate COMPRESSIVE STRESS, light lines indicate TENSILE STRESS, and broken lines indicate ZERO STRESS for the given position of the applied loads.

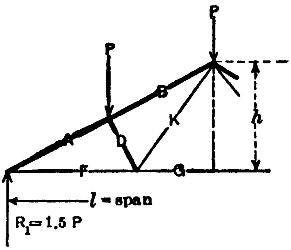
When stresses are to be computed for a number of trusses of a type of truss or a system of loading not given in the following tables, considerable time may be saved by preparing a unit stress coefficient table similar to those illustrated.

Stress coefficient tables may be prepared for wind-loads and any assumed distribution of horizontal reactions by constructing a unit wind panel-load stress-diagram for the assumed reaction conditions and tabulating the stresses as scaled, in a manner similar to that illustrated in the examples given in Chapter XXV.

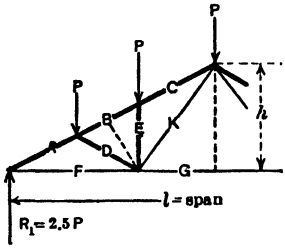
Table XI gives coefficients which are general for any span and depth for eight-panel roof-trusses with the Howe and Pratt types of bracing. Tables XII and XIII give formulas for COMPUTING the stresses in symmetrical Howe and Pratt trusses which are symmetrically loaded. The coefficients are given for trusses having an odd number of panels. For the Howe truss with an even number of panels the coefficients for the center load on the top chord are each divided by two. For the center load on the bottom chord the coefficients are also divided by two, except that for the center vertical, which remains unity. For the Pratt truss with an even number of panels the coefficients are divided by two for the center loads for all pieces, except that for the center vertical for loads on the top chord, the coefficient remains unity. For the young architect or engineer these tables will be found useful in furnishing a check upon stresses determined by ALGEBRAIC OR GRAPHICAL METHODS.

Table VII. Coefficients for Determining the Stresses in Simple Fink and Fan Trusses

WHEN PANEL-LOADS ARE ALL EQUAL



Simple Fink Truss



Simple Fan Truss

To find the stress in any member, multiply its factor by the panel-load, P

SIMPLE FINK TRUSS

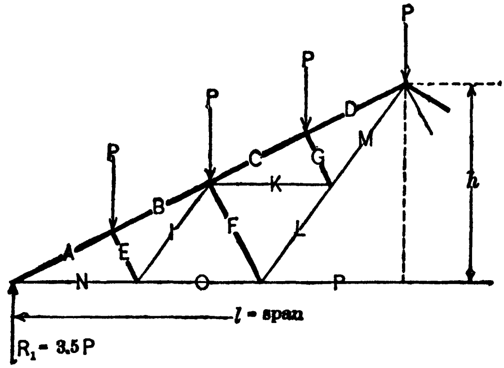
Member	Kind of stress	$l/h = 3$	$l/h = 3.464 = 30°$	$l/h = 4$	$l/h = 5$
A.....	Compression	2.70	3 00	3 35	4.04
B.....	Compression	2.15	2.50	2.91	3.67
D.... .	Compression	0.83	0.87	0 89	0.93
F... . .	Tension	2.25	2 60	3 00	3.75
G	Tension	1.50	1 73	2.00	2.50
K....	Tension	0.75	0 87	1.00	1.25

SIMPLE FAN TRUSS

Member	Kind of stress	$l/h = 3$	$l/h = 3.464 = 30°$	$l/h = 4$	$l/h = 5$
A...... .	Compression	4.51	5.00	5.59	6.73
B..... .	Compression	3.54	4 00	4.55	5.59
C..........	Compression	3.40	4 00	4.70	5 99
D.... ...	Compression	0.93	1.00	1.08	1.21
E..... ..	Compression	0.93	1 00	1.08	1.21
F.........	Tension	3.75	4.33	5 00	6.25
G..... .	Tension	2.25	2.60	3.00	3.75
K....	Tension	1.50	1.73	2 00	2.50

Table VIII. Coefficients for Determining the Stresses in an Eight-Panel Fink Truss

WHEN PANEL-LOADS ARE ALL EQUAL



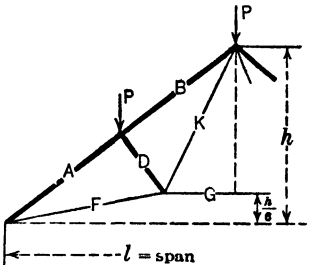
Eight-panel Fink Truss

To find the stress in any member, multiply its factor by the panel-load, *P*

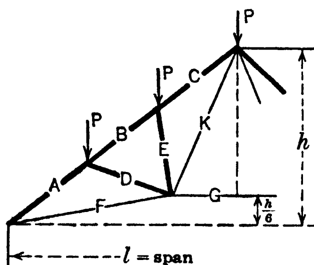
Member	Kind of stress	$l/h = 3$	$l/h = 3.464 = 30°$	$l/h = 4$	$l/h = 5$
A.......	Compression	6.31	7 00	7.83	9 42
B.........	Compression	5.76	6.50	7.38	9.05
C.........	Compression	5.20	6.00	6.93	8 68
D.........	Compression	4.65	5 50	6 48	8.31
E.........	Compression	0.83	0.87	0.89	0.93
F.........	Compression	1.66	1.73	1.79	1 86
G.........	Compression	0.83	0.87	0 89	0.93
I.........	Tension	0 75	0 87	1.00	1 25
K.........	Tension	0.75	0.87	1.00	1 25
L.........	Tension	1.50	1 73	2.00	2.50
M.........	Tension	2.25	2.60	3.00	3.75
N.........	Tension	5.25	6.06	7.00	8.75
O.........	Tension	4.50	5 19	6.00	7.50
P.........	Tension	3.00	3.46	4.00	5.00

Table IX. Coefficients for Determining the Stresses in Cambered Fink and Fan Trusses

WHEN PANEL-LOADS ARE ALL EQUAL AND THE CAMBER EQUALS ONE-SIXTH THE RISE



$R_1 = 1.5P$ Fig. A



$R_1 = 2.5P$ Fig. B

To find the stress in any member, multiply its factor by the panel-load, P

TRUSS LIKE FIG. A

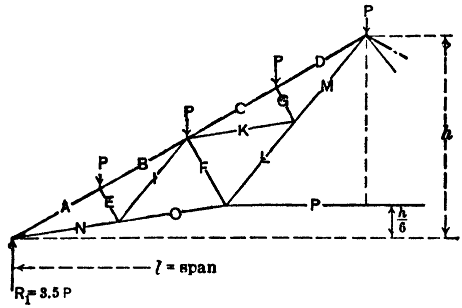
Member	Kind of stress	$l/h=3$	$l/h=3.464$ $=30°$	$l/h=4$	$l/h=5$
A.. .	Compression	3 64	4 13	4 70	5.78
B.....	Compression	3 09	3 63	4 25	5.41
D....	Compression	0 83	0 87	0.89	0 93
F ..	Tension	3 07	3 62	4 24	5.40
G .	Tension	1 80	2 08	2.40	3 00
K...	Tension	1.43	1.69	1 98	2 52

TRUSS LIKE FIG. B

Member	Kind of stress	$l/h=3$	$l/h=3.464$ $=30°$	$l/h=4$	$l/h=5$
A. .	Compression	6 09	6 88	7 83	9 64
B...	Compression	4 89	5 63	6 48	8.10
C.	Compression	4.96	5 88	6 93	8 89
D...	Compression	1.04	1 15	1.26	1 49
E ..	Compression	1.04	1 15	1 26	1.49
F .	Tension	5 12	6.03	7.07	9.01
G	Tension	2 70	3.12	3.60	4.50
K.........	Tension	2.66	3.13	3.67	4.69

Table X. Coefficients for Determining the Stresses in an Eight-Panel Cambered Fink Truss

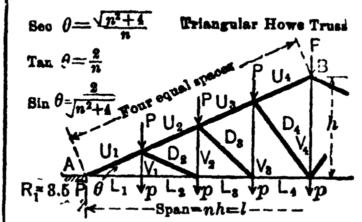
WHEN PANEL-LOADS ARE ALL EQUAL AND CAMBER EQUALS ONE-SIXTH THE TOTAL RISE

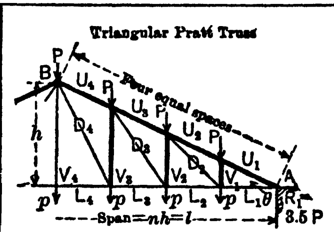


To find the stress in any member, multiply its factor by the panel-load, *P*

Member	Kind of stress	$l/h=3$	$l/h=3.464$ $=30°$	$l/h=4$	$l/h=5$
A	Compression	8 49	9 63	10.96	13.49
B	Compression	7 94	9 13	10 51	13.11
C	Compression	7 39	8 63	10 06	12 74
D	Compression	6 83	8 13	9 61	12 37
E	Compression	0 83	0 87	0.89	0 93
F	Compression	1 66	1 73	1 79	1 86
G	Compression	0 83	0 87	0 89	0.93
I	Tension	1 02	1 21	1.41	1 80
K	Tension	1 02	1 21	1 41	1 80
L	Tension	2 87	3 37	3.96	5 04
M	Tension	3 89	4 58	5.37	6 85
N	Tension	7 17	8 44	9 90	12 61
O	Tension	6 15	7 23	8.48	10 81
P	Tension	3.60	4.16	4.80	6.00

Table XI. Coefficients for Eight-Panel Roof-Trusses



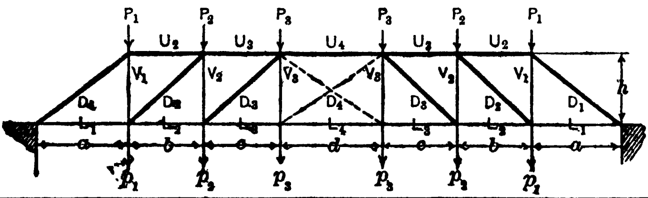


Member	Howe: Roof-loads, P	Howe: Ceiling-loads, p	Howe: Length of member	Pratt: Roof-loads, P	Pratt: Ceiling-loads, p	Pratt: Length of member
U_1	$1.75\sqrt{n^2+4}$	$1.75\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$	$1.75\sqrt{n^2+4}$	$1.75\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$
U_2	$1.50\sqrt{n^2+4}$	$1.50\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$	$1.75\sqrt{n^2+4}$	$1.75\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$
U_3	$1.25\sqrt{n^2+4}$	$1.25\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$	$1.50\sqrt{n^2+4}$	$1.50\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$
U_4	$1.00\sqrt{n^2+4}$	$1.00\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$	$1.25\sqrt{n^2+4}$	$1.25\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$
L_1	$1.75\,n$	$1.75\,n$	$0.125\,nh$	$1.75\,n$	$1.75\,n$	$0.125\,nh$
L_2	$1.75\,n$	$1.75\,n$	$0.125\,nh$	$1.50\,n$	$1.50\,n$	$0.125\,nh$
L_3	$1.50\,n$	$1.50\,n$	$0.125\,nh$	$1.25\,n$	$1.25\,n$	$0.125\,nh$
L_4	$1.25\,n$	$1.25\,n$	$0.125\,nh$	$1.00\,n$	$1.00\,n$	$0.125\,nh$
V_1	0	1.0	$0.25\,h$	1.0	0	$0.25\,h$
V_2	0.5	1.5	$0.50\,h$	1.5	0.5	$0.50\,h$
V_3	1.0	2.0	$0.75\,h$	2.0	1.0	$0.75\,h$
V_4	3.0	4.0	$1.00\,h$	0	1.0	$1.00\,h$
D_2	$0.25\sqrt{n^2+4}$	$0.25\sqrt{n^2+4}$	$0.125h\sqrt{n^2+4}$	$0.25\sqrt{n^2+16}$	$0.25\sqrt{n^2+16}$	$0.125h\sqrt{n^2+16}$
D_3	$0.25\sqrt{n^2+16}$	$0.25\sqrt{n^2+16}$	$0.125h\sqrt{n^2+16}$	$0.25\sqrt{n^2+36}$	$0.25\sqrt{n^2+36}$	$0.125h\sqrt{n^2+36}$
D_4	$0.25\sqrt{n^2+36}$	$0.25\sqrt{n^2+36}$	$0.125h\sqrt{n^2+36}$	$0.25\sqrt{n^2+64}$	$0.25\sqrt{n^2+64}$	$0.125h\sqrt{n^2+64}$

Stress = coefficient $\times P$ or p

For a half-truss supported at A and B, reduce all top-chord coefficients by $\sqrt{n^2+4}$ and all bottom-chord coefficients by n. The coefficients for the web-members used remain unchanged.

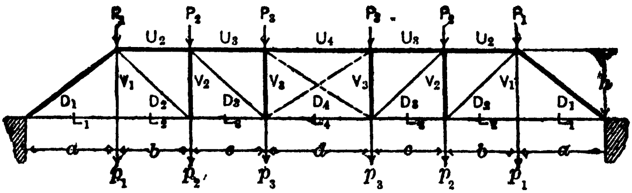
Table XII. Coefficients for Howe Trusses which are Symmetrical About the Center of the Span and Symmetrically Loaded



Member	7 panels			5 panels		3 panels
	P_1	P_2	P_3	P_1	P_2	P_1
L_1 and U_2	$a\div h$	$a\div h$	$a\div h$	$a+h$	$a\div h$	$a+h$
L_2 and U_3	$a\div h$	$(a+b)\div h$	$(a+b)+h$	$a+h$	$(a+b)\div h$	.. .. ..
L_2 and U_4	$a\div h$	$(a+b)\div h$	$(a+b+c)\div h$	... .	.... ..	.... . .
L_4. ..	$a+h$	$(a+b)\div h$	$(a+b+c)\div h$	.... .	....	.
D_1 .	$\sqrt{a^2+h^2}+h$	$\sqrt{a^2+h^2}+h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$
D_2...	0	$\sqrt{b^2+h^2}\div h$	$\sqrt{b^2+h^2}+h$	0	$\sqrt{b^2+h^2}+h$	. ...
D_3..	0	0	$\sqrt{c^2+h^2}+h$	0	0	. ...
D_4.	0	0	0	.	....	. ...
V_1..	0	1 0	1 0	0	1.0	0
V_2. .	0	0	1 0	0	0	....
V_3.. .	0	0	0	. .. .	.. . . .	.... . .
	p_1	p_2	p_3	p_1	p_2	p_1
V_1.	1 0	1.0	1.0	1 0	1.0	1.0
V_2 .	0	1.0	1 0	0	1.0	... ..
V_3	0	0	1.0	.. .	. .. . .	.

For loads p_1, p_2, etc., the coefficients for the chords and diagonals are the same as given for the loads P_1, P_2, etc. The coefficients for the verticals for loads p_1, p_2, etc, are given in the supplementary table below the general table. Tension is indicated in the truss diagram by light lines.

Table XIII. Coefficients for Pratt Trusses which are Symmetrical About the Center of the Span and Symmetrically Loaded



Member	7 panels			5 panels		3 panels
	P_1	P_2	P_3	P_1	P_2	P_1
L_1 and L_2	$a-h$	$a\div h$	$a\div h$	$a\div h$	$a\div h$	$a\div h$
L_3 and U_2	$a\div h$	$(a+b)\div h$	$(a+b)\div h$	$a\div h$	$(a+b)\div h$	..
L_4 and U_3	$a\div h$	$(a+b)\div h$	$(a+b+c)\div h$	.		..
$U_4=L_4$..	.		.			
D_1	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$	$\sqrt{a^2+h^2}\div h$
D_2. . . .	0	$\sqrt{b^2+h^2}\div h$	$\sqrt{b^2+h^2}\div h$	0	$\sqrt{b^2+h^2}\div h$	. . . . .
D_3. . . .	0	0	$\sqrt{c^2+h^2}\div h$	0	0	. . . .
D_4. . . .	0	0	0			. .
V_1. . . .	0	0	0	0	0	0
V_2. . . .	0	1.0	1.0	0	1.0	. .
V_3.	0	0	1.0			
	p_1	p_2	p_3	p_1	p_2	p_1
V_1. . .	1.0	0	0	1.0	0	1.0
V_2	0	0	1.0	0	0	. .
V_3. . . .	0	0	0	. .	. . . .	. . .

For loads p_1, p_2, etc., the coefficients for the chords and diagonals are the same as given for the loads P_1, P_2, etc. The coefficients for the verticals for loads p_1, p_2, etc., are given in the supplementary table below the general table. Tension is indicated in the truss-diagram by light lines.

Examples Showing Use of Tables in Stress-Computations

Simple Fan Truss. Example 1. In this example a simple fan truss of 36-ft span is considered. The distance on centers of trusses is 12 ft. The height of truss is 9 ft, or $l/h = 4$. The total load per square foot of roof is 40 lb. The length of rafter is 20 ft, nearly. The panel-load, $P = {}^{20}\!/_{3} \times 12 \times 40 = 3\,200$ lb. Then from Table VII.

Stress in lower end of rafter $A = 3\,200 \times 5.59 = 17\,888$ lb
Stress in ends of main tie $F = 3\,200 \times 5.00 = 16\,000$ lb
Stress in center of main tie $G = 3\,200 \times 3.00 = 9\,600$ lb
Stress in braces D and $E = 3\,200 \times 1.08 = 3\,456$ lb
Stress in tie $K = 3\,200 \times 2 = 6\,400$ lb

Five-Panel Howe Truss. Example 2. (Table XII.) A five-panel Howe truss is considered, for which $h = 6$ ft, $a = 9$ ft, $b = 10$ ft and $c = 12$ ft. Let the trusses be spaced 10 ft on centers, the roof-load be 40 lb per sq ft and the ceiling-load 15 lb per sq ft. The panel-loads become:

$$\left.\begin{aligned} P_1 &= \tfrac{1}{2}(\ 9 + 10)\,(10 \times 40) = 3\,800 \text{ lb} \\ p_1 &= \tfrac{1}{2}(\ 9 + 10)\,(10 \times 15) = 1\,400 \text{ lb} \end{aligned}\right\} = 5\,200 \text{ lb}$$

$$\left.\begin{aligned} P_2 &= \tfrac{1}{2}(10 + 12)\,(10 \times 40) = 4\,400 \text{ lb} \\ p_2 &= \tfrac{1}{2}(10 + 12)\,(10 \times 15) = 1\,700 \text{ lb} \end{aligned}\right\} = 6\,100 \text{ lb}$$

L_1 and $U_2 = {}^{9}\!/_{6} \times 5\,200 + {}^{9}\!/_{6} \times 6\,100 = 17\,000$ lb
L_2 and $U_3 = {}^{9}\!/_{6} \times 5\,200 + {}^{19}\!/_{6} \times 6\,100 = 27\,100$ lb
$D_1 = 10.82/6\,(5\,200 + 6\,100) = 20\,400$ lb
$D_2 = 11.66/6 \times 6\,100 = 11\,900$ lb
$V_1 = 4\,400 + 1\,400 + 1\,700 = 7\,500$ lb
$V_2 = 1\,700$ lb

In the above results all values between 50 and 100 have been considered 100.

3. General Arrangement of Roof Framing Plan

The general arrangement of the roof-construction should be considered by the architectural designer when the preliminary studies are being made for any building. The framing of the roof is an important factor both as to cost and general appearance of the building. A scheme which is dictated by architectural requirements without due consideration of structural procedure may give rise to an uneconomical and inefficient structural design.

The architectural designer should provide satisfactory depths, spacings and supports for the roof-trusses.

The four fundamental items which must be considered in connection with the structural roof plan are:

(1) The spacing of trusses.
(2) The spacing of purlins.
(3) The type of truss.
(4) Provisions for support.

Spacing of Trusses. The most economical spacing of trusses, considering the complete roof framing system, depends upon relative costs of the component parts. The unit cost of trusses, per square foot of horizontal area, decreases as the spacing of trusses is increased. The unit cost of purlins, however, increases as the spacing of the trusses is increased.

The unit cost of trusses per square foot of horizontal projection is, in gen-

eral, several times that of purlins; hence, it is usually the cost of trusses which controls the economy of the design.

It is generally desirable, for truss-spans under 100 ft, to use simple, rather than trussed or built-up, sections for purlins, and the maximum truss-spacing may be governed by this condition.

For long spans the spacing of trusses may be increased, and rafters supported by heavy purlins may be introduced to provide the direct support for the roof-covering.

The spacing of trusses is often governed by local conditions, such as the positions of special loads which are to be supported, the location of piers or columns, or by other requirements fixed by the architectural design of the building. Considering the spacing from an economical standpoint, for roofs of the more permanent type of construction, the values given in Table XIV will probably result in the most favorable economy.

Table XIV. Economical Spacing of Roof-Trusses

Span of truss in feet	Spacing of trusses in feet
20 to 40	12 to 16
40 to 60	16 to 18
60 to 80	18 to 20
80 to 100	20 to 22
100 and over	22 to 24

Spacing of Purlins. The maximum spacing of purlins is limited by the allowable span of the structural roof-covering. Whenever possible, the purlins should be placed at the upper chord panel-points. In cases where the type of truss is such that the allowable span of the roof-covering is less than the panel-point distance, intermediate purlins may be used. Such an arrangement subjects the supporting chord-member to a bending as well as a direct stress, since the chord must act as a beam in addition to its function as a member of the truss.

In general, the panel lengths of the supporting chord are made equal to, or somewhat less than, the limiting span of the structural covering; however, the introduction of intermediate purlins may, in some cases, be an economical procedure, since the saving in weight of the smaller and more closely spaced purlins may offset the increased weight of the supporting chord-members.

When roof-coverings are laid on sheathing supported by rafters parallel to the upper chords of the trusses, the purlin spacing is governed by the limiting span length of the rafters.

In calculating the safe span of the structural covering both the FLEXURAL STRENGTH and the probable DEFLECTION should be investigated. When the weathering surface of the roof is tile or slate, the deflection of the sheathing should be limited to about $\frac{1}{360}$ part of the span in order to avoid cracking the material. Roofs covered with materials not subject to cracking under excessive deflection, present a much better appearance when the deflection has been kept within this same limit. Assuming that the structural

covering is continuous in its span over several purlins, the moment coefficient may be taken at $\frac{1}{10}$ and the deflection coefficient at about $\frac{2}{384}$ for uniformly distributed loads.

Let w = the design loading in pounds per square foot of roof-surface;
L_p = the spacing of purlins in feet;
A = the angle of the roof slope with the horizontal, in degrees;
t = the thickness of continuous sheathing;
I = the moment of inertia of a cross-section of sheathing one foot wide, inches^4;
E = modulus of elasticity of sheathing material in pounds per square inch;
f = allowable unit stress in flexure, in pounds per square inch.

Then, for the maximum spacing of purlins as limited by the flexural strength of the sheathing of homogeneous material:

$$\frac{w\,(\cos A)\,L_p{}^2 \times 12}{10} = \frac{fI}{c} = \frac{f}{6} \times 12\,t^2$$

whence

$$L_p = 1.29\,t\sqrt{\frac{f \sec A}{w}} \tag{6}$$

For a limiting value of Δ = 1/360 part of the span:

$$\Delta = \frac{2}{384} \cdot \frac{w\,(\cos A)\,L_p{}^4 \times \overline{12}^3}{EI} = \frac{L \times 12}{360}$$

and

$$I = \frac{1}{12} \cdot 12 \cdot t^3 = t^3$$

whence

$$L_p = \frac{t}{6.46}\sqrt[3]{\frac{E \sec A}{w}} \tag{7}$$

If the structural slab is a nailing concrete the value of I, for the usual specifications, may be taken as about 0.6 the value of a homogeneous section of the dimensions of the section of the nailing concrete slab.

The maximum purlin spacing will be limited by the flexural strength of the slab, or:

$$\frac{w\,(\cos A)\,L_p{}^2 \times 12}{10} = 0.6\left(\frac{f}{6} \times 12 \times t^2\right)$$

whence

$$L_p = t\sqrt{\frac{f \cdot \sec A}{w}} \tag{8}$$

The limiting spans for wood sheathing as determined by Equations (6) and (7) are given in Table XV, and the limiting spans for nailing concretes in common use, as determined by Equation (8) are given in Table XVI.

From the above equations it is evident that limiting spans both in flexure and deflection vary directly as the thickness of the structural covering, hence Tables XV and XVI may be used for various thicknesses by applying the proper ratios.

Table XV. Limiting Spans for One-Inch Sheathing for Various Design Loads and Roof-Slopes

$f = 1\,200$ lb per sq in. $E = 1\,200\,000$ lb per sq in.
(values given in feet)

Slope of roof	Unit vertical design load in pounds per square foot of roof-surface				
	20	30	40	50	60
Flat	10 0 (6.0) *	8.1 (5.3)	7.0 (4.8)	6.3 (4.4)	5.8 (4.1)
15°	10 2 (6 1)	8.3 (5 4)	7 2 (4 9)	6 4 (4 5)	5.9 (4.2)
25°	10 5 (6 2)	8 6 (5 5)	7 4 (5 0)	6 6 (4 6)	6.1 (4.3)
30°	10 7 (6 3)	8.8 (5.6)	7 6 (5 0)	6 8 (4 7)	6.2 (4 4)
35°	11.1 (6 5)	8.9 (5.7)	7.8 (5 2)	7 0 (4 8)	6.4 (4 5)
40°	11 4 (6.6)	9.3 (5.8)	8.1 (5.3)	7.2 (4.9)	6.6 (4.6)
45°	11.9 (6 8)	9.7 (5.9)	8.4 (5.4)	7.5 (5.0)	6.9 (4.7)

* Limiting values for deflection in parenthesis. Values for thicknesses other than one inch can be determined by direct proportion.

Table XVI. Limiting Spans for 3-In Nailing Concrete Roof-Slabs for Various Design Loads and Roof-Slopes

$f_c = 225$ lb per sq in. $E_c = 1\,000\,000$ lb per sq in.
$f_s = 14\,000$ lb per sq in. $n = 30$
(values given in feet)

Slope of roof	Unit vertical design load in pounds per square foot of roof-surface				
	30	40	50	60	70
Flat	8.2	7 1	6.3	5 7	5 5
15°	8.4	7.2	6.5	5 9	5.6
25°	8.6	7.4	6.7	6.1	5.7
30°	8 8	7 6	6 9	6.3	5.8
35°	9 1	7.9	7 1	6 5	5.9
40°	9.4	8.2	7.3	6.7	6.2
45°	10.0	8.5	7.6	6.9	6.5

Values for thickness other than 3 in can be determined by direct proportion.

4. Design of Rafters, Hips and Valleys

Rafters and Hip and Valley Beams may be treated as horizontal beams, so far as the component of the vertical loading normal to slope is concerned.

The component of the vertical loading which is parallel to the inclined beam will produce a direct compression throughout the beam if resisted at the lower end, or a direct tension throughout the beam if resisted at the upper end, or compression on the lower portion and tension on the upper portion of the beam if resisted at both ends.

If the parallel components of the applied loads are axial, the average unit stress caused by them will be added to the flexural stress of the same character at the section where $f_a + f_b$ is maximum. The cross-section of the member may be tentatively selected by considering the maximum moment due to the normal components. Later this section may be corrected to satisfy the combined stress $f_a + f_b$. If the parallel components of the applied loads are not axial their eccentricity may increase or diminish the bending moment of the normal components.

In practical design the parallel components of the end reactions of an inclined beam may be distributed as determined by the following assumptions:

(1) That the reaction at the upper support is horizontal.

(2) That the total parallel component is resisted at the lower support.

(3) That the reactions at each support are vertical.

These three cases are illustrated in Fig. 1.

Case 1. The reaction at the upper support is assumed to be horizontal, and since the three forces producing equilibrium must meet in a common point, the direction of the reaction at the lower support can be found as shown at (*a*). The magnitude of the upper reaction may be determined by taking moments about the lower support: $\Sigma M = 0$; $-6.93\,R_2 + 1\,800 \times 6 = 0$; whence $R_2 = 1\,560$ lb, and R_1 may be found from $\Sigma V = 0$. The direction of R_1 makes an angle, with the vertical, the tangent of which is $\dfrac{6.0}{6.93} = 0.866$, or $\beta = 40° - 55'$. Then $R_1 \cos\beta = 1\,800$, or $R_1 = 2\,380$ lb. The reactions may also be found by constructing a force-polygon as shown at (*c*). Each reaction is then resolved into components normal and parallel to the slope of the inclined beam with the following results:

$$\begin{array}{l} R_1 \\ 2\,380 \text{ lb} \end{array} \left\{ \begin{array}{l} N_1 = 780 \text{ lb} \\ P_1 = 2\,250 \text{ lb} \end{array} \right.$$

$$\begin{array}{l} R_2 \\ 1\,560 \text{ lb} \end{array} \left\{ \begin{array}{l} N_2 = 780 \text{ lb} \\ P_2 = 1\,350 \text{ lb} \end{array} \right.$$

The maximum bending moment, at the mid-point of span, is:

$$M_{\max} = \frac{1\,560 \times 13.86 \times 12}{8} = 32\,400 \text{ lb-in}$$

If the allowable unit flexural stress is 1 200 lb per sq in the tentative section is found from:

$$\frac{1\,200\,bd^2}{6} = 32\,400$$

or

$$bd^2 = 162$$

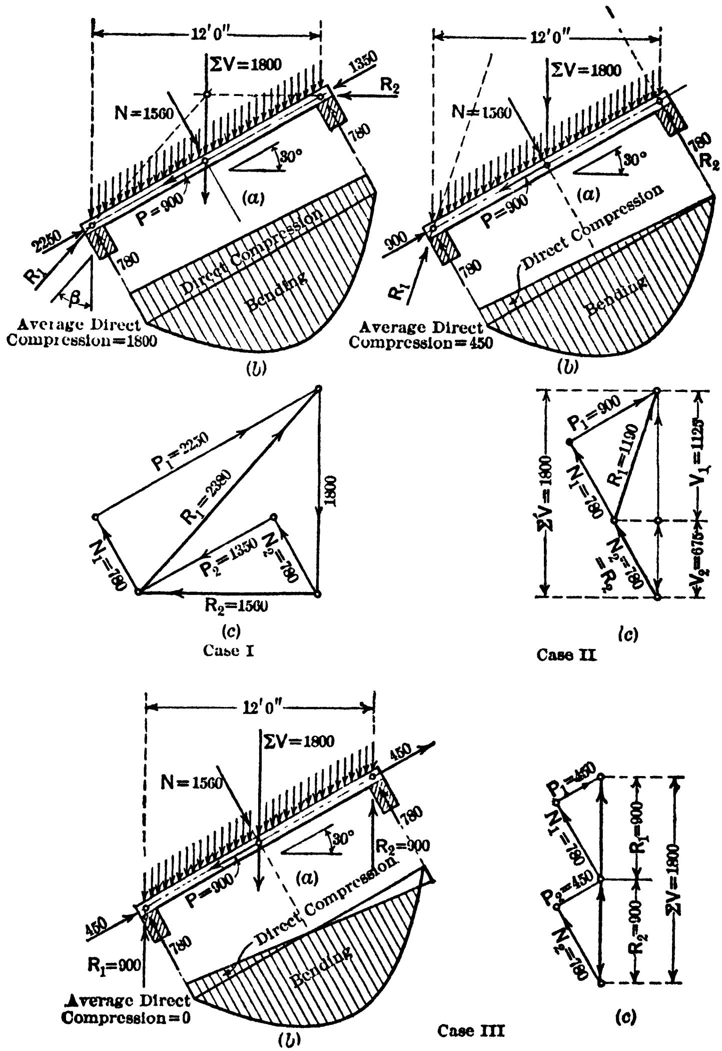


Fig. 1. Design of Rafters, Hips and Valleys

A nominal $2\frac{1}{2} \times 10$-in timber section provides a bd^2 value $= 2.125 \times \overline{9.5}^2 = 191$. The average direct compression, at the mid-point of span, is $\frac{1}{2}(2\,250 + 1\,350) = 1\,800$ lb, then:

$$f_a + f_b = \frac{1\,800}{2.125 \times 9.5} + \frac{32\,400 \times 6}{2.125 \times 9.5^2}$$

$$= 89.5 + 1\,018.5 = 1\,108 \text{ lb per sq in}$$

The permissible working unit stress is an indeterminate value between the values for flexure and the value for axial compression. If the inclined beam is stayed laterally by sheathing its L/d ratio is $\frac{13\,86 \times 12}{9.5} = 17.5$, and if the material is common grade, Southern yellow pine, the allowable axial unit stress is:

f_a (Table XXI) $= 836$, and f_b (Table XX) $=$ **1 200 lb per sq in**

The allowable combined unit stress is:

$$\frac{89.5}{1\,108} \times 836 = 67.0$$

$$\frac{1\,018.5}{1\,108} \times 1\,200 = \frac{1\,100.0}{1\,167.0} \text{ lb per sq in}$$

The section, as selected, is, therefore, satisfactory.

Case 2. The entire parallel component of the applied load may be assumed to be resisted at the lower support as shown in Case II, Fig. 1. The reactions and their components normal and parallel to the inclined beam are found by constructing the force-polygon as shown at (*c*). The results are:

$$\begin{array}{cl} R_1 & \left\{ \begin{array}{l} N_1 = 780 \text{ lb} \\ P_1 = 900 \text{ lb} \end{array} \right. \\ 1\,190 \text{ lb} & \end{array}$$

$$\begin{array}{cl} R_2 & \left\{ \begin{array}{l} N_2 = 780 \text{ lb} \\ P_2 = 0 \end{array} \right. \\ 780 \text{ lb} & \end{array}$$

The maximum bending moment, at the center of the span, as in Case I, is:

$$M_{\max} = 32\,400 \text{ lb-in}$$

The average direct compression, at the mid-point of span, is $\frac{1}{2} \times 900 = 450$ lb. Assuming the same section as in Case I:

$$f_a + f_b = \frac{450}{20.19} + \frac{32\,400 \times 6}{191}$$

$$= 1\,041 \text{ lb per sq in}$$

The allowable unit stress is 1 115 lb per sq in, found in the same manner as in Case 1.

Case 3. If the reactions are assumed to be vertical, then the normal and parallel components are equal, as shown at (*c*), Case III, Fig. 1. In this assumption the parallel components of the loads on the upper half of the inclined beam are resisted by a parallel reaction, acting away from the end of the beam at the upper support. The parallel components of the loads on the lower half are resisted by a parallel reaction, acting against the end of the beam at the lower support.

The axial stress at the mid-point of span is zero. The assumptions made in this case are, therefore, equivalent to an assumption of a horizontal span length equal to the horizontal projection of the inclined beam and loaded with the total vertical load.

The maximum unit stress is the flexural stress at the mid-point of span, which for the cross-section adopted in the preceding cases is:

$$f_b = \frac{32\,400 \times 6}{191} = 1\,018.5 \text{ lb per sq in}$$

The full allowable flexural stress as specified for horizontal beams should not be permitted, but should be reduced, for timber beams, making an angle A with the horizontal, as follows:

$$f' = f_c \left(1 - \frac{L}{60\,d} \cdot \sin A\right) \tag{9}$$

in which d = dimension of timber perpendicular to the axis of bending under column action, f_c = allowable unit stress in flexure, and L = the unsupported length with reference to column action, in inches.

Then for the case at hand

$$f' = 1\,200 \left(1 - \frac{17.5}{60} \times \frac{1}{2}\right)$$

$$= 1\,020 \text{ lbs per sq in}$$

5. Design of Purlins

The Purlin Load, when common rafters are employed to support the sheathing, consists of a series of uniformly spaced concentrated loads, of a number such that the total load may usually be considered as uniformly distributed without serious error. When the sheathing or other continuous structural covering is supported directly by the purlins, the loading is uniform and is equal to one panel-load.

PURLINS are sometimes placed with their principal axes in a horizontal and vertical position, without regard to the slope of the roof. Such an arrangement provides the greatest resistance to vertical loads; however, it may be quite unfavorable for wind-loads.

The usual method, and the one which provides for the most satisfactory details, is to place the purlins so that their principal axes are parallel and perpendicular to the roof-slope.

In any type of construction a purlin on a sloping roof presents a problem of OBLIQUE LOADING.

When the plane of loading does not coincide with one of the principal axes of the section, the purlin does not deflect in the plane of the loading and the neutral axis is oblique to that plane.

At a point such as O, Fig. 2, the coördinates of which are x and y with reference to axes X and Y, the stress may be found by resolving the bending moment, due to the applied loads, into its components along the principal axes.

The stress at point O, due to each component of the moment, is found separately and the values added algebraically. The stress at point O, Figs. 2(a) and 2(b), is, therefore:

$$f_0 = \frac{M\,(\cos A)\,y}{I_x} + \frac{M\,(\sin A)\,x}{I_y} \tag{10}$$

It may sometimes be necessary to find the direction of the true neutral axis for oblique loading, in order to locate the extreme, or most highly stressed, fiber. Since there is zero stress at all points along the neutral axis, x and y become the coordinates of one point on that axis when $f_0 = 0$, in Equation (10). The ratio y/x then becomes the tangent of angle Θ, which the neutral axis makes with principal axis X, Fig. 2, or:

$$\tan \Theta = \frac{y}{x} = \frac{I_x}{I_y} \tan A \tag{11}$$

For a given roof-slope A, and a given purlin-section, the maximum unit fiber-stress for vertical loading, as given by Equation (10), is.

$$f_{\max} = M_v \left(\frac{c_1 \cos A}{I_1} + \frac{c_2 \sin A}{I_2} \right) \tag{12}$$

in which M_v = the moment of the vertical loads supported by the purlin;
A = the angle of the roof-slope with the horizontal;
I_1 = moment of inertia of the purlin-section referred to the axis parallel to the roof-slope (axis 1–1);
c_1 = the extreme fiber distance with reference to axis (1–1);
I_2 = moment of inertia of the purlin-section referred to the axis perpendicular to the roof-slope (axis 2–2);
c_2 = the extreme fiber distance with reference to axis (2–2).

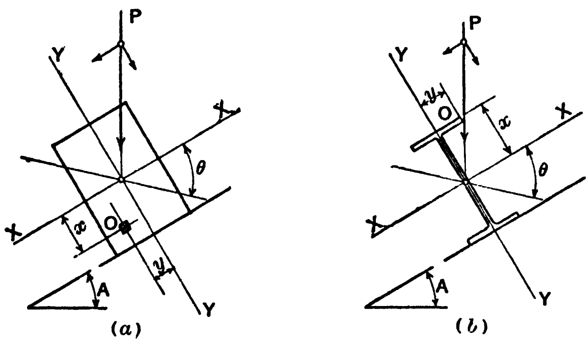


Fig. 2

The MAXIMUM FIBER-STRESS increases as angle A increases, the increase being due to the component of the moment parallel to the roof, or the bending about axis 2–2.

The stress resulting from this component of the bending moment may be materially reduced by the use of a rigid structural covering, or by employing tie-rods at intermediate points of the purlin-span. When the structural covering is rigid, as in the case of a concrete slab, a large portion of the parallel component of the vertical roof-load is transferred to the trusses directly through the beam action of the slab in the plane of the roof. A structural covering of sheathing supported by rafters, or supported directly by the purlins, will provide an appreciable amount of stiffness and beam action in the plane of the roof. The value of a structural covering in reducing the bending stress about the axis of the purlin, normal to the roof, is indeterminate, and

even with a fairly rigid covering the use of tie-rods may sometimes be advisable.

With an arrangement as shown at (*a*), Fig. 3, each tie-rod between purlins P_1 and P_2 is assumed to support one-third of the parallel component tributary to purlin P_1; each tie-rod between purlins P_2 and P_3 will support the load from the one tie-rod in panel ① plus one-third of the parallel component tributary to purlin P_2. The load on each successive set of tie-rods above the eave panel is, therefore, increased by one-third of the parallel component of one panel-load. The ridge-purlin in such an arrangement is subjected to two concentrated loads, at the points of tie-rod connections, each equal to the vertical component of the accumulated tie-rod load for each slope of the roof.

For a symmetrical roof the vertical concentrated load at each one-third point of the ridge is equal to $\frac{1}{3}(S \cdot L \cdot w \sin^2 A)$, where w is the unit vertical roof-load in pounds per square foot of horizontal projection, S = the spacing of trusses in feet, and L = the span of trusses in feet. It is important to note

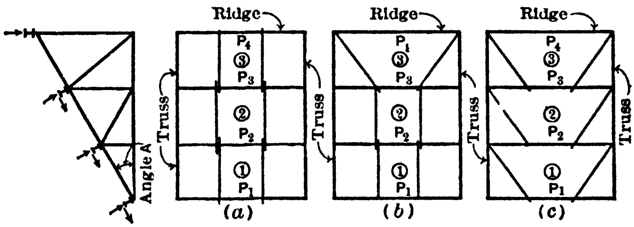


Fig. 3. Arrangements of Tie-rods for Supporting Purlins

that for unsymmetrical roof-slopes or for an unsymmetrical distribution of roof-loads, an unbalanced horizontal component will exist at the ridge-purlin.

If the distribution of the roof-loads to the truss is consistent with the assumptions relative to the tie-rod action as described, there will be transmitted to the peak of a symmetrical truss, the vertical component of two-thirds the total parallel component of the vertical roof-load, or $\frac{2}{3}(S \cdot L \cdot w) \sin^2 A$, in addition to the half-panel-load on each side of the ridge-purlin. If the roof or roof-loads are unsymmetrical, the ridge-purlin must transfer the vertical components of the tie-rod loads plus the unbalanced horizontal component to the peak of the supporting truss.

The reactions of the intermediate purlins on the truss would consist of the normal component of one panel-load plus one-third of the parallel component of one panel-load.

In calculating the panel-loads for the design of trusses, when tie-rods have been employed as at (*a*), Fig. 3, the distribution of load is generally assumed to be uniform, the previously assumed tie-rod loading being disregarded.

Such an inconsistency can be justified only on the grounds that a considerable portion of the parallel component of the roof-loads is transferred to the trusses directly through the structural covering.

The method of placing tie-rods as shown at (*b*), Fig. 3, relieves the ridge-purlin of the concentrated loads by transferring the tie-rod load directly to the peak of the truss.

The theoretical truss loading for the tie-rod arrangement at (*b*) is exactly the same as for the arrangement at (*a*).

The arrangement of tie-rods as shown at (*c*), Fig. 3, eliminates the additional load at the peak of the truss by transferring the full parallel component of each panel-load directly to the corresponding panel-point of the truss.

The action of the various parts of the roof-structure and the distribution of the parallel components of the roof-loads, when a rigid structural covering

Table XVII. Coefficients of M_v for Total Maximum Purlin Stress in Equation $f_{max} = M_v \cdot K$

For Tie-rods at the One-third Points, or Equivalent

Section	Slope of Roof = Angle A in Degrees							
	10°	15°	20°	25°	30°	35°	40°	45°
7″-⊔- 9.8#	0.1874	0.1966	0.2043	0.2101	0.2146	0.2167	0.2180	0.2173
7″-⊔-12.25#	0.1648	0.1734	0.1796	0.1848	0.1885	0.1906	0.1920	0.1914
8″-⊔-11.5#	0.1415	0.1492	0.1550	0.1597	0.1636	0.1659	0.1674	0.1672
8″-⊔-13.75#	0.1279	0.1348	0.1404	0.1148	0.1448	0.1507	0.1522	0.1522
9″-⊔-13.4#	0.1094	0.1158	0.1206	0.1250	0.1284	0.1302	0.1318	0.1320
9″-⊔-15.0#	0.1028	0.1090	0.1139	0.1180	0.1215	0.1234	0.1252	0.1257
7″-I-15.3#	0.1056	0.1089	0.1110	0.1125	0.1134	0.1129	0.1121	0.1103
7″-I-17.5#	0.0981	0.1014	0.1035	0.1049	0.1059	0.1054	0.1048	0.1028
10″-⊔-15.3#	0.0863	0.0916	0.0958	0.0990	0.1019	0.1038	0.1050	0.1053
10″-⊔-20.0#	0.0744	0.0793	0.0832	0.0867	0.0895	0.0913	0.0928	0.0934
8″-I-18.4#	0.0773	0.0790	0.0820	0.0834	0.0842	0.0843	0.0840	0.0828
8″-I-20.5#	0.0731	0.0758	0.0776	0.0791	0.0799	0.0798	0.0795	0.0784
8″-I-23.0#	0.0688	0.0713	0.0730	0.0744	0.0752	0.0752	0.0750	0.0740
12″-⊔-20.7#	0.0553	0.0589	0.0619	0.0646	0.0668	0.0683	0.0696	0.0702
9″-I-21.8#	0.0586	0.0609	0.0624	0.0636	0.0644	0.0637	0.0644	0.0637
9″-I-25.0#	0.0547	0.0569	0.0586	0.0596	0.0605	0.0607	0.0607	0.0599
12″-⊔-25.0#	0.0493	0.0526	0.0553	0.0577	0.0597	0.0611	0.0622	0.0626
8″-C-24.0#	0.0498	0.0499	0.0499	0.0498	0.0490	0.0479	0.0466	0.0448
10″-C-23.0#	0.0435	0.0444	0.0450	0.0452	0.0451	0.0441	0.0437	0.0425
8″-C-27.0#	0.0439	0.0443	0.0443	0.0442	0.0435	0.0425	0.0413	0.0396
10″-C-26.0#	0.0386	0.0394	0.0399	0.0400	0.0400	0.0394	0.0388	0.0377
9″-C-29.0#	0.0374	0.0380	0.0382	0.0381	0.0378	0.0369	0.0361	0.0348
12″-C-25.0#	0.0354	0.0364	0.0373	0.0378	0.0379	0.0378	0.0374	0.0367
10″-C-30.0#	0.0333	0.0341	0.0344	0.0346	0.0345	0.0340	0.0334	0.0325
9″-C-32.0#	0.0339	0.0343	0.0345	0.0345	0.0340	0.0334	0.0325	0.0314
12″-C-28.0#	0.0302	0.0311	0.0316	0.0319	0.0320	0.0316	0.0313	0.0305
12″-C-32.0#	0.0265	0.0272	0.0275	0.0277	0.0279	0.0276	0.0272	0.0267
12″-C-36.0#	0.0235	0.0241	0.0245	0.0247	0.0248	0.0245	0.0242	0.0237

is used without tie-rods, is probably not dissimilar to the conditions existing when a tie-rod system such as illustrated at (*c*) is employed with a non-rigid covering.

When a purlin is supported by tie-rods at the one-third points of its span, L, the unsupported length is reduced to one-third L, and the three spans become a partially continuous beam with respect to moments parallel to the

roof-slope. The coefficients of wL^2 for both positive and negative moments may be taken as $\frac{1}{10}$, and Equation (12) becomes:

$$f_{max} = M_v\left(\frac{c_1 \cos A}{I_1} + \frac{8}{90}\,\frac{c_2 \sin A}{I_2}\right) \tag{13}$$

For a given roof-slope and a given purlin-section the quantity within parenthesis in Equation (13) is a constant, or:

$$f_{max} = M_v \cdot K \tag{14}$$

The values of K, Equation (14), for various rolled-steel sections commonly used as purlins, are given for various roof-slopes, in Table XVII.

The stress coefficients for timber purlins, assuming no support in the plane of the roof, are given in Table XVIII.

With the usual methods employed in timber roof-construction, where the continuous sheathing is supported by rafters spanning between purlins, or where the sheathing is supported directly by the purlins, the bending stress due to the moment in the plane of the roof is somewhat reduced. This stress may be assumed as equivalent to that produced by the component of the roof-loads in the plane of the roof when the purlin has a support, in the plane of the roof, at its mid-point of span.

The stress coefficients for this case are given in Table XIX.

Example 1. Purlin Design. Let it be required to design a steel-beam purlin for the following data:

(1) Spacing of trusses = 18 ft 0 in.
(2) Equivalent uniform design load carried by purlin = 68 lb per sq ft of roof.
(3) Spacing of purlins along the slope of the roof = 8 ft 6 in.
(4) Slope of roof (Angle A) = 30°.
(5) Allowable working unit stress = 18 000 lb per sq in.
(6) Structural covering consists of a nailing concrete slab.

$$M_v = \tfrac{1}{8} \times 68\,(18 \times 8.5) \times 18 \times 12 = 280\,900 \text{ lb-in}$$

From Equation (14),

$$K = \frac{18\,000}{280\,900} = 0.0641$$

From Table XVII, a 9 in –I– 21.8 lb, or any section having a smaller coefficient than 0 0641 in the column for 30° slope, will be satisfactory.

Example 2. Let it be required to design a timber purlin for the following data:

(1) Spacing of trusses = 16 ft 6 in.
(2) Equivalent uniform design load = 38 lb per sq ft of roof.
(3) Spacing of purlins = 7 ft 8 in.
(4) Slope of roof (Angle A) = 33°.
(5) Allowable working unit stress = 1 200 lb per sq in.
(6) No support assumed in the plane of the roof.

$$M_v = \tfrac{1}{8} \cdot 38\,(16.50 \times 7.67)\,16.50 \times 12 = 118\,900 \text{ lb-in.}$$

From Equation (14),

$$K = \frac{1\,200}{118\,900} = 0.0101$$

From Table XVIII, interpolating between the values for 30° and 35°, it is found that the proper purlin size is a nominal 8 × 12-in timber.

Table XVIII. Timber Purlins. Coefficients of M_v for Total Maximum Purlin Stress in Equation $f_{max} = M_v \cdot K$

No Support in the Plane of the Roof

Nominal Size (Inches)	Slope of Roof = Angle with Horizontal in Degrees							
	10°	15°	20°	25°	30°	35°	40°	45°
4 x 4	0.1459	0.1544	0.1619	0.1675	0.1722	0.1756	0.1778	0.1784
4 x 6	0.0656	0.0715	0.0769	0.0817	0.0859	0.0893	0.0923	0.0945
4 x 8	0.0396	0.0443	0.0486	0.0525	0.0561	0.0591	0.0618	0.0640
4 x 10	0.0264	0.0302	0.0337	0.0370	0.0400	0.0426	0.0450	0.0470
4 x 12	0.0192	0.0224	0.0254	0.0282	0.0308	0.0330	0.0351	0.0369
6 x 6	0.0418	0.0443	0.0463	0.0480	0.0495	0.0503	0.0509	0.0511
6 x 8	0.0237	0.0256	0.0273	0.0287	0.0301	0.0311	0.0319	0.0325
6 x 10	0.0155	0.0171	0.0185	0.0198	0.0210	0.0219	0.0227	0.0234
6 x 12	0.0111	0.0124	0.0137	0.0149	0.0158	0.0166	0.0174	0.0181
6 x 14	0.0084	0.0096	0.0107	0.0116	0.0126	0.0133	0.0140	0.0146
8 x 8	0.0165	0.0175	0.0182	0.0189	0.0195	0.0198	0.0201	0.0202
8 x 10	0.0107	0.0115	0.0122	0.0128	0.0133	0.0137	0.0140	0.0142
8 x 12	0.0076	0.0083	0.0089	0.0094	0.0099	0.0103	0.0106	0.0108
8 x 14	0.0057	0.0063	0.0068	0.0073	0.0078	0.0081	0.0085	0.0087
8 x 16	0.0045	0.0050	0.0055	0.0059	0.0063	0.0067	0.0070	0.0072

Table XIX. Timber Purlins. Coefficients of M_v for Total Maximum Purlin Stress in Equation $f_{max} = M_v \cdot K$

Support in the Plane of the Roof Equivalent to a Center Tie-rod

Nominal Size (Inches)	Slope of Roof = Angle with Horizontal in Degrees							
	10°	15°	20°	25°	30°	35°	40°	45°
4 x 4	0.1295	0.1296	0.1295	0.1275	0.1249	0.1213	0.1170	0.1115
4 x 6	0.0549	0.0558	0.0560	0.0560	0.0555	0.0544	0.0532	0.0514
4 x 8	0.0316	0.0325	0.0328	0.0332	0.0332	0.0329	0.0324	0.0316
4 x 10	0.0201	0.0208	0.0214	0.0217	0.0219	0.0219	0.0218	0.0214
4 x 12	0.0140	0.0147	0.0152	0.0156	0.0159	0.0162	0.0160	0.0159
6 x 6	0.0368	0.0372	0.0370	0.0365	0.0360	0.0347	0.0335	0.0318
6 x 8	0.0203	0.0205	0.0205	0.0204	0.0202	0.0197	0.0191	0.0184
6 x 10	0.0128	0.0130	0.0132	0.0132	0.0131	0.0129	0.0126	0.0123
6 x 12	0.0089	0.0091	0.0092	0.0093	0.0093	0.0092	0.0091	0.0089
6 x 14	0.0065	0.0067	0.0069	0.0070	0.0070	0.0070	0.0069	0.0068
8 x 8	0.0146	0.0148	0.0146	0.0144	0.0137	0.0137	0.0132	0.0126
8 x 10	0.0092	0.0093	0.0093	0.0092	0.0091	0.0088	0.0086	0.0083
8 x 12	0.0063	0.0064	0.0065	0.0065	0.0064	0.0063	0.0061	0.0059
8 x 14	0.0047	0.0048	0.0048	0.0048	0.0047	0.0047	0.0046	0.0045
8 x 16	0.0036	0.0037	0.0037	0.0038	0.0037	0.0037	0.0036	0.0036

6. Details of Timber Construction

Working Unit Stresses. The allowable WORKING UNIT STRESSES for various structural timbers, to be used in dry locations, and where the deflection is not to increase with time, are given in Tables XX and XXI.

Table XX. Allowable Unit Working Stresses for Structural Timber in Dry Locations

Kind of Timber	Standard Grade	Allowable Unit Stress. Lb per Sq In			
		Flexure		Compression	
		Extreme Fiber	Horizontal Shear	Parallel to Grain	Normal to Grain
Douglas Fir (Oregon and Washington)	Select	1600	90	1175	345
	Common	1200	72	880	325
Douglas Fir (Rocky Mtn. Region)	Select	1100	85	800	275
	Common	880	68	640	
Hemlock (West Coast)	Select	1300	75	900	300
	Common	1040	60	720	
Hemlock (Eastern)	Select	1100	70	700	300
	Common	880	56	560	
Oak (White and Red)	Select	1400	125	1000	500
	Common	1120	100	800	
Pine (Southern Yellow)	Select	1600	110	1175	345
	Common	1200	88	880	325
Pine (Northern White and Western Yellow)	Select	900	85	750	250
	Common	720	68	600	
Pine (Norway)	Select	1100	85	800	300
	Common	880	68	640	
Redwood	Select	1200	70	1000	250
	Common	960	56	800	
Spruce Red and White)	Select	1100	85	800	250
	Common	880	68	640	
Fir Commercial White)	Select	1100	70	700	300
	Common	880	56	560	

The working unit stress for axial tension may be taken as the values given for extreme fiber stress in flexure For buildings in which the timber is protected from the elements and not subject to vibrations of impact loads, the allowable working unit stresses may be increased 20%.

Where a moderate INCREASE IN DEFLECTION after first placement of the load is not objectionable, the compressive and extreme fiber-stresses given in the tables may be increased 20%. When the structure is subject to VIBRATIONS or IMPACT, the stresses should not be increased beyond the tabular values. The stresses as given were compiled from the recommendations of the United States Forest Service and the Forest Products Laboratory.

Accessory Material. STEEL in the form of rods, plates, bolts, screws, and nails; and CAST IRON in the form of washers and bearing-plates, are not only convenient and valuable auxiliary materials used in conjunction with timber construction, but are often necessary adjuncts in most timber structures. Timber is capable of resisting tensile stress to an amount equal to the allow-

able unit stress in flexure; however, it is difficult to make satisfactory tension connections for timber-members. Steel rods, on account of the simplicity of connection details, provide the most satisfactory and at the same time the most economical forms for web tension-members, in timber-roof-trusses.

Table XXI. Allowable Unit Working Stresses for Timber Columns in Dry Locations

Values in Pounds per Square Inch

Kind of Timber	Standard Grade	Ratio of Length to Least Dimension = L/d										
		10 or Less	12	14	16	18	20	25	30	35	40	45
Pine (Northern White and Western Yellow)	Select	750	733	718	695	663	617	438	304	224	171	110
	Common	600	590	583	572	556	532	434				
Douglas Fir (Ore. and Wash.) Pine (Southern Yellow)	Select	1175	1149	1127	1093	1045	975	702	487	358	274	175
	Common	880	870	861	847	826	796	675				
Douglas Fir (Rky. Mtn. Reg.) Spruce (Red and White) Pine (Norway)	Select	800	786	774	753	726	688	526	365	268	206	132
	Common	640	632	627	617	602	582	500				
Hemlock (West Coast)	Select	900	885	872	852	823	783	614	426	313	240	153
	Common	720	712	706	696	680	660	573				
Hemlock (Eastern) Fir (Commercial White)	Select	700	689	678	664	641	611	482	335	246	188	121
	Common	560	554	549	542	530	515	449				
Oak (White and Red)	Select	1000	982	967	943	908	860	658	457	336	257	164
	Common	800	790	783	771	753	728	625				
Redwood	Select	1000	972	947	910	856	781	526	365	268	206	132
	Common	800	786	773	754	726	688					

For Buildings in which timber is protected from the elements and not subject to vibration or impact loads, the allowable unit stresses may be increased 20%.

Washers. The allowable compression normal to the grain of the various kinds of timber used in building-construction is less than the allowable compressive stress parallel to the grain. For this reason it is necessary to use WASHERS under the heads and the nuts of steel rods and bolts to properly distribute the loads to the timber surface.

It is evident from the range of allowable bearing values normal to the grain for various kinds of timber, that the term STANDARD WASHER has no meaning in so far as timber structures are concerned. The types of washers generally used in timber construction are OGEE CAST-IRON WASHERS, Fig. 4(*a*), or rectangular STEEL-PLATE WASHERS.

All bolts and rods used in timber construction should be provided with washers of ample area to conform to the allowable bearing unit stress normal to the grain of the timber, regardless of what may be termed standard washers. In a well-designed detail, both the diameter and thickness of an ogee washer depend upon the allowable bearing unit stress for the timber and the total stress in the rod or bolt.

A related system of dimensions for CAST-IRON OGEE WASHERS, so established that both the bearing stress on the timber and the flexural stress in the washer, for assumed uniform pressure, reach their full allowable values simultaneously, is shown in Fig. 4.

Example. To illustrate the use of the diagrams shown in Fig. 4, let it be required to design a cast-iron ogee washer for the following data:

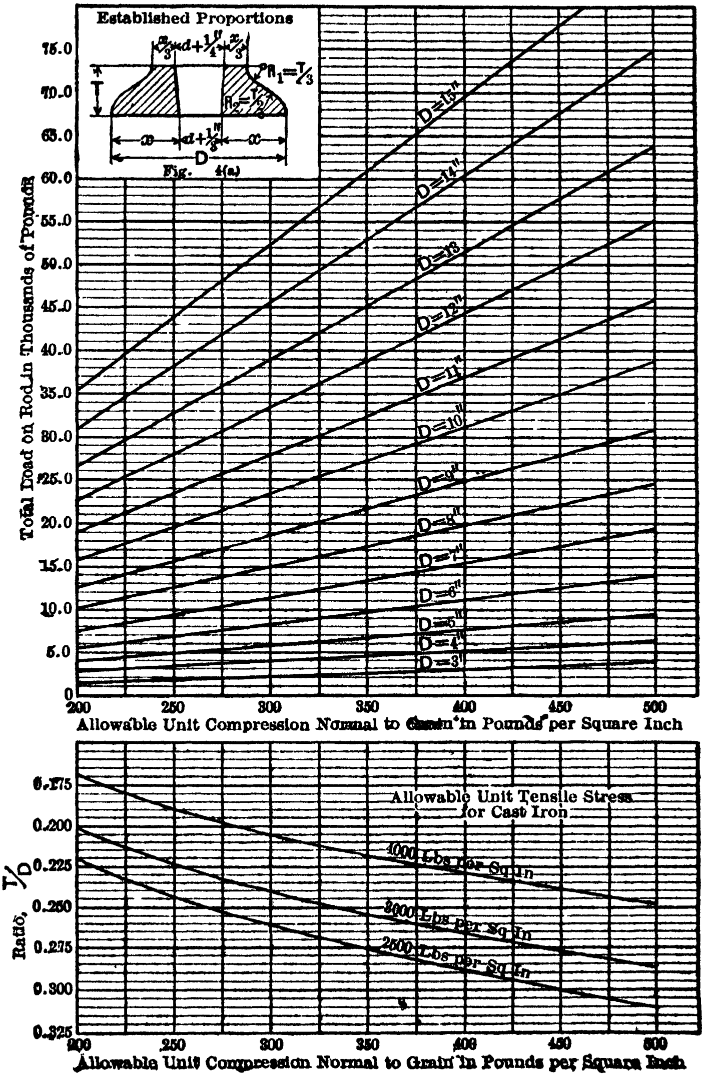


Fig. 4. Washers

(1) Tension on $1\frac{1}{8}$-in upset rod = 15 600 lb.
(2) Allowable unit bearing normal to grain of timber = 345 lb per sq in.
(3) Allowable flexural unit stress in cast iron = 3 000 lb per sq in.

Enter the upper portion of the diagram on the horizontal line, for a total load on the rod of 15 600, and at the intersection of this line and a vertical through $f_b = 345$, find the value of $D = 7.5$ in.

In the lower portion of the diagram, at the intersection of the ordinate through $f_b = 345$ and the curve for an allowable tension of 3 000 lb per sq in in cast iron, find the ratio $T/D = 0.253$. The required dimensions of the washer are:

$$D = 7.5 \text{ in;}$$

$$T = 7.5 \times 0.253 = 1.897 \quad \text{or} \quad 1\tfrac{7}{8} \text{ in;}$$

$$R_1 = \frac{T}{3} = 0.632 \quad \text{or} \quad \tfrac{5}{8} \text{ in;}$$

$$R_2 = \frac{T}{2} = 0.948 \quad \text{or} \quad \tfrac{15}{16} \text{ in;}$$

$$x = \tfrac{1}{2}(7.5 - 1.5) = 3 \text{ in.}$$

The diameter of the outer face of washer is:

$$2\left(\tfrac{3}{8}\right) + (1.5 + 0.25) = 3\tfrac{3}{4} \text{ in.}$$

The approximate dimensions of cast-iron ogee washers, in terms of the diameter of the bolt or rod, are given in Table XXII, Fig. 5.

When the loads to be transferred are large it is sometimes necessary to use cast-iron washers of the RIBBED type such as illustrated at (*a*), (*b*) and (*c*), Fig. 5.

For connections in which the bolt or rod is inclined to the bearing surface of the timber, BEVELED cast-iron washers such as illustrated at (*d*) and (*e*), Fig. 5, may be employed.

In designing a beveled washer it is necessary to provide sufficient resistance parallel to, as well as normal to, the grain of the timber. The component of R, parallel to the grain, Fig. 5(*e*), must be resisted by a compression on the ends of the fibers along AB of an amount equal to component H of the stress R. The component T, normal to the fibers, must be resisted along the surface AC.

The resultant pressure on each resisting surface must intersect at a common point on the action line of the stress in the rod if a uniform distribution of pressure on the two surfaces is to be obtained. The size and thickness of steel-plate washers for various sizes of rods and various allowable bearing values are given in Table XXIII. The dimensions of the SIDES of the washers are given to the nearest $\frac{1}{8}$ in, and the required THICKNESSES, assuming a uniform distribution of pressure, are given to the nearest $\frac{1}{16}$ in. The thickness of flat-plate washers should never be less than 0.6 times the root diameter of the bolt or rod with which they are used.

Lag-Screws. LAG-SCREWS are essentially large screws with a head similar to that of a bolt. They vary in size from $\frac{1}{4}$ in $\times$ $1\frac{1}{2}$ to 1 in $\times$ 12 in.

Perhaps the most valuable experimental data relative to the value of lag-screws in timber details were reported by Mr. H. D. Dewell in Engineering News, July 20, 1916. The safe working values recommended for the sizes of lag-screws most commonly employed in building-construction are given as follows:

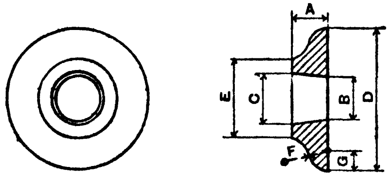
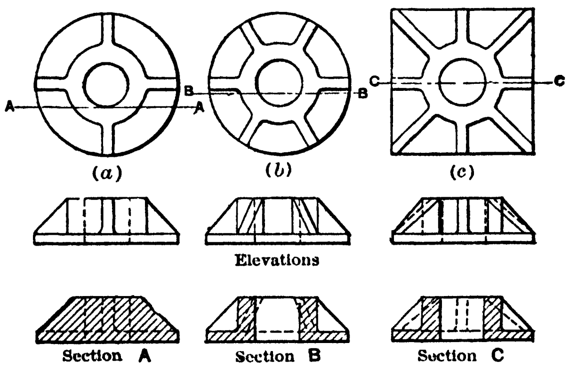


Fig. 5 (a)

Ogee Cast Iron Washers
Table XXII

Diameter of Bolt Inches.	A	B	C	D	E	F	G	Weight per 100. — Pounds —
d	d	$d + \frac{1}{8}''$	$d + \frac{1}{4}''$	$4d$	$2.25d$	$0.75d$	$0.50d$	$3d^3$

Special Cast Iron Washers



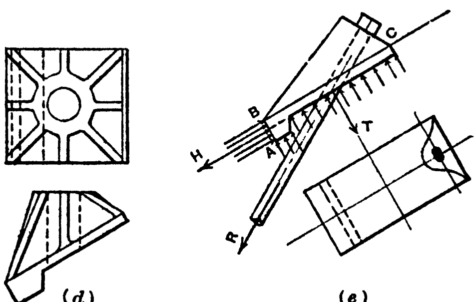


Fig. 5 (b)

Fig. 5. and Table XXII. Cast-iron Washers

Table XXIII. Dimensions of Steel Plate Washers for Various Sizes of Rods and Bolts and Various Bearing Values *

Dimensions of Plates in Inches

Bolt or Rod		Total Allowable Stress in Pounds		(Dimensions of Steel Plate Washers (Inches)					
				f_s=16000 Lb per Sq In			f_s=18000 Lb per Sq In		
Diam. In	Net Area Sq In	f_s=16000 Lb per Sq In	f_s=18000 Lb per Sq In	f_b=345 Lb per Sq In	f_b=300 Lb per Sq In	f_b=250 Lb per Sq In	f_b=345 Lb per Sq In	f_b=300 Lb per Sq In	f_b=250 Lb per Sq In
5/8	0.202	3 230	3 640	3 1/8x3 1/8x7/16	3 3/8x3 3/8x7/16	3 5/8x3 5/8x7/16	3 3/8x3 3/8x7/16	3 1/2x3 1/2x7/16	3 7/8x3 7/8x7/16
3/4	0.302	4 830	5 440	3 7/8x3 7/8x1/2	4 1/8x4 1/8x1/2	4 1/2x4 1/2x1/2	4 1/8x4 1/8x1/2	4 3/8x4 3/8x1/2	4 3/4x4 3/4x1/2
3/4 Upset	0.442	7 070	7 960	4 5/8x4 5/8x5/8	5x5x5/8	5 3/8x5 3/8x5/8	4 7/8x4 7/8x5/8	5 1/4x5 1/4x5/8	5 3/4x5 3/4x5/8
7/8	0.419	6 700	7 540	4 1/2x4 1/2x5/8	4 7/8x4 7/8x5/8	5 1/4x5 1/4x5/8	4 3/4x4 3/4x5/8	5 1/8x5 1/8x5/8	5 5/8x5 5/8x5/8
7/8 Upset	0.601	9 620	10 820	5 3/8x5 3/8x13/16	5 7/8x5 7/8x13/16	6 3/8x6 3/8x3/4	5 3/4x5 3/4x13/16	6 1/8x6 1/8x13/16	6 3/4x6 3/4x3/4
1 Upset	0.785	12 560	14 130	6 1/4x6 1/4x15/16	6 5/8x6 5/8x7/8	7 1/4x7 1/4x7/8	6 1/2x6 1/2x15/16	7x7x7/8	7 1/2x7 1/2x7/8
1 1/8 Upset	0.994	15 900	17 890	7x7x1	7 3/8x7 3/8x1	8 1/8x8 1/8x1	7 3/8x7 3/8x1	7 7/8x7 7/8x1	8 1/2x8 1/2x1
1 1/4 Upset	1.227	19 630	22 090	7 3/4x7 3/4x1 3/16	8 1/4x8 1/4x1 1/8	9x9x1 1/8	8 1/8x8 1/8x1 1/8	8 3/4x8 3/4x1 1/8	9 1/2x9 1/2x1 1/8
1 3/8 Upset	1.485	23 760	26 730	8 1/2x8 1/2x1 1/4	9x9x1 1/4	10x10x1 1/4	9x9x1 1/4	9 5/8x9 5/8x1 1/4	10 1/2x10 1/2x1 1/4
1 1/2 Upset	1.767	28 270	31 800	9 3/8x9 3/8x1 1/2	10x10x1 7/16	10 7/8x10 7/8x1 3/8	9 7/8x9 7/8x1 7/16	10 1/2x10 1/2x1 7/16	11 1/2x11 1/2x1 3/8

* The allowable unit-bearing values, Table XX, may be increased 20% when the washer does not cover more than 80% of width of timber.

Metal plate lagged to timber:

3/4 × 4 1/2-in lag-screws	1 030 lb per screw
7/8 × 5-in lag-screw	1 200 lb per screw

Wooden plate lagged to timber:

3/4 × 4 1/2-in lag-screws	900 lb per screw
7/8 × 5-in lag-screws	050 lb per screw

Bolts. BOLTS employed in timber construction are classified as: (*a*) COMMON, OR ROUGH; (*b*) MACHINE, OR FINISHED; (*c*) DRIFT-BOLTS.

COMMON BOLTS are manufactured either as rough or finished, the former being the type more generally used in timber construction. Finished bolts are obtained by turning rough bolts to exact dimensions, and are more generally used in connecting metal parts of machines.

DRIFT-BOLTS are either round or square in section, with or without heads or points, and are driven as a spike.

Tension web-members in timber roof-trusses are generally long steel rods with heads of sufficient thickness to develop the safe net tensile strength of the rod. The other end of the rod is threaded to receive a nut. Such members are in reality long bolts, and the effective cross-section is the net section at the root of the threads. These values are to be found in the manufacturers' handbooks.

The excess metal required throughout the length of a tension rod to provide a sufficient area at the root of the thread can be reduced by the use of upset screw-ends. The diameter of a portion of the rod at the threaded end is enlarged so that the net section, after the threads are cut, is equal to or greater than the gross section of the body of the rod.

Economical design requires that tension rods of a diameter of 1 in or more be upset for lengths greater than 10 to 12 ft.

Allowable Compression on Surfaces Inclined to the Direction of the Grain. Joint details in timber construction frequently require the members to be cut or notched at an angle with the grain of the various timbers to be joined. The allowable unit compression on an inclined surface is somewhat indeterminate.

Professor Henry S. Jacoby, in Structural Details,* has developed the following general equation:

$$c = p \sin^2 \Theta + n \cos^2 \Theta \tag{15}$$

in which c = allowable unit compression on inclined surface;
p = allowable unit compression parallel to the grain;
n = allowable unit compression normal to the grain;
Θ = angle of inclined surface with the grain.

On the basis of a number of tests on various kinds of structural timber, Professor M. A. Howe developed the following equation for allowable unit compression on inclined surfaces: †

$$c = n + (p - n) \left(\frac{\Theta^\circ}{90^\circ}\right)^{5/2} \tag{16}$$

where the notation is the same as in Equation (15).

Resistance of Timber to Pressure from Metal Pins and Bolts. The bearing of a cylindrical pin or bolt in a close-fitting hole is a special case of compression on inclined surfaces. Each differential surface around the periphery of the cylinder will have an allowable unit compression depending upon the angle which the tangent to the cylinder, at that point, makes with the grain of the timber.

The average unit stress on the diametral plane of the cylindrical body for load applied parallel to the direction of the grain is given by the following equation:

$$B = \tfrac{2}{3} p + \tfrac{1}{3} n \tag{17}$$

in which B = the allowable average unit compression on the diametral plane of bolt or pin;
p = allowable unit compression parallel to the grain;
n = allowable unit compression normal to the grain.

The total resistance of the bolt or pin, when the bending in the bolt or pin is not the limiting factor, is:

$$B \cdot d \cdot l = (\tfrac{2}{3} p + \tfrac{1}{3} n)\, d \cdot l \tag{18}$$

where d = diameter of bolt or pin and l = the length subjected to load.

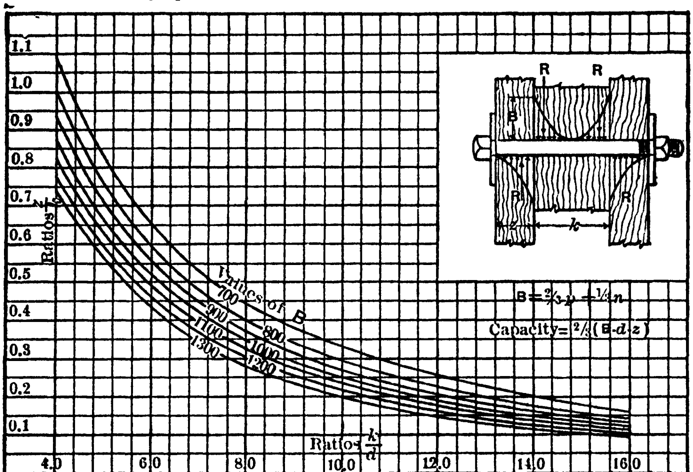
Bending Stresses in Bolts and Pins in Timber Joints. Bolted joints in timber construction may be made with wood splice-pads, as illustrated in Fig. 6, or with steel splice-plates as shown in Fig. 7.

Neglecting the friction on the contact surfaces of the splicing members and the members spliced, each bolt in the connection must act as a beam, being loaded throughout its central portion by the bearing of the main member, and supported at each end by bearing on the splicing member. The actual distribution of the bearing throughout the length of the bolt is indeterminate,

* Structural Details or Elements of Design in Timber Framing, by H. S. Jacoby; John Wiley & Sons, Inc., 1921.

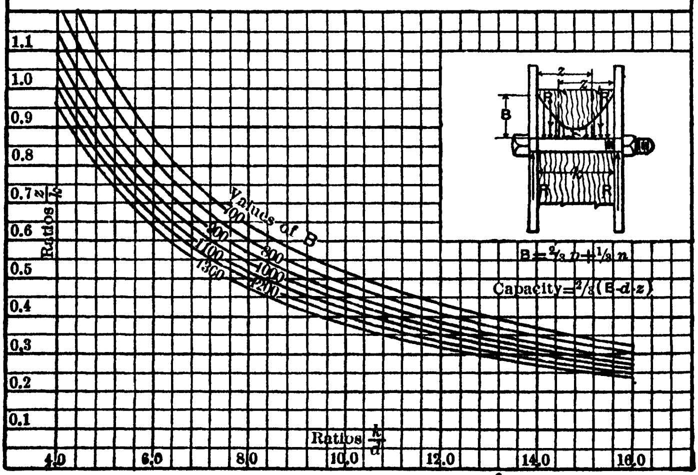
† Eng. News, Vol. 68, No. 5 and No. 10.

on account of the deflection of the bolt and the friction in the joint resulting from the drawing up of the bolts.



Capacity of Bolts used with Wood Splice Pads. f_s =16000 Lbs per Sq in

Fig. 6



Capacity of Bolts used with Steel Splice Plates. f_s =16000 Lb per Sq in

Fig. 7

Figs. 6 and 7. Stresses in Bolts and Pins in Timber Joints

In Fig. 6, the distribution of pressure upon the bolt from the main member is assumed to vary from maximum at the contact surfaces with the splice-

pads to a minimum at the center of the bolt-span, where the deflection is the greatest. The variation is assumed to follow the law of the parabola. The bearing of the bolt on the fibers of the splice-pad is assumed to follow the same law.

If the resisting moment of the bolt is equal to the bending moment:

$$M = \frac{B \cdot d \cdot z}{3}\left(\frac{z}{4} + \frac{k}{8}\right) = \frac{f_s \cdot I}{c} \tag{19}$$

in which B = safe unit bearing value on the cylindrical bolt;
d = the diameter of the bolt in inches;
z = length over which the bearing is acting (being equal for both splice-pad and main member) in inches;
k = breadth of main member in inches;
f_s = allowable flexural unit stresses in bolt, in pounds per square inch;
$\frac{I}{c}$ = section-modulus of bolt cross-section in inches^3.

If the value of z is such that with the maximum allowable bearing on bolt, the allowable flexural unit stress is just developed:

$$z = \sqrt{\left(\frac{k}{4}\right)^2 + \frac{3}{8}\frac{f_s \pi d^2}{B}} - \frac{k}{4} \tag{20}$$

For all values of $z \leqq \frac{k}{2}$, the load distribution on the bolt is as shown in Fig. 6. For values of $z > \frac{k}{2}$, but not exceeding k, the pressure volume will be assumed to be folded back at $\frac{k}{2}$ as shown in Fig. 7.

Equation (20) shows that for all usual values of f_s and B, z exceeds $\frac{k}{2}$ only when the ratio $\frac{k}{d}$ is comparatively small, that is, when the diameter of the bolt is large compared to its length, and in such cases a more uniform distribution of pressure is to be expected.

It is also evident that the assumed folding back of the pressure volumes results in a slightly smaller moment. In any case, therefore, for all values of z up to $z = k$, provided the thickness of wood splice-pads is equal to z up to values of $z = \frac{k}{2}$ and equal to $\frac{k}{2}$ for values of z greater than $\frac{k}{2}$, the capacity of any bolt may be expressed as twice the reaction on one splice-pad, or:

$$C = \left(\frac{B \cdot d \cdot z}{3}\right) 2 \tag{21}$$

where B is the value given by Equation (17) and C = the capacity of the bolt in pounds. The value of z is given by Equation (20). The relations between the ratios $\frac{z}{k}$ and $\frac{k}{d}$ for usual values of B and for an allowable working unit stress of 16 000 lb per sq in for bolts in flexure are shown graphically in Fig. 6.

When the main members are spliced by steel splice-plates, as shown in Fig. 7, the span and deflection of the bolt are reduced, thereby increasing the

capacity of the bolt. Using the same notation and assumptions as before, except that in this case the pressure of the bolt upon the steel splice-plates is assumed to be concentrated at the center of the plate:

$$z = d\sqrt{\frac{3 f_s \pi}{8 \cdot B}} \tag{22}$$

and

$$C = \left(\frac{B \cdot d \cdot z}{3}\right) 2 \tag{23}$$

The relations between $\frac{z}{k}$ and $\frac{k}{d}$ for various values of B, and for $f_s = 16\,000$ lb per sq in = allowable flexure in bolts, are shown graphically in Fig. 7. Under favorable conditions and with good workmanship the capacities of bolts as determined from the diagrams in Figs. 6 and 7 may be increased 20%.

The manner in which these diagrams are used will be explained by the following numerical examples:

Example 1. Let it be required to design a bolted joint to splice a nominal 8 × 8-in southern yellow pine timber, using 4 × 8-in wooden splice-pads. The stress to be provided for is 22 500 lb total tension.

The following working unit stresses are taken from Table XX:

Compression parallel to grain	=	1 175 lb per sq in
Compression normal to grain	=	345 lb per sq in
Then $B = \frac{2}{3}(1\,175) + \frac{1}{3}(345)$	=	900 lb per sq in

If 1-in bolts are to be used, $\frac{k}{d} = 7.5$, and in Fig. 6, the ordinate through this value of k/d intersects the curve for $B = 900$ at $z/k = 0.42$, whence $z = 0.42 \times 7.5 = 3.15$.

The capacity of one 1-in bolt is:

$$C = \tfrac{2}{3}(900 \times 1 \times 3.15)\,1.2 = 2\,260 \text{ lb}$$

The number of bolts required on each side of the splice is 22 500/2 260 or ten 1-in bolts.

Example 2. Let it be required to design the joint in the preceding example using steel splice-plates. From Fig. 7, the value of $k/d = 7.5$ intersects the curve for $B = 900$ at $z/k = 0.62$, whence $z = 0.62 \times 7.5 = 4.65$. The capacity of one 1-in bolt is:

$$C = \tfrac{2}{3}(900 \times 1 \times 4.65)\,1.2 = 3\,340 \text{ lb}$$

The number of bolts required on each side of the splice is 22 500/3 340 or seven 1-in bolts.

7. Detailed Design of Timber Roof-Construction

The structural design of a timber roof includes the design of the SHEATHING, RAFTERS (if used), PURLINS, CEILING-BEAMS (if the ceiling is supported by the trusses) and the TRUSSES. In order to illustrate fully the general method of procedure a roof of timber construction will be completely designed for the following data:

Given Data. (1) The finished roof surface is to be ¼-in slate laid 4 in to the weather over one layer of slater's felt on tight sheathing.

(2) An investigation will determine the relative economy of sheathing supported directly by the purlins and the use of rafters to support the sheathing.

(3) Slope of the roof .... 30°

(4) Trusses to be supported on masonry walls; span, center to center of bearing .... 44 ft 0 in

(5) Spacing of trusses .... 18 ft 0 in

(6) Material for sheathing, rafters, and purlins will be common southern yellow pine and the timber-members of the truss will be southern yellow pine of the select grade.

(7) Allowable unit stresses will be taken from Tables XX and XXI with the recommended increase of 20%. Tension in timber will be taken equal to the allowable bending in the extreme fiber, without increase.

(8) The allowable unit stresses for steel will be:

Tension	16 000 lb per sq in
Shear	10 000 lb per sq in
Flexure (extreme fiber)	18 000 lb per sq in
Bearing	20 000 lb per sq in

(9) The allowable unit stresses for cast iron will be:

Flexure (extreme fiber)	3 000 lb per sq in
Compression..............	12 000 lb per sq in
Shear	2 000 lb per sq in

The following unit loads, taken from Tables I to V, inclusive, will be adopted:

(1) Roof-covering	9 lb per sq ft of roof
(2) Snow-load	20 lb per sq ft of roof
(3) Ice and sleet	10 lb per sq ft of roof
(4) Ceiling plus framing	15 lb per sq ft of roof
(5) Wind-load	24 lb per sq ft of roof

Design of Sheathing. The superimposed load on the sheathing, in pounds per square foot of roof, is:

Finished roof	9 lb
Sheathing (assumed)	5 lb
Equivalent uniform vertical live load (Table VI) ..	24 lb
Total	38 lb

It is evident from an inspection of the span that the purlin spacing will be about 8.5 ft.

The required thickness of sheathing, determined by the limitation for deflection, is given by Table XV, as:

$$t = \frac{8.5}{5.1} \times 1 = 1.66 \quad \text{or} \quad 1\tfrac{5}{8} \text{ in}$$

If rafters are used to support the sheathing at 24-in centers the sheathing may be made a minimum thickness or $\frac{7}{8}$ in. The superimposed unit load on the rafter will be practically the same as in the preceding investigation.

Assuming nominal 2 × 6-in rafters ($1\frac{5}{8} \times 5\frac{5}{8}$), $\frac{I}{c} = 8.57$ and $A = 9.14$. The total vertical load per rafter span is $2 \times 8.5 \times 38 = 646$ lb. From the

method of design given in Case 2, Article 4, the average direct compression on the rafter is $\frac{1}{2}(646) \sin 30° = 162$ lb. The maximum moment at the mid-point of span is:

$$M_{max} = \frac{646 \times 8.5 \times 12}{8} = 8\,250 \text{ lb-in}$$

and

$$f_{max} = \frac{8\,250}{8.57} + \frac{162}{9.14} = 983 \text{ lb per sq in}$$

It is more economical to use the thinner sheathing supported by rafters than to use the thicker sheathing supported directly by the purlins, and since there is no limitation on the vertical dimensions of the structure in this case, the more economical method will be used.

Design of Purlins. The dead load supported by the purlins, in pounds per square foot of roof, is:

Finished roofing	9.0 lb
Sheathing	3.0 lb
Rafters	1.5 lb
Purlins (estimated)	3.5 lb
	17.0 lb

The equivalent uniform live load from Table VI is 24.0 lb per sq ft of roof-surface. The total design load is $(17 + 24) = 41$ lb per sq ft of roof.

The total purlin moment due to the vertical loading is:

$$M_v = \frac{41\,(8.5 \times 18)\,18 \times 12}{8} = 170\,000 \text{ lb-in}$$

If the working unit stress be taken at 20% more than that given in Table XX, or $1.2 \times 1\,200 = 1\,440$, the maximum stress coefficient is:

$$K = \frac{1\,440}{170\,000} = 0.0085$$

Assuming no intermediate purlin support in the plane of the roof (Table XVIII), the size of purlin required is a nominal 8×14-in timber. If the rafter and sheathing construction is assumed to provide support in the plane of the roof equivalent to a single center support (Table XIX), the required nominal size of purlins is 8 in $\times$ 10 in.

Since rafter and sheathing construction cannot be assumed to provide as rigid a support in the plane of the roof as solid sheathing, the purlins in this case will be taken as the average required by Tables XVIII and XIX, or a nominal size of 8 in $\times$ 12 in.

Design of Ceiling-Beams. The ceiling will be supported by horizontal beams, framed between and supported by the truss at the lower chord panel-points. The ceiling-joists will be nominal 2 in $\times$ 4 in at 16-in centers framed between the ceiling-beams to support the plaster base and plaster. The unit loading for the ceiling-construction is:

Plaster base and plaster	10.0 lb per sq ft
Ceiling-beams (assumed)	1.25 lb per sq ft
Ceiling-joists (assumed)	1.25 lb per sq ft
Total	12.5 lb per sq ft

The total load on each ceiling-beam, assuming six spans along the lower chord, is:

$$18 \times 7.33 \times 12.5 = 1\,650 \text{ lb}$$

$$M = \frac{1\,650 \times 18 \times 12}{8} = 44\,600 \text{ lb-in}$$

Assume a width of beam of 4 in (actual 3⅝ in), then:

$$\frac{f}{6}bd^2 = \frac{1\,440}{6}(3\tfrac{5}{8})\,d^2 = 44\,600$$

whence $$d = 7.2 \text{ in}$$

The nominal size beam required by flexure is, therefore, 4 in × 8 in ($I/c = 33.98$), and the extreme fiber-stress = 1 310 lb per sq in.

The maximum deflection (Table VIII, Chapter XVII) is:

$$\Delta = \frac{1\,310}{1\,000} \times \frac{6\,075}{7.5} = 1.14 \text{ in}$$

The limit for the deflection of beams supporting plastered ceilings is $L/360$, or in this case 0.6 in.

It is evident, therefore, that a beam of about a 10-in depth must be used to provide sufficient stiffness. Assume a nominal 4×10-in beam $\left(\frac{I}{c} = 54.53\right)$, then:

$$f = \frac{44\,600}{54.53} = 820 \text{ lb per sq in}$$

and $$\Delta = \frac{820}{1\,000} \times \frac{6.075}{9.5} = 0.525 \text{ in}$$

The allowable unit horizontal shear is 1.2 × 88 = 106 lb per sq in. The maximum unit horizontal shear for the 4 × 10-in beam (actual area = 34.45 in²), is:

$$v = \frac{3}{2}\,\frac{825}{34.45} = 36 \text{ lb per sq in}$$

The check for horizontal shear might also be accomplished by referring to Table V, Chapter XVII. The horizontal shear does not govern in southern yellow-pine beams when

$$\frac{L}{d} > 13.0$$

Design of Trusses. The various factors governing the selection of the type of truss for any given case are:

(1) The slope of the roof;

(2) The available depth for the trusses;

(3) The relative positions of upper and lower chord loads;

(4) The material of which the truss is to be constructed;

(5) The span and spacing of trusses and the magnitude of loads to be supported.

The advantages of the Howe type of truss for timber construction have been discussed in Chapter XXV. The only disadvantage of this type for symmetrical triangular roof-trusses is that it necessitates long compression web-members in the panels adjacent to the center of the span.

In timber construction this objection is of less consequence than in steel construction. The number of panels for a triangular roof-truss of a given span depends upon the satisfactory span of the structural covering and an economical value for the unsupported length of the upper chord members of the truss. The most economical arrangement in timber construction provides for upper chord panel lengths not less than about 6 ft and not greater than about 10 ft for the usual types of roof-construction. A six-panel Howe truss, as shown in Fig. 8, will be selected for the design under consideration.

Weight of Truss. The total load to be supported by the truss is:

Upper chord:	
18 ft (44 ft Sec. 30°) 41	= 37 600 lb
Lower chord:	
18 ft × 44 ft × 12.5	= 9 900 lb
Total	47 500 lb

The approximate weight of the truss as given by Equation (1) is:

$$W = \frac{47\,500}{65}\left(1 + \frac{44}{30} + \frac{44}{5\sqrt{18}}\right)$$
$$= 2\,600 \text{ lb}$$

This weight will be distributed to the panel-points of both upper and lower chords as follows:

To upper chord: 1.5 lb per sq ft of roof-surface.

To lower chord: 1.5 lb per sq ft of horizontal projection.

The final corrected unit loads are:

Upper chord:

Dead loads:

(1) Finished roofing	9.00 lb per sq ft of roof
(2) Sheathing	3.25 lb per sq ft of roof
(3) Rafters	1.50 lb per sq ft of roof
(4) Purlins	3.00 lb per sq ft of roof
(5) Truss	1.50 lb per sq ft of roof
Total	18.25 lb per sq ft of roof
Snow-load (max.)	22 50 lb per sq ft of roof
Snow-load (min.)	10.00 lb per sq ft of roof
Wind-load	24.00 lb per sq ft of roof

Lower chord:

(1) Ceiling-beams	2.0 lb per sq ft
(2) Ceiling-joists	1.5 lb per sq ft
(3) Plaster base and plaster	10.0 lb per sq ft
(4) Truss	1.5 lb per sq ft
Total	15.0 lb per sq ft

The total panel loads are:

Upper chord:

(1) Dead load	**= 18 × 8.47 × 18.25 = 2 780 lb**
(2) Max. snow-load	**= 18 × 8.47 × 22.50 = 3 430 lb**
(3) Min. snow-load	**= 18 × 8 47 × 10.00 = 1 530 lb**
(4) Wind-load	**= 18 × 8.47 × 24.00 = 3 660 lb**

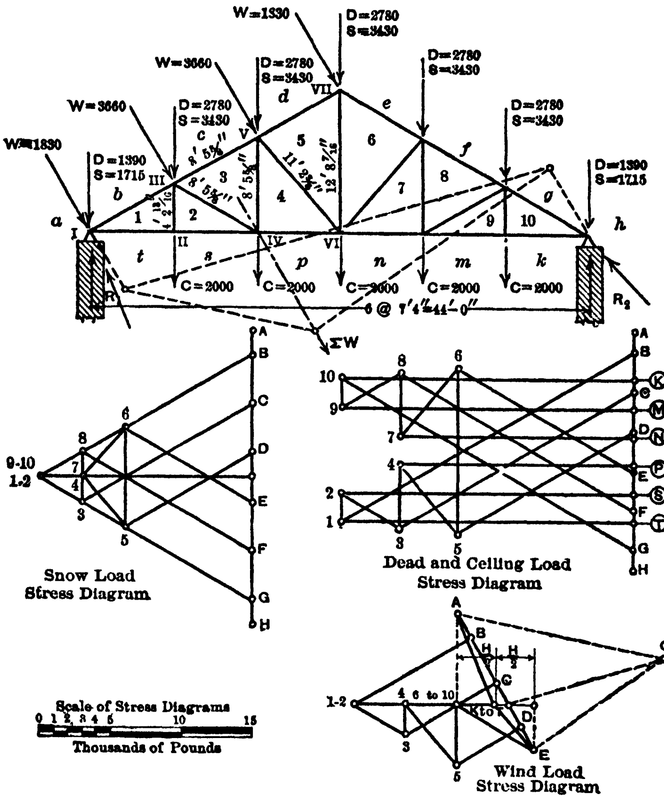


Table XXIV. Maximum Stresses and Required Sections

Member	Stresses in Thousands of Pounds: Dead and Ceiling ①	Maximum Snow ②	Minimum Snow ③	Wind ④	Maximum ① ②	Maximum ①③④	Nominal Size	Ratio $\frac{L}{d}$	Allowable Unit Stress Lbs. per Sq. In.	Section Areas Square Inches: Required	Furnished
b-1	−23.85	−17.15	−7.63	−8.50	−41.00	−39.98	6″x 8″	18.5	1230	33.40	41.25
c-3	−19.60	−13.70	−6.10	−7.38	−33.30	−33.08	do.	18.5	1230	27.00	do.
d-5	−14.35	−10.30	−4.57	−5.35*	−24.65	−24.27*	do.	18.5	1230	20.20	do.
1-t	+20.70	+14.85	+6.60	+10.00	+35.55	+37.25	do.		1600	(net) 23.40	(gross) do.
2-s	+20.70	+14.85	+6.60	+10.00	+35.55	+37.25	do.		1600	(net) 23.40	(gross) do.
4-p	+16.60	+11.87	+5.26	+6.40	+28.47	+28.26	do.		1600	(net) 17.80	(gross) do.
1-2	+2.00	0	0	0	+2.00	+2.00	⅝″ Φ Rod		16000	0.13	0.20
2-3	−4.80	−3.43	−1.52	−4.22	−8.23	−10.54	4″x 6″	28.0	750	14.10	20.39
3-4	+4.40	+1.72	+0.76	+2.10	+6.12	+7.27	1″ Φ Rod		16000	0.46	0.55
4-5	−6.40	−4.53	−2.01	−5.60	−10.93	−14.10	6″x 6″	24.5	870	16.30	30.25
5-6	+11.60	+6.86	+3.05	+4.25	+18.46	+18.90	1¼″ Φ Rod		16000	1.18	1.29

Maximum Leeward Stress = −6.40; Maximum Combined Stress = −25.32

Fig. 8 and Table XXIV. Six-panel Howe Truss

Lower chord:

(1) Dead load $= 18 \times 7.34 \times 15.00 = 2\,000$ lb

The truss will be considered as anchored at each bearing and the horizontal component of the wind-load will be assumed to be equally divided to the two supports.

The necessary stress-diagrams for determining the maximum stresses in the members of the truss are shown in Fig. 8 and the results are given in Table XXIV.

Design of Members. UPPER CHORD. The upper chord will be made of a uniform section throughout. Splices are undesirable in chord members and the possible saving of material is offset by the sacrifice in stiffness. The maximum design stress is 41 000 compression. The L/d ratio (assuming a least dimension of $5\frac{1}{2}$ in) is 18.5 and the allowable unit compression (values given in Table XXI increased by 20%) is (1 025) 1.2 = 1 230 lb per sq in.

The required area is $\frac{41\,000}{1\,230} = 33.4$ sq in.

The nearest size of standard-dimensioned timber meeting this requirement is a nominal 6×8 in ($5\frac{1}{2} \times 7\frac{1}{2}$ in) with a gross-sectional area of 41.25 sq in. An excess of 15% to 20% in compression-members is not undesirable in the design of details, as will be seen.

LOWER CHORD. The gross-sectional area of the tension chord should be from 40% to 60% greater than the required net area, since notches, bolt-holes and other reductions of the section on account of joint details will materially reduce the gross area. Moreover, an excess of area in both chords is desirable on account of the secondary stresses resulting from unavoidable eccentricity in the joint details.

The net area required for the lower chord in the end panels is 37 250/1 600 = 23.4 sq in. A nominal 6×8-in timber will provide about 50% excess section and will be used for the lower chord member throughout. It will be necessary to splice this member and the splice will be made between joints IV and VI. The maximum stress in this section is 28 470 lb tension. One-inch bolts and $\frac{5}{16}$-in steel splice-plates will be used. The number of bolts required is determined as follows:

$$B = [\tfrac{2}{3}\,(1\,175) + \tfrac{1}{3}\,(345)]\,1.2 = 1\,080 \text{ lb per sq in}$$

From the diagrams in Fig. 7, $\frac{z}{k} = 0.77$ for the lesser width of the timber, or $z = 0.77 \times 5.5 = 4.25$.

The capacity of one bolt is:

$$C = (1\,080 \times 1 \times 4.25) = 4\,600 \text{ lb}$$

The number of bolts required on each side of the splice is 28 470/4 600 = 6.2 or 6 bolts. If the splice is made with the bolts through the larger dimension of the timber, $z = 0.57 \times 7.5 = 4.27$, and the same number of bolts are required as before.

Members 2–3 and 4–5. The length, center to center of end details, of member 2–3 is 102 in. Assuming a nominal 4×6-in ($3\frac{5}{8} \times 5\frac{1}{2}$-in) timber, $L/d = \frac{102}{3.63}$, and the allowable unit stress is $630 \times 1.2 = 750$ lb per sq in. The required net area is 14.10 sq in and the area provided is 20.39 sq in.

The ratio L/d for member 4–5 is $\frac{134}{5.5} = 24.5$, assuming a 6×6-in timber.

The allowable unit stress is 725 × 1.2 = 870 lb per sq in. The net area required is 16.3 sq in and the gross area furnished by a nominal 6 × 6-in timber is 30.25 sq in. An investigation of smaller sections will readily show that the size of member 4–5 is governed by the L/d ratio and that a 6 × 6-in timber is the smallest that may be used.

Tension Web-Members. The tension web-members will be plain round rods throughout. A slight saving in weight could be made by using an upset rod for member 5–6, but the saving is too small in this case to justify the change.

As an example of the design of the tension web-member, consider member 5–6. The maximum stress is 18 900 lb tension. The net area required is 18 900/16 000 = 1.18 sq in. The net area, at the root of the thread of a 1½-in rod, is 1.29 sq in. A 1¼-in upset rod provides a net area of 1.22 sq in and requires an additional 4 in in length for the upset. The saving in weight, using the upset rod, would be 12.7 × 6.0 − 13.08 × 4.17 = 21 lb.

Design of Joints. The design of the joint details, in timber construction, is of greater importance than the design of the members. The requisites of a good joint are:

(1) Capability of transferring stress directly.

(2) Simplicity of detail.

(3) Avoid the use of different kinds of auxiliary material, with different physical characteristics, in the transfer of stresses.

(4) The efficiencies of the various parts should be the same.

End-Joint. The various methods which may be used in the design of the end-joint can be classified as follows:

(*A*) Notched types.

(*B*) Gusset-plate, plain or tabled.

(*C*) Bearing-plate or sole-plate.

Each of these types has numerous variations; however, the fundamental principles of each may be demonstrated by a few typical examples.

Joint I. Type A. *Fig.* 9. When the cornice detail is such that it may be made to enclose the projection of the lower chord beyond the exterior wall the detail shown in Fig. 9 provides a simple and economical method for Joint I.

The upper chord stress is transmitted to the lower chord through direct bearing on planes normal to the upper chord. The total bearing area required is determined by the allowable bearing on the planes at 60° to the direction of the grain of the lower chord, Equation (15) or (16).

From Equation (15):

$$c = 1\,175\,(\sin^2 60°) + 345\,(\cos^2 60°) = 1\,160 \text{ lb per sq in}$$

Assuming the chord members to be placed with their longer dimensions horizontal, the total required length of planes A–B and B'–C is:

$$\frac{41\,000}{1\,160 \times 7.5} = 4.7 \quad \text{or} \quad 4\tfrac{3}{4} \text{ in}$$

Make plane A–B = 2.00 in
Make plane B'–C = 2.75 in

$$\text{Compression on plane } A\text{–}B = 17\,300 \text{ lb} = \left(\frac{2}{4.75} \times 41\,000\right)$$

$$\text{Compression on plane } B'\text{–}C = 23\,700 \text{ lb} = \left(\frac{2.75}{4.75} \times 41\,000\right)$$

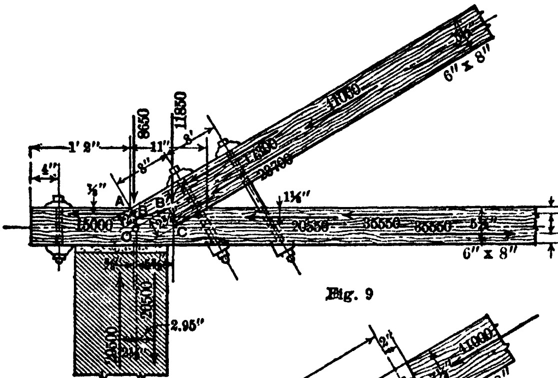


Fig. 9

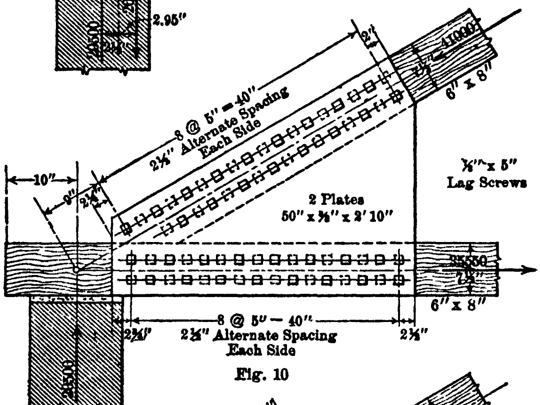


Fig. 10

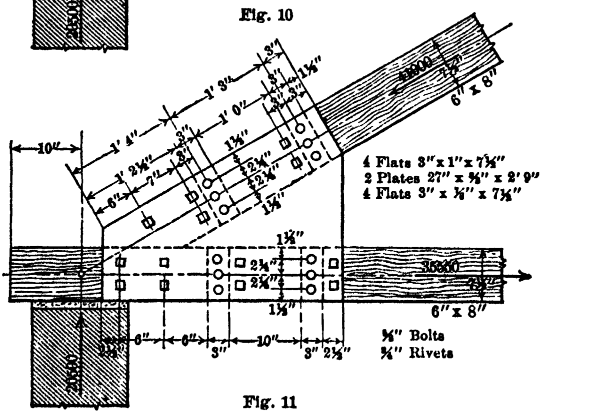


Fig. 11

Figs. 9, 10 and 11. Details of Joints at Wall

The horizontal projection of the lower chord beyond B must be sufficient to develop the necessary horizontal shear. The required projection is:

$$\frac{17\ 300\ (\cos 60^\circ)}{7.5\ (110)\ 1.2} = 15 \text{ in}$$

The required projection beyond C is:

$$\frac{23\ 700\ (\cos 60^\circ)}{7.5\ (110)\ 1.2} = 21 \text{ in}$$

A projection of 1 ft 2 in beyond point A satisfies these requirements. The horizontal dimensions from point O to action lines of the vertical components of the compression on planes A–B and B'–C are $\frac{3}{8}$ in to the left and $5\frac{3}{8}$ in the right, respectively. The resultant vertical component is, therefore,

$$\frac{(-8\ 650)\ \frac{3}{8} + (11\ 850)\ 5\frac{3}{8}}{20\ 500} = 2.95 \text{ in}$$

to the right of point O, as shown in Fig. 9.

The resultant of the horizontal components of the compression on planes A–B and B'–C is

$$\frac{15\ 000 \times \frac{7}{8} + 20\ 550 \times 1\frac{1}{8}}{35\ 550} = 1.02 \text{ in}$$

below the upper face of the lower chord. The net depth of the lower chord at point $C = 3\frac{1}{8}$ in. The total tension in the lower chord, therefore, acts at 1.56 in above the lower face of the chord, and the moment arm of the horizontal couple in the lower chord is 2 92 in, as shown in Fig. 9.

The magnitude of the horizontal couple is:

$$M = 35\ 500 \times 2.92 = 104\ 000 \text{ lb-in}$$

To neutralize this moment the center line of the supporting wall will be moved to the left of point O a distance such that the arm of the vertical couple is just sufficient to produce a clockwise moment equal to the counter-clockwise moment of the horizontal components and the tension in the lower chord.

The required arm is:

$$\frac{104\ 000}{20\ 500} = 5.08 \text{ in}$$

Since the resultant of the vertical component has been found to be 2.95 in to the right of point O, the center line of the wall must be placed at $5.08 - 2.95 = 2\frac{1}{8}$ in to the left of point O.

The maximum unit stress on the net section of the lower chord occurs under combined dead, ceiling, minimum snow and wind-loads and is:

$$\frac{37\ 250}{3.13 \times 7.5} = 1\ 585 \text{ lb per sq in}$$

The maximum bearing of the lower chord on the masonry wall is:

$$\frac{20\ 500}{7.5 \times 12.5} = 220 \text{ lb per sq in}$$

which is well within the allowable compression normal to the grain.

It will be assumed that the character of the masonry is such that the bearing is satisfactory.

Two $\frac{3}{4}$-in bolts will be provided, as shown, to hold the chord members securely in position. They will assist, to an indeterminate extent, in the transfer of chord stresses; however, this value is properly neglected in the design of the joint.

Joint I. Type B. Method 1. Fig. 10. The gusset, or steel fish-plate detail, illustrated in Fig. 10, is one of simplest and most satisfactory types of end-joints.

The size and shape of the steel plates, used on each side of the chord members, are determined by the number and spacing of lag-screws. The usual sizes of lag-screws employed in joint details of timber roof-trusses are $\frac{3}{4}$ in $\times$ $4\frac{1}{2}$ in and $\frac{7}{8}$ in $\times$ 5 in. The allowable capacities of lag-screws are given in Article 6.

In this detail the chord members will be placed with their lesser dimensions horizontal and $\frac{7}{8} \times$ 5-in lag-screws will be used. The number of lag-screws required is:

$$\text{Upper chord} = \frac{41\,000}{1\,200} = 34$$

$$\text{Lower chord} = \frac{37\,250}{1\,200} = 31$$

The lag-screws will be staggered on each face of each chord member, as shown. The thickness of the steel gusset-plates should not be less than $\frac{5}{16}$ in.

The greatest unsupported dimension of the gusset-plates should not exceed 60 times the thickness, or in this case

$$t = 1/60 \times 22 = 0.366$$

The plates will be made $\frac{3}{8}$ in thick. This detail is especially well adapted to trusses which are supported on timber columns, in which case the plates may be extended to provide for the connections to the columns.

Through bolts should never be used in a detail of this type, since it is practically impossible to line up the borings in the timbers with the holes in the plates. The maximum unit tensile stress on the net section of the lower chord is $\frac{37\,250}{5.5\,(7.5 - 1.75)} = 1\,180$ lb per sq in.

If the bearing on the masonry wall becomes excessive a steel bearing plate of sufficient size to distribute satisfactorily the vertical reaction should be used.

Joint I. Type B. Method 2. Fig. 11. The detail illustrated in Fig. 11 is known as a tabled fish-plate joint. Steel flats, riveted to the gusset-plates, are assumed to take the entire stress in each chord member and transfer it directly through the steel plate. The bearing area required for the tables in the upper chord is:

$$\frac{41\,000}{(1\,175)\,1.2} = 29.0 \text{ sq in}$$

If two tables are assumed on each side of the chord, the required thickness of each table is:

$$\frac{29.0}{4 \times 7.5} = 0.97 \quad \text{or} \quad 1.0 \text{ in}$$

The tables will be made 3 in wide to allow suitable edge distance for ¾-in rivets. The minimum distance between tables is determined by horizontal shear, and is

$$\frac{41\,000}{4\,(110 \times 1.2)\,7.5} = 10.35 \text{ in}$$

The maximum stress in the lower chord is 37 250, and the required thickness of tables, assuming two on each side, is:

$$\frac{37\,250}{4\,(1\,175 \times 1.2)\,7.5} = 0.88 \quad \text{or} \quad \tfrac{7}{8} \text{ in}$$

The minimum distance between the tables in the lower chord is:

$$\frac{37\,250}{4\,(110 \times 1.2)\,7.5} = 9.4 \text{ in}$$

The spacing of the tables along the upper chord will be made 12 in and along the lower chord 10 in as shown in the detail.

The steel plates must be of sufficient thickness to transmit the compression from the upper chord over the length between tables without buckling, or, $L/r \leqq 120$; whence:

$$t = \frac{12\sqrt{12}}{120} = 0.346, \quad \text{or} \quad \tfrac{3}{8} \text{ in}$$

(the least radius of gyration being $t/\sqrt{12}$)

The compression on each upper chord table is:

$$\frac{41\,000}{4} = 10\,250 \text{ lb}$$

applied at ½ (1 + 0.375) = 0.688 in from the mid-point of the steel plate. The moment due to eccentricity is

$$M = 10\,250 \times 0.688 = 7\,050 \text{ lb-in}$$

Two ⅝-in bolts, adjacent to each table, will be employed to counteract the rotational tendency due to this eccentric moment.

The center lines of the bolts will be placed 3 in from the center lines of the tables and the tension on each bolt will be:

$$\frac{7\,050}{2 \times 3} = 1\,175 \text{ lb per bolt}$$

The value of one ⅝-in bolt in tension is 3 200, based on the net area at the root of the thread which will provide an excess to permit the bolts to be drawn up tightly in order to insure close contact between the tables and the timber.

The same arrangement will be used for the lower chord, and in addition four ⅝-in bolts will be placed near the outer end to hold the plates firmly in place.

The maximum unit stress on the net section of the lower chord is:

$$\frac{37\,250}{(5.5 - 2 \times \tfrac{7}{8})\,7.5} = 1\,325 \text{ lb per sq in}$$

Joint I. Type C. Fig. 12. The detail illustrated in Fig. 12 is known as a toe-plate joint. The inclined bolts are not considered to assist in the transfer of stress. They will, however, be subjected to some tension due to the tendency of the upper chord to rotate about point B. When the fibers of the upper chord are confined by an enveloping toe-plate the unit compressive stress may be increased 25% in addition to increases otherwise permitted. At 60° to the direction of the grain:

$$C = [(1\,175)\sin^2 60° + (345)\cos^2 60°]\,1.50$$
$$= 1\,450 \text{ lb per sq in}$$

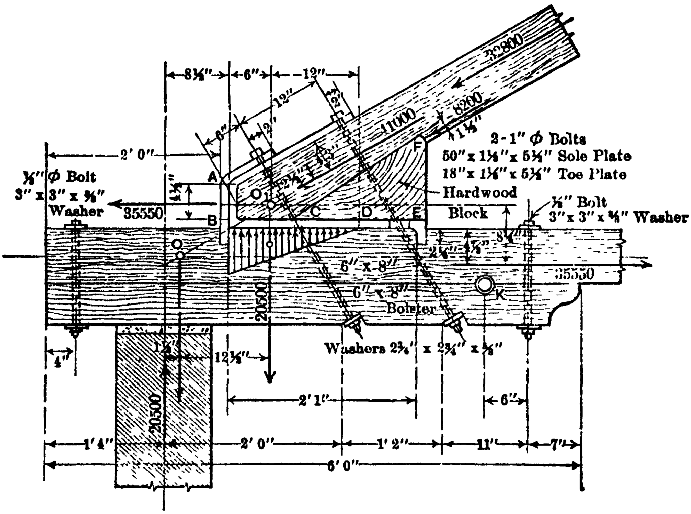


Fig. 12. Detail of Joint at Wall

The required depth of the toe cut is:

$$\frac{35\,550}{1\,450 \times 5.5} = 4.49 \quad \text{or} \quad 4\tfrac{1}{2} \text{ in}$$

A hardwood block is employed in the angle between the chord members to insure rigidity when the inclined bolts are drawn up. The block is LET INTO the upper chord $1\frac{1}{2}$ in. The stress on the block at F is $\frac{1.5}{7.5} \times 41\,000 =$ 8 200 lb, which is delivered to the plane C–E in an indeterminate manner. The resultant pressure on plane C–E may be assumed to act at $\frac{1}{3}$ to $\frac{1}{6}$ C–E from point C, and will be taken as 4 in in this design.

The resultant compression at the toe will, therefore, act at a distance of 4.3 in from the upper surface of the upper chord. At point O_1 the total compression is divided into its vertical and horizontal components. The intensity of the vertical pressure on plane B–D will not be uniform since there is a rotational tendency of the upper chord about point B. The assumed distribution of pressure is indicated by the stress volume shown in Fig. 12.

The distance B–D must be three times the horizontal dimension from O_1 to plane A–B to place the action line of the resultant pressure on a vertical through O_1.

The maximum unit pressure on the upper chord is twice the average pressure over the length B–D, or

$$\frac{2\ (20\ 500)}{18 \times 5.5} = 414 \text{ lb per sq in}$$

The allowable unit pressure normal to the grain is $1.2 \times 345 = 414$ lb per sq in.

The tension on the net section of the lower chord acts at $4\frac{7}{8}$ in below the upper surface of the chord. The vertical distance between this action line and the resultant pressure on the plane A–B is $8\frac{1}{4}$ in.

The horizontal moment produced through the transfer of the horizontal component of the upper chord stress is counter-clockwise, and equal to $M = 35\ 550 \times 8.25 = 293\ 300$ lb-in. The action line of the vertical component of the upper chord stress is $12\frac{1}{2}$ in from point O, and the vertical, clockwise moment is $M = 20\ 500 \times 12.5 = 256\ 300$ lb-in. The unbalanced moment is $293\ 300 - 256\ 300 = 37\ 000$ lb-in, counter-clockwise.

To neutralize this moment the wall reaction will be placed $\frac{37\ 000}{20\ 500} = 1.81$ or $1\frac{7}{8}$ in to the left of a vertical through point O. The depth of the lugs at B and E is determined by the allowable compression parallel to the grain, and assuming each lug to take one-half of the total horizontal component of the upper chord stress required depth of each is:

$$\frac{35\ 550}{2\ (1\ 175 \times 1.2)\ 5.5} = 2.29 \quad \text{or} \quad 2\tfrac{1}{4} \text{ in}$$

The thickness of the steel plates will be governed by the required thickness of the lugs which is determined by the bending moment on these lugs.

$$M = \frac{35\ 550}{2} \times \frac{2\ 25}{2} = 20\ 000 \text{ lb-in}$$

and

$$M = \frac{f}{6} b \cdot t^2 = \frac{f}{6} \times 5.5\ t^2$$

whence

$$t = \sqrt{\frac{20\ 000 \times 6}{18\ 000 \times 5.5}} = 1.1 \quad \text{or} \quad 1\tfrac{1}{8} \text{ in}$$

The required distance between lugs is determined by the allowable horizontal shear, and is:

$$\frac{35\ 550}{2\ (110 \times 1.2)\ 5.5} = 24.4 \quad \text{or} \quad 25 \text{ in}$$

A bolster is added to the under side of the lower chord, as shown in the detail, to provide additional area from which the notches for the washers used at the lower ends of the inclined bolts may be cut. The bolster also materially stiffens the joint. A cylindrical key is placed at K, designed to transmit the horizontal component of the full allowable tension in the inclined bolts, which for 1-in ϕ bolts is:

$$(0.55 \times 16\ 000)\ 2 \sin 30° = 8\ 800 \text{ lb}$$

The allowable bearing on the cylindrical key is given by Equation (17) as:

$$[\tfrac{2}{3}(1\,175) + \tfrac{1}{3}(345)]\,1.2 = 1\,160 \text{ lb per sq in}$$

on the plane of the diameter.

Since the total pressure must be transmitted by each half of the key, the required diameter is:

$$\frac{5.5\,(1\,160)\,d}{2} = 8\,800$$

or

$$d = 2.77 \text{ in}$$

A nominal $2\frac{1}{2}$ steel pipe (outside diameter $2\frac{7}{8}$ in) having a thickness of 0.276 in, will be used for the key.

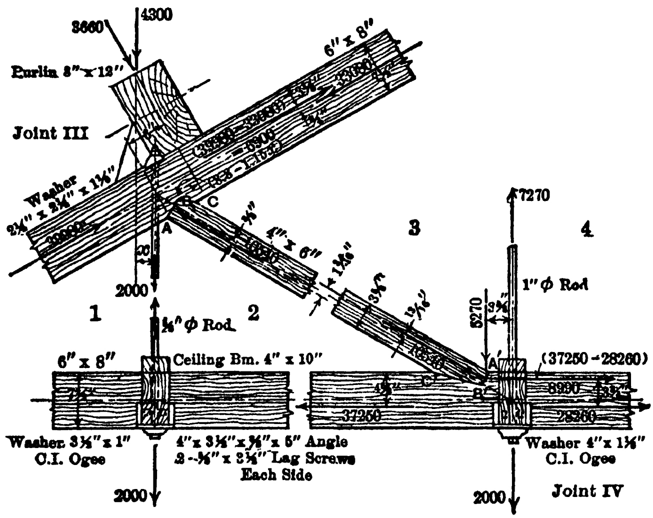


Fig. 13. Details of Joints

The rotational tendency of the upper chord about the toe is $M = 41\,000 \times 2.5 = 102\,500$ lb-in. If T_1 and T_2 are the tension values for the bolts at 6 and 18 in, respectively, from the center of the toe cut:

$$6\,T_1 + 18\,T_2 = 102\,500$$

but

$$T_1 = \tfrac{6}{18}\,T_2$$

whence

$$20\,T_2 = 102\,500$$

or

$$T_2 = 5\,125 \text{ lb}$$

and

$$T_1 = 1\,708 \text{ lb}$$

The one-inch bolts, with a net area of 0.55 sq in, are therefore satisfactory for the detail.

Joints III and IV. Member 2–3. Method 1. In the joint details illustrated in Fig. 13 the bearing surfaces A–B and A'–B' are at right-angles

to the axis of member 2–3; therefore the entire stress in the member is transmitted on these planes.

The angle of inclination of plane A–B with the grain of the upper chord is 30°, and the inclination of plane A'–B' with the grain of the lower chord is 60°.

The allowable unit compression on these planes and their required lengths are:

Plane A–B: $c = (560)\ 1.2 = 672$ lb per sq in

$$\text{length} = \frac{10\ 540}{(672)\ 5.5} = 2.85 \quad \text{or} \quad 2\tfrac{7}{8} \text{ in}$$

Plane A'–B': $c = (970)\ 1.2 = 1\ 160$ lb per sq in

$$\text{length} = \frac{10\ 540}{(1\ 160)\ 5.5} = 1.66 \quad \text{or} \quad 1\tfrac{5}{8} \text{ in}$$

The length of plane A'–B' will be made 2 in in order to reduce the eccentricity in member 2–3. The eccentric moment in member 2–3, due to the transfer of stress at its upper and lower ends, is:

$$M = 10\ 540 \times 1\tfrac{3}{16} = 12\ 500 \text{ lb-in}$$

If the joints are true and tight fitting, this moment, which tends to rotate member 2–3 in a counter-clockwise sense, may be satisfactorily resisted by friction on the planes A–B and A'–B'. However, in anticipation of imperfections in workmanship and shrinkage of timber, TOE-NAILING should always be employed in a joint of this type.

The design of the joint details is governed by the loading which produces maximum stress n member 2–3 ($D + C + \text{Min}\cdot S + W$). The maximum combined stress in member 2–3 due to direct compression and eccentricity is:

$$f_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{10\ 540}{20.39} + \frac{12\ 500 \times 6}{5.5\ (3.63)^2}$$

$$= 518 + 1\ 038 = 1\ 556 \text{ lb per sq in}$$

The allowable unit stress is:

$$\frac{518}{1\ 556} \times (750)\ 1.2 + \frac{1\ 038}{1\ 556} \times (1\ 600)\ 1.2 = 1\ 580 \text{ lb per sq in}$$

At joint IV the moment of the horizontal couple is:

$$M_h + 8\ 990 \times 3.75 + 33\ 700 \text{ lb-in}$$

The moment of the vertical couple is:

$$M_v = 5\ 270 \times 3.73 = 19\ 100 \text{ lb-in.}$$

The unbalanced moment, which produces bending stress in the lower chord, is $33\ 700 - 19\ 100 = 14\ 600$ lb-in. The net minimum net section of the lower chord occurs on a vertical plane through point B', and the maximum combined unit stress on this section is:

$$f_{\max} = \frac{37\ 250}{5.5 \times 5.8} + \frac{14\ 600 \times 6}{5.5 \times (5.8)^2}$$

$$= 1\ 166 + 474 = 1\ 640 \text{ lb per sq in}$$

The allowable unit stress is:

$$\frac{1\ 166}{1\ 640} \times 1\ 600 + \frac{474}{1\ 640} \times (1\ 600)\ 1.2 = 1\ 694 \text{ lb per sq in}$$

The bending stress due to eccentricity in the upper chord may be eliminated by placing the purlin in the proper position, with respect to the center of the joint, to neutralize the rotational tendency.

If the dead plus minimum snow-load (4 300), and the wind-load (3 660), transmitted to the truss by the purlin, are assumed to act at the mid-point of the upper surface of the purlin, as shown in Fig. 13:

let x = the horizontal distance from the action line of the vertical load to the center of the joint detail;

then $(8.8 - 1.15\,x)$ = the distance from the center of the joint detail to the normal center line of the purlin.

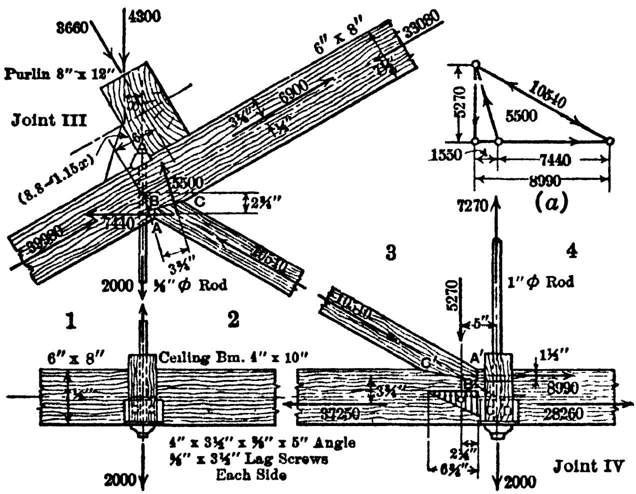


Fig. 14. Details of Joints

For $\Sigma M = 0$

$$-4\,300\,x + 3\,660\,(8.8 - 1.15\,x) + (6\,900)\,\tfrac{3}{4} + 10\,540 \times \tfrac{3}{8} = 0$$

or $x = 4.75$ in

and $(8.8 - 1.15\,x) = 3.3$ in

For the condition of loading which produces maximum stress in the upper chord $(D + C + S)$, the vertical panel-load is 2 780 + 3 430 = 6 210 lb.

For $\Sigma M = 0$

$$-6\,210\,x + (41\,000 - 33\,300)\,\tfrac{3}{4} + 8\,230 \times \tfrac{3}{8} = 0$$

or $x = 1.43$ in

and $(8.8 - 1.15\,x) = 7.25$ in

Considering both possible conditions of loading, the purlin will be placed, as shown, with its normal center line 6 in above the center of the joint detail.

Joints III and IV. Member 2–3. Method 2. The essential difference between the details illustrated in Fig. 14 and those of the preceding design

is in the end cuts of member 2–3. Both planes of the end cuts are assumed to function in the transfer of stress. By employing two bearing surfaces the cuts in the chord members will be shallower than in the preceding design. The only method of procedure in proportioning the two bearing surfaces is the CUT AND TRY method since many combinations are possible and satisfactory.

In this example the planes A–B and A'–B' will be made vertical and the inclination of the planes B–C and B'–C' will depend upon the required depths of the vertical planes.

If the length of plane A–B is made 1¼ in, plane B–C makes an angle of 16° 30′ with the direction of the grain of the upper chord, and its length is 3.75 in.

The component of the total stress on each plane is shown at (*a*), Fig. 14, to be:

On plane AB = 7 440 lb
On plane BC = 5 500 lb

The allowable unit bearing on these planes is:

$$\text{On plane } AB,\ C = 1\,175\,(\sin^2 60^\circ) + 345\,(\cos^2 60^\circ) = 1\,160 \text{ lb per sq in}$$

$$\text{On plane } BC,\ C = 1\,175\,(\sin^2 16^\circ\,30') + 345\,(\cos^2 16^\circ\,30') = 475 \text{ lb per sq in}$$

The total allowable bearing is:

On plane AB = $1.25 \times 5.5 \times 1\,160 = 7\,995$ lb
On plane BC = $3.75 \times 5.5 \times 475 = 9\,800$ lb

Since no cuts less than 1¼ in should be specified, the cuts will be made as assumed.

At joint IV, if plane A'–B' is made 1½ in, plane B'–C' is 7½ in in length and makes an angle of 12° with the direction of the grain of the lower chord.

The horizontal component to be provided for on plane A'–B' is 8 990 lb. The unit compression is $\frac{8\,990}{1.5 \times 5.5} = 1\,090$ lb per sq in and the allowable value of 60° to the grain is 1 160 lb per sq in.

When the angle between the two planes of an end cut is less than 90°, as at joint IV, the component of the stress on the plane making the smaller angle with the supporting member should be taken at right-angles to the component on the plane making the larger angle.

In such a case the intensity of pressure on the longer plane will vary as indicated in Fig. 14. The pressure on B'–C' will be assumed to vary from maximum at B' to zero at a distance equal to three times the distance of the action line of the vertical component from B'. The maximum pressure on B'–C' is, therefore:

$$2 \times \frac{5\,270}{5.5 \times 6.75} = 285 \text{ lb per sq in}$$

The allowable unit compression on a plane at 12° with the grain is given by Equation (15) (increased by 20%) as 452 lb per sq in.

The moments due to the eccentric transfer of stress at joint IV are:

$$M_h = 8\,990 \times 3.75 = +\,33\,700$$
$$M_v = 5\,270 \times 5.00 = -\,26\,350$$

The unbalanced moment is 7 350 lb-in (clockwise).

The maximum combined unit stress in the lower chord is:

$$f_{max} = \frac{37\,250}{5.5 \times 6.0} + \frac{7\,350 \times 6}{5.5\,(6.0)^2} = 1\,352 \text{ lb per sq in}$$

The allowable unit stress, determined in the same manner as in the preceding design, is 1 650 lb per sq in.

The position of the purlin to neutralize the moment at joint III is determined as before:

From $\Sigma M = 0$:

$$-4\,300\,x + 3\,660\,(8.8 - 1.15\,x) + (6\,900)\,\tfrac{1}{2} + (7\,440)\,2\tfrac{3}{4} - (5\,500)\,3\tfrac{3}{4} = 0$$

or $$x = 4.2$$

and $$(8.8 - 1.15\,x) = 4 \text{ in}$$

The value of x required to neutralize the moment under the loading which produces maximum stress in the upper chord is:

From $\Sigma M = 0$:

$$6\,210\,x + (7\,700)\,\tfrac{1}{2} + (5\,800)\,2\tfrac{3}{4} - (4300)\,3\tfrac{3}{4} = 0$$

or $$x = 0.60$$

and $$(8.8 - 1.15\,x) = 8 \text{ in}$$

The purlin will be placed at 6 in as in the preceding design.

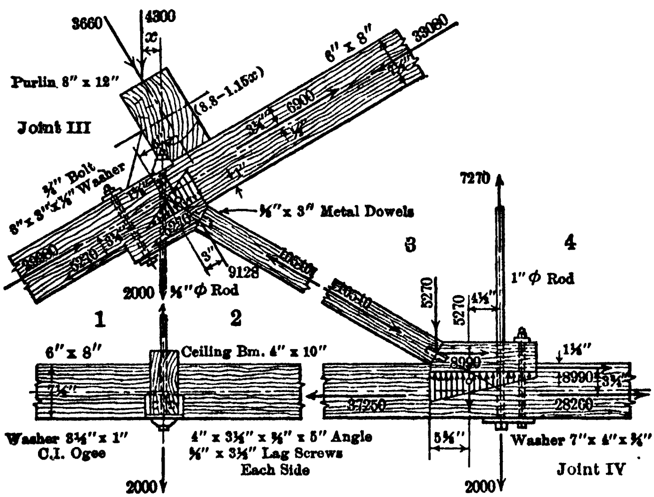


Fig. 15. Details of Joints

Joints III and IV. Member 2–3. Method 3. The detail illustrated in Fig. 15 is known as the BEARING-BLOCK detail. The required lengths of the bearing-blocks are determined by horizontal shear, and the depths of the DAPS in the chord members are determined by the allowable bearing parallel to the grain. The bolts near the outer ends of the bearing-blocks are employed to hold the block securely in position. The bearing of the blocks on

the chord members is not uniformly distributed, and the maximum intensity at the inner end, of each block, must always be provided for.

At joint III:

$$\text{Minimium length of block} = \frac{5\,270}{5.5\ (110)\ 1.2} = 7.25 \text{ in}$$

$$\text{Minimium depth of dap} = \frac{5\,270}{5.5\ (1\,175)\ 1.2} = 0.68 \text{ in}$$

The resultant pressure of the bearing-block on the upper chord is assumed to act through the intersection of the center line of member 2–3 and the bearing plane of the chord member and the block. The unit pressure at this point is $\frac{2}{3}$ the maximum pressure since the pressure volume is a triangular wedge. The maximum allowable pressure is (345) 1.2 = 414 lb per sq in.

The total pressure to be transferred is 9 128 lb and the required length of the bearing plane is:

$$\frac{2 \times 9\,128}{5.5\ (414)} = 8.05 \text{ in}$$

The required distance between the action line of the resultant pressure and the inner end of the block is $\frac{8.05}{3} = 2.68$ in. If this distance is made 3 in and the dap in the chord made 1 in, the block may be made of a $3\frac{1}{2} \times 5\frac{1}{2}$-in section. The length of the block will be made $12\frac{1}{2}$ in in order to provide clearance between the bolt at the outer end and member 1–2.

The position of the purlin to neutralize the moment resulting from the transfer of stress is determined as before:

From $\Sigma M = 0$:

$$-4\,300\,x + 3\,660\,(8.8 - 1.15\,x) + (6\,900)\,\tfrac{1}{2} + (5\,270)\,3\tfrac{1}{4} - (9\,128)\,1\tfrac{5}{8} = 0$$

or $$x = 4.45 \text{ in}$$

and $$(8.8 - 1.15\,x) = 3.7 \text{ in}$$

The required position of the purlin to neutralize the moment for stresses due to combined dead and maximum snow-loads is found to be $7\frac{3}{4}$ in above the center of the detail.

The purlin will be placed with its normal center line 6 in above the center of the joint detail, as in the preceding designs.

At joint IV:

$$\text{Minimum length of block} = \frac{8\,990}{5.5\ (110)\ 1.2} = 12.4 \text{ in}$$

$$\text{Minimum depth of dap} = \frac{8\,990}{5.5\ (1\,175)\ 1.2} = 1.16 \text{ in}$$

The block will be made 15 in long and the dap will be made $1\frac{1}{8}$ in. The section of the block will be $4\frac{1}{4}$ in $\times$ $5\frac{1}{2}$ in. The resultant pressure on the chord acts at a distance of $5\frac{5}{8}$ in from the inner end of the block and is, therefore, within the kern. The maximum unit bearing is:

$$\frac{5\,270}{5.5 \times 15} + \frac{5\,270\,(7.5 - 5.63) \times 6}{5.5\,(15)^2} = 112 \text{ lb per sq in}$$

A flat plate washer will be designed for member 3–4 and the toe-bolt, the latter being assumed to be drawn up with an initial tension of 3 000 lb. The

total area required is $\frac{7\,270 + 3\,000}{(345)\,1.2} = 25$ sq in.

The washer will be made 4 in × 7 in to allow for the reduction of area due to bolt-holes.

The projection of the washer beyond the center line of member 3–4 will be made $2\frac{1}{2}$ in. The moment at the center line of member 3–4 is:

$$\frac{10\,270}{28 - 1.6} \times 2.5 \times 4 \times \frac{2.5}{2} = 4\,800 \text{ lb-in}$$

The required thickness is

$$t = \sqrt{\frac{6\,M}{f \cdot b}}, \quad \text{or} \quad t = \sqrt{\frac{6 \times 4\,800}{18\,000 \times 2.88}} = 0.74 \quad \text{or} \quad \tfrac{3}{4} \text{ in.}$$

The moments in the lower chord due to the transfer of stress are:

$$\begin{aligned} M_h &= 8\,990 \times 3.75 &&= +\ 33\,700 \text{ lb-in} \\ M_v &= 5\,270 \times 4.50 &&= -\ \underline{23\,600} \text{ lb-in} \\ \text{Unbalanced moment} & &&= +\ 10\,100 \text{ lb-in} \end{aligned}$$

The maximum combined unit stress on the lower chord section is:

$$f_{\max} = \frac{37\,250}{5.5 \times 6.38} + \frac{10\,100 \times 6}{5.5\,(6.38)^2} = 1\,330 \text{ lb per sq in,}$$

which is satisfactory.

The design of joint V is similar to that of joint III and will not be taken up in detail.

Joints VI and VII. Method I. The design of joint VI may be in accordance with any of the methods previously explained.

The compression-members make an angle of 49° 10′ with the horizontal and 40° 50′ with the vertical. The inclined surfaces of the bearing-block, shown in Fig. 16, make angles of 40° 50′ with the direction of the grain of the block, and the allowable unit compression is:

$$c = [(1\,175) \cdot \overline{0.65}^2 + (345) \cdot \overline{0.76}^2]\,1.2 = 835 \text{ lb per sq in}$$

The maximum unit pressure occurs under dead, minimum, snow, and wind-loads, and is:

$$\frac{14\,100}{5.5 \times 5.5} = 466 \text{ lb per sq in}$$

The dap in the lower chord must be sufficient to provide for the greatest possible difference in the horizontal components of the two compression-members. This maximum difference occurs under the loading for maximum stress in one of the compression-members. The maximum stress in the windward member is 14 100 lb, and the simultaneous stress in the leeward member is 6 400 lb, as shown in Table XXIV.

The maximum horizontal component to be provided for is:

$$(14\,100 - 6\,400) \cos 49^\circ\,10' = 5\,030 \text{ lb}$$

The minimum required depth of the dap is:

$$\frac{5\,030}{5.5\,(1\,175)\,1.2} = 0.65 \text{ in}$$

The dap will be made one inch and the section of the block will be 5 in × $5\frac{1}{2}$ in. The horizontal bearing surface of the block must be sufficient to

provide for the maximum vertical components of the two compression-members. The maximum simultaneous vertical components occur under dead and maximum snow-loads, and their sum is:

$$2\ (10\,930) \sin 49° \ 10' = 16\,460 \text{ lb}$$

The minimum required length of the block for bearing is:

$$\frac{16\,460}{5.5\ (345)\ 1.2} = 7.05 \text{ in}$$

The minimum length of the block for horizontal shear is:

$$\frac{5\,030}{5.5\ (110)\ 1.2} = 6.95 \text{ in}$$

The minimum length of block to provide full bearing for the web-members is 14 in, as shown in Fig. 16.

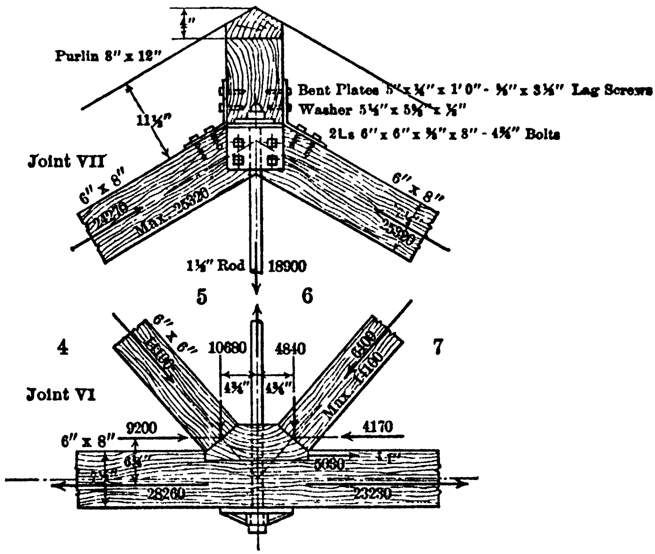


Fig. 16. Details of Joints

The maximum combined unit stress in the lower chord occurs with the maximum difference of stress in the two compression-members, 4–5 and 6–7. The moments due to stress transfer are:

$$\begin{aligned} M_h &= (\ 9\,200 - 4\,170)\ 6\tfrac{1}{4} &&= +\ 31\,400 \text{ lb-in} \\ M_v &= (10\,680 - 4\,840)\ 4\tfrac{3}{4} &&= -\ \underline{26\,600} \text{ lb-in} \\ \text{Unbalanced moment} & &&= +\ \ \ 4\,800 \text{ lb-in} \end{aligned}$$

and

$$f_{\max} = \frac{28\,260}{5.5 \times 6.5} + \frac{4\,800 \times 6}{5.5 \times (6.5)^2} = 916 \text{ lb per sq in}$$

At joint VII, the upper chord members will be cut at the peak to provide a horizontal bearing surface for the purlin. The chord members will be held

in position by a $6 \times 6 \times \frac{3}{8} \times 8$-in angle, each side and the purlin will be held in place by two bent-plates 5 in $\times \frac{1}{4}$ in $\times$ 12 in, as shown in Fig. 16. The net bearing area required for the plate washer for member 5–6 is:

$$\frac{18\,900}{(560)\,1.2} = 28.2 \text{ sq in}$$

where the value 560 is the allowable compression at 30° to the grain. The gross area required is $28.2 + 2.4 = 30.6$ sq in. The washer will be made $5\frac{1}{2}$ in $\times$ $5\frac{5}{8}$ in. The size of the standard nut for the $1\frac{1}{2}$-in ϕ rod, is $2\frac{3}{8}$-in square. The moment at the center line is:

$$\frac{18\,900}{2} \times \frac{1}{4} \cdot (5\tfrac{5}{8} \text{ in} - 2\tfrac{3}{8} \text{ in}) = 7\,650 \text{ lb-in}$$

and the required thickness is:

$$t = \sqrt{\frac{7\,650 \times 6}{18\,000\,(5.5 - 1.75)}} = 0.825 \quad \text{or} \quad \tfrac{7}{8} \text{ in}$$

The area of the vertical bearing plane between the chord members, deducting for a $1\frac{9}{16}$-in hole, is 25.5 sq in. The allowable unit bearing at 60° to the grain is 1 160 lb per sq in.

The required area for maximum chord stress is $\dfrac{(24\,650)\,0.866}{1\,160} = 18.4$ sq in. The area provided is, therefore, satisfactory.

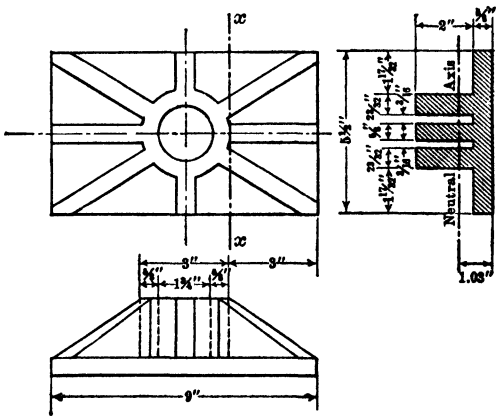


Fig. 17. Details of Washer

Design of Special Cast-Iron Washer at Joint VI. The opening through the washer to receive member 5–6 will be made $\frac{1}{4}$ in larger than the diameter of the rod. The walls of the center spool, the base and the webs, will be made $\frac{5}{8}$ in, the minimum thickness for cast iron.

The required area in bearing to provide for the maximum stress in member 5–6 is:

$$\frac{18\,900}{(345)\,1.2} = 45.6 \text{ sq in}$$

The area deducted for the rod opening is 2.4 sq in, hence the gross area must be 45.6 + 2.4 = 48 sq in. If the width of the washer is made the full width of the chord, the length required is 48/5.5 = 8.92 or 9 in. The resisting moment, at the face of the spool, plane xx, is determined as follows:

(1) Location of neutral axis:

$$y = \frac{2\,(2.06)\,1.63 + 0.63\,(5.5)\,0.315}{4.12 + 3.46} = 1.03 \text{ in}$$

(2) Moment of inertia about the neutral axis:

$$I = \frac{(2.06)\cdot 2^3}{12} + 4.12 \times \overline{0.60}^2 + \frac{(5.5)\cdot\overline{0.63}^2}{12} + 3.46 \times \overline{0.71}^2 = 4.77 \text{ in}^4$$

The resisting moment is:

$$\frac{fI}{c} = \frac{3\,000 \times 4\,77}{1.03} = 13\,900 \text{ lb-in}$$

The maximum bending moment at section xx is:

$$\tfrac{3}{9} \times 18\,900 \times \tfrac{3}{2} = 9\,450 \text{ lb-in}$$

The resulting maximum tensile stress on the extreme fiber of the cast iron is only 2 000 lb per sq in; however, a reduction of dimensions is inadvisable and the washer will be made as shown in Fig. 17.

Joints VI and VII. Method 2. A second method for framing joints VI and VII is shown in Fig. 18.

At joint VI the two compression members are LET INTO the lower chord.

The maximum horizontal component to be provided for is 5 030 lb; therefore, the maximum depth of the vertical cut at the toe is:

$$\frac{5\,030}{5.5\,(970)\,1.2} = 0.79 \text{ in}$$

The cut will be made $1\tfrac{1}{4}$ in to enter the chord $\tfrac{1}{2}$ in.

The inclined bearing surfaces make angles of 9° with the lower chord and 50° with the direction of the grain of the web-members. The maximum allowable unit compression on the lower chord is, by Equation (15), 420 lb per sq in. The resultant pressure on the chord will be assumed to be vertical and its action line is 3 in from the center line of the detail as shown in Fig. 18. The resultant is within the kern of the bearing area, hence the maximum unit compression is:

$$\frac{10\,680}{6.75 \times 5.5} + \frac{(10\,680)\,\tfrac{3}{8} \times 6.0}{5.5 \times (6.75)^2} = 384 \text{ lb per sq in}$$

The unbalanced moment due to the transfer of stress in the lower chord is:

$$\begin{aligned} M_h &= (\,9\,200 - 4\,170)\,3.75 = +\,17\,000 \text{ lb-in} \\ M_v &= (10\,680 - 4\,840)\,3.00 = -\,17\,500 \text{ lb-in} \\ \text{Unbalanced moment} &= \quad 500 \text{ lb-in} \end{aligned}$$

The hole for the $1\tfrac{1}{2}$-in rod will be bored one-sixteenth of an inch larger than the diameter of the rod, hence the net width of the lower chord 3.94 in, and

$$f_{max} = \frac{28\,260}{3.94 \times 7} + \frac{500 \times 6}{3.94\,(7)^2} = 1\,046 \text{ lb per sq in}$$

At joint VII, the end cuts of the upper chord members and the washer for member 5–6 are the same as in the preceding design. The chord members

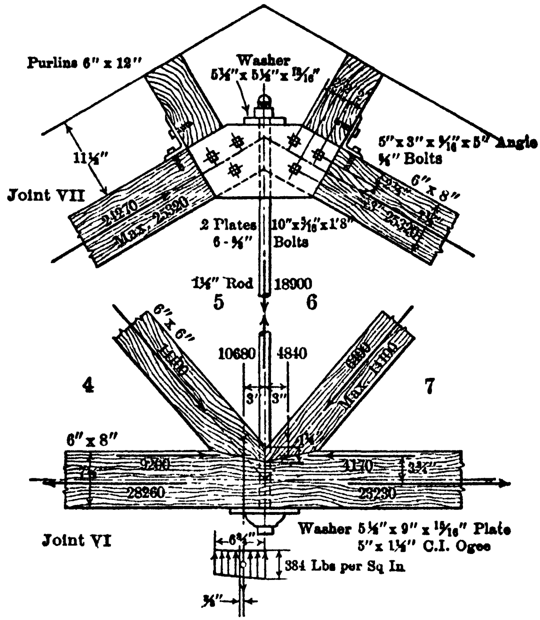


Fig. 18. Details of Joints

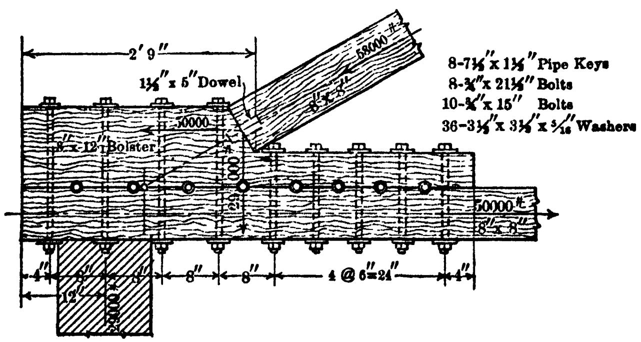


Fig. 19. Detail of Joint

are held in position by steel plates on each side of the joint, bolted to the chord members with three ⅝-in bolts on each side of each chord. The entire stress is assumed to be transferred through bearing on the vertical plane

between the chord members; hence, the holes for the bolts will be bored ¼ in larger than the diameter of the bolts.

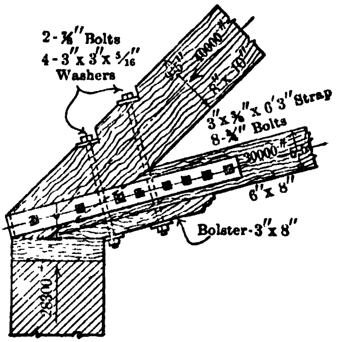


Fig. 20

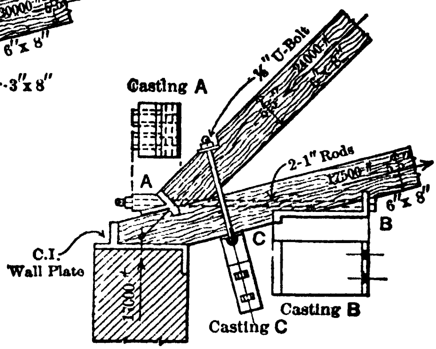


Fig. 21

Figs. 20 and 21. Details of Joints

Miscellaneous Details. The various methods of analysis and design in the preceding examples have been completely explained in order to guide the designer in the procedure to be employed in analyzing the various types of joints encountered in timber roof-truss construction.

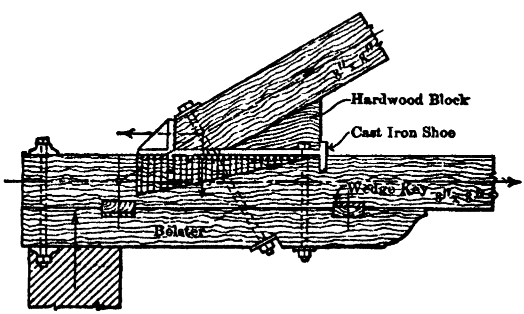


Fig. 22. Detail of Joint

Figs. 19 to 24 inclusive illustrate other types of joints and joint details that may be analyzed and designed by the same principles and methods which have been explained in detail.

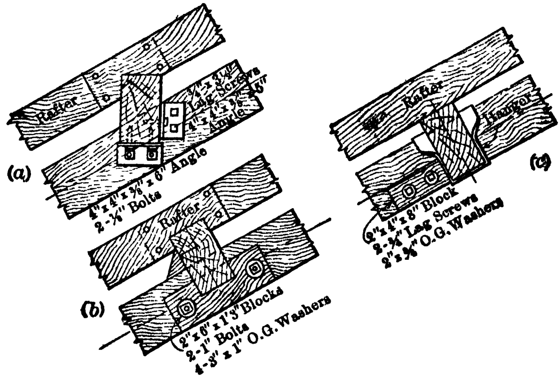


Fig. 23

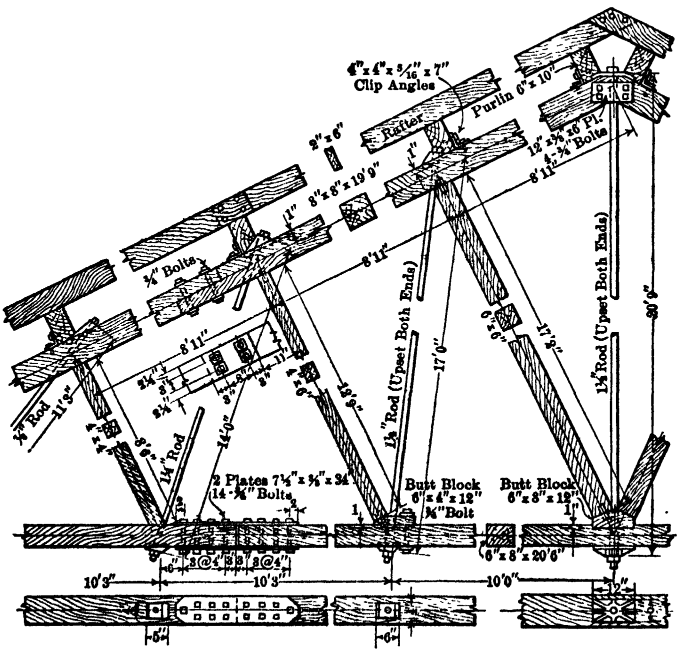


Fig. 24

Figs. 23 and 24. Miscellaneous Truss Details

8. Steel Frame Roof-Construction

General. In steel frame roof-construction the rafters, purlins, trusses, and ceiling-beams are made of standard rolled sections. Rafters, when used, are generally channel or beam-sections, although with some types of pre-cast roof slabs angles may be used as rafters or sub-purlins.

The principal purlins and ceiling-beams are usually channel, I-beam, Bethlehem-beam, or Carnegie-beam sections.

Steel roof-trusses may be classified according to the manner in which their members are connected, as (1) PIN-CONNECTED, and (2) RIVETED; however, trusses are seldom constructed with pin-connected joints except in some cases where the span is great, since pin-joints, although theoretically the ideal type of joint, do not provide the desirable stiffness for relatively light roof-truss construction. In RIVETED roof-trusses, both the tension and compression-members are generally made up of angles, or combinations of plates and angles. The top and bottom chords of trusses for usual spans generally consist of two angles back to back with the gusset-plates for web-member connections between the parallel legs of the angles.

The web-members may also be made of two angles back to back, or in the case of small stresses they may consist of a single angle.

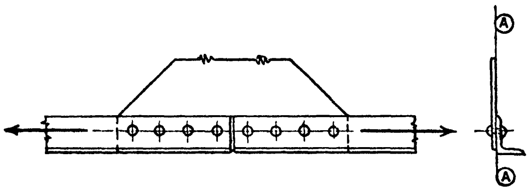


Fig. 25. Single-shear Joint

Riveted Joints. The function of a riveted joint is to transmit the stress from one member or element to one or more other members or elements.

The stresses in the main members can be fairly well determined by the various methods of analysis, but the distribution of stress in joint details is, for the most part, practically indeterminate, and good design of such details requires considerable experience and judgment.

A well-designed riveted joint should provide for the TRANSFER OF STRESS without overstressing any member or element, and the entire detail should be free from all tendency to permanent distortion.

The distortion of a riveted joint does not vary uniformly with the increase of load for the usual values of safe loads, due to the friction developed as a result of the clamping action of the rivets. With good design and good workmanship friction in a riveted joint is rarely overcome, except in the case of hand-driven rivets in joints subject to alternating stresses. In such cases the working unit stresses should be made less than for joints secured by power-driven rivets.

Riveted joints may be divided into two general classes:

(1) SINGLE-SHEAR, single-bearing joints.

(2) DOUBLE-SHEAR, double-bearing joints.

In the SINGLE-SHEAR JOINT, Fig. 25, the principal stresses on the rivets, neglecting friction, are a shearing stress on plane AA and bearing stresses on the body of the rivet in contact with the angle leg and the gusset-plate.

If the thickness of the gusset-plate is less than the thickness of the angle, the bearing on the gusset-plate controls the allowable bearing stress.

When the rivets of a joint are subject to single shear the intensity of bearing both on the connection-plate and the connected piece is greater at the contact surfaces and less at the outer surfaces which bear against the rivet-heads. In a DOUBLE-SHEAR JOINT, Fig. 26, the bearing may be assumed to be uniformly distributed over the connection-plate, and the average working unit stress in bearing may, therefore, be taken at a larger value than for single-shear joints. The rivets are subjected to shear on the two sections AA and BB.

Size of Rivets. The proper size of rivet to use in a given joint depends to some extent upon the thickness of the pieces to be connected. When the total thickness of metal necessitates the use of a rivet whose grip exceeds four to five times its diameter, bending stresses of considerable importance may be developed,* and larger rivets, or a greater number than required for

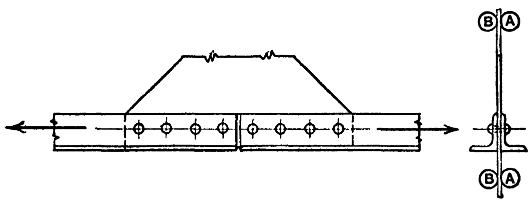


Fig. 26. Double-Shear Joint

shear and bearing, may be necessary to provide sufficient resistance. Long rivets are not usually encountered in roof-truss details and the governing factors are, therefore, SHEAR and BEARING. In a SINGLE-SHEAR JOINT, the allowable average unit bearing is taken at about 1 7 times the allowable unit shear and for equal resistance to both shear and bearing:

$$\frac{\pi d^2}{4} \cdot f_s = d \cdot t \cdot f_b$$

where d = diameter of rivets, t = least thickness of connection-plate or piece connected, f_s = allowable unit shear, and f_b = allowable unit bearing, then

for $$\frac{f_b}{f_s} = 1.7, \quad d = 2.16\, t$$

In a DOUBLE-SHEAR JOINT the ratio $\frac{f_b}{f_s}$ is generally taken at about 2 to 2.2 and $d = \frac{4.4\, t}{\pi} = 1.4\, t$.

The DIAMETER OF RIVETS for roof-truss details is generally taken at about twice the thickness of the average gusset-plate; or conversely, the average thickness of gusset-plates to be employed is made about one-half the diameter of the rivets to be used.

For light structural work, such as roof-trusses not exceeding 100 ft in span, gusset-plates are generally made from ¼ to ½ in thick and rivets are ⅝ to ⅞

* When the grip of a rivet exceeds five times its diameter, the number of required rivets as calculated for shear or bearing shall be increased 1% for each ⅒ in of grip beyond five diameters.

in in diameter. The average conditions encountered are most satisfactorily met by using $\frac{5}{16}$ or $\frac{3}{8}$-in gusset-plates with $\frac{3}{4}$-in rivets. The values of the usual sizes of rivets employed in building-construction, together with the bearing values, both for single and double-shear joints, for usual thicknesses of plates are given in Tables XXV and XXVI.

Table XXV. Working Values for Hand-driven Rivets

Unit Shear = 10 000 lb per sq in

Unit Bearing { Single = 16 000 lb per sq in; Double = 20 000 lb per sq in }

Size of Rivet	$\frac{5}{8}$ In		$\frac{3}{4}$ In		$\frac{7}{8}$ In		1 In		$\frac{1}{8}$ In	
Area Sq In	0.3068		0.4418		0.6013		0.7854		0.9940	
Single Shear Lb	3070		4420		6015		7855		9940	
Double Shear Lb	6140		8840		12030		15710		19880	
Bearing	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb
Thickness of Plate in Inches $\frac{1}{4}$	2500	3130	3000	3750	3500	4380	4000	5000	4500	5630
$\frac{5}{16}$	—	3910	3750	4690	4380	5470	5000	6250	5630	7030
$\frac{3}{8}$	—	4690	—	5630	5250	6560	6000	7500	6750	8440
$\frac{7}{16}$	—	5470	—	6560	—	7660	7000	8750	7880	9840
$\frac{1}{2}$	—	—	—	7500	—	8750	—	10000	9000	11250
$\frac{9}{16}$	—	—	—	8840	—	9850	—	11250	—	12660
$\frac{5}{8}$	—	—	—	—	—	10940	—	12500	—	14060

Bearing values not given beyond point where shear governs.

Table XXVI. Working Values for Power-driven Rivets

Unit Shear = 13 500 lb per sq in

Unit Bearing { Single = 24 000 lb per sq in; Double = 30 000 lb per sq in }

Size of Rivet	$\frac{5}{8}$ In		$\frac{3}{4}$ In		$\frac{7}{8}$ In		1 In		$\frac{1}{8}$ In	
Area Sq In	0.3068		0.4418		0.6013		0.7854		0.9940	
Single Shear Lb	4140		5960		8120		10600		13420	
Double Shear Lb	8280		11930		16240		21200		26840	
Bearing	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb
Thickness of Plate in Inches $\frac{1}{4}$	3750	4690	4500	5630	5250	6560	6000	7500	6750	8440
$\frac{5}{16}$	—	5860	5630	7030	6560	8200	7500	9380	8440	10550
$\frac{3}{8}$	—	7030	—	8440	7880	9840	9000	11250	10130	12660
$\frac{7}{16}$	—	8200	—	9840	—	11480	10500	13130	11810	14770
$\frac{1}{2}$	—	—	—	11250	—	13130	—	15000	—	16880
$\frac{9}{16}$	—	—	—	—	—	14770	—	16880	—	18980
$\frac{5}{8}$	—	—	—	—	—	—	—	18750	—	21090

Bearing values not given beyond point where shear governs.

Rivet Spacing. The MINIMUM DISTANCE between centers of rivet-holes is three diameters of the rivet; but the PREFERABLE DISTANCE is not less than 4 in for $1\frac{1}{8}$-in rivets, $3\frac{1}{2}$ in for 1-in rivets, 3 in for $\frac{7}{8}$-in rivets, and 2 in for $\frac{5}{8}$-in rivets. The maximum PITCH in the line of stress is 16 times the thinnest outside plate or shape connected, or 20 times the thinnest enclosed plate or shape.

For 7/8-in and 3/4-in rivets used in roof-truss joints the maximum pitch is preferably 6 in, and for 5/8-in rivets a maximum of 4 in should not be exceeded.

For angles with two gauge-lines and rivets staggered the maximum pitch in the line of stress on either gauge line should not exceed 24 times the thinnest plate or section, with the preferable maximums limited to twice the above values.

In compression-members composed of two angles, STITCH-RIVETS through washers should be employed to maintain a constant spacing. The distance between such rivets should be such that the slenderness-ratio L/r for each angle is not more than that of the entire member, with a maximum of about 2 ft 6 in. Stitch-rivets should be employed in tension-members at intervals not to exceed about 4 ft.

The minimum distance from the center of any rivet-hole to a sheared edge is 2 in for 1 1/8-in rivets, 1 3/4 in for 1-in rivets, 1 1/2 in for 7/8-in rivets, 1 1/4 in for 3/4-in rivets, and 1 1/8 in for 5/8-in rivets.

Truss connections carrying calculated stresses should never have less than two rivets through any member transferring stress.

Riveting Details. Rivets cannot be driven closer to the web of a beam, or the leg of an angle, than the minimum distance required to operate the die which is used to form the rivet-head. The sizes of standard dies for various sizes of rivets are given in Table XXVII. The usual rule for calculating clearance is "one-half the diameter of the rivet-head plus three-eighths of an inch."

There are two kinds of rivets used in structural steel framing, classified according to the type of head: (1) BUTTON HEADS, and (2) COUNTERSUNK HEADS.

There is no fixed standard for the proportions of button heads; however, the dimensions given in Table XXVIII * are used with slight modifications by all manufacturers.

It is sometimes necessary to FLATTEN rivet-heads in order to provide clearance for adjacent members. This flattening may vary from a slight reduction of the full head to a flush or countersunk head. The dimensions of countersunk heads are given in Table XXVIII. Rivets with countersunk heads

Table XXVII. Standard Rivet Dies

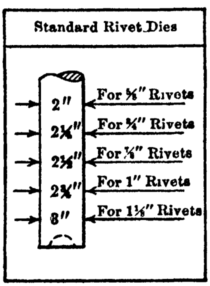
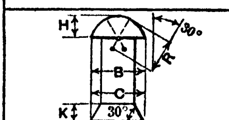


Table XXVIII. Dimensions of Rivets

Dimensions of Rivets



Diam. Ins.	Button Heads B	Button Heads H	Button Heads R	Counter Sunk C	Counter Sunk K
5/8	1 1/16	7/16	11/16	1	5/16
3/4	1 1/4	1/2	13/16	1 3/16	3/8
7/8	1 7/16	5/8	15/16	1 3/8	7/16
1	1 5/8	11/16	1	1 9/16	1/2
1 1/8	1 13/16	3/4	1 1/8	1 3/4	9/16

* American Institute of Steel Construction Standards.

are not as strong as rivets with button heads, and are much more expensive on account of the REAMING necessary to form the head.

GAUGE-LINES are those lines parallel to the length of a member, on which the rivets are placed. The term GAUGE is used to designate the normal distance between gauge-lines or between a gauge-line and a surface or edge of the member.

The standard gauge dimension for structural-steel angles are given in Table XXIX.

The PITCH of rivets is the distance center to center of adjacent rivets whether they are on the same or different gauge lines.

Table XXIX. Gage Dimensions for Standard Angles

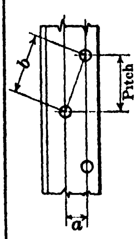
Gage Dimensions — For Standard Angles

Leg	2	2½	3	3½	4	5	6	7	8
g_1	1⅛	1⅜	1¾	2	2½	3	3½	4	4½
g_2	---	---	---	---	---	2	2¼	2½	3
g_3	---	---	---	---	---	1¾	2¼	3	3
Max. Rivet	⅝"	¾"	⅞"	⅞"	⅞"	⅞"	⅞"	1"	1⅛"

The minimum pitch of rivets to maintain the minimum distance of three diameters between centers of adjacent rivets is given for various gauges, and the usual sizes of rivets, in Table XXX.

Design of Tension-Members. The strength of a tension-member depends

Table XXX. Minimum Pitch to Maintain Three Diameters Center to Center of Rivets *

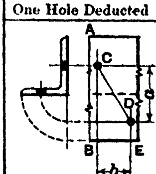


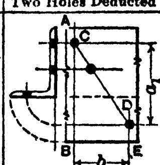
Diam of Rivet	Min Value b	Gage Line Distance a, Inches: 1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
⅝"	1⅞	1⅝	1⅜	1⅛	⅝	0	---	---	---	---	---
¾"	2¼	2	1⅞	1⅝	1⅜	1	0	---	---	---	---
⅞"	2⅝	2½	2⅜	2⅛	2	1¾	1⅜	¾	0	---	---
1"	3	2⅞	2¾	2⅝	2½	2¼	2	1⅝	1⅛	0	---
1⅛"	3⅜	3¼	3⅛	3	2⅞	2¾	2½	2¼	2	1½	⅞

* From A.I.S.C. "Steel Construction"

Table XXXI. Stagger of Rivets to Maintain Net Section

To Maintain the Equivalent of — One Hole Deducted / Two Holes Deducted





Values for ⅝" Rivets = ⅛" Less than for ¾" Rivets. Values for 1" Rivets = ⅛" More than for ⅞" Rivets.

a	¾" Rivet b	⅞" Rivet b	a_1	¾" Rivet b	⅞" Rivet b
1	1¾	1¾	5	3 1/16	3 5/16
1½	1⅞	2	5½	3¼	3½
2	2 1/16	2¼	6	3⅜	3⅝
2½	2¼	2 7/16	6½	3½	3¾
3	2 7/16	2⅝	7	3⅝	3⅞
3½	2 9/16	2 13/16	7½	3¾	4
4	2 11/16	3	8	3⅞	4⅛
4½	2 15/16	3 3/16	8½	4	4¼

upon its net cross-sectional area, and the arrangement of rivets should be such that the necessary punching will not reduce the cross-section more than is absolutely necessary.

Table XXXII. Allowable Tension Values for One Angle with One Rivet Hole Deducted*

Values in Thousands of Pounds for Allowable Unit Stresses of 16 000 and 18 000 lb per sq in

Size of Angle In.	Weight Pounds per Ft	Gross Area Sq In.	Allowable Net Stress 16000 Lb per Sq In ⅞ Rivets	16000 Lb per Sq In ¾ Rivets	18000 Lb per Sq In ⅞ Rivets	18000 Lb per Sq In ¾ Rivets	Size of Angle In.	Weight Pounds per Ft	Gross Area Sq In.	Allowable Net Stress 16000 Lb per Sq In ⅞ Rivets	16000 Lb per Sq In ¾ Rivets	18000 Lb per Sq In ⅞ Rivets	18000 Lb per Sq In ¾ Rivets
6 x 6 x ⅞	33.1	9.73	141.6	143.4	159.4	161.3	4 x 4 x ⅝	15.7	4.61	63.7	65.0	71.7	73.1
13/16	31.0	9 09	132.5	134.1	149.0	150.8	9/16	14.3	4.18	57.9	59.0	65.1	66.4
¾	28.7	8.44	123.0	124.5	138.4	140.1	½	12.8	3.75	52.0	53.0	58.5	59.6
11/16	26.5	7.78	113.4	114.9	127.6	129.2	7/16	11.3	3.31	45.9	46.9	51.7	52.7
⅝	24.2	7.11	103.7	105.0	116.7	118.1	⅜	9.8	2.86	39.7	40.5	44.7	45.6
9/16	21.9	6.43	93.9	95.0	105.6	106.9	5/16	8.2	2.10	33.4	34.1	37.6	38.3
½	19.6	5.75	84.0	85.0	94.5	95.6	4 x 3½ x ⅝	14.7	4.30	58.8	60.0	66.1	67.5
7/16	17.2	5.06	73.9	74.9	83.2	84.2	9/16	13 3	3.90	53.3	54.5	60.0	61.3
⅜	14.9	4.36	63.7	64.5	71.7	72.6	½	11.9	3.50	48.0	50.0	54.0	55.1
6 x 4 x ⅞	27.2	7.98	113.6	115.4	127.9	129.8	7/16	10.6	3.09	42.4	43.3	47.7	48.7
13/16	25.4	7.47	106.6	108.2	119.8	121.6	⅜	9.1	2 67	36.7	37.4	41.3	42.1
¾	23.6	6.94	99.0	100.5	111.4	113.1	5/16	7.7	2.25	31.0	31.6	34.9	35.6
11/16	21.8	6.40	91.4	92.8	102.8	104.3	4 x 3 x 9/16	12.4	3.62	49.0	50.0	55.0	56.3
⅝	20.0	5.86	83.7	85.0	94.2	95.6	½	11.1	3.25	44.0	45.0	49.5	50.6
9/16	18.1	5.31	76.0	77.1	85.4	86.7	7/16	9.8	2.87	38.9	39.8	43.8	44.8
½	16.2	4.75	68.0	69.0	76.5	77.6	⅜	8.5	2.48	33.6	34.4	37.9	38.7
7/16	14.3	4.18	59.8	60.8	67.3	68.3	5/16	7.2	2.09	28.5	29.1	32.0	32.7
⅜	12.3	3.61	51.7	52.5	58.2	59.1	3½ x 3½ x 9/16	12.4	3.62	49.0	50 1	55.0	56.3
6 x 3½ x ⅞	26.7	7.55	106.8	108.5	120.1	122.1	½	11.1	3.25	44.0	45.0	49.5	50.6
13/16	24.0	7.06	99.9	101.6	112.4	114.3	7/16	9.8	2.87	38.9	39.8	43.8	44.8
¾	22.4	6.56	93.0	94.5	104.6	106.3	⅜	8.5	2.48	33.6	34.4	37.9	38.7
11/16	20.6	6.06	85.9	87.3	96.7	98.2	5/16	7.2	2.09	28.5	29.1	32.0	32.7
⅝	18.9	5.55	78.8	80.0	88.6	90.0	3½ x 2½ x ½	9.4	2.75	36.0	37.0	40.5	41.6
9/16	17.1	5.03	71.5	72.6	80.4	81.7	7/16	8.3	2.43	31.8	32.8	35.4	36.8
½	15 3	4.50	64.0	65.0	72.0	73.1	⅜	7.2	2.11	27.7	28.5	31.2	32.1
7/16	13.5	3.97	56.5	57.4	63.6	64.6	5/16	6.1	1.78	23.5	24.2	26.4	27.1
⅜	11.7	3.42	48.7	49.4	54.8	55.6	¼	4.9	1.44	19.0	19.5	21.4	22.0
5 x 3½ x 11/16	18.3	5.37	75.2	76.3	84.3	85.8	3 x 3 x 7/16	8.3	2.43	31.8	32.8	35.4	36.8
⅝	16.8	4.92	68.6	69.9	77.3	78.7	⅜	7.2	2.11	27.7	28.5	31.2	32.1
9/16	15.2	4.47	62.6	63.7	70.3	71.6	5/16	6.1	1.78	23.5	24.2	26.4	27.1
½	13.6	4.00	56.0	57.0	63.0	64.1	¼	4.9	1.44	19.0	19.5	21.4	22.0
7/16	12.0	3.53	49.4	50.4	55.6	56.6	3 x 2½ x 7/16	7.6	2.22	28.3	29.3	31.9	32.9
⅜	10.4	3.05	42.7	43.5	48.1	49.0	⅜	6.6	1.92	24.6	25.4	27.7	28.6
5/16	8.7	2.56	36.0	36.6	40.4	41.1	5/16	5.6	1.62	21.0	21.6	23.6	24.2
5 x 3 x ⅝	15.7	4.61	63.7	65.0	71.7	73.1	¼	4.5	1.31	17.0	17.4	19.1	19.6
9/16	14.3	4.18	57.9	59.0	65.1	66.4	2½ x 2½ x ⅜	5.9	1.73		22.4		25.2
½	12.8	3.75	52.0	53.0	58.5	59.6	5/16	5.0	1.47		19.2		21.5
7/16	11.3	3.31	45.9	46.9	51.7	52.7	¼	4.1	1.19		15.5		17.5
⅜	9.8	2.86	39.7	40.5	44.7	45.6	2½ x 2 x 5/16	4.5	1.31		16.6		18.7
5/16	8.2	2.40	33.4	34.1	37.6	38.3	¼	3.62	1.06		13.4		15.1

* For single angle tension members allow 80% of tabular values.

The punching of structural steel injures, to some extent, the metal immediately surrounding the hole. Rivet-holes are punched 1/16 in larger than the nominal diameter of the rivet to allow the hot rivet to be entered. In computing the NET AREA of a tension-member, the diameter of the rivet-hole is

considered to be ⅛ in larger than the nominal diameter of the rivet, thereby making an allowance for the injury to the metal due to punching. The AREA to be deducted for one hole is, therefore (d + ⅛ in)t, where d = diameter of rivet and t = thickness of metal. For countersunk rivets the diameter of the holes is usually taken as ¼ in greater than the nominal diameter of the rivet when the thickness of the metal is ⅝ in or less.

Rivet-holes in angles can usually be arranged so that only one hole need be deducted when two or three gauge-lines are used, or so that only two holes need be deducted when three or four gauge-lines are used. The pitch of staggered rivets, Table XXXI, will provide for equal net areas on sections A–B and A–C–D–E.

The TOTAL ALLOWABLE TENSION VALUES for one angle, assuming that angle to be one of a pair, with one rivet-hole deducted, and for allowable working unit stresses of 16 000 and 18 000 lb per sq in of net cross-section, are given in Table XXXII.

For tension-members composed of single angles the values given in Table XXXII should be reduced 20%.

Design of Compression-Members. A compression-member of a truss is subjected, primarily, to stresses in the direction of its longitudinal axis which tend to shorten the member. When the member is short and possesses a relatively high lateral strength the fiber-stresses are closely proportional to the load, but, for a given cross-section, as the length increases the maximum fiber-stresses increase at a higher rate than the rate of increase of load. This phenomenon is referred to as COLUMN ACTION.

No exact theoretical formula has yet been found which will give the STRENGTH of compression-members such as are employed in structures. The RESISTANCE of a compression-member to the axial compression, together with the bending induced by this compression, depends upon the area of the member, the disposition of the area in section, and the length of the member. The MOMENT OF INERTIA of the cross-section is, therefore, one of the important factors in calculating the strength of compression-members. For the purposes of computation, however, it is much more convenient to use the RADIUS OF GYRATION as a measure of the moment of inertia.

The ratio L/r, where L = the unsupported length of the compression-member in inches and where r = the least radius of gyration in inches, is called the slenderness-ratio. The unsupported length is determined by the distance between such points of support as will prevent deflection in the direction corresponding to the particular radius of gyration under consideration. In modern practical design the two following general types of column formulas are employed:

$$(1)\quad \frac{P}{A} = \frac{f}{1 + \dfrac{l^2}{\alpha r^2}} \qquad \text{(Gordon-Rankine)}$$

$$(2)\quad \frac{P}{A} = f - c\frac{L}{r} \qquad \text{(Straight Line)}$$

in which P = total axial compression;
A = area of cross-section;
f = maximum allowable intensity of compression on short blocks;
L = length;
r = least radius of gyration;
α and c are constants determined by experiment.

The preceding column formulas expressed in numerical values that represent modern specifications are as follows:

$$\frac{P}{A} = \frac{18\,000}{1 + \dfrac{L^2}{18\,000\,r^2}} \tag{24}$$

$$\frac{P}{A} = 19\,000 - 100\,\frac{L}{r} \tag{25}$$

$$\frac{P}{A} = 16\,000 - 70\,\frac{L}{r} \tag{26}$$

in which P = total axial compression;
A = the area of the member in square inches;
L = the length of the member in inches;
r = the least radius of gyration of the cross-section in inches.

In the design of compression-members the axial compression and the length are usually known. The type of section is then assumed and, by trial, a combination of the two factors A and r is found such that the equation is satisfied.

The diagrams shown in Fig. 27 will materially shorten the cut and try process usually involved in the design of compression-members. The use of the diagrams in Fig. 27 can best be explained by the following example:

Example 1. Required the design of a compression-member for the following data:

L = 8 ft 9 in = 105 in
P = 72 000 (total compression)

Member to be two angles back to back
Gusset-plates 3/8 in
Referring to the properties of two angles, back to back, Chapter X, and simultaneously referring to the upper portion of Fig. 27, it is seen that two angles 5 in × 3 in × 5/16 in (long legs back to back) have a least radius of gyration of 1.22.

Tracing horizontally from L = 105 on the axis of ordinates to intersect the radical line r = 1.22 and projecting vertically to the axis of abscissae L/r = 86, and continuing vertically to the curve for column equation (3), P/A = 12 800. The required area should be $\frac{72\,000}{12\,800} = 5.65$ sq in. The area furnished is 4.8. The next size angles of the series 5 in × 3 in × 3/8 in provide A = 5.7 and a least r = 1.23. Proceeding as before, L/r = 85 and P/A = 12 850. The required area is 5.6 and the angles are therefore satisfactory.

Example 2. Let it be required to design the member of the preceding example, using column equation (2).

It is evident from the diagrams that a larger member will be necessary than in the preceding design. From an inspection of the table of properties of angles back to back, assume two angles 5 in × 3½ in × 3/8 with a value of least r = 1.46, read L/r = 72 and project to intersect the graph for formula (2) at P/A = 11 800. The area required is $\frac{72\,000}{11\,800} = 6.1$, which is the area furnished by the angles selected.

Design of Riveted Joint Details. STRAIN, or distortion of the various parts and elements, is a very important factor in the rational design of riveted

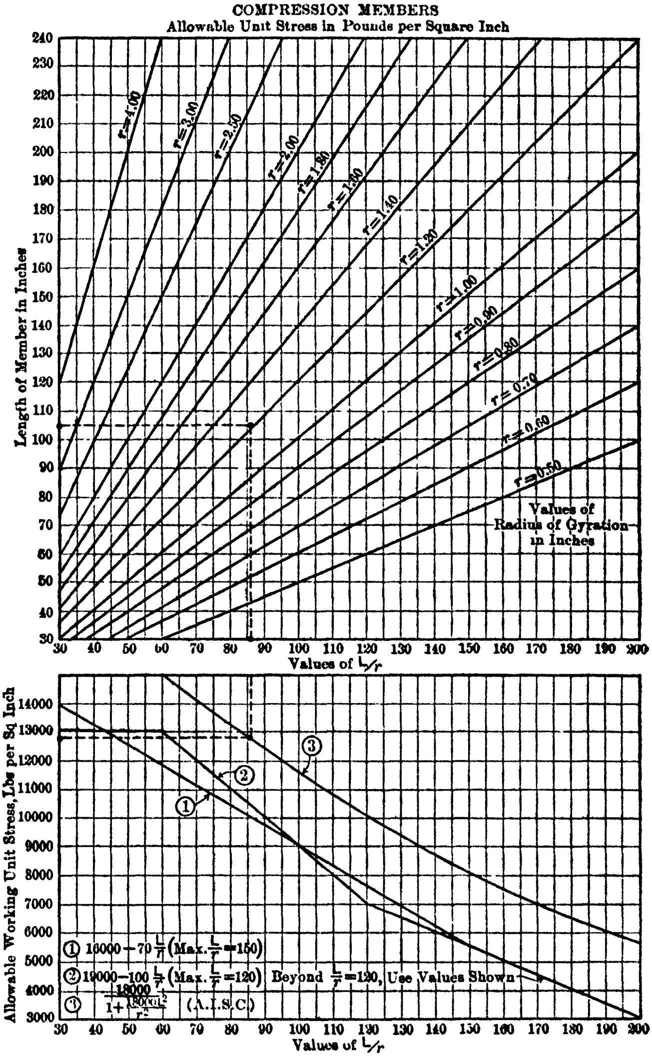


Fig. 27. Compression Members. Allowable Unit Stress per Square Inch

joints. An exact analysis, if such were possible, would be too complex for practical design.

The following assumptions are, therefore, generally made in the design of riveted joints:

(1) That the direct stresses transmitted by any member are equally proportioned to the rivets connecting that member.

(2) That the friction resulting from the clamping action of the rivets is neglected in calculating the resistance of rivets.

(3) The bending stresses in the rivets are neglected when the rivet grip is less than five times the diameter of the rivet.

(4) That each rivet completely fills the hole into which it is entered.

(5) That rivets in compression-members replace the metal lost in punching and that no deduction need be made from the gross area of the member.

(6) That the net section of a tension-member is capable of offering the same unit resistance as the gross section befoie punching.

(7) That stresses are uniformly distributed over the net sections of tension-members and the gross sections of compression-members.

Consistency in the design of riveted joints based on the preceding assumptions requires that all details should be arranged and proportioned to provide, as far as possible, for a uniform distribution of stress in the members and the various elements of the joint. Uniform stress in the main members requires that the gravity axis of all members at the joint shall meet in a common point.

UNIFORMITY OF STRESS in the rivets and connecting plates is difficult, and often impossible, to secure; however, the nearer it is approached the more permanently elastic will be the structure. The number of rivets required for the connection of any given member depends upon the stress transmitted to the joint by that member.

When a member ends at a joint the entire stress in the member must be transferred by the rivets connecting the member to the joint detail. When a member is continuous through a joint the stress transferred by that member is the difference in the stresses on each side of the joint.

Eccentric Connections in Riveted Joints. ECCENTRICITY in riveted joints may result from either or both of the following causes:

(1) When the action line of the resultant stress in any member, at the joint, does not pass through the center of gravity of the rivets connecting that member.

(2) When the gravity axes of all the members joined do not intersect at a common point.

Any moment due to eccentricity in a joint detail will be taken partly by each member and by the rivets in each member. The most rigid member and its connecting rivets will resist the most moment.

The DISTRIBUTION OF THE MOMENT to the various members involved will depend upon their relative stiffness, which may be determined as follows:

(1) The deflection of any member subjected to bending under the action of a load W, is:

$$\Delta = \frac{WL^3}{\alpha EI} \quad \text{or} \quad W = \frac{\alpha \Delta EI}{L^3} \tag{a}$$

in which α = a constant depending upon the manner in which the member is supported and the type of loading;

E = modulus of elasticity of material in inches²;

I = moment of inertia of the cross-section of the member referred to the axis about which bending takes place in inches⁴;

L = length of the member in inches;

Δ = deflection of neutral axis in inches.

(2) The general expression for the bending moment, using the preceding notation, is:

$$M = \frac{WL}{\beta} \quad \text{or} \quad W = \frac{\beta M}{L} \tag{b}$$

in which β = a constant depending upon the manner in which the member is supported and the type of loading.

Equating the values of W as given by Equations (a) and (b) and solving for a simultaneous value of M;

$$M = \frac{\alpha}{\beta} \cdot \frac{EI\Delta}{L^2} \tag{c}$$

The angular displacement of each member, due to the rotational tendency, is $\frac{\Delta}{L/2}$ and is equal for all members; hence, α, B E, and $\frac{\Delta}{L/2}$ are constants for each member when the physical characteristics and methods of connection are the same for all members. The moment resisted by any member, therefore, varies directly as the moment of inertia of that member with respect to the axis about which bending takes place, and inversely as the length of the member; then if

$\Sigma\frac{I}{L}$ = the sum of the stiffness factors of all members involved at the joint;

and $\frac{I_x}{L_x}$ = the stiffness factor of member x;

$\frac{I_x/L_x}{\Sigma I/L} \cdot M$ = the moment to be assigned to member x, where M represents the total moment at the joint.

The rivets connecting any member to the gusset-plate should be proportioned to transmit safely the moment stress assigned to that member in addition to the direct or axial stress. The moment stress on any rivet connecting any member varies directly as its distance from the center of gravity of the group of rivets. The moment stress on the outermost rivet of the group is determined as follows:

$$k_0 = \frac{M r_0}{\Sigma r^2} \tag{d}$$

in which k_0 = the moment stress on the outermost rivet;
r_0 = the distance of the outermost rivet from the center of gravity of the group;
Σr^2 = the sum of the squares of the distance of each rivet in the group from the center of gravity of the group;
M = the moment resisted by the group of rivets.

Example 1. The standard gauge-lines for structural angles do not coincide with the gravity axis of the angle; consequently some eccentricity exists in the connection of each of the members of a detail such as shown in Fig. 28. The gravity axis of member 3–4 is at 0.91 in from the back of the outstanding legs. The standard gauge-line distance is 1.75 in, and under the assumption that the axial stress is uniformly distributed over the cross-section of the member, the eccentricity is 1.75 − 0.91 = 0.84 in.

The moment due to this eccentricity is:

$$M = 21\,200 \times 0.84 = 17\,800 \text{ lb-in}$$

In a detail such as shown it may be assumed that the rivets develop the resistance to the moment and that the stress in the member at section AA, and beyond, is uniformly distributed over the cross-section.

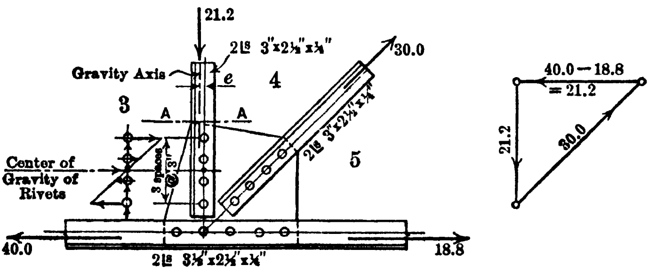


Fig. 28. Detail of Joint

The stress on the outermost rivets of the group connecting member 3–4 to the gusset-plate may then be determined as follows:

$$\Sigma r^2 = 2(\overline{1.5}^2 + \overline{4.5}^2) = 45 \text{ in}^2$$

$$r_o = 4.5 \text{ in}$$

whence
$$k_o = \frac{17\,800 \times 4\,5}{45} = 1\,780 \text{ lb}$$

The stress on each rivet of the group due to the direct compression in the member is:

$$k_1 = \frac{21\,200}{4} = 5\,300 \text{ lb}$$

The combined moment and direct stress on the outermost rivets of the group (top and bottom rivets) is:

$$k = \sqrt{(1\,780)^2 + (5\,300)^2} = 5\,600 \text{ lb}$$

Example 2. When the action lines of the stresses of all members at a given joint do not intersect in a common point the resulting moment must be resisted by each member and its connections.

Two common cases of such eccentricity found in roof-truss details are illustrated in Fig. 29(a) and (b). Let it be required to design the end joint detail shown at (a), Fig. 29. The moment may be calculated by combining any two of the non-concurrent forces and finding the magnitude of the resultant couple:

$$M = 20\,000 \times 12 = 40\,000 \times 6 = 34\,640 \times 6.93$$
$$= 240\,000 \text{ lb-in}$$

The lengths of the members, measured center to center of gauge-line intersections, are:

member a–1 = 8 ft 4 in = 100 in
member 1–x = 9 ft 8 in = 116 in

Since it is necessary to know the sections of the members before the moment can be distributed a tentative design must be made.

Assume the following tentative distribution of moments:

$$\text{To member } a\text{–}1;\ 60\%\cdot M = 144\,000 \text{ lb-in}$$
$$\text{To member } 1\text{–}x:\ 40\%\cdot M = \underline{96\,000} \text{ lb-in}$$
$$\text{Total} = 240\,000 \text{ lb-in}$$

For member a–1, try two angles 6 × 4 × 5/8 in, with the long legs back to back:

$$A = 11.72 \text{ sq in}$$
$$S = 10.62 \text{ in}^3$$
$$\text{Least } r = 1.67 \text{ in and } L/r = 60$$
$$I = 42.14 \text{ in}^4$$

Then
$$f_{max} = \frac{40\,000}{11\,72} + \frac{144\,000}{10\,62}$$
$$= 3\,420 + 13\,600 = 17\,020 \text{ lb per sq in}$$

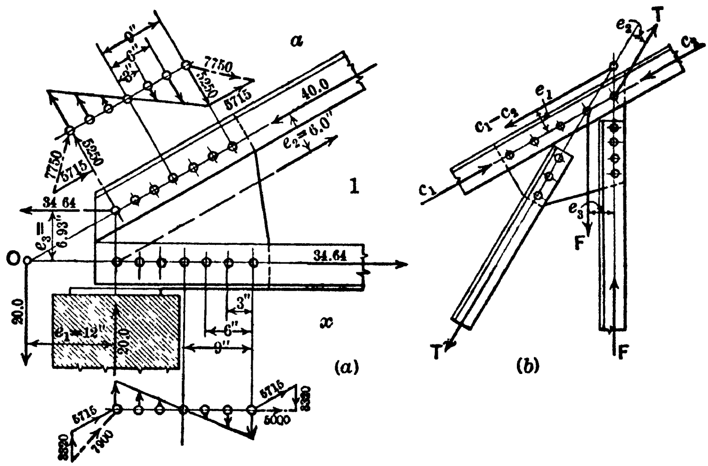


Fig. 29. Details of Joints

The allowable unit stress (since for $L/r = 60$ the allowable unit stress for direct compression is 15 000 lb per sq in) is:

$$\frac{342}{1\,702}(15\,000) + \frac{1\,360}{1\,702}(18\,000) = 17\,400 \text{ lb per sq in}$$

For the tentative design of member 1–x, try two angles 6 × 4 × 7/16 in with the long legs back to back:

$$A \text{ (net for one } 3/4\text{-in rivet)} = 7.59 \text{ sq in}$$
$$S = 7.66 \text{ in}^3$$
$$I = 30.92 \text{ in}^4$$

Then
$$f_{max} = \frac{34\,640}{7.59} + \frac{96\,000}{7.66}$$
$$= 4\,560 + 12\,500 = 17\,060 \text{ lb per sq in}$$

The allowable unit stress is 18 000 lb per sq in. The stiffness factors for the members as tentatively designed are:

$$\text{member } a\text{–}1 = \frac{42.14}{100} = 0.4\,214$$

$$\text{member } 1\text{–}x = \frac{30.92}{116} = 0.2\,670$$

$$\text{Total} = 0.6\,884$$

The distribution of moments is:

$$M \text{ member } a\text{–}1 = \frac{4\,214}{6\,884} \times 240\,000 = 147\,000 \text{ lb-in}$$

$$M \text{ member } 1\text{–}x = \frac{2\,670}{6\,884} \times 240\,000 = 93\,000 \text{ lb-in}$$

$$\text{Total} = 240\,000 \text{ lb-in}$$

The assumed distribution of moments and the distribution resulting from the tentative design are sufficiently close to warrant a final check on the stresses in the members as tentatively selected.

The stresses in the members as selected are as follows:

The combined unit stress, member a–1, is:

$$f_{max} = \frac{40\,000}{11.72} + \frac{147\,000}{10.62}$$

$$= 3\,420 + 13\,850 = 17\,270 \text{ lb per sq in}$$

(The allowable unit stress, determined as before, is 17 480 lb per sq in.)

The combined unit stress, member 1–x, is:

$$f_{max} = \frac{34\,640}{7.59} + \frac{93\,000}{7.66}$$

$$= 4\,560 + 12\,100 = 16\,660 \text{ lb per sq in}$$

(The allowable unit stress is 18 000 lb per sq in.)

The above results are satisfactory, and it should be remembered that any revision in the design of either member will affect the distribution of the moment stress in both members.

The maximum combined stress on the rivets connecting each member to the gusset-plate must be investigated.

The number of rivets required must be estimated and the resulting stresses calculated. The number or the spacing or both may then be revised and the stresses rechecked until a satisfactory result is obtained.

For member a–1, try seven ¾-in rivets spaced at 3 in on centers, as shown in Fig. 29(a). The moment to be resisted by these rivets is the moment assigned to member a–1, or 147 000 lb-in:

$$\Sigma r^2 = 2(3^2 + 6^2 + 9^2) = 252 \text{ in}^2$$

$$r_o = 9 \text{ in}$$

Then

$$k_o = \frac{147\,000 \times 9}{252} = 5\,250 \text{ lb}$$

The stress on each rivet from the direct compression in the member is:

$$k_1 = \frac{40\,000}{7} = 5\,715 \text{ lb}$$

The combined stress on the two end rivets of the group is:

$$k = \sqrt{(5\,250)^2 + (5\,715)^2} = 7\,750 \text{ lb}$$

For member 1–x, assume seven ¾-in rivets spaced at 3 in on centers, then:

$$\Sigma r^2 = 252 \text{ in}^2 \text{ and } r_0 = 9 \text{ in}$$

as before.

The moment stress on each end rivet of the group is:

$$k_o = \frac{93\,000 \times 9}{252} = 3\,320 \text{ lb}$$

Each rivet of this group must provide a resistance to the tendency of the lower chord to move upward due to the fact that the reaction is transmitted directly to the outstanding legs of the lower chord member. The direct stress to be resisted is, therefore, the resultant of the tension, 34 640 lb, and the vertical reaction, 20 000 lb, or 40 000. The resultant of any two of the three forces being equal and opposite to the third force, then:

$$k_1 = \frac{40\,000}{7} = 5\,715 \text{ lb}$$

Since the moment stress and the direct stress are not at right-angles to each other the easiest method of finding the resultant stresses is by means of a force-polygon as shown in Fig. 29(a). The maximum combined stress is found to be 7 900 lb as shown.

Clip-Angle Connections. Angles connected by one leg only are subjected to bending due to the fact that their gravity axis does not coincide with the line of action of the stress in the connecting rivets.

Tests * of angles connected by one leg show results of 75% to 80% of the full value of the net cross-section. This is increased 5% to 10% by the use of CLIP-ANGLES providing a connection for both legs as illustrated in Fig. 30(a).

Approximately the same results were found for two angles connected as shown at (b), Fig. 30.

Specifications usually require that angles connected on one side of a gusset-plate as at (a) and (b), Fig. 30, be designed for an efficiency of 125% on the basis of their net areas in order to provide for the increase in stress due to eccentricity.

End Joint Details. The usual methods employed for end joint details of steel roof-trusses are shown in Fig. 31.

When the trusses are supported on masonry walls provision must be made for EXPANSION and CONTRACTION due to temperature changes, and for spans of more than 40 to 60 ft certain types of trusses require a provision for LATERAL DISPLACEMENT due to deformation. (See Chapter XXV.)

For spans up to 80 ft expansion details are usually of the type shown at (a), Fig. 32.

* Engineering News, Vol. 56, page 14.

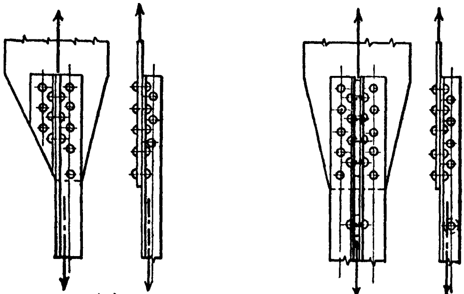
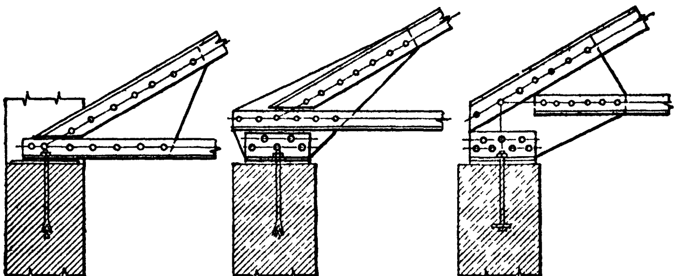
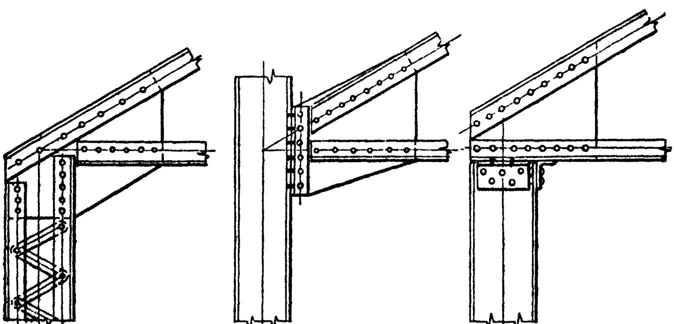


Fig. 30. Clip-angle Connections



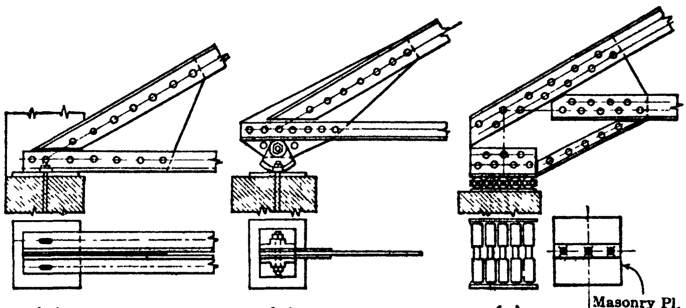
Typical End Joint Details for Trusses Supported on Masonry Walls



Typical End Joint Details for Trusses Supported on Columns

Fig. 31. End Joint Details

Slotted holes are provided in the member of the truss which delivers the reaction to the bearing-plate, and the ANCHOR-BOLTS extend upward through the slotted holes. The nuts on the ends of the anchor-bolts hold the truss in position while the slotted holes allow the truss a more or less free longitudinal movement. The slotted holes should provide for a movement of $\frac{1}{8}$ in for every 10 ft of span. The full provision should be made on each side of the center line of the anchor-bolt. Anchor-bolts should never be less than $\frac{5}{8}$ in in diameter and should be bedded in the masonry at least 8 in.



(a) (b) (c)

Fig. 32. Details of Joints for Expansion

When the span of the truss exceeds 80 ft, and when the roof-loads are relatively large, a rocker bearing, Fig. 32(*b*), or a roller bearing, Fig. 32(*c*), may be used.

The vertical webs of cast-iron ROCKERS are designed for an allowable unit stress of $9\,000 - 40\,L/r$, where L = the height and r = the least radius of gyration, of the vertical web. The minimum thickness of any part should be $\frac{5}{8}$ in. The allowable unit bearing for cast-iron rockers is $300\,d$ per lin in, where d = the diameter of the curved contact surface in inches.

ROLLERS for roof-truss bearing are solid steel cylinders and should not be less than $2\frac{1}{2}$ in in diameter. The allowable unit bearing is $600\,d$ per lin in of roller, where d = the diameter of rollers in inches.

Bearing Details. The average allowable working unit stress in bearing on masonry walls and piers in pounds per square inch, is as follows:

Granite masonry, Portland-cement mortar	500 to 600
Concrete, 2 000 lb per sq in Ult	450
Concrete, 2 500 lb per sq in Ult	560
Concrete, 3 000 lb per sq in Ult	675
Limestone masonry, Portland-cement mortar	400 to 500
Sandstone masonry, Portland-cement mortar	300 to 400
Hard-burned brick, Portland-cement mortar	250 to 300
Medium-burned brick, Portland-cement mortar	175 to 200

The distribution of pressure under bearing-plates in details as shown in Figs. 31(*a*), (*b*), (*c*), and 32 is an indeterminate function of the stiffness of the members delivering the truss reaction to the bearing-plate, the stiffness of the bearing-plate and the relative elasticity of superimposed detail and the masonry upon which it bears.

The working unit values given above are allowable average values and the size of the bearing-plate is determined by dividing the total vertical reaction by the allowable unit stress.

It must be realized that such a procedure is merely an assumption and that the projection of shoe or flange angles as well as the projections of the bearing-plate must be kept within certain limits if excessive bearing stresses are to be avoided.

The distribution of pressure over the bearing legs of the shoe or flange angles varies from a maximum at the center of the detail to a minimum toward the outer ends of the angle legs. The variation probably follows somewhat that which is indicated in Fig. 33(a).

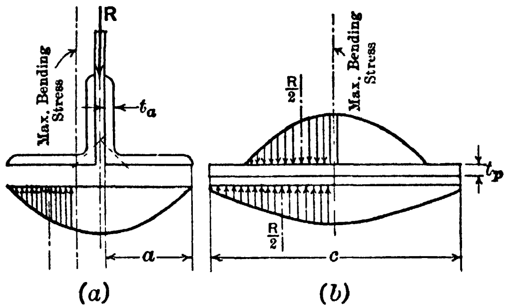


Fig. 33. Distribution of Pressure

The moment which produces maximum bending stresses in the bearing-leg of the shoe or chord angle may be taken as equivalent to two-thirds of the load assigned to the outstanding leg multiplied by one-fourth of the length of that leg, or:

$$M_a = \frac{R \cdot a}{12} \tag{28}$$

and the required thickness of the angle leg is:

$$t_a = \sqrt{\frac{1}{2} \cdot \frac{Ra}{f \cdot b}} \tag{29}$$

in which R = the maximum vertical reaction;
f = allowable unit stress in flexure;
a = width of projecting leg of shoe or flange angles;
b = length of projecting leg of shoe or flange angles in bearing perpendicular to the wall.

Following the same general assumptions, and using the same moment on the angle leg as in (28), the maximum bending moment in the bearing-plate, Fig. 33(b), is:

$$M_p = \frac{R}{12}(c-a) \tag{30}$$

and the required thickness of the bearing-plate is:

$$t_p = \sqrt{\frac{1}{2} \cdot \frac{R}{f} \frac{(c-a)}{b}} \tag{31}$$

in which c = the width of the bearing-plate parallel to the wall;
b = the width of the bearing-plate perpendicular to the wall;
R, f and a are as in Equation (29).

Example. Let it be required to design the bearing detail, shown in Fig. 31(*b*), for the following given data:

Shoe angles, 6 in × 4 in (long legs back to back).

Maximum vertical reaction	= 38 000 lb
Allowable average unit bearing on masonry	= 200 lb per sq in
Allowable extreme fiber-stress in bending	= 18 000 lb per sq in

The required area of the bearing-plate is $\frac{38\,000}{200} = 190$ sq in, and will be made 14 in × 14 in = 196 sq in. The required thickness of the shoe angle, Equation (29), is:

$$t_a = \sqrt{\frac{1}{2} \cdot \frac{38\,000}{18\,000} \times \frac{4}{14}} = 0.55 \text{ in}$$

and will be made $\frac{9}{16}$ in.

The required thickness of the bearing-plate, Equation (31), is:

$$t_p = \sqrt{\frac{1}{2} \cdot \frac{38\,000}{18\,000} \times \frac{14-4}{14}} = 0.866 \text{ in}$$

and will be made $\frac{7}{8}$ in.

If 6 × 6-in angles were to be used instead of 6 × 4-in angles, the thickness of the angles would be $\frac{11}{16}$ in and the thickness of the bearing-plate would be $\frac{13}{16}$ in.

9. Detailed Design of Steel Roof-Construction

General. An example of the complete design of the steel roof-construction for a building, beginning with the layout from the architectural drawings, will be presented in the following articles.

The roof-plan, showing the location of pilasters and a section with the established roof and ceiling lines, is shown in Fig. 34.

The specified roof-construction is a slate weathering-surface laid over a properly prepared base as required for slopes of less than 5 in 12. The structural covering will be a NAILING CONCRETE slab. The concrete will envelop the sides of the purlins and the thickness on each side of the web will equal the flange projection.

The roof is to be designed for an equivalent uniform live load of 30 lb per sq ft of roof-surface, and this live load may cover the entire roof or only one-half of the roof. The masonry bearing-plates will have their outer edges 4 in from the outer face of the supporting piers in order to provide ample protection from the weather.

A preliminary investigation of the roof slab thickness, purlin size and ceiling-construction, indicates that about 2 ft should be deducted from the vertical dimension, as established by the architectural section, in fixing the vertical dimension between gauge-lines of the chord members at the center of span.

The span of the trusses, center to center of bearing, will depend upon the required size of the bearing-plate, which a preliminary calculation indicates to be about 16 in × 16 in. The span will, therefore, be taken as 68 ft 0 in

center to center of bearing, as shown, and the working space for the truss layout is as shown on the architectural section, Fig. 34(*b*).

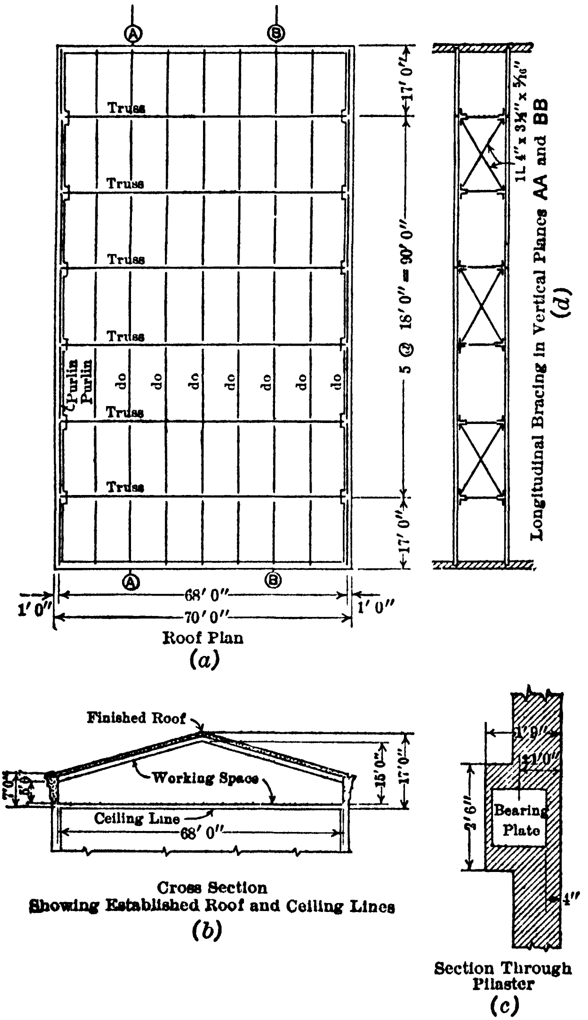


Fig. 34

Working Unit Stresses. The following working unit stresses will be adopted:

I. Nailing concrete	
(1) Flexure	300 lb per sq in
(2) Shear (diagonal tension)	20 lb per sq in
II. Steel	
(1) Flexure	18 000 lb per sq in
(2) Tension (net section)	18 000 lb per sq in
(3) Compression (column members)	$\dfrac{18\,000}{1 + \dfrac{L_2}{18\,000\, r^2}}$
(a) Maximum compression	15 000 lb per sq in
(b) Maximum L/r (principals)	120
(c) Maximum L/r (bracing)	200
(4) Shear	
(a) On power-driven rivets	13 500 lb per sq in
(b) On hand-driven rivets	10 000 lb per sq in
(5) Bearing	
(a) On power-driven rivets	
Single	30 000 lb per sq in
Double	24 000 lb per sq in
(b) On hand-driven rivets	
Single	20 000 lb per sq in
Double	16 000 lb per sq in
III. Masonry	
(1) Average bearing	250 lb per sq in

All rivets are to be ¾ in with 13/16-in open holes. The smallest allowable angle is 2½ in × 2 in × ¼ in. Gusset-plates, in so far as is practical, are to be ⅜ in in thickness.

Design of Structural Covering. The slope of the upper chord is 19/64 or 16° 24′ with the horizontal.

$$\text{Sine } A = 0.2823$$
$$\text{Cos } A = 0\ 9593$$
$$\text{Sec } A = 1.0424$$

In accordance with the general considerations outlined in Article 3, the truss will be divided into eight panels. The lower chord panel length will be 68/8 = 8 ft 6 in, and the upper chord panel length will be 8.5 × 1.0424 = 8.860 or 8 ft 10 5/16 in. The spacing of trusses is fixed by the location of the pilasters and is as shown in Fig. 34(*a*).

The unit loading for the design of the roof slab, in pounds per square foot of roof-surface, is:

Dead load:

(1) Slate and base	12.0
(2) Slab (assumed)	30.0
	42.0
Live load	30.0
Total	72.0

From Equation (8), Article 4, the required thickness of the slab is:

$$t\sqrt{\frac{300 \times 1.042}{72}} = 8.86$$

or

$$t = 4.26 \text{ in}$$

If the slab is made $4\frac{1}{2}$ in, the weight of the slab per square foot of roof-surface is 33 lb, the total design load is $72 + 3 = 75$ lb per sq ft of roof-surface, and the required thickness of the slab is 4.33 in.

The slab will be made $4\frac{1}{2}$ in and the weight of purlins plus the covering around the purlins will be assumed as equal to an additional 5 lb per sq ft of roof-surface.

The total design load for purlins will, therefore, be taken as 80 lb per sq ft of roof-surface.

Design of Purlins. The uniformly distributed vertical load on each typical purlin is:

$$8.86 \times 18 \times 80 = 12\,750 \text{ lb}$$

and

$$M_v = \frac{12\,750}{8} \times 18 \times 12 = 344\,300 \text{ lb-in}$$

The stress coefficient is:

$$\frac{18\,000}{344\,300} = 0.0523$$

There is no value in Table XVII which corresponds exactly with this stress coefficient for the given roof-slope. The tabular values do indicate, however, that a 10 in–C–21.0 lb is the most likely section.

Assume this section, then, from Equation (13), Article 4:

$$f_{\max} = 344\,300\left(\frac{0.959}{21.73} + \frac{8}{90}\cdot\frac{0.282}{4.0}\right)$$

$$= 344\,300\ (0.0506) = 17\,400 \text{ lb per sq in}$$

An investigation of other available sections shows that the section selected is the most economical; however, if it were necessary to conserve space an 8 in–C–24.0 (Table XVII) could be used and $f_{\max} = 344\,300\ (0.0499) = 17\,200$ lb per sq in. The 10-in section is lighter and provides more rigidity in the plane of the roof and will be used throughout.

The average weight of purlins plus covering is approximately 5 lb per sq ft of roof-surface, as assumed.

Design of Ceiling-Beams. Ceiling-beams will be supported at each lower chord panel-point, their span being equal to the truss spacing. Furring-angles will support the metal lath and plaster over the span of 8 ft 6 in between ceiling-beams. The unit load of the ceiling construction is:

Metal lath and plaster.	10 lb per sq ft
Ceiling beams and furring.................	2 lb per sq ft
Total	12 lb per sq ft

The uniformly distributed load on each ceiling beam is:

$$8.5 \times 18 \times 12 = 1\,800$$

and

$$M = \frac{1\,800}{8} \times 18 \times 12 = 48\,600 \text{ lb-in}$$

Assume a 6 in–I–12.5 lb (section-modulus $= 7.27$ in^3), then:

$$f = \frac{48\,600}{7.27} = 6\,700 \text{ lb per sq in}$$

If the ceiling-beams are laterally stayed at their mid-points of span, $\frac{L}{b} = 33.4$ and from Table IV, Chapter XVIII, the allowable unit stress is $0.74 \times 18\,000 = 13\,300$ lb per sq in.

The deflection coefficient for an 18-ft span, Table II, Chapter XVIII, is 6.033, for $f = 18\,000$ lb per sq in, whence:

$$\Delta = \frac{6\,700}{18\,000} \times \frac{6.033}{6} = 0.375 \text{ in}$$

The maximum allowable deflection, 1/360 of the span, is 0.600 in.

Since the stiffness of the construction in the plane of the lower chord will be largely dependent upon the ceiling-beams it is not advisable to use a smaller section, and the beam assumed will be used throughout.

Estimated Weight of Trusses. The loading for one typical truss is:

Upper chord:

$$8\,(8.86 \times 18)\,80 = 102\,000 \text{ lb}$$

Lower chord:

$$7\,(1\,800) = 12\,600 \text{ lb}$$

$$\text{Total} \quad 114\,600 \text{ lb}$$

From Equation (2), Article 1, the estimated weight of the truss is:

$$W = \frac{114\,600}{110}\left(1 + \frac{68}{30} + \frac{68}{5\sqrt{18}}\right)$$
$$= 6\,600 \text{ lb}$$

The following distribution of the weight of the truss to the various panel-points will be made:

Add 450 lbs to each upper chord panel-point.
Add 400 lb to each lower chord panel-point.

The total panel-loads for design are:

Upper chord panel-loads $= 12\,750 + 450 = 13\,200$ lb.
Lower chord panel-loads $= 1\,800 + 400 = 2\,200$ lb.

Calculations of Truss Stresses. The outline of a typical truss as fixed by the established dimensions of the architectural section, is shown at (*a*), Fig. 35.

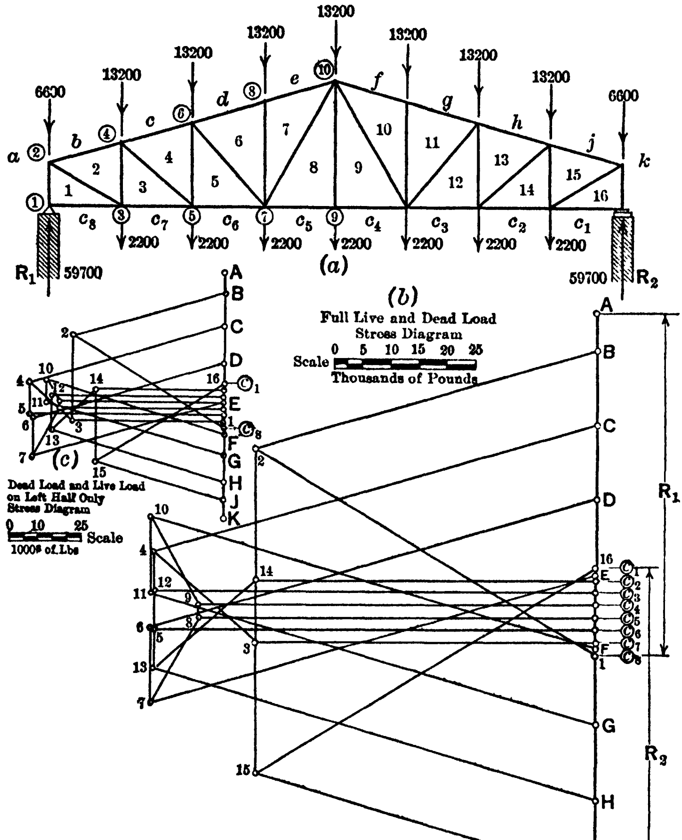
Various combinations of the web-members are possible; however, the arrangement shown was selected, after a preliminary study of different arrangements, as that which will give the best and most economical results.

The stress-diagram for the full live and dead load is shown at (*b*), Fig. 35. It should be noted that the stresses in members 5–6 and 11–12 are very small for the maximum load, being less than 1 000 (tension) each.

It might be possible, under partial loading, for the character of stress in these members to be reversed.

When an equivalent live load is used in design, it is advisable to construct a stress-diagram assuming the live load on one side of the roof only. This condition of loading might occur frequently either with maximum snow, or with ice and sleet plus wind-loads.

A stress-diagram for full dead load on both upper and lower chord panels and with live load on the left half of the truss is shown at (*c*), Fig. 35.



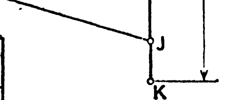


Table XXXIII. Design Stresses and Sections

Member	Design Stress 1000 Lbs.	Length Ft. Ins.	Section	Member	Design Stress 1000 Lbs.	Length Ft. Ins.	Section
a-1	−60.50	5-0	2 L^s 4 × 3 × $\frac{5}{16}$	1-2	+70.50	9-10$\frac{11}{32}$	2 L^s 4 × 3$\frac{1}{2}$ × $\frac{5}{16}$
b-2	−62.80	8-10$\frac{5}{16}$	2 L^s 5 × 3$\frac{1}{2}$ × $\frac{3}{8}$	2-3	−34 00	7-6	2 L^s 3$\frac{1}{2}$ × 2$\frac{1}{2}$ × $\frac{1}{4}$
c-4	−81.50	do	do	3-4	+24 00	11-4$\frac{1}{32}$	2 L^s 2$\frac{1}{2}$ × 2 × $\frac{1}{4}$
d-6	−82.00	do	do	4-5	−13.80	10-0	2 L^s 3$\frac{1}{2}$ × 2$\frac{1}{2}$ × $\frac{1}{4}$
e-7	−82 00	do	do	5-6	− 2.00*	13-1$\frac{1}{8}$	2 L^s 4 × 3 × $\frac{5}{16}$
c_8-1	0.0	8-6	2 L^s 2$\frac{1}{2}$ × 2 × $\frac{1}{4}$	6-7	−13 20	12-6	2 L^s 4 × 3 × $\frac{5}{16}$
c_7-3	+60.00	do	2 L^s 4 × 3$\frac{1}{2}$ × $\frac{7}{16}$	7-8	+17 70	17-2$\frac{7}{8}$	1 L 3 × 2$\frac{1}{2}$ × $\frac{5}{16}$
c_6-5	+78.20	do	do	8-9	+2.20	15-0	1 L 2$\frac{1}{2}$ × 2 × $\frac{1}{4}$
c_5-8	+70 30	do	2 L^s 4 × 3$\frac{1}{2}$ × $\frac{3}{8}$	* D. + L. L. on Left Half Only			

Fig. 35. Calculation of Stresses in Truss

This diagram shows that, under the loading condition assumed, the stress in member 5–6 is 2 000 (compression). The amount of stress is small; however, the fact that under certain conditions the character of the stress changes from tension to compression is very important.

The design stresses taken from (*a*) and (*b*), Fig. 35, and the lengths of the members calculated from tables of squares, are shown in Table XXXIII.

Design of Tension-Members. The limiting dimensions of a truss which may be riveted up completely in the shop, and delivered as a unit at the building site, depend upon the methods of transportation. When the truss is to be shipped by rail, the vertical dimension, as the piece is loaded, is generally limited to 10 or 12 ft. The facilities for erection may, in some cases, govern the size of a single unit which can be erected.

The trusses for the building under consideration will be shipped in two principal parts, SPLICES being made at joint (10) and at each joint (7).

When chord members are continuous through several panels it is usually more economical to use an unspliced member, designed for the maximum stress. The lower chord will be made continuous from joint (3) to joint (7), and the length between joint (7) left and joint (7) right as well as member 8–9 will be shipped as separate units and their connections will be field-riveted when the truss is assembled. The maximum design stress for the lower chord section C_7–3 and C_6–5 is 78 200 lb (tension). Referring to Table XXXII, the most economical section available is one composed of two $4 \times 3\frac{1}{2} \times \frac{3}{8}$-in angles providing for an allowable tension of $2 \times 42.1 = 84\ 200$ lb, and weighing $2 \times 9.1 = 18.2$ lb per lin ft. If, however, in making the splicing detail at joint (7), it becomes necessary to allow for the deduction of two rivet-holes from the gross area of each angle, it will be necessary to use a larger section. For this case, assume two $4 \times 3\frac{1}{2} \times \frac{7}{16}$-in angles:

Gross area, 2×3.09	$= 6.18$ sq in
Deduct $4 \times (\frac{3}{4} + \frac{1}{8}) \times \frac{7}{16}$	$= 1.53$ sq in
Net area	4.65 sq in

The total allowable tension on the net section is $4.65 \times 18\ 000 = 83\ 700$ lb.

The end section of the lower chord is, in reality, member 1–2, and member C_8–1 might be considered unnecessary since its stress, from the diagrams in Fig. 35, is zero. The member is, however, of considerable value in stiffening the end detail and, moreover, for a case of inclined loading, member C_8–1 will be subjected to a primary stress and that stress may be either tension or compression depending upon the direction of the reaction of the inclined load. The member should, therefore, be designed to fulfill the requirements of compression-member of a bracing system, with a minimum L/r of 200.

The length of the member is 102 in, and the least radius of gyration should, therefore, be $\frac{102}{200} = 0.51$. This requirement may be provided by a single angle $3 \times 2\frac{1}{2} \times \frac{1}{4}$ in, weighing 4.5 lb per lin ft, or by two $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in angles weighing 7.2 lb per lin ft. The two angles are preferable, and in view of their greater stiffening value, the relatively small difference in total weight is more than offset.

The stress in member 7–8 is 17 700 lb (tension) and, from Table XXXII, the allowable value is supplied by a single angle $3 \times 2\frac{1}{2} \times \frac{1}{4}$ in which, with one hole deducted, provides for 19 600 lb. In selecting single angles, however, it must be borne in mind that only 80% of the tabular values may be used, as explained in Article 8.

The value of the angle selected is, therefore, $0.80 \times 19.6 = 15\ 680$ lb.

In selecting single angle members it is convenient to multiply the actual design stress by 1.25 and use the resulting value as a basis for selecting the member.

Thus for member 7–8, an angle having an allowable tabulated value of 17 700 × 1.25 = 22 125 must be selected. The angle required is a 3 × 2½ × 5⁄16 in, with a capacity of 0.80 × 24 200 = 19 600 lb.

The remaining tension-members are selected in the same manner as shown by the preceding examples and the final sections selected are given in Table XXXIII.

Design of Compression-Members. The upper chord on each slope will be made of the same section throughout. This is good practice for it will probably be about as economical as to provide a splice at those joints where the section might be changed and it also results in a more rigid structure.

The length of the upper chord divisions are 106 5⁄16 in. The purlins and the roof-construction will provide ample lateral support for the upper chord, and the unsupported length is, therefore, the length of the upper chord panel. In designing compression chord members for the usual types of roof-construction it will be found that for panel lengths of from 7 to 10 ft, the average allowable unit stress for members composed of two angles will range from 12 000 to 15 000 lb per sq in.

The maximum stress in the upper chord is 82 000 (compression), and assuming f allowable at about 13 500 lb per sq in, the L/r ratio, Fig. 27, is about 75 and for $L = 106$ in r is shown to be about 1.4.

Select two angles 5 × 3½ × ⅜ in with the long legs back to back and, assuming a ⅜-in gusset-plate, the least r, as given by the tables of properties, is 1.46, whence $L/r = 73$, and the allowable unit stress, from Fig. 27, is 13 850 lb per sq in. The area of the two angles selected is 6.10 sq in, and the total allowable compression is 84 480 lb.

The sections of compression web-members are often governed by the slenderness-ratio rather than the stress in the member. The average allowable unit compression values will vary from about 8 000 to 12 000 lb per sq in.

The length of member 2–3 is 7 ft 6 in = 90 in. The design stress (Table XXXIII) is 34 000 lb, and, from Fig. 27, if the allowable unit stress be assumed as 12 000, $L/r = 95$ and $r = 0.95$. Assume two 3½ × 2½ × ¼-in angles. $A = 2.62$ sq in and the least r (⅜-in gusset-plate) is 0.95. From Fig. 27, the allowable unit compression is 12 100 lb per sq in and the total allowable compression for the member selected is:

$$12\,100 \times 2.62 = 31\,700 \text{ lb}$$

The area of the member selected is insufficient, and the next lightest weight of two angles, long legs back to back, which provides a greater area with a slightly greater value of the least radius of gyration, is two 3½ × 2½ × ¼-in angles. $A = 2.88$ and the least $r = 1.09$. Then $L/r = 90/1.09 = 82.5$ and the allowable unit compression is 13 000 lb per sq in. The total allowable compression for the member selected is 2.88 × 13 000 = 37 400 lb. This member provides a greater capacity than required but is the most economical section available.

The length of member 4–5 is 120 in and, hence, the least radius of gyration permissible is 1.0. This is furnished most economically by the section selected for the member (2–3) just designed. In this case $L/r = \dfrac{120}{1.09} = 110$, and from Fig. 27, the allowable unit compression is 10 750 lb per sq in. The capacity of the member is 2.88 × 10 750 = 30 960 lb, while the stress in the

member is only 13 800 lb. The section of the member is, therefore, governed by the slenderness-ratio. The same is true of members 5-6 and 6-7.

The sections selected for the various members, together with the design stresses and lengths computed to the nearest thirty-second of an inch, are given in Table XXXIII.

Design of Joint Details. Gusset-plates will be made $\frac{3}{8}$ in throughout, except at the joints where the splices occur, where $\frac{7}{16}$-in plates will be used in order to reduce the number of field-rivets required.

Riveted joints in roof-truss construction should be as compact as practicable and in no case should an unsupported dimension of the gusset-plate exceed 60 times the thickness of the plate, unless stiffener angles are used, as shown in Fig. 32(*c*).

The number and arrangement of rivets will be determined by the values given in Tables XXV to XXXI, inclusive. The size and shape of each gusset-plate will depend upon the number and spacing of rivets in the various members connected. The width of a gusset-plate measured at right-angles to any member should increase, from the outermost rivet in the member, toward the center of the detail, in order to provide for increasing stress from the successive rivets.

When a member such as the upper or lower chord is continuous through the joint, the stress transferred to the joint is the difference in the chord stress on each side of the joint. This amount may be relatively small, and the gusset-plate dimension along the chord as required for the accommodation of the web-members may be relatively large. In such a case it is necessary to provide more rivets than the number theoretically required, in order that the maximum spacing be not exceeded. At the peak joint it is generally advisable to use the same number of rivets on the shop side of the joints as required on the field side in order to provide for symmetry in details and shop work.

The stresses to be transferred by the members, the governing rivet value, the number of rivets required, and the number used at each joint, are given in the table Rivets Required.

Design of Splices. The detail of the splice at joint (7) is shown at *c*, Fig. 36. The gusset-plate extends below the chord member to provide for a standard end connection of the ceiling-beam. The splice detail can be made either by a direct transfer of the total stress in each chord member to the gusset-plate by means of rivets through the vertical legs of the angles, or by the use of splice-plates on the horizontal legs in conjunction with the connections of the vertical legs.

When the stresses to be transferred are relatively large the first method requires a long joint detail.

If, in the detail under consideration, the stress in the chord members is to be transferred directly to a $\frac{7}{16}$-in gusset-plate by rivets in the vertical legs, the number of shop-rivets required for member c_6–5 is 78 200/9 840 = 8, and the number of field-rivets required for the member is 70 300/6 560 = 11.

The total length of plate required, using $2\frac{1}{2}$-in rivet-pitch, $1\frac{1}{2}$-in edge distance and allowing $\frac{1}{4}$-in clearance between the ends of the two chord members, would be $48\frac{3}{4}$ in.

When the number of rivets on any one gauge-line is 10 or more, the rivets near the ends of the group will receive more stress than those nearer the center of the group. As a general rule, when more than 10 rivets are required for the connection at the end of any member, a thicker gusset-plate or double gauge-lines should be used. A more satisfactory detail at joint (7) can be made by using horizontal splice-plates to transfer a part of the stress, thereby

reducing the amount of stress to be transferred by the rivets through the vertical legs to the gusset-plate.

Rivets Required

Joint	Members	Total stress (Thousands of pounds)	Allowable rivet value, lb	Number of rivets	
				Required	Used
1	c_8–1	0	. .	. . .	2
	a–1	60 50	8 440	7	7
2	a–1	60 50	8 440	7	7
	b–2	62 80	8 440	8	8
	1–2	70.50	8 440	9	9
3	c_8–1	0		. .	2
	1–2	70 50	8 440	9	9
	2–3	34 00	8 440	4	4
	c_7–3	60.00	8 440	7	7
4	(c–4)–(b–2)	81.50–62.80	8 440	2	3
	2–3	34 00	8 440	4	4
	3–4	24.00	8 440	3	3
5	(c_6–5)–(c_7–3)	78 20–60 00	8 440	2	4
	3–4	24 00	8 440	3	3
	4–5	13 80	8 440	2	3
6	(d–6)–(c–4)	82 00–81.50	8 440	2	3
	4–5	13.80	8 440	2	3
	5–6	2 00	8 440	2	3
7	c_6–5	78 20	* Detail	. .	
	5–6	2 00	9 840	2	3
	6–7	13.20	9 840	2	3
	7–8	17 70	5 960	3	3
	c_5–8	70.30	* Detail		.
8	(d–6)–(e–7)	82.00–82.00	.	. . .	2
	Purlin Load	13 20	8 440	2	.
	6–7	13 20	8 440	2	2
9	(c_5–8)–(c_4–9)	70.30–70.30	. . . .	. . . . .	. .
	Ceiling Load	2.20	8 440	2	2
	8–9	2.20	4 420	2	2
10	e–7	82.00	* Detail	.	. .
	f–10	82.00	* Detail	. . . .	. .
	7–8	17.70	5 960	3	4
	8–9	2.20	4 420	2	2
	9–10	17.70	4 420	4	4

* See Design of Splices.

In general not more than about 50% of the stress in either of the members should be transferred through the splice-plates.

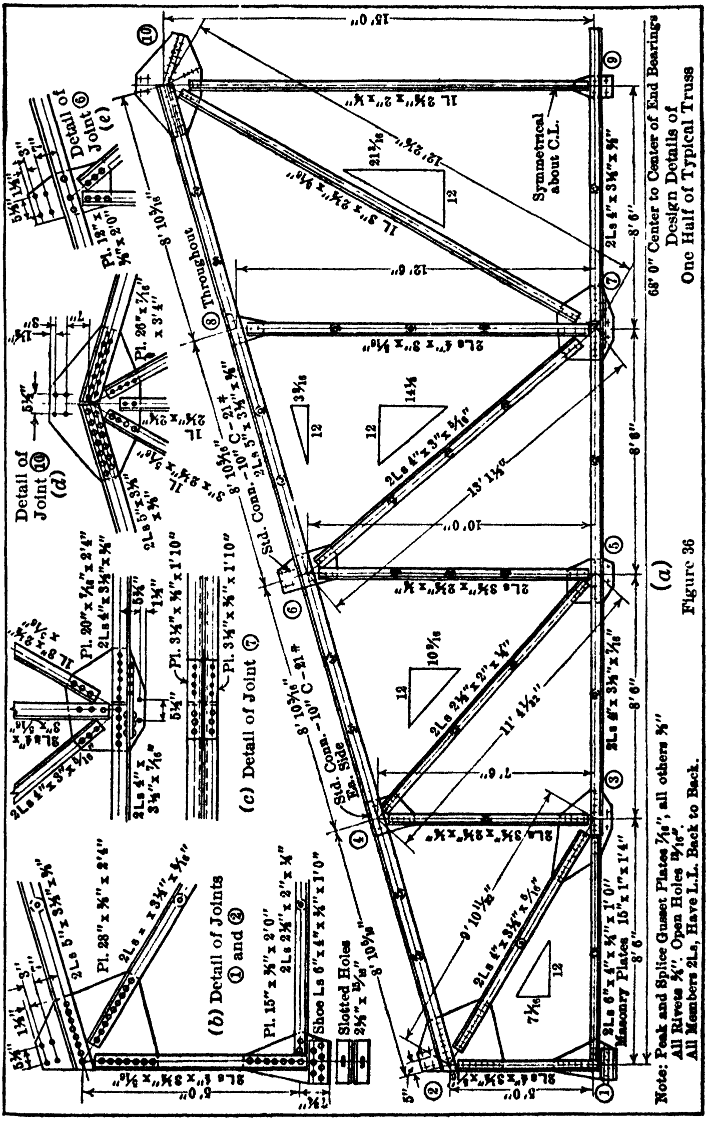


Fig. 36. Details of Roof Truss

The rivet values, allowable pounds per rivet, are:

Shop side		Field side	
Bearing $\frac{7}{16}$-in pl.	= 9 840	Bearing $\frac{7}{16}$-in pl.	= 6 560
Single shear	= 5 960	Single shear	= 4 420

Since the gusset-plate is to extend below the angles, in this case, two splice-plates, each $3\frac{1}{2}$ in wide, will be used. Assume four rivets in each splice-plate on the field-connected side. Their value, determined by single shear, is:

$$2 \times 4 \times 4\,420 = 35\,360 \text{ lb}$$

The number of field-rivets required to connect the vertical legs to the gusset-plate, on the field side, is:

$$(70\,300 - 35\,360)/6\,560 = 5.35 \quad \text{or} \quad 6$$

Since no more stress can be transferred by the splice-plate than that determined by the rivet value on the side of the splice having the least allowable value, the rivets connecting the horizontal legs to the splice-plates on the shop side cannot transfer more than 35 360 lb.

The number required is 35 360/5 960 = 5.9 or 6 rivets, and the number required to connect the vertical legs to the gusset-plate on the shop side is (78 200 − 35 360)/9 840 = 4.35 or 5 rivets.

The transfer of stress is, then, as follows:

Shop side		Field side	
Horizontal legs	= 35 360 lb	Horizontal legs	= 35 360 lb
Vertical legs	= 42 840 lb	Vertical legs	= 34 940 lb
Total	= 78 200 lb	Total	= 70 300 lb

The difference between the total stress transferred on the two sides of the joint is:

$$78\,200 - 70\,300 = 7\,900 \text{ lb}$$

This difference in stress which tends to move the gusset-plate horizontally toward the left is just neutralized by the horizontal components of the stresses in the web-members 5–6 and 7–8, as is shown by the stress-diagrams in Fig. 35. The minimum thickness of the splice-plates is determined by the allowable unit tension on the net section. Each plate will be subjected to a tension of 35 360/2 = 17 680 lb. The minimum thickness required, allowing for the deduction of one rivet-hole $\frac{1}{8}$ in larger than the diameter of the rivet, is:

$$t = \frac{17\,680}{18\,000\,(3\frac{1}{2} - \frac{7}{8})} = 0.375 \quad \text{or} \quad \tfrac{3}{8} \text{ in}$$

When rivet-holes occur in both legs of an angle, the pitch of rivets to maintain a net section equal to that section with one hole deducted is given in Table XXXI.

The gauge distance on the 4-in leg of the chord member is $2\frac{1}{2}$ in, and on the $3\frac{1}{2}$-in leg the gauge is 2 in. The value of a, Table XXXI, is $2\frac{1}{2}$ in + 2 in $= 4\frac{1}{2}$ in, and b, the minimum pitch, is $2\frac{15}{16}$ in, which would require the rivets connecting the vertical legs to be placed at $2 \times 2\frac{15}{16} = 5\frac{7}{8}$ in apart, and the use of a splice-plate detail to reduce the length of the connection would not be accomplished.

Provision was therefore made in the design of the chord members for a deduction of two rivet-holes, which will allow the usual spacing of 2½ in in both legs of the angles.

Peak Splice. Members *e*–7 and 7–8 will be shop-riveted to the gusset-plate at the peak and all other members will be field-riveted, as shown at (*d*), Fig. 36.

An investigation of the number of rivets required for the field-connection of member *f*–10, assuming a ⅜-in gusset-plate, shows that 82 000/5 630 = 14.6 or 15 rivets would be necessary. These rivets could be placed on two gauge-lines; however, a more satisfactory detail is possible with a 7/16-in gusset-plate.

The number of rivets required for the connection of member *f*–10 is, then, 82 000/6 560 = 12.5 or 13 rivets. Two gauge-lines will be used, with 7 rivets on the gauge-line nearer the outstanding legs and 6 rivets on the other gauge-line.

To promote symmetry in detailing and shop work, the same number and arrangement of the shop-rivets for the connection of member *e*–7 will be used as required for the field-connection of member *f*–10.

The number of field-rivets required for the connection of member 9–10 is: 17 700/4 420 = 4, and the same number of shop-rivets will be used in the connection for member 7–8. The stress in member 8–9 is 2 200 lb. The member is field-connected, and the rivets being in single shear, the number required is 2 200/4 420 = 0.5 rivet. Since two rivets is the minimum number which should be used in any member subjected to a calculated stress, the minimum requirement governs for this connection and all others where the calculated number of rivets is less than two.

End Bearing Detail. The detail of the end joint is shown at (*b*), Fig. 36.

The size of the masonry bearing-plate is governed by the magnitude of the maximum vertical reaction and the allowable unit bearing value specified for masonry:

$$\text{Area of plate} = \frac{59\,700}{250} = 238.8 \text{ sq in}$$

The dimension of the bearing-plate perpendicular to the wall is more or less fixed by the pilaster dimension shown in Fig. 34(*c*), as about 16 in. The required width of the plate parallel to the wall is (238 8)/16 = 15 in.

The number of rivets required to transfer the reaction to the shoe angles is (59 700)/8 440 = 7 rivets. The vertical legs of the shoe angles will be made 6 in, and the seven rivets required will be arranged on two gauge-lines as shown in the detail, Fig. 36(*b*).

If the outstanding legs of the shoe angles are made 6 in, the bending moment governing the thickness of metal required is given by Equation (28) as:

$$M_a = \frac{59\,700}{12} \times 6 = 29\,850 \text{ lb-in}$$

If the dimension of the shoe angle perpendicular to the wall is made 12 in as required to accommodate the seven rivets, the thickness of metal required as given by Equation (29), is:

$$t_a = \sqrt{\frac{1}{2} \cdot \frac{59\,700 \times 6}{18\,000 \times 12}} = 0.91 \quad \text{or} \quad {}^{15}\!/_{16} \text{ in}$$

The thickness of the masonry plate with 6 × 6-in shoe angles, with a thickness as determined above, is given by Equation (31), as:

$$t_p = \sqrt{\frac{1}{2}\cdot\frac{59\,700}{18\,000}\cdot\frac{15-6}{16}} = 0.97 \quad \text{or} \quad 1 \text{ in}$$

If the shoe angles be made 6 in × 4 in × 12 in long, the required thickness of the angles is:

$$t_a = \sqrt{\frac{1}{2}\cdot\frac{59\,700}{18\,000}\cdot\frac{4}{12}} = 0.74 \quad \text{or} \quad \tfrac{3}{4} \text{ in}$$

and

$$t_p = \sqrt{\frac{1}{2}\cdot\frac{59\,700}{18\,000}\cdot\frac{11}{16}} = 1.06 \quad \text{or} \quad 1 \text{ in}$$

The latter combination provides a more economical detail and will be used.

The LATERAL EXPANSION resulting from probable temperature changes and displacement due to deformation will be provided for by slotted holes in the horizontal legs of the shoe angles at one support. The diameter of the anchor-bolts will be $\tfrac{7}{8}$ in, and full provision for probable lateral movement will be made on each side of the anchor-bolts, as shown at (*b*), Fig. 36.

As specified in Article 8, the allowance will be $\tfrac{1}{8}$ in for each 10 feet of span. The required length of the slotted holes is:

$$\tfrac{7}{8} \text{ in} + 2\left(\frac{6.8}{8}\right) \text{ in} = 2.57 \quad \text{or} \quad 2\tfrac{5}{8} \text{ in}$$

Lateral Bracing. The LATERAL RIGIDITY of the entire roof-construction is an important item and one that is too often neglected. When the roof-construction is supported by end walls which may be subjected to wind-loads, the LATERAL BRACING serves to stiffen the roof system, thereby preventing excessive stresses due to a possible displacement of the trusses. Many cases have been recorded of the failure of a steel roof-frame, during construction, due to the absence of proper bracing. Very often the unsymmetrical loads resulting from material and equipment incidental to construction cause serious displacements of the members of the framing system. Such conditions might be impossible in the completed roof braced by a rigid and continuous structural covering; however, in any case, the small additional expense of bracing is justified in any important project.

All trusses supported on masonry walls, where the span is greater than 40 ft, should at least be braced in pairs either by SWAY BRACING in the planes of the chords, or by LATERAL BRACING extending the depth of the trusses in planes at right-angles to the trusses. No well-defined mathematical procedure for the design of such bracing is possible since the forces involved are of an indefinite nature. Experience and judgment are the all important factors in the design of lateral bracing systems.

For the roof under consideration the PURLIN CONNECTIONS at joints (2), (6) and (10) will be made of standard end connection details to the gusset-plates which are extended above the chord member for that purpose. A series of such connection details provides a considerable amount of lateral rigidity in the plane of the upper chord before the structural covering is in place.

In addition to the rigid purlin connections, the trusses will be braced in pairs in the planes of members 4–5 and 12–13, as shown in Fig. 34(*d*).

When TENSION-MEMBERS are used in inclined positions in bracing systems their slenderness-ratio should be limited to about 250 to 300.

The length of the diagonal bracing members between trusses, Fig. 34(d), is $\left(\sqrt{18^2 + 10^2}\right) 12 = 247$ in. The least radius of gyration should be about 1.0 and is supplied by a single angle $4 \times 3\frac{1}{2} \times \frac{5}{16}$ in.

The ceiling-beam at joint (5) will act as the bottom strut of the lateral frame and the purlin at joint (6) will act as the top strut.

The maximum allowable slenderness-ratio for struts in bracing systems is 200. The least radius of gyration of the 10 ft-C-21-lb purlin section is 1.39, and prior to the placing of the structural covering this value will be considered as the factor governing the stiffness of the purlin. The length is $18 \times 12 = 216$ in and $L/r = \frac{216}{1.39} = 155$, which is satisfactory. The least radius of gyration of the 6 in-I-12.5-lb ceiling-beam is 0.72, and $L/r = \frac{216}{0.72} = 300$, which is excessive.

The ceiling-beams may be supported laterally at their mid-points of span by an angle framed across their upper flanges and extending between the outer ceiling-beams at joints (3) right and left. This angle may be the smallest angle permissible or a $2\frac{1}{2} \times 2 \times \frac{1}{4}$ in, and the slenderness-ratio of the ceiling-beam at joints (5) right and left may then be taken as $L/r = \frac{9 \times 12}{0.72} = 150$, which is satisfactory.

Design Detail. The complete design of the truss is shown in Fig. 36(a), which illustrates the general type of drawing prepared by the designer and from which the shop drawings can be intelligently made.

The drawing shows the sizes and arrangement of members, the thickness and general proportions of gusset-plates, the number of rivets at each joint, and the overall dimensions.

In the case of special details it is sometimes necessary to show by a larger-scale drawing the particular features which the designer has incorporated in the detail.

CHAPTER XXVII

ELEVATOR SERVICE IN BUILDINGS *

1. General Considerations

Quality of Service. Requirements of passenger elevator service vary greatly in different types of buildings and even in buildings of the same type in different localities. The QUANTITY of service necessary depends on the PEAKS of traffic during the arrival in the morning, the luncheon period, extending in some cases from noon until 2:30 P.M., and in the evening when emptying the building. A sufficient quantity of elevator service must be provided so that in the morning peak the tenants will not be delayed too long in reaching the floors on which they work, and particularly, so that there shall not be congestion in the lower halls. The QUALITY of elevator service is also important, and the INTERVAL, which is the average time between cars, one-half of which the passengers must wait on the average for an elevator, must not be too long. It is also desirable to have smooth and rapid operation, with acceleration and retardation that is not uncomfortable for passengers. Automatic operation insures smooth performance, including proper acceleration and retardation, and practically eliminates complaints about operators, no skill being required. The lanterns over the doors that indicate the approach of the elevator and the direction in which it will travel, should light only when the car is certain to stop at the landing. Having determined the TRAFFIC PEAK, which is the maximum number of passengers per unit of time, the elevators must have a traffic handling capacity equal to this requirement, and must be arranged in suitable groups so as to obtain a satisfactory interval and a suitable grouping arrangement. The groups of elevators should not be too long, preferably not more than 30 ft, and should, wherever possible, be located on private halls which do not have through traffic.

In general, buildings requiring passenger elevator service may be divided into two classes: (1) rented office-buildings having DIVERSIFIED TENANCY, and (2) SPECIAL-PURPOSE buildings.

Office Buildings Having Diversified Tenancy. This is the usual type of office-building, and the elevator traffic characteristics follow a rather definite trend. The PEAKS of traffic are lower the greater the TIME-SPREAD of the traffic waves. The diversified tenancy causes the tenants to start and stop work at various times instead of simultaneously, as is the case in certain special-purpose office-buildings. In a diversified-tenancy office-building, for example, the SPREAD OF ARRIVAL may be about one hour, with the peak, in the larger cities, occurring near 9:00 A.M., and amounting usually to from 10 to 12% of the population of the building in five minutes. Five minutes is taken as the unit upon which the quantity of elevator service is based, because otherwise there will be an objectionable accumulation in the hall at the lower terminal loading floor. The occupants of the building will arrive in a wave which will gradually rise to a peak about the time that the majority of people start work, and then drop rather quickly. During the arrival wave, the passenger traffic will be practically all in the "up" direction.

* The information contained in this Chapter has been compiled by E. W. Yearsley through the courtesy of the Otis Elevator Company, New York City.

There may, however, be special conditions requiring people to report first at certain floors and then go to other floors on which they work.

After the morning peak of arrival, the traffic of those who leave the building to look after outside business will commence, and shortly after this there will be people arriving and departing who have business with the occupants of the building. From about 10 o'clock until noon the traffic up and down is likely to be nearly equal and usually the combined up and down traffic is not as great as that of the arrival peak. At about 12 o'clock some of the occupants go to lunch, and there is a heavy down peak of traffic. There is usually heavy traffic both up and down from a little after noon to at least 2:00 P.M. The peak of this traffic is generally at about 1:00 P.M., when there is about an equal number of people returning from and going to lunch. In a diversified-tenancy office-building, the five-minute peak at 1 o'clock will vary from about 12 to 16% of the population, with about 60% of the traffic up and 40% down. After about 2:30 P.M. the traffic will be reduced and will be nearly equal up and down, and this condition will continue until about 4:30 P.M., when the traffic is usually considerably reduced until about 5:00 P.M., when the peak of emptying the building begins. This is usually heavy immediately after 5:00 P.M. (See Fig. 1.)

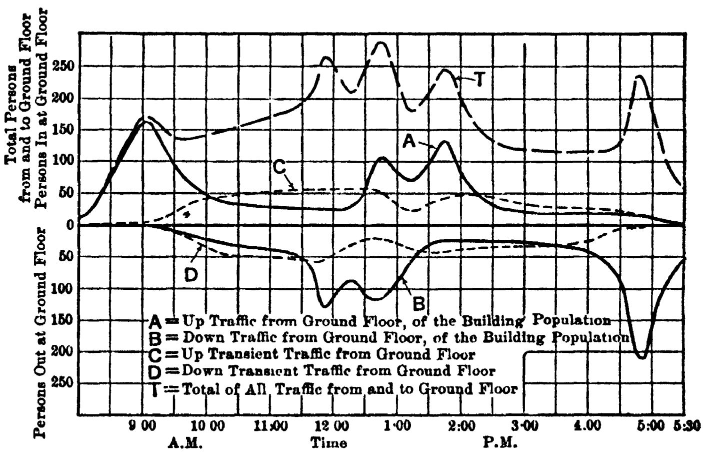


Fig. 1. Curve of Typical Traffic Loads

The peak of arrival is influenced by the horizontal transportation facilities and may be considerably increased if the building is immediately adjacent to terminals at which trains are arriving, as a single train may discharge a great number of people who are occupants of the building. The evening traffic peak when emptying the building is apt to be very high because the people are in the building and a large number will reach the elevators at about the same time after they have stopped work. This peak, however, is not so important as the morning peak, because waiting passengers are divided among the elevator hallways above the lower terminal.

Inter-floor Traffic is usually comparatively small in a diversified-tenancy office-building but may be very high in a special-purpose building. In most cases the inter-floor traffic is not great in the peak traffic periods, although

if the luncheon period is protracted, it will probably be necessary to consider inter-floor traffic. Tests of various diversified-tenancy office-buildings show inter-floor traffic in the morning peak when filling the building to be 1 to 4% of the passengers handled. In the noon peak period of buildings of this type, the inter-floor traffic may be from 3 to 8%. For special-purpose buildings the inter-floor traffic in the peaks varies widely, but seldom exceeds 10% when filling the building, and may be from 5 to 15% at the luncheon peak.

Special-purpose Buildings. Special-purpose office-buildings are those occupied by a single company or by several large concerns. The particular characteristic of these buildings is that salaried people form a large percentage of the population, who are compelled to start and stop work promptly at stated times. In the largest cities, for example, the usual starting time is 9:00 A.M., and in some cases employees must record their arrival by a time-clock. The result of this condition is extremely high peaks of arrival, and it is not at all uncommon, in special-purpose buildings, for as much as one-third or even more of the population to arrive at the elevators between 8·55 and 9:00 A.M. An example of this condition is a building entirely occupied by a life insurance company or by the offices of a railroad company. Numerous tests of buildings of this type have shown FIVE-MINUTE ARRIVAL PEAKS of one-quarter to one-third or more of the population. (See Fig. 2.)

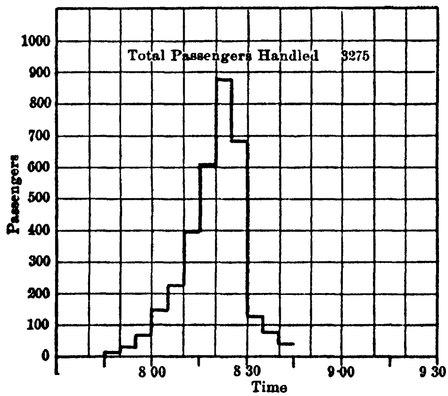


Fig. 2. Chart Showing Passengers carried Up by Elevators of a Large "Single-purpose" Building During Morning Peak Period.

NOTE: Population of building is 3450 (occupied by a Life Insurance Company). Clerks start work in this building at 8:30 A.M.

In order to reduce the number of elevators required, it has been arranged in some buildings of this type to divide the employees into groups which start and stop work at different times. Generally it is not desirable to base the elevators on any less requirement for traffic than that corresponding to a division of the employees into two nearly equal groups, differing at least twenty minutes in time of starting in the morning and stopping in the evening. Any division, of course, interferes to some extent with the efficiency of the business. While in some cases it has been necessary to divide into three groups, this is usually unsatisfactory.

Special-purpose office-buildings are very apt to be provided with RESTAURANTS OF CAFETERIAS, and the usual practice in the luncheon period is to

divide the employees into a number of groups, extending, in some cases, from 11:00 A.M. to after 2:00 P.M., and allowing from twenty to thirty minutes between groups in order to permit the elevators to serve the luncheon group and also in consideration of the capacity of the restaurants or cafeterias. The location of restaurant or cafeteria must also be taken into account; if it is located on a floor above the second, the employees will frequently go to lunch from their offices by elevator, then back to the office, then down to the street and back to the office, so that the elevators have a great deal of traffic in the luncheon period.

There is usually a considerable amount of INTER-FLOOR TRAFFIC in special-purpose buildings. This inter-floor traffic may have a considerable influence on the elevator requirements and greatly increases the number of stops made by the elevators. Generally, in special-purpose buildings, there is less traffic between peaks than in diversified-tenancy buildings, although this is not always the case. Sometimes there are offices in special-purpose buildings which a large number of people visit at certain times for some special purpose. In some cases, four or five large companies will entirely occupy an office-building, and the conditions of traffic will be quite similar to those existing when one company occupies the building. When this is the case, it is usually more difficult to divide into groups starting and stopping at different times. It will undoubtedly be found desirable in some special-purpose buildings to have the elevator-service supplemented by ESCALATOR-SERVICE for a certain number of the lower floors. In some types of special-purpose office-buildings, for example those occupied by the offices of utility companies, the morning peaks may run from one-fifth to one-seventh of the population in five minutes. In such buildings there are often different departments having different working conditions, which tends to increase the spread of the traffic peaks.

The Density of Population is based on the net rentable or usable area, which is the inside area of the building, less the space occupied by utilities and corridors. This working-area is usually from 65 to 75% of the gross area inside the walls of the building. In general, for diversified-tenancy office-buildings, the AVERAGE DENSITY is one person per 100 sq ft of net area for buildings in the large cities. In smaller diversified-tenancy office-buildings the average is about one person per 110 to 125 sq ft of net area. In very large diversified-tenancy office-buildings the density on large floors is apt to run higher and may run from one person per 75 to one person per 90 sq ft net area. With these densities the peaks are higher and run from one-seventh to one-eighth of the population arriving in five minutes.

In special-purpose buildings the density of population varies considerably, with a maximum of about one person per 50 sq ft net, running from this to one person per 80 or more sq ft net area. A table of densities for various types of occupancy is shown by Table I.

Table I. Density of Population

Type of occupancy	Average density (sq ft per person)
Open office space with two people at a desk	60 to 70
Private offices	100 to 125
Average small offices	95 to 115
Stenographers (separately grouped)	50 to 60
File space	130 to 160
Clerks at large tables	40 to 50
Drafting-rooms, normal	55 to 60
Restaurants and cafeterias	8 to 12
Auditoriums and theaters (exclusive of platform area)	5 to 6

2. Types of Buildings Requiring Elevators

Hotels. The elevator requirements of hotels vary considerably with the type of the hotel and particularly with the location of restaurants, assembly-rooms, etc. The large hotels, having a considerable amount of commercial trade, have the most severe requirements for elevator-service and the most definite requirements. Such hotels usually have a fixed CHECKING-OUT HOUR at sometime from 6 to 7 P.M. This increases the peak of elevator service, and the evening peak is also heavy, owing to guest traffic, arrivals and departures, and special restaurant and assembly requirements. The conditions are most severe when the hotel is the HEADQUARTERS OF A CONVENTION, as there are then a maximum number of guests and also a great many visitors, particularly when special functions are held. The registration or number of guests at a given time is, of course, dependent on the number of single and double rooms and may generally, in busy seasons, be taken as an average of 1.2 to 1.4 persons per bedroom. During convention periods the average reaches as high as 1.8 persons per bedroom. The elevators must carry from 9 to 12% of the registered number of guests in five minutes with the traffic nearly equally divided between up and down, and in addition to this the traffic to and from banquets and other special functions must be handled. The traffic from BANQUETS has a very high peak, often as much as one-third or more of the people attending coming to the elevators within five minutes. There is also apt to be heavy traffic in the hotel at about 8:30 A.M. when the guests are going to breakfast. Account must be taken of the location and traffic of restaurants, cafeterias, roof-gardens, assembly-rooms, ball-rooms and other public rooms, the worst condition existing when there is a great amount of space in the upper part of the building where banquets or assemblies will be held, or where there is very heavy restaurant service. Basement service and the arrangement of the lower floors must also be taken into account. In some large hotels, for example, the lobby and office floor, with restaurants, is an entire floor, anywhere from twelve to twenty feet above the street level, and on the street level there are shops and other public rooms. A full floor below the street there may be grills and cafeterias of large size, so that in this case the elevators must stop on every trip at each of these floors when ascending and descending. Such an arrangement greatly reduces the handling capacity of the elevators for the ordinary guest-room traffic.

Apartment-Houses. In large apartment-houses and apartment-hotels, the heaviest traffic is usually in the early evening about dinner-time, and the elevators may have to handle about 6 to 8% of the tenants in five minutes, both up and down. The requirements of apartment-houses vary considerably, depending on the type of occupancy, and may, particularly in the smaller apartment-houses, be less than the above. In certain types of apartment-houses there is a traffic peak when occupants are leaving in the morning. This is the case where a considerable number of people have to take certain trains. The occupants may also be working people who have to reach their business offices promptly, this having a tendency to increase the morning traffic peak.

Department-Stores. The elevator requirements of department-stores vary considerably with the size of the store as well as with its height and particularly with the character of the business done.

Escalators. Those department-stores doing the largest volume of business and catering generally to a large percentage of the public, where the pur-

chases are numerous and where there are many customers, have to be provided with a great amount of VERTICAL TRANSPORTATION. To handle this traffic, escalators and elevators are necessary. ESCALATORS are of great advantage in handling the basement and second-floor traffic, and in the larger stores they also handle the inter-floor traffic to a great extent throughout most of the building. They are also excellent for handling the additional traffic on heavy business days, particularly during the Christmas and the Spring shopping periods. (See Table II.) The amount of vertical trans-

Table II. Capacity of Escalators

Angle of inclination is usually 30°

Width, ft	Speed, ft per min	Capacity, Passengers per hour
2	90	4 000
3	90	6 000
4	90	8 000

portation required in a department-store must be based on the number of customers in a given time. In large department-stores it has been found that an average of one passenger per hour to and from the ground-floor per 25 to 35 sq ft of net sales area is the usual basis for traffic requirements. In some cases, however, it has been found that this runs to one person per 20 sq ft or less, and in other cases to one person per 40 sq ft. For basement traffic one person per 6 to 10 sq ft of sales area per hour is a good basis. There will be a large amount of inter-floor traffic carried by the elevators and this must, of course, be taken into account in calculating their performance. It is now usual in large department-stores to provide up and down escalator service for any where from 50 to 65% of the traffic, and the basements are best served by escalators only, stopping the elevators at the ground floor.

Professional Buildings. It has become modern practice to provide professional buildings particularly for the use of DOCTORS and DENTISTS. In such buildings, the elevators must be based on the traffic of patients, who very often have friends accompanying them. When doctors are receiving patients in regular office-hours they can, in some cases, handle as high as one patient in fifteen minutes. Dentists will handle patients on an average of between one-half hour and one hour. The conditions vary greatly in these professional buildings, but the elevators must generally handle at least one patient up and down per doctor, dentist or assistant in one-half hour, and in some buildings the traffic is more than double this amount.

Hospitals. The maximum traffic in hospitals is usually caused by the visitors, and the rules usually require visitors to come within a specified hour or two hours. The usual basis averages about one to two visitors per bed per hour. To this must be added the normal traffic of patients and doctors in attendance, and particularly the traffic to clinics, which is sometimes considerable.

Schools and Colleges. The traffic requirements of comparatively high buildings used for schools and colleges are apt to be so severe that special arrangements of operation, such as having the elevators stop at every other floor or every third floor, must be resorted to. Generally a large number of

people must be carried in fifteen-minute periods between classes, and there is also very heavy traffic at arrival and departure and in the luncheon period. The requirements between classes are very difficult to determine, as they depend on the number of people who must change floors and the regulations requiring certain people to walk.

3. Types and Capacities

Capacity and Speed of Passenger Elevators. For office-buildings in general, the LOAD CAPACITY of passenger elevators varies from 2 500 to 3 500 lb, but 4 000 to 5 000-lb capacities are used in some cases. The capacity to be selected depends on the interval that can be obtained; where the characteristics of the building are such that the interval with large cars is 20 to 25 seconds in the morning peak, it is desirable to use these. While in some cases it is necessary to use elevators of less capacity than 2 500 lb, these cars are usually too small for office-building service. The rated speeds that should be selected depend of course on the rise. Considering modern office-buildings, which are usually 20 stories or more in height, the rated speeds of the passenger elevators should not be less than 600 ft per min with manual operation, or 700 ft per min with automatic operation. For buildings 30 stories in height, the express elevators should operate at 800 ft per min and have automatic control. This speed is too high for satisfactory manual control. For 50-story buildings, elevator speeds of 1 000 ft per min are desirable for the high-rise express group, employing automatic control. For 75-story buildings, the automatic elevators which serve the upper part of the tower should be 1 200 ft per min. Elevators are safe, comfortable and efficient at 1 200 ft per min, the maximum acceleration and retardation being only a little more than for 600 to 700 ft per min. The elevator cars used in department-stores are usually from 3 000 to 5 000-lb capacity, with a rated speed of from 400 to 500 ft per min. These cars generally make stops either at every floor, or at a very large percentage of the floors, so that higher speeds have little advantage. Department-store elevators usually have two-speed center-opening doors with a 5-ft, or more, opening.

Size of cars. It is very desirable to employ cars having platforms 7 ft wide, and two of these cars will go into an 18 by 19-ft bay, which is about the most efficient length of bay for an office-building in consideration of division of rentable space. A car of this width permits the use of a 3-ft 6-in opening and single-leaf center-opening doors, which are the quickest operating and most efficient for power operation. There is also space to put the control panel at the side of the doors and on the front of the car, in which location it can be handled to best advantage by the operator. (See Fig. 3.)

Platform-Area. The platform-area of passenger elevators should be proportioned on a definite basis in relation to the capacity. The following data give the loading per square foot and the net platform-size for various capacities. As the size of the car increases, the pounds per square foot also increase, owing to the fact that the passengers pack more closely into a large car. (See Fig. 4.)

Grouping of Elevators. Generally in office-buildings the elevators should be arranged in groups of preferably not less than four, nor more than eight, except in very high buildings. Groups of six, three opposite three, are often used. For the high-rise express-elevators of a 70 to 75-story tower, the

group of elevators should not be less than eight. Elevators should be centralized as far as possible in an office-building and should not be located

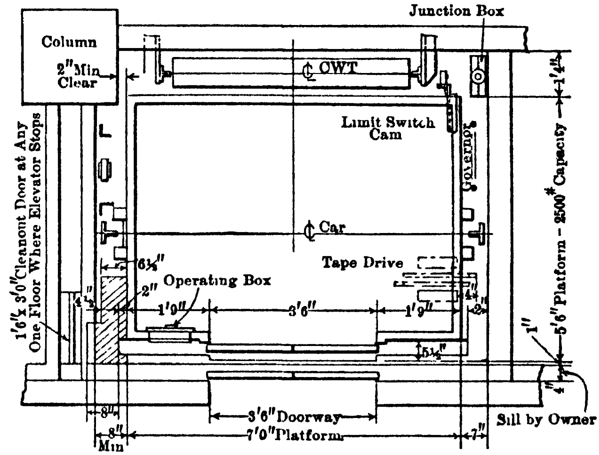


Fig. 3. Typical Hatch for Signal-control Elevators

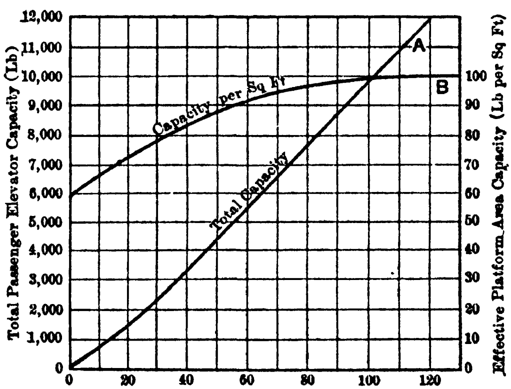


Fig 4.* Chart Showing Passenger-elevator Carrying Capacity Corresponding to Effective Platform-area.

NOTE: For passenger elevators having effective platform areas above 120 sq ft, the rated capacity shall not be less than 100 lb per sq ft.

* Taken from "A Safety Code for Elevators."

so as to face on thoroughfare halls. The various groups of elevators are run to various heights in accordance with the requirements and the grouping that will give a satisfactory interval. Usually, a building 20 stories high will have

LOCAL and EXPRESS ELEVATORS. Higher and larger buildings will usually have several groups of elevators. Duplication of service of certain floors by two or more groups is generally undesirable, but the usual practice is to provide a floor that is served in common by two groups so as to permit the passengers to change from one group to the other for inter-floor service. Elevators are sometimes operated without providing these change floors. In some special cases where a satisfactory interval cannot be obtained with two groups, unless an excessive number of elevators is employed, it is desirable to operate all the elevators as locals, even in buildings as high as 30 to 35 stories. While a few large buildings are provided with a group of elevators serving all floors to take care of inter-floor traffic, this is not economical as these elevators are not used to good advantage. In some special-purpose buildings it is frequently desirable to arrange elevators so that they may serve all floors between peaks, but dividing them into express and local groups during the peaks, morning, noon and evening.

Types of Elevators. From an operating standpoint, elevators are divided into two classes, AUTOMATIC and MANUAL. It is evident that an automatic elevator can give the maximum amount of safe service. The most expert operator cannot operate the car at any time as well as the automatic, and the operator cannot be depended upon to maintain high working efficiency. Considerable skill is required of operators who handle high-speed elevators, whereas the attendant who polices the car and regulates the closing of the doors for an automatic elevator needs no skill and has very little manual labor to perform. Automatic elevators for SELF-SERVICE which have no attendant in the car are used extensively for apartments and for private residences. While usually of slow speed, they may be safely and efficiently operated at high speeds.

There are three types of automatic elevators. The first is called from a single button in each hallway and answers that particular call and is exclusively in the service of the people calling until they have completed their trip. This type of elevator gives privacy of service, but its handling capacity is reduced because it does not stop for waiting passengers on the floors it passes. Another type of push button elevator is that which is called by one of two buttons in each hallway for passengers to go up or down. In this case, the elevator usually operates to go to the furthest call in the trip where a button has been pressed and then travels in the opposite direction until all passengers have been discharged, stopping for any passengers who have pressed buttons in the halls at floors it will pass and for the direction in which it is moving. Still another type has up and down calling buttons in each hallway and makes a complete trip, answering all calls. Elevators of this type are used for automatic operation in office-buildings, the hall buttons being arranged to stop automatically the first car of a group that approaches the landing, in the required direction, from which the call has been sent. With elevators of this type there is an attendant in the car who initiates the closing of the doors after the passenger transfer at the landings has been completed. This attendant also polices the car, prevents crowding and also presses the car buttons in accordance with the instructions of passengers as to the floors at which they desire to alight. These car buttons, one for each floor served, may be momentarily pressed at any time after the passenger enters the car until the car has reached a point a sufficient distance from the stop to allow the elevators to slow down properly. With elevators of this type the car and hatchway doors are usually power operated and the elevators also level automatically at the land-

ings so that the car floor is always practically level with the stationary floor while the passenger-transfer at the landing is taking place. This eliminates tripping hazard and increases the speed of movement of passengers into and out of the cars.

Performance and Traffic-Handling Capacity of Elevators. In service, elevators have to run various distances from one floor, which may be from 10½ to 12½ ft or more to the entire run, which may be as high as from 75 to 100 stories. Time must be allowed to accelerate and retard, at rates which the passengers can comfortably stand and which the apparatus can satisfactorily accomplish. Provision must be made for making satisfactory landings. In order to obtain the best results, the acceleration and retardation should be as nearly constant as possible, and the rate of change of acceleration and retardation when going from start into acceleration, from acceleration into practically constant speed, from constant speed to retardation, and from retardation to stop, should also be practically constant. High-speed elevators during the acceleration may average about 6½ ft per second per second, and may retard at an average of about 8½ ft per second per second. If this acceleration and retardation is smooth, it will not be uncomfortable for passengers. About ¼ of a second is usually sufficient for the periods when acceleration and retardation are changing, although a somewhat longer time is usually required to change from the constant accelerating period to constant full-speed running.

In order to stop an elevator accurately, it must be brought to a very slow speed before the power is cut off and the brake applied. The minimum time for slowing down and stopping the elevator is obtained when the main controller functions with a high retardation, bringing the elevator close to the landing without delay, and the elevator is then automatically leveled while the doors are opening. This leveling should have a definitely limited slow speed and should operate within a definitely limited zone on either side of the landing. Full protection should be provided when leveling, by flush hatchway construction and aprons on the cars, which arrangements prevent passengers getting their feet caught. It is satisfactory to have higher rates of retardation than acceleration, as it has been found that the passengers are less sensitive to retardation. Having a certain definite elevator apparatus including the controlling devices, it is possible to determine how long it will take the elevator to run various distances including acceleration and retardation. By the use of mathematics, and in conjunction with various traffic tests, it is possible to determine how many stops an elevator will be expected to make on the average in the peak periods, morning, luncheon and evening. When the number of probable stops is established, it may be assumed that the elevators will, in a typical peak service trip, make equal runs corresponding with the number of stops in the local zone, and run the express zone at full speed. In the morning peak it may be assumed that each elevator makes the down trip without intermediate stops, and in the evening peak makes the up trip without intermediate stops. During the luncheon peak there will be a considerable number of stops made in both directions. To the time required for the elevator to run the various distances, including acceleration and retardation, in a typical peak round trip, the time for making landings must be added, the time for operating doors and gates, and also the time required for passengers to enter and leave the car. Finally, an allowance must be made for contingencies and dispatching. Determining the typical round-trip time that is to be expected fixes the number of passengers that the elevator can handle in a given time, the standard being a five-minute

Table III. Elevator Installations

Type of Building	Rentable area above first floor, sq ft	Number of stories	Elevator Equipment: Number of cars in group	Floors served	Type of elevators
Office	241 000	22	4	2 to 12	Gearless traction, automatic signal control
			4	12 to 22	
Office	205 000	25	5	2 to 15	Gearless traction, manual rheostatic control, without automatic leveling
			5	15 to 25	
Office	1 003 000	30	8	2 to 7	Gearless traction, automatic signal control
			8	8 to 14	
			8	15 to 22	
			8	22 to 30	
			2	2 to 30	Gearless traction, manual voltage control without automatic leveling (freight cars)
Office	1 026 100	40	8	2 to 10	Gearless traction, manual rheostatic control, without automatic leveling
			8	11 to 19	
			8	20 to 24	
			8	25 to 32	
			8	32 to 40	
			8	1 to 40*	
Office	710 600	71	8	2 to 12	Gearless traction, automatic signal control
			8	12 to 26	
			6	26 to 44	
			6	44 to 57	
			2†	57 to 71	

* Interfloor service. † Shuttle elevator

period. Having determined from the characteristics of the building in connection with various traffic tests of similar buildings in similar locations, what the requirement in number of passengers handled in five minutes is for the arrival, departure and luncheon peak periods, the number of elevators of that particular kind, capacity and speed that will be required to give adequate service is determined. Twenty-five-hundred-pound capacity elevators of a given type, may, for example, serve in a diversified-tenancy office-building from about 15 000 to 35 000 sq ft of net rentable area. For buildings of different types the number of square feet served per elevator will vary greatly, even with the same load-capacity, and the load-capacity to be employed is dependent on the grouping and the interval.

The determination of the number and grouping of elevators for a building requires the best judgment, based on experience and a great amount of experimental data. Also every possible effort should be made to obtain information that will fix, as definitely as possible, the service required, not only when the building is new, but throughout its expected life. There is no general rule or formula that is even reasonably satisfactory that can be applied to such determination.

In order to give an idea of elevators in diversified-tenancy office-buildings, a list of such elevators, including grouping, speed, capacity, rise and number of floors served, is given in Table III. These data are from buildings, in large cities, that have been proved by a considerable period of service to have a reasonably adequate amount of elevator service. (See Table III.)

4. Elevator Apparatus

Geared and Gearless Elevators. At the present time, practically all elevators are ELECTRIC. A few HYDRAULIC elevators, usually of the plunger type for short rises, are still occasionally employed. Electric elevators may be divided into two general classes, GEARLESS and GEARED. The gearless elevator has the driving-sheave directly mounted on the armature-shaft. The elevator motor is of slow speed, usually from 60 to 125 revolutions per minute, and the speed of elevators for 1 : 1 roping is from 600 to 1 200 ft per min, and for 2 : 1 roping from 300 to 500 ft per min. Practically all electric elevators are of the TRACTION TYPE, which is much safer and better than the DRUM TYPE and which is adaptable to buildings of any height. With this type of elevator the traction is relieved when either the car or counterweight bottoms, so that the driving-sheave may rotate without moving the elevator. This eliminates the danger of excessive strains obtained when drum machines accidentally overtravel.

The higher-speed elevators generally employ DOUBLE WRAP with round grooves in the driving-sheave. With this type, the hoisting ropes pass once over the driving-sheave, then around an auxiliary sheave, and a second time over the driving-sheave. For elevators of medium and slow speeds SINGLE WRAP is generally employed, in which the ropes pass only once over the driving-sheave, and it is necessary to provide grooves that will pinch the ropes to some extent in order to secure sufficient traction. The most satisfactory sheave of this type is one having round grooves which are undercut so that the rope does not come into contact with the bottom of the groove. This type of groove prevents excessive wear of sheave and rope, insures that the sheaves will not wear eccentric, and that they will maintain a smooth contact surface if the apparatus is properly designed. "V" grooves have not been found satisfactory, as they wear rough and eccentric, because of excessive pressure between rope and sheave.

Geared elevators are generally of the WORM-GEAR type, which is sometimes supplemented by additional SPUR-GEAR reduction, and also by 2 : 1, 3 : 1, or even 4 : 1 roping. With 1 : 1 roping the speed of the car and the counterweight is the same as the speed of the driving-sheave at the pitch diameter; with 2 : 1 roping the speed of car and counterweight is one-half the speed of the sheave at the pitch diameter, etc. This speed reduction is obtained by the usual method of arranging sheaves and rope-hitches. With geared elevators the motors generally run at about 700 to 900 revolutions per minute. It is not generally desirable to run geared elevators above 400 ft per min, and above 300 ft per min it is usually more satisfactory to use gearless elevators with 2 : 1 roping. The advantages of gearless elevators are the slow-speed motors and the absence of gears and thrust bearings. Geared elevators develop lost motion in the gears, owing to wear, with consequent rough operation. It is impossible to adjust satisfactorily for this wear, which gradually gets worse until the gears have to be renewed. For high-speed elevators there is also apt to be vibration from the gears that is felt unpleasantly in the cars. The thrust bearings also require considerable attention.

Ropes. High-speed gearless elevators, with 1 : 1 roping, usually have six to eight ropes, depending on the load. A driving-sheave of 36-in diameter with 5/8-in ropes and double wrap is frequently employed for these elevators. For 2 : 1 gearless elevators, five or six ropes are often used with 26-in to 30-in diameter driving-sheaves and 1/2-in or 5/8-in ropes. For geared elevators of the usual capacities 1/2 to 5/8-in ropes are used, generally varying in number from 3 to 8.

The FACTOR OF SAFETY of the ropes used with traction elevators is high. (See Fig. 5.) For this reason, and because of the multiple ropes, the danger

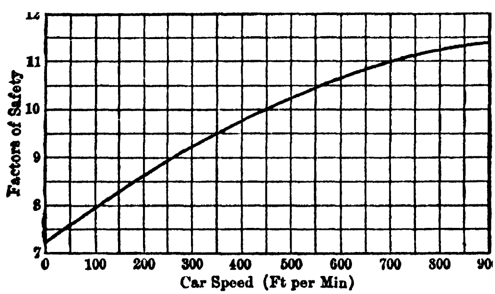


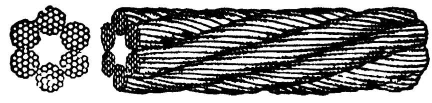
Fig. 5.* Factors of Safety for Hoisting and Counterweight Cables for Passenger Elevators

NOTE: It is recommended that traction-machine passenger elevators use factors of safety 10% in excess of the values given in this curve to allow for wear on the cables.

* Taken from "A Safety Code for Elevators."

of ropes parting and causing an elevator accident is so extremely small with traction elevators that there is practically no chance of such an accident occurring. Also, owing to the characteristics of the traction drive, there is practically no chance of pulling the ropes out of the fastenings if the fastenings are reasonably well made. Plenty of warning is given by broken wires, which will be evident by visual inspection, so that the ropes of an elevator

are changed some time before there is any danger. The standard hoisting cable is shown by *A*, Fig. 6. A specially designed traction rope, known as the SEALE TYPE (see *B*, Fig. 6), is used where the conditions are severe. The larger wires in the outside strands provide more wear and are an advantage in increasing the life of cables under severe conditions. Wire ropes are usually laid up without preforming of wires of strands, but ropes are available with wires and strands PREFORMED. Up to the present time, however, tests have not shown that commercial ropes that are preformed have any advantage over ropes of the usual type, and they are apt to have some disadvantages, one of these being that they stretch considerably after installation. Conditions vary so greatly that it is very difficult to give even general figures as to the life of ropes in elevator-service.



A Standard Hoisting Cable

B. Special Traction Rope

Fig. 6. Elevator Cable and Rope

Mechanical Safeties. Except for very slow speeds and short rises, all elevators are required to have MECHANICAL SAFETIES arranged to stop and hold the car on the guides in case of accidental overspeed or in case the ropes should part. These safeties are applied by a CENTRIFUGAL GOVERNOR, usually located at the top of the hoistway, which is driven by a loop of wire rope attached to a point on the car, the tension being taken up by a weight and floating sheave in the pit. When the speed for which the governor has been set is reached, it trips and applies friction jaws to the governor rope. This causes the RELEASING CARRIER, which is a friction device on the car, to be pulled out, and the further movement of the car applies the safety jaws to the rails under sufficient pressure to stop and hold the car.

There are two principal types of these mechanical safeties. The first, known as the FLEXIBLE GUIDE CLAMP type, acts almost instantly at governor-tripping speed to apply the safety jaws to the guides by means of a wedge and roller device. The second, known as the WEDGE CLAMP type, applies the jaws to the guides by screw and toggle action and usually requires from four to eight feet travel of the car after governor-tripping speed is reached before sufficient pressure is applied to slow down the car. The FLEXIBLE GUIDE CLAMP type, as the name implies, applies flexible pressure by spring action while the safety is operating. This type, therefore, tends to prevent excessive retardations due to variations in rail joints and other variations. The WEDGE CLAMP type has only the elasticity of its parts, and with it the retardations obtained when the safety acts are sometimes higher than with the flexible guide clamp type. It also has the defect of requiring a considerable movement of the car to apply it while the flexible guide clamp type requires only a few inches of movement to apply.

For very slow-speed elevators a type of safety using a SOLID WEDGE AND ROLLER is sometimes employed. As this safety has very little elasticity, it stops the elevator almost instantly when applied, and it can therefore be used only for very slow speeds.

Safeties are located under the car platform in a beam forming a part of the car sling.

It is not only important that the mechanical safety should stop and hold the car in case of accident, but it must do so without high retardations that will be dangerous for passengers. In order to insure safe retardations, it is desirable to employ a device for anchoring the tension sheave in the pit over which the compensating ropes pass, so that it cannot move up but is free to move down. This device ties together the car and counterweight below, and as these are tied together above by the hoist ropes, the masses of the system are all linked together and must travel at the same speeds, no matter what the retardation. With this device the retardation is never much more than gravity, either when the mechanical safeties or the buffers are operating, while without it the retardation will probably be somewhat more than twice gravity when the mechanical safety operates with light loads in the car. Since this device prevents the car and counterweight from jumping, in case of retardation above gravity, due to safety or buffer operation, it is very desirable, as otherwise there will be excessive strains on fastenings and ropes, due to the dropping back of car or counterweight after jumping. Cars are provided with mechanical safeties in almost all cases, and counterweights should be provided with mechanical safeties when there is accessible space underneath the counterweight.

Buffers. OIL BUFFERS are usually provided for high-speed elevators, the car buffers being located in the pit and the counterweight buffers attached to the counterweight frame. The stroke of these buffers should be equal to, or greater than, the distance required to retard from governor-tripping speed to zero speed with gravity retardation. By employing a device which will apply the mechanical safety in case the elevator does not slow down properly when approaching the terminals, it is possible to reduce considerably the buffer-stroke from the above and still have a safe elevator. This device is particularly advantageous when there is but little space available underneath the elevator for the pit and also when the over-travel at the top must be reduced to a minimum.

SPRING BUFFERS are used for the slower-speed elevators, but they are not as satisfactory as oil buffers and are not generally used beyond about 250 ft per min car speed.

Counterweighting and Compensating. Traction elevators are COUNTERWEIGHTED so that the counterweight is heavier than the empty car by 40 to 42½% of the rated load which is the capacity of the elevator. With such counterweights, the average load of the elevator is nearly balanced and the maximum net load that must be lifted is from 57½ to 60 per cent of the rated load of the elevator. Except for short rises, the weight of the hoisting cables must be COMPENSATED by compensating cables attached to car and counterweight and passing over a floating tension sheave with weight in the hoistway pit. Or for some elevators, particularly of the slower speeds, CHAIN COMPENSATION may be used, the chain usually being attached to car and counterweight and looping through the pit space or in some cases being fastened at the center of the hoistway and to the bottom of the car. Chains are noisier and less effective than ropes for compensating purposes.

Elevator Motors. All gearless motors are of the multipolar direct-current type, provided either with shunt fields only, or with compound fields. These motors generally operate on a potential of 220 to 300 volts and have a rated speed of 60 to 125 revolutions per minute. The alternating-current motor is not adapted to gearless-elevator operation. Geared-elevator motors of the direct-current type are usually compound wound and run at speeds of 600 to 900 revolutions per minute, with 800 revolutions per minute the usual

standard. These motors may have shunt-field control giving 2 : 1 or even 3 : 1 speed variation. It is not usually desirable to provide more field variation than the above because the motors are too sluggish and their kinetic energy becomes too great.

Alternating-current motors are of the INDUCTION TYPE, single speed for geared elevators up to 100 to 200 ft per min, and two speed for geared elevators up to 400 to 450 ft per min. The induction motors employed usually have SQUIRREL-CAGE ROTORS, with winding having sufficient resistance to give suitable starting torque characteristics. In some cases SLIP-RING MOTORS are used, but these are generally not so dependable as squirrel-cage motors of a suitable design. The slip-ring motor requiring a polar rotor winding is usually noisier than the squirrel-cage motor and also usually has considerably more kinetic energy. Slip-ring rotors are particularly objectionable for two-speed motors. The slip-ring motor has the inherent objection that high currents may be obtained with low torque, which is not the case with squirrel-cage motors, unless a phase is open, in which case both types lose their torque. Alternating-current motors have the objection that the speed control must be obtained by changing the number of poles, either providing two separate windings with different numbers of poles or a single winding with connections arranged to change the pole number. The control of alternating-current motors in acceleration and retardation is not as good as that of direct-current motors. The alternating-current motors also have much more kinetic energy than direct-current motors, and the power consumption of elevators using alternating-current motors is generally from 1.5 to 3 times the power consumption of elevators having the same duty operated by direct-current motors.

Voltage Control. By employing voltage control, it is possible to provide the same elevators, capable of the same performance, when the supply is alternating current as when it is direct current. With this type of control an individual generator is provided for each elevator, which is usually driven by an individual motor. Where the supply is direct current, the driving motor is of the compound type; where the supply is alternating current a high-efficiency squirrel-cage induction motor is used. The speed of these motor-generator sets is usually about 1 200 revolutions per minute, though sometimes higher or lower speeds are employed. For alternating-current supply, EXCITERS must be provided. These are sometimes directly driven by the motor-generator sets and in other cases a single motor-generator set, supplying several elevators, is provided for excitation purposes. The exciter is used to provide direct current for field excitation and controller and brake operation. With voltage control the unit motor-generator runs at practically constant speed while the elevator is in service. The voltage of the generator, which provides voltage for the armature of the elevator motor, is regulated so that it is gradually increased during acceleration and reduced during retardation by varying the excitation of the individual generator field. This type of control is inherently smooth because, owing to the field induction, the armature voltage does not change very rapidly as the field excitation is changed. This control is also very accurate as it provides definite voltage control at all speeds and throughout the range of load, whereas with rheostatic control speed varies greatly with load, at the lower speeds. Elevators operated on voltage control perform equally well whether the supply is direct or alternating current, and the power consumption is no higher with alternating current and may, in fact, be lower if a device is used for reducing the excitation of the driving motor of the motor-generator set during

the time that the elevator is stopped at landings for the transfer of passengers. Voltage control is particularly adapted to the operation of high-speed elevators and high and medium-speed automatic elevators.

Rheostatic Control. Rheostatic control may be employed both for direct and alternating-current elevator motors. With this type of control, direct-current motors are regulated in acceleration and retardation by series and parallel armature rheostats, and by field rheostats, and rheostats and pole-changing devices supply the regulation for alternating-current motors. This type of control is inherently less smooth than voltage control, because the current and the torque change very rapidly as the main current controller switches operate, and this results in shocks that are noticeable to passengers in the car. The acceleration and retardation are not nearly so constant with rheostatic control as with voltage control, and this is also unpleasant for passengers. Since at the lower speeds the elevator does not have constant speed regulation for different loads up and down, a rheostatically controlled elevator cannot be so accurately regulated and stopped, and there will be considerably more time lost in making landings with this type of control than with voltage control.

Magnet Controllers. Magnet controllers, using magnet-operated switches and brakes either with direct-current or split or multiphase alternating-current magnets, are almost entirely used for elevator control. In some cases motor-operated controls are employed. Alternating-current magnets are always less satisfactory than direct-current magnets, particularly for operating brakes Direct-current brake magnets are controlled by series and parallel resistances without the use of dash pots. Alternating-current brake magnets use air or oil dash pots to regulate their application, and dash pots are usually used in connection with alternating-current magnet switches and alternating-current torque motors used for controller and brake operation. In many cases it is desirable to use direct-current magnets for elevator control when the supply is alternating current. Such magnets are always used with voltage control, and for rheostatic control it is often found desirable to use a small motor-generator set with direct-current supply for operating the control magnets and the brake.

5. Operating Devices

Operating Handle. Modern elevator-operating devices are generally of the flush type with operating handle extending about 3 in. This handle is arranged to be normally held by a spring in center position and in some cases has indications of the various operating positions.

For MANUAL OPERATION, the lever is usually thrown to full-speed position, which is the limit of its travel, in order to start the car. It is held in this position while the car is running and until the elevator has reached the point where it is necessary to slow down in order to make a stop. The higher the speed of the elevator, the further will this point be from the landing. With manual operation, the operating lever is usually returned part way toward the stop position in order to slow down the car. When close to the landing it is returned to the stop position, when current is cut off and the brake applied. In case the resulting stop is not satisfactory, it is necessary to INCH by moving the lever part way from the stop position and returning it to stop position.

With AUTOMATIC OPERATION, the lever is thrown to the limit of its travel in order to start the doors to close. The doors close automatically and may be reversed by returning the lever to door-opening position. After the door is

closed and locked, the car starts and the elevator proceeds until acted upon by the automatic control so that it comes down to a slow speed within the leveling zone. The control is then automatically turned over to the definitely limited slow-speed leveling control and the car levels, the doors being so synchronized that when the car is level they are opened just sufficiently for the passengers to enter or leave. In the automatic elevator-operating panel, there is a bank of push buttons, one for each floor, and the momentary pressure of these buttons, at any time before the car has reached the slowing-down point of the corresponding stop, insures that the car automatically makes the stop indicated by the button.

Safety Switch. Both manual and automatic controls have a SAFETY SWITCH in the car for use in emergency, which, when opened, applies all the available electrically operated slowing-down and stopping devices and thus brings the elevator to a stop in case of failure of the normal controlling devices. Controlling panels are usually provided with a switch covered by glass with a hammer attached for breaking the glass in case of emergency. With the glass broken, the car may be operated independently of the door and gate interlocks. SELF-LEVELING ELEVATORS are usually provided with switches which, when held in, will operate the car at slow speed and under the leveling control. This is advantageous for emergency and inspection contingencies. In some cases, a switch is provided in the controlling panel for reducing the running speed of the elevator. Automatic elevators have a switch in the controlling panel, which, when held in operating position, will eliminate the hall stops for that particular elevator, these stops, however, remaining in force until another elevator has answered them. This BY-PASS SWITCH in the car is used principally when the cars are loaded to capacity, in which case they should not stop for passengers waiting in the hall, as they cannot accommodate them. Upon taking the hand from the by-pass switch, it automatically returns to normal position and the car immediately assumes the hall stops again. With automatic elevators, a switch is provided in the control panel for reversing the elevator at any point in its travel. The usual reversal acts automatically at the terminal landings.

Voltage-control elevators are usually provided with a key switch in the operating panel for starting and stopping the motor-generator set. An illuminated jewel is provided which shows that the motor-generator set is running.

Both manual and automatic elevator-operating devices have the levers arranged so that they return to stop position in case the hand is removed. In some cases, the operating lever is provided with an additional device that operates an emergency switch in case the operator's hand is removed from the operating lever. This is principally used in connection with automatic operation.

Interlocks. Modern passenger elevators are provided with full INTERLOCKS so arranged that all hatchway doors must be closed and locked before the elevator can start, and the opening of any hatchway door will stop the elevator if it is running. Doors or gates on the cars are provided which have contacts so arranged that these doors or gates must be closed before the elevator can start. Opening of door and gate contacts will stop the elevator when it is running, except when it is automatically leveling and is within a zone a few inches either way from the landing. While elevators are sometimes operated without proper interlocks, they cannot be considered reasonably safe. The statement has been made that interlocks on elevators reduce their traffic-handling capacity. This, however, is not the case if the elevators

are safely operated, because the only gain that can be made by eliminating interlocks is by operating the elevators under unsafe conditions with the doors open. Most elevator accidents are due to elevators operating without proper interlocks.

Doors and Door-operating Devices. The most satisfactory doors for hatchway openings are hollow metal with flush outside surface. Sliding-doors are usually used for elevators, and these should be provided with suitable tracks and hangers. For high-speed operation, particularly with power, ball-bearing sheave-hangers should be employed, substantially built and rigidly attached to the doors. The hangers should be arranged to allow a little flexibility perpendicular to the direction of motion of the doors, with a minimum of lost motion between tracks and hangers. Adjustable sheaves should be provided below the tracks in order to take up lost motion. The sheaves should be hardened and ground, and the track should be straight and carefully machined and must be substantially supported. The doors should be guided at the bottom by shoes, operating in carefully machined sill slots so arranged that they will be self-cleaning, the dirt falling out through the bottom of the slot into pockets arranged so that they can be readily cleaned.

For passenger elevators in office-buildings, a door-opening 3 ft 6 in wide is most satisfactory. This opening will allow two people to pass abreast. Any smaller opening will interfere with the passenger transter at landings, and a larger opening is not desirable because the doors become too heavy and take too long to operate. Openings of 5 ft or more are used for department-stores where a large number of people transfer at a large percentage of the landings. Such doors are generally power operated.

For manually operated elevators, it is generally the practice to provide gates on the car. Automatic elevators are usually provided with solid flush doors on the cars which operate in unison with the hatchway doors. These are safer, quieter and more substantial than gates. With automatic elevators it is not necessary or even desirable that the operator should be able to look out of the car, except when it is at landings. With manual operation, the operator must have good visibility out of the car in order to make even reasonably satisfactory stops. Even with this visibility, the speed of the elevator is limited to 600 to 700 ft per min, as the operator cannot successfully gauge the position of the car in relation to the floors, or read the floor numbers, if the rated speed of the elevator is higher.

For power operation, CENTER-PARTING doors are preferable to TWO-SPEED doors if the car is sufficiently wide to permit 3 ft 6 in of opening and single-leaf doors. The so-called two-speed door, in which one panel moves at twice the speed of the other panel, has 2.5 times the kinetic energy of the center-parting door, with the same opening and the same time of door opening or closing. The center-parting doors are therefore safer, faster operating and require less maintenance than two-speed doors. They are particularly adapted to power operation. For hand operation, the two-speed door is usually as good as, or better than, the center-parting door.

Power door-operating devices have generally been pneumatically operated, but electric operators are now extensively used. PNEUMATIC OPERATORS have separate engines for every door, which means that they may require considerable attention and frequently cause shut-downs in service periods. Inspection is likely to be neglected because of overtime required. The traveling hose necessary for operating the car door or gate is also objectionable. ELECTRIC OPERATORS with a single motor and engine for operating the car

and all the hatchway doors of an elevator are very satisfactory. They require a minimum of attention and are not likely to cause shut-downs. Very little maintenance and inspection are required. It is decidedly advantageous to have power door-operators close by SPRING ACTION and not by power in order to guard against injury from passengers accidentally being struck by a closing door.

6. Cost of Elevator Service

The Cost of Elevator Service must be computed for each individual case: it includes not only the item for power consumption but the cost for operators, maintenance and insurance charges, interest and amortization, and also consideration of the number of elevators required to give a satisfactory amount of service, and the space occupied by these elevators. Where the service is intensive, it will be found that the cost of elevator-service, if all charges are taken into account and consideration given to the space saved, will be less with equipment that gives more service per unit, although the first cost of the elevators complete may be higher for the better equipment. Under favorable conditions, the cost of an automatic-elevator plant will be but little more than when manually operated elevators are used, because more elevators of the latter type will be required to give an equal amount of service to the former. In many cases it will be found practically impossible to obtain a satisfactory amount of rental space without the use of automatic elevators. The quality of automatic-elevator service is superior, which not only helps to keep the building rented but which also increases the rental value of the space. The use of automatic elevators therefore means a saving in the actual cost of the elevator-service.

7. Construction and Design of Elevator-Cabs

The design of elevator-cabs varies considerably for different conditions and in accordance with different tastes. In the modern buildings it is usual to select a style of design for the main floor hallways and to have the cabs correspond to this. METAL CABS are frequently used with baked enamel or lacquer finish. HARDWOOD CABS are used to a considerable extent for passenger elevators in high-class buildings. A combination of metal and wood is also often used. It is generally desirable to have the sides of the cab flush, and sanitary corners are used to some extent. The sides and backs of the cabs are usually provided with brass hand-rails. This is convenient for passengers and protects the finish.

The VENTILATION of cabs is important and it is usual to supply adjustable louvers in connection with openings near the bottom of the sides and back, and a grille around the top, generally of bronze or wood. It is also desirable to have fans in the top of the cab, or above it, for ventilating the car.

It is becoming common practice to provide FLOOR INDICATORS of the illuminated type in elevator cars to show the position of the car in the hatchway and the floors at which it is stopping. These are convenient for passengers and avoid mistakes and delays.

8. Signal Systems

Automatic signal-control elevators have no operator's signal in the car. The floor indicator shows the operator and the passengers the location of the car. With manually operated elevators FLASH SIGNALS are used for passenger elevators to notify the operator that a stop must be made. When

cars are close together, the signal to stop will be received by two or more cars and all these cars may stop, resulting in unnecessary delay, or the operator may neglect to stop, expecting another car to answer the call. Some separate signal systems have an automatic transfer which keeps the signal set until a car has stopped for the waiting passenger. Automatic signal-control elevators insure that the first car will stop that can accommodate the waiting passenger, and the signal always remains in force until a car has stopped to collect waiting passengers.

LIGHT AND DROP ANNUNCIATORS are used for manually operated elevators where the service is not very busy. They are also used in some cars provided for night and Sunday service so as to make it unnecessary for the elevator to make a full trip.

LANTERNS are generally provided over each hatchway door with different colored lamps for up and down indication. At the ground floor a single lantern is usually provided. These lanterns are lighted to indicate the approach of a car that is going to stop. TAP BELLS, sounding as the lantern lights, have been found advantageous in calling attention of the waiting passengers so that they will be ready to enter the car without delay when the doors open. All lanterns should give fairly strong illumination so as to call the attention of passengers promptly to the approaching car. With automatic signal-control it is always certain that when a hall lantern lights, the corresponding car will stop.

In some cases mechanical floor indicators are used, particularly at the ground floor and sometimes at other landings, to indicate the location of the car. For busy elevators with several elevators in the group this is not the usual practice at the present time, even for the ground floor. A wall panel is usually provided which indicates the position of all elevators by illuminated signals, for the information of the starter. Frequently there is provided in this panel, for each group of elevators, a row of up and down signals, one pair for each floor, which indicate the floors at which passengers are waiting; thus informing the starter of the condition of the service. In this panel, push buttons are also usually provided by means of which the starter can signal the operator with a buzzer in the car. In some cases telephones are also provided in each of the cars, at the starter's location, in the pent-houses, and sometimes in the superintendent's office.

9. Dispatching Devices

Automatic Dispatching Devices are frequently installed in busy office buildings. These devices automatically give a signal to the operator at the terminal landings to indicate when a car should leave. They are given at regular intervals in consideration of the service conditions, and the dispatching device is arranged so that these intervals may be adjusted. The type of dispatching device that gives signals for cars to leave the terminals without regard to their position in the group has the advantage that it gives a maximum traffic-handling capacity, because the whole system is not held up if one car is delayed. It also permits any car to be taken out of service without interfering with the operation of the scheduling device. There is also an advantage in providing the scheduling device with an arrangement which causes it to delay when there is no car at the landing to receive the signal, that is, in case the interval has been set too short. After the car arrives, a sufficient time is allowed to load and then it is immediately dispatched, the scheduling device afterward proceeding at the regular speed of operation.

CHAPTER XXVIII

SPECIFICATIONS * FOR THE STRUCTURAL STEEL-WORK OF BUILDINGS. DATA ON STRUCTURAL STEEL

By

ROBINS FLEMING

OF THE AMERICAN BRIDGE COMPANY, NEW YORK, N. Y.

1. General

(1) **Drawings.** The drawings forming a part of these specifications are [give number, maker, title, and date of each drawing].

(2) **Classification.** For the purpose of classification buildings are divided into two classes:

I. Mill-Buildings

II. Office-Buildings

Under Class I are included manufacturing plants, machine-shops, power-houses, rolling-mills, foundries, forge-shops, pattern and template-shops, train-sheds, pier-sheds, car-barns, roundhouses, electric-light stations, armories, and buildings of a similar character.

Under Class II are included office-buildings, hotels, apartment-houses, dwellings, public buildings (hospitals, libraries, schools, court-houses, jails), places of public assembly (churches, theaters, halls), stores, warehouses, garages, and buildings of a similar character.

(3) **Scope of Work.** It is intended that these specifications and drawings cover the structural steelwork complete for the building. Cast-iron bases are included with the structural steel. The steel-erector shall erect in place the steel framework on foundations furnished by others. Anchor-bolts, loose lintels, and material not connected with main frame of structure, are to be delivered at the site, but put in place by other contractors.

(4) **Materials to be Furnished for Buildings of Class I.** Unless specified otherwise in contract, the materials to be furnished for buildings of Class I include steel trusses, columns, purlins, bracing, floor-framing, crane-girders with rails, trolley-beams, lintels, girts, framing around door and window-openings, beams supporting tanks, elevator-framing, stair-framing, floor-plates, bunker-framing and steel lining, stairs and railings unless of an ornamental character, cast-iron bases, grillage-beams, and anchor-bolts.

The materials not furnished include ornamental ironwork and steelwork, masons' anchors, carpenters' anchors and irons, elevator sheave-beams, switches for trolley-beams, steel stacks, steel tanks, and steel reinforcement for concrete.

* The various values used in these Specifications agree generally with those used throughout the book. Any slight variations in values are due to recognized and allowable differences in engineering judgment, differences in Building Codes, etc.

(5) **Materials to be Furnished for Buildings of Class II.** Unless specified otherwise in contract, the MATERIALS TO BE FURNISHED for buildings of CLASS II include steel columns, cast-iron bases, rolled and cast-steel slabs, grillage-beams, anchor-bolts, floor-framing, roof and ceiling-framing, purlins, cornice-supports, supports for tanks, penthouse framing, bracing, and lintels.

The MATERIALS NOT FURNISHED include ornamental ironwork and steelwork, masons' anchors, terra-cotta anchors, carpenters' anchors and irons, stair-work, elevator-framing, elevator sheave-beams, steel stacks, steel tanks, light shapes supporting metal ceiling-lath, cast-iron sills and similar work, and steel reinforcement for concrete.

(6) RIVETS AND BOLTS for fastening steel to steel (but not for connecting the work of other trades) shall be furnished by the steel-contractor. Fitting-up bolts for erection are to be furnished by the contractor for erection as a part of his equipment.

(7) As soon as possible a COLUMN-FOOTING PLAN shall be sent to the purchaser, showing the location, elevation, and dimensions of all column-bases, with the location, elevation, size, and length of all anchor-bolts. The loads coming upon the column-footings from the columns shall also be given.

(8) Crane-clearance diagrams showing the CLEARANCES assumed for traveling cranes shall be furnished the purchaser at an early date.

(9) **Substitution of Material.** If the contractor wishes to substitute OTHER SHAPES OR SIZES for those called for on the drawings he may do so, subject to the approval of the engineer, provided the architectural features are maintained and proposed sections are sufficient to carry the required loads.

(10) **Work of Other Trades.** Holes conforming to the usual standards of fabrication shall be punched in the steel for attaching the work of other trades, provided their location is given while the working drawings are being made.

(11) **Working Drawings.** WORKING OR SHOP DRAWINGS shall be made by the steel-contractor, and when requested, prints in duplicate sent to the purchaser or his engineer for approval. The engineer's approval of drawings shall cover general design, strength and type of details. The engineer shall not be held responsible for the fit of work at the site. If, to expedite delivery, or for any other reason, he waives the approval of drawings, the contractor will not be relieved of responsibility for errors or omissions due to neglect or oversight on his (the contractor's) part.

(12) All work shall conform to local or state ORDINANCES AND REGULATIONS.

2. Material *

(13) **Properties and Tests.** All parts of the metallic structure shall be of ROLLED STEEL, except column-bases, bearing plates, or minor details, which may be of CAST IRON or CAST STEEL.

(14) Structural steel may be made by either the BESSEMER PROCESS or the OPEN-HEARTH PROCESS; except that rivet-steel, and steel for plates or angles over ¾ in in thickness which are to be punched, shall be made by the open-hearth process.

* The requirements for material are taken, by permission, from the following Standard Specifications of the American Society for Testing Materials, Philadelphia, Pa.: Standard Specifications for Structural Steel for Buildings (A9–29), Standard Specifications for Gray Iron Castings (A48–29), and Standard Specifications for Steel Castings (A27–24).

(15) Structural steel, if made by the Bessemer process, shall contain not more than 0.10%; and if by the open-hearth process, not more than 0.06% of PHOSPHORUS. Rivet-steel shall not contain more than 0.06% of PHOSPHORUS nor more than 0.045% of SULPHUR.

(16) Structural steel shall have an ULTIMATE TENSILE STRENGTH of from 55 000 to 65 000 lb, and rivet-steel from 46 000 to 56 000 lb per sq in of cross-section. The MINIMUM YIELD-POINT as determined by the drop of the beam of the testing-machine shall be one half of the ultimate tensile strength.

(17) The MINIMUM PERCENTAGE OF ELONGATION in 8 in shall be 1 400 000 divided by the ultimate tensile strength. For structural steel over ¾ in in thickness, a deduction of 0.25 from the above percentage of elongation in 8 in shall be made for each increase of 1/32 in in thickness above ¾ in, to a minimum of 18%. For structural steel under 5/16 in in thickness, a deduction of 1.25 from the above percentage of elongation shall be made for each decrease of 1/32 in in thickness below 5/16 in.

(18) TEST-SPECIMENS for plates, shapes, and bars shall bend cold through 180° without cracking on the outside of the bent portion, as follows: For material ¾ in or under in thickness, flat on itself; for material over ¾ in, to and including 1¼ in in thickness, around a pin the diameter of which is equal to the thickness of the specimen; and for material over 1¼ in in thickness, around a pin the diameter of which is equal to twice the thickness of the specimen. The TEST-SPECIMEN for rivet-steel shall bend cold through 180°, flat on itself, without cracking on the outside of the bent portion.

(19) IRON CASTINGS shall be of tough gray iron, true to pattern, free from cracks, flaws, and excessive shrinkage. The SULPHUR-CONTENTS shall be not over 0.08% for light castings, 0.10% for medium castings, and 0.12% for heavy castings.

A TEST-BAR 1.2 in in diameter and 18 in long, placed upon supports 12 in apart and tested under a centrally applied load, shall conform to the following requirements:

MINIMUM APPLIED LOAD, 1 500 lb for light, 1 750 lb for medium, and 2 000 lb for heavy castings;

MINIMUM DEFLECTION AT CENTER, 0.20 in.

Light castings shall be able to withstand an ultimate tension of 18 000, medium castings 21 000, and heavy castings 24 000 lb per sq in. Castings having any section less than ½ in thick shall be known as LIGHT CASTINGS. Castings in which no section is less than 2 in thick shall be known as HEAVY CASTINGS. MEDIUM CASTINGS are those not included in the above classification.

(20) STEEL CASTINGS for ordinary use, not annealed, shall contain not more than 0.45% of CARBON, nor more than 0.06% of PHOSPHORUS. They shall substantially conform to the sizes and shapes of the patterns, be made in a workmanlike manner, and be free from injurious defects.

3. Loads

(21) **Roof-Loads.** ROOF-TRUSSES AND COLUMNS shall be designed to carry a uniform load per square foot of exposed roof-surface, applied vertically. This load includes the weight of the structure, the snow, and the wind. For spans up to and including 90 ft, and in climates corresponding to that of New York, the total minimum uniform load in pounds per square foot of roof-surface for different kinds of covering shall be taken as follows:

	lb per sq ft
Corrugated metal	40
Gravel or composition on wood sheathing	50
Slate on boards	50
Tile on steel purlins	55
Gravel or composition on cinder concrete	65
Gravel or composition on stone concrete	75
Slate or tile on cinder concrete	70
Slate or tile on stone concrete	80
Fire-proof buildings of CLASS II, where slope is less than 2 in per ft	90

(22) For roof-spans over 90 ft, the above-cited loads shall be increased 1% for every 2-ft increase of span.

(23) For roofs in climates where snow is excessive, from 5 to 10 lb per sq ft shall be added, and in climates where there is not liable to be snow, 10 lb per sq ft may be deducted from the foregoing loads.

(24) If a ceiling is carried by the roof-framing, the ceiling-load shall be assumed at not less than 10 lb per sq ft.

(25) If SHAFTING is carried by the bottom chord, the load at the shaft shall be assumed at not less than 2 000 lb for light shafting, 4 000 lb for ordinary shafting, and 6 000 lb for heavy shafting. Unless the shafting is definitely located these loads shall be considered as liable to be concentrated at any point of the bottom chord.

(26) In designing PURLINS carrying roof-covering only, the loads in Paragraphs (21) and (23) may be decreased 5 lb and considered normal to the roof. When the pitch of the roof is more than from 2½ in to 1 ft, tie-rods shall be used between the purlins.

(27) SPECIAL LOADINGS, such as tanks or elevator-supports above the roof, and hoists or trolleys on the bottom chord, shall be taken into consideration.

(28) FLAT ROOFS used as places of public assembly or for storage-purposes shall be considered as floors.

(29) **Floor Loads.** Floor loads consist of DEAD LOADS and LIVE LOADS. The dead load is composed of the weight of the floor-construction and of any permanent wall resting upon it. In designing floor-beams and girders for fire-proof construction the dead load shall be assumed at not less than 70 lb per sq ft. Partitions of wooden studding or of hollow tile, not more than 4 in thick, may be considered as part of the live load.

(30) Unless governed by a local or state building code, buildings of CLASS I shall be designed for MINIMUM LIVE LOADS, in pounds per square foot of floor-area, as follows:

	lb per sq ft
Mold-lofts, pattern and template-shops	60
Machine-shops, light machinery	120
Machine-shops, heavy machinery	150 to 200
Factories	150 to 200
Foundries, charging-floor	300 to 800
Power-houses	200

(31) BUILDINGS FOR SPECIAL INDUSTRIES shall be designed for the loadings incident to those industries.

(32) Provision shall be made for the SUPPORT OF MACHINERY, engines, boilers, tanks, and other concentrated loads, when carried by the steel construction.

(33) **Crane-Loads.** Loads due to electric TRAVELING CRANES shall be increased 25% to provide for the effects of impact. For hand-power cranes the impact may be taken at 10%. For two cranes in action on the same girder, no impact need be added, provided the stress obtained is larger than the stress due to a single crane with impact. In addition to the vertical loads the top flanges of crane-girders shall be designed to resist a transverse horizontal thrust on each carriage, applied at the wheels, of 10% of the lifting capacity of the crane. The breaking-force due to stopping the crane shall be assumed at 20% of each wheel-load and may be considered as distributing itself along the entire length of the runway.

(34) **Coal-Bunkers.** COAL-BUNKERS shall be assumed to be surcharged when it is possible for them to be so loaded. The weight of anthracite coal shall be taken at not less than 55 lb per cu ft, and the angle of repose assumed to be 30°. The weight of bituminous coal shall be taken at not less than 50 lb per cu ft, and the angle of repose assumed to be 35°.

(35) Buildings of CLASS II shall be designed for minimum live loads, in pounds per square foot of floor-area as follows:

	lb per sq ft
Dwellings (private residences), first floor	40
Dwellings (private residences), upper floors	30
Apartment-houses, first floor	50
Apartment-houses, upper floors	40
Hotels, first floor	80
Hotels, upper floors	40
Office-buildings, first floor	100
Office-buildings, upper floors	50
School-buildings, class-rooms	50
School-buildings, assembly-rooms	75
Churches and theaters	75
Places of public assembly, where floors are used for drilling or dancing	120
Places of public assembly, where floors are not used for drilling or dancing	100
Retail stores, ordinary	100
Warehouses	200 to 500
Private garages, pleasure vehicles only	60
Public garages, pleasure vehicles only	90
Garages, motor trucks, from 1 to 3 tons capacity	150
Garages, motor trucks, from 3½ to 5 tons capacity	200

(36) CONCENTRATED LOADS shall be taken into consideration. Every steel beam in any floor used for business purposes shall be capable of sustaining a live load, concentrated at the middle, of not less than 3 000 lb. Every steel beam in the floor of a garage shall be capable of sustaining a concentrated live load of 2 000 lb, if a private garage storing pleasure vehicles only; of 3 000 lb, if a public garage storing pleasure vehicles only; of 8 000 lb, if motor trucks of from 1 to 3 tons capacity are stored; and of 12 000 lb, if motor trucks of from 3½ to 5 tons capacity are stored. Structural members carrying elevators and elevator-machinery shall be proportioned to carry twice the actual moving dead and live loads.

(37) Reduction of Live Load. The full live FLOOR LOAD shall be used in proportioning all parts of buildings designed for warehouses, and such buildings as are likely to be loaded on all floors at the same time. In other buildings the specified live load may be reduced 10% for girders carrying 200 sq ft or more of floor. For COLUMNS the load on the top floor may be assumed at 90% of the specified live load, the live load on the floor next below the top floor at 85%, and on each succeeding lower floor at correspondingly decreasing percentages, provided that on no floor shall less than 50% of the specified live load be used, and that for the lower floor the full specified live load shall be used. No reduction shall be made for any ROOF LOAD.

(38) In calculating COLUMN LOADS no reduction of floor-area shall be made for stair-wells. Stairways shall be proportioned for not less than 75 lb per sq ft of horizontal projection.

(39) Wind-Pressure. Wind shall be assumed blowing horizontally in any direction. The surface exposed to WIND-PRESSURE shall be measured vertically from the ground to the top of the structure, including the roof.

(40) When the OVERTURNING MOMENT due to wind-pressure exceeds 75% of the moment of stability the structure shall be securely anchored.

(41) All steel buildings shall be designed to carry wind-pressure to the ground by steel framework. For buildings belonging to CLASS I the wind-pressure shall be assumed at not less than 15 lb per sq ft on the side or end surface and the normal pressure on the roof corresponding to a horizontal pressure of 20 lb per sq ft wind blowing horizontally.

(42) The steel framework of fire-proof buildings belonging to CLASS II shall be designed to resist a wind-pressure of not less than 20 lb per sq ft on the sides, and the corresponding normal pressure on the roof for heights up to 300 ft. For heights greater than 300 ft 2 lb shall be added for each 100 ft or fraction thereof on the portion above the 300-ft level.

(43) The normal pressure, P_n, in pounds per square foot, on a surface inclined θ degrees to the horizontal for a horizontal wind-pressure, P, of 20 lb per sq ft, according to the DUCHEMIN FORMULA,

$$P_n = P \frac{2 \sin \theta}{1 + \sin^2 \theta}$$

is as follows:

θ	P_n lb per sq ft	Slope	θ	P_n lb per sq ft
5°	3 46	1 in to 1 ft	4° 45′ 49″	3.30
10°	6 76	2 in to 1 ft	9° 27′ 45″	6 39
15°	9 63	3 in to 1 ft	14° 2′ 10″	9 14
20°	12 25	4 in to 1 ft	18° 26′ 6″	11.50
25°	14 35	5 in to 1 ft	22° 37′ 12″	13 42
30°	16 00	6 in to 1 ft	26° 33′ 54″	14.88
35°	17 28	7 in to 1 ft	30° 15′ 24″	16 06
40°	18 20	8 in to 1 ft	33° 41′ 25″	16.95
45°	18 88	. .	.	.
45° to 90°	20 00	. . . .	. . .	.

For a pressure other than 20 lb per sq ft these values are to be changed proportionately.

Only the excess of the wind-stresses, determined by the data of this paragraph, over the wind-stresses according to Paragraph (21), need be considered. In arriving at this excess, the wind included in the total uniform roof-loads designated in Paragraph (21), shall be assumed at 5 lb per sq ft for slopes of 3 in per ft and less, and 10 lb for slopes more than 3 in per ft.

(44) CIRCULAR STEEL CHIMNEYS and TANKS shall be designed to resist a wind-pressure of not less than 20 lb per sq ft on the projected area, that is, the diameter multiplied by the height.

(45) SKY-SIGNS on tops of buildings shall be designed to withstand a wind-pressure of not less than 30 lb per sq ft of surface.

4. Stresses

(46) **Working Stresses.** In proportioning structural steel for stresses due to the COMBINED DEAD AND LIVE LOADS together with IMPACT, the working stresses in pounds per square inch of sectional area shall be not more than the following:

	lb per sq in
Tension, net section, rolled steel	18 000
Direct compression, rolled steel and steel castings	18 000
Bending on extreme fibers of rolled shapes, built sections, girders, and steel castings, net section	18 000
Bending, on extreme fibers of pins	27 000
Shear, on power-driven rivets and pins	13 500
Shear, on hand-driven rivets	10 000
Shear, on unfinished bolts	9 000
Shear, average, on webs of plate girders and rolled beams, gross section	12 000
Bearing pressure, on power-driven rivets and pins	27 000
Bearing, on hand-driven rivets	20 000
Bearing, on unfinished bolts	18 000
Tension, in rivets	12 000
Tension, in field-bolts (not anchor-bolts)	12 000
Axial compression, on gross section of columns and struts	$18\,000-70l/r$
where l is the effective length of the member, in inches, and r is the least radius of gyration of section, in inches, with a maximum of	14 000

(47) For COMBINED STRESSES due to wind and other loads the unit stresses in Paragraph (46) may be increased 33⅓% except for Paragraph (44), provided the section thus obtained is not less than that required if wind-forces are neglected.

(48) When the laterally unsupported length, l, of the compression-flange of beams and girders exceeds 12 times its width, b, the UNIT STRESS IN THE COMPRESSION-FLANGE, shall not exceed $19\,000-250l/b$.

(49) COUNTERSUNK RIVETS in plates of thickness equal to or greater than one half the diameter of rivet shall be assumed to have three fourths the value of rivets with full heads. In plates of thickness less than one half the diameter of rivet their values shall be taken as three eighths that of full-headed rivets. RIVETS WITH FLATTENED HEADS of height not less than ⅜ in, or one half the diameter of the rivet for ⅝-in rivets and less, may be assumed to have the value of corresponding rivets with full heads. Rivets with heads flattened

to less than these heights shall have countersunk holes and be regarded as countersunk rivets.

(50) The allowable pressure of COLUMN-BASES and BEARING-PLATES on masonry shall not exceed, in pounds per square inch, the following.

	per lb sq in
On brickwork, cement mortar	250
On brickwork, lime mortar	150
On Portland-cement concrete, 1 : 2 : 4 mixture	600
On Portland-cement concrete, 1 : 3 : 5 mixture	400
On rubble masonry, cement mortar	250
On rubble masonry, lime mortar	120
On first-class dimension sandstone	400
On first-class limestone	500
On first-class granite	600

5. Design

(51) **General Design-Requirements.** TRUSSES shall be riveted structures. TENSION-MEMBERS as well as COMPRESSION-MEMBERS shall be composed of rolled shapes or built-up sections. Flat bars with riveted ends shall not be used.

(52) In calculating TENSION-MEMBERS, net sections shall be used. The diameters of rivet-holes shall be assumed to be ⅛ in larger than the nominal size of the rivet. In single angles connected by one leg, the net area of the connected leg and one half that of the outstanding leg shall be considered effective.

(53) The NOMINAL SIZES OF RIVETS shall be used in calculations of their values.

(54) In proportioning COLUMNS provision shall be made for eccentric loading.

(55) COLUMNS AND STRUTS with direct loads of 40 000 lb or less, when spliced, shall have the entire load transmitted through splice-plates.

(56) COLUMN-SPLICES shall be designed to resist the bending-stresses, and to make the columns practically continuous for their whole length.

(57) Members subject to REVERSAL OF STRESS from moving loads shall be proportioned for the stress requiring the larger section, but their connections shall be proportioned for the larger stress plus one half the smaller.

(58) The EFFECTIVE LENGTH OF MAIN COMPRESSION-MEMBERS shall not exceed 120 times their least radius of gyration, and for secondary members 160 times their least radius of gyration. Any portion of the cross-section of a compression-member may be neglected in computing the radius of gyration, provided that portion is neglected in the design of the member.

(59) WHEEL-LOADS OF CRANES shall be assumed to be distributed on the top flanges of runway girders over a distance equal to the depth of the girder, with a maximum of 30 in.

(60) PLATE GIRDERS shall be proportioned either by the moment of inertia of their net section, or upon the assumption that the bending-stresses are resisted by the flanges concentrated at their centers of gravity, and that the shear is resisted by the web. When the second method is used one-eighth of the gross section of the web, if properly spliced, may be used as flange-section.

(61) WEB-PLATES OF GIRDERS shall have a thickness of not less than $\frac{1}{160}$ of the unsupported distance between flange-angles.

(62) COVER-PLATES OF GIRDERS to have their full sectional area included in flange section shall not project beyond the outer line of rivets more than 6 in or more than 8 times the thickness of the thinnest plate. Cover-plates exceeding these limits not more than 50% may have one-half the excess section included in flange section. No part of a projection beyond the outer line of rivets more than 9 in or more than 12 times the thickness of the thinnest plate shall be considered as part of the flange section.

(63) WEB-STIFFENERS, in pairs, shall be placed over bearings, at points of concentrated loadings and at intermediate points, usually not farther apart than the full depth of the girder, when the thickness of the web is less than $\frac{1}{60}$ of the unsupported distance between flange-angles.

(64) STIFFENERS under concentrated loads and over bearings shall be designed as columns, with a length equal to one-half the depth of the girder, and shall have enough rivets to properly transmit the shear. When loads are transmitted through the bearing of stiffeners, the bearing value may be assumed at 27 000 lb per sq in of section, excluding the area of the chamfered portion over fillets of flange-angles.

(65) THE DEPTH OF GIRDERS AND ROLLED BEAMS in floors shall be not less than $\frac{1}{24}$ of the span, and if used as roof-purlins shall be not less than $\frac{1}{40}$ of the span. In case of floors subject to shocks and vibrations the depth shall be limited to $\frac{1}{16}$ of the span.

(66) STEEL PURLINS shall be single rolled shapes, plate girders or lattice girders.

(67) Lateral, longitudinal, and transverse BRACING in all structures shall preferably be composed of rigid members, and shall be designed to withstand wind and other lateral forces when building is in process of erection as well as after erection.

(68) WIND-BRACING shall be provided for tall buildings by making the connection-joint between girders and columns sufficient for the bending due to side pressure as well as for the vertical load. The shear due to side pressure shall be carried to column footings. Gusset-plates, knee-braces or diagonal bracing may be used as is best suited for the building under consideration.

(69) No steel in any structural member subject to stress shall be less than $\frac{1}{4}$ in thick, except the webs of rolled beams and channels. Steel subject to the action of harmful gases or severe atmospheric conditions shall be not less than $\frac{5}{16}$ in thick.

6. Details

(70) **General Detail Requirements.** DETAILS throughout shall conform to first-class standard practice.

(71) No connection except lattice-bars shall have less than two rivets, preferably three, for better handling in fabrication.

(72) In cases where it is necessary to carry loads subject to shock by bolts in tension, CHECK-NUTS shall be used. When bolts go through beveled flanges, BEVELED WASHERS to match shall be used so that head and nut are parallel. In general, rivets and bolts in tension shall be avoided as far as practicable.

(73) ABUTTING JOINTS in compression-members faced for bearing shall be spliced sufficiently to hold the connecting members accurately in place.

(74) When two or more rolled beams are used to form a girder, they shall be connected by BOLTS AND SEPARATORS at intervals of not more than 6 ft. All beams having a depth of 12 in and more shall have at least two bolts to each separator.

(75) The MINIMUM DISTANCE BETWEEN CENTERS OF RIVET-HOLES in compression-members shall be three diameters of the rivet, and the maximum distance in the line of stress 16 times the thickness of the plate with a maximum of 6 in for plates under ½ in thick and 9 in for plates ½ in and over.

(76) The MINIMUM DISTANCE FROM THE CENTER OF ANY RIVET-HOLE TO A SHEARED EDGE shall be 1½ in for ⅞-in rivets, 1¼ in for ¾-in rivets, 1⅛ in for ⅝-in rivets, and 1 in for ½-in rivets; and to a rolled edge, 1¼, 1⅛, 1, and ⅞ in, respectively.

(77) The MAXIMUM DISTANCE FROM THE CENTER OF ANY RIVET-HOLE TO ANY EDGE shall be eight times the thickness of the plate.

(78) The PITCH OF RIVETS AT THE ENDS OF BUILT COMPRESSION-MEMBERS shall not exceed four diameters of the rivets for a length equal to one-and-one-half times the maximum width of the member.

(79) The LATTICING OF COMPRESSION-MEMBERS shall be proportioned to resist a shearing-stress equal to 2½% of the direct stress. TIE-PLATES shall be provided at each end and at intermediate points where latticing is interrupted. In main members carrying calculated stresses, the end tie-plates shall have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than half this distance. Their thickness shall be not less than 1/60 of the same distance.

7. Workmanship

(80) **General Requirements.** All WORKMANSHIP shall be first-class in every respect.

(81) Material shall be thoroughly STRAIGHTENED before being worked, by methods that will not injure it.

(82) SHEARING shall be done accurately, and all portions of the work exposed to view neatly finished.

(83) ABUTTING SURFACES OF COMPRESSION-MEMBERS, except where joints are fully spliced, shall be planed to an even bearing so as to give close contact throughout.

(84) PUNCHING shall be done accurately, but occasional inaccuracies in matching of holes may be corrected with a reamer. The diameter of the punch shall be not more than 1/16 in larger, nor that of the die ⅛ in larger than the diameter of the rivet. Rivets shall be driven by pressure-tools wherever possible.

(85) HOLES IN MATERIAL of same thickness as diameter of punch plus ⅛ in may be punched full size. Holes in greater thicknesses shall be either sub-punched and reamed or drilled from the solid.

(86) WEB-STIFFENERS OF PLATE GIRDERS under concentrated loads shall have the ends milled.

8. Painting

(87) **General Painting Requirements.** Cast iron need not be painted at the shop. Steelwork for foundations to be entirely embedded in concrete shall not be painted, but must be free of dirt, grease, or other matter which

would impair the bond of the concrete. Other steelwork shall be thoroughly cleaned and given one coat of paint before shipment. Surfaces that are in contact after members are shop-riveted need not be painted.

(88) Machine-finished surfaces shall be coated with white lead and tallow before shipment.

(89) After erection all structural metalwork shall be cleaned of dirt and rust and given one coat of paint of a color or shade different from that of the shop-coat.

(90) All painting at the shop and site shall be done by hand when the surface of the metal is perfectly dry. Painting shall not be done in freezing weather.

(91) Paint shall be a good quality of red lead or graphite, ground in pure linseed oil or its equivalent.

9. Inspection

(92) **General Requirements.** All inspection and tests shall be made at the option and expense of the purchaser.

(93) If material is tested at the mills, the necessary number of test-pieces and the use of a testing-machine shall be furnished free of charge by the steel-contractor.

(94) The purchaser or his representative shall have free access at all times to the mills where material is rolled and to the shops where it is fabricated. In ample time for his needs he shall be given dates of mill and shop-operations and furnished with complete working drawings.

10. Erection

(95) **General Requirements.** The structural steel and iron, except anchor-bolts, loose lintels, and material not connected with the main frame of the structure, shall be erected by the steel-contractor on foundations furnished by the purchaser.

(96) Care shall be taken that all steelwork is level and plumb before bolting or riveting.

(97) Proper provision shall be made for resisting stresses due to erection operations.

(98) In general, field-connections shall be riveted, but connections of the following classes may be bolted:

(*a*) Light subordinate framing, such as purlins, monitor and skylight-framing, girts, platforms, stair-framing, partitions, ceilings, and penthouses;

(*b*) Ordinary framing of beams to beams, and beams to girders;

(*c*) Connections not subject to direct shearing-stress.

All connections, however, affected by loads that cause undue vibration, shall be riveted. One-story buildings, not subjected to excessive wind-pressure or not supporting heavy concentrated loads, shafting, or moving loads, may be bolted throughout. The threaded part of a bolt shall not be so long that the bearing value of the unthreaded portion is reduced to less than the shearing value of the bolt. Washers shall be used under nuts wherever needed.

(99) Drift-pins shall be used only to bring parts together. Unfair holes shall be made to match by reaming.

(100) After finishing the work the erector shall remove his equipment and all rubbish resulting from his operations.

DATA ON STRUCTURAL STEEL *

Estimating the Cost of Structural Steel for Buildings

Structural steel for buildings is commonly made up of I beams, channels, angles and plates, which may be used as single beams or braces, or built into riveted girders, columns, or trusses. Rolled columns are largely replacing built-up columns. The cost of the completed steelwork is made up of the following items:

(1) Cost of the plain steel at the mill, plus freight and dealers' profits.

(2) Extras for cutting, punching, fitting and assembling into girders, columns, or trusses.

(3) Cost of the fittings, such as connection-angles, gusset-plates, etc.

(4) Shop-painting.

(5) Cost of erection at the building.

(6) Painting after erection.

Base-Price of Steel For orders of any considerable size, the cost of plain steel is based on the price at the mills plus the freight to the point of delivery.

The BASE-PRICE, free on board cars at Pittsburgh, Pa. (1930), is about $1.95 per 100 lb for I beams and channels 15 in and less, and for angles from 3 to 6 in.

I beams over 15 in, cost 10 cts per 100 lb extra, and tees over 3 in, 5 cts extra.

For angles and channels under 3 in, the base is $1.95 at Pittsburgh.
For angles, over 6 in, $1.95 + $0.10.†
For H beams, $1.95 + $0.10.
For bulb angles, $1.95 + $0.30.‡
For rolled floor plate, $1.95 + $1.75.§
For plates, structural, the base is, $1.95.
For rolled columns and girders, base price, $1.95 plus extras, $0.05 to $0.20.
For plates, flange, the base is $1.95 + $0.15.¶
For corrugated steel, painted, No. 22, $2.95 + $0.25.¶
For corrugated steel, galvanized, No. 22, $3.60.
For steel sheets, black, Nos. 10 and 11, $2.20.
For steel sheets, galvanized, Nos. 10 and 11, $3.05.
For steel sheets, black, No. 22, $2.90.
For steel sheets, galvanized, No. 22, $3.55.
For bar-iron, the base is $1.95.
For rivets, $2.50.
For steel bars, $1.95.

* Valuable data were contributed for this section by Associate Editor Robins Fleming.

† $1.95 + $0.10 means a base-price of $1.95 and an extra $0.10.

‡ $1.95 + $0.30 means a base-price of $1.95 and an extra of $0.30.

§ $1.95 + $1.75 means a base-price of $1.95 and an extra of $1.75; the same with $1.95 + $0.15, etc. Corrugated steel, painted, is usually quoted at a base-price plus an extra for painting. At present (1930) it is $3.00 + $0.25 for gauge No. 24.

¶ $1.95 + $0.15 means a base-price of $1.95 and an extra of $0.15; the same with $2.95 + $0.25, etc. Corrugated steel, painted, is usually quoted at a base-price plus an extra for painting. At present (1930) it is $3.00 + $0.25.

Freight-Rates (November, 1929) in car-load lots are, per hundred pounds:

Pittsburgh	to Albany, N. Y.	34.0 cts
	to Baltimore	31.0 cts
	to Boston	36.0 cts
	to Buffalo, N. Y.	27.0 cts
	to Chicago	34.0 cts
	to Cincinnati	27.0 cts
	to Cleveland	19.0 cts
	to Columbus, O.	25.5 cts
	to Denver	115.0 cts
Pittsburgh	to Louisville	32.0 cts
	to New York	34.0 cts
	to Norfolk, Va.	38.0 cts
	to Philadelphia	32.0 cts
	to Richmond, Va.	38.0 cts
	to Rochester, N. Y.	26.5 cts
	to St. Louis	40.5 cts
	to Washington, D. C.	31.0 cts

On account of the expense of carrying beams in stock, local dealers usually charge from ½ to 1½ cts a pound, extra, on orders supplied from stock.

Standard Classification of Extras. These lists are for STEEL BARS AND SMALL SHAPES, and the extras are added to the BASE-PRICES for each 100 pounds. This standard classification was adopted in 1928, by the Carnegie Steel Company.

Specification and Inspection

Hull-material, subject to United States Navy Department specifications for medium or soft steel	$0.10
High-tensile hull-steel (except rivet-rods) subject to United States Navy Department specifications	1.00

Charges for other than mill-inspection, such as Lloyd's or American Bureau of Shipping, for buyer's account.

Quantity-Differentials

All specifications for less than 2 000 lb of a size will be subject to the following extras, the total weight of a size ordered to determine the extra, regardless of length and regardless of exact quantity actually shipped:

Quantities less than 2 000 lb, but not less than 1 000 lb	$0.15
Quantities less than 1 000 lb	0.35

Straightening

Machine-straightening prices furnished on application.

Machine-Cutting to Specified Lengths, Rounds and Squares, 1½ Inches and Larger

Machine-cutting to lengths over 48 in	$0.20
Machine-cutting to lengths over 24 in to 48 in, inclusive	0.30
Machine-cutting to lengths over 12 in to 24 in, inclusive	0.40
Machine-cutting to lengths of 12 in and less, extra will be furnished on application, but will not be less than	0.70

The above extras apply only to 0.50% carbon and under. Extras for machine-cutting over 0.50% carbon will be furnished on application.

Extras for machine-cutting Rounds and Squares under 1½ in, Flats, etc., will be furnished on application.

Cutting to Specified Lengths, Other than Machine-Cutting

Cutting to lengths of over 60 in	No charge
Cutting to lengths over 48 in to 60 in, inclusive	$0.10
Cutting to lengths over 24 in to 48 in, inclusive	0.15
Cutting to lengths over 12 in to 24 in, inclusive	0.30
Cutting to lengths of 12 in and less, extra will be furnished on application, but will not be less than	0.40

Cost of Erecting. For erecting ordinary beams and columns in buildings having masonry walls the cost of erection should not exceed $20 per ton when there are bolted connections, and it will sometimes be as low as $13 per ton. For erecting the steelwork of skeleton buildings having riveted connections it is customary to allow $18 per ton.

Cost of Painting. The usual charge for shop-painting is about $3 per ton, but if done in accordance with the specification in Article 8 it would exceed this amount. For painting one additional coat after erection, allow about $3.50 per ton.

Roof-Trusses. In lots of at least six, the shop-cost of ordinary roof-trusses in which the ends of the members are cut off at right-angles is about as follows: * Trusses weighing 1 000 lb each, from $2.00 to $3.50 per 100 lb; trusses weighing 1 500 lb each, from $2.00 to $2.50 per 100 lb; trusses weighing 2 500 lb each, from $1.50 to $2.50 per 100 lb; and trusses weighing from 3 500 to 7 500 lb, from $1.25 to $2.00 per 100 lb. Pin-connected trusses cost from 10 to 30 cts per 100 lb more than riveted trusses.*

Steel Mill-Buildings. The average shop-cost for the frames of steel mill-buildings, including draughting, is about $30 per ton, and the cost of erection from $20 to $35 per ton.*

Cost of Drafting. Details for church and court-house roofs having hips and valleys cost from $10 to $20 per ton; details for ordinary mill-buildings cost from $6 to $12 per ton. The cost of making shop-drawings varies greatly with the character of the construction of the buildings, and with the accuracy of the architect's drawings. The average costs per ton of steel, for making shop-drawings are about as follows:

For entire skeleton construction, in which the loads are all carried to the foundations by the steel columns, $4.00.

For the interior parts which are supported on steel columns, when the outside walls carry the floor-loads and their own weight, $3.50.

For the interior parts which are supported on cast-iron columns, when the outside walls carry the floor-loads and their own weight, $2.50.

For construction without columns, and in which the floor-beams rest on masonry walls, $2.50.

For buildings in which roof-trusses supported by columns comprise the greater part of the construction, $7.00.

For buildings in which roof-trusses on masonry walls comprise the greater part of the construction, $4.00.

For mill-buildings, average, $9.00.

* If there is little duplication of parts or if manual labor enters into the fabrication to any great extent the costs given will be increased.

For manufacturing or shop-buildings, with flat roofs, and one story in height, $3.00.
For alterations, additions, remodeling, which require measurements before details and shop-drawings can be made, $12.00.*

Weights of Steel in Buildings †

Factors Affecting the Weights of Steel Structures are many. Municipal building codes specify loads and working stresses which vary greatly. The architectural features to be followed also play an important part. In mill buildings the weight is affected by the kind of roofing and siding used, capacity of cranes, spacing of trusses and columns, special loadings and often the allowable minimum thickness of metal.

The weight per square foot of floor-surface or per cubic foot of volume of a structure already built should not be assumed as the weight of a proposed structure unless all the conditions which govern the one are found in the other. Approximate estimates are often wanted before plans or designs have been started. These should be made only by experienced engineers.

Weights of Steel in a Number of Structures are given in the following notes: Among prominent New York buildings the Woolworth Building towering 792 ft above the sidewalk with a ground-area of 31 000 sq ft weighs 3.00 lb per cu ft of volume; the Equitable Building, 159 ft by 308 ft, 41 tiers of framed beams, weighs 37 lb per sq ft of framed area and 2.55 lb per cu ft of volume; the 39-story Bankers' Trust Building with an area of 9 000 sq ft, 3.1 lb per cu ft; the 25-story Municipal Building with an area of 42 700 sq ft, 3.6 lb; the 25-story Hotel McAlpin with an area of 31 000 sq ft, 2.0 lb. Of recent buildings, the New York Telephone Company Building, 192 ft 4 in by 254 ft between enclosure walls, 29 floors above street-level, with numerous offsets, 34.3 lb per sq ft of framed area; the New York Life Insurance Company Building, 179 ft by 183 ft, 30 stories above first floor and 3 basements below, 32.9 lb per sq ft of framed area and 2.35 lb per cu ft of volume. Of the tall apartment-houses springing up on every side, one, quite complicated in design, 20 stories high and about 40 000 sq ft of ground area, weighs 1.6 lb per cu ft. An apartment hotel of nearly the same area, simple in design and 28 stories high, weighs 1.5 lb per cu ft.

The Fidelity-Philadelphia Trust Company, in Philadelphia, 30 stories above the street and 3 basements below, weighs 2.48 lb per cu ft for its 11 500 000 cu ft of volume. The 16-story Benjamin Franklin Hotel of the same city, 185 ft by 230 ft, weighs 20.4 lb per sq ft of framed area and 1.62 lb per cu ft of volume. The 22-story Rittenhouse Plaza Apartments, 127 ft by 136 ft, weighs 17.15 lb per sq ft and 1.77 lb per cu ft.

The Convention Hall of Atlantic City, one of the largest buildings of its type in the world, finished in 1929, is 350 ft by 675 ft. The trusses have a clear span from wall to wall with center pins 135 ft above the floor. It is claimed that 40 000 persons can be seated in the auditorium and 5 000 in the ball-room. The weight of steel was 11 050 tons, or 93.2 lb per sq ft of ground area.

For buildings higher than 8 or 10 stories the total weight of steel in floors will increase in direct proportion to the stories, while the weight of columns will increase more rapidly. In office buildings of 6 to 12 stories the average

* This cost of $12.00 includes the cost of taking measurements. This generally has to be done by the contractor.

† From notes by Robins Fleming.

weight of the steel framing is probably 1.8 lb per cu ft of volume; in hotels 1.4 lb. In higher buildings the average weight per cubic foot will be more. Buildings with setbacks require heavy girders, and it is impracticable to give a probable weight except for each individual case.

The following data were collected for the previous edition of the Handbook; it is reprinted as there given. Under recent specifications and codes a saving of from 2% to 6% could be effected in some instances due to lighter loads and higher unit stresses being allowable.

Armories. The three-hinged arches with roof-framing of an armory in Brooklyn, 191 by 300 ft in area, weighs 15 5 lb per sq ft of ground area. An armory in Buffalo, 233 by 335 ft, weighs 18 3 lb. The steelwork of the Kingsbridge Armory, New York City, 289 by 590 ft, said to cover the largest drill-hall in the world, weighs about 90 lb per sq ft, of which one-half is roof and one-half floor and miscellaneous framing.

Boiler-Shops. Sizes and weights per square foot of a few boiler-shops are as follows: 167 by 336 ft, three aisles, floor in center and cranes in outer aisles, concrete roof and sides, steel purlins and girts, 23.9 lb; 124 by 300 ft, three aisles with 15, 25 and 50-ton cranes, respectively, steel purlins and brick walls between columns, 36 lb; 74 by 160 ft, 10-ton crane in center aisle, single beams over side aisles to carry roof, galvanized corrugated-steel covering and siding, 16.15 lb; 85 by 140 ft, two aisles, one with crane, 20.8 lb; 94 by 97 ft, two aisles, one with crane, 26.3 lb.

Car-Barns. The steel roof-trusses and bracing of a car-barn 100 by 154 ft, wood purlins, brick walls, weighs 6.2 lb per sq ft. Another car-barn, 44 by 270 ft, corrugated-steel roof, and sides on steel purlins and girts, 9.15 lb. Another, 100 by 154 ft, four aisles, concrete roof on steel purlins, 11.8 lb.

Cement-Plants. Four cement-plants with ground-areas of 58 000, 73 000, 83 000 and 128 000 sq ft, respectively, weigh, respectively, 23.6, 22.0, 23.5, and 17.5 lb. These weights are the averages of the buildings that usually form a cement-plant. The individual buildings vary from 10 lb for an engine-room to 36.7 lb for a clinker-grinding room.

Coal-Bunkers. The weights of six coal-bunkers of the suspended type and with capacities of from 350 to 1 000 tons, range from 128 to 234 lb per ton of capacity, the average being 204 lb. A system of rectangular pockets to store 7 500 tons (10 ft 6 in from ground to valves) weighs 158.3 lb per ton of capacity. In all cases the weights of supports but not of roofs are included. A 35 by 70-ft coal-bin supported on plate girders with a capacity of 1 000 tons weighs 240 lb per ton of capacity, including the roof-trusses that carried the conveyor.

Forge-Shops. The steel framing for the roof of a forge-shop 83 by 126 ft, with no columns and no cranes, covered with corrugated steel on steel purlins, weighs 11.1 lb per sq ft of ground-area. A forge-shop 220 by 240 ft, four aisles, each with crane-runways, composition roofing, concrete sides, steel purlins and girts, weighs 24.6 lb. A forge-shop 110 by 425 ft for heavy work, 47 ft 6 in to bottom chord, two aisles each with a 50-ton crane, tile roof, glass and brick sides, weighs 40 lb.

Foundries. A pipe-foundry, 50 by 150 ft, slate covering, wooden purlins, brick walls, 15-ton crane, weighs 11.35 lb per sq ft. A similar one for the same company, 45 by 82 ft, with a 30-ton crane, weighs 17.23 lb. A foundry, 71 by 180 ft, one center aisle, with light crane, lean-to each side, corrugated-steel roof and sides, weighs 14.8 lb. A foundry, 150 by 290 ft, for a pump-company, four aisles with 20-ton crane in one aisle. wooden purlins. two 40

by 50-ft charging-floors of concrete on steel beams, weighs 13.9 lb. A foundry, 116 by 252 ft, equipped for heavy work, 60-ft center aisle, two side aisles, 28-ft charging-floor, storage-platform, weighs 38.9 lb.

Machine-Shops. A machine-shop, 90 by 328 ft, for heavy work, one center aisle 40 ft wide with 25-ton crane, each side aisle 25 ft wide with gallery-floor and 5-ton crane underneath, tile roof on steel purlins, brick and glass sides, weighs 43 lb per sq ft of ground-area. A two-story machine-shop, 69 by 422 ft, three aisles, light cranes in lower story, composition roof, steel purlins, concrete sides, weighs 35.15 lb. A one-story building, 75 by 300 ft, 20 ft to bottom chord, shafting, corrugated-steel roofing and siding, weighs 13.0 lb. Another one-story building, 70 by 100 ft, 18 ft to bottom chord, shafting, concrete roof on trusses 10 ft apart, no purlins, weighs 13.88 lb. In addition, the steel framing for the Hy-rib sides of this building weighs 3.44 lb per sq ft of vertical surface. A machine-shop, 116 by 252 ft, 60 ft center aisle, with upper 10-ton-crane runway and lower 25-ton-crane runway, two side aisles 28 ft wide with traveling jib-cranes, weighs 33 lb.

Rolling-Mills. A rolling-mill, 93 by 186 ft, corrugated-steel roof and sides, weighs 17.6 lb per sq ft. Another, 170 by 384 ft, two aisles each with 5-ton cranes, saw-tooth roof-trusses on longitudinal girders, concrete slabs on steel purlins, brick walls between columns, weighs 17.5 lb. A similar building for shop-purposes weighs 18.62 lb.

Paper-Mills. The entire structural steel for three paper-mills weighs respectively, 18.4, 20.6 and 21.4 lb per sq ft of area. All roof-trusses are of the flat type, spaced 8 ft apart in the first and third, and 16 ft in the second.

Power-Houses. A power-house, 44 by 186 ft, 49 ft to bottom chord, 60-ton crane, tile roof on steel purlins, brick walls between columns, weighs 50 lb per sq ft. Another, 53 by 270 ft, 33 ft to bottom chord, 20-ton crane, tile roof on steel purlins, brick walls and sash between columns, weighs 39.6 lb. Another, 120 by 96 ft, one aisle for boiler-room and one with 10-ton crane for engine-room, steel purlins for concrete roof-covering, brick walls between columns, weighs 17.8 lb.

Train-Shed. The train-shed, 390 by 815 ft, of the Central Railroad of New Jersey in Jersey City, is a series of concrete and steel umbrellas, of the Bush-type. The structural steel weighs 17 lb per sq ft of area.

Three Industrial Plants. In one of the plants of a great industrial corporation a two-story shop, 51 by 380 ft, weighs 28 lb per sq ft of ground-area; a three-story shop, 80 by 420 ft, 37.9 lb; a three-story shop, 80 by 300 ft, 46.3 lb; a three-story shop, 80 by 630 ft, 67.5 lb; a four-story shop, 77 by 140 ft, 66.6 lb; a foundry, 121 by 150 ft, 40 5 lb. In another plant of the same corporation, a three-story machine-shop, 80 by 510 ft, weighs 84.3 lb; a five-story office-building, 49 by 243 ft, 70.3 lb; a power-house, 55 by 120 ft, 37.5 lb; a blacksmith-shop, 81 by 200 ft, 15 6 lb. In a plant of another corporation, a boiler-house, 50 by 94 ft, weighs 23.3 lb; a furnace-building, 60 by 160 ft, 25.1 lb; a rolling-mill, 80 by 80 ft, 24.4 ft; a rod-mill, 243 by 220 ft, 28.1 lb.

Cost of Merchant Steel. The cost of merchant iron and steel of all kinds is based on a certain size of each particular shape, which is taken as the BASE, and the price of all other sizes is figured at a certain extra rate above the base according to a standard CARD OF MILL-EXTRAS. The BASE-PRICE may fluctuate and be changed without notice, but the extras remain constant, and are the same in all localities. The following tables include the standard classification of extras on iron and steel bars:

Standard Classification * of Extras on Iron and Steel Bars

Adopted January 2, 1928

Rounds and Squares

Sizes	Extra per 100 lb	Sizes	Extra per 100 lb
¾ to 3 1/16 in	Base	7/32 in	$1.50
⅝ to 11/16 in	$0.10	3/16 in	2.00
9/16 in	0.15	3⅛ to 3 9/16 in	0.10
½ in	0.20	3⅝ to 4 1/16 in	0.15
7/16 in	0.30	4⅛ to 4 9/16 in	0.25
⅜ in	0.40	4⅝ to 5 1/16 in	0 35
11/32 in	0.55	5⅛ to 5 9/16 in	0.45
5/16 in	0.70	5⅝ to 6 1/16 in	0.55
9/32 in	0.85	6⅛ to 6 9/16 in	0 65
¼ in	1.00	6⅝ to 7¼ in	0 75

Flats—Extras per 100 lb

Width in inches	Thickness in inches 4 to 3 1/16	3 to 2 1/16	2 to 1 1/16	1 to 13/16	¾ to ½	7/16 to ⅜	5/16 to ¼
⅜	.	.	.	.			$1.40
7/16	..	..	.	..	..	$1.00	1.20
½	.	.	.	.		0 70	0 90
9/16	..	..	..	..	$0 50	0 50	0 75
⅝ to 11/16	.	.	.	..	0 40	0 40	0 60
¾ to 13/16	....	.	.	..	0.30	0.30	0.40
⅞ to 15/16	...	.	.	$0.20	0 20	0 20	0 30
1 to 1⅜	..	.	$0.10	base	base	base	0.15
1 7/16 to 2	..	.	0.10	base	base	base	0.15
2 1/16 to 2½	..	$0.20	0 10	base	base	base	0.15
2 9/16 to 3	..	0 20	0 10	base	base	base	0.15
3 1/16 to 3½	$0.30	0 20	0.10	base	base	base	0 15
3 9/16 to 4	0.30	0.20	0.10	base	base	base	0 15
4 1/16 to 5	0.30	0.20	0 10	base	base	base	0.15
5 1/16 to 6	0.30	0 20	0 10	base	base	base	0.15

Standard Classification † of Angles, Channels and Tees

Angles

Sizes	Extra per 100 lb
1½×1½ in and wider, but under 3 in × 3/16 in and over	$0 15
1½×1½ in and wider, but under 3 in × ⅛ in	0.25
1 ×1 to 1¼×1¼ in × 3/16 in and over	0 25
1 ×1 to 1¼×1¼ in × ⅛ in	0 30
⅞×⅞ in × 3/16 in	0.35
⅞×⅞ in × ⅛ in	0 40
¾×¾ in × 3/16 in	0.45
¾×¾ in × ⅛ in	0 60
⅝×⅝ in × ⅛ in	1.50
⅝×⅝ in × 3/32 in	2 00
½×½ in × ⅛ in	2 20
½×½ in × less than ⅛ in	2.50
3×3 in × 3/16 in	0.35
3×3 in × ⅛ in	0.50

Unequal-leg angles are subject to special prices, which will be furnished on application

* Intermediate sizes take the next higher extra. It is not customary to enforce more than one-half the "standard-card extras" for round and square bars.

† Intermediate sizes take the next higher extra.

Standard Classification * of Angles, Channels and Tees (Concluded)

Channels

Sizes	Extra per 100 lb
$1\frac{1}{2}$ in and wider, but under 3 in × $\frac{3}{16}$ in and over...	$0 25
$1\frac{1}{2}$ in and wider, but under 3 in × $\frac{1}{8}$ in	0.40
1 to $1\frac{1}{4}$ in × $\frac{3}{16}$ in and over.	0.40
1 to $1\frac{1}{4}$ in × $\frac{1}{8}$ in	0.50
1 to $1\frac{1}{4}$ in × $\frac{7}{64}$ in	0 70
$\frac{3}{4}$ and $\frac{7}{8}$ in × $\frac{3}{16}$ in and over.....	0.50
$\frac{3}{4}$ and $\frac{7}{8}$ in × $\frac{1}{8}$ in.	0.60
$\frac{3}{4}$ and $\frac{7}{8}$ in × $\frac{7}{64}$ in	0.80
$\frac{5}{8}$ in × $\frac{1}{8}$ in and over..	1.70
$\frac{5}{8}$ in × $\frac{3}{32}$ in	2.00
$\frac{1}{2}$ in × $\frac{7}{64}$ in and over	2 50
$\frac{1}{2}$ in × $\frac{5}{64}$ in...	3 00

Tees

Sizes	Extra per 100 lb
$1\frac{1}{2}$×$1\frac{1}{2}$ in and wider, but under 3 in × $\frac{3}{16}$ in and over	$0 30
1×1 to $1\frac{1}{4}$×$1\frac{1}{4}$ in × $\frac{3}{16}$ in and over..... .	0 55
1×1 to $1\frac{1}{4}$ in × $\frac{1}{8}$ in	0.70
$\frac{7}{8}$×$\frac{7}{8}$ in × $\frac{3}{16}$ in	0.70
$\frac{7}{8}$×$\frac{7}{8}$ in × $\frac{1}{8}$ in.....	0.60
$\frac{3}{4}$×$\frac{3}{4}$ in × $\frac{3}{16}$ in.....	0.90
$\frac{3}{4}$×$\frac{3}{4}$ in × $\frac{1}{8}$ in.....	1.10
$\frac{5}{8}$×$\frac{5}{8}$ in × $\frac{1}{8}$ in.....	1.80
$\frac{1}{2}$×$\frac{1}{2}$ in × $\frac{1}{8}$ in.....	2.50

Unequal-leg tees are subject to special prices, which will be furnished on application.

* Intermediate sizes take the next higher extra.

The base for car-load lots for any city may be obtained by adding the freight-rates to the base prevailing at the mills.

CHAPTER XXIX

DOMICAL AND VAULTED STRUCTURES*

By

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1. Domes *

Classification: Domical structures may be considered under two main divisions: (1) Smooth-shell domes, and (2) ribbed domes. The first division may again be divided into (*a*) domes with shells of uniform thickness, and (*b*) domes with shells of uniformly varying thickness The materials of construction of division (1) are brick, stone, concrete, and tile; and of division (2), steel, concrete, and wood. A dome may be constructed with or without a LANTERN, or with or without an OCCULUS or EYE; and in the case of ribbed domes, they may have either circular or polygonal bases.

(1) Smooth-Shell Domes

Mechanical Principles. Under this heading are considered both (*a*) domes with shells of uniform thickness, and (*b*) domes with shells of uniformly varying thickness, and also domes with or without lanterns and eyes. A dome whose shell tapers toward the top is the more stable dome. It is evident that the upper part, or crown, tends to fall in and thereby push out the lower portion; hence the lighter the upper part is in relation to the lower part, the more stable the dome. The exact actions of the INTERNAL STRESSES in a dome are difficult to determine, but a very practical solution can, however, be developed after assuming that the stresses are parallel to a surface midway between the outer and inner surfaces of the dome.

General Analysis. A dome may be imagined to consist of a number of horizontal rings of decreasing diameter, each one laid on top of another. Since the upper part tends to fall in and push out the lower part, there must be a tendency to contract each ring in the upper part and to expand each ring in the lower part. That is, there must be END-COMPRESSION on all stones (imaginary divisions in concrete) of the upper part, and END-TENSION on all stones of the lower part. The dividing line or horizontal joint between these upper and lower parts of the dome is called the JOINT OF RUPTURE. The angle made by the joint of rupture with the vertical (center of dome as apex of angle) is known as the CRITICAL ANGLE. It is evident, then, that the determination of the JOINT OF RUTPURE and the CRITICAL ANGLE determines also the points below which there is tension in the rings. By reinforcing the lower part with steel bands or rods to resist this tension, the dome can be made secure. If the dome is a FLAT DOME, that is, one in which the angle the base makes with the vertical is less than the CRITICAL ANGLE, the tension-steel must be placed at the base of the dome to resist the outward push or thrust.

* See, also, Chapters VII and VIII.

Notation and Theory (see Fig. 1):

r = mean radius of dome;
a = thickness of shell at crown;
t = thickness of shell or of ring at any point;
α = angle made with the vertical by radius to lantern-ring. Center of dome is the apex of angle. In a dome without lantern, $\alpha = 0$;
θ = angle made with the vertical by radius passing through any point in shell (in equations angles are in radians);
c = constant of variation of t with respect to arc θ;
$\frac{cr}{a}$ = a constant (based on above notation) for any dome;
ϕ = crititical angle, that is, the angle made with the vertical by the joint of rupture;
w = weight of cubic unit of masonry;
V = volume of shell of complete dome above any ring;

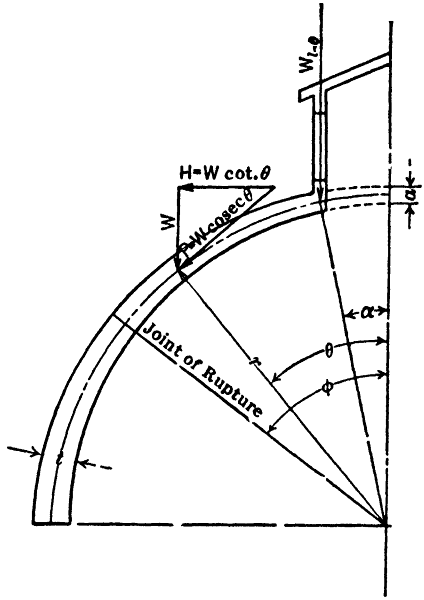


Fig. 1. Smooth-shell Concrete Dome. Analysis

$W_a = wV$ = total weight of complete shell above any ring (including the part removed for eye);
W_{l-o} = weight of lantern minus weight of shell removed for occulus or eye (W_{l-o} may be either positive or negative);
$W = W_a + W_{l-o}$;
$n = \frac{W_{l-o}}{2\pi war^2}$, a constant for any dome;
P = total tangential pressure for any ring, due to lantern and shell above that ring;
U = tangential pressure per unit-length of ring;
H = total radial horizontal pressure on any ring, due to outward push or thrust of shell above that ring;
T = hoop-tension or hoop-compression in ring, due to H;

Using this analysis and notation, the following equations are developed:

Equation (1) $t = a + cr\theta$

Equation (2) $V = 2\pi r^2[a(1 - \cos\theta) + cr(\sin\theta - \theta\cos\theta)]$

Equation (3) $W_a = wV = 2\pi war^2\left[(1-\cos\theta) + \frac{cr}{a}(\sin\theta - \theta\cos\theta)\right] = war^2z,$

in which

$$s = 2\pi\left[(1 - \cos\theta) + \frac{cr}{a}(\sin\theta - \theta\cos\theta)\right]$$

Equation (4)

$$P = 2\pi\, war^2\left[(n \operatorname{cosec}\theta + \operatorname{cosec}\theta - \cot\theta) + \frac{cr}{a}(1 - \theta\cot\theta)\right]$$

Equation (5)

$$U = war\left[\frac{n}{\sin^2\theta} + \frac{\operatorname{cosec}\theta - \cot\theta + \frac{cr}{a}(1 - \theta\cot\theta)}{\sin\theta}\right]$$

$$= war\,(S_1 + S), \quad \text{in which}$$

$$S_1 = \frac{n}{\sin^2\theta} \quad \text{and}$$

$$S = \frac{\operatorname{cosec}\theta - \cot\theta + \frac{cr}{a}(1 - \theta\cot\theta)}{\sin\theta}$$

Equation (6)

$$T = \frac{H}{2\pi} = war^2\left[n\cot\theta + (1 - \cos\theta)\cot\theta + \frac{cr}{a}(\sin\theta - \theta\cos\theta)\cot\theta\right]$$

$$= war^2(Y_1 + Y), \quad \text{in which}$$

$$Y_1 = n\cot\theta \quad \text{and}$$

$$Y = \left[(1 - \cos\theta)\cot\theta + \frac{cr}{a}(\sin\theta - \theta\cos\theta)\cot\theta\right]$$

Equation (7)

$$\frac{cr}{a} = \left[n \operatorname{cosec}^2\theta - \frac{\cos\theta - \sin^2\theta}{1 + \cos\theta}\right] \div \left[\theta\cos\theta - \frac{1 - \theta\cot\theta}{\sin\theta}\right]$$

Design and Investigation of Smooth-Shell Circular Domes. By the use of the foregoing equations any CIRCULAR DOME can be designed or investigated. The computations, however, connected with some of these equations are long and tedious, and are simplified by using curves plotted from the solutions found, after giving different values to some of their elements or factors. (See Plates I, II, III, and IV.)

Equation (7) is represented by the curves in Plate I. By the use of these curves the position of the JOINT OF RUPTURE for any dome is found by inspection when the values of $\frac{cr}{a}$ and n are known. The value of $\frac{cr}{a}$ is easily determined, as c can be found by using Equation (1) after determining or assuming a, the thickness at the crown, and t, the thickness at the base; and the value of n is found from the ratio $n = \frac{W_{l-o}}{2\pi\, war^2}$.

Equation (3) is represented by the curves in Plate II. From these curves the weight of any shell is determined.

Equation (5) is represented by the curves in Plate III. Knowing the values of n and $\frac{cr}{a}$, the values of S_1 and S are found by inspection, and hence U is easily computed.

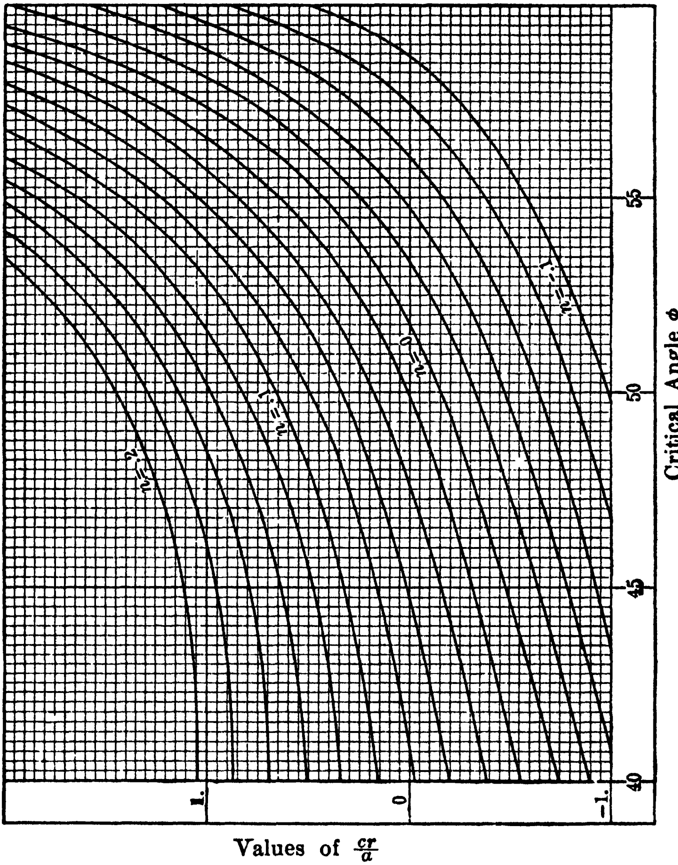


Plate I. Curves for Determination of Joint of Rupture of Domes. Based on Equation (7)

Equation (6) is represented by the curves in Plate IV. Knowing the values of n and $\frac{cr}{a}$, the values of Y_1 and Y are found, and T computed for any ring. When n equals zero, Y_1 equals zero, and the value of T depends upon Y as given in the lower curves. It will be noticed that Y increases as θ increases until the CRITICAL ANGLE for a dome without lantern, or eye ($n = 0$), is reached; that is, each successive ring increases the outward thrust, and at the CRITICAL ANGLE there is a maximum value of Y, and hence a **MAXIMUM HOOP-TENSION** T. After the CRITICAL ANGLE is passed the rings are in tension, and therefore T and Y are reduced by the tension required of the ring or masonry.

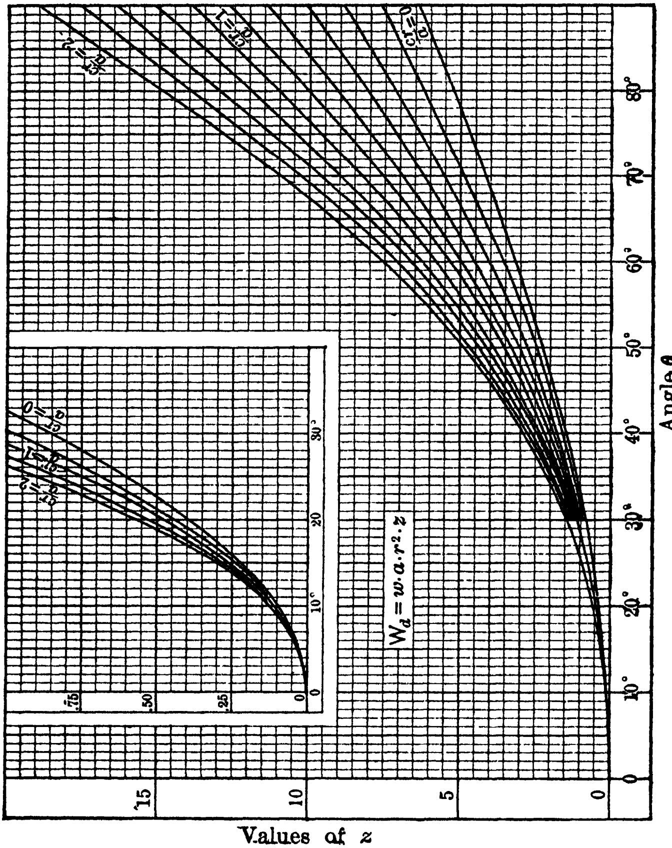


Plate II. Curves for Determination of Weight of Shell of Domes. Based on Equation (3)

The curves also indicate that the stability of a dome with a shell of uniform thickness and with no lantern is not affected by the thickness of the shell, since $c = 0$, and therefore $\frac{cr}{a} = 0$, regardless of the value of a.

Example. It is required to design a SMOOTH-SHELL REINFORCED CONCRETE DOME of 45-ft radius, and with a lantern of 10-ft radius, weighing 50 000 lb. The shell within the lantern is to be removed, forming an eye. (Fig. 2.)

Solution. Assume a crown-thickness, a, of 5 in, and a thickness, t, at the base of 8 in.

From Equation (1)

$$t = a + cr\theta, \quad \text{or} \quad \frac{cr}{a} = \frac{t - a}{a\theta}$$

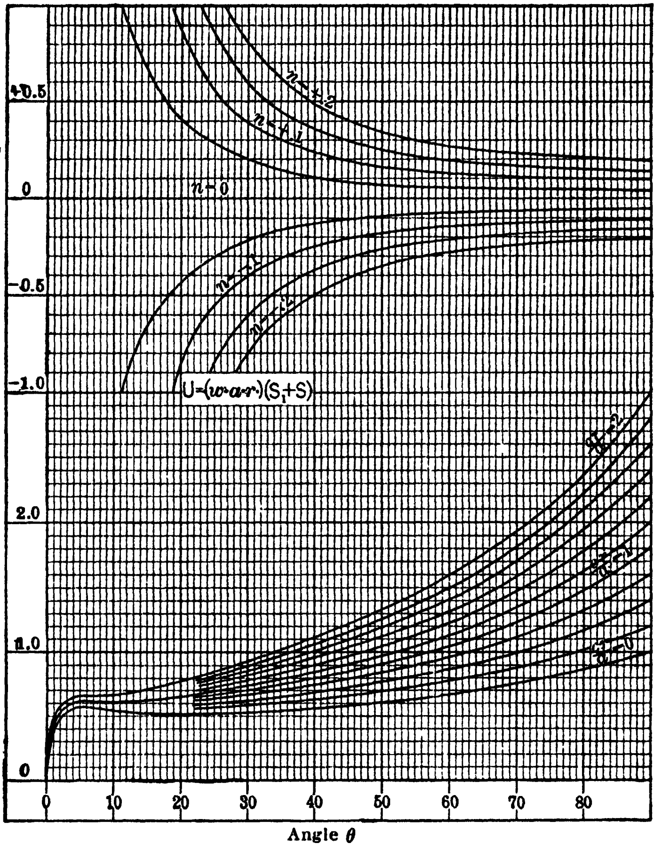


Plate III. Curves for Determination of Tangential Stress per Unit-length of Dome-ring. Based on Equation (5)

For the dome without wind-loads or snow-loads

$$\frac{cr}{a} = \frac{\frac{8}{12} - \frac{5}{12}}{\left(\frac{5}{12}\right) 1.396} = 0.43$$

The angle $\alpha = \sin^{-1} \frac{10}{45} = 12° 50'$.

From Plate II the weight of the shell removed for the eye is

$$150 \left(\frac{5}{12}\right) (45)^2 (0.165) = 20\,883 \text{ lb}$$

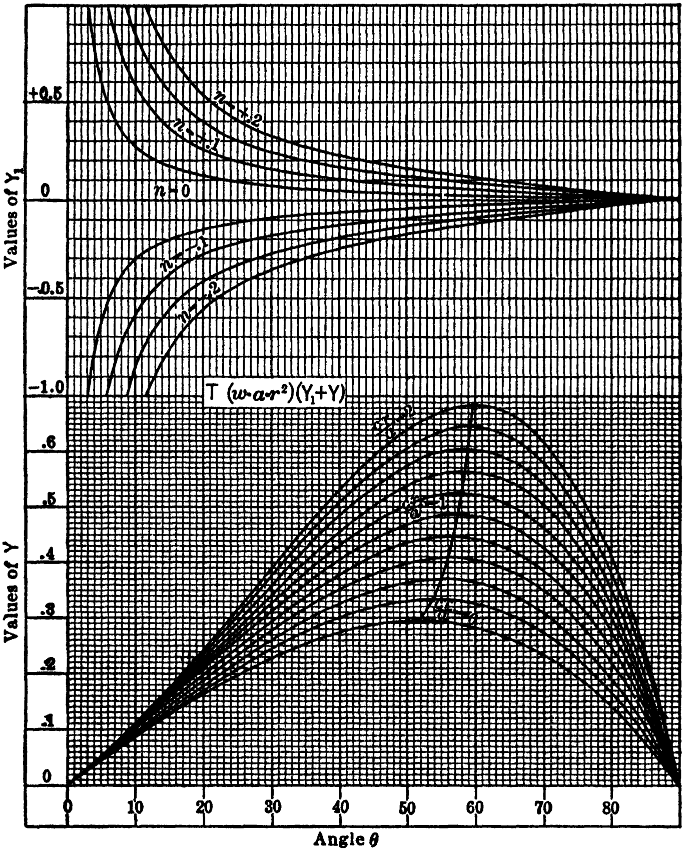


Plate IV. Curves for Determination of Hoop-tension or Hoop-compression in Dome-ring. Based on Equation (6)

Therefore

$$W_{l-o} = 50\,000 - 20\,883 = 29\,117 \text{ lb}$$

For wind-loads and snow-loads a simple and safe method of procedure is to allow a uniform load over the surface of the dome, since this load can be translated into its equivalent in inches of masonry and hence the same equations and curves used. A wind-load, for example, of 25 lb per sq ft, is equivalent to 2 in of concrete weighing 150 lb per cu ft. Hence the new a and t equal 7 and 10 in, respectively. Hence from Equation (1)

$$\frac{cr}{a} = \frac{\dfrac{10}{12} - \dfrac{7}{12}}{\left(\dfrac{7}{12}\right) 0.1396} = 0.307$$

and

$$n = \frac{W_{l-o}}{2\pi\, w a r^2} = \frac{29\ 117}{2\pi\ (150)\left(\frac{7}{12}\right)(45)^2} = 0.026^*$$

From Plate I, with $\frac{cr}{a} = 0.307$, and $n = 0.026$, the CRITICAL ANGLE is found to be 52° 35′.

From Plate IV, at the CRITICAL ANGLE for the dome with snow-load and wind-load

$$T = 150\left(\frac{7}{12}\right)(45)^2(0\ 020+0.352)$$
$$= 65\ 914 \text{ lb tension}$$

This must be resisted by steel reinforcing rods. Allowing a unit tensional stress of 16 000 lb per sq in in the steel, a total of $\frac{65\ 914}{16\ 000} = 4.12$ sq in sectional area of steel is required. At the base ($\theta = 80°$)

$$T = 150\left(\frac{7}{12}\right)(45)^2(0.005+0.186)$$
$$= 33\ 843 \text{ lb}$$

The total cross-sectional area of steel in tension at the base is $\frac{33\ 843}{16\ 000} = 2.12$ sq in, given by five round rods, each ¾ in in diameter.

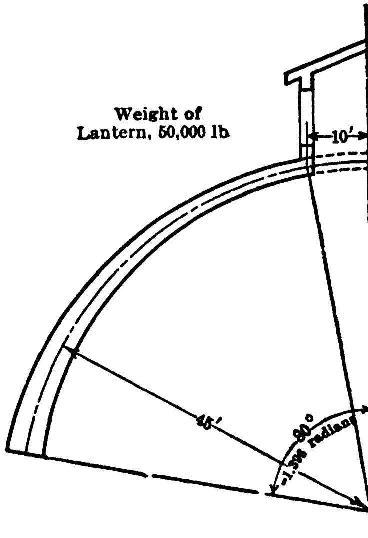


Fig. 2. Smooth-shell Concrete Dome with Lantern and Eye. See Example

The remaining required sectional area of steel, 4.12 − 2.12 = 2 sq in, must be spaced in the lower part of the dome over an angular distance of (80° − 52° 35′) = 27° 25′ = 0.4785 radian, or 0.4785 × 45 = 21.53 ft up the surface of the dome.

The assumed thickness of shell at the base was 8 in, and the thickness at the lantern-ring will be, from Equation (1),

$$t = a + cr\theta = a + a\frac{cr}{a}\theta$$

or

$$5 + 5\ (0.43)\ (0.2239 = 5.48 \text{ in}$$

Allowing 0.2 per cent of steel cross-section, horizontally and meridionally, for SECONDARY STRESSES caused by temperature-changes and possible unequal snow-loads and wind-loads, there should be 8 × 12 × 0.002 = 0.19 sq in of steel cross-section per running foot at the base, and 5.48 × 12 × 0.002 = 0.13 sq in per running foot at the lantern-ring. The spacing of the horizontal

* The snow-load on the top of the lantern is taken care of because snow-loads and wind-loads over the entire dome were included, and only the actual masonry of the eye was subtracted.

reinforcing is best found as indicated in Fig. 3. Curve *A* gives the total amount of steel necessary for SECONDARY STRESSES above any point in the cross-section of the shell. Curve *B* gives the necessary tensional resistance,

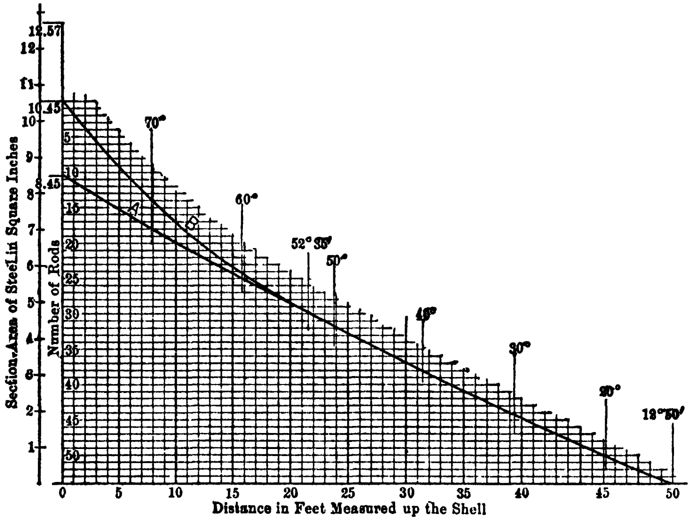


Fig. 3. Diagram for Determination of Amount of Horizontal Steel Reinforcing in Concrete Domes

using Curve *A* as a starting-point for the ordinates. Various points on the curve are easily determined. For example, for $\theta = 70°$

$$t = 5 + 5\,(0.43)\,(1.2217) = 7.63 \text{ in}$$

The cross-section of temperature-steel per foot at 70° is

$$7.63 \times 12 \times 0.002 = 0.18 \text{ sq in}$$

The total cross-section of temperature-steel above 70° is then

$$\frac{0.18 + 0.13}{2} \times (1.2217 - 0.2239)\,45 = 6.96 \text{ sq in}$$

The tension-steel cross-section (Plate IV) above 70° is

$$4.12 - \frac{(150)\,(7/12)\,(45)^2\,(0.298 + 0.009)}{16\,000} = 0.72 \text{ sq in}$$

With points determined in this manner the curves are developed.

The total cross-section of horizontal steel in the entire cross-section of the shell is

$$8.45 + 2.00 = 10.45 \text{ sq in}$$

with an additional 2.12 sq in of tension-steel at the base. If ½-in round rods are used, there will be $\dfrac{10.45}{0.1963} = 54$, required in the shell. By dividing the area

below the curve (Fig. 3) into 54 parts, the distance up from the base, where each rod should be placed, is determined. The meridional steel should be such that there will be 0 19 sq in of cross-section per foot of circumference at the base, and 0.13 sq in per foot of circumference at the lantern-ring; that is, if ½-in round rods are used, they should be spaced $\frac{0.1963}{0.19} = 1.03$ ft at the base, and $\frac{0.1963}{0.13} = 1.51$ ft at the lantern-ring. The punching-shear at the lantern-ring is equal to

$$\frac{50\,000}{(5.48)\,(2\pi\,10 \times 12)} = 12.1 \text{ lb per sq in}$$

This is well within the limit of 40 lb per sq in.

(2) Ribbed Domes

General Principles. The following discussion applies to domes of either circular or polygonal horizontal cross-sections. All steel domes are ribbed domes, and usually have from six to twenty-four ribs resting against a LANTERN-RING or SPIDER at the top. The ribs may have solid webs, perforated webs, or latticed webs, with angle or channel-flanges. The latticed angle-ribs are preferable because of their lightness. The tension-rings and compression-rings may be built similar to the main ribs, and should brace the latter through rigid gusset-connections. The diagonals are usually rods with turnbuckles for adjustment. CONCRETE-RIBBED DOMES OR WOODEN-RIBBED DOMES may be designed according to the same general principles followed for steel domes, but the diagonals are omitted and dependence for rigidity is placed on the slab-fillings between the ribs.

The Schwedler Method for the Design of Steel Domes. W. Schwedler has by simple resolution of the forces derived equations for domes, based on the forms of SURFACES OF REVOLUTION. These equations are easily checked when the forces acting through a RIB (the rib acting as a strut between the joints) and through a RING at a joint are considered. The following laws may be stated:

(1) The ribs are in maximum stress when the whole dome is loaded;

(2) A ring is in maximum tension when all of the dome above the ring is fully loaded, and in maximum compression when all of the dome below the ring and the ring itself is fully loaded;

(3) The DIAGONALS are not stressed when the dome is symmetrically loaded. The diagonals in a panel are in maximum stress when the dome on one side of a meridional plane passed through the center of that panel is fully loaded and the other side unloaded.

In Fig. 4 let

α_1, α_2, α_3, etc. = angles made by rib-sections with the horizontal;
β_1, β_2, β_3, etc. = angles made by diagonals with the ribs;
P_1, P_2, P_3, etc. = dead loads at ends of rib-sections;
L_1, L_2, L_3, etc. = live loads at ends of rib-sections;
D_1, D_2, D_3, etc. = stresses in rib-sections;
T_1, T_2, T_3, etc. = stresses in rings;
N_1, N_2, N_3, etc. = stresses in diagonals;
n = number of ribs.

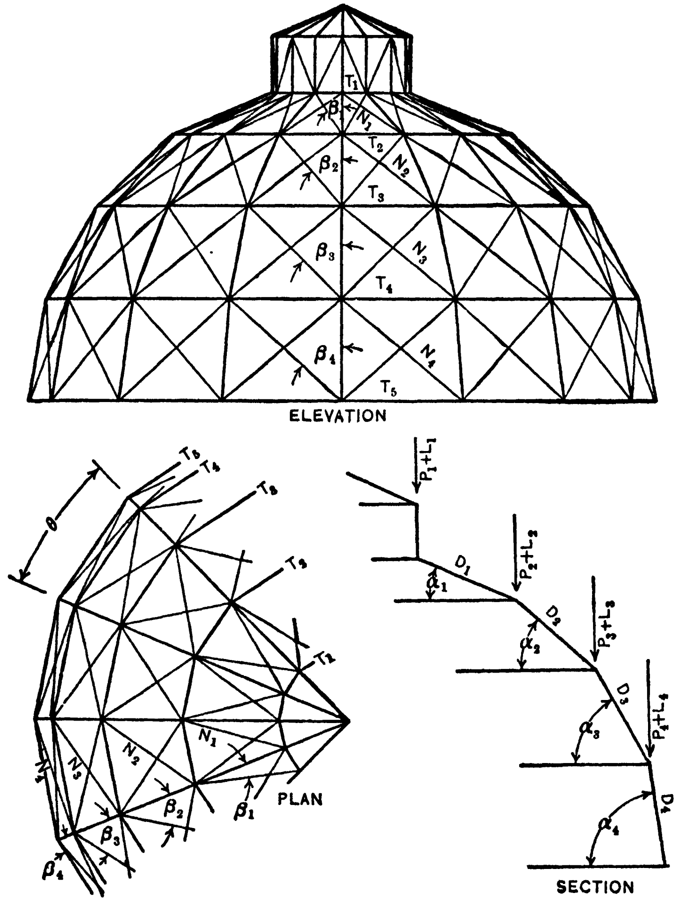


Fig. 4. Schwedler Ribbed Dome

Then

$$D_1 = \frac{P_1 + L_1}{\sin \alpha_1} \quad D_2 = \frac{(P_1 + L_1) + (P_2 + L_2)}{\sin \alpha_2}$$

$$D_3 = \frac{(P_1 + L_1) + (P_2 + L_2) + (P_3 + L_3)}{\sin \alpha_3}, \text{ etc.}$$

$$T_1 = -\frac{(P_1 + L_1) \cot \alpha_1}{2 \sin \frac{\pi}{n}} = -\frac{D_1 \cos \alpha_1}{2 \sin \frac{\pi}{n}}$$

(when the result is negative the stress is compressive).

$$\left\{\begin{array}{l} \text{Maximum } T_2 = \dfrac{(P_1 + L_1)\cot\alpha_1 - (P_1 + L_1 + P_2)\cot\alpha_2}{2\sin\dfrac{\pi}{n}} \\[2ex] \text{Minimum } T_2 = \dfrac{(P_1 \cot\alpha_1 - (P_1 + P_2 + L_2)\cot\alpha_2}{2\sin\dfrac{\pi}{n}} \end{array}\right.$$

$$\left\{\begin{array}{l} \text{Maximum } T_2 = \dfrac{(P_1+L_1+P_2+L_2)\cot\alpha_2 - (P_1+L_1+P_2+L_2+P_3)\ \cot\alpha_3}{2\sin\dfrac{\pi}{n}} \\[2ex] \text{Minimum } T_3 = \dfrac{(P_1 + P_2)\cot\alpha_2 - (P_1 + P_2 + P_3 + L_3)\cot\alpha_3}{2\sin\dfrac{\pi}{n}}, \text{ etc.} \end{array}\right.$$

$$N_1 = \frac{L_1}{2\sin\alpha_1\cos\beta_1} \quad N_2 = \frac{L_1 + L_2}{2\sin\alpha_2\cos\beta_2} \quad N_3 = \frac{L_1 + L_2 + L_3}{2\sin\alpha_3\cos\beta_3}, \text{ etc.}$$

For the stresses in the diagonals the factor 2 is introduced because Muller-Breslau found, by exact analysis, stresses only one half as large as those determined by the simple resolution of forces. The diagonals are stressed under a wind-load, and this is resisted by assuming a vertical live load equal to from 20 to 30 lb per sq ft of HORIZONTAL PROJECTION.

A GRAPHICAL METHOD, developed by E. Schmidt, for determining the stresses D_1, D_2, D_3, T_1, T_2, T_3, etc., is shown in Fig. 4A.

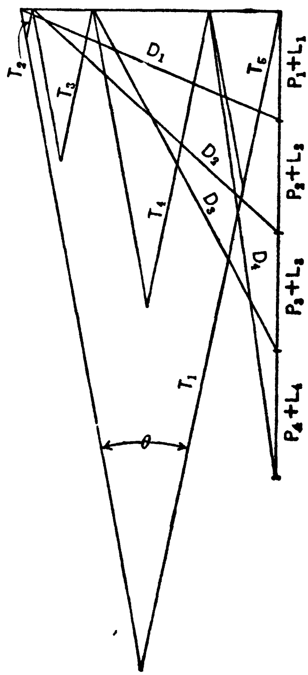


Fig. 4A. Graphical Determination of Stresses in Ribbed Domes

Weights of Steel Domes. It was found by Scharowsky, from calculations made for a large number of Schwedler FLAT DOMES varying in span from 60 to 180 ft, that the weight of the lantern and steel skeleton per sq ft of projected (covered) area is

$$w = 0.0156\,S + 4$$

where w = pounds per square foot of projected area, and S = the span, in feet. For preliminary calculations on FULL HEMISPHERICAL DOMES, the weight found by this equation should be increased from two and a half to three times.

Steel Dome of the Horticulture Palace, San Francisco, Cal. This is a SCHWEDLER HEMISPHERICAL DOME of 152-ft span, with twenty-four latticed

ribs, 36 in deep, carrying a LANTERN-RING or SPIDER at the top, and connected by eleven horizontal rings The lantern-ring is 6 ft in diameter, 36 in deep, with a solid web, and braced twice diametrically. The ribs are constructed of two 4 by 4 by 5/16-in angles at the top, two 3 by 3 by 5/16-in angles at the bottom, and a 2½ by 2½ by ¼-in angle single-lattice web. The dome-steel weighs about 17 lb per sq ft of projected area.

Concrete Ribbed Domes. In a REINFORCED-CONCRETE RIBBED DOME the number of ribs, varying from eight upward, is determined by the substructure and the size of the dome. The different steps in designing a ribbed reinforced-concrete dome are: (1) the determination of the number of ribs and rings; (2) the determination of the loading, per rib, using the required shell-thickness and the assumed rib-sizes and ring-sizes for preliminary calculations; (3) the finding of the forces acting on the ribs by the use of Schwedler's formulas; (4) the drawing of the ELASTIC CURVE for the ribs; (5) the determination of the stresses and necessary reinforcement in the ribs, rings, and slabs; (6) the adjustment of sizes and loads, so as to be on the side of safety; and (7) the reworking of the preliminary computations for the final design. The ELASTIC CURVE should always remain in the MIDDLE HALF * OF THE RIB, and should never be so far away from the center of gravity of the rib-section that the maximum compressive stress of 500 lb per sq in in the outer fiber of the rib is exceeded. The reinforcement in the RIBS should be sufficient to resist the flexural stresses due to the ECCENTRICITY OF THE ELASTIC CURVE. The reinforcement in the RINGS should be sufficient to resist the tensile stresses, and should be as straight as possible in order to avoid a sidewise stress or movement. The rings must be reinforced to resist their FLEXURE, as beams. The panel-slabs, if domical (see Smooth-Shell Domes), should be reinforced for SHRINKAGE-STRESSES and TEMPERATURE-STRESSES, in addition to the reinforcement for tension below the critical angle. If the slabs are straight they should be designed as floor-slabs, and by similar methods.

Example.† It is required to build a dome (Fig. 5) with a span of 132 ft and a rise of 31 ft 6 in. This makes the radius 85 ft. The eye is to be 12 ft in diameter. The outer surface of the dome is to be a domical slab on ribs, carrying a suspended plastered ceiling forming the inner surface.

Solution.‡ To obviate the necessity of building a complete domical form, supported from the floor below, the decision is to build a RIBBED DOME as follows: It is decided to build a central tower to temporarily carry the upper ends of the ribs, to precast these ribs, raise them into position, cast the ring of the eye, suspend the ring-forms from the ribs, pour the rings in place, and then fill in the slab-panels on forms supported from the rings and ribs.

Since the SPAN of the dome is 132 ft, the CIRCUMFERENCE at the base is 414.7 ft. Because of the suspension of the panel-forms, it is well to keep the PANEL-WIDTHS within about a 20-ft limit. Hence twenty ribs are necessary. With three INTERMEDIATE RINGS, the lower panels are approximately square. The rings are not to show below the ceiling, and hence a narrower spacing toward the top is unnecessary for appearance. For preliminary calculations,

* The usual practice has been to keep the resistance-line within the MIDDLE THIRD in arches generally. In reinforced-concrete arches and domes it may depart a small distance outside the middle third, but there should be sufficient steel to resist any tension developed. The ribs of domes differ from ordinary arches as they are rigidly braced by the rings and the slab-panels.

† The dome of this example is similar to the dome over the Hippodrome at Copenhagen, Denmark, by Christiani and Nielsen.

‡ In the solution of this example all calculations have been made with the slide-rule.

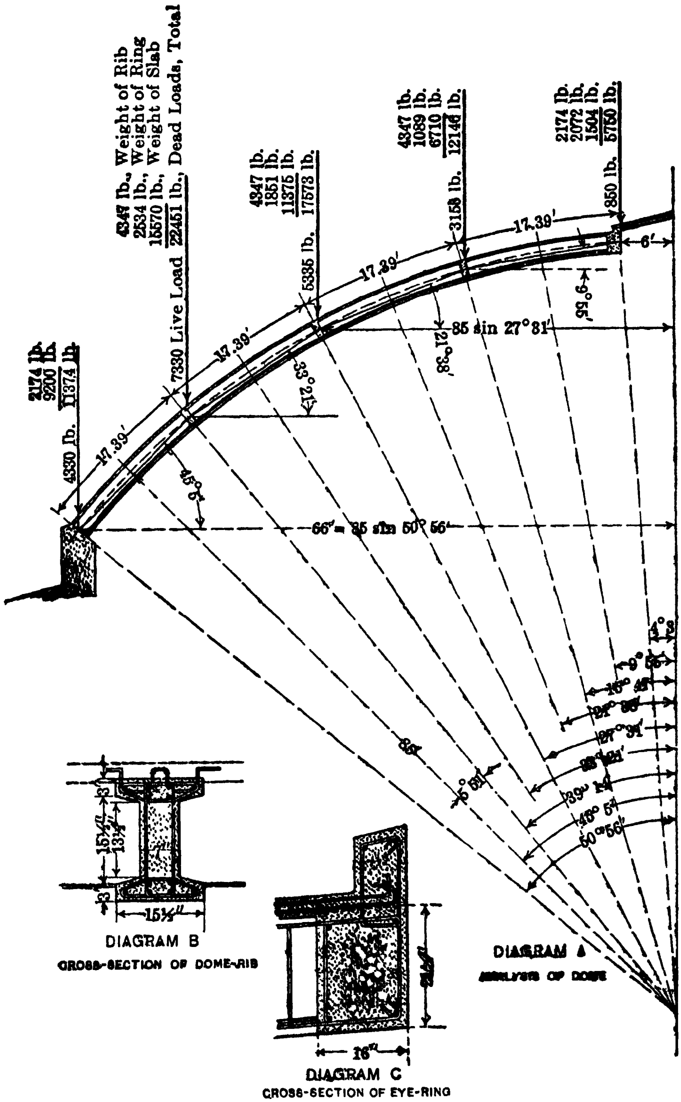


Fig. 5. Segmental Concrete Ribbed Dome

allowing 1 100 lb per lin ft for the eye-ring and the load due to a glass covering over the eye, 250 lb per ft for the weight of the ribs, and 150 lb per ft for the weight of the intermediate rings; and assuming a slab-thickness of $3\frac{1}{2}$ in, a suspended plaster ceiling $\frac{3}{4}$ in thick, and 25 lb per sq ft of surface for snow-loads and wind-loads (or an equivalent of a 2-in thickness of concrete), the loading on the ribs is as shown in Diagram A of Fig. 5. To illustrate the METHOD OF DETERMINING THE LOADS, the calculations for the loading at the lower intermediate ring are given. The WEIGHT OF THE RIB is $250 \times 17.39 = 4\,347$ lb. The WEIGHT OF THE RING is $150 \dfrac{2 \times \pi \times 85 \times \sin 39° 14'}{20} = 2\,534$ lb. The WEIGHT OF THE SHELL AND CEILING between $\theta = 33° 21'$ and $\theta = 45° 5'$ is, from the curves in Plate II, with $\dfrac{cr}{a} = 0$,

$$\frac{150\left(\dfrac{3\frac{1}{2}+\frac{3}{4}}{12}\right)(85)^2\,(1.84) - 150\left(\dfrac{3\frac{1}{2}+\frac{3}{4}}{12}\right)(85)^2\,(1\ 03)}{20} = 15\,570 \text{ lb}$$

The total dead load is

$$4\,347 + 2\,534 + 15\,570 = 22\,451 \text{ lb}$$

The total live load is

$$\frac{15\,570 \times 2}{3\frac{1}{2} + \frac{3}{4}} = 7\,330 \text{ lb}$$

The stress D_4 (see method in Fig. 4A), in the lower section of the rib is the largest, and according to Schwedler's formulas, is

$$\frac{6\,600 + 15\,301 + 22\,908 + 29\,781}{\sin 45° 5'} = 105\,400 \text{ lb}$$

The ECCENTRICITY of the stress D_4 is

$$85 - (85 \cos 5° 51') = 0.445 \text{ ft}$$

The moment due to the ECCENTRICITY of D_4 is

$$105\,400 \text{ lb} \times 0.445 \text{ ft} = 46\,903 \text{ ft-lb}, \quad \text{or} \quad 562\,836 \text{ in-lb}$$

To resist the COLUMN-LIKE COMPRESSION of D_4 there is required a cross-sectional area of rib of

$$\frac{105\,400}{500} = 211 \text{ sq in of concrete}$$

To resist the effect of the ECCENTRICITY of the stress D_4, it is necessary to insert enough steel so that the total stress in it, multiplied by the distance between the top and bottom steel reinforcements, is equal to the moment, 562 836 in-lb, already found.

Since the ECCENTRICITY of D_4 is 0.445 ft, and the line of action of the thrust is to be kept within the MIDDLE HALF OF THE RIB, the rib will be

$$4 \times 0.445 = 1.78 \text{ ft} = 21\tfrac{1}{2} \text{ in in depth}$$

Allowing $1\frac{1}{2}$ in of concrete for steel-protection at the top and bottom of the rib, the distance between the inner and outer reinforcements is

$$21\tfrac{1}{2} - 3 = 18\tfrac{1}{2} \text{ in}$$

Therefore a stress of

$$\frac{562\,836}{18\frac{1}{2}} = 30\,424 \text{ lb}$$

is to be resisted by the steel at the top and bottom. Since there is steel in both COMPRESSION and TENSION, the allowable UNIT STRESS in it is

$$\left(\frac{650 \times 18\frac{1}{2}}{21\frac{1}{2}}\right)(15 - 1) = 7\,830 \text{ lb per sq in}$$

This is because the allowable compressive unit stress in the outer fibers of concrete beams is 650 lb per sq in; the ratio of the MODULUS OF ELASTICITY of the steel and of concrete, 15; and the distance between the inner and outer steel reinforcements, and the distance of the rib-depth, $18\frac{1}{2}$ and $21\frac{1}{2}$ in, respectively. The 1 in the expression $(15 - 1)$ is to take care of the stress carried by the concrete replaced by the steel.

The TOTAL CROSS-SECTION of steel necessary at both the top and bottom of the rib is, therefore,

$$\frac{30\,424}{7\,830} = 3.89 \text{ sq in}$$

furnished by four $1\frac{1}{8}$-in round rods. The best arrangement of the 211 sq in of concrete, and the steel, results in a cross-sectional shape shown in Diagram B of Fig. 5. The STIRRUPS should be spaced not more than three fourths of the distance between lines of longitudinal steel, or $\frac{3}{4} \times 18\frac{1}{2} = 12$ in (approximately), and they should be made from $\frac{3}{8}$-in round rods. Because of the TIES in the flanges, it is advisable to use small $\frac{1}{4}$-in rods as STIFFENERS at the intersections of the ties and stirrups. Projecting LOOPS should be left for fastening the panel-slabs. The actual weight per linear foot of the ribs is

$$\frac{211}{144} \times 150 = 220 \text{ lb}$$

for the concrete, plus

$$(4 + 3.38) + 11 = 25 \text{ lb}$$

for the steel, equal to a total of 245 lb, as against 250 lb per lin ft previously allowed in the calculations.

As the ribs are to be precast and raised into place, it is necessary to determine whether they are of sufficient strength for this, and whether they will stand, unsupported by the rings, without breaking under their own weight. By considering the ribs to be simple arches, and testing them by determining the line of thrust, it is found that they are amply safe. In order to resist the thrusts or stresses developed by raising the ribs into place, it is necessary to tie the ends together with bow-string rods.

The stresses in all of the rings, except the FOOTING-RING, are compressive. This is because they are all above the CRITICAL ANGLE. (See Smooth-Shell Domes.) Therefore, in determining the stresses by Schwedler's formulas, only the equations for the MINIMUM VALUES of T_1, T_2, T_3, and T_4 need be used.

The stress in the EYE-RING is

$$-\frac{(5\,750 + 850) \cot 9° 55'}{2 \sin \frac{\pi}{20}} = -120\,600 \text{ lb}$$

The stress in the FIRST IMMEDIATE RING IS

$$\frac{(5\,750 \cot 9° 55') - (5\,750 + 12\,146 + 3\,155) \cot 21° 38'}{2 \sin \frac{\pi}{20}} = -65\,000 \text{ lb}$$

The stress in the SECOND INTERMEDIATE RING is

$$\frac{\left\{\begin{array}{c}(5\,750 + 12\,146)\cot 21^\circ\,38' - (5\,750 + 12\,146 \\ + 17\,573 + 5\,335)\cot 33^\circ\,21'\end{array}\right\}}{2\sin\dfrac{\pi}{20}} = -\,53\,900 \text{ lb}$$

The stress in the THIRD INTERMEDIATE RING is

$$\frac{\left\{\begin{array}{c}(5\,750 + 12\,146 + 17\,573)\cot 33^\circ\,21' - (5\,750 + 12\,146 \\ + 17\,573 + 22\,451 + 7\,330)\cot 45^\circ\,5'\end{array}\right\}}{2\sin\dfrac{\pi}{20}} = -\,35\,600 \text{ lb}$$

The stress in the FOOTING-RING is tensile, and hence the equation for the MAXIMUM VALUE of T_5 gives

$$\frac{\left\{\begin{array}{c}(5\,750 + 850 + 12\,146 + 3\,155 + 17\,573 + 5\,335 \\ + 22\,451 + 7\,330)\cot 45^\circ\,5'\end{array}\right\}}{2\sin\dfrac{\pi}{20}} = 238\,000 \text{ lb}$$

The EYE-RING should have $\frac{120\,600}{500} = 242$ sq in of concrete, but for appearance it should be as wide as or wider than the ribs; hence it is made 21½ in high and 16 in wide. This size allows, also, a firm ANCHORAGE for the rib-reinforcing. With 1% of reinforcing it requires four 1⅛-in round rods. (See Diagram C, Fig. 5.) The first intermediate ring should be

$$\frac{65\,000}{500} = 130 \text{ sq in}$$

in cross-section, requiring a 7-in width and a 18½-in height, to resist the load, as a COLUMN. As the ring must also act as a BEAM, carrying its own weight, the weight of half the slab, and the live load (the forms taking the place of the live load during construction), steel must be added to resist the BENDING MOMENT

$$\frac{wl^2}{12} = \frac{\left(\dfrac{130}{144}\times 150\right)\left(\dfrac{2\pi\,85\sin 15^\circ\,47'}{20}\right)^2 + \left(\dfrac{6\,710 + 3\,155}{2}\right)\left(\dfrac{2\pi 85\sin 15^\circ\,47'}{20}\right)}{12}$$
$$= 3\,570 \text{ ft-lb} = 42\,840 \text{ in-lb}$$

Sufficient steel, in TENSION and COMPRESSION, must be added to keep the additional stress in the concrete, due to this moment, down to 150 lb per sq in, since 500 + 150 = 650 lb per sq in is the MAXIMUM ALLOWABLE COMPRESSIVE STRESS in the concrete of the beam. From standard formulas for Reinforced Concrete.

$$K = \frac{42\,840}{7 \times (17)^2} = 21.2$$

and when $K = 21.2$ and $S_t = 16\,000$ lb per sq in; $S_c = 245$ lb per sq in, $p = 0.0014$, and $x = 0.185$. Since S_c must not exceed 150 lb per sq in, it is necessary to add COMPRESSION-STEEL to resist a stress of

$$\left(\frac{245 - 150}{2}\right)(7)(0.185 \times 17) = 1\,040 \text{ lb}$$

The allowable stress in the COMPRESSION-STEEL, less the stress already allowed for the concrete which is replaced by the steel, if placed 1½ in from the outside, is

$$\left(\frac{650}{0.185 \times 17}\right)\left((0.185 \times 17) - 1.5\right)(15 - 1) = 4\ 770 \text{ lb per sq in}$$

The amount of COMPRESSION-STEEL is, therefore,

$$\frac{1\ 040}{4\ 770} = 0.22 \text{ sq in, cross-section}$$

The tensile-steel necessary is

$$0.0014 \times 7 \times 17 = 0\ 16 \text{ sq in}$$

but because of the NEGATIVE MOMENT at the ribs, the same cross-sectional area is used as for COMPRESSION, that is, 0.22 sq in, furnished by two ⅜-in round rods. The UNIT-SHEAR is

$$\frac{\left(\frac{130}{144} \times 150\right)\left(\frac{2\pi\ 85 \sin 15° 47'}{20}\right) + \left(\frac{6\ 710 + 3\ 155}{2}\right)}{7 \times 17 \times \left(1 - \frac{0\ 185}{3}\right) \times 2} = 18.6 \text{ lb per sq in}$$

No STIRRUPS are necessary to resist shear, but stirrups made from ¼-in round rods should be spaced about 18 in on centers, to tie the panel-slabs securely to the ring.

The SECOND INTERMEDIATE RING, if made the same size as the first, will have a stress of

$$\frac{53\ 900}{7 \times 18½} = 416 \text{ lb per sq in}$$

The MOMENT will be

$$\frac{\left(\frac{130}{144} \times 150\right)\left(\frac{2\pi\ 85 \sin 27° 31'}{20}\right)^2 + \left(\frac{11\ 375 + 5\ 335}{2}\right)\left(\frac{2\pi\ 85 \sin 27° 3'}{20}\right)}{12}$$

$$= 10\ 300 \text{ ft-lb} = 123\ 600 \text{ in-lb}$$

From standard formulas for Reinforced Concrete, $K = 61\ 2, S_c = 455$ lb per sq in, $p = 0.0043$, and $x = 0.3$. Since S_c cannot exceed $650 - 416 = 234$ lb per sq in, the COMPRESSION-STEEL must resist

$$\left(\frac{455 - 234}{2}\right)(7)(0.3 \times 17) = 3\ 930 \text{ lb}$$

The section-area of the COMPRESSION-STEEL is, therefore,

$$\frac{3\ 930}{\left(\frac{650}{0.3 \times 17}\right)\left((0.3 \times 17) - 1.5\right)(15 - 1)} = 0.62 \text{ sq in}$$

furnished by two ¾-in round rods at top and bottom. The UNIT SHEAR is

$$\frac{\left(\frac{130}{144} \times 150\right)\left(\frac{2\pi 85 \sin 27° 31'}{20}\right)^2 + \left(\frac{11\ 375 + 5\ 335}{2}\right)}{7 \times 17 \times \left(1 - \frac{0\ 3}{3}\right) \times 2} = 46.9 \text{ in per sq in}$$

It is therefore necessary to resist 46.9 − 40 = 6.9 lb per sq in of shear, with STIRRUPS, that is, with two $\frac{3}{8}$-in round-rod stirrups, spaced 12 in apart at the ends, and the others 18 in apart through the remaining distances.

The THIRD INTERMEDIATE RING is 7 by $18\frac{1}{2}$ in in section, with two $\frac{7}{8}$-in round rods at top and bottom, and with two $\frac{3}{8}$-in round-rod STIRRUPS, spaced 9 in apart at the ends, two more, spaced 12 in, and the rest spaced 18 in.

The MOMENT due to the ECCENTRICITY of the COLUMN-LIKE THRUST, that is, the longitudinal horizontal compressive stress, in the rings is resisted by the slabs. A more exact analysis may be made by considering only the NORMAL COMPONENTS OF THE LOADS on the rings in determining these moments.

The FOOTING-RING must have enough tensile-steel to resist the outward PUSH or THRUST of the ribs, that is $\frac{238\,000}{16\,000} = 14.9$ sq in of steel cross-section. In addition to this, if the ring acts as a BEAM, there must be sufficient steel to resist the moment due to the combined weights of the dome and the ring itself.

The PANEL-SLABS being domical and above the critical angle, are in compression, and should be designed as illustrated in the discussion of Smooth-Shell Domes.

2. Vaults *

Classification. Vaults may be conveniently considered under the following divisions: (1) Barrel vaults, (2) Groined vaults, and (3) Ribbed vaults (Masonry, Tile, or Framed).

General Considerations. A knowledge of the ELASTIC THEORY OF ARCHES and the stability of buttresses is necessary in a rigid investigation of vaults, since their design involves the application of the principles of that theory. (See, also, Chapters VII and VIII.) In any vault, lines of action of the stresses or thrusts must pass through the material between certain limiting lines; otherwise the vault may fail. These thrusts are brought to the grade-line, or to foundations, by walls, often buttressed in the case of barrel vaults, and by piers and buttresses in the case of groined and ribbed vaults. By building vaults of light materials, such as hollow bricks or hollow tiles, the magnitude of the thrusts are decreased, and lighter walls, piers, or buttresses can be used.

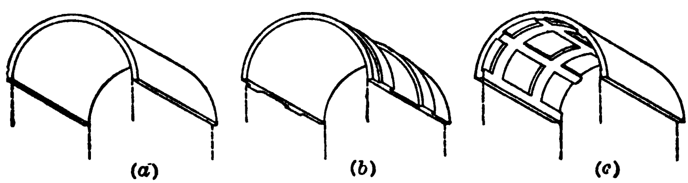


Fig. 1. Three Methods of Building Barrel Vaults

Barrel Vaults. Fig. 1, (a), (b), and (c), illustrates three methods of building BARREL VAULTS. In (c) the longitudinal ribs are merely for appearance, as they do not strengthen the vault. The diagrams (a) and (b), Fig. 2, illustrate two methods of disengaging the masonry of barrel vaults from the walls. Diagram (b) is the better method, and improves the appearance of the vault

* A full treatment of this subject may be found in the Handbuch der Architektur and Breyman's Baukonstructions Lehre.

on the inside. Diagrams (*c*) and (*d*) illustrate the use of stone skewbacks for segmental vaults.

Strength of Barrel Vaults. Barrel vaults may be considered as a SERIES OF ARCHES set next to each other; and hence if a section one unit long is found safe when investigated as an arch, the vault itself is considered safe. By building the wall and the vault together as a unit, to a point on the arc 60° from the vertical or crown, that is, to a point on the intrados one third of the distance from the horizontal spring-line, the actual span is materially decreased. With the spring-line at 60°, the line of thrust in an unloaded arch or barrel vault of an equal thickness throughout, will remain within a strip whose radial thickness or width is about one forty-second of the radius. If the line of thrust is to remain within the middle third of the arch-ring or vault-ring, t should be $(r/42) \times 3 = r/14$. If it is to remain within the middle half, t should be $(r/42) \times 2 = r/21$. In the following example, the theory of the middle half will be followed, in which $t = r/21$. If it were assumed that $t = r/14$, the line of thrust being kept within the middle third, the span of the vault in the example would have to be changed from 21 to 14 ft. If built, then, as described (Fig. 3), the minimum thickness of the unloaded vault-shell is about one twenty-first of the vault-radius, that is,

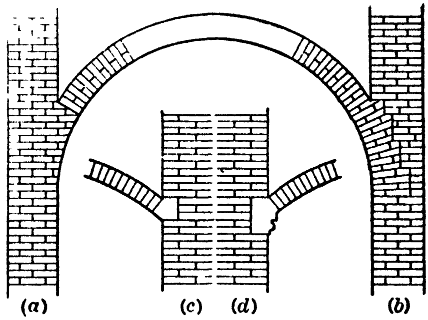


Fig. 2. Methods of Joining Barrel Vaults to Walls

$$t = r/21$$

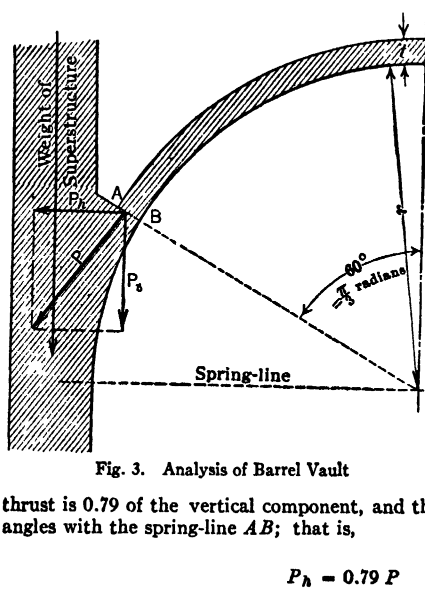


Fig. 3. Analysis of Barrel Vault

The VERTICAL COMPONENT P_v of the thrust P is equal to the weight of half the free vault, that is, of the section $ABCD$. It can be shown that the HORIZONTAL COMPONENT P_h of the thrust is 0.79 of the vertical component, and that the thrust is not at right-angles with the spring-line AB; that is,

$$P_h = 0.79\,P$$

Example. It is required to construct a barrel vault over a corridor 21 ft wide. The vault-radius is $10\frac{1}{2}$ ft, and the minimum thickness of the shell is $10.5/21 = 0.5$ ft $= 6$ in. If built of bricks it is cheaper to build a ribbed vault, as the unit-dimensions of bricks are approximately 4 in, 8 in, 12 in, etc. Referring to Fig. 4, it is found that a 4-in vault with ribs 4 by 8 in every 3 ft

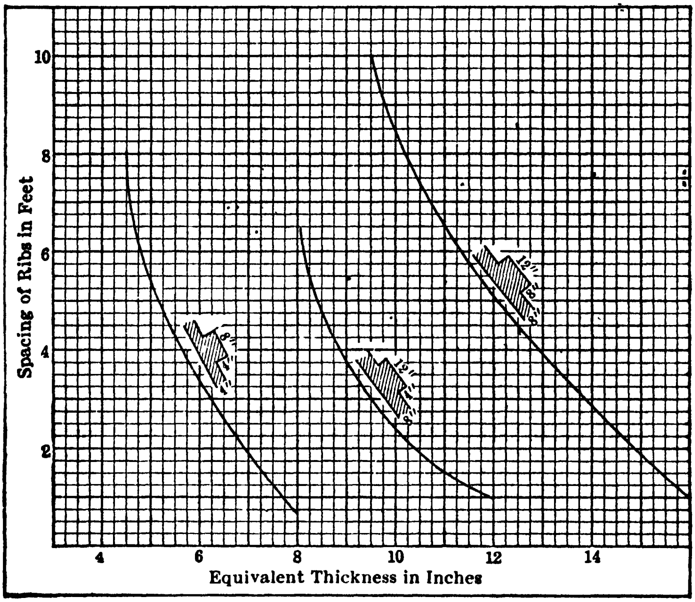


Fig. 4. Barrel Vaults, Ribbed and Non-ribbed. Equivalent Thicknesses

3 in, is equivalent to a 6-in vault, and hence would be used. The brick masonry weighs 125 lb per cu ft.

The VERTICAL COMPONENT P_v of the thrust is

$$\frac{\left\{ \begin{array}{c} [4/12 \times 3\frac{1}{4} \times 1/3\pi \times 10.5 \times 125] + [4/12 \times 8/12 \\ \times\ 1/3\pi\,(10.5 + 0.33) \times 125] \end{array} \right\}}{3\frac{1}{4}} = 557 \text{ lb per lin ft}$$

The HORIZONTAL COMPONENT P_h of the thrust is $0.79 \times 557 = 440$ lb per lin ft. The supporting wall must be thick enough, buttressed enough, or loaded sufficiently from above, to take care of this horizontal component of the thrust.

Fig. 5 is a graphical analysis of the stresses in this vault. It will be noticed in Fig. 5 that the line of pressure remains in the MIDDLE HALF of the vault-thickness. Scheffler, after numerous tests of vaults, stated * that if one fourth the vault-thickness is deducted at the extrados, and one fourth at the intrados, and that if the line of pressure found according to the elastic theory

* Theorie der Gewölbe.

of arches is confined to the remaining portion, that is, the MIDDLE HALF, then the vault may be considered safe. Fig. 6 shows the resistance-line passing slightly outside the middle third. It illustrates the less conservative theory

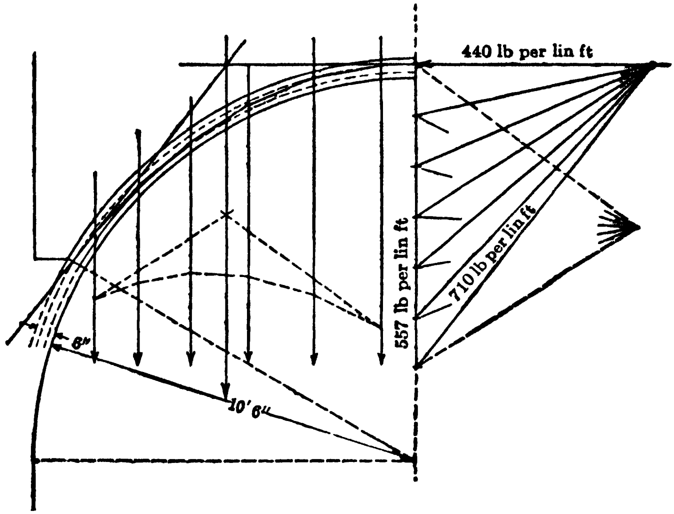


Fig. 5. Graphical Determination of Stresses in a Barrel Vault

that the resistance-line might in some cases pass near the outside of the middle half. The arch or vault in Diagram (*b*) of Fig. 6 would have a greater tendency to fail according to the middle-third theory, because the line of pressure or resistance-line passes outside of the middle third. Diagram (*a*)

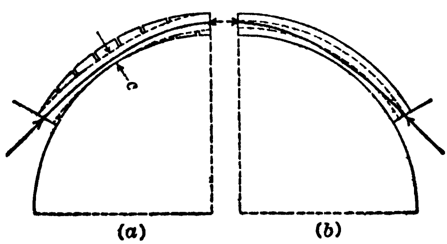


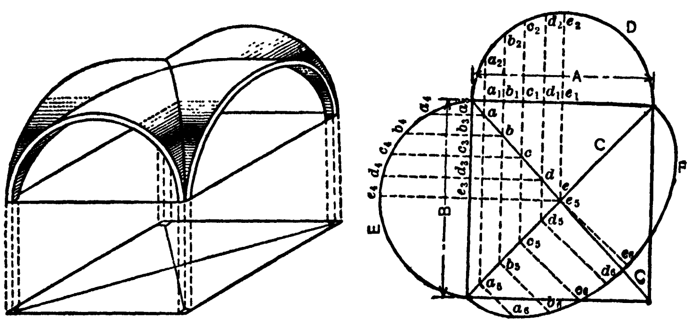
Fig. 6. Line of Pressure through Vault-thickness

of Fig. 6 shows the same arch or vault with the shell cut so that the line of pressure passes down the exact center of the uncut portion. This results in a sort of theoretical or ideal arch-form. Of course the thickness of part *c* must be sufficient to develop a safe compressive resistance in the material, and it is advisable to add sufficient steel to take care of any tension in the parts farthest from the resistance-line. Vaulted construction is often rela-

tively protected and free from the live loads and moving loads to which arches are generally subjected; and for such construction Scheffler's conclusions are considered valid.

Groined Vaults. A GROINED VAULT is formed by the intersection of two BARREL VAULTS. (See (*a*), Fig. 7.) By using groined vaults it is possible to bring the tops of windows and doors above the spring-lines of the vaults, and to concentrate the pressures or thrusts on piers or columns.

Groins. The intersections of two vaults, called GROINS, are straight lines in horizontal projection, only when they are of the same curvature and height. If the vaults are of different widths, it is best to make one semicircular, draw the horizontal projections of the groins as straight lines, and then determine



(*a*) Perspective, Showing Penetrations and Intersections

(*b*) Intersecting Vaults of Different Widths

Fig. 7. Groined Vault

the contour of the other vault. This is illustrated in Fig. 7 (*b*). Vault *A* is semicircular and has a span *A*. Vault *B* has a span *B*. *CC* are the GROINS, and *D* is the circular contour of the narrow vault. Any points, *a*, *b*, *c*, etc., are chosen at random, and lines a–a_2, b–b_2, c–c_2, etc., and a–a_4, b–b_4, c–c_4, etc., drawn parallel to the axes of the respective vaults. The line a_3–a_4 is laid off equal to a_1–a_2; b_3–b_4, equal to b_1–b_2, etc. The smooth curve connecting a_4, b_4, c_4, etc., is the contour *E* of the vault *B*. In like manner the contour *F* of the groins is found by similarly laying off a_5–a_6, b_5–b_6, etc., equal to a_1–a_2, b_1–b_2, etc.

The VAULT-SHELLS, at the intersections or groins, should never have what might be called MITER-JOINTS. The vaults should be monolithic or there should be concealed ribs to carry the vault-shells and transmit the thrusts to the piers. If the intersecting vaults are of stone, and of the same diameter, the groins may be built as shown in Fig. 8 (*a*) for small vaults, or as in Fig. 8 (*b*) for larger vaults. In Fig. 8 (*a*) the GROIN-STONES are ⌊-shaped and are cut so as to carry the stone courses of one vault around to the other vault. The stone shown in Diagram (*a*) of Fig. 8 is shown in plan at *b*, with two views at *c* and *d*. A better method is shown in Fig. 8 (*b*). Here the groin-stones are cut so that the joints are normal to the groins, thus forming concealed ribs. This bearing-surface is obtained as follows. Point *a*, the intersection of an extended vault-joint and the groin-edge, is projected

down to a' and b', the intersections of the projecting line and the assumed side and center lines of the rib. Point b' is projected up to b'', a point on the center line or edge of the rib. From b'' a horizontal line intersects with a line projected up from a' to give c'' a point on the joint, which is drawn

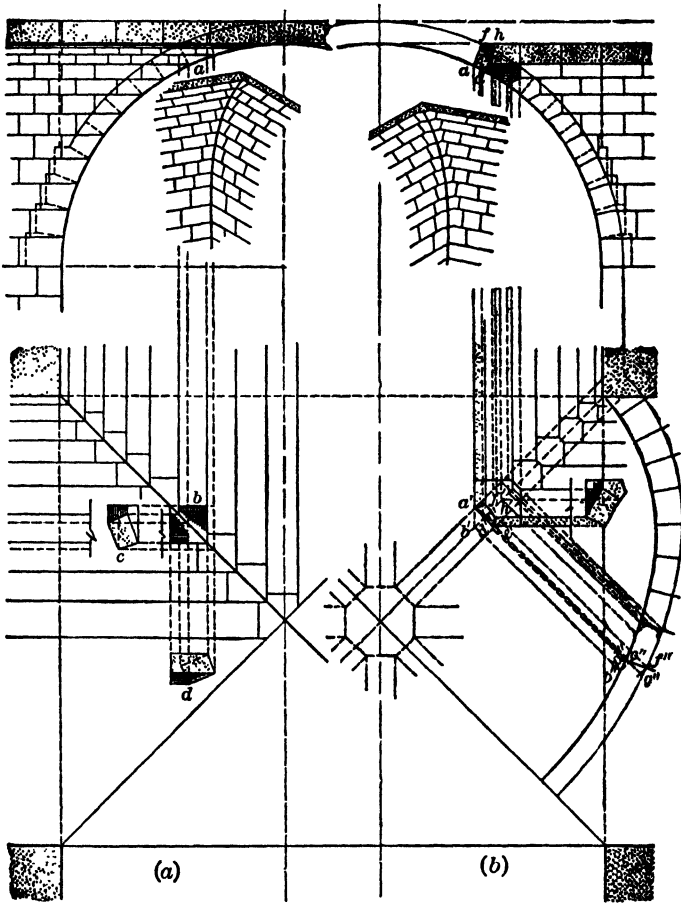


Fig. 8. Groin-details for Stone Vaults of the Same Diameter

normal to the groin. The intersection d'' of this joint with the groin-edge is projected down to d' on the center line of the rib. By connecting a' and d', and d' and e' (the point opposite a' on the other side of the rib) with a curved line, the lower edge of the bearing-surface is determined.

Points d' and e' projected up determine d and e, the same points in elevation. Other points on these curved lines can be found by choosing points between a and d and projecting them in the same way as in the method used

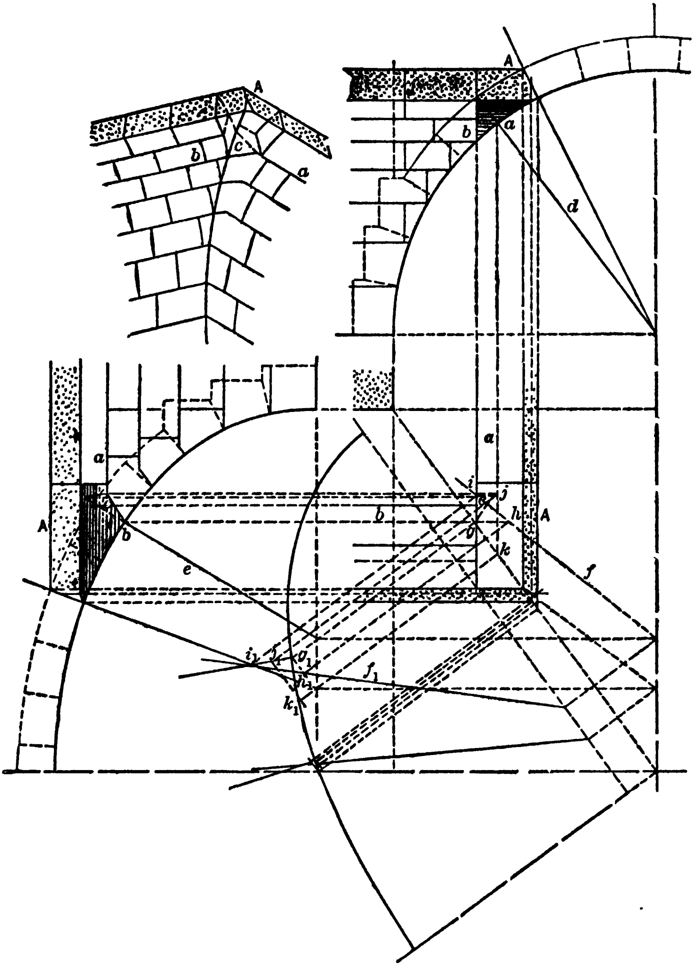


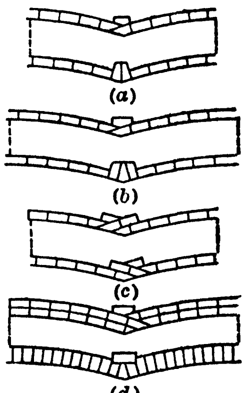
Fig. 9. Groin-details for Stone Vaults of Different Diameters

to find the projections of f. The procedure is as follows. The point f is projected down to the center line of the rib, locating g'. Then g' is projected up to g'', the intersection with the line representing the joining of the upper

surfaces of the vaults. A horizontal line is projected over from g'' to f'', the point of intersection with the normal joint. The point f'' is projected down to f', on the projecting line from f. By connecting f' and h' (h' is opposite f' and equidistant from the center line of the rib) with a straight line, the upper edge of the bearing-surface is determined. The point h is found by projecting up from h'. By connecting a' and f', and e' and h'; also a and f, and e and h, the side edges of the bearing-joint are located. The lower bearing-surface of a stone, or the upper bearing-surface of the next lower stone, is found in a similar manner.

If the vaults are not of the same diameter, either of two methods may be used. The number of stone courses in both vaults may be made the same, thus making the courses in the wide vault wider than those in the narrow vault, and the method of finding the shape of the groin-stones is similar to that shown in Fig. 8 (*b*); or the stones may be the same width, thus making a greater number of courses in the wide vault than in the narrow vault. In the latter case the groin-stones are determined as in Fig. 9. To take care of the different number of courses in the two vaults, one course in the narrow vault is sometimes made to receive two courses in the wide vault, as shown by stone *A* in Fig. 9. Because the joint *a* is higher than the joint *b*, there results a peak toward the side of the groin-line. This is cut off at right-angles to the groin, thus making the bearing-surface *c*. This surface is determined as follows. The intersection of the joint-planes *d* and *e* is at *f*. The vertical projection f_1, of *f*, is drawn through g_1, found by projecting up *h* and *g*, and a horizontal line from g_1. The intersection of f_1 and a line through g_1, normal to the groin-curve gives i_1, which, projected to *i*, gives the intersection of the sides of the bearing-surface *c*.

The point *j* is found by projecting up *k*, the intersection of *a* and the diagonal, to k_1; then projecting k_1 to j_1, the intersection with the normal line; and then projecting j_1 to *j*. By connecting *g* and *j* with a curved line (other points of which are determined by drawing lines parallel to *a* and proceeding by the method used in finding *j*); and *g* and *i*, and *j* and *i*, with straight lines; the sides of *c* are determined.



(*a*) (*b*) (*c*) (*d*)

Fig. 10. Groins of Brick Vaults

If the vaults are built of brick, it is better to run the courses at right-angles to the groins, thus giving a chance for the bricks to overlap, as shown in Fig. 10. If the brick courses are to run parallel to the center line of the vaults, it is necessary to use stone ribs to carry the shell.

Determination of the Stresses in Groined Vaults. The problem of a groined vault spanning a RECTANGULAR AREA which is not square, is here considered, as a vault spanning a SQUARE AREA offers fewer difficulties and can be worked out on the same principles. The problem is to span an area, whose half-length of the short diameter is *a*, and whose half-length of the long diameter is *b*, Fig. 11 (*a*). In order to obtain a more stable construction, the point of intersection of the crowns of the vault is raised a distance $cd = c'd$, thus giving the crown of the long-span vault a slope *ce* and the crown of the short-span vault a slope *c'f*. The vault is divided into strips *A*, *B*, *C*, etc.,

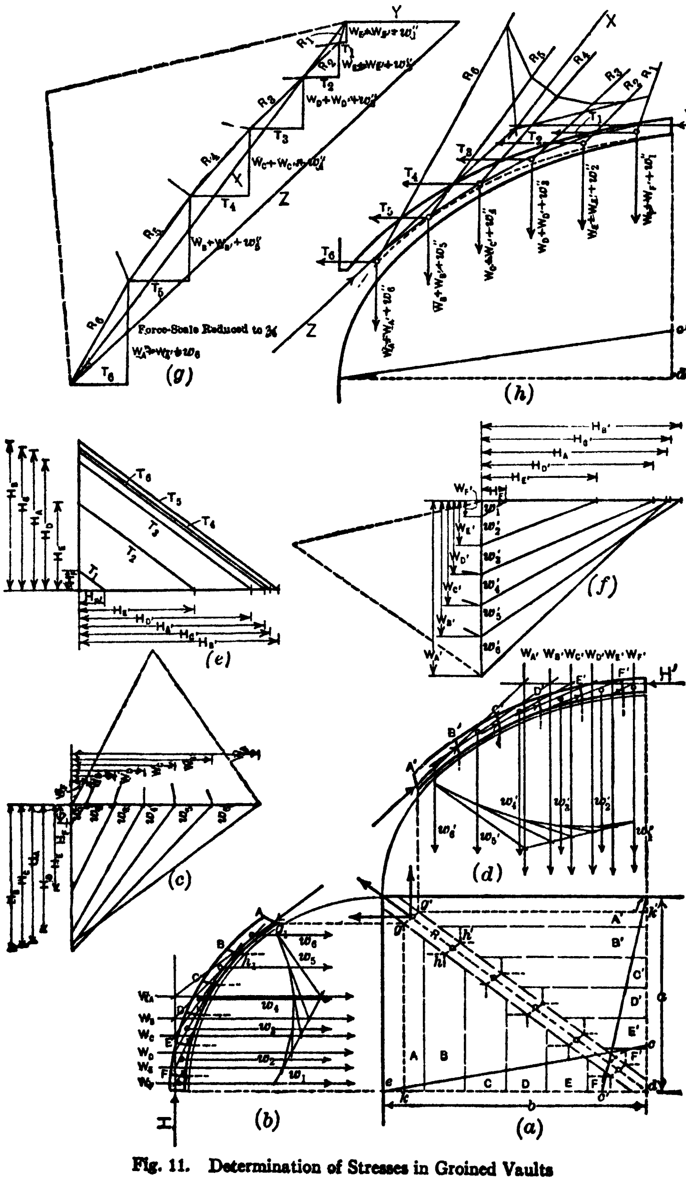


Fig. 11. Determination of Stresses in Groined Vaults

and A', B', C', etc., from the rib R, as shown in the projected area in Fig 11 (a). The rib R is given a width equal to the assumed width of the supporting diagonal concealed arch, and the widths A, B, C, etc., and A', B', C', etc., are obtained by dividing the two vaults into the same number of equal parts. These strips are considered as adjacent arches resting on the rib R. For simplification the line of pressure or resistance-line of each strip is placed in the center of that strip as gk in A and $g'k'$ in A'. The error in this is on the side of safety.

Even though the projected areas of the two intersecting vaults are the same, the actual surface-area of the smaller-span vault is slightly larger than that of the longer-span vault. Therefore, if the vaults are of the same thickness, the shorter-span vault is slightly heavier than the larger-span vault. In order to have the resultants of the horizontal components of the thrusts from strip A and strip A', strip B and strip B', etc., parallel to the direction of the rib R, the procedure is as follows.

The thrusts of the strips on the heavier side, that is of strips A, B, C, D, E, and F, are determined as shown in Fig. 11 (b) and (c). The curvature of the strips being the same, the work can be considerably lessened by dividing the arch into sections of unequal lengths for weight-determinations. The dividing line for the sections is found, by projecting up the point of intersection of the line of pressure of each strip and the side of the rib R, as g to g', h to h', etc. The weights w_1, w_2, w_3, etc., of each section are then determined and the composite load-line drawn as in Fig. 11 (c). The positions of W_A, W_B, W_C, etc., in Diagram (b) are determined by the usual STRESS-POLYGON. H is then drawn so as to be at the upper limit, and the different thrusts so as to act near the lower limit of the middle half of the vault-thickness.* Lines drawn in Fig. 11 (c) parallel to these thrusts, determine their values, and the values of the horizontal components H_A, H_B, H_C, etc. The weights w'_1, w'_2, w'_3, etc., in Diagram (d), are found in the same way, the load-line in Diagram (f) drawn, and the positions of W_A', W_B', W_C', etc., found as before. H' in Fig. 11 (d) is drawn, at the upper limit of the middle half in this demonstration.* H_A', H_B', H_C', etc., however, must have such values that the resultants of H_A, and H_A', H_B and H_B', etc., are parallel to R. The required values of H_A', H_B', H_C', etc., are found as in Fig. 11 (e), by laying off H_A, H_B, H_C, etc., and drawing T_1, T_2, T_3, etc., parallel to R. The resulting values of H_A', H_B', H_C', etc., are then laid off in Fig. 11 (f) and the thrusts drawn. When drawing the thrusts in Fig. 11 (d) through the intersection, of H' and W_A', H' and W_B', etc., parallel to their directions in Fig. 11 (f), it is found that they act very slightly above the lower edge of the middle half.

The rib R is then drawn as in Fig. 11 (h) and the points of application of the loads located. The LOAD-POLYGON is drawn as in Fig. 11 (g). The RESULTANTS R_1, R_2, etc., are drawn in both Diagrams (h) and (g) of Fig. 11, the position of X in Diagram (h) found by the usual STRESS-POLYGON, and the THRUSTS Z and Y determined. The point through which Z, Diagram (k), passes at the spring of the rib, should be so chosen that the LINE OF PRESSURE remains at least within the middle half of the rib; or the more usual and conservative limits of the middle third may be used. In the case of brick vaults the strips A, B, C, etc., are taken at right-angles to the groin, resulting in vertical loads, only, on the assumed rib.

Ribbed Vaults. In RIBBED VAULTS the ribs are designed to be built first,

* The theory of the middle third is the one usually followed, as it is the most conservative and results in a larger factor of safety.

to be free-standing, and of sufficient strength to support the shell when it is placed over them. To simplify the construction, all the rib-arcs are ordinarily made with the same radius, thus making all the ribs disengage each other at the same height. This makes the narrower rib-arches pointed, and the diagonal rib-arches semicircular, but they are all constructed of similar stones with cross-sections of the same shape. To determine the points *A* and *B* (Fig. 12), at which the ribs become independent of each other and of the wall, the proceeding is as follows. In plan the clustered ribs are shown just above the column-capitals, with the diagonal ribs extending into the wall a distance *ab*. To find the height at *A*, draw an arc through *a* with the same curvature as that of the diagonal rib, and draw at right-angles to the ribs, in plan, a line from *b*, until it cuts this arc at *c*. The height *cb* is the height at *A*. The

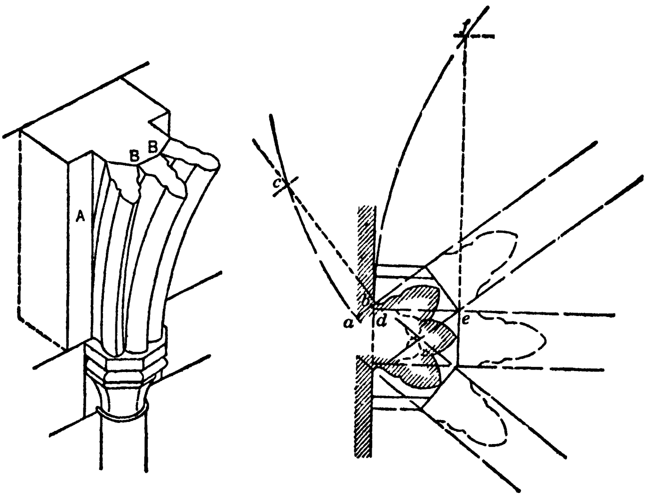


Fig. 12. Vault-rib Construction

height at *B*, equal to *fe*, is found in the same way. The WEBS, or parts of the vault-shell supported by the ribs, are usually shallow arches in cross-section, and are SPHERICAL TRIANGLES, that is, they are DOMICAL.

In order to use to the fullest advantage the finished lower portions of the vault as supports for the upper courses as laid, the courses of the VAULT-SHELL, or WEB are laid in planes normal to the wall and the transverse ribs. This is shown in horizontal projection in Fig. 13 I. The web being arched in both directions, the thrusts act in two directions, as in domes. From the study of the theory of domes it is found that the THICKNESS OF THE SHELL in a dome has NO EFFECT ON ITS STABILITY. The web in ribbed vaults being domical, can be made relatively thin, but for stone or brick vaults it should not be less than about 4 in thick for spans up to 35 ft.

The ribs are designed as arches, loaded with the thrusts of the web supported. These thrusts are determined as illustrated in Fig. 13 II. The vaulting resting on the half-wall, or transverse rib *A*, and the half-diagonal rib *B*, is divided into any number of equal LUNES, or figures bounded by the

two intersecting arcs, and radiating from the AXIS OF THE DOME of which that part of the vaulting is a SPHERICAL TRIANGLE. This axis is found by projecting, at right-angles from the ribs A and B, lines starting at the center of curvature of the ribs and intersecting at the point e, which is the projection of the axis of the dome. The RADIUS OF THE DOME is then R_1 in

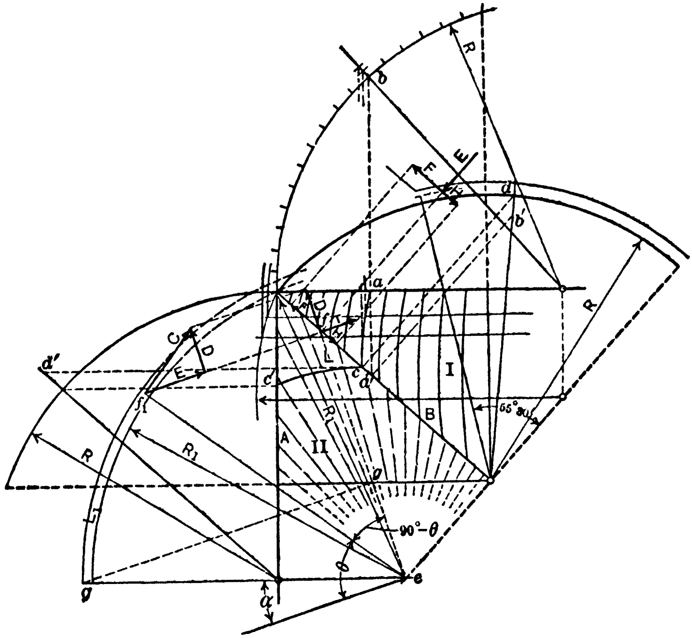


Fig. 13. Determination of Stresses in Vault-ribs

Fig. 13 II, equal to the distance from e to the spring of the diagonal rib B. The thrust of each LUNE on the ribs is then found as shown for lune L.

Example. Let the radius R_1, Fig. 13 II, be 25 feet, and let the shell be 4 in thick, constructed of stone, and weighing 125 lb per cu ft. The angle θ (by measurement) is 54° 30′, and the angle α is 18° 30′. These are found by projecting up from the point of intersection f of the center line of the lune L and the center of the rib B, and the intersection g of the center line of the lune L and the crown-line of the vault, to f_1 and g_1, respectively, on the vertical projection L_1 of the lune L.

Using the same notation, equations, and curves as were derived for SMOOTH SHELL DOMES (Article 1), it is found from Plate II, with $\frac{cr}{a} = 0$ and $\alpha = 18°$ 30′, that

$$W_{l-o} = -125\,(4/12)\,(25)^2\,(0.33) = -8\,594 \text{ lb}$$

and that

$$n = \frac{-8\,594}{2\pi\,(125)\,(4/12)\,(25)^2} = -0.0525$$

From Plate I it is found that the CRITICAL ANGLE, for values of $\frac{cr}{a} = 0$ and $n = -0.0525$, is 55° 30′, and the vaulting should be back-filled as high as this, as shown.

From Plate III, with $\frac{cr}{a} = 0$. and $n = -0.0525$, it is found that at $\theta = 54°$ 30′

$$U = (125 \times 4/12 \times 25)(-0.079 + 0.63) = 573 \text{ lb}$$

By measurement, the width of LUNE L at f is 2 ft; hence the total TANGENTIAL PRESSURE C (Fig. 13), is $2 \times 573 = 1\,146$ lb. The HORIZONTAL COMPONENT D of this is 6 655 lb, and the COMPONENT F (along the rib B) of D is 5 750 lb. The VERTICAL COMPONENT E of C is 9 339 lb.

The VALUE OF T is found from Plate IV to be $125 \times 4/12 \times (25)^2 \times (-0.004 + 0.295) = 7\,570$ lb, and the COMPONENT H (along the rib B) of T is 3 630 lb.

The THRUSTS acting on the rib B of the other LUNES above L are found in the same way, and the portion of rib above the back fill investigated as an arch. In Fig. 13 that portion of the rib below the web is not indicated.

For vaults with semicircular diagonals of about 33-ft span, the ribs should be from 7 to 10 in wide and from 10 to 14 in in total height, and the minimum dimensions of the projecting portions of the ribs below the webs, for smaller vaults, should be 3½ in width and 6 in in height.*

Tile Vaults. TILE VAULTS, as built by the R. Guastavino Company, are constructed of tiles, from 6 by 12 to 24 in in plan, and 1 in in thickness, and laid in several layers so as to make a solid, thin shell that is both light and strong. Because of the overlapping of the tiles, the shell has considerable TENSILE RESISTANCE, and the vaults are practically MONOLITHIC. It is due to this and to the lightness of the construction that the thrusts and the weight of the entire structure are materially reduced. Ordinarily a finished ACOUSTIC TILE, backed by rough CONSTRUCTIONAL TILE, is used for the exposed surfaces.

Framed Vaults. Vaulting in buildings of moderate cost is frequently constructed by suspending from the roof-trusses STEEL OR WOODEN FRAMES carrying lath and plaster. The roof-trusses must in this case be designed to carry the direct loads of the FRAMED VAULTING, which must be of the required strength and shape to carry and fit the plastered surfaces.

* Handbuch der Architektur.

CHAPTER XXX

HEATING AND VENTILATION OF BUILDINGS *

By
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Physical Units and the Measurement of Heat

System of Units. In this country the system of units in general use by engineers is known as the FOOT-POUND-SECOND SYSTEM, and the following definitions and examples will show the significance of each.

Definition of Units Employed. The UNIT OF TIME is the second, which is equal to $\frac{1}{86\,400}$ part of the mean solar day. t = time. Time is also expressed in minutes and hours.

L = length. The UNIT OF LENGTH is the foot = 0.3048 meter.

W = weight. The UNIT OF WEIGHT is the pound = 0 4532 kilogram.

A = area. The UNIT OF AREA is the square foot. The unit often used is the square inch.

V = volume. The UNIT OF VOLUME is the cubic foot. Volume = area × length = $A \times L$.

Example. The volume displaced per stroke by the plunger of a pump, if the diameter is 6 in and the stroke is 12 in, is $\frac{1}{4} \times \pi \times 6^2 \times 12 = 339.29$ cu in, or 0 196 cu ft.

If the plunger makes 30 WORKING strokes (not revolutions) per minute, then the plunger-DISPLACEMENT per minute is 0.196 × 30 = 5.88 cu ft. One United States gallon = 231 cu in = 0.1336 cu ft. This pump will therefore theoretically deliver 5.88/0.1336, or 44 gal per minute. The actual delivery of the pump will be 10 to 15% less, owing to the SLIP, which is the leakage back through the pump-valves, around the plunger, and that due to imperfect filling of the pump-cylinder on the suction-stroke.

Density. D = density. The weight of a unit volume (1 cu ft) of a substance is called its DENSITY. The density of water at 70° F. is 62.3 lb per cubic foot. The density of air at 70° is 0.075 lb per cubic foot. The pump in the preceding example would, therefore, handle 5.88 × 62.3 or 366 lb of water per minute.

If the water-end of the pump is operated by a steam-cylinder having a displacement of 0.349 cu ft per stroke, and takes steam at the same pressure for the full stroke as in the DIRECT-ACTING type and if we assume that the steam-pressure is 100-lb gauge, we find from the steam-table (Table I), that the density of steam at this pressure is 0.2565 lb. The STEAM-CONSUMPTION of the pump, therefore, would be 0.2565 × 0.349 × 30 × 60 = 161.6 lb per hour, theoretically. A fan handling 10 000 cu ft per minute of air at 70° F, delivers 10 000 × 0.075 = 750 lb per minute.

* The data of this section have been largely condensed from Vol. I of Mechanical Equipment of Buildings, by Harding and Willard, published by John Wiley & Sons, Inc.

Velocity. v = velocity. The RATE OF MOTION of a body is measured by the distance passed over in a unit time. Velocity is expressed in feet per second.

Energy or Work. U = ENERGY or WORK. The UNIT OF WORK is the foot-pound, and is the quantity of energy expended or the work performed by a force of 1 lb moving through a distance of 1 ft in the line of action of the force.

Power is the RATE OF DOING WORK. Note that POWER involves the factor TIME and is equal to the amount of work done divided by the time required to do this work.

Horse-Power. h.p. = HORSE-POWER. The UNIT OF POWER is the horse-power and is the performance of work at the rate of 550 ft-lb per second, or 33 000 ft-lb per minute.

Example. Required the theoretical work and horse-power developed by the water-end of the pump in the preceding example, assuming that the head or the height pumped against is 200 ft, and that no frictional resistance is to be overcome.

The work Um performed per minute is the lifting of the weight of water, W, 366 lb per min, through a height of 200 ft and is $Um = 366 \times 200 = 73\,200$ ft-lb per min. The h.p. $= Um/33\,000 = 73\,200/33\,000 = 2.22$.

The actual power required will be somewhat greater, as the force required to overcome frictional resistance, etc., has been neglected.

Equivalent Values of Electrical and Mechanical Units

1 horse-power	746 watts 0 746 kilowatt 33 000 ft-lb per min 550 ft-lb per sec 2 546 Btu per hr	1 kilowatt	1 000 watts 1.34 h.p. 44 220 ft-lb per min 737 6 ft-lb per sec 3 412 Btu per hr

Measurement of Pressure. It is customary to measure PRESSURE by means of GAUGES which, in reality, only indicate the difference between the pressure being measured and the pressure of the atmosphere, BAROMETRIC PRESSURE, at the same time and place. These gauges may indicate either a higher or lower pressure than that of the atmosphere; in the former case they are known as PRESSURE-GAUGES and in the latter as VACUUM-GAUGES or DRAFT-GAUGES.

Pressure-Gauges and Vacuum-Gauges. The most common type of pressure-gauge (Fig. 1) is provided with a flexible hollow brass tube of oval cross-section known as a BOURDON TUBE. When subjected to pressure, this tube tends to straighten out; and this causes a sector of a gear to mesh with a small pinion, which is on the same shaft with the indicating hand or pointer, and rotate the latter a corresponding amount. The pointer is placed just in front of a graduated dial, not shown in the figure, from which the pressure may be read in suitable pressure-units, such as pounds per square inch. These gauges may also be used for indicating vacuum, or a pressure less than that of the atmosphere.

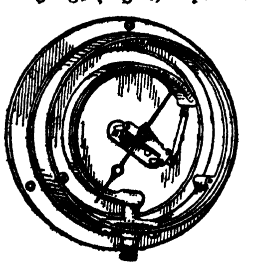


Fig. 1. Single-spring Pressure-gauge. Interior View.

Draft-Gauges. The measurement of pressures but slightly above or below the atmospheric pressure, barometric pressure, is usually accomplished by the use of a

DRAFT-GAUGE. This is essentially a U tube, containing either water, kerosene, alcohol, or mercury, mounted upon a graduated scale, and reading either in inches of fluid or in pounds or ounces per square inch. Since the pressure indicated is a differential one, due to the left-hand leg being open to the air, the reading must be obtained by adding the depression in the left-hand leg to the elevation in the right-hand leg, using zero as the reference-point in both cases.

Barometers. The PRESSURE OF THE ATMOSPHERE is usually measured by a MERCURIAL BAROMETER (Fig. 2), which, in its simplest form, consists of a glass tube about 3 ft long, closed at one end. After being filled with mercury it is inverted in a shallow bath of mercury. The pressure of the atmosphere at sea-level maintains the mercury-column in the tube about 30 in above the level in the bath or cistern. The barometric height or length of this column of mercury varies with the altitude above or below sea-level. When the mercury in the tube falls, that in the cistern rises in corresponding proportion, and vice versa, so that there is an ever-varying relation between the level of the mercury in the tube and the mercury in the cistern, which affects the accuracy of the readings. It is, therefore, necessary, before reading the height of the mercury-column on the stem of the barometer by means of a movable vernier, to adjust the level of the mercury in the cistern. All standard or observatory-barometers of the mercurial type have this adjustment. Barometers of other types, such as the ANEROID BAROMETER, must be frequently compared with a standard mecurial barometer in order to check the accuracy of their readings.

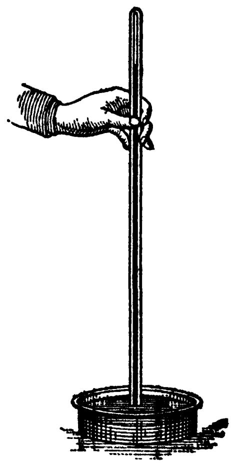


Fig. 2. Simple Barometer

Barometric Pressure. By BAROMETRIC HEIGHT is meant the height of a column of pure mercury at 32° F. which just balances the pressure of the atmosphere at the time and place of the observation. The STANDARD or NORMAL BAROMETRIC PRESSURE is defined as the pressure of a column of pure mercury 760 mm (29.92 in) high at 32° F. This is the normal barometric pressure at latitude 45° and at sea-level. Since the weight of 1 cu in of mercury under these same conditions is 0.491 lb, the normal barometric pressure = height of mercury-column × weight per cubic inch = 29.92 × 0.491, or 14.7 lb per sq in. This pressure of 14.7 lb per sq in is known as the ABSOLUTE PRESSURE of the atmosphere at latitude 45° and at sea-level. Now, since the ordinary pressure-gauge measures only pressures above or below that of the atmosphere, it is necessary to ADD THE BAROMETRIC PRESSURE at the place in question TO THE GAUGE-READING TO OBTAIN THE TOTAL ABSOLUTE PRESSURE corresponding to the pressure indicated by the gauge. That is, absolute pressure = barometric pressure + gauge-pressure.

Heat

Definition of Heat. HEAT IS A FORM OF ENERGY. It is, in fact, the kinetic and potential energy of the molecules of which all substances, whether solid, liquid, or gaseous, are composed. Whenever the vibratory motion of the molecules composing a body of given mass is increased from any cause the THERMAL

KINETIC ENERGY is increased. The temperature of the body rises, its SENSIBLE HEAT increases, and the body feels warmer.

Measurement of Temperature. Thermometry. INTENSITY OF HEAT is measured by THERMOMETERS and PYROMETERS, the latter being used for high temperatures above from 400° to 500° F. In engineering work mercurial thermometers are very largely employed. These depend upon the uniform expansion of mercury to indicate changes in temperature. The UNIT OF MEASUREMENT is called a DEGREE, and is capable of very exact determination, provided that two points, at which the heat-intensity is always constant, can be used as bases or references for calibration. The melting-point of ice and boiling-point of water at atmospheric pressure are usually selected as bases, and the uniform expansion of the mercury between these two points is indicated on a scale divided into 180, 100, or 80 divisions. (Fig. 3) Each of these divisions is known as a DEGREE and the scales used are known respectively as FAHRENHEIT, CENTIGRADE or CELSIUS, and REAUMUR. The Fahrenheit scale is used almost exclusively in engineering in this country.

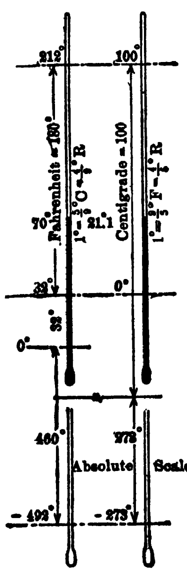


Fig. 3. Fahrenheit and Centigrade Thermometers

Absolute Temperature. In addition to the three temperature-scales already described, physicists employ what is known as the ABSOLUTE SCALE OF TEMPERATURES, based on the so-called ABSOLUTE ZERO OF TEMPERATURE, at which point no molecular vibration exists. This zero is conceived as 491.6° F. below the melting-point of ice (32° F.), it having been discovered that an ideal perfect gas would change in volume by 1/491.6 of its volume at 32° F. for each 1° change in its temperature, at constant pressure. Thus, if 491.6 cu ft of gas, measured at 32° F., is cooled 20° F. at constant pressure, the new volume will be 471.6 cu ft. It is only necessary to add 491.6 − 32, or 459.6, to the actual thermometer-reading to get the absolute temperature. That is, $T = t + 459.6$, where T = absolute temperature, and t = actual thermometer-reading on the Fahrenheit-scale. For engineering-work, 460° is used rather than 459.6°. For the Centigrade scale the relation is $T = t + 273.1$.

Measurement of Heat-Quantity. Calorimetry. HEAT MAY BE MEASURED, since it is a form of energy, in any of the usual energy-units, as the JOULE, FOOT-POUND, or HORSE-POWER HOUR. It is the custom, however, to use for this purpose a special unit more readily applicable to heat-changes. This unit in the English system is known as the BRITISH THERMAL UNIT (Btu), and is the amount of heat required to raise 1 lb of water from 63° to 64° F. For all practical purposes in ordinary calculations, a Btu is the amount of heat required to raise 1 lb of water 1° F.

Specific Heat. It is a well-known fact that equal quantities of heat will raise equal weights of different substances a different number of degrees, depending on the nature of the substances. This property of matter is known as SPECIFIC HEAT, and for any substance can be expressed as the number of Btu required to raise or lower the temperature of 1 lb 1° F., at some given temperature. It is also customary to make use of the mean or average value for a certain temperature-interval. Two specific heats are recognized, one known

as the TRUE SPECIFIC HEAT, measured at the temperature stated, and the other as the MEAN SPECIFIC HEAT, which is the average value between the temperatures under consideration. The specific heat of air at constant pressure is 0.24.

Relation between Units of Energy and Power. Since the various forms of energy, heat, mechanical energy, electrical energy, etc., are mutually convertible, there must be definite numerical relations between the various units used to express energy. As determined by various physicists the relation between the Btu and the ft-lb is

$$1 \text{ Btu} = 777.64 \text{ ft-lb}$$

The number 777.64 is called the MECHANICAL EQUIVALENT OF HEAT and is denoted by J. For ordinary use the value 778 may be taken. Another convenient relation is:

$$1 \text{ h.p} = 2\,546 \text{ Btu per hr}$$

Steam

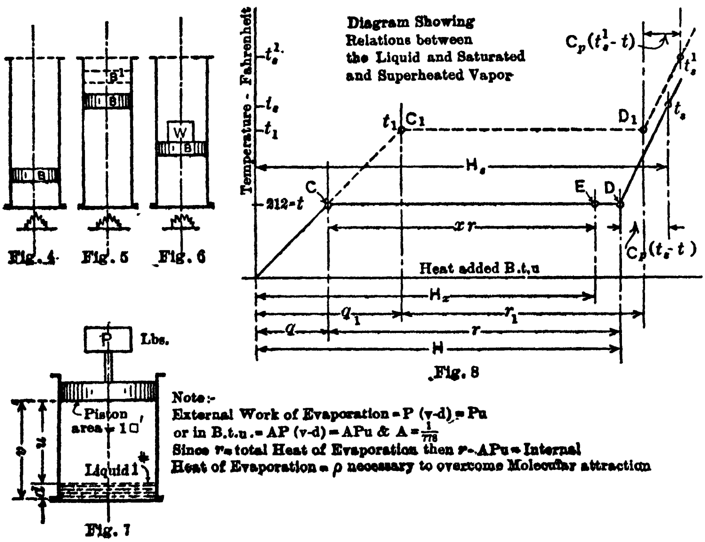
Properties of Steam. Steam is water-vapor, which exists in the vaporous condition because sufficient heat has been added to the water, from which the steam has been formed, to supply the latent heat of evaporation, and to change the liquid into a vapor. This change of state takes place at a definite and constant temperature, which is determined solely by the pressure of the steam. A change in pressure will always be accompanied by a change in the temperature at which ebullition or boiling will occur, and there will be a corresponding change in the latent heat. The properties of steam, together with other characteristics, are tabulated in the steam-tables. (See Table I.) Steam in contact with the water from which it has been generated is known as SATURATED STEAM, and may be known as DRY SATURATED STEAM, or as WET SATURATED STEAM. The latter contains more or less actual water in the form of mist or PRIMING, as it is called. If dry, saturated steam be heated, and the pressure maintained the same as when it was vaporized, its temperature will increase and it will become SUPERHEATED; that is, its temperature will be higher than that of saturated steam at the same pressure.

Sensible and Latent Heat. Whenever heat is added to a substance, without change of state, its temperature is raised, and the heat thus added is known as SENSIBLE HEAT, as, for example, the heat added to water the temperature of which is between 50° and 140° F. Sensible-heat changes, as already stated, are measured by the thermometer. Heat may be added to a body without any change of temperature provided a change of state from solid to liquid or from liquid to vapor takes place, and the heat thus added is known as LATENT HEAT. When the change is from solid to liquid, as from ice to water, this heat is known as the LATENT HEAT OF FUSION. At atmospheric pressure ice melts at 32° F., and the latent heat is 144 Btu per lb. When the change is from liquid to vapor, as from water to steam, the heat required to effect the change is known as the LATENT HEAT OF EVAPORATION. At atmospheric pressure water evaporates at 212° F., and the latent heat is 971.7 Btu per lb. A conception of the relation between the properties or characteristics of steam, and the manner in which the changes of state, temperature and pressure are brought about, is described in the following paragraphs.

Generation of Steam. Consider a frictionless cylinder (Fig. 4), containing 1 lb of water at 32° F. Also consider the pressure of the atmosphere to be 14.7 lb per sq in and to be replaced by that of the piston B. When heat is applied to the cylinder the temperature of the water rises until the boiling-point, 212° F., is reached. The heat necessary to raise the temperature from

32° F. to the boiling-point is known as the HEAT OF THE LIQUID or SENSIBLE HEAT, and is denoted by the symbol Q. This condition is denoted in Fig. 8 by the point C. The average specific heat of water between 32° F. and 212° F. is 1; hence the number of British thermal units (Btu) necessary to raise the temperature of the water this amount is 212 − 32 or 180 Btu.

When more heat is added the water begins to evaporate and expand at constant temperature until, as in Fig. 5, the water is entirely changed into steam. This condition is also shown in Fig. 8, by the point D. The heat thus added is known as the LATENT HEAT OF EVAPORATION and is denoted by the symbol r. This heat r is subdivided into two parts. (See Fig. 7.) First the attraction between the molecules must be broken down. This is known as the INTERNAL LATENT HEAT and is denoted by the symbol ρ. Next the external resistance



Figs. 4 to 8. Diagrams Explaining the Generation of Steam

must be overcome, the weight P being raised against gravity. The heat thus added is known as EXTERNAL LATENT HEAT and is designated by the symbols APu, where u is the change in volume, in cu ft of 1 lb of water, A is 1/778, and P is the pressure of the atmosphere in pounds per square feet (barometic pressure). It is evident then that the latent heat

$$r = \rho + APu, \quad \text{or} \quad \rho = r - APu$$

The term APu is the heat-equivalent of the work performed for the change in volume from water to steam.

The heat added from the starting-point (32° F.), is known as TOTAL HEAT (H), or $q + r = H$. If more heat is added, the pressure remaining constant, the temperature of the steam rises and the steam becomes what is known as SUPERHEATED STEAM. The heat added is equal to the MEAN SPECIFIC HEAT (C_p) of the steam, times the change in temperature ($t_s - 212$). The specific

Table I. Properties of Saturated Steam *

G. A. Goodenough

Absolute pressure		Temperature, deg. F.	Volume, cu ft per lb	Weight, lb per cu ft	Heat-content in Btu		Latent heat in Btu	
Inches of mercury	Lb per sq in				of liquid	of vapor	of vaporization	Internal
	p	t	v	d	q	H	r	ρ
4 072	2	126.10	173.6	0.00576	94 02	1 116.2	1 022.2	957.9
8 144	4	152.99	90.6	0 01104	120.9	1 127.9	1 007.0	939.9
12.216	6	170.07	62.0	0.01614	137.9	1 135.0	997.1	928.2
16.29	8	182.87	47.35	0.02112	150.8	1 140.3	989.5	919.4
20.36	10	193.21	38.43	0 02602	161.1	1 144.4	983.3	912.2
24.43	12	201.96	32.41	0 03086	169.9	1 147.9	978 0	906 0
30	14.74	212.13	26.75	0.03739	180.1	1 151.8	971.7	898.8
. .	16	216 3	24.76	0.04038	184.3	1 153.4	969.1	895.8
....	18	222.4	22.18	0 04508	190 5	1 155.7	965.2	891.4
. .	20	228.0	20.10	0.04976	196.0	1 157.7	961.7	887.3
..	22	233 1	18 38	0 0544	201.2	1 159.6	958.4	883.6
.	24	237.8	16.95	0.0590	206.0	1 161.3	955.3	880.1
.	26	242.2	15.73	0.0636	210.4	1 162.8	952.4	876.8
.	28	246.4	14.67	0.0681	214.6	1 164.3	949.7	873.7
.	30	250.3	13.76	0 0727	218.6	1 165.7	947.1	870.7
.....	32	254.0	12.95	0 0772	222.4	1 166.9	944.6	867.9
.. .	34	257.6	12.24	0.0818	225 9	1 168.1	942.2	865.2
..	36	260.9	11.60	0 0862	229 4	1 169.2	939.9	862.7
..	38	264.2	11.03	0 0907	232.6	1 170.3	937.7	860.2
.	40	267.2	10.51	0 0951	235.8	1 171.3	935.5	857.8
..	50	281.0	8.53	0.1173	249.8	1 175.6	925.9	847.1
.	54	285.9	7.93	0.1261	254.7	1 177.1	922.4	843.2
.	60	292.7	7.18	0.1392	261.7	1 179.1	917.4	837.8
...	64	296.9	6.76	0.1479	266.1	1 180.3	914.3	834.3
	70	302 9	6.22	0.1609	272.2	1 182.0	909 8	829.5
.	74	306 7	5.90	0.1695	276.1	1 183.0	906.9	826.4
.	80	312 0	5.48	0 1824	281.6	1 184.4	902.8	821.9
. .	84	315 4	5.23	0 1910	285.1	1 185.3	900.2	819.1
.	90	320 3	4.905	0.2039	290.1	1 186.5	986.4	815.0
. ..	94	323.3	4.709	0.2124	293.3	1 187.3	894.0	812.4
.	100	327.8	4.442	0.2251	297.9	1 188.4	890.5	808.6
.	104	330 7	4.279	0 2337	300.9	1 189.0	888.2	806.1
. .	110	334.8	4.057	0.2465	305.1	1 190.0	884.8	802.6
.	114	337.4	3.921	0.2550	307.9	1 190.6	882.7	800.3
	120	341 3	3.735	0.2678	311.9	1 191.4	879.5	796.9
.	124	343.7	3.620	0.2762	314.4	1 191.9	877.5	794.8
... .	130	347.4	3.461	0.2889	318.2	1 192.6	874.4	791.6
... .	134	349.7	3.363	0.2973	320.6	1 193.1	872.5	789.5
.. .	140	353.1	3.226	0.3100	324.2	1 193.7	869.6	786.4
.. .	144	355 3	3.140	0.3184	326.5	1 194.1	867.7	784 5
...	150	358.5	3.020	0.3311	329.8	1 194.7	864.9	781.6
.. .	154	360.5	2.945	0.3396	332.0	1 195.1	863.1	779.7
.. .	160	363.6	2.839	0.3522	335.2	1 195.7	860.5	776.9
... .	164	365.6	2.773	0 3606	337.3	1 196.0	858.7	775.1
.. .	170	368.5	2.679	0.3733	340.3	1 196.5	856.2	772.4
.....	174	370.4	2.620	0.3817	342.3	1 196.8	854.5	770.6
.....	180	373.1	2.536	0.3943	345.2	1 197.2	852.0	768.0
.....	190	377.6	2.408	0.4154	350.0	1 197.9	847.9	763.9
....	200	381.9	2.292	0.4364	354.5	1 198.5	844.0	759.8

* Condensed from original tables published by John Wiley & Sons, Inc.

heat of steam is the Btu, or heat, required to raise the temperature of 1 lb of the steam 1° F. Since the specific heat of steam is less than that of water, the slope of this line becomes greater than that of the water-line. The point is now located at t_s (Fig. 8), and the steam has increased in volume in the cylinder (Fig. 5), until the piston occupies the dotted position B'.

If instead of the above condition of pressure, additional pressure is added, as shown by the weight W in Fig. 6, the temperature of the boiling-point will be raised from the temperature of 212° F. to some other point, as t_1 (Fig. 8). As may be seen by this figure, the sensible heat q has been increased to q_1. When more heat is added the water is evaporated at the temperature t_1, and if heat again be added the SATURATED STEAM will become SUPERHEATED STEAM.

Quality of Steam. The proportion of the DRY STEAM, per pound of steam delivered by the boiler, is known as the QUALITY OF THE STEAM and is represented by the symbol x, and the heat (Hx) contained in the steam above 32° F. is $q + xr$; the state-point is located at E (Fig. 8).

Specific Volume and Density. The volume of a pound of steam is known as the SPECIFIC VOLUME v, and as may be seen by comparing Figs. 5 and 6, decreases as the pressure increases. The reciprocal of this, or the weight of steam per cubic foot, is known as the DENSITY, and is denoted by d or $1/v$.

Entropy. Another quantity known as ENTROPY is made use of in calculations relating to steam-engines and turbines, and is defined as the ratio obtained by dividing the quantity of heat added to a substance by the absolute temperature at which it is added.

The Total Heat, H, of a dry, saturated vapor for any pressure and temperature is the sum of the heats required to raise the temperature of one pound of the liquid from the freezing-point to the given temperature and corresponding pressure and ENTIRELY VAPORIZE IT at this pressure. For this case $x = 1$, and consequently

$$H = (p + APu) + q = r + q$$

The total heat (H_x) of wet vapor at any pressure and temperature is

$$H_x = xr + q$$

It is manifestly incorrect to say that this is the heat in the vapor, as the APu is not the heat in the vapor, but the external work performed by the vapor while evaporating.

Superheated Steam or Vapor. Superheated steam is defined as water-vapor which has been heated out of contact with its liquid, until its temperature is higher than that of saturated vapor at the same pressure.

The heat-content of superheated steam or vapor may be expressed by the equation

$$H_s = q + r + C_p\ (t_s - t) = H + C_p\ (t_s - t)$$

where t_s is the temperature of superheated vapor, t the temperature of saturated vapor at the corresponding pressure, q the heat of the liquid at t, and r the heat of vaporization at temperature t. C_p is the mean specific heat of superheated vapor (approximately 0.50), H the total heat of 1 lb of dry saturated steam, and H_s the total heat of 1 lb of superheated steam.

Properties of Air

Charles' Law. Charles' Law refers to the relation between pressure, volume and temperature of a gas, and may be stated as follows. The volume of a given weight of gas varies directly as the absolute temperature at constant pressure, and the pressure varies directly as the absolute temperature at con-

stant volume. Hence, when heat is added at constant volume V_c, this equation results:

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

or for the same temperature-range, at constant pressure P_c, the relation is

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

In general, for any weight of gas M, since volume is proportional to weight at any given volume and temperature,

$$PV = MRT$$

which is the characteristic equation for a perfect gas. In this formula

P = the absolute pressure of the gas in pounds per square foot = 2116.8 (atmospheric pressure);
V = the volume of the weight M in cubic feet;
M = the weight in pounds of the gas taken;
R = a constant depending on the nature of the gas = 53.37 for air;
T = the absolute temperature in degrees Fahrenheit (t + 459.6).

Table II. Properties of Dry Air

Barometric pressure, 29.921 in. Specific heat, 0 24

Temperature in degrees Fahrenheit	Weight per cubic foot in pounds	Per cent of volume at 70° Fahrenheit	Btu absorbed by one cubic foot dry air per degree Fahrenheit	Cubic feet of dry air warmed one degree per Btu
0	0 08636	0.8680	0 02080	48.08
10	0.08453	0.8867	0.02039	49 05
20	0.08276	0.9057	0.01998	50.05
30	0 08107	0.9246	0 01957	51.10
40	0 07945	0.9434	0 01919	52.11
50	0 07788	0 9624	0.01881	53 17
60	0 07640	0.9811	0.01846	54 18
70	0 07495	1 0000	0.01812	55.19
80	0.07356	1 0190	0 01779	56.21
90	0 07222	1 0380	0.01747	57 25
100	0 07093	1.0570	0 01716	58 28
110	0 06968	1 0756	0 01687	59.28
120	0 06848	1.0945	0 01659	60 28
130	0 06732	1 1133	0 01631	61 32
140	0 06620	1 1320	0 01605	62 31
150	0.06510	1.1512	0.01578	63.37
160	0 06406	1 1700	0.01554	64 35
170	0 06304	1.1890	0 01530	65 36
180	0 06205	1.2080	0 01506	66 40
190	0 06110	1.2270	0 01484	67.40
200	0 06018	1.2455	0.01462	68.41
240	0 05673	1.3212	0 01380	72.46
300	0.05225	1.4345	0.01274	78 50
350	0.04903	1.5288	0.01197	83.55
400	0.04618	1 6230	0.01130	88.50
450	0 04364	1.7177	0.01070	93.46
500	0 04138	1 8113	0.01018	98.24
550	0.03934	1.9060	0.00967	103.42
600	0.03746	2.0010	0.00923	108.35
700	0.03423	2.1900	0.00847	118.07

A PERFECT GAS conforms exactly to the above equation, and while no gases are PERFECT in this sense, they conform so nearly that the above equation applies to most engineering-computations. The volume of 1 lb of air, known as the SPECIFIC VOLUME, at any temperature and pressure, can be found at once by the equation

$$V = (53.37 \times T)/P$$

Estimating Heating Requirements of Buildings

Heat Required and Supplied. The amount of heat, measured in Btu, to be supplied by the heating-apparatus to a building to maintain the inside temperature above that of the outside, commonly termed HEAT-LOSSES, is:

(*a*) The heat required to offset the heat-transmission of the walls, ceiling or roof, and floor. This loss of heat depends upon the type and materials of construction used and the temperature-difference to be maintained between the inside and the outside of the building.

(*b*) The heat required to warm the air entering the building from the outside, either by infiltration or purposely introduced for ventilation.

(*c*) The heat supplied by persons, lights, machinery and motors, which may be deducted from the sum of items (*a*) and (*b*) to obtain the net amount of heat to be supplied by the heating-apparatus. (Item (*c*) is usually not considered.)

It is customary in all calculations connected with the design of heating-installations to base the estimate on the amount of heat per hour to be supplied by the apparatus. The total heat to be supplied per hour is H = [(item *a*) + (item *b*) − (item *c*)] Btu. The method in use for the calculation of the various items above mentioned will now be taken up and discussed in the order given.

Temperatures. The inside temperature to be maintained and the air required for ventilation for various classes of work are discussed under Ventilation, to which the reader is referred. The outside temperature for which the heating-installation should be designed is fixed by the lowest outside temperature that is liable to continue for several days during the heating-season.

Usual Inside Temperature Specified

Kind of buildings	Degrees F.
Public buildings	68–72
Factories	65
Machine-shops	60–65
Foundries, boiler-shops, etc	50–60
Residences	70
Bath-rooms	85
Schools	70
Hospitals	72–75
Paint-shops	80

In designing the heating-system a temperature of from 10° to 15° F. higher than the lowest recorded temperature is recommended to be used for the outside temperature. (See Table III.)

Heat-Transmission of Walls, Ceilings, Roofs, Floors, etc. (*a*) The heat-loss through building-construction is dependent upon the character of the material, thickness and character of the surfaces, and the velocity of the air over the surfaces. Numerous tests have been conducted by various experimenters to determine accurately the heat-transmission of various types of

Table III. Outside Temperatures

Lowest and Average Temperatures in the United States. All stated in Fahrenheit degrees and compiled from United States Weather Bureau Records

State	City	Lowest	Average*	State	City	Lowest	Average*
Ala.	Mobile	−1	57.7	Neb.	North Platte	−35	34.6
	Montgomery	5	56.1		Lincoln	−29	35.8
Ariz	Flagstaff	−21	34.8	Nev.	Carson City	−22	
	Phoenix	22	58.9		Winnemucca	−28	37.9
Ark	Fort Smith	−15	49.5	N. H.	Concord	−35	33.1
	Little Rock	−12	52.0	N. J.	Atlantic City	−7	41.6
Cal.	San Diego	32	57.2	N. Y.	Saranac Lake	−38	34.1
	Independence	10	48.7		New York City	−6	40.1
Col	Denver	−29	38.4	N. M.	Roswell	−14	48.9
	Grand Junction	−16	39.2		Santa Fé	−13	38.0
Conn.	Southington	−19	36.3	N. C.	Hatteras	8	53.3
D. C.	Washington	−15	42.9		Charlotte	−5	49.8
Fla	Jupiter	24	69.8	N. D.	Devil's Lake	−51	18.9
	Jacksonville	10	60.9		Bismarck	−44	23.5
Ga.	Savannah	8	57.2	Ohio	Toledo	−16	36.8
	Atlanta	−8	51.4		Columbus	−20	39.8
Idaho	Boise	−28	39.6	Okla	Oklahoma	−17	47.1
	Lewiston	−18	42.5	Ore.	Baker City	−20	34.1
Ill.	Chicago	−23	35.9		Portland	−2	45.4
	Springfield	−22	39.0	Pa.	Pittsburgh	−20	40.8
Ind.	Indianapolis	−25	40.4		Philadelphia	−6	41.8
	Evansville	−15	44.1	R. I.	Providence	−9	37.5
Ia.	Sioux City	−31	32.1		Rock Island	−4	39.7
	Keokuk	−26	37.6	S. C.	Charleston	7	56.9
Kan.	Dodge City	−26			Columbia	2	53.5
	Wichita	−22	42.9	S. D.	Huron	−43	25.9
Ky.	Louisville	−20	45.0		Yankton	−32	31.2
La.	New Orleans	7	60.5	Tenn	Knoxville	−16	47.0
	Shreveport	−5	55.7		Memphis	−9	50.7
Me.	Eastport	−21	31.1	Tex	Corpus Christi	11	62.7
	Portland	−17	33.5		Fort Worth	−8	49.5
Md.	Baltimore	−7	43.3	Utah	Salt Lake City	20	39.7
Mass	Boston	−13	37.2	Vt	Northfield	−32	27.8
Mich	Alpena	−27	29.1	Va.	Cape Henry	5	48.6
	Detroit	−24	35.3		Lynchburg	−5	45.2
Minn	Duluth	−41	25.5	Wash.	Seattle	3	44.3
	Minneapolis	−33	28.4		Spokane	−30	37.0
Miss.	Meridian	−6	53.9	W. Va	Parkersburg	−27	41.9
	Vicksburg	−1	56.0		Elkins	−21	38.8
Mo.	Springfield	−29	43.0	Wis.	La Crosse	−43	31.2
	Hannibal	−20	39.7		Milwaukee	−25	32.4
Mont	Havre	−55	27.7	Wyo	Cheyenne	−38	33.7
	Helena	−42	30.9		Lander	−36	29.0

* Average is taken from October 1 to May 1.

construction. Tests are usually conducted to determine the conductivity (c) of the material, which when combined with the surface coefficients (k_1 and k_2) in the transmission formula gives the unit heat-transmission (U) for the construction under consideration.

Table IV. Coefficients of Transmission *U*

Btu per sq ft per hr per ° F diff. in air temperatures

WOOD WALLS

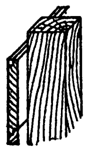
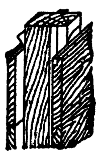
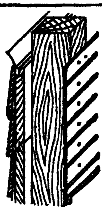
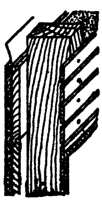
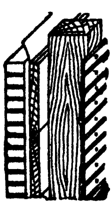
	⅞-in sheathing and building paper	0.50
	1⅝-in sheathing and building paper	0.40
	⅞-in sheathing, building paper and air space	0 24
	Clapboards, sheathing, paper	
	+ ½-in plaster on wood lath	0.25
	+ ¾-in plaster on metal lath	0.26
	+ ½-in plaster on ⅜-in plaster board	0 25
	+ ½-in plaster on ½-in rig. ins. board	0.19
	+ ½-in plaster on 1½-in corkboard	0.11
	+ ½-in plaster on 2-in corkboard	0.10
	+ ½-in plaster on wood lath + 3⅝-in flaked gypsum fill	0.09
	Ditto with 3⅝-in rock wool fill	0.07
	Stucco-Outside Finish	
	+ ½-in plaster on wood lath	0 30
	+ ¾-in plaster on metal lath	0.31
	+ ½-in plaster on ⅜-in plaster board	0.30
	+ ½-in plaster on ½-in rig. ins. board	0.22
	+ ½-in. plaster on 1½-in corkboard	0.12
	+ ½-in plaster on wood lath + gypsum fill	0.10
	Ditto with rock wool fill	0.07
	Brick Veneer	
	+ ½-in plaster on wood lath	0.27
	+ ¾-in plaster on metal lath	0.28
	+ ½-in plaster on ⅜-in plaster board	0.27
	+ ½-in plaster on ½-in rig. ins. board	0 20
	+ ½-in plaster on 1½-in corkboard	0.12
	+ ½-in plaster on wood lath + gypsum fill	0.10
	Ditto with 3⅝-in rock wool fill	0.07

Table IV (Continued). Coefficients of Transmission *U*

Btu per sq ft per hr per ° F diff. in air temperatures

COMMON BRICK WALLS

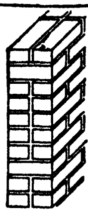
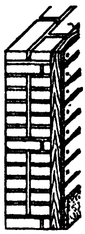
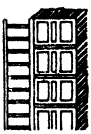
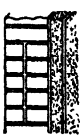
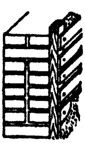
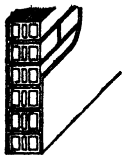
Thickness of brick wall (X)			4 in	8 in	12 in	16 in
	Plain brick wall (4-in face brick)—no interior finish		...	0 50	0.36	0.28
	Brick wall with ½-in plaster inside		....	0 46	0.34	0.27
	Brick Wall Furred					
	½-in plaster—wood lath		...	0 30	0.24	0.20
	¾-in plaster—metal lath		....	0.32	0.25	0.21
	½-in plaster on ⅜-in plaster board		....	0.25	0.21	0.18
	Brick Wall Furred					
	½-plaster on ½-in rigid insulating board		...	0 22	0.19	0.16
	½-in plaster on plaster board, furred		. ..	0 30	0 24	0.20
	4-in brick wall with tile backing, plastered inside	Tile thickness				
		6″	0 34	...	....	....
		8″	0.33	...	...	....
		10″	0 32	.	...	....
		12″	0.26	...	....	....
	Brick wall insulated with corkboard, plastered inside	Corkboard thickness				
		1″	....	0.17	0.15	0.14
		2″	....	0.11	0.10	0.10
		3″	....	0.08	0.07	0.07
		4″	....	0.06	0.06	0.06
	½-in plaster, wood lath, 1 ⅝-in furring strips with flake gypsum filling for insulation		....	0.17	0.15	0.14
	Rock wool filling		....	0.13	0.12	0.11

Table V. Coefficients of Transmission U

Btu per sq ft per hr per ° F diff. in air temperatures



Tile Walls—Thickness tile	8 in	10 in	12 in
Tile walls with 2-in stucco finish outside plain	0.40	0.39	0.30
+½-in plaster finished inside...	0 37	0.37	0.29
+¾-in plaster on metal lath....	0 27	0.27	0 22
+½-in plaster—wd. lath finished	0 26	0.26	0 22
+1-in corkboard +½-in plaster finish.	0.17	0.16	0.15
+2-in corkboard +½ in plaster finish..	0.11	0.11	0.10
+3-in corkboard +½-in plaster finish.	0 08	0.08	0.07
+4-in corkboard +½-in plaster finish	0 06	0 06	0 06
Plain tile faced with 4-in stone	0 36	0.35	0 28
+½-in plaster finish on tile	0 34	0 33	0 26
+¾-in plaster finish on metal lath	0 25	0.25	0.21
+½-in plaster finish on wood lath	0 24	0 24	0 20
Concrete Walls—Thickness	**6 in**	**8 in**	**10 in**
Plain concrete walls..............................	0 79	0 62	0 48
+½-in plaster on concrete	0 70	0 57	0 44
+¾-in plaster on metal lath	0 42	0 37	0 31
+½-in plaster on wood lath, furred...............	0 39	0 34	0 29
+1-in corkboard +½-in plaster finish..............	0 21	0.20	0.19
+2-in corkboard +½-in plaster finish..............	0 12	0.12	0 11
+3-in corkboard +½-in plaster finish..............	0 09	0 08	0.08
+4-in corkboard +½-in plaster finish..............	0 07	0.07	0 07
Concrete Floors on Ground *			
Bare concrete	0 90	0 79	0.70
+Y. p. fl'g on sleepers...........................	0 48	0 45	0.41
+Y. p. fl'g on sleepers + fin. maple fl'g............	0.36	0.34	0 32
Windows, single............	.....	......	1 1
Windows, double (with air space).................	...	......	0 45
Corrugated sheet steel.........................	. ...	......	1.5
⅜-in corrugated asbestos.........................	...	......	1.4

* It is customary to assume ground temperature as 55° F with only inside surface coefficient, $K_1 = 1.6$.

Table V (Continued). Coefficients of Transmission U

Btu per sq ft per hr per ° F diff. in air temperatures

Flat Roofs with Built-Up Roofing	
7/8-in wood	0.50
1 5/8-in wood	0 40
4-in concrete	0.72
6-in concrete	0.64
4-in concrete + 1-in corkboard insulation	0 21
6-in concrete + 1-in corkboard insulation	0 21
1 5/8-in precast tile	0 85
2 3/8-in gypsum fiber concrete	0.40
3 3/8-in gypsum fiber concrete on plaster board	0 32
Flat metal roofs—no insulation	0 95
Flat metal roofs + 1/2-in rigid insulating board	0.39
Flat metal roofs + 1-in rigid insulating board	0 25
Flat metal roofs + 1-in corkboard insulation	0.23
Flat Roofs + Built-Up Roofing with Metal Lath and Plaster Ceilings	
7/8-in wood + roofing—no insulation	0.32
1 5/8-in wood + roofing—no insulation	0.26
4-in concrete + roofing—no insulation	0.40
6-in concrete + roofing—no insulation	0 37
1 5/8-in precast tile—no insulation	0 43
2 3/8-in gypsum fiber concrete—no insulation	0 27
3 3/8-in gypsum fiber concrete—no insulation	0 23
Flat metal roofs—no insulation	0 46
Flat metal roofs + 1/2-in rigid insulating board	0.27
Flat metal roofs + 1-in rigid insulating board	0.19
Flat metal roofs + 1-in corkboard insulation	0.18
Pitched Roofs (Ceiling applied direct to rafters)	
Wood shingles on wood strips (no ceiling)	0.48
Wood shingles + metal lath, plaster ceiling	0.30
Wood shingles + plaster on plaster board ceiling	0 29
Wood shingles + 3 5/8-in flaked gypsum fill with metal lath and plaster ceiling	0.10
Ditto for 3 5/8-in rock wool fill	0.07

It is sufficiently accurate to assume the above values of U for asphalt or asbestos shingles, composition roofing, and slate or tile roofing on wood sheathing.

Space limitation does not permit the inclusion of more complete heat-transmission tables. See the latest edition of the Am. Soc. H. and V. E. Guide.

Heat-Transmission Coefficients (U) by Computation. The following formula may be employed to determine the heat-transmission of various constructions of building materials and insulators:

Let U = unit heat-transmission of a wall, floor or roof under consideration. Btu per degree difference in temperature per hour between the inside and outside air temperatures per sq ft per hour;

c_1, c_2, c_3, etc., = conductivity of the various materials employed in the construction per 1 in thickness (see Table VI);

x_1, x_2, x_3, etc., = thickness of materials measured in inches corresponding to values of c;

k_1 = combined radiation and convection coefficient for inside wall surface (see Table VI);

$k_2 = 3.63\ k_1$ = combined radiation and convection coefficient for outside wall surface.

$$U = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{x_1}{c_1} + \frac{x_2}{c_2} + \frac{x_3}{c_3}, \text{ etc.}}$$

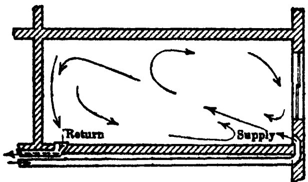
The following examples, Fig. 9, illustrate the method employed in calculating the heat-transmission coefficient for two types of walls.

Heat-Transmission of Roofs and Floors. The temperature of the air in contact with the under side of a ceiling or roof is found to be higher than the temperature maintained at the breathing-line, at which point the temperature is usually measured; and this is due to the natural tendency of the warmer or less dense air to rise. It is recommended that an increase of approximately 15% be made to the specified inside temperature for the temperature at the ceiling for ceiling or wall-heights not exceeding 15 ft, and 30% for ceiling-heights of 20 ft or more, in estimating the heat-loss of roofs. Thus, if 65° F. is the specified inside temperature to be maintained in a room the height of which is 20 ft, the temperature of the air in contact with the under side of the roof may be assumed to be 65° + 30%, or 85° F. The loss of heat through the ceiling of a room over which a large air-space exists, through partitions between a heated and a cold room, or through the first floor to the cellar, may be estimated on the assumption that the warmed rooms give off sufficient heat to

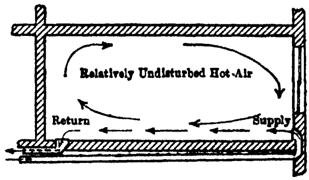
Table VI. Recommended Conductivities of Building Materials and Insulation (*c*)

Material	Conductivity
Brick, common	5.00
Brick, face	9.2
Cement mortar	12.00
Cinder concrete	5.20
Cinder blocks † (8 in)	0.62*
Cinder blocks† (12 in)	0.51*
Concrete blocks † (8 in)	1.0*
Haydite concrete	1.62
Concrete	12.00
Gypsum fiber concrete	1.66
Hollow clay tile (4 in)	1.00*
Hollow clay tile (6 in) ‡	0.64*
Hollow clay tile (8 in) ‡	0.60*
Hollow clay tile (10 in) ‡	0.58*
Hollow clay tile (12 in) ‡	0.40*
Hollow gypsum tile (4 in)	0.46*
Insulations:	
Corkboard	0.30
Flexible	0.27
Flaked gypsum (24 lb)	0.48
Rigid insulation	0.33
Rock wool	0.30
Plaster (gypsum)	3.3
Plaster board (3/8 in)	3.73*
1-in fir sheathing and building paper	0.82*
Yellow pine lap siding	1.28*
Yellow pine or fir	0.80
Maple or oak	1.15
Shingles, wood	1.28*
Air spaces	1.10*
Surfaces, still air (k_1)	1.65*
Surfaces, 15 mph (k_2)	6.00*
Plaster board (1/2 in)	2.82*
Roofing:	
Asbestos shingles	6.00*
Asphalt or composition roofing	6.50*
Built-up, 3/8-in thick	3.53*
Slate shingles	10.37
Wood shingles (see woods).	
Stone	12.50
Stucco	12.00
Tile or terrazzo	12.00
Wood lath and plaster	2.50*
Woods:	
1-in fir sheathing, building paper and yellow pine lap siding	0.50*

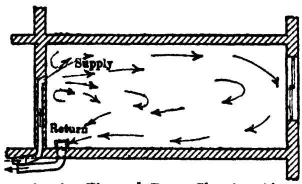
* For thickness or condition stated, not per 1 in.
† One air cell in the direction of heat flow.
‡ The 6-in, 8-in, and 10-in hollow tile figures are based on two cells in the direction of heat flow.



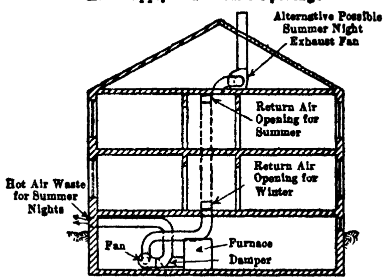
Section Through Room Showing Air Circulation when Heating with Low-Supply and Return Openings



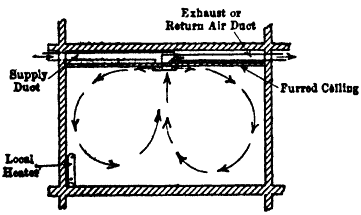
Section Through Room Showing Air Circulation when Cooling with Low-Supply and Return Openings



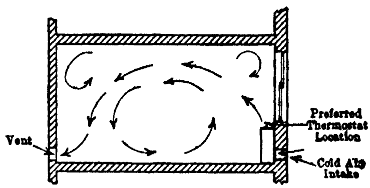
Section Through Room Showing Air Circulation when Cooling with High-Supply Opening and Low-Return Openings



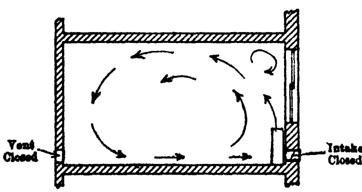
Section Through an Elemental Mechanical Warm Air Heating-Cooling System. The Attic Fan is Alternative



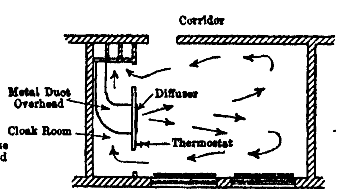
Section Through a Radiator-Heated Room



Section Through a Unit Ventilator-Equipped Room when Heating



Section Through a Unit Conditioner-Equipped Room when Cooling

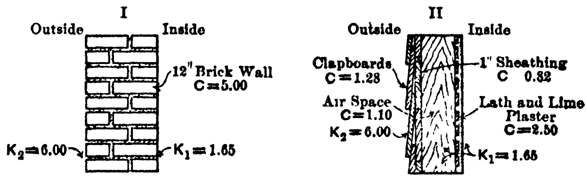


Plan of a Classroom in a School Ventilated by a Central Fan

Typical Air Distribution in Rooms with Various Types of Systems. (Taken from "American Society of Heating and Ventilating Engineers' Guide.")

maintain the temperature of these colder spaces according to the following schedule:

Closed attics under metal or slate roofs	14° F.
Closed attics under tile, cement, tar, or gravel roofs	23° F.
Cellars and rooms kept closed	35° F.



$$U = \frac{1}{1/1.65 + 1/6.00 + 12/5} = 0.32 \qquad U = \frac{1}{1/1.65 + 1/6.00 + 1/1.28 + 1/0.82 + 1/1.10 + 1/2.50} = 0.25$$

Fig. 9. Examples in the Calculation of Heat Transmission of Various Constructions

The heat-transmission of floors that are laid directly upon the ground may be estimated on the assumption that the ground in contact with the under side of the floor has an approximate temperature of 50° F. Thus the estimated heat-loss through a 6-in concrete floor laid directly upon the ground, assuming an inside temperature of 65° F., is

$$0.563\,(65 - 50) \quad \text{or} \quad 8.4 \text{ Btu per square foot per hour}$$

Table VII. Heat Equivalent of Air Entering by Infiltration

The heat (h) required to warm the air entering building is:

Q = cu ft per hr entering air based on inside temp.
0.24 = sp ht air
d = density (wt cu ft) = 0.075 (appx.)
t, t_0 = inside and outside air temp, ° F.
$h = 0.24 \times 0.075\,(t - t_0)\,Q$ Btu per hr.

Heat Required per Cu. Ft. Infil. per Hr.

Inside temp, ° F	Outside temp, ° F		
	−10	0	+10
50	1 12	0.94	0.75
60	1.28	1.09	0 91
70	1.44	1.26	1.08
80	1.60	1.42	1.22

Table VIII

Air Change Method

Assume number of air changes from following table and multiply by cubic contents of room for estimate of Q.

	No. Air Changes
Rooms exposed one side	1
" " two sides	2
" " three-four sides	2½
Rooms, no windows or outside doors	½
Entrance halls, drug stores	2½
Bath rooms	2
Churches	¾
Factory buildings (large areas)	¾ to 1

Crack Method (preferred method)

Q is determined by taking the linear feet crack times the cubic feet per hour from Table IX.

For double-hung windows take perimeter of sash for length of crack, plus meeting rail. Add perimeter of frame in masonry wall, if not calked.

For continuous factory steel sash take sides and top in contact with steel mullions and lintel for each sash unit, plus perimeter of ventilating section.

For single sash unit set in masonry wall take top width, plus perimeter of ventilating section. The infiltration through well-constructed walls is small and is customarily neglected.

Table IX. Air Leakage in Cubic Feet per Hour and Btu Necessary to Heat Such Leakage from 0° to 70° F for Cracks Indicated

Type of window		Wind velocity, miles per hour					
		5	10	15	20	25	30
		cubic feet per foot of crack per hour					
Double-hung wood sash windows (unlocked)	Around sash, 1/16-in crack and 3/64-in clearance. Plain non-stripped	39.3	84.9	124	161	....	233
	Average weatherstripped	3	11.7	22.9	34.9	....	59.6
	Around uncalked frame	1.4	11.3	22.6	31.1	...	53.6
Double-hung metal windows	Non-weatherstripped, locked	20	45	70	96	125	154
	Non-weatherstripped, unlocked	20	47	74	104	137	170
	Weatherstripped, unlocked	6	19	32	46	60	76
Rolled section steel sash windows	Industrial pivoted, 1/16-in crack.	52	108	176	244	304	372
	Architectural projected, 3/64-in. crack	20	52	88	116	152	208
	Residential casement, 1/32-in crack	14	32	52	76	100	128
	Heavy casement section, projected, 1/32-in crack	8	24	38	54	72	96

Example. An office 14 by 16 by 10-ft-high ceiling has two unstripped 3 by 7-ft wooden-sash windows. The maintained inside temperature is 70°, and the outside temperature 0° F. Required the heat-loss by infiltration, wind velocity assumed 15 miles per hour.

Solution. By the first method, assuming two air-changes per hour, the loss is estimated as:

$$b = 1.26 \times 2 \times (14 \times 16 \times 10) = 5\,645 \text{ Btu per hr}$$

By the second or crack-method this loss is estimated as:

$$b = 1.26 \times 2(3 + 3 + 3 + 7 + 7 \text{ perimeter}) \times 124 = 7\,187 \text{ Btu per hr}$$

It is assumed that the frames have been calked.

Increase in Calculated Heat-Losses. It is advisable to increase the calculated heat-losses by approximately 15 to 20% for walls that are exposed to the prevailing winds.

Heat Supplied by Persons, Lights, Motors, Machinery, etc. (c) The quantity of heat emitted by persons is ordinarily not of sufficient importance to be taken into account, except in cases of assembly-halls and theaters. The following allowances may be made when required:

(1) Persons (sensible heat):

Man at rest	300 Btu per hour
Man at work	500 Btu per hour

The heat introduced by lights is as follows:

(2) Lights:

Electric lamps:

Btu per hour equals total watts of lamps × 3.415

Gas-lighting:

1 cu ft producer gas	150 Btu
1 cu ft illuminating gas	700 Btu
1 cu ft natural gas	1 000 Btu

A Welsbach burner averages 3 cu ft of gas per hour and a fish-tail burner 5 cu ft per hour.

(3) Motors. Motors and the machinery which they drive, if both are located in the room, convert all of the electrical energy supplied into heat, which is retained in the room if the product being manufactured is not removed until its temperature is the same as the room-temperature.

$$\text{Btu supplied per hour} = \frac{\text{motor horse-power}}{\text{efficiency of motor}} \times 2\,546$$

in which 2 546 is the Btu equivalent of 1 horse-power hour. In high-powered mills this is the chief source of heating and is sometimes sufficient to overheat the building even in zero weather, thus requiring cooling by ventilation the

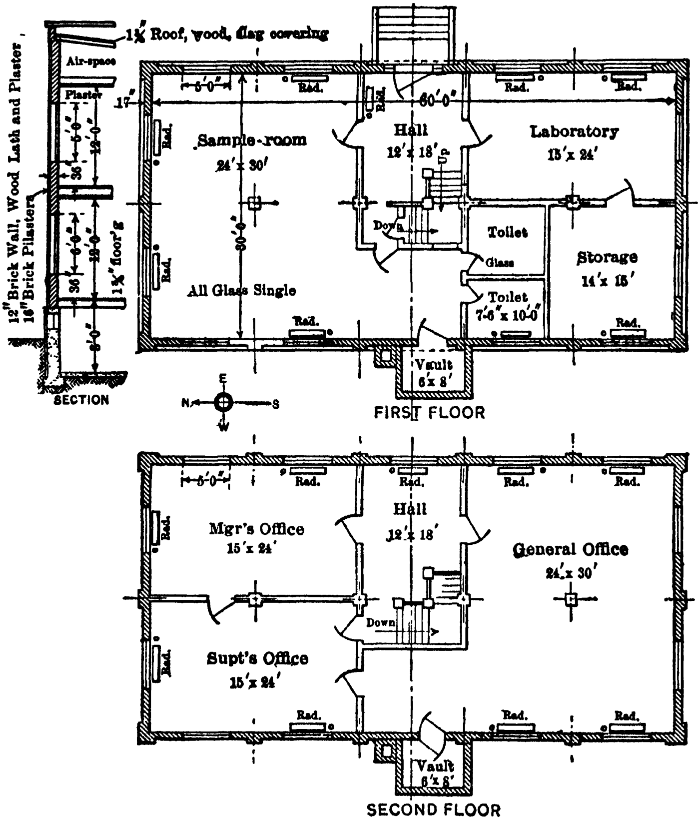


Fig. 10. Floor-plans and Section of Building. See Table X

year round. The lighting load in modern offices is sometimes nearly sufficient to balance the heat-loss.

Short Rules for Estimating the Heat-Loss of Buildings. There is a great variety of RULE-OF-THUMB METHODS for estimating the heat-loss H for proportioning the heating-surface required when direct radiation is to be used.

These so-called practical rules are intended to be based on average building-construction and on the ratio of wall- and glass-surface to the cubical contents as found in buildings of the class to which they refer. These rules when modified for unusual conditions and applied by engineers of long experience in the proportioning and design of heating systems produce satisfactory results. They are, however, rapidly being discarded except as rough checks on the more refined methods of calculation.

Carpenter's Rule. The following formula, or rule, which has been widely used for many years in this country, was proposed by R. C. Carpenter. It is not intended to be applied to buildings covered with corrugated sheet steel or metal lath and plaster walls, unless the wall-constant is changed to suit the condition.

By reference to Tables IV and V, it will be noted that a fair average value for the heat-transmission of the usual well-constructed building-wall is approximately 0.25 Btu and for glass 1.0 Btu per degree difference between the inside and outside temperature per hour.

Professor Carpenter stated that usually we may, with sufficient accuracy, neglect all inside walls, floors and ceilings and consider only the outside walls.

Let C = cubical contents of room in cubic feet;
n = number of air-changes per hour;
0.02 = Btu to raise 1 cu ft of entering air 1° F.;
W = net wall-surface in square feet;
G = glass-surface in square feet;
$(t - t_0)$ = temperature-difference between inside and outside;
H = total heat to be supplied per hour in Btu;

$$H = (0.02\,nC + G + \tfrac{1}{4}W)(t - t_0).$$

Calculating the Heat-Loss of a Building. The following example (Table X) will serve to illustrate the method employed in calculating and tabulating the heat-loss of a typical building, the floor-plans and section being shown in Fig. 10. The heating requirements are for a temperature of 70° F. with zero weather. The unit heat-transmission (U) for the outside walls per square foot is taken from Table IV for a temperature-difference of 70° or 70 × 0.24 = 16.8 Btu sq ft hr. The heat-loss through the first floor is based on a temperature-difference of 70 − 35 or 35°. The heat-transmission per square foot per 1° difference in temperature per hour for 1¾-in wood is 0 37; hence for 35° it is 0.37 × 35 = 13 Btu per hour. The heat-loss through the ceiling of the second floor is based on a temperature-difference of 70 − 23 = 47°, 23° being the assumed temperature of the attic with zero weather. The heat-transmission per square foot per hour is therefore 47 × 0.62 = 29 Btu. The infiltration-loss is, in this example, based on an estimated number of air-changes per hour as indicated.

By Carpenter's rule the heat-loss of this building based on two air-changes per hour, is

$$[0.02 \times 2 \times 41\,730 + (3\,464/4) + 730] \times 70 = 228\,564 \text{ Btu per hour}$$

It is recommended that the heat-transmission of each wall be separately calculated in order that the percentage, usually 15%, may be added to the calculated transmission of the north and west walls as previously mentioned under the heading Increase in Calculated Heat Losses.

Table X. Tabulation of Heat-Losses for Building Shown by Fig. 10

Room-designation	Net volume, cu ft	Net wall-area, sq ft	Floor or ceiling, sq ft	Glass-area, sq ft
1	2	3	4	5
First floor:				
Sample-room	10 080	852	864	180
Hall........	2 595	99	216	45
Laboratory..	4 320	378	360	90
Office.......	2 520	288	210	60
Toilet......	900	90	75	30
Second floor:				
Mgr's office	4 320	393	360	75
Hall........	2 595	119	216	25
Gen'l office..	10 080	852	864	150
Sup't's office	4 320	393	360	75
Totals....	41 730	3 464	....	730

Room-designation	Transmission-loss, Btu per hour			Infiltration loss, Btu per hour		Total heat-loss, Btu per hour
	Wall-loss 16.8× col. 3	Floor or ceiling-loss, 13×col. 4 29×col. 4	Glass-loss, 78.8× col. 5	Assumed no. air changes per hour	Infiltration-loss, 1.26×col. 2×col. 9	
1	6	7	8	9	10	11
First Floor:						
Sample-room	14 314	11 232	14 184	1	12 700	52 430
Hall........	1 663	2 808	3 556	3	10 809	18 836
Laboratory.	6 350	4 680	7 112	2	10 886	21 936
Office.......	4 838	2 730	4 728	2	6 350	18 646
Toilet......	1 512	975	2 364	2	2 268	7 119
Second floor:						
Mgr's office.	66 327	10 440	5 910	2	10 886	33 868
Hall........	1 989	6 264	1 970	3	10 809	21 032
Gen'l office.	14 134	25 056	11 820	2	25 400	76 410
Sup't's office.	6 630	10 440	5 910	2	10 886	33 866
Totals....	... ..	....	....	......	100 994	284 143

Radiation

Direct Radiation. Steam or hotwater radiators placed in the room to be heated are termed DIRECT RADIATORS OR DIRECT RADIATION. Common types of direct radiators are shown in Figs. 11, 12, 13 and 14. The column-type radiators, Figs. 11 and 12, are obsolete, being superseded by the tubular type, Fig. 14.

Indirect Radiation. Radiators used to warm the air passed over them, the heating of the building being accomplished by warm air, are termed INDIRECT

RADIATORS OF INDIRECT RADIATION. (See Figs. 40 and 41.) This type of radiation is frequently used for installations in which provision must be made for ventilation as well as heating, as in the case of schools, public buildings, etc. Indirect radiation is also used to some extent in high-grade residence-heating where direct radiation may be thought unsightly, particularly for the first floor. Direct radiation is ordinarily employed for the floors above the first floor. The principal use of indirect radiators is in connection with the HOT-BLAST SYSTEM of heating including unit-heaters, described later, in which a fan is used to circulate the air over the radiator.

Direct-Indirect Radiation. DIRECT-INDIRECT RADIATORS (Fig. 15) are radiators placed in the rooms to be heated and furnished with a cold-air connection through the outside wall, the air being circulated by gravity. It serves the purpose of providing tempered-air ventilation.

Materials and Connections of Radiators. Radiators are constructed of cast iron, brass or copper, pressed steel or pipe-coils. The sections for one-pipe steam systems are connected only at the bottom. The sections for hot-water radiators and two-pipe steam systems are connected at both top and bottom. The latter is known to the trade as HOT-WATER RADIATION

Pressure in Radiation. Cast-iron radiators should not be operated above 15 lb-per-sq-in pressure. Standard pipe-coil direct radiation, up to 125 lb per sq in.

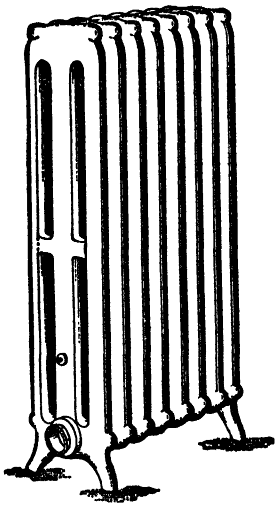


Fig. 11. Rococo Three-column Radiator*

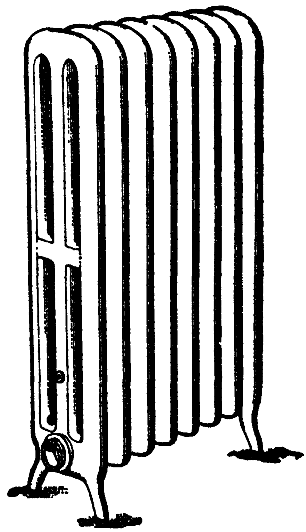


Fig. 12. Peerless Three-column Radiator*

Rating of Direct Radiators. Radiators are rated by the manufacturer according to nominal heating-surface in square feet (equivalent direct radiation, e.d.r.), based on a heat emission of 240 Btu per square foot of nominal surface per hour for steam radiators and 150 Btu per square foot of nominal surface for hot-water radiators. Cast-iron direct radiators are built up of sections. The

* The column type radiator is no longer manufactured, the tubular type (Fig. 13) having taken its place.

amount of nominal heating-surface per section of cast-iron radiators for the various standard heights manufactured is given in manufacturers' tables.

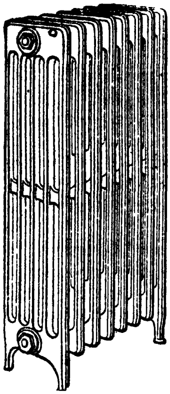


Fig. 13. Tubular Type Radiator, 5-tube—8 Sections

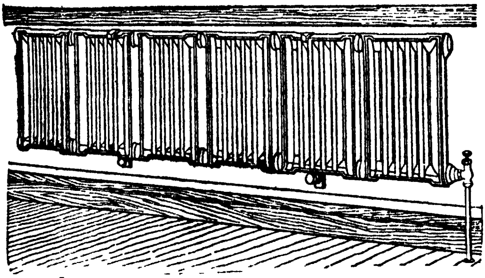


Fig. 14. Typical Installation of Rococo Wall-radiators in Single Tier on Adjustable Brackets

Direct Pipe-Coil Radiation was formerly largely used in manufacturing establishments and is usually made up of 1¼ or 1½-in pipe screwed into cast-iron manifolds as shown in Fig. 16. The unit heater is now largely employed for industrial heating.

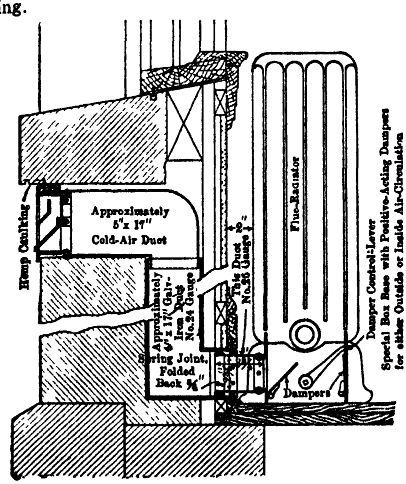
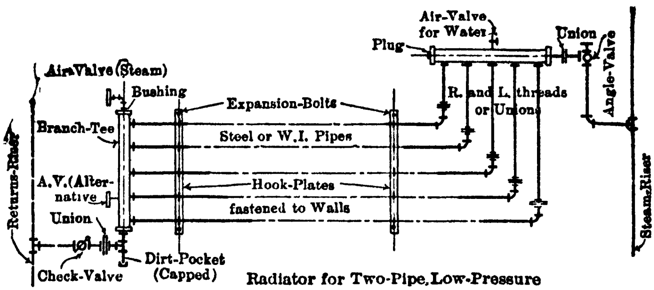


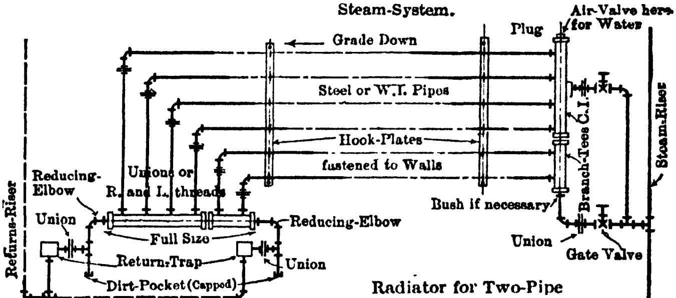
Fig. 15. Direct-indirect Radiator-installation

Invisible or Concealed Radiators for Use in Walls and Cabinets. An entirely new type of radiator heating has been developed in recent years in which the heating elements (Fig. 18) have EXTENDED SURFACES of copper,

Pipe-Coil Radiators and Connections

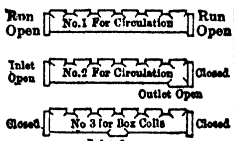


Radiator for Two-Pipe, Low-Pressure Steam-System.



Radiator for Two-Pipe Vacuum Steam-System, Divided Surface.

Branch-Tees (Crane Co.)



Order by Size and Number

Linear Feet of Pipe per 1□′ H.S.

2.9′ of 1″ Pipe = 1□′ H.S.
2.3′ of 1¼″ " = 1□′ H.S.
2.0′ of 1½″ " = 1□′ H.S.
1.6′ of 2″ " = 1□′ H.S.

TABLE

1″ Branch-Tees 2½″ c–c			1¼″ Branch-Tees, 3″ c–c			1½″ Branch-Tees, 3½″ c–c			2″ Branch-Tees, 4½″ c–c		
Runs			Runs			Runs			Runs		
1″–1¼″	1½″	2″	1¼″–1½″	2″	2½″	1½″–2″	2½″	3″	2″	2½″–3″	3½″
No of Branches			No. of Branches			No of Branches			No. of Branches		
2 to 9	2 to 16	2 to 16	3 to 16	3 to 16	3 to 16	3 to 12	3 to 12	3 to 12	3 to 10	3 to 10	3 to 10
Inside Diams.			Inside Diams			Inside Diams.			Inside Diams.		
1⅜′	2⅛″	2⅛″	2½″	2½″	2½″	2¾″	2¾″	2¾″	3½″	3½″	3½″

NOTES. All openings in Branch-Tees for circulation are tapped right hand.

Branch-Tees for Box Coils are always tapped left hand in branches and right hand in back inlet.

The run and back opening of Branch-Tees are tapped the same size as branches, unless otherwise ordered.

Fig. 16. Pipe-coil Radiation-data

brass, aluminum or other metals. These radiators are always installed at the base of a concealed flue space in one of the walls of the room to be heated or are placed in the bottom of a visible sheet-metal cabinet. Suitable inlets and outlets must be provided and the higher the column of heated air above the heating unit the greater will be the heating effect.

It will be evident that this is essentially an indirect system of heating in which the air is recirculated from the floor of the room and delivered back again through the outlets at the top of the flue or cabinet at a temperature sufficiently great so that in cooling to the inlet of floor temperature it gives up enough heat to supply the heat-loss from the room.

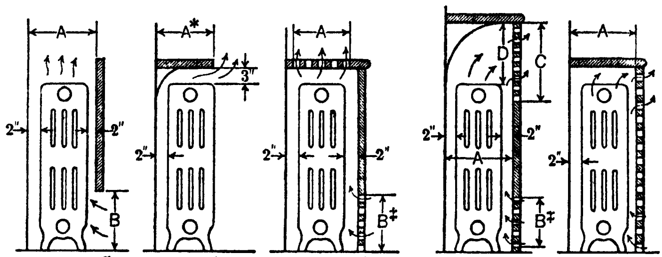


Fig. 17. Radiator Enclosures

Heat-Emission of Direct Radiation. The unit heat-transmission K, or the Btu transmitted by one square foot of actual measured surface of direct radiation per hour per degree difference between the heating medium and the temperature of the air in the room, varies somewhat with the type of radiator, height, temperature, etc.

Table XI is based on the average performance of direct cast-iron column type steam radiators standing exposed in still air at 70° F. with steam at 220° F., or 2-lb pressure, with a standard temperature-difference of 150° between the heating medium and the air in the room. In order to apply the coefficients given in Table XI to conditions other than standard, it is only necessary to know the variation in K for a given increase or decrease in the temperature-range above or below 150°, the standard range. An examination of test-data so far available indicates that this variation is nearly 0.2%* per degree above or below the standard range of 150°. Thus, if a three-column, 38-in high, direct radiator is to be used in a room kept at 60° F., with steam at 230°, we would have a temperature-range of 170° or 20° above standard, and the value of K would become

$$K = (1.55 + 0.002 \times 20 \times 1.55) = 1.61$$

and each square foot of radiation would give off $1.61 \times 170 = 274$ Btu per hr.

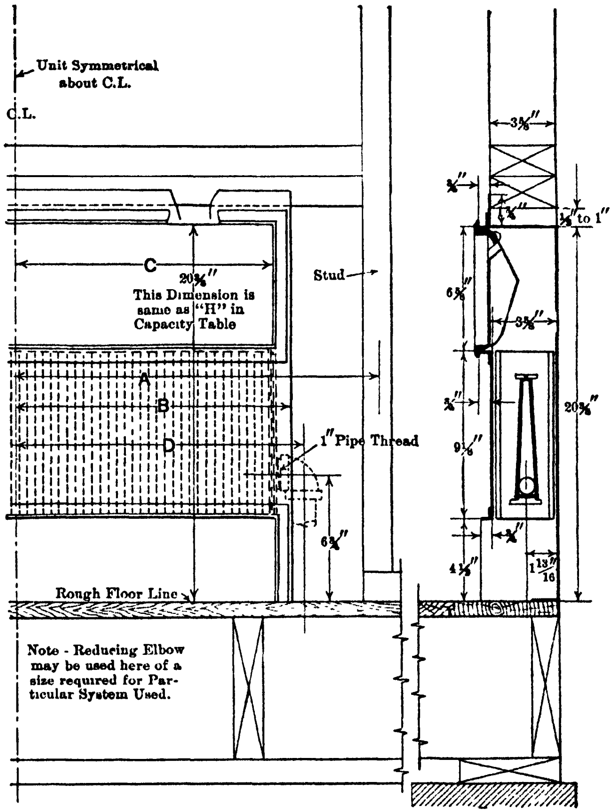


Fig. 18. Concealed Radiator Installed

Table XI. Values of K for Direct Radiators

Type of radiator	Height of radiator			
	20 and 22 in	26 in	32 in	38 in
One column	1.95	1 90	1 85	1 80
Two columns	1.80	1.75	1 70	1 65
Three columns	1.70	1.65	1 60	1 55
Four columns	1.60	1.55	1.50	1 45
Flue, 42 sq ft		...	...	1.57*
Window	1.85	...		......
Wall (horizontal)	1.95	...	...	....
Wall (vertical)	1.90	.....	.....	......
Pipe-coils	2.00	.	.	....

* Air entering flues at 70° F. and leaving same at 152° F. Allen.

K increases (1) as height of radiator is reduced and (2) as number of columns or width of radiator decreases.

Coefficients of Transmission for Direct Hot-Water Radiators. Table XI may be used for values of K for hot-water radiators of the same type as there listed, but allowance should be made for the lower temperature-range in hot-water heating. Thus, with a room usually at 70° F., and water at 180° entering and at 160° leaving the radiator, the temperature-range is only 100°, or 50° less than the standard range. Then for a two-column 26-in high direct radiator, the value of K becomes

$$K = (1.75 - 0.002 \times 50 \times 1.75) = 1.58$$

and each square foot of this radiation gives off, $1.58 \times 100 = 158$ Btu per hr

Concealed Radiators. Fig. 17 gives the approximate percentage to add or deduct to the computed amount of radiation to allow for enclosures of various kinds. Concealed radiators are not generally looked upon with favor from a strictly sanitary point of view.

The Usual Assumptions Made for the Heat-Transmission of Direct Radiation is 240 Btu per sq ft per hr for low-pressure steam (2 lb) cast-iron radiators, and 150 Btu per sq ft per hr for cast-iron hot-water radiators with the water at 180°. The square-foot rating of heating-boilers is based on the above figures. For more exact values use the data given in Table XI. Accordingly, a hot-water installation requires 60% more radiation than a low-pressure steam system.

Example. It is required to determine the amount (R) of direct cast-iron radiation, low-pressure steam and hot water, to supply a heat-loss of

$$H = 10\,000 \text{ Btu per hr}$$

Solution. For the direct steam system

$$R = H/240 = 42 \text{ sq ft}$$

and for a direct hot-water system

$$R = H/150 = 66\tfrac{2}{3} \text{ sq ft}$$

If a three-column cast-iron radiator, 38 in high, is to be used, the heating-surface of which is 5 sq ft per section, it will require $40/5 = 8$ sections for the steam-job, making the length of radiator equal to $8 \times 2\tfrac{1}{2} = 20$ in.

Fuels and Combustion

Classification of Fuels. Fuels are generally classified as solid, liquid, and gaseous. SOLID FUELS are coal, wood, and wastes. LIQUID FUELS are petroleum and its products. GASEOUS FUELS are natural and artificial gas.

Coal Fields in the United States. Most of the anthracite is found in beds of less than 500 sq miles in area located in eastern Pennsylvania. The principal deposit of semibituminous coal is about 300 miles long by 20 miles wide and lies along the eastern edge of the Northern Appalachian field. The bituminous coals extend from this deposit westward. A little graphitic coal is found in Rhode Island.

Composition of Coal. The uncombined carbon in coal is known as FIXED CARBON. Some of the carbon-constituent is combined with hydrogen, and this, together with other gaseous substances driven off by the application of heat, form that portion of the coal known as the VOLATILE MATTER. The fixed carbon and the volatile matter constitute the COMBUSTIBLE. The oxygen and

nitrogen contained in the volatile matter are not combustible, but custom has applied this term to that portion of the coal which is dry and free from ash, thus including the oxygen and nitrogen in the combustible.

Classification of Coals. Coals may be classified according to the percentages of fixed carbon and volatile matter contained in the combustible.

Table XII. Classification of Coals (Kent)

Name of coal	Percentages of combustible		Btu per pound of combustible
	Fixed carbon	Volatile matter	
Anthracite	97 0 to 92 5	3 0 to 7 5	14 600 to 14 800
Semianthracite .	92 5 to 87 5	7 5 to 12 5	14 700 to 15 500
Semibituminous .	87 5 to 75 0	12 5 to 25 0	15 500 to 16 000
Bituminous, East...	75 0 to 60 0	25 0 to 40 0	14 800 to 15 300
Bituminous, West..	65 0 to 50 0	35 0 to 50 0	13 500 to 14 800
Lignite. . .	50 0 and under	50 0 and over	11 000 to 13 500

Calorimetric Determinations. The only accurate and reliable way to determine the heating-value of a fuel is to do so experimentally with a calorimeter. For solid fuels, the BOMB-CALORIMETER is the most practical. The various types on the market include the Mahler, the Hempel, the Atwater and the Emerson. These consist essentially of a tight vessel containing a weighed sample and oxygen under pressure. This receptacle is placed within another vessel containing a known weight of water and surrounded by heat-insulating material to minimize radiation. The sample is EXPLODED electrically, and the heat absorbed by the surrounding water is determined by means of a very accurate thermometer reading hundredths of a degree. Correction has to be made for the heat absorbed by the instrument itself, and for radiation.

For a complete description of calorimeters and their use, see Carpenter and Diederich's Experimental Engineering.

Calorific Value by Formula. The following expression, known as DU LONG'S FORMULA for heating-value per pound of coal, can be used if the ultimate chemical analysis of the fuel is known:

$$F = 14\,600\,C + 62\,000\,(H - \tfrac{1}{8}O) + 4\,000\,S$$

where C, H, O, and S represent the proportionate parts of each element per 1 lb of fuel, and F denotes the heat-value in Btu per pound due to combustion. This formula does not apply when the fuel contains carbon monoxide, CO, but can be made to apply by adding a term, 10 150 C, in which C is the proportionate part of carbon burned to the monoxide.

Example. The application of the formula to a coal of ultimate analysis as here given follows:

Analysis (based on fuel as received)

C	74.79%
H	4.98
O	6.42
N	1.20
S	3.24
H_2O	1.55
Ash	7 82
	100.00%

Then by Du Long's formula, $14\,600 \times 0.7479 + 62\,000\,(0.0498 - 0.0642/8) + 4\,000 \times 0.0324 = 13\,650$ Btu per 1 lb of coal.

A bomb-calorimeter test showed 13 480 Btu for this coal. The formula fails to allow for evaporating and superheating the moisture present in the fuel.

Combustion of Fuel. Combustion, as used in steam-engineering, signifies a rapid chemical combination between oxygen, and the carbon, hydrogen, and sulphur composing the various fuels. This combination takes place usually at

Table XIII. Theoretical Amount of Air Required for Combustion

Fuel	Composition by weight			Lb of air per lb of fuel
	% C	% H	% O	
Wood-charcoal	93 0		. .	11.16
Peat-charcoal	80.0	.. .	......	9.6
Coke	94 0	.		10 8
Anthracite coal	91 5	3 5	2.6	11 7
Bituminous coal, dry	87 0	5 0	4 0	11.6
Lignite	70 0	5.0	20.0	8.9
Peat, dry	58.0	6.0	31.0	7.68
Wood, dry	50.0	6.0	43 5	6.00
Mineral oil	85 0	13 0	1 0	14 30

high temperature with the evolution of light and heat. The substance combining with the oxygen is known as the COMBUSTIBLE, and if it is completely burned or oxidized the combustion is PERFECT, that is, no more oxygen can be taken up by the products of the reaction. The combustion is IMPERFECT or incomplete when carbon burns to form carbon monoxide, CO, instead of the dioxide, CO_2, since the former may be further burned to form carbon dioxide if the necessary oxygen is supplied. It is necessary to provide for an EXCESS OF AIR when burning coal under either natural or forced draft, amounting to approximately 50 to 100% of the net calculated amount, or about 18 to 24 lb per lb of coal. Less air results in IMPERFECT COMBUSTION and smoke, while an excess cools the fire and setting and carries away a large percentage of the heat in the flue-gases.

Table XIV. Weight and Calorific Value of Various Gases

At 32 Degrees Fahrenheit and Atmospheric Pressure, with Theoretical Amount of Air Required for Combustion

Gas	Symbol	Cubic feet of gas per pound	Btu		Cubic feet of air required per cubic foot of gas
			Per pound	Per cubic foot	
Hydrogen	H	178 0	62 000	348	2 408
Carbon monoxide	CO	12.81	4 380	342	2.388
Methane	CH_4	22.4	23 842	1 065	9.57
Ethane	C_2H_6	12.0	22 400	1 865	16.74
Ethylene	C_2H_4	12 8	21 430	1 675	14.33
Acetylene	C_2H_2	13.79	21 430	1 555	11 93

Fuel-Storage. Space for fuel-storage must be based on fuel-consumption per season as estimated under Fuel-Consumption, and in government build-

ings it is customary to proportion the storage-space on the basis of 8 sq ft of floor-area per ton, the storage-space being made ample to hold an entire season's supply.

The following volumes per ton of 2240 lb of coal are given for proportioning storage-space: bituminous coal, 41 to 45 cu ft, and may run as high as 49 cu ft; anthracite coal, 34 to 41 cu ft; charcoal, 123 cu ft; coke, 70.9 cu ft.

This is based on fuel broken down ready for market. Also 1 bushel hard coal = 86 lb and 1 bushel soft coal = 76 lb.

Steam- and Hot-Water Heating Boilers

Pressures, Attention, and Materials. Heating-boilers usually operate under much LOWER PRESSURE than do power-boilers, and in most cases receive far less attention. The steam-boilers are usually designed to operate on from 2 to 5 LB STEAM-PRESSURE, and the WATER-BOILERS or hot-water heaters are seldom subjected to a hydrostatic head in excess of 50 FT when in operation. The ATTENTION given these boilers is of such an INTERMITTENT CHARACTER that they must carry the heating-load of comparatively long periods without firing. These periods may range from 6 to 10 hrs and in consequence the combustion-rate is low, and relatively large grates and fire-pots are necessary. The MATERIALS employed for constructing heating-boilers are CAST IRON or STEEL, cast iron being more common for the smaller sizes, although boilers of nearly 100 equivalent steam-boiler horsepower are made of this same material, and steel being more generally used in the larger sizes. The government departments usually specify steel heating-boilers, and they are used extensively in office and loft-buildings as well.

Boiler Heating-Surface. The CAPACITY of any boiler or water-heater depends on the amount of, and the temperatures on the opposite sides of, the heat-transmitting surfaces in contact with the water in the boiler on one side, and the fire or hot gases on the other. It is most important that a rapid circulation of water and the hot gases shall take place over these surfaces, and preferably in opposite directions. Two kinds of surface are distinguished in boiler-practice, and known as direct and indirect surface. DIRECT SURFACE is that on which the fire shines, and INDIRECT that in contact with the flue-gases only. All such surface must have water on the opposite side. In some boilers the hot gases are allowed to come in contact with the boiler-surface above the water-line so that there is only steam in contact with this surface on the inner side. Such surface is known as SUPERHEATING-SURFACE in order to distinguish it from ordinary heating-surface. Direct surface is the more valuable of the two, per square foot, as it is usually subjected to a higher temperature, and furthermore because the intensity of radiation from an incandescent surface appears to vary as some power of the temperature of that surface, either the third or fourth.

Equivalent Evaporation. The equivalent evaporation of a boiler is the pounds of water the boiler would evaporate per pound of coal burned if it received the feed-water at 212°, and evaporated it into steam at this same temperature and pressure, so that the evaporation would take place FROM AND AT 212° F. In practice the feed-water is usually below this temperature and evaporation actually takes place at some higher temperature than 212°. Hence, to find the EQUIVALENT EVAPORATION it is always necessary to make use of the following relation:

$$E = \frac{(x_2r_2 + q_2 - q_1)}{971.7} \times P$$

where the fractional part of the expression is known as the FACTOR OF EVAPORATION; so that

$$E = \text{factor of evaporation} \times P$$

E = equivalent evaporation from and at 212° F., in pounds;
x_2 = quality of steam as actually evaporated;
r_2 = latent heat of steam as actually evaporated;
q_2 = heat of the liquid as actually evaporated;
q_1 = heat of the liquid as actually fed to boiler;
P = actual evaporation in pounds per pound of fuel burned;
971.7 = latent heat of steam at 212° F.

Boiler Horse-Power. A boiler horse-power is the energy required to evaporate 34.5 lb of water at 212° F. into DRY STEAM of 212° F., or

$$971.7 \times 34.5 = 33\,524 \text{ Btu}$$

The HORSE-POWER RATING of a boiler is always measured in terms of the equivalent evaporation. Thus, if we divide the EQUIVALENT EVAPORATION of a boiler by 34.5 we obtain the boiler horse-power developed.

Boiler-Efficiencies. Heating-boilers, operated at their rated capacity, will show an EFFICIENCY of from 60 to 75%. This efficiency is the ratio of heat absorbed per pound of dry coal by the water and steam in the boiler to the actual heat-value of one pound of the coal, and is the COMBINED EFFICIENCY of the boiler and furnace.

Rates of Combustion for Heating-Boilers. Combustion-rates for varying sizes of grates are given in Table XV:

Table XV. Rates of Combustion for Heating-Boilers

Based on anthracite coal 13 000 Btu per lb; boiler and grate efficiency, 65%

Equivalent direct steam radiation, sq ft (240 Btu per sq ft), Design load	Combustion rate lb per sq ft of grate per hour for design load	Probable maximum combustion rate for starting-up load	Assumed Ratio $\frac{\text{Starting up load}}{\text{Design load}}$
Up to 420	4 85	8	1.65
420 to 840	5.00	8	1 60
840 to 2 500	5.80	9	1 55
2 500 to 5 000	6.70	10	1.50
5 000 to 7 500	7.50	11	1.45
7 500 to 12 000	8.60	12	1.40
12 000 to 16 000	10.00	14	1.40

The DESIGN LOAD referred to in Table XV is the normal load to be carried by the boiler for which the radiation was calculated and is the equivalent radiation based on a heat emission of 240 Btu per sq ft, or, in other words, is the total Btu to be supplied by the boiler divided by 240.

The boiler ratings given in manufacturers' catalogues usually refer to the DESIGN LOAD and not the MAXIMUM OR STARTING-UP LOAD, and it is assumed that, with the size of chimney specified, the boiler is capable of developing the additional percentage of rating required for starting up with cold radiation. Some manufacturers prefer to give their ratings based on the maximum output as determined by tests for the conditions of operation as to firing period,

etc. In this case it is necessary to multiply the design load by the factors given in Table XV to arrive at the corresponding manufacturers' rating or divide the manufacturers' ratings by these factors to arrive at the corresponding design load rating.

It is essential that the designer of the heating system know the method of rating employed by the manufacturer of the boiler to be used in order that he may select the proper size. It is recommended that he always check the grate-area and fuel-holding capacity of the boiler selected by means of formulas given later in the text. It will be later noted that steel heating-boilers are rated by the manufacturer on a basis of grate-area and minimum heating-surface.

Rating of Heating-Boilers. Standard Conditions. It is the general custom of American manufacturers of heating-boilers to rate their boilers in terms of the number of square feet of standard direct cast-iron radiating-surface which the boiler is capable of supplying under the following conditions:

(1) Steam boilers; steam-pressure 2-lb gauge at boiler.

(2) Hot-water boilers; water-temperatures: 180° F. leaving, and 160° F. entering boiler.

(3) Fuel; anthracite coal of stove-size.

(4) Draft as supplied by the size of chimney recommended.

The RATE OF COMBUSTION, or amount of coal necessary per hour for the boiler to develop its rating has, until recently, seldom been given; and the method of determining the rating has varied with different makers and is seldom stated. Moreover, it is possible for a boiler to be placed on the market and assigned a certain rating although such rating has never been actually checked by test. It therefore becomes most important to not only establish STANDARD CONDITIONS FOR RATING-TESTS, but to require the manufacturers to be in a position to produce certified test-sheets of such tests for his line of boilers. The STANDARD CONDITIONS under which a boiler should be tested to develop its rating are generally understood by the manufacturers at the present time to be as follows:

(1) Pressure, temperature and fuel as stated above.

(2) Fuel-capacity to be sufficient to carry the boiler from 6 to 8 hr on one charge and leave 20% reserve for igniting fresh charge.

(3) Draft of sufficient intensity to burn the fuel at the required rate. A chimney not less than 40 ft in height is recommended.

(4) Each square foot of direct radiation has a heat-emission value of 240 Btu, and 150 Btu per sq ft per hour for steam and water-radiators respectively. Radiation calculations based on these values is termed EQUIVALENT DIRECT RADIATION.

(5) The condensation from steam-radiators returns to the boiler at the same temperature as the steam, or without loss of heat, so that the boiler simply supplies the latent heat of evaporation at 2 lb pressure, or 971.7 Btu per lb evaporated.

(6) The water from hot-water radiators returns to the boiler at 160°, allowing a 20° drop in the radiators, so that there is no loss in temperature allowed in the return-main.

(7) Suitable heat-allowance must be made for all connecting piping and boiler-surface, and such surface must be figured as radiating-surface or its equivalent. A general rule for obtaining the design load is to add, for an ordinary installation, about 25% to the sq ft of radiation installed, in calculating the total load on the boiler, with either anthracite or bituminous fuel, to allow for radiation-loss of piping and boiler.

(8) It is assumed that the amount of radiation installed is sufficient to maintain the desired room-temperature for the outside temperature employed in the heat-loss calculations.

Equivalent Boiler Horse-Power Rating of Heating-Boilers. The capacities of heating-boilers may be stated in boiler horse-power, and the equivalent of same in square feet of standard radiation may be easily determined as follows:

Since 1 boiler horse-power is equal to 34.5 lb of water evaporated per hour, from and at 212° F., the boiler must deliver

$$34.5 \times 971.7 \text{ (latent heat at 212° F.)} = 33\,524 \text{ Btu per hr}$$

Now since 1 sq ft of standard cast-iron steam-radiation transmits 240 Btu per hour,

$$1 \text{ boiler horse-power} = 33\,524/240 = 139.7 \text{ sq ft of this radiation.}$$

It also follows that the equivalent boiler horse-power rating of a hot-water heater is

$$33\,524/150 = 223.5 \text{ sq ft of direct cast-iron hot-water radiation.}$$

Grate-Surface. It is always advisable to check THE GRATE-AREA REQUIRED for heating-boilers, especially if the total heat-loss to be supplied by the boiler is known. This total heat-loss must include not only the calculated loss, due to transmission through walls and glass, for which the radiation is proportioned, but also about 25% additional for heat-losses from the piping system, boiler, etc. So that, if H is the building-loss in Btu, $1.25\ H$ = total Btu-loss to be supplied by the boiler at normal or design load.

Then

$$G = 1.25H/(C \times F \times E)$$

where C = rate of combustion in pounds of dry coal per square foot of grate-area per hour (see Table XV), F = calorific value of fuel in Btu per pound of dry coal (12 000 is the usual assumption for anthracite coal), and E = the combined efficiency of boiler and grate (60% is the usual assumption). G is the grate-area sq ft and the boiler selected should have not less than this grate-area. The grate-area is usually given by manufacturers' tables. Special attention is called to the distinction between GRATE-AREA and FIRE-BOX or FUEL-POT AREA as explained below under Depth of Fuel-Pot.

The grate-area required for the design load as determined by the code for rating low-pressure heating-boilers adopted by the Steel Heating Boiler Institute (1929) is as follows:

$$G = \sqrt{\frac{\text{catalogue rating (in sq ft steam-radiation)} - 200}{25.5}}$$

for boilers with ratings from 300 to 4 000 sq ft of steam-radiation.

$$G = \sqrt{\frac{\text{catalogue rating (in sq ft steam-radiation)} - 1\,500}{16.8}}$$

for boilers with ratings of 4 000 sq ft of steam-radiation and larger.

This code further states that the boiler heating-surface in square feet must not be less than one-fourteenth of the rating in square feet of equivalent direct radiation. Manufacturers of steel boilers give both boiler heating-surface

and grate-area in their rating tables. Manufacturers of cast-iron boilers do not give boiler heating-surface in their literature.

The above ratio ($\frac{1}{14}$) is based on the builders' rating for power boilers or 10 sq ft of heating-surface per boiler horse-power. Therefore, 10 ÷ 139.7 (sq ft radn. per b.h.p) = $\frac{1}{14}$ (approximately).

Depth of Fuel-Bed. The average of the fire-box area is usually somewhat larger than the grate-area in sectional boilers, while it may be less than the grate-area in certain types of round boilers. In any event the capacity of the fire-box or fuel-pot from grate to middle of fire-door for boilers of 2 000 sq ft rating and below should be sufficient to hold all the coal required for an 8-hr firing-period, plus at least 20% reserve to be used for igniting a fresh charge.

The following method is used to determine the depth of pot or the firing-period as the case may be. Let G = grate-area in sq ft, C = rate of combustion, A = average area of fire-pot, h = firing-period in hours, W = weight of fuel per cu ft (50 lb for anthracite and 40 lb for bituminous), D = depth of fuel-bed in ft. Then $(GCh) + 20\%$ (allowance to ignite fresh charge) = total weight of one charge; also, AWD = total weight of one charge. Hence

$$D = 1.2GCh/AW, \text{ or } h = AWV/1.2GC$$

As noted above, D is measured from grate to center of fire-door, which varies with different boilers. This formula allows for the greater bulk of soft coal.

Example. Given a boiler with grate-area of 8 sq ft, average area fire-pot 9 sq ft, height to center of fire-door = 18 in, rate of combustion = 6 lb per sq ft of grate for anthracite coal. Required the number of hours this boiler will carry its load on one charging.

Solution.

$$h = (9 \times 50 \times 1.5)/(1.2 \times 8 \times 6) = 11.7 \text{ hours}$$

This item is stated in heating literature as either attention, firing period or fuel available in hours.

Effects of Fuels on Ratings. All ratings are based on ANTHRACITE COAL OF STOVE-SIZE unless otherwise stated. In case bituminous coal is used and the boiler is selected by catalogue-rating, a boiler with fire-pot having at least 25% greater capacity should be selected, for the same weight of coal occupies 25% more space. With SOFT COAL additional heating-surface is also required as the accumulation of soot from such coal renders the heating-surface less effective than when hard coal is used. Boilers for PEA-COAL should also have a larger fire-pot than those for stove or furnace-coal. The SMALL SIZES OF ANTHRACITE contain far more ash than the larger sizes, and hence have a greater bulk for the same heating effect; so that larger fuel-pots for the same capacity are required. FIRING-PERIODS, differing from the one on which the boiler is rated, will also affect the fuel-holding capacity. For example, if it is required to operate a certain line of boilers designed for an 8-hr period on a 12-hr basis, at least 50% greater fuel-holding capacity will be necessary and a larger boiler must be selected, as shown by the formula already given for the depth of the fuel-pot.

Equivalent Rating for Conditions Other than Standard. It often happens that the load connected to a steam or hot-water boiler may not be operated under the standard conditions previously assumed as a basis of rating. In this case tables of ratings cannot be used until the EQUIVALENT VALUE of this load in terms of square feet of standard cast-iron radiation has been determined.

The following relations show a method for finding such equivalent values:

Let R = sq ft of equivalent direct radiation = 240 Btu per sq ft for steam, and 150 Btu per sq ft for water. Also let

r = actual sq ft of radiation to be supplied;
K = coefficient of transmission for this radiation;
t_s or t_w = temperature of steam or average temperature of hot water in the radiator;
t_a = temperature of air surrounding radiator;
$K(t_s - t_a)$ = radiation-factor or Btu given off per sq ft per hr;

Then

$$R_s = r_1 \times K_1(t_s - t_a)/240, \text{ and } R_w = r_2 \times K_2(t_w - t_a)/150$$

Example. (Steam-heating) Required the size of boiler (rating in sq ft of standard cast-iron radiation) to suppiy 1 000 sq ft of direct pipe-coil radiation. Steam-pressure = 5-lb gauge. Air = 65° F. K (by test) = 2.42 Btu. From steam-tables, $t_s = 227.14$, $R = 1\,000 \times 2\,42(227.14 - 65)/240 = 1\,000 \times (2.42 \times 162.14)/240 = 1\,634$ sq ft. To this add 25% for pipe and boiler-radiation, or, $1.25 \times 1\,634 = 2\,042$ sq ft, practically a 2 000-sq-ft-capacity boiler will be required to carry the design load. The boiler must be capable of developing approximately 60% more rating (see Table XV for ratio of starting-up load to design load) to provide for the additional tax placed on the boiler for starting up with the chimney specified. The grate-area should be checked by calculation previously given to ascertain minimum size.

$$G = 1.25H/(C \times F \times E)$$

Example. (Water-heating.) Let Q = total number of gal of water to be heated in h hours.

$W = (8\frac{1}{3} \times Q)/h$ = weight of water to be heated per hour
t_1 = initial temperature of water, t_2 = final temperature of water

Then $W\,(t_2 - t_1)$ = Btu to be supplied per hour. Hence $W\,(t_2 - t_1)/150$ = hot-water-heater rating required. $W\,(t_2 - t_1)\,1240$ = steam-boiler rating required.

Example. A swimming pool contains 50 000 gal of water, and this water is heated by being passed through a hot-water heater in four hours. Entering-temperature = 50° F. and final temperature = 75° F. Hot-water radiation reduced to equivalent standard value = $[(50\,000 \times 8\frac{1}{3})/(4 \times 150)] \times (75 - 50)$ = 17 350 sq ft = rating of hot-water heater, to which must be added 25% for losses from piping, plus the estimated heat-loss from the pool.

Fuel-Consumption. The ESTIMATED FUEL-CONSUMPTION FOR HEATING-BOILERS per heating-season may be based on grate-areas, square feet of radiation installed, or cubic contents of building to be heated. The United States Treasury Department allows 5 tons of coal per sq ft of grate-area per season of 240 days, or 1 lb of coal per cu ft of contents of building for the same period. This applies to government buildings. The district steam-heating companies estimate 500 lb of steam per sq ft of direct steam-radiation per season, which is practically the same as 70 lb of coal of good quality. This is approximately equivalent to assuming that one-third of the radiation installed is in operation continuously for 240 days. In other words, the coal required for a heating-season is about one-third the quantity that would be used if all the radiation were in constant use every hour of the day and night. This statement applies to the northern states. The amount of coal for maximum conditions is determined as follows:

Since each foot of direct steam-radiation or its equivalent will emit 240 Btu per hour under conditions of 2 lb (220°) pressure at boiler, and 70° air surrounding the direct radiators (the piping on the average job may be roughly taken as 25% of the direct radiation); and since for approximation we may assume 8 000 Btu per pound of anthracite coal burned; we can readily estimate the amount of coal per hour if R = amount of direct radiation in square feet:

$$(1.25 \times R \times 2.50)/8\,000 = \text{coal per hour in pounds}$$

In a heating-season of 7 months or 210 days of 24 hours each, there would be burned under maximum conditions during the entire period

$$(1.25 \times R \times 240 \times 210 \times 24)/(8\,000 \times 2\,000) = 0.103\,R \text{ ton of coal}$$

the actual consumption being about one-third of the maximum possible, or 0.034 R ton of coal for the heating-season For hot-water heating the fuel-consumption for the entire season is approximately 0.021 R tons.

Types of Heating-Boilers. Cast-iron steam-heating boilers are designed to be operated at a maximum pressure of 15 lb per sq in, and the sections are

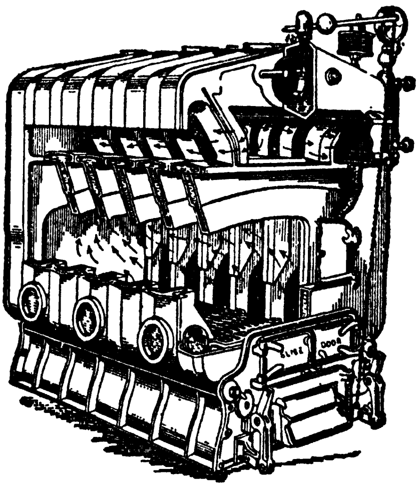


Fig. 19. Sectional Type of Cast-iron Boiler

tested by the manufacturer to about 100 lb per sq in, hydrostatic pressure. Cast-iron boilers are constructed of sections, which are connected by means of nipples of either the push or screw-type. The sections are held in place by means of long bolts. Round-type boilers have horizontal sections surrounding the fire-pot, and in the sectional type the sections are placed vertically. (See Figs. 19, 20, and 21.) The maximum size of round-type boilers manufactured is rated at about 1 400 ft. Sectional boilers are obtainable up to a 10 000-sq ft rating. (See manufacturers' catalogues for capacities, dimensions, etc.)

Fire-box type of steel heating boilers, in sizes comparable with cast-iron boilers, are now manufactured by a number of concerns.

Down-draft smokeless type steel boilers may be obtained with two sets of grates, the upper being an inclined water-grate and the lower the usual type of rocking-grate.

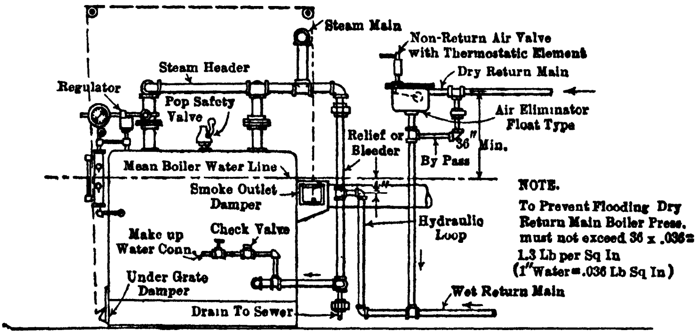


Fig. 20. Trimmings and Connections for Sectional Steam-heating Boiler. (Vapor System)

Minimum allowable heights and diameters or areas of chimneys are specified and insisted upon by the manufacturers of the boilers.

Smokeless Cast-Iron and Steel Boilers. Boilers having specially designed furnaces to provide for the volume of preheated air for the complete combustion of the volatile combustible matter given off in large quantities in the burning of soft coal, are now being made for use with free-burning soft coal, where local smoke ordinances would not permit the use of such fuel in ordinary boilers.

Steel Heating-Boilers. There are two general types of all-steel boilers used for

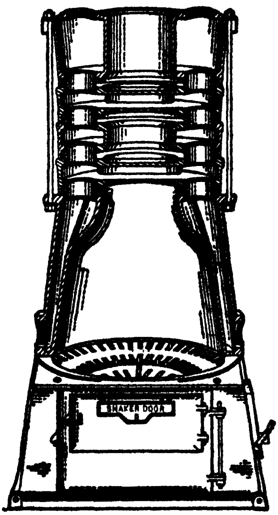


Fig. 21. Section of Round-type Boiler

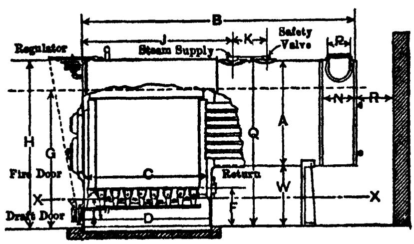


Fig. 22. Portable Type Steel Heating Boiler

heating work, the FIRE-BOX TYPE (Fig. 22) and the RETURN TUBULAR TYPE.

In the fire-box type the grate and combustion-chamber are surrounded by an extension of the steel shell which is water-jacketed. The products of combustion pass directly through the tubes to the smoke-flue located in the rear.

In the return tubular type, the boiler consists of a shell with tubes set in a brick setting, the grate and combustion-chamber being directly under the front portion of the shell. The products of combustion in this case pass under and around the shell to the rear of the boiler, and then through the tubes to the front into the smoke-box.

Fire-box type boilers may be obtained in capacities ranging from 350 to 32 000 sq ft of direct steam radiation. Detailed information as to capacities, dimensions, etc., may be obtained fron the makers' catalogues.

Selection of Heating-Boilers. Rating of Heating-Boilers. The rating is based on the amount of connected load (output Btu per hr or sq ft of equivalent direct radiation e.d.r.) which the boiler is capable of supplying at the boiler outlets under conditions of firing period, fuel, etc. specified: 2 lb gauge pressure for steam boilers; 180° F water leaving boiler for hot-water boilers. Boiler to be connected with the size and height of chimney recommended by manufacturer.

Connected or Design Load (*H* Btu per hr). Take the sum of the following items: (*a*) Estimated heat emission of connected direct and indirect radiation as determined by heat-loss computations. (*b*) Heat required for water-heating or other heat-consuming apparatus. (*c*) Heat-loss from piping connecting boiler with radiation and other apparatus.

Starting-Up Load ($H \times$ factor). The connected load is to be multiplied by the following factors to allow for heating up cold radiation in a reasonable time. This is the maximum load which the boiler is assumed to carry. (A.S.H.V.E. Code Min. Req'ts)

Sum of Items a+b+c	Factor
Up to 100 000 Btu hr......	1.65
100 000 to 200 000 Btu hr	1.60
200 000 to 600 000 Btu hr......	1 55
600 000 to 1 200 000 Btu hr......	1 50
1 200 000 to 1 800 000 Btu hr	1.45
Above 1 800 000 Btu hr	1 40

Boiler Output Capacity to be Installed. Install a boiler provided with an output rating (Btu hr or e.d.r.) corresponding to the design load for the firing period (attention) hr desired. Then be sure that the maximum output rating of boiler with a short firing period (maximum rating stated by manufacturer) is not less than the estimated starting-up load.

Heat-loss from piping (*c*) (Btu per sq ft pipe surface per hr).

	Uncovered	¾-in. Covering
Low-pressure steam (approximate).....	360	90
Hot water (approximate)..............	252	63

Surrounding air assumed as 45° F. The frequent assumption for this item is: Add 25% for steam and 35% for water.

Chimneys for Heating-Boilers. (See, also, under Chimneys.) In order to produce an intensity of draft sufficient to properly operate low-pressure heating-boilers, hot-water boilers, and hot-air furnaces up to their rated capacity, the chimney should not be less than 40 ft in height, measured from the grate. No flue should be less than 8 × 8 in. The failure of many heating-installations may be traced to insufficient draft to burn the fuel at the rate required to run the boiler or furnace to rated capacity. The temperature of flue-gases leaving the boiler ordinarily ranges between 400° and 600° F. when the apparatus is worked at its rated capacity. The chimney should be so located with reference to any higher buildings nearby that wind-currents will not form eddies and force the air downward in the shaft, as shown in Fig. 23. The flue should run as nearly straight as possible from the base to the top outlet. The outlet must not be capped so that its area is less than

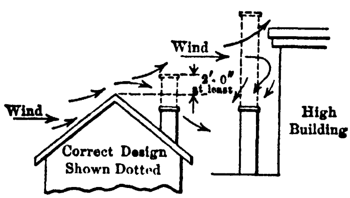


Fig. 23. Relation of Height of Chimney to Draft

the area of the flue. The flue should have no opening into it other than the boiler smoke-pipe. Sharp bends and offsets in the flue often reduce the area and choke the draft, and the flue must be free of any feature which prevents the full area for the passage of smoke. If the flue is made of tile, the joints must be well cemented, or all space between the tile and brickwork filled in tightly. There must be no open crevices into the flue where the tile sections meet, otherwise the draft will be checked. If the flue is made of brick, the stack should have outside walls at least 8 in thick to insure safety. The inside joints should be well struck, and each course should be well bedded and free from surplus mortar at the joints. The exposed bricks at the top of a brick chimney should be laid in cement mortar to prevent the acid fumes and rain from cutting out the joints. This will happen if lime mortar is used. The most desirable location for a chimney is near the center of the building, as all walls are then kept warm. If there is a soot-pocket in the flue below the smoke-pipe opening, the clean-out door should always be tightly closed. If this soot-pocket has other openings into it from fireplaces or other connections, these openings check the draft and prevent the best results. The smoke-pipe should not extend into the flue beyond the inside surface of the latter. If it does extend beyond, its end cuts down the area of the flue. The joints, where the smoke-pipe fits the smoke-hood of the boiler, or where the pipe enters the chimney, should be made tight with boiler-putty or asbestos cement. Fire-clay flue-linings are used in the best practice for small and medium-sized flues. Rectangular flue-linings are rated by outside dimensions and round linings by inside dimensions. The reader is advised to obtain a copy of Chimney Ordinance of the National Board of Fire Underwriters.

Table XVI. Standard Dimensions of Rectangular Fire-Clay Flue-Linings

(Eastern Clay Products Association)
No allowance for radial corners

Inside dimensions of flue-linings, in	Inside cross-sectional area of flue-linings, sq in	Thickness of shell, in	Outside dimensions of flue-linings, in	Outside cross-sectional area of flue-linings, sq in	Cross-sectional area of shell, sq in	Length, ft	Approximate weight per foot, lb	Approximate number of flue-linings per car
3¼ × 7¼	23 56	⅝	4½ × 8½	38 25	14 69	2	14	1953
3¼ × 11¾	38 19	⅝	4½ × 13	58 5	20.31	2	19	1322
6¼ × 6¼	39 06	⅝	7½ × 7½	56 25	17.19	2	14	1784
7¼ × 7¼	52 56	⅝	8½ × 8½	72 25	19.69	2	18½	1335
7 × 11½	80 5	¾	8½ × 13	110 5	30	2	28	892
6¾ × 16¼	109 69	⅞	8½ × 18	153	43.31	2	37	675
11¼ × 11¼	126 56	⅞	13 × 13	169	42.44	2	35½	709
11¼ × 16¼	182 84	⅞	13 × 18	234	51.16	2	52	482
15¾ × 15¾	248 06	1⅛	18 × 18	324	75 94	2	69	367
17¼ × 17¼	297 56	1⅜	20 × 20	400	102.44	2	102½	241
17 × 21	357	1½	20 × 24	480	123	2	115	166
21 × 21	441	1½	24 × 24	576	135	2	125	201

Note. The standard sizes shown above are also manufactured with round openings for flue-rings. The diameter of the opening is cut to suit the outside diameter of the proper size flue-ring and allow ¼-in clearance. Rectangular openings are not standard but can be furnished to order.

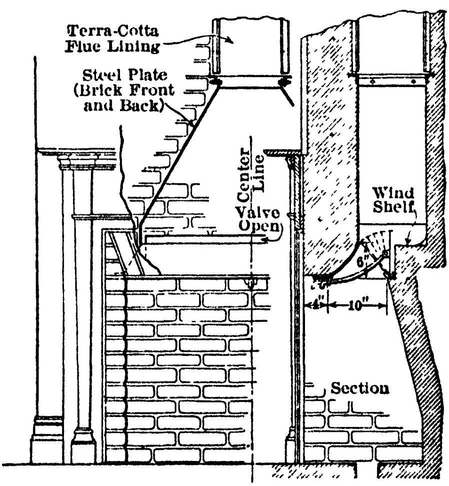


Fig. 24. Details of Construction of Fireplace 20 in Deep with "Covert" Series E Improved Damper and Steel Smoke Chamber

Flues for Kitchen Ranges and Fireplaces. For a kitchen range an $8\frac{1}{2}$ by $8\frac{1}{2}$-in tile flue is ordinarily sufficient, but an $8\frac{1}{2}$ by 13-in is better. For fireplaces the sectional area of the flue for burning wood or bituminous coal should be from $\frac{1}{10}$ to $\frac{1}{8}$ the area of the fireplace-opening for a rectangular flue, and $\frac{1}{12}$ for a circular flue. For burning anthracite coal the areas may be reduced to $\frac{1}{12}$ and $\frac{1}{16}$, respectively.

Table XVII. Covert Fireplace Throats and Dampers

Series	Damper number	Front width of fireplace, in	Base opening of throat not including flange, in			Proper flue lining exterior dimensions, in
			Front	Rear	Depth	
D*	424	24	24	18	8	$8\frac{1}{2} \times 8\frac{1}{2}$
	430	30	30	24	8	$8\frac{1}{2} \times 13$
	432	32	32	26	8	$8\frac{1}{2} \times 13$
	436	36	36	30	8	$8\frac{1}{2} \times 13$
	442	42	42	36	8	13×13
	448	48	48	42	8	13×13
	454	54	54	48	8	13×18
	460	60	60	54	8	13×18
E*	524	24	24	18	10	$8\frac{1}{2} \times 8\frac{1}{2}$
	530	30	30	24	10	$8\frac{1}{2} \times 13$
	532	32	32	26	10	$8\frac{1}{2} \times 13$
	536	36	36	30	10	$8\frac{1}{2} \times 13$
	542	42	42	36	10	13×13
	548	48	48	42	10	13×13
	554	54	54	48	10	13×18
	560	60	60	54	10	13×18

* Series D is 8 in deep (inside measure) and is for fireplaces 18 in deep or under. Series E is 10 in deep (inside measure) and is for fireplaces 20 in deep or under. The above flue-sizes apply only to fireplaces having ordinary height. Smoke Chamber for $8\frac{1}{2} \times 13$-in flue is reversible and will properly connect, whether the shorter or longer dimension runs parallel with the breast. For 13×18-in flue it is necessary to know which dimension runs parallel with the breast.

Selection of Chimney-Flues. (See, also, under Chimneys.) The selection of chimney-flues for heating-boilers must depend upon the judgment of the heating-engineer, but it is believed that Table XVIII, will assist the engineer and architect in selecting flues. It is necessary that AREA and HEIGHT, THICKNESS OF WALLS, GENERAL STRUCTURE, and the POSITION OF THE TOP OUTLET with reference to the building and other buildings near by should be carefully noted and observed in the selecting or building of a flue. The figures given under the varying heights of chimneys are diameter-measurements in inches, or, the side of a square, the theory being that the spirally ascending column of smoke and gases will make a 12 by 12-in flue no more effective in practical working-area than a 12-in round flue. Rectangular shapes may be used if the area is equal and the difference in width and breadth is not extreme. A maximum ratio of 2 : 1 for the internal dimensions should not be exceeded.

Table XVIII. Flue-Sizes and Chimney Heights

Warm-air furnace capacity, square inches of leader pipe	Steam boiler capacity, square feet of radiation	Hot-water heater capacity, square feet of radiation	Rectangular flue			Round flue		Height in feet from grate
			Nominal dimensions of fire-clay lining, in	Actual inside dimensions of fire-clay lining, in	Actual area, sq in	Inside diameter of lining, in	Actual and effective area, sq in	
790	590	973	8½×13	7 ×11½	81			35
1 000	690	1 140				10	79	
	900	1 490	13 ×13	11¼×11¼	127			
	900	1 490	8½×18	6¾×16¼	110			
	1 100	1 820				12	113	40
	1 700	2 800	13 ×18	11¼×16¼	183			
	1 940	3 200				15	177	
	2 130	3 520	18 ×18	15¾×15¾	248			
	2 480	4 090	20 ×20	17¼×17¼	298			45
	3 150	5 200				18	254	50
	4 300	7 100				20	314	
	5 000	8 250	24 ×24	21 ×21	441			55
	4 600	7 590		20 ×24	480			
	5 570	9 190		24 ×24*	576			60
	5 580	9 200				22	380	
	6 980	11 500				24	452	65
	7 270	12 000		24 ×28	672			
	8 700	14 400		28 ×28	784			
	9 380	15 500				27	573	
	10 150	16 750		30 ×30	900			
	10 470	17 250		28 ×32	896			
	11 800	19 500				30	707	70
	14 700	24 300				33	855	
	17 900	29 500				36	1 018	

* Dimensions below are larger than those in which rectangular fire-clay flue-linings are commercially available and hence are for unlined rectangular flues—requiring thicker walls than when lined.

Stacks for Tall Buildings are special cases and may be designed by methods used in the design of chimneys for power-boilers. (See Vol. 1, Mechanical Equipment of Buildings, by Harding and Willard. See, also, under Chimneys.)

Direct Steam Heating

Systems of Direct Steam Heating in Use. Piping systems in connection with direct radiation may be divided into two general classes: (1) GRAVITY CIRCULATING SYSTEMS and (2) MECHANICAL CIRCULATING SYSTEMS. The distinguishing characteristic is the manner in which the water of condensation is returned to the boiler.

In gravity systems the condensate is returned to the boiler by gravity due to the STATIC HEAD OF WATER in the return-mains, the system being a closed circuit. The elevation of the boiler water-line must consequently be below the lowest radiation with a gravity-return system.

In the mechanical systems the condensate is generally returned by gravity to a receiver. ordinarily vented to the atmosphere, from which it flows to

a PUMP or RETURN-TRAP and is forced into the boiler. This is not a closed circuit and the relative elevation of the radiation and boiler water-line is not important. In the case of VACUUM SYSTEMS, a vacuum pump is attached to the return-main and may discharge the condensate directly into the boiler if low pressure or into a receiver located above the boiler, or into a feed-water heater with a high-pressure boiler installation. It should be understood that any gravity type of system hereinafter described may be changed into a mechanical system by the addition of a pump attached to the return-main.

In general it may be stated that a mechanical system is required only for the following conditions:

(*a*) When the elevation of the boiler in reference to the radiation is such as will not permit of the return of the condensate by gravity;

(*b*) When the boiler pressure is greater than the pressure supplied the radiation, as for example, a high-pressure boiler installation supplying steam through a reducing-valve to the radiation.

The following types of steam-heating systems are in common use:

ONE-PIPE CIRCUIT SYSTEMS
ONE-PIPE RELIEF SYSTEMS
TWO-PIPE SYSTEMS
AIR-LINE SYSTEMS
VAPOR or AIR RETURN SYSTEMS (two-pipe)
VACUUM SYSTEMS (two-pipe)

In all of these systems provision must be made to maintain the water in the boiler at the normal water-line level.

One of the most prolific causes of CRACKING of the sections in a cast-iron boiler is the lowering of the water-line thereby uncovering heating-surface which is practically in contact with the fire.

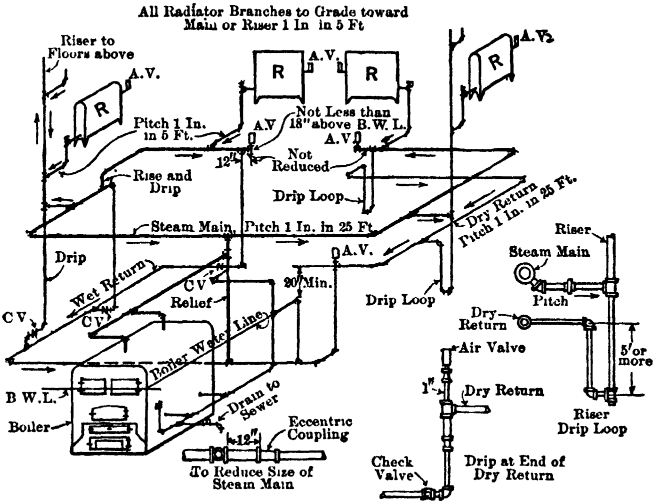
Owing to the loss of pressure in a gravity-return system caused by the frictional resistance of the piping, valves, etc., a static head of water must exist in the return piping, above the boiler water-line, equivalent to this pressure-loss (30 in per lb loss in pressure). When the system is started up with cold radiation a greater volume of steam is moved through the piping with a consequently greater loss in pressure and therefore a larger withdrawal of water from the boiler than is necessary during the normal heating period to create the necessary static head in the return piping. It is during this period that the cracking of cast-iron boiler sections sometimes occurs.

If the return connection to the boiler is made at the normal water-line and a connection made to the steam space of the boiler or delivery line, it is evident that the lowering of the water-line will cease when the level of the water reaches the point where the return connection is made. Check-valves on the returns at the boiler are not required with this arrangement of return piping. The size of the steam connection to the equalizing pipe for various grate-areas is given by Fig. 25. This method of connecting the boiler returns is recommended by the National Boiler & Radiator Manufacturers Association, and the Boiler Insurance Companies.

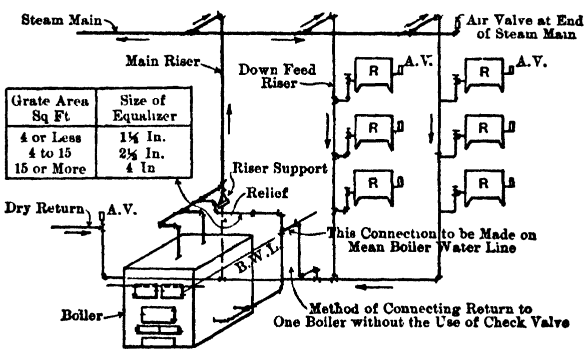
One-pipe Relief System. This system (Fig. 25) has been, in the past, perhaps the most popular of all types of steam systems and is applicable to multi-story installations. Only one pipe-connection is made to each radiator. Each radiator is equipped with an air-valve or connected with an air-exhaust system described under Air Line Systems.

The steam-main, in this system, is reduced in size by the use of ECCENTRIC COUPLINGS in proportion to the radiation supplied, as the riser-connections

are taken off, and is uniformly graded away from the boiler not less than 1 in in 40 ft and preferably 1 in in 25. RISER-CONNECTIONS that are dripped are taken off from the bottom of the main at 45° and pitched toward the riser.



One-pipe Relief System



Grate Area Sq Ft	Size of Equalizer
4 or Less	1¼ In.
4 to 15	2½ In.
15 or More	4 In

Mills System

Fig. 25. One-pipe Relief System and Mills System

Each riser is dripped into the return-main which, if run below the boiler water-line, is termed a WET RETURN and if run above the boiler water-line is termed a DRY RETURN. A dry return may be changed into a wet return by the installation of a false water-line as indicated by Fig. 26.

False Water Line. Returns that must run above the boiler water-line, and therefore would naturally be dry, may be changed to wet returns, if necessary, by the use of a FALSE WATER-LINE which is established at the end of the return just before it drops to enter the boiler. This inverted SEAL or DAM should be protected against the possibility of freezing, and should be provided with a by-pass valve, as shown in Fig. 26. It is usually possible to locate the seal in the boiler-room, near the boiler. It will be noted that the new water-line on the branch return is now at a distance D above the actual boiler water-line, which is still the water-line of all other returns except this one.

A WET RETURN is always considered more satisfactory whenever the building arrangement will permit. A swing check-valve placed on the horizontal run of drip-connection to wet return is used and recommended by many engineers. Drips into a dry return should be made through a water-sealed drip loop 5 ft or more in depth (or 30 in per lb pressure to be carried). No branch or lateral larger than the supply to a single radiator should be pitched against the steam-flow and drained back to the steam-main.

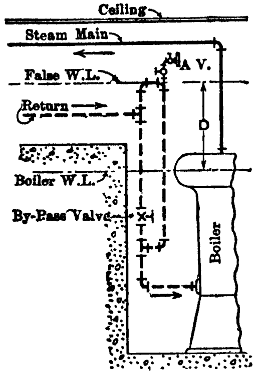


Fig. 26. False Water Line

The elevation of the end of the steam-main should not be less than 18 to 20 in above the mean boiler water-line and should be dripped into the return-line and provided with an automatic air-valve preferably of the thermostatic non-return type as shown. The end of a dry return should be at least 18 to 20 in above boiler water-line as indicated (Fig. 25).

Dry-return mains are pitched toward the boiler with a uniform grade or pitch of not less than 1 in in 40 ft and preferably 1 in in 25 ft.

Mills System. A more satisfactory arrangement (Fig. 25) for tall buildings and factory work is to run the steam-main near the ceiling of the top floor or the roof and install DOWN-FEED RISERS to serve the radiation. This arrangement of the piping is commonly known as the MILLS SYSTEM. In this case the steam and condensate flow in the risers in the same direction and higher velocities and consequently smaller pipe may be used than with an up-feed system. The risers are dripped at the bottom into the return as previously indicated. These systems are ordinarily operated with 2 to 5-lb boiler pressure at normal load. The steam-piping is usually designed for a loss in pressure of approximately 1 oz per 100 ft of run including allowances for ells and other fittings.

Two-Pipe Gravity System. This system (Fig. 27) is adapted for use with large installations and for all work where indirect radiation is installed. The system consists of an INDEPENDENT STEAM-MAIN, either run in the basement or overhead as with the Mills system, INDEPENDENT RETURN-MAIN and a SEPARATE STEAM and RETURN CONNECTION to each radiator, which is of the hot-water type if cast iron (loops connected top and bottom). Each radiator is provided with an automatic air-valve or connected to a system of air-exhaust piping later described under Air Line Systems.

Modifications of the two-pipe system where traps are used on the return end of the radiators, as with the VAPOR and VACUUM SYSTEMS, are later described.

The STEAM-MAIN is reduced in size in proportion to the radiation supplied as the riser-connections are taken off. Eccentric couplings should always be used at the reduction points to avoid water-pockets. Riser connections

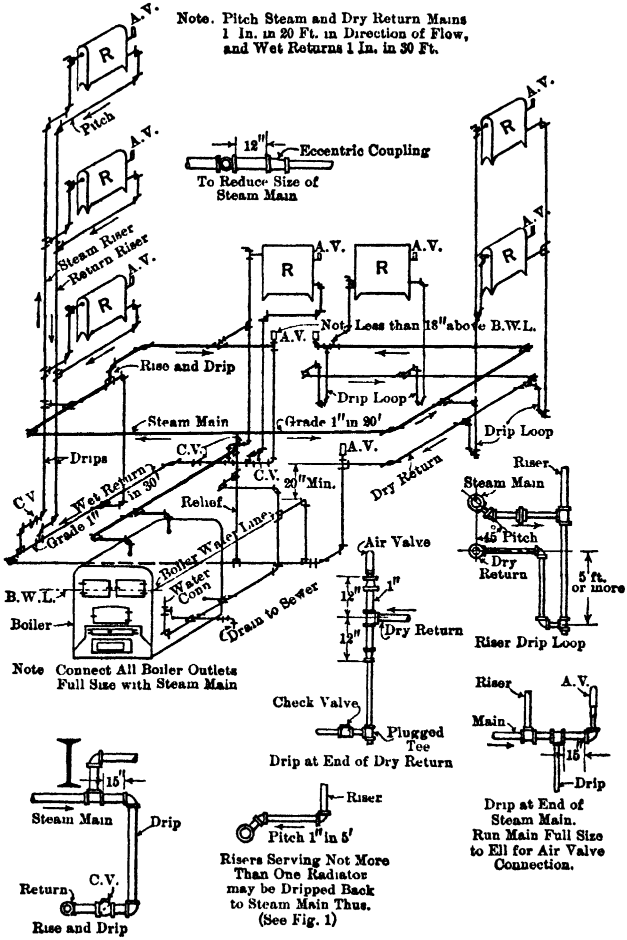


Fig. 27. Two-pipe Gravity System

that are not dripped should be taken off from the side of the main and it dripped from the bottom of the main. No branch or lateral larger than the supply to a single radiator should be pitched back and drained into the steam main. The steam-main in this case does not carry any condensation from the radiation except as noted in the preceding statement. The steam-main

at the end of run is dripped into the return main as indicated for the ONE-PIPE RELIEF SYSTEM. Drips on the steam-main, except at points where a DRIP AND RISE is necessary to clear obstructions, are not recommended as the

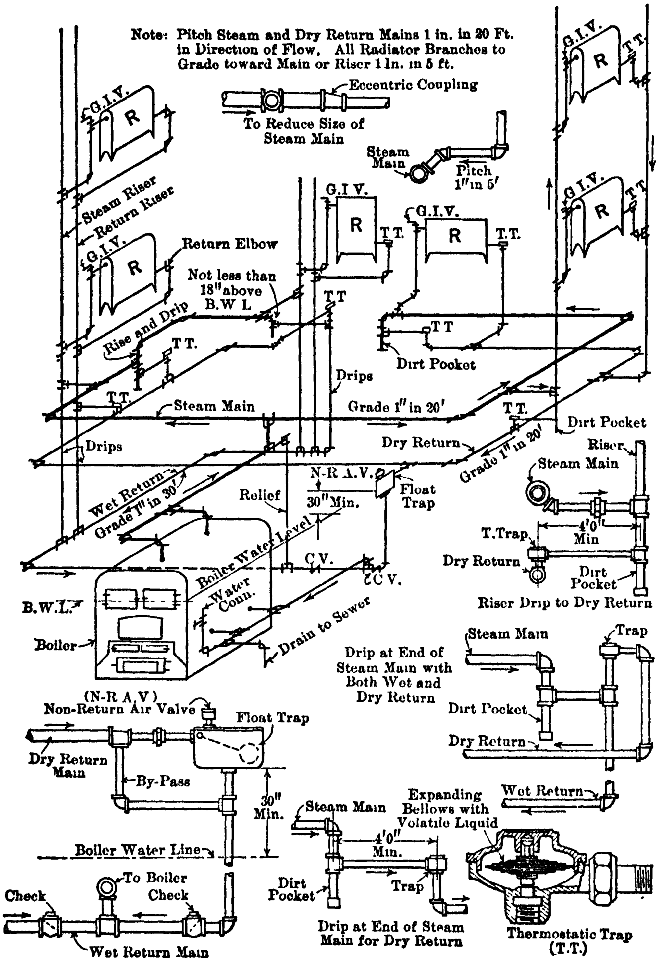


Fig. 28. Vapor System

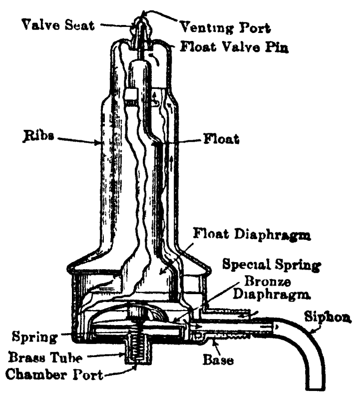
velocity of the steam is ordinarily so high that little or no water will be removed by the drips.

The RETURN may be run either wet or dry. If run dry the return-risers must be provided with either a trap or drip-loop, as previously explained

under the One-Pipe Relief System. The elevation of a dry return should not be less than 20 in above the mean boiler water-line. A dry return must never be trapped. The elevation of the steam-main at the last riser-connection should be at least 18 in above the normal boiler water-line and will of course be more if the return is run dry if the end of the dry return at the boiler is 20 in or more above the boiler water-line. An automatic air-valve of the non-return type should be installed at the end of the steam-main and dry return-main, as indicated. The grade of steam and dry return-mains should be at least 1 in in 40 ft in the direction of flow and preferably 1 in in 25 ft.

These systems are ordinarily operated with 2 to 5-lb boiler pressure normal load, and the steam-piping is ordinarily designed for a loss in pressure of 1 oz per 100 ft of run including allowance for fittings. (See Tables XXII and XXIV for PIPING SIZES.)

Automatic Radiator Air-Valves for Gravity Systems. The AUTOMATIC REMOVAL OF AIR from steam-radiators must be provided for if the highest efficiency of the radiating-surfaces is to be realized in gravity circulating systems. Manually controlled air-valves or cocks are usually neglected, and are seldom used for steam-radiators although their use is quite general for hot-water radiators. Fig. 29 shows a float-type of automatic air-valve. Thermostatic air-valves are finding great favor in this field. The proper LOCATION OF THE AIR-VALVE on a steam-radiator is at the end of the radiator opposite the steam-inlet, and as near the bottom of the radiator as possible, since air is heavier than steam at the same temperature. In practice, however, the manufacturer of radiators usually places the air-valvetapping about two-thirds the height of the radiator from the floor in order to prevent possible flooding of the valve.



Fit. 29. Hoffman Non-return Type Air-valve

Air-Line System. The AIR-LINE SYSTEM (Fig. 30) may be attached to any one or two-pipe gravity system, and is applied by connecting the automatic air-valve of each radiator with small-size piping to an exhauster which maintains a slight vacuum in the air-piping and effectually removes the accumulation of air in the radiators. As this scheme is a positive means of air-removal its application to the ordinary one or two-pipe gravity system will improve its operation. The original air-line system is known as the PAUL SYSTEM. The exhauster used for less than 2 500 sq ft is a water-driven vacuum-pump, with a pressure of at least 20 lb per sq in. Larger systems use a high-pressure steam-jet (see above), or if steam is not available, a motor-driven vacuum-pump of about ¼ horse-power, 1 in air-mains in basement, and a gate valve on each air-riser are used. The steam used varies from 1 to 5% of the total condensation. All radiator-connections are made as shown.

Vapor or Air Return Systems (Figs. 20 and 28). All so-called VAPOR or AIR-RETURN SYSTEMS are essentially two-pipe systems designed to operate, normally, at or slightly above atmospheric pressure. In all air return systems the air passes down the return-pipes with the condensate and is afterward forced out of the return-main at one or several points.

The vapor systems find their widest application in buildings covering a moderate area, as residences, stores, etc. Vapor or air return systems are designated by various manufacturers of specialties as MODULATING SYSTEM (Warren Webster Co.) RETURN HEATING SYSTEM (C. A. Dunham Co.), etc. Water-type radiation, if cast iron, is usually employed as with all two-pipe systems, but no air-valves on the radiators are required or installed. Steam-type radiation may be used if the inlet is placed at the top of the first section.

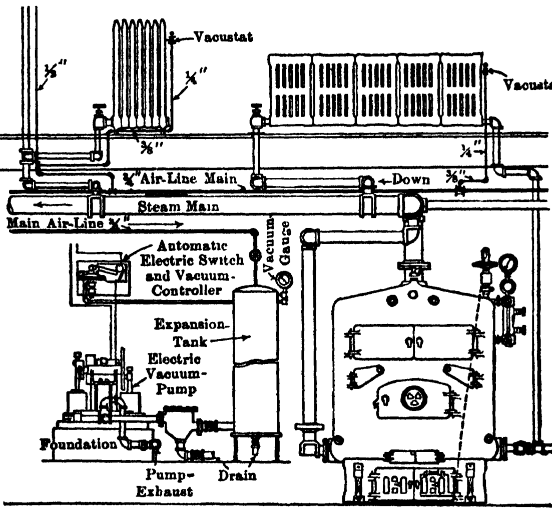


Fig. 30. Air-line System

The steam-supply to each radiator is controlled by means of a graduated or fractional inlet valve (also termed modulating). This control permits a partial heating of the radiator in mild weather by admitting less steam than the radiator is capable of condensing for the condition of operation. There are several general types of vapor systems in use. The most common type vapor system (TYPE *a*) is a CLOSED-CIRCUIT GRAVITY-SYSTEM employing thermostatic traps on the return-ends of the radiators, or one thermostatic trap for each return-riser as indicated. The condensation from the riser drips into a wet return-main and the accumulated air from the system is passed by the traps into a dry return-main. The arrangement of piping in this case is shown on the left-hand side of Fig. 28. The end of the dry return-main, at the drop, is equipped with a combination air-valve of the float type and a non-return air check-valve or vent. Normally, the vent opening in the float-chamber is open to the atmosphere and the air is expelled through the non-return check-valve, but should the water back up in the return the float closes the air-vent opening and prevents the water from escaping. A com-

bination float, thermostatic element and non-return check-valve is manufactured by several well-known manufacturers. The addition of the thermostatic element prevents the escape of steam should any of the thermostatic traps leak steam into the return.

It is preferable to drip the steam-main and steam-risers directly into a wet return-main, as shown by the piping-arrangement on the left-hand side (Fig. 28). No thermostatic trap is in this case required at the base of steam-riser. One thermostatic trap may be employed for each return-riser in place of one trap for each radiator. In this case the dry return handles only the accumulated air in the system and vapor that leaks by the traps on the return riser-connections. If, owing to the building arrangement, the installation of a wet return is impossible, then the arrangement of piping with dry return, as shown on the right, may be used, in which case each radiator is equipped with a thermostatic trap on the return end. The steam-risers are in the latter case dripped through a thermostatic trap to the dry return and the return-riser connected to the dry return. The dry return handles all of the condensation as well as the air. Whenever a thermostatic trap is employed to drip a steam-riser or main it should be connected at least 4 ft from the pipe being dripped and the connection to the trap left uncovered so that the conduction of heat from the pipe being dripped will not affect the action of the thermostatic element. Unless this is done the conducted heat may be sufficient to prevent the trap from opening.

The piping-system is ordinarily sized so that at normal or designed load, the pressure required at the boiler does not exceed approximately 8 oz or ½ lb per sq in to circulate the steam to the most distant radiator. In mild weather the pressure required is lower and depends entirely on the heating load. The thermostatic traps used with this system pass air and water and only a slight amount of vapor into the dry return which is condensed in the return-piping. Practically no pressure, therefore, exists in the dry return-main so that the boiler pressure will be balanced by a column of water in the return-piping. The boiler safety-valve is usually set to blow at 1 lb per sq in pressure. In starting up the plant the damper regulator, which should be of the sensitive type, designed for the very low pressures used with this system, frequently permits the boiler pressure to exceed 1 lb per sq in before the fire can be sufficiently checked. The water will, in this case, stand in the return-piping approximately 30 in for each pound pressure per square inch above the boiler water-line. The end of the dry return-main, at the drop, should, therefore, not be placed less than 24 in and more if possible above the normal boiler water-line as indicated by Fig. 20. In normal operation the water in the return-piping will ordinarily not stand more than 12 to 15 in above the mean boiler water-line. The steam and return-mains are pitched in the direction of the flow, the same as for the two-pipe system, not less than 1 in in 40 ft and preferably 1 in in 25 ft. The steam riser-connections are pitched toward the riser and taken off the bottom of the main at 45° except for a branch connecting a single radiator in which case the branch is taken off at the side and pitched back toward and drained into the steam-main.

Grade or pitch branch-connections approximately 1 in in 5 ft. The steam-piping in the system as with the other types is ordinarily designed for 1 oz loss per 100 ft of run including allowance for fittings. If the runs are long, a careful estimate should be made of the friction pressure loss through the system and such pipe sizes chosen as will prevent the loss during the starting-up period from exceeding approximately 1 lb per sq in. (See example under Pipe Sizes for Direct Steam Heating.)

TYPE *b* vapor system (not illustrated) is similar to Type *a* described except that a POSITIVE RETURN FLOAT-TYPE TRAP is employed. In this case when the float rises and closes the air outlet it also acts to admit steam from the

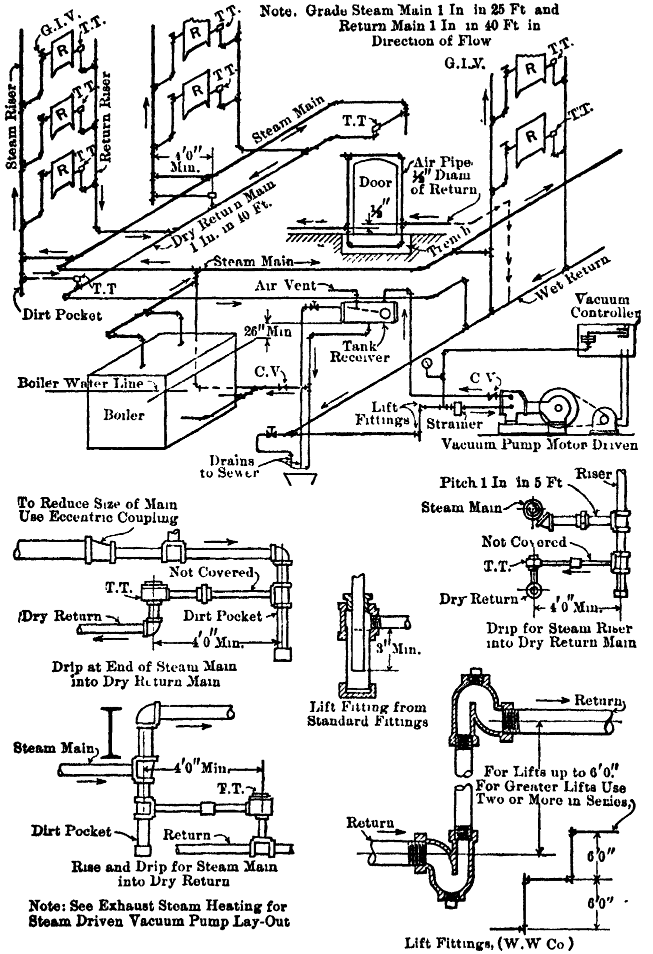


Fig. 31. Vacuum System

boiler into the trap, balancing the pressure, and allowing the water in the trap to return to the boiler due to the static head of water in the vertical return-connection to the trap.

It is evident that with the positive return type of trap the boiler-pressure is not limited to a relatively small amount as with Type *a* system.

Vacuum Systems (Fig. 31). The mechanical vacuum system is a two-pipe system employing a thermostatic trap on the return-end of each radiator and a vacuum-pump on the main return. The thermostatic trap serves the same purpose here as indicated under Vapor Systems, namely, to freely allow the escape of air and condensation, but prevent the escape of steam to the return-line. The vacuum-pump delivers the condensate and air to an air-liberating tank with float which closes the air-vent when the tank becomes half full of condensate.

Further accumulation of air and water creates additional pressure in the tank until the boiler pressure is exceeded and the condensate flows through the tank outlet to the boiler until the tank is about half full, when the float opens the vent and permits the accumulated air to escape.

The modern vacuum-pumps are usually of the rotary type with the water impeller and air rotor located on one shaft directly connected, through a flexible coupling, with the motor. The air liberating tank, strainer, vacuum-breaker and vacuum-controller are all an integral part of the pump and mounted on a cast-iron base. These pumps are fitted with dual automatic control so arranged that the pump will be

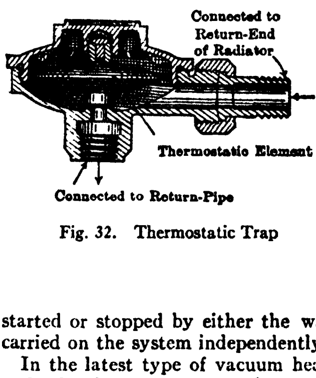


Fig. 32. Thermostatic Trap

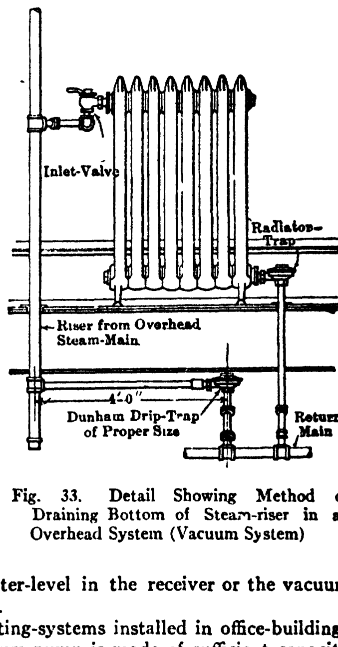


Fig. 33. Detail Showing Method of Draining Bottom of Steam-riser in an Overhead System (Vacuum System)

started or stopped by either the water-level in the receiver or the vacuum carried on the system independently.

In the latest type of vacuum heating-systems installed in office-buildings and other large structures, the vacuum-pump is made of sufficient capacity and size to carry a relatively high partial vacuum on the complete system, including piping, steam and return-lines and the boiler. If the vapor-pressure in the radiators be varied, the steam temperature varies and consequently the heat-emission of the radiators is also varied.

If the partial vacuum carried on the system be varied automatically as the outside temperature varies so that the heat emission of the radiation balances the heat loss of the rooms, a room-temperature control is obtained which approaches that of automatic control. This arrangement prevents overheating in mild weather and results in a saving of fuel. In this system the partial vacuum on the boiler is varied from 0 in to 20 in of mercury with an average for the entire heating season of approximately 15 in, corresponding to approxi-

mately 7½ lb per sq in below atmospheric pressure and a corresponding steam temperature of 180° F.

It is evident that, with the arrangement of control as outlined above, this type of steam system closely corresponds in operation and results obtained to those of a hot-water installation.

The Dunham Differential System and the Warren Webster Moderator System are examples of this arrangement of control and may be applied to buildings requiring 5 000 sq ft of direct radiation and above.

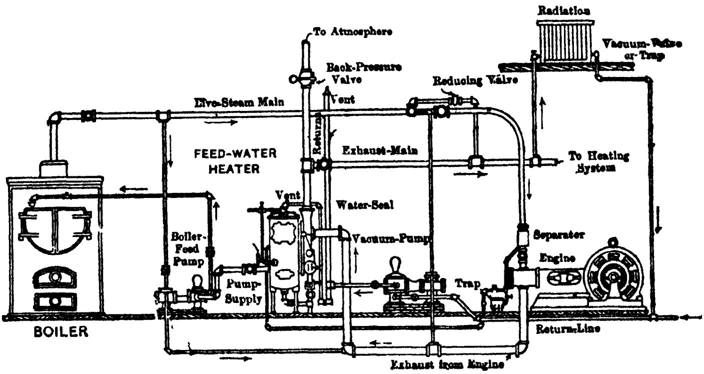


Fig. 34. Exhaust Steam-heating Vacuum System

Table XIX. Capacities of Dunham Vacuum-Traps

Number	Size, in	Capacity, direct radiation, sq ft	Pipe-connection, in	Weight, lb	Diameter of port, in	Lift, in
1	½	100	½	1½	..	.
2	½	350	½	2½	⅜	⅛
3	. ..	450	¾	.	..	.
B.T.	¾	1 500	¾	13	¾	3/16
B.T.	1	3 000	1	21	1	. ....

These traps are designed for steam-pressures not in excess of 10-lb gauge. For main and riser-drips, use no smaller trap than the No. 3, and install trap as per details.

Care must be exercised in selecting a trap or traps of the proper size for hot-blast heating-coils. The capacity-ratings for all traps are in terms of direct cast-iron radiation, on a condensation-basis of approximately 0.25 lb per sq ft per hr. Every unit of blast-coil must be reduced to that basis before trap-sizes are chosen and specified. (See Hot-Blast Heating for further details in reference to rating of vacuum-traps for hot-blast coils.)

Design of Low-Pressure Steam-Heating Systems

Amount of Radiation Required. The heat-loss, H, of the various rooms is calculated as previously indicated, and H is divided by 240. The result is the amount of equivalent direct radiation in square feet, nominal rating, required.

The heat-emission of cast-iron radiation for pressures up to 5 lb per sq in may be assumed as 240 Btu for all practical purposes of calculation.

Rating of Boiler Required. If anthracite coal is to be used for fuel add not less than 25% to the total amount of direct radiation to be installed to allow for radiation-loss of boiler, mains, returns, etc. The steam-mains and risers should always be covered. See Rating of Heating-Boilers for further data.

Size of Mains, Branches and Return-Pipes. Steam-mains in low-pressure gravity systems are generally so proportioned that the loss in pressure, due to pipe-friction, does not exceed approximately 1 oz or 0.062 lb per sq in, per 100 ft of run. For long runs a larger drop in pressure is permissible. The reason for thus limiting the pressure-loss is apparent from an inspection of Fig. 35.

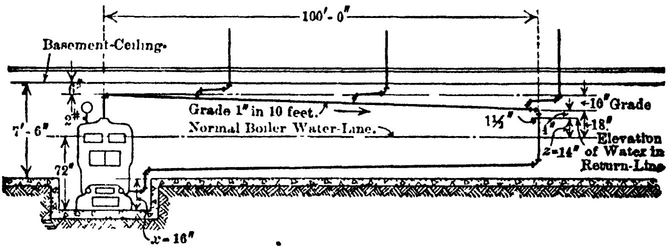


Fig. 35. Location of Boiler and Arrangement of Pipes for Low-pressure Steam-heating Systems

Owing to the fact that the steam is losing pressure as it flows through the main, it follows that the pressure at the last riser will be lower than in the boiler. The difference in pressure, or pressure-loss, P, causes the water in the return-main to stand higher than the water-line of the boiler. The added height Z is equal to the height of a column of water which pressure P will support. Thus, if the boiler-pressure is 2 lb per sq in and the pressure at the far end of the main is, say $1\frac{1}{2}$ lb, with water weighing 61 lb per cu ft, or 0.035 lb per cu in, the water in the return will stand $(2 - 1.50) \div 0.035$, or $Z = 14$ in above the water-line of the boiler for a $\frac{1}{2}$-lb loss in pressure between the boiler and the end of main. It is apparent in this instance that unless the water-line of the boiler is about 18 in or more below the last riser. or radiator-connection, water is quite likely to flood the steam-main and to be accompanied by a hammering and a poor circulation in the radiators located at or near the end of the run. Steam-mains are graded in the direction of flow approximately 1 in in 10 ft.

Referring to Fig. 35, and assuming a 7 ft-6 in, or 90 in, clear height of basement, and a boiler having a 72-in water-line and a length of steam-main of 100 ft, it is evident that the boiler must be located in a pit, the depth, X, of which is

$$6 + 10 + 18 + 72 - 90 = 16 \text{ in}$$

in order to maintain 18 in between the water-line in the boiler and the end of steam-main. The distance in practice should not be made less than from 18 to 24 in. The extreme pressure-load stated, ½ lb, in this illustration, is never approached in normal operation, when the mains are designed for 1-oz drop per 100 ft, but may approach the value stated when the system is being started up with cold radiators, when the rate of condensation is very much higher.

Allowable Pressure-Drop in Low-Pressure Vapor and Vacuum Steam-Heating Mains. It has been customary practice for a number of years to proportion steam-heating mains within buildings on a basis of 1-oz drop in pressure per 100 ft of run regardless of the length of the run. It is now considered good practice to limit the total drop in pressure from the boiler to the farthest radiator about as follows for the various systems stated in Table XX.

Table XX

	Total Drop
One-pipe low-pressure gravity systems equivalent length run 200 ft or less	2 oz
Two-pipe low-pressure gravity systems equivalent length run 200 ft or less..	2 oz
Two-pipe vapor systems equivalent length run 200 ft or less	2 oz
One-pipe low-pressure gravity systems equivalent length run 200 ft to 600 ft	4 oz
Two-pipe low-pressure gravity systems equivalent length run 200 ft to 600 ft...	4 oz
Two-pipe vapor systems equivalent length run 200 ft to 400 ft	2 oz
Two-pipe vapor systems equivalent length run 200 ft to 600 ft.	4 oz
Vacuum-pump systems equivalent length run 200 ft to 600 ft	4 oz
Vacuum-pump systems equivalent length run 200 ft to 1200 ft.	8 oz

To find the allowable pressure loss per 100 ft of run, divide the total allowable pressure-drop as given above by the length of run in feet by 100. See Table XXI.

Table XXI

Total drop, oz per sq in	Pressure-drop per 100 ft of run, oz per sq in and equivalent length of run, ft									
	100	200	300	400	500	600	700	800	900	1 000
2	2	1	0 67	0 50	0 40	0 33	.	.	..	..
4	4	2	1 33	1	0.80	0 67	....		.	.
8	..	4	2 66	2	1.6	1 33	1.14	1	0 89	0.80

The pressure-loss in a pipe flowing full of steam may be approximated by Babcock's formula

$$W = 87.5 \sqrt{\frac{ypd^5}{L \times \left(1 + \frac{3.6}{d}\right)}}$$

in which W is the weight of steam flowing per minute in pounds, L the length of pipe in feet, d the diameter of pipe in inches, y the density of the steam and p the loss in pressure in pounds per square inch. See Table XXV for friction allowance of valves and ells.

One square foot of direct radiation will condense, under normal conditions of operation, 0.25 lb per hr, and the density of steam, y, is 0.043 lb for a 2.3-lb pressure. The sizes of steam-mains given in Table XXII were calculated by the above formula. To allow for the fittings approximately twice this, or 1/8 lb per sq in per 100 ft of pipe may be assumed. The pressure-loss per 100 ft of

run serving various amounts of radiation is readily determined by means of the charts, Figs. 36 and 38.

Pipe Sizes for Low-pressure Steam, Vapor and Vacuum Systems. (Table XXII.) This table may be employed for sizing the piping within buildings for all types of low-pressure-steam and vapor systems and represents modern practice. The rating given for the steam-mains is based on a

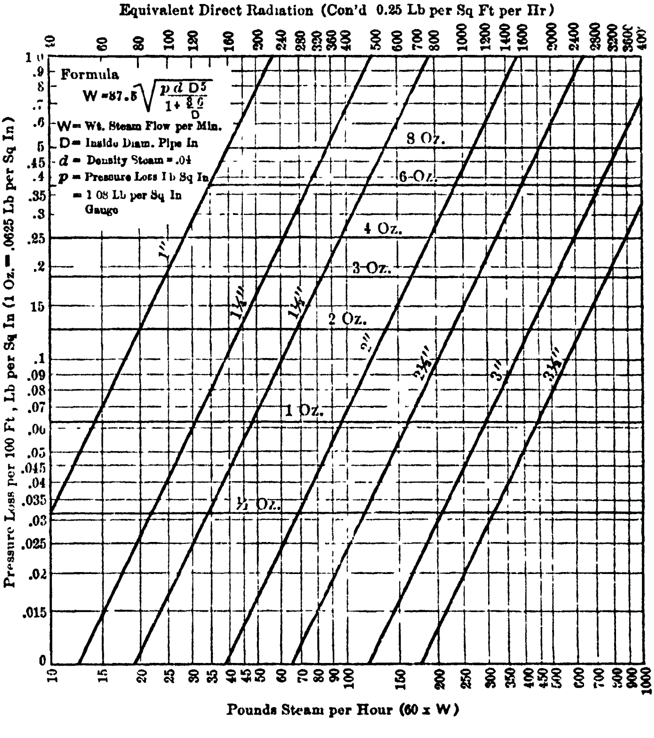


Fig. 36. Friction Pressure-Chart for Low-pressure Steam-heating. (Pipe Sizes 1 in to 3½ in)

pressure-loss of 1 oz, 2 oz, and 3 oz per 100 ft of run. If it is desired to design the steam-main for a fixed total loss in pressure (P) for its length (L), proceed as follows. Determine the pressure-loss per 100 ft of run which will be equal to $P \times L/100$, locate this pressure-loss on the charts, Figs. 36 and 38, and determine the nearest size of pipe required from the intersection of the horizontal pressure-loss line with the vertical line corresponding to the weight of steam to be carried by the pipe per hour or equivalent direct radiation. It is always advisable in any large gravity steam system to check the total loss in pressure in the system by making use of the charts, Figs. 36 and 38.

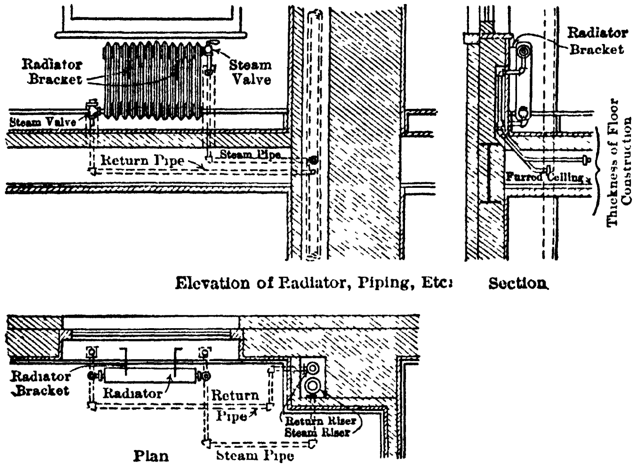


Fig. 37. Details of Piping Connecting with Radiators

Table XXII. Capacities of Steam-Mains, Branches and Risers

Capacities Stated in Equivalent Square Feet of Direct Radiation. One Square Foot of Equivalent Radiation Assumed to Condense 0.25 Lb per Hour

Nominal pipe size, in	Steam mains and down-feed risers dripped; branches to risers dripped; steam and condensate flowing in same direction. Pressure loss, oz per 100 ft: ½	¾	1	Branches to risers not dripped *: One-pipe gravity systems	Two-pipe gravity vapor and vacuum systems	Up-feed supply risers: One-pipe gravity systems†	Two-pipe gravity vapor and vacuum systems‡
¾	§	§	§	..	......	25	30
1	38	47	55	20	26	45	55
1¼	86	105	120	55	58	98	120
1½	135	163	190	80	95	152	190
2	275	325	385	165	195	288	385
2½	440	540	635	260	395	464	635
3	800	1 000	1 165	475	700	799	1 165
3½	1 200	1 500	1 735	745	1 150	1 144	1 735
4	1 750	2 100	2 460	1 110	1 700	1 520	2 460
5	3 200	4 000	4 545	2 180	3 150	....	....
6	5 000	6 300	7 460	......	....	....	....
8	11 000	14 000	15 335	....	.....	......	......
10	20 000	24 100	28 345	.....	.....	....	....
12	32 000	40 000	45 490	...	....	....	....

* Radiator branches more than 8 ft long to be one pipe size larger than table.
† Based on tests by A.S.H. & V.E. Research Laboratory.
‡ Based on 1-oz pressure-loss per 100-ft run.
§ See charts, Figs. 36, 38.

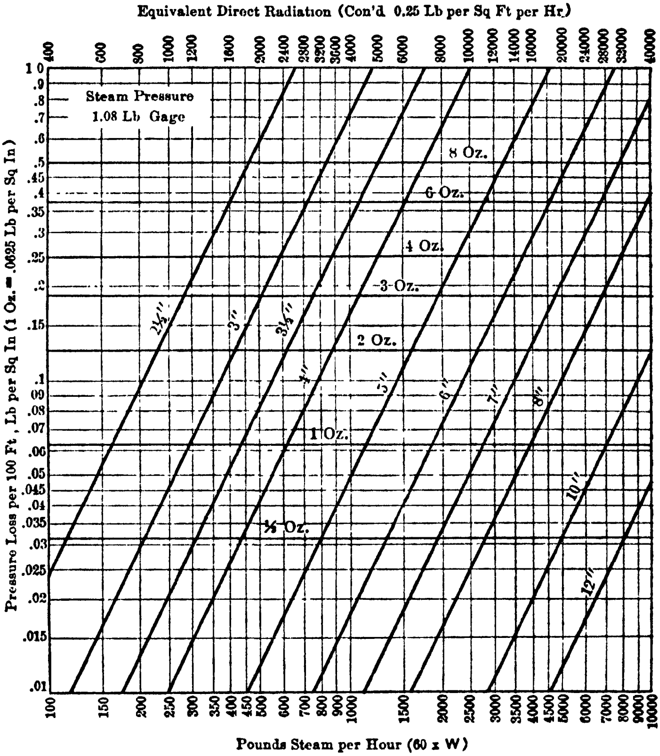


Fig 38. Friction Pressure-loss Chart for Low-pressure Steam-heating (Pipe Sizes $2\frac{1}{2}$ in to 12 in)

Table XXIII. Recommended Radiator Valve Capacities and Vertical Connections

Size, in	Single-pipe gravity systems	Two-pipe gravity systems		Vapor and vacuum systems
		Radiator supply valve	Radiator return valve	
¾	.	30	120	Use manufacturers' listed capacities of valves and return-traps Vertical connections to be same size as valve and trap employed. Return horizontal runout to be not less than ¾ in
1	20	55	190	
1¼	55	120	385	
1½	81	190	.	
2	165	385	.	

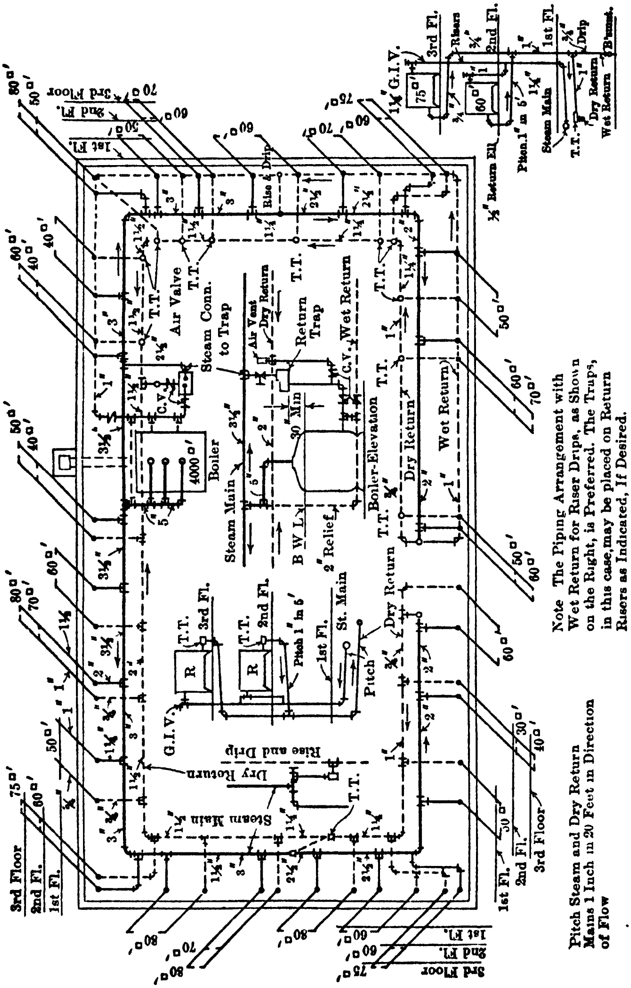


Fig. 39. Example in Pipe Sizing, Vapor System (see Tables XXII, XXIII and XXIV)

Table XXIV. Capacities of Dry and Wet Return-Mains

Capacities Stated in Equivalent Square Feet of Direct Radiation

Nominal pipe size, inches	Dry return-mains					
	One- and two-pipe gravity and vapor systems up to 200 ft *	One- and two-pipe gravity systems exceeding 200 ft in length *			Two-pipe vapor systems exceeding 200 ft in length *	
		Length, L, feet				
		300	400	600	300	400
¾						
1	320	370	320	275	285	250
1¼	670	770	670	480	595	520
1½	1 058	1 210	1 058	757	945	820
2	2 300	2 640	2 300	1 630	2 140	1 880
2½	3 800	4 380	3 800	2 770	3 470	3 040
3	7 000	8 000	7 000	5 000	6 250	5 480
3½	10 000	11 500	10 000	7 200	[illegible] 800	7 880
4			15 000	10 700	13 400	11 700

Nominal pipe size,	Vacuum system			Wet return-mains			
	Return-mains and return-risers *			Gravity and vapor systems, pressure-loss ½ in water per 100 ft run †			
	Length, L, feet						
	100	300	600	100	200	400	600
¾	800	462	326				
1	1 400	810	570	1 525	1 083	762	625
1¼	2 400	1 387	976	3 255	2 311	1 627	1 335
1½	3 800	2 195	1 547	4 541	3 224	2 270	1 862
2	8 000	4 622	3 256	8 450	6 000	4 425	3 465
2½	13 400	7 745	5 453	13 176	9 355	6 588	5 402
3	21 400	12 360	8 710	21 122	15 000	10 511	8 660
3½	32 000	18 490	13 020	32 500	23 075	16 250	13 325
4	44 000	25 430	17 910	45 077	32 000	22 538	18 482

* Recommendations of Joint Committee, A.S.H. & V.E. and H.P.C.N.A.; also A.S.H. & V E. Minimum Requirements Code.

† Calculated from formula proposed by Dr. Biel.

NOTE. For capacities for any length of run L' divide capacities given in table in the column under length L by $\sqrt{\dfrac{L \text{ (Length of Run at Top of Column)}}{L' \text{ (Actual Length of Run)}}}$.

Minimum grade for steam and dry return-mains 1 in in 40 ft.

Minimum grade for horizontal branches to radiators 1 in in 20 ft.

Above table applies to pipes properly reamed and first-class workmanship.

Table XXV. Allowance for Friction of Ells and Valves

(Add to measured length of pipe the equivalent length given by table)

Pipe size, inches	Add for each globe-valve, feet	Add for each ell, feet	Pipe size, inches	Add for each globe-valve, feet	Add for each ell, feet
1	2	1.5	4	20	13
1¼	4	3.0	4½	24	16
1½	5	3.5	5	28	19
2	7	5	6	37	24
2½	10	6	8	53	35
3	14	9	10	70	47
3½	17	11	12	86	58

The loss by entrance to steam-line from boiler is assumed the same as for a globe-valve.

Gravity Indirect Heating

General Description. A satisfactory means of providing for the heat-loss in a room, and, at the same time, supplying air-ventilation, is accomplished by this system. The radiators properly encased with a sheet-metal casing, covered with insulation, are ordinarily hung from the basement-ceiling by means of light angle-iron or strap-iron, as shown in Fig. 40. Each radiator is ordinarily provided with a fresh-air inlet and hot-air duct connecting the radiator with the room-register. A recirculating duct may be provided, as indicated, in order to economize on the heating in extremely cold weather if desired. There should be a separate vertical hot-air duct for each register to be supplied, connected with its own indirect radiator. Attempting to supply more than one register from an indirect radiator is not usually successful, or recom-

Table XXVI. Final Temperature of Air Passing Over Indirect Radiation. Extended-Surface Type. Initial Temperature of Air, 0° F. Heater, One Stack in Depth. Four-Inch Spacing of Sections

Velocity of air through free area of heater, in ft per min, v*	Final temperature, t_2, in degrees Fahrenheit	
	Hot water, 180° F.	Steam at 2-lb pressure
50	122	147
100	100	127
125	95	120
150	90	113
175	86	106
200	82	102

* Measured at 70° F. For first-floor registers a velocity of 150 ft per min through free area and 150 ft per min for second-floor registers is the usual assumption.

mended, unless a fan system is employed to give a positive air-flow as with the hot-blast system, described later. The hot-air ducts for the upper floors, for

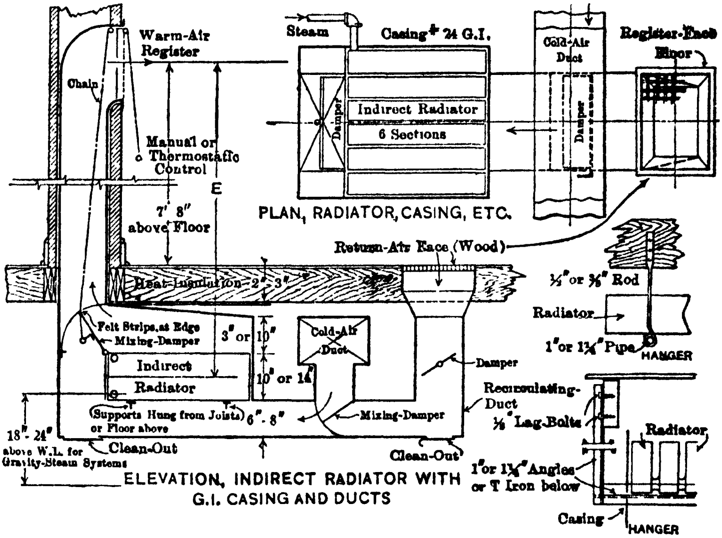


Fig. 40. Indirect Radiators, Casings, Connections, etc.

best results, should be double pipe as later shown under furnace-heating. The indirect radiator is designed to present a maximum of heating-surface to the air passing over same. Among the various standard types for gravity indirect heating may be mentioned the Indirect Pin Radiator (Fig. 41), Excelsior, Sterling and Vento. Indirect radiators are now rated according to the temperature-increment, or rise, which they are capable of giving to the air passing between and over the sections of the heater for various velocities of air, initial temperature, and temperature or pressure of the steam, or temperature of the hot water. The velocities stated are, for convenience in rating, based on air at 70°. The free or unobstructed area means the net area between heater-sections after deducting the area of the projecting surfaces from the gross area. Limitations of space prevent giving more than these data for one type of indirect radiator.

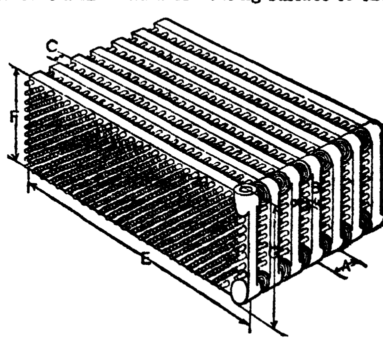


Fig. 41. School Pin Indirect Radiator for Steam or Water

Table XXVI gives the results of tests made on Vento, indirect radiation, American Radiator Company. (See Fig. 42.)

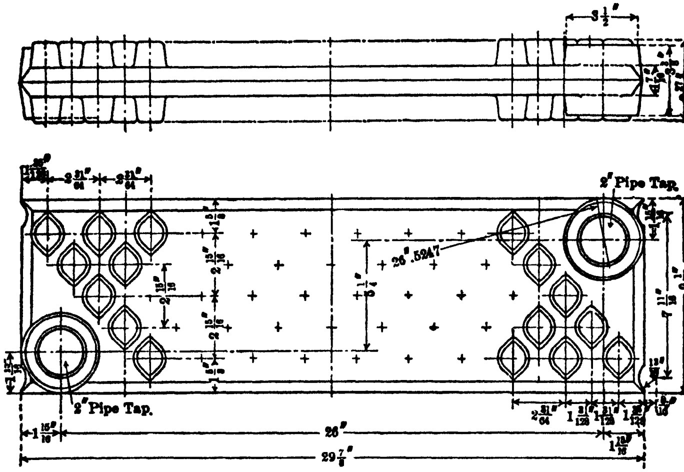


Fig. 42. Vento Thirty-Inch Indirect Heater-Section

Table XXVII. Dimensions, etc., Vento Indirect Radiation

Size	Heating-surface, s, sq ft	Height, in	Width, in	Free area between sections, (a), sq ft
30-in section	8.00	29⅞	9⅛	0 256
40-in section .	10.75	41 1/64	9⅛	0.35
50-in section.....	13.50	50 29/32	9⅛	0.428
60-in section ...	16.00	60 11/16	9⅛	0.511

Spacing of sections, 4 in on centers for gravity air-circulation.

Weight of Air to be Circulated per Minute, W; t_1 = temperature of air leaving register; t = temperature of room. 0.24 = sp heat of air; H = heat-loss of room to be warmed in Btu per hour; V = volume of air in cubic feet per minute measured at 70°.

Then $W = H/[60 \times 0.24(t_1 - t)]$, lb of air per min
$V = W/(0.075$ = the density of the air at 70°)

Area of Indirect Heating-Surface Required, S; t_0 = initial temperature of air entering indirect heater; t_2 = final temperature air leaving heater = t_1 + 5° (assumed temperature-loss in hot-air duct); V = velocity through free area of heater in feet per minute, measured at a temperature of 70°; F = total fire-area required in heater, in square feet; a = free area for one section of heater in square feet. n = number of sections required.

Then $F = V/v = W/0.75v$, $n = F/a$ and $S = n \times s$

Example. Required the amount of indirect low-pressure steam-surface of the extended-surface type, the number and size of sections, and the over-all dimensions of an indirect radiator to supply the necessary heat to warm a first-floor room, the heat-loss of which is $H = 20\,000$ Btu per hour. All the air is to be taken from the outside, the temperature of which is $t_0 = 0°$ F. The inside temperature to be maintained is $t = 70°$ F.

Solution. It is first necessary to assume a temperature, t_1, for the air entering the room in order to calculate the amount or weight, W, of air required to be circulated to convey the heat required to make up the heat-loss H.

Assume $t_1 = 95°$ and $t_2 = t_1 + 5$ (loss) $= 100°$; and $v = 100$ ft per min from Table XXVI.

Then $W = 20\,000/[60 \times 0.24(100 - 70)] = 46.3$ lb per min

and $V = 46.3/0.75 = 617$ cu ft per min, measured at 70° F.

Assume 40 in as the length of section desired in this installation,

$a = 0.35$ (Table XXVII);
$F = 617/100 = 6\ 17$, the total square feet of free area required;
$n = 617/0\ 35 = 18$, the number of sections of 40 in.

Vento is, therefore, required, giving a total heating surface of

$$S = 10.75 \times 18 = 193\ 5 \text{ sq ft}$$

Dividing this equally between two indirect radiators the width of each heater is equal to 9 (sections) $\times$ 4 (spacing-sections) $= 36$ in.

Low-Pressure Boiler-Rating Required for Gravity Indirect Radiation. The amount of heat given up to the radiator is

$$h = 0\ 24W(t_2 - t_0) \times 60 \text{ Btu per hr}$$

The equivalent rating in square feet of direct radiation is therefore $R = h/240$, plus 25% for radiation of mains, returns, etc.

Example. Required the equivalent low-pressure boiler-rating to supply the indirect radiation in preceding example

Solution.

$$h = 0.24 \times 46.3\ (100 - 0) \times 60 = 66\,672 \text{ Btu per hr}$$
$$R = (66\,672/240) \times 1.25 = 347 \text{ sq ft}$$

Table XXVIII. Theoretical Velocity (V) of Air, in Feet per Second, Due to Natural Draft

Height of flue in feet, E	Excess of temperature in flue above external air							
	10°	15°	20°	25°	30°	50°	100°	150°
1	1 1	1 4	1.6	1.8	2.0	2 5	3.6	4.4
5	2 5	3 1	3 6	4 0	4 5	5.6	8 1	9 9
10	3 6	4 4	5.1	5 7	6 6	8 1	11.4	14.0
15	4 4	5 4	6 3	7 0	7 7	9.9	14.0	17.1
20	5 1	6.3	7.2	8.1	8 8	11 4	16.1	19.8
25	5 7	7.1	8 1	9 0	9 9	12 8	18 0	22.1
30	6.3	7 8	8.8	9 9	10 8	14.0	10.8	24.2
35	6.8	8.4	9.5	10.7	11.7	15.1	22.3	26.1
40	7.3	8.9	10.2	11 4	12.5	16.1	22.8	27.9

Area of Hot-Air Ducts for Gravity-Circulation. A velocity of approximately one third of the theoretical velocity attainable by natural draft, due to the smaller density of the heated air, is assumed in practice in proportioning the area of the hot-air ducts.

Example. Required the size of hot-air duct for each of the indirect radiators in the preceding examples for a first-floor installation, the effective height E being 5 ft.

Solution. The excess of temperature in the flue above the external air is $100 - 0 = 100°$. The theoretical velocity in the duct, from Table XXVIII, is $8.1 \times 60 = 486$ ft per min. The actual velocity is approximately one third of this, or 162 ft per min. The weight of air per minute passing through the flue is 23 lb, or

$$23/[0.071 \text{ (density at } 100°)] = 324 \text{ cu ft}$$

The required area of the flue is therefore

$$324/162 = 2 \text{ sq ft} = 288 \text{ sq in}$$

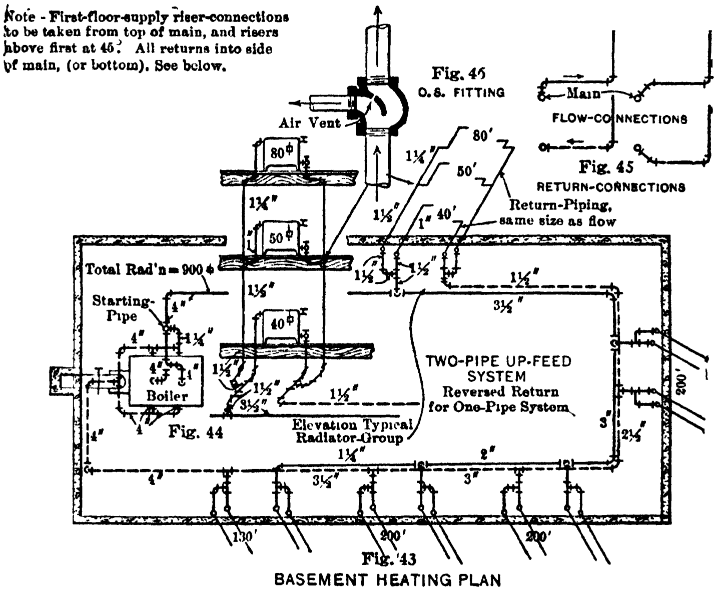
The gross area of the register-face must be approximately 1.8 this amount or 518 sq in, to obtain the same free area through the register-grill as exists in the flue or duct. Sizes of standard registers are given in the section on Furnace-Heating.

Direct Hot-Water Heating

Systems in Use. Systems for heating with direct hot-water radiators, like the direct steam heating systems, may be divided into two general classes, the first of which includes all those systems OPERATING BY GRAVITY ONLY, depending on the difference in density of the water-columns in the flow and return-lines to cause circulation. The second class includes those systems in which a FORCED CIRCULATION is maintained by means of a pump placed on the return-line just before it enters the boiler or heater. These latter systems are employed usually only in large installations or in district-heating service.

Gravity Hot-Water Heating Systems. The gravity systems are divided into the UPFEED SYSTEMS, using basement-mains, and the DOWNFEED SYSTEMS, using overhead or attic-mains. The upfeed systems may have either a one-pipe basement-main or two-pipe basement-mains, and the latter type may have either a DIRECT or a REVERSED return-main. (See Figs. 43 and 44 for reversed return.) The downfeed systems may have either SINGLE OR DOUBLE RISERS. Either system may be operated with an OPEN or CLOSED EXPANSION-TANK, as shown in Figs. 43 to 46. In general, the downfeed or overhead systems are more positive, permit the use of smaller mains and risers, and provide for the automatic removal of air from the radiators and piping. It is necessary, however, that the head room or clear space in the attic should be at least 4 or 5 ft if the overhead mains and branches are to be properly installed. It is sometimes possible to run the overhead mains at the ceiling of the top floor, and in such cases the above restriction does not apply. Mains run in attics must be well insulated to prevent freezing. The underfeed systems are used where basement-space is available, and of little or no value, and the radiation is located on two or more floors; or where attic-space is so limited that it would not be possible to install overhead mains and branches. Underfeed systems are liable to prove unsatisfactory in buildings less than two stories in height, as the motive head with radiators on the first floor only is so slight that faulty or deficient circulation is quite likely to result.

The Upfeed One-Pipe System. The upfeed one-pipe system is in general use today, and is employed almost exclusively by the United States Treasury and War Departments whenever upfeed hot-water systems are to be installed. In this system, as shown in Figs. 48 and 50, the supply-main rises close to the basement-ceiling just above the boiler and grades down in the direction of flow, with a uniform grade of ¾ in in 10 ft. Branches are taken from the top of this main for supplying flow-risers and the return-branches are made into the side or bottom. (Fig. 45.) Flow-connections should always be made from the top, or at an angle of 45° in the case of branches near the boiler,



Figs. 43, 44, 45 and 46. Hot-water Heating. Two-pipe Up-feed System

or for branches supplying only upper-floor radiators. It will be seen that in the case of branches supplying radiators on all floors the upper-floor radiators may be made to PULL or augment the circulation of the first-floor radiators by taking the basement-branch for the former from the side of the branch running to the latter radiator. The first-floor branch is usually run full size all the way to favor the lowest radiator, as shown in Fig. 50. After having served all the radiator-branches the main drops and returns to the boiler, continuing the same size for the entire circuit. Connections (Fig. 48) to radiators should be made at the TOP on the supply-end, using a union elbow, and at the BOTTOM on the return-end, using a quick-opening hot-water radiator-valve with union connection. By this arrangement only one valve is required to control the radiator. Since the temperature of the water in the one-pipe main gradually drops, due to the return of water at a lower temperature from the radiators served in the course of the main around the building, it is advisable to increase the last radiators on the main from 5 to 10% in area and to increase the size of

HOT-WATER HEATING - EXPANSION TANKS
(OPEN AND CLOSED SYSTEMS)

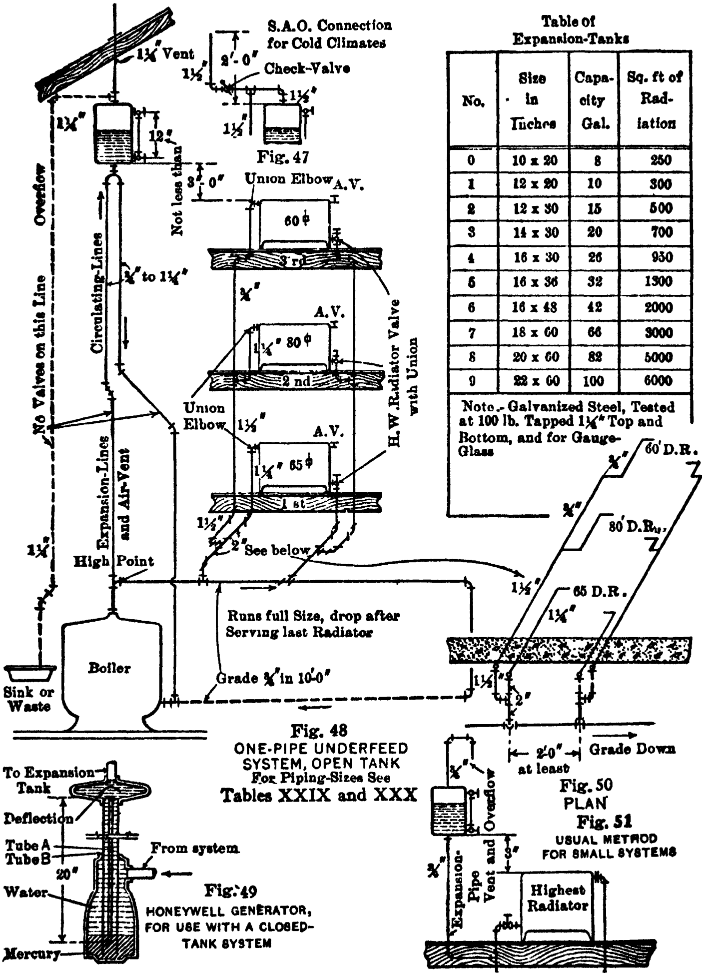


Table of Expansion-Tanks

No.	Size in Inches	Capacity Gal.	Sq. ft of Radiation
0	10 x 20	8	250
1	12 x 20	10	300
2	12 x 30	15	500
3	14 x 30	20	700
4	16 x 30	26	950
5	16 x 36	32	1300
6	16 x 48	42	2000
7	18 x 60	66	3000
8	20 x 60	82	5000
9	22 x 60	100	6000

Note.- Galvanized Steel, Tested at 100 lb. Tapped 1¼" Top and Bottom, and for Gauge-Glass

Figs. 47, 48, 49, 50 and 51. Hot-water Heating. Expansion-tanks. One-pipe Underfeed System

branch and riser-connections at the end of the main by a one-pipe size. Pipe-sizes may be taken from Tables XXIX and XXX. In using the tables all mains must be measured back to the boiler, and risers to any floor are propor-

tioned to supply all the radiation above that floor as well as the radiator actually installed on that floor, as shown in Figs. 48 and 50.

Table XXIX. Hot-Water Heating. Piping-Sizes. Open-Tank (Upfeed with Basement-Mains)

Mains up to 100 ft

Pipe-size in inches	Direct radiation in square feet	Indirect radiation in square feet
1¼	135	100
1½	220	135
2	350	225
2½	460	320
3	675	500
3½	850	650
4	1 100	850
4½	1 350	1 050
5	1 700	1 350
6	3 600	2 900
7	4 800	3 900
8	6 200	5 000
9	7 700	6 300
10	9 800	7 900
12	14 000	11 400

Table XXIX was compiled by J. J. Hogan, and is to be used for either one or two-pipe work.

Length of main must be measured back to boiler.

For mains over 100 ft, reduce capacity in the ratio of $\sqrt{\frac{100}{L}}$.

Table XXX. Hot-Water Heating. Piping-Sizes. Open-Tank (Upfeed with Basement-Mains)

Branches and risers

Pipe-size in inches	Floor: First	Floor: Second	Floor: Third	Floor: Fourth
	Direct radiation in square feet			
¾	30	45	55	70
1	60	75	85	95
1¼	110	120	135	150
1½	180	195	210	230
2	290	320	350	370
2½	400	490	525	550
3	620	650	690	730
3½	820	870	920	970
4	1 050	1 120	1 185	1 250
4½	1 325	1 400	1 485	1 560

Table XXX was compiled by J. J. Hogan, and is to be used for either one or two-pipe work.

The Upfeed Two-Pipe System. The upfeed two-pipe system is also in very general use, and if installed with a REVERSED RETURN, as shown in Figs. 43 and 44, will give good results. If a DIRECT RETURN is used so that the water circulates first through the radiators nearest the boiler, and then through each succeeding group in turn, the ends of mains will be slow in warming up, the last radiators may be cold, and the system prove unsatisfactory. With the REVERSED-RETURN SYSTEM each group of radiators has exactly the same length of water-travel, and hence the resistance to be overcome is practically the same, irrespective of the distance of the radiator-group from the boiler. It will be noted that the return begins at the first radiator served and flows in the SAME DIRECTION as the flow-main, increasing in size while the latter decreases. The flow-main grades UP uniformly ¾ in in 10 ft, and the return grades DOWN toward the boiler with the same pitch. Pipe-sizes may be taken from Tables XXIX and XXX as in one-pipe work, and the main-size reduced or increased as rapidly as the change in radiation supplied will permit. It is also customary in government work to install a STARTING-PIPE (Fig. 43), between the main flow and return at the boiler, in underfeed systems. This pipe ranges from 1¼ to 2½ in in size, depending upon the capacity of the boiler, and is intended to assist in the establishment of an initial circulation between flow and return-headers, even before the water in the mains is moving.

Equalization-Table. In Federal-building work N. S. Thompson makes use of the following Equalization-Table in proportioning mains and risers serving more than one radiator in both upfeed and downfeed systems. The equalizing-numbers represent the relative capacities of the different sizes of pipes for

Table XXXI. Equalization-Table for Mains and Risers

in	in	in	in	in
½ = 2	¾ = 5	1 = 10	1¼ = 20	1½ = 30
2 = 60	2½ = 110	3 = 175	3½ = 260	4 = 380
5 = 650	6 = 1 050	7 = 1 600	8 = 2 250	

Example. A 1¼-in, 1½-in and 2-in pipe have a total value of 110 units, and hence are equivalent in carrying capacity to a 2½-in main.

the same friction-pressure loss per 100 ft of run, and are proportional to the 5/2 powers of the diameters. Thus the weight of water flowing varies as shown by the relation, $W = Kd^{5/2}$, in which W = weight, I = a constant, and d = pipe-diameter.

Details of Piping Systems and Connections for Direct Hot-Water Heating. The distinctive piping-details of each system of hot-water heating have been discussed under that system, as described in the preceding paragraphs. In general all main piping and branches must be UNIFORMLY GRADED, as already indicated, and ample provision made for EXPANSION and CONTRACTION, and the ready REMOVAL OF AIR from all parts of the system. Air-traps or pockets in a hot-water system are fully as serious as water-pockets in a steam system. Hence a hot-water main grading down in the direction of flow cannot be relayed unless an air-outlet is provided at the top of the relay. If the main is reduced in size at any point an ECCENTRIC FITTING must be used to keep the TOP of the large and small main in the same plane and avoid an air-pocket. Not only must all the piping be designed to permit the removal of air, but FREE AND COMPLETE DRAINAGE of water must be provided for as well, so that when the drain or blow-off cock is opened at the boiler the entire system can be emptied of water. If branch-mains are taken from a HEADER at the boiler

they must all rise to the SAME ELEVATION so that the tops of all the branches will lie in the same plane as they start away from the boiler. The fittings on all main piping and branches must be of the long-sweep pattern, and all pipe should be carefully reamed to remove burrs and sharp edges. Where the same riser supplies radiators on two or more floors the branches to the radiators on the intermediate floors may be connected with special tees (Fig. 46) known as O. S. FITTINGS, with a deflector arranged to divert the current of flow into the outlet of the tee, and thus favor the radiators on the intermediate or lower floors. By using TOP-FLOW and BOTTOM-RETURN CONNECTIONS at each radiator it is possible to positively control each unit by a single valve, except for the slight circulation intended to prevent freezing, which takes place through the $\frac{1}{16}$-in-diameter hole drilled in the valve-disc or sleeve, when the valve is closed. If both connections are made at the bottom tappings, and only one valve is used, it is entirely possible that the radiator may still be supplied with hot water through the unvalved connection even when the valve is closed.

Air-Removal in Hot-Water Systems. Suitable provision must be made for the removal of air from all hot-water radiators, wherever an upfeed system is installed. Usually small air-cocks are attached to the highest point of each radiator and are periodically opened to relieve any accumulation of air. If these cocks are forgotten a radiator may become air-bound and fail to heat because of faulty circulation; hence automatic air-valves are sometimes installed for this purpose. The automatic air-valve for hot-water radiators is not very generally used, due to its liability to pass water as well as air; but a standard type, made by the Monash-Younker Company, may be mentioned.

Expansion-Tanks for Hot-Water Heating. Open-Tank Systems. The low-pressure system of hot-water heating is not a closed system, as provision must be made for expansion and contraction of the water within the system. An open tank is provided at a suitable elevation, not less than 3 ft above the highest radiator, and connection made to the nearest return-riser; or preferably a separate expansion-line is run to the flow or return-main in the basement. The SIZE OF THE EXPANSION-TANK varies with the amount of water in the system, and also with the range in temperature of same, and its capacity is determined as follows:

The increase in volume of a given weight of water heated from 32° to 212° is about $\frac{1}{23}$, or approximately 4.33%; so that for every 23 gal in the system at 32°, an allowance of 1 gal must be made in the expansion-tank when the water in the system is raised to 212°. Cast-iron radiators have an internal volume of $1\frac{1}{2}$ pints per sq ft, while steel radiators and 1-in pipe hold about 1 pint per sq ft. Assuming the internal volume of the radiators to be about 50% of the entire system, we have for 3 000 sq ft of actual radiation, $3000 \times 2 \times \frac{1}{8}$ gal $= 750$ gal of water. This water will increase $\frac{1}{23} \times 750 = 33$ gal on being heated from 32° to 212°. Hence an expansion-tank of $2 \times 33 = 66$-gal capacity is necessary, the tank being made double the theoretical volume for practical considerations.

A list of expansion-tank capacities and dimensions is given in a table (included with Figs. 47 to 51), from which a commercial tank may be readily selected for systems under 6 000 sq ft. For larger systems the size of tank should be separately determined and the nearest commercial tank-size, as taken from the manufacturer's list, should be specified. These tanks should have 1 or $1\frac{1}{4}$-in top and bottom tappings with $\frac{1}{2}$-in water gauge tappings, for connecting a gauge-glass, at least 12-in long, on the side of the tank as shown in Fig. 43. The tank must be securely supported well above the highest radiator in the system, and in the larger installations special framing must often be

designed to carry the weight of tank and water. Automatic expansion-tanks equipped with a ball-cock and overflow are sometimes installed, and the altitude-gauge on boiler, and the gauge-glass and fittings on tank omitted. These tanks may be covered with hardwood and varnished if it is necessary to place them in a finished room or apartment.

Expansion-Tank Connections. The most approved method of connecting an expansion-tank to a low-pressure one-pipe system is shown in Fig. 48, where an expansion-and-vent line is run from the top of the main, at the high point just above the boiler, and connected to a return-bend just beneath the tank. A return circulating-line is taken from the other side of this bend and connected with the return-main at the boiler. The circulation of water in this loop will prevent freezing at the tank. From the top of the tank a 1¼-in vent-line is taken through the roof, and a 1¼-in overflow is taken out of this vent-line at a tee just above the tank. This overflow should discharge into an open sink or drain near the boiler so that it will be immediately evident to a person in the boiler-room, filling the system, just when the water has risen to the overflow above the tank. The movable hand on the boiler altitude-gauge can then be set to correspond with the middle of the gauge-glass, and the water-level brought to this point with the system cold. No valves should ever be installed on either the expansion or the overflow-lines, and in case the system is valved at the boiler the expansion-line must be connected on the boiler-side of this valve; and where two boilers are installed this line must be carried to a point above the waterline in the expansion-tank to prevent siphoning the water out of the entire system in case it is necessary to drain only one boiler. Expansion or vent-pipe connections must always be so made to main-piping in basement so that all air will be automatically removed from high points. Wherever possible risers or branches to risers may be used for relieving any accumulation of air in the main-piping. In SMALL INSTALLATIONS the expansion-line may be connected to the return-riser of one of the highest radiators, and no overflow other than the vent need be provided for, as shown in Fig. 51. This is a cheap method, and should not be resorted to unless extreme economy must be practised. The tank must be in the same room with the radiator to prevent freezing, as no circulation is provided for, and the overflow is simply discharged out of doors and usually upon the roof. The usual result is that an unsightly appearance is soon created.

The United States Treasury Department employs a special vent and overflow connection (Fig. 47) in cold climates, where there is liability of the vent-line freezing up if run out through the roof, due to the condensation and freezing of vapor passing out through this line. The vent-line is made only 2 ft high above the tank so that it is kept within the building, and it is equipped with a check-valve to prevent the escape of water through the same in case the tank should suddenly overflow. The closing of the check-valve will compel the excess water to pass down the overflow, and prevent the flooding of the building.

Closed-Tank Systems. The PERMISSIBLE TEMPERATURES in any hot-water system are limited by the pressure on the system, which latter factor determines the point at which boiling will take place. The pressure at any elevation in an OPEN-TANK hot-water system will vary directly with the distance below the level of the water in the expansion-tank, and hence it will be possible theoretically to carry the water in the boiler at a temperature corresponding to the hydrostatic pressure at the boiler before boiling would occur. The relation between hydrostatic head, pressure and boiling-point are given in the following table:

Table XXXII. Relation between Hydrostatic Head, Pressure and Boiling-Point

Hydrostatic head in feet.	0	12	24	37	49	61	74	87	100	113	125
Pressure in pounds per square in.....	0	5	10	15	20	25	30	35	40	45	50
Boiling-point......	212	227	239	250	259	268	274	281	287	292	298

Practically it would be quite impossible to carry temperatures in excess of 212° in any part of an open-tank system, as the high-temperature water would immediately rise into the open tank and boil. In order to overcome the limitations of the open-tank system, in which water will always boil as soon as a temperature of 212° F. is reached, various means of increasing the pressure in these systems have been resorted to in the attempt to carry a higher water-temperature in the radiators in very cold weather than would be possible with an open-tank system. These devices have usually been installed on the expansion-line, either at the boiler or else just below the expansion-tank and the static head increased by interposing a column of mercury in the path of the expanding water as it flows into the expansion-tank.

A common form of the apparatus, known as the HONEYWELL HEAT-GENERATOR, is shown in Fig. 49, in which it is seen that water entering the generator from the system will force the mercury up the inner tube *A* until a head of 20 in or 10 lb is established, at which time the entrance to this tube will be uncovered by the mercury and water or air may enter it and pass to the expansion-tank. Any excess of mercury above that required to just fill tube *A* is returned by tube *B* to the reservoir in the base. When the system cools off water can flow back down tube *A* as soon as the mercury-column drops in it, and the slight head of mercury then existing at the outlet of this tube is easily overcome by the head of water in the expansion-tank above this point. This increase of 10 lb in static pressure makes it possible to carry a maximum water-temperature of 240°, nearly 30° higher than would be possible in an open-tank system. While a temperature as high as this could theoretically be carried AT THE BOILER in an open-tank system with a static head of 24 ft, just as soon as this water rose in the system it would boil, and escape from the expansion-tank, at the same time emptying the system of water. In fact with the open-tank system the water is liable to be driven out at a temperature of 212° F. The use of pressure-generators similar to the above makes it possible to use smaller radiators in the heated rooms, as it is entirely possible to maintain steam-temperatures in the radiators whenever desired. Since higher temperatures are used, the difference between

Table XXXIII. Riser-Sizes for Honeywell System

Pipe-size inches	Capacity in square feet of hot-water radiation		
	1st floor	2nd floor	3rd floor
½	30	40	50
¾	75	100	125
1	75 up	100 up	125 up

flow and return-riser temperatures becomes greater than in the open-tank system, and hence a greater motive head exists and smaller mains and risers may be used with this system. The HONEYWELL COMPANY recommends the schedule of radiator-tappings shown in Table XXXIII.

It should be remembered that since radiators and pipes are smaller in this system there is much less water than in the open-tank system, making it more sensitive in warming up and also in cooling off. The GENERATOR should not be placed close under the expansion-tank. Otherwise than this its location may be anywhere in the expansion-line, as the same hydrostatic head is always acting in addition to the head of mercury-column.

Warm-Air Furnace Heating

The Furnace and Its Location. The method of warming or heating a building by what is generally known as a warm-air furnace is termed FURNACE HEATING. The furnace may be of a cast-iron or steel construction or a com-

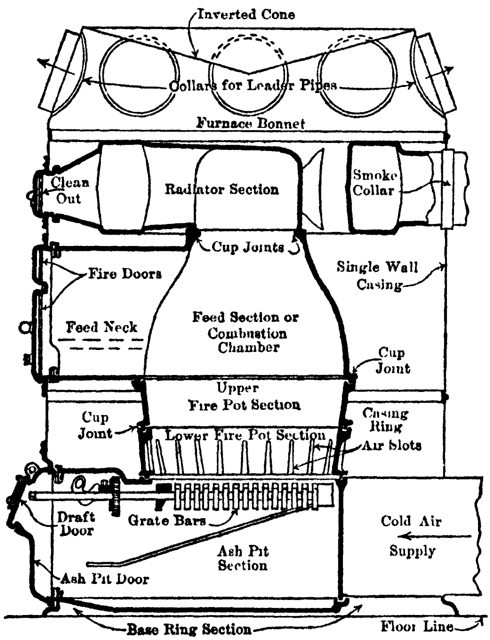


Fig. 52. Section of a Majestic Cast-iron Circular Radiator Furnace and Casing

bination of the two (Figs. 52 and 53), containing a COMBUSTION-CHAMBER, FIRE-POT and GRATE. Furnaces for soft coal are usually designed with a secondary air-supply or OVERDRAFT for admitting heated air just at the surface of the fire in order to produce a more perfect combustion of the volatile combustible gases which are liberated from this fuel immediately after firing.

This overdraft should be under positive control so that it may be checked or closed after the fuel has been coked. Soft coal may also be burned efficiently in the underfeed-type of furnace in which coal is fed from below by means of a plunger operating in a feed-chute discharging through the center of the grate. The furnace should be located in the basement in an approximately central position with reference to the rooms to be heated, and preferably toward the side or sides from which the prevailing winds blow in the wintertime. This arrangement not only favors the more exposed rooms on the floors above by shortening the leaders to these rooms, but also makes it possible to reduce the length of the cold-air duct, if one is employed, which should be run from the exposed side of the building to the cold-air pit below the furnace. Usually, however, the air is all recirculated, the recirculation register being located in the hall as shown by Figs 54 and 67, passed through the space between

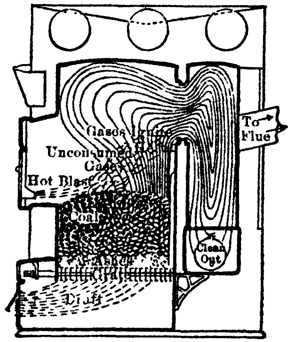


Fig. 53. Section of a Steel Furnace with Radiator and Casing

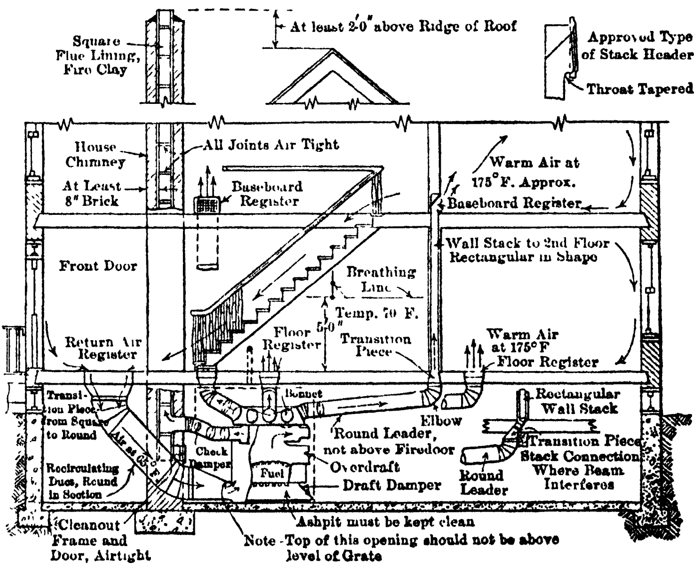


Fig. 54. Section through a Residence Showing Details of a Warm-air Heating System (See Fig. 68 for plan)

the heater and its jacket, and warmed by coming in contact with the outside heated surface of the combustion-chamber and the radiator, which is

usually just above the combustion-chamber. It is then discharged through flues connected at the top of the JACKET, FURNACE CAP or BONNET to the rooms to be warmed.

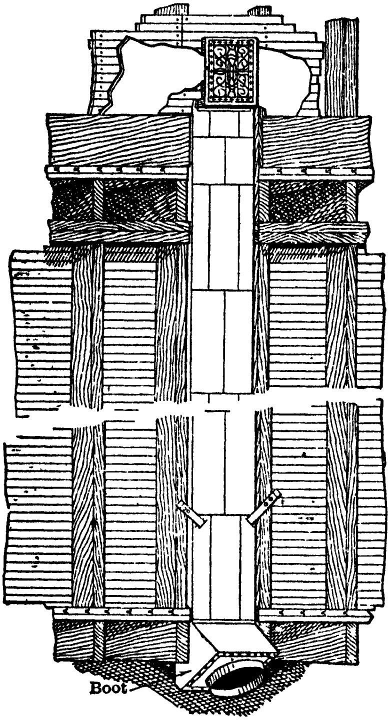


Fig. 55. Vertical Stack with Side-wall Register

Leaders and Stacks. These connecting flues are made up of two sections, (1) the nearly horizontal round pipes in the basement, known as LEADERS (Figs. 58 and 59), which connect to the COLLARS on the top or conical sides of the bonnet, and (2) the vertical rectangular pipes called STACKS (Fig. 55), which connect the BOOT at the outer end of the leader, with the double-walled REGISTER-BOX (Fig. 56) into which the REGISTER-GRILLE covering the opening into the room, is fitted. The leaders should have an upward pitch toward the base of the stack of at least 1 in per foot, and for the best results they should not be more than from 12 to 15 ft in length. The boots are made in a great variety of shapes to suit actual conditions, and are simply ADAPTERS for the purpose of changing from round leaders to rectangular stacks. The stacks are usually run between the studding of interior walls or partitions (Figs. 55 and 56), since if they are placed in outside walls the cooling effect reduces their efficiency not only in temperature of air, but also in velocity of flow. The METAL used for leaders and stacks is usually bright IX tin, although for leaders larger than 12 in, galvanized steel of No. 26 United States Standard gauge is usually employed. Since the stacks must frequently run in a 4-in stud-

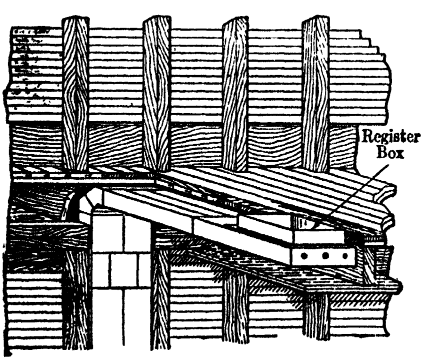


Fig. 56. Vertical Stack and Register-box

ding-space, with a net depth of about 3¾ in, every effort must be made to keep them as deep as possible; and steel lathing or expanded metal should be used in front of all such stacks, which ordinarily have only a single layer of asbestos-paper covering. A more effective insulation may be provided

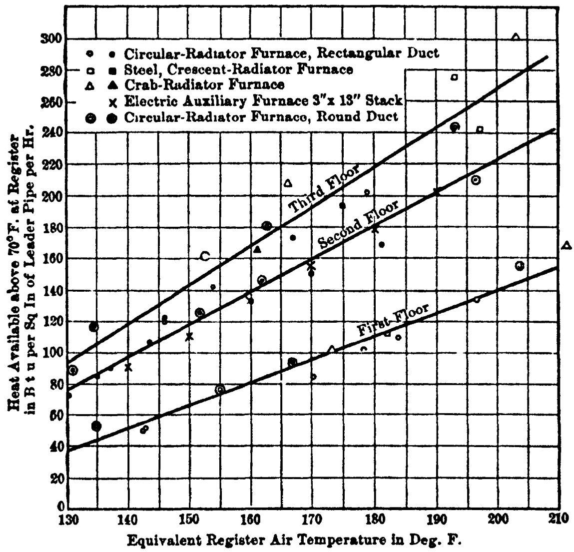


Fig. 57. Value of Square Inch of Leader Pipe Area for First, Second and Third Floors

by using a DOUBLE-WALL STACK, in which there is an air-space between the inside and outside pipes, and no asbestos covering is used. Attention of the architect is here called to the fact that in the case of large second-floor rooms to be warmed by one register, 6-in stud partitions are generally required for the first floor.

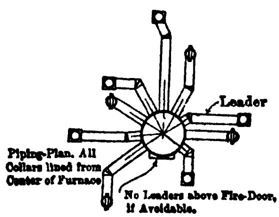


Fig. 58. Warm-air Furnace-leaders with Elbows

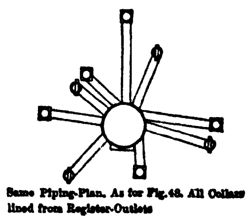


Fig. 59. Warm-air Furnace-leaders without Elbows

Commercial Ratings of Furnaces. Manufacturers formerly rated their furnaces according to the amount of space, cubical contents, in the ordinary residence-construction they will heat to 70° F. in zero weather. The actual

size of the furnace naturally depends upon the heat-transmission of the walls, floors and roofs, plus the infiltration-losses, as already explained. The claim, however, is made that these "in turn bear a reasonably uniform relation to the cubical contents of the ordinary house," with the usual proportions and ratios of wall to glass-surface, and that therefore the rating, as given, is justifiable. Tables XXXIV and XXXV were taken from the Warm Air Furnace Hand-

Table XXXIV. Approximate Capacity of Warm-Air Furnaces for Residences of Ordinary Construction in Cubic Feet of Space Heated

Divided space in cubic feet			Fire-pot		Undivided space in cubic feet		
+10°	0°	−10°	Diameter, in	Area, sq ft	+10°	0°	−10°
12 000	10 000	8 000	18	1.8	17 000	14 000	12 000
14 000	12 000	10 000	20	2.2	22 000	17 000	14 000
17 000	14 000	12 000	22	2.6	26 000	22 000	17 000
22 000	18 000	14 000	24	3.1	30 000	26 000	22 000
26 000	22 000	18 000	26	3.7	35 000	30 000	26 000
30 000	26 000	22 000	28	4.3	40 000	35 000	30 000
35 000	30 000	26 000	30	4.9	50 000	40 000	35 000

Table XXXV. Air-Heating Capacity of Warm-Air Furnaces Based on Total Cross-Section of Leader Pipes

Fire-pot		Casing*	Total cross-sectional area of leader pipes, a sq in	Number and size of leader pipes that may be supplied
Diameter, in	Area, sq ft	Diameter, in		
18	1.8	30–32	180	3–9″ or 4–8″
20	2.2	34–36	280	2–10″ and 2–9″ 2–9″ and 2–8″
22	2.6	36–40	360	3–10″ and 2–9″ 4–9″ and 2–8″
24	3.1	40–44	470	3–10″ and 1–9″ 2–10″ and 5–8″
26	3.7	44–50	565	5–10″, 3–9″ 3–10″, 4–9″ and 2–8″
28	4.3	48–56	650	2–12″, 3–10″ and 3–9″ 5–10″, 3–9″ and 2–8″
30	4.9	52–60	730	3–12″, 3–10″ and 3–8″ 5–10″, 5–9″ and 1–8″

* The casing-diameter should be such that the minimum cross-sectional area M, between casing and radiator, will be at least 20% greater than the total cross-sectional area of all the heat-pipes, a, or $M = 1.2 \times a$ sq in.

book, published by the Federal Furnace League, an association of United States furnace-manufacturers. This association is no longer in existence. If the majority of the basement or leader-pipes exceed 12 ft in length or have less

than 1 in rise to the foot, or if more than one sixth of the outside surface of the building is glass, then the furnace should be increased one or more sizes. The size of the furnace required is now quite generally determined by the combined area of the cross-sections of the warm-air pipes. Tables XXXIV and XXXV are intended to be employed as only a rough approximation. Furnaces should always be designed on the basis of the heat-loss calculations as given in the text following.

The Design of a Gravity Warm-Air Furnace Heating-System

Recommended Procedure to be Followed. The rational design of a furnace heating-system involves the determination of the following items:

(*a*) Heat loss in Btu per hour from each room in the building.

(*b*) Area and diameter in inches of warm-air pipes in basement known as leaders.

(*c*) Area and dimensions in inches of vertical wall pipes known as stacks.

(*d*) Free and gross area and dimensions in inches of warm-air registers.

(*e*) Area and dimensions of (1) recirculating or (2) outside air supply-ducts in inches. There may be one or more of each.

(*f*) Free and gross area and dimensions in inches of recirculating registers.

(*g*) Size of furnace necessary to supply the warm air required to overcome the heat loss from the building. This size should include square inches of leader pipe area which the furnace must supply. It is also desirable to specify a minimum bottom fire-pot diameter in inches, which is the nominal grate-diameter in the case of furnaces burning solid fuel. The kind of fuel and its approximate heat value should be known.

(*h*) Area and dimensions in inches of chimney-lining and smoke-pipe. If an unlined chimney is to be used, that fact should be made clear. The height of chimney must be known as it may affect the area, and chimneys under 30 ft in height may prove unsatisfactory.

(a) Heat Losses from Building. The heat which will be required for each room in the building depends on (1) the HEAT-TRANSMISSION LOSSES through walls and glass as well as through floors and ceilings when the latter two are next to unheated spaces, and (2) the INFILTRATION OF COLD AIR through the cracks around outside windows and doors. Calculations for the heat required in Btu per hour should be made as previously indicated.

(b) Leader Sizes. A building is heated by a furnace heating-system with warm air introduced into the various rooms at some temperature higher than that to be maintained at the breathing-level, which is usually 70° F. If the difference between the entering-air temperature and the room-temperature is small, the operating head will be small and large pipes and registers will be needed, but if this difference is large the operating head will be increased almost proportionally to the new temperature difference, and smaller pipes and registers than in the first case may be used. As a result, in a gravity circulating warm-air furnace system, the size of the leader to a given room depends on the temperature of the warm air entering the room at the register. A reasonable air temperature at the registers must therefore be agreed upon before the system can be designed. The National Warm Air Heating Association has approved as satisfactory an average air temperature of 175° F. at the registers. At this temperature the heat-carrying capacity (heat available above 70° F.) per square inch of leader pipe per hour for first, second or third floors is shown by Fig. 57 at 175° F. to be 104, 172 and 208 Btu respec-

tively, based on tests at the University of Illinois. For average calculations,* the values 111, 167 and 200 will simplify the work and may be satisfactorily substituted for these heat-carrying capacities. If H represents the total heat to be supplied to any room, the resulting equations are:

$$\text{Leader areas for first floor, square inches} = \frac{H}{111} = \text{approximately } 0.009\,H \quad (1)$$

$$\text{Leader areas for second floor, square inches} = \frac{H}{167} = \text{approximately } 0.006\,H \quad (2)$$

$$\text{Leader areas for third floor, square inches} = \frac{H}{200} = \text{approximately } 0.005\,H \quad (3)$$

If it is desired to design for a lower warm-air register temperature, say 160° F., the original values 104, 172 and 208 become 82, 140 and 168 (Fig. 57 at 160° F.), and the resulting equations are:

$$\text{Leader areas for first floor, square inches} = \frac{H}{82} = \text{approximately } 0.012\,H \quad (4)$$

$$\text{Leader areas for second floor, square inches} = \frac{H}{140} = \text{approximately } 0.007\,H \quad (5)$$

$$\text{Leader areas for third floor, square inches} = \frac{H}{168} = \text{approximately } 0.006\,H \quad (6)$$

These equations are applicable to straight leaders from 6 to 8 ft in length. Longer leaders must be very thoroughly covered or else the vertical wall stacks must be increased in area above approximately 75% of leader area, as discussed under Stack Sizes. If some provision is not made for these longer leaders, the register air temperature may be much lower than anticipated and the room may not be properly heated.

While Fig. 57 takes care of the drop of temperature in straight leaders up to 8 ft in length connected to stacks having about 75% the area of the leader, the designer must make allowances for all other conditions.

In addition to the effect of leader length and register air temperature the drop in temperature between furnace bonnet and register face is affected by the stack-leader ratio and the stack construction, whether single or double wall, for various stacks. The value of high stack-leader ratios and double-wall pipe to upper floors is very apparent in its effect on reducing the temperature drop between bonnet and register face.

Leader sizes should in general be not less than obtained by Equations (1) to (3), nor should leaders less than 8 in in diameter be used. It is not considered good commercial practice to specify diameters except in whole inches, although there is no real reason for not using half inches if necessary. The tops of leaders should be at the same elevation as they leave the furnace bonnet, and from this point there should be a uniform up-grade of 1 in per foot of run in all cases. Leaders over 12 ft in length are to be avoided or receive very special attention.

The leader capacity curves already described under Leader Sizes are based on stack-leader ratios of approximately 70%, but frequently the designer will find it desirable or necessary to use larger or smaller ratios and hence Figs. 60

* See the Standard Code Regulating the Installation of Warm-Air Heating Systems in Residences.

and 61 showing the performance of a great variety of stack-leader combinations for 8-in and 10-in leaders have been given for second-floor systems.

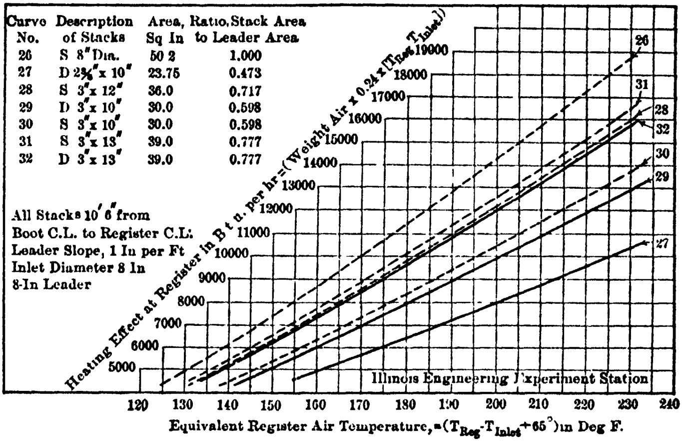


Fig. 60. Heating Effect at Registers for Various Stacks with 8-in Leader

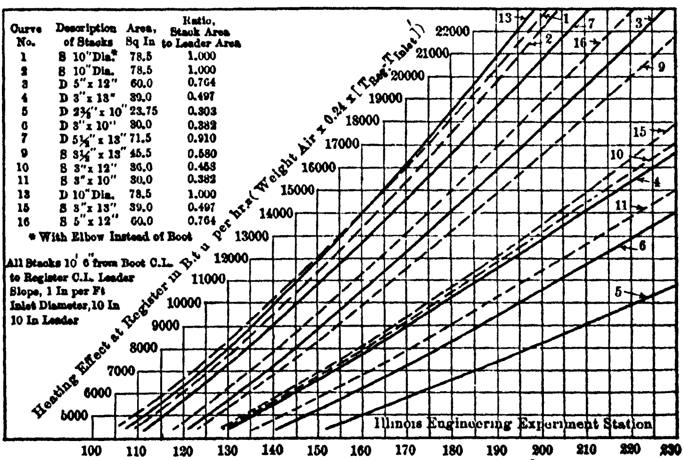


Fig. 61. Heating Effect at Registers for Various Stacks with 10-in Leader

The leaders may be laid off (Fig. 58) by dividing up the circumference of the bonnet into areas proportional to the amount of air to be distributed by each leader, and then connecting collar and leader radially to furnace-cap,

making one or more elbows in the leader, if necessary, to connect with stack. Another method is to run practically all leaders direct from the furnace to the foot of stack (Fig. 59) and cut the collars in on the angles at which they intersect the casing. The former method is recommended, and requires less skill and special fitting of leaders in installation.

The basement heating plan (Fig. 68) for an actual installation is shown with radial leader connections to the bonnet.

(c) **Stack Sizes.** The wall pipe or stack for an upper floor should be made not less than 70% of the area of the leader. So long as the leader is short and straight, as was the case for Fig. 57, such a practice is probably justified since the loss in capacity occasioned by the smaller stack is not very serious for ratios above 70%. For leaders over 8 ft in length or for leaders which are not straight, the ratio of stack-area to leader-area should be greater than 70% in order to offset the greater temperature losses in the longer leader. In gravity circulating systems, this STACK-LEADER RATIO is a very important consideration. Specific data for a variety of cases are presented in Figs. 60 and 61, and the designer should check his stack-leader combinations with the nearest comparable case as shown in these figures. Any second-floor stack supplying heat to a room requiring 10 000 Btu per hour will need a 6-in studding space at a register air temperature not in excess of 175° F. The desirability of using uniformly high stack-leader ratios throughout the installation cannot be too strongly recommended, since if a uniform stack-leader ratio is not maintained the temperature losses for the smaller ratios will be excessive and the rooms served by them will be underheated. See Tables XXXVII and XXXVIII for commercial sizes of stacks.

(d) **Warm-Air Register Sizes** (Figs. 62 and 63). After the sizes and locations of all leaders and stacks have been determined, it is a simple matter to

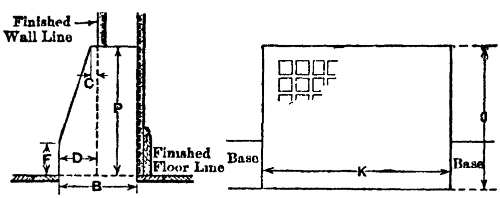


Fig. 62. Baseboard Stackhead and Register Dimensions (See Table XLI)

select suitable types and sizes of warm-air registers, from tables of commercial types of registers. See Tables XXXIX to XLI.

The registers used for discharging warm air into the rooms should have a FREE OR NET AREA not less than the area of the leader in the same run of piping. The free area should be at least 70% of the gross area of the register. No upper-floor register should be wider horizontally than the wall stack, and it should be placed either in the baseboard or side-wall, and not in the floor. First-floor registers may be of the baseboard or floor type with the former location preferred. No first-floor register should require a register-box more than 14 in wide, although it may be longer than 14 in.

All warm-air registers should be located in or close to inside partitions as attempts to run stacks in cold outside walls or extend long, first-floor leader runs to cold walls almost always result in failure. Registers should, of course, be located so as to interfere as little as possible with rugs, furniture, and

passageways, which means they must be kept out of the floor. Experimental data indicate that, in general, the BASEBOARD location gives better heating effects for both first- and second-floor registers, although the particular combinations of fittings used in entering the register-box has a marked effect on the relative merits of the two locations. The results of tests indicate that choice between floor and wall types of register combinations should favor that system which offers the least resistance due to elbows or angles.

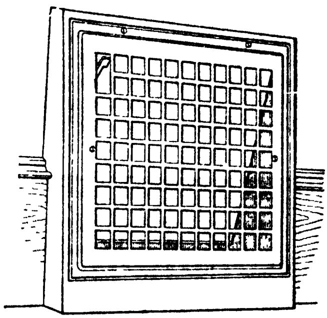


Fig. 63. Baseboard Register with Single Valve

NOTE: Baseboard register used on First Floor takes the supply from a flue 8⅝ in deep or 5 in deeper than the studding.

(e) **Air-Supply Duct Sizes** (Fig. 67). Ducts for recirculating air from the house or for bringing in outside air should be as short and direct as possible. The areas of such ducts should never be less than the combined areas of all warm-air leaders, and ducts of the recirculating type may be made even larger than the total leader-area. It is usually sufficient to base the size of either recirculating or outside-air ducts on leader-area, although it is also desirable to check this size against velocity of flow which should not exceed 4 ft per second in short straight ducts and 3 ft per second or less in long ducts or ducts with several turns. The air-volume and velocity should be based on density of air at 65° F.

The superior performance of the round duct using two 45° instead of two 90° elbows is very apparent, and the leader capacity curves (Fig. 57) used for designing a gravity furnace system apply to such an installation.

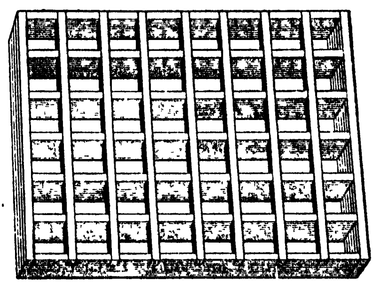


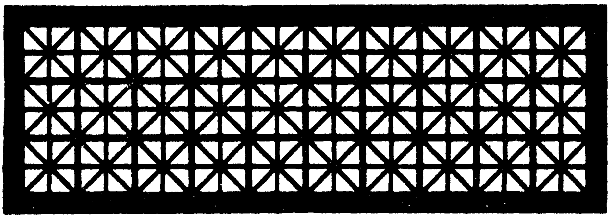
Fig. 64. Wooden Cold-Air Face.—Floor Type

Outside-air duct connections, if used, should be made to a window-frame the full area of duct and such window should be in a wall exposed to prevailing winter winds. The inside type of recirculating duct or ducts is always preferred for residence installations.

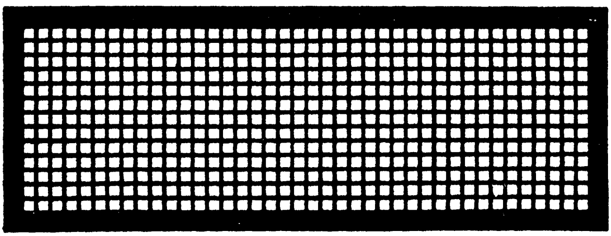
(f) **Recirculating Register Sizes.** (Fig. 64, Table XLII.) The number and location of recirculating registers, as in the case of the recirculating ducts, must be considered in arriving at the sizes, although it is usually quite sufficient to make the free area of such registers equal to the duct-area. If a single recirculating connection is used, the register through which the air in the building is returned to the furnace should always be placed in a central position in the first floor, usually in the main hall if one exists. Air from the upper floors must have free access to this register through the stairway of the building. Sometimes more than one return-air register is found desirable, and such multiple

returns are often justified provided the total cross-sectional area of ducts and registers is increased materially over that for a single, return-duct system.

(g) **Furnace Size.** The furnace should be selected on the basis of warm-air operation so that it will never be necessary to supply air at the register faces at a temperature above 175° F. even on the very coldest day. Furnaces at present are rated on the basis of the number of square inches of basement leader-pipe which they will supply. This rated area should always be approximately equal to the sum of the areas of all the warm-air leaders to which the furnace is connected. It is therefore possible simply to add up the areas of the basement leader-pipes as determined by the factors taken from Fig. 57 at a register air temperature of 175° F., and then select a furnace having not less than this rated capacity in square inches of leader-pipe. (See Tables



Grecian 3 In.—47% Open Area



Square—55% Open Area

Fig. 65. Two Types of Metal Grills

XLIV and XLV.) It is unwise to increase this value by taking the next size larger furnace.

Selecting a furnace on the basis of LEADER-PIPE AREA alone is a quick and simple procedure, but most designers will insist on a method for further checking the selection. In the investigation of furnace performance at the University of Illinois, it was found that, for any given type of furnace having a constant heating surface-grate surface ratio, the grate-area required to provide a given heating capacity depends on several factors. First, the AIR TEMPERATURE at the register for which the plant has been designed must be determined. Usually this temperature is taken as 175° F. Second, the COMBUSTION-RATE must always correspond with the register air temperature.

For 175° register warm-air temperature the combustion rate is 7.5 lb of coal per sq ft of grate per hour. The third factor is the EFFICIENCY, which is

about 57.5% and must correspond to the first two factors. The fourth factor is the HEAT VALUE per pound of fuel burned, which was 12 790 Btu.

Let H_b = heat available at bonnet in Btu per hour;
H_h = actual total heat loss from house in Btu per hour;
1.33 = provision for a 33% heat-loss between furnace bonnet and register (in some cases, under very favorable conditions this value may be 1.25);
G = grate-area in square feet bottom firepot area;
E = efficiency of the furnace (0.55 to 0.62);
F = fuel value of the coal in Btu per pound (usual assumption 12 000 to 13 000);
C = pounds of coal burned per square foot of grate surface per hour (usual assumption 5 5 to 7 5).

Then
$$H_b = 1.33\, H_h = G \times C \times E \times F \tag{7}$$

The equation may be written to express GRATE-AREA direct as,

$$G = \frac{1.33\, H_h}{C \times E \times F} \tag{8}$$

Assuming $E = 0.575$, $F = 12\,800$ and $C = 7.5$, then
$$G = H_h \div 41\,500 \tag{9}$$

Having found the desired grate-area, it is necessary to select a commercial furnace having the corresponding grate and casing diameters as given by manufacturers' data. The leader-area as calculated should also be again checked against the makers' data.

Suppose it is desired to select a furnace to deliver air to the rooms at a register temperature approximating 160° rather than 175° F., the relation is —combustion-rate 5.5 lb, register air temperature 160° F., and efficiency of the furnace 62%.

(h) Chimney Sizes. The area and internal dimensions of the chimney may be taken from Table XVIII.

Rating a Warm-Air Furnace. It is also possible on the basis of the performance data for leaders and stacks, and Equations (1) and (4), to set up a RATING EQUATION OF FORMULA for any common type of warm-air furnace. Simply divide the heat available at the registers by the average leader carrying-capacity based on a two-story installation.

Thus, from Fig. 57, at 175° F. register air temperature the average leader capacity per square inch is $(104 + 172) \div 2 = 138$ Btu. Hence the rating of a furnace (A) in square inches of leader pipe is, under the conditions itemized below (see Equation (7) for literal values):

$$A = \frac{0.75\, H_b}{138} = \frac{0.75\, G \times C \times E \times F}{138}\,[1 + 0.02\,(R - 20)] \tag{10}$$

and substituting the same values for C, E and F as in Equation (7) for a register air temperature of 175° F., the rating of a furnace in square inches of leader pipe is:

$$A = 300 \times G\,[1 + 0.02\,(R - 20)] \tag{11}$$

Note that $300 = \dfrac{7.5 \times 0.575 \times 12\,800 \times 0.75}{138}$, and in case G is in square inches instead of square feet then

$$A = 2.08\, G\,[1 + 0.02\,(R - 20)] \tag{12}$$

Equations (11) and (12) apply to commercial furnaces when operating under the following conditions:

(1) Fuel, stove-size anthracite coal of 12 800 Btu heat value per pound as fired.

(2) The chimney, 12 in in diameter and 35 ft high, and ample to create the draft necessary to burn 7.5 lb coal per square foot of grate per hour. On some tests, this chimney supplied sufficient intensity of draft to burn over 13 lb of coal per square foot of grate per hour.

(3) The cold-air duct, of the recirculating type with large flat shoe and a single register at first-floor level.

(4) Casing, of best diameter for each furnace tested, with a black iron liner and 1-in air space. The bonnet, conical, without radiation shield.

(5) Leaders, stacks, and registers supply air to three floor-levels.

Table XXXVI.—Sizes of Commercial Round Pipe-Leaders and Ducts, 2-Piece 30° and 45° Angles, 3-Piece 60° Angles, and 4-Piece 90° Angles, of IC, and IX Tin and 26 and 28 United States Standard Gauge, Diameter, Inches

5	6	7	8	9	10	12	14	16	18	20	22	24	26	28	30	32	34	36

Table XXXVII. Sizes of Single-Wall Pipe and Fittings of IC, and IX Tin and 26 and 28 United States Standard Gauge, Inches

Pipe.....	3×10	3⅝×10	3×12	3⅝×12	3×13	4⅜×13	5×13⅝
Size of collar in boots..	8	9	8 and 9	9	10	10	12

Table XXXVIII. Sizes of Double-Wall Pipe and Fittings of IC and IX Tin, Inches

Inside..	2⅝×10	3 ×10	2⅝×12	3 ×12
Outside.......	3¼×10⅝	3⅝×10⅝	3¼×12⅝	3⅝×12⅝
Size of collar in boots.....	8–9	8–9	8–9	8–9–10
Inside..	3 ×13	4⅜×13	4⅜×14	.....
Outside.....	3⅝×12⅝	5 ×13⅝	5 ×14⅝	....
Size of collar in boots......	9–10	10–12	10–12	....

Note. The commercial wall pipe sizes are in general much too small to develop the full carrying capacities of leaders to fit the size of collar in boots. In only a few cases is the wall pipe area equal to 75% of the collar or leader-area.

Standard Register Sizes. The commercial sizes of floor, wall and ceiling registers range from 4 in × 6 in to 38 in × 42 in, varying by even inches with a few odd sizes.

In order to simplify and standardize the selection of warm-air registers for the average residence installation, a Joint Committee on Standardization of Warm Air Registers, composed of register manufacturers, made the following recommendation at a meeting on September 4, 1923.

Table XXXIX. Standard Register Sizes for Warm-Air Furnace Heating, Inches

Leader pipe diameter	Register size	Stack head, outside dimensions	Base extension
First-floor baseboard registers*			
8	8×10	6⅛×10⅝	2¼
9	9×12	6⅛×12⅝	2¼
10	10×12	7⅛×12⅝	3¼
12	11×13	9⅛×13⅝	5¼
12	12×14	9⅛×14⅝	5¼
Second-floor baseboard registers			
8	8×10		Min 1 in, max 1¼ in
8	8×10	.	Min 1 in, max. 1¼ in
9	9×12		Min 1 in, max 1¼ in
Floor registers			
8	8×10		.
9	9×12	.	. . . . .
10	10×12		. . . . . .
12	12×14		. . . . .
14	14×18		. . . .
16	16×22		. . . .

* No recommendation for 14-in diameter pipe Bottom of register from 2 to 2¼ in above finished floor.

The use of a FLOOR REGISTER may be permitted in an entrance hall for drying shoes and garments, but it is most unsanitary and cannot fail to collect dirt and filth of all kinds, and is subject to the further objection that its use frequently requires the cutting of carpets. In case such registers are used, however, suitable register-boxes must be provided, and are preferably constructed with double walls and tapered from the round collar to the top of the rectangular box.

Table XL. Dimensions of Register-Boxes, Inches

6×8	9×12	12×14	14×18	16×18	16×30	18×30	20×30	24×27	27×38	36×36
8×10	10×12	12×15	14×20	16×20	18×21	20×24	21×29	24×30	30×30	38×42
8×12	10×14	14×16	14×30	16×24	18×24	20×26	24×24	27×27	30×36	..

Registers with a high percentage of FREE AREA are desirable, since smaller commercial units may be used, and provide the same free area as larger sizes. The frictional losses through the register grilles are an insignificant item in any gravity circulating warm-air furnace plant, provided the registers have a free area above 70% of gross area and not less than leader or return-duct area. All registers on the warm-air side of the system should have SHUT-OFF VALVES or LOUVERS for controlling air-flow.

Table XLI. Dimensions of Stackheads and Baseboard Registers Inches (See Fig. 62)

(Tuttle & Bailey Manufacturing Company)

Depth of Head, *B*	Use With Register Number		Arrangement	Stackhead Dimensions *C*	Stackhead Dimensions *D*	Stackhead Dimensions *F*	Stackhead Dimensions *P*	Register Dimensions Outside Height *J*	Register Dimensions Outside Width *K*
3 5/8	3 5/8	8×10	ONE REGISTER	1	1	1 1/4	9 1/4	10 3/4	13
3 5/8	3 5/8	8×12	ONE REGISTER	1	1	1 1/4	9 1/4	10 3/4	15
3 5/8	3 5/8	8×13	ONE REGISTER	1	1	1 1/4	9 1/4	10 3/4	16
3 5/8	3 5/8	9×12	ONE REGISTER	1	1	1 1/4	10	11 1/2	15
5 5/8	5 5/8	8×10	ONE REGISTER	1	1 1/8	2	10	11 1/2	13
5 5/8	5 5/8	8×12	ONE REGISTER	1	1 1/8	2	10	11 1/2	15
5 5/8	5 5/8	9×12	ONE REGISTER	1	1 1/8	2	11	12 1/2	15
6 1/8	6 1/8	8×10	ONE REGISTER	1	1 7/8	2	10	11 1/2	13
6 1/8	6 1/8	8×12	ONE REGISTER	1	1 7/8	2	10	11 1/2	15
6 1/8	6 1/8	8×13	ONE REGISTER	1	1 7/8	2	10	11 1/2	16
6 1/8	6 1/8	9×12	ONE REGISTER	1	1 7/8	2	11	12 1/2	15
6 1/8	6 1/8	10×12	ONE REGISTER	1 1/4	1 7/8	2	12	13 1/2	15
6 1/8	6 1/8	10×13	ONE REGISTER	1 1/4	1 7/8	2	12	13 1/2	16
6 5/8	6 5/8	7×10	ONE REGISTER	1	2 3/8	2 3/4	9 1/2	11	13
6 5/8	6 5/8	7×12	ONE REGISTER	1	2 3/8	2 3/4	9 1/2	11	15
6 5/8	6 5/8	8×10	ONE REGISTER	1	2 3/8	2 3/4	10 1/2	12	13
6 5/8	6 5/8	8×12	ONE REGISTER	1	2 3/8	2 3/4	10 1/2	12	15
6 5/8	6 5/8	8×13	ONE REGISTER	1	2 3/8	2 3/4	10 1/2	12	16
6 5/8	6 5/8	9×12	ONE REGISTER	1	2 3/8	2 3/4	11 1/2	13	15
6 5/8	6 5/8	10×12	ONE REGISTER	1	2 3/8	2 3/4	12 1/2	14	15
6 5/8	6 5/8	10×13	ONE REGISTER	1	2 3/8	2 3/4	12 1/2	14	16
6 5/8	6 5/8	10×14	ONE REGISTER	1	2 3/8	2 3/4	12 1/2	14	17
7 1/8	7 1/8	12×13	ONE REGISTER	1	2 3/4	2 3/4	14 1/2	16	16
7 1/8	7 1/8	12×14	ONE REGISTER	1	2 3/4	2 3/4	14 1/2	16	17
8 5/8	8 5/8	10×12	ONE REGISTER	1	4 1/2	2 3/4	12	13 1/2	15
8 5/8	8 5/8	10×13	ONE REGISTER	1	4 1/2	2 3/4	12	13 1/2	16
8 5/8	8 5/8	12×13	ONE REGISTER	1	4 1/2	2 3/4	14 1/2	15 3/4	16
8 5/8	8 5/8	12×14	ONE REGISTER	1	4 1/2	2 3/4	14 1/2	15 3/4	17
8 5/8	8 5/8	12×15	ONE REGISTER	1	4 1/2	2 3/4	14 1/2	15 3/4	18
7 5/8	5 5/8	8×10	TWO REGISTERS FOR DOUBLE HEADER	1	1 1/8	2	10		
7 5/8	5 5/8	8×12	TWO REGISTERS FOR DOUBLE HEADER	1	1 1/8	2	10		
7 5/8	5 5/8	9×12	TWO REGISTERS FOR DOUBLE HEADER	1	1 1/8	2	11		
8 5/8	6 1/8	8×10	TWO REGISTERS FOR DOUBLE HEADER	1	1 7/8	2	10		
8 5/8	6 1/8	8×12	TWO REGISTERS FOR DOUBLE HEADER	1	1 7/8	2	10		
8 5/8	6 1/8	8×13	TWO REGISTERS FOR DOUBLE HEADER	1	1 7/8	2	10		
8 5/8	6 1/8	9×12	TWO REGISTERS FOR DOUBLE HEADER	1	1 7/8	2	11		
8 5/8	6 1/8	10×12	TWO REGISTERS FOR DOUBLE HEADER	1 1/4	1 7/8	2	12		
8 5/8	6 1/8	10×13	TWO REGISTERS FOR DOUBLE HEADER	1 1/4	1 7/8	2	12		
9 5/8	6 5/8	7×10	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	9 1/2		
9 5/8	6 5/8	7×12	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	9 1/2		
9 5/8	6 5/8	8×10	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	10 1/2		
9 5/8	6 5/8	8×12	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	10 1/2		
9 5/8	6 5/8	8×13	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	10 1/2		
9 5/8	6 5/8	9×12	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	11 1/2		
9 5/8	6 5/8	10×12	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	12 1/2		
9 5/8	6 5/8	10×13	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	12 1/2		
9 5/8	6 5/8	10×14	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/8	2 3/4	12 1/2		
10 5/8	7 1/8	12×13	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/4	2 3/4	14 1/2		
10 5/8	7 1/8	12×14	TWO REGISTERS FOR DOUBLE HEADER	1	2 3/4	2 3/4	14 1/2		
13 5/8	8 5/8	10×12	TWO REGISTERS FOR DOUBLE HEADER	1	4 1/2	2 3/4	12		
13 5/8	8 5/8	10×13	TWO REGISTERS FOR DOUBLE HEADER	1	4 1/2	2 3/4	12		
13 5/8	8 5/8	12×13	TWO REGISTERS FOR DOUBLE HEADER	1	4 1/2	2 3/4	14 1/2		
13 5/8	8 5/8	12×14	TWO REGISTERS FOR DOUBLE HEADER	1	4 1/2	2 3/4	14 1/2		
13 5/8	8 5/8	12×15	TWO REGISTERS FOR DOUBLE HEADER	1	4 1/2	2 3/4	14 1/2		

Warm-air registers may be placed in the floor, but preferably they should be located in or against inside partitions, for first-floor rooms. By using the modern BASEBOARD REGISTER it is usually possible to secure the required capacity without resorting to floor registers. These baseboard registers can be connected to a flue from 3 to 4½ in deeper (Table XLI) than the studding. This has been accomplished by making the special baseboard register so that it projects 2 in into the room at the floor line, necessitating the floor being cut out, and also utilizing the space of about 1 in occupied by the lath and plaster, or a total increase in depth of flue of about 3 in. For special conditions where ordinary baseboard register cannot be set in the partition, an OUT-O-WALL BASEBOARD TYPE may be used successfully. For upper-floor rooms registers should be placed in inside partition walls, using BASEBOARD or CONVEX REGISTERS for shallow stacks. As a general rule warm-air registers should be so placed as to shorten leader and stack connections as much as possible.

Table XLII. Dimensions of Wood Cold-Air Faces for Floors (Fig. 64)

Rock Island wood faces

Round pipe size, in	Face size, in	Free air opening, sq in	Round pipe size, in	Face size, in	Free air opening, sq in
12	12×20	135	22	20×36	403
12	14×16	133	22	24×30	403
14	12×24	161	22	28×28	439
14	14×18	151	24	22×36	440
14	14×20	157	24	22×40	484
14	16×20	179	24	24×36	484
16	12×30	202	26	30×30	504
16	14×24	188	28	30×36	605
16	14×26	203	30	36×36	725
16	14×30	220	Special narrow sizes		
16	16×24	215			
18	16×30	269	24	72×12	458
18	18×24	242	26	72×14	534
18	20×24	269	27	72×16	610
18	20×26	291	28	72×18	687
20	14×40	314	30	72×20	763
20	18×30	303	34	72×24	916
20	20×30	336	36	72×30	1 145
20	24×24	323	40	72×36	1 374

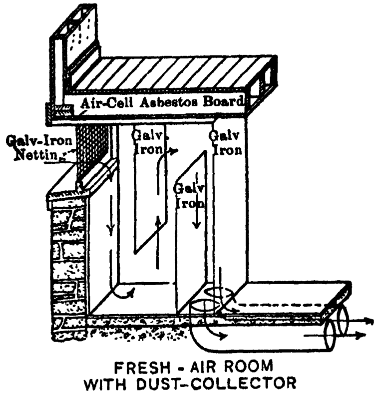


Fig. 66. Fresh-air Room with Dust-collector

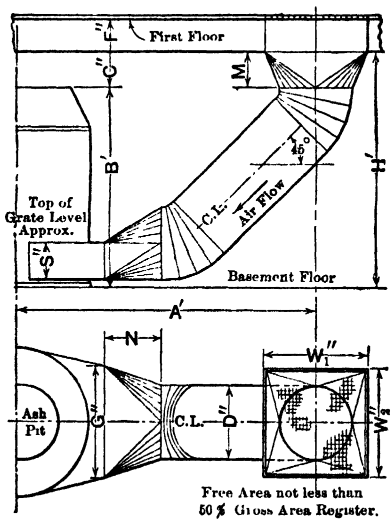


Fig. 67. Plan and Elevation of Recirculating Duct and Shoe

NOTE: For average conditions, make
D = approximately 1/2 casing diameter
G = approximately 3/4 casing diameter
S = approximately 1/4 casing diameter

Table XLIII. Capacities and Dimensions of Fresh-Air Ducts, Rooms, etc.

Size of horizontal portion of rectangular fresh-air duct, in	Size of horizontal portion of round fresh-air duct, in	Cross-section area of horizontal portion of fresh-air duct, in	Size of fresh-air; room-length and width (height same as depth of cellar), in	Size of fresh-air intake (area of woven-wire netting, not including frame), in
8×18	1–14	144	18×48	12×16
8×21	1–15	168	21×48	14×16
8×24	1–16	192	24×48	16×16
10×21	1–16	210	21×60	14×20
10×24	1–18	240	24×60	16×20
10×27	2–13	270	27×60	18×20
10×30	2–14	300	30×60	20×20
12×27	2–14	324	27×72	18×24
12×30	2–15	360	30×72	20×24
12×33	2–16	396	33×72	22×24
12×36	2–17	432	36×72	24×24
12×39	2–17	468	39×72	24×26
14×36	2–18	504	36×84	24×28
14×39	2–19	546	39×84	26×28
14×42	2–19	588	42×84	28×28
14×45	2–20	630	45×84	28×30
14×48	2–21	672	48×84	28×32
14×51	2–21	714	51×84	28×34
16×48	2–22	768	48×96	32×32
16×51	2–23	816	51×96	32×34
16×54	2–24	864	54×96	32×36
16×57	2–24	912	57×96	32×38
16×60	2–25	960	60×96	32×40

Table XLIV. Dimensions and Capacity Data for "Majestic" Furnaces (Fig. 52)

(The Majestic Co)

	Diameter of firepot, in	Depth of firepot, in	Diameter of radiator, in	Diameter of grate, in	Height of ashpit, in
Standard furnace:					
2142	21	11	32	18	14
2447	24	12	36	21	14
2652	26	12	36	22½	14
2856	28	14½	44	24½	15
Down draft:					
221	21	11	. .	18	14
224	24	12	.	21	14
226	26	12		22½	14
228	28	14½	.	24½	15
"1200" series:					
1220	20	12	30	16¾	12
1222	22	12	32	18½	13
1224	24	12	35	20½	13
Heavy duty heater:					
200	21–29½	15½		18½×27¾	14

	Height of furnace casting, in	Diameter of casing, in	Height of casting to hood, in	Height of hood standard 12-in pipe, in	Diameter of smoke collar, in
Standard furnace:					
2142	53	44	52½	13½	8
2447	55½	48	55¼	13½	9
2652	55½	52	55¼	13½	9
2856	60	56	61	14	10
Down draft:					
221	53	42×48	53	13	8
224	55½	46×51	56½	13	9
226	55½	48×52	56½	13	9
228	60	52×56	64	14	9
"1200" series:					
1220	47	40	50	13	8
1222	48	44	50½	13	8
1224	50½	48	52½	13	9
Heavy duty heater:					
200	55	45×79	55	......	10

Table XLIV (Continued). Dimensions and Capacity Data for "Majestic" Furnaces (Fig. 52)

(The Majestic Co)

	Height of bottom smoke collar, in	Size of feed door, in	Size of ashpit, door, in	Grate area, sq in	Heating surface, sq in
Standard furnace:					
2142	44	11 ×15	21 ×14	259	5 600
2447	45	13 ×15	24 ×14	346	6 972
2652	45	13 ×15	24 ×14	397	7 211
2856	49	13 ×15	27 ×14	462	9 191
Down draft:					
221	44	11 ×15	21 ×14	259	7 588
224	45	13 ×15	24 ×14	346	8 342
226	45	13 ×15	24 ×14	397	8 512
228	49	13 ×15	27 ×14	462	9 909
" 1200 " series:					
1220	38	11½× 8½	16½×11	224	4 464
1222	39	12½× 9½	18 ×12	276	5 010
1224	42	12½× 9½	18 ×12	330	5 935
Heavy duty heater:					
200	43	12½×14½	11½×17¾	501	.

	Heating surface to grate-area, ratio	Free air area, sq in	Pipe-area, sq in	Shipping weight, lb
Standard furnace:				
2142	21 6	678	468	1 325
2447	20 1	958	607	1 565
2652	18 2	1 043	670	1 805
2856	19.9	1 166	807	2 625
Down draft:				
221	29 3	1 348	538	1 520
224	24 1	1 512	656	1 690
226	21 4	1 595	715	1 920
228	21.4	1 341	832	2 625
" 1200 series "				
1220	19 9	520	391	1 000
1222	18 2	646	465	1 140
1224	18 0	910	554	1 280
Heavy duty heater:				
200	. . .	.	1 600	2 600 casting only

Table XLV. Dimensions and Capacity Data for "Front Rank" Furnaces (Fig. 53)

(Langenberg Manufacturing Co.)

Number of furnace	381	45XL	455	51XL
Diameter of casing, inches......	38	45	45	51
Stretch out of casing, feet and inches......	9–11⅛	11–9⅜	11–9⅜	13–4⅛
Width lower section casing, inches	23	23	24	23
Width middle section casing, inches	30	26	30	26
Width canopy not less than, inches	12	15	15	15
Height cased not less than, inches*	68	67	72	67
Diameter drum, inches......	18	22	22	26
Height drum, inches.....	58	52	58	53
Diameter radiators, inches..	9	10	10	11
Height radiators, inches.......	37	29	34	29
Diameter smoke-pipe, inches....	8	9	9	9
Size feed-door opening, inches.	10 ×12	8¾×12½	12¼×14	8¾×12½
Size ashpit opening, inches. .	8¼×11½	10 ×15½	10 ×15½	10 ×15½

Number of furnace	515	575	635	665
Diameter of casing, inches......	51	57	63	66
Stretch out of casing, feet and inches....	13–4⅛	14–11¼	16–5⅝	17–1⅝
Width lower section casing, inches	24	24	24	26
Width middle section casing, inches	30	30	30	36
Width canopy not less than, inches	16	18	18	24
Height cased not less than, inches*	73	75	75	89
Diameter drum, inches....	26	29	32	32
Height drum, inches.......	59	62	62	69
Diameter radiators, inches..	11	13	15	18
Height radiators, inches....	35	38	38	42
Diameter smoke-pipe, inches....	9	10	10	10
Size feed-door opening, inches. .	12¼×14	12¼×14	12¼×14	12¼×14
Size ashpit opening, inches ..	10×15½	10 ×17	10 ×17	10 ×17

* Lowest point of canopy must be at least 8 in above drum head.

Certified Measurements and Ratings

Heating surface, square inches...	6 395	6 671	7 378	7 763
Grate-area, square inches...	177	258	258	388
Rating of leader-pipe supplied by furnace*...	350	505	529	679
Heating surface, square inches...	8 490	10 258	11 678	13 877
Grate-area, square inches.......	388	471	578	578
Rating of leader-pipe supplied by furnace*................	705	853	1 015	1 093

The heating surfaces and grate-areas in the above table are hereby certified to as correct and are on file in the office of the National Warm Air Heating Association, Columbus, Ohio.

* The ratings as given above have been determined by using the furnace rating formula in the Fifth Edition of the Standard Code.

Table XLVI. Capacities and Dimensions of Chimney-Flues for "Front Rank" Furnaces

Diameter of furnace smoke-pipe, inches	Size of chimney flue inside, inches			Proper inside diameter of standard flue-linings, inches	Proper height of chimney above grate, feet
	Proper diameter, round chimney	Proper size of unlined square chimney	Proper size of unlined rectangular chimney		
8	10	9½×9½	8×12	7×11½	34
9	11	10½×10½	10×12	7×11½	36
10	12	11½×11½	10×14	11¼×11¼	38
12	14	13½×13½	12×16	11¼×16¼	40
14	16	15½×15½	14×18	15¾×15¾	40
15	17	16½×16½	14×20	17¼×17¼	40
Flue Dimensions where Two Connections are Made to the Same Flue					
2- 8	14	13½×13½	12×16	11¼×16¼	40
2- 9	15	14½×14½	14×16	15¾×15¾	40
2-10	16	15½×15½	14×18	15¾×15¾	40

The above sizes are the correct ones to use in new construction.

Design of a Warm-Air Furnace Installation. (Tables XLVII and XLVIII). Table XLVII is an example of an actual installation and serves to illustrate the method employing the data given in this section.

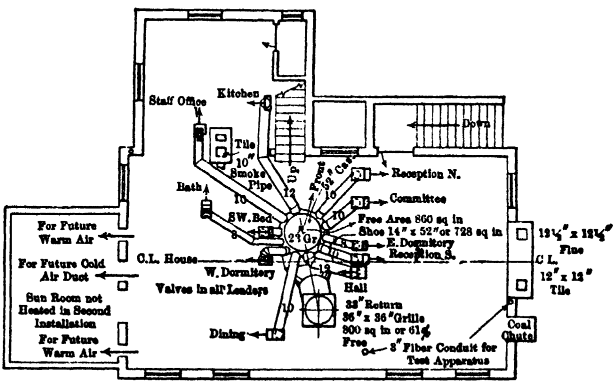


Fig. 68. Plan of Warm-air Heating Plant with One Recirculating Duct. (See Fig. 54 for section)

Table XLVII. Example: Heat-Losses, Leader-Pipes, Wall Stacks and Register Sizes (Figs. 54 to 68)

Floor	Room	(1) Heat loss, Btu per hour (70°–0° F.), H	(2) Calculated area, square inches	
			Leaders H ÷ {111, 167, 200}	Stacks, L. A. × 0.75 *
First.....	Living-room, N......	18 340 (with S.)	83	62
	Living-room, S.... .		83	62
	Hall	5 855	155	116
	Dining-room.. ..	9 460	84	63
	Kitchen	8 428	101	76
	Kitchen alcove.	2 800	In kitchen	In kitchen
Second.	E. bedroom.	19 213	115	86
	S. W. bedroom.. ..	9 175	55	41
	Bathroom .	3 117	19	14
	N W bedroom ..	13 340	80	60
	Hall, add to 1st	5 640	See 1st floor	
Third	E. dormitory......	8 441	42	31
	W. dormitory. .	8 654	43	32
	Hall, add to 1st.	5 737	See 1st floor	
Total	All halls	17 232		
First. ..		44 883	506	379
Second.		50 485	269	201
Third .		22 832	85	63
Total		118 200	860	643

Floor	Room	(3) Net volume, cubic feet; basis for recirculation estimates	(4) Leaders (actual)	
			Diameter, inches	Area, square inches
First....	Living-room, N . .	2 410 (with S.)	10	78.5
	Living-room, S..			
	Hall... .	1 445	10	78.5
	Dining-room..	1 710	12	113
	Kitchen	1 375	10	78.5
	Kitchen alcove .	420	12	113
Second...	E. bedroom. .	2 144	10	78.5
	S. W. bedroom..	1 471	9	64
	Bathroom.. . .	472	8	50
	N. W bedroom	1 388	10	78.5
	Hall, add to 1st	1 549		
Third..	E dormitory.......	868	8	50
	W. dormitory .	831	8	50
	Hall, add to 1st...	513		
First....	....	7 360		461.5
Second..	.. ..	7 024	. . ...	271
Third	. . . .	2 212	. ..	100
Total.		16 596		832.5

* In the case of first-floor baseboard registers, "stack-area" is in reality throat-area.

Table XLVII (Continued). Example: Heat-Losses, Leader-Pipes, Wall Stacks and Register Sizes (Figs. 54 to 68)

		(5) Stacks (actual)*		
		Size, inches	Type	Area, square inches
First.....	Living-room, N..... Living-room, S..	5½ ×13	Double	71 5
	Hall..	5½ ×13	Double	71.5
	Dining-room.....	7⁹⁄₁₆ ×14	Single	106
	Kitchen	5½ ×13	Double	71.5
	Kitchen alcove..	7 ×14	Single	98
Second...	E. bedroom...	5 ×12	Single	60
	S. W bedroom..	3½ ×12	Single	42
	Bathroom..	3 ×10	Double	30
	N. W. bedroom	5½ ×13	Double	71.5
	Hall, add to 1st			
Third..	E. dormitory ...	3 ×10	Single	30
	W dormitory	3 ×10	Double	30
	Hall, add to 1st			
First	....		..	417.5
Second	....			203.5
Third	....			60
Total	..			681

		(6) Warm-air register †		
		Size, inches	Free area, square inches	Free area, per cent of total
First.....	Living-room, N . Living-room, S.....	10×12	83.5	70
	Hall ..	10×12	83.5	70
	Dining-room. ...	12×14	120.5	72
	Kitchen .	10×12	83 5	70
	Kitchen alcove..	12×14	120 5	72
Second.	E bedroom .	10×12	83 5	70
	S. W. bedroom....	9×12	74	69
	Bathroom..	8×10	53	67
	N. W. bedroom....	10×12	83 5	70
	Hall, add to 1st.			
Third..	E. dormitory.......	8×10	53	67
	W. dormitory .	8×10	53	67
	Hall, add to 1st .			
First.....	....	..	491.5	......
Second..	....	...	294	....
Third...	....	......	106	....
Total.	....	.....	891.5	....

* In the case of first-floor baseboard registers, "stack-area" is in reality throat-area.

† Baseboard registers in all rooms except E. and W. dormitories and bath with side wall type.

Total heat loss with sun-porch, 141 805 Btu, with dining-room = 6 615 Btu, and sun-porch = 26 450 Btu.

Size of furnace is based on equations (1) and (3) in section on Furnace Size and is calculated as follows:

$$G = \frac{1.33H}{C \times E \times F} = \frac{1.33 \times 118\,200}{7.5 \times 0.575 \times 12\,790} = 2.85 \text{ sq ft}$$

G in square inches = 144 × 2.85 = 411, or nearest diameter = 23 in at 415 sq in

(1) The heat loss for each room was calculated for a mean breathing-level temperature of 70° F. inside in all heated rooms, and for outside weather conditions of 0° F. and a wind movement of 15 miles per hour. The sun-porch and an unequipped bath on the third floor were not heated, and the temperature of attic spaces was assumed as 35° F. In all respects, the residence is of standard frame dwelling construction, with the single exception of the studding, which is 2 in by 6 in instead of the usual 2 in by 4 in This, according to the commonly accepted theory of heat transmission of walls, operates to increase the wall heat-loss, but permits the use of larger wall stacks than could be used in 2 by 4-in construction. The wall section is as follows: weather boarding, building paper, ship-lap siding on 2 by 6-in studding, lath, and plaster with rough sand finish. The coefficient of heat-transmission for this wall section is 0.20 Btu per sq ft per hour per 1° F., at a wind velocity of 15 miles per hour. The walls were not insulated, nor was weather stripping used at the windows and doors. Interlocking copper shingles were used on the roof.

(2) The calculated areas of leaders were found by dividing the heat-loss from each room by the heat-carrying capacity of each square inch of leader, for which, at a register air temperature of 175° F., the values are derived from Fig. 54.

1 sq in of leader to first floor = 111 Btu per hour
1 sq in of leader to second floor = 167 Btu per hour
1 sq in of leader to third floor = 200 Btu per hour

Since these values apply only when wall stack areas are about 75% of leader-areas, the calculated stack-areas were found by multiplying leader-areas by 0.75.

(3) The net internal volumes of all heated rooms were totaled as a basis for making an estimate of the number of recirculations required per hour as shown in the following calculation.

The weight of air (W) to be circulated per hour must be sufficient to supply the heat-loss (H) when the air temperature drops from 175° to 70° F.

$$H = W \times 0.24 \times (175 - 70) \quad \text{or} \quad W = 4\,690 \text{ lb per hour}$$

and the air volume (Q) at 70° F., corresponding to this weight, is

$$Q = \frac{W}{0.075} = \frac{4\,690}{0.075} = 62\,600 \text{ cu ft per hour}$$

which gives as the number of recirculations (N) in zero weather:

$$N = \frac{62\,600}{16\,596} = 3.75$$

(4) The actual leader-areas and diameters were selected as nearly as possible in accordance with the calculated areas, adjusting somewhat for long and short runs, and using pipe-diameters in whole inches. The only exceptions made to this precedure were in the case of the leader-pipe to the combined halls and to the east bedroom, second story. The total hall requirements call for 155 sq in, whereas only 113 sq in or a 12-in diameter leader was installed. The location of the recirculating register in the first-story hall floor was regarded as justification for this reduction. In the case of the east bed oom, the short direct run taken off near the furnace front was regarded as very favorable, and, as an experiment, the leader size was reduced from 12 in to 10 in. The results were quite satisfactory in this particular installation, but such a large reduction would not be advisable except under very special conditions. No leader less than 8 in in diameter nor more than 12 in in diameter was used.

(5) The actual stack or throat-areas were selected to agree as nearly as practicable with the calculated areas, but at the same time careful consideration was given to the installa-

tion conditions affecting each stack leader system. Stack-areas were kept in excess of 75% of leader-areas unless especially favorable conditions existed. This was not done in the case of the third-story dormitories, which have a stack leader ratio of 0.60 instead of 0.75, and these rooms are somewhat underheated as a result. These stacks should have been made 3 in × 12 in or possibly 3 in × 13 in, instead of 3 in × 10 in.

(6) The warm-air registers were so selected that in each case the free area through the grilles would be not less than the leader pipe area on the same run. Registers of the baseboard type having free areas of at least 70% of the gross or commercial areas were chosen and no register had either dimension more than 14 in. Three side-wall registers with free areas slightly under 70% of gross areas were used in the two third-story dormitories and the bathroom on the second floor.

Table XLVIII. Cold-Air Return Duct Size

Location	(1) Free area of grilles *	(2) Area of ducts	(3) Area of shoes
Single return:			
Center of front hall..	800 sq in	854 sq in	728 sq in
(Dimensions)...	(36 in × 36 in)	(33 in diameter)	(14 in × 52 in)

* The free area of the grille is 61.5% of the total area

(1) The total cross-sectional area of the recirculating duct system was made not less than the total leader-area of 832.5 sq in. A 36 × 36-in grill (free area at 61.5% = 800 sq in) was selected for the front hall, although slightly under size. The small increase in friction to air-flow through the grille was compensated for by the short direct duct.

(2) A round duct with well-tapered connections at register-box and furnace-shoe was selected with total area slightly in excess of total leader-area.

(3) Shoe-areas are limited to an extent by two items: the height of grate above top of base ring and, in the case of the single-return, the diameter of the casing. In the present case, the former dimension is 12 in and the latter 52 in. Shoes wider than casings are unsightly and expensive, hence for the single-return system a compromise was made, and a 14 by 52-in shoe selected. This shoe had 12.5% less area than the total leader area.

Central Fan Heating

General Features. The mechanical indirect method of heating, commonly known as the CENTRAL FAN SYSTEM, particularly adapted to the warming and ventilating of large structures, is made up of three units: (1) A HEATER constructed of pipes, tubes, or cast-iron sections, through which steam, hot water or hot gas may be passed. (2) A FAN OR BLOWER to circulate air over the heater surfaces, the air acting as a heat carrier or medium of heat-transfer. (3) A SYSTEM OF DUCTS OR PIPES to convey the heated air from the heater to points where heat may be required. When the heater is located between the fan and main duct, the combination is termed BLOW THROUGH, and when the fan is installed between the heater and the duct, the arrangement is known as DRAW THROUGH. These two arrangements are shown in Fig. 70. The DRAW-THROUGH combination is more often used for shop and factory installations where compactness is desirable, the BLOW-THROUGH combination being used principally for HOT-AND-COLD systems as installed in schools and public buildings.

Advantages of the Blower or Hot-Blast System. The advantages of the blower or hot-blast system over those of direct radiation, briefly summarized, are:

(1) When ventilation is a requirement in order to maintain a healthful atmosphere, this method affords a positive means of accomplishing this

particularly desirable result, which is entirely independent of the changing climatic conditions.

(2) When a standard humidity of the air is to be maintained, a feature which is becoming to be more generally recognized as desirable in any heating-and-ventilating installation, and quite essential to the successful manufacture of some materials, the humidifying-apparatus may readily be made an integral part of the system.

(3) A much smaller amount of radiating-surface is required to perform an equal heating-duty, with a consequent reduction in the number of steam-tight joints, unions and valves to keep in repair.

(4) The air-leakage being mostly outward, the building will in general be freer from drafts and more uniformly heated. If the air is simply recirculated, no fresh air being taken into the heating system from the outside, the above statement does not apply. The pressure of the air in the building, even when all of the air is taken into the heating system from the outside, is comparatively feeble, and some air will enter by infiltration through the window and door-cracks on the windward side of the building, although the statement is often made that the leakage being all outward, prevents the infiltration of cold air from the outside.

(5) This system is more easily regulated, and readily responds to changing outside temperatures.

(6) The air entering for ventilation may be conveniently cooled in summer, either by the circulation through the heater of cold water or of brine previously cooled by mechanical refrigeration.

(7) Simply running the fan will in itself greatly relieve the oppressiveness in hot sultry weather, and when cold water is circulated through the coils the difference is very noticeable.

Typical Arrangements. When ventilation is not a requirement, or when it is relatively unimportant, as is frequently the case in shop or factory-heating where the number of persons vitiating the air is small compared with the cubical contents of the building, the air may be simply RECIRCULATED, sufficient fresh air for ventilation being supplied by infiltration. The amount of heat to be supplied the heater in this case is the same as would be required for a direct-radiation installation. When ventilation is a requirement to be met a COLD-AIR INTAKE is provided. Since the amount of air necessary for heating is generally in excess of the amount required for ventilation considerable economy may be effected by recirculating a portion of the air. In this case only sufficient fresh air is drawn into the system from the outside to meet the ventilation requirement and the remainder of the air necessary for heating, is recirculated. This may be readily effected by an arrangement of ducts and dampers on the suction-side of the fan. If the fresh air introduced is to be washed or conditioned the washer or humidifier and tempering-coil may be added between the inlet for the recirculated air and the fresh-air intake.

Amount of Air to be Circulated for Heating. The weight of air to be circulated per hour for heating a room or building is found by dividing the heat-loss (H) by the amounts of heat given up by 1 lb of air in cooling from the temperature at the duct-outlets to the mean room-temperature.

Let H = heat-loss of room, Btu per hr;
M = weight of air to be introduced in room per hour;
t = mean inside temperature;
t_d = temperature of air leaving duct-outlets.

Then $$M = H/[0.24\,(t_d - t)]$$

The temperature t_d depends upon the temperature of the air entering the heater, the velocity through the clear area, the amount of heating-surface and the temperature of the steam. This temperature in practice ordinarily ranges from 125° to 150° F. and may be readily determined for any specified condition by the data given later under Hot-Blast Heaters. The temperature of the air leaving the duct-outlets for ordinary installations, when the ducts are not run underground or in outside walls, may be assumed to be the same as the temperature (t_2) of the air leaving the heater. Any loss in temperature in this case goes toward heating the building and is therefore not a direct loss. If, however, the ducts are run underground or in outside walls, a considerable loss in temperature may occur, which is a direct loss, and must be provided for by INCREASING THE TEMPERATURE OF THE AIR LEAVING THE HEATER by an amount equal to the estimated temperature-drop in the ducts.

Temperature of Air Entering Heater.

Let t_1 = temperature of air entering heater;
t_0 = outside temperature;
t = mean inside temperature;
t_2 = temperature of air leaving heater;

(*a*) When the air is all recirculated, $t_1 = t$;
(*b*) When fresh air only is circulated, $t_1 = t_0$;
(*c*) When a portion of the air is recirculated the resulting temperature of the mixture of fresh and recirculated air may be found by the METHOD OF MIXTURES.

Let M_v = weight of fresh air, pounds required per hour for ventilation (30 cu ft per min per person);
= 0.075 × 1 800 × number of persons (usual requirements);
M_r = weight of air that may be recirculated;
$M = M_v + M_r$;
$H = 0\ 24\ (M_v + M_r)\ (t_d - t)$.

Having assumed or fixed the value of t_d, the only unknown quantity is M_r.

$$M_r = H/[0.24\ (t_d - t)] - M_v$$

The temperature t_1 may then be found as follows:

$$M_v \times (t_0 + 460) = A$$
$$M_r \times (t + 460) = B$$
$$(M_v + M_r)\ (t_1 + 460) = A + B$$

or
$$t_1 = (A + B)/(M_v + M_r) - 460$$

Example. The heat-loss H for a certain factory-building is 70 600 Btu per hr. The number of men employed is 50. Mean inside temperature t = 65° F. Outside temperature t_0 = 0° F. Ventilation is to provided at the rate of 1 800 cu ft of fresh air per hour per person. Assumed temperature of air leaving duct-outlets is 135° F.

Solution. M_v = 0.0075 × 1 800 × 50 = 6 750 lb per hr fresh air for ventilation. The weight of air that may be be recirculated is

$$M_r = 706\ 000/[0.24\ (135 - 65)] - 6\ 750 = 35\ 273 \text{ lb per hr}$$

The temperature of the air entering the heater will be:

$$6\ 750 \times (\ 0 + 460) = 3\ 105\ 000$$
$$35\ 273 \times (65 + 460) = 18\ 516\ 750$$

$$t_1 = (21\ 621\ 750/42\ 023) - 460 = 55° \text{ F.}$$

If FRESH AIR ONLY is to be used, as in school-house and public-building heating, the weight of air to be circulated is determined directly by the ventilation requirement.

Then $M = M_v = H/[0.24\,(t_d - t)]$, or $t_d = (H + 0.24\,M_v t)/0.24\,M_v$

Temperature of Air at Duct-Outlets. When heating, by the hot-blast system, a building containing a number of rooms having different heat-losses and ventilation requirements, it is obviously impossible to maintain the desired temperature by controlling the temperature (t_2) of the air leaving the heater at one point. The temperature t_d of the air leaving the duct-outlets will ordinarily be different for each room in the building, as shown in the following example. This result is accomplished by the double plenum-chamber system described later.

Example. Let it be required to determine the temperature of the entering air (t_d) to offset the heat-loss and provide ventilation for the several rooms as given in Table XLIX. Inside temperature (t) to be maintained is 70°.

Table XLIX. Data for Example

Room-number	Number of occupants, n	Ventilation		Heat-loss, H	Temperature of entering air, t_d (see formula)
		Cubic feet per hour, at 70°, 1 800 × n	Weight per hour, M_v		
A-1	50	90 000	6 750	32 000	89 5
A-2	53	95 400	7 125	21 000	82.2
Hal'	. .... .	30 000	2 250	4 000	77.4

Temperature of Air Leaving Heater. If all of the air is first warmed by the TEMPERING-COIL to 70° F., and a mixture of approximately $(1 - x)$ parts of tempered air and x parts of hot air is to be used, then the required temperature of the hot air leaving the heater may be determined, for any particular case, by the METHOD OF MIXTURES previously given; or, assuming this temperature, the proportions of hot and tempered air may be determined.

Example. Required the temperature of the hot air (t_h) leaving the heater for room A-2 (Table XLIX), if the mixture entering the room is made up of one half tempered air at 70° and one half hot air. The total weight of air entering the room is 7 125 lb per hr, or 3 562.5 lb of tempered air and 3 562.5 lb of hot air.

Solution. $3\,562.5 \times (70 + 460) + 3\,562.5 \times (t_h + 160) = 7\,125 \times (82.2 + 460)$. Hence $t_h = 94$.

Assuming a temperature of $t_h = 120°$, it is required to determine the relative proportions, by weight, of the mixture required.

Let x = parts of hot air in mixture. Then $(1 - x)$ = parts of tempered air.

$$x\,(120 + 460) + (1 - x) \times (70 + 460) = (82.2 + 460)$$

Then

$$x = 0.244 \quad \text{and} \quad (1 - x) = 0.756$$

Air Supplied for Ventilating Purposes Only. Split System. A combination to direct radiation, to offset the heat-loss H, and a hot-blast system, to supply the fresh air needed for ventilation, termed a SPLIT SYSTEM, is sometimes

installed. In this case it is customary to install a heater of sufficient capacity to warm the air for ventilation to about 80°. The heater used for this purpose is made three sections deep and is termed a TEMPERING-COIL.

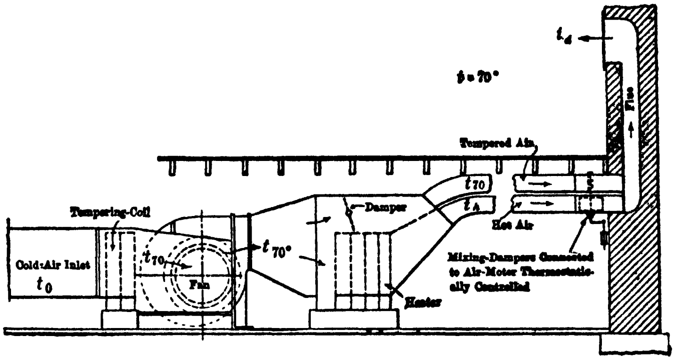


Fig. 69. Double-duct System

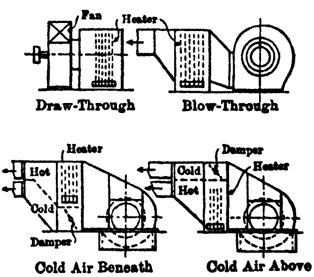


Fig. 70. Types of Heater-jackets

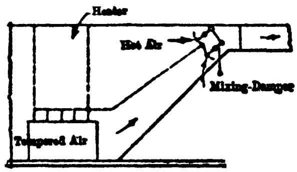


Fig. 71. Single-duct System

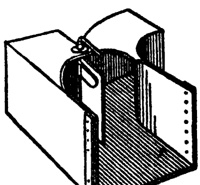


Fig. 72. Deflecting-damper for Branch-duct

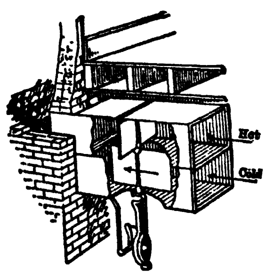


Fig. 73. Thermostatically-controlled Mixing-damper

Figs. 69 to 73. Details of Ducts for Hot-and-cold Heating Systems

Hot-and-Cold Systems. In order to accomplish the results required in the preceding example, the so-called HOT-AND-COLD SYSTEM or DOUBLE-PLENUM-CHAMBER SYSTEM is used. All of the air drawn into the system from the outside is first passed through a TEMPERING-COIL, which is designed to heat the

air to approximately 70°. A portion of the tempered air is then passed through a heater and raised to 125° to 150°. Then if varying proportions of the hot and tempered air are correctly mixed the resulting temperature (t_d) is readily controlled without varying the quantity of air discharged, which evidently must remain constant on account of the ventilation requirement. There are two methods of distribution used, as shown in Figs. 69 and 71. Referring to Fig. 71 it is seen that the hot and tempered air meet at the end of the plenum-chamber at the entrance to the ducts, and the temperature of the mixture is controlled by the MIXING-DAMPERS, which may either be hand-operated or placed under automatic thermostatic control. It will be observed that the plenum-chamber is divided, and that each duct serving a room has its own independent set of mixing-dampers. This method of distribution is known as the SINGLE-DUCT SYSTEM and is frequently employed where the installation of the DOUBLE-DUCT SYSTEM, as described below, is not feasible or is undesirable. Fig. 69 shows a double set of ducts run from the plenum-chamber to the base of each vertical flue, one carrying the hot air and the other the tempered air, the mixing being done at the base of the flue as shown. The mixing dampers (Fig. 73) may be controlled by hand by means of a chain carried up the flue and run into the room at a point several feet above the floor-line, or placed under automatic thermostatic control through the medium of a compressed-air-operated damper.

Hot-Blast Indirect Heaters

Copper-Tube Heaters. (Fig 74.) There are a number of spirally wound extended-surface copper-tube heaters available for hot-blast installations,

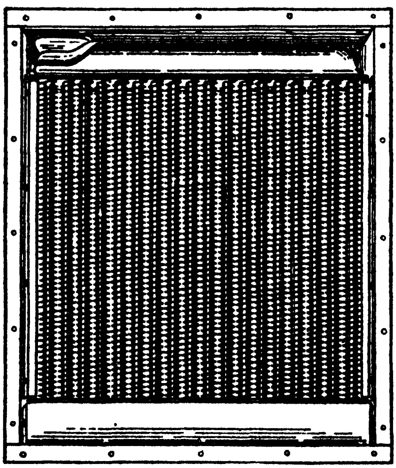


Fig. 74. Aerofin Low-pressure Blast Heater

as for example AEROFIN, ARCO BLAST (Am. Rad. Co.) and SUPERFIN (York Heating and Ventilating Corp.). Fig. 74 shows the low-pressure type as manufactured by the Aerofin Corp.

AEROFIN is made up of straight seamless copper or brass tubing about which

is wound a thin narrow helical extended surface, also of copper or brass made metallically integral with the tube by means of a mechanically effected solder bath. The helical extended surface is applied to the tubes by means of a machine which automatically corrugates the inner edge of the metal ribbon, and winds the ribbon uniformly about the tube, so that the pitch of the helix is uniform and the extended surface is at right-angles to the tube. As the metal ribbon is wound about the tube the machine treats the surfaces with acid and applies the molten solder while the tube and ribbon are held firmly in position. The solder is flowed over the tube and the extended surface so that both are entirely covered, forming a permanent metallic unit which is extremely effective for the transmission of heat.

The finished tubes are then mechanically pressed into thin flexible tube plates of copper, brass, or other non-corrosive metal, all of the tubes being pressed into both top and bottom plates in one operation. The punched holes in the tube plates are slightly smaller than the tubes themselves, so that a strong and tight joint is effected when the tubes are forced into the plates. The tube plate, when punched, is also cupped, or dished, around each opening, and formed so that there is a deep collar about each tube when pressed into the plate. This construction not only provides a great contact area between the tube and the plate, but also renders the plate flexible at the point of juncture with the tube, thus compensating for any movement of the tube due to expansion or contraction.

The SHELT-METAL JACKET is an integral part for all types of copper-tube heaters. The complete assembled blast heater is made up of a number of units similar to Fig. 74. The manufacturers furnish performance tables similar to the Vento tables, appearing later in the text, giving the temperature rise of the air, condensation and friction based on various air velocities through the heater. Copper-tube type heaters for pressures of 25 lb per sq in and above are usually constructed without the header, the pipe coil being of the return-bend type.

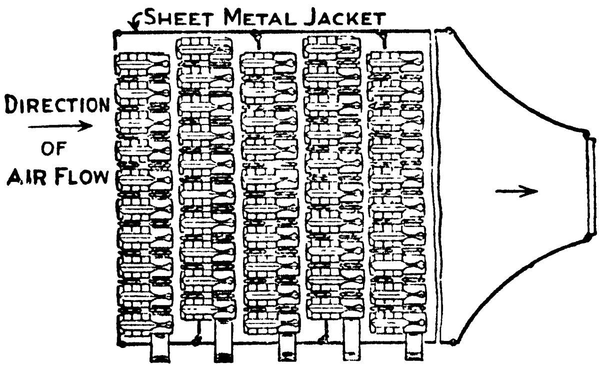


Fig. 75. Plan of the Assembly of a Vento Heater

Cast-Iron Indirect Heaters. Cast-iron sections for indirect heaters are often used, and Fig. 75 shows a cast-iron heating-unit or section, named Vento, manufactured by the American Radiator Company, which is quite widely used in this class of work. A STACK made up of several sections has a smaller

number of joints than a pipe-coil section of equal heating-surface. The deterioration of the cast-iron sectional type of heater is practically nothing except for the right-hand and left-hand hexagonal nipples connecting the units which go to make up a stack. There are three standard lengths of Vento heater-sections as indicated in Table L, which also includes other data required by the designer.

Table L. Vento Hot-Blast Heater-Data

Length of section in inches	Heating-surface, regular sections, in square feet	Free area in square feet, per section, inches on centers			Ratio, square feet, heating-surface to free area for 5 in on centers
		4⅝	5	5⅜	
40	10 75	0 52	0 62	0 72	17.34
50	13.50	0 65	0 77	0 91	17.53
60	16.00	0.78	0 92	1.08	17.39

Selection of Hot-Blast Heaters. General Conditions. In selecting the size of a heater for any particular service the choice is based on the final temperature desired and the FREE AREA required for a certain allowable velocity for Vento and the FACE AREA for Aerofin and similar type copper-tube heaters. That is, for any specified initial and final temperature desired, and a certain number of sections, a final temperature results when the velocity has been fixed in advance. Good practice limits the velocity to the values given by the following tables. High velocities are objectionable in public-building work on account of the resulting noise. The resistance through the heater increases in proportion to the square of the velocity, which adds to the power required to move the air as the velocity is increased, as will be noted later.

Rating of Hot-Blast Heaters. The RATING of an assembled heater of several sections or stacks is based on the TEMPERATURE-RISE of the air passing over the heating-surface for certain velocities through the free or unobstructed area of the heater-face for Vento and face area for Aerofin and Superfin. The VELOCITY is based on the volume of air at an assumed temperature of 70° for convenience in rating.

Let M = weight of air to be circulated through heater per hour;
0.075 = density of air at 70°;
A = either free area or face area of heater in square feet;
V = velocity of air in feet per minute through free area based on 70° temperature.

Then

$$A = M/(60 \times 0.075 \times V) = M/4.5\, V \text{ sq ft.}$$

The following tables will serve as guides in selecting the VELOCITY V and the NUMBER OF SECTIONS OR STACKS for various purposes.

Temperature-Rise. The TEMPERATURE-RISE of the air passing through hot-blast heaters of various types has been well established by experiment, the various manufacturers having published the results in the form of bulletins and catalogues.

Example. It is required to determine the size of a Vento hot-blast heater to supply the necessary heat for a public building, the calculated heat-loss of

which is $H = 1\ 420\ 000$ Btu per hr for 70° inside and 0° outside temperature. The temperature of the air entering the rooms, t_d, is to be approximately

Table LI. Allowable Velocities of Air through Free-Area Vento Heaters *

Referred to a temperature of 70° F.

Number of stacks deep, regular 5 in on centers	Public-building work, velocity in feet per minute	Factory work, velocity in feet per minute
4	1 000 to 1 500	1 200 to 1 600
5	1 000 to 1 300	1 200 to 1 600
6	1 000 to 1 200	1 200 to 1 600
7	900 to 1 100	1 200 to 1 500
8	800 to 1 000	1 200 to 1 400

* The allowable velocities for heaters rated on a based or *face area* will be approximately one-half the values given in table for Aerofin and Superfin heaters.

Table LII. Number of Stacks of Vento Ordinarily Required

Service	Number of sections or stacks	Number of rows of 1-in pipe
Public buildings, fresh air, exhaust-steam....	5	20
Industrial buildings, fresh air, 4-lb gauge .	6	24
Industrial buildings, fresh air, exhaust-steam. .	7	28
Industrial buildings, recirculation, 5-lb steam. .	5	20
Tempering-coils, fresh air, exhaust-steam .	3	12

120°. The steam-pressure is 5-lb gauge. The temperature of the air entering the heater is 0°.

Solution. First determine from Table LIII the number of stacks deep required for F.T. at 120° and entering air at 0°, using a velocity of 1 000 ft per min. This condition calls for a 5-stack-deep heater. Then determine the weight of the air to be circulated per hour.

$$M = H/[0.24\,(t_d - t)] = 1\ 420\ 000/[0.24\,(120 - 70)] = 118\ 333 \text{ lb}$$

The free area required is

$$A = 118\ 333/(4.5 \times 1\ 000) = 25.1 \text{ sq ft}$$

Referring to Table L and choosing a 60-in length of units, 5 in on centers, it is found that the free area per section is 0.92 sq ft. The number of sections required across the face of the heater is

$$A/0.92, \quad \text{or} \quad 25.1/0.92 = 27$$

The heating-surface per section is 16 sq ft. The total heating-surface is therefore

$$S = 5 \times 27 \times 16 = 2\,160 \text{ sq ft}$$

and the condensation per hour is

$$2\,160 \times 1.56 \text{ (Table LIII)} = 3\,370 \text{ lb}$$

to be supplied by the boiler, or by exhaust-steam, at 5-lb pressure.

Table LIII. Final Temperatures and Condensations, Vento Heaters

Regular section, Standard spacing, 5-in center to center, of loops Steam, 5-lb gauge. C. is the condensation in pounds per hour per square foot of heating-surface. F. T. is the final temperature of air leaving heater

Velocity through heater in feet per minute, measured at 70°									
Number of stacks deep	Temperature of entering air	1 000		1 200		1 400		1 600	
		F.T.	C.	F.T.	C.	F.T.	C.	F.T.	C.
1	20	51	1 99	49	2 23	47	2.42	45	2 56
	30	60	1 92	58	2 17	56	2 33	54	2.46
	40	68	1 80	66	2 00	64	2 16	62	2 26
	60	84	1 54	82	1.69	81	1.89	80	2 05
	70	92	1 41	90	1.54	89	1 71	88	1.85
2	20	76	1 80	72	2.00	69	2.20	66	2 36
	30	83	1.70	79	1 89	76	2.06	73	2 21
	40	90	1.60	86	1.77	83	1.93	81	2.10
	60	103	1.38	100	1.54	98	1.71	96	1.85
	70	110	1.28	107	1.42	105	1.57	103	1.69
3	20	97	1.65	92	1 85	88	2 06	85	2 22
	30	103	1.56	98	1.75	94	1.91	91	2.08
	40	109	1.47	104	1.64	100	1.79	97	1 95
	60	120	1 28	116	1.44	113	1.58	110	1 71
	70	126	1.20	122	1.34	119	1.46	116	1 57
4	20	115	1.52	110	1.73	105	1.91	101	2.08
	30	120	1.44	115	1.63	110	1.80	106	1.95
	40	124	1.35	119	1.52	115	1.68	111	1 82
	60	134	1.19	129	1.33	125	1 46	122	1.59
	70	138	1.09	134	1 23	131	1.37	128	1.49
5	20	130	1.41	124	1.60	119	1.78	114	1 93
	30	134	1 33	128	1.51	123	1 67	118	1.80
	40	138	1.26	132	1.42	127	1.56	123	1 70
	60	145	1.09	140	1.23	136	1 36	133	1.50
	70	149	1 01	144	1.14	141	1 27	138	1.40
6	20	142	1 30	136	1.49	130	1 65	126	1.81
	30	145	1 23	139	1.40	134	1.56	130	1.71
	40	148	1 15	143	1.32	138	1.47	134	1 60
	60	155	1.02	150	1.15	146	1.29	142	1.40
	70	.		.	.	. .		. . .	. .
7	20	152	1.21	146	1.39	141	1.55	136	1.70
	30	155	1.15	149	1.31	144	1.46	139	1.60
	40	158	1.08	153	1.24	148	1.39	143	1.51

Table LIII (Continued). Final Temperatures and Condensations, Vento Heaters

Number of stacks deep	Temperature of entering air	Velocity through heater in feet per minute, measured at 70°: 1 000		1 200		1 400		1 600	
		F.T.	C.	F.T.	C.	F.T.	C.	F.T.	C.
1	− 20	.				.	.	. .	. .
	− 10	. .				.	.	.	. .
	0	35	2.24	32	2.46			.	.
2	− 20	49	2.22	44	2.46	40	2.69	37	2.92
	− 10	56	2.12	51	2.35	47	2.56	44	2.77
	0	62	1.99	58	2.23	54	2.42	51	2.62
3	− 20	75	2.03	69	2.28	64	2.51	59	2.70
	− 10	80	1.92	75	2.18	70	2.39	66	2.60
	0	86	1.84	81	2.08	76	2.27	72	2.46
4	− 20	96	1.86	90	2.12	84	2.34	78	2.51
	− 10	101	1.78	95	2.02	89	2.22	84	2.41
	0	106	1.70	100	1.92	95	2.13	90	2.31
5	− 20	114	1.72	107	1.95	100	2.15	94	2.34
	− 10	118	1.64	111	1.86	105	2.06	99	2.24
	0	122	1.56	115	1.77	109	1.96	104	2.14
6	− 20	129	1.59	121	1.81	115	2.02	110	2.22
	− 10	132	1.52	125	1.73	119	1.93	114	2.12
	0	135	1.44	129	1.65	123	1.84	118	2.02
7	− 20	141	1.47	134	1.69	128	1.90	122	2.08
	− 10	144	1.41	137	1.62	131	1.81	126	1.99
	0	147	1.35	140	1.54	135	1.73	130	1.90
8	− 20	151	1.37	144	1.58	138	1.77	133	1.96
	− 10	153	1.31	147	1.51	141	1.69	136	1.87
	0	156	1.25	150	1.44	144	1.62	139	1.78

Design of Air-Ducts

Pressure-Loss. The frictional resistance of air flowing through smooth sheet metal ducts, commonly termed PRESSURE-LOSS, measured in inches of water, for 70° air and for a length of duct equal to 100 ft, is given by the following formula:

$$h = 0.000136 \times (R/A) \times v^2$$

in which R is the perimeter of the duct in feet, A the area of duct in square feet, v the velocity of the air in feet per second, and h the pressure-loss measured in inches of water-column. For round ducts the above formula reduces to

$$h = 0.00055\, v^2/D$$

in which D is the diameter of the duct in feet.

The diagrams in Figs. 76 and 77 are based on this formula, from which the DIAMETER OF A ROUND DUCT for various velocities, and the PRESSURE-LOSS OR RESISTANCE for various quantities of air flowing, may be found without solving the above equation.

Example. What should be the size of a round duct required to convey 1 500 cu ft of air per minute with a velocity of 1 800 ft per min; and what is the pressure-loss per 100 ft of duct.

Solution. Locate 1 500 on the upper side of the pipe-diagram in Fig. 76, and pass horizontally downward until the 1 800-ft-velocity diagonal line is inter-

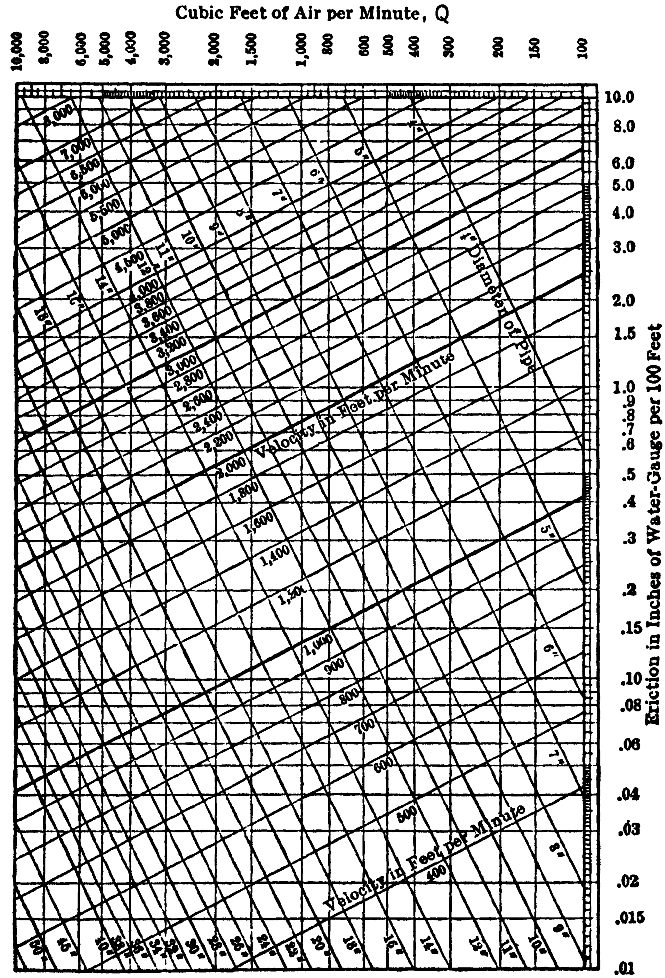


Fig. 76. Diagram of Friction-pressure Loss

sected. The duct which comes nearest to the required size has a diameter of 12 in. At this intersection pass to the right side to the base-line and read 0.48-in water-pressure loss.

Allowable Velocity of Air in Ducts and Flues. In order to limit the resistance or pressure-loss in the duct system the designer should, in general, keep

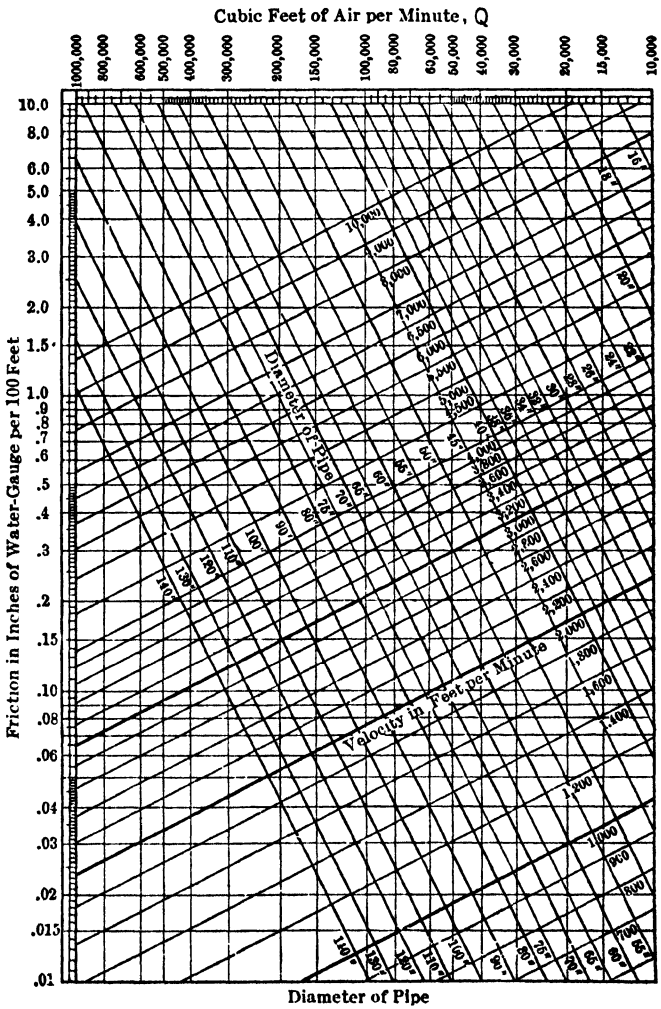


Fig. 77. Diagram of Friction-pressure Loss

the velocities within the limits stated in Table LIV. In public-building work the air should be delivered to a room at a velocity that will insure its movement to the desired points in the room without objectionable draft or noise in passing through the register-grills.

Table LIV. Allowable Velocities in Hot-Blast Systems

Types of buildings	Allowable velocity in feet per minute
Public buildings	
Through free area of wall-registers	400– 500
Through free area of floor-registers	200– 300
Vertical flues to registers	600– 750
Connections to base of flues	800–1 000
Main horizontal distributing ducts	1 500–2 500
Manufacturing plants	
In plants where the occupation is more or less sedentary and the employe sits all day feeding automatic machinery:	
Main ducts	1 200–1 500
Branches	600– 900
In plants where the employe stands all day, as in machine-shops, foundries, etc.:	
Main ducts	1 500–2 400
Branches	900–1 500

The velocity through the fan-outlet, under the ordinary conditions that obtain in heating work, varies from 1 500 to 2 500 ft per min.

Metal Ducts Construction. The accompanying Table LV gives the minimum thickness of metal for circular and rectangular air-ducts in heating and ventilating work.

Table LV. Guages of Galvanized Iron or Steel to be Used for Ducts, for Outside Air Intake

Heating and ventilating			
Round ducts diameter, in	Gauge	Rectangular ducts width, in	Gauge
6 to 10	26	Up to 12	26
11 to 29	24	13 to 30	24
30 to 39	22	31 to 60	22
40 to 49	20	61 to 118	20
50 and above	18	118 and above	18

Thus, a round duct 11 to 29-in diameter should be made of No. 24 gauge sheet steel, and a rectangular duct 31 to 60 in wide should be made of No. 22 gauge.

Pressure-Loss of Rectangular Ducts. The simplest method of determining this is to proportion the system for ROUND DUCTS throughout, and then transfer to RECTANGULAR SIZES giving equal pressure-losses (not equal areas) by means of Table LVI.

Table LVI. Round and Rectangular Ducts of Equal Pressure-Losses

Side of rectangular duct in inches	4	6	8	10	12	14	15	16	18	20	22	24
	Equivalent diameters in inches											
4	4 4											
5	4 9											
6	5 4	6.6										
7	5.8	7.0										
8	6 1	7.6	8.8									
9	6 5	8 0	9 3									
10	6 8	8 4	9.8	11 0								
11	7.1	8 8	10.2	11.5								
12	7.4	9.2	10.7	12 0	13 2							
13	7 6	9.6	11.1	12 5	13 7							
14	7 6	9 9	11.5	12.9	14 3	15 4						
15	8 2	10 2	11.9	13.4	14.7	16 0	16.5					
16	8 4	10 5	12.3	13.8	15.2	16 5	17.1	17 6				
17	8 6	10 8	12.6	14 2	15.7	17 0	17 6	18 2				
18	8 9	11.1	13.0	14.6	16.1	17.4	18 1	18 7	19.8			
19	9.1	11.4	13.3	15.0	16 5	17.9	18.6	19 2	20.4			
20	9.3	11.6	13.6	15.4	17.0	18.4	19 0	19 7	20 9	22 0		
22	9 7	12.1	14.2	16 1	17 8	19.2	19.9	20.6	21 9	23.1	24 2	
24	10 0	12.6	14.8	16.8	18.5	20.0	20.8	21 5	22 8	24.0	25 2	26.4
26	10 4	13 1	15 4	17.3	19.2	20.8	21.6	22.3	23.8	25.1	26.3	27 5
28	10 8	13.5	15.9	18 0	19.8	21.5	22.4	23.1	24.6	26 0	27 3	28 5
30	11.0	13 9	16 4	18.5	20.5	22.2	23.1	23.9	25 4	26.8	28.2	29.5
32	11.3	14 3	16 9	19.1	21.1	22.9	23.8	24 6	26 2	27.7	29 1	30.5
34	11.6	14 7	17.3	19 6	21 6	23.5	24.4	26.3	26 9	28.5	30 0	31.3
36	11.9	15.1	17 7	20.1	22 2	24.2	25.1	26 0	27 7	29.3	30.8	32.2
38	12.2	15 4	18.2	20.6	22 8	24.8	25.8	26.7	28.4	30.0	31.5	33.1
40	12 5	15.7	18.6	21.1	23.3	25.4	26.4	27 3	29 1	30.8	32.4	33.9
42	12.7	16.1	19.0	21.6	23.8	25 9	26.9	27.9	29.8	31.4	33.0	34.5
44	13 0	16.4	19 4	22.0	24 3	26.5	27.5	28 5	30.3	31 2	33.7	35 3
46	13 3	16 7	19.8	22.4	24.8	27.0	28.1	29.1	31.0	32.8	34.6	36.2
48	13 5	17.0	20.1	22.8	25.2	27.5	28.6	29.6	31.6	33.4	35.2	37.0
50	13 7	17.3	20 4	23.2	25.7	28 0	29.2	30.3	32.2	34.1	35.9	37.6
52	13.9	17.6	20 8	23 6	26.2	28.5	29.6	30.7	32.9	34.7	36.5	38 3
54	14 1	17 9	21.1	24.0	26 6	29 0	30.1	31.2	33.4	35 3	37.2	38.9
56	14.3	18.2	21.5	24.4	27.0	29.5	30.6	31 7	33.9	35 9	37.8	39.6
58	14 6	18 4	21.8	24.7	27.4	30.0	31.1	32 2	34.4	36.4	38.4	40.3
60	14 7	18.7	22.1	25 1	27.8	30.5	31.6	32.7	34 9	37 1	39.1	40 9
62	15.0	19.0	22.4	25 5	28.2	30 9	32 1	33.2	35.4	37.7	39.6	41.6
64	15.1	19.2	22.7	25 9	28.6	31.3	32 6	33.7	35 9	38.2	40.2	42 2
66	15 3	19.5	23.0	26 2	29.0	31.7	33.0	34.2	36.4	38.7	40.8	42.8
68	15 5	19.7	23.3	26.5	29.4	32.1	33 4	34 7	36 9	39.2	41.4	43.4

Example. What is the width of a rectangular duct 6 in high equivalent to the pressure-loss for a duct 12 in in diameter? **Solution.** 22 in.

Table LVII. Friction Pressure-Loss of 90° Elbows

Radius of throat in diameters of pipe	¼	½	¾	1	1¼	1½	2	3	4	5
Number of diameters of straight pipe of equivalent pressure-loss . .	67	30	16	10	7.5	6	4 3	4.8	5.2	5.8

Example. A duct 12 in in diameter and 120 ft long contains two 90° elbows. The ratio of the radius of throat to pipe-diameter is 3. The amount of air flowing is 1 500 cu ft per min and the velocity 1 800 ft per min.

Solution. The total equivalent length of duct is

$$120 + (2 \times 4.8) = 129.6 \text{ ft}$$

The pressure-loss, from the diagram of Fig. 64, is 0.48 in per 100 ft. The loss is

$$0.48 \times (129.6/100) = 0.62 \text{ in of water}$$

The pressure-loss through register-grills may be taken at 0.023 in for a velocity of 400 ft per min through free area. The gross area of registers is twice the free area. The pressure-loss in standard air-washers for a velocity of 400 ft per min through the free area may be assumed to be 0.15 in of water. In the case of humidifiers, in which the spray is directed against the flow of air, a pressure-loss of 0.55 in of water may be assumed for preliminary estimates. The values assumed for this loss vary with different manufacturers. The pressure-loss through hot-blast heaters may be taken from Table LVIII.

Table LVIII. Friction of Air through Vento Heaters

Friction-loss, in inches of water, due to air passing through Vento stacks, based on free area of heater. Regular section. Standard 5-in spacing of loops. Air-temperature 70° F.

Velocity through free area in feet per minute	One stack	Two stacks	Three stacks	Four stacks	Five stacks	Six stacks	Seven stacks
800	0.037	0.070	0 103	0.135	0.167	0.200	0 232
900	0 047	0.088	0 129	0 170	0 211	0 252	0 293
1 000	0.059	0.109	0.160	0.211	0.262	0.313	0 364
1 100	0.071	0 132	0.193	0 255	0.316	0.377	0 438
1 200	0.084	0 157	0.230	0 303	0.376	0.449	0 522
1 300	0.099	0 185	0.271	0.356	0 442	0 528	0 614
1 400	0.115	0.214	0 314	0 414	0 513	0.612	0 712
1 500	0.132	0 246	0.360	0.474	0.588	0 702	0 816
1 600	0.150	0.280	0.410	0 540	0.670	0.800	0 930
1 700	0.169	0.316	0.463	0 609	0.756	0 903	1 049
1 800	0.190	0 354	0.518	0.683	0.848	1.012	1.177

Effect of Temperature on Pressure-Losses. The preceding data on pressure-losses in ducts, registers and heaters are based on an air-temperature of 70°. For other temperatures, the pressure-losses are to be multiplied by the ratio, density of air at actual temperature to density at 70°. These ratios are given in Table LIX. For heaters use the average temperature of the air passing through the heater.

Design of Duct Systems. There are two schemes used in PROPORTIONING AIR-DUCTS: (1) the velocity method, and (2) the method of equal friction pressure-loss per foot of length. The first method involves the fixing of the velocities (see Table LIV) in the various sections, and the gradual reduction of the

Table LIX. Ratios of Density of Air at Actual Temperature to Density at 70° F.

Temperature	Factor	Temperature	Factor
100	0 945	140	0.880
120	0.910	150	0 865
130	0.890	160	0 850

velocity from the beginning of duct to the point of discharge. In this case the pressure-loss is computed separately for each section having a different velocity and the various pressure-losses added together to obtain the total loss in pressure. The second method is used principally in the design of duct systems for factory-heating. The velocity in the outlet farthest from the fan is fixed and the area and diameter of this branch are determined by the volume of air to be delivered. The friction pressure-loss per 100 ft of a duct of this size is determined by the diagrams in Figs. 76 and 77. The remainder of the main duct is then proporticned for this same pressure-loss per 100 ft.

Example. The first method is illustrated in Fig. 78, showing a single-duct system. The risers are figured for a velocity of 600 ft per min, or 10 ft per sec;

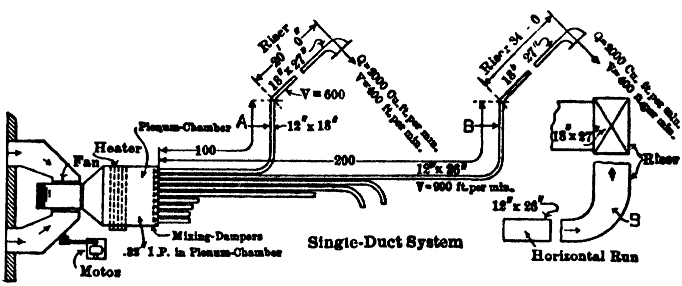


Fig. 78. Single-duct System

and 400 ft per min, or 6.6 ft per sec; through free area of register-grill. The velocity in the longest main, *B*, is 900 ft per min, the volume of air to be delivered 2 000 cu ft per min, and the temperature 120°.

Solution. The area of the riser required is

$$2\ 000/600 = 3\tfrac{1}{3} \text{ sq ft, or } 480 \text{ sq in}$$

An 18 by 27-in riser, giving 486 sq in area, is used. The area of main, *B*, is

$$2\ 000/900 = 2.22 \text{ sq ft, or } 320 \text{ sq in}$$

The size of the duct is 12 by 27 in. The diameter of a round duct for the same friction-loss for the riser, from Table LVI, is 24 in, and for the main, *B*, 19.5 in.

Referring to the diagram in Fig. 76, the pressure-loss for the riser is 0.032 in per 100 ft. For the main the pressure-loss is 0.09 in per 100 ft. The main has one elbow and the riser one elbow, and the ratio of radius at throat to diameter will be assumed to be 3, in both cases. The equivalent length of main, *B*, is therefore

$$200 + (4.8 \times 20/12) = 208 \text{ ft}$$

Table LX. Speeds, Capacities and Horse-Powers, American Sirocco Single-Inlet, Standard-Width Fans

American Blower Company

Air-temperature, 70° F. Ratio of total to static pressure, 1.15. Ratio of velocity to static pressure, 0.15. Fifty-per-cent opening

Fan-number	Diameter of wheel in inches	Maintained resistance or static pressure in inches of water-gauge	¼	½	¾	1	1¼	1½	2
4	24	C. F. M.* R P. M. B H. P.	2 425 225 .244	3 480 320 .526	4 260 391 97	4 920 453 1.485	5 500 505 2.08	6 020 554 2.725	6 945 640 4.19
4½	27	C. F. M. K. P. M. B H. P.	3 070 200 308	4 410 284 .664	5 400 348 1.215	6 205 402 1.86	6 970 449 2.62	7 620 492 3 44	8 800 568 5.29
5	30	C F M. R. P. M. B H. P.	3 790 180 .378	5 450 256 .811	6 650 313 1.492	7 690 362 2.295	8 600 403 ʒ.225	9 416 443 4.22	10 870 512 6.48
6	36	C. F M. R P. M. B. H. P.	5 460 150 .541	7 835 214 1.06	9 580 260 2.14	11 060 302 3.28	12 350 336 4.6	13 540 369 6.01	15 630 427 9.27
7	42	C. F. M. R. P M. B H. P.	7 425 129 .731	10 670 183 1.57	13 050 223 2.9	15 070 259 4.44	16 800 288 62.2	18 425 316 8.15	21 260 336 12.55
8	48	C F. M. R P. M. B. H. P.	9 720 112 .954	13 920 160 2 045	17 000 196 3 76	19 700 226 5.78	22 000 252 8.14	24 100 277 10.61	27 820 320 16.32
9	54	C. F. M. R. P. M. B. H. P.	12 250 100 1.206	17 615 142 2 58	21 500 174 4 73	24 860 201 7 26	27 800 224 10 22	30 440 246 13.38	35 140 285 20 5
10	60	C F. M. R. P. M. B. H. P.	15 150 90 1 473	21 760 128 3.17	26 500 156 5.81	30 750 181 8.95	34 300 202 12.55	37 650 222 16.48	43 400 256 25.25

11	66	C F M.	18 350	26 350	32 200	37 200	41 500	45 530	52 550
		R P. M.	82	117	142	165	184	202	233
		B H. P.	1.78	3 82	7 05	10.78	15.15	19 85	30 45
12	72	C. F. M.	21 800	31 350	38 300	44 240	49 400	54 130	62 500
		R. P. M.	75	107	130	151	168	185	214
		B. H. P.	2.11	4.54	8.35	12.81	18.0	23 55	36 2
13	78	C. F. M.	25 600	36 800	45 000	52 000	58 100	63 600	73 500
		R. P M.	69	99	120	140	155	171	197
		B. H. P.	2 465	5 31	9 8	15.0	21.1	27 6	42.5
14	84	C. F M.	29 650	42 650	52 100	60 200	67 300	73 700	85 000
		R P M.	64	92	112	130	144	158	183
		B. H. P.	2 87	6.15	11.33	17.4	24.45	32.0	49.2
15	90	C F. M.	34 100	49 000	59 900	69 230	77 500	84 700	97 800
		R P M.	60	86	104	121	135	148	171
		B. H. P.	3 27	7 06	13 0	19.93	28 0	36 6	56.4
16	96	C. F. M.	38 800	55 900	68 100	78 750	88 400	96 500	111 800
		R. P. M.	56	80	98	112	126	139	160
		B. H. P.	3.73	8 0	14.8	22.65	31 8	41.6	65.1
17	102	C. F. M.	43 800	63 100	77 000	89 000	99 900	109 000	126 000
		R P M.	53	75	92	106	119	130	150
		B H. P.	4 21	9 03	16.75	25 6	35 9	46 9	73 5
18	108	C. F. M.	49 200	70 750	86 400	99 900	112 000	122 000	141 500
		R P M.	50	71	87	100	112	123	142
		B. H. P.	4 72	10 15	18 8	28 65	40.3	52 6	82 5
19	114	C. F. M.	54 750	78 750	96 200	111 000	124 500	136 000	157 200
		R. P M.	47	67	83	95	106	117	134
		B. H P	5.25	11 3	20 9	31 9	45 0	58 6	91 8
20	120	C. F. M.	60 600	87 300	106 800	123 000	138 000	150 800	174 500
		R. P M.	45	64	78	91	101	111	128
		B. H P.	5 82	12 5	23 1	35 4	49 7	65 0	101.8

Double-inlet fans have approximately double the capacities of single-inlet fans and require for their operation, under the same conditions, approximately twice the power. The "per-cent opening" refers to the opening used for the rating given from the test-data.

*** C. F. M. denotes cubic feet per minute; R. P. M., revolutions per minute; and B. H. P., brake horse-power.**

and the pressure-loss is

$$0.09 \times (208/100) = 0.187 \text{ in}$$

The equivalent length of riser is

$$34 + (4.8 \times 24/12) = 44 \text{ ft}$$

and the pressure-loss is

$$0.032 \times (44/100) = 0.014 \text{ in}$$

The pressure-loss through the register-grill is 0.023 in. The total resistance of the duct system is therefore

$$0.0187 + 0.014 + 0.023 = 0.224 \text{ in}$$

Assuming that a five-section Vento heater is employed, with a velocity (figured through free area) of 1 200 ft per min, the pressure-loss through the heater is 0.376 in (Table LVIII). The total resistance against which the fan must operate is

$$0.224 \text{ in} + 0.376 \text{ in} = 0.60 \text{ in}$$

based on 70° air. Assuming the temperature of the air to be 120°, the resistance is

$$0.60 \times 0.91 \text{ (Table LIX)} = 0.55 \text{ in}$$

The second method of duct-design is illustrated by the example given under Application of Hot-Blast Heating Data.

Ventilating Fans

Steel-Plate Fan. The standard type of fan that was used for a number of years in hot-blast work is known as the STEEL-PLATE FAN, the construction of the wheel being shown in Fig. 79. As the name implies, the wheel and casing of this fan are constructed of steel plate and light structural sections, the wheel having eight to twelve blades, straight or slightly curved at the periphery, and in a direction opposite to the rotation. Steel-plate fans are designated by number, this number being the approximate height of the fan-casing in inches.

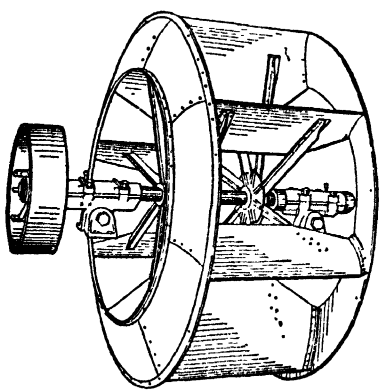


Fig. 79. Standard Steel-plate Fan-wheel

Multibladed Fan. The new type of fan wheel, known as the MULTIBLADE or TURBINE-TYPE IMPELLER, Fig. 80, is now used almost exclusively in the field of heating and ventilating, and on account of its higher efficiency, quieter running, and relatively smaller size for the same capacity than the steel-plate fan, has supplanted the latter. The higher efficiency is accounted for by the material reduction of the air-resistance or pressure-head loss by friction through the fan, due to the shorter blades and the larger inlet, which is of practically the same diameter as the wheel itself.

Rating of Fans. The volume of air at 70° which a fan will deliver (cubic feet per minute) varies with the resistance against which it operates. In order to choose a fan from Table LX, the resistance (static pressure) must first be determined by the duct-design, and after the size of the heater has been chosen and its resistance determined. The speed and brake horse-power required to drive the fan are also stated in the tables. The temperature of the air handled by the fan with DRAW-THROUGH apparatus is higher than 70°, except for a fan which is connected ahead of a tempering-coil, usually a two-section-deep heater. The tabulated speed, volume and brake horse-power to maintain the pressure must be multiplied by the factors given in Table LXI for temperatures other than 70°. The above factors in this table are the square roots of the ratios of the density of the air at 70° F. to its density at the temperature stated.

Table LXI. Factors for Speed, Volume and Brake Horse-power

Temperature in degrees Fahrenheit	Factors
0	0 932
100	1 028
120	1 046
130	1 055
140	1.064
150	1.073

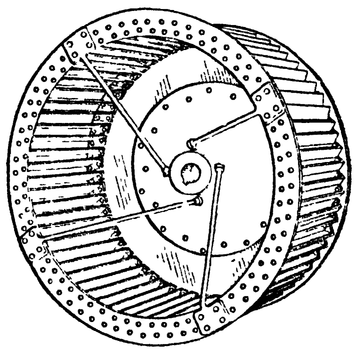


Fig. 80. Multiblade Wheel

Application of Hot-Blast Heating-Data to an Industrial Building

The Application of the Foregoing Data on hot-blast heating to an industrial building follows (see Fig. 81). The calculated heat-loss is 1 423 920 Btu per hr.

Conditions. Air recirculated and inside temperature maintained at 60°. Velocity of air through heater, from 1 000 to 1 200 ft per min. Velocity of air at last outlet in duct system, 1 275 ft per min. Temperature of air delivered by heater, 145°.

Weight of Air to be Circulated per Minute. This is

$$1\,423\,920/[0.24\,(145 - 60)\,160] = 1\,163 \text{ lb}$$

Size of Vento Heater. This condition, 60° initial and 145° final temperature, requires a heater five stacks deep (Table LIII), and a velocity of 1 000 ft per min through free area at a temperature of 70°. The volume of air per minute measured at 70°, to be handled by the heater and fan is

$$1\,163/0.075 = 15\,506 \text{ cu ft}$$

The FREE AREA required is:

$$15\,506/1\,000 = 15.5 \text{ sq ft}$$

Assuming a 60-in length of section with loops 5 in on centers, the free area per section is 0.92 sq ft (Table L). The number of sections per stack is

$$15.5/0.92 = 17$$

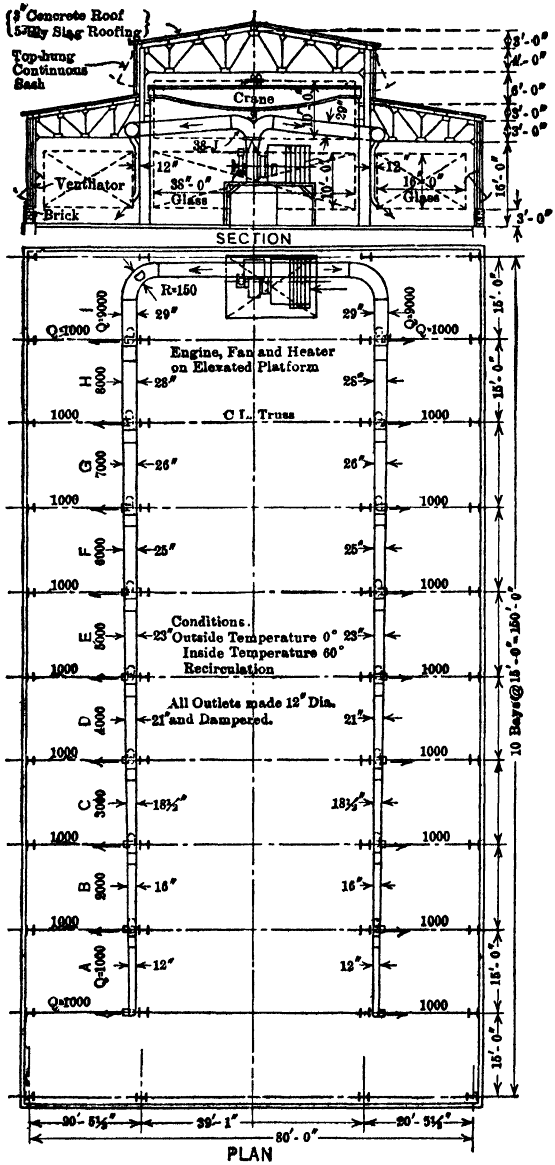


Fig. 81. Hot-blast Heating for a Factory Building. (See Example for Computations)

The total number of sections required is

$$5 \times 17 = 85$$

The total heating-surface is

$$85 \times 16 = 1\,360 \text{ sq ft}$$

Weight of Steam or Condensation per Hour. This is

$$1\,360 \times 1.09 \text{ (Table LIII)} = 1\,482 \text{ lb}$$

The equivalent amount of direct radiation is

$$1\,482/0.25 = 5\,929 \text{ sq ft}$$

Design of Duct System. The round ducts will be designed for equal-friction pressure-loss per foot of length. The final velocity at the last or most remote outlet from fan will be taken at 1 275 ft per min. The friction pressure-loss for this velocity, as read from the diagram in Fig. 76, is 0.25 in of water per 100 ft of length. There are to be eighteen outlets. The total volume of air to be discharged, measured at 145° F., is

$$1\,163/0.065 = 18\,000 \text{ cu ft per min}$$

or

$$18\,000/18 = 1\,000 \text{ cu ft per min per outlet}$$

The cross-sectional area of the outlet or last section is 1 000/1 275 sq ft, corresponding to a circular section with a diameter of 12 in. The branch-outlets may all be made the same size and provided with dampers to adjust or equalize the flow. The friction pressure-loss in the duct system is therefore

$$(212/100) \times 0.25 = 0.53 \text{ in of water}$$

The size of each section of duct is determined by locating the quantity of air at the right of the diagram and passing horizontally to the intersection with the 0.25-in pressure-loss line.

Table LXII. Data for Design of Ducts in Fig. 81

Section	Quantity of air in cubic feet per minute 145° F.	Duct-diameter in inches	Velocity in feet per minute	Measured length plus allowance for ells, in feet
A	1 000	12	1 275	$25 + [1 \times (6+3)] = 34$
B	2 000	16	.	15
C	3 000	18½		15
D	4 000	21	..	15
E	5 000	23	. ...	15
F	6 000	25	....	15
G	7 000	26	......	15
H	8 000	28	.	15
I	9 000	29		$35 + [2.4(6+10)] = 73$
J	18 000	38	2 285	..
				Total length = 212

Selection of Fan for Draw-Through Arrangement. The static pressure rating required, referred to a temperature of 70°, is.

Pressure-loss in heater (data from Table LVIII) = 0.26 in
Pressure-loss in duct (data from chart, Fig. 76) = 0.53 in
Total = 0.79 in

The actual pressure-loss will be somewhat less, owing to the fact that the air-temperature is higher (145° F.) and the density less than for air at 70° F. The actual estimated pressure-loss is therefore assumed to be $\frac{3}{4}$ in.

The volume of air the fan must handle in this example is 18 000 cu ft per min, measured at 145° F. As stated under Rating of Fans, to maintain a constant pressure the tabulated speed, volume and horse-power must be multiplied by the square root of the ratio of densities, or

$$\sqrt{0.075/0.066} = 1.07 \text{ (nearly) (Table LIX)}$$

We therefore select from Table LX a fan having a capacity, measured at 70° F., equal to

$$18\,000/1.07 = 16\,822 \text{ cu ft per min (approximately 17 000)}$$

when operating with a static pressure of $\frac{3}{4}$ in. A No. 8 Sirocco fan fulfills this requirement. The tabulated speed and horse-power when multiplied by the factor 1.07 gives

$$196 \times 1.07 = 210 \text{ r.p.m.}$$

and

$$3.76 \times 1.07 = 4.02 \text{ brake horse-power}$$

To allow for the contingency of possible overload add 20%, making the required horse-power equal to 4.82, say, a 5 horse-power motor.

Selection of Fan for Blow-Through Arrangement. In this case the fan may be called upon to handle air at a temperature of 0° F., or lower. Assuming the same weight of air, or 70 000 lb per hr, to be handled by the fan at a static pressure of $\frac{3}{4}$ in, the volume at 0° is

$$70\,000/(0.086 \times 60) = 13\,566 \text{ cu ft per min}$$

Referring to Table LXI, the ratio between the speed, volume and power necessary to produce the same pressure for air at 0° and air at 70°, is found to be 0.932. We therefore choose a fan with a capacity of

$$13\,566/0.932 = 14\,557 \text{ cu ft of air at } 70°$$

and with a static pressure of $\frac{3}{4}$ in.

Fan-Engine. When high-pressure steam is available an automatic high-speed engine is frequently employed for fan-driving, and the exhaust from the engine is used in the first section of the heater.

Selection of Motor for Fan-Driving. It is considered good practice to add from 15 to 20% to the brake horse-power, as determined from the fan-tables, for the rating of the motor, to allow for a possible overload due to the fact that the fan may not be operated under exactly the same conditions as to pressure and speed as those under which it was originally rated.

Additional Heating Requirement. It is frequently desirable to proportion the heating-apparatus large enough so that the fan may be shut down at night and started up about two hours before the shop or factory is opened in the morning. In this event it may be safely assumed that the temperature of the air in the building will not be below 30° F. when the fan is started, and that the air is all recirculated. The fan and heater must be of sufficient capacity to take care of the heat-loss from the building, including the infiltration, and in addition to warm up the contained air from 30° to 60° in two hours. Assuming the same data as given in the preceding example, the additional heat required will be, if the cubic contents of the building are 328 000 cu ft,

$$(328\,000 \times 0.08 \times 0.24 \times 30)/2 = 94\,464 \text{ Btu per hr}$$

This amounts to an increase of approximately 7% in the heating requirements as previously calculated, and is readily provided for by increasing the steam-pressure carried in the heater to approximately 10-lb gauge. Catalogues, bulletins, etc., on the subject of hot-blast heating, air-washing and humidification may be obtained from the American Blower Company, the B. F. Sturtevant Company, the Buffalo Forge Company, and the Carrier Air Conditioning Company.

Example.* **Schoolhouse Ventilating System** (see Fig. 82). It is here assumed that ventilation is to be supplied at the rate of 30 cu ft of air per minute per pupil. The heating requirements are to be taken care of by the installation of direct radiation (split system).

The air quantities and velocities assumed for the various sections of the trunk-line duct system are given by Table LXIII.

Table LXIII. Air Schedule for Typical School Layout

Room number	Cubic content	Dimensions, ft	Floor, sq ft	Number of pupils	Air required per min, cu ft	Size of riser, in	Velocity, ft per min	Size of diffuser, in	Diffuser velocity, ft per min
1	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
2	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
3	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
4	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
5	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
6	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
7	8 720	33 ×22×12	726	50	1 500	16×24	563	24×30	300
8	8 720	33 ×22×12	726	50	1 500	16×24	563	24×30	300
9	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
10	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
11	5 800	22 ×22×12	483	33	990	16×16	556	16×30	297
12	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
13	3 360	21½×13×12	280	20	600	12×16	450	16×20	270
14	3 360	21½×13×12	280	20	600	12×16	450	16×20	270
15	17 500	54 ×27×12	1 460	100	3 000	2–16×24	564	24×30	300
16	6 480	20 ×27×12	540	37	1 110	16×20	503	20×26	289
17	7 000	26½×22×12	583	40	1 200	16×20	540	20×30	288
Auditorium	37 600	65 ×34×17	2 220		2 500	2–16×20	565	20×30	301
Office	3 700	14 ×22×12	308		250	8×12	375	12×12	250

The pressure losses (Table LXIV), for the various sections of the duct were determined in a manner similar to the method previously given.

The bottom of all diffusers located in the walls at the end of supply risers to be 8 ft above floor-line. The vent openings to be located at the floor-line.

Unit heaters are construed to mean any combination of an indirect radiator and fan or blower, constructed or manufactured to be sold and erected as a unit having a common enclosure or casing. A motor or other auxiliary apparatus, such as valves, bypass dampers, louvers, screens, controls, etc., may or may not be included as a part of the unit, as may be specified.

* C. A. Criqui.

Table LXIV. Trunk-Line Distribution for Typical School-Building (See Fig. 82)

Section,	Velocity, ft per min	Volume, cu ft per min	Area, sq ft	Size, ft-in	Length, ft	Equivalent, round duct diameter, in	Pressure loss, inch of water
A......	1 300	26 455	20.35	10- 6×2- 0	14	57	0 0070
B..	1 150	13 815	12.01	6- 0×2- 0	10	44	0 0057
C .	1 100	13 541	12 3	6- 0×2- 0	12	44	0 0060
D....	1 000	10 510	10 5	5- 3×2- 0	6	41	0 0030
E........	900	6 550	7 3	3- 8×2- 0	15	36	0 0048
F ..	850	5 335	6 27	3- 1×2- 0	8	33	0 00288
G... .	800	2 640	3 3	1- 8×2- 0	20	26½	0 0056
Riser S-2-9	540	1 200	2 22	0-16×0-20	20	19½	0 0056
Diffuser.	288	1 200	4 2	0-20×0-30	..		.

Total loss of straight ducts	= 0 040
Allow for diffuser and regulating damper	= 0 100
1 elbow in B	= 0 042
2 elbows in G	= 0 007
2 elbows in riser	= 0 003
Total for duct system.	= 0 192
Pressure loss, fresh air intake, 1 5 × 0 05 v p. Velocity 900 ft per min)	= 0 075
Pressure loss tempering coil, 1 000 ft per min velocity . .	= 0 100
Pressure loss through air washer 500 ft per min	= 0 230
Pressure loss through reheater 1 000 ft per min velocity. .	= 0 100
Pressure loss, fan connection to duct system, 0 48 × 0 16 v p. at fan outlet (1 600 ft per min)	= 0 076
Total pressure loss....	= 0.773

The heating and ventilating of buildings is frequently accomplished by means of unit heaters which incorporate in a SINGLE UNIT all the apparatus necessary for providing, directing and controlling the necessary volume of air heated to the proper temperature for the purpose. For schoolhouse purposes, the unit heater frequently provides for the ventilation requirements only, the heating being taken care of by direct radiation.

Unit heaters generally have no air ducts attached either to the outlets or to the inlets, the units being complete heating batteries in themselves.

They may be located in the various rooms or parts of a building for discharging warmed air into the room or building, and either entirely or partially recirculating the air, or else having a fresh-air connection for special ventilation purposes. These units, in the case of a schoolroom and sometimes in factories, are set on the floor, but more frequently in factories suspended from the roof-trusses so as to leave the floor-spaces unobstructed.

The schoolroom ventilating units are usually located under a window so that fresh air may be taken into the unit over the window-sill and under the bottom sash of the raised window. Otherwise special openings through the walls are required.

In the case of the factory units, the feature of noise produced by the air passing through the fans, heaters and ducts is not particularly important, but a specification for the schoolroom unit should state that the specified air quantities be delivered into the schoolroom without any objectionable noise

The engineering information given throughout this chapter is applicable to the calculations of heating and ventilating installations where unit heaters are to be used. The data as to sizes, capacities, etc , of the various makes of units must necessarily be obtained from the manufacturers.

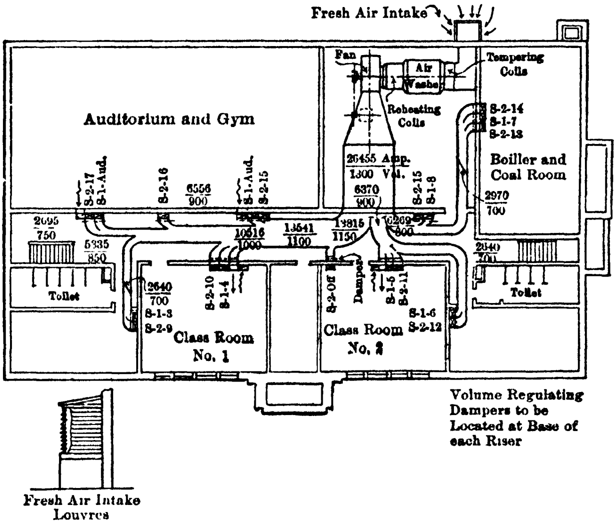


Fig 82. Metal Duct Distribution. Typical School Layout. Basement Plan.

Classification of Unit Heaters. Unit heaters may be broadly classified in accordance with the type of building they are designed to serve: (1) SCHOOLS and PUBLIC BUILDINGS, (2) INDUSTRIAL and FACTORY-BUILDINGS.

School and Public-Building Class. This class, usually located under a window, consists of a small ventilating fan or fans of the enclosed multivane type, fan motor and indirect heating surface of either copper or cast iron all enclosed in a finished sheet-steel casing provided with an outside air intake and recirculation intake. Each intake is provided with a damper and the dampers are usually so connected that the full opening of one closes the other. The fan draws in either all outside air, all recirculated room air, or a mixture of the two as may be desired.

One type of unit heater in this class is shown by Figs. 83–84.

The weight of air M to be circulated per hour is fixed by the ventilating requirements, usually 25 to 30 cu ft per minute per occupant of the room measured at 70° F.

If N = number of occupants;

then $M = N \times 30 \times 0.075 \times 60 = 135\,N$

Case I. If no direct radiation is installed, the entire heating and ventilating requirements must be taken care of by the unit heaters.

Let H = heat loss of room Btu per hour;

t = temperature to be maintained in the room;

t_y = temperature of air leaving the unit (see manufacturers' tables).

$$H = 0.24\,M\,(t_y - t)$$

$$t_y = \frac{H}{0.24\,M} + t$$

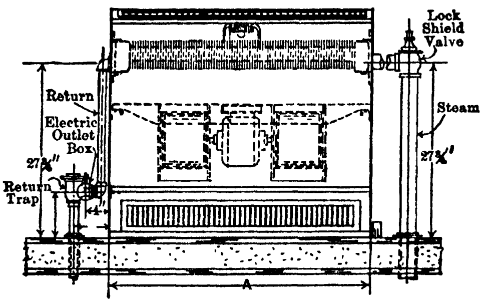


Fig. 83. Unit-heater, Type A Peer Vent

The amount of heat to be supplied by the heater will depend entirely on the amount of outside air and its temperature that must be circulated in accordance with the ventilating requirements.

Let t = temperature of outside air.

If all outside air must be employed and H_1 = heat to be supplied by the indirect heater in Btu per hour, then

$$H_1 = 0.24\,M\,(t_y - t_0) \tag{1}$$

If all of the air is recirculated,

$$H_1 = 0.24\,M\,(t_y - t) \tag{2}$$

This is the same amount as would be required from the direct radiation.

The heating surface to be installed is based on Equation (1) and as the heat loss H of the room varies, the bypass damper, under thermostatic control, automatically bypasses more or less air around the heater, thus controlling the temperature of the air t_y entering the room and consequently the heat which this air gives up to balance H.

Case II. Heat loss of the room, H, is to be taken care of by the installation of direct radiation and the ventilating requirements by unit heaters.

The area in square feet of direct cast-iron radiation to be installed = $\frac{H}{R}$, in which R = Btu emitted per square foot per hour by cast-iron radiator (usually assumed 240). In this case t_y should be equal to or slightly higher than the room temperature, t, to be maintained, usually 70°.

Then $H_1 = 0.24\,M\,(70 - t_0)$. The total heat to be supplied in this case will be $H + H_1$ Btu per hour.

We have assumed in the above formula a unit heater having the actual capacity required with $t_y = 70°$. Actually, however, it is necessary to employ a standard unit heater. These units are rated on a basis of cubic feet per minute C delivered at a certain temperature t_y with an initial temperature t_0. Choose the nearest size heater having a capacity C above 30 N cubic feet per minute, measured at 70°, the temperature t_y being stated in the table for assumed outside air temperature, t_0.

The heat available from the unit heater for heating the room, H_h, will be

$$H_h = 0.24 \times 60 \times C \times .075 \times t_y - t) \text{ Btu per hour}$$
$$(0.075 = \text{density of air at } 70°)$$

Subtract this amount from H in determining the direct radiation to be installed; or the square feet of direct radiation actually to be installed will be $\dfrac{H - H_h}{R}$.

The heat to be supplied to the ventilating unit will be

$$H_v = 0.24 \times 60 \times C \times 0.075\,(t_y - t_0) \text{ Btu per hour}$$

The total heat to be supplied both the direct and indirect radiation will therefore be

$$(H - H_h) + H_v \text{ Btu per hour.}$$

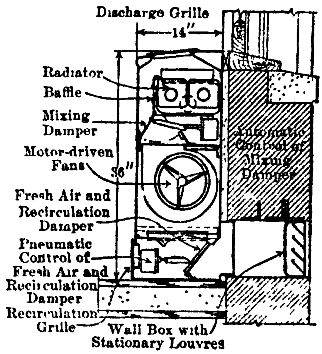


Fig 84. Section Through Peer Vent Unit-heater

Example, Case I. Assume a schoolroom designed for 40 pupils with a heat loss $H = 30\,000$ Btu per hour, both heating and ventilating requirements to be taken care of by unit heaters.

$t = 70°$. $t_0 = 0$. All fresh air to be circulated at the rate of 30 cu ft per minute per pupil

$$M = 40 \times 30 \times 0.075 \times 60 = 5\,400 \text{ lb per hour}$$

$$t_y = \frac{30\,000}{0.24 \times 5\,400} + 70 = 93.1°$$

$$H_1 = 0.24 \times 5\,400\,(93 - 0) = 115\,506 \text{ Btu per hour}$$

Assuming 2-lb gauge steam-pressure, and corresponding latent heat of 970 Btu per lb, the condensation per hour will be $\dfrac{115\,506}{970} = 119.1$ lb.

Example, Case II. Assume the same data as in the preceding example. The volume of air required for ventilation based on 30 cu ft per min per pupil is $30 \times 40 = 1\,200$ cu ft per min.

The equivalent amount of direct radiation required is $30\,000 \div 240 = 125$ sq ft.

Referring to a manufacturer's rating table, we find that the amount of direct radiation which a certain unit will replace or that can be omitted is

74 sq ft; therefore the actual direct radiation to be installed will be approximately 51 sq ft.

Factory-Type Unit Heaters. The complete unit consists of a fan or fans, indirect heating surface of pipe coil, cast-iron sections or copper sections, all enclosed in a sheet-metal jacket. Factory heaters are of various types, designed to be hung from the roof structure, placed on a wall-bracket, or set on the floor.

Generally, the air passing through the unit is all recirculated air as sufficient air for ventilating purposes, in the average industrial plant, is provided by the infiltration, unless odors, fumes, etc., given off by the process require outside air to be drawn into the building. If the HUMIDIFICATON of the air is a requirement, combination unit heaters are on the market in which there is an air washer, provided with a water heater, capable of saturating the air passing through the unit. A fresh-air intake is usually provided, in this case, so that cooling of the air for ventilation during the summer months may be accomplished.

There are a large number of factory-type unit heaters on the market but space will not permit of illustrating any unit heaters of this class.

The installation of floor-type unit heaters with recirculation in single-story industrial buildings will effect a saving in the fuel required for heating variously estimated from 10 to 30% as compared with direct radiation. The initial cost of the unit-heater installation is less than the equivalent direct-radiation installation.

The saving in fuel is effected by recirculating the air at a lower level, which produces lower air temperatures at the higher wall elevations and under the roof deck than is obtained with direct radiation, the heat loss of the building thus being reduced.

This class of heater is rated by the manufacturer on a basis of Btu output with recirculated air at 60° – 65° F., the number of units required of any certain size being determined by dividing the calculated heat-loss of the building (H) by the Btu output of the unit selected. Consult manufacturers' literature for further information in reference to unit heaters and their rating.

Ventilation

Natural and Mechanical Ventilation. Ventilation, whether NATURAL or MECHANICAL, consists in the displacement of vitiated air from an apartment and its replacement by fresh air. To state that the air in an apartment is renewed any given number of times per hour is not strictly accurate, as a positive change does not actually occur; the incoming air mixes with and dilutes the foul air to a point suitable for healthful respiration. In NATURAL-VENTILATION systems the movement of the air in flues, ducts, etc., is induced solely by the thermal head produced by the difference between the density of the column of air in the ducts and that of the outside atmosphere; the higher the temperature in the ducts the more powerful the draft. The direction and velocity of the wind materially affect the natural ventilation, retarding or accelerating the movement of the air through ducts and flues, according to the exposure of the building and the position of inlets and outlets. In MECHANICAL VENTILATION the movement of air is positively maintained by means of various types of fans, driven by an engine or electric motor. With fans of known efficiencies the results can be accurately estimated. The principal advantages of the use of mechanical systems of heating and ventilation have already been stated under Hot-Blast Heating.

Systems of Ventilation. Ventilation systems are also broadly divided into two general classes known as the UPWARD SYSTEM and the DOWNWARD SYSTEM. The UPWARD SYSTEM (Fig. 85) is sometimes used for audience-rooms where

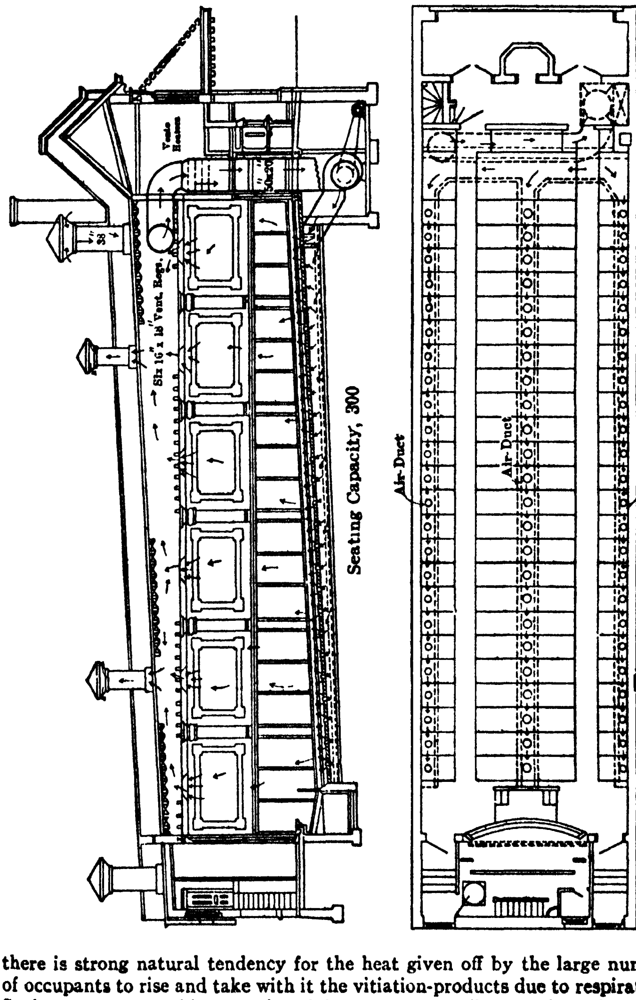
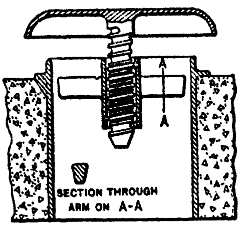


Fig. 85. Small Moving-picture Theater. Upward System Ventilating

there is strong natural tendency for the heat given off by the large number of occupants to rise and take with it the vitiation-products due to respiration. Such systems are seldom employed for summer ventilation when air cooled below the average desired room temperature must be introduced. Upward systems are not always a success for winter ventilation and many designers prefer a downward system for both summer and winter conditions. The

AIR IS SUPPLIED NEAR THE FLOOR-LINE through mushroom ventilators in the floor, or through the hollow pedestals of the chairs themselves, or through low registers. The VITIATED-AIR OUTLETS ARE IN OR NEAR THE CEILING. This system makes it rather difficult to heat the room in advance of the arrival of the audience as the outlets allow the warmed air to escape almost as rapidly as it can be introduced. The DOWNWARD SYSTEM (Figs. 87–88) is very generally used in school-rooms, hospitals, institutions, and audience-rooms, as theaters, moving-picture houses, etc. The occupants are not as closely placed in a school-room as in a theater, and a more even distribution of air and more uniform heating can be secured when the AIR IS SUPPLIED EIGHT FEET OR MORE ABOVE THE FLOOR, and the VITIATED AIR REMOVED AT OR NEAR THE FLOOR-LINE. On account of the elevation of the inlets above the heads of the occupants there is little liability of drafts, and if the outlets are on the same side wall as the inlets there is very little opportunity for short-circuiting between inlet and outlet, since the incoming air must flow out across the room to the cold outside wall before it can cool and drop to the floor-level. It is, however, necessary in the downward system, to overcome the natural tendency of the heated air from the bodies of the occupants to rise



SIZES OF "ABC" MUSHROOM VENTILATOR

Size.	Approximate Inside Diameter.	Approximate Weight
4	$4\frac{1}{8}$	6 lb
5	$5\frac{1}{8}$	10 lb
6	$6\frac{5}{8}$	15 lb

Fig. 86. Section through A B C Mushroom Ventilator

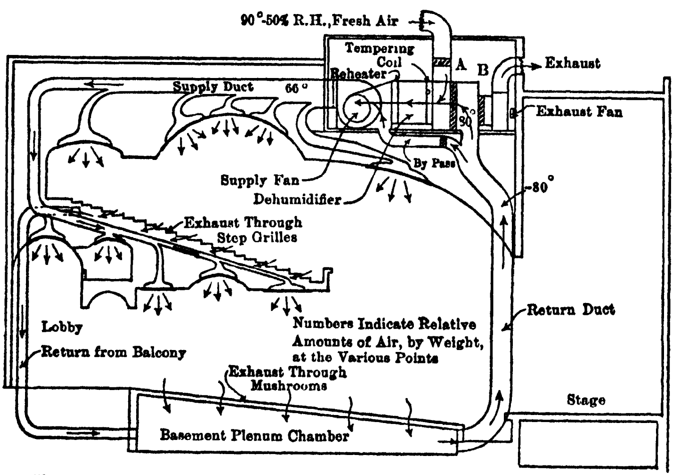


Fig. 87. Section through Theater Showing Cooling Plant. Downward System

and oppose the uniform downward tendency of the incoming fresh air. The selection of either system must depend entirely on the conditions to be met. These have been outlined in the above paragraphs.

In some recent theater and ball-room installations the EJECTOR SYSTEM (Fig. 88) has been employed. Air is introduced at a high velocity at the rear wall well above the audience through outlets unobstructed by grilles or any ceiling beams. A more vigorous circulation is possible with this arrangement than with any other method. This system is somewhat difficult to design as the size of the outlets and velocity of air must be carefully proportioned to to obtain satisfactory results.

The Effect of Vitiated Air. The amount of CARBON DIOXIDE present in vitiated air has been, until recently, quite generally understood to be the element of danger that should be kept within safe limits. Dr. Ira Remsen has pointed out that the presence of carbon dioxide in itself is not dangerous to health except that it reduces the supply of oxygen by displacing it. Carbon dioxide is not poisonous, but the ORGANIC IMPURITIES that are exhaled at the same time with other gases that are given off may prove a menace to health. The ill effects of breathing air in a poorly ventilated room are due to the small

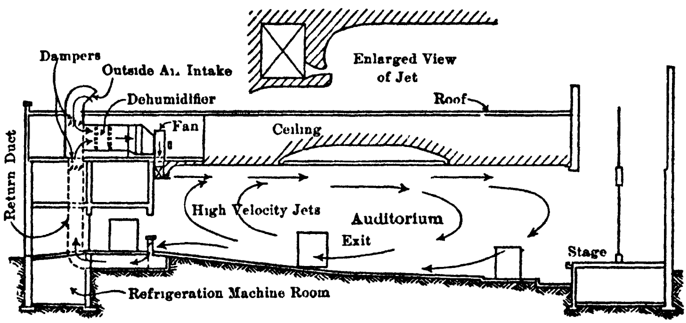


Fig. 88. Section through Theater Showing Ejector System

quantities of decomposing organic matter and unhealthful gases. The carbon dioxide generated by the lungs and given off at the same time as the other impurities serves more or less as an indicator of the presence of the real danger. Any lowering of the oxygen-supply that is actually required for the proper and necessary transformation of the potential heat-value of the food into the physical and nervous energy required to keep the human machine running, and to readily supply the additional demand made upon that machine to perform external work, means that industrial workers who perform their duties in a vitiated atmosphere do so at the expense of a lowered vitality, and are naturally less productive. Satisfactory ventilation consists not only in constantly supplying, in a pure condition, fresh air FREE FROM ODORS, DUST AND OTHER IMPURITIES, at the proper temperature and WITH THE PROPER AMOUNT OF MOISTURE PRESENT, but also in efficiently removing the vitiated air. This cannot be positively or satisfactorily accomplished during the heating-season by simply opening the doors and windows. Some mechanical means must be employed.

Relation between Humidity and Temperature. The proper and healthful RELATIVE HUMIDITY OF THE AIR in buildings has only in recent years been given the thought and attention it rightfully deserves. Heated or warmed air, whether purposely introduced into a building for warming, or naturally entering by infiltration, on being expanded by heat, has its percentage of moisture

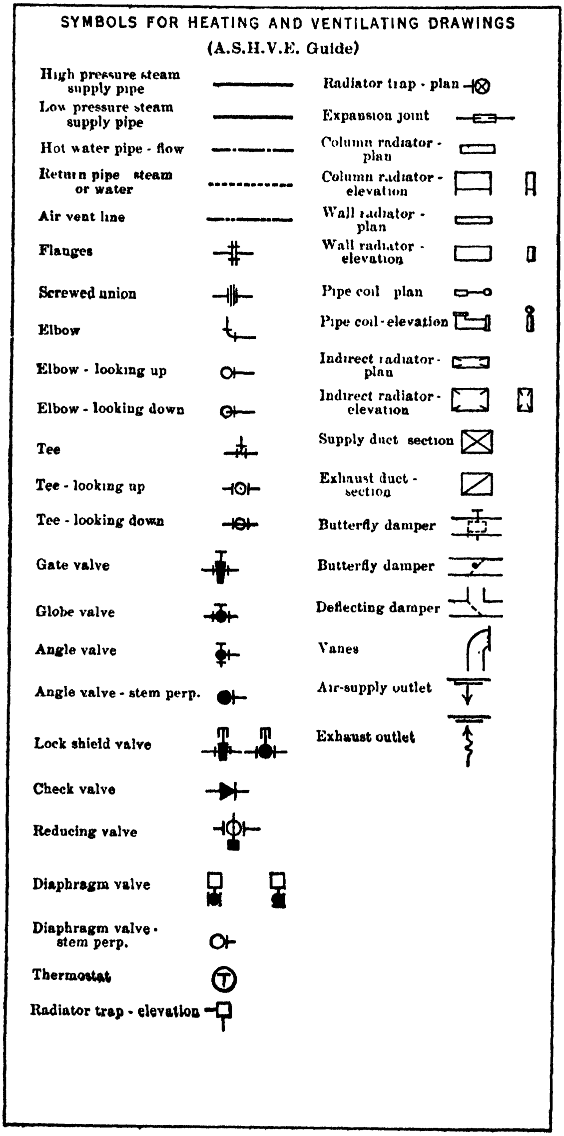


Fig. 89. Heating and Ventilating Symbols

or relative humidity lowered, and consequently its CAPACITY FOR ABSORBING MOISTURE greatly increased. There is, therefore, experienced the sensation due to so-called DRY HEAT. This causes an excessive and unnatural evaporation of moisture from the skin and from the membranes of the respiratory organs. Evaporation takes place by the direct application of heat and is essentially a refrigerating or cooling process. The abstraction of heat from the body for this purpose naturally tends to lower the surface-temperature, and one feels several degrees cooler than the temperature recorded by the thermometer in the room. Human comfort as affected by TEMPERATURE, HUMIDITY, and AIR MOTION has been a subject of research for a number of years by the research staff of the American Society of Heating and Ventilating Engineers. It has become standard practice, in this country, to install mechanical ventilating systems equipped with air-washers, heaters and refrigerating apparatus in all important moving-picture theaters and assembly-halls, the control of the temperature and relative humidity being made automatic. A STANDARD RELATIVE HUMIDITY may be obtained when mechanical ventilation is used by the addition of a HUMIDIFIER to the system.

Ventilation and Air-Conditioning Standards

Extract from A.S.H.V.E. Ventilation Code (1934)

Air Quantity. Not less than 30 cu ft per min of air to be circulated through room per occupant.

Not less than 10 cu ft per min of air to be drawn into the system from outside per occupant.

Temperature and Humidity. The maintained inside relative humidity (R.H.) shall not be less than 30% or more than 60% when heating is required. The effective temperature range shall be between 64° and 69° F, and 69° to 73° F when cooling is required. (See comfort zone—Comfort Chart).

Air Motion. The air velocity through room measured 36 in above floor line shall not exceed 50 ft per min.

Air Distribution. The air shall be distributed and circulated so that the variation in carbon dioxide concentration shall not exceed one part in 10,000 measured 36 in above floor line. The data following are not a part of this Code.

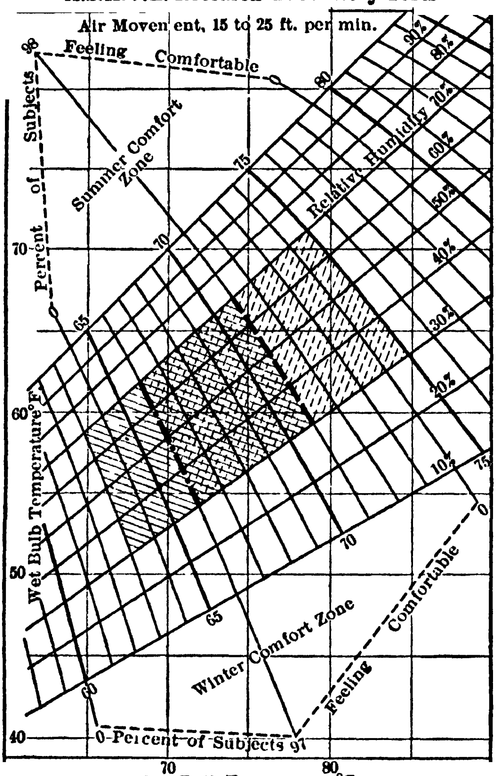
Table LXV. The Usual Requirements for Air Supplied per Person are as Follows (G. D. Small)

HOSPITALS	Ordinary	from 35 to 40 cu ft per min
	Epidemic	80 cu ft per min
		Air-change
	Detention-rooms	6 min
	Toilet-rooms	6 min
	Bath-rooms and duty-rooms	8 min
	Kitchens	3 min
	Serving-rooms	10 min
	Fumigating-rooms	10 min
WORKSHOPS	25 cu ft per min	
PRISONS	30 cu ft per min	
THEATERS	from 20 to 30 cu ft per min	
MEETING-HALLS	20 cu ft per min	
SCHOOLS	30 cu ft per min per child and 40 cu ft per min per adult	

Air-Conditioning for Comfort. The A.S.H.V.E. Research Laboratory conducted a series of experiments to determine various combinations of temperature, relative humidity, and air motion to be maintained in an occupied space, which produce a feeling of comfort for the occupants.

COMFORT CHART

A.S.H.V.E. Research Laboratory Tests



Average Winter Comfort Zone
Average Winter Comfort Line
Average Summer Comfort Zone
Average Summer Comfort Line

The results of these experiments have been embodied in a series of charts based on persons normally clothed for either the winter and summer season and for various velocities over the subjects.

Ventilation-Laws. The number of VENTILATION-LAWS has increased very rapidly in the last few years, not only as regards the number of states which have added such laws to their codes, but also as to the scope and effectiveness of these statutes. In many cases a special ventilation-officer or commission has been appointed to see to the enforcement and extension of the requirements for compulsory ventilation, so that it behooves the architect or engineer to become thoroughly familiar with the law of the state or states wherein he practices. These laws in many cases attempt to provide definite regulations for heating and ventilating all classes of public and school buildings. Future legislation will undoubtedly take a more specific form, establishing complete and definite codes for the heating and ventilation not only of public buildings but of workshops, factories and mercantile establishments as well.

Air-Conditioning. Under this heading belong the problems relating to air-washing, humidification, dehumidification, cooling and drying.

A clear understanding of the principles and laws governing air and vapor mixture, termed PSYCHROMETRY, is essential to solve satisfactorily air-conditioning problems. By air-conditioning is meant present day methods and appliances for controlling air cleansing, humidification, heating and cooling, to provide throughout the year conditions within buildings satisfactory to human comfort and health Space limitations do not permit treating this somewhat complicated subject in this chapter.

Typical Specification for a Low-Pressure, Vapor or a Vacuum System of Heating

(NOTE: Use only such description and headings in the following specification as apply to the system contemplated.)

General. This Contractor shall provide all the necessary tools, appliances, construction equipment, temporary scaffolding, supports, etc., as may be required to install the complete heating system hereinafter described.

This Contractor will be bound by the provisions of the detail general conditions, which form a part of the contract documents for the general contract for the building and which will also be made a part of the contract documents for the sub-contract for heating. (If the contract for heating is to be let direct and is not a part of the general contract, it is recommended that the Architect include with this specification such part or parts of "The General Conditions of the Contract for the Construction of Buildings" standard form of the American Institute of Architects, as he may desire to incorporate as part of the contract documents for this work. It is recommended that the standard printed form be employed with such deletions and changes as the Architect may desire).

This Contractor shall do all excavation work, cutting, drilling and patching as may be required to install the heating system and when completed shall remove all temporary scaffolding, debris, etc., resulting from his operations, from the building and property.

This Contractor shall in no case cut or drill any structural member in the building, which would tend to weaken the structure, without first securing permission in writing from the Architect.

This Contractor shall be held responsible for the safety and condition of all materials delivered to the site as required for this operation and all work done until such time as the complete system has been accepted by the Architect.

All materials furnished shall be new and in first-class condition. The work shall be done in a neat and first-class workmanship manner.

Any materials or appliances not fully conforming to the requirements of this specification that may have been incorporated in the work shall, without delay or further notice, be removed and replaced with materials that do conform to the specification.

This Contractor shall repair any and all damage, of any kind, nature or description, to the building that may result from his operations, in a manner that is satisfactory to the Architect before the work will be accepted.

Description of Work. This Contractor shall install a complete (low-pressure, vapor or vacuum) system strictly in accordance with the heating plans, details for same and the following specification:

This system has been designed and apparatus selected to maintain an average breathing line temperature within the building of 70° F. when the outside temperature is (....°) and when operating with a steam pressure of .... lb per sq in gauge at boiler.

Boiler. The boiler to be a hand-fired, low-pressure (cast-iron round or sectional type or steel type) provided with a minimum grate-area of .. sq ft, as manufactured by (list the names of manufacturers acceptable here) designed to burn (anthracite, bituminous, coke, gas or oil) as fuel. (State here character of solid fuel, if same is to be employed, Btu value of gas or oil. If solid fuel is to be employed state the "firing period" in hours desired when the boiler is carrying the load for the temperature conditions for which system is designed.)

This boiler shall be guaranteed by the manufacturer to have been constructed and tested in accordance with the A.S.M.E. Boiler Construction Code.

This boiler shall be connected to a chimney or stack .... in × .. in inside dimensions by . .. ft in height provided by others.

The boiler shall be equipped with rocking grates suitable for fuel specified for solid-fuel boiler, damper in smoke outlet and undergrate damper; 1 accurate pressure gauge ...., in diameter (brass or nickel-plated) with scale reading up to 30 lb per sq in with non-syphon connection to boiler; 1 water column with gauge glass and two trycocks; 1 pop safety-valve set to relieve at . . lb per sq in, complying with A.S.M.E Standards (for vapor systems the valve is usually set to blow at 3 lb per sq in); 1 brass draw off cock; 1 all-metal-type steam-regulator with bellows-type diaphragm for damper control; a full set of firing and cleaning tools supplied by the boiler manufacturer as standard equipment.

Boiler Foundation and Ash-Pit. This Contractor shall construct a concrete boiler foundation and ash-pit, including all excavation and back-filling required, in accordance with the detail for same as furnished by the manufacturer of the boiler.

Smoke-Flue. Connect boiler smoke-outlet to chimney or stack with a round flue the same size as boiler smoke-outlet. Provide a damper with segment-type handle in this flue. Flue to be constructed of galvanized sheet steel (or iron).... U. S. gauge. This flue and its connection with the chimney to be made tight.

Pipe and Fittings. All pipe employed must be new, free from scale or rust, full weight (mild steel or genuine wrought iron). All screwed piping must be fitted with occasional flange unions. Straighten all pipe, ream all ends to remove the burrs and remove all dirt or other material from pipe, fittings and valves before erection.

All fittings employed shall be standard-weight gray iron, recessed type, straight, true, accurately threaded, free from blow-holes and other defects. Where pipe size is reduced in a line, reducing couplings or reducing fittings are to be employed; bushings will not be permitted in these cases.

All runs to be straight and parallel with building lines and all risers to be plumb and straight.

Steam and return-mains shall be graded in the direction as indicated by the drawings and with the minimum grades, as indicated, which are 1 in in 40 ft for steam and dry return-mains and 1 in in 100 ft for wet return-mains. If it is found impossible to obtain the grade in mains as called for by the plans then a "rise and drip" shall be provided at suitable points connecting with a wet return or connected with a dry return through a thermostatic trap.

Pipe-Supports. The steam and overhead dry return-mains shall be supported at intervals not exceeding 10 ft

The hangers employed shall be expansion or adjustable (style and manufacturer's name).

Blow-Down Connection. Install a blow-down connection run to floor-drain in boiler-room, if practical. This connection to be provided with brass blow-down valve and brass cock,

Feed-Water Connection. Connect boiler with outlet in water-main, left by plumber for this purpose, with ¾-in galvanized pipe provided with one brass stop-cock and one brass globe-valve.

Boiler Connections to Steam and Return-Main. Connect all steam outlets on boiler, without reduction, to steam-main as indicated by the plan. Suitable provision has been made to provide for expansion.

(If more than one boiler is to be installed an iron body globe-valve should be specified to be installed in each boiler-lead The main return-connection to each boiler must also be provided with a valve in order to isolate the boiler from the system)

The boiler lead shall be provided with a tee the outlet of which is connected to the steam-main, the lead to be continued beyond this tee and terminated in an ell connected with the wet return to provide for relief. (See Fig. 20.)

The main return shall be connected to the boiler through a hydraulic loop. Top of loop to be 4 in below the mean water-line of boiler.

Supply and Return-Risers and Branches. The supply and return-risers are to be run as indicated by the plans Risers are to be run (exposed or concealed) and are to be of pipe sizes indicated by the plans.

All horizontal branch radiator-connections are to be graded back to risers with as much grade as is possible to obtain and not less than 1 in in 5 ft.

All connections are to be made with ample provision for expansion and IN NO CASE IS A BRANCH CONNECTION TO HAVE A POCKET.

Floor and Ceiling-Plates. Nickeled plates are to be provided wherever an exposed pipe passes through a floor, ceiling or partition. These plates are to be (style and name of manufacturer).

Direct Radiation. Direct radiation shall be installed at locations shown by the floor-plans. All radiation to be either (names of acceptable manufacturers).

(Give a list of radiation to be installed, including the floor and room, type, number of sections, height and manufacturer's rated surface for each radiator.)

Radiator Enclosures. (Give a list with location of each radiator to be provided with enclosures. State material, style, manufacturer's name and finish desired.)

Radiator Inlet Valves. All radiators shall be provided with nickel plates, rough body, graduated or modulating type inlet valve with wood handle. These valves are to be (manufacturer's name). The size of valves to correspond to the following schedules:

¾ in not exceeding 30 sq ft	1¼ in not exceeding 120 sq ft
1 in not exceeding 55 sq ft	1½ in not exceeding 190 sq ft

Orifices or Regulating Plates. (For Vapor or Vacuum System.) All radiator inlet valves to be provided with orifice plates. The diameter of the orifice in each case to be proportioned to the amount of radiation served by the radiator valve for the pressure to be carried.

Air-Valves. (For Low-Pressure Steam Systems, either one-pipe relief or two-pipe low-pressure systems) Each radiator shall be provided with a nickel-plated air- and vacuum-valve of an approved make, guaranteed to eliminate the air from the radiator, not bellow steam or water and to prevent the return of air to the radiator (type or style and name of manufacturer).

(It is very essential that only first-class air-valves be employed to obtain satisfactory heating results)

The ends of the steam-mains are to be provided with the same type of non-return air-valve as installed on the radiators.

Radiator-Traps. (For Vapor and Vacuum Systems.) The return end of each radiator shall be provided with a (style and manufacturer's name) nickel-plated, rough body, thermostatic-type trap.

The size of the trap in each case must correspond to the manufacturer's listed rating for the amount of direct radiation to which the trap is connected.

Drip-Traps. (For Vapor and Vacuum Systems.) Thermostatic-type traps are to be provided for dripping risers, etc., where called for by the plans. These traps are to be of the same make as provided for the radiators.

Dirt-Traps. (Vacuum and Vapor Systems.) The bottom of all steam-risers that are dripped into a dry return-main through a thermostatic-trap must be provided with a dirt-pocket. The drip from steam-mains, connecting with a dry return through a thermostatic-trap must also be provided with at dirt-pocket.

Lift-Fittings and Strainer. (Vacuum System.) The suction line to the vacuum-pump to be provided with lift-fittings and a strainer (if required), as indicated by the drawings.

Vacuum-Pump and Receiver. (Vacuum System.) This contractor shall furnish and install one (rated size or number and manufacturer's name) motor-driven vacuum-pump. The current supplied is (....volts direct or ....phase ....cycle ....volts alternating current).

The motor shall be of ample capacity to operate continuously without undue heating or overload. Pump to be bronze fitted throughout, including the water-impeller and air-rotor.

The control shall be dual automatic so arranged that the pump will be started or stopped by either the water-level in the receiver or by the vacuum carried on the system.

The pump shall be provided with a strainer, vacuum gauge, vacuum regulator and relief-valve. The size installed must conform to the manufacturer's listed rating for the amount of installed radiation. The Contractor shall provide the Owner with a copy of the manufacturer's guarantee for this pump. The piping connections to pump must be installed strictly in accordance

with the manufacturer's recommendations. This Contractor shall install the necessary wiring run in metal conduit between the motor and starting-switch.

(In case some radiation is located below the strainer connection of pump, it becomes necessary to install an auxiliary tank, provided with float switch on the return line near pump.)

Insulation and Pipe-Covering. Before any insulation or pipe-covering is installed, the system shall be tested as specified under Tests.

All steam and dry return-mains and steam-risers shall be covered with (specify thickness and type of covering desired, usually 1-in thickness or 4-ply asbestos air-cell sectional covering with canvas jacket secured to the pipe with lacquered metal bands).

Cover all fittings and valves, except radiator valves and return traps, with asbestos cement, approximately 1 in in thickness, troweled smooth and finished with a canvas jacket smoothly pasted on the asbestos. The canvas to have the same weight as that used on the pipe-covering.

The boiler shall be covered first with 1 in thickness of magnesia blocks or (.. ..) insulating blocks securely wired in place with 2-in hexagonal wire mesh stretched tight and finished smooth and neat with ½ in thickness hard-finish plastic asbestos cement, the total thickness of insulation to be approximately 1½ in. Cover insulation with a canvas jacket smoothly pasted on same.

(If the boiler is provided with an insulated metal jacket by the manufacturer obviously no further insulation will be required)

Pipe-Sleeves. This Contractor shall furnish and set a pipe-sleeve in the floor or partition construction for every pipe passing through same.

Painting. The exposed surface of all radiators shall receive two coats of special radiator enamel paint applied by this Contractor (or painting contractor). The paint and color for same to be selected by the Architect. All paint shall be delivered to the job in the original package and to be applied strictly in accordance with the paint manufacturer's specification. Before the paint is applied, the exposed surface of all radiators shall be thoroughly cleaned of dirt and rust scale by means of a wire brush.

The finished paint must be smooth, neat and clean before it will be accepted by the Architect.

(If the pipe covering is to be painted include specification for same under this heading)

Automatic Temperature Control. (If automatic temperature control is contemplated, a complete description of the system and apparatus desired should be included under this heading.)

Cleaning Boilers. The safety-valve is to be removed and a temporary line connected to this opening, or highest opening on the boiler, and discharging into sewer. Place a sufficient quantity of soda ash inside the boiler to cause saponification of the oil and grease when mixed with boiling-hot water. A moderate fire to be kept in the boiler sufficient to cause foaming and removal of the grease and oil through the temporary waste-connection as fresh water is gradually fed to the boiler through the boiler feed-line.

After the above-mentioned treatment, the boiler shall be thoroughly washed out with fresh water to remove the dirt, scale, and chemicals, the wash water to leave boiler through the blow-off line or from the lowest opening in the boiler. If foaming should occur later with boiler in operation the above treatment must be repeated by this Contractor until the cause has been removed.

Blowing-Out System. All piping and radiation shall be thoroughly blown out with live steam after having first removed the thermostatic element from the return radiator and drip-traps and the system operated several days with the condensation being wasted to the sewer. This work shall be done before any tests are applied to the system.

Temporary Setting of Direct Radiators. Each bidder shall state in his proposal his charge for connecting each radiator required for temporary heat, including the disconnecting of the radiator, and cleaning of it. Temporary radiators shall be supported from the floor on wood blocks and placed at a sufficient distance from the wall so that the wall finish, millwork, etc., may be applied without interference by the radiator.

The Architect's representative in charge of the work will designate what radiators are to be temporarily connected by this Contractor.

Fuel and Attendance for Temporary Heat. The cost of fuel and attendance required for temporary heat shall be paid for by the (Owner or Contractor). If Owner is to pay for this service the cost is to be determined on a basis of the cost to the Contractor for fuel, water, supplies, and electric power, plus the labor cost including labor insurance plus ...... per cent for overhead supervision and profit.

The Contractor shall submit original invoices showing cost of the fuel delivered on site, supplies, water cost, electric power cost, cost of removal of ashes and original payroll sheets, which must be approved by the Architect's representative in charge of the work. No costs are to be included by the Contractor except those enumerated.

(If Contractor is to include the cost of temporary heat in the contract price, then the specifications should clearly state what is to be required, and the length of time temporary heat is to be supplied. In either case the Contractor is to assume full responsibility for the operation and condition of the heating system, at all times, until final acceptance of the completed contract.)

Request for Samples.—The following samples of materials are hereby requested to be submitted by successful bidder before the contract will be awarded (list to follow):

Tests. (Economy tests for heating boilers and other heating equipment are not usual. If any tests of this character are required the conditions under which the tests are to be conducted must be clearly outlined, covering all points involved. (See A.S.H.V.E. Heating Boiler Test Code.) The following tests are, however, usual)

All concealed low-pressure piping shall be tested and remain tight under a hydraulic pressure of 50 lb per sq in before being covered in. The entire system shall be tested under ...... (usually 10) lb steam-pressure. All such tests to be made in the presence of the Owner's representative.

Material and Workmanship Guarantee. The work herein specified shall be done by competent workmen in a satisfactory manner acceptable to the Architect's representative in charge of the work.

The Contractor shall, upon notice given by the Architect's representative, replace any man or men in the employ of the Contractor on this work, who, in his opinion, are detrimental to the best interests of either the Owner or Contractor in completing the work herein specified in a manner which strictly complies with the specifications and time limit set for completion.

The entire system when completed shall be tested in the presence of the Architect's representative as outlined under Tests.

The Contractor shall make good at his own expense any defects in mate-

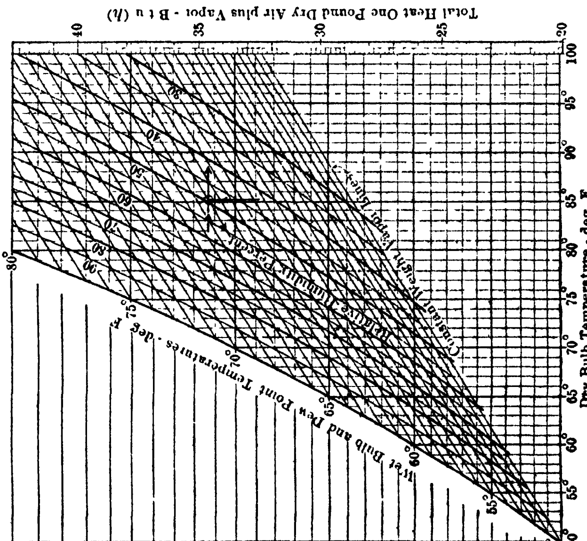
rials, furnished under this specification, or workmanship which may develop for a period of one year from the date of completion of the entire contract.

Any leaky joints found shall be made good without calking Split or defective pipe shall be removed and replaced by good pipe. Defective castings shall be removed and replaced by good castings. Any part of any piece of apparatus which is found to be defective shall be replaced in the system by the Contractor free of expense to the Owner.

The Contractor, if requested, is to furnish the Owner with a duplicate of the manufacturer's guarantee of any piece of apparatus which the Contractor may install under this contract.

PSYCHROMETRIC CHART

(Goodenough's Values for Saturated Air)



Temp. w.b. or dp. °F.	Total Heat at w.b.temp	Wt Vapor per lb. Air lb.	Diff in wt. per deg.
80	42 64	.02226	
79	41 63	.02152	.00740
78	40 64	02080	.00720
77	39 67	02010	.00441
76	38 73	01942	.00076
75	37 81	01877	00068
74	36 91	01813	00065
73	36 03	.01751	00064
72	35 17	01692	.00062
71	34 33	01634	00059
70	33.51	01578	.00053
69	32 71	01524	.00054
68	31 92	01471	.00053
67	31 15	01420	.00051
66	30.39	.01370	00050
65	29 65	01323	00047
64	28.93	.01276	00047
63	28 22	01231	00045
62	27.52	01188	00043
61	26.84	01146	.00042
60	26 18	.01105	.00041
59	25 52	.01066	00039
58	24 88	01028	.00038
57	24 25	.00991	.00037
56	23 64	.00955	.00035
55	23 04	.00920	.00035
54	22 45	.00887	.00033
53	21 87	.00855	.00032
52	21.30	.00823	.00032
51	20.74	.00798	.00030
50	20.19	.00768	

CHAPTER XXXI

CHIMNEYS *

By

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Draft. To burn a fuel at a given rate (pounds per square foot of grate-surface per hour) requires a definite weight of air to be supplied for combustion The air passes under the grate and through the fuel-bed and meets with considerable resistance in its flow, not only through the fuel-bed, but through or around the boiler-tubes and smoke-flue or breeching. The motive force causing the air-flow in a natural-draft plant is supplied by the chimney. The difference between the atmospheric pressure and the pressure existing at any point in the furnace or in the flue is termed the DRAFT at that particular point. This pressure is ordinarily measured by means of a U tube filled with water, the draft being recorded in inches of water, and is the difference in the heights of the water-columns in the two legs of the U tube.

Height. The INTENSITY OF DRAFT that a chimney is capable of producing at the base is a function of its height, the temperature of the flue-gases, and the temperature of the outside air, which is generally assumed to be 60°. The temperature of the flue-gas is ordinarily assumed to be 550°. The intensity of draft produced, per foot height, measured in inches of water is

$$H = 0.0071\,L$$

L = height of chimney above grate, in feet. The flue-gas temperature is taken at 550° and the outside temperature at 60°. Ordinarily 0 8 H is taken as representing the AVAILABLE DRAFT, in order to allow for the cooling of the chimney gases. Then 0.8 H must be equal to or greater than the sum of the expected draft-losses as given in the following paragraphs.

Draft-Losses. The DRAFT-LOSSES through the fuel-bed depend upon the rate of combustion required and the kind of fuel. This loss may be approximated by using the data in Table I.

The LOSS OF DRAFT between the grate or furnace and a point just beyond the damper-box of a boiler is about as shown in Table II when the boilers are operated at normal rating; bituminous coal burned at the rate of from 25 to 30 lb per sq ft of grate-surface per hour.

The loss of draft through the boiler will depend largely upon the method of baffling employed, and increases with the per-cent rating at which the boiler is operated. The precipitating-figures should be increased by approximately 55% when the boiler is operated at 150% of its rated capacity, and by 75% when it is run at 200% rating.

Velocity of Gases through Flue and Chimney. In preliminary estimates 5 lb coal per boiler horse-power developed, and 24 lb air per lb of coal is usually

* See, also, Flue Sizes under Heating and Ventilation of Buildings.

assumed. The customary allowable VELOCITIES OF GASES in chimneys, when the design is based on 120 lb of the flue-gas per hour per rated boiler horsepower, varies from 17 ft per sec for a diameter of stack equal to 24 in, to 31 ft per sec for a 72-in or larger diameter. These figures correspond to a weight of 0.68 and 1.10 lb per sq ft of area. The formula that is supposed to give the most economical diameter for an unlined steel chimney or stack, and used by many engineers in this country, is $d = 4.68\sqrt[5]{(\text{h p})^2}$, in which d is the inside diameter in inches and h.p. is the rated capacity of the boilers served.

The following figures are frequently used by engineers for approximating the loss of draft in flues or breechings:

Table I. Loss of Draft between Furnace and Ash-Pit to Burn Coal

Kind of coal	Combustion-rate, *R*, in pounds of dry coal per square foot of grate per hour						
	15	20	25	30	35	40	45
	Force of draft in inches of water						
Ill., Ind., Kan , bituminous .	.14	20	26	33	.40	48	.57
Ala., Ky., Pa , Tenn , bituminous	.16	.23	31	40	49	60	.72
Md., Pa , Va., W. Va , semibituminous	.18	.26	35	.45	57	71	87
Anthracite pea. . . .	.30	45	64	88	1.23	..	..
Anthracite buckwheat No. 1. . .	.43	.68	1 00	1.50	.	.	..

Table II. Loss of Draft between Grate or Furnace and a Point Just Beyond Damper-Box

Horizontal return tubular. .	25 to 30 in of water
Babcock & Wilcox . .	.20 to .35 in of water
Stirling . . .	51 in of water
Vertical tubular .	.43 in of water

(1) Horizontal flues, square or rectangular, from 0.13 to 0.15 in of water per 100 ft. Increase these values 50% for brick-lined flues. Loss of draft for easy right-angle bends, 0.05 in of water.

(2) When economizers are to be installed the temperature of the flue-gas is reduced to from 250° to 325°, and the total head, H, should be calculated on a basis of these temperatures.

(3) The loss of draft through the economizers should not be figured less than 0.3 in of water.

(4) The turns which the flue makes in leaving the damper-box of the boiler, where it enters the main flue and at the stack, should be considered and allowed for.

(5) It is customary to make the flue or breeching approximately from 10 to 15% greater in area than the stack to which it connects. The cross-section is reduced in proportion to the volume of gas to be handled as the flue passes the boilers in succession. The width of the flue or breeching, where it enters the

chimney, should never exceed one third the outside diameter of the chimney at its base.

Example. The method of procedure in determining the dimensions of a chimney and breeching is explained in the following example.

Three 150-h.p. return tubular boilers with a total of 1 500 sq ft of heating-surface are to be served. The total area of the grate-surface is 90 sq ft. The measured length of the breeching is 40 ft. The gas makes two right-angle turns, one at the entrance to the breeching, and one on entering the chimney. The fuel assumed is Pennsylvania bituminous coal. If 5 lb of coal per boiler horse-power per hour is assumed as the fuel-consumption, the rate of combustion is $(3 \times 150 \times 5)/90 = 25$ lb per sq ft of grate-surface per hour.

The weight of flue-gas per second is

$$(3 \times 120 \times 150)/(60 \times 60) = 15 \text{ lb}$$

Assuming a temperature of 550°, the volume of the flue-gas per second is $15/0.0393 = 382$ cu ft. Assuming an allowable velocity through the chimney-area of 25 ft per sec, the required area is,

$$382/25 = 15.3 \text{ sq ft}$$

corresponding to 54-in diam, approximately. The area of the flue is to be 15% greater, or

$$15.3 \times 1.15 = 17.6 \text{ sq ft}$$

at the last boiler next to the chimney. The chimney must produce sufficient draft to overcome the following resistance. The loss of draft through fuel-bed based on a rate of combustion of 25 lb per sq ft per hr (Table I) is 0.31 in. The loss of draft through return tubular boilers (Table II) is 0.27 in. The loss of draft through the breeching is

$$0.15 \times 40/100 = 0.06 \text{ in}$$

The loss of draft occasioned by two turns is

$$2 \times 0.05 = 0.10 \text{ in}$$

The total loss is

$$0.31 + 0.27 + 0.06 + 0.10 = 0.74 \text{ in}$$

Then

$$H = 0.74/0.8 = 0.92 \text{ in}$$

or approximately 1 in.

Substituting this value of H in the equation

$$H = 0.0071\,L$$

the height, L, of the stack is

$$1/0.0071 = 140 \text{ ft}$$

measured above the grate.

Kent's Chimney-Formulas. The following chimney-formulas by William Kent are largely used by engineers in this country: The formula is based on the assumption that the friction-head in the chimney is considered equivalent to a diminution of the area by an amount equal to a lining of inert gas, 2 in in thickness.

If A = the actual area in square feet;
E = the effective area in square feet;
D = the diameter in feet;

Then $E = A - 0.60\sqrt{A}$.

The draft-power of a chimney varies directly as the effective area, E, and as the square root of the height, L. The formula for the horse-power of a chim-

Table III. Size of Chimneys for Steam-Boilers

Kent's Formula

Diameter, in	Area, sq ft	Effective area, $E = A - 0.6\sqrt{A}$ sq ft	Height of chimney in feet: 50	60	70	80	90	100	110	125	150	175	200	225	250	300	Equivalent square chimney. Side of square, $\sqrt{E}+4$ in
			Commercial horse-power of boiler *														
18	1.77	.97	23	25	27	29											16
21	2.41	1.47	35	38	41	44											19
24	3.14	2.08	49	54	58	62	66										22
27	3.98	2.78	65	72	78	83	88										24
30	4.91	3.58	84	92	100	107	113	119									27
33	5.94	4.48		115	125	133	141	149	156								30
36	7.07	5.47		141	152	163	173	182	191	204							32
39	8.30	6.57			183	196	208	219	229	245							35
42	9.62	7.76			216	231	245	258	271	289	316						38
48	12.57	10.44				311	330	348	365	389	426						43
54	15.90	13.51					427	449	472	503	551	595					48
60	19.64	16.98					536	565	593	632	692	748					54
66	23.76	20.83						694	728	776	849	918	981				59
72	28.27	25.08						835	876	934	1 023	1 105	1 181	1 253			64
78	33.18	29.73							1 038	1 107	1 212	1 310	1 400	1 485	1 565		70
84	28.48	34.76							1 214	1 294	1 418	1 531	1 637	1 736	1 803	2 005	75
90	44.18	40.19								1 496	1 639	1 770	1 893	2 008	2 116	2 318	80
96	50.27	46.01								1 712	1 976	2 027	2 167	2 298	2 423	2 654	86
102	56.75	52.23								1 944	2 130	2 300	2 459	2 609	2 750	3 012	91
108	63.62	58.83								2 090	2 399	2 592	2 771	2 939	3 098	3 393	96
114	70.88	65.83									2 685	2 900	3 100	3 288	3 466	3 797	101
120	78.54	73.22									2 986	3 226	3 448	3 657	3 855	4 223	107
132	95.03	89.18									3 637	3 929	4 200	4 455	4 696	5 144	117
144	113.10	106.72									4 352	4 701	5 026	5 331	5 618	6 155	128

* Based on a consumption of 5 lb of fuel per boiler horse-power. For any other rate, multiply the tabular figure by the ratio of 5 to the maximum expected coal-consumption per horse-power per hour.

ney will take the form, h.p. $= CE\sqrt{L}$, in which C is a constant. The value of C as obtained by Kent from an examination of a large number of chimneys is 3.33 when 5 lb of coal is burned per boiler horse-power per hour.

The formula for the horse-power rating of a chimney is, therefore,

$$\text{h.p.} = 3.33\,E\sqrt{L} = 3.33\,(A - 0.6\sqrt{A})\sqrt{L}$$

or

$$E = 0.3\ \text{h.p.}/\sqrt{L}$$

The Babcock & Wilcox Company recommend that when the fuel used is low-grade bituminous coal of the Middle or Western States, the sizes given in Table III be increased from 25 to 60%, depending upon the nature of the coal and the capacity desired. If the gas makes more than two turns it is advisable to increase the diameter given in the table by one size. The height must be increased at least 30% if economizers are used. Table III may be applied to heating-boilers, the equivalent rating in square feet of direct radiation being approximately equal to the horse-power rating × 100.

Chimneys for Tall Office and Loft-Buildings. The chimney or stack for a tall building is a special case in which the height is frequently fixed by the height of the structure itself or the height of the adjoining buildings. In this case a diameter is assumed and the method outlined in the preceding example applied.

General Formulas for the Design of Brick Chimneys. See Fig. 1

Let P = horizontal wind-pressure in pounds per square foot, ordinarily assumed as 25 lb per sq ft for round chimneys

xx = any section distant z from top of chimney

$z\left(\frac{d + d_1}{2}\right)$ = projected area above xx

R = horizontal wind-load in pounds

$= Pz\left(\frac{d + d_1}{2}\right)$

y = distance from xx to center of gravity of portion above xx

M = wind-moment in foot-pounds

$= Pzy\left(\frac{d + d_1}{2}\right)$

PROPERTIES OF SECTION

d_1 = outside diameter

d_2 = inside diameter

$c = d_1/2$

I = moment of inertia of section

A = area of section in square feet

$= 0.7854\,(d_1^2 - d_2^2)$

$\frac{I}{c}$ = section-modulus

$$\frac{I}{c} = \frac{0.0982\,(d_1^4 - d_2^4)}{d_1}$$

W = weight of chimney above xx, in tons

S_1 = compressive stress at edge on leeward side due to W, in tons per square foot

S_2 = compressive stress at edge on leeward side due to M, in tons per square foot

$$S_1 = +\frac{W}{A} \qquad S_2 = \pm\frac{Mc}{I}$$

$$\text{Windward side, } S_w = \frac{W}{A} - \left[\frac{Mc}{I}\text{ (tension)}\right]$$

$$\text{Leeward side, } S_l = \frac{W}{A} + \left[\frac{Mc}{I}\text{ (compression)}\right]$$

S_w and S_l should not exceed the following values, in tons per square foot, for Radial Brick Chimneys:

Maximum tension		Maximum compression	
Below 150 ft. . .	2 to 2½	200 ft and below.	19
From 150 to 200 ft.	1 to 1½	Above 200 ft	21
Above 200 ft.	0		

Foundations. Calculate wind-moment, M_1 for chimney above ground-line.

$$M_1 = Phy_1\left(\frac{d + d_3}{2}\right)$$

l = length of side of square base in feet

$A_1 = l^2$ = area of base in square feet

$\frac{I}{c} = \frac{l^3}{6}$ = section modulus of base

W_1 = combined weight of chimney and foundation

Example. It is required to determine the maximum compression, in tons per sq ft at the base of the column, for the chimney shown in Fig. 2, and also the maximum soil-pressure in tons per sq ft. The assumed wind-pressure is 25 lb per sq ft. (See General Formulas for the Design of Brick Chimneys, and Fig. 1.)

The area of section at base, $A = 0.7854\,(16^2 - 12.3^2) = 80.9$ sq ft.

The section-modulus at base, $I/c = [0.0982(16^4 - 12.3^4)]/16 = 257$.

The total weight of brick column (Table V) is W = 495 tons (interpolated).

The projected area of column is $\frac{1}{2} \times (8.75 + 16) \times 180 = 2\,228$ sq ft.

The horizontal wind-load, $R = 2\,228 \times 25 = 55\,700$ lb = 27.8 tons.

The moment-arm of R is $y = \frac{1}{3} \times 180[(2 \times 8.75) + 16]/(8.75 + 16) = 81$ ft.

The wind-moment, $M = 81 \times 27.8 = 2\,252$ ft tons.

$S_1 = 495/80.9 = 6.2$ tons per sq ft.

$S_2 = \pm\, 2\,252/257 = 8.7$ tons per sq ft.

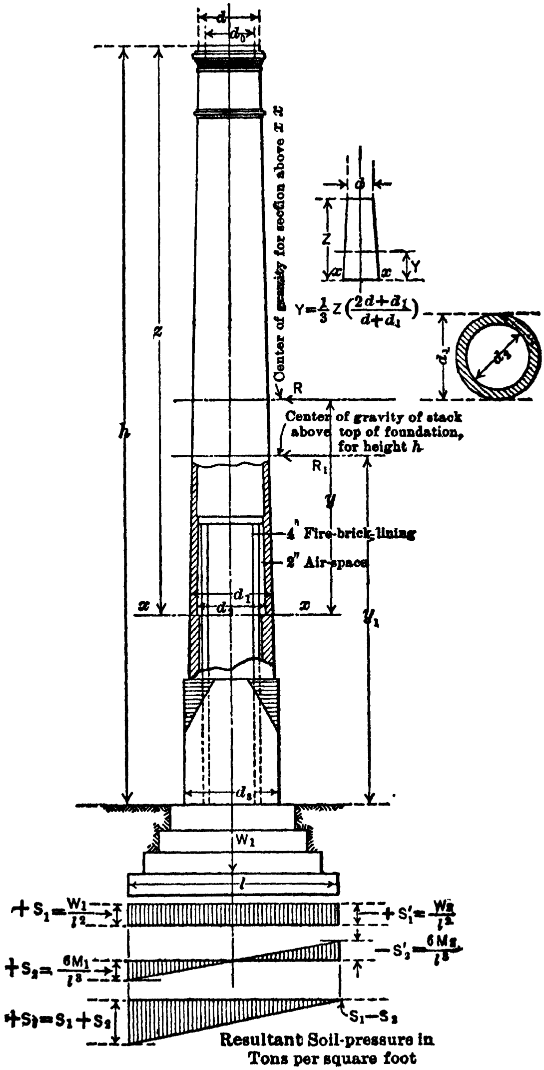
The maximum compression on the leeward side, $S_1 + S_2 = 6.2 + 8.7 = 14.9$ tons per sq ft. The maximum tension on the windward side, $S_1 - S_2 = -\,2.2$ tons per sq ft. The following computations are for a square base:

Foundation. The length of base, $l = 25.5$ ft, $A_1 = l^2 = 650$ sq ft, $I/c = 25.5^3/6 = 814$. The weight of foundation, based on 1.9 tons per cu yd, is 266 tons. The weight of the 4½-in lining is $36 \times 11 \times 0.063 = 25$ tons. The total weight of column, lining and foundation, is $W_1 = 495 + 25 + 266 = 786$ tons.

The moment-arm for R may be assumed the same as before, or 81 ft. Then $M = 2\,252$ ft-tons. The section-modulus of the base, $I/c = l^3/6 = 814$.

$S_1 = 786/650 = 1.2$ tons per sq ft. $S_2 = 2\,252/814 = 2.8$ tons per sq ft.

The maximum soil-pressure, $S_1 + S_2 = 1.2 + 2.8 = 4$ tons per sq ft.



$$S_1 = \frac{W_1}{l^2} = \text{compression per sq ft due to } W_1$$

$$S_2 = \frac{6\,M_1}{l^3} = \text{compression per sq ft due to } M_1$$

Fig. 1. Details of Construction of Tall Brick Chimney

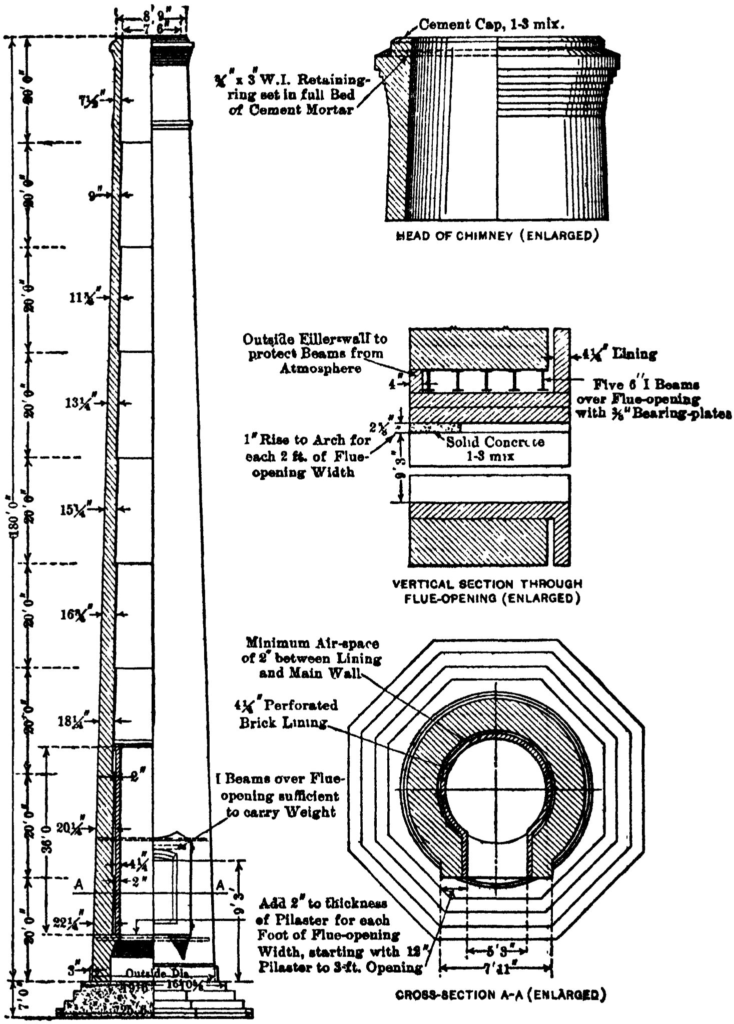


Fig. 2. Details of Tall Radial-brick Chimney

Table IV. Bottom Diameters of Radial-Brick Chimneys

Height of chimney in feet	Internal diameter at top														
	3′	3′ 6″	4′	4′ 6″	5′	5′ 6″	6′	6′ 6″	7′	7′ 6″	8′	8′ 6″	9′	9′ 6″	10′
	Diameters at bottom of column, in feet														
75	7.42	7.69	7.96	8.46	8.96	9.46	9.96								
80	7.80	8.04	8.27	8.70	9.13	9.58	10.02								
85	8.18	8.38	8.58	8.95	9.31	9.70	10.08								
90	8.57	8.73	8.88	9.18	9.48	9.81	10.13								
95	8.95	9.07	9.19	9.43	9.66	9.93	10.19								
100	9.33	9.42	9.50	9.67	9.83	10.04	10.25	10.75	11.25	11.75	12.25				
105	9.70	9.78	9.85	10.03	10.21	10.38	10.55	11.03	11.50	11.95	12.40				
110	10.06	10.13	10.20	10.40	10.60	10.73	10.85	11.30	11.75	12.15	12.55				
115	10.43	10.49	10.55	10.77	10.98	11.07	11.15	11.58	12.00	12.35	12.70				
120	10.79	10.85	10.90	11.14	11.37	11.41	11.45	11.85	12.25	12.55	12.85				
125	11.16	11.21	11.25	11.50	11.75	11.75	11.75	12.13	12.50	12.75	13.00	13.50	14.00	14.50	15.00
130			11.65	11.88	12.10	12.12	12.13	12.47	12.80	13.09	13.37	13.80	14.22	14.69	15.15
135			12.05	12.25	12.45	12.48	12.51	12.81	13.10	13.42	13.73	14.08	14.43	14.87	15.30
140			12.45	12.63	12.80	12.85	12.90	13.15	13.40	13.75	14.10	14.38	14.65	15.05	15.45
145			12.85	13.00	13.15	13.22	13.26	13.49	13.70	14.08	14.46	14.66	14.86	15.23	15.60
150			13.25	13.38	13.50	13.58	13.66	13.83	14.00	14.42	14.83	14.96	15.08	15.42	15.75
155			13.58	13.73	13.87	13.97	14.06	14.18	14.30	14.68	15.06	15.19	15.31	15.61	15.91
160			13.92	14.08	14.23	14.35	14.46	14.53	14.60	14.95	15.30	15.43	15.55	15.81	16.07
165			14.25	14.43	14.60	14.73	14.86	14.88	14.90	15.22	15.53	15.66	15.78	16.00	16.22
170			14.59	14.78	14.96	15.11	15.26	15.23	15.20	15.49	15.77	15.90	16.02	16.20	16.38
175			14.92	15.13	15.33	15.50	15.66	15.58	15.50	15.75	16.00	16.13	16.25	16.40	16.54
180									15.80	16.05	16.30	16.40	16.50	16.65	16.80
185									16.10	16.35	16.60	16.68	16.75	16.91	17.06
190									16.40	16.65	16.90	16.95	17.00	17.16	17.31
195									16.70	16.95	17.20	17.23	17.25	17.41	17.57
200									17.00	17.25	17.50	17.50	17.50	17.67	17.83
205													17.80	17.98	18.16
210													18.10	18.30	18.50
215													18.40	18.62	18.83
220													18.70	18.94	19.17
225													19.00	19.25	19.50

Table V. Dead-Load of Radial-Brick Chimneys in Tons of 2 000 Pounds *

Height in feet	Inside diameter at top, in feet							
	3	4	5	6	7	8	9	10
90	90	98	110	122	.	.	.	.
100	110	120	131	143	161	180	.	.
110	138	143	155	167	188	206	.	.
120	160	170	185	198	218	237	.	.
130	.	202	218	231	252	273	295	318
140	.	237	256	270	290	310	337	357
150	.	277	296	311	330	353	375	400
160	.	317	340	357	375	402	423	447
170		362	388	410	425	454	475	500
180			.		480	510	537	555
190	.	..		.	535	570	592	617
200	..	.	.		600	632	657	685
210	..	.	.	.	.	.	727	760
220	..	.	.	..	.	.	804	843

* These values are interpolated from curves by the M. W. Kellogg Company, and are for round radial-brick chimneys exclusive of the weight of the foundation.

Table VI. Width of Foundations at Base for Radial-Brick Chimneys *

Height in feet	Inside diameter at top, in feet							
	3	4	5	6	7	8	9	10
	ft in	ft in	ft in	ft in	ft in	ft in	ft in	ft in
90	11 6	12 0	13 0	13 9	.	.		
100	12 6	13 0	14 0	14 8	15 6	16 0	.	..
110	13 6	14 3	15 0	15 6	16 6	17 0	.	.
120	14 6	15 3	16 0	16 6	17 6	18 0	.	.
130	15 6	16 6	17 3	17 8	18 8	19 3	20 0	21 3
140	..	17 9	18 6	18 10	19 9	20 3	21 0	22 3
150	..	19 0	19 9	20 0	21 0	21 6	22 3	23 6
160	..	20 6	21 0	21 6	22 6	23 0	23 6	24 9
170		22 0	22 6	23 0	23 9	24 3	25 0	26 3
180		.	.	.	25 6	26 0	26 6	27 6
190	.	.		.	27 0	27 6	28 3	29 3
200		.	.	.	28 6	29 0	29 9	30 9
210	.	.	.	.		.	31 6	32 6
220	..	.	.	..	.	..	33 0	34 3
250	.	.	.	.	.	.	.	37 0

* These values are interpolated from curves by the M. W. Kellogg Company. The maximum unit soil-pressure at the outer edge of the foundation, due to dead and wind-loads, does not exceed 2 tons per square foot.

Reinforced-Concrete Chimneys. Area of Steel Reinforcement Required. The following formulas are used by one concern in the design of reinforced-concrete chimneys:

M = wind-moment at section considered, in inch-pounds;
W = weight of shell above section considered, in pounds;
D = outside diameter of shell, in feet:

d = inside diameter of shell, in feet;
R = radius of steel circle, in inches;
r = radius of neutral core = $\frac{1}{8}[D1 + (d/D)_2]$ feet;
WR = moment of stability from weight of shell, in inch-pounds;
S = 16 000, the allowable fiber-stress in the steel in pounds per square inch;
A = total cross-sectional area of steel rods required at section considered, in square inches;
$A = 2(M - Wr)/SR$ = number of bars × cross-sectional area of one bar.

The bars used are ordinarily ¾-in square, twisted bars, each with a cross-sectional area of 0 5625 sq in. The thickness of the shelf, if made without a taper, is ordinarily 6 in, and if constructed with a taper the shell is made 5 in in thickness at the top and increased ¼ in in thickness for each 5 ft in height. The maximum compression due to the wind-moment and dead load in the concrete, at the base, is ordinarily limited to 350 lb per sq in. The same formulas apply in this case as in the design of brick chimneys (Fig. 1).

Example. A reinforced-concrete chimney has the following dimensions: 150 ft high; no taper; 9 ft inside diameter; thickness of shell 6 in; outside diameter 10 ft; weight of shell 335 250 lb. It is required to determine the total cross-sectional area of reinforcement.

$M = 25 \times 10 \times 150 \times 150/2 = 2\,812\,500$ ft-lb $= 33\,750\,000$ in-lb;
$r = 10/8 \times [1 + (9/10)^2] = 2.3$ ft $= 27.6$ in;
$R = 58$ in;
$A = 2[33\,750\,000 - (335\,250 \times 27.6)]/16\,000 \times 58) = 53$ sq in;

If ¾-in square bars are used, it requires 53/0.5625 = 94 bars.

Table VII. Reinforced-Concrete Chimneys. Dimensions. (Fig. 3)

Height above grade, ft	Inside diameter, ft	Depth below grade, ft	Height double shell, ft	Height single shell, ft	Total height $A+B+C$, ft	Maximum outside diameter, ft in	Width of square foundation, ft
H	*G*	*A*	*B*	*C*	*D*	*E*	*F*
100	4	5	33	67	105	6 4	12
100	5	5	33	67	105	7 4	12
125	5	5	42	83	130	7 4	15
125	6	5	42	83	130	8 4	16
150	6	6	48	102	156	8 4	18
150	7	6	48	102	156	9 4	18
150	8	6	48	102	156	10 4	19
175	8	7	57	118	182	10 6	22
175	9	7	57	118	182	11 6	.22
175	10	7	57	118	182	12 6	23
200	10	7	66	134	207	12 6	25
200	11	7	66	134	207	13 6	25
200	12	7	66	134	207	14 6	26
225	12	8	69	156	233	14 8	29
225	13	8	69	156	233	15 8	29
225	14	8	69	156	233	16 8	30
250	14	8	81	169	258	16 8	32
250	15	8	81	169	258	17 8	33
250	16	8	81	169	258	18 8	34

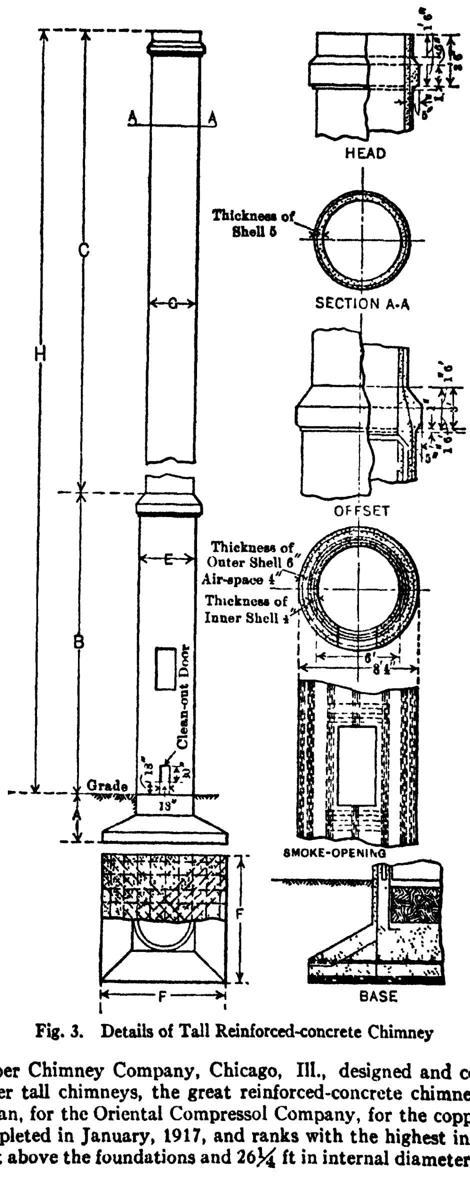


Fig. 3. Details of Tall Reinforced-concrete Chimney

The Weber Chimney Company, Chicago, Ill., designed and constructed, among other tall chimneys, the great reinforced-concrete chimney at Saganoseki, Japan, for the Oriental Compressol Company, for the copper smelter. It was completed in January, 1917, and ranks with the highest in the world, being 570 ft above the foundations and 26¼ ft in internal diameter at the top.

Self-Sustaining Steel Chimneys* are largely used, especially for tall chimneys of iron-works and power-houses from 150 to 300 ft in height.

"The advantages claimed are: Greater strength and safety; smaller space required; smaller cost by 30 to 50% as compared with brick chimneys; avoidance of infiltration of air and consequent checking of the draught, common in brick chimneys. They are usually made cylindrical in shape, with a wide curved flare for 10 to 25 ft at the bottom. A heavy cast-iron base-plate is provided, to which the chimney is riveted, and the plate is secured to a massive foundation by holding-down bolts. No guys are used."†

A large SELF-SUSTAINING STEEL CHIMNEY is that built by the Chicago Bridge and Iron Works at the plant of the United Verde Copper Company, Clarkdale, Arizona. It is 30 ft 9½ in in diameter and 400 ft in height. The thickness of plates varies from ¼ in at the top to 11/16 in for the bell-shaped portion at the bottom. The weight of steel is 800 000 lb. The stack is anchored to the foundation by thirty-six bolts, each 4 in in diameter, upset, and spaced equidistant in a bolt-circle of 25 ft 4½ in radius.

Table VIII. Sizes of Foundations for Self-Sustaining Steel Chimneys, Half-Lined ‡

Diameter, clear, in feet	3	4	5	6	7	9	11
	ft in	ft in	ft in	ft in	ft in	ft in	ft in
Height .	100	100	150	150	150	175	225
Least diam of foundation	15 9	15 3	20 4	21 10	22 7	25 9	29 11
Least depth of foundation	6 6	7	9	8	9	10	13
Height	...	125	200	200	250	275	300
Least diam of foundation	..	17 6	23 8	25 0	29 8	33 6	36 0
Least depth of foundation	..	7 6	10	10	12	12	14

The governing feature in the design of a SELF-SUSTAINING STEEL CHIMNEY OR STACK is the force of the wind. The cylinder above any horizontal plane section may be assumed to act as a cantilever beam in which the bending moment, in foot-pounds, is

$$M = HD \times P \times \tfrac{1}{2}H = \tfrac{1}{2}H^2DP$$

in which H is the height in feet above the section considered, D the diameter in feet and P the assumed pressure of the wind in pounds per square ft on a vertical cross-section. The fiber-stress S, in pounds per square inch, according to the formula for flexure, is $S = Mc/I$. For hollow cylinders of large diameter and small thickness, the moment of inertia $I = \pi R^3 t$, in which R = mean radius in feet (equivalent to c in the flexure formula) and t = thickness of shell in inches. Hence

$$S = MR/12\pi R^3 t = 0.106\ M/D^2t, \text{ and } t = 0.106\ M/SD^2.$$

The stress S is tensile on the windward side and compressive on the leeward side. There is also a small compressive stress due to the weight of the stack. The value of P may be taken at 25 lb per sq ft and of S at 16 000 or 18 000 lb per

* Compiled from data furnished by Robins Fleming.

† Mechanical Engineers' Pocket Book. Kent.

‡ These dimensions were taken from a pamphlet published by the Philadelphia Engineering Works.

sq in. As stacks are built for durability as well as strength it is often advisable to increase the theoretical thickness of the shell. No plate should be used with a thickness less than ¼ in. It is important that the stack be securely anchored to the foundation. Many methods have been proposed for determining stresses in anchor-bolts. As the problem depends for its solution on the physical conditions of stack, base and bolts, no exact analysis is possible. The most severe assumption is that the bolts are screwed up with a high initial tension. The anchor-bolt ring can then be considered in the same way as a ring of the cylinder. The maximum stress at any point of the bolt-circle is developed when the wind is blowing parallel to the radius through that point. The stress for each circumferential inch is $0.106\,M/(2\,R_1)^2$, $2R_1$ being the diameter of the bolt-circle. Let b be the circumferential distance in inches between adjacent anchor-bolts, N the number of bolts equidistant on the bolt-circle and W the weight of the stack. For the anchor-bolt on the windward side there is a tensile stress, S_w, due to the wind,

$$S_w = 0.106\,bM/(2R_1)^2$$

Since

$$b = (2R_1 \times 12 \times \pi)/N$$

$$S_w = 2\,M/R_1N$$

Deducting the weight of the portion of the stack between adjacent bolts, the maximum tensile stress in any anchor-bolt may be expressed by the equation

$$S_w = (2\,M/R_1N) - W/N$$

Radial-Brick Chimneys. These chimneys are built with special blocks formed to suit the circular and radial lines of each section of the chimney so that the finished brickwork has joints of an even thickness throughout and a perfectly smooth surface. The blocks being much larger than common bricks, there are only from one third to one half as many joints. Radial-brick chimneys are always circular in plan above the base. The best form of base is octagonal in cross-section so as to permit the breeching to enter the chimney at a flat surface and at the same time comply best with the rules of stability. Except for chemical-works, refineries, furnaces, etc., radial-brick chimneys are built with a single shell, a lining only being provided in the immediate vicinity of the flue-entrance. All radial bricks are perforated vertically and this insures thorough burning and allows the mortar to enter the perforations, thus forming a vertical anchorage.

Radial blocks for chimney-construction have been used extensively in England, Germany, France and Russia since 1870. They were not introduced into this country, however, until 1898. In 1869 or 1870 Alphons Custodis, of Dusseldorf, Germany, originated a method of building tall chimneys of perforated radial blocks, made from selected clays and burned at a very high temperature, and in 1898 an American company * was formed for the purpose of erecting chimneys by this method of construction. Since that time the company through various agencies has built more than six thousand chimneys in all parts of the world. A very tall chimney, 585 feet high and 60 ft in internal diameter at the top, was built by this company in 1918 for the Anaconda Copper Company, at Anaconda, Mont.

Mr. H. R. Heinicke,† of Chemnitz, Germany, builder of the 460-ft stack at Halsbrücke, Germany, has employed radial bricks made especially for each

* Alphons Custodis Chimney Construction Company, New York City.

† H. R. Heinicke, Incorporated, New York City.

chimney. This firm through long and costly research has done much to make chimney-building a science. The chimney at Halsbrücke is a very remarkable one on account of its proportions. In a height of 460 ft, the diameter at the top is only 8 ft, whereas the 585-ft stack at Anaconda, Mont., has a diameter of 60 ft at the top.

The Heine Chimney Company * has erected many important high chimneys. The essential difference in the methods of construction used by this company from those of the other chimney-constructors is that the Heine Chimney Company uses perforated, INTERLOCKING, radial bricks. It is claimed that this interlocking-feature has an advantage over the straight-sided bricks in acting as a preventive of deep weathering of the joints and of air-leaks. In addition to this it is claimed that the circumferential strength of the walls when built of this type of brick is considerably greater than when built with plain-sided or corrugated bricks. The perforations in these bricks are fewer but larger than those of some of the other constructors. The brick-work is laid on full-mortar beds with SHOVED joints. These large perforations allow the mortar to rise in them, thus forming PINS which give the walls great strength and enable them to withstand the stresses due to expansion caused by the high temperature of the flue-gases. In walls more than one brick thick, the bricks are laid up in English bond, that is, with alternate header and stretcher-courses. This company advocates this method of construction even in chimneys built with the ordinary straight-sided common building-bricks. Among the many important chimneys constructed by the Heine Chimney Company is the one erected at the St. Joseph Lead Company's plant, at Herculaneum, Mo. The height of this chimney is 350 ft and the inside diameter at the top 20 ft.

The M. W. Kellogg Company † has designed and built many radial-brick chimneys for power-plants, chemical-works and other purposes. Several of the important chimneys put up by them are mentioned in the list of tall chimneys at the end of this chapter. Some of the details of construction differ from those of the other companies mentioned. One of the points of difference is the detail relating to the corrugations on their bricks. These corrugations are 3/8 in wide and 1/8 in deep and are placed along the vertical sides of the bricks as they lie in the wall. The adhesion between the bricks and mortar is increased by this increased area. It is claimed that tests made show that this is the case. On account of these corrugations it is not considered necessary to embed any ironwork in these chimneys to prevent the development of cracks due to heat-expansion. Ironwork has sometimes been inserted when plain-sided bricks have been used. It is claimed that this design is somewhat heavier than that employed by some other constructors, this company holding that it is not safe to figure on wind-pressure of less than 25 lb per sq ft of projected area. Among the many tall chimneys erected by this company may be mentioned especially the chimney at Douglas, Ariz., erected for the Copper Queen Consolidated Mining Company.

There are other reliable companies which design and construct tall chimneys. Those mentioned here were the pioneers in this work.

* The Heine Chimney Company, Chicago, Ill.
† The M. W. Kellogg Company, New York City

Partial List of Tall Chimneys Over 300 Feet in Height

It is to be noted that this list is constantly added to from year to year.

	Height, ft	Diam. inside at top, ft
* Anaconda, Mont., Anaconda Copper Co. (1918)	585	60
* Tacoma, Wash., American Smelting & Refining Co. (1917)..	573	25
† Saganoseki, Japan, Oriental Compressol Co. (1917)...	570	26¼
* Great Falls Mont., Boston & Montana Consolidated Copper and Silver Mining Co. (1907)....	506	50
‡ Freiberg, Saxony, Germany, Halsbrucke Foundry.........	460	..
Glasgow, Port Dundas, Scotland, F. Townsend	454	..
Glasgow, St. Rollox, Scotland, Tenant & Co	436½	..
* Jerome, Ariz., United Verde Extension Mining Co. (1918)...	425	30
Creusot, France, Messrs. Musprath Chemical Works	406	..
§ Clarkdale, Ariz., United Verde Copper Co.....	400	30¾
* El Paso, Tex., Consolidated Kansas City Smelting & Refining Co. (1916)....	400	30
* Hayden, Ariz., American Smelting & Refining Co. (1911) ...	400	25
* East Helena, Mont., American Smelting & Refining Co. (1917)	400	16
Halifax, Dean Clough Mill, Scotland, Messrs. Crossley's...	381	..
* Easton, Pa., C. K. Williams & Co. (1911)	375	7
Bolton, Lancashire, England, Dobson & Barlow	367	..
* Rochester, N. Y., Eastman Kodak Co. (two) (1906, 1911) .	366,	9 and 13
* Constable Hook, N. J., Orford Copper Co. (two) (1900, 1910)	365	10
* Garfield, Utah, Garfield Smelting Co. (1913)	350	22
‖ Herculaneum, Mo., St. Joseph Lead Co . . .	350	20
Boston, Mass., Fall River Iron Co	350	11
* Newark, N. J., Heller Merz Co. (1904)....	350	8
East Newark, N. J., Clark Thread Co..	335	..
Barmen, Prussia, Germany, Wessenfield & Co.	331	..
Edinburgh, Scotland, Gas-Works..........	329	..
‡ Copper Hill, Tenn., Tennessee Copper Co.	325	20
‡ Indianapolis, Ind., Indianapolis Traction Co	320	13
Huddersfield, England, Brook & Son, Fire-clay Works. ...	315	..
Smethwick, England, Adams Soap-Works......	312	..
* Providence, R. I., Rhode Island Suburban Railway Co......	308	16
* New York City, N. Y., New York Steam Co. (1904)	308	15
Carlisle, England, P. Dickon & Son	300	..
Bradford, England, Mitchell Brothers....	300	..

* Constructed by the Alphons Custodis Chimney Construction Company, New York City.

† Reinforced concrete, The Weber Company, Chicago, Ill.

‡ Constructed by H. R. Heinicke, Incorporated, New York City.

§ Self-sustaining steel chimney.

‖ Constructed by The Heine Chimney Company, Chicago, Ill.

Partial List of Tall Chimneys over 300 Feet in Height (Continued)

	Height, ft	Diam. inside at top, ft
* Garfield, Utah, American Smelting and Refining Co. (1905).	300	30
* Hayden, Ariz., American Smelting and Refining Co . . .	300	25
† Douglas, Ariz., Copper Queen Consolidated Mining Co.....	300	22
‡ Tacoma, Wash., Tacoma Smelting Co....................	300	18
‡ McGill, Nev., Steptoe Valley Traction Co.................	300	15
* Brooklyn, N. Y., Nichols Chemical Co. (1905)	300	12
* Claymont, Del., General Chemical Co. (1912).............	300	8

* Constructed by the Alphons Custodis Chimney Construction Company, New York City.

† Constructed by The M. W. Kellogg Company, New York City.

‡ Reinforced concrete, The Weber Chimney Company, Chicago, Ill.

§ Constructed by H. R. Heinicke, Incorporated, New York City.

CHAPTER XXXII

HYDRAULICS, PLUMBING AND DRAINAGE, ILLUMINATING-GAS AND GAS-PIPING

By
J. J. COSGROVE
CONSULTING ENGINEER

1. Hydraulics

Water is practically an imcompressible liquid, weighing, at the average temperature of 62° F., 62.355 lb to the cu ft and 8.335 lb to the gallon. These figures change slightly with changes in temperature and atmospheric pressure, and a slight variation for the same temperature will be found in different works. 62.4 lb per cu ft is the weight of water used for ordinary computations.

Pressure of Water. The pressure of still water in pounds per square inch against the sides of any pipe or vessel of any shape whatever is due alone to the HEAD, or height of the surface of the water above the point considered pressed upon, and is equal to 0 433 lb per sq in for every foot of head at 62° F. The fluid-pressure per square inch is equal in all directions. To find the total pressure of quiet water against and perpendicular to any surface, whether vertical, horizontal, or inclined at any angle, whether it be flat or curved, multiply together the area in square feet of the surface pressed, the vertical depth of its center of gravity below the surface of the water, and the constant 62.4. The product will be the required pressure in pounds. This may be expressed by formula as follows:

$$P = 62.4\,AD$$

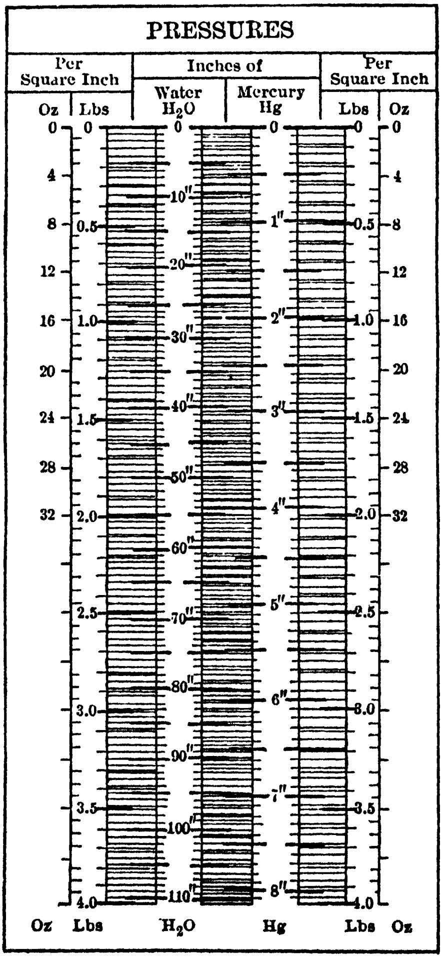
in which P = the pressure in pounds of quiescent water on the surface considered;
A = the area pressed upon in square feet; and
D = the vertical depth in feet of center of gravity of surface considered.

Table I. Pressure in Pounds per Square Inch for Different Heads of Water in Feet

Head, ft	0	1	2	3	4	5	6	7	8	9
0		0 433	0 866	1 299	1 732	2 165	2.598	3 031	3 464	3 897
10	4 330	4 763	5 196	5.629	6 062	6.495	6.928	7.361	7.794	8 227
20	8 660	9.093	9.526	9.959	10 392	10.825	11.258	11 691	12 124	12 557
30	12 990	13 423	13 856	14.289	14.722	15.155	15.588	16.021	16 454	16 887
40	17.320	17 753	18 186	18 619	19 052	19.485	19 918	20.351	20 784	21.217
50	21.650	22 083	22 516	22.949	23 382	23 815	24 248	24 681	25 114	25 547
60	25 980	26 413	26.846	27.279	27.712	28.145	28 578	29.011	29.444	29.877
70	30 310	30 743	31.176	31.609	32 042	32.475	32 908	33.341	33.774	34.207
80	34.640	35.073	35.506	35.939	36.372	36.805	37.238	37.671	38.104	38.537
90	38.970	39.403	39.836	40.269	40.702	41.135	41.568	42.001	42.436	42.867

The pressure for greater heads can be readily found by multiplication or addition; thus, the pressure for a head of 110 ft is ten times that for 11 ft. The pressure for 118 ft is equal to the pressure for 110 ft plus that for 8 ft.

Table II. Pressure Conversion



Pressures of water are measured in inches of water or in inches of mercury. In Table II the corresponding pressures are given in pounds or ounces, inches of water and inches of mercury.

Flow of Water in Pipes. Owing to the many practical and variable conditions which affect the flow of water in pipes, such as the smoothness of the pipe, number and character of the joints, bends and valves in the pipe, to say nothing of the size and length of the pipe, all formulas for the velocity and discharge of water in and through pipes can only be considered as approximate. The following formulas and data are taken largely from the National Tube Company's Book of Standards. They agree fairly well with similar tables by Kent and Trautwine, both of whom devote much space to this subject. The QUANTITY OF WATER passing through a given pipe is governed by the sectional area of the pipe or outlet and the mean VELOCITY. The velocity depends primarily upon the PRESSURE or HEAD, and is greatly affected by FRICTION, which again varies with the smoothness of the bore, the diameter and length of the pipe, and whatever obstructions there may be in the pipe. The HEAD is the vertical distance from the surface of the water in the reservoir to the center of gravity of the lower end of the pipe when the discharge is into the air, or to the level surface of the lower reservoir when the discharge is under water. When the pressure is produced by mechanical means, the head of water in feet may be readily determined by the following table:

Table III.* For Converting Pressure in Pounds per Square In into Head of Water in Feet

Pressure	0	1	2	3	4	5	6	7	8	9
0	..... .	2.309	4.619	6.928	9.238	11.547	13.857	16 166	18.476	20.785
10	23.0947	25.404	27.714	30.023	32.333	34.642	36.952	39.261	41.570	43.880
20	46.1894	48 499	50.808	53.118	55.427	57.737	60 046	62.356	64.665	66.975
30	69.2841	71.594	73.903	76.213	78.522	80.831	83.141	85.450	87.760	90.069
40	92.3788	94.688	96 998	99 307	101.62	103.93	106.24	108.55	110.85	113.16
50	115.4735	117.78	120.09	122.40	124 71	126.02	129.33	131.64	133 95	136.26
60	138.5682	140.88	143.19	145 50	147.81	150.12	152.42	154.73	157.04	159.35
70	161.6629	163.97	166.28	168 59	170.90	173.21	175 52	177.83	180.14	182.45
80	184.7576	187.07	189.38	191.69	194 00	196 31	198.61	200.92	203.23	205.54
90	207.8523	210.16	212.47	214.78	217.09	219.40	221.71	224.02	226.33	228.64

* Tables I and III are exact for water at 62° F. and for atmospheric pressure at 14.7 lb per sq in.

To find the velocity of water discharged from a pipe-line longer than four times its diameter, knowing the head, length and inside diameter, use the following formula:

$$v = m\sqrt{\frac{hd}{L + 54d}}$$

in which v = approximate mean velocity in feet per second;
m = coefficient from the table below;
d = diameter of pipe in feet;
h = total head in feet;
L = total length of line in feet.

The following coefficients are averages deduced from a large number of experiments. In most cases of pipes carefully laid and in fair condition, they should give results varying not more than from 5 to 10%.

Values of Coefficient *m*

$\sqrt{\frac{hd}{L+54d}}$	Diameter of pipe in feet							
	0 05	0 10	0.50	1	1 5	2	3	4
	m	*m*	*m*	*m*	*m*	*m*	*m*	*m*
0.005	29	31	33	35	37	40	44	47
0.01	34	35	37	39	42	45	49	53
0.02	39	40	42	45	49	52	56	59
0.03	41	43	47	50	54	57	60	63
0.05	44	47	52	54	56	60	64	67
0.10	47	50	54	56	58	62	66	70
0.20	48	51	55	58	60	64	67	70

Example. Given the head, $h = 50$ ft; the length, $L = 5\,280$ ft and the diameter, $d = 2$ ft; to find the velocity and quantity of discharge.

Substituting these values in the foregoing formula, we get

$$\sqrt{\frac{d \times h}{L + 54\,d}} = \sqrt{\frac{2 \times 50}{5\,280 + 108}} = \sqrt{\frac{100}{5\,388}} = 0.136$$

In column headed $\sqrt{\frac{hd}{L + 54\,d}}$ find 0.10, which is the value nearest to 0.136, and look along this line until column headed 2 is reached; then read 62 as the value of coefficient *m*.

Then $v = 62 \times 0.136 = 8.432$ ft per sec, the velocity required.

To find the discharge in cubic feet per second, multiply this velocity by area of cross-section of pipe in square feet.

Thus, $3.1416 \times (1)^2 \times 8.432 = 26.49$ cu ft per sec.

Since there are 7.48 gal in a cubic foot, the discharge in gallons per second = $26.49 \times 7.48 = 198.2$.

The above formula is only an approximation, since the flow is modified by bends, joints, incrustations, etc.

To find the head in feet necessary to give a stated discharge in cubic feet, use the formula

$$h = \frac{0.000704\,Q^2\,(L + 54\,d)}{d^5}$$

in which h = total head in feet;
L = total length of line in feet;
d = diameter of pipe in feet;
Q = quantity of water in cu ft per second.

Example. Given the diameter of pipe, $d = 0.5$ ft; the length of pipe, $L = 20$ ft; and the quantity of water to be discharged, $q = 3.07$ cu ft per sec; to find the necessary head.

Substituting these values in the above formula, we get

$$h = \frac{0.000704 \times 9.4 \times (20 + 27)}{(0.5)^5}$$

$$= \frac{0.000704 \times 9.4 \times 47}{0.03125} = 9.95 \text{ ft, the required head.}$$

The following formula is simpler and can be used when 54 d in relation to L is so small as to be negligible:

$$h = \frac{0.000704\, Q^2 \times L}{d^5}$$

If the pipe instead of being straight has easy curves (say, with radius not less than five diameters of the pipe) either horizontal or vertical, the discharge will not be materially diminished so long as the total heads and total actual lengths of pipe remain the same, but it is advisable to make the radius as much more than five diameters as can conveniently be done.

To find the diameter of a pipe of given length to deliver a given quantity of water under a given head use the following,

$$d = 0.234 \sqrt[5]{\frac{Q^2 L}{h}}$$

in which d = diameter of pipe in feet;
Q = cubic feet per second delivered;
L = length of line in feet;
h = head in feet.

Example. Given the head, h = 700 ft; the length of pipe, L = 3 000 ft; the quantity to be delivered, Q = 4 cu ft per sec; required the diameter of pipe necessary.

Substituting these values in the foregoing formula, we get:

$$d = 0.234 \sqrt[5]{\frac{16 \times 3\,000}{700}} = 0.234 \sqrt[5]{68.57} = 0.545 \text{ ft} = 6\ 54 \text{ in}$$

To find the diameter of pipe required to deliver a given quantity of water with a given head.

Rule. (1) Reduce the head to feet per 100 ft; (2) from Table IV, find the discharge for the head thus obtained through a pipe 1 ft in diameter; (3) divide the required discharge by that obtained from Table IV; look for the quotient in the column of Table V, headed Ratio of Discharge, etc., and opposite it, in the adjoining columns of the table, will be found the required diameter.

Note. The use of Tables IV and V gives results sufficiently correct for pipes less than 700 diameters in length.

Example. If the head of water from a reservoir to the point of delivery is 20 ft in a distance of 1 860 ft, what is the diameter of a pipe required to deliver 6 cu ft of water per second?

20 ft head in 1 860 ft = 20/18.60 ft in 100 ft, or 1.075 ft in 100.

From Table IV we find that the discharge per second with a head of 1.136 is 3 989 cu ft; for a head of 1.075 it would be about 3.8 cu ft. Dividing the required discharge 6, by 3.8 cu ft per sec, we have 1.58. From Table V the diameter of pipe having a ratio of discharge equal to 1.58 is found to be about 14½ in; therefore we must use a 15-in pipe to obtain the required discharge. If the required discharge is in gallons, divide by 7.5 to reduce to cubic feet. If in cubic feet per minute, divide by 60 to reduce to feet per second.

Table IV. Velocities and Discharges Through a Straight, Smooth Pipe One Foot in Diameter and One Mile, or 5 280 Diameters, in Length

Head in feet per 100 ft	Head in feet per mile	Velocity in feet per sec	Discharge in cubic feet per sec	Discharge in cubic feet per 24 hours
0.0568	3	1.13	0.8914	76 982
0.0758	4	1.31	1.028	88 862
0.0947	5	1.47	1.150	99 403
0 1136	6	1.61	1.264	109 209
0.1325	7	1.74	1.366	118 022
0 1514	8	1.86	1.455	125 740
0.1703	9	1.96	1.539	132 969
0.1894	10	2.08	1.633	141 145
0.2273	12	2.27	1.782	153 964
0.2652	14	2.45	1.924	166 233
0 3030	16	2 62	2.057	177 724
0.3409	18	2.78	2.183	188 611
0.3788	20	2.93	2.301	198 806
0 4735	25	3 28	2.572	222 156
0.5682	30	3.59	2.819	243 604
0.6629	35	3.88	3.047	263 260
0 7576	40	4.15	3.267	282 228
0.8523	45	4.40	3.451	298 209
0.9470	50	4 64	3.638	314 352
1.136	60	5.08	3.989	344 649
1.326	70	5.49	4.311	372 470
1.515	80	5.85	4.602	397 613
1.704	90	6.23	4.900	423 435
1.894	100	6 56	5.144	444 312
2.083	110	6.87	5.395	466 128
2.272	120	7.18	5.639	487 209
2.462	130	7.47	5.866	506 822
2.652	140	7.76	6.094	526 521
2.841	150	8 05	6.322	546 048
3.030	160	8.30	6.534	564 576
3.219	170	8.55	6.715	580 176
3.408	180	8.80	6.903	596 418
3.596	190	9.04	7.100	613 440
3.788	200	9.28	7.276	628 704
4.261	225	9.84	7.696	664 848
4.735	250	10.4	8.168	705 728
5.208	275	10.8	8.482	732 844
5.682	300	11 3	8.914	769 824
6.629	350	12.3	9.621	831 168
7.576	400	13.1	10.28	888 624
8 532	450	13 9	10.91	943 056
9.47	500	14 7	11.50	994 032
10.41	550	15.4	12.09	1 044 576
11.36	600	16.1	12.64	1 092 096
12.30	650	16.7	13.11	1 132 704
13.25	700	17.4	13.66	1 180 224
14.20	750	18.0	14.13	1 220 832
15.15	800	18.6	14.55	1 257 408
16.09	850	19.1	15.00	1 296 000
17.04	900	19.6	15.39	1 329 696
17.99	950	20.3	15.94	1 377 216
18.94	1 000	20.8	16.33	1 411 456
22.73	1 200	22.7	17.82	1 539 648
26.52	1 400	24.5	19.24	1 662 336
30.30	1 600	26.2	20.57	1 777 248
34.08	1 800	27.8	21.83	1 886 112
37.87	2 000	29.3	23.01	1 988 064
47.35	2 500	32.8	25.72	2 221 560
56.81	3 000	35.9	28.19	2 436 040

Table V. Diameters of Pipes and Ratio of Discharge

Diameter of pipe, in	Diameter of pipe, ft	Ratio of discharge to that through a 1-ft pipe with the same head per mile	Diameter of pipe, in	Diameter of pipe, ft	Ratio of discharge to that through a 1-ft pipe with the same head per mile
1	0.0833	0 0020	12½	1.042	1 106
1½	0 1250	0 0055	13	1.083	1 221
2	0.1667	0 0113	14	1.167	1.470
2½	0.2083	0 0198	15	1 250	1.746
3	0 2500	0 0310	16	1.333	2 053
3½	0.2917	0 0458	17	1.417	2 388
4	0 3333	0 0643	18	1.5	2.754
4½	0.3750	0 0857	19	1.583	3 153
5	0 4167	0 1119	20	1.667	3 585
5½	0.4583	0.1422	21	1.75	4 051
6	0.5	0 1767	22	1.833	4.551
6½	0 5417	0 2159	23	1 917	5.084
7	0 5833	0 2600	24	2	5.649
7½	0 6250	0 3090	24⅝	2.052	6.000
8	0 6667	0 3631	26	2 167	6.912
8½	0 7083	0 4220	28	2.333	8.319
9	0 75	0.4871	30	2 5	9.822
9½	0 7917	0 5575	30¼	2 521	10.0
10	0 8333	0 6337	32	2 667	11.6
10½	0 8750	0 7157	34	2 833	13.5
11	0 9167	0 8044	36	3	15.5
11½	0.9583	0 8987	38	3 167	17.8
12	1	1	40	3 333	20.2

This table shows, also, the relative discharging capacities of long pipes. Thus, one 12-in pipe is equal to two 9-in pipes, to nearly six 6-in pipes, or to thirty-three 3-in pipes.

Table VI. Flow of Water in House Service-Pipes

Thomson Meter Company

To find the discharge in gallons, multiply by 7.47

Condition of discharge	Pressure in main, lb per sq in	Discharge in cubic feet per minute from the pipe — Nominal diameters of iron or lead service-pipe in inches: ½	⅝	¾	1	1½	2	3	4	6
Through 35 ft of service-pipe; no back-pressure	30	1.10	1.92	3 01	6.13	16.58	33 34	88.16	173 85	444 63
	40	1.27	2 22	3.48	7.08	19.14	38.50	101.80	200.75	513.42
	50	1.42	2.48	3.89	7.92	21.40	43.04	113.82	224.44	574 02
	60	1.56	2.71	4.26	8 67	23.44	47.15	124.68	245.87	628 81
	75	1.74	3.03	4.77	9 70	26.21	52.71	139.39	274 89	703.03
	100	2.01	3 50	5.50	11.20	30.27	60.87	160.96	317.41	811.79
	130	2.29	3.99	6.28	12.77	34.51	69.40	183.52	361.91	925.58
Through 100 ft of service-pipe; no back-pressure	30	0.66	1 16	1.84	3.78	10.40	21.30	58.19	118.13	317.23
	40	0.77	1.34	2.12	4.36	12.01	24.59	67.19	136.41	366.30
	50	0.86	1.50	2.37	4.88	13.43	27.50	75.13	152.51	409.54
	60	0.94	1.65	2.60	5.34	14.71	30.12	82.30	167.06	448.63
	75	1.05	1.84	2.91	5.97	16.45	33 68	92.01	186.78	501.58
	100	1.22	2.13	3.36	6.90	18.99	38.89	106.24	215.68	579.18
	130	1.39	2 42	3 83	7.86	21.66	44.34	121.14	245.91	660.36
Through 100 ft of service-pipe and 15-ft vertical rise	30	0.55	0.96	1.52	3.11	8.57	17.55	47.90	97.17	260.56
	40	0.66	1.15	1.81	3.72	10.24	20.95	57.20	116.01	311.09
	50	0.75	1.31	2.06	4.24	11.67	23.87	65.18	132 20	354.49
	60	0.83	1.45	2.29	4.70	12.94	26.48	72.28	146.61	393.13
	75	0 94	1.64	2.59	5.32	14 64	29 96	81.79	165.90	444.85
	100	1.10	1.92	3.02	6.21	17.10	35.00	95.55	193.82	519.72
	130	1.26	2.20	3.48	7.14	19.66	40.23	109.82	222.75	597.31
Through 100 ft of service-pipe and 30-ft vertical rise	30	0.44	0.77	1.22	2.50	6.80	14.11	38.63	78 54	211.54
	40	0.55	0 97	1.53	3.15	8 68	17.79	48.68	98.98	266.59
	50	0 65	1.14	1.79	3.69	10.16	20.82	56.98	115.87	312.08
	60	0.73	1.28	2.02	4.15	11.45	23 47	64.22	130.59	351.73
	75	0.84	1.47	2.32	4.77	13 15	26.95	73.76	149.99	403.98
	100	1.00	1.74	2.75	5.65	15.58	31 93	87.38	177.67	478.55
	130	1.15	2.02	3.19	6.55	18.07	37.02	101.33	206 04	554.96

Table VI may also be used when the pressure is in feet-head of water by reducing the head in feet to pounds per square inch by Table I. Thus, if we wish the discharge per minute through a ¾-in pipe 100 ft long with a head of 70 ft, we find from Table I that a head of 70 ft corresponds to a pressure of 30 lb per sq in, and from Table VI we find the discharge thrcugh a ¾-in pipe 100 ft long with a pressure of 30 lb to be 1.84 cu ft per minute.

Table VII. Friction of Water in Pipes Based on Ellis and Howland's Experiments

The following table gives the friction-loss in pounds-pressure per square inch for EACH 100 ft of length in clean iron pipes of different sizes, discharging given quantities of water per minute. This friction-loss is greatly increased by bends or irregularities in the pipe.

To find the friction-head in feet, multiply by 2.3

Gallons per minute	Sizes of pipes, inside diameter							
	¾ in	1 in	1¼ in	1½ in	2 in	2½ in	3 in	4 in
5	3.3	0.84	0.31	0.12				
10	13.0	3.16	1.05	0.47	0.12			
15	28.7	6.98	2.38	0.97	0.26			
20	50.4	12.3	4.07	1.66	0.42			
25	78.8	19.0	6.40	2.62	0.64	0.21	0.10	0.27
30		27.5	9.15	3.75	0.91			
35		37.0	12.4	5.05	1.22			
40		48.0	16.1	6.52	1.60		0.20	
45			20.2	8.15	2.02			
50			24.9	10.0	2.44	0.81	0.35	0.09
75			56.1	22.4	5.32	1.80	0.74	0.23
100				39.0	9.46	3.20	1.31	0.33
125					14.9	4.89	1.99	0.49
150					21.2	7.00	2.85	0.69
175					28.1	9.46	3.85	0.94
200					37.5	12.47	5.02	1.22
250						19.66	7.76	1.89
300						28.06	11.2	2.66
350							15.2	3.65
400							19.5	4.73
450							25.0	6.01
500							30.8	7.43
600								9.54
700								14.32

Friction in the water-supply system is not confined to the pipe and fittings; a proportionately large head is lost driving water through filters and other apparatus. Meters also consume considerable head.

Safe Pressures and Equivalent Heads of Water for Cast-Iron Pipes of Different Sizes and Thicknesses

Calculated by F. H. Lewis from Fanning's Formula

Thickness, in	Size of pipe, in											
	4		6		8		10		12		14	
	Pressure, lb	Head, ft	Pressure, lb	Head, ft	Pressure, lb	Head, ft	Pressure, lb	Head, ft	Pressure, lb	Head, ft	Pressure, lb	Head, ft
7/16	112	258	49	112	18	42						
1/2	224	516	124	280	74	171	44	101	24	55		
9/16	336	774	199	458	130	300	89	205	62	143	42	97
5/8			274	631	186	429	132	304	99	228	74	170
11/16							177	408	137	316	106	244
3/4							224	516	174	401	138	316
13/16									212	488	170	392
7/8									249	574	202	465
15/16											234	538
1											266	612
	16		18		20		24		30		36	
5/8	56	129	41	95								
11/16	84	194	66	152	51	118	30	69				
3/4	112	258	91	210	74	170	49	113	24	55		
13/16	140	323	116	267	96	221	68	157	39	90		
7/8	168	387	141	325	119	274	86	198	54	124	32	74
15/16	196	452	166	382	141	325	105	242	69	159	44	101
1	224	516	191	440	164	378	124	286	84	194	57	131
1 1/8			216	497	209	481	161	371	114	263	82	180
1 1/4					256	589	199	458	144	332	107	247
1 3/8							237	546	174	401	132	304
1 1/2									204	470	157	362
1 5/8									234	538	182	419
1 3/4											207	477

Weights of Lead and Gaskets for Pipe-Joints

Dennis Long & Company

Diameter of pipe, in	Lead, lb	Gasket, lb	Diameter of pipe, in	Lead, lb	Gasket, lb
2	2.5	0.125	12	15	0.250
3	3.5	0.170	14	18	0.375
4	4.5	0.170	16	22	0.500
6	6.5	0.200	18	26	0.500
8	9.0	0.200	20	33	0.625
10	13.0	0.250			

Cast-iron bell and spigot-pipe is known in some localities as CORPORATION PIPE. It is made in four grades, known as Class A, Class B, Class C, and Class D, according to the head or pressure for which it is intended. Class B is the grade most used. Table VIII gives useful information about standard makes of commercial bell and spigot-pipe. The weights are approximate and the weight per foot includes an allowance for the bell. A standard pipe is 12 ft long, although shorter lengths in units of one foot can be had without a bead on the end.

Table VIII. Sizes, Weights and Thickness of Cast-Iron Bell and Spigot-Pipe for Various Pressures

Nominal inside diameter, in	Class A		Class B		Class C		Class D	
	Gas, 100 ft head, 43 lb pressure		Standard, 200 ft head, 86 lb pressure		Medium heavy, 300 ft head, 130 lb pressure		Extra heavy, 400 ft head, 173 lb pressure	
	Thickness, in	Weight per ft-lb	Thickness, in	Weight per ft-lb	Thickness, in	Weight per ft-lb	Thickness, in	Weight per ft-lb
3	.39	14 5	.50	18.0	.53	19 0	.48	18.0
4	.42	20.0	.45	21.7	.48	23.3	.52	25.0
6	.44	30 8	.48	33 3	.51	35.8	.55	38.3
8	.46	42.9	.51	47.5	.56	52.1	.60	55.8
10	.50	57 1	.57	63.8	62	70.8	.68	76.7
12	.54	72.5	.62	82 1	.68	91 7	.75	100.0
14	.57	89.6	.66	102 5	.74	116.7	.82	129.2
16	.60	108 3	.70	125.0	.80	143.8	.89	158.3
18	.64	129.2	.75	150 0	.87	175.0	.96	191.7
20	.67	150.0	.80	175.0	92	208.3	1.03	229.2
24	.76	204.2	.89	233 3	1 04	279.2	1.16	306 7
30	.88	291 7	1.03	333 3	1 20	400.0	1.37	450.0
36	.99	391.7	1.15	454.2	1 36	545.8	1.58	625.0
42	1.10	512.5	1 28	591 7	1.54	716.7	1.78	825.0
48	1.26	666 7	1 42	750 0	1 71	908.3	1.96	1 050.0
54	1.35	800 0	1.55	933 3	1 90	1 141.7	2.23	1 341.7
60	1.39	916.7	1.67	1 104 2	2 00	1 341.7	2.38	1 583.3
72	1.62	1 283 4	1 95	1 545 8	2.39	1 904 2	. .	.....
84	1.72	1 633.4	2 22	2 104 2			.	.....

Table IX. Approximate Cost of Laying Cast-Iron Water Pipe with Lead Joints

Size of pipe	Total cost of excavating, back filling, laying pipe including labor, fittings and joint materials, but not the pipe. Per linear foot									Cost of pipe per linear foot at $56 per net ton for 4-in and $52 for larger sizes F. O. B. factory, plus $2.60 per ton freight, and $2 for hauling			
Depth of trench	Excavation	Cost per linear foot with labor per day			Cost per linear foot		Linear total cost per foot less pipe at			Class A for 100-foot head	Class B for 200-foot head	Class C for 300-foot head	Class D for 400-foot head
		$3.00	$5.00	$7.00	Lead and hemp	Fittings	$3.00	$5.00	$7.00				
4-in 5 ft	Easy	.255	.40	.56	.05	.05	.355	.50	.66	$0.606	$0.658	$0.706	$0.7575
	Medium	.29	.50	.70			.39	.60	.80				
	Hard	.42	.72	1.00			.52	.82	1.10				
	Rock	2.60	4.20	5.50			2.70	4.30	5.60				
6-in 5 ft	Easy	.27	.42	.58	.07	.075	.415	.565	.725	.872	.943	1.01	1.09
	Medium	.315	.52	.72			.46	.665	.865				
	Hard	.44	.74	1.02			.585	.885	1.165				
	Rock	2.69	4.34	5.68			2.835	4.485	5.825				
8-in 5 ft	Easy	.34	.52	.73	.087	.11	.537	.717	.927	1.22	1.35	1.48	1.58
	Medium	.40	.66	.92			.597	.857	1.117				
	Hard	.57	.95	1.31			.767	1.147	1.507				
	Rock	2.21	5.29	6.81			3.407	5.49	7.01				
10-in 5 ft	Easy	.37	.56	.78	.11	.15	.63	.82	1.04	1 62	1.81	2.00	2.17
	Medium	.43	.71	1.00			.69	.97	1.26				
	Hard	.62	1.03	1.42			.88	1.29	1.68				
	Rock	3 47	5.61	7.36			3.73	5.87	7.62				
12-in 5 ft	Easy	.39	.60	.84	.13	.19	.71	92	1.16	2.05	2.33	2.60	2.83
	Medium	.46	.76	1.06			.78	1 08	1.38				
	Hard	67	1.11	1 53			.99	1.43	1.85				
	Rock	3.75	6.03	7.91			4.05	6.35	8.23				
16-in 6 ft	Easy	.54	.85	1.18	.20	.37	1.11	1 42	1.75	3 07	3.54	4.07	4.48
	Medium	.63	1.06	1.47			1.20	1 63	2.04				
	Hard	88	1.48	2.05			1.45	2.06	2.62				
	Rock	5 36	8 69	11 40			5 93	9 26	11.97				

Where trench bracing is necessary, allow $3 per linear foot for 100 ft less salvage value of lumber.

Table IX (Continued). Approximate Cost of Laying Cast-Iron Water Pipe with Lead Joints

Size of pipe / Depth of trench	Total cost of excavating, back filling, laying pipe including labor, fittings and joint materials, but not the pipe. Per linear foot								Cost of pipe per linear foot at $56 per net ton for 4-in and $52 for larger sizes F. O. B. factory plus $2.60 per ton freight and $2 for hauling			
	Excavation	Cost per linear foot with labor per day			Cost per linear foot		Linear total cost per foot less pipe at			Class A for 100-foot head	Class B for 200-foot head	Class C for 300-foot head	Class D for 400-foot head
		$3.00	$5.00	$7.00	Lead and hemp	Fittings	$3.00	$5.00	$7.00				
20-in 6 ft	Easy	.76	1.06	1.50	.25	.41	1.36	1.72	2 16	$4.25	$4.95	$5.90	$6.49
	Medium	.81	1.21	1.87			1 47	1.87	2 53				
	Hard	1.15	1.91	2.65			1.81	2.57	3 31				
	Rock	6.45	10.40	13.64			7.11	11.06	14 30				
24-in 6½ ft	Easy	.93	1.42	2.00	.30	58	1 81	2 30	2.88	5 78	6 60	7.90	8.68
	Medium	1.10	1.81	2.52			1 98	2.69	3.40				
	Hard	1.52	2.61	3.60			2 45	3 49	4.48				
	Rock	8.44	13.84	18.16			9 32	14.72	19.04				
30-in 7 ft	Easy	1.19	1 78	2.54	.37	.82	2 38	2.97	3 73	8.26	9.43	11.32	12.74
	Medium	1.41	2.32	3 23			2 60	3 51	4 42				
	Hard	2.02	3.36	4.63			3.21	4 55	5 82				
	Rock	10.69	17.28	22.67			11.88	18.47	23.86				
36-in 7½ ft	Easy	1 55	2.33	3 28	.44	1.13	3.12	3 90	4.85	11.09	12.86	15.45	17.69
	Medium	1.84	3.47	4.19			3 41	5 04	5.76				
	Hard	2.65	4.39	6.00			4 22	5.96	7.57				
	Rock	13.87	22.39	29.38			15 44	23.96	30.95				
48-in 8½ ft	Easy	2.23	3 43	4.76	.58	2.35	5 16	6.36	7 69	18.87	21.23	25.71	29.72
	Medium	2.64	4 40	6.07			5 57	7.33	9.00				
	Hard	3.81	6 40	8.76			6.74	9.33	11.69				
	Rock	19.22	31 12	40.77			22 15	34 05	43 70				

Where trench bracing is necessary, allow $3 per linear foot for 100 ft less salvage value of lumber.

For preliminary estimates, Table IX, giving the approximate cost of laying cast-iron water pipes with lead joints, will be found safe as well as convenient. From the table a unit price per foot for each size of pipe can be established and then the totals multiplied by those units.

Private Water-Supply, Pumps

Private Water-Supplies. The architect is frequently required to furnish a water-supply for isolated buildings, and even in cities it is becoming quite common for manufacturing establishments and large buildings to have their own water-supply; so that some knowledge of the various methods of supplying water is requisite. Power-pumps are of so many kinds and so intricate in construction that no attempt will be made to describe them.

The Hydraulic Ram. Where a small stream of water having a fall of 2 ft or more flows near the premises, an hydraulic ram may be used to great advantage to furnish water for domestic purposes, or even for irrigation. The ram is operated by the momentum of the water flowing through the drive-pipe and delivers water into an open tank. Water can be conveyed by a ram 13 000 ft when elevated 500 ft, provided there is sufficient fall. The drive-pipe supplying the ram should be 30 or 40 ft long to give the necessary momentum. The use of the ram is the most economical method of pumping water, as there is no expense for maintenance except for repairs, and the cost of installation, also, is small.

The Capacities of the Rife Rams are given in the following table. The capacities are determined from the table by multiplying the available supply of water per minute, or the rated amount of water a Rife ram will use, by the factor found in the table at the intersection of the line giving the fall available, for the drive-pipe, and the column showing the height the water is to be elevated. The factor for a 10-ft fall and 50-ft discharge is 192, and this multiplied by the supply of water per minute will give the delivery per day. This is shown by the example worked out in the corner of the table. These capacities are based on efficiencies dependent on the ratio of fall to lift. A fall of 10 ft and a lift of 50 ft give a ratio of 1 to 5, and an efficiency of 66⅔%. The efficiencies of Rife rams based on various ratios are also given in the table.

Deep Wells and Plunger-Pumps. The common method of obtaining a private water-supply is to drive a deep well until a sufficient supply of water is obtained. The depth to which a well must be driven will, of course, depend upon the locality, and can only be determined by drillings. As the well is driven, a large wrought-iron pipe is sunk to form the casing. Casings are seldom less than 6 or more than 10 in inside diameter, 8 in being the common size. When the water-pocket has been reached, the water will usually rise and stand in the pipe. The cost of drilling deep wells, per foot of depth, INCLUDING THE CASING, differs, of course, with the strata, location and other local conditions. For raising the water into an open tank a single-acting pump consisting of a working-head, (Fig. 1), which operates a piston located in a cylinder placed in a smaller pipe lowered into the well through which the water is raised is commonly employed. The cylinder should preferably be placed below the water-line in the well, and is usually connected with the working-head by wooden sucker-rods. The working-head may be operated by hand, or by a crank-rod attached to a pumping-jack, windmill or engine. With a single-acting pump the plunger is raised and lowered once with every revolution of

the driving-wheel, the principle of operation being the same as in an ordinary hand suction-pump. Fig. 2 shows a simple arrangement for operating a working-head by belt-power. This is known as a deep-well power working-head. A DEEP-WELL PUMP (Fig. 2) differs from a SUCTION-PUMP in that it will raise water from any depth, whereas a suction-pump in practice will raise water only about 25 ft. A suction-pump may be placed at any point in relation to the well, and will draw the water any reasonable horizontal distance. The deep-well pump, on the other hand, must be set directly over the well, but

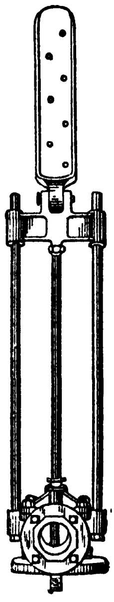


Fig. 1. Working-head for Deep-well Pump

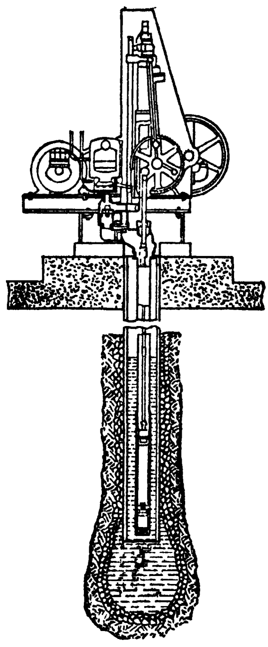


Fig. 2. Deep-well Pump

it will then deliver the water at any desired point. The amount of water pumped in a minute by any single-acting pump is determined by the diameter of the suction-cylinder, the length of stroke, and the number of strokes per minute. The table following gives the capacity per stroke for cylinders of different diameters, and for strokes of different lengths. To find the capacity per minute, multiply the values given in the table by the revolutions per minute. The usual speed of single-acting working-heads and pumping-jacks is from 25 to 30 revolutions per minute. Cylinders over 2¾ in in diameter should have a substantial iron working-head.

Table X. Water Required for Rife Rams

Number	Dimensions						Size of drive-pipe, in	Size of delivery-pipe, in	Gallons per minute required to operate engine, gal	Least number of feet of fall recommended, ft	Weight, lb
	Height, ft	in	Length, ft	in	Width, ft	in					
10	2	1	3	2	1	8	1¼	¾	3 to 6	3	150
15	2	1	3	4	1	8	1½	¾	5 to 12	3	175
*20	2	3	3	8	1	9	2	1	10 to 18	2	225
25	2	3	3	9	1	9	2½	1	11 to 24	2	250
30	2	7	3	10	1	10	3	1¼	15 to 35	2	275
40	3	3	4	4	2	0	4	2	30 to 75	2	600
80	7	4	8	4	2	8	8	4	150 to 350	2	2 500
*120	.	..				.	12	5	375 to 750	2	3 000
†120	8	9	9	6	3	8	12 (two)	6	750 to 1 500	2	5 500

* Single. † Duplex.

Table XI. Capacities of Rife Rams

Power-head or fall in ft	Height or head in feet the water is to be delivered																
	4	10	15	20	30	40	*50	60	70	80	90	100	120	140	160	180	200
2	540	192	128	96	64	43	29	24	..	.	.	.	..	..	.	.	.
3	.	301	192	144	96	72	58	43	37	27	24	..	.	..	.	.	.
4	..	432	256	192	128	96	77	64	55	43	38	29	24	.	.		
5	.	540	345	240	160	120	96	80	69	60	53	43	30	26	.		.
6	..	.	432	302	192	144	115	96	82	72	64	57	43	31	27	24	.
7	..	..	505	378	235	168	134	112	96	84	75	67	50	36	31	28	25
8	..	..	...	432	270	192	154	128	110	96	86	77	64	55	43	38	29
9	..	..	...	485	300	216	173	144	124	108	96	86	72	62	54	43	39
*10	..	..	...	540	360	252	*192	160	137	120	107	96	80	68	60	53	43
12	.	..	..	...	430	301	230	192	165	144	128	115	96	82	72	64	57
14		.	.	.	505	353	270	224	192	168	150	135	112	96	84	75	67
16						432	323	257	220	192	171	154	128	110	96	85	77
18						486	390	303	247	216	192	173	144	124	108	96	86
20						540	430	336	288	240	214	192	160	137	120	107	96
22						..	475	370	303	264	235	212	176	151	132	118	105
24						..	520	405	346	288	256	230	192	164	144	128	115
26						..	..	470	375	328	278	250	208	178	156	139	125
28						...	...	505	430	354	300	269	224	192	168	149	134
30						..	.	540	465	405	336	288	240	206	180	160	144

Example

With a supply of 1 400 gal. per min. 10-ft fall, 50-ft elevation, No. 120 engine will deliver 268 800 gal per day
$1400 \times 192 = 268800$

* Multiply factor opposite POWER-HEAD and under PUMPING-HEAD by the number of gallons per minute USED by the engine; the result will be the number of gallons DELIVERED per day.

The efficiency developed is governed by the ratio of fall to pumping-head.

The efficiency of rife rams is based on......
- 75% for a ratio of 1 to 2½
- 70% for a ratio of 1 to 3
- 66⅔% for a ratio up to 1 to 18
- 60% for a ratio up to 1 to 23
- 50% for a ratio up to 1 to 30

Table XII. Showing Capacity of Single-Acting Pumps of Given Diameter and Length of Stroke

Diameter of cylinder in inches	Length of stroke in inches								
	6	8	10	12	14	16	18	20	24
	Capacity per stroke in gallons								
1¼	0.0319	0.0425	0.0531	0.0637	0.0743	0.0848	0.0955	0.1062	0.1274
1⅜	0.0385	0.0513	0.0642	0.0770	0.0890	0 1027	0.1156	0.1280	0.1541
1½	0.0459	0.0612	0.0765	0.0918	0.1071	0.1224	0.1377	0.1530	0.1836
1¾	0.0625	0.0833	0.1041	0.1249	0.1457	0.1666	0.1874	0.2082	0.2499
2	0.0816	0.1088	0.1360	0.1632	0.1904	0.2176	0.2448	0.2720	0.3264
2¼	0.1033	0.1377	0.1721	0.2063	0.2410	0.2754	0.3096	0.3442	0.4128
2½	0.1275	0.1700	0.2125	0.2550	0.2975	0.3400	0.3825	0.4250	0.5100
2¾	0.1543	0.2057	0.2571	0.3085	0.3598	0.4114	0.4626	0.5142	0.6170
3	0.1836	0.2448	0.3060	0.3672	0.4284	0.4896	0.5508	0.6120	0.7344
3¼	0.2154	0.2872	0.3594	0.4312	0.5030	0.5748	0.6466	0.7182	0.8624
3½	0.2499	0.3332	0.4165	0.4998	0.5831	0.6664	0.7497	0.8330	0.9996
3¾	0.2868	0.3824	0.4780	0.5736	0.6692	0.7648	0.8605	0.9561	1.1470
4	0.3264	0.4352	0.5440	0.6528	0.7616	0.8704	0.9792	1.0880	1.3056
4¼	0.3684	0.4912	0.6141	0.7368	0.8596	0.9824	1.1050	1.2280	1.4730
4½	0.4131	0.5508	0.6885	0.8262	0.9639	1.1016	1.2393	1.3770	1.6524
4¾	0.4602	0.6136	0.7671	0.9204	1.0730	1.2270	1.3800	1.5340	1.8400

Table XIII. Action of Wind and Capacities of Pumping Windmills

Velocity per hour in miles	Pressure per square foot in pounds	Description of wind	Action of wind and windmills
3	0.045	Just perceptible. . .	Windmills will not run
5	0.125	Pleasant wind	Might start if lightly loaded
8	0.33	Fresh breeze.	Will start pumping
10	0.5	Average wind. . . .	Pumps nicely if properly loaded
15	1.125	Good working wind .	Does excellent work
20	2	Strong wind. . . .	Gives best service
25	3.125	Very strong wind.	Maximum results secured
30	4.5	Gale.	Should be furled out of wind
40	8	Storm.	Well-constructed mills and towers safe if properly erected
50	12.5	Severe storm	
60	18	Violent storm.	Buildings, trees, etc., might be injured
80	32	Hurricane.	Buildings, trees, etc., would be injured
100	50	Tornado.	Ruin

From the above table it will be seen that the only available winds are those blowing with a velocity of from 8 to 25 miles per hour, and that a 15-mile wind

can be utilized to the best advantage. It is therefore advisable to LOAD a windmill for a 15-mile wind. It then starts pumping in an 8-mile wind, does excellent work in a 15-mile wind and reaches the maximum results in a 25-mile wind.

Table XIV. Capacity of the Windmill

Diameter of mill wheel, ft	Velocity of wind in miles per hour	Revolutions of wheel per minute	Gallons of water raised per minute to an elevation of						Equivalent actual useful h.p. developed
			25 ft	50 ft	75 ft	100 ft	150 ft	200 ft	
8½	16	40 to 50	6 192	3 016					0.04
10	16	35 to 40	19.179	9 563	6.638	4.750			0 12
12	16	30 to 35	33.941	17.952	11.851	8.435	5.680		0 21
14	16	28 to 35	45 139	22.569	15.304	11.246	7.807	4.998	0 28
16	16	25 to 30	64 600	31 654	19.542	16.150	9.771	8 075	0.41
18	16	22 to 25	97 682	52.165	32.513	24 421	17.485	12 211	0 61
20	16	20 to 22	124.950	63.750	40.800	31.248	19.284	15.938	0 78
25	16	16 to 18	212.381	106.964	71.604	49.725	37.349	26.741	1 34

Windmills. The horse-power of windmills is proportional to the squares of their diameters and inversely as their velocities; for example, a 10-ft mill in a 16-mile breeze will develop 0.15 horse-power at 65 revolutions per minute; and with the same breeze:

a 20-ft mill, at 40 revolutions per minute, 1 horse-power;
a 25-ft mill, at 35 revolutions per minute, 1¾ horse-power;
a 30-ft mill, at 28 revolutions per minute, 3½ horse-power.

The wheels of very few windmills are larger than 25 ft in diameter. There are no pumps which will enable the user of a windmill to utilize the increased power obtained from winds of high velocity, so that in practice the amount of water pumped by windmills in high winds is but little more than is pumped by the same mills in winds having velocities of from 12 to 18 miles per hour. For this reason it is customary to regulate windmills to govern at about 25 miles an hour.

Air-Lift Process. Compressed air is used to an increasing extent for raising water from artesian wells. The process in general consists of submerging a discharge-pipe in a closed well, with a smaller pipe inside delivering compressed air into it at the bottom. The compressed air by its inherent expansive force lifts a column of mingled air and water which is conveyed to an open tank, to permit of the escape of the air. If desired the water may then be conveyed by gravity into a series of closed tanks, and forced by air-pressure to different parts of a building, the only machinery required being an air-compressor and power for driving it. The slip of the bubble constitutes the chief loss of energy in the air-lift. The method of piping a well differs according to its general conditions and the quantity of water to be pumped. "No two wells are alike, and consequently the method of piping which might be applied to one would be unsuited to another."

Advantages of the Air-Lift Process. From two to six times as much water may be obtained from a given diameter of well as with any other known sys-

tem, because there are no valves, cylinders, or rods to hinder the rapid discharge of water. One air-compressor operates any number of wells, which may be any distance apart so as not to affect one another. There is nothing outside the engine-room to look after or wear out. Nothing but common pipe in the wells. Sand or gravel does no harm. The COST of raising 1 000 gal of water by this method, including fuel, labor, oil, interest on cost of well, boiler, compressor, foundations, pipes, real estate, erection and taxes, including 15% for depreciation, runs from 4 cts down to .8 ct, according to the size of the plant, height of lift, and other local conditions. With the average outfit of medium or small size, it is usually under 2½ cts. The air-lift process is now extensively used in ice-works, breweries, cold-storage houses, textile mills, dye-works, etc., and a great variety of industrial plants, and for the water-supply of quite a number of the smaller cities. In Newark, N. J., pumps of this type are at work having a total capacity of 1 000 000 gal daily, lifting water from three 8-in artesian wells.*

Pneumatic Water-Supply Systems. A closed water-tight tank of iron or steel is used with the pneumatic system, and this tank may be located at any level, for the water is forced from it by means of compressed air confined in the top of the tank. This fact makes it possible to bury the tank in the ground below the frost-line, away from the heat of the sun, and where the water will have an almost uniform temperature the year round. The water is protected from possible contamination from insects, rats, birds, dust, or other agencies, while the tank takes up no valuable space above ground, imposes no weight upon the attic-floor of a building, and does not disfigure the landscape. The principle of operation is this: Air is compressible, while water is not. If, then, water is pumped into a closed tank at the bottom, it will trap the air within, and the more water pumped in, the greater the compression of the air. The elasticity of the air, then, will force the water out again, whenever a faucet is opened, and the water will continue to flow as long as the air is under sufficient pressure in the tank. In practice the air would become absorbed by the water in the tank, and in a short time become exhausted, if it were not supplied as fast as used. This is accomplished by injecting a proportionate amount of air with each stroke of the pump, by means of a SNIFTER-VALVE air-compressor, or other device. All connections to the tank are taken from the bottom, to prevent the escape of air which would occur if the connections were taken from the top of the tank.

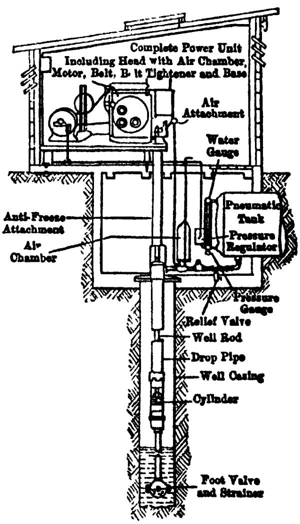


Fig. 3. Typical Installation of Deep-well Pump for Pneumatic Water Supply

A pneumatic system of water-supply used in connection with a deep well is shown in Fig. 3. The pumping-head is located directly over the well for

* Kent.

deep-well pumping, although in shallow-well pumping it may be located at any convenient place not necessarily over the well. In cold climates all parts of the pump containing water are located in a pit below frost-line, an anti-freezing attachment being used for the purpose.

Pneumatic systems are generally operated at pressures varying from 25 to 50 lb. That is, when the pressure of air and water within the tank drops to 25 lb the pump starts automatically and continues pumping until the pressure rises to 50 lb, when the pump automatically cuts out. This range of pressures helps determine the size of tank required.

The following table shows approximately the amount of water that can be drawn from a tank with the pressure starting at 50 lb and dropping to 25 lb, the proportion of air in the tank being $\frac{1}{3}$ to $\frac{2}{3}$ water.

Capacity of tank, gal	Air in tank, gal	Water in tank, gal	Water that can be drawn without pump operating, gal
25	$8\frac{1}{3}$	$16\frac{2}{3}$	5
40	$13\frac{1}{3}$	$26\frac{2}{3}$	8
80	27	53	16
120	40	80	24
220	73	147	44
315	105	210	63
525	175	350	105
1 000	333	666	200

The use of the water likewise helps determine the size of tank to install. For domestic purposes, a comparatively small tank might be desirable to insure frequent changes and keep the water from becoming stale. Where an uninterrupted supply is a necessity, on the other hand, an extra large tank might be desirable to tide over breakdown or repair periods.

Horse-Power Required to Raise Water to Different Heights

General Principles. The power required to raise a certain quantity of water to a certain height varies directly with the quantity to be raised, and also with the height. For instance, it requires twice as much power to raise 200 gal per minute 10 ft high as it does to raise 100 gal to the same height and in the same time; and to raise 100 gal 20 ft high requires twice as much power as it does to raise 100 gal 10 ft high. To find the theoretical horse-power necessary to elevate water to a given height, multiply the number of gallons per minute by 8.335, the weight of 1 gal, and this result by the total number of feet the water is raised, that is, from the surface of the water to the highest point to which the water is raised, and the result gives the power in foot-pounds; divide by 33 000, and the quotient is the horse-power. To the theoretical power a liberal allowance must be made for the inefficiency of the pump. For a cylinder-pump add from 75 to 100%. To the actual height to which the water is to be raised add the friction-loss in feet, given in Table VII, when the discharge is to be piped any distance.

Example. Find the theoretical horse-power required to raise 100 gal per minute 120 ft high, through a 3-in pipe, 200 ft long.

Solution. From Table VII the friction-head for 100 gal per min in a 3-in pipe, 100 ft long, is 1.31×2.3 or 3 ft. For 200 ft it will be 6 ft, which, added to 120, gives 126 ft for the height. Then theoretical horse-power $= 100 \times 8.35 \times 126/33\,000 = 3.2$ h.p. The actual horse-power required will probably vary from 5 to 6, according to the efficiency of the pump. The mistake of using too small a discharge-pipe can easily be seen from Table VII. For instance, if one attempted to force 100 gal per minute through 100 ft of 2-in pipe, the back-pressure would be equivalent to raising the water 22 ft high. The fuel used would be correspondingly increased. Right-angle turns are to be avoided, as the friction is very materially increased, being practically equal to the friction of 25 ft of straight pipe.

Pump Piping. Both the operating economy and the efficiency of a pumping plant depend much upon the manner in which the piping is proportioned and installed. FRICTION is the great power consumer so that suction and discharge lines must be of ample size, as straight as possible and free from unnecessary bends or fittings. Whenever possible LONG RADIUS BENDS are used and GATE-VALVES in preference to GLOBE-VALVES.

When installing suction lines it is well to SLOPE them towards the source of water-supply. The slope permits draining the line and avoids air-pockets.

For reciprocating pumps on long suction-lines an AIR-CHAMBER near the pump is recommended. For any type of pump the suction-line must be tight. If the suction-pipe leaks, air will be drawn in, causing a reciprocating pump to pound and a centrifugal pump to stop pumping and simply churn.

The SUCTION-LIFT of a pump depends upon the temperature of the water to be lifted and its elevation with relation to sea-level. Table XV gives the safe suction-lifts for pumps under various conditions of elevation and temperature.

When a battery of pumps all draw their supply of water from one source, it is advisable to run independent suction-lines to each pump. When the source of supply is above the level of the pump, however, so that water flows to the pumps by gravity, the suctions can all connect to a common header, as is commonly done in practice. Under such conditions each suction-pipe should be VALVED.

Table XV. Safe Suction-Lifts of Pumps

Elevation above sea-level, feet	Atmospheric pressure, pounds per square inch	Barometric pressure, inches of mercury	Temperature of water to be pumped, degrees Fahrenheit				
			60	90	120	150	180
			Safe suction-lift of pumps in feet				
10 000	10.107	20.582	17.0	16.4	14.7	11.3	4.7
5 000	12.224	24.890	20.5	19.9	18.2	14 8	8.2
4 000	12.689	25.837	21.5	20 9	19.2	15.8	9.2
3 000	13.169	26.813	22.4	21 8	20 2	16.8	10.2
2 000	13.665	27.824	23.2	22 6	21 0	17.6	11.0
1 000	14 174	28 861	24.1	23 5	21 9	18.5	11.9
Sea-level	14.696	29.925	25.0	24.4	22.8	19.4	12.8

Water of temperatures higher than 180° F. cannot successfully be raised by suction but for the best operation must flow to the pump by GRAVITY.

Waters of temperatures lower than 180° but over 100° F. can be raised by suction, but are more successfully handled when they flow by gravity to the pumps.

The size of discharge-pipe from a pump affects greatly the cost of operation. A small pipe will cost less to install, but the initial cost will be more than offset by the cost of operation, as the following example shows.

Table XVI. Theoretical Horse-Power Required to Pump Water Through 1 000 Feet of New Pipe

Discharge in gallons per minute	Diameter of discharge pipe in inches							
	1¼	1½	2	2½	3	4	5	6
	Horse-power required							
50	5.4	2.5	0.9					
75		8.0	2.8	0.9				
100		18.3	6.4	2.1	0.9			
150			20.3	7.1	2.9	0.6		
200				15.5	6.3	1.5		
300					20.5	5.0	1.7	
400						11.3	3.8	1.6
500						21.6	7.3	2.9
600							12.2	5.1

Take for comparison a 5-in and a 6-in pipe, each discharging 500 gal of water per minute. The 5-in pipe requires 7.3 theoretical horse-power; the 6-in pipe requires only 2.9 theoretical horse-power.

Assuming a 70% pump efficiency and 86% motor efficiency, the over-all efficiency would be $0.70 \times 0.86 = 0.60 = 60\%$. The actual electrical input to the motors therefore would be 12.1 horse-power for the 5-in pipe and 4.8 horse-power for the 6-in pipe, or 9 and 3.6 kw's, respectively. Based on an 8-hour day and 2 cts per kw hour, the approximate costs for a year would be $540 for the 5-in discharge-pipe and $220 for the 6-in discharge-pipe.

Table XVII. Effective Fire-Streams

Using 100 ft of 2½-in ordinary best-quality rubber-lined hose between nozzle and hydrant or pump

Smooth nozzle	¾ in					⅞ in				
Pressure at hydrant, lb	32	54	65	75	86	34	57	69	80	91
Pressure at nozzle, lb....	30	50	60	70	80	30	50	60	70	80
Vertical height, ft..	48	67	72	76	79	49	71	77	81	85
Horizontal distance, ft.	37	50	54	68	62	42	55	61	66	70
Gal discharged per min .	90	116	127	137	147	123	159	174	188	201

Smooth nozzle	1 in					1⅛ in				
Pressure at hydrant, lb.	37	62	75	87	100	42	70	84	98	112
Pressure at nozzle, lb....	30	50	60	70	80	30	50	60	70	80
Vertical height, ft.......	51	73	79	85	89	52	75	83	88	92
Horizontal distance, ft...	47	61	67	72	76	50	66	72	77	81
Gal discharged per min..	161	208	228	246	263	206	266	291	314	336

Fire-Streams. The following is an extract from a paper read by John R. Freeman at a meeting of the New England Waterworks Association, entitled Some Experiments and Practical Tables Relating to Fire-Streams: "When unlined linen hose is used the friction or pressure-loss is from 8 to 60%, increasing with the pressure. This kind of hose is best for inside use in short lengths. Mill-hose is better than unlined linen hose for long lengths, but ordinarily the best quality of smooth rubber-lined hose is superior to the mill-hose, having less frictional resistance. The ring-nozzle is inferior to the smooth nozzle and actually delivers less water than the smooth nozzle. For instance, the 7/8-in ring-nozzle discharges the same quantity of water as a 3/4-in smooth nozzle, and a 1-in ring-nozzle the same as a 7/8-in smooth nozzle. Two hundred and fifty gallons per minute is a good standard fire-stream at 80-lb pressure at the hydrant; 100-lb pressure should not be exceeded except for very high buildings or lengths of hose exceeding 300 ft."

Notes on the Construction of Cylindrical Wooden Tanks

Material should be either cedar, cypress, juniper, fir, yellow pine, or white pine, free from imperfections and thoroughly air-dry. Clear Louisiana red, Gulf cypress makes the most durable tanks.

Staves and Bottom of tanks of greater capacities than 15 000 gal should be made of 2½-in, dressed to about 2¼ in, stock for tanks 12 ft and not exceeding 16 ft diameter or 16 ft deep. For larger tanks 3-in, dressed to about 2¾ in, stock should be used. For smaller tanks 2-in stock may be used. Staves should be connected about one-third the distance from the top by a 5/8-in dowel to hold them in position during erection. The bottom planks should be dressed on four sides, and the edges of each plank should be bored with holes not over 3 ft apart for 5/8-in dowels.

Taper. The batter to each side should not be less than ¼ in nor more than ½ in per ft.

Hoops should be of ROUND wrought iron or mild steel of good quality. Wrought iron is preferable because it does not rust as easily as steel. There should be no welds in any of the hoops. Where more than one length of iron is necessary, lugs should be used to make the joints; and when more than one piece is necessary the several pieces constituting one hoop should be tied together in preparing for shipment. Hoops for fire-tanks should be of such size and spacing that the stress in no hoop will exceed 12 500 lb per sq in when computed from the area at root of thread. For general purposes, a stress of 15 000 lb per sq in is permissible. On account of the swelling of the bottom planks, the hoops near the bottom may be subjected to a stress greater than that due to the water-pressure alone; additional hoops, therefore, should be provided. For tanks up to 20 ft in diameter, one hoop of the size used next above it should be placed around the bottom opposite the croze and not counted upon as with standing any water-pressure. For tanks 20 ft or more in diameter, two hoops, as above, should be used. Hoops with UPSET ends must not be used. The top hoop should be placed within 2 in of the top of staves, so that the overflow-pipe may be inserted as high as possible. Hoops should be so placed that the lugs will not be in a vertical line. No hoop should be less than ¾ in in diameter. All should be cleaned of mill-scale and rust and painted one coat of red lead, lampblack and boiled oil before erecting.

Note. The strength of a tank depends chiefly on its hoops. Round hoops are specified because they do not rust rapidly; a slight amount of rust does

not have the same weakening effect as on a flat hoop, and round hoops are not likely to burst when the tank swells, as they will sink into the wood.

Spacing of Hoops. The hoops should be spaced so that each one will have the same stress per square inch, and no space should be greater than 21 in. To meet this requirement the hoops must be spaced quite close together at the bottom, the space between them gradually increasing towards the top.

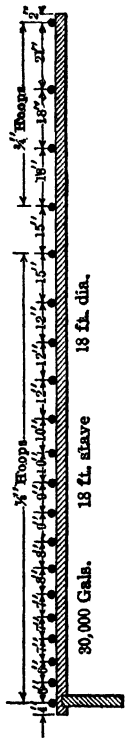
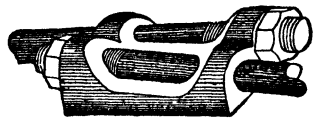
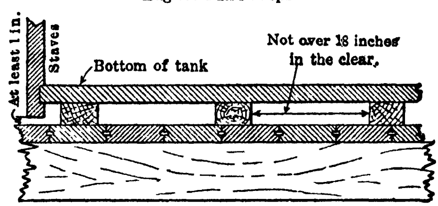


Fig. 4. Diagram of Hoop-spacing for Tanks



Lug for Tank-hoops



Support for Bottom of Tank

Fig. 5. Wooden Tank Details

Fig. 4 shows the proper spacing of hoops for a tank 18 ft in diameter, with 18-ft staves. The spacing for seven other sizes of tanks is given in the pamphlet referred to. It may be computed by the following formula:

$$\text{Spacing of hoops in inches} = \frac{\text{strength}}{2\ 6 \times \text{diameter in feet} \times H}$$

For strength of a $\frac{3}{4}$ 9-in rod use 3 750; of a $\frac{7}{8}$-in rod, 5 250; of a 1-in rod, 6 875; and of a $1\frac{1}{8}$-in rod, 8 625.

H is the distance from surface of the water to center of hoop in feet.

Example. How far apart should 1-in hoops be placed, at 15 ft 2 in from top of tank, on a tank 20-ft diameter?

$$\textbf{Solution.}\quad \text{Spacing} = \frac{6\ 875}{2.6 \times 20 \times 15} = 8\tfrac{3}{4}\ \text{in}$$

Lugs should be as strong as the hoops. A lug similar to Fig. 4 is simple and fulfils the requirement for strength. Malleable lugs are required.

Support. The weight of the tank should be supported entirely from its bottom; and in no event should any weight come on the bottom of the staves. The planks upon which the tank-bottom rests should cover at least one-fifth the area of the bottom, should be not over 18 in apart, and of such thickness that the bottom of the staves will be at least 1 in from the floor (see Fig. 5).

The Discharge-Pipe should preferably leave the bottom of the tank at its center and extend up inside of the tank 4 in, to allow the sediment to collect in the bottom of the tank.

The Overflow-Pipe should be placed as near the top of the tank as possible, discharging either through side or bottom, as may be desired. An overflow is much to be preferred to a telltale, as the latter is liable to get out of order.

Heating. Tanks of moderate size need to be provided with some means to prevent freezing. When a tank is in an enclosed room, as in a mill-tower, the best method is to keep the room warm by a coil of steam-pipe with a return to the boiler-room. A covered tank out of doors may often be similarly heated by placing the steam-pipe in the bottom of the tank. With a tank located on a high trestle, or at a distance from the steam-supply, it is often impracticable to arrange a return-pipe. In this case steam may be blown directly into the water in the tank. A 1-in pipe is generally sufficient for this purpose. It should be carried to the top of the tank and there bend over and dip downwards, so that its outlet is about 1 ft below the high-water line. A check-valve should be placed in this steam-pipe, near its point of discharge, to prevent water being drawn back by siphon-action when the steam is shut off. The water in fire tanks must be kept from freezing by means of a water-heater which either heats a coil in the tank, or circulates a current of water through the tank.

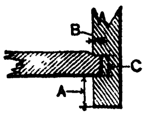
Frostproofing for Pipes. The discharge-pipe from a tank on a trestle, or from one elevated above a roof, must be protected from freezing. The common practice is to enclose the pipe in a double, triple, or quadruple box made of boards and tarred paper, as shown in Fig. 6. If steam is supplied to the tank, the steam-pipe is carried inside the box. In New England, New York State and Canada the quadruple boxing is generally used, whereas in the milder regions to the south triple or double boxing is used. The boxing should always be carried down into the ground below the frost-line, and a good tight joint made at the underside of the tank.

Fig. 6. Method of Frostproofing Pipes

Covers. For economy in heating and to prevent birds, leaves, etc., from getting into the water, all out-of-door tanks should be covered. A double cover is recommended consisting of a tight flat cover made of matched boards supported by joists which span the top of the tank, and above this a shingled, conical roof. To prevent the covering from being blown off, it should be firmly fastened to the top of the tank by straps of iron. In order to keep out the wind particular attention should be given to making a tight joint where the roof rests on the top of the staves.

Scuttles should be arranged in both the conical and flat covers to give access to the inside of the tank and a substantial, permanent ladder erected to give easy access to the top of the tank.

Table XVIII. Dimensions of Tanks of Standard Sizes



Approximate net capacity, gal	Size, outside dimensions: Average diameter, ft in	Size, outside dimensions: Length of stave, ft	Thickness of lumber after being machined: Staves, in	Thickness of lumber after being machined: Bottom, in	A, in	B, in	C, in	Hoops: Number of	Hoops: Size, in
10 000	13 4	12	2¼	2¼	3½	⅝	2⅛	11	¾
15 000	14 6	14	2¼	2¼	3½	⅝	2⅛	14	¾
20 000	15 6	16	2¼	2¼	3½	⅝	2⅛	5 11	¾ ⅞
25 000	17 6	16	2¾	2¾	3½	¾	2⅝	4 12	¾ ⅞
30 000	18 0	18	2¾	2¾	3½	¾	2⅝	4 16	¾ ⅞
50 000	22 0	20	2¾	2¾	3½	¾	2⅝	4 19	⅞ ⅞
75 000	24 6	24	2¾	2¾	3½	¾	2⅝	4 6 21	⅞ 1 1⅛
100 000	28 6	24	2¾	2¾	3½	¾	2⅝	5 29	1 1⅛

Pumps for Fire-Streams. The dimensions of steam-pumps for fire-protection in buildings, approved by the Board of Underwriters, can be found in Table XIX.

Table XIX. National Standard Steam Fire-Pump Sizes

Direct-acting non-flywheel type

Pump sizes; cylinders: Steam, diameter	Pump sizes; cylinders: Water, diameter	Pump sizes; cylinders: Stroke	Ratio of piston areas, about	Capacity at 100 lb at pump: Number of 1⅛-in streams	Capacity at 100 lb at pump: Nominal gallons per minute	Capacity at 100 lb at pump: Actual gallons per minute as per following note	Boiler power required: Horse-power	Boiler power required: Steam pressure in pounds at pump	Full speed: Revolutions per minute	Full speed: Piston travel, feet per minute
14 12	7 7¼	12 12	4 to 1	2	500	483 520	100	40	70	140
16	9	12	3 to 1	3	750	806	115	45	70	140
18 18½	10 10¼	12 12	3 to 1	4	1 000	999 1 650	150	45	70	140
20	12	16	2¾ to 1	6	1 500	1 655	200	50	60	160

Capacity of Standard Fire Pump Sizes. Plunger-diameter alone will not tell how many gallons per minute a pump can deliver, and it is not reasonable to continue the oldtime notion of estimating capacity on the basis of 100 ft per minute piston travel.

Careful experiments on a large number of pumps of various makes at full speed show that in a new pump with clean valves, and an air-tight suction-pipe, and less than 15-ft lift, the actual delivery is only from 1½ to 5% less than plunger-displacement. This slip will increase with wear, and for a good average pump in practical use, probably 10% is a fair allowance to cover slip, valve leakage and slight short-stroke.

Largely from tests, but partly from practical experience and recognizing that a long-stroke pump can run at a higher rate of piston travel in linear feet per minute than a short-stroke pump, and that a small pump can make more strokes per minute than a very large one, the speeds given in Table XIX have been adopted as STANDARDS IN FIRE SERVICE for direct acting (non-fly-wheel) steam-pumps, which have the large steam and water passages herein noted.

National Board of Fire Underwriters' Suggestions for Centrifugal Fire-Pumps and Steam-Driven Pumps

"**The centrifugal pump** is well adapted for driving by an ELECTRIC MOTOR or by a STEAM TURBINE, as the speed of the pump and the prime mover may readily be made the same, thus permitting direct connection. Whether or not the motor drive may be accepted in any case is largely a question of the reliability of the electric current in the event of a fire in the property itself or in adjoining buildings which might threaten the property. To make a motor-driven pump acceptable, the electric power supply should be as reliable as the steam-supply on the average property for the ordinary steam fire-pump. Each case must therefore be specially considered and decided upon its merits. Other methods of driving, such as the GASOLINE ENGINE or the STEAM TURBINE, may be employed, and where reliable will be acceptable.

"Pump equipment to be satisfactory shall furnish its rated capacity at the top sprinklers at a working pressure of at least 25 lb; when hose streams are to be supplied, the pump should furnish rated capacity at not less than 100 lb total discharge pressure, and at least 50 lb at the topmost 2½-in standpipe outlet or most remote 2½-in hydrant outlet, while water is being discharged through 50 ft of 2½-in cotton rubber-lined hose, and 1⅛-in nozzle."

The six standard centrifugal pumps are the 500-gal, 750-gal, 1 000-gal, 1 500-gal, 2 000-gal, and 2 500-gal sizes. The rated speed of those pumps will be about as follows:

Electric-motor driven	1 800 r.p.m.
Steam-turbine driven	2 500 r.p.m.
Gasoline-engine driven	1 600 r.p.m.
Chain driven	900 r.p.m.

Steam-Driven Pumps. "The BOILER-POWER is that required for continuous running at full speed and pressure. It is, however, often best to put in a larger pump than the existing boilers could drive at full capacity, as a small boiler will drive a 750-gal pump at the 500-gal speed with very nearly as much economy as it can drive a 500-gal pump at full speed. The pump then does not have to be changed when the plant is enlarged and the boiler-power increased.

"It is proper to run fire-pumps at the highest speed that is possible without causing violent jar or hammering within the cylinders. Considerations of

wear do not affect the brief periods of fire service or test, hence these speeds are greater than allowable for constant daily duty.

"AIR and VACUUM-CHAMBERS in accordance with the sizes given in the following table must be provided with all pumps. If the air-chamber is of cast iron, the pump manufacturer must warrant that it has been subjected to a hydraulic test of 400 lb per sq in before it is connected to the pump."

Size of Vacuum and Air Chambers

Capacity of pump, gallons per minute	Vacuum-chamber is to contain	Air-chamber is to contain
500	13 gal	17 gal
750	18 gal	25 gal
1 000	24 gal	30 gal
1 500	30 gal	40 gal

House Tanks. House tanks for large buildings may be divided into SUCTION-TANKS and STORAGE-TANKS. Suction-tanks may be either cylindrical and closed or rectangular and open. Storage or supply-tanks are generally rectangular and open, but provided with a cover.

Cylindrical tanks are as a rule built with heads dished out. The radius of the dished heads is equal to the diameter of the tanks.

For tanks up to 36 in in diameter the heads are made of steel 1/16 in thicker than the shell-plates. Tanks of from 36 to 96 in in diameter have heads 1/8 in thicker than the shells. Tanks of greater diameter than 96 in have heads 1/4 in thicker than the shells.

Sometimes a cylindrical tank is made with one head dished in. When that is the case, for diameters up to 72 in, the dished-in head is made 1/8 in thicker than if it were dished out. For diameters from 72 to 96 in the dished-in head is increased in thickness 1/4 in. Above 96 in in diameter reversed heads should not be used.

The FORCE TENDING TO DISRUPT a cylindrical tank is the internal unit pressure acting over the diameter, or projected area, of the tank. Opposed to that force is the STRENGTH OR THICKNESS OF METAL in the two sides of the tank.

The INTENSITIES OF STRESSES in lb per sq in found in water towers are as follows: A tensile stress due to hydrostatic pressure at any vertical joint or section of the shell of a tank filled with water,

$$S = 62.5\,HD/(2 \times 12\,t) = 2.6\,HD/t$$

A compressive stress at any horizontal joint or section, due to the weight of the stack,

$$S = W/(\pi D \times 12\,t) = 0.026\,W/Dt$$

A stress at any horizontal joint or section, tensile on the windward side and compressive on the leeward side, due to the wind, $S_2 = 0.106\,M/D^2t$. In the above equation, H = height of tank in ft above section considered, D = diameter in ft, t = thickness of shell in in, W = weight of tank in lb, and M = bending moment in ft-lb. The conditions for overturning from wind are most severe when the tank is empty.

Rectangular house tanks have condensation-pans usually of 1/16 in steel, 4 in deep, extending 6 in outside of the tank. A condensation-pan 4 in deep requires no top-edge stiffening. The tank is provided with an iron ladder and, if 6 ft or under, is generally made of 1/4-in plates; if from 6 to 8 ft deep, 5/16-in plates; from 8 to 10 ft deep, 3/8-in plates.

Tanks are ELEVATED a distance of about 18 in by means of I beams resting

on the condensation-pan. This gives access to the bottom of the tank for painting and provides the necessary space for pipe-connections.

House tanks should be painted both inside and outside with Bitumastic or some similar solution applied while hot. The condensation pan should be treated to a like coat to prevent rusting.

BRACES of $2\frac{1}{2} \times \frac{1}{16}$-in steel on edge are used as tie-rods to stiffen a rectangular tank and an angle iron is used as a top-edge stiffener.

2. Plumbing and Drainage

Definitions of Terms

HOUSE-SEWER is that portion of the drainage system extending from the building line to the street-sewer.

HOUSE-DRAIN is the system of horizontal piping inside the building that extends to and connects with the house-sewer. All soil, waste and leader stacks, also yard, area and sometimes floor-drains are connected to the house-drain.

STACKS are the vertical lines of soil, waste and vent-pipes which extend from the house-drain up to and through the roof. Up to the highest fixture on the line drainage pipes conveying fluids are either SOIL or WASTE-STACKS. Above the highest fixture they become VENT-STACKS.

SOIL-STACKS are the vertical lines which receive the discharge from water-closets. Soil-stacks may, and generally do, receive the discharge from other fixtures also, such as from lavatories, sinks or bath-tubs.

WASTE-STACKS are the vertical lines which receive the discharge of fixtures other than water-closets.

VENT-STACKS receive no discharges. They supply the drainage system with air to prevent syphonage of traps, and relieve pressure within the system. A vent-stack accompanies every soil or waste-stack. The vent-stack is connected to the base of the accompanying stack by means of a "Y" below the lowest fixture, and to the upper part of the stack above the highest fixture by means of a "T" fitting, or it may extend independently through the roof. When a vent-stack is connected to a soil or waste-stack above the highest fixture discharging into the line, from that point to the roof the continuation is known as a VENT-STACK.

SOIL-PIPE is the system of piping at the various floors which connects water-closets to the soil-stack.

WASTE-PIPE is the system of piping at the various floors which connects fixtures other than water-closets to the soil or waste-stack or to a soil-pipe.

VENT-PIPE is the system of piping at the various floors which connects the soil or waste-pipe to the corresponding vent-stacks.

TRAPS. A trap is a device which permits the free passage of liquids through it, and also of any solid matters that may be carried by the liquid, while at the same time preventing the passage of air or gas in either direction. Traps used for plumbing purposes are shaped so that an amount of water sufficient to close the passage and prevent the passage of air will stand in them at all times. The principle of the common trap is shown in Fig. 7. The pipe T receives the waste from a sink or lavatory, while the lower end B connects with the drainage system. Sewer-gas rises in pipe B, but is prevented from passing to the fixture by the water which stands in the trap. The depth of water through which gas must pass to effect a passage is termed the WATER-SEAL. The water-seal in the trap, Fig. 7, is the distance S. All plumbing-pipes which connect with a drainage-system require to be trapped to prevent

sewer-gas from passing through them into the room in which the fixture is located.

Ventilation of Traps. When a considerable volume of water rushes down a pipe it forms a suction, and if the pipe is made air-tight, this suction

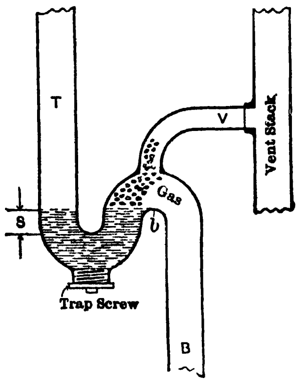


Fig. 7. Water-seal of Trap

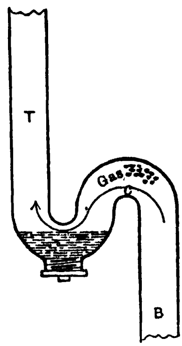


Fig. 8. Water-trap Unsealed

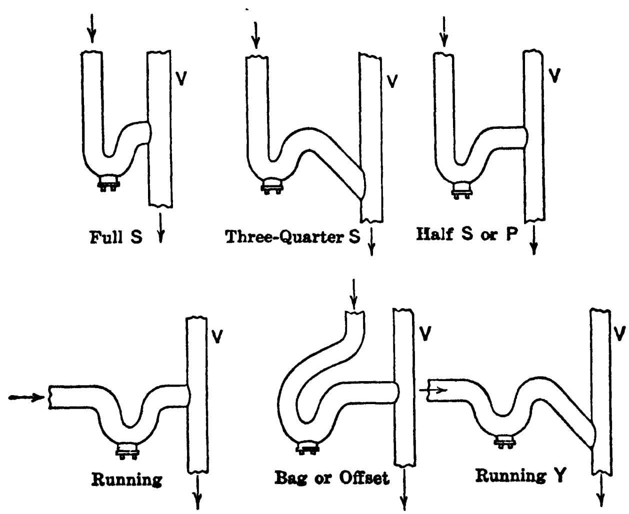


Fig. 9. Types of Traps

is often sufficient to prevent enough water remaining in the trap to form a seal, thus leaving an opening for the passage of sewer-gas, as in Fig. 8. By connecting the upper bend of a trap with the outside air by means of a pipe, as at *V*, Fig. 7, syphonic action will be broken, and the water in the pipe *T* will

not fall below the level of the outlet at *b*. There are also several varieties of back-pressure traps, designed to prevent the sewage from flowing back into the house-drain. These are in the nature of check-valves, and are used principally in seaport-towns where tide-water might possibly force the sewage back.

The more common shapes of lead traps used in plumbing, with their trade names, are shown in Fig. 9. The same shapes are also made of cast iron and brass. The pipes marked *V* are the vent-connections. The traps for water-closets are commonly formed in the fixture.

Drainage System

The various parts of a drainage system are shown in Fig. 10. That portion of the horizontal drain extending from the street-sewer to within a few feet of the foundation wall of a building is the HOUSE-SEWER. Continuation of this horizontal drain within the building is the MAIN HOUSE-DRAIN. Just inside of the foundation wall is the MAIN DRAIN-TRAP and a FRESH-AIR INLET leading to the atmosphere outside.

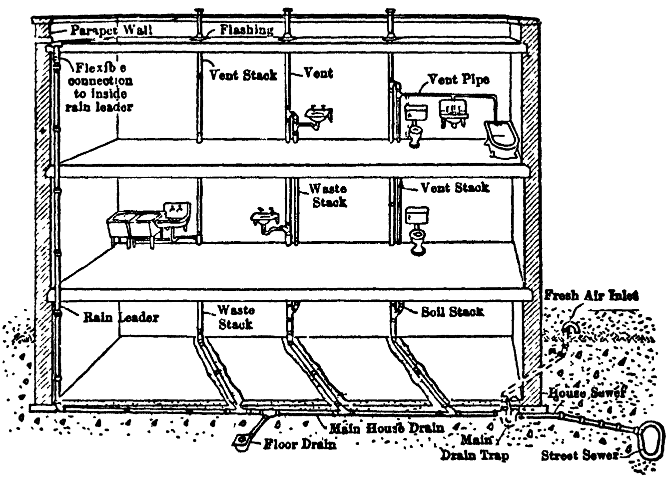


Fig. 10. Drainage System for a Residence

A FLOOR-DRAIN is shown in the cellar floor. Floor-drains are advisable only where a water-seal in the trap will be assured. Unless water is to be splashed, poured or spilled on a floor it is better to omit the floor drain or run the waste into a rain-leader, yard or area-drain on the house side of the trap. Rain-water will then insure the trap being sealed.

Branches to waste and soil lines are taken off with Y or T-Y fittings. If the runs are horizontal, long turn or long radius T-Y's are used. Short turn T-Y's are permissible for vertical stacks.

All vertical stacks extend through the roof where they open to the atmosphere. This enables air to circulate through the system, tending to keep the pipes free from the accumulation of objectionable odors or gases.

Materials for the Drainage System. The drainage system is made up of CAST-IRON SOIL-PIPE, GALVANIZED WROUGHT IRON, STEEL or LEAD PIPE. LEAD PIPE is now used principally for short connections between water-closets and soil-pipe. Sometimes it is used for other fixture connections. The lead pipe usually used in a drainage system is known as D pipe, and has the following weights per linear foot:

Table XX. Weight of Lead Pipe

Inside diameter of pipe	Weight per foot
1¼-in pipe	2½ pounds
1½-in pipe	3 pounds
2-in pipe	4 pounds
3-in pipe	6 pounds
4-in pipe	8 pounds

CAST-IRON SOIL-PIPE is commonly used for the house-drain, stacks of soil, waste, vent, and leader-lines in buildings of moderate height. In the tall steel skeleton type of building, cast-iron soil-pipe is used for all underground work, and galvanized iron or steel pipe with recessed drainage fittings for the soil, waste, and leader-stacks. Ordinary galvanized fittings are used on the vent-stacks.

Cast-iron hub and spigot-pipe is made in two grades or weights, known as STANDARD and EXTRA HEAVY. Extra heavy soil-pipe is used in the better grade of buildings. The weights of various sizes of soil pipe and their fittings are given in Table XXI.

Standard-weight GALVANIZED WROUGHT IRON or STEEL PIPE is also used for drainage-pipe.

Proportions of the Drainage System. The main house-drain and the house-sewer are preferably of one size. In cities having plumbing laws or regulations, the size of house-drain is given in the code. Where there are no regulations the house-sewer and house-drain are determined by the volume of sewage to be handled.

When rain-water discharges into the drainage-system the volume of rain-water is so much greater than the volume of domestic sewage that ordinarily if the sewers are large enough to take care of the rainfall they will be big enough for domestic purposes. In exceptional cases the domestic sewage might have to be added to the rainfall in determining the volume of liquid to be cared for. It is seldom that rain falls at a greater rate than 6 in per hour, and that for a period not longer than ten minutes at a time.

Table XXII gives the precipitation in various parts of the United States.

Table XXI. Approximate Weight in Pounds of Soil Pipes and Fittings

Grade	Standard							
Size, inches ..	2	3	4	5	6	8	10	12
Pipe, per length of 5 ft ..	18	26	35	45	55	85	115	165
Quarter bends..	4½	7	10½	13	16½	33	54	58½
Sixth bends.....	4	6½	9	11½	14	30	39	48
Eighth bends..	3½	6	8½	10½	13	24	38	42½
T-branch	7	11	14½	18½	23	33	42	102
Sanitary-T......	7½	12	16	20	24	49	100	.
Crosses	9½	14	19	23½	28½	40	70	100
Sanitary cross..	10	15½	20½	25½	31½	52	..	
Y-branch...	7½	12	17	22	28	58	106	158
Half Y-branch.	7	11	15	19	24	..		.
Double Y-branch	10½	16	22½	30	36½	77	121	.
Double hubs ..	4	5¾	7½	9	10½	16	21	35
S-traps .. .	8½	14	21	28½	37½	94		..
P-traps. .	7	12	17½	24½	31½	80		.
Running-traps..	8½	13½	19½	26	34	54		
Reducers . .		4½	6	7½	9	18	21½	28

Grade	Extra-heavy							
Size, inches..	2	3	4	5	6	8	10	12
Pipe, per length of 5 ft... ...	27	47	65	85	100	170	225	270
Quarter bends...	6¾	10½	15	19	23½	52	80	95
Sixth bends	6	9¼	13	16½	20	44	50	90
Eighth bends .	5½	8½	12¼	15¼	18¼	40	56	80
T-branch	10¼	15¼	21	26½	32½	60	75	166
Sanitary-T .	11	16¼	22½	28	34½	80	135	210
Crosses... .	13¼	19¾	27	33½	40½	92	116	176
Sanitary cross .	14½	22	29½	36½	44½	79	161	
Y-branch	11	17	24	31½	39½	91	126	218
Half Y-branch.	10¼	15½	21½	27½	34	90		
Double Y-branch	14¾	23	32	41½	52	100	153	266
Double hubs .	5¾	8¼	10¾	12¾	14¾	30	35	45
S-traps .	12	20	30	40½	53½	123		...
P-traps	10½	17¼	25½	34½	45	100		. .
Running-traps..	11¾	19	27	37	48	110		
Reducers. . .	..	6¼	8½	10¾	12¾	15	28½	48

Table XXII.* Intensity of Rainfall

Locality	Maximum rate in feet per hour for 5 minutes	Rate of downfall in inches per hour for		
		5 minutes	10 minutes	60 minutes
	Ft	In	In	In
Bismarck, S. D.	.75	9 00	6.00	2 00
St. Paul, Minn	.70	8.40	6.00	1.30
New Orleans, La.	.68	8 16	4 86	2.18
Milwaukee, Wis.	.65	7 80	4 20	1.25
Kansas City, Mo	.65	7 80	6 60	2 40
Washington, D C.	63	7.50	5.10	1 78
Jacksonville, Fla.	62	7 44	7 08	2.20
Detroit, Mich.	.60	7 20	6 00	2 15
New York City	.60	7 20	4 92	1 60
Boston, Mass.	56	6 72	4 98	1 68
Savannah, Ga.	55	6.60	6 00	2 21
Indianapolis, Ind.	.55	6 60	3 90	1 60
Memphis, Tenn	.55	6 60	4 80	1 86
Chicago, Ill	55	6 60	5 92	1 60
Galveston, Tex.	.54	6 48	5 58	2 55
Omaha, Neb.	.50	6.00	4 80	1.55
Dodge City, Iowa.	50	6.00	4 20	1 34
Norfolk, Va.	.48	5.76	5 46	1.55
Cleveland, Ohio	.47	5.64	3 66	1.12
Atlanta, Ga.	.46	5.46	5 46	1 50
Key West, Fla.	.45	5 40	4 80	2.25
Philadelphia, Pa.	45	5.40	4 02	1 50
St. Louis, Mo.	.40	4 80	3.84	2 25
Cincinnati, O.	.38	4.56	4 20	1.70
Denver, Col.	.30	3.60	3 30	1 18
Duluth, Minn.	.30	3.60	2.40	1.35

* Compiled by the Weather Bureau from records covering a long term of years.

In Table XXIII can be found the areas of roof or other pitched surface that can be cared for by house-drains of various diameters laid at given grades. The areas given are conservative.

Table XXIII. Size of House-Drains

Diameter of drain, inches	Areas the pipes will drain when laid at grades of	
	¼ inch per foot	½ inch per foot
3	1 200 sq ft	1 500 sq ft
4	2 500 sq ft	3 200 sq ft
5	4 500 sq ft	6 000 sq ft
6	8 000 sq ft	10 000 sq ft
7	12 400 sq ft	15 000 sq ft
8	18 000 sq ft	22 500 sq ft
9	25 000 sq ft	31 500 sq ft
10	41 000 sq ft	59 000 sq ft
12	69 000 sq ft	98 000 sq ft

When the volume of sewage added to the rainfall is large, the size of house-sewer and house-drain can be determined by this empirical rule:

Rule. Allow 1 sq in in sectional area of the drain for each 2 cu ft or 15 gal of sewage to be discharged per minute.

Table XXIV. Carrying Capacity of Sewer-Pipe

Gallons per minute

Size of pipe, in	Fall per 100 ft							
	1 in	2 in	3 in	6 in	9 in	1 ft	2 ft	3 ft
3	13	19	23	32	40	46	64	79
4	27	38	47	66	81	93	131	163
6	75	105	129	183	224	258	364	450
8	153	216	265	375	460	527	750	923
9	205	290	355	503	617	712	1 006	1 240
10	267	378	463	755	803	926	1 310	1 613
12	422	596	730	1 033	1 273	1 468	2 076	2 554
15	740	1 021	1 282	1 818	2 224	2 464	3 617	4 467
18	1 168	1 651	2 022	2 860	3 508	4 045	5 704	7 047
24	2 396	3 387	4 155	5 874	7 202	8 303	11 744	14 466
27	4 407	6 211	7 674	10 883	13 257	15 344	21 771	26 622
30	5 906	8 352	10 223	14 298	17 714	20 204	28 129	35 513
36	9 707	13 769	16 816	23 763	29 284	33 722	47 523	58 406

Table XXV. Quantities of Cement, Sand and of Cement Mortar for Sewer-Pipe Joints

Prepared by J. N Hazlehurst

For each 100 ft of sewer (with Portland cement, 375 lb net per bbl)

Size of pipe, in	Length, ft	Mortar, cu yd	Proportions: 1 cement to					
			1 sand			2 sand		
			Cement, bbl	Sand, cu yd	Pipe per bbl cement, lin ft	Cement, bbl	Sand, cu yd	Pipe per bbl cement, lin ft
6	2½	0 003	0 01248	0.00201	803	0.00855	0.00252	1 168
8	2½	0.038	0.15808	0.02546	633	0.10830	0.03192	923
10	2½	0.058	0.24128	0 03886	410	0.16530	0.04872	605
12	2½	0.089	0.37024	0.05963	270	0.25365	0.07476	394
15	2½	0.123	0.51268	0.08241	195	0.35055	0.10332	285
18	2½	0.167	0.69472	0.11189	144	0.47595	0.14018	210
20	2½	0.237	0.98592	0.15879	101	0.67545	0.19908	148
24	2½	0.299	1.24384	0 20033	80	0.85215	0.25116	117
27	3	0.492	2.04672	0 32964	49	1.40220	0.41328	71
30	3	0.548	2.27968	0.36716	44	1.56180	0.46032	64
36	3	0.849	3.53184	0.56883	29	2.41965	0.71316	41

Rain-Leaders are preferably not smaller than 2½ in in diameter. Smaller pipes are liable to clog with leaves. They are never larger than 8 in in diameter, for when an area requires a large leader it is better to provide several leaders of smaller diameters.

Size of Leaders. The Barrett Company, from their extensive experience in roofing, sum up their conclusions in the two following paragraphs and Table XXVI of leader sizes:

"For roof-areas of more than 4 000 sq ft, it is considered the best practice to drain to different points, using smaller leaders, rather than to one outlet of larger size, although in some cases the arrangement of leader-lines makes the latter method compulsory.

"A standard table not always being available, an easy and reliable formula to use is as follows: For roofs covered with gravel or slag, with an incline not exceeding ¼ in per ft, allow 300 sq ft of roof-surface to each square inch of leader opening; for roofs of greater incline or saw-tooth roof-construction, allow 250 sq ft of roof-surface to each square inch of leader opening; for metal, tile, slate or similar roofs of any incline, allow 200 sq ft of roof-surface to each square inch of leader opening. Table XXVI can be depended upon in estimating roof drainage, it being based on the heaviest storm conditions recorded."

Table XXVI. Size of Leaders

Kind of roof	Diameter of leader in inches					
	2½	3	4	5	6	8
	Square feet the leader will drain					
Covered with gravel, slag or other similar material with incline ¼ in in 1 ft	1 800	1 800 to 2 200	2 200 to 3 600	3 600 to 5 600	5 600 to 8 000	8 000 to 14 000
Same, with incline ½ in or more in 1 ft, and saw-tooth	1 200	1 200 to 1 700	1 700 to 3 100	3 100 to 4 900	4 900 to 7 000	7 000 to 12 000
Metal, tile, slate or similar roofs of any incline	1 000	1 000 to 1 400	1 400 to 2 500	2 500 to 3 900	3 900 to 5 600	5 600 to 10 000

Table XXVII. Areas of Pipes

Diameter of pipes in in...	2	2½	3	4	4½	5
Areas of pipes in sq in..	3 14	4 9	7 06	12.57	15.9	19.63
Areas of pipes in sq ft	.022	.034	.05	.087	.11	.136

Diameter of pipes in in..	6	7	8	9	10	12
Areas of pipes in sq in..	28.27	38 48	50.25	63.61	78.54	113.1
Areas of pipes in sq ft...	.196	.266	.35	.44	.54	.785

In the densely built-up districts of many cities the public sewers are overtaxed during heavy rainstorms, and water from the sewers backs up into the lower floors of the building through the plumbing fixtures. It is well, therefore, when planning the plumbing for a large building of any kind subject to this flood condition, to discharge all fixtures on the ground floor as well as those at a lower level into a SEWAGE EJECTOR. If for any reason this cannot be done, a BACK-WATER TRAP should be installed in the main house-drain, and a quick-closing lever handle valve installed next to the back-water trap so it can be shut in case of threatened flood.

Materials for Industrial Wastes. In chemical laboratories, chemical works, storage-battery rooms, dye-works and in buildings used for other industrial purposes, the waste fluids are often of such a nature that they will corrode and destroy in a very short time the usual materials of a drainage system.

There are three chemical-resisting materials on the market made up into pipe and fittings with which a drainage system can be installed. They are HIGH-SILICON CAST-IRON pipe and fittings; FIBER CONDUIT; and INDURATED RUBBER.

DURIRON is a commercial example of high-silicon pipe. It is a cast metal, extremely hard and close grained. Owing to its high percentage of silicon, 14.5%, Duriron is very brittle, taking on some of the properties of glass, so it has to be worked and handled with care. Even when calking, the danger of cracking a hub is great unless the lead is calked with great care. The pipe is easily cut with an ordinary pipe-cutter. Duriron soil-pipe and fittings can be had in all sizes of ordinary extra-heavy cast-iron pipe; also a special size 1½ in in diameter with fittings to match may be had. The pipes are put together with a lead-calked joint, but instead of oakum, asbestos rope is used as a packing Duriron floor-drains, sinks, vats, kettles and other acid-resisting containers are also made. High-silica pipe and fittings will resist all commercial acids, but will not withstand the destructive effect of sodas or other alkalies

Lead pipe has been used to some extent for acid wastes, as have also lead-lined iron pipes. Lead is suitable for only a few acids, however, and there is a weak spot at every joint of lead-lined iron pipe and where lead pipes are joined, unless the lead is burned, not soldered. Concentrated nitric acid will completely dissolve lead pipe in a short time, while concentrated hydrochloric acid will destroy lead pipe in a couple of years.

FIBER CONDUIT, made up in the form of pipe and fittings, is used in chemical laboratories and industrial works where both acid and alkaline wastes have to be taken care of. The fiber conduit is resistant to almost every waste encountered in industry.

INDURATED RUBBER is made up in the form of pipe, fittings and valves. There are many places where rubber supply-pipes, pumps and wastes are preferable to either metal or fiber conduits.

Water Consumed in Buildings

Estimate of Water Needed. If the amount of water that would be required in a new building were known, it would be a simple matter to proportion the piping to supply the necessary quantity. As a guide in estimating the water-supply for a building the following tables will prove helpful. In Table XXVIII is given the average amount of water used per day in several well-known New York hotels and office-buildings. The cubes of the building are given, the number of occupants and the number of plumbing fixtures.

Table XXVIII. Consumption of Water in Hotels and Office Buildings

Building	Cube in feet	Number of occupants	Number of bath-rooms	Number of plumbing fixtures	Water used per day, gal (average)	Storage capacity section-tank, gal	House pumps
Ritz-Carlton... ..	3 746 373	900 guests 300 employees*	428	1 617	270 000	15 000	2—150 g p m
N. Y. Ambassador	4 680 290	800 guests 265 employees	366	1 288	270 000	26 000	2 Worthington, 5¼×3½
Equitable Building (120 Broadway)	24 918 736	12 000 tenants	.	1 500 approx	105 905	50 000	8 Worthington: 4—14×20–9½×18 2—10×17–8½×15 2— 9×14–8½×15
Woolworth Building...	13 110 942	12 000 tenants	..	2 300 approx	175 905	88 000	8 pumps, combined capacity 2 000 g p m; average 250 g p m
Municipal Building. .	19 000 000	4 363 tenants	..	.	254 737	68 000	3—500 g p m, 400 ft head; 2—300 g p m 600 ft head

* Number of employees assumed, allowing one employee to three guests.

In the case of hotels, the cube seems to bear no relation to the water-consumption. nor does the number of fixtures installed. The NUMBER OF OCCUPANTS bears a reasonable relation, however, the water-consumption in one of the hotels listed averaging 225 gal per occupant per day, while the average daily consumption in the second hotel was approximately 250 gal per occupant per day. In both of the hotels steam turbine generators were installed which required water for condensation. As 20 to 30 gal of water are required to condense $8\frac{1}{3}$ lb of steam, the per capita consumption would be cut down considerably where current is obtained from the public service corporation.

In the case of the two office-buildings tabulated, each having the same tenancy, there is seemingly a great difference in the amount of water consumed per day. However, while both buildings have a large number of non-tenants visiting them daily, the Woolworth has the larger number because of its law school and sight-seers. These swell the total water-consumption.

The Municipal Building shows up even better than the Woolworth the difference in the water-consumption due to tenants, and that due to callers. The Municipal Building has only two-thirds the tenants of either of the two other buildings, still the water-consumption is much greater. This is largely due to the number of people having business with the City Departments who flock to the Municipal Building every day throughout the year.

When an office-building is erected it is impossible to foretell what the occupancy will be, therefore the water-supply must be large enough to take care of the maximum demand.

In some of the larger cities the quality of the water supplied is such that TUBURCULATION takes place inside of the pipes, or there may be CORROSION and a partial clogging from rust. In such localities an additional area must be provided in the water-pipes to allow for loss of capacity, or pipes of materials that will not be affected by the water must be used.

The MONTHLY TOTALS of water consumed do not vary much in large buildings throughout the year. This is shown by Table XXIX, giving the water-consumption for one year in the Woolworth Building and the Municipal Building. Readings in the Municipal Building were taken three times a month, approximately every ten days.

More important than the daily, monthly or yearly consumption of water, from a pipe-size standpoint, is the HOURLY RATE OF FLOW of water. It is found in large office-buildings that there are two periods of excess consumption. One is from 9:00 to 10:00 A.M., just after the tenants arrive; the other during the noon-hour. In the Municipal Building it was found that the average daily consumption of water was 33 965 cu ft, while the hourly consumption between 9:00 and 10.00 A.M. was 1 690 cu ft, or about one-twentieth the average daily consumption.

The water-supply for a building must be proportioned to take care of the maximum demand while at the same time maintaining a reasonable reserve capacity for a still further demand. When considering the maximum demand the occupancy of a building must be considered. A study of the foregoing tables will show that in two buildings of the same class having an equal number of tenants, the difference in occupancy increased the water-consumption over 66% in one building, while compared with a public building the difference in consumption was almost as 1 to 7.

Supply-Pipes. The materials best suited for water supply-pipes can be determined only in the localities where they are to be installed. No set rule or law can be laid down that will be right in all cases. There are some localities where LEAD pipes are destroyed in a very short time on account of the

chemical composition of earth or water. In other localities BRASS pipes will not stand up under use. Still other localities are extremely hard on IRON and STEEL whether GALVANIZED or PLAIN. The only rule therefore is the LAW OF EXPERIENCE. Brass and galvanized wrought-iron or steel are the materials commonly used. Lead pipe is used in some localities, while block-tin pipe is used for soda-fountain work, beer pump-lines and for conducting chemical preparations in industrial plants.

Table XXIX. Monthly Water-Consumption of Buildings

Month	Consumption of water in cubic feet			Month	Consumption of water in cubic feet		
	Woolworth Building	Municipal Building			Woolworth Building	Municipal Building	
		Every 10 days	Total for month			Every 10 days	Total for month
Jan.	618 000	314 670 266 830 289 290	870 790	July	651 300	364 130 342 750 355 400	1 062 280
Feb.	687 000	290 900 350 260 223 410	864 570	Aug.	774 000	334 980 329 700 393 160	1 057 840
Mar	666 000	328 520 292 330 386 180	1 007 030	Sept.	785 200	320 240 374 190 364 290	1 058 720
April	718 000	304 290 337 560 327 810	969 660	Oct	798 600	352 800 352 890 346 540	1 052 230
May	661 000	385 010 346 080 407 170	1 138 260	Nov.	716 400	306 200 392 490 392 650	1 091 340
June	774 700	384 550 340 900 268 270	993 720	Dec.	704 600	296 900 351 980 361 560	1 010 440

Designation of Lead Pipe. The different thicknesses of lead pipe were formerly designated by letters as in Table XXXI, but are now more commonly designated as in Table XXX, which may be considered as generally accepted by dealers.

Coils of supply-pipe weigh about 200 lb; aqueduct about 90 lb; suction-pipe, 100 to 180 lb each.

Block-tin pipe is stronger for a given weight per foot than lead pipe or tin-lined lead pipe. As compared with lead pipe its strength is as 3½ to 1.

Tin-lined and lead-lined iron pipe is made with inside diameters of ½, ¾, 1

Table XXX. Weights and Sizes of Lead Pipe

Caliber	Weight per foot, lb	Weight per foot, oz	Caliber	Weight per foot, lb	Weight per foot, oz
¼-in Tubing		6	1½-in Aqueduct	3	
Fish seine	1	4	Extra light	3	8
⅜-in Aqueduct		8	Light	4	
Extra light		9	Medium	5	
Light		12	Strong	6	
Medium	1		Extra strong	7	8
Strong	1	8	Extra extra strong	9	
Extra strong	2		1¾-in Extra light	3	12
½-in Aqueduct		10	Light	4	8
Extra light		12	Medium	5	8
Light	1		Strong	6	8
Medium	1	4	Extra strong	8	
Strong	1	12	2-in Waste	3	
AA	2		Extra light	4	
Extra strong	2	8	Light	5	
Extra extra strong	3		Medium	7	
⅝-in Aqueduct		12	Strong	8	
Extra light	1	4	Extra strong	9	
Light	1	12	Extra extra strong	10	8
Medium	2		2½-in Waste	4	
Strong	2	8	Light	6	
Extra strong	3		Medium, 3/16 thick	8	
Extra extra strong	3	8	Strong, ¼ thick	11	
¾-in Aqueduct	1		Extra strong, 5/16		
Extra light	1	8	thick	14	
Light	2		Extra extra strong,		
Medium	2	4	⅜ thick	17	
Strong	3		3-in Waste	4	
Extra strong	3	8	Light	6	3
Extra extra strong	4		Medium, 3/16 thick	9	
⅞-in Aqueduct	1	8	Strong, ¼ thick	12	
Extra light	2		Extra strong, 5/16		
Light	2	8	thick	16	
1-in Aqueduct	1	8	Extra extra strong,		
Extra light	2		⅜ thick	20	
Light	2	8	3½-in Waste	5	
Medium	3	4	Strong, ¼ thick	15	
Strong	4		Extra strong, 5/16		
Extra strong	4	12	thick	18	
Extra extra strong	5	8	4-in Waste	5	
1¼-in Aqueduct	2		Medium	10	
Extra light	2	8	Strong, ¼ thick	16	
Light	3		Extra strong, 3/16		
Medium	3	12	thick	22	
Strong	4	12	Extra extra strong,		
Extra strong	6		⅜ thick	25	
Extra extra strong	6	12	5-in Waste	8	

1¼, 1½ and 2 in, and in 10-ft lengths, threaded without couplings. Tin-lined and lead-lined fittings are also made.

Weights and Sizes of Sheet Lead

Thickness, in.	1/24	1/20	1/16	1/16 full	5/64	3/32	1/8	1/6	1/5	1/5 full	1/4	1/3	3/8
Lb per sq ft. .	2½	3	3½	4	5	6	8	10	12	14	16	20	24

Table XXXI. Thickness and Strength of Lead Pipes

Caliber	Mark	Weight per foot	Thickness	Mean bursting pressure	Safe working pressure	Caliber	Mark	Weight per foot	Thickness	Mean bursting pressure	Safe working pressure
in		lb oz	in	lb	lb	in		lb oz	in	lb	lb
3/8	AAA	1 12	0.18	1 968	492	1	A	4 0	0 21	857	214
3/8	AA	1 5	0 15	1 627	406	1	B	3 4	0 17	745	186
3/8	A	1 2	0.13	1 381	347	1	C	2 8	0.14	562	140
3/8	B	1 0	0.125	1 342	335	1	D	2 4	0.125	518	129
3/8	C	0 14	0.11	1 187	296	1	E	2 0	0.10	475	118
3/8	. ..	0 10	0.087	1 085	271	1		1 8	0 09	325	81
7/16	. ..	0 9½	0 08	775	193	1¼	AAA	6 12	0.275	962	240
½	AAA	3 0	0.25	1 787	446	1¼	AA	5 12	0 25	823	205
½	...	2 8	0.225	1 655	413	1¼	A	4 11	0.21	685	171
½	AA	2 0	0.18	1 393	343	1¼	B	3 11	0.17	546	136
½	A	1 10	0 16	1 285	321	1¼	C	3 0	0 135	420	105
½	B	1 3	0.125	980	245	1¼	D	2 8	0.125	350	87
½	C	1 0	0 10	782	195	1¼		2 0	0.095	322	80
½	D	0 9	0 065	468	117	1½	AAA	8 0	0.29	742	185
½	....	0 10	0.07	556	139	1½	AA	7 0	0.25	700	175
½	...	0 12	0.09	625	156	1½	A	6 4	0.22	628	157
5/8	AAA	3 8	0 23	1 548	387	1½	B	5 0	0.18	506	126
5/8	AA	2 12	0 21	1 380	345	½	C	4 4	0.15	430	107
5/8	A	2 8	0.18	1 152	288	½	D	3 8	0.14	315	78
5/8	B	2 0	0.16	987	246	1½	. ..	3 0	0 12	245	61
5/8	C	1 7	0.117	795	198	1¾	B	5 0	...	.	116
5/8	D	1 4	0.10	708	177	1¾	C	4 0			93
¾	AAA	4 14	0.29	1 462	365	1¾	D	3 10	0.125	318	79
¾	AA	3 8	0.225	1 225	306	2	AAA	10 11	0.30	611	152
¾	A	3 0	0.19	1 072	268	2	AA	8 14	0 25	511	127
¾	B	2 3	0.15	865	216	2	A	7 0	0 21	405	101
¾	C	1 12	0.125	782	195	2	B	6 0	0 19	360	90
¾	D	1 3	0.09	505	126	2	C	5 0	0.16	260	65
1	AAA	6 0	0 30	1 230	307	2	D	4 0	0.09	200	50
1	AA	4 8	0.23	910	227	..	...	....	.	:	..

Water Required for Various Purposes. For sprinkling 100 sq ft of lawn, about 1 cu ft, or 7 to 8 gal. For soaking 100 sq ft of lawn, about 2½ cu ft, or from 15 to 16 gal.

To flush closets, 5 to 6 gal at each flush. The actual rate of discharge from water-closets is about 1¼ gal per second. To fill the bowl of an ordinary lavatory, about 1½ gal. To fill a bath-tub, about 20 gal. Only about 8 gal are used for a bath.

Table XXXII. Weight and Sizes of Pure Block-Tin Pipe

Size inside diameter, in	Weight per foot, oz	Size inside diameter, in	Weight per foot, lb
3/16	4	3/4	9, 12, 16
1/4	4, 5, 6	1	12, 16
5/16	4, 5, 6, 8	1 1/4	20, 28
3/8	4, 5, 6, 8	1 1/2	24 and upwards
1/2	5, 6, 8, 10	2	32 and upwards
5/8	9, 12, 16	....	....

The consumption of water by farm animals varies greatly, depending upon the season of the year, the age and individual habits of the animals. The following can be accepted as extremes: Horse, 5 to 10 gal per day; cattle, 7 to 12 gal per day; hogs, 1½ to 2½ gal per day; sheep, 1 to 2 gal per day.

For mixing concrete 6 to 8 gal of water are required for each 94-lb sack of cement.

Each developed boiler horse-power requires 34.5 lb or 4.1 gal of water.

Compound engines will develop an engine horse-power on 15 lb of water evaporated to steam.

Single condensing engines will develop an engine horse-power on 18 to 22 lb of water.

Automatic non-condensing engines will develop an engine horse-power on 28 to 32 lb of water.

A side-valve throttle-governing engine will develop an engine horse-power on 62½ lb or 1 cu ft of water.

Locomotives average a consumption of 3 000 gal of water for every 100 miles

Forty to 60 lb of condensing water are required for each lb of steam condensed to moderate temperature; 60 to 100 lb of water per lb of steam when the water used is at the higher temperature common to cooling-tower practice.

Consumption of water in military camps is about 10 gal per capita daily.

Equation of Pipes. In a water-supply distributing system numerous branches are taken off from the main lines. The question which rises constantly is, "How many branches can be supplied by the main?". The number can be determined by Table XXXIII.

The statement is often made that doubling the diameter of a pipe increases its capacity four times. That statement, however, is only partly true. At the same velocities of flow, doubling the diameter of a pipe increases its capacity four times. But the same head or pressure will produce different velocities in pipes of different sizes or lengths, and doubling the diameter of a pipe when the pressure remains constant increases the capacity of the pipe almost six times.

In Table XXXIII figures above the diagonal line refer to standard-weight wrought-iron and steel pipes, the diameters of which vary a little from the actual diameters given. In the lower part of the table the figures refer to pipes of the actual sizes given. Extra strong and double extra strong pipes have smaller bores than those of standard wrought-iron pipes. The outside diameters of all pipes of corresponding sizes are the same for threading standardization. As the walls of the extra strong and double extra strong pipes are thicker, the pipe bores are of necessity smaller.

Table XXXIII. Proportioning Mains and Branches

Diameters of standard water steam and gas pipes

Diameters	½	¾	1	1½	2	2½	3
½	. .	2 27	4 88	15 8	31 7	52 9	96 9
¾	2 60	. .	2 05	6 97	14 0	23 3	42 5
1	7 55	2 90	. .	3 45	6 82	11 4	20 9
1½	24 2	9 30	3 20	.	1 26	3 34	6 13
2	54 8	21 0	7 25	2 26		1 67	3 06
2½	102	39 4	13 6	4 23	1.87	......	1.83
3	170	65 4	22 6	7 03	3 11	1 66	
4	376	144	49 8	15 5	6 87	3 67	2 21
5	686	263	90 9	28 3	12 5	6 70	4 03
6	1 116	429	148	46 0	20 4	10 9	6 56
7	1 707	656	226	70 5	31 2	16 6	10 0
8	2 435	936	322	101	44 5	23 8	14 3
9	3 335	1 281	440	137	60 8	32 5	19 5
10	4 393	1 688	582	181	80 4	42 9	25 8
11	5 642	2 168	747	233	103	55 1	33.1
12	7 087	2 723	938	293	129	69 2	41 6
13	8 657	3 326	1 146	358	158	84 5	50 7
14	10 600	4 070	1 403	438	193	103	62 2
15	12 824	4 927	1 698	530	234	125	75 3
16	14 978	5 758	1 984	619	274	146	88 0
17	17 537	6 738	2 322	724	320	171	103
18	20 327	7 810	2 691	840	371	198	119
20	20 676	10 249	3 532	1 102	487	260	157
24	42 624	16 376	5 644	1 761	778	416	250
30	75 453	28 990	9 990	3 117	1 378	736	443
36	120 100	46 143	15 902	4 961	2 193	1 172	705
42	177 724	68 282	23 531	7 341	3 245	1 734	1 044
48	249 351	95 818	33 020	10 301	4 554	2 434	1 465

Diameters	4	5	6	7	8	9	10
½	205	377	620	918	1 292	1 767	2 488
¾	90 4	166	273	405	569	779	1 096
1	44 1	81 1	133	198	278	380	536
1½	13 0	23 8	39 2	58 1	81 7	112	157
2	6 47	11 9	19 6	29 0	40 8	55 8	78.5
2½	3.87	7 12	11.7	17 4	24 4	33.4	47.0
3	2 12	3.89	6 39	9 48	13 3	20 9	23.7
4	......	1.84	3 02	4 48	6 30	8.61	12 1
5	1 83	. .	1.65	2 44	3 43	4 69	6 60
6	2 97	1 63	......	1 48	2 09	2.85	4 02
7	4 54	2 49	1 51	. .	1.41	1 93	2 71
8	6 48	3 54	2 18	1 43	.	1 35	1 93
9	8.85	4.85	2 98	1 95	1 37		1 41
10	11.7	6.40	3.93	2 57	1 80	1.32	.

Actual Internal Diameters

Table XXXIII (Continued). Proportioning Mains and Branches

Diameters of standard water steam and gas pipes

Diameters	4	5	6	7	8	9	10
11	15 0	8 22	5 05	3 31	2 32	1 70	1.28
12	18 8	10 3	6 34	4 15	2 91	2 13	1.61
13	23 0	12 6	7 75	5 07	3 56	2 60	1 98
14	28 2	15 4	9 48	6 21	4 35	3 18	2.41
15	34 1	18 7	11 5	7 52	5 27	3 85	2 92
16	39 9	21 8	13 4	8 78	6 15	4 51	3.41
17	46 6	25 6	15 7	10 3	7 20	5 27	3 99
18	54 1	29 6	18 2	11 9	8 35	6 11	4.63
20	70.9	38 9	23 9	15 6	10 9	8 02	6 07
24	113	62 1	38 2	25.0	17 5	12 8	9.70
30	201	110	67 6	44 2	31 0	22 7	17 2
36	319	175	108	70 4	49 3	36 1	27.3
42	473	259	159	104	73 0	53 4	40 5
48	663	363	223	146	102	75 0	56 8

Diameters	11	12	13	14	15	16	17
½	3 014	3 786	4 904	5 927	7 321	8 535	9 717
¾	1 328	1 668	2 161	2 615	3 226	3 761	4 282
1	649	815	1 070	1 263	1 576	1 837	2 092
1½	190	239	310	375	463	539	614
2	95 1	119	155	187	231	269	307
2½	56 9	71 5	92.6	112	138	161	184
3	31.2	39 1	50.6	61 1	75 5	88 0	100
4	14 7	18 5	23 9	28 9	35 7	41 6	47.4
5	8.00	10 0	13 0	15 7	19.4	22.6	25.8
6	4 86	6 11	7 91	9 56	11 8	13 8	15.6
7	3 28	4 12	5 34	6 45	7 97	9 31	10 6
8	2 33	2 92	3 79	4 57	5 67	6 60	7.52
9	1 71	2 14	2 77	3 35	4 14	4 83	5.50
10	1 21	1 52	1 97	2.38	2 94	3 43	3.91
11	...	1 26	1 63	1.88	2 43	2 83	3 22
12	1 26	.	1.30	1 57	1 93	2.26	2.58
13	1 53	1 22	.	1 21	1 49	1 74	1 98
14	1 88	1 50	1 22	. .	1 24	1 44	1.64
15	2 27	1 81	1 48	1 21		1.17	1.35
16	2 66	2 12	1 73	1 42	1 18	. . .	1.14
17	3 11	2.47	2.03	1 66	1 37	1.17	......
18	3 60	2 87	2 35	1 92	1 59	1 36	1 16
20	4.73	3 76	3 08	2 52	2 08	1 78	1 52
24	7.55	6 01	4 92	4 02	3 32	2 84	2 43
30	13.4	10 7	8 72	7 14	5 88	5 03	4 30
36	21.3	16 9	13 9	11 3	9 37	8 01	6.85
42	31 5	25 1	20 5	16 8	13 9	11.9	10.1
48	44.2	35.2	28 8	23.5	19 4	16.6	14.2

Actual Internal Diameters

Use of Table XXXIII. The table is used in the following manner. If it is desired to know the number of ¾-in pipes that will equal in discharging capacity a 2-in pipe, glance down the column headed 2 to the intersection of the line in the first column marked ¾. This shows that it requires fourteen ¾-in pipes to equal one 2-in pipe.

To find the number of pipes of one size that equal the discharging capacity of another in the lower part of the table, follow down the column the size of the smaller pipe until it intersects the line of the larger one. Thus, to find the number of 2-in pipes that equal in discharging capacity a 10-in pipe, follow down the column marked 2 to where it intersects the line marked 10 and it will be found that 80.4 2-in pipes are equal to one 10-in pipe.

In like manner the size of main required to serve several branches of different sizes can be determined. Suppose it is necessary to find the diameter of main that will serve one 2-in, one 3-in and one 4-in pipe. Use the capacity of the smallest pipe as a measure Then, one 2-in pipe has the capacity of one 2-inch pipe; one 3-in pipe has the capacity of 3.06 2-in pipes; one 4-in pipe has the capacity of 6.47 2-in pipes. The combined capacity of the pipes equals 10.53 2-in pipes. From the table it is found that a 5-in pipe has the capacity of 11.9 2-in pipes, so would be more than large enough to serve all the branches.

Hot-Water Supply. Gas, steam and coal are generally used for HEATING WATER. In the larger buildings steam is used. In medium-sized buildings coal heats the water. In residences gas is generally the fuel for water-heating; it is also being used to an increasing extent in hotels and office-buildings of medium size.

The consumption of hot water in large office-buildings is not so large as might be expected. In the Equitable Building of New York City, having a tenancy of 12 000 people, the average monthly consumption does not vary much from about 15 000 cu ft. That is at the low rate of 1.25 cu ft or 9.35 gal per person per month, a little over 1 qt per day. The water is heated, however, to a temperature of 120° F. Water of 100° is as hot as can comfortably be used, so to cool 9.35 gal of 120° water to about 100° would require the admixture of 6½ gal of 60° water. That would give each tenant almost 16 gal of hot water for a working month of about 24 days. To heat this water requires 480 000 lb of steam, or 1 lb of steam for every 2 lb of water heated.

In the following tables are given, respectively, the hot water consumed daily for one week and the gas required to heat the water in a hotel-building and in several apartments. The data are obtained from meter readings by the American Gas Association.

Boiling-Point of Water. The TEMPERATURE at which water boils varies with the PRESSURE. In a vacuum of one pound absolute pressure water boils

Table XXXIV. Water-Heating Data for Office-Buildings

System	Fixtures	Monday to Friday, inclusive	Ordinary day	Entire week
31-section boiler; Thermo control; 1 000-gal storage; instantaneous hot-water service boiler	250 hot-water fixtures	Hot water used, gal	6 395	35 200
		Gas consumed, cu ft	11 311	69 200
		Temperature rise, deg. F.	65.6	65.8
		Temperature of water delivered	135.9	136.8
		Cu ft gas per gal	1.76	1.96

Table XXXV. Water-Heating Data—Apartments

Apartment No. 1

Building	System	Hot-water fixtures
Six-story, basement and cellar. Two-room and bath per story	Six-section hot-water boiler; 220-gal tank circulating system	12 baths, 12 basins

	Consumption per day		
	Maximum	Minimum	Entire week
Hot water used, gal	405	339	2 437
Gas consumed, cu ft	1 300	1 169	8 300
Temperature rise, deg F	75	76	75.6
Temperature of water delivered, deg. F	125	124	124.4
Cu ft gas per gal	3.2	3.4	3.4

Apartment No 2

Building	System	Hot-water fixtures
Six-story and basement. Twelve 9-room apartments	Eleven-section hot-water boiler; Thermo control, circulating system, 250-gal tank	25 baths 13 basins 11 showers 12 sinks 26 laundry tubs 53 people

	Consumption per day		
	Maximum	Minimum	Entire week
Hot water used, gal	3 000	2 485	17 910
Gas consumed, cu ft	4 700	4 050	29 000
Temperature rise, deg F	93	91	91.5
Temperature of water, deg. F	132	131	131
Cu ft gas per gal	1.56	1.63	1.62

at a temperature of 102.018° F. At 15.31 lb gauge pressure, water boils at 250.293° F. The relation between the boiling temperature of water and the pressure is absolute; pressure cannot be increased without increasing the temperature, nor can the temperature of the boiling-point of water be increased without increasing the pressure.

The temperatures and corresponding pressures of boiling water can be found in Table XXXVI.

Expansion of Water. When water at or above the temperature of 39.1° F. is heated it increases in VOLUME. The temperature 39.1° F. is known as the POINT OF MAXIMUM DENSITY. When the water is at a lower temperature the application of heat causes it to contract and decrease in bulk, and the withdrawal of heat causes it to expand and increase in volume.

Table XXXVI. Boiling-Point of Water

Absolute pressure in pounds per square inch	Boiling-point of water, deg. F.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density	Absolute pressure in pounds per square inch	Boiling-point of water, deg. F.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density
1	102 018	20 623	46	275 704	563.0
2	126.302	10 730	48	278 348	540.9
3	141.654	7 325	50	280 904	520 5
4	153.122	5 588	52	283.381	501.7
5	162.370	4 530	54	285.781	484 2
6	170 173	3 816	56	288 111	467.9
7	176.945	3 302	58	290.374	452.7
8	182 952	2 912	60	292.575	438.5
9	188 357	2 607	62	294 717	425.2
10	193 284	2 361	64	296.805	412 6
11	197 814	2 159	66	298.842	400 8
12	202.012	1 990	68	300.831	389 8
13	205.929	1 845	70	302.774	379.3
14	205 604	1 721	72	304.669	369 4
14.69	212.000	1 646	74	306.526	360 0
15	213.067	1 614	76	308.344	351.1
16	216.347	1 519	78	310.123	342.6
17	219.452	1 434	80	311 866	334 5
18	222.424	1 359	82	313.576	326.8
19	225.255	1 292	84	315.250	319 5
20	227 964	1 231	86	316.893	312 5
22	233 069	1 126	88	318 510	305 8
24	237.803	1 038	90	320.094	299 4
26	242.225	962 3	92	321.653	293 2
28	246 376	897 6	94	323 183	287 3
30	250.293	841 3	96	324.688	281 7
32	254.002	791.8	98	326.169	276.3
34	257.523	748 0	100	327.625	271.1
36	260 833	708 8	105	331.169	258.9
38	264.093	673.7	110	334 582	247.8
40	267 168	642.0	115	337 874	228.3
42	270.122	613 3	120	341 058	. . . .
44	272.965	587.0			

The expansion, weight, density and comparative volume of pure water at different temperatures can be found in Table XXXVII.

Transmission of Heat to Water. Heat must be transmitted from the heating-medium to water through the walls of a COIL OR CONTAINER. The RATE at which heat can be transmitted depends upon the material through which the heat must pass. Copper, brass, wrought iron or steel and cast iron are the only materials used to any extent in the manufacture of heaters for water. The RATE OF HEAT-TRANSMISSION for these materials can be found in Table XXXVIII. The table is based on the assumption that the transmitting surface is clean and free from ash or incrustation of any kind.

The QUANTITY of heat transmitted depends upon the difference in tem-

Table XXXVII. Expansion of Water

Volume and Density of Water at Different Temperatures Calculated by Means of Rankin's Approximate Formula (D. K Clark)

Temperature, deg. F.	Comparative volume, water at 32° = 1	Comparative density water at 32° = 1	Weight of 1 cu ft, pounds	
32	1 00000	1.00000	62 418	Freezing point
35	0 99993	1 00007	62.422	
39 1	0 99989	1 00011	62.425	Point of maximum density
40	0 99989	1 00011	62.425	
45	0 99993	1 00007	62.422	
46	1.00000	1.00000	62.418	Same volume and density as at the freezing-point
50	1.00015	0 99985	62.409	
52 3	1.00029	0 99971	62.400	Weight taken for ordinary calculations
55	1.00038	0.99961	62.394	
60	1.00074	0 99926	62.372	
62	1.00101	0 99899	62.355	Standard temperature
65	1 00119	0 99881	62.344	
70	1.00160	0 99832	62.313	Cold baths
75	1.00239	0.99771	62.275	
80	1.00299	0 99702	62.240	Tepid baths
85	1.00379	0 99622	62.182	
90	1.00459	0 99543	62.133	Warm baths
95	1 00554	0 99449	62.074	
100	1 00639	0 92365	62.022	
105	1 00739	0 99260	61.960	Temperature of condenser water. Hot baths
110	1 00889	0.99119	61.868	
115	1.00989	0 99021	61.807	
120	1 01139	0 98874	61.715	
125	1 01239	0 98808	61.654	
130	1 01390	0 98630	61.563	
135	1 01539	0 98484	61.472	
140	1 01690	0 98339	61.381	
145	1 01839	0 98194	61.291	
150	1 01889	0 98050	61 201	
155	1.02164	0 97882	61.096	
160	1 02340	0 97714	60.991	
165	1 02589	0 97477	60 843	
170	1 02690	0.97380	60.783	
175	1 02906	0.97193	60.665	
180	1.03100	0 97006	60 548	
185	1 03300	0 96828	60.430	
190	1.03500	0.96632	60.314	
195	1.03700	0.96440	60.198	
200	1.03889	0 96256	60.081	
205	1 0414	0 9602	59.93	
210	1.0434	0 9584	59.82	
212	1.0444	0.9575	59.76	Boiling-point by formula
212	1.0466	0.9555	59.64	Boiling-point by direct measurement
230	1 0529	0 9499	59.26	
250	1 0628	0 9411	58.75	
270	1 0727	0 9323	58.18	
290	1 0838	0 9227	57.59	
298	1.0899	0.9175	57.27	Temperature of steam of 50 lb effective pressure per sq in
338	1 1118	0.8994	56.14	Temperature of steam of 100 lb pressure per sq in
366	1.1301	0.8850	55.29	Temperature of steam of 150 lb pressure per sq in
390	1.1444	0.8738	54.54	Temperature of steam of 205 lb effective pressure per sq in

perature between the heating-medium and the absorbing water, the thickness of the walls of the vessel and the material of which it is made.

Table XXXVIII. Comparison of Heat-Transmitting Surfaces

Material	Heat transmitted per square foot of heating surface per hour for each degree F. difference between the heating medium and the water
Copper plate.... . . .	275 Btu
Copper or brass pipe..	300 Btu
Wrought-iron or steel pipes	200 Btu
Cast-iron surface..	80 Btu

Capacity of Coal-Burning Water-Heaters. The capacity of a water-heater depends upon the amount of coal it can burn efficiently during a given period of time, the thickness and conductivity of the walls of the fire-box. Boiler iron is a better conductor of heat than cast iron; therefore a heater made of boiler iron of given surface will heat more water in an hour than will a cast-iron heater of equal surface, the amount of coal burned and the intensity of the fire being equal in both cases.

The amount of coal economically burned in a heater depends upon the area of grate and the size of the smoke-flue. Heaters burn from 3 to 6 lb, and will probably average 4 lb of coal per hr per sq ft of grate-surface. The total heat of combustion of a pound of anthracite coal of average composition is 14 133 Btu. Of this amount, however, a large percentage passes up the chimney as hot gases, and is otherwise lost, so that under ordinary conditions only about 8 000 Btu are actually available for transmission to the water.

Commercially, water-heaters are rated on a certain firing. That is, it is assumed that the heater will be charged with coal once in a given number of hours. Eight hours is assumed as the IDEAL FIRING-PERIOD. This means that the boiler will need attention only three times in 24 hr. The combustion-chamber is made large enough to hold the necessary coal for the 8-hr firing-period, and the grate-surface large enough to burn the coal. The heater then possesses a large reserve capacity. By cutting down the firing-period to 4 hr, the amount of water that can be heated will be almost doubled.

For this reason, the rating of a water-heater means little or nothing unless the firing-period is stated. One boiler might be heating a given quantity of water on an 8-hr firing. A smaller heater would heat the same quantity of water through the same range of temperature operating on a 4-hr firing-period. Both heaters could claim the same rating if the firing-period were not considered, although one of the heaters would have only about 50% the capacity of the other.

The HEAT-TRANSMISSION to water through coils or other heating surface is based on the assumption that the surfaces on both sides of the transmitting material are CLEAN and free from INCRUSTATION OF LIME OR MAGNESIA. However, no surface is free from incrustation after having been in service for some time. All water, even the softest, generally contains a small proportion of lime or magnesia, and it may be assumed that the softest of water is at least one degree hard. The lime or magnesia causing hardness of water, deposits on the water-heating coils, building up from year to year in proportion to the

hardness of the water. This scale will cut down the heat-transmission of the coils as follows:

$\frac{1}{16}$-in lime scale will cut down the heat transmission 13%
$\frac{1}{8}$-in lime scale will cut down the heat transmission 22%
$\frac{1}{4}$-in lime scale will cut down the heat transmission 38%
$\frac{3}{8}$-in lime scale will cut down the heat transmission 50%
$\frac{1}{2}$-in lime scale will cut down the heat transmission 60%
$\frac{3}{4}$-in lime scale will cut down the heat transmission 90%

In view of the progressive loss of heat-transmissions of coils and heaters, allowance must be made of sufficient excess capacity when new to offset the loss of capacity when old, and provision made to clean or replace the coils before the loss becomes excessive.

Temperature of Water Required. The TEMPERATURE OF WATER required for various classes of service may be found in Table XXXIX.

Table XXXIX. Temperatures of Water for Various Services

Class of service	Temperature required, deg. F.	
	Minimum	Maximum
Garages (for washing cars)	80	100
General domestic use	130	160
Laundry (hand work)	115	212
Laundry (machine work)	180	212
Barber shop (non-sterilizing)	115	150
Bar and soda fountains (hot drinks)	175	212
Lavatory and cleaning uses	115	150
Baths only	110	150
Shower-baths	110	150
Swimming-pools	80	212
Baptistries	80	212
Dishwashing (hand work)	130	212
Dishwashing (machine)	180	212
Milk dealers (not sterilizing or pasteurizing)	115	150

Heating Water with Steam. Water for use in large quantities is generally heated by means of steam by the aid of STEAM-COILS. Generally speaking, coils of brass or copper are preferable to iron for this purpose, although there are uses for which copper or brass pipe could not be employed. The rate of transmission of heat is 50% greater through brass or copper coils than through equal sizes of iron or steel coils.

The SIZE OF STEAM-COIL in square feet required to heat a certain quantity of water in a given time can be found by the following rule:

Rule. Multiply the weight of water in pounds by the number of degrees temperature F. the water is to be raised, and divide the product by (300, if the coil is brass or copper; 200, if the coil is iron or steel) times the difference between the temperature of the steam and the average temperature of the water.

In computing the HEATING-SURFACE of coils the INNER SURFACE of the pipe is taken, as that is the real heating-surface to which heat is applied. The

average temperature of the water in contact with the coil is taken as the temperature of the water.

Example. How many square feet of heating surface will be required in a copper coil to heat 300 gal of water per hour from 50° to 200° F. with steam 15-lb gauge pressure?

Solution. $300 \times 8.3 = 2\,490$ lb of water to be heated $200° - 50° = 150°$ rise in temperature of water. $(150° \div 2) + 50 = 125°$ = average temperature of water. $250°$ = temperature of steam at 15-lb gauge pressure. $250 - 125 = 125$ = the difference between temperature of steam and average temperature of water. Then:

$$\frac{2\,490 \times 150}{300 \times (250 - 125)} = 9.9 \text{ sq ft of copper coil}$$

The following constants will be found convenient in proportioning steam-coils for heating water:

W = gallons of water to be heated.
$W \div 10$ = sq ft of iron pipe-coil required for exhaust-steam.
$W + 15$ = sq ft of copper pipe-coil required for exhaust-steam.
$W \times 0.07$ = sq ft of iron pipe-coil for 5 lb pressure-steam.
$W \times 0.045$ = sq ft of copper pipe-coal for 5 lb pressure-steam.
$W \times 0.05$ = sq ft of iron pipe-coil for 25 lb steam-pressure.
$W \times 0.035$ = sq ft of copper pipe-coil for 25 lb steam-pressure.
$W \times 0.04$ = sq ft of iron pipe-coil for 50 lb steam-pressure.
$W \times 0.25$ = sq ft of copper pipe-coil required for 50 lb steam-pressure.
$W \times 0.03$ = sq ft of iron pipe-coil required for 75 lb steam-pressure.
$W \times 0.02$ = sq ft of copper pipe-coil required for 75 lb steam-pressure.

Drinking-Water Supply for Buildings. Drinking-water, filtered and cooled, has become an almost indispensable item of equipment for large buildings. Such a system is shown diagrammatically in Fig. 11. It consists of a grid of riser-branches indicated by solid lines, and returns or circulation-pipes indicated by dotted lines.

Pumps are necessary to keep the water in circulation, and the drinking-fountain branches are taken off the vertical risers so that the water delivered at the faucets will be direct from the cooling-coils. The system consists essentially of a set of water-filters, a cooling-tank, circulating-pumps and a balancing-tank. Water is drawn by the pumps through the filters and cooling-tank and is forced upward to the balancing-tank (Fig. 12), from which it overflows back to the cooling-tank to be recooled and recirculated. Make-up water flows from the house-tank to the balancing-tank, where it is controlled by a ball-cock which keeps the system full of water at all times.

This method of make-up necessitates the use of tank-water which might or might not be filtered. An alternate method, therefore, is to discharge from the filter into a second balancing-tank located near the cooling-tank. The supply to this balancing-tank is controlled by a ball-cock, so that as water is drawn at a drinking-fountain the balancing-tank is automatically supplied with filtered water. The overflow from the upper balancing-tank flows back into this make-up balancing-tank. The upper balancing-tank is located at a level of 8 ft or more above the level of the highest fixture to be supplied with drinking-water.

In the diagram is shown a cooling-tank. This tank is provided with brine coils for cooling the water. The brine for the coils can be supplied from a general brine circulating system, or a special refrigerating unit provided for the purpose. In practice cross-connections and by-passes would be provided.

For instance, there would be two filters with a by-pass, and the filters would be cross-connected so that either, both, or none could be in service at one time. In like manner the lower balancing-tank could be by-passed.

In proportioning a drinking-water system, a per capita allowance of 1/4 to 1/8 gal capacity in the water-cooling tank will be found satisfactory for most installations.

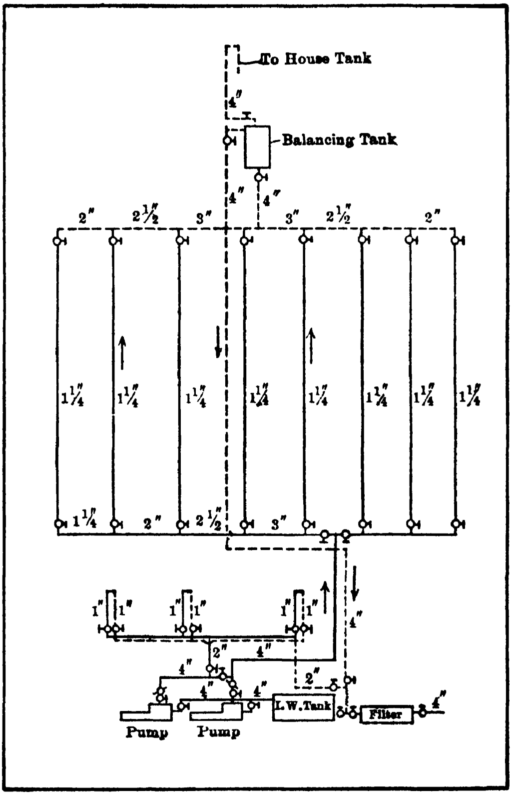


Fig. 11. Drinking-Water Supply for Buildings

It is necessary to INSULATE thoroughly all supply and return drinking-water pipes, otherwise they will condense the moisture in the atmosphere, thereby SWEATING and wetting ceilings over which they pass. It is equally necessary to cover the drinking-water pipes to prevent the water absorbing heat. Wastes from drinking-fountains ought likewise to be covered with anti-sweat covering.

Swimming-Pools are installed in club buildings, natatoriums, gymnasiums, and like buildings. They are approximately 25 by 75 ft in plan, or large enough for playing water-polo and similar games.

The pools are built with SLOPING BOTTOMS so that one portion of the tank has a depth of water from 4 to 5 ft and the deeper portion a depth of from 7 to 8 ft.

Provision is made to HEAT and STERILIZE the water in the pool. This is done by means of circulating-pumps which draw water from outlets at the bottom of the pool, pass it through a large strainer, a heater, filter, and steri-

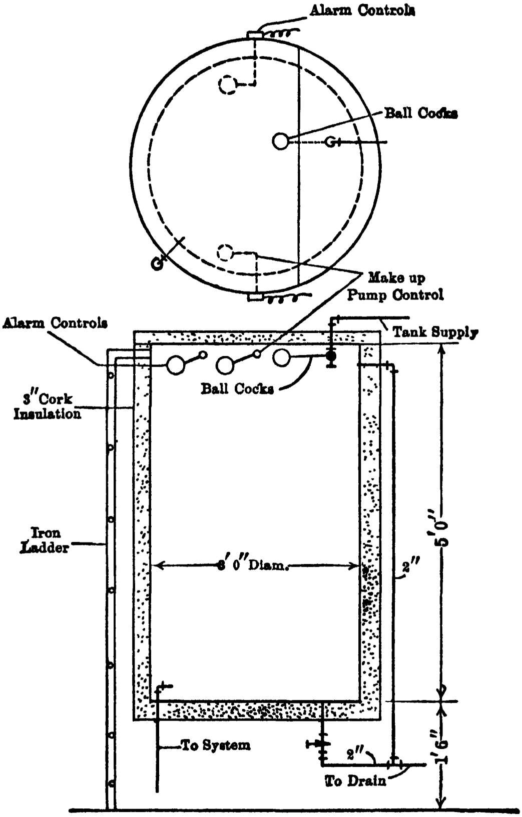


Fig. 12. Drinking-Water Balance Tank

lizer, and then back to the pool. The water returned to the pool flows in through a number of inlets spaced equally about the tank. An instantaneous steam water-heater is the type most favored for heating the water where a supply of steam is available throughout the year. A pressure type of SAND-FILTER is provided to remove all suspended matter in the water, and the sterilizing apparatus is used to rid the water of most of the bacteria. The two

types of sterilizing apparatus most in favor for sterilizing purposes are the ULTRA-VIOLET-RAY APPARATUS and CHLORINATING APPARATUS. The water in a swimming-pool ought to contain not more than 1 000 colonies of bacteria per cc on agar, incubated at 37.5° C., and a *B. coli* count of 1 per cc. The water should be taken from any part of the pool and examined within 48 hours.

In pools that are not treated with the chlorination process or ultra-violet rays, as many as 216 000 colonies of bacteria per cc have been found, and all samples showed gas indicating *B. coli*. A cubic centimeter is equal to about 15 drops.

When pools are so located that they cannot be emptied by GRAVITY, PUMPS must be provided with sufficient capacity to empty the pool in the shortest time allowable. Where revenue from the pool must be considered, the pumps ought to be able to empty the pool in 4 hr or less. Whatever repairs needed then can be made and the pool refilled without being out of service more than 8 or 10 hr.

For cleaning the walls and bottom of the pool without emptying the tank, a pool suction-pump, hose, and suction-nozzles similar to those used for vacuum-cleaning can be provided.

The large strainers on the recirculating line are better provided in duplicate, so that one can be cleaned while the other is in service.

Value of Pipe-Covering. There are many different uses for pipe-covering in plumbing practice. Cork covering or asbestos-sponge felted covering is used for drinking-water pipes; for hot-water pipes, 85% magnesia or air-cell asbestos is used, according to conditions. Anti-sweat covering should be used for wastes from drinking-fountains, cold-water pipes over ceilings, rain-leaders over finished ceilings, or other pipes that might condense moisture and allow it to drip where it would cause damage. Finally there is the frost-proofing of water-pipes exposed in cold places where they are liable to freeze.

Insulation for Hot-Water Pipes. Hot-water pipes and hot-water tanks when uncovered lose by radiation from their surface. To prevent this loss of heat and consequent extra consumption of coal, hot-water pipes, circulation-pipes and hot-water tanks in large institutions are generally covered with some non-heat-conducting material. The value of pipe-covering is not proportional to its thickness. Sectional pipe-coverings average about $1\frac{3}{8}$ in in thickness and reduce the loss by radiation about 90%. Doubling the thickness of pipe-covering saves only about another 5% of heat-loss. In specifying covering for pipes and boilers, therefore, a thickness of $1\frac{3}{8}$ in will be sufficient. Carbonate of magnesia is a very poor conductor of heat. Therefore, it is a good material for covering hot-water pipes. Carbonate of lime, on the other hand, is not a good covering material, although it often masquerades as carbonate of magnesia. When magnesia pipe-covering is specified, therefore, it is well to require a composition containing from 80 to 90% of magnesia, and require a test to be made at the expense of the contractor, but by a chemist named by the architect.

Expansion of Hot-Water Pipes. In all tall buildings expansion-loops ought to be placed in both the hot-water and the circulation-pipes, to permit the expansion and contraction of the lines without injury to the system. These loops are usually from 6 to 8 ft long, made up with elbows, and extend into the floor of the building. Generally the hot-water and circulation-pipes are supported midway between loops so that they can expand both up and down. The length that water-pipes will expand depends upon the degree to which they are heated, and the materials of which the pipes are made. The first of the

following four tables gives the expansion of cast-iron pipes, the second the expansion of wrought-iron pipes, the third the expansion of steel pipes, and the fourth the expansion of brass pipes.

Expansion of Cast-Iron Pipe

Temperature of air when pipe is fitted, deg. F.	Length of pipe when fitted, ft	Length of pipe when heated to							
		215° F.		265° F.		297° F.		338° F.	
		ft	in	ft	in	ft	in	ft	in
0	100	100	1 59	100	1.96	100	2.20	100	2.50
32	100	100	1 36	100	1 65	100	1 96	100	2.27
64	100	100	1 12	100	1 43	100	1 73	100	2 00

Expansion of Wrought-Iron Pipe

Temperature of air when pipe is fitted, deg. F.	Length of pipe when fitted, ft	Length of pipe when heated to							
		215° F.		265° F.		297° F.		338° F.	
		ft	in	ft	in	ft	in	ft	in
0	100	100	1.72	100	2.21	100	2.31	100	2.70
32	100	100	1.47	100	1 78	100	2.12	100	2.45
64	100	100	1.21	100	1 61	100	1.87	100	2.19

Expansion of Steel Pipe*

Temperature of air when pipe is fitted, deg. F.	Length of pipe when fitted, ft	Length of pipe when heated to							
		215° F.		265° F.		297° F.		338° F.	
		ft	in	ft	in	ft	in	ft	in
0	100	100	1 57	100	1 93	100	2.17	100	2.47
32	100	100	1 33	100	1 70	100	1.93	100	2.31
64	100	100	1 10	100	1 47	100	1.70	100	2.00

* Table computed by Jones and Laughlin Steel Corporation.

Expansion of Brass Pipe

Temperature of air when pipe is fitted, deg. F.	Length of pipe when fitted, ft	Length of pipe when heated to							
		215° F.		265° F.		297° F.		338° F.	
		ft	in	ft	in	ft	in	ft	in
0	100	100	2.58	100	3.18	100	3.56	100	4.05
32	100	100	2 19	100	2 79	100	3.18	100	3.67
64	100	100	1.81	100	2.41	100	2.79	100	3.28

Liquid-Soap Systems. In large buildings, soap is conveyed to the various lavatories on the many floors as a liquid, through a system of piping.

Fig. 13 shows diagrammatically the way the system is installed.

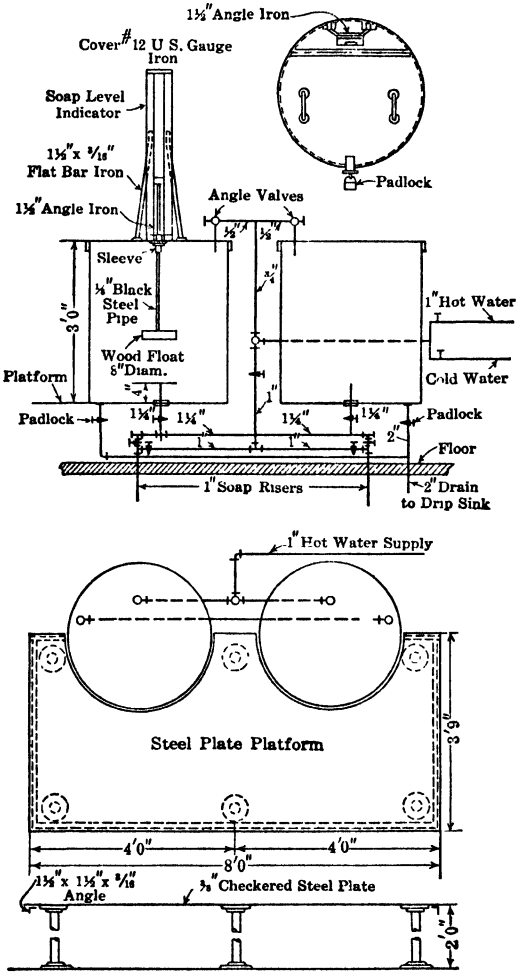


Fig. 13. Liquid Soap Tank

A battery of two SOAP-TANKS is provided, each tank having a capacity for a few weeks' supply of soap. Each tank is provided with hot and cold water for cleaning the tanks and also for mixing and thinning the soap solution

when necessary. Branches from the hot and cold-water pipes are connected to the soap-main, so that the main soap-pipe and its various branches can be flushed out whenever necessary. As there is a possibility of the soap sludging in the pipes, thereby stopping the flow if the liquid is not of the proper consistency, the FLUSHING CONNECTIONS are quite necessary. An iron platform with steps leading thereto is provided for convenience of the maintenance crew in charging the tanks

Black iron pipe is used for the soap-distributing system, and recessed drainage fittings are best for the mains and all branches 1 in in diameter and larger. They are equally desirable for the smaller sizes of pipe, but are not made in sizes smaller than 1 in.

At each battery of fixtures supplied with liquid soap, it is well to provide a PLUGGED CLEAN-OUT in the line (Fig. 14), so that in case the small pipe becomes clogged with soap it can be rodded out and flushed through the supply faucets.

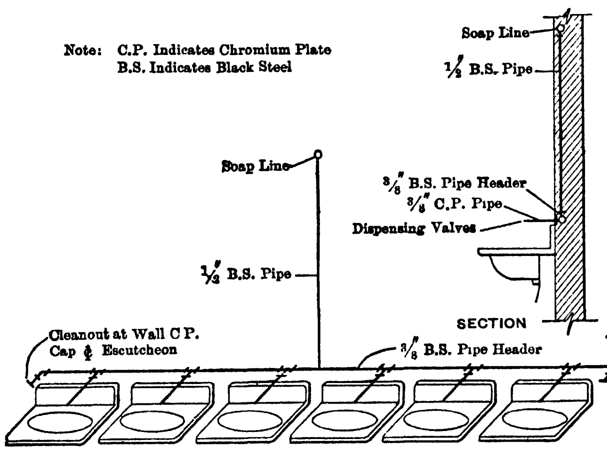


Fig. 14. Typical Soap Dispensing System

A $\frac{3}{8}$-in pipe is of sufficient size to supply a battery of fixtures. Generally the branch from battery supply to individual fixtures is $\frac{1}{4}$ in in diameter. As the consumption of liquid soap is small, the volume to be delivered can be taken care of by relatively small pipes. Where a battery of two tanks is installed, a header of $1\frac{1}{4}$-in pipe between the two tanks with a main drop riser of 1 in will be found of sufficient size. A 1-in drop riser will take care of 75 fixtures where the building is tall. In low buildings covering a large area it might be necessary to increase the soap-main one size.

Mixing and storage space in the liquid-soap tank of from 2 to 3 qt per fixture will be found a good proportion. A battery of two tanks is always convenient from an operating standpoint, as this enables a reserve tank of soap to be held ready for use at a moment's notice. A SOAP-LEVEL INDICATOR is provided at the tank so that the amount of unused soap in the tank can be seen at a glance.

It is good practice to continue the lower end of a drop riser, or the extreme

end of a long horizontal run, to a nearby slop-sink and control it there with a plugged valve. This will enable the operating force to flush out the pipes whenever necessary or desirable.

Testing Plumbing Systems

Testing Water-Supply Pipes. The water-distribution system of small buildings is generally tested by turning on the city water after plugging all the outlets. In tall buildings where the work is concealed and built in before the house-tank is ready to be filled, the system is tested by pumping it full of water and keeping it under pressure for 24 hours. Even when a tank supply is available it is well to test all lines to about 200-lb pressure per sq in.

Testing Drainage System. The drainage system is tested by plugging all outlets and filling the pipes with water. In tall buildings the work has to be tested in sections to keep up with the progress of the other trades. Special test fittings are provided for this purpose in the stacks at various floors, as for instance, the 12th and 24th floors of a 30-story building. The top section is tested first, the intermediate next, and the lowest section last. When applying a test to the lower section the water is allowed to rise to above the level of the test fittings so that all joints will have been under water.

The Smoke Test. A smoke test sometimes is applied as a final test. It consists, briefly, in pumping the system full of a dense pungent smoke under a slight pressure by means of a smoke-machine. The pressure should be only sufficient to balance a column of water one inch high. This is sufficient pressure to disclose any leaks, but is not great enough to force the seal in traps. The smoke test is of considerable value in disclosing leaks in small buildings, but of proportionally less value as the size of the building and drainage system increase. However, the smoke test is the best available for a final test and will have to continue in use until something better is evolved.

3. Illuminating-Gas and Gas-Piping

Varieties of Gas. There are five different kinds of gas used for domestic purposes. They are coal-gas, oil-gas, water-gas, natural gas, and acetylene-gas.

Coal-Gas is produced by the distillation or driving off of the light hydrocarbons, or gas, from bituminous coal. Nothing is added to the gas, and the by-products of distillation are coke, tar, and ammonia. Very little coal-gas is now used commercially.

Oil-Gas is made in much the same manner as coal-gas, by the process known as destructive distillation. This consists of heating the oil to a very high temperature, thereby causing the heavy hydrocarbons to break up into lighter or gaseous form. Animal and vegetable fats and oils, waste fats that occur in the manufacture of woolens, and ordinary rosins are used, as well as petroleum, in the manufacture of oil-gas.

Water-Gas, which is probably more extensively manufactured than any other kind of gas for general distribution, is made commercially by the contact of steam with incandescent carbon in the form of anthracite coal or coke. The steam is decomposed, the hydrogen being separated from the oxygen. The oxygen then takes up carbon from the coal or coke. This gas would burn with a non-luminous flame and would be useless for lighting except with incandescent mantles, so in practice the water-gas is enriched with oil-gas which

furnishes the hydrocarbon necessary to make a luminous flame, but this adds very little to the heating quality of the gas.

Natural Gas is obtained from drilled wells. In localities where it can be obtained it furnishes cheap light and fuel. The natural gas obtained in the hard-coal regions develops more heat per cubic foot in burning than any other kind of gas except acetylene. Natural gas is usually delivered under greater pressure in the street-mains and house-pipes than manufactured gas.

Acetylene-Gas is formed by bringing water and calcium carbide in contact. Calcium carbide is produced by the electrical fusion of coke and lime. It is a very hard substance like dark granite, has a very slight odor, will not burn or explode, and can be handled in any quantity with perfect safety. The fact that carbide begins to disintegrate and give off acetylene at the slightest touch of moisture makes it practicable to generate the gas in small quantities.

Process of Generating Acetylene-Gas. The satisfactory production of acetylene-gas requires a generator which shall feed carbide of sufficient size and weight to be plunged a sufficient depth under the water in the generator-chamber to insure coolness and proper washing. The carbide-chamber must be so arranged and protected that no gas can return to it to be wasted when the chamber is refilled. It must feed carbide loosely and in very small quantities to provide for perfect coolness by free access of water to all of the carbide. It must work automatically and with absolute certainty. Acetylene-gas to be pure must be thoroughly washed. Impure acetylene, as with any other illuminating-gas, means a discoloration of the flame, diminished illuminating power, clogging of pipes and burners with carbon and other foreign matter, and smoky burners, causing blackening of ceilings and tarnished and soiléd woodwork and upholstery. It is now generally agreed that the requirements above outlined can be attained only by a generator of the plunger-type. Portable generators are made in sizes of 5, 10, 15, 20 and up to 500-lights capacity. In all machines dropping carbide into water there should be a connection open from the carbide-holding receptacle to the safety-vent run out of doors from the gasometer. To develop the full illuminating power of the gas it is necessary to use a burner-tip having the thinnest slit obtainable, the illuminating power of the gas being about fifteen times that of coal-gas, for the same consumption. The light is a clear white, very nearly resembling sunlight in color and diffusiveness, with none of the red of the incandescent lamp, the orange of the ordinary gas-flame, or the green tone of the incandescent mantle; and it possesses the quality, unique among artificial illuminants, of reproducing even the most delicate shades of color as faithfully as sunlight. Even when used with mantle-burners, as it may be with great economy, acetylene-light presents a strong dissimilarity from ordinary gas under the same conditions. Acetylene corrodes silver and copper, but does not affect brass, iron, lead, tin, or zinc.

Piping a House for Gas

General Principles and Requirements. Ordinary wrought-iron or steel pipe is used in piping for all kinds of gas. Galvanized malleable-iron fittings are generally used in preference to plain iron fittings. Leaks should never be made tight with gas-fitters' cement. It cracks off easily and will melt in warm places. When pipes under floors run across floor-joists, the joists should be notched near their ends, or where supported by partitions, and not near the middle of the spans. It is evident that a 10-in joist, notched 2 in in the middle of the span, is no stronger than an 8-in joist. The whole system

of piping must be free from low places or traps and should pitch toward the main rising pipe. The riser should run up in a partition as near the center of the building as is practicable. It is obvious that where gas is distributed from the center of a building, smaller running lines of pipe will be needed than when the main pipe runs up on one end. Hence joists will not require as deep cutting, and the flow of gas will be more regular and even. For the same reason, in large buildings more than one riser may be advisable. Drip-pipes in a building should always be avoided. The whole system of piping should be so arranged that any condensed gas will flow back through the system and into the service-pipe in the ground. All outlet-pipes should be so securely and rigidly fastened in position that there will be no possibility of their moving when the fixtures are attached. Center pipes should rest on a solid support fastened to the floor-joists near their tops. The pipe should be securely fastened to the support to prevent lateral movement. The drop-pipe must be perfectly plumb, and pass through a guide fastened near the bottom of the joists, which will keep it in position and not be easily displaced by lathers, masons, or other workmen. In the absence of express directions to the contrary, outlets for brackets should generally be 5 ft 6 in high from the floor, except that it is usual to put them 6 ft high in halls and bath-rooms. The upright pipes should be plumb, so that the nipples that project through the walls will be level. The nipples should project not more than ¾ in from the face of the plastering. Laths and plaster together are usually ¾ in thick; hence the nipples should project 1½ in from the face of the studding. Drop center pipes should project 1½ in below the furring, or joists if there is no furring, where it is known that there will be no stucco or centerpieces used. Where centerpieces are to be used, or where there is a doubt whether they will be or not, then the drop-pipes should extend about a foot below the furring. All pipes being properly fastened, the drop-pipe can be safely taken out and cut to the right length when fixtures are put on. Gas-pipes should never be placed on the bottoms of floor-joists that are to be lathed and plastered, because they are inaccessible in the contingency of leakage, or when alterations are desired. The whole system of piping should be tested under a pressure of air that will raise a column of mercury 6 in high in a glass tube. The pipes are either tight or they leak. There is no middle ground. If they are tight the mercury will not fall a particle. A piece of paper should be pasted on the glass tube, even with the mercury, to mark its height while the pressure is on. The system of piping should remain under test for at least a half-hour.

Rules and Table for Proportioning Sizes of House-Pipes for Gas *

Rules Governing Sizes of Gas-Pipes. Table XL is based on the well-known formula for the flow of gas through pipes. The friction, and therefore the pressure necessary to overcome the friction, increases with the quantity of gas that goes through, and as the aim of the table is to have the loss in pressure not exceed 1/10 in water-pressure in 30 ft, the size of the piping increases in going from an extremity toward the meter, as each section has an increasing number of outlets to supply. The quantity of gas the piping may be called on to pass through is stated in terms of ⅜-in outlets, instead of cubic feet, outlets being used as a unit instead of burners, because at the time of first inspection the number of burners may not be definitely determined. In making the table, each ⅜-in outlet was assumed to require a supply of 10 cu ft per hour. In using the table observe the following rules:

* The Denver Gas and Electric Company.

(1) No house-riser shall be less than ¾ in. The house-riser is considered as extending from the cellar to the ceiling of the first story. Above the ceiling the pipe must be extended of the same size as the riser, until the first branch line is taken off.

(2) No house-pipe shall be less than ⅜ in. An extension to existing piping may be made of ¼-in pipe to supply not more than one outlet, provided said pipe is not over 6 ft long.

(3) No gas-range shall be connected with a smaller pipe than ¾ in.

(4) In figuring out the size of pipe, always start at the extremities of the system, and work TOWARD the meter.

(5) In using the table, the lengths of pipe to be used in each case are the lengths measured from one branch or point of juncture to another, disregarding elbows or turns. Such lengths will be hereafter spoken of as SECTIONS. No change in size of pipe may be made except at branches or outlets, each section therefore being made of but one size of pipe.

(6) If any outlet is larger than ⅜ in it must be counted as more than one, in accordance with the schedule below:

Size of outlet, inches........	½	¾	1	1¼	1½	2	2½	3	4
Value in table................	2	4	7	11	16	28	44	64	112

(7) If the exact number of outlets given cannot be found in the table, take the next larger number.

(8) If, for the number of outlets given, the exact length of the section which feeds these outlets cannot be found in the table, the next larger length, corresponding to the outlets given, must be taken to determine the size of pipe required. Thus, if there are eight outlets to be fed through 55 ft of pipe, the length next larger than 55 in the eight-outlet line in the table is 100, and as this is in the 1¼-in column, that size pipe would be required.

(9) For any given number of outlets, do not use a smaller size pipe than the smallest size that contains a figure in the table for that number of outlets. Thus, to feed 15 outlets, no smaller size pipe than 1 in may be used, no matter how short the section may be.

(10) In any piping-plan, in any continuous run from an extremity to the meter, there may not be used a longer length of any size pipe than found in the table for that size, as 50 ft for ¾ in, 70 ft for 1 in, etc. If any one section would exceed the limit length, it must be made of larger pipe. Thus, 6 outlets could not be fed through 75 ft of 1-in pipe, but 1¼ in would have to be used. When two or more successive sections work out to the same size of pipe and their total length or sum exceeds the longest length in the table for that size pipe, make the section nearest the meter of the next larger size. For example, if we have 5 outlets to be supplied through 45 ft of pipe and these 5 and 5 more, making 10 in all, through 30 ft of pipe, we should find by the table that 10 outlets through 30 ft would require 1-in pipe, and that 5 outlets through 45 ft would also require 1-in pipe, but as the sum of the two sections, 30 plus 45 equals 75 ft, is longer than the amount of 1 in that may be used in any continuous run, the 30-ft section, being the one nearer the meter, must be made of 1¼-in pipe. The application of the limit in length of any one size in a continuous run may also be shown as follows· Eight outlets will allow of 13 ft of ¾-in pipe in the section between the eighth and ninth outlets (counting from the extremity of the system toward the meter), provided that this 13 ft added to the total length of ¾-in pipe that may have been used between the extremity of the run and the eighth outlet does not exceed 50 ft, which, according to the table, is the greatest length of ¾ in allowable in any one branch of the system. Therefore, up to the eighth outlet, 37 ft of ¾-in pipe could have

Table XL. Showing the Correct Sizes of House-Pipes for Different Lengths of Pipes and Number of Outlets

Number of ⅜-in outlets	Lengths of pipes in feet									
	⅜-in pipe	½-in pipe	¾-in pipe	1-in pipe	1¼-in pipe	1½-in pipe	2-in pipe	2½-in pipe	3-in pipe	4-in pipe
1	20	30	50	70	100	150	200	300	400	600
2		27	50	70	100	150	200	300	400	600
3		12	50	70	100	150	200	300	400	600
4			50	70	100	150	200	300	400	600
5			33	70	100	150	200	300	400	600
6			24	70	100	150	200	300	400	600
7			18	70	100	150	200	300	400	600
8			13	50	100	150	200	300	400	600
9				44	100	150	200	300	400	600
10				35	100	150	200	300	400	600
11				30	90	150	200	300	400	600
12				25	75	150	200	300	400	600
13				21	60	150	200	300	400	600
14				18	53	130	200	300	400	600
15				16	45	115	200	300	400	600
16				14	41	100	200	300	400	600
17				12	36	90	200	300	400	600
18					32	80	200	300	400	600
19					29	73	200	300	400	600
20					27	65	200	300	400	600
21					24	58	200	300	400	600
22					22	53	200	300	400	600
23					20	49	200	300	400	600
24					18	45	190	300	400	600
25					17	42	175	300	400	600
30					12	30	120	300	400	600
35						22	90	270	400	600
40						17	70	210	400	600
45						13	55	165	400	600
50							45	135	330	600
65							27	80	200	600
75							20	60	150	600
100								33	80	360
125								22	50	230
150								15	35	160
175									28	120
200									21	90
250									14	59

been used, and yet allow 13 ft of ¾ in to be used in the section between the eighth and ninth outlets. If more than 37 ft had been used, then the entire 13 ft between the eighth and ninth outlets would have to be of 1-in pipe.

(11) Never supply gas from a smaller size of pipe to a larger one. If we have 25 outlets to be supplied through 200 ft of pipe, and these 25 and 5 more, making 30 in all, through 100 ft of pipe we should find by the table that 25 outlets through 200 ft would require 2½-in pipe, and 30 outlets through 100 ft would require 2-in piping, but as under this condition a 2-in pipe would be supplying a 2½-in pipe, the 100-ft section must be made 2½ in. The sizes of pipes in Fig. 15 are in accordance with the foregoing rules and the table.

Heat Units in Gas. There is no one definite number of heat units that can be given as the heat value of gas. As already pointed out, there are three processes of manufacture of artificial gas, all of which are used throughout the country. Of the manufactured gases, two or more kinds are sometimes combined, for instance, when oil-gas is added to water-gas. Then, too, natural gas is now being mixed with manufactured gas, so that the exact

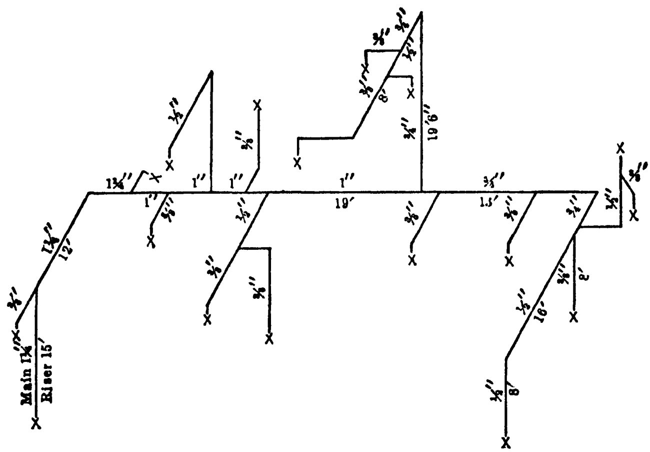


Fig. 15. Diagram of Gas-piping

amount of heat given off by a cubic foot of gas will depend upon the standard of the local gas company.

Manufactured gas is generally said to contain 700 heat units, and natural gas from 740 to 1 117, or an average of 950, heat units per cu ft, according to the territory where the natural gas is produced. As a matter of fact, the heat contained in manufactured gas varies from 500 to 600 Btu. It seldom exceeds 650. In some states the heat value of gas is governed by laws, and the tendency seems towards establishing a standard of 525 Btu per cu ft of manufactured gas.

CHAPTER XXXIII

ILLUMINATION OF BUILDINGS

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General Aims. The illumination of buildings is assuming a position of ever-increasing importance. The old method of installing a few lights more or less at random has been superseded by the inclusion of lighting as an integral part of the building design. Only in this way is a thoroughly pleasing and adequate installation possible. There has also been a breaking away from the stereotyped forms of lighting fixtures and a great influx of new ideas leading to various types of luminaires and luminous panels. There is also an increase in the use of artificial skylights and of cove lighting. All these must be built in and cannot be added readily as an afterthought.

Interiors should be adequately illuminated to minimize eye-strain, reduce accidents, and give a pleasing and cheerful effect. Even more important is the requirement that there shall be freedom from glare, nothing being more fatiguing than facing a glaring light source, either directly or by reflection in a polished desk-top. The aim should be to produce a lighting installation which will give soft, well-diffused, glareless but adequate illumination.

Of the result, the eye is the final judge, and on its verdict the lighting system stands or fails. The esthetic elements are of prime importance in many cases; they are, of course, hardly amenable to calculation. There remain, however, a great many factors which can be calculated easily, and such calculations will often save costly mistakes. It is the purpose in the following pages to give general design principles for various types of installations and to provide tables showing modern practice.

Nature of Light. Electromagnetic waves are used almost continuously during the waking moments of every normal human being. Comparatively long waves are utilized in RADIO; the broadcast band, for instance, running from wave-lengths of 200 to 550 meters (about 650 to 1 800 ft). A band of shorter waves is known as the INFRA-RED, and is responsible for most of the heat we receive from the sun as well as for the energy radiated from all hot bodies here on earth. We use extremely short electromagnetic waves in the form of X-RAYS, while somewhat longer waves are known as ULTRA-VIOLET. But of greatest importance to the human race is a band having wave-lengths from about 0.00004 to 0.00007 cm. These are important, not because of any inherent characteristic of the waves themselves, but due to a peculiarity of the human eye which responds to electromagnetic radiation of only these lengths.

It has been found that the sun produces practically all wave-lengths, not only in the visible range but also in the near-by ultra-violet and infra-red. In the visible region, differences in length are perceived as different COLORS. The approximate wave-lengths corresponding to the colors are as follows:

Violet	0.000040 to 0.000045 cm
Blue	0.000045 to 0.000049 cm
Green	0.000049 to 0.000055 cm
Yellow	0.000055 to 0.000059 cm
Orange	0.000059 to 0.000063 cm
Red	0.000063 to 0.000070 cm

Light from the sun contains all these colors and gives the sensation of WHITE LIGHT. A light source which gives out a preponderance of long waves will be expected to have a REDDISH or ORANGE color. For instance, the well-known neon sign emits nearly all its energy at long wave-lengths and gives a characteristic red-orange light.

Some objects reflect light-waves of a certain length only, and absorb all the rest. It is this property that gives color to objects. Suppose, for instance, that a piece of cloth is receiving light from the sun, all of which is absorbed except the waves of proper length to cause a sensation of green. The green waves only would then come from the cloth to the eye, all the rest being absorbed, and the cloth would appear green. If it absorbed waves of all lengths, it would appear black, because no light would be reflected from it to the eye. If now the piece of cloth, which absorbs all wave-lengths except that of green, were exposed to a source of light which was emitting all colors EXCEPT GREEN, there being no green waves to be reflected from it, the cloth in this light would appear black. Suppose a piece of cloth absorbed all colors but two, say violet and red. When light having all wave-lengths fell upon it, it would absorb all the waves except violet and red. These two the cloth would reflect as a mixture and would appear purple. If, however, the source of light contained no violet waves, it could only reflect the red waves and appear red. This light, then, would not cause the cloth to show its normal color. So in choosing an artificial source of light, it is necessary to select one which will send out all wave-lengths, if we wish to have the different objects appear in their normal colors

Experiment has shown that though incandescent lamps emit all wave-lengths in the visible region, they do not give them in the correct proportions to produce white light. The old carbon lamp gave a light which was distinctly reddish in color. The modern tungsten lamp, particularly in the largest sizes, is much better in this respect, though the light is still far from white. Thus where it is necessary to match colors, special filters must be used in front of the lamps. These cut out some of the red rays and give a nearly white light at the expense of efficiency. Except for color matching, the color of the clear or frosted Mazda lamp is usually considered satisfactory.

Vision. As previously stated, the eye is only sensitive to a certain limited band of wave-lengths and is blind to electromagnetic waves of other lengths. It sees the waves of lengths from about 0.00004 to 0.00007 cm as different colors, ranging from a deep violet at the former to a deep red at the latter limit. But it does not see all colors equally well. It favors the yellow-green, and becomes less and less sensitive as the ends of the visible spectrum are approached. This is shown in Table I, which gives the internationally accepted values of relative visibility. These results are an average of tests made by a large number of observers.

Table I

	Wave-length	Relative visibility		Wave-length	Relative visibility
Violet . . .	0.000040 cm	0 0004	Yellow. .	0.000056 cm	0 995
	41	0 0012		57	0 952
	42	0 0040		58	0.870
	43	0 0116		59	0.757
	44	0 023			
	45	0.038	Orange. .	60	0 631
				61	0 503
Blue .	46	0 060		62	0 381
	47	0 091		63	0 265
	48	0.139			
			Red. . .	64	0 175
Green .	49	0 208		65	0 107
	50	0 323		66	0 061
	51	0 503		67	0 032
	52	0 710		68	0 017
	53	0 862		69	0 0082
	54	0 954		70	0.0041
	55	0.995			

Luminous Flux. Quantity of light is measured in LUMENS.* Exactly how the lumen is measured and how it is standardized need not concern us here. For this information the reader is referred to any text on illumination. Suffice it to say that the total amount of light emitted by light sources is measured in lumens, values being furnished by the lamp manufacturers. In the earlier days of the industry, incandescent lamps were rated in CANDLES but this has been superseded by the specification of lumens. Thus the luminous flux from an ordinary 100-watt frosted-globe tungsten lamp is 1 340 lumens. This can be redirected by various kinds of reflectors or luminaires, and finally it can be spread over a portion of a room to give a satisfactory illumination.

Illumination. By the ILLUMINATION of a surface we mean the amount of light falling upon a unit area of that surface. Illumination is usually expressed in LUMENS PER SQUARE FOOT or FOOT-CANDLES. The two terms are synonymous, but the former is the preferable one and will be used in this treatise.

For instance, a desk-lamp using a 25-watt bulb and directing 200 lumens of light onto a desk 3 ft $\times$ 4 ft will produce an average illumination of

$$\frac{200}{3 \times 4} = 16.7 \text{ lumens/ft}^2$$

Intensity of a Point Source. A point source of light is said to have an INTENSITY OF ONE CANDLE if it illuminates a sphere of one-foot radius to one lumen/ft^2 when the light is placed at the center of this sphere. For engineering purposes it is usually permissible to consider incandescent lamps and even luminaires as point sources.

Brightness. For light sources which are not point sources, the term BRIGHTNESS may be used. The brightness of a luminous surface is defined

* The internationally accepted definition of *light* or *luminous flux* is the rate of flow of radiant energy evaluated with respect to the visual sensation. It is measured in lumens.

as the intensity per unit of projected area, and is often expressed in CANDLES PER SQUARE FOOT.

Measurement of Illumination. The eye is notoriously poor as an instrument for the measurement of illumination. Like the ear, it is capable of detecting small DIFFERENCES but cannot give ABSOLUTE VALUES with any degree of accuracy. Thus to determine whether the illumination produced by an existing lighting installation is sufficient or satisfactory as regards uniformity, some form of measuring instrument is necessary. Two types are in use today—the FOOT-CANDLE METER and the MACBETH ILLUMINOMETER.

The FOOT-CANDLE METER is shown in Fig. 1. A series of translucent openings in the top panel is lighted from within by a small incandescent lamp

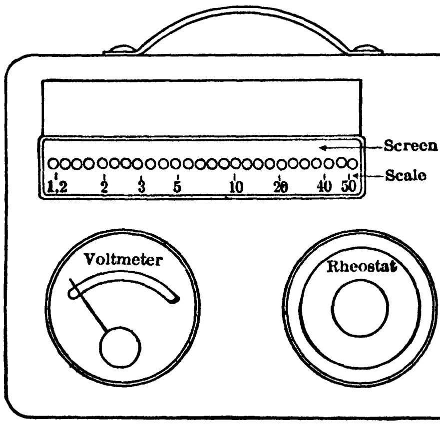


Fig. 1. Foot-candle Meter

which is so arranged that the openings constitute a series of spots varying from very low brightness on the left to great brightness on the right. The internal lamp is operated from a flash-light battery within the case, and the rheostat (lower right) is adjusted to give constant voltage across the lamp as indicated by the voltmeter (lower left). To determine the illumination at any point, on a desk, for instance, the meter is placed on the desk at that point, the rheostat is adjusted to give rated voltage, and the illumination is read off the scale of the instrument by selecting the particular spot which appears to be the same brightness as its background.

The MACBETH ILLUMINOMETER (Fig. 2) is a much more accurate and versatile instrument. Besides the instrument proper shown in Fig. 2, there is included a battery box with rheostats and ammeter for supplying the lamp, portable standard lamp, test-plate, filters, test-plate tripod, etc. The instrument is generally used with a special glass TEST-PLATE about 6 in in diameter, which is placed where the illumination is to be measured. The illuminometer is then sighted on the test-plate and the internal lamp is moved back and forth until the two comparison surfaces appear to be of the same brightness. The illumination (lumen/ft^2 or foot-candles) is then read directly on a scale attached to the movable lamp.

If the illumination is greater than 25 or less than 1 lumen/ft², filters can be used to extend the range. For daylight illumination a special filter is used to give a good color match in order to facilitate the measurements

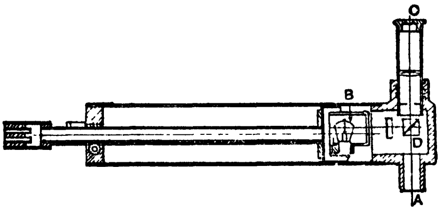


Fig. 2. Section of Macbeth Illuminometer *

* From Westinghouse Lamp Co. Bulletin E-101

Light Sources

The Sun. The sun is still used for interior illumination more than any artificial source. It has a diameter of 865 000 miles, is at a distance of approximately 93 000 000 miles, and has a brightness of the order of 200 000 candles per cm². This brightness is greatly in excess of anything obtainable on the earth. As noted from Table II, the crater of the carbon arc is only about 1/12 as bright. The light received from the sun contains all colors but is somewhat stronger in the center of the spectrum (yellow-green) than at the ends of the spectrum. It is an interesting fact that our eyes are also most sensitive to light of this color, which would seem to indicate that by a process of evolution the eye has fitted itself particularly to sunlight.

Incandescent Lamps. Since the first incandescent lamp of 1879 there has been continuous progress leading to greater efficiency and whiter light. (See Table III.) This has been brought about by improvements in the technique of manufacture and by the use of filament materials which can be operated at higher temperatures. As the temperature of the filament is raised the amount of light increases very rapidly and becomes more nearly white in color.

The light given off by a lamp is measured in LUMENS, while the electrical input is expressed in WATTS. Thus the efficiency is usually given in LUMENS PER WATT. Efficiency can also be expressed in per cent by using the experimentally determined relation that 670 lumens = 1 watt. For instance, the ordinary 100-watt lamp has an efficiency of 13.8 lumens/watt. In per cent this is $\frac{13.8}{670} = 2.0\%$ efficiency. Expressed in this way, the best of modern lamps show themselves to be distressingly inefficient. A large proportion of the energy given off by incandescent lamps is in the form of long infra-red rays which give no sensation of light but heat any objects on which they fall. That this heat must be taken into account in some cases is shown by the fact that in the ballroom of the St. George Hotel, Brooklyn, N. Y., 2 500 sq ft of direct heating surface were eliminated because of the large number of lamps used for lighting.

Mazda B and Mazda C Lamps. Two types of lamps are made today—the VACUUM LAMP (Mazda B) and the GAS-FILLED LAMP (Mazda C). The aim of the lamp manufacturers is to operate filaments at the highest temperature

consistent with a life of 1 000 hours. In a vacuum lamp this temperature is limited not by the melting-point of tungsten but by the large amount of tungsten which is evaporated from the filament at high temperatures. This evaporated tungsten deposits on the bulb in the form of a black powder which cuts down the life. Thus in a vaccum lamp it is necessary to operate the filament far below the melting-point to prevent blackening of the bulb.

Table II. Brightness of Some Light Sources

Source	Temperature, deg. *K*	Brightness, candles/cm²
Sun	....	200 000
Clear sky (average)	. .	0.4
Searchlight arc	... .	50 000
Crater of solid carbon arc	....	16 000
Mercury vapor lamp	. .	2.3
900-watt projection lamp	3 290	2 630
1 000-watt tungsten lamp	2 990	1 210
500-watt tungsten lamp	2 930	1 000
100-watt tungsten lamp	2 760	579
40-watt tungsten lamp	2 465	203
Metalized-carbon lamp (G. E. M.)	2 195	78.1
Treated-carbon lamp	2 165	70.6
Untreated—carbon lamp	2 080	54.9
Candle flame	. .	0.5
Welsbach gas-mantle	1 900	5

It has been found, however, that by filling the bulb with an inert gas the evaporation of the filament is greatly reduced, and thus it is possible to operate the filament at a higher temperature. At the present time all 115-volt lamps above 40 watts are gas-filled. With small lamps the loss in heat by convection currents in the gas more than balances the gain due to higher filament temperature, so that vacuum lamps are more efficient than gas-filled lamps in the small sizes.

Table III. Improvements in Incandescent Lamps

Date	Lamp	Approximate efficiency, lumens/watt
1879	Edison's first successful lamp (carbonized thread filament)	1.4
1881	First commercial lamps (carbonized bamboo)	1.6
1893	Carbonized cellulose filament	3.4
1905	Metallized carbon filament (G. E. M.)	4.2
1906	Tantalum filament	4.8
1907	Pressed tungsten filament	8
1911	Drawn tungsten filament (vacuum)	10
1913	Drawn tungsten filament (gas-filled lamp)	20

Effect of Line Voltage. It is of the utmost importance that the lamps be correctly chosen as to voltage. If the actual voltage is 115, lamps rated at 115 volts should be used. The use of 120-volt lamps will give illumina-

tion which is poor both in amount and color. It seems better to err by using lamps rated at too low a voltage rather than the reverse. While it is true that this will reduce the life of the lamps, the efficiency will be considerably increased, which may more than compensate economically for the increased cost of lamp renewals.

A rough rule applicable over a small change of voltage is as follows:

A 1% increase in voltage causes a 1.5% increase in watts, a 3.5% increase in lumens, a 1.8% increase in efficiency and a 13% decrease in life.

More accurate formulas are:

$$\frac{W_1}{W_2} = \left(\frac{V_1}{V_2}\right)^n$$

$$\frac{F_1}{F_2} = \left(\frac{V_1}{V_2}\right)^k$$

$$\frac{L_1}{L_2} = \left(\frac{V_2}{V_1}\right)^d$$

in which W_1 = watts input at voltage V_1;
W_2 = rated watts;
F_1 = lumens at voltage V_1;
F_2 = rated lumens;
L_1 = life at voltage V_1;
L_2 = rated life;
V_1 = voltage at which lamp is operated;
V_2 = rated voltage.

The constants n, k, and d are as follows:

Type of Lamp	n	k	d
Vacuum	1.54	3.38	13.1
Gas-filled	1.58	3.51	13.5

Example. A 300-watt 120-volt lamp is operated at 112 volts. What are the characteristics?

Reference to Table IV shows that the standard 300-watt lamp emits 5 400 lumens and has a life of 1 000 hours at rated voltage.

At 112 volts, the input will be

$$W_1 = 300\left(\frac{112}{120}\right)^{1.58} = 270 \text{ watts}$$

The luminous output will be

$$F_1 = 5\,400\left(\frac{112}{120}\right)^{3.51} = 4\,260 \text{ lumens}$$

The life will be

$$L_1 = 1\,000\left(\frac{112}{120}\right)^{13.5} = 2\,490 \text{ hours}$$

The efficiency will be

$$F_1/W_1 = 15.8 \text{ lumens/watt.}$$

Thus the life is more than doubled but the efficiency has been reduced from 18.0 to 15.8 lumens per watt.

Table IV. Data on Some Standard Mazda Lamps

(Data compiled from Manufacturers' Schedules, Large Mazda Lamps, National Lamp Works.)

Watts	Bulb	Type	Service	Initial lumens	Initial lumens /watt	Rated life, hr
10	S–14 clear	B	Sign....	76	7.6	1 500
15	A–17	B	General.....	138	9.2	1 000
15	G–16½ white	B	Decorative..... ...	122	8.1	600
25	A–19	B	General..... ...	250	10.0	1 000
25	A–19 daylight	B	Sign...	166	6.6	1 000
25	G–18½ white	B	Decorative..	230	9.2	750
40	A–19	C	General......... ...	424	10.6	1 000
40	T–8 clear	B	Limited......... .. .	400	10.0	1 000
40	G–25 white	B	Decorative....... . .	400	10.0	750
50	A–21	C	General....... .	560	11.2	1 000
50	T–8 clear	C	Projection	725	14.5	50
60	A–21	C	General....... ...	714	11.9	1 000
60	A–21 daylight	C	General.....	464	7.7	1 000
75	A–23	C	General.....	968	12.9	1 000
100	A–23	C	General..... .	1 400	14.0	1 000
100	A–23 day	C	General..... ...	910	9.1	1 000
100	T–8½ clear	C	Projection..... .	1 780	17.8	50
150	PS–25 clear	C	General...... . ..	2 355	15.7	1 000
150	PS–25 daylight	C	General..	1 531	10.2	1 000
200	PS–30 clear	C	General......	3 380	16.9	1 000
200	PS–30 W. B.*	C	General.....	.	.	1 000
200	PS–30 daylight	C	General	2 197	11.0	1 000
250	G–30 clear	C	Floodlight..... . .	3 625	14.5	800
300	PS–35 clear	C	General...........	5 490	18.3	1 000
300	PS–35 W. B.*	C	General.............	.		1 000
300	PS–35 daylight	C	General......	3 569	11.9	1 000
500	G–40 clear	C	Floodlight..........	8 300	16.6	800
500	PS–40 clear	C	General..... ..	9 750	19.5	1 000
500	PS–40 W. B.*	C	General..... .. .	. .	.	1 000
500	PS–40 daylight	C	General.	6 338	12.7	1 000
750	PS–52 clear	C	General	14 700	19.6	1 000
750	PS–52 W. B.*	C	General.	..	.	1 000
1 000	PS–52 clear	C	General	20 600	20.6	1 000
1 000	PS–52 W. B.*	C	General..... . .	. . .	.	1 000

* White bowl.

All the above lamps are for 110, 115, or 120 volts. Type "B" is vacuum type; type "C" is gas-filled. Sizes below 250 watts have medium screw base; lamps above 250 watts are usually fitted with mogul base.

Lamp Data. Table IV gives a list of the lamps most frequently used for interior illumination and for building floodlighting. The bulbs are designated by a letter and a number, the former referring to the shape of the globe and the latter to its diameter. P is pear-shaped, S is straight-side, T is tubular, G is spherical. The number gives the largest diameter of the bulb in eighths of an inch. Thus a T–8 lamp has a tubular bulb 8 eighths or one inch in diameter.

Mercury Vapor Lamps. Another type of lamp used in industrial lighting is the mercury vapor lamp manufactured by the General Electric Vapor

Lamp Co. Fig. 3 shows the general appearance of the unit. Its characteristics are as follows:

Watts	=	450
Lumens	=	5 625
Lumens per watt	=	12.5

The light is a blue-green color; and, because of the entire absence of red or orange, cannot be used where colors must be seen in their true values. However, it is finding wide use in certain industrial applications such as machine shops wood-working plants, textile mills, drafting offices, and newspaper composing rooms. It is particularly fitted for inspection of many products since the almost monochromatic nature of the light promotes high visual acuity.

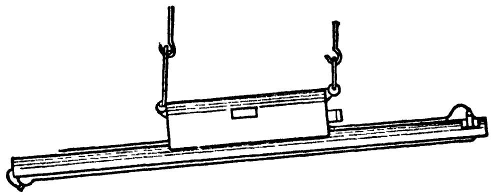


Fig. 3. Mercury-vapor Lamp

Calculation of Illumination

The Ideal Case. Consider first the IDEAL CASE of a perfect luminaire which absorbs no light, perfectly reflecting walls and ceiling, and an absorbing floor. Then all the light emitted by the lamp will be absorbed ultimately by the floor, no matter in what direction it is emitted by the luminaire or how many times it is reflected by walls or ceiling. Thus if the total light emitted by the lamp is F lumens, all this light must be distributed over the floor and the AVERAGE ILLUMINATION is

$$E = \frac{F}{A} \text{ (lumens/ft}^2\text{)} \tag{1}$$

where A is the floor area in sq ft.

The Coefficient of Utilization. In the actual case, some light is always absorbed by the luminaire, and if part of the output of the luminaire is directed at the ceiling or walls, a portion will be absorbed by them. So the illumination on the working plane will be reduced below the value (1). It is usual to introduce the COEFFICIENT OF UTILIZATION (k_u) to take care of this absorption of light. Thus in the actual room,

$$E = k_u \frac{F}{A} \tag{2}$$

k_u is always less than unity and approaches *one* the more closely the room and luminaire approach the ideal. k_u is a sort of efficiency—it tells how well the installation utilizes the light produced by the lamps.

It is evident that anything which affects the light distribution of the luminaire or the amount of light absorbed by the walls and ceiling will affect the coefficient of utilization. Thus k_u is a function of (1) The luminaire; (2) shape of room; (3) reflection factor of walls; (4) reflection factor of ceiling. The results of a large number of illumination measurements made with various-shaped rooms and different reflection factors of walls and ceiling

resulted in a set of tables by which the coefficient of utilization can be found in any ordinary case.

Table V.* Room Index for Narrow and Average Rooms

		Feet						
For indirect lighting use ceiling height		9 and $9\frac{1}{2}$	10 to $11\frac{1}{2}$	12 to $13\frac{1}{2}$	14 to $16\frac{1}{2}$	17 to 20	21 to 24	25 to 30
		Feet						
For direct lighting use mounting height		7 and $7\frac{1}{2}$	8 and $8\frac{1}{2}$	9 and $9\frac{1}{2}$	10 to $11\frac{1}{2}$	12 to $13\frac{1}{2}$	14 to $16\frac{1}{2}$	17 to 20
Room width, ft	Room length, ft	Room index						
9 ($8\frac{1}{2}$–$9\frac{1}{2}$)	8–10	1.0	0.8	0.6	0.6			
	10–14	1.0	0.8	0.8	0.6			
	14–20	1.2	1.0	0.8	0.6	0.6		
	20–30	1.2	1.2	1.0	0.8	0.6	0.6	
	30–42	1.5	1.2	1.0	0.8	0.6	0.6	0.6
	42–up	2.0	1.5	1.2	1.0	0.8	0.6	0.6
10 ($9\frac{1}{2}$–$10\frac{1}{2}$)	10–14	1.2	1.0	0.8	0.6	0.6		
	14–20	1.2	1.0	0.8	0.6	0.6	0.6	
	20–30	1.5	1.2	1.0	0.8	0.6	0.6	
	30–42	1.5	1.2	1.2	1.0	0.8	0.6	0.6
	42–60	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	60–up	2.0	1.5	1.5	1.0	1.0	0.8	0.6
12 (11–$12\frac{1}{2}$)	10–14	1.2	1.0	0.8	0.8	0.6	0.6	
	14–20	1.5	1.2	1.0	0.8	0.6	0.6	
	20–30	1.5	1.2	1.2	1.0	0.8	0.6	0.6
	30–42	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	42–60	2.0	1.5	1.5	1.2	1.0	0.8	0.6
	60–up	2.0	2.0	1.5	1.2	1.0	0.8	0.6
14 (13–$15\frac{1}{2}$)	14–20	1.5	1.2	1.0	1.0	0.8	0.6	0.6
	20–30	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	30–42	2.0	1.5	1.5	1.2	1.0	0.8	0.6
	42–60	2.0	2.0	1.5	1.5	1.0	0.8	0.6
	60–90	2.5	2.0	2.0	1.5	1.2	1.0	0.6
	90–up	2.5	2.0	2.0	1.5	1.5	1.2	0.8
17 (16–$18\frac{1}{2}$)	14–20	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	20–30	2.0	1.5	1.5	1.2	1.0	0.8	0.6
	30–42	2.5	2.0	1.5	1.2	1.0	1.0	0.6
	42–60	2.5	2.0	2.0	1.5	1.2	1.2	0.8
	60–110	2.5	2.0	2.0	1.5	1.2	1.2	0.8
	110–up	3.0	2.5	2.0	2.0	1.5	1.2	1.0
20 (19–$21\frac{1}{2}$)	20–30	2.5	2.0	1.5	1.2	1.0	0.8	0.6
	30–42	2.5	2.0	2.0	1.5	1.2	1.0	0.8
	42–60	2.5	2.5	2.0	2.0	1.5	1.2	0.8
	60–90	3.0	2.5	2.0	2.0	1.5	1.2	1.0
	90–140	3.0	2.5	2.5	2.0	1.5	1.5	1.0
	140–up	3.0	2.5	2.5	2.0	1.5	1.5	1.0
24 (22–26)	20–30	2.5	2.0	2.0	1.5	1.2	1.0	0.8
	30–42	3.0	2.5	2.0	1.5	1.2	1.2	0.8
	42–60	3.0	2.5	2.5	2.0	1.5	1.2	1.0
	60–90	3.0	2.5	2.5	2.0	1.5	1.5	1.0
	90–140	3.0	3.0	2.5	2.0	2.0	1.5	1.2
	140–up	3.0	3.0	2.5	2.0	2.0	1.5	1.2

* From Westinghouse Lamp Co. Bulletin E-108.

Table V* (Continued). Room Index for Narrow and Average Rooms

		Feet						
For indirect lighting use ceiling height		9 and 9½	10 to 11½	12 to 13½	14 to 16½	17 to 20	21 to 24	25 to 30
For direct lighting use mounting height		7 and 7½	8 and 8½	9 and 9½	10 to 11½	12 to 13½	14 to 16½	17 to 20
Room width, ft	Room length, ft	Room index						
30 (27–33)	30–42	3.0	2.5	2.5	2.0	1.5	1.2	1.0
	42–60	3.0	3.0	2.5	2.5	1.5	1.5	1.0
	60–90	4.0	3.0	3.0	2.5	2.0	1.5	1.2
	90–140	4.0	3.0	3.0	2.5	2.0	2.0	1.5
	140–180	4.0	3.0	3.0	2.5	2.0	2.0	1.5
	180–up	4.0	3.0	3.0	2.5	2.0	2.0	1.5
36 (34–39)	30–42	4.0	3.0	2.5	2.0	1.5	1.5	1.0
	42–60	4.0	3.0	3.0	2.5	2.0	1.5	1.2
	60–90	5.0	3.0	3.0	3.0	2.0	2.0	1.5
	90–140	5.0	4.0	3.0	3.0	2.5	2.0	1.5
	140–200	5.0	4.0	3.0	3.0	2.5	2.0	1.5
	200–up	5.0	4.0	3.0	3.0	2.5	2.0	1.5
40 or more	42–60	5.0	4.0	3.0	These values are given on the next page			
	60–90	5.0	4.0	4.0				
	90–140	5.0	4.0	4.0				
	140–200	5.0	5.0	4.0				
	200–up	5.0	5.0	4.0				

Room Index for Large High Rooms

		Feet							
For indirect lighting use ceiling height		14 to 16½	17 to 20	21 to 24	25 to 30	31 to 36	37 to 50		
For direct lighting use mounting height		10 to 11½	12 to 13½	14 to 16½	17 to 20	21 to 24	25 to 30	31 to 36	37 to 50
Room width, ft	Room length, ft	Room index							
14 (13–15½)	14–20	1.0	0.8	0.6	0.6	...	...	...	...
	20–30	1.0	0.8	0.6	0.6	...	...	...	...
	30–42	1.2	1.0	0.8	0.6	0.6	...	...	...
	42–60	1.5	1.0	0.8	0.6	0.6	0.6	...	...
	60–90	1.5	1.2	1.0	0.6	0.6	0.6	...	...
	90–up	1.5	1.5	1.2	0.8	0.6	0.6	...	...
17 (16–18½)	14–20	1.0	0.8	0.6	0.6	...	...	...	...
	20–30	1.2	1.0	0.8	0.6	...	...	...	...
	30–42	1.2	1.0	1.0	0.6	0.6	0.6	...	...
	42–60	1.5	1.2	1.2	0.8	0.6	0.6	0.6	...
	60–110	1.5	1.2	1.2	0.8	0.6	0.6	0.6	...
	110–up	2.0	1.5	1.2	1.0	0.8	0.6	0.6	...

* From Westinghouse Lamp Co. Bulletin E-108.

Table V* (Continued). Room Index for Large High Rooms

		Feet							
For indirect lighting use ceiling height		14 to 16½	17 to 20	21 to 24	25 to 30	31 to 36	37 to 50		
		Feet							
For direct lighting use mounting height		10 to 11½	12 to 13½	14 to 16½	17 to 20	21 to 24	25 to 30	31 to 36	37 to 50
Room width, ft	Room length, ft	Room index							
20 (19–21½)	20–30	1.2	1.0	0.8	0.6	0.6			
	30–42	1.5	1.2	1.0	0.8	0.6	0.6	.	.
	42–60	2.0	1.5	1.2	0.8	0.6	0.6	0.6	. .
	60–90	2.0	1.5	1.2	1.0	0.6	0.6	0.6	.
	90–140	2.0	1.5	1.5	1.0	0.8	0.8	0.6	0.6
	140–up	2.0	1.5	1.5	1.0	1.0	0.8	0.6	0.6
24 (22–26)	20–30	1.5	1.2	1.0	0.8	0.6	0.6	.	.
	30–42	1.5	1.2	1.2	0.8	0.6	0.6	.	.
	42–60	2.0	1.5	1.2	1.0	0.8	0.6	0.6	.
	60–90	2.0	1.5	1.5	1.0	0.8	0.6	0.6	0.6
	90–140	2.0	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	140–up	2.0	2.0	1.5	1.2	1.0	0.8	0.8	0.6
30 (27–33)	30–42	2.0	1.5	1.2	1.0	0.8	0.6	0.6	.
	42–60	2.5	1.5	1.5	1.0	1.0	0.8	0.6	.
	60–90	2.5	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	90–140	2.5	2.0	2.0	1.5	1.2	1.0	0.8	0.6
	140–180	2.5	2.0	2.0	1.5	1.2	1.0	0.8	0.6
	180–up	2.5	2.0	2.0	1.5	1.2	1.0	0.8	0.6
36 (34–39)	30–42	2.0	1.5	1.5	1.0	0.8	0.8	0.6	
	42–60	2.5	2.0	1.5	1.2	1.0	0.8	0.6	0.6
	60–90	3.0	2.0	2.0	1.5	1.0	1.0	0.6	0.6
	90–140	3.0	2.5	2.0	1.5	1.2	1.0	0.8	0.6
	140–200	3.0	2.5	2.0	1.5	1.5	1.2	1.0	0.8
	200–up	3.0	2.5	2.0	1.5	1.5	1.2	1.0	0.8
42 (40–45)	42–60	3.0	2.0	1.5	1.2	1.0	0.8	0.8	0.6
	60–90	3.0	2.5	2.0	1.5	1.2	1.0	0.8	0.6
	90–140	3.0	2.5	2.5	2.0	1.5	1.2	1.0	0.6
	140–200	3.0	2.5	2.5	2.0	1.5	1.2	1.0	0.8
	200–up	3.0	2.5	2.5	2.0	1.5	1.5	1.2	0.8
50 (46–55)	42–60	3.0	2.5	2.0	1.5	1.2	1.0	0.8	0.6
	60–90	3.0	3.0	2.5	1.5	1.5	1.2	1.0	0.6
	90–140	3.0	3.0	2.5	2.0	1.5	1.5	1.2	0.8
	140–200	3.0	3.0	2.5	2.0	2.0	1.5	1.2	0.8
	200–up	3.0	3.0	2.5	2.0	2.0	1.5	1.2	1.0
60 (56–67)	60–90	4.0	3.0	2.5	2.0	1.5	1.2	1.0	0.8
	90–140	4.0	3.0	3.0	2.5	2.0	1.5	1.2	1.0
	140–200	4.0	3.0	3.0	2.5	2.0	1.5	1.5	1.0
	200–up	4.0	3.0	3.0	2.5	2.0	2.0	1.5	1.0
75 (68–90)	60–90	5.0	4.0	3.0	2.5	2.0	1.5	1.2	0.8
	90–140	5.0	4.0	3.0	2.5	2.0	1.5	1.5	1.0
	140–200	5.0	4.0	4.0	3.0	2.5	2.0	1.5	1.2
	200–up	5.0	4.0	4.0	3.0	2.5	2.0	1.5	1.2
90 or more	60–90	5.0	4.0	3.0	2.5	2.0	1.5	1.2	1.0
	90–140	5.0	5.0	4.0	3.0	2.5	2.0	1.5	1.2
	140–200	5.0	5.0	4.0	3.0	2.5	2.0	1.5	1.2
	200–up	5.0	5.0	4.0	3.0	3.0	2.5	2.0	1.5

* From Westinghouse Lamp Co. Bulletin E-108.

Room Index. The effect of the second factor—shape of room—is evaluated from Table V. This gives what is known as the ROOM INDEX, a factor which is made equal to 1 00 for a cubical room. For instance, a room 14 × 14 × 14 ft has a room index of 1.0. It will also be noted that a high, narrow room has a small index caused by the comparatively large amount of light absorbed by the walls, while a large room with low ceiling has a large index because of the slight effect of the walls.

For instance, a room 24 ft high and 12 × 25 ft has a room index of 0.6, while a room 15 ft high and 60 × 100 ft has a room index of 4.0.

Calculation. Knowing the room index, the type of luminaire to be used, and the kind of walls and ceiling, one can refer to Table VI and obtain the coefficient of utilization k_u. The illumination is then determined very easily by use of Equation (2). This gives the average illumination on the working plane on the assumption of new lamps, clean luminaires, and freshly finished ceiling. To allow for a certain amount of dust, blackening of lamp bulbs, etc., DEPRECIATION FACTORS are given in Table VI. The actual average illumination which would be obtained some time after installation would be

$$E = \frac{F}{A} k_u \cdot k_d \tag{3}$$

in which k_d is this depreciation factor.

The use of Table VI requires an approximate knowledge of the reflection factor of walls and ceiling. This can be obtained from Table VII.

Example. What is the average illumination on the desk level 30 in above the floor in an office 15 × 25 ft having a ceiling height of 12 ft? It is lighted by two 500-watt lamps in prismatic glass inclosing units (No. 8, Table VI) which are hung 9½ ft above the floor. The walls and ceiling are fairly light (50% reflection factor).

First obtain the room index from Table V. Looking in the column of 9 and 9½ ft mounting height, one finds a ROOM INDEX of 1.2.

Table VI is next used to determine the coefficient of utilization and the depreciation factor. The COEFFICIENT OF UTILIZATION corresponding to this room index and this kind of walls and ceiling is seen to be 0.40. This means that 40% of the light emitted by the lamp actually gets down on the working plane. The DEPRECIATION FACTOR under average conditions is 0.70. Reference to Table IV shows that the 500-watt clear lamp gives 9 850 lumens.

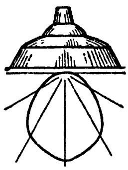
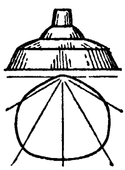
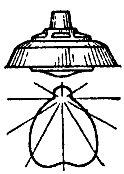
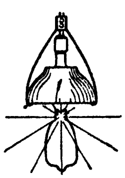
Thus the illumination on the working plane is

$$E = \frac{9\,850 \times 2}{15 \times 25} \times 0.40 \times 0.70$$

$$= 14.7 \text{ lumens/ft}^2$$

Design of a Lighting Installation. (1) The first step is to decide upon the ILLUMINATION desired. Representative values are given in Tables VIII and IX. It should be realized, of course, that the value to be used is a matter of judgment on the part of the lighting engineer. The table, however, represents good practice at the present day. These values are much higher than were used a few years ago, the general tendency being to use higher levels of illumination as such become economically feasible.

Table VI.* A Guide to the Selection of Reflecting

Lighting unit		Efficiency based upon: Illumination on horizontal	Efficiency based upon: Illumination on vertical	Appearance of lighted room	Direct glare	Reflected glare	Shadows	Maintenance
Direct Lighting—General Industrial Reflectors								
1 RLM Dome White Bowl Lamp 90° to 180°—0% 0° to 90°—66%		A Ex- cel- lent	B Good	B Good	B+ Very good	B Good	B+ Very good	A− Very good
2 RLM Dome Clear Lamp 90° to 180°—0% 0° to 60°—76%		A+ Ex- cel- lent	B+ Very good	C+ Very fair	C Fair	D Unsatis- factory above polished surfaces	C+ Very fair	A+ Ex- cel- lent
3 Glassteel Diffuser Clear Lamp 90° to 180°—7% 0° to 90°—60%		A− Very good	B Good	A− Very good	A− Very good	B+ Very good	A Ex- cel- lent	B Good
High Mounting—Industrial Reflectors								
4 Concentrated Prismatic Reflector Aluminum Cover Clear Lamp 90° to 180°—14% 0° to 90°—65%		A+ Ex- cel- lent	B Good	B Good	C+ Very fair	D Unsatis- factory above polished surfaces	C Fair	A Ex- cel- lent

* From Westinghouse

Equipment and Coefficients of Utilization

Probable average illumination—as fraction of initial illumination: Clean conditions	Average conditions	Dirty conditions	Ceiling	Very light (70%)			Fairly light (50%)			Fairly Dark (30%)	
			Walls	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly dark (30%)	Very dark (10%)
			Room index	Coefficients of utilization							
Calculation Data—General Units											
.80	75	.65	0.6	32	28	25	32	28	25	27	25
			0 8	40	.36	34	.39	.35	.33	.35	.33
			1.0	.43	.39	.37	.42	.39	.37	.39	.37
			1.2	.46	.43	41	.45	.43	.41	.43	.41
			1 5	.48	.45	.43	.47	.45	.43	.45	.43
			2 0	52	50	48	.51	.49	.47	.49	47
			2 5	56	.54	52	.55	.53	.51	.53	.51
			3.0	.57	55	53	56	.54	.52	.54	.52
			4.0	.60	58	.56	.59	.57	.55	.57	.55
			5 0	61	.59	57	.60	.58	.57	.58	.56
.80	75	70	0.6	34	.29	24	.34	.29	.24	.28	24
			0.8	.42	.38	.34	.42	.37	.33	.37	33
			1 0	.46	.43	.39	45	.42	.39	.42	.39
			1.2	.50	.47	.43	.49	.46	.43	.45	.42
			1 5	.53	.50	.46	.52	.49	.46	.48	.45
			2 0	.58	.55	.51	.57	.54	.51	.53	.51
			2 5	.62	59	.56	.61	.58	.56	.58	.56
			3 0	.64	61	.58	.63	.60	.58	.60	.58
			4 0	.67	65	.63	.66	64	.62	.63	.61
			5.0	.69	.67	65	.67	.66	.64	.65	.63
.75	.70	.60	0 6	29	25	21	.28	.24	.21	.23	21
			0 8	.36	.32	.29	.35	.31	.28	.31	28
			1.0	39	36	.33	.38	.35	.33	34	.32
			1 2	.42	.39	.36	.41	.38	.36	.37	.35
			1.5	.45	.42	.39	.43	.40	.38	.39	.38
			2.0	.49	.46	.43	.48	.45	.43	.44	.42
			2.5	.53	.50	.47	.51	.49	.47	.47	.46
			3 0	.54	.52	.49	.52	.50	.49	.49	.47
			4.0	.57	.55	.53	.55	.53	.51	51	.50
			5 0	58	.56	54	.56	.54	53	52	51
Calculation Data—High Bay Units											
.80	.75	.65	0 6	.40	37	.36	.39	.37	.35	.38	.34
			0 8	.48	.46	.45	.46	.45	.44	.44	.42
			1 0	.52	.50	.49	.50	.48	.48	.48	.46
			1 2	.55	.54	.53	.54	.52	.50	.51	.48
			1.5	.58	.57	.55	.55	.54	.53	.53	.50
			2 0	.61	.60	.58	.59	.58	.56	.56	.52
			2 5	.65	.62	.61	.62	.60	.59	.58	.55
			3.0	.66	.65	.63	.63	.62	.60	.59	.56
			4.0	.68	.66	.65	.64	.63	.62	.60	.57
			5 0	.70	68	.66	66	.64	.63	.61	.58

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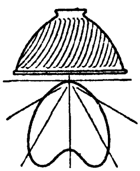
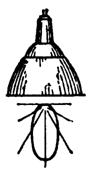
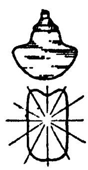
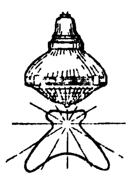
Table VI* (Continued). A Guide to the Selection

Lighting unit		Efficiency based upon: Illumination on horizontal	Efficiency based upon: Illumination on vertical	Appearance of lighted room	Direct glare	Reflected glare	Shadows	Maintenance
High Mounting—Industrial Reflectors (continued)								
5 Mirrored Glass Reflector Clear Lamp 90° to 180°—0% 0° to 60°—70%		A+ Excellent	B Good	B Good	C+ Very fair	D Unsatisfactory above polished surfaces	C Fair	A Excellent
6 Polished Aluminum Reflector Clear Lamp 90° to 180°—0% 0° to 90°—72%		A+ Excellent	B Good	B Good	C Very fair	D Unsatisfactory above polished surfaces	C+ Fair	A− Very good
Store and General Utility Units								
7 White Glass Enclosing Globe 90° to 180°—35% 0° to 90°—45%		B+ Very good	B+ Very good	A Excellent	B Good	B+ Very good	A− Very good	B+ Very good
8 Prismatic Glass Enclosing Unit 90° to 180°—27% 0° to 90°—53%		A− Very good	B+ Very good	A Excellent	B Good	B− Very fair	B Good	B Good

* From Westinghouse.

of Reflecting Equipment and Coefficients of Utilization

Probable average illumination—as fraction of initial illumination			Ceiling	(Very light (70%)			Fairly light (50%)			Fairly dark (30%)	
Clean conditions	Average conditions	Dirty conditions	Walls	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly dark (30%)	Very dark (10%)
			Room index	Coefficients of utilization							
Calculation Data—High Bay Units (continued)											
.80	.70	.60	0.6	.40	.38	.36	.39	.38	.36	39	.36
			0.8	.48	.46	.45	.47	.45	.44	45	.43
			1.0	.51	.50	.49	.50	.49	.48	50	.48
			1.2	.54	.53	.52	.53	.52	51	52	.50
			1.5	.57	.56	.54	.56	55	54	.54	53
			2.0	.60	.59	.57	.59	.58	.57	57	.56
			2 5	.63	.61	.60	.61	.60	.59	59	58
			3.0	.64	.63	.61	.63	.61	.60	60	59
			4.0	65	.64	.63	.64	.62	.61	61	.60
			5.0	.67	.65	64	.64	63	.62	.62	61
.80	.70	.60	0 6	.41	.39	.37	40	.39	37	40	37
			0.8	.49	.47	.47	48	.47	.46	.47	44
			1.0	.52	.52	.51	.52	.51	.50	.51	50
			1.2	.55	54	.54	.54	.54	.52	.54	52
			1.5	59	.57	.56	.57	.56	55	.56	54
			2 0	61	60	.59	.60	.60	.58	.59	.57
			2.5	.65	.63	.62	.63	.62	61	.61	60
			3.0	.66	65	.63	.64	.63	.62	62	61
			4.0	.67	65	.65	.65	64	.63	63	62
			5 0	69	67	65	66	65	64	.64	63
Calculation Data—Enclosing Units											
.80	.75	.65	0 6	.22	.17	14	20	16	13	14	.12
			0 8	.27	.22	.19	.25	.21	18	19	17
			1.0	31	.26	.23	.28	.24	21	22	19
			1.2	.35	.30	.26	31	27	24	.25	22
			1.5	.38	.33	.29	.34	.30	27	.27	.24
			2.0	.42	38	.33	.38	.34	.31	.31	.28
			2.5	.46	.41	.37	.41	.37	34	34	.31
			3.0	.49	45	.40	.43	.39	.36	.36	.33
			4.0	.53	.48	.44	.47	43	.40	.38	.36
			5.0	.55	.51	.47	.49	.45	.42	40	.38
.80	.70	.60	0.6	.28	.22	.18	.26	.21	.17	.19	.16
			0 8	.35	.29	25	.33	.28	.24	.26	.23
			1 0	.38	.33	.29	.36	.32	.28	.30	.27
			1.2	.43	.37	.33	.40	.35	.31	.33	30
			1.5	.46	.41	.36	.43	.38	.34	.35	.33
			2.0	.51	.46	.42	.47	.43	.40	.40	.38
			2.5	.55	.51	.46	.51	.47	.44	.44	.42
			3.0	.58	.54	.50	.54	.50	.47	.46	.44
			4.0	62	.58	.55	.57	.54	.51	50	.48
			5 0	65	.61	57	.60	56	.53	52	.50

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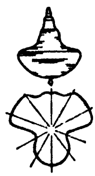
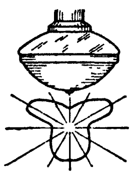
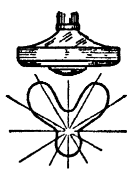
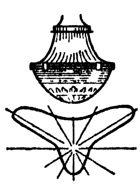
Table VI* (Continued). A Guide to the Selection

Lighting unit		Efficiency based upon: Illumination on horizontal	Efficiency based upon: Illumination on vertical	Appearance of lighted room	Direct glare	Reflected glare	Shadows	Maintenance
Semi-Indirect and Indirect Lighting Units								
9 Enclosed Semi-Indirect Enameled Bottom Etched Top 90° to 180°—50% 0° to 90°—27%		B− Very fair	B− Very fair	A Ex-cel-lent	A− Very good	B+ Very good	A− Very good	B Good
10 Enclosed Semi-Indirect Enameled Bottom Etched Top 90° to 180°—48% 0° to 90°—32%		B Good	B− Very fair	A Ex-cel-lent	A− Very good	B+ Very good	A− Very good	B Good
11 Enclosed Semi-Indirect Cased-Glass Bottom Etched Top 90° to 180°—51% 0° to 90°—21%		C+ Very fair	C+ Very fair	A Ex-cel-lent	A Ex-cel-lent	A Ex-cel-lent	A Ex-cel-lent	B− Very fair
12 Enclosed Semi-Indirect Prismatic Glass 90° to 180°—69% 0° to 90°—17%		B− Very fair	C+ Very fair	A Ex-cel-lent	A Ex-cel-lent	A Ex-cel-lent	A Ex-cel-lent	B Good

* From Westinghouse

of Reflecting Equipment and Coefficients of Utilization

Depreciation Factor: Clean condition	Depreciation Factor: Average condition	Depreciation Factor: Dirty condition	Ceiling / Walls / Room index	Very light (70%): Fairly light (50%)	Very light (70%): Fairly dark (30%)	Very light (70%): Very dark (10%)	Fairly light (50%): Fairly light (50%)	Fairly light (50%): Fairly dark (30%)	Fairly light (50%): Very dark (10%)	Fairly dark (30%): Fairly dark (30%)	Fairly dark (30%): Very dark (10%)
				Coefficients of Utilization							
Calculation Data—Semi-Indirect and Indirect Units											
.75	.70	.	0 6	17	13	10	14	11	09	08	.07
			0.8	21	.17	14	18	14	12	.12	10
			1 0	24	.20	17	21	.17	15	.14	.12
			1 2	28	23	20	.23	.19	17	.16	.14
			1 5	31	.26	.23	.26	22	19	.18	.16
			2.0	35	30	27	29	25	22	20	.18
			2 5	38	34	.30	32	.28	.25	23	20
			3 0	41	37	33	.34	30	.27	.25	.22
			4 0	.45	41	37	37	34	31	27	.25
			5 0	47	43	40	39	36	33	29	27
.75	.70		0.6	19	14	.11	16	12	10	.10	.08
			0 8	24	19	16	21	16	14	.14	.12
			1 0	27	22	19	23	.19	17	.16	.14
			1 2	30	25	22	.26	.22	19	18	.16
			1 5	34	29	25	29	25	22	.20	.18
			2 0	38	33	29	32	28	.25	23	21
			2 5	41	37	33	.35	32	28	26	.24
			3 0	44	40	36	38	34	31	28	.26
			4 0	.49	44	40	41	37	35	31	29
			5.0	51	47	.43	43	39	37	33	.31
.75	.65		0 6	16	12	10	13	.10	.08	08	07
			0 8	20	16	14	17	14	11	.11	.09
			1 0	23	19	17	19	16	14	.13	11
			1 2	.26	22	19	22	18	16	.14	.13
			1 5	29	.25	21	24	20	19	.16	.14
			2 0	32	28	25	27	23	21	.18	.17
			2 5	35	31	28	29	26	24	.20	19
			3 0	.38	.34	.31	31	28	.26	.22	.21
			4 0	.41	.38	35	34	.31	.29	.24	.23
			5 0	43	39	37	36	33	31	.26	24
.75	.70		0 6	18	14	11	.14	11	.09	07	.06
			0 8	22	18	15	18	14	.12	10	.09
			1 0	.25	21	18	20	17	.14	.12	10
			1.2	29	24	21	.23	19	.17	.14	.12
			1.5	.33	28	24	26	22	.19	.16	.14
			2 0	36	32	.28	29	.25	.22	.18	.16
			2.5	.40	35	.32	.31	.28	25	.20	.18
			3 0	.43	.38	35	33	.30	.27	22	.20
			4 0	.47	.43	.39	37	.34	.31	.24	.23
			5.0	49	45	.42	39	36	.33	.26	.24

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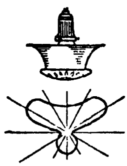
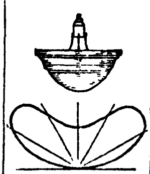
Table VI* (Continued). A Guide to the Selection

Lighting Unit		Efficiency based upon: Illumination on horizontal	Efficiency based upon: Illumination on vertical	Appearance of lighted room	Direct glare	Reflected glare	Shadows	Maintenance
Semi-Indirect and Indirect Lighting Units (continued)								
13 Open Semi-Indirect Enameled Deflector Dense Glass Bottom Plate 90° to 180°—54% 0° to 90°—18%		B− Very fair	C Fair	A Ex-cel-lent	A− Very good	B+ Very good	A− Very good	C Fair
14 Open Indirect 90° to 180°—80% 0° to 90°—0%		C+ Very fair	C Fair	B+ Very good	A+ Ex-cel-lent	A+ Ex-cel-lent	A+ Ex-cel-lent	C Fair

* From Westinghouse

of Reflecting Equipment and Coefficients of Utilization

Depreciation Factor: Clean condition	Depreciation Factor: Average condition	Depreciation Factor: Dirty condition	Ceiling	Very light (70%)			Fairly light (50%)			Fairly dark (30%)	
			Walls	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly light (50%)	Fairly dark (30%)	Very dark (10%)	Fairly dark (30%)	Very dark (10%)
			Room index	Coefficients of utilization							
Calculation Data—Semi-Indirect and Indirect Units (continued)											
70	60		0.6	.18	15	.13	15	12	.10	10	.08
			0 8	.22	19	17	.19	16	.14	.13	.11
			1.0	.25	.22	20	.21	18	.16	.15	.13
			1 2	.28	25	22	.24	21	.19	.16	.15
			1 5	.31	27	24	.26	22	.21	.17	.16
			2 0	.34	31	.28	28	25	.24	.20	.19
			2.5	.37	.34	.32	.30	28	26	22	.21
			3 0	.39	.36	34	32	29	.28	.23	.22
			4 0	43	.40	37	.34	32	.31	.25	.24
			5 0	44	41	40	.36	34	32	.26	.25
.70	60	.	0 6	.15	.12	10	.11	.09	07	.05	.04
			0 8	18	.15	.13	.13	.11	09	.07	.06
			1 0	22	19	.16	.15	.13	.11	.08	.07
			1 2	.25	.22	19	18	.15	.13	09	.08
			1.5	.27	.24	.21	20	17	15	.10	.09
			2 0	.30	.27	.25	.22	.19	.17	.11	.10
			2 5	.34	31	.28	.24	.22	.20	.13	.12
			3 0	.36	.33	.30	.26	.24	.22	.14	.13
			4 0	.40	.37	.34	.28	26	.24	15	.14
			5 0	42	39	37	30	28	26	17	.15

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Table VII. Reflection Factors for Typical Walls and Ceilings

Surface	Class	Color	Reflection factor
Paint	Very light	White	0.81
		Ivory	0.79
		Cream	0.74
Paint	Fairly light	Buff	0.63
		Light green	0.63
		Light gray	0.58
		Light blue	0.58
Paint	Fairly dark	Tan	0.48
		Dark gray	0.26
		Olive green	0.17
Wood	Fairly dark	Light oak	0.32
		Dark oak	0.13
		Mahogany	0.08
Cement	Fairly dark	Natural	0.25
Brick		Red	0.13

Table VIII. Present Standards of Illumination for Industrial Interiors *

	Lumens/ft² recommended	
	Good practice	Minimum
Aisles, Stairways, Passageways	3	2
Assembling:		
Rough	8	5
Medium	12	8
Fine	20	12
Extra fine	50–100	25
Automobile Manufacturing:		
Automatic screw machines	15	10
Assembly line	15	10
Frame assembly	12	8
Tool-making	20	12
Body Manufacturing—Assembly, finishing and inspecting	50–100	25
Bakeries	12	8
Book-binding:		
Folding, assembling, pasting, etc	8	5
Cutting, punching and stitching	12	8
Embossing	15	10
Candy-making	12	8
Canning and Preserving	12	8
Chemical Works:		
Hand furnaces, boiling tanks, stationary driers, stationary or gravity crystallizing	5	3
Mechanical furnaces, generators and stills, mechanical driers, evaporators, filtration, mechanical crystallizing, bleaching	6	4
Tanks for cooking, extractors, percolators, nitrators, electrolytic cells	10	6
Clay Products and Cements:		
Grinding, filter presses, kiln rooms	5	3
Molding, pressing, cleaning and trimming	8	5
Enameling	10	6
Color and glazing	15	10
Cloth Products:		
Cutting, inspecting, sewing—Light goods	15	10
Dark goods	50–100	25
Pressing, cloth treating (oilcloth, etc.)—Light goods	12	8
Dark goods	20	12
Coal Breaking and Washing, Screening	5	3
Construction—Indoor general	5	3
Dairy Products	12	8
Electric Manufacturing:		
Storage battery, molding of grids, charging room	10	6
Coil and armature winding, mica working, insulating processes	20	12
Elevator—Freight and Passenger	8	5
Engraving	50–100	25
Forge Shops and Welding	10	6

* From Westinghouse Lamp Co. Bulletin E-108.

Table VIII (Continued). Present Standards of Illumination for Industrial Interiors *

	Lumens/ft² recommended	
	Good practice	Minimum
Foundries:		
Charging floor, tumbling, cleaning, pouring and shaking out	8	5
Rough molding and core-making	10	6
Fine molding and core-making	15	10
Garage—Automobiles:		
Storage—Dead	3	2
Live	8	5
Repair department and washing	15	10
Glass Works:		
Mix and furnace-rooms, pressing and Lehr, glass-blowing machines	10	6
Grinding, cutting glass to size, silvering	12	8
Fine grinding, etching and decorating	15	10
Glass cutting (cut glass)	25–50	15
Fine polishing and inspecting	25–50	15
Glove Manufacturing:		
Light Goods—Cutting, pressing, knitting	12	8
Sorting, stitching, trimming and inspecting	15	10
Dark Goods—Cutting, pressing, knitting	15	10
Sorting stitching, trimming and inspecting	50–100	25
Hat Manufacturing:		
Dyeing, stiffening, braiding, cleaning and refining—		
Light	10	6
Dark	15	10
Forming, sizing, pouncing, flanging, finishing, ironing—		
Light	12	8
Dark	15	10
Sewing—Light	15	10
Dark	50–100	25
Ice-Making:	10	6
Engine and compressor room	10	6
Inspecting:		
Rough	10	6
Medium	15	10
Fine	25	15
Extra fine	50–100	25
Glossy or polished surfaces	Special lighting	glinting
Jewelry and Watch Manufacturing	50–100	25
Laundries and Dry Cleaning	12	8
Leather Manufacturing:		
Vats	5	3
Cleaning, tanning and stretching	6	4
Cutting, fleshing and stuffing	10	6
Finishing and scarfing	15	10

* From Westinghouse Lamp Co. Bulletin E-108.

Table VIII (Continued). Present Standards of Illumination for Industrial Interiors *

	Lumens/ft² recommended	
	Good practice	Mini-mum
Leather Working:		
Pressing, winding and glazing—Light	12	8
Dark	15	10
Grading, matching, cutting, scarfing, sewing—Light	15	10
Dark	50-100	25
Locker Rooms	6	4
Machine Shops:		
Rough bench and machine work	10	6
Medium bench and machine work, ordinary automatic machines, rough grinding, medium buffing and polishing	15	10
Fine bench and machine work, fine automatic machines, medium grinding, fine buffing and polishing	20	12
Extra fine bench and machine work, grinding (fine work)	50-100	25
Meat Packing:		
Slaughtering	8	5
Cleaning, cutting, cooking, grinding, canning, packing	12	8
Milling—Grain Foods:	-	
Cleaning, grinding and rolling	8	5
Baking or roasting	12	8
Flour grading	25	15
Offices:		
Private and general—Close work	15	10
No close work	10	8
Drafting-room	25	15
Packing:		
Crating	6	4
Boxing	10	6
Paint Manufacturing	10	6
Paint Shops:		
Dipping, spraying, firing	8	5
Rubbing, ordinary hand-painting and finishing	12	8
Fine hand-painting and finishing	15	10
Extra fine hand painting and finishing (automobile bodies, piano cases, etc)	50-100	25
Paper Box Manufacturing:		
Light	10	6
Dark	12	8
Storage of stock	5	3
Paper Manufacturing:		
Beaters, machine grinding	6	4
Calendering	10	6
Finishing, cutting and trimming	12	8
Plating	8	5
Polishing and Burnishing	12	8

* From Westinghouse Lamp Co. Bulletin E-108.

Table VIII (Continued). Present Standards of Illumination for Industrial Interiors *

	Lumens/ft² recommended	
	Good practice	Minimum
Power Plants, Engine Rooms:		
Boilers, coal and ash handling, storage, battery rooms. . .	5	3
Auxiliary equipment, oil switches and transformers .	8	5
Switchboard, engines, generators, blowers, compressors. .	10	6
Printing Industries.		
Matrixing, and casting, miscellaneous machines, presses	12	8
Proof-reading, lithographing, electrotyping. .	15	10
Linotype, monotype, typesetting, imposing stone, engraving. . .	50-100	25
Receiving and Shipping	6	4
Rubber Manufacturing and Products:		
Calenders, compounding mills, fabric preparation, stock-cutting, tubing-machines, solid tire operations, mechanical goods, building, vulcanizing	12	8
Bead building, pneumatic tire building and finishing, inner-tube operation, mechanical goods trimming, treading	15	10
Sheet Metal Works:		
Miscellaneous machines, ordinary bench work. . . .	12	8
Punches, presses, shears, stamps, welders, spinning, fine bench work	15	10
Tin plate inspection .	25	15
Shoe Manufacturing:		
Hand turning, miscellaneous bench and machine work	12	8
Inspecting and sorting raw material, cutting, lasting and welting (light) .	15	10
Inspecting and sorting raw material, cutting, stitching (dark).	50-100	25
Soap Manufacturing:		
Kettle-houses, cutting, soap-chip and powder	8	5
Stamping, wrapping and packing, filling and packing soap-powder. .	10	6
Steel and Iron Mills, Bar, Sheet and Wire Products:		
Soaking pits and reheating furnaces.	3	2
Charging and casting floors	6	4
Muck and heavy rolling, shearing, rough by gauge, pickling and cleaning	8	5
Plate inspection, chipping. .	25	15
Automatic machines, red, light and cold rolling, wire-drawing, sheering, fine by line	12	8
Stone Crushing and Screening:		
Belt conveyor tubes, main line shafting, spaces, chute rooms, inside of bins	3	2
Primary breaker room, auxiliary breakers under bins. .	5	3
Screen rooms.	8	5

* From Westinghouse Lamp Co. Bulletin E-108.

Table VIII (Continued). Present Standards of Illumination for Industrial Interiors *

	Lumens/ft² recommended	
	Good practice	Mini-mum
Store and Stock Rooms:		
Rough bulky material	3	2
Medium or fine material requiring care	8	5
Structural Steel Fabrication	10	6
Sugar Grading	25	15
Testing:		
Rouhg	8	5
Fine	15	10
Extra fine instruments, scales, etc	50–100	25
Textile Mills:		
Cotton—Opening and lapping, carding, drawing-frame, roving, dyeing	8	5
Spooling, spinning, drawing-in, warping, weaving, quilling, inspecting, knitting, slashing (over beam end)	12	8
Silk—Winding, throwing, dyeing	12	8
Quilling, warping, weaving and finishing—Light goods	15	10
Dark goods	20	15
Woolen—Carding, picking, washing and combing	6	4
Twisting and dyeing	10	6
Drawing-in, warping—Light goods	10	6
Dark goods	15	10
Weaving—Light goods	12	8
Dark goods	20	12
Knitting machines	15	10
Tobacco Products:		
Drying, stripping, general	3	2
Grading and sorting	25	15
Toilet and Wash-rooms	6	4
Upholstering:		
Automobile, coach and furniture	15	10
Warehouse	3	2
Woodworking:		
Rough sawing and bench work	8	5
Sizing, planing, rough sanding, medium machine and bench work, gluing, veneering, cooperage	12	8
Fine bench and machine working, fine sanding and finish.	15	10

* From Westinghouse Lamp Co. Bulletin E-108.

Table IX. Present Standards of Illumination for Commercial Interiors *

	Lumens/ft² recommended: Good practice	Lumens/ft² recommended: Minimum
Armories:		
Drill sheds	10	6
Exhibition halls	12	8
Art Galleries:		
General	5	3
On paintings	25–100	10
Auditoriums	5	3
Automobile Show Rooms	15	10
Bank:		
Lobby	10	6
Cages and offices	15	10
Barber Shop	15	10
Base-Ball—Indoor Game	15	10
Basket-Ball	15	10
Bowling:		
On alley, runway and seats	8	5
On Pins	25	15
Billiards—General	6	4
On table	25	15
Cars:		
Baggage, day coach, dining, Pullman	8	5
Mail—Bag racks	12	8
Letter cases	15	10
Storage	6	4
Street railway and subway	10	6
Churches:		
Auditorium	3	2
Sunday-school room	8	5
Pulpit or rostrum	12	8
Art glass windows	25–50	15
Club-Rooms:		
Lounge	5	3
Reading-room	12	8
Court-Rooms	10	6
Dance-Halls	6	4
Dental Offices:		
Waiting-room	6	4
Operating office	12	8
Dental chair	50	25
Depot Waiting-Room	8	5
Drafting-Room	25	15
Elevator—Freight and passenger	6	4
Fire Engine House:		
When alarm is turned in	8	5
At other times	3	2
Garage—Automobiles:		
Storage—Dead	3	2
Live	8	5
Repair department and washing	15	10

* From Westinghouse Lamp Co. Bulletin E-108.

Table IX (Continued). Present Standards of Illumination for Commercial Interiors *

	Lumens/ft² recommended Good practice	Minimum
Gymnasiums:		
Main exercising floor.	12	8
Swimming-pool	8	5
Shower-rooms	6	4
Locker-rooms	6	4
Fencing, boxing, wrestling	12	8
Halls, Passageways in Interiors.	3	2
Handball	25	15
Hospitals:		
Lobby and reception room	6	4
Corridors	3	2
Wards (with local illumination).	5	3
Private rooms	8	5
Night illumination	0.2	0.1
Operating-table	100–200	75
Operating-room	15	10
Laboratories	15	10
Hotels:		
Lobby	8	5
Dining-room	6	4
Kitchen	10	6
Bed-rooms	8	5
Corridors	3	2
Writing-room	12	8
Library:		
Reading-rooms	12	8
Stack-room	6	4
Lodge-Rooms	6	4
Lunch-Room.	12	8
Market	12	8
Moving-Picture Theater:		
During intermission	5	3
During pictures		0.1
Museum:		
General	8	5
Special exhibits	25–100	10
Office-Buildings:		
Private and general offices—Close work	15	10
No close work	10	8
File-room	6	4
Vault.	6	4
Reception room	6	4
Post Office:		
Lobby	10	6
Work room—Sorting, mailing, etc.	15	10
Storage	10	6
Private and general offices	15	10
File-room and vault	6	4
Corridors and stairways	3	2

* From Westinghouse Lamp Co. Bulletin E-108.

Table IX (Continued). Present Standards of Illumination for Commercial Interiors*

	Lumens/ft² recommended	
	Good practice	Minimum
Railway:		
Depot—waiting-room	8	5
Ticket offices	12	8
Rest room—smoking room	8	5
Baggage room—Checking office	12	8
Storage	6	4
Concourse	6	4
Train platform	4	2
Restaurants	8	5
Racquet	25	15
Schools:		
Auditorium	8	5
Class-rooms, library and office	12	8
Corridors and stairways	5	3
Drawing	25	15
Laboratories	12	8
Manual training	12	8
Sewing rooms	25	15
Study room—Desks	12	8
Blackboards	12	8
Show Cases	Two to four times that of the store proper	
Show Windows:		
Large cities—Brightly lighted districts	150	100
Secondary business locations	75	50
Neighborhood stores	50	30
Medium cities—Brightly lighted districts	75	50
Neighborhood stores	50	30
Small cities and towns	50	30
Lighting to reduce daylight window reflections	200–1 000	..
Skating-Rink (Indoor)	8	5
Squash	25	15
Stores:		
Large specialty and department stores—		
Main floors	15	10
Other floors	12	8
Basement store	15	10
Small stores—		
Art	12	8
Automobile supply	12	8
Bake-shop	12	8
Book	12	8
China	12	8
Cigar	15	10
Clothing	15	10
Confectionery	12	8

* From Westinghouse Lamp Co. Bulletin E-108.

Table IX (Continued). Present Standards of Illumination for Commercial Interiors *

	Lumens/ft² recommended	
	Good practice	Minimum
Stores (Continued):		
Small stores (Continued):		
Dairy products	12	8
Decorator	12	8
Drug	15	10
Dry goods	15	10
Electrical supply	15	10
Florist	12	8
Furrier	15	10
Grocery	12	8
Haberdashery	15	10
Hardware	12	8
Hat	15	10
Jewelry	15	10
Leather, handbags and trunks	12	8
Meat	12	8
Millinery	15	10
Music	12	8
Notions	12	8
Piano	12	8
Shoe	15	10
Sporting goods	12	8
Tailor	15	10
Tobacco	15	10
Variety store	15	10
Telephone Exchange:		
Operating-rooms	8	5
Terminal rooms	12	8
Cable Vaults	6	4
Tennis (Indoor)	25–50	15
Theaters:		
Auditorium	3	2
Foyer	8	5
Lobby	12	8
Toilet and Wash-rooms	6	4

* From Westinghouse Lamp Co. Bulletin E-108.

A point often overlooked in this connection is that even with the highest values now in use the illumination is much less than what we are accustomed to get out of doors in the shade during the day, and is even considerably less than we get indoors during the day. The former value is of the order of 1 000 lumens/ft², and the latter may be from about 20 to 100 lumens/ft² under ordinary conditions. Thus even in a good modern lighting installation the illumination is below what we are accustomed to in the daytime.

(2) The second step is to decide upon the TYPE OF LUMINAIRE. This is entirely a matter of judgment. Such factors as esthetics, convenience, freedom from glare, cost, ease of cleaning, etc., should enter. To aid in the selec-

tion of the best type for the purpose, the left side of Table VI may be found useful.

Three General Systems of Lighting. Three systems of lighting are available, each having its own types of luminaires.

(a) Direct Lighting. A system is designated as DIRECT when more than one-half the light reaches the area to be illuminated by coming directly from the light-source, without being reflected from the ceiling or walls. This includes all systems using lamps with clear, frosted, translucent, or opalescent globes, or reflectors, in which the light is reflected downward. It is the most efficient system, was the first to be used, and is still the most common. The color of the walls or ceiling has less effect in this system than in the others.

(b) Indirect Lighting. A system is designated INDIRECT when all the light is thrown first on the ceiling and walls, and reflected from these to the surface to be illuminated. Any system which conceals the source of light by opaque reflectors is thus INDIRECT. Light finish must always be used on the walls and ceiling with this system. Even then, the efficiency is usually lower than that of a direct system, but the total absence of glare and shadows and the even distribution of light make this a popular scheme in restaurants, show-rooms, etc., where decorative lighting is desired.

(c) Semi-indirect Lighting. This system throws most of the light to the walls and ceiling, but allows a small percentage to be diffused through the reflector straight to the area to be illuminated. This system is rapidly coming

Table X. Spacing of Outlets *

Ceiling height (or height in the clear)	Spacing between outlets		Spacing between outside outlets and wall		Approximate area per outlet (at usual spacings)
	Usual	Maximum (for units at ceiling)	Aisles or storage next to wall	Desks, work-benches, etc., against wall	
ft	ft	Not more than †		Not more than †	sq ft
8	7	7½		3	50–60
9	8	8		3	60–70
10	9	9		3½	70–85
11	10	10½		3½	85–100
12	10–12	12	Usually one-half actual spacing between units	3½–4	100–150
13	10–12	13		3½–4½	100–150
14	10–13	15		4–5	100–170
15	10–13	17		4–5	100–170
16	10–13	19		4–6	100–170
18	10–20	21		4–6	100–400
20	18–24	24		5–7	300–500
22	20–25	27		5–7	400–600
24	20–30	30		6–8	400–900
26	25–30	33		8–9	600–900
30 and up	25–30	40		8–10	600–900

* From Westinghouse Lamp Co. Bulletin E-108.

† Where it is definitely known that some form of indirect lighting will be used, the maximum spacing between outlets may be increased about 2 feet, and the distance from the outside outlets to the wall may be increased by 1 ft.

into favor because apparently we have become accustomed to looking for the source of light and miss it when it is concealed as in the indirect system. The totally indirect fixtures often show up rather unpleasantly as a dark spot against a light background. This is avoided in the semi-indirect system. The slightly higher efficiency of this system is another advantage over the indirect. Any given room may usually be lighted by any one of the three systems, although it is generally true that conditions are such as to make one of the three more desirable than either of the other two.

Table XI. Mounting Height of Lighting Units

Direct lighting units				Semi-indirect and indirect lighting	
Actual spacing between units	Distance of units from floor not less than	Desirable mounting height in industrial interiors	Desirable mounting height in commercial interiors	Actual spacing between units	Recommended suspension length (top of bowl to ceiling)
ft	ft			ft	ft
7	8	12 ft above floor if possible—to avoid glare, and still be within reach from step-ladder for cleaning		7	1–3
8	8½			8	1–3
9	9			9	1–3
10	10		The actual hanging height should be governed largely by general appearance, but particularly in offices and drafting-rooms, the minimum values shown in column 2 should not be violated.	10	1½–3
11	10½			11	2–3
12	11			12	2–3
14	12½			14	2½–4
16	14			16	3–4
18	15	Where units are to be mounted much more than 12 ft it is usually desirable to mount the units at ceiling or on roof-trusses.		18	3–4
20	16			20	4–5
22	18			22	4–5
24	20			24	4–6
26	21			26	4–6
28	22			28	5–7
30	24			30	5–7

(3) The LOCATION OF OUTLETS is the next step. Table X will be found useful here. The object is to put the lights close enough together so that the illumination is uniform. If the luminaires are too far apart the illumination directly beneath them may be good but the values between will be poor. A general rule is to use a spacing between outlets approximately equal to the height of unit above the floor. The spacing between an outlet and the wall is usually made about half the spacing between two outlets.

A very common mistake in direct lighting is to use a mounting height which is too low. The use of a high mounting height (even higher perhaps than the recommended values of Table XI) has the following advantages:

(a) Raising the luminaire causes little change in the average illumination but makes the lighting more uniform.

(b) A greater mounting height allows greater spacing between units which reduces the cost of wiring.

(c) The use of a smaller number of luminaires of large size allows the use of larger lamps which are more efficient.

For indirect or semi-indirect lighting, most of the light comes from the ceiling, and the mounting height of the units has very little effect on the uniformity of illumination.

(4) Knowing the size of room and the finish of walls and ceiling, we next obtain the room index (Table V) and the coefficient of utilization (Table VI). The TOTAL LUMINOUS OUTPUT of the lamps must then be

$$F = \frac{A \times E}{k_u \times k_d} \tag{4}$$

in which A = floor area;
E = average illumination desired (Table VIII);
k_u = coefficient of utilization (Table VI),
k_d = depreciation factor (Table VI).

(5) The luminous output of each lamp is found by dividing F by the number of lamps. The nearest LAMP SIZE is then selected from Table IV. This completes the design of the lighting system for the room.

The Foregoing Rules Indicate the General Practice in planning the illumination of a room. It must be said, however, that this set of rules must not be followed too slavishly. In illumination no rules can take the place of judgment and intelligence. Each project must be considered more or less as a problem by itself, for which previous experience and former installations should be made to furnish data and to suggest methods. It is well, therefore, when planning the illumination of a room, to visit as many similar rooms as possible, note the effect of the systems in use and obtain data as to their efficiency, cost, etc. The most successful scheme may then be used as the basis for planning the desired installation.

Example. To design the lighting system for a drafting-room 25 × 63 ft with a 13-ft ceiling.

(1) According to Table IX, good practice is to use 25 lumens/ft² for drafting-rooms.

(2) It is necessary that the light be unusually well diffused so that annoying shadows will not be cast on the work. For this reason we shall use a totally indirect system. The unit decided upon is No. 14 (Table VI).

(3) Since the ceiling height is 13 ft, a spacing between units of 13 ft is permissible (Table X). Using 12½ ft spacing and 6½ ft between outlets and walls, we get 10 outlets in 2 rows of 5 each.

(4) The room index, according to Table V, is 2.5. The walls and ceiling must be light to get good results with an indirect system. Using a very light ceiling and fairly light walls, we obtain 0.34 for the coefficient of utilization and 0.60 for the average depreciation factor (Table VI).

Thus the total luminous output of the lamps must be by Equation (4):

$$F = \frac{25 \times 63 \times 25}{0.34 \times 0.60} = 193\,000 \text{ lumens}$$

(5) Since there are 10 lamps, the luminous output of each one must be 19 300 lumens. The 1 000-watt lamp gives 20 400 lumens (Table IV), which should be satisfactory. Thus we use 10 indirect units with 1 000-watt lamps and obtain slightly over 25 lumens/ft² of very even and well-diffused light.

A Few Modern Designs. With modern high levels of illumination, direct lighting of the ordinary type is very likely to produce glare as well as harsh shadows and uneven illumination. These defects can be eliminated by using

DIFFUSING GLASS of sufficient area in front of the lamps. Many modern installations of the highest class use large WALL-PANELS or SKYLIGHTS of etched glass, or other diffusing glass, and obtain a very pleasing effect and absence of glare.

COVE LIGHTING is also frequently used to supply a major part of the light This can be supplemented if desired by ornamental luminaires hung from the ceiling or by torchiers on the walls, the purpose of these being to add a touch of color or to give variety to the lighting rather than to add materially to the level of illumination. In this way all luminous surfaces are of low brightness and all traces of glare are banished.

An example of the latter method is the FUR SALON of L. BAMBERGER & COMPANY, Newark, N. J. Here cove lighting is used, 25-watt daylight lamps being placed on 9-in centers and concealed behind a serrated molding near the ceiling. Modernistic etched-glass wall-panels and luminaires by Siegel of Paris give a decorative effect. A total of 11 000 watts of daylight lamps produces an average illumination of 5 lumens/ft². The room is 27 × 63 ft.

THE QUOTATION-BOARD ROOM OF THE SAN FRANCISCO STOCK EXCHANGE is another example of thoroughly modern treatment. This room is 122 ft long, 68 ft wide, and has a 40-ft ceiling. There are no pendant fixtures, but in the center of the ceiling there is a panel 109 × 39 ft made up of metal and hammered glass. Above this panel are fifty-five 750-watt lamps and twenty 300-watt lamps in prismatic glass reflectors. The resulting illumination in the room is 30 lumens/ft², and is very well diffused The board itself is illuminated to 75 lumens/ft² by a special trough containing 150-watt lamps on 8½-in centers.

Daylighting of Interiors

Until recently the problem of daylight illumination of buildings received but little attention. Too often windows were placed purely by guess, and the resulting illumination was highly uneven and inadequate. Mistakes of this type are difficult to remedy.

Within the last few years, however, considerable research has been made on the calculation of daylighting and on the best size and location of windows. The method has been used recently in the design of new buildings for General Motors, the General Electric Company, Western Electric Company, and others, and has resulted in a marked improvement in the quality and quantity of illumination.

Case I. Window Perpendicular to Working-Plane. It has been shown by Higbie * that a window of uniform brightness of B candles/ft² will produce an illumination at a point P of

$$E = \frac{B}{2}\left[\tan^{-1}\frac{m/f}{\cot\gamma} - \frac{\cot\gamma}{\sqrt{\cot^2\gamma+1}}\tan^{-1}\frac{m/f}{\sqrt{\cot^2\gamma+1}}\right] \quad \text{(lumens/ft}^2\text{)}$$

in which m = width of window (feet);
f = height of window (feet);
γ = vertical angle subtended by window.

Values of illumination are given in Table XII for a window brightness of 200 candles/ft². This is a representative value for a dull day. The table applies directly only when the bottom of the window is on the level of the working-plane and the point P is on a perpendicular to the window as shown in Fig. 4 By a slight modification, however, the table can be applied to any case.

* See Bibliography at end of chapter.

Table XII. Illumination from Rectangular Window

Perpendicular to Working-Plane

Lumens/ft² for uniform window brightness of 200 candles/ft²

γ	m/f = 10	m/f = 3	2	1.5	1.0	0.7
5°	0 61			. ..	..	. .
10	2.33	2 10	1 4	. ..	. .	. .
15	5 36	4 10	3 2	2 70	1.76	1 20
20	9 50	8 0	6 6	5.6	4 2	2.92
25	14 7	13 2	11.6	10.0	7.7	5 6
30	21.0	19 6	18 4	16 4	12.8	9 6
35	28 5	27 6	26 0	23 6	19 6	14 8
40	36 8	36 0	34 2	32.0	27 4	21.6
45	46 2	44 8	43 0	40 6	35 2	28.8
50	56 2	54 4	52.4	50.0	46 0	38 2
55	67 0	65 6	63.0	60.6	57 0	49.0
60	78.4	77 0	75 0	73.0	70.0	61.6
65	90 6	89 6	88 4	86 0	83 0	75 0
70	103 6	102 8	101 8	100.0	96 6	90 0
75	116 6	116 4	116 0	114 0	111 0	106 0
80	129 8	129.4	128 8	127 2	125 2	121 6
90	157 0	157 0	157 0	157 0	157 0	157 0

γ	0.5	0.4	0.3	0.25	0.2	0.1
15°	0 90	. .	.	.	.	.
20	2 2	1 70	1 2		.	.
25	4 2	3 3	2 6	2 0	1 6	.
30	7 2	5 8	4 4	3 6	2.8	1 6
35	11.2	9 2	6 8	5 6	4 4	2.4
40	16 0	13 6	10 2	8 2	6 8	3 6
45	21.4	18 6	14 4	12 0	10 0	5 0
50	30 0	25 2	20 2	16 2	13 0	6.2
55	40 0	33 6	27 0	22 0	18 0	8 8
60	51 0	44 0	35 6	30.0	24 0	12 2
65	64 0	56 8	46.4	40 2	33 0	17 0
70	80 0	72 0	60 8	54 0	44.0	24 0
75	97.0	89 6	78 2	70 0	60 0	34 0
80	114.4	108 2	98 4	90 0	80 2	50 0
90	157 0	157.0	157.0	157 0	157.0	157.0

Case II. Window Parallel to Working-Plane. Here the point P is directly beneath one corner of the window (Fig. 5).

$$E = \frac{B}{2}\left[\frac{m/f}{\sqrt{(c/f)^2 + (m/f)^2}}\tan^{-1}\frac{1}{\sqrt{(c/f)^2 + (m/f)^2}}\right.$$
$$\left. + \frac{1}{\sqrt{(c/f)^2 + 1}}\tan^{-1}\frac{m/f}{\sqrt{(c/f)^2 + 1}}\right] \quad \text{(lumens/ft}^2\text{)}$$

in which E = illumination at P;
B = brightness of window (candles/ft²);
m = larger dimension of window (feet);
f = smaller dimension of window (feet);
c = height of window above working plane (feet).

Table XIII. Illumination from Rectangular Window

Parallel to Working-Plane

Lumens/ft² for uniform window brightness of 200 candles/ft²

c/f	m/f = 10	m/f = 7	5	4	3	2.5	2.0	1.5	1.2	1.0
0 1	156 2	156 2	156.2	156 2	156 2	156 2	156 2	156 0	155 8	155 6
0.2	154 0	154 0	154 0	154.0	154 0	154 0	153 6	153.6	153 0	152.4
0.5	140 0	140.0	140.0	140.0	140 0	140.0	139 8	136.6	133 8	132.2
0.6	134.9	134 9	134.9	134.6	134.3	133 5	132.4	129.7	126 2	121 2
0.8	122.8	122.8	122.8	122.0	121.4	120.0	118.6	112.0	110.0	103 4
1.0	111.0	110 8	110 4	110.0	109.0	107.6	105.0	99.8	93 6	87 0
1.5	87.5	87.0	86.0	84.8	83.4	78.8	73.8	73.4	61 6	56 4
2.0	70.0	69 4	68.4	67.0	65.8	60.9	55 8	49.0	42.9	37 6
3.0	49.4	48.5	46.5	44.3	41.8	37.4	33.0	27.2	23 4	19.5
5.0	28.8	28.0	25.2	22.7	20.4	16 9	14.1	11.3	9.5	7.6
8.0	17 2	15.0	12 5	10.7	8 5	7.3	5 9	4.5	3.67	3 06
10.0	13 3	11 8	8 5	7.1	5.5	4.3	3 8	2.83	2.36	1.97

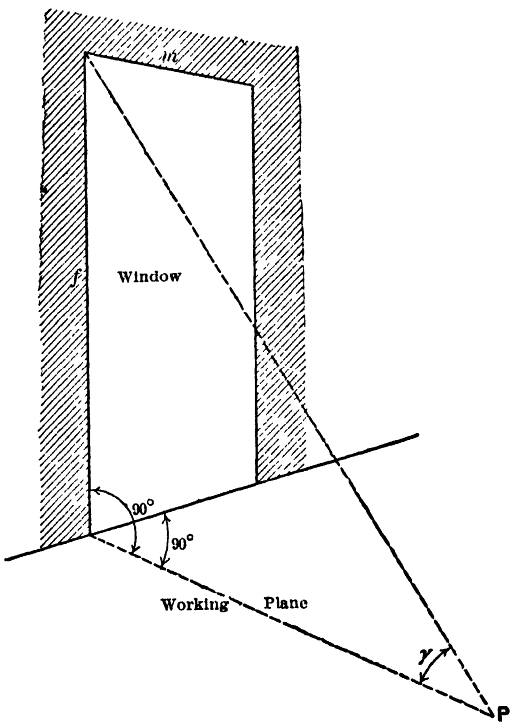


Fig. 4. Illumination from Vertical Window

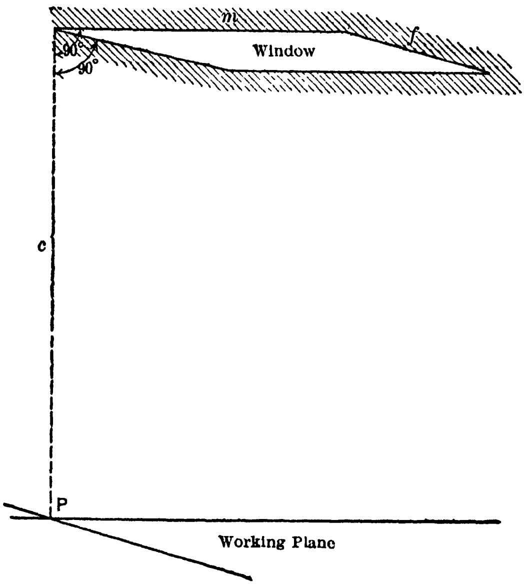


Fig 5 Illumination from Horizontal Window

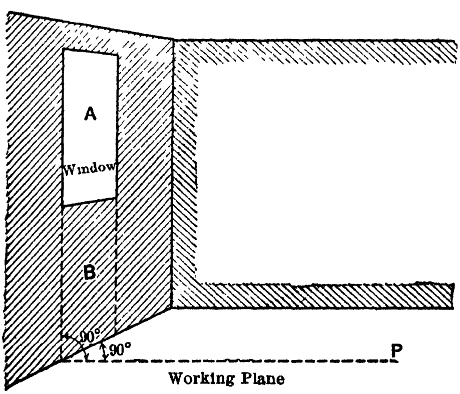


Fig. 6. Illumination from Window A.

Table XIII gives the illumination for any size window of brightness 200 candles/ft².

Other Positions of Windows. Tables XII and XIII apply directly only to the illumination at points on a perpendicular erected at a corner of the window. The question immediately arises as to the illumination at other points. This can be found very easily as shown below.

Consider the illumination at P (Fig. 6) where the bottom of the window A is not on the level of the working-plane. First take the fictitious window AB. Table XII applies, and the illumination is quickly obtained. Next the value for the fictitious window B is determined. The true illumination at P is then the difference of the two, or:

(Illumination at P due to A)

= (Illumination due to AB) − (Illumination due to B).

Similarly for the window AB of Fig. 7:

(Illumination at P due to AB)

= (Illumination due to BD) − (Illumination due to D)

+ (Illumination due to AC) − (Illumination due to C).

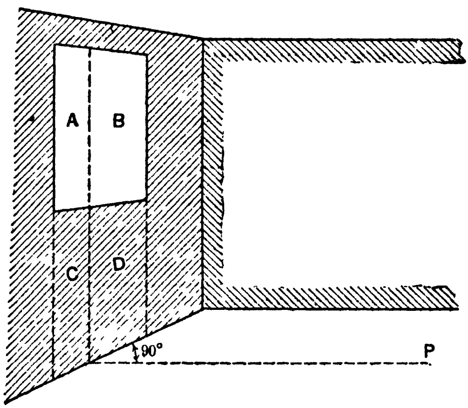


Fig. 7. Illumination from Window AB

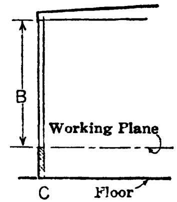
If much work of the kind is to be done, it is advisable to plot a set of curves from Tables XII and XIII. Then from a blue-print of the room, the angle γ can be read by means of a protractor, the values of m and f can be scaled off, and the illumination at any point determined.

Artificial Windows. The method is also applicable to artificial lighting from skylights, modernistic panels, indirectly lighted ceilings, etc. In such cases the values of the table must be multiplied by the ratio of the actual brightness of the source to 200 Thus if the source has a brightness of 50 candles/ft², the values of the table must be multiplied by 50/200 = 0.25.

Example. A room is illuminated by an artificial skylight 8 × 16 ft made of diffusing glass and evenly illuminated from above by incandescent lamps to a brightness of 100 candles/ft². What is the illumination on a desk directly beneath the center of the skylight and 12 ft below it?

Table XIV. Illumination from Vertical Side-Wall Windows *

Feet from plane of window	Window height B — Number of 20-in high panes														
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
5	50.0	59.5	69.5	77.0	83.5	88.5	93.0	96.8	100.0	103.0	105.4	107.6	110.0	111.9	114.0
10	26.2	32.7	40.0	48.5	56.0	63.8	70.0	75.0	80.0	84.0	87.0	90.0	93.0	95.5	99.0
15	15.0	19.5	24.5	31.0	38.0	43.5	48.5	54.0	59.5	66.0	68.5	72.3	76.0	80.0	85.0
20	9.6	12.4	16.0	20.5	26.8	32.0	36.0	41.8	45.0	52.0	55.0	59.0	63.0	66.8	72.0
25	6.6	8.5	11.0	14.5	19.2	23.0	27.5	32.0	36.1	41.0	44.2	47.5	51.5	55.0	60.0
30	4.7	6.3	8.2	11.0	14.5	17.5	21.0	24.7	28.7	33.0	36.3	39.5	42.5	46.2	50.0
35	3.5	4.75	6.3	8.4	11.0	13.5	16.2	19.5	23.0	26.5	29.4	32.0	35.0	38.1	41.5
40	2.72	3.65	4.85	6.5	8.8	11.0	13.0	15.6	18.4	21.2	24.1	26.6	29.0	32.0	34.8
45	2.12	2.9	3.9	5.1	7.0	8.6	10.3	12.7	14.8	17.2	19.2	21.5	24.2	26.5	29.0
50	1.7	2.3	3.15	4.2	5.6	6.8	8.4	10.0	12.0	14.0	16.0	17.9	20.2	22.5	24.8
55	1.41	1.9	2.58	3.44	4.57	5.6	6.9	8.3	9.8	11.5	13.2	15.0	17.0	18.6	20.5
60	1.2	1.58	2.12	2.82	3.75	4.6	5.8	6.8	8.1	9.5	11.1	12.7	14.3	15.9	17.5
65	1.0	1.33	1.79	2.30	3.15	3.9	4.7	5.75	6.8	8.0	9.25	10.6	12.2	13.6	15.0
70	0.86	1.14	1.5	2.0	2.7	3.4	4.15	5.0	5.91	6.9	7.9	9.2	10.1	11.7	13.0
75	0.74	0.98	1.3	1.7	2.33	2.96	3.50	4.3	5.05	5.9	6.9	8.0	9.1	10.1	11.3
80	0.63	0.85	1.12	1.5	2.03	2.55	3.1	3.75	4.4	5.15	6.1	7.0	8.0	8.95	9.9
85	0.56	0.75	1.0	1.31	1.8	2.2	2.7	3.3	3.9	4.52	5.25	6.1	7.0	7.8	8.7
90	0.49	0.67	0.9	1.18	1.58	1.95	2.4	2.9	3.5	4.1	4.85	5.5	6.2	6.9	7.6
95	0.44	0.6	0.8	1.05	1.4	1.7	2.1	2.5	3.05	3.65	4.23	4.9	5.6	6.2	6.8
100	0.4	0.54	0.72	0.94	1.3	1.53	1.9	2.3	2.75	3.3	3.8	4.4	5.0	5.55	6.15
105	0.36	0.49	0.66	0.86	1.2	1.4	1.7	2.1	2.5	3.0	3.5	4.0	4.5	5.05	5.6
110	0.33	0.44	0.6	0.8	1.12	1.3	1.58	1.9	2.25	2.71	3.2	3.7	4.2	4.6	5.07
115	0.3	0.41	0.55	0.74	1.05	1.2	1.48	1.6	2.0	2.5	2.9	3.4	3.75	4.1	4.7
120	0.27	0.38	0.51	0.7	0.99	1.1	1.38	1.5	1.8	2.3	2.6	3.1	3.4	3.8	4.3
125	0.25	0.35	0.48	0.66	0.93	1.05	1.3	1.45	1.7	2.1	2.4	2.8	3.1	3.5	4.0
130	0.23	0.32	0.45	0.63	0.88	1.0	1.23	1.4	1.6	1.96	2.2	2.5	2.82	3.2	3.7
135	0.21	0.3	0.43	0.6	0.84	0.98	1.17	1.35	1.55	1.81	2.0	2.3	2.6	3.0	3.45
140	0.2	0.28	0.4	0.57	0.8	0.96	1.14	1.3	1.5	1.7	1.9	2.12	2.4	2.75	3.2



20-in high glass

Panes		Ft	In
3	=	5	2
4	=	6	10
5	=	8	6
6	=	10	3
7	=	11	11
8	=	13	8
9	=	15	4
10	=	17	1
11	=	18	9
12	=	20	5
13	=	22	2
14	=	23	11
15	=	25	7
16	=	27	4
17	=	29	0

* Taken from Randall & Martin—Trans. I E S **25**, 1930, p 262.

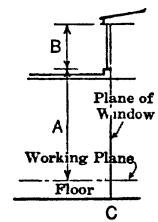
Table XV. Illumination from Vertical Windows *

Feet from plane of window	Vertical Height above Working Plane A: 15 ft 0 in — Window height B				
	3 ft 6 in	5 ft 2 in	6 ft 10 in	8 ft 6 in	10 ft 3 in
5	6.0	8.0	10.2	12.0	13.3
10	11.6	14.8	19.6	22.4	24.0
15	13.2	16.0	21.2	25.6	27.6
20	12.0	15.3	19.6	24.1	27.2
25	10.2	13.6	17.0	21.2	24.4
30	8.6	11.6	14.9	18.4	21.3
35	7.2	9.6	12.7	15.6	18.1
40	6.1	8.0	10.8	13.2	15.6
45	5.0	6.8	8.9	11.0	13.1
50	4.1	5.6	7.4	9.3	11.2
55	3.3	4.8	6.0	7.7	9.5
60	2.5	4.0	5.0	6.5	8.0
65	2.2	3.3	4.2	5.5	6.8
70	1.8	2.8	3.6	4.8	5.9
75	1.5	2.3	3.0	4.1	5.0
80	1.2	2.0	2.6	3.6	4.4
85	1.0	1.7	2.3	3.1	4.0
90	0.9	1.5	2.0	2.7	3.5
95	0.85	1.2	1.8	2.4	3.1
100	0.8	1.1	1.6	2.15	2.8
105	0.7	1.0	1.4	1.8	2.5
110	0.6	0.9	1.1	1.6	2.2
115	0.55	0.79	1.08	1.4	1.95
120	0.5	0.71	0.95	1.25	1.74
125	0.46	0.64	0.85	1.1	1.55
130	0.42	0.58	0.77	1.0	1.4
135	0.4	0.53	0.70	0.9	1.26
140	0.37	0.49	0.64	0.82	1.16

Feet from plane of window	Vertical Height above Working Plane A: 25 ft 0 in — Window height B				
	3 ft 6 in	5 ft 2 in	6 ft 10 in	8 ft 6 in	10 ft 3 in
5	2.3	3.0	3.6	4.2	5.0
10	4.4	6.0	7.1	8.6	9.4
15	5.9	8.8	10.3	13.0	14.2
20	7.0	10.2	12.5	15.5	17.9
25	7.6	10.4	13.2	16.6	19.5
30	7.2	10.0	13.1	16.7	20.0
35	6.8	9.2	12.3	15.5	18.3
40	6.2	8.3	11.0	13.9	16.2
45	5.6	7.4	9.8	12.4	14.5
50	5.1	6.5	8.7	11.0	13.0
55	4.5	5.8	7.7	9.8	11.6
60	4.0	5.1	6.8	8.7	10.3
65	3.5	4.6	6.1	7.7	9.1
70	3.0	4.1	5.4	6.7	8.0
75	2.7	3.7	4.8	6.0	7.0
80	2.4	3.2	4.3	5.3	6.2
85	2.1	2.9	3.8	4.7	5.5
90	1.8	2.5	3.3	4.1	4.9
95	1.6	2.2	2.9	3.6	4.4
100	1.3	1.8	2.4	3.0	3.9
105	1.0	1.5	2.0	2.7	3.5
110	0.8	1.2	1.7	2.3	3.1
115	0.73	1.07	1.46	2.03	2.7
120	0.63	0.91	1.25	1.78	2.4
125	0.54	0.79	1.09	1.52	2.2
130	0.47	0.7	0.94	1.33	1.95
135	0.42	0.61	0.83	1.16	1.71
140	0.38	0.54	0.74	1.02	1.51

Feet from plane of window	Vertical Height above Working Plane A: 35 ft 0 in — Window height B				
	3 ft 6 in	5 ft 2 in	6 ft 10 in	8 ft 6 in	10 ft 3 in
5	1.2	1.7	2.2	2.7	3.2
10	2.5	3.7	4.5	5.7	7.2
15	3.8	5.4	6.6	8.6	10.4
20	4.75	6.6	8.0	10.8	13.2
25	5.45	7.2	9.0	11.8	14.5
30	5.7	7.4	9.2	12.0	15.0
35	5.4	7.1	9.0	11.8	14.9
40	4.9	6.7	8.6	11.5	14.2
45	4.5	6.1	8.0	10.6	13.6
50	4.15	5.6	7.3	9.8	12.8
55	3.8	5.1	6.6	8.9	11.7
60	3.45	4.6	6.0	8.1	10.8
65	3.1	4.2	5.4	7.4	9.6
70	2.8	3.8	4.8	6.7	8.5
75	2.5	3.5	4.35	6.0	7.7
80	2.3	3.15	4.0	5.4	6.9
85	2.12	2.85	3.65	4.8	6.2
90	2.0	2.6	3.35	4.4	5.5
95	1.85	2.4	3.05	4.0	5.0
100	1.7	2.25	2.8	3.65	4.65
105	1.63	2.1	2.6	3.35	4.2
110	1.52	2.0	2.48	3.1	3.95
115	1.45	1.92	2.35	2.9	3.7
120	1.38	1.83	2.25	2.75	3.45
125	1.33	1.76	2.18	2.61	3.21
130	1.28	1.69	2.1	2.5	3.03
135	1.24	1.64	2.04	2.41	2.05
140	1.21	1.6	2.0	2.37	2.72

Feet from plane of window	Vertical Height above Working Plane A: 45 ft 0 in — Window height B				
	3 ft 6 in	5 ft 2 in	6 ft 10 in	8 ft 6 in	10 ft 3 in
5	0.86	1.7	2.8	3.9	5.8
10	2.4	3.4	4.8	6.5	8.3
15	3.4	4.9	6.6	8.1	10.2
20	4.5	6.4	8.0	9.5	11.5
25	5.3	7.6	9.1	10.7	12.5
30	5.7	8.3	9.8	11.5	13.1
35	5.5	7.9	9.7	11.3	12.8
40	5.1	7.4	9.2	10.9	12.3
45	4.7	6.9	8.7	10.4	11.7
50	4.3	6.3	8.0	9.8	11.0
55	4.0	5.7	7.4	9.0	10.5
60	3.7	5.2	6.9	8.3	9.8
65	3.3	4.7	6.3	7.8	9.2
70	3.0	4.3	5.8	7.2	8.6
75	2.7	3.9	5.3	6.7	8.1
80	2.4	3.6	4.8	6.2	7.6
85	2.23	3.3	4.45	5.7	7.1
90	2.05	3.0	4.15	5.3	6.6
95	1.9	2.75	3.9	4.9	6.1
100	1.75	2.5	3.7	4.6	5.7
105	1.60	2.35	3.4	4.3	5.3
110	1.5	2.2	3.15	4.1	4.95
115	1.41	2.05	2.9	3.87	4.6
120	1.33	1.94	2.7	3.65	4.3
125	1.27	1.84	2.5	3.45	4.15
130	1.21	1.73	2.35	3.22	3.86
135	1.17	1.64	2.21	3.05	3.68
140	1.13	1.57	2.10	2.9	3.5



20-in high glass

Panes		Ft	In
2	=	3	6
3	=	5	2
4	=	6	10
5	=	8	6
6	=	10	3

* Taken from Randall & Martin—Trans. I. E. S. **25**, 1930, p. 262.

Table XVI. Illumination from 30° Sloping Windows

Feet from plane of window	Slant height above working plane A: 15 ft 0 in — Window height B: ft 3	ft 6	ft 9	ft 12	25 ft 0 in — Window height B: ft 3	ft 6	ft 9	ft 12	35 ft 0 in — Window height B: ft 3	ft 6	ft 9	ft 12	45 ft 0 in — Window height B: ft 3	ft 6	ft 9	ft 12
5	6.2	10.2	13.0	15.5	2.4	3.9	6.0	8.3	1.8	3.0	4.2	5.8	0.9	1.7	2.3	3.1
10	19.0	33.0	42.5	67.5	6.2	9.3	14.0	18.5	3.4	6.1	8.9	11.7	2.0	4.0	5.3	7.0
15	28.0	55.0	73.5	85.0	10.0	16.5	25.3	29.5	5.0	9.2	13.5	18.0	3.3	6.5	9.0	11.5
20	19.6	40.5	65.0	77.2	13.0	24.8	35.6	40.5	6.6	12.2	18.3	24.0	4.8	9.5	13.0	17.0
25	13.8	27.2	47.5	64.0	15.5	28.5	38.4	46.5	8.1	15.3	22.6	30.0	6.3	12.5	17.8	23.1
30	9.8	19.0	31.0	51.0	14.0	27.0	38.0	47.5	9.6	18.5	26.8	35.5	7.8	15.5	23.0	29.5
35	7.2	15.0	25.3	39.2	12.0	24.3	35.4	46.8	11.0	21.2	30.3	40.0	9.2	18.0	27.0	34.5
40	5.1	11.9	20.5	31.0	10.4	21.5	32.3	44.3	11.4	22.5	32.5	42.5	9.9	18.9	28.2	36.3
45	4.0	9.5	17.1	24.8	8.9	18.8	28.8	39.0	10.6	20.7	30.3	39.8	9.6	18.8	28.1	36.5
50	3.1	7.7	14.3	21.0	7.7	16.0	25.0	34.2	9.2	18.5	27.5	36.2	9.0	18.0	27.4	35.7
55	2.5	6.4	11.9	17.8	6.6	13.8	21.0	29.8	8.0	16.2	24.6	33.0	8.3	16.8	26.0	34.3
60	2.1	5.4	9.8	15.2	5.6	11.8	18.0	25.7	6.9	14.2	22.0	30.1	7.6	15.5	24.0	32.0
65	1.8	4.6	8.3	12.9	4.7	10.0	15.3	22.0	5.9	12.2	19.5	27.2	7.0	14.2	22.0	29.4
70	1.5	3.9	6.9	10.8	4.0	8.4	12.9	18.7	5.0	10.5	17.1	24.4	6.3	12.9	19.8	26.7
75	1.4	3.3	5.8	9.0	3.4	6.9	10.7	15.8	4.3	9.1	15.0	21.5	5.6	11.6	17.8	24.3
80	1.3	2.6	4.9	7.5	2.8	5.6	8.8	13.2	3.7	7.9	12.9	19.0	5.0	10.4	15.9	22.0
85	1.2	2.3	4.1	6.3	2.4	4.6	7.4	10.9	3.2	6.9	11.0	16.3	4.4	9.0	14.0	19.7
90	1.1	1.9	3.5	5.4	2.0	4.0	6.2	9.0	2.8	6.0	9.6	13.9	3.8	8.0	12.5	17.5
95	1.0	1.7	2.9	4.5	1.7	3.4	5.3	7.6	2.5	5.2	8.4	11.8	3.4	7.0	11.2	15.6
100	0.9	1.5	2.4	3.9	1.5	2.8	4.6	6.5	2.1	4.5	7.4	10.2	3.0	6.3	10.1	14.0
105	0.8	1.3	2.0	3.2	1.3	2.5	4.0	5.8	1.9	4.0	6.5	9.0	2.8	5.6	9.1	12.7
110	0.8	1.1	1.8	2.8	1.1	2.3	3.5	5.0	1.7	3.5	5.7	7.8	2.5	5.1	8.3	11.5
115	0.7	1.0	1.6	2.4	1.0	2.0	3.2	4.5	1.5	3.1	5.0	7.0	2.3	4.8	7.5	10.5
120	0.7	1.0	1.5	2.1	0.9	1.8	2.9	4.0	1.4	2.9	4.5	6.3	2.0	4.3	7.0	9.6
125	0.6	0.9	1.4	1.9	0.8	1.6	2.5	3.6	1.2	2.6	4.0	5.7	1.8	4.0	6.3	8.8
130	0.6	0.9	1.3	1.7	0.8	1.5	2.3	3.3	1.0	2.4	3.6	5.2	1.6	3.6	5.7	8.0
135	0.6	0.8	1.2	1.6	0.7	1.5	2.1	3.0	0.9	2.2	3.4	4.7	1.5	3.4	5.2	7.2
140	0.5	0.8	1.1	1.5	0.7	1.4	2.0	2.8	0.8	2.0	3.0	4.3	1.3	3.1	4.7	6.4

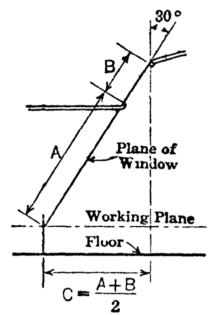
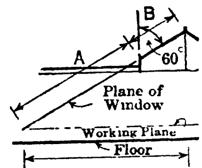


Table XVII. Illumination from 60° Sloping Windows

Feet from plane of window	Slant height above working-plane A															
	30 ft 0 in				50 ft 0 in				70 ft 0 in				90 ft 0 in			
	Window height B				Window height B				Window height B				Window height B			
	ft 2	ft 4	ft 6	ft 8	ft 2	ft 4	ft 6	ft 8	ft 2	ft 4	ft 6	ft 8	ft 2	ft 4	ft 6	ft 8
5	0.85	1.4	2.4	4.0	0.22	0.23	0.24	0.25	0.16	0.18	0.19	0.2	0.12	0.14	0.16	0.17
10	2.75	4.8	7.7	9.9	0.53	0.63	0.7	0.77	0.25	0.34	0.39	0.43	0.16	0.21	0.25	0.29
15	5.5	10.0	14.5	17.5	1.2	1.5	1.9	2.25	0.4	0.65	0.8	0.93	0.21	0.31	0.41	0.49
20	9.0	17.0	23.3	27.5	2.3	3.1	4.2	5.4	0.68	1.2	1.5	1.95	0.3	0.45	0.67	0.83
25	13.0	24.5	33.2	40.5	4.0	5.4	7.7	10.7	1.18	1.98	2.55	3.6	0.41	0.7	1.05	1.35
30	16.0	30.5	42.5	52.0	5.9	8.8	12.3	18.0	1.9	3.0	4.2	5.9	0.57	1.08	1.6	2.2
35	14.1	28.2	44.2	59.0	8.0	13.0	18.5	25.5	2.75	4.4	6.4	9.1	0.82	1.6	2.55	3.5
40	10.8	23.0	39.5	55.5	10.0	18.5	25.0	34.5	3.8	6.3	9.0	13.2	1.2	2.25	3.45	4.6
45	8.0	16.5	29.0	43.5	11.8	23.5	33.0	42.0	5.0	8.4	12.2	18.0	1.7	3.2	4.65	6.5
50	5.8	11.5	20.4	31.0	13.0	27.0	38.5	48.5	6.4	11.0	16.0	23.0	2.3	4.3	6.3	8.6
55	4.2	8.2	14.4	22.0	12.8	26.8	40.5	53.0	7.9	13.8	20.0	28.5	3.2	5.7	8.1	11.0
60	3.2	6.0	10.4	16.2	11.2	24.0	38.0	53.0	9.3	17.1	23.8	34.5	3.9	7.2	10.2	13.8
65	2.4	4.6	7.8	12.0	9.0	20.0	32.0	45.5	10.7	20.0	28.0	40.0	4.7	8.7	12.5	16.8
70	1.9	3.6	6.0	9.2	7.1	15.5	24.5	39.0	11.2	21.4	31.5	41.5	5.55	10.3	14.8	19.8
75	1.46	2.9	4.7	7.2	5.4	12.2	19.5	31.0	10.8	20.5	31.0	40.9	6.25	12.0	17.0	22.5
80	1.2	2.4	4.0	5.8	4.2	9.4	15.0	24.0	9.9	18.6	28.0	38.5	6.8	13.2	19.4	25.0
85	0.99	2.06	3.35	4.65	3.4	7.4	11.8	19.8	8.4	16.0	24.5	34.5	7.1	14.2	21.0	27.8
90	0.83	1.75	2.75	3.95	2.6	5.8	9.2	15.8	7.0	13.8	21.0	30.0	7.1	14.8	22.0	29.5
95	0.7	1.5	2.3	3.35	2.1	4.5	7.3	12.6	5.8	11.7	18.0	25.2	6.9	14.5	21.8	29.1
100	0.6	1.28	2.0	2.8	1.7	3.6	5.7	10.0	4.7	9.6	14.8	21.2	6.4	13.7	20.5	27.5
105	0.51	1.1	1.7	2.38	1.35	2.85	4.5	7.9	3.85	7.7	12.2	17.5	5.9	12.6	18.8	25.0
110	0.44	0.94	1.5	2.05	1.1	2.3	3.7	6.3	3.05	6.4	10.0	14.0	5.35	11.2	16.6	23.0
115	0.39	0.81	1.28	1.75	0.91	1.85	3.95	5.0	2.4	5.0	8.0	11.5	4.75	9.9	14.8	21.0
120	0.34	0.7	1.1	1.5	0.78	1.5	2.4	4.1	1.9	4.0	6.4	9.3	4.2	8.6	13.0	19.0
125	0.3	0.6	0.94	1.28	0.66	1.25	2.0	3.4	1.5	3.1	5.2	7.6	3.65	7.5	11.3	16.8
130	0.27	0.52	0.82	1.1	0.56	1.03	1.7	2.8	1.2	2.4	4.2	6.2	3.1	6.5	9.8	14.8
135	0.23	0.45	0.71	0.92	0.47	0.86	1.4	2.4	0.91	1.95	3.45	5.0	2.6	5.4	8.4	12.8
140	0.21	0.4	0.6	0.8	0.41	0.75	1.2	2.05	0.74	1.5	2.75	4.1	2.2	4.6	7.3	11.3



$C = 0.87\,(A + B)$

Here the window must be considered as made up of four equal parts in order to get the desk beneath the corner of each part. For one section:

$$\begin{aligned} m &= 8 \\ f &= 4 \\ m/f &= 2.0 \\ c &= 12.0 \\ c/f &= 3.0 \end{aligned}$$

Table XIII gives 16.5 as the illumination due to such a 4 × 8-ft window with a brightness of 200 candles/ft². For the entire 8 × 16-ft window, the illumination on the desk would be four times this or 66.0 lumens/ft². But the actual brightness is not 200 candles/ft² = it is only 100. Thus the actual illumination on the desk is

$$66.0 \times 100/200 = 33.0 \text{ lumens/ft}^2$$

The Fenestra Method. The above is a general method applicable to many problems. For the special cases most used in practical daylighting, Randall and Martin * have formulated tables which give directly the daylight illumination due to windows of brightness 200 candles/ft².

Table XIV gives the illumination on a working plane which is at the level of the bottom of the window. It applies to steel-sash windows with clear or hammered glass having a transmission factor of 80%. The windows should have an almost unobstructed length of 200 ft or more. For any number of panes 20 in high, and for any distance from the window, the illumination is read directly from the table. In case the window is less than 200 ft long, a correction is applied (Fig. 8). Also if the window is dirty another correction (Fig. 9) is used.

Table XV gives the illumination from vertical monitor windows. It is used exactly like Table XII.

Table XVI and **Table XVII** are for sloping monitor windows. Here it should be noted that the distances are measured from the PLANE OF THE WINDOW; that is, from the point C on the working plane.

Example. A steel-sash window is 6 panes high (10 ft 3 in) and 20 ft wide. What is the illumination at a point P on the level of the bottom of the window and 15 ft from it?

From Table XIV the illumination from a 200-ft window is found to be 31.0 lumens/ft². However, the window is only 20 ft wide, so Fig 8 must be used. This gives a factor of 0.38. So the actual illumination at P is

$$31.0 \times 0.38 = 11.8 \text{ lumens/ft}^2$$

This is for a clean window. After two months' use, we could expect

$$11.8 \times 0.72 = 8.5 \text{ lumens/ft}^2$$

by use of Fig. 9.

It will be noted that the Higbie method and the Fenestra method are based upon slightly different postulates. The Higbie method depends upon a theoretical treatment which assumes that the windows are of uniform brightness throughout, while the Fenestra method is based largely upon actual measurements made in buildings. In case prismatic glass is used, both methods will probably give values which are low for long distances from the

* See Bibliography at end of chapter.

window, since it is the function of such glass to throw the light far into the room.

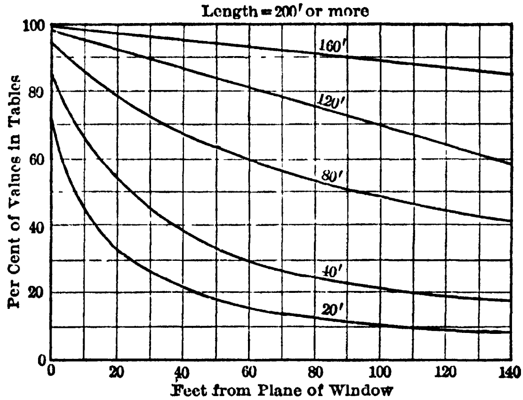


Fig. 8. Corrections for Window Length *

* From Randall and Martin, I.E.S. Trans. 25, 1930, p. 262

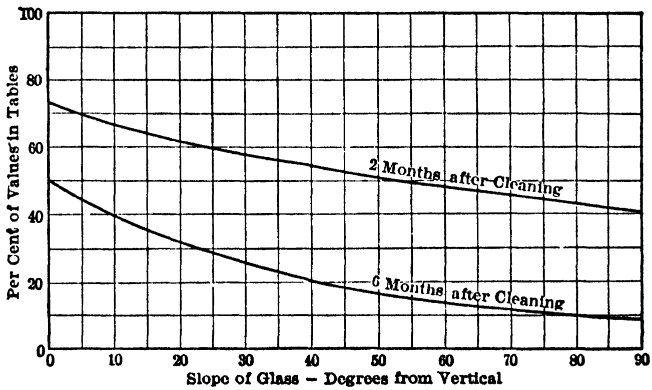


Fig 9. Corrections for Dirt on Glass *

* From Randall and Martin.

The Diffusion of Light through Windows*

Tests on the Diffusion of Light by Glass.† The results of tests on a score or more of different glasses may be stated briefly. We may increase the light in a room 30 ft or more deep to from three to fifteen times its present value by

* See, also, the subjects Pressed Prism-Plate Glass and Prism Glass.
† From Report No. III, Insurance Engineering Experiment Station.

using FACTORY-RIBBED GLASS instead of PLANE GLASS in the upper sashes. By using prisms we may, under certain conditions, increase the effective light fifty times. The gain in effective light on substituting ribbed glass or prisms for plane glass is much greater when the sky-angle is small, as in the case of windows opening upon light-shafts or narrow alleys. The increase in the strength of the light directly opposite a window in which ribbed glass or prisms have been substituted for plane glass is at times such as to light a desk or table 50 ft from the window better than one 20 ft from the window had previously been lighted.

The Kinds of Glass Tested were as follows:

(1) Ground glass of different degrees of fineness.

(2) Rough plate or hammered glass

(3) Ribbed or corrugated glass, with five, and eleven and twenty-one ribs to the inch, the corrugations being sinusoidal in outline, as in *A*, Fig. 10, and the back of the plate smooth.

(4) Glass known as MAZE, FLORENTINE or FIGURED, in which a raised pattern is worked upon one side, practically roughening the whole surface.

(5) Wash-board glass, corrugated, with twenty-one ribs to the inch on one side and five ribs to the inch on the other side, the ribs being parallel.

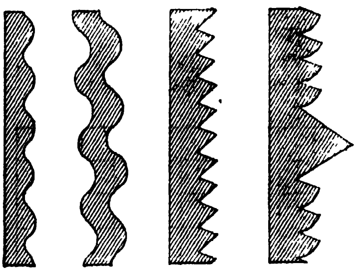


Fig. 10. Types of Ribbed or Prism-glass

(6) Skylight-glass, which has five ribs to the inch on each side, the groove on one side being opposite the rib on the other, giving a sinuous section *B*, Fig. 10.

(7) Ripple-glass, with rippled surfaces on both sides; of very beautiful appearance and a clear white color.

(8) Glass ribbed on one side and figured on the other.

(9) Ribbed glass with a wire net pressed into it, to increase its resistance to fire.

Of these several specimens, one or two may be dismissed with brief mention. Ground glass is of little value, except as a softening medium for bright sunlight. Its rapidly increasing opaqueness with moisture and dust makes it undesirable as a window-glass. The common rough plate has very little action as a diffusing-medium, giving no perceptible change in the effective light. Ripple-glass has great value as a diffusing-medium in small rooms with nearly open horizon. Of the ribbed glasses, the fine Factory-Ribbed, with twenty-one ribs to the inch, is distinctly the best, not in all probability because of the fineness, but because of the greater sharpness of the corrugations. The Ribbed wire-glass is about 20% less effective than the ordinary Factory-Ribbed glass. The addition of a second corrugation upon the back of the plate giving the Skylight and Wash-Board glass is of no apparent value. The raised pattern imprinted upon one surface of the glass, as in the case of the Maze glass, gives the widest diffusion, especially in bright sunlight. A raised figure, when worked upon the back of the Ribbed glass, renders it less offensive to the eye in bright sunlight, but less effective in deep rooms. The only glasses of this group which it is worth while, then, to discuss further are the Factory-Ribbed and the Maze glass. The second group comprises various prism glasses (Fig. 10).

Conclusions. (1) The conditions in a room less than 15 ft deep are such that, except with a skylight of less than 45°, it is not advisable to alter the general course of the light by using a prismatic or ribbed glass. A nearly hemispherical diffusion, such as is given by the Maze or Ripple-glass, is ordinarily preferable.

(2) When a room is from 20 to 60 ft deep, or even more, and has a skylight of 60° or less, the ribbed and prismatic glass results in a very great gain in effective light. The gain in brilliancy is such as to make a basement with prism-canopies as light as a second story with plane glass.

Rooms with windows opening upon light-shafts and narrow alleys with very limited openings to the sky, where the available light is now small, may have the light 20 ft back from the window increased ten or twenty times by using prisms; and, by using canopies of prisms, it is sometimes possible to strengthen the light fifty to one hundred times. With sky-angles of 30°, or less, and in deep rooms, the relative efficiency of the prism tile increases greatly. The refraction of the incident ray in a case of the ribbed glass and prism is shown in Fig. 11. Ribbed and maze glass are of very great value in softening the light, especially in the case of such windows as are exposed to the direct sun, aside from their effectiveness in strengthening the light at distant points. With the Maze glass, the artist may have, in all weather and in all directions, what is in effect a much-desired NORTH LIGHT. The photographer may have in this way as well diffused a light as he now has with cloth screens or shades, and with a much greater intensity. To be efficient in rooms 20 ft deep or more, ribbed glass should be set with its ribs horizontal, and where the sunlight falls upon it, it should be provided with thin white shades. All inferences drawn from the test are made upon the assumption that the windows are to be glazed with diffusing glass only in the upper half, which is the common practice. If the lower sash is to be glazed with diffusing glass as well, a further increase of about 25% may be expected.

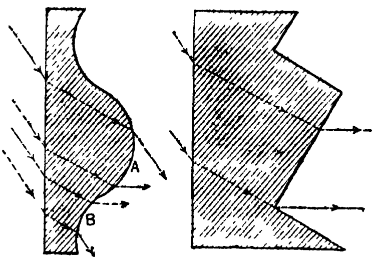


Fig. 11.—Refraction of Light in Ribbed and Prism-glass

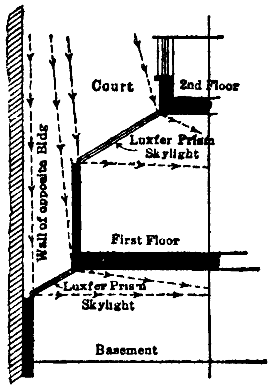


Fig. 12. Basement and First Story Lighted from Court

Considering both expense and efficiency, the following general suggestions are given: Use Maze or Ripple-glass in small rooms or offices not more than 15 or 20 ft deep; use Factory-Ribbed glass in rooms from 30 to 50 ft deep, with sky-angles of 60° or more; use prisms or Factory-Ribbed glass, in sheets, in all vertical windows in rooms more than from 50 to 60 ft deep, with sky-angle of less than 45°. With a sky-angle of less than 30° use prisms in canopies. Fig.

12 shows an effective method of lighting the basement and first story where the light must come from a court.

Floodlighting of Buildings

Floodlighting provides a means of revealing and emphasizing architectural beauty which would otherwise be hidden at night. The method has been made economically feasible by modern high-efficiency lamps.

For buildings of simple construction it is usually most effective to illuminate the entire front more or less UNIFORMLY. Any marked variation in illumination from point to point (SPOTTY illumination) must be avoided. This means that the number of floodlight projectors must be adequate, the beams must not be too concentrated, and the projectors must be carefully adjusted to give the best effect. A GRADUAL INCREASE in illumination from bottom to top, so that the upper part of the building stands out in contrast with the background, is effective. Usually the projectors can be placed on nearby buildings at a distance of 50 to 200 ft.

For the setback type of construction floodlighting seems particularly adaptable. The projectors are placed just inside the parapet and a foot or two above the roof level. The units can be arranged to give either a uniform illumination over the setback section or to give a gradual variation in illumination from top to bottom. The projectors should be carefully adjusted after installation to give the most pleasing result. Different colors can be employed advantageously on the different sections by the use of colored glasses in the floodlight units.

Equipment. Floodlight projectors may be divided into two general classes, the NARROW-BEAM type, using concentrated-filament projection lamps and carefully designed reflectors; and the MEDIUM AND BROAD BEAM types, using ordinary lamps. The former finds its principal use in lighting statues, columns, and other features where a broad beam would result in much wasted light. For most purposes, however, the ordinary type of lamp in a medium-beam reflector is recommended.

The control of light is done almost entirely by the REFLECTOR, the front glass being used largely to make the unit water-tight and dust-proof. Reflectors are made of various materials, as shown in Table XVIII.

Table XVIII. Reflection Factors of Various Metals

	Per cent
Silver plate	92
Silvered glass	82–88
Chromium plate	65
Polished aluminum	62
Nickel plate	55

Silver-plated metal reflectors are the most efficient when new, but are never used in floodlights because they quickly tarnish. Silvered glass is very efficient and is widely used. Chromium is finding considerable use because of its freedom from corrosion and its ability to stand rough usage which would ruin glass reflectors. However, it will be noted that it reflects much less light than silvered glass.

COVER-GLASSES are usually of clear heat-resisting glass which is clamped between asbestos gaskets to make a water-tight unit. In some cases, lightly

stippled glasses are used to give a more diffused beam. COLORED GLASS FILTERS are available in red, amber, green, and blue; and by combination of projectors with glasses of different colors any tint can be obtained on the floodlighting surface.

Design of a Floodlight Installation. Table XIX gives RECOMMENDED VALUES OF ILLUMINATION to be used in floodlighting installations for down-town buildings in cities of various size. It will be noted that in small cities a lower level of illumination is permissible than in larger cities. This is due to the lower illumination usually found in streets and store windows in the smaller cities and the smaller number of electric signs. If the building to be floodlighted is in an outlying section of a large city, the conditions are much as they are in smaller cities, and the column for the next smaller size city should be used.

Table XIX applies to white light. If the lighting is to be done in COLOR, the wattage must be increased to take into account the absorption of light by the colored glass. The watts should be increased approximately as follows:

Amber	50%	increase in watts
Red	100%	increase in watts
Green	200%	increase in watts
Blue	400%	increase in watts

Table XX gives the LUMINOUS OUTPUT of various floodlight projectors. The results are given in lumens in the main beam of light, the spill light being neglected. For most purposes the medium-beam projectors using general-service lamps are best.

Table XXI gives the luminous outputs of various lamps used in flood-lights. The general-service lamps have a life of 1 000 hours, and may be burned in any position, while the floodlight lamps have a life of 800 hours and must not be burned within 45° of the vertical, base up.

Table XIX. Recommendations for Floodlighting *

Representative building materials	Initial reflection factors, per cent	Foot-candles for down-town † buildings in cities of population: 50 000 or over	50 000 to 5 000	Under 5 000
White terra-cotta Cream terra-cotta Light marble	60–80	10	8	6
Light gray limestone Bedford limestone Buff limestone Smooth buff face brick	40–60	15	12	8
Briar hill sandstone Smooth gray brick Medium gray limestone Common tan brick	20–40	20	15	10
Dark field gray brick Common red brick Brownstone	10–20	30	20	15

* Taken from Edison Lamp Works Bulletin LD-159.

† For buildings in outlying districts use the foot-candles recommended for down-town buildings in cities of the next smaller classification.

NOTE. Buildings composed of material having a reflection factor less than about 20% cannot economically be floodlighted unless there is a large amount of light trim.

Table XX. Beam Lumens of Typical Floodlighting Units *

Beam spread	Projectors designed for floodlight lamps †		Projectors designed for general-service lamps		
	Lamp size in watts	Average beam lumens	Lamp size in watts	Average beam lumens: Reflector diameter 12 to 16 in	Average beam lumens: Reflector diameter 18 to 24 in‡
Narrow....			300	1 400	....
	250	1 100	500	2 500	... . .
			750		5 500
	500	2 600	1 000	. .	7 800
			1 500		10 500
Medium			300	1 700	.
	250	1 150	500	3 000	
			750	4 900	6 000
	500	2 800	1 000	7 000	8 500
			1 500		12 500
Broad...... ...			300	1 900	.....
	250	1 200	500	3 400	. .. ..
			750	5 200	6 200
	500	2 900	1 000	7 400	8 800
			1 500	.	13 000

* Taken from Edison Lamp Works Bulletin LD-159.

† These lamps have concentrated filaments and can be burned in any position except within 45° of the vertical, base up

‡ These large units are recommended for long throws, or where the installation will be kept in operation for at least five years, or where there are unusually severe operating conditions.

Table XXI. Lumen Outputs of Mazda Lamps for Floodlighting Service

	General-service lamps (rated life 1000 hr)					Floodlight lamps (rated life 800 hr)	
Lamp watts .. .	300	500	750	1 000	1 500	250	500
Approximate lumens.....	5 400	9 850	14 700	20 400	32 400	3 575	8 300
Lumens per watt. .	18 0	19.7	19 6	20 4	21 6	14.3	16.6

Procedure in Design. (1) Decide upon how much of the building is to be floodlighted and where projectors are to be placed.

(2) Obtain lumens/ft² from Table XIX.

(3) Assume size of projectors (500 watt is used perhaps more than any other size). Obtain beam lumens from Table XX.

(4) Determine the number of projectors by use of the formula:

$$\text{Number of projectors} = \frac{(\text{lumens/ft}^2) \times (\text{area to be lighted})}{0.7 \times (\text{beam lumens})}$$

where (lumens/ft²) is obtained from Table XIX and (beam lumen) is obtained from Table XX. The factor 0.7 is to allow for light spilled around the edges of the building.

In most cases this will give a large number of units so that there is no question but that the projectors can be adjusted to give uniform illumination over the area in question. A method of checking for uniform coverage is given in the Edison Lamp Works Bulletin LD–159, from which the above data were obtained and to which the reader is referred for more information.

Example. The top setback section of a building in a city of 200 000 is to be floodlighted. The building is of common tan brick and the section is 30 ft high and 100 ft long on a side.

Table XIX gives 20 lumens/ft² as the illumination to be used. The area to be lighted on each side of the building is $30 \times 100 = 3\ 000$ sq ft. It is decided to use 300-watt medium-beam units with general-service lamps. Table XX gives 1 700 beam lumens as the light emitted by such a projector. Thus,

$$\text{Number of projectors} = \frac{3\ 000 \times 20}{0.7 \times 1\ 700} = 50$$

Use 50 projectors on each side of the building (200 total), and mount them just inside the parapet.

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CHAPTER XXXIV

ELECTRIC WIRING

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General Considerations and Definitions. Electrical energy is now in common use, furnishing power, heat and light, operating bells and buzzers, and transmitting messages by telephone and telegraph. In order to accomplish these results, a current of electricity must flow around an electric circuit. The nature of electricity is not known, but the flow of it through an electric circuit is analogous to the flow of water through a system of pipes.

Current. Amperes. The flow of water is measured in GALLONS PER SECOND. The flow of electricity is measured in AMPERES. An ampere-flow of electricity is analogous to a gallon-per-second flow of water. The amperes thus indicate the quantity of electricity flowing through an electrical appliance in one second. About ½ ampere is flowing through an ordinary 60-watt, 115-volt incandescent lamp. An arc-lamp usually requires from 5 to 10 amperes.

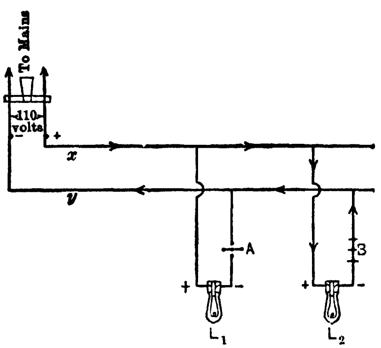


Fig. 1. Current always Flows from (+) to (−)

Pressure. Volts. When a current of water flows from one point to another in a pipe-system, it is always because there is a hydraulic pressure present causing it to flow. This pressure is usually measured in pounds per square inch. Similarly, when a current of electricity flows from one point to another in an electric circuit, it is because there is an electric pressure present which causes it to flow. This electric pressure is measured in VOLTS. An electric pressure of 1 VOLT is analogous to a hydraulic pressure of 1 lb per sq in. The pressure which causes the ½-ampere current to flow through an incandescent lamp is usually 115 volts. The electric company installs at least two wires in a residence and then maintains an electric pressure of 115 volts between them just as the water company maintains a pressure in the water-pipes. This electric pressure is at all times tending to force electricity from one wire to the other wire across the space between the two wires, just as the water-pressure

tends to force the water out from the pipe. The rubber insulation is put on to prevent this flow, very much as the strength and compactness of the iron prevents the flow of water through the walls of the pipe. But when one terminal of a lamp is connected to one wire and the other terminal to the other wire, the electric pressure tending to send a current from one wire to the other, sends a current through the lamp and causes it to glow. We mark the wire bringing the current to the lamp (+). The wire taking the current away, we mark (−). Thus in Fig. 1, if the current comes in on the wire marked (x), this wire is (+) and the wire (y) is (−). A pressure of 115 volts is maintained which tends to cause a current to flow across from the wire (x) to the wire (y). No current can flow, however, unless some path is afforded between the two wires. For instance, no current is flowing through lamp L_1, because the open switch A makes a gap across which the current cannot pass. Switch B, however, is closed, thus allowing the pressure to force a current from the wire (x) through the lamp L_2 to the wire (y) and back into the street-mains. Of course the electric company maintains the 115-volt pressure between the wires (x) and (y) whether any current is drawn from the wires or not, just as a water company maintains the pressure in the water-mains whether any water is drawn from the pipes or not.

Resistance. Ohms. The fact that a current of only ½ ampere flows through an incandescent lamp when a pressure of 115 volts is applied to it, is due to the RESISTANCE of the fine filament. This resistance of the filament is analogous to the resistance which a pipe of small bore offers to the flow of water. The resistance of an electrical appliance is merely the ratio of the pressure to the current which that pressure can force through it. As an equation, it is expressed

$$\text{Resistance} = \frac{\text{pressure}}{\text{current}}$$

When the pressure is measured in volts and the current in amperes, the resistance is then in ohms. Thus

$$\text{Ohms} = \frac{\text{volts}}{\text{amperes}}$$

Thus, since a pressure of 115 volts forces ½ ampere through an ordinary incandescent lamp, the resistance of the lamp is $115/\frac{1}{2} = 230$ ohms.

Ohm's Law. This relation between pressure, current and resistance is called OHM'S LAW. It is written in symbols in the three forms

$$R = E/I$$
$$E = IR$$
$$I = E/R$$

where R = resistance in ohms;
E = pressure in volts;
I = current in amperes.

Example. An electric flat-iron has a resistance of 35 ohms. What current will flow through it when it is put across a 110-volt circuit?

$$I = E/R = 110/35 = 3.14 \text{ amperes}$$

Example. An electric toaster takes 5 amperes when on a 115-volt circuit. What resistance does it have?

$$R = E/I = 115/5 = 23 \text{ ohms}$$

Insulators and Conductors. In order that practically no current may leak from one wire to the other, the wires are covered with rubber. This rubber covering offers such high resistance to the flow of an electric current that, although two wires may lie very close to one another with only this rubber between them, practically no current leaks through the rubber from one wire to the other. Materials such as rubber, glass, porcelain, dry wood, etc., have this resisting property and are said to be INSULATORS. Metals, on the other hand, offer very little resistance to the flow of an electric current and are called CONDUCTORS. A copper wire $\frac{1}{10}$ in in diameter has a resistance of only $\frac{1}{1000}$ of an ohm per foot. Accordingly, because of their low resistance, copper wires are generally used to carry electric currents, and because of its high resistance, rubber is generally used as a covering of the copper wires to prevent leakage from one wire to another. Wire, approved by the National Board of Fire Underwriters and installed according to their rules, will have the proper insulating covering for each installation.

Power. Watts. The flow of an electric current has been likened to the flow of water through a pipe. A current of water is measured by the number of gallons, or pounds, flowing per minute; a current of electricity is measured by the number of amperes. The power required to keep a current of water flowing is the product of the current in POUNDS PER MINUTE by the head, or pressure, in FEET. This gives the power in FOOT-POUNDS PER MINUTE. To reduce to horse-power, it is necessary merely to divide by 33 000. Thus

$$\frac{(\text{pounds per minute}) \times (\text{feet})}{33\,000} = \text{horse-power}$$

In exactly the same way, the POWER required to keep a current of electricity flowing is the product of the current in AMPERES by the pressure in VOLTS. This gives the power in WATTS.

$$\text{Watts} = \text{amperes} \times \text{volts}$$

The term WATT is merely a unit of power, and denotes the power used when one volt causes one ampere of current to flow. The watts consumed when any given current flows under any pressure can always be found by multiplying the current in amperes by the pressure in volts. Thus, if an incandescent lamp takes 0.5 ampere when burning on a 110-volt line, the power consumed equals

$$0.5 \times 110 = 55 \text{ watts}$$

That is,

$$\text{Power} = \text{current} \times \text{pressure}$$

or

$$\text{Watts} = \text{amperes} \times \text{volts}$$

Example. What power is consumed by a motor which runs on a 220-volt circuit, if it takes 4 amperes.

$$\text{Watts} = \text{amperes} \times \text{volts} = 4 \times 220$$
$$\text{Power} = 880 \text{ watts}$$

Incandescent lamps are rated as to the voltage of the line on which they can run, and also as to the amount of electric power it takes to keep them glowing. Thus, a lamp may be rated as a 110-volt, 50-watt lamp. Another lamp may be rated as a 110-volt, 25-watt lamp. This means that both lamps are intended to run on a 110-volt circuit, but the first takes twice as much power as the second.

The Power-Equation. The above relation between volts, amperes and watts is usually expressed in the form of an equation:

$$P = IE$$
$$I = P/E$$
$$E = P/I$$

in which P = power in watts;
I = current in amperes;
E = pressure in volts.

Example. What current does a 40-watt tungsten-lamp take when running on a 115-volt circuit?

$$I = P/E = 40/115 = 0.348 \text{ ampere}$$

Power. Kilowatt and Horse-Power. Because the watt is so small a unit of power, being only 0.74 ft-lb per second, a larger unit, the kilowatt, is generally used in connection with machines, etc.

$$1 \text{ kilowatt} = 1\ 000 \text{ watts} = 1\tfrac{1}{3} \text{ horse-power}$$

Thus a motor drawing 10 amperes from a 220-volt line would take $10 \times 220 = 2\ 200$ watts $= 2\ 200/1\ 000 = 2.2$ kilowatts.

At 80% efficiency this motor would give out 80% of $2.2 = 1.76$ kilowatts $= 1.76 \times 1\tfrac{1}{3} = 2\tfrac{1}{3}$ horse-power.

Horse-Power-Hour. Kilowatt-Hour. When a man buys mechanical power to run machinery, he has to pay not only according to the horse-power he uses but also according to the number of hours he uses the power. For instance, he may use 40 horse-power for 1 hour and pay $1.20 for it, that is, at the rate of 3 cts for each horse-power-hour. If he uses 40 horse-power for 2 hours he would have to pay twice as much, because he has used the same power twice as long. Another way of stating the same fact is to say that he used twice as many horse-power-hours. For in the first instance he used 40×1, or 40-horse-power-hours, and in the second 40×2, or 80 horse-power-hours. In other words, he did twice as much work in the second case as he did in the first, or received twice as much energy. The unit of work or energy, then, is the HORSE-POWER-HOUR, and is the work done in 1 hour by a 1-horse-power machine.

Example. How much work is done by a machine delivering 15 h.p. when it is run for 8 hours?

1 h.p. in 1 hr does 1 h.p.-hr
15 h.p. in 1 hr does 15 h.p.-hr
15 h.p. in 8 hr does 8×15, or 120 h.p.-hr

That is

$$\text{Work} = \text{horse-power} \times \text{hours}$$

or

$$15 \times 8 = 120 \text{ h.p.-hr}$$

Similarly, electric power is sold by the KILOWATT-HOUR. This unit is the work or energy delivered in one hour by a 1-kilowatt machine.

For lighting purposes electrical energy is usually sold for from 6 to 10 cts per kilowatt-hour. Thus at 10 cts per kw-hr the monthly bill for burning a 40-watt lamp on an average of 5 hours per day would be computed as follows:

For 1 month of 30 days the lamp is burning

$$30 \times 5 = 150 \text{ hours}$$

To use a 40-watt lamp 150 hours consumes

$$40 \times 150 = 6\,000 \text{ watt-hours} = 6\,000/1\,000 = 6 \text{ kilowatt-hours}$$

At 10 cts per kw-hr, 6 kw-hr cost

$$6 \times 10 = \$0.60$$

An instrument called a KILOWATT-HOUR METER is placed in each house to measure the number of kilowatt-hours which each customer consumes. See *M* in Fig. 11 for location of Kilowatt-hour meter, and Fig. 16 for method of connection in typical installation.

Heating-Effect of Current. An electric current always heats the material through which it passes. Examples of this are the incandescent lamp, in which the current heats the fine tungsten wire until it glows; the electric heaters for chafing-dishes, toasters, etc. Even the wires carrying the current to and from the lamps are heated by the passage of the current through them. But since the heating effect for a given current is directly proportional to the resistance of the conductor, and the conductors always have very little resistance, the heating here is very slight indeed. If conductors of smaller size, and therefore of a higher resistance, were used, the heating would be very pronounced; in fact, it would soften the rubber insulation and might even produce a temperature high enough to set fire to the building. For this reason The National Board of Fire Underwriters issues a table specifying the size of wire which must be used for each amount of current. If smaller wire is used, the resistance of it might be great enough to raise the temperature to a dangerous degree. On the other hand, if a greater current than allowed by this table is sent over the wire, the temperature will also rise, because the heating of a current is also directly proportional to the SQUARE OF THE CURRENT. Thus, doubling the current which a certain wire is carrying will quadruple the amount of heat which the wire must dissipate. For this see Table III and IV.

Fuses and Circuit-Breakers. Use is made of the heating effect of a current to protect a circuit against too much current, very much as a boiler is protected by a safety-valve against too much pressure. A small piece of fusible metal, generally a mixture of lead and bismuth, is inserted in the circuit in such a way that all the current which passes through the circuit must also pass through this piece of metal. This device is called a FUSE. Any current which would be dangerous to the circuit melts this fuse, opens the circuit at this point, and thus protects the rest of the circuit from the effects of the current. The cause of the large current may be then removed and a new fuse inserted in place of the old one. CIRCUIT-BREAKERS are also used to protect a circuit against too much current. They are AUTOMATIC SWITCHES controlled by an electro-magnet and are made in a variety of styles. They operate upon the principle that when an electric current passes through a coil of wire it makes a magnet of the coil. The coil is so adjusted that when a current of a certain number of amperes passes through it, it attracts to itself a small piece of iron. The motion of this piece of iron opens the circuit. Fuses and circuit-breakers are thus AUTOMATIC SAFETY-DEVICES required for the protection of all constant-potential systems whatever the voltage. Both are for the purpose of protecting the wires from damage due to the presence of too much current from any cause whatever. A FUSE CUT-OUT or CIRCUIT-BREAKER is required at or near the place where the wires enter a building, and every branch circuit must be protected by a cut-out. Circuit-breakers are more expensive than fusible cut-outs, and are generally used only on SWITCHBOARDS for large installations.

Dynamo-Electric Machines. There are three classes of dynamo-electric machines:

(1) GENERATORS for generating an electric current.
(2) MOTORS for converting electrical into mechanical energy.
(3) TRANSFORMERS and ROTARY CONVERTERS.
  (*a*) Transformers for converting one voltage into a higher or lower voltage. Converters and transformers belong to the same class.
  (*b*) Rotary converters for changing alternating currents to direct currents or vice versa.

GENERATORS are of two general classes, namely, continuous-current and alternating-current machines; the latter are commonly called ALTERNATORS. Generators and motors of all kinds vary in voltage, current and speed, according to the purpose for which they are designed. A TRANSFORMER consists essentially of two coils of wire wound upon an iron core. Its function is to convert electrical energy from one voltage to another. If it reduces the voltage it is known as a STEP-DOWN transformer, and if it raises it, it is known as a STEP-UP transformer. A transformer has no moving parts and requires no attendant.

Kinds of Currents Produced. There are two kinds of electrical currents commonly used for light and power in building: (1) DIRECT CURRENTS, and (2) ATERNATING CURRENTS.

"A direct current is uniform in strength and direction, while an alternating current rapidly rises from zero to a maximum, falls to zero, reverses its direction, attains a maximum in the new direction and again returns to zero. A complete set of these changes is called a CYCLE. The number of times the current goes through these changes during each second is called the FREQUENCY of the current. The frequency commonly used for incandescent lighting is 60 cycles per second; that is, the current goes through the above changes in value 60 times per second. A frequency of 25 cycles is also used, especially for motors, although it is not so satisfactory for use with incandescent lights. If a direct current is likened to the steady flow of water through a pipe-system an alternating current may be likened to the rapid surging back and forth of water in a pipe-system. The advantages of alternating over direct currents are: (1) Greater simplicity of dynamos and motors, no commutators being required in some types; (2) the feasibility of obtaining high voltages by means

Table I. Average Current Taken by Direct-Current Motors

Horse-power	Amperes on 110-volt line	Amperes on 220-volt line	Horse-power	Amperes on 110-volt line	Amperes on 220-volt line
¼	3	1.5	25	186	93
½	5.4	2.7	30	222	111
1	9	4.5	35	260	130
2	17	8.5	40	296	148
3	25	12.5	50	.....	185
5	40	20	60	...	220
7½	58	29	75	...	275
10	76	38	85	....	312
15	114	57	100	.....	366
20	150	75	....		....

of transformers for cheapening the cost of transmission; (3) the facility of transforming from one voltage to another, either higher or lower, for different purposes." *

The current taken by single-phase alternating-current motors can be found by noting the current taken by a direct-current motor of the same size and voltage, and dividing this current by the power-factor of the alternating-current motor. To find the current taken by each terminal of a three-wire, three-phase alternating-current motor, divide the current taken by a single-phase alternating-current motor of the same size and voltage by 1.73.

Example. What current is taken by a 5-horse-power, alternating-current, 220-volt, induction-motor of 80% power-factor?

Solution. A 5-horse-power, direct-current, 220-volt motor takes 20 amperes. A single-phase, 5-horse-power, 220-volt motor of 80% power-factor takes 20/ .80 = 25 amperes.

Electric-Lighting Systems Commonly Used for Supplying the Electrical Energy to Lamps

Direct-Current, Constant-Potential Systems. The systems most used in America are:

(1) Two-WIRE SYSTEM largely used for incandescent lighting from small plants, as for a large office-building or factory. It is usually operated at 115 volts.

(2) THREE-WIRE SYSTEM used in small towns for the lighting of buildings from the public mains, usually operated at 230 volts.

Alternating-Current, Constant-Potential Systems. There are two systems:

(1) SINGLE-PHASE SYSTEM. The term PHASE is used in connection with alternating-current systems only in the sense of CIRCUIT. Thus a single-phase system means an alternating-current system sending out power from one circuit only of the generator. A three-phase system has three circuits.

(2) THREE-PHASE SYSTEM. Three or four wires are used. This system is used almost universally by large generating stations because it is the most economical in wire. (See Table II.) For a comparison of a three-wire direct current with a three-phase, three-wire alternating current, see text under this heading. An alternating current may be changed to a direct current at a substation by a rotary converter or by a mercury-arc rectifier. The latter is very generally used in garages in order to convert an alternating current into a direct current for charging storage-batteries.

Methods of Connecting Lamps. There are three ways of connecting lamps to the distribution-wires: (1) in series; (2) in parallel; and (3) in parallel series.

(1) **Lamps in Series.** Lamps are said to be connected in series when they are arranged one after the other, so that the same current flows through all the lamps. The most common example of this system is the lighting of electric cars and the stations on an electric-railway line. The voltage of such lines is usually 550 volts. Since the ordinary incandescent lamp requires but 110 volts, five of these are placed in series as in Fig. 2. Each lamp now has a pressure of 110 volts across it, and the set of five lamps requires 550 volts across it, and so can be placed across the railway supply-wires. When lamps are arranged in series the total resistance of the circuit is the sum of the resist-

* Adapted from Kent's Pocket Book

ances of the several parts, and the voltage required to force the current through a number of lamps in series is the sum of the voltages required for the separate lamps. Thus the voltage required to supply the proper current for four 52-volt lamps is 4 × 52 = 208 volts The series system requires that the same current flow through each lamp, and if one lamp burns out the circuit is broken and all of the lamps will go out, unless some provision is made for maintaining the circuit around the dead lamps.

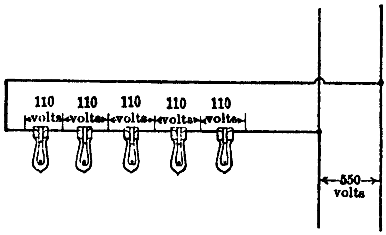


Fig. 2. Five Lamps in Series on a 550-Volt Line. Each Lamp has a Voltage of 110 Volts across It

(2) **Lamps in Parallel.** This is the common method of connecting incandescent lamps. It is illustrated in Fig. 3. With this system the pressure in each lamp is the same as in the distributing lines, and any lamp may be turned on or off without affecting the other lamps. For this system the PRESSURE or voltage must be kept constant, while the current or quantity of electricity flowing in the lines will depend upon the number of lamps that are burning. Thus if twelve 110-volt lamps are connected in parallel and each lamp takes 0.51 ampere, the total current will be 6 12 amperes, and the power required will be 673.2 watts.* With but one lamp burning, a current of only 0.51 ampere will flow. The voltage, however, must be the same for one lamp as for the twelve. For lamps in parallel, therefore, a CONSTANT-POTENTIAL system is required. The current for lamps in parallel may be turned on or off at the lamp, or a switch-loop may be run any distance and the contact made by a switch (S) as for the lower lamp (Fig. 3).

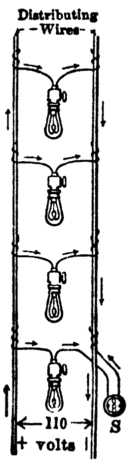


Fig. 3. Four Lamps in Parallel. Each Lamp Has the Full-line Pressure of 110 Volts across It

(3) **Lamps in Parallel Series.** This method is a combination of the other two. Parallel lines are run as in the parallel system, but two or more lamps are connected in series between them as in Figs. 4 and 5. This method of connecting lamps is used principally in places where it is desired to operate lamps on a power system. Fig. 4 shows a series of five lamps operated on a 500-volt system and Fig. 5 a series of two lamps on a 220-volt system using 110-volt lamps. Any number of series may be connected across the mains, each series being independent of the others. But in each series if one light burns out, the others in the same series will be useless, and one lamp alone cannot be used.

The Edison Three-Wire System. Figs. 3, 4, and 5 are examples of the two-wire system of distribution, which is the system recommended for average-sized office-buildings, apartment-houses, theaters and stores. Where power for motors is to be taken from the same plant as the lighting current and where the power is not too great, this two-wire system may also be used. Separate mains, however, should under all circumstances be run for the motors, as

* Watts being equal to amperes times voltage.

otherwise the variation in motor load will cause a very appreciable fluctuation in illumination. Where comparatively long lines are required and the current to be supplied is large the THREE-WIRE SYSTEM is used. By this system two voltages such as 115 and 230 can be supplied, the 115-volt circuit supplying

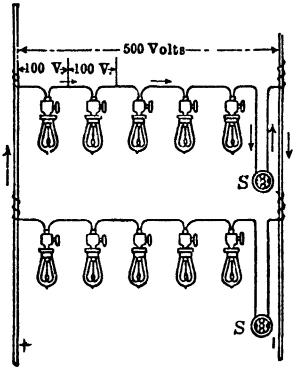


Fig. 4. Lamps in Parallel Series

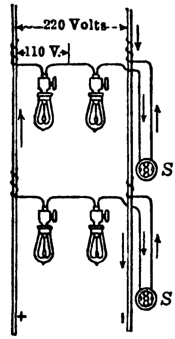


Fig. 5. Lamps in Parallel Series

the incandescent lights and the 230-volt circuit the motors. Fig. 6 shows how the wires are run and connections made. The pressure between the two outside wires is the full voltage transmitted from the generator. The current in these two wires flows in opposite directions. The middle wire, called the

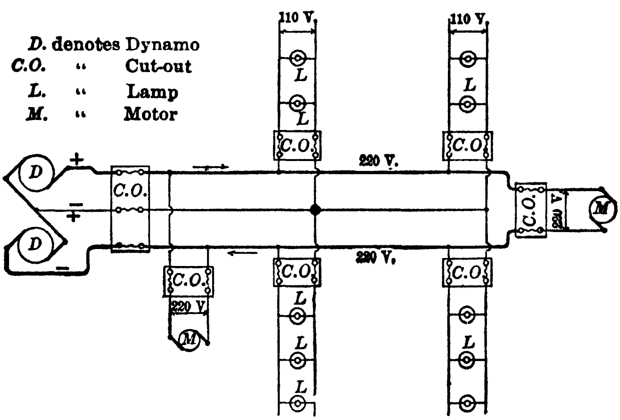


Fig. 6. Example of Three-wire System of Wiring

NEUTRAL WIRE, forms one side of two circuits, the current from one circuit tending to flow in one direction and that from the other circuit in the opposite direction; consequently, when currents of the same strength, in amperes, are flowing in both circuits they neutralize each other in the middle wire and

there will be no current flowing in this wire. With a current of 10 amperes flowing in one circuit and one of 6 amperes in the other circuit, the current flowing in the neutral wire will be 4 amperes. To obtain the greatest benefit from this system, it should always be installed so that there will be nearly the same load or number of lamps on each side of the neutral wire. Even then there will be times when more lamps will be burning on one side than on the other, so that it is necessary to give some size to the neutral wire The neutral wire is seldom made less than one-half the cross-section of the outer wires. For distributing mains in buildings having a lamp load only, the neutral wire should be of the SAME SIZE as the outer wires. From Table II it will be seen that the three-wire system effects a considerable saving in copper, amounting to fully 60% of the ordinary two-wire 110-volt system. As a rule, in supplying current for light and power from one plant, the main wires only are arranged on the three-wire system and the distributing wires are run on the two-wire system as in Fig. 6. When using the three-wire system for lighting only, the three wires are usually run no farther within the building than to the centers of distribution, and from these centers two wires are run for each circuit, the circuits being divided as equally as possible on the two sides of the three-wire system as shown by Fig. 7. Three-wire mains are now very commonly used where the current exceeds 100 amperes. When motors are operated from the three-wire system they are usually connected only to the outside wires. Motors used on three-wire incandescent-lighting systems should be wound for 230 volts.

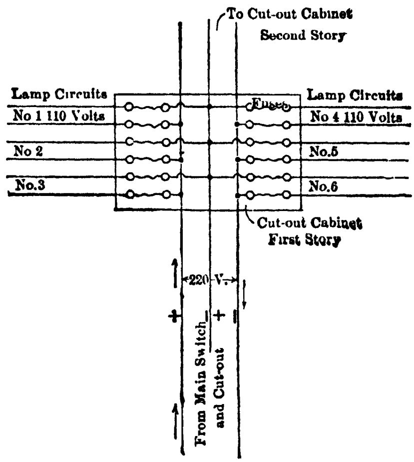


Fig. 7. The Wiring of a Cabinet

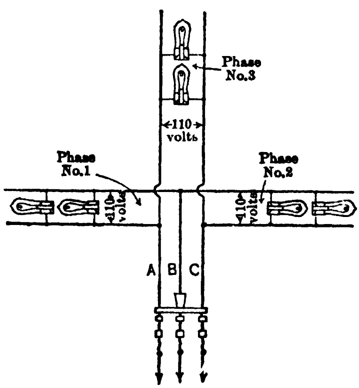


Fig. 8. Three-phase, Three-wire System, Alternating Current. Compare with Fig. 9

Comparison of the Three-Phase and Three-Wire Edison Systems. The wiring for the Edison three-wire direct-current system is the same as that

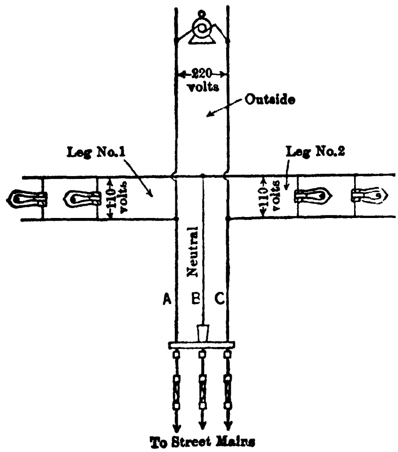


Fig. 9. Three-wire System, Direct Current. Compare with Fig 8

for the three-wire, three-phase alternating-current system, the only difference being that the voltage BETWEEN ANY TWO WIRES of a three-phase system is the same. Thus in Fig. 8 which represents a three-wire. three-phase system the voltage between the wires *A* and *B* (phase No. 1) is 110 volts; between *B* and *C* (phase No. 2) is 110 volts; and between *A* and *C* (phase No. 3) is 110 volts. But in Fig. 9, which represents a three-wire direct-current system, in which the voltage across *A* and *B*, and *B* and *C*, is 110 volts, the voltage across *A* and *C* is 220 volts or twice that across either leg.

Table II. Relative Weight of Copper Required in Different Systems for Equal Effective Voltage

Direct-current, ordinary two-wire system	1 000
Direct-current, three-wire system, all wires of same size.......	0 375
Direct-current, three-wire system, neutral, one-half size...	0 313
Alternating-current, single-phase two-wire system. . .	1 000
Three-phase three-wire.	0 750
Three-phase four-wire	0 333

Wire-Calculations

Wire-Gauges.* As the diameter of wires is ordinarily designated by the number of a wire-gauge, and as there are a number of wire-gauges in common use, some knowledge of those used for copper wire is necessary. The Brown & Sharpe, or B. & S., gauge is almost exclusively used in America in connection with electrical work, except where the size of the wire is designated in circular mils. The sizes of wire given by this gauge range from No. 0000 (0.46 in) to No. 40 (0.0031 in), but No. 14 is the smallest size permitted for interior wiring. The No. 10 wire has a diameter of about $1/10$ in and its resistance per 1 000 ft is very nearly 1 ohm. For any given number of this gauge a wire three numbers higher has very nearly half the cross-section, and one three numbers lower has twice the cross-section; thus a No. 13 wire has very nearly one-half the cross-section of a No. 10 wire, and a No. 7 has twice the cross-section of a No. 10, or four times that of a No. 13.

The Circular-Mil Wire-Gauge. In practice the B. & S. gauge is commonly used for designating wires up to No. 0 or No. 00, and all wires above that size

* For other gauges, see index.

are designated by circular mils (c.m.). The size of wire required is often determined in circular mils and designated by the corresponding B. & S. gauge-number, which is readily done by means of Table III.

The basis of the circular-mil gauge is the area of a wire $\frac{1}{1000}$ in in diameter (1 mil = 0.001 in); consequently, 1 c.m. = 0.0000007854 sq in. As the areas of circles vary as the squares of their diameters, it follows that the sectional area of a wire 2 mils in diameter = 4 c.m., of a wire 10 mils in diameter 100 c.m., and so on.

When wires are designated by circular mils, the SECTIONAL AREA and not the diameter is generally given, c.m. always referring to sectional area. The diameter of a wire in MILS OR IN THOUSANDTHS OF AN INCH = square root of its area in CIRCULAR MILS.

Thus the diameter of a wire of 3 600 c.m. = 60 mils, or 0.060 in.

The diameter of a wire 14 400 c.m. = 120 mils = 0.12 in.

The area of a wire 0 162 in in diameter, or 162 mils $= 162^2$ = 26 244 c.m.

To reduce circular mils to square inches. Multiply by 7 854 and point off ten places of decimals. Thus, 5 000 c.m. = 7 854 × 5 000 = 0.0039270000 sq in.

To obtain the sectional area of a square or rectangular bar in circular mils. Multiply together its dimensions in mils and the product by 1.273.

Example. What is the sectional area in circular mils of a bar $\frac{1}{8}$ in × $\frac{1}{4}$ in?

Solution. $\frac{1}{8}$ in = 0.125 in = 125 mils, $\frac{1}{4}$ in = 0.250 in = 250 mils; 125 × 250 × 1.273 = 39 781 25 c m.

The weight of bare copper wire per 1 000 ft = c.m. × 0.003027 lb. Thus the weight of 1 000 ft of copper wire having a sectional area of 2 000 c.m. = 0.003027 × 2 000 = 6.054 lb. Table IV gives the dimensions and weights of bare copper wire from No. 18 to No. 0000 B. & S.

Carrying Capacity of Copper Wire. The safe carrying capacity of copper wire for interior wiring is practically fixed by the underwriters, and if the capacity limits given in the table published by them are exceeded it would tend to destroy the right to recover insurance in case of fire. The safe carrying capacity of rubber-covered and weather-proof wires given by the National Board of Fire Underwriters is shown by Table III. The lower ampere-capacity assigned to rubber-covered wires is due to the fact that the rubber insulation would deteriorate in quality under a temperature as high as that allowed for weather-proof wire. No wire smaller than No. 14 may be used under insurance-rules, except that No. 16 may be used for flexible cord and No. 18 for fixture-wiring. Nos. 13, 11, 9 and 7 are not usually carried in stock and can only be purchased on special order. Rubber-covered wire must be used for service-wires, for molding-work and in damp places; it is more expensive than weather-proof wire. The latter wire may be used in open or exposed places and for outside line-wires.

Drop of Potential. When an electric current flows through a wire of any appreciable length the pressure becomes reduced by the resistance of the wire, so that if the current enters the wire at, say, 110 volts, at the extreme end of the circuit it will be somewhat less, depending upon the length and sectional area of the wire. This loss in voltage is called DROP OF POTENTIAL. Drop of potential corresponds to LOSS OF HEAD in hydraulics. As a drop of voltage materially below that for which the lamps are designed means diminished light, it is very important that the wires be proportioned so that the drop shall not be sufficient to affect the illumination. The table for safe carrying capacity for wires has nothing to do with the drop of potential which these

currents will cause in the wires. Accordingly, mains and distributing wires may be capable of carrying the number of amperes in accordance with Table II, and yet cause a drop of potential of such magnitude that the most distant lamps will burn only at a dull red. It is therefore necessary, in computing the size of these mains and distributing wires, to consider two things:

(1) That the wire is large enough, according to the underwriters' table, to carry the current safely.

(2) That the potential drop from the generator to the farthest lamp shall not be excessive.

The drop in volts (not in percentage) = current in line × resistance of line, or drop in volts = amperes × ohms.

Example. What will be the drop in a circuit of No. 14 copper wire 280 ft long, supplying nine lamps, requiring 4.5 amperes?

Solution. From Table IV it is found that the resistance of No. 14 wire is 2.527 ohms per 1 000 ft; hence for 280 ft it will be $2.527 \times 0.280 = 0.7075$ ohm, and drop in volts $= 4.5 \times 0.7075 = 3.1837$ volts. The voltage for this current (0.5 ampere per lamp) will be about 110; consequently, the percentage of drop $= 3.1837/110 = 2\frac{9}{10}\%$, nearly. A 2% drop on a pressure of 110 volts is 2.2 volts.

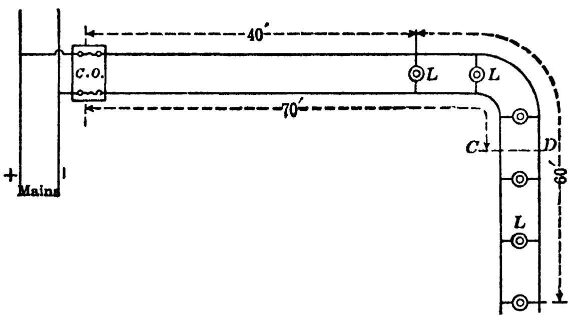


Fig. 10. The Point *D* is the Load-center

Load-Center. The meaning of this term may best be illustrated by an example. Let Fig. 10 represent a circuit carrying six lamps, the first lamp being 40 ft from the cut-out, or source of supply. The whole of the current must be transmitted through this 40 ft, but from that point it will gradually fall off, and the average current will only extend to the point *CD*, half way between the extreme lamps. Or, in other words, the load-center is analogous to the center of gravity of the lamps on the circuit. The load-center determines the length of the line in the rules for finding the necessary size of wire.

Calculations for Size of Wire for Incandescent Lighting. The sizes of wires for interior lighting are or should be always determined on a basis of a fixed drop of potential, usually 2 volts on the distributing circuit and from 2 to 3 volts on the feeders or mains.* The size of wire may be determined either in terms of its sectional area in circular mils or in terms of its resistance in ohms per 1 000 ft. Knowing the sectional area in circular mils, one may

* Many municipal lighting companies require that there shall be no more than 2% total drop in the wiring for interior lighting.

find the corresponding gauge-number from Table III, or if the resistance in ohms per 1 000 ft is known, the corresponding gauge-number may be found from Table IV.

Table III. Carrying Capacity of Wires and Cables

FOR INTERIOR CONDUCTORS, ALL VOLTAGES

From the National Electrical Code

No. of wire, B. & S. gauge*	Circular mils	Capacity in amperes	
		Rubber-covered	Weather-proof
18	1 624	3	5
16	2 583	6	10
14	4 107	15	20
12	6 530	20	25
10	10 380	25	30
8	16 510	35	50
6	26 250	50	70
5	33 100	55	80
4	41 740	70	90
3	52 630	80	100
2	66 370	90	125
1	83 690	100	150
0	105 500	125	200
00	133 100	150	225
000	167 800	175	275
0000	211 600	225	325
Cables	200 000	200	300
"	300 000	275	400
"	400 000	325	500
"	500 000	400	600
"	600 000	450	680
"	700 000	500	760
"	800 000	550	840
"	900 000	600	920
"	1 000 000	650	1 000
"	1 100 000	690	1 080
"	1 200 000	730	1 150
"	1 300 000	770	1 220
"	1 400 000	810	1 290
"	1 500 000	850	1 360
"	1 600 000	890	1 430
"	1 700 000	930	1 490
"	1 800 000	970	1 550
"	1 900 000	1 010	1 610
"	2 000 000	1 050	1 670

* For other gauges, see index

The formula for circular mils is as follows:

$$\text{Circular mils} = \frac{10.4 \times 2d \times N \times I}{v} \qquad (1)$$

The formula for resistance per 1 000 ft of wire is

$$\text{Resistance} = \frac{1\,000\,v}{N \times I \times 2d} \qquad (2)$$

Table IV. Dimensions, Weights and Resistances of Copper Wire

Gauge-number, B. & S.	Diameter in mils	Area in cir. mils	Area in sq in	Weight in lb per 1 000 ft		
				Bare wire	Weather-proof* wire	Ohms per 1 000 ft at 20° C. or 68° F.
0000	460	211 600	0.166190	640.73	800	0.04893
000	410	167 800	0.131790	508.12	666	0.06170
00	365	133 100	0.104520	402.97	500	0 07780
0	325	105 500	0.082887	319.74	363	0 09811
1	289	83 690	0.065732	253 43	313	0 1237
2	258	66 370	0.052128	200.98	250	0 1560
3	229	52 630	0.041339	159.38	200	0.1967
4	204	41 740	0 032784	126.40	144	0 2480
5	182	33 100	0.025999	100.23	125	0.3128
6	162	26 250	0 020618	79.49	105	0 3944
7	144	20 820	0 016351	63 03	87	0 4973
8	128	16 510	0.012967	49.99	69	0.6271
9	114	13 090	0.010283	39.65	. .	0 7908
10	102	10 380	0 008155	31.44	50	0 9972
11	91	8 234	0 006466	24.93		1 257
12	81	6 530	0.005129	19.77	31	1.586
13	72	5 178	0.004067	15.68	. .	1.999
14	64	4 107	0.003225	12.44	22	2.527
15	57	3 257	0.002558	9.86	. .	3.179
16	51	2 583	0 002028	7.82	14	4 009
17	45	2 048	0.001608	6.20	. .	5 055
18	40	1 624	0.001275	4.92	11	6.374

* Approximate weight of weather-proof line-wire for outdoor work is 10% less than here given.

In both these formulas d = distance in feet, one way, from cut-out to load-center for distributing wires, or from entrance cut-out or source of current to distributing center for main lines or feeders. I = current in amperes PER LAMP. N = number of lamps supplied. v = drop in volts. Both formulas apply to any voltage and to any two-wire system. To use these formulas for the ordinary three-wire system, let N = maximum number of lamps on ONE SIDE of the neutral wire and DOUBLE THE DROP IN VOLTS. The neutral or middle wire should be of the same size as the outside wires.

Example. The distance from the cut-out to load-center of a circuit carrying sixteen 40-watt, 110-volt lamps is 50 ft. What size of wire should be used for a drop of 2 volts?

Solution. $d = 50$; $N = 16$; $I = 40/110 = 0.364$; and $v = 2$.

By Formula (1),

$$\text{Circular mils} = \frac{10.4 \times 100 \times 16 \times 0.364}{2} = 3\,030$$

Table III shows that the next larger size of wire is 4 107 c.m., equivalent to a No. 14 wire.

By Formula (2),

$$\text{Resistance per 1 000 ft} = \frac{1\,000 \times 2}{12 \times 0.364 \times 100} = 4.59$$

which we see from Table IV, is about the resistance of a No. 16 wire; but as No. 14 is the smallest wire permitted that size must be used.

To find the smallest size of wire that will comply with the underwriters' rules it is only necessary to compute the total current in amperes, and from Table III select the wire having a capacity equal to or next above the required number of amperes.

Formulas (1) and (2) may also be used for MOTOR-WIRING, if the required current in amperes is known, by substituting the given number of amperes for $N \times I$.

Table V. Wire Size Required *

(Length of wire for a circuit is double the length of run)

WATTS PER CIRCUIT

LENGTH OF RUN (PANEL BOX TO OUTLET)	100	150	200	300	400	500	600	700	800	900	1000	1200	1400	1600	1800	2000	2200	2400	2600	2800	3000	3200
30	14	14	14	14	14	14	14	14	14	14	14	14	14	12	12	12	12	10	10	10	10	10
40	14							14	14	14	14	12	12	12	10	10	10	10	10	8	8	8
50	14							14	14	12	12	12	12	10	10	10	10	8	8	8	8	8
60	14							14	12	12	12	10	10	10	10	8	8	8	8	8	6	6
70	14	14	14	14	14	14	14	12	12	12	12	10	10	8	8	8	8	8	6	6	6	6
80	14	14	14	14	14	14	12	12	12	10	10	10	8	8	8	8	8	6	6	6	6	6
90	14	14	14	14	14	12	12	12	10	10	10	10	8	8	8	6	6	6	6	6	6	4
100	14	14	14	14	14	12	12	12	10	10	10	8	8	8	6	6	6	6	6	6	4	4
110	14	14	14	14	14	12	12	10	10	10	10	8	8	8	6	6	6	6	4	4	4	4
120	14	14	14	14	12	12	10	10	10	10	8	8	8	6	6	6	6	4	4	4	4	4
130	14	14	14	14	12	12	10	10	10	8	8	8	6	6	6	6	4	4	4	4	4	4
140	14	14	14	14	12	12	10	10	8	8	8	8	6	6	6	6	4	4	4	4	4	4
150	14	14	14	12	12	10	10	10	8	8	8	6	6	6	6	4	4	4	4	4	4	2
160	14	14	14	12	12	10	10	8	8	8	8	6	6	6	4	4	4	4	4	4	2	2
170	14	14	14	12	12	10	10	8	8	8	8	6									2	2
180	14	14	14	12	10	10	10	8	8	8	6	6									2	2
190	14	14	14	12	10	10	8	8	8	8	6	6									2	2
200	14	14	14	12	10	10	8	8	8	6	6	6									2	2
210	14	14	14	12	10	10	8	8	8	6	6	6									2	2
220	14	14	14	12	10	10	8	8	8	6	6	6									2	2
230	14	14	12	12	10	8	8	8	6	6	6	6									2	1
240	14	14	12	10	10	3	8	8	6	6	6	4									1	1
250	14	14	12	10	10	8	8	8	6	6	6	4	4	4	4	2	2	2	2	2	1	1

The difference in the cost of wiring with No. 12 instead of No. 14 is so slight, and the advantages generally sufficient, that many electrical men specify No. 12 as the minimum size for branch circuits.

This table shows the wire size required for 115-volt circuits of various lengths of run, based on a drop of two volts between the panel board and the outlets.

It is difficult in branch circuits, at least with present wiring methods, to use larger than No. 8 wire because of the difficulty of handling in conduit. Where the proper wire size for a proposed installation, according to this table, is larger than No. 8, it is usually best either to provide more circuits with less load, or to relocate distribution centers to decrease lengths of run. The dotted line shows the usual practical limits for circuit load and length of run.

Note. These recommendations on wiring are based on the allowances of The National Code: i. e. circuits equipped with medium screw sockets limited to 15 amperes and not more than 12 outlets per circuit; mogul sockets—limited to 40 amperes and 8 outlets per circuit. Present wiring practice is usually well within the limit allowed by the code. In some cases it is necessary to meet other requirements of local codes.

* From National Lamp Works Bulletin No. 41-D.

Table VI. Recommended Capacity for Outlet *

Actual floor area per outlet Square feet	Class A Installations (Such as offices, drafting-rooms, factories, etc.) Wattage capacity per outlet	Class B Installations (Such as stores, school-rooms) Wattage capacity per outlet	Class C Installations (Such as neighborhood stores, storage areas in factories† and basements) Wattage capacity per outlet	Class D Installations (Such as storage areas in garages and unimportant basements) Wattage capacity per outlet
65–75	300	200	150	100
75–85	300	250	150	100
85–95	350	250	200	100
95–110	400	300	200	100
110–125	450	350	250	150
125–140	500	400	250	150
140–160	600	450	300	150
160–190	700	500	350	200
190–220	800	600	400	200
220–260	950	700	450	250
260–300	1 100	800	550	300
300–340	1 250	950	650	300
340–390	1 450	1 100	750	350
390–440	1 650	1 250	850	400
440–500	. .	1 400	950	450
500–560	.	1 600	1 050	500
560–630		1 800	1 200	550
630–710	. .	.	1 350	650
710–800	. . . . .	. . . .	1 500	750
800–900	. . . .	. . . . . .	1 700	850

* From Westinghouse Lamp Co., Bulletin E-108.

† In factories it is often desirable to convert storage areas into work places to meet immediate production needs. For this reason, it is recommended that storage areas be wired according to Class B Specifications.

Example. What should be the size of the wires to be run to a motor that requires 30 amperes at 220 volts and is situated 200 ft from the distributing pole, the drop in volts not to exceed 2 %?

Solution. Using Formula (1), and substituting 30 for $N \times I$, we have

$$\text{Circular mils} = \frac{10.4 \times 400 \times 30}{4.4} = 28\,400$$

which requires a No. 5 wire. Either the watts or the current in amperes is stamped on every motor. If watts are given, the current in amperes may be found by dividing the watts by the voltage. If kilowatts are given, multiply by 1 000 and then divide by the voltage.

Wiring-Tables. Several forms of wiring-tables which are very useful to electricians are published in various books on electricity. For ordinary interior wiring Table V will show at a glance the number of wire, B. & S. gauge, required.

Simple Example of Wiring. To show the method of wiring an ordinary building for incandescent lighting we will take a two-story building having a floor-plan as shown in Fig 11. Most of the light-outlets are on the ceiling and are indicated by a small circle. The outlet marked *E* is a special outlet for heating, etc., and must be described in the specifications. Let us assume it is to take 320 watts. This is equivalent to adding eight 40-watt lamps to this circuit. *F* and *G* are wall-outlets. The meanings of the symbols used are explained under Standard Wiring Symbols. The numbers 1 and 2 inside the circles denote the number of 60-watt lamps to the outlet. The same number of 25-watt or 40-watt lamps may always be used without overloading the circuits. See Standard Wiring Symbols. The service wires should connect to the main fuse-block and switch, which should be in a small cabinet, and to the meter (*M*). The distribution-cabinet should be located near the center of the building, say at *DC*, and there should be a cabinet on each floor. From this cabinet we will run four circuits for each floor, which are indicated by the letters *A*, *B*, *C* and *D*. Circuit *A* shows the wires run for a switch on the wall near the door of each of four rooms to control the lights in those rooms. The lights on circuits *B* and *D* are not switched, except the outlet at head of stairs, which is controlled by a snap or push-button switch at *S*. For a first-class job all of the four circuits would be controlled by knife-switches in the cabinet, as shown in Fig. 12; but this is not absolutely necessary.

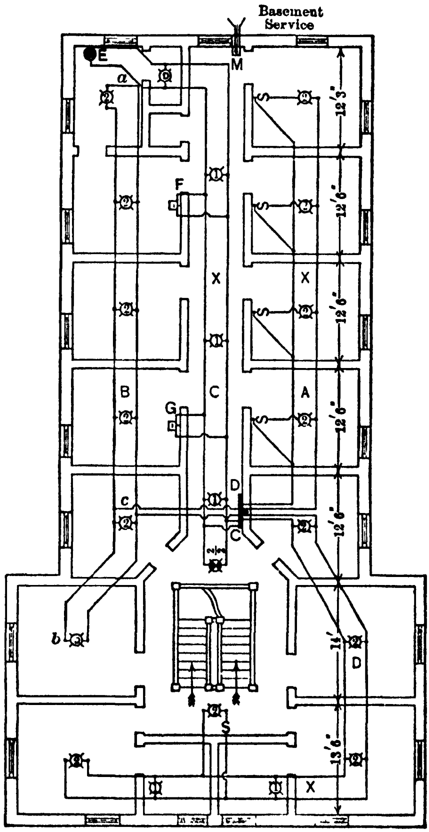


Fig. 11. Wiring-diagram for Second Story

Size of Wires. The load-center of circuits *A*, *C*, and *D* would be at about the points marked *X* (Fig. 11). For circuit *B* take one-half the distance *ab* and add to it the distance from *c* to the cabinet. In figuring the length of

line, 6 ft should be added for the drop from ceiling to the cabinet. In computing the current taken by each lamp it is assumed that no smaller than a 40-watt tungsten is used. The number of 40-watt lamps and length of wire for each circuit are as follows:

Circuit *A*, 8 lights, 41 ft one way to load-center.
Circuit *B*, 11 lights, 52 ft one way to load-center.
Circuit *C*, 16 lights, 37 ft one way to load-center.
Circuit *D*, 12 lights, 59 ft one way to load-center.
Total number of lamps, 47.

From Table V we see that all of the lamp-circuits can be No. 14 wire, which is the smallest size permitted.

Feeders. These should be run on the three-wire system. Allowing for 2 × 47 or 94 lamps in first and second stories and eight in basement, the feeders must be capable of supplying 102 lamps. Each 40-watt lamp would take 40/110 = 0.364 ampere. The distance from outside the building to distribution-cabinet is about 72 ft, allowing for three drops. Using Formula (1), and assuming that there will be fifty-one lamps on each side of the three-wire system, and doubling the drop in volts, gives

Circular mils

$$= \frac{10.4 \times 144 \times 0.364 \times 51}{4}$$

$$= 6\,960 \text{ c.m.}$$

which calls for No. 11 wire; but as this size is not carried in stock we must use No. 10. From the second story to the third No. 12 wires could be used. For almost all buildings lighted from a central station the lamp-circuits will not usually require a wire larger than No. 14, so that about the only wires which the architect needs to look after are the wires which run to the distribution-cabinets.

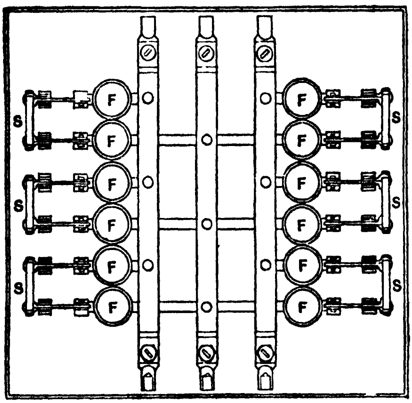


Fig. 12. Cabinet-wiring for Knife-switch Control

Switches. A switch is a device for opening or closing a circuit at will either at the fixture or at any other point. In the better class of buildings the majority, if not all, of the ceiling lights are controlled by switches placed at a convenient place on a side wall. Lights may be controlled at any distance from the fixture by running a switch-loop. For controlling either a single lamp or fixture, or any number of lamps, a switch-loop is run as shown on circuits *A* and *C*, as in Fig. 11. As shown also in Fig. 3, one side of the loop must be connected with one of the distributing wires and the other side to the lamp. When a number of lamps are to be controlled by one switch, as in the case of hall-lights, and the lamps in large rooms, such as churches, theaters, concert-halls, etc., a separate circuit is usually run for those lamps, and a

switch anywhere in one of the distributing lines will turn on or off all of the lights on that line. It is also practicable to control one lamp from two or three places. Thus by a duplex or three-point switch and proper wiring, a lamp may be lighted or turned off from either the first or second story at will. By means of two three-point switches and one four-point switch a first-story hall-lamp may be controlled at will from either the first, second or third stories. Fig. 13 shows the method of control from any number of points, since any number of 4-point snap-switches, such as *B*, *C* and *D*, can be inserted between the 3-point switches *A* and *E* if more points of control are needed. Fig. 14 shows one method of wiring for controlling a hall-light from first and second stories by means of two 3-point switches. With the switches in the position shown the circuit is broken, as there is no connection between the lamps and line *B*. By turning either switch a connection is made with line *B* and the current will flow.

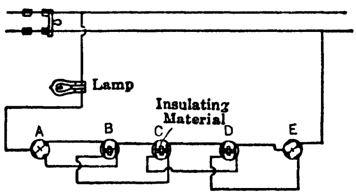


Fig. 13. The Lamp May be Turned Off or On From Any of the Five Points, *A*, *B*, *C*, *D*, or *E*

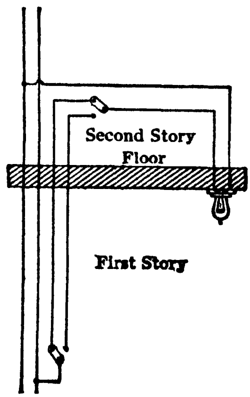


Fig. 14. The Lamp May Be Turned Off or On From Either the First or Second Story

Kinds of Switches. For controlling lamps from one point three kinds of switches are used, namely, SNAP-SWITCHES, FLUSH or PUSH-BUTTON SWITCHES and KNIFE-SWITCHES. When less than eight lamps are controlled by the switch, a flush or push-button switch is commonly used where a neat appearance is desirable, and in places where this is of no importance, a snap-switch is used, as it is the cheaper. Where a circuit of twelve or more lamps is controlled by a switch, a double-pole (d.p.) knife-switch (Fig. 15) is commonly used, being generally placed in a cabinet. Knife-switches should always be used on main wires. Snap and push-button switches are made both single and double pole. A SINGLE-POLE switch opens only one side of the circuit and a DOUBLE-POLE switch both sides. A double-pole knife-switch necessarily opens both sides. A switch used on a three-wire system may have three poles.

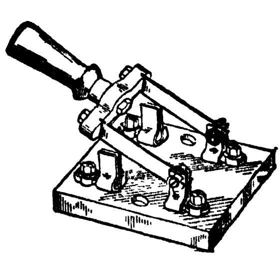


Fig. 15. Common Knife-switch

Methods of Wiring

Various wiring methods have been developed to meet different conditions:

(1) OPEN WIRING is probably the cheapest method and is still used to some extent in industrial work. In dry places and for voltages not in excess of 300, the wires must be separated $2\frac{1}{2}$ in from each other and at least $\frac{1}{2}$ in from the surface wired over. In damp locations the spacing to the surface must be increased to one inch and rubber-covered wire must be used.

(2) CONCEALED KNOB-AND-TUBE WIRING was once the favorite method of house wiring, but has now been largely superseded by armored cable and other high-class methods. The conductors are supported on porcelain knobs and are run in the hollow spaces of walls and ceilings. Wires must be rubber-covered. They must be separated at least 5 in and must be maintained one inch from the surface wired over. Where the 5-in spacing cannot be maintained, the wires are encased in lengths of approved flexible tubing.

(3) RIGID CONDUIT. Since weather-proof and rubber-covered wire cannot be run in brick walls or floors of brick, terra-cotta, or concrete without some protection other than the covering of the wires, it is necessary in such places to run the wires in tubes or conduits. In fire-proof buildings, all the wires are run in conduits or in armored cable.

For rigid conduit systems only mild steel piping of the same thickness as ordinary gas-piping is approved by the underwriters. The conduit must be continuous from outlet to outlet and must properly enter and be secured to all fittings. It is installed in the same manner as a good job of gas-fitting, except that conduit may be bent to a curve and no elbow can be used having less than a $3\frac{1}{2}$-in radius for the inner edge. Wherever branches are taken off, junction boxes must be provided, and every outlet must have an approved outlet box and plate. The wires must be No. 14 or larger, rubber-covered. The conduit system must be grounded.

Rigid conduit is undoubtedly the best form of wiring, though it is also the most expensive on account of the labor cost for bending, threading, etc.

(4) FLEXIBLE ARMORED CONDUIT is made of steel ribbon wound spirally. Its advantage over rigid conduit is that it is so much easier to install. The same outlet boxes and fittings are used as for rigid conduit, and the complete conduit system is grounded.

(5) METAL RACEWAYS are not approved for wires larger than No. 8 and must not be fused for more than 30 amps. Rubber-covered wire must be used and must be continuous from outlet to outlet. Metal raceways must be grounded.

(6) FLEXIBLE ARMORED CABLE consists of two or three rubber-covered wires protected by a spiral steel tape. It may be used in open or concealed wiring in dry locations. It may be fished and may be embedded in the plaster finish. For very damp locations, a special cable having a lead sheath under the protective armor is available. Steel covering must be grounded.

(7) NON-METALLIC SHEATHED CABLE is used only in residences or offices in residential neighborhoods. It must not be installed in masonry, concrete, or where exposed to the weather. It is comparatively cheap and easy to install.

National Electrical Code. The National Board of Fire Underwriters has prepared a code of rules and requirements for the installation of electrical lighting which is the generally recognized standard and with which all interior wiring must comply if it is desired to obtain insurance on the building. This code has also been made a part of the ordinances of most of the larger cities. The National Board of Underwriters also publishes, semi-annually, a SUP-

PLEMENT to the National Electrical Code which contains a list of all articles that have been examined and approved for use in connection with the code, together with the names of the manufacturers. Articles not included in this list will not be passed by the inspectors. Copies of the code and supplement can be obtained from the nearest Underwriters' Inspection Bureau, or by writing to the Underwriters' Laboratories, 207 E. Ohio St., Chicago; 109 Leonard St., New York; or 205 Merchants' Exchange, San Francisco. The following requirements apply to almost every installation, and every architect should be conversant with them.

Extracts from the National Electrical Code *

(1) All wire for concealed work must be of the best approved rubber-covered brands No wire smaller than No. 14 B. & S. gauge is to be used. All wire run in conduits must have double-braid covering.

(2) Where wires are concealed and run parallel to joists they must be supported on porcelain knobs which hold the wires at least 1 in from woodwork or surface wired over. Knobs must be SECURELY FASTENED and MUST BE PLACED EVERY 4½ FT. Where wires are run through joists they must be bushed with porcelain tubes the entire width of joists. All wires must be drawn tight, so as to have all slack removed.

(3) In concealed work all wires MUST BE SEPARATED FROM EACH OTHER BY AT LEAST 5 IN. Where wires run down partitions, especially partitions formed by 2 by 4-in studs, the wires must be so supported as to run in the middle of partition. If more than two wires are run down partition between studs, they must be separated by at least 5 in.

(4) Where wires pass through floors they must be protected from the floor up to a point 5 ft above the floor with conduit or with boxing. There must always be a space of 1 in between the wires and the boxing.

(5) All joints must be securely soldered and taped. A splice to be approved must be both mechanically and electrically secure without solder, but must be soldered unless made with some form of APPROVED splicing-device. Joints to be properly taped require, where rubber-covered wire is used, first to be taped with rubber tape and then with friction-tape. The insulation of a joint must equal that on the conductors.

(6) Where wires enter the building they must be provided with drip-loops.

(7) There must be a MAIN CUT-OUT AND SWITCH installed in an easily accessible place, as near as possible to the point where the wires enter the building.

General Suggestions for Electric Work †

General Principles and Recommendations. In all electric-work conductors, however well insulated, should always be treated as bare, to the end that under no conditions, existing or likely to exist, can a grounding or short circuit occur, and so that all leakage from conductor to conductor, or between conductor and ground, may be reduced to the minimum. In all wiring special attention must be paid to the mechanical execution of the work. Careful and neat running, connecting, soldering, taping of conductors, and securing and attaching of fittings, are specially conducive to security and efficiency, and

* The numbers here given do not correspond with those in the code, and several of the rules are much abridged. They are intended to give the substance, rather than the exact language.

† Preface to the National Electrical Code.

will be strongly insisted on. In laying out an installation, except for constant-current systems, the work should, if possible, be started from a center of distribution, and the switches and cut-outs, controlling and connected with the several branches, be grouped together in a safe and easily accessible place, where they can be readily got at for attention or repairs. The load should be divided as evenly as possible among the branches, and all complicated and unnecessary wiring avoided. The use of wireways for rendering concealed wiring permanently accessible is most heartily indorsed and recommended; and this method of accessible concealed construction is advised for general use. Architects are urged, when drawing plans and specifications, to make provi-

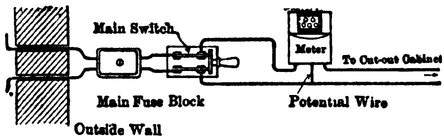


Fig. 16. Main Switch, Fuse-block and Meter Located Near the Point of Entrance of the Service-wires

sion for the channeling and pocketing of buildings for electric-light or power-wires. Fig. 16 shows a common arrangement of main cut-out, switch and meter.

Specifications for Interior Wiring *

Specifications for Interior Wiring should provide:

(1) That the wiring shall be installed in accordance with the latest rules and requirements of the National Board of Fire Underwriters, the local ordinances, and the rules of the local electric light company, where current is to be taken from the public mains.

(2) No electrical device or material of any kind to be used that is not approved by the Underwriters' National Electric Association, and all articles must have the name or trade-mark of the manufacturer and the rating in volts and amperes or other proper units marked where they may readily be observed after the device is installed.

Requirements (1) and (2) are sufficient to insure a SAFE installation.

(3) Contractor must obtain a satisfactory certificate of inspection from the city inspector or from the inspector of the local board of fire-underwriters.

(4) If the wires are to run in a conduit system it should be so specified. When a conduit system is used, THE WIRES SHOULD NOT BE DRAWN IN until all mechanical work as far as possible is completed. It is best to wait until after the plastering is dry. All conduit systems must be GROUNDED.

(5) Size of Wires. The best method is to specify the size of all wires, no wire to be less than No. 14 B. & S. gauge; but if the architect does not care to do this, the following clause is sufficient, provided he can have confidence that the contractor will comply with it: "All wires must be of such size that the drop in potential at farthest light-outlet shall not exceed 2% under maximum load."

(6) Cut-out cabinets and where they are to be placed; also location of mainline cut-out and fuse. For buildings containing not more than forty

* Wiring specifications for buildings having their own generating plant should be prepared by an expert.

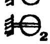
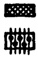
lights, one distributing point is generally sufficient, although in large houses it is often convenient to have a cut-out cabinet on each floor

(7) Number and kind of switches. All outlets should be marked on the plans, and the number of lights indicated by figures 1, 2, 3, 4, etc., as in Fig. 11. See Standard Symbols. The location of all switches for controlling lights should also be indicated on the plans.

Approximate Cost of Wiring for Incandescent Lighting. Approximate estimates of the cost of wiring buildings for electric lighting are usually based on the number of outlets (not lamps). The actual cost will depend upon the number of pounds of wire required, the kind and number of switches, character of cut-out cabinets, etc., and the time required to do the work, so that a close estimate cannot be made without plans and specifications. Again, wages and prices of material vary to a considerable extent in different parts of the country, so that an estimate that would be about right for one locality would not suffice for another.

Standard Symbols for Wiring Plans

As recommended and adopted by the Association of Electragists, International, The American Institute of Architects and the American Institute of Electrical Engineers, and approved by the American Engineering Standards Committee.

	Ceiling outlet.
	Ceiling outlet (gas and electric).
	Ceiling lamp receptacle. Specifications to describe type such as key, keyless or pull chain.
	Ceiling outlet for extensions.
	Ceiling fan outlet.
	Wall bracket.
	Wall bracket (gas and electric).
	Wall outlet for extensions.
	Wall fan outlet.
	Wall lamp receptacle. Specifications to describe type such as key, keyless or pull chain
	Single convenience outlet.
	Double convenience outlet.
	Junction box.
S^1	Local switch—single pole.
S^2	Local switch—double pole.
S^3	Local switch—3 way.
S^4	Local switch—4 way.
S^D	Automatic door switch.
S^K	Key push button switch.
S^E	Electrolier switch.
S^P	Push button switch and pilot.
	Motor.
	Motor Controller.
	Lighting panel.
	Power panel.
	Heating panel
	Pull box.
	Cable supporting box.
	Meter.
	Transformer.
———	Branch circuit, run concealed under floor above.
- - - -	Branch circuit, run exposed.
———	Branch circuit, run concealed under floor.

Standard Symbols for Wiring Plans (Continued)

As recommended and adopted by the Association of Electragists, International, The American Institute of Architects and the American Institute of Electrical Engineers, and approved by the American Engineering Standards Committee.

II	This character marked on tap circuits indicates 2 No. 14 conductors in ½-inch conduit.
I II	Indicates 3 No. 14 conductors in ½-inch conduit.
II II	Indicates 4 No. 14 conductors in ¾-inch **conduit** unless marked ½-inch.
I II II	Indicates 5 No. 14 conductors in ¾-inch conduit.
II II II	Indicates 6 No. 14 conductors in 1-inch conduit unless marked ¾-inch.
I II II II	Indicates 7 No. 14 conductors in 1-inch conduit.
II II II II	Indicates 8 No. 14 conductors in 1-inch conduit.

NOTE. If larger conductors than number 14 are used, use the same symbols and mark the conductor and conduit size on the run.

———	Feeder run concealed under floor above.
----	Feeder run exposed.
— —	Feeder run concealed under floor.
	Annunciator.
	Interior telephone.
	Public telephone.
	Clock (secondary).
	Clock (master).
	Time stamp.
	Electric door opener.
	Watchman station.
	Watchman central station detector.
	Public telephone—PBX switchboard.
	Interconnection telephone central switchboard.
	Interconnection cabinet.
	Telephone cabinet.
	Telegraph cabinet.
	Special outlet for signal system. As described in specifications.
	Battery.
—·—	Signal wires in conduit. Concealed under floor.
—··—	Signal wires in conduit. Concealed under floor above.

CHAPTER XXXV

ACOUSTICS OF BUILDINGS

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Introduction. The modern science of the ACOUSTICS OF BUILDINGS began with the work of Wallace C. Sabine about 1895. In a series of fundamental, painstaking experiments, he lifted the fog of mystery that hung over the subject, and showed that action of sound in buildings could be expressed in terms of well-known scientific laws. Because of his pioneer work, and later investigations, it is now possible to prescribe conditions for securing good acoustics in any room; also, to construct walls necessary for adequate sound-proofing with some degree of success. The architectural engineer thus has available a considerable body of data for guidance in the control of acoustics in buildings. For architects, there remains an undeveloped problem in the design of auditoriums which will give conditions favorable for performers in the generation of sound and at the same time will insure comfortable listening for auditors. There are thus two chief problems in the acoustics of buildings: the ACOUSTICS OF ROOMS and SOUNDPROOFING OF ROOMS.

1. Acoustics of Auditoriums

Action of Sound in a Room. When a speaker or musician appears before an audience, the sounds he creates proceed outward in spherical waves until they strike the boundaries of the room. Here they are reflected, transmitted

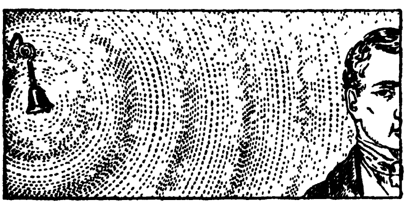

Fig. 1. Diagram Showing How Sound Travels in Compressional Waves from the Source to the Listener

and absorbed in varying amounts, depending on the character of the walls. Fig. 2 pictures a pulse of sound in a room 60 ft long and 40 ft wide, $\frac{1}{60}$ second after it has left the speaker at S. The pulse travels rapidly—as fast as a rifle bullet, about 1 120 ft per second at ordinary temperatures—so that, by successive reflections, it quickly fills the room. Fig. 3 pictures the same pulse $\frac{1}{60}$ second later than the position in Fig. 2, and shows the increasing reflections and crossings of the sound. The imagination readily supplies the details after $\frac{1}{10}$ second has elapsed, when sound has been reflected many times

not only from the walls as pictured in Figs. 2 and 3, but also from the ceiling

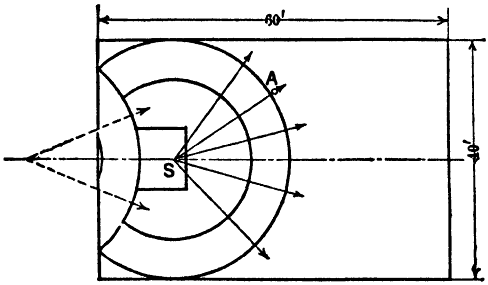

Fig. 2. Pulse of Sound in a Room, 1/60 of a Second after Leaving the Source, *S*

and floor, so that every element of volume in the room is filled with waves proceeding in every direction. This means that the sound has the same average LOUDNESS for all auditors, even those in remote corners.

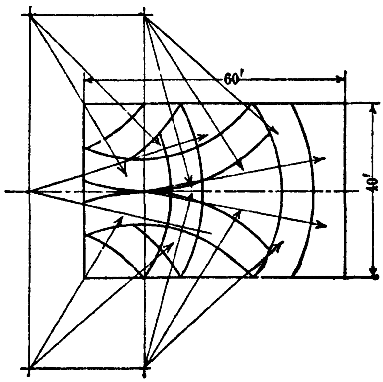

Fig. 3. Pulse of Sound, 3/60 Second after Leaving Source

Sound Waves in an Auditorium. The pulse shown in Figs. 2 and 3 is of much shorter duration than those usually met with in auditoriums. A simple musical tone, for instance, consists of a series of compressions and rarefactions that follow each other regularly. Fig. 4 is a photographic attempt to illustrate a musical sound in an auditorium. The waves pictured are really water waves. A thin metal strip, shaped so as to give a miniature outline of an auditorium, was laid flat on the glass bottom of a tank of shallow water. By directing puffs of air on the water surface, waves were generated that proceeded outward in circles and were reflected from the metal outline. Flashes of light coming up through the glass bottom of the tank cast a shadow of the waves on a screen and allowed a photograph to be taken. These water waves are quite similar in their action to sound waves and present a convenient means for studying the acoustic effect of proposed architectural features in an auditorium, so that objectionable constructions may be redesigned before the final plans are completed.

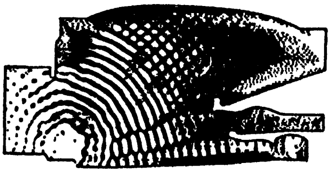

Fig. 4. Photograph Showing How Sound Waves Will Act in an Auditorium

Defects Due to Reflected Sound. While the reflection of sound has the advantage of increasing the loudness somewhat, it is also responsible for most of the acoustics defects in a room, such as ECHOES, RESONANCE and REVERBERATION. It appears of practical advantage to eliminate or reduce the reflected sound, as explained later under Ideal Auditorium Acoustics.

Echoes. When sound is reflected from a surface in the room, particularly a curved surface, auditors will get a distinct confusing repetition of the direct sound that is called an ECHO. This occurs when the time interval between

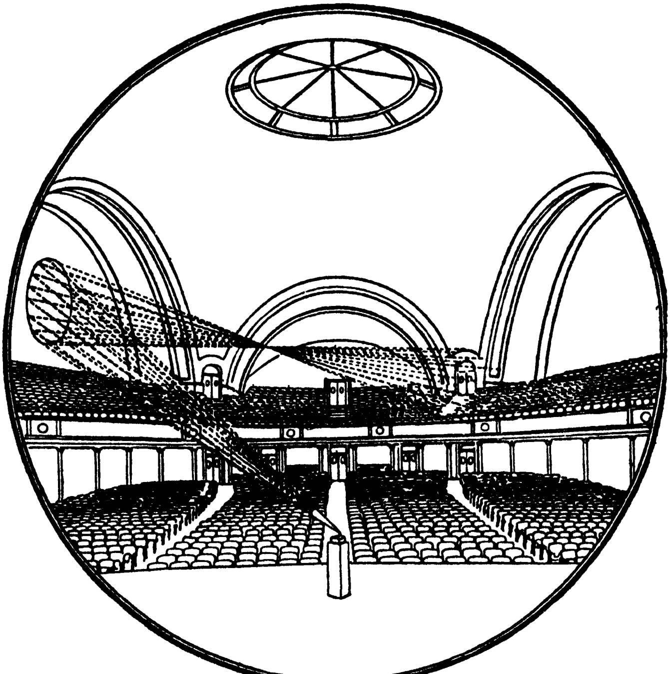

Fig. 5. Echo Set up by Reflected Sound in University of Illinois Auditorium

the direct and reflected sounds is about $\frac{1}{15}$ second or more, which corresponds to a difference in path of these two sounds of about 75 ft. Fig. 5 pictures such an echo. Curved walls are a menace to good acoustics. Fig. 6 shows a concentration of sound due to a curved wall, which will be annoying to auditors at C and D, although the difference in path between direct and reflected sound may not be great enough to produce a distinct echo. The effect of curved walls may be minimized by inserting grills in them, or by coffering them with highly absorbent material, so that the reflected sound is broken up.

Reverberation. The chief acoustic defect in rooms is a reverberation, or undue prolongation of sound. For example, when sound waves strike

plaster walls, only about 3% of the energy is absorbed, so that 97% is reflected. At the next reflection the same action follows, so that as many as 300 to 400

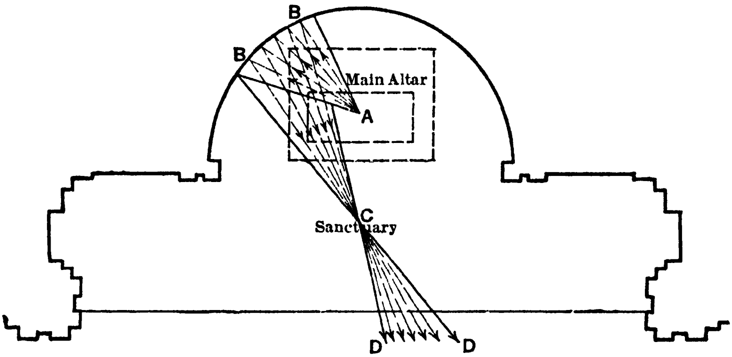

Fig. 6. Objectionable Focusing of Sound by Curved Wall

reflections may take place in a room before the sound energy is used up. The correction for this defect lies in the installation of suitable materials

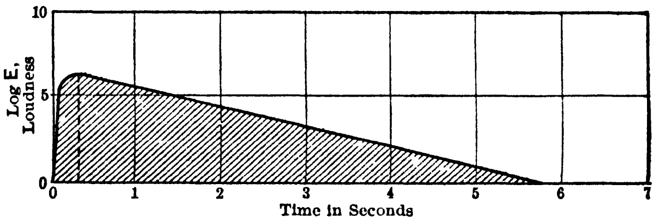

Fig. 7. Objectionable Persistence of Sound in a Reverberant Room

which absorb more energy at each reflection and cause the sound to die out more quickly. Fig. 7 gives a diagram to illustrate, and shows that the loud-

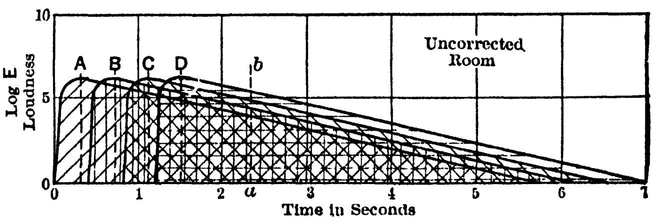

Fig. 8. Confusing Overlap of Four Words of a Speech in a Reverberant Room

ness of a spoken sound rises quickly to a maximum value in about 0.3 second, but that about 5 seconds are taken for it to die out. Fig. 8 shows the con-

fusing overlap of four words in a speech and makes clear why people are unable to understand the jumble of sounds sét up

Fig. 9 shows the same words in the same room, Fig. 8, after it has been corrected with absorbing materials.

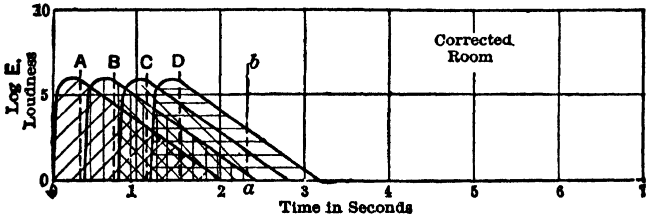

Fig. 9. Diagram Showing Beneficial Reduction of Overlap in Speech Sounds in a Corrected Room

Correction of Reverberant Rooms. The problem of correcting a reverberant room involves a calculation of the volume and absorbing values which are substituted in an equation developed by Wallace C. Sabine:

$$t = 0.05\ V/as$$

in which t = time in seconds for a standard sound to die out;
0.05 = a constant for standard loudness;
V = volume of the room in cubic feet;
as = absorption of all the surfaces and objects in the room.

The factor as is found by adding the absorptions of the different surfaces in the room as indicated by the equation:

$$as = a_1s_1 + a_2s_2 + a_3s_3 + \ .$$

where the surfaces s_1, s_2, s_3, . . . and likewise the coefficients a_1, a_2, a_3, . . . refer to the different materials. When the absorption as becomes large so that the average coefficient a exceeds the value 0 25, it is replaced by the factor $-\log_e (1 - a)$ as required by Eyring's equation. This means that the desired time of reverberation may be brought about by using a smaller amount of absorbing material.

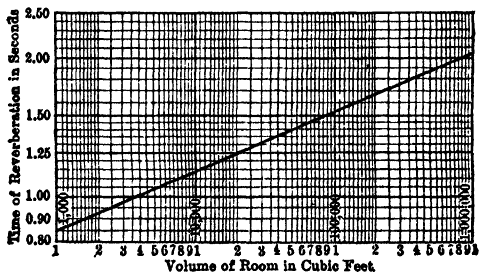

Fig. 10. Optimum Time of Reverberation for any Room

Optimum Time of Reverberation. A study of a large number of auditoriums gave data leading to the graph drawn in Fig. 10, which gives average values for the time of decay of a sound of the frequency 512 cycles per second for audi-

toriums of different volumes. It should be noted that the time in any auditorium should be short, not exceeding 2 seconds for a large volume of 1 000 000 cu ft.

Optimum Time of Reverberation for Different Types of Rooms *

Volume (cu ft)	Churches	Theaters and School auditoriums	Sound pictures
100 000 – 200 000	1.4 ±0.3 sec	1 2 ±0 3	1.2 ±0.2
200 000 – 400 000	1.6 ±0.3 sec	1.3 ±0.3	1.3 ±0.2
400 000 – 800 000	1 75 ±0.3 sec	1 4 ±0.3	1 4 ±0.2
800 000–1 000 000	1.85 ±0.3 sec	....	1.6 ±0 2

These values are for the frequency of 512 cycles per second, with capacity audiences present.

Table I.† Coefficients of Sound Absorption for the Average Pitch of 512 Vibrations per Second

Material	Coefficient per sq ft
Brick wall, painted	0 017
Brick wall, unpainted	0 03
Carpet, unlined	0 15–0.20
Carpet, felt-lined	0.20–0.35
Fabrics, hung straight:	
Light, 10 oz per sq yd	0.11
Medium, 14 oz per sq yd	0.13
Heavy, draped, 18 oz per sq yd	0.50
Openings:	
Stage, depending on furnishings	0 25–0 75
Deep balcony, upholstered seats	0 50–1 00
Grills, ventilating	0 15–0 50
Plaster, gypsum or lime, smooth finish on tile or brick	0 025
Plaster, gypsum or lime, smooth finish on lath	0 03–0.04
Plaster, gypsum or lime, rough finish on lath	0 06
Glass	0 03
Marble or glazed tile	0 01
Wood panelling	0.06
Floors:	
Concrete or terrazzo	0 015
Wood	0 03
Linoleum, asphalt, rubber, or cork tile on concrete	0.03–0.08
Seats and Audience	
Metal or wood chairs (units per seat)	0 17
Auditorium chair, wood veneer seat and back	0.25
Wood pews	0.4
Pew cushions	1.45–1.90
Theater chairs, upholstered in leatherette	1.6
Theater chairs, heavily upholstered, plush or mohair	2.6–3.00
Seated audience, per person (depending on character of seats)	3.0–4.3

* *Theory and Use of Architectural Acoustical Materials*, published by the Acoustical Materials Association, and republished with their permission.

† Copied from Bulletin II of the Acoustical Materials Association.

Absorbing Coefficients. Before installing absorbing materials to reduce the time of reverberation, it is necessary to know the absorbing value of various products that may be used in a room. A large number of these have been determined experimentally by Wallace Sabine and later investigators as shown in Table I. A perfect absorber is one that absorbs all the sound falling on it, so that its coefficient is 1. A material with the coefficient 0.50 will absorb half the sound incident upon it, or 50%.

Inspection of Table I shows that plaster, brick, wood and glass—the materials most commonly used for the interior surfaces of rooms—have small absorbing values, and account for the defective reverberation so prevalent in modern buildings. Porous materials such as hairfelt, carpets, fiber boards, cork, etc., as well as certain porous acoustic plasters, are usually good absorbers. The sound energy is converted into heat by friction in the pores of materials. An audience is a good absorber of sound on account of the clothing worn, and must always be considered in the acoustic correction of rooms.

Numerical Example of Acoustic Adjustment of a Room. Consider a room of 44 000 cu ft volume, for which optimum acoustics are desired when an average audience of 100 people is present. The optimum time of reverberation for a volume of 44 000 cu ft (Fig. 10) is 1.37 seconds, so that the absorption a needed according to Sabine's equation is:

$$1\ 37 = 05 \times 44\,000/a$$

from which, a = 1 610 units. The absorption of the room is as follows:

Open windows	30 sq ft at 1 00 =	30 units
Plaster on tile walls	6 000 sq ft at 0 025 =	150 units
Plaster on lath	3 000 sq ft at 0.033 =	99 units
Wood floor (varnished)	3 000 sq ft at 0.03 =	90 units
Seats 300	at 0.15 =	45 units
Total empty room		414 units
One-third audience, 100 at (4.7–0.15) *		455 units
		869 units

* An auditor adds only the difference between his absorption, 4.7 units, and the seat he covers, 0.15 unit, which has already been counted.

The room thus has 870 units, in round numbers, and will be too reverberant. It will be necessary to add 740 units of absorption to give the 1 610 units desired for optimum acoustics. The acoustic adjustment may be made in several ways, depending on the circumstances.

Solution 1. Install 1 580 sq ft of material at 0.5 = 740 units. This could be applied in decorative panels on the ceiling.

Solution 2. Install 3 700 sq ft of acoustic plaster at 0.2 = 740 units. Plastering the ceiling and the upper side walls would give the desired absorption.

Solution 3. Install 300 upholstered seats at 2.5 = 750 units.

Solution 4. (*a*) 300 upholstered seats at 1.6 = 480
(*b*) 1300 sq ft acoustic plaster at .2 = 260
— 740 units.

It is usually better to spread the absorption over some area, rather than concentrate a highly absorbent material on a small area. Upholstered sets make a room less dependent on the audience; if an upholstered seat is unoc-

cupied, it presents considerable absorption; if it is occupied the auditor adds the absorption of his clothing.

Selection of Absorbing Materials. When selecting a material for the acoustic adjustment of a room, several considerations should be held in mind.

(1) **The Absorbing Value of the Material.** To be effective, the material should be porous, preferably with rather large openings at the surface that allow sound to penetrate to the interior, where absorption takes place in the fine pores by reason of friction between the vibrating air particles and the walls of the pores. The absorption depends on the pitch of the sound, being a maximum usually for 1 000 to 2 000 vibrations per second. In computing the acoustic correction of rooms, the coefficients for the pitch 512 are used, as shown in Table I, since this value is found to be a satisfactory average of the coefficients for all the pitches. When comparing the cost of different materials, the absorption coefficient should be considered For example, 1 000 sq ft of a material of coefficient 0.5 gives 500 units of absorption ($1\ 000 \times 0.5 = 500$), while a material of 0.25 coefficient would require 2 000 sq ft to give the same number of units ($2\ 000 \times 0.25 = 500$).

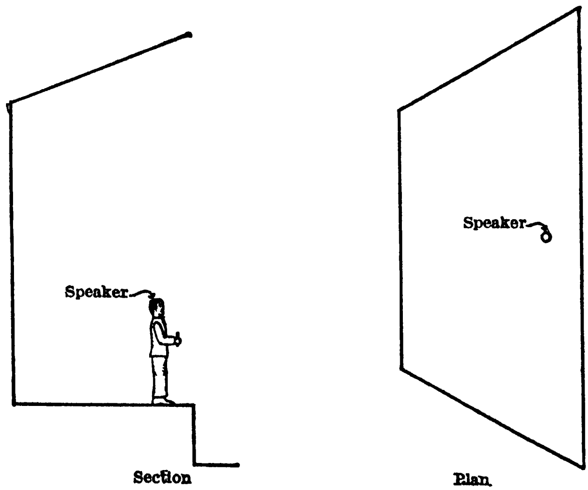

Fig. 11. Plan and Section of Reflecting Walls about Speaker or Musician to Give Perfect Generation of Sound

(2) **Painting or Redecorating Materials.** It is necessary to keep the pores of the material open if it is to retain its acoustic value. Materials with large channels or perforations can be painted without acoustic loss if these holes are not closed. Otherwise it is necessary to use special sprays in decoration.

(3) **Standard Materials.** Materials made at a factory and shipped for installation are more likely to have standard values than those made up on the job, where the skill of the workmen determines the efficiency.

Ideal Auditorium Acoustics. There is a need and desire for perfect acoustics in auditoriums, but the construction of such rooms is not common

because of lack of the necessary guidance. It has already been shown that the reflection of sound to auditors in auditoriums is responsible for most of the acoustic defects. The radical question then arises, is it possible to get rid of the reflected sound? The answer is, yes, as illustrated by an open-air theater; and these theaters always have good acoustics. The problem then is to make the indoor theater as dead as outdoors by the installation of sound-absorbing materials on the walls and ceilings, with upholstered seats on the floor. But will this not make the room too dead? No, provided certain reflecting walls are provided for the performers. Recent experiments show that speakers and musicians find it easy to talk or sing if they can "hear themselves" by the sound reflected from nearby walls. These walls should be as substantial as possible (but heavy painted canvas surfaces can be used) and should preferably not be more than 25 ft distant from the performer. Fig. 11 gives the plan and section of a suggested simple stage for satisfactory generation of sound by a performer.

There are two requirements for an ideal auditorium:

(1) For perfect GENERATION of sound, the performer should have reflecting walls near by to amplify the sound and allow him to "hear himself." Additional loudness, if necessary, can be obtained by the usual electric loud-speaker amplification.

(2) For perfect RECEPTION of sound by listeners, the reflection by walls and ceiling of a room should be reduced to a minimum by the use of sound-absorbing materials, so that the auditorium will be like an outdoor theater.

2. Soundproofing in Buildings

Need for Soundproofing. The problem of soundproofing in buildings has received an increasing amount of attention in recent years, because of the insistence on the reduction of noise. The demand for quiet rooms in hospitals, hotels, and office-buildings, the desirability of insulating music studios and rooms where disturbing sounds are produced, and the necessity for solving other problems for the control of noise have led to repeated inquiries by architects and builders for reliable information about effective methods of insulating sound.

Sound Insulation a Matter of Uncertainty. A number of perplexing problems are involved in the practical insulation of sound in buildings. The theory of the subject is incomplete, and practical attempts to secure effective soundproofing are not always attended with success, even though the constructions used are in accord with the theory and apparently have the elements of adequate insulation. Sound progresses with facility through the different solid materials in a building structure in paths not easy to trace, and may be heard in positions quite remote from the source. This action, together with the extreme sensitivity of the ear, explain why the insulation of sound is a difficult matter.

Two Types of Sound to be Considered. The general problem of soundproofing involves a consideration of two types of sound. First, there are those, such as violin sounds, that are generated in the air and progress in the air. These are quite effectively stopped by fairly rigid walls that do not have openings for pipes or ventilators. Second, there are vibrations, set up by an unbalanced motor that is rigidly attached to the building structure, that are very difficult to stop, and special devices are required to insulate the motor from the structure.

When sound is incident upon a partition, it is reflected, absorbed and trans-

mitted, the exact amount of each depending on the character of the partition and the incident sound. Fig. 12 pictures these effects.

Transmission of Sound Through a Partition. The sound emitted from the source S fills the room quickly where part of it is absorbed by the walls and part is transmitted into the second room. The transmission through the dividing partition can take place in three ways.

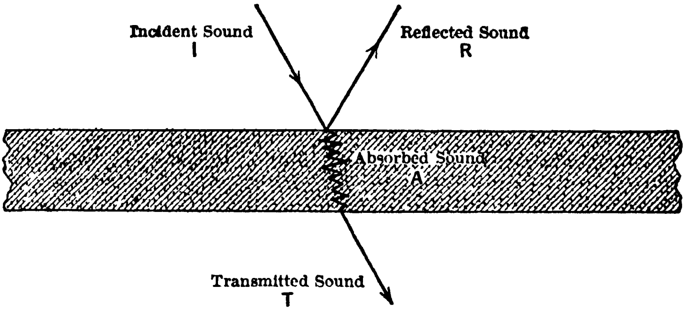

Fig. 12. The Reflection, Absorption and Transmission of Sound by a Partition

(1) It can pass easily through ventilating ducts, through cracks around doors or through other similar openings. Unless these air passages are stopped or guarded in some way, it is useless to build any "sound-proof" construction.

(2) The partition is set into forced vibrations by the rapid pushes and pulls of the sound compressions and rarefactions, so that it sets up sound waves in

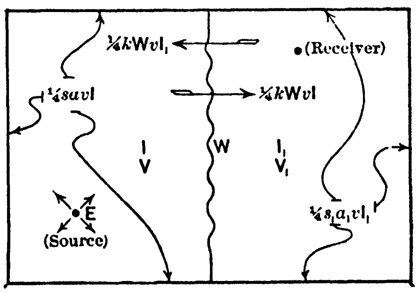

Fig. 13. Diagram Showing How Sound in One Room is Absorbed and Transmitted to Another

the room opposite the source. These vibrations, while very small, averaging about $\frac{1}{2500}$ in in amplitude, are yet sufficient to create audible sounds. It should be remembered that a vibratory motion of air particles of about $\frac{1}{250}$ in corresponds to the passage of a loud sound.

(3) Sound waves can go directly through the partition as an elastic-wave motion in which the pressures and rarefactions are transmitted from particle to particle in the partition and are communicated to the air on the farther

side. But the amount of sound thus transmitted through plaster partitions is very small, since more than 95% of the incident sound is reflected, and only a small fraction remains for possible transmission.

Experience shows that the transmission of sound is greatest through air passages; next in importance comes the vibration of the partition, while least of all is the transmission by elastic waves in the solid material in the partition. A continuous partition, with no cracks or openings, that is heavy and stiff, offers the best possibility of stopping sound. In building-constructions, however, it is desired that partitions be light in weight, easily constructed and cheap; so that the question arises as to how far one can depart from heavy, stiff constructions, and still get adequate soundproofing.

Musical sounds are more difficult to insulate than speech sounds. The tones of music are usually prolonged somewhat, which gives time to set up vibrations of walls and floors. Radio music has introduced a difficult problem of sound insulation in modern dwellings. If neighboring apartments are to be free from disturbing sounds, either the radio must be operated softly, or else the floors and walls must be made more robust.

Loudness of Sounds. To understand the experimental values obtained for different types of partitions, it is first desirable to explain the LOUDNESS of sound as perceived by people The ear is very sensitive to faint sounds, but is equipped with a protective mechanism that makes it more and more insensitive as the sound gets louder. In other words, the ear responds to the logarithm of the intensity rather than the intensity. For example while 100 violins would be 10 times as INTENSE as 10 violins, they would sound only twice as LOUD ($\log 100 = 2$, $\log 10 = 1$).

The loudness values are expressed by acoustical engineers as DECIBELS (abbreviated to db), which are obtained by multiplying the logarithm of the intensity by 10. Thus a standard sound, 1 000 000 times as intense as a sound barely audible, would have a loudness of 60 decibels ($10 \times \log 1\,000\,000 = 60$ db).

A scale of loudness is shown in Fig. 13, which gives the values of some usual sounds in terms of decibels. The zero of this scale is the loudness of sound that is barely perceptible; ordinary conversation between persons about 3 ft distance from each other varies from 35 to 65 db, an airplane has the loud sound of about 95 db, while a sound of 108 db is the loudest that the ear can stand without pain. This scale is based on an average complex sound given by the Western Electric 3A audiometer. For sounds of very low or very high pitch, the scale would not be applicable

Experiments on Transmission of Sound through Partitions. Experimentally, in determining the transmission efficiency, the procedure is to measure the intensity I' of the incident sound in front of the partition and also the intensity I'' on the farther side after the sound is transmitted. Results are expressed either as the REDUCTION FACTOR, I'/I'', or as the logarithm of I'/I''. To be effective, a partition should reduce the intensity at least 1 000 fold, so that an incident sound of intensity $I' = 1\,000\,000$ should be reduced to $I'' = 1\,000$, after transmission. The logarithm I'/I'' is thus 3, or 30 decibels reduction. Referring to Fig. 14, this would mean that a sound of 60 decibels would be reduced to 30 db, or 50 decibels to 20, etc.

Transmission of Sound by Floors. It appears that sound is more easily transferred through floors than through partitions. This is due to actual contact of sound-producing bodies with the floor, such as pianos, radios, persons walking, etc. Tapping a wall or floor with a solid object will produce a sound that is easily heard in the room below, while talking or clapping one's

hands generates a sound in air that has greater difficulty in penetrating the wall or floor. For this reason, it is desirable that the floor be made of more robust construction than walls, and fortunately this is usually the case, because the floor must be built strong enough to carry the weight put upon it.

In this connection, it is interesting to note that in buildings constructed of concrete and masonry, the floors are strong and heavy, while the partitions which are carried by the floors are as light in weight as possible. For this reason, the problem of soundproofing in such a building is more concerned

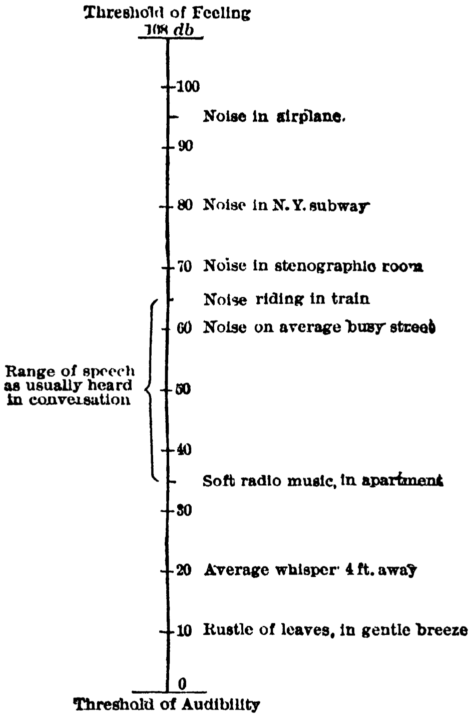

Fig. 14. A Scale of Loudness (Waterfall)

with the partitions than with the floor. With buildings constructed of wood the opposite is true, for in this case the partitions carry the floors, and there is more trouble with transmission through floors than through partitions.

Numerical Example of Transmission of Sound. The effectiveness of a partition in reducing sound may be expressed in terms of the loudness scale. Suppose that conversation of 60 db loudness takes place in a room. The absorption of the walls and furnishings may reduce this 5 db to 55 db. In passing through an effective partition, the loudness may be reduced, say, 30 db, to the value 25 db. The absorption of the second room will further

reduce the sound 5 db to a final value of 20 db, which is so weak that it is scarcely noticed. If, however, the original sound was due to a loud radio of say 80 db, the reduction by the rooms and the partition would be the same number of db as before $(5 + 30 + 5 = 40)$, but the residual loudness would now be 40 db, which would be objectionable.

The quieting effect of materials on the walls of a room is given in an interesting experiment by Chrisler.* "A practical demonstration of the above statement was made by lining a large box with highly absorbing material. The average reduction factor of this box was slightly under 40 sensation units. The box was placed in a very reverberant room. A person inside the box could understand everything said by a person outside, while the person outside was unable to understand a single word that was spoken by the person inside unless he raised his voice. The reason for this was that the absorbing material in the box lowered the intensity of sound originating within to such an extent that it was no longer sufficient to carry through and be audible after transmission."

This example shows that the reduction of sound depends on the absorption of the walls, as well as on the effectiveness of the partition. Davis † has given a formula: $I''/I' = \dfrac{kW}{a_1s_1}$ where k is the transmission per sq ft of the partition; W, the area of the partition; and a_1s_1, the absorption of the room into which the sound is transmitted. Numerical data obtained by different laboratories for I'/I'' for the same type of partition do not agree, probably because the areas of the partitions and the absorption of the receiving-rooms are different. The value of k, however, should have the same value for any condition of test.

Experimental Values of Sound Reduction by Partitions. A great amount of data on sound reduction by partitions has been obtained in different laboratories, but the values of log I'/I'' do not always agree, as just explained. For example, in the following results for wood-stud partitions, the values obtained by the Bureau of Standards with a reverberant receiving-room are about double the Riverbank values where the receiving-room was quite dead acoustically.

Table II. Reduction of Sound by Wood-Stud Partitions *

Lath	Plaster	Coats	Lb per sq ft	10 × log reduction factor	
				Riverbank	Bur. standards
Metal	Gypsum	S & B	17 4	27 0	53 7
Wood	Gypsum	S & B	18.0	27 3	46 9
Wood	Gypsum	S, B, F	18.6	28 0	42 7
Wood	Lime	S, B	17.4	35.4	69.1
Wood	Lime	S, B, F	18.0	35.2	60.8

* P. E. Sabine, Amer. Arch't., August 5, 1926.

Riverbank values are averages for 17 tones from 128 to 4 096 vibrations

* Bureau of Standards, Research Paper No. 48.

† Philosophical Magazine, Vol. 50, page 75, 1925; Vol 2, page 543, 1926

per second. Bureau of Standards values are averages for four frequency bands, 250 to 3 470 vibrations per second.

Some conclusions drawn (P. E. Sabine) are:

There is no important difference in the sound insulation afforded by gypsum plaster laid on metal lath and the same material laid on wood lath;

Compared with similar tests on masonry partitions, the wood-stud construction gives the same sound reduction as an all-masonry wall of equal weight;

Oral tests showed that speech of conversational loudness could be heard and understood easily through all these partitions.

Sound transmission measurements were taken by Wallace Waterfall * on actual partitions in apartments, as well as on the same types of construction in the laboratory. Measurements were taken by the ear and also by instruments. The following table gives one set of results obtained for wood-stud partitions showing the relative values obtained by three methods: reverberation (ear), audiometric (ear), and instrumental.

Reduction of Sound by Wood-Stud Partitions (Waterfall)

Method	Pitch of sound				
	256	512	1024	2048	Average
Reverberation (P. E Sabine)	22 6	34	40 8	36 6	33 5
Audiometric (ear)	29	34	37	42	36
Instrumental *	29	30	36	43	35

* In this method, the sound was "wavered" on either side of the pitch indicated. The agreement of the average values is gratifying.

The following table † gives data on plaster partitions, for range of pitch 128 to 4 096 vibrations per second. The partitions were 8 ft 2 in by 6 ft 2 in

Table III. Sound Reduction for Plaster Partitions (P. E. Sabine)

Partition	Lb per sq ft	10 × Av. log I′/I″
2-in Gypsum tile, unplastered	10 4	23 6
3-in Hollow tile, unplastered	11 1	24 2
1½-in Plaster on metal lath	13 9	25 3
3-in Solid gypsum tile, unplastered	14 2	26.7
2-in Solid gypsum and 1½-in plaster	15 0	27 2
4-in Hollow clay tile, unplastered	17 0	28 3
2-in Gypsum and 1-in plaster	19 6	29 5
2-in Gypsum and 1¼-in plaster	21 4	30 5
4-in Clay tile and ½-in plaster	22 0	30.7
2½-in Plaster on metal lath	23 2	32 4
3-in Solid gypsum and 1¼-in plaster	25 4	32 8
4-in Hollow clay tile and 1-in plaster	27 0	33 6
4-in Hollow clay tile and 1¼-in plaster	28 8	34 0
3½-in Plaster on metal lath	32.5	36.0
4½-in Plaster on metal lath	41.8	38.2

* Journal of Acoustical Society, Jan., 1930.
† P. E. Sabine. Amer. Arch't, July 4, 1923.

in area, built in between two entirely separate rooms. It is to be noted that the reduction of sound (10 log I'/I'') is proportional to the weight of the partition per sq ft, the heavier partitions thus stopping more sound.

Experimental Work at Bureau of Standards. The transmission of sound through a large number of structures has been measured.* The lightest material tested was wrapping paper, while the heaviest structure was a combination tile floor with a cinder fill and concrete finish, weighing 109 lb to the square foot. For structures that are approximately homogeneous, such as masonry partitions the log I'/I'' is proportional to the logarithm of the weight per unit area. This conclusion, which was also found by P. E. Sabine, means that the sound reduction is more effective with the heavier partitions.

The following table gives some of the results obtained in the investigation just mentioned. (Original paper should be consulted for details.)

Table IV. Sound Reduction by Walls and Floors
(Chrisler & Snyder)

Material	Lb per sq ft	10 × Av. log I'/I''
Hollow clay tile, 3 in thick, plastered both sides	28	44 4
Hollow clay tile, 4 in thick, plastered both sides	29	43 5
Hollow clay tile, 6 in thick, plastered both sides	37	44.6
Hollow clay tile, 8 in thick, plastered both sides	48	49.8
Hollow clay tile, double 3 in partition, 1¾ in air space containing 1 in Flaxlinum.	50	59.2
Brick panel, 8 in thick, plastered both sides .	97	56 7
Single sheet of galvanized iron, 0 03 in thick	1.2	28 3
Single sheet plate glass, ¼ in thick	3.5	32.7

Practical Results Obtained by Various Partitions. As already explained, the numerical results for log I'/I'' obtained by different laboratories do not agree. Some comparison of results is obtained by the following explanations.

At the Bureau of Standards, the data are classified in four groups, as follows:

"**Panels Whose Reduction Factors are Over 60 Sensation Units.** Conversation carried on in an ordinary tone of voice is reduced to inaudibility. If there is external noise in the listening-room, a shout on the other side of the panel would be practically unnoticeable.

"**Panels Whose Reduction Factors Lie between 50 and 60 Sensation Units.** Conversation in ordinary tones heard through the panel is barely audible but unintelligible.

"**Panels Whose Reduction Factors Lie between 40 and 50 Sensation Units.** Conversation in ordinary tones heard through the panel is quite audible but difficult to understand. If the voice is raised, it becomes intelligible.

"**Panels Whose Reduction Factors Are Less Than 40 Sensation Units.** Conversation in ordinary tones heard through the panel is distinctly audible and intelligible."

At Riverbank Laboratories, P. E. Sabine states (see Table III) "speech of conversational loudness could be heard and understood easily through all these partitions." The partitions were wood-stud with sound reduction of 27 to 35 sensation units (decibels).

* V. L. Chrisler and W. F. Snyder. Research Paper No. 48.

Waterfall * gives the following observations:

"With an average reduction factor of 25 db, normal speech can be understood quite easily and distinctly through the partition.

"With an average reduction factor of 30 db, loud speech can be understood fairly well if conditions are quiet.

"With an average reduction factor of 35 db, loud speech is audible, but not easily intelligible under quiet conditions.

"With an average reduction factor of 40 db normal speech is not audible, loud speech can be faintly heard, but not easily understood, and for all practical purposes the partition can be considered as 'sound proof.' Separating partitions between apartments should have a factor of about 40.

"For piano-rooms, organ-rooms, etc., a better partition is sometimes needed. Higher factors than 40 are desirable for such cases."

Acoustic Control of Ventilation System. One of the greatest sources of acoustic trouble in buildings is the ventilating system. Sound travels easily from room to room through these "speaking tubes." An effective means of reducing this transfer of sound is to use small, individual pipes that extend from the supply chamber to each room without the usual large main duct. Large pipes can be padded on the inside to reduce sound. Baffles of sound-absorbing material help. A mushroom cap over open ends of pipes in attics has proved beneficial. Low-velocity fans, with balanced motors, create less disturbing noises than small high-velocity machines.

Vibrations in Buildings. Vibrations may be set up in the building itself by motors, elevators, etc., or they may be originated outside by street-cars, trucks, and trains. Such vibrations are likely to disturb occupants in buildings because of mechanical vibration of the floors, or by the annoying sounds set up by vibrating partitions, by rattling windows and other loose constructions.

These vibrations will be minimized in buildings that are massive in construction. It is helpful to have balanced motors and other quiet-running machinery. Padding placed under machines is helpful, but the vibrations are complex so that successful results are not always obtained.

References for Further Reading

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Speech and Hearing, Harvey Fletcher.
Text-book of Sound, E. H. Barton.
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* Technical Note No. 10a, The Celotex Company.

CHAPTER XXXVI

ARCHITECTURAL SHADES AND SHADOWS

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1. Relation to Architectural Drafting

Relation to Architectural Drafting. The science of all drafting is called DESCRIPTIVE GEOMETRY. It has sometimes been called the grammatical construction of the language of graphics. All the real progress in the growth of that language began with the formulation of this science about the beginning of the nineteenth century. The function of descriptive geometry, as its name implies, is to describe (in the language of graphics) geometric forms in space. Applied to architecture, its general function is to describe, upon a plane surface, the three dimensions of buildings, their details and entourages. In this application it is called ARCHITECTURAL DRAFTING.

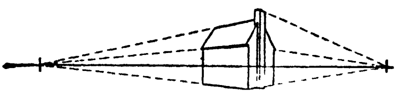


Fig. 1. A Perspective Drawing

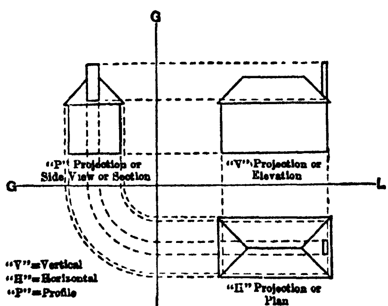


Fig. 2. Ortho-graphic Projection

Architectural Drafting. To accomplish its purpose, ARCHITECTURAL DRAFTING resorts to either of two conventions, called PROJECTIONS, wherein, imaginary projecting lines, called VISUAL RAYS, are conceived as drawn from the lines of the object in space to imaginary receiving planes, and the resulting image is drawn on paper.

These two systems of projections, employed in architectural drafting, are in their chronological order:

(1) CONIC PROJECTION, or PERSPECTIVE DRAWING (Fig. 1). (Isometric, oblique and cabinet drawing, etc., are modifications of perspective drawing which find but little use in architectural drafting.)

(2) ORTHO-GRAPHIC PROJECTION, commonly called MECHANICAL DRAWING (Fig. 2).

A brief description of these two systems will help to explain the statement of the function and importance of each. They differ only in the assumed location of the observer's eye, from which point the imaginary projecting lines, or visual rays, are conceived to be drawn.

Conic projection or perspective drawing, although conventional drafting, makes some concessions to the natural laws of vision. The IMAGE-RECEIVING PLANE is conceived to be a single plane (the picture plane), and the observer's eye to be located at a reasonable finite distance (Fig. 3). The projecting lines (or visual rays) then converge to a point (the observer's eye). When the resulting picture plane projection is drawn, it is called a perspective drawing (Fig. 1). Man's very first attempts to draw were efforts at natural representation, i.e., perspective drawing.

Ortho-graphic projection, on the other hand, is the extreme of conventional drafting. The interesting thing is, that perspective should have developed so far before this system originated. In this system the image-receiving planes are conceived to be the three planes of projection, called the coordinate planes, VERTICAL, HORIZONTAL and PROFILE, in a tri-rectangular relation, and the observer's eye to be located at an infinite distance. The projecting lines, or visual rays, to each coordinate plane, are parallel to each other and

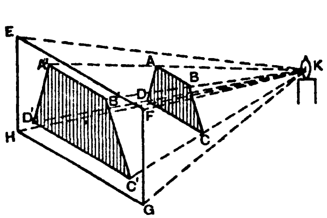


Fig. 3. Perspective or Conic Projection

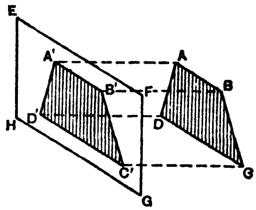


Fig. 4. Ortho-graphic Projection

perpendicular to their particular plane of projection (Fig. 4). When the resulting vertical or V, horizontal or H, and profile or P projections are drawn upon a plane surface, they are called, respectively, ELEVATION, PLAN, and SIDE-VIEW or SECTION (Fig. 2). The terms V projection and elevation, H projection and plan, P projection and side view, are therefore used interchangeably.

Working Drawings and Rendered Drawings. Generally speaking, in the architectural drafting of today, these plans, elevations and perspectives may be divided into WORKING DRAWINGS and RENDERED DRAWINGS.

Working drawings can be dismissed from our consideration here, for no matter how intricate the working drawing may become, it has but one supreme function, that is, to convey information. It must always remain the extreme of careful, conventional drafting for two reasons: (1) to eliminate the least possibility of misinterpretation, and (2) to have the effect of inspiring neat and careful results in work done under its guidance.

Rendered drawings, on the contrary, are intended to convey both information and an impression, and may vary, according to their exact purpose, all the way from simple washes applied to conventional plans and elevations, to the less conventional but more elaborate presentation of a rendered perspective. Indeed, because linear perspective is already a concession to the natural laws of vision, its rendering, in its tendency toward natural representation, may very easily overstep the bounds of conventional architectural drawing. It then becomes a picture instead of a drawing, for its main purpose is to convey an impression rather than to convey information. Aside from the technique, the difference between painting a picture and making a drawing is extremely fundamental. In painting a picture the artist hopes to express

and convey a definite mood, such as awe inspired by the grandeur and majesty of a storm, or serenity created by the peace and beauty of a rural scene. In his picture he is not so much concerned with the outline of a wave or of a tree, or even with the reproduction of what are regarded as exact natural colors. But he is interested with imparting to the beholder the mood which has been created within himself. The degree of success which he attains in this purpose is alone the measure of the artist's ability. A draftsman may be skilled in the technique of pencil and brush and yet lack this final requirement to be an artist.

The Function of Shades and Shadows in Rendered Drawings. However, regardless of how much rendering is applied to architectural plans and elevations to express plane values, textures, colors, etc., they must remain, in their nature, descriptive, conventional drawings conveying accurate information. That is, they must remain the linear drawings of either ortho-graphic or perspective projection, and when, for instance (Fig. 5), in rendering the elevation of a building, the projections of the shade and shadow surfaces of the various architectural features are added, their purpose is not primarily to make a picture, but to express the facade's third dimension. If these shade and shadow surfaces are to fulfil this primary purpose, their outlines must be obtained according to certain fixed rules and conventions, so that their graphic language may also have constant significance and be universally understood. These fixed rules and conventions constitute the SCIENCE OF SHADES AND SHADOWS.

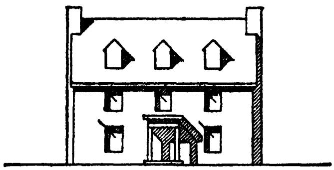


Fig. 5. An Elevation

2. Character and Scope

Character and Scope. The science of shades and shadows, as employed by geometricians, may be said to have two principal purposes: (1) to furnish exercise in developing the mental ability to completely visualize and realize geometric description (sizes, shapes and relative positions) of geometric surfaces in space; (2) to express this mental concept in graphics, by devising means of determining, both accurately and quickly, the outlines of the conventional light, shade and shadow parts of surfaces.

This abstract study of the theory of the science of shades and shadows, as an application of descriptive geometry, is admittedly of greatest fundamental importance to the architectural student, as elementary training in the creative processes of the mind. To be of practical value to him, however, the concrete result of such a study must be the acquisition of a vocabulary of projections of shade lines and shadow lines of constantly recurring architectural forms. Complete understanding of the derivation of this vocabulary assures its intelligent use, and both the rendering and the reading of architectural plans and elevations are greatly facilitated. The development of this vocabulary is really the distinguishing characteristic of that application of the science of shades and shadows which is called ARCHITECTURAL shades and shadows.

The Art of Rendering Architectural Shades and Shadows. In conventional architectural drafting the SCIENCE OF DETERMINING architectural shades and shadows is so closely associated with the ART OF RENDERING architectural shades and shadows that they are very frequently confused. At this point therefore, it is important that their fundamental difference be clearly under-

stood. The division, by lines, of all surfaces into their light, shade and shadow parts, according to certain conventions, is a matter of mathematical precision. The science, the function of which is to devise the processes for determining these lines is called the SCIENCE of shades and shadows. It is an application of descriptive geometry. The realistic modeling of these separate light, shade and shadow parts of surfaces (with their outlines thus carefully determined) to resemble natural light effects is a function of the ART of architectural rendering. No matter how realistic this rendering may become, the original lines of demarcation between the light, shade and shadow parts must be maintained as long as the subject is a conventional drawing and not a picture.

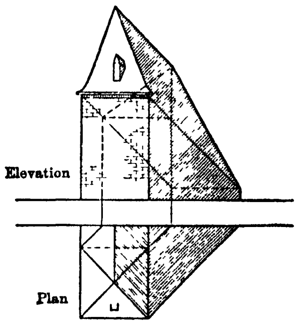


Fig. 6. Prism and Pyramid

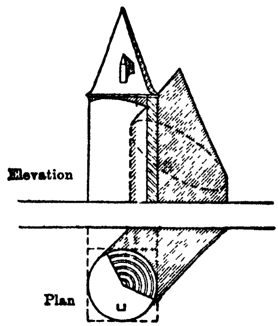


Fig. 7. Cylinder and Cone

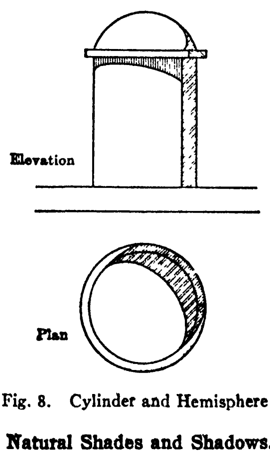


Fig. 8. Cylinder and Hemisphere

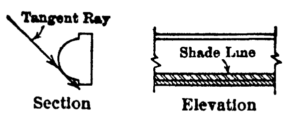


Fig. 7A. Half Round or Bead Molding

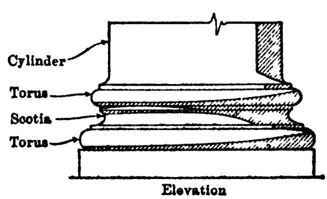


Fig. 9. Attic Base

Natural Shades and Shadows. In nature, a surface becomes visible only when rays of light reflect from it to an observer's eye. Thus it is reflected light which reveals the sizes, shapes and locations of surfaces in space. Hence, any surface exposed to light immediately becomes another source of light.

This reflected light, although less intense than the original direct light (on account of the absorption and refraction of some rays for color, etc.), is misleading and tends to render the perception of the true sizes, shapes and relative positions of adjacent surfaces difficult and sometimes even impossible. Surfaces, IN NATURE, are classified as (1) HIGH LIGHTS, (2) LIGHTS, (3) SHADES and (4) SHADOWS, with all the intermediate gradations which only the human visual mechanism can detect and register. In an effort to express in drawings only the true size and relative positions of adjacent surfaces, and that as emphatically as possible, CONVENTIONAL shades and shadows are employed.

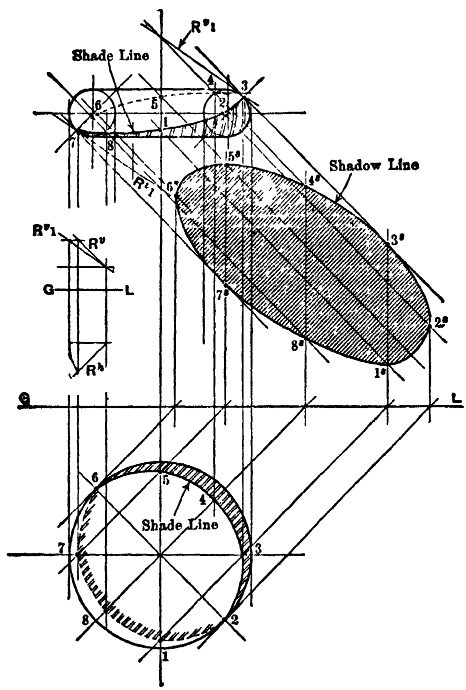


Fig. 9A. Shade Line and Shadow Line of a Torus

Conventional Shades and Shadows. The shades and shadows which are supposed to result when objects are conceived as being exposed to an assumed direct light, which is in a certain fixed position, are called CONVENTIONAL shades and shadows. The amount and direction of the light are then assumed to be constant. The light in conventional shades and shadows is assumed to have but one direct source, i.e., the sun, located at infinite distance, so that the rays may be taken as parallel to each other. It is assumed to be of constant intensity and location, and arbitrarily divides all surfaces, regardless of their geometrical description, into light, shade or shadow surfaces, recognizing no intermediate gradations.

Light, Shade and Shadow Surfaces. Light, shade and shadow being attributes of geometric surfaces when exposed to light, an understanding of the fundamental principles of geometry is essential. It must be remembered at the outset that the points, lines, surfaces and solids of plane and solid geometry are purely ideal. A solid in common language is something real, i.e., a limited portion of space filled with matter; but a geometrical solid is purely an ideal, i.e., a limited portion of space which may be occupied by a physical body, or marked out in some other way. The surface of a solid is simply the boundary of the solid, i.e., that which separates it from surrounding space. The surface is no part of a solid, and has but two dimensions, length and breadth, but no thickness.

Although the general science of shades and shadows considers every conceivable solid of pure geometry, its limited application to architecture must of necessity be restricted to consideration of the simple geometric solids which represent an analysis of architectural detail.

Tabulation. The geometrical forms to which constantly recurring architectural detail (both casting and receiving shadows) can be reduced, by convention, are tabulated in the table and illuminated in Figs. 6, 7, 7A, 8, 9, 9A, 10, 10A, 10B, 10C, 10D, and 11.

	Kind of solid	Kind of surface	Examples of occurrence in architecture
1	Prism	Plane	Plane wall buildings (Fig 6)
2	Pyramid	Plane	Plane roofs (Fig 6)
3	Convex right circular cylinder	Convex single curved	Column shafts, towers, moldings etc. (Figs 7, 7A and 9)
4	Convex right circular cone	Convex single curved	Conical roofs (Fig. 7)
5	Sphere	Convex double curved	Domes, balls, etc. (Fig 8)
6	Torus	Convex double curved	Moldings such as column bases (Figs. 9 and 9A)
7	Hollow right circular cylinder	Concave single curved	Niches, moldings, etc. (Figs. 10 and 10A)
8	Hollow right circular cone	Concave single curved	Moldings and ornament (Fig. 11)
9	Hollow hemisphere	Concave double curved	Moldings and ornament (Figs 10 and 10B)
	Hollow scotia	Concave double curved	Moldings and ornament (Fig, 10D)
	Hollow torus	Concave double curved	Moldings and ornament (Fig. 10C)
	Scotia	Concave double curved	Moldings such as column bases (Fig. 9)

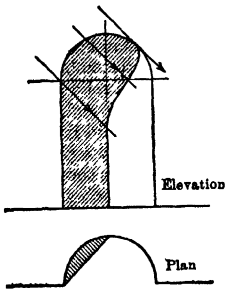


Fig. 10 Hollow Cylinder and Hollow Quarter Sphere

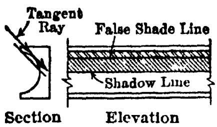


Fig. 10A. Half Hollow or Cavetto

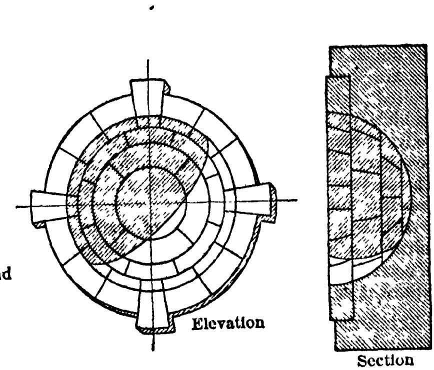


Fig. 10B. Hollow Hemisphere

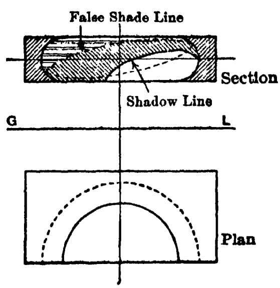


Fig. 10C. Hollow Torus

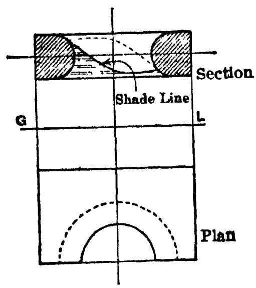


Fig. 10D. Hollow Scotia or Gorge

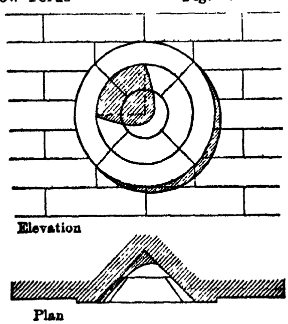


Fig. 11. Hollow Cone

3. Conventions

Conventions. It has been shown that the science of shades and shadows ignores completely the geometrical classifications of surfaces and recognizes but three classes of surfaces: (1) light, (2) shade and (3) shadow (Fig. 14).

In the study of light, shade and shadow surfaces, with reference to their cause, the one outstanding fact of importance to remember is that LIGHT MUST TRAVEL IN A STRAIGHT LINE. Then, however light or dark a surface may appear depends upon its inclination toward the light, and reveals its size and shape. Its position relative to other surfaces is disclosed by the shadows which it receives and casts. Shadows in nature contribute to revealing size, shape and position not only of the objects casting the shadows, but also of the surfaces receiving the shadows.

Source of Light. In its effort to express upon architectural drawings only the true sizes, shapes and relative positions of surfaces, and that as emphatically as possible, SHADES AND SHADOWS assumes but one source of direct light, i e , the sun. This avoids the confusion and difficulties encountered in natural vision caused by various other sources of light (reflected, diffused, or artificial).

Ray of Light. In its effort to standardize the projections of the outlines of the light, shade and shadow surfaces, and thus make their language universal, the convention of shades and shadows assigns a fixed location to the sun, so

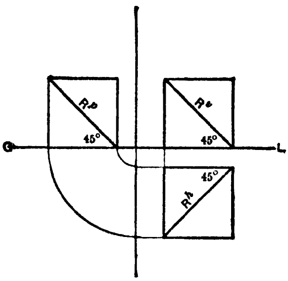


Fig. 12. Projections of the Conventional Ray of Light

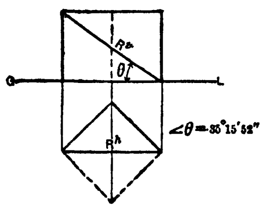


Fig. 13. True Angle of the Conventional Ray of Light

that the conventional rays of light, besides being assumed parallel to each other, arrive at the object in space from a constant direction and at a constant angle. The assumed direction and angle of the conventional ray of light have so many practical advantages that they are, with rare exceptions, universally accepted. In architectural shades and shadows they are practically invariable. It is universal to assume the rays of light of conventional shades and shadows as parallel lines coming over the left shoulder and sloping down toward the right with their projections making 45° with the *GL* (ground line), in other words, parallel to that diagonal of a cube which slopes *d–b–r*, i.e., downward, backward, to the right, when the cube has two faces parallel to each of the three coordinate planes (Fig. 12). The actual angle which the ray of light makes with any one coordinate plane is of course the angle of the diagonal of any cube with its base, 35° 15′ 52″ (Fig. 13). The practical advantage in drafting of having all three projections of the ray of light at 45° with the *GL* is self-evident. Other advantages will become immediately apparent in the later study of the shadows of certain straight lines and circles

4. Definitions and Principles

Definitions and Principles. Fig. 14 illustrates the principles and terms employed in ARCHITECTURAL shades and shadows. An object in space, such as a sphere, is conceived to be exposed only to the direct light of the sun which is assumed in CONVENTIONAL shades and shadows. It is said to cast a shadow upon the surface or part of the surface of a second object when the first object is so situated as to prevent light rays from reaching the surface of the second object The object excluding the light may be entirely external (as in this instance), or it may be an attached projection (as a pilaster or a cornice) (Fig. 15), but it received the light which the shadow surface otherwise would receive Depending upon its position relative to the assumed

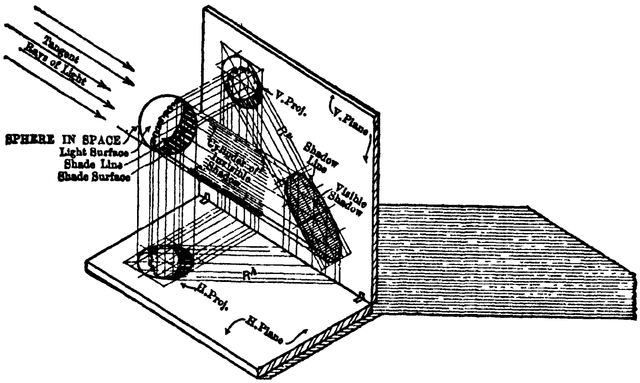


Fig. 14. Light, Shade and Shadow of a Sphere

light and adjacent surfaces, every surface is distinctly classified as light, shade or shadow.

(1) **A light surface** is that portion of the surface of a solid which, because of its shape and position, is reached by direct light (Fig. 14).

(2) **A shade surface** is that portion of the surface of a solid which, because of its shape and position, is not reached by direct light (Fig. 14).

(3) **A shadow surface** is that portion of the surface of a solid which, although faced directly toward the light, receives no direct light because of the intervention of some surface nearer the sun (Fig. 14). (A plane surface to which rays of light are exactly tangent is called a SURFACE OF LIGHT AND SHADE and is treated as a shade surface, the theory being that the first line of the surface is the only one in a position to receive direct light) (Fig. 16).

(4) **Shade Line.** The line dividing the light surface from the shade surface upon a plane or convex curved surface solid is called its SHADE LINE. Obviously it is formed by adjacent points of tangency of successive rays of light (Fig. 14). The shade line of a concave surface is called a FALSE SHADE LINE, because although it is formed by tangent rays of light it does not cast shadows. It divides shade from shadow (Fig. 17).

(5) **Invisible Shadow.** The rays of light coming from the sun are tangent to the object along its shade line. They continue on beyond the object, forming the surface of a geometric solid or INVISIBLE SHADOW (an absence of light in space) (Fig. 14).

(6) **Shadow Line and Visible Shadow.** When this geometric solid of invisible shadow with its surface composed of light rays, reaches the next

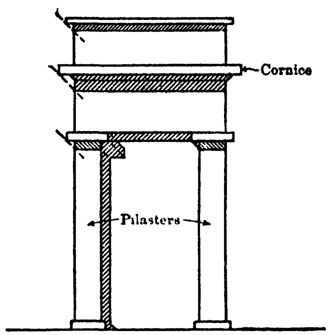


Fig. 15. Shadows Cast by Projecting Parts

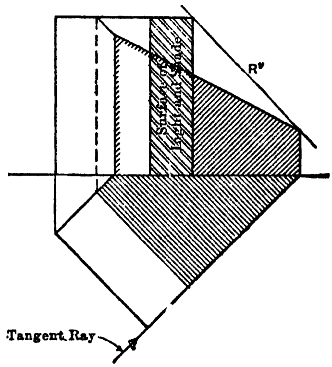


Fig. 16. A Surface of "Light and Shade"

material surface, the outline of their intersection is called the SHADOW LINE and the intersection is called a cast VISIBLE SHADOW (Fig. 14).

Distinction between Actual Shade and Shadow Lines in Space and Their Geometric Projections upon Architectural Drawings. Although we almost invariably, in practice, use the terms SHADE LINE and SHADOW LINE to describe them, actually, in theory, it is the geometric projection of the shade line or of the shadow line which is shown upon architectural plans and elevations. This is the same thing only when they lie in plane surfaces identical with the coordinate planes of projection. Otherwise it must be constantly remembered that, whereas the shade and shadow surfaces are actually part of objects in space, the shade and shadow lines are parts of the projected drawing. To illustrate, a shadow line may be an ellipse in space but a circle in projection (see Figs. 36 and 37), or it may be a broken line in space and projected as a straight line (Figs. 32–33–34–35). Similarly, a shade line may be a circle in space but an ellipse in projection (as the shade line of a sphere, Fig. 14). When the architectural drawings are geometrical plans and elevations, it is, of course, the geometric projections of the outlines of the light, shade and shadows surfaces which we want. When these surfaces, so outlined, are rendered, they express a third dimension upon an otherwise two-dimensional

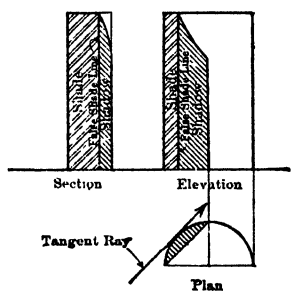


Fig. 17. "False" Shade Lines

drawing. Therefore, since the information which they convey is definite, it is extremely important that they be both accurately found, and clearly and accurately drawn. There can be no compromise; they must be correct. The correct outlines of the shade surfaces (in projection), and the correct outlines of the shadow surfaces (in projection), are just as vital to a rendered architectural drawing as the correct outlines of the geometric surfaces (in projection). They are, in fact, integral lines in the drawing. If the architectural drawing is a perspective, the shade lines and shadow lines are drawn in perspective just as any other line of the drawing.

Fundamental Principles. From the above explanations and definitions, and study of Fig. 14, it is extremely important to realize the following axioms. In casting shadows they should continually be borne in mind, as they are the very fundamental principles.

(1) The SHADE LINE is determined by tangent rays of light; and the SHADOW LINE is determined by incident rays of light.

(2) The SHADOW LINE of any object whatever upon any surface is the shadow of its SHADE LINE.

(3) SHADOWS always fall upon surfaces faced toward the sun, i e., light surfaces.

(4) Surfaces which are turned away from the sun are SHADE SURFACES and incapable of receiving shadow.

(5) Any surface or portion of a surface already in shade or shadow cannot cast shadow because of the absence of light.

(6) For any surface in light there is a shadow.

(7) The shape of the VISIBLE SHADOW which is cast by any object in space upon any surface depends upon three factors alone, as follows: (*a*) the shape of the surface of the object which is casting the shadow; (*b*) the shape of the surface of the object receiving the shadow; (*c*) the relation of both these surfaces to the direction of the assumed source of light.

5. Theory of Casting Shadows

Theory of Casting Shadows. In attempting to cast ARCHITECTURAL shades and shadows it must, above all, be clearly understood that the SHADOW LINE of any solid, whether its surface be plane or curved, is actually nothing more than the shadow of a certain line in that surface, i.e., its SHADE LINE (Fig. 14). Then it can readily be seen that the work of finding the shadow of any solid is equivalent to finding the shadow of a certain limited portion of its surface. The SHADOW LINE of a surface or part of a surface is obviously the shadow of its boundary line, but the shadow of any line is only the shadows of the points in that line. Therefore, in the final analysis, the important fundamental step to which all shadow-casting is finally reduced is FINDING THE SHADOWS OF POINTS. Equipped with the knowledge of how to find the shadow of any point upon any surface, one is prepared to cast the shadow of any line upon any surface, and finally therefore of any surface upon any other surface.

Methods of Casting Shadows. The shadow of a point upon any surface whatever is never anything but the point where a ray of light drawn through that point in space pierces the surface whereon the shadow falls (Fig. 18). There are but two methods of finding the shadows of points (and hence of lines and surfaces): first, the method of OBLIQUE PROJECTION and second, the SLICING method.

When the surface receiving the shadow is of such kind and position that it can be projected as line, i.e., plane or single curved and perpendicular to V, H or P, then the method is an elementary problem in descriptive geom-

etry. It is called the method of OBLIQUE PROJECTION, and it never varies. When, however, the plane or single curved surface receiving the shadow is oblique and cannot be projected as a line in V, H or P, or is a double curved surface which can never be projected as a line, then the SLICING method must be used.

To Find the Shadow of a Point by Oblique Projection. This method consists simply of passing an OBLIQUE line through the point, parallel to the conventional ray of light, and the point where this line strikes the intercepting

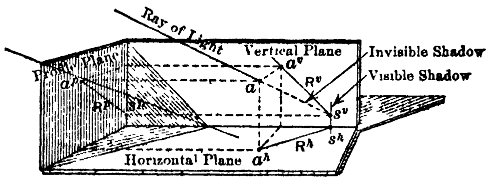


Fig. 18. The Shadow of a Point

or receiving surface is the visible shadow of the point (Fig. 18). The part of the line between the point in space and its visible shadow is called its invisible shadow. The line which is the projection of the shadow receiving surface becomes the GROUND LINE (GL) for the shadows.

Geometric Demonstration. Definition. The shadow of a point is the trace of a ray of light drawn through that point.

The simplest illustration of this method of finding the shadow of any point upon any surface is found in the shadow of a given point upon either the

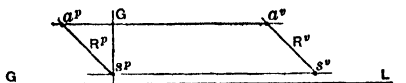


Fig. 19 Using the "P" and "V" Projections

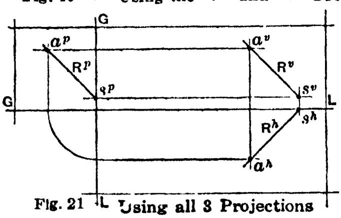


Fig. 21 Using all 3 Projections

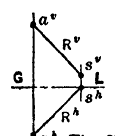


Fig. 20 Using the "H" and "V" Projections

Figs. 19, 20, 21. The Shadow of a Point by "Oblique Projections"

V or H coordinate planes, while using these two projections to obtain the shadow.

Assume a point a, located by its projections, a^h and a^v and a^p to be 1 in from V and $1\frac{1}{2}$ in from H.

Being nearer to V than H, its shadow will fall upon the vertical coordinate, and either the P and V (Fig. 19), or H and V projections (Fig. 20), could be used to obtain the shadow. For this demonstration the H and V projections are used (Fig. 20).

Draw the ray R through the point. R^v will pass through the V projection

of the point. R^h will pass through the H projection of the point. R^p will pass through the P projection of the point (Fig. 21).

(1) The shadow, being a point in V, has its H projection in the ground line, (the GL is the H projection of every point in V).

(2) The H projection of the shadow point must also be in R^h. (By definition, the shadow point is a point in the ray of light; therefore, its H projection s^h must be in R^h).

(3) The H projection of the shadow point must lie at the intersection of GL and R^h. (Since a point which is common to two lines can only be their intersection)

(4) The V projection s^v of the shadow point must lie in a perpendicular to the GL at s^h. (The two projections of any point lie in the same line perpendicular to the GL)

(5) The V projection of the shadow point must lie in R^v. (Reason similar to (2) above.)

(6) Therefore, the V projection of the shadow point must lie at the intersection of the perpendicular to the GL at s^h and R^v. (Reason same as (3) above)

Now a point in a plane is its own projection in the plane, and s^v is the shadow of the point a.

Although the above demonstration of the finding of the shadow of a point upon a coordinate plane presents the simplest illustration of the method of OBLIQUE PROJECTION, the procedure for the shadow of any point in space upon any surface, where this method applies, is exactly the same.

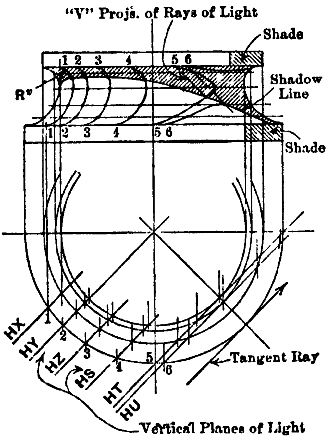


Fig. 22. The Slicing Method of Finding the Shadows of Points

To Find the Shadow of a Point by Slicing (Fig. 22). This method is quite simple, and is sufficiently explained by the diagram. The line, which is a circle, forming the top edge of the scotia, casts a shadow upon the surface of the scotia. The shadows of six points are found in order to plot the shadow line. The method is as follows: (*a*) Cut the line casting the shadow and the surface receiving the shadow with vertical PLANES OF LIGHT. (A plane which is assumed to be made up entirely of rays of light is called a plane of light.)

(*b*) Draw rays of light from the points, 1, 2, 3, 4, 5 and 6, which are cut on the line casting the shadow until they strike the slices made upon the surface of the scotia by the auxiliary planes. These are the shadow points of points 1, 2, 3, 4, 5 and 6. The curved line which joins them is the SHADOW LINE.

The slices are the traces of planes of light upon the surface of the scotia, and the shadow points are traces of rays of light in those planes, because a line lying in a plane has its traces in the traces of that plane.

The Shadow of a Line. Since a line is mathematically defined as the path of a point in motion, it follows that to find the shadow of any line upon any

surface implies simply the finding of the shadows of a sufficient number of successive positions of the point to plot the shadow of the line. In other words, find either by OBLIQUE PROJECTION or by SLICING the shadows of sufficient points in the line to plot the shadow of the line. Obviously the number and position of these points would depend upon, first, the kind of line casting the shadow, and second, the kind of surface receiving the shadow. For example:

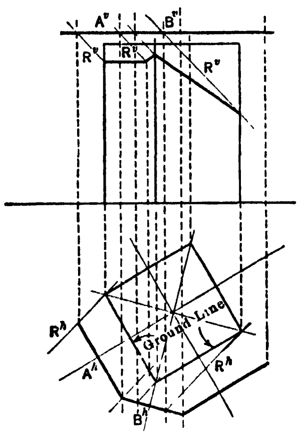


Fig. 23. The Shadow of a Straight Line upon a Plane Surface

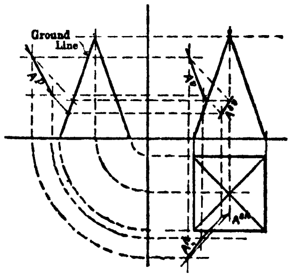


Fig. 24. The Shadow of a Straight Line upon a Plane Surface

(1) **To Find the Shadow of any Straight Line upon Any Plane Surface Which Can Be Projected as a Line** (Figs. 23 and 24), it is necessary to find

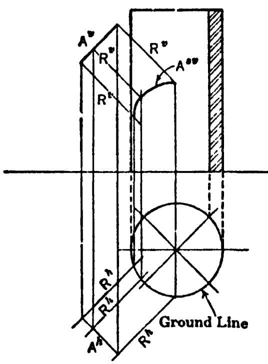


Fig. 25. The Shadow of a Straight Line upon a Single Curved Surface

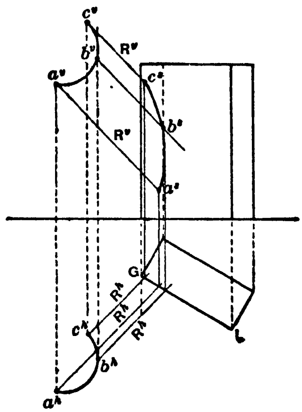


Fig. 26. The Shadow of a Curved Line upon a Plane Surface

the shadow of two points in the line by OBLIQUE PROJECTION and join them, since it is obvious that the shadow must be a straight line.

(2) To Find the Shadow of Any Straight Line upon any Single Curved Surface When the Surface Can Be Projected as a Line (Fig. 25), it is only necessary to find the shadow of at least three points by OBLIQUE PROJECTION to obtain the sense of the shadow line.

(3) To Find the Shadow of Any Curved Line upon Any Plane or Single Curved Surface When This Surface Can Be Projected as a Line (Fig. 26) would also necessitate more than two shadow points found by oblique projection.

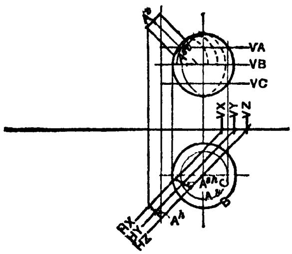


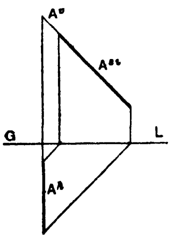
Fig. 27. The Shadow of a Straight Line upon a Double Curved Surface

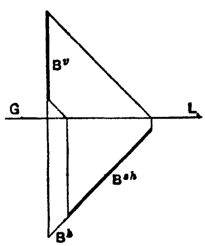
(4) To Find the Shadow of Any Straight or Curved Line upon Any Double Curved Surface or Any Other Surface Which Cannot Be Projected as a Line (Fig. 27) compels the use of the slicing method already described.

The fundamental thing which underlies the finding of the shadow of any line upon any surface by any method is, in the last analysis, the shadow of a point.

6. Standard Architectural Shadows

Standard Architectural Shadows. Certain straight lines in certain positions relative to the coordinates are constantly recurring as shade lines in architectural drawings. They occur so frequently and their shadows are so critical and expressive that they should be instantly recognized and their shadows correctly drawn. The circle, either in whole or in part, also occurs with great frequency in the shade lines of solids casting shadows. Like the straight line, it is constantly recurring in certain fixed positions relative to the coordinates,





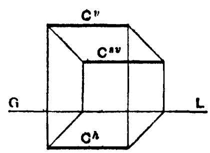


Fig. 28 Fig. 29 Fig. 30

Figs. 28, 29, 30. Projections of Lines, *A*, *B* and *C*, and Their Shadows upon the Co-ordinates

and if these shadows are at instant command the work of casting shadows is greatly facilitated.

The Shadows of Three Certain Straight Lines. For the purposes of explanation, these lines will be designated as follows: (1) Line *A* which is perpendicular to *V* and parallel to *H* (Fig. 28). (2) Line *B* which is perpendicular to *H* and parallel to *V* (Fig. 29). (3) Line *C* which is parallel to both *V* and *H* (Fig. 30).

Since A and B are, in effect, the same line, what is true of the shadows of A upon each of the two coordinate planes is simply reversed for B. However, for the sake of absolute clarity, they are considered as separate and different lines.

Fig. 31 illustrates the occurrence of these lines A, B and C in an architectural drawing. The shadows of these lines, when plotted carefully upon a drawing, convey much information, and are quickly and easily found.

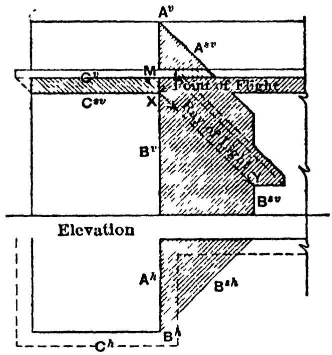


Fig. 31. Lines A, B and C, as they occur in Architectural Drawings

(1) The Shadow of a Line A which is Perpendicular to V, and Parallel to H (Figs. 28 and 31). As SEEN IN V PROJECTION, the shadow of this line is always a 45° line coincident in direction with the ray of light, regardless of what surfaces the shadow line crosses.

Actually, of course, this shadow line in space takes the contour of the surfaces which it crosses (Fig. 34). But the shadow line is the line of intersection of a plane of light through the line A and the shadow receiving surface. Since the line A is perpendicular to V, the plane of light through it is also perpendicular to V,

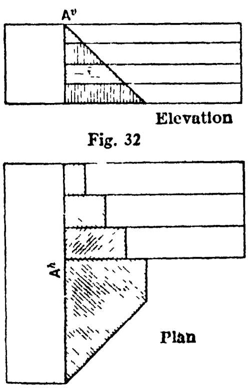


Fig. 32

Fig. 33

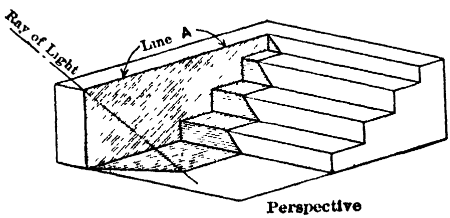


Fig. 34

Figs. 32, 33 and 34. The Shadow of the Line A upon a Flight of Steps

and any line in it will be projected in V coincidentally with the V projection of the conventional ray of light. Therefore, in elevation, the shadow line appears as an unbroken 45° line d-r (Fig. 32).

As SEEN IN H PROJECTION the shadow of the line A will express the contour of the surfaces it crosses (Fig. 33). The line A is parallel to the H coordinate,

and in that projection the relation to the coordinate is the same as in the case of line *C*. In Fig. 33 the surface is a flight of steps, and the rise and tread of the steps are expressed by the shadow line as exactly as a cross-section would show them.

(2) **The Shadow of a Line B which is Perpendicular to H, and Parallel to V** (Figs. 29 and 31). As SEEN IN *H* PROJECTION, the shadow of this line will be a 45° line *b–r* for the same reasons as the *V* projection of line *A*.

As SEEN IN *V* PROJECTION, the shadow of the line, *B*, like both *A* and *C*, will express the exact contour of whatever surface it crosses.

(3) **The Shadow of a Line *C*, which is Parallel to Both *V* and *H*** (Figs. 30 and 31), will express the contour of the surfaces which it crosses, as exactly as a cross-section, WHEN SEEN IN EITHER *V* or *H* PROJECTION.

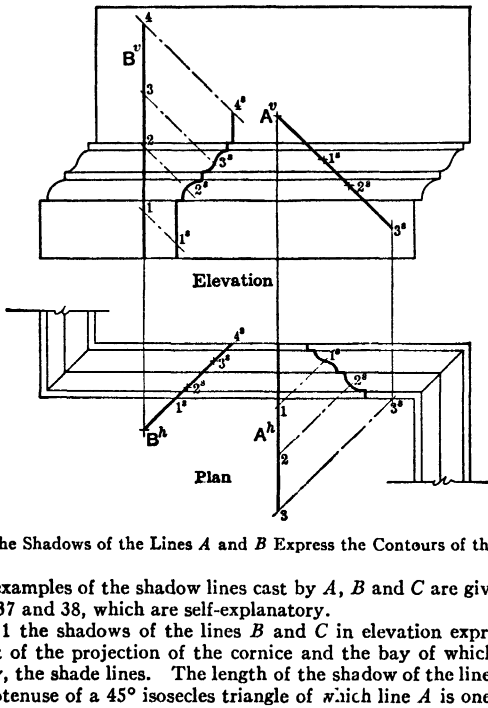


Fig. 35. The Shadows of the Lines *A* and *B* Express the Contours of the Moldings

Specific examples of the shadow lines cast by *A*, *B* and *C* are given in Figs. 31, 35, 36, 37 and 38, which are self-explanatory.

In Fig. 31 the shadows of the lines *B* and *C* in elevation express exactly the amount of the projection of the cornice and the bay of which they are, respectively, the shade lines. The length of the shadow of the line *A* is equal to the hypotenuse of a 45° isosceles triangle of which line *A* is one leg.

Fig. 35 illustrates how the shadows of line *A* and *B* may be used to express the profiles of moldings.

Fig. 36 shows the shadow of a line *C*, parallel to both *V* and *H* as it falls across the surface of a convex right circular cylinder, whose axis is perpendicular to *H*. The shadow which a square abacus casts upon the cylindrical shaft of a column is a typical example. The shadow itself in space is actually an ellipse, being an oblique section of a right circular cylinder, but it is so projected in *V* as to have equal axes. It is therefore a circle in *V* projection. The cen-

ter of the circle is a distance down upon the axis of the cylinder equal to the distance which the line C lies in front of the axis of the cylinder. This is best observed in P projection. Fig. 37 is similar to Fig. 36 except that the cylinder is now concave instead of convex. The lower half of the circle becomes the shadow of line C instead of the upper half, as in the case of the convex cylinder.

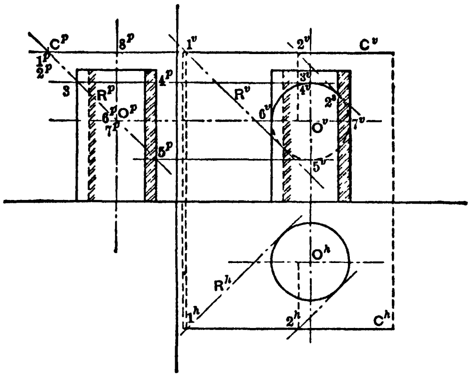


Fig. 36. The Shadow of a Line Which is Parallel to both V and H as it Falls across a Convex, Right Circular Cylinder, the Axis of Which is Perpendicular to H

Fig. 38 illustrates one of the most expressive of shadow lines. The shadow of this line B expresses exactly the slope of the roof, whatever it may be. If there are changes in the slope of the roof, as in a gambrel roof, this shadow will show it. Obviously, the shadow of a chimney will express upon the ele-

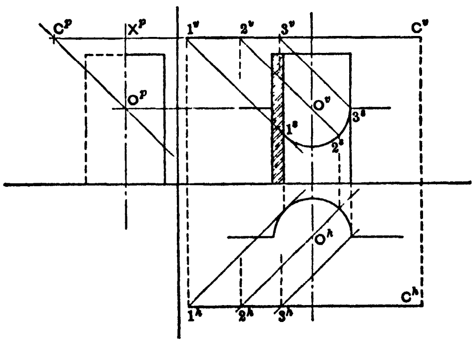


Fig 37. The Shadow of a Line Which is Parallel to both V and H as it Falls across a Concave, Right Circular Cylinder, the Axis of Which is Perpendicular to H

vation of a roof its exact slope so that a careful observer need not refer to any other drawing. No shadow better illustrates the importance of accurately cast shadows.

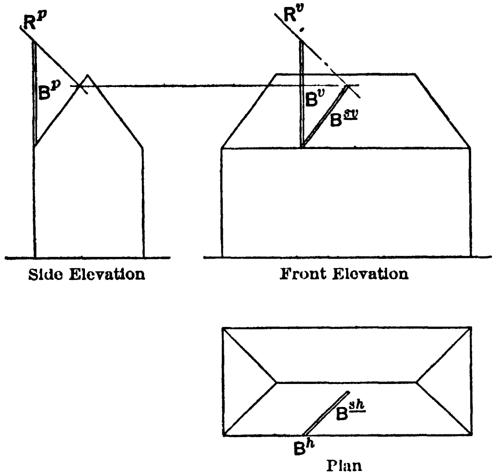


Fig. 38. The Shadow of a Vertical Line upon a Roof Plane

The Shadows of Nine Certain Circles. The shadows of nine circles, the planes of which lie in certain positions relative to the coordinates, follow:

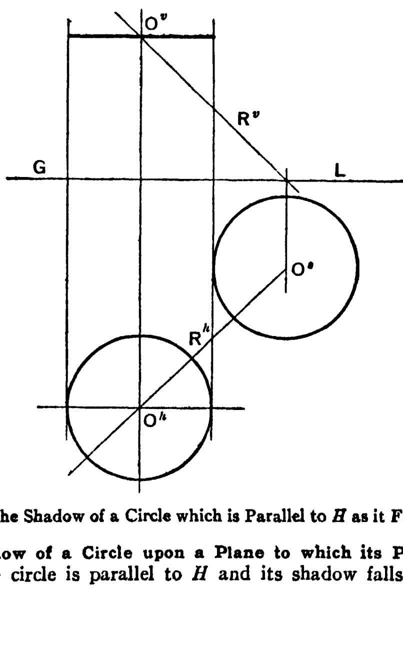


Fig. 39. The Shadow of a Circle which is Parallel to *H* as it Falls upon *H*

(1) The Shadow of a Circle upon a Plane to which its Plane is Parallel (Fig. 39). The circle is parallel to *H* and its shadow falls upon *H*. The

shadow in this instance will be another circle of the same radius. It is only necessary to find the shadow O^s of the center O. It will be noted, that the distance O-O^s, along R^h is equal to the diagonal of a square of which the distance O^v lies above the GL is a side.

(2) **The Shadow of a Circle, the Plane of Which is Parallel to One Coordinate as it Falls upon the Opposite Coordinate** (Fig. 40). The circle is parallel to H, and its shadow falls upon V. The shadow, in this instance, will be an ellipse, and it is most quickly and accurately plotted by the method of 8 POINTS AND 8 TANGENTS. A square $ABCD$ is circumscribed about the circle and its shadow found. The diagonals A^s C^s and B^s D^s are then drawn, obtaining the center O^s, from which points 1^s, 3^s, 5^s and 7^s are easily obtained. This gives 4 points and 4 tangent lines. The points 2^s, 4^s, 6^s and 8^s are then

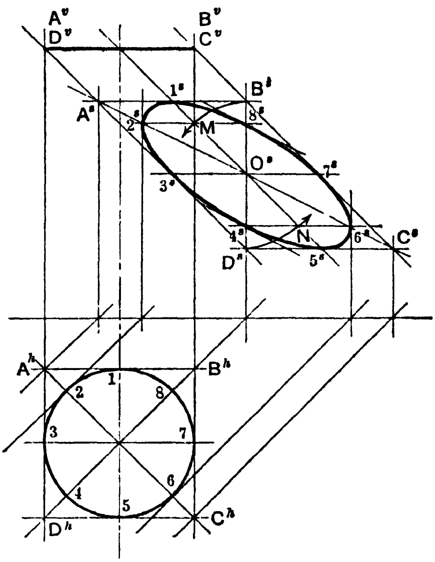


Fig. 40. The Shadow of a Circle Which is Parallel to H as it Falls upon V. Shadow Found by Method of "8 Points and 8 Tangents"

found as shown, and tangent lines, parallel to the diagonals A^s C^s and B^s D^s, are drawn. No attempt should be made to draw the ellipse until all 8 points and 8 tangents are plotted. This shadow can be found without the use of a plan, as indicated. With O^s as a center and O^sB^s as a radius, locate the point M, from which points 2^s and 8^s are obtainable, etc.

(3) **The Shadow of a Circle which Lies in a Profile Plane as it Falls upon V or H** (Fig. 41). The shadow, in this instance, is again an ellipse, which is also found by circumscribing a square about the circle and using 8 POINTS AND 8 TANGENTS. The method and its short-cut is made sufficiently clear by the drawing and reference to the preceding problem.

(4) **The Shadow of a Circle Which Lies in a Plane Parallel to H as it Falls upon a Vertical Plane Which is 45° h–l** (Fig. 42). In this instance, there is a difference between the shadow in space and its V projection which we obtain

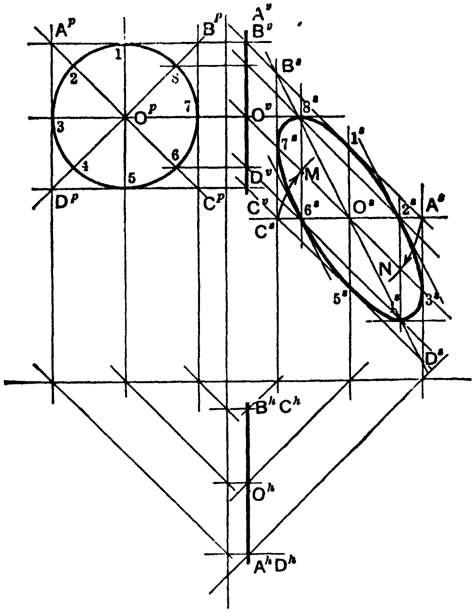


Fig. 41. The Shadow of a Circle Which is Parallel to *P* as it Falls upon *V*. Shadow Found by Method of "8 Points and 8 Tangents"

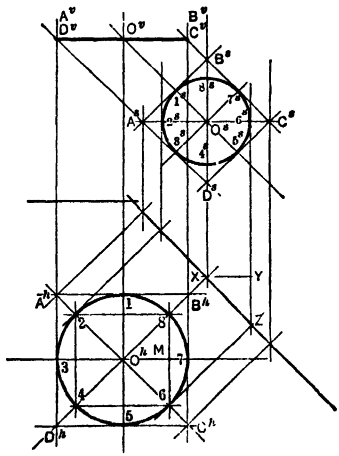


Fig. 42. The Shadow of a Circle upon a Vertical Plane (45° B–L)

Actually the shadow is an ellipse. But its horizontal axis is so foreshortened (in projection) as to make the ellipse appear in V projection as a circle whose radius is equal to $\frac{1}{2}$ the side of the inscribed square 2–4–6–8. The proof for this last statement becomes evident when Fig. 42 is studied. Isosceles triangles O^h M–6 and X–Y–Z are equal. From this it can be seen that $O^hM = XY = O^s6^s$.

(5) The Shadow of that Great Circle of a Sphere which Lies in a Plane Perpendicular to the Ray of Light as it Falls upon Either Coordinate Plane (Fig. 43). This circle is at once recognized as the shade line of a sphere (Fig. 14). Its two projections are ellipses and the shadow is an ellipse. The shade line is obtained by a variation of SLICING, called the METHOD OF SYMMETRY FOR SPHERES. (See, also, Figs. 67 and 68) Slice the sphere by 5 planes:

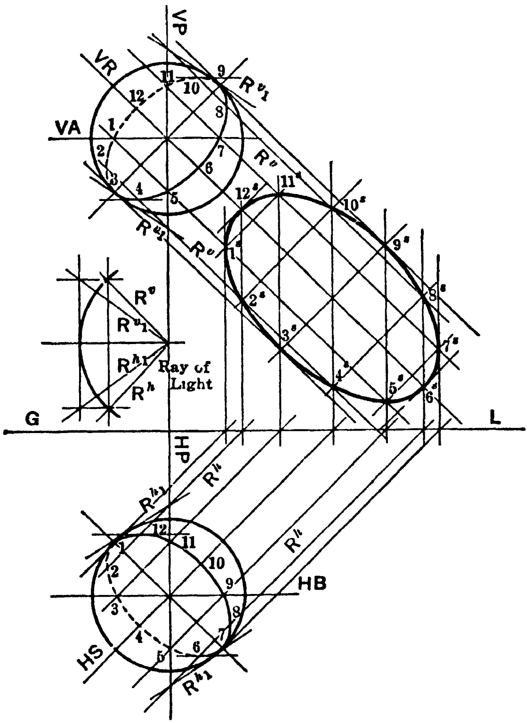


Fig. 43. "Shade Line" Obtained by the Method of "Symmetry for Spheres" "Shadow Line" Found by Casting the Shadows of 12 Points

(1) the profile plane P, (2) the horizontal plane A, (3) the vertical plane B, (4) the auxiliary plane of light R which is perpendicular to V, (5) the auxiliary plane of light S which is perpendicular to H. Tangent rays are then drawn (in projection) to sections P, A and B and the points, 5 and 11, 1 and 7, 3 and 9 thus obtained. Tangent rays (revolved), drawn to the sections R and S when revolved parallel to H and V, respectively, obtain the points 12 and 6,

10 and 4, respectively. These four points are critical points in the shade line, inasmuch as 10 is the highest point, 4 the lowest, 12 nearest to V and 6 furthest from V. The other two points 2 and 8 are plotted by symmetry.

The shadow of the shade line is most quickly found by finding the shadows of the twelve points and joining them.

It will be noticed that the major axis of the ellipse of shade casts the minor axis of the ellipse of shadow and is equal to it. The minor axis of the ellipse of shade casts the major axis of the ellipse of shadow and is equal to $\frac{1}{3}$ its length.

The Shadows of Circles upon Surfaces Other than Plane Surfaces. In the practical work of casting architectural shadows, it frequently happens that we are confronted with the problem of casting the shadow of a circle or part of a circle upon surfaces other than plane surfaces (see Fig. 102). But almost without exception the problem presented is one which is constantly recurring, and therefore, if once understood, is quickly solved. Examples of such problems will now be given.

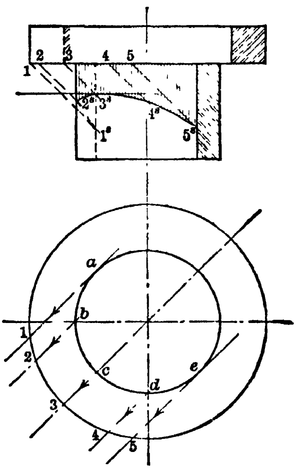


Fig. 44. The Shadow Cast by a Cylindrical Cap upon a Cylindrical Shaft below

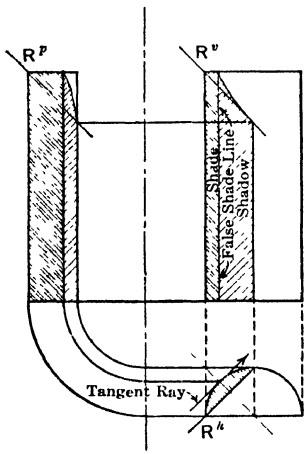


Fig. 45. The Shadow within a Hollow Cylinder

(6) The Shadow of a Circle upon the Surface of a Convex Right Circular Cylinder (Fig. 44). This problem occurs with great frequency as the shadow of a cylindrical cap overhanging a cylindrical shaft. The method of solution has been explained elsewhere. (See Fig. 52.) A projecting band of molding upon a cylindrical wall surface would be a typical example. The shadow is extremely useful in expressing the curvature of the surface.

(7) The Shadow of a Circle upon a Concave Right Circular Cylinder (Figs. 45 and 46). This problem arises in the shadow in a hollow cylinder (Fig. 45), and the shadow within a barrel vault (Fig. 46).

(8) The Shadow of a Circle upon the Surface of a Hollow Cone (Fig. 47). Variations of this problem are of frequent occurrence. The solution is similar to the first method for the head of a niche (Fig. 49).

(9) The Shadow of a Circle upon the Surface of a Hollow Hemisphere (Fig. 48). The most common occurrence of this shadow is that cast within

a spherical-headed niche by the circular rim of the niche. When Figs. 45 and 48 are combined we have the entire shadow which occurs within a spher-

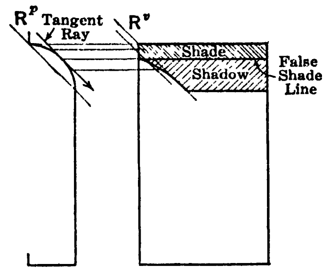


Fig. 46. The Shadow within a Barrel Vault

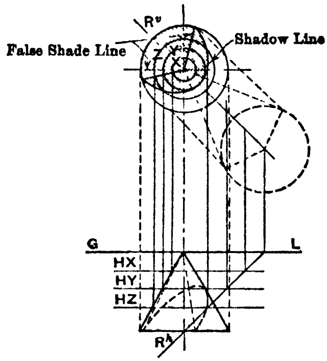


Fig. 47. The Shadow within a Hollow Cone

ical-headed niche. Figs. 49, 50 and 51 illustrate three methods whereby the shadow in the head of a niche may be obtained. They are all applications of

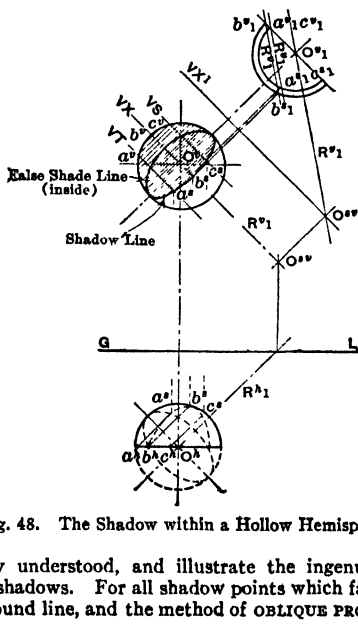


Fig. 48. The Shadow within a Hollow Hemisphere

principles already understood, and illustrate the ingenuity which can be shown in finding shadows. For all shadow points which fall below the spring line we have a ground line, and the method of OBLIQUE PROJECTION will serve.

First Method Auxiliary Planes Parallel to the Line Casting the Shadow (Fig. 49) shows a method whereby auxiliary vertical planes X, Y and Z are used to cut lines x, y and z from the niche. The shadow of the rim upon each of these planes is a circle since the rim is a circle parallel to V. The points where these circles cut the lines x, y and z are points in the shadow of the rim

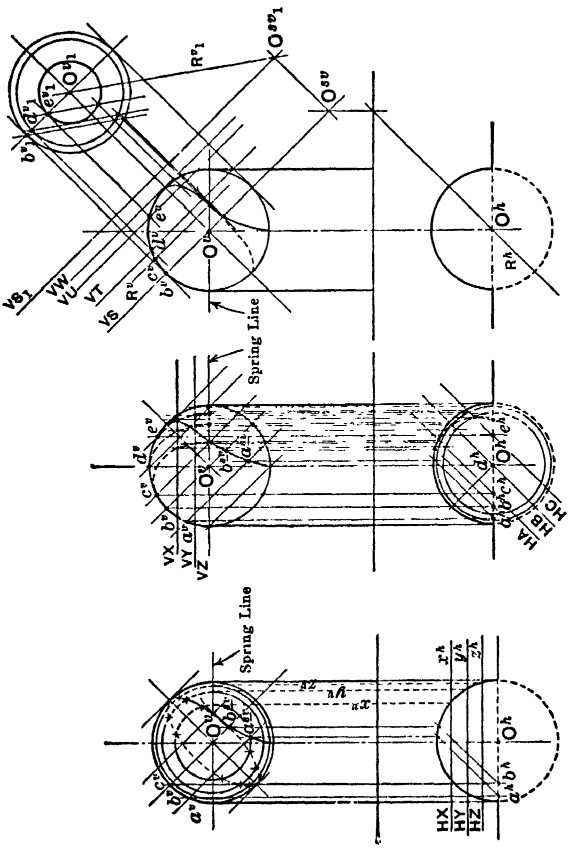


Fig. 49

Fig 50

Fig. 51

Figs. 49, 50, 51. The Shadow Line within a Spherical-Headed Niche

and in the head of the niche since the lines x, y and z are in the head of the niche.

Second and Third Methods: Slicing (Figs. 50 and 51). These are two variations of the SLICING METHOD, already explained, of finding this shadow. In Fig. 50 the rays of light (in projection) are immediately applied to the projected sections, and shadow points obtained. In Fig. 51 both the slices and the ray of light are revolved into the coordinate plane before the shadow points are obtained. (See Fig. 48.) They are then counter-revolved. This method

is sometimes called the REVOLVED PLANE method, but is really but a variation of the slicing method.

7. Practice of Casting Shadows

Practice of Casting Shadows. The ultimate aim of the science of shades and shadows is to cast the shadow of any geometric surface upon any other geometric surface. Since architectural design is fundamentally the art of arranging geometric surfaces in such combined relations that their light effects (lights, shades and shadows) will be pleasing, it appears that each rendered drawing presents new problems to be solved. Equipped with all the necessary knowledge, it now becomes necessary to be resourceful. Every drawing to be rendered will contain objects the shadows of which cannot be cast either by oblique projection or slicing alone. Even the shadows of single objects may be most advantageously cast by the use of both methods applied to different parts (as in column caps or bases). There are no fixed rules. The Fundamental Principles outlined in Part 4 must be thoroughly assimilated. The Standard Shadows described in Part 6 must be at instant command. Even then, after the Theory of Casting Shadows (explained in Part 5) is completely understood, the practical work of casting shadows may be greatly simplified by ingenious short-cuts or expedients invented to meet new conditions. They do not involve any new method of casting shadows, but may reduce some complex shadow to several simpler ones (Figs. 55A and 55B), or may simplify the procedure, as in Fig. 57. Examples given below are but a few of the better-known and generally used.

(1) Dispensing with One Projection. In the method of oblique projection for the shadow of a point shown in Fig. 20, it is obviously only necessary to have the V projection and know how far the point is from V. The plan is unnecessary, since the shadow is a distance horizontally to the right of the point equal to the distance the point lies in front of V, and of course upon the ray.

Similarly, for a shadow upon H, the elevation is unnecessary. The convenience of this in actual practice where plan and elevation may be upon different sheets and at different scales is self-evident.

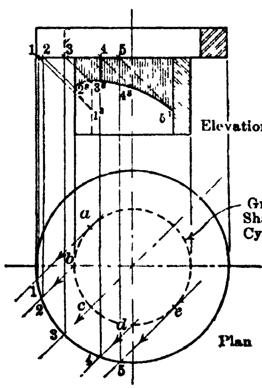


Fig. 52. The Shadow of a Cylindrical Cap Overhanging a Cylindrical Shaft by "Critical Rays"

(2) Critical Rays (Figs. 52 and 53) are used to illustrate a device used in many instances to simplify the work of finding shadows of lines. In Fig. 52 it is required to find the shadow of a cap overhanging a cylindrical shaft. The lower edge of the upper cylinder is, of course, the shade line for the shadow upon the shaft. Instead of taking points at random and finding their shadows, in order to plot the shadow line, certain rays are drawn backward from the CRITICAL SHADOW POINTS a, b, c, d and e. It is found that the

points 1, 2, 3, 4 and 5 are the points in the shade line which cast shadow points *a*, *b*, *c*, *d* and *e*. Their *V* projection is then obtained and shadow points are thus obtained at 1^s and 5^s which are on the shade lines of the cylindrical shaft, at 2^s which is on the contour element, at 4^s the *V* projection of the axis of the cylindrical shaft, and at 3^s which point is cast by the ray which would pass through the axis of the cylinder. This shadow point 3^s is the

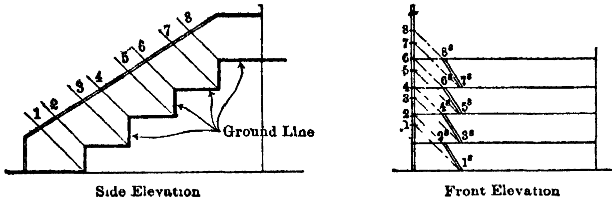


Fig. 53. The Shadow of a Handrail upon a Flight of Steps by "Critical Rays"

highest point in the curve of the shadow line because that ray of light travels the shortest distance. A ray slightly to either side would travel further and the shadow point be lower. It will be noted that points 1^s and 5^s are at the same level, and 2^s and 4^s are also a pair.

Fig. 53 illustrates how this same device of CRITICAL RAYS may be used in determining shadow lines which break across several adjacent surfaces. The example given is the shadow of a sloping hand-rail beside a flight of steps. In this instance, both the treads and the risers are projected as lines at the same time only in the side-view or *P* projection, so that projection is most convenient to use as a ground line for the shadows. Only the critical shadow points 1 to 8 inclusive are found. In the practice of casting shadow many opportunities are presented for the exercise of ingenuity as shown by this method of critical rays.

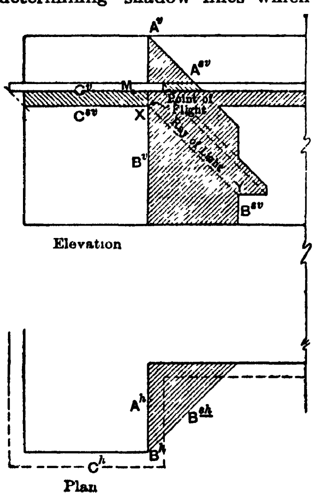


Fig. 54 The Point of Flight

(3) **The Point of Flight.** Fig. 54 illustrates another device which is exceedingly useful in the practical work of casting shadows. The POINT OF FLIGHT is the point where the shadow of line *C* jumps from one surface to another. The ray of light passing through the point of flight *x*, which is the last shadow point of the line *C* upon the first surface, obtains the point *y* in the silhouette of the shadow. The point *y* is the first point of the shadow of the line *C* upon the second surface. In other words, the ray of light from the point *M* in the line *C*, touches the vertical edge *B* at *x*, the point of flight, and continues on to strike the wall surface at *y*. The figure illustrates a characteristic use of the ray of light through the point of flight which has many obvious advantages. There is no more important step to be learned in practical shadow-casting than frequent use of this device. It simplifies the work tremendously.

(4) **Auxiliary Shadows** (Figs. 55A and 55B). In architectural drawing a large group of figures to be represented are solids of revolution with their axes commonly perpendicular to H or V or parallel to GL It is therefore possible, without much difficulty, to determine their shadows by constructing

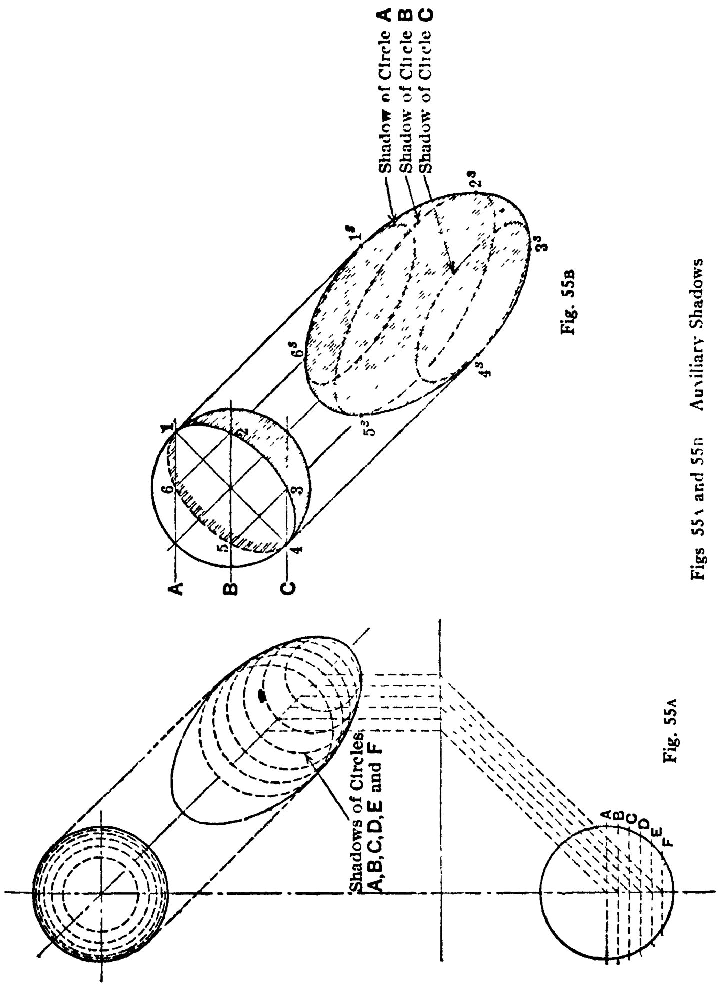


Fig. 55B

Fig. 55A

Figs 55A and 55B Auxiliary Shadows

the shadows of a few principal cross-sections, since every cross-section would be circular. Figs. 55A and 55B are simple illustrations of the possibilities of this method. Its application facilitates the work by use of the auxiliary lines whose shadows may be cast readily and accurately. The principles upon which the method depends are made sufficiently clear by the drawings.

(5) Circumscribing or Inscribing Surfaces (Figs. 56 and 57). This application depends upon the principle that, at a point of tangency of two surfaces, whatever is true of one surface is also true of the other for such a point is common to both. In Fig. 56 the shade line of the sphere is found by circumscribing cones, and in Fig. 57 the shadow of the sphere is found by inscribing a cube. The drawings are sufficient explanation.

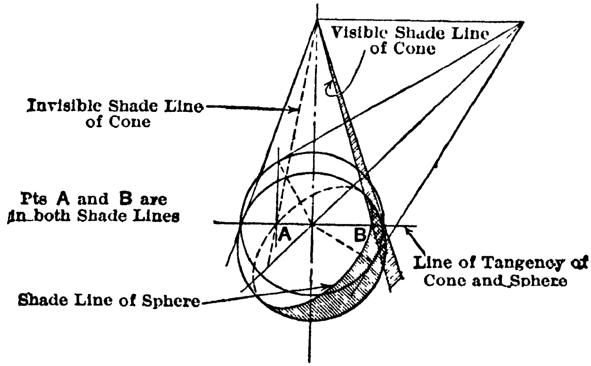


Fig. 56. A Cone Circumscribed about a Sphere

(6) Pyramids and Cones. Fig. 58 shows how finding the shade and shadow of a pyramid is simplified by producing the pyramid until its base rests in *H*. The new base is then its own shadow and need not be cast. The shade surfaces of the original pyramid are the same as those of the new one. The shadow of the base of the first pyramid is then easily found. The same would

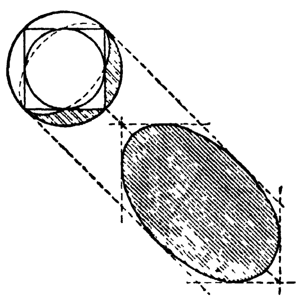


Fig. 57. A Cube Inscribed within a Sphere

apply to cones (Fig. 59). Fig. 56 shows a quick way of finding the shade line of a cone by the SHADOW COMPLETE method.

(7) Niches (Figs. 49, 50, 51). The three methods already explained, showing how the shadow within a spherical-headed niche is found, are further examples of the ingenuity which must be shown in the practice of casting shadows.

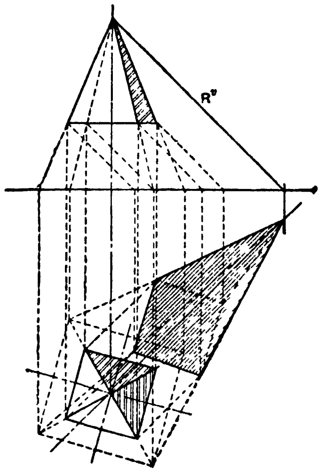


Fig. 58. Producing a Pyramid

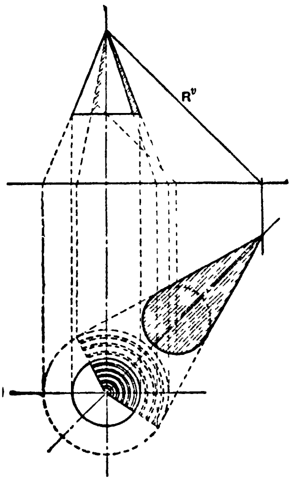


Fig. 59. Producing a Cone

8. Determining and Forecasting Shade Lines

Determining and Forecasting Shade Lines. The primary and most important function of the science of shades and shadows is that of determining shade lines. It has been shown that, in the final analysis, the shadow line of any object upon any surface is cast by the shade line of that object (Fig. 14). Therefore, it follows that it is only necessary to be able to find the shadows

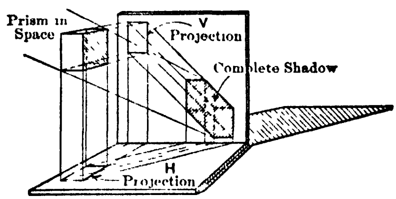
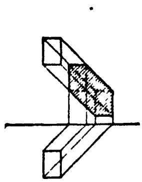


Fig. 60. Casting the "Shadow Complete"

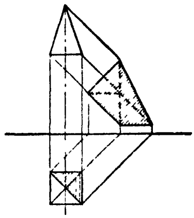
of points in order to find any shadows, once the shade line has been determined. Of course the shadow line of any object upon any surface could be approximated by casting the shadows of sufficient random points in its surface and joining the outermost by a line (Fig. 60). But the labor of finding a great number of unnecessary shadow points and the uncertainty of the accuracy of the results for more complex solids compel a more scientific procedure. Therefore, determining the shade line of an object not only definitely outlines the light and shade surfaces of the object itself, but if the shade line can possibly be forecast, the work of finding the shadow line becomes greatly simplified.

Shadow Complete Method (Figs. 61, 62, 63). The simplest method of determining the shade line of any object is to trace back from the outline of its COMPLETELY CAST SHADOW, and discover which points in the surface of the object cast the shadow line. Theoretically it will apply to any solid, and its only use is to determine the light and shade portions of the surface of the object, since the shadow is already obtained. This method is particularly applicable to cones (Fig. 63).



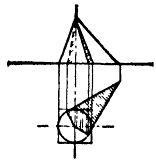
Prism

Fig. 61



Pyramid

Fig. 62

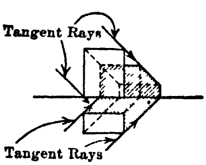


Cone

Fig. 63

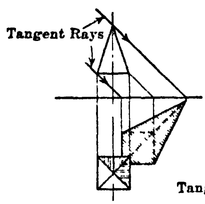
Figs. 61, 62 and 63. "Shadow Complete" Method of Determining Shade Line as Applied to Prisms, Pyramids and Cones

Tangent Ray Method (Figs. 64, 65, 66). Any application of the science of shades and shadows which seeks to predetermine or forecast the shade lines of objects before their shadow is cast, must devise ways for finding sufficient points upon their surfaces where rays of light are tangent, and connect those points of tangency and thus plot the shade lines. Sometimes this is so simple and direct as to amount practically to inspection (Figs. 64, 65, 66). It is then called the TANGENT RAY method. It consists of applying the pro-



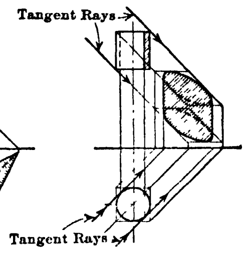
Prism

Fig. 64



Pyramid

Fig. 65



Right Cylinder

Fig. 66

Figs. 64, 65 and 66. "Tangent Ray" Method of Predetermining Shade Line as Applied to Prisms, Pyramids and Cylinders

jections of rays of lights to surfaces, on the coordinate where those surfaces are projected as lines. It will be apparent at once whether the surface is reached by light or not. The shade line divides the surfaces of light from the surfaces of shade thus determined.

Obviously, this method will apply only to solids the surfaces of which can somewhere be projected as lines.

Slicing and Revolved Plane Methods (Figs. 67 and 68). In both of these methods, auxiliary planes of light are used to cut critical sections from the surface of the solid. These sections are either projected (SLICING method,

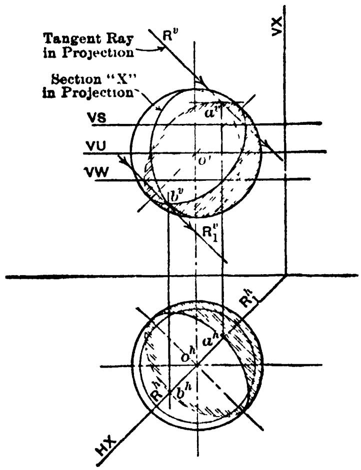


Fig. 67. The Slicing Method

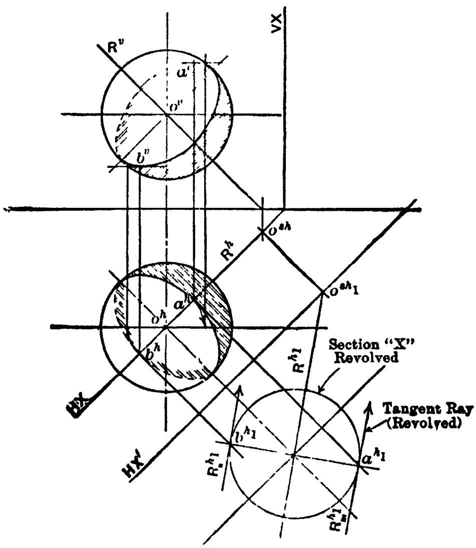


Fig. 68. The Revolved Plane Method

Fig. 67), or revolved into a coordinate (REVOLVED PLANE method, Fig. 68). The tangent ray is then drawn to the section of the surface (ray in projection in the slicing method and revolved ray in the revolved plane method). This obtains successive points of tangency of rays, hence points in the shade line. Although separate names have been given these methods, they are really one method, the slicing method. Both slicing and revolved planes are applicable to any solid whatever, but the revolved plane method is particularly useful when the object is a double curved surface of revolution. The sections cut by the auxiliary planes then are all circles, and when revolved into one of the

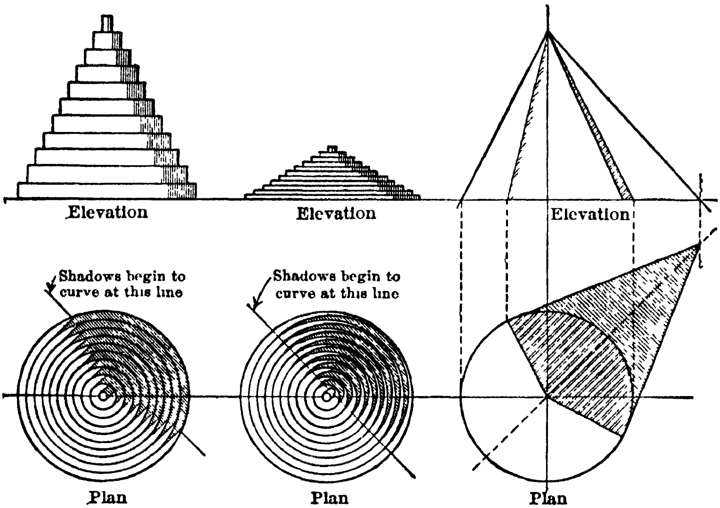


Fig. 69 Fig. 70 Fig. 71

Figs. 69, 70 and 71. The Paradox of Shade Line

coordinate planes, can be drawn with a compass. Fig. 67 illustrates the slicing method as applied to a sphere, and Fig. 68 illustrates the revolved plane method also applied to a sphere. In practice, the complete shade line of a sphere is found by the revolved planes method of slicing. (See Fig. 43) Because symmetrical points are found, this variation of the slicing method is called the METHOD OF SYMMETRY FOR SPHERES.

The Tangent Planes Method, applicable to all convex double curved surfaces such as spheres, torii, etc., consists briefly in finding where planes of light are tangent to the surface. If a plane of light is tangent to the surface, it is a geometrical truth that a ray of light is tangent at the same point, hence that point is in the shade line of the object. A succession of such points constitutes the shade line. But since the slicing method applies in every case, this method is rarely used.

The Paradox of Shade Line (Figs. 69, 70 and 71). An interesting paradox arises concerning the shade lines of right circular cylinders and right circular cones (both axes perpendicular to *II*). The shade line of the cylinder is determined by the tangent ray in *H* projection (Fig. 66). The shade line of the

cone must be found by the shadow complete method (Fig. 63). The tangent ray method does not apply to the cone. If a number of cylinders are piled up in a conical manner (Figs. 69 and 70), the shade line upon each will be on the corner, and the complete shade line of the pile will be a broken line or series of steps at the corner, irrespective of its steepness and the size of the steps. Even if they are so small that the surface looks almost smooth (Fig. 70), and the pile of cylinders looks like a cone, the shade line will not change, although the shade line upon the cone (Fig. 71), which the pile of cylinder simulates, may vary anywhere from being a single line where a ray of light is exactly tangent up to the two corner elements, according to the slope of the cone. In the latter case the cone has become a 90° cone or a right circular cylinder.

9. Vocabulary

Vocabulary. Repeatedly to work out the shade lines and shadow lines of the elements constantly found in architectural drawings would only involve a considerable amount of unnecessary labor, once their derivations are understood. Since much architectural detail is vocabulary, and can be reduced by convention to certain definite geometric forms already tabulated (Figs 6 to 11, inclusive), we have constant repetition of certain objects casting shadows and certain surfaces receiving shadows.

It is, therefore, of the greatest fundamental importance to acquire from a study of shades and shadows a vocabulary of projections of shade lines and shadow lines of these constantly recurring architectural forms, so that both the rendering and the reading of architectural plans and elevations may be facilitated.

These surfaces which are constantly casting and receiving shadow, alone and in combinations, occur in column caps and bases, cornices, pediments, moldings, brackets, modillions, balusters, etc. Their shade lines and shadow lines together with those of the orders (Doric, Ionic, Corinthian, and Composite) compose an extensive vocabulary. Intelligent use of this vocabulary presupposes a complete understanding of its derivation. In the use of this vocabulary there can be no compromise. The shade lines and shadow lines must be correctly drawn, in order that they fulfil their primary purpose, i.e., to convey accurate information. Although an approximation may seem to suffice, only absolute accuracy will be true. The critical and important points in the shade and shadow lines may be so subtle that, although the memory may be relied upon for an approximation, when absolute accuracy is sought either the problem should be correctly solved or visual reference made to a previous correct solution.

Following is a reference vocabulary of miscellaneous architectural shades and shadows.

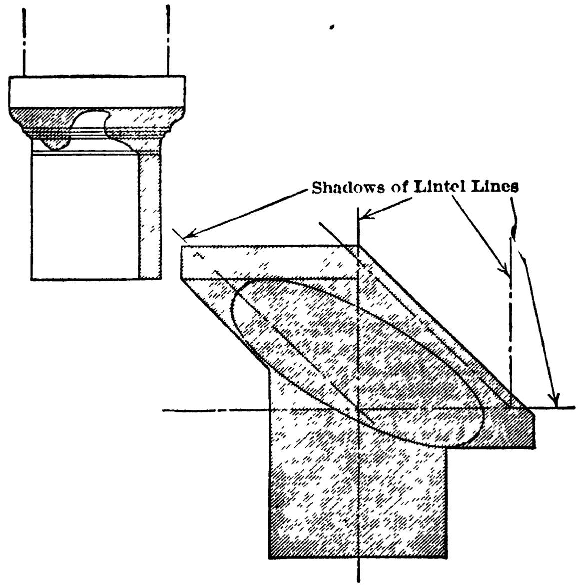


Fig 72. Greek Doric Capital

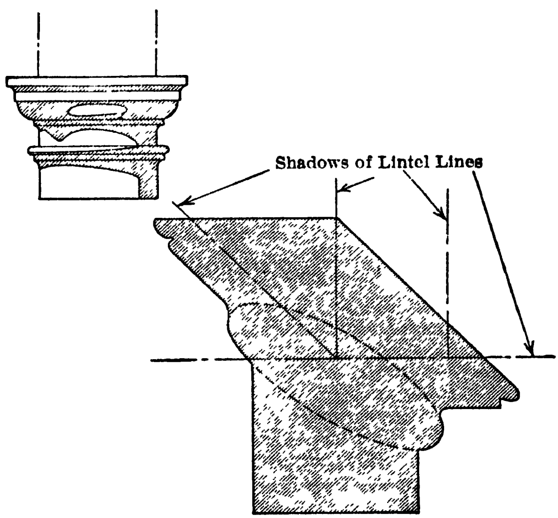


Fig. 73. Tuscan Capital

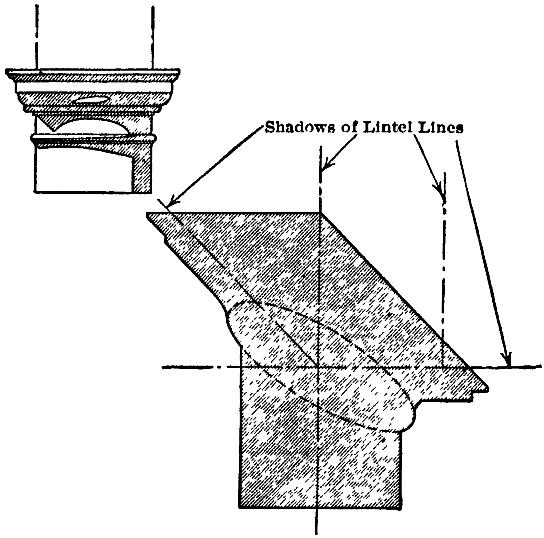


Fig. 74. Roman Doric Capital

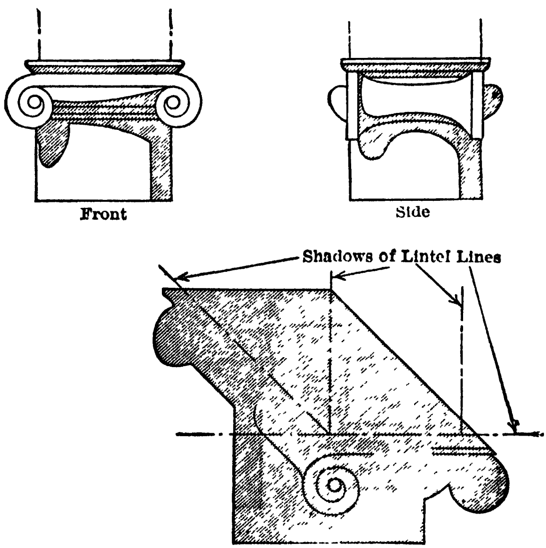


Fig. 75. Ionic Capital

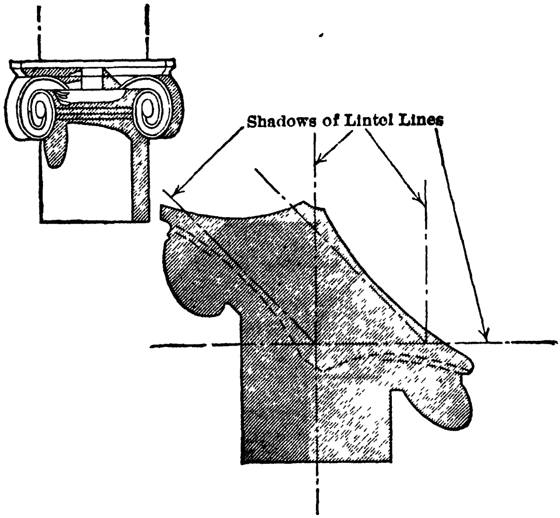


Fig. 76. Angular Ionic Capital

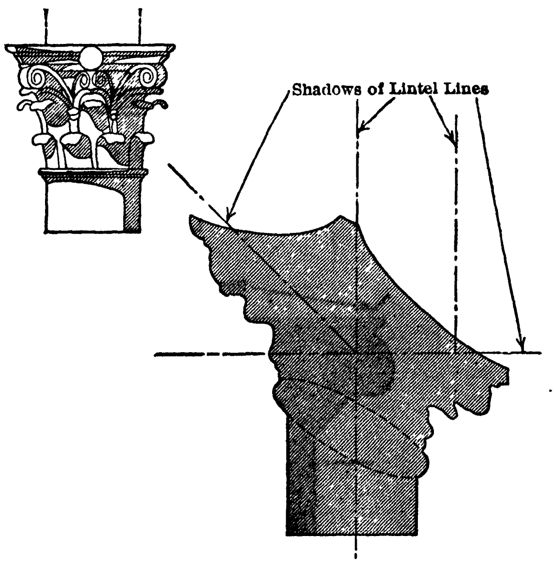


Fig. 77. Corinthian Capital

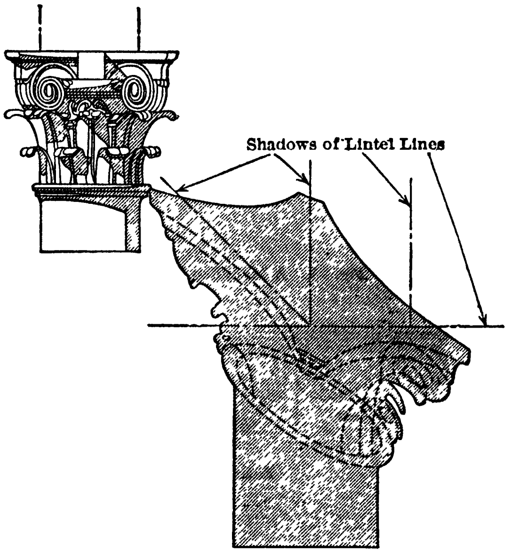


Fig. 78. Composite Capital

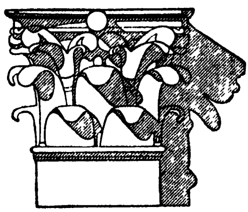


Fig. 79. A Corinthian Pilaster Cap in Block

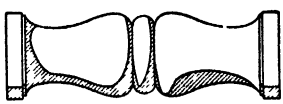


Fig. 80. The Scrolls of an Ionic Capital

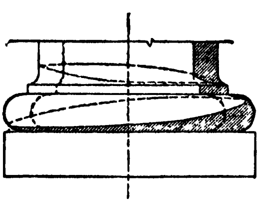


Fig. 81. A Tuscan Base

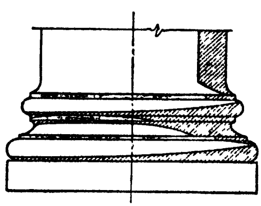


Fig. 82. An Attic Base

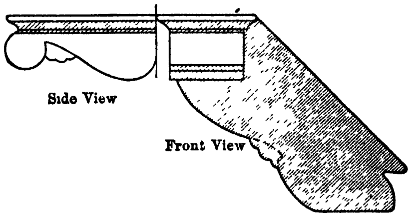


Fig. 83. A Modillion in Block

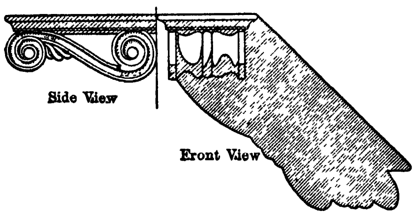


Fig. 84. A Modillion in Detail

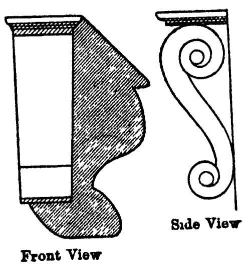


Fig. 85. A Console in Block

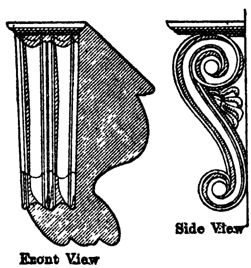


Fig. 86. Block Console in Detail

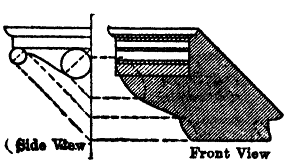


Fig. 87. A Modillion in Block

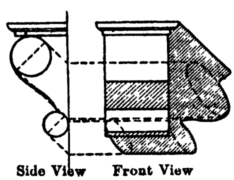


Fig. 88. A Console in Block

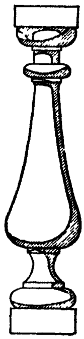


Fig. 89.
A Baluster

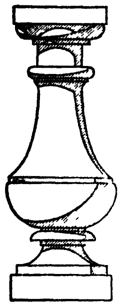


Fig 90
A Baluster

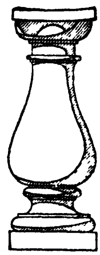


Fig. 91.
A Baluster

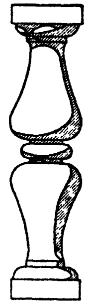


Fig. 92.
A Double Baluster

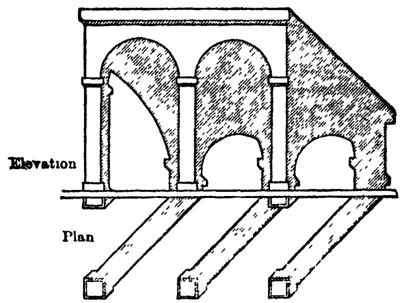


Fig 93. In This Case the Arches are Parallel to the Wall. The Outer Edge of the Intrados Casts a Semicircular Shadow, Which Intersects the Similar Shadow of the Inner Edge of the Intrados

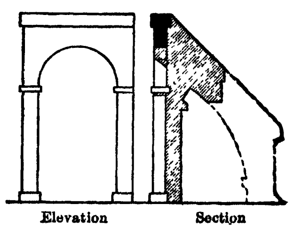


Fig. 94. In the Same Way as in Fig. 93, the Shadows of the Outer and Inner Edges of the Arch Which is in Plane Perpendicular to the Wall are Intersecting Ellipses

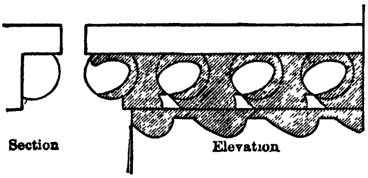


Fig. 95. Beads and Fillet

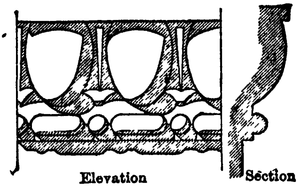


Fig 96. Eggs and Darts

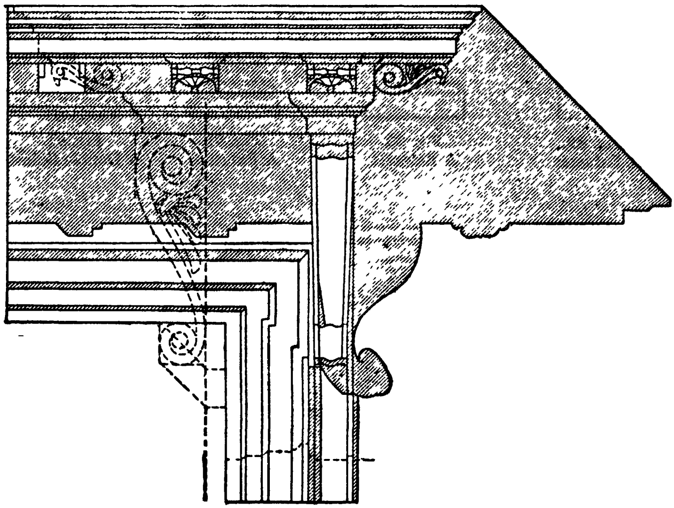


Fig. 97. Shades and Shadows of a Cornice over a Doorway

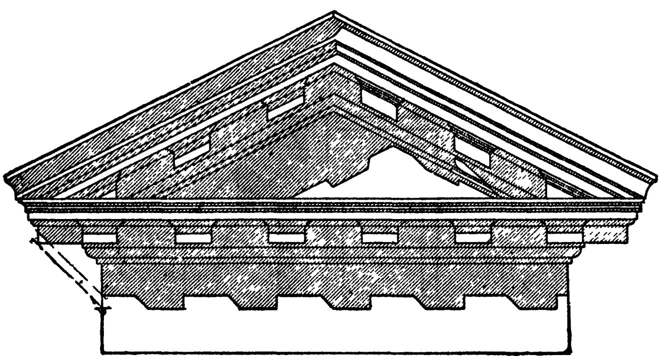


Fig. 98. A Pediment. Shadows upon the Raking Moldings, the Tympanum, and the Frieze below

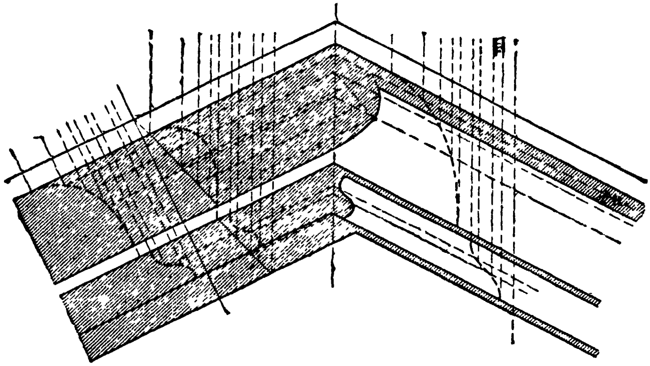


Fig. 99. Detail of Shadows upon the Raking Mouldings of Pediment (Fig 98)

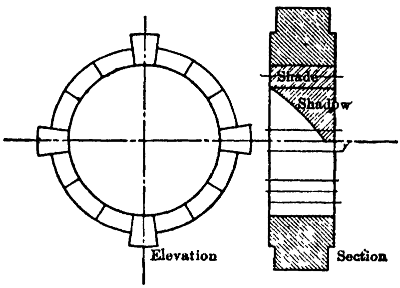


Fig. 100. A Circular Opening in a Wall (a Barrel Vault)

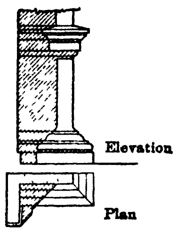


Fig. 101. (See also Fig 35) When a Line is Parallel to a Co-ordinate Plane, that Projection of its Shadow Line Expresses Exactly the Contours of the Surfaces Which the Shadow Crosses

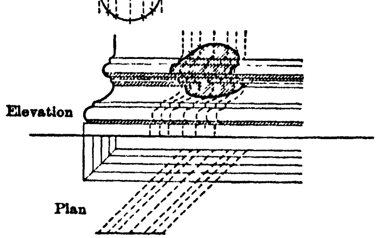


Fig. 102 (see Fig. 101). A Method of Finding the Shadow of a Plane Figure upon a Surface Other than a Plane Surface

PART III

MISCELLANEOUS DATA

REVISED AND NEW MATERIAL ADDED

By

J. HORACE FRANK, A.I.A.

Specific Gravities and Weights.
Wire Gauges and Metal Data.
Nails and Screws.
Excavating.
Stonework.
Brick and Brickwork.
Lime.
Sand and Gravel.
Lathing and Plastering.
Lumber and Carpenterwork.
Building Papers, Felts and Insulators.
Paint and Varnish.
Window Glass and Glazing.
Roofing.
Tiling.
Asphaltum.
Mineral Wool.
Estimating the Cost of Buildings
The Quantity System.
Dimensions and Data for Architects' Drawings and Specifications
Athletic Fields and Courts
Mail-Chutes
Refrigeration.
Tower Clocks.
Library Book-stacks.
Classical Moldings.
The Classical Orders.
Lightning Conductors.
Vacuum Cleaning.
Waterproofing for Foundations.
Force of the Wind.
Wind-stresses in Tall Buildings.
Horse-power, Pulleys, Gears, Belting and Shafting.
Chain-blocks, Hoists and Hooks.
Bells.
Symbols for the Apostles and Saints.
Documents of the American Institute of Architects.
Glossary of Terms Used by Architects and Builders.
Architectural Terms as Defined in Various Building Codes.

SPECIFIC GRAVITY

The Specific Gravity of a substance is the number which expresses the ratio that the weight of a given volume of the substance bears to the weight of the same volume of distilled water at a temperature of 62° F.; or, the specific gravity of a body is equal to its weight divided by the weight of an equal volume of water. The specific gravity of a substance, multiplied by the weight of a cubic foot of water, will gi e the weight of a cubic foot of the given substance. The weight of a cubic foot of water, at 62° F. and at the sea-level, is about 62.355 lb.* The specific gravity of a solid substance may be determined by first weighing a portion of it in air and then in water and dividing the weight in air by the loss of the weight in water; the quotient is the specific gravity required.

Example. A piece of granite weighs 5 32 lb. in air; when immersed in water it weighs 3 32 lb.

Solution. Weight in air (5.32 lb) divided by loss of weight in water (2 lb) = 2.66, the specific gravity.

$$2.66 \times 62.355 \text{ lb} = 165.84 \text{ lb} = \text{weight per cubic foot}$$

Note. 1 cu ft = 7.48 gal.

* The textbooks differ slightly in regard to this value.

Specific Gravities and Weights per Cubic Foot of Various Substances *

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Agate . . . 2.5 to 2.8	2.6	162.1
Air, atmospheric at 60° F, under pressure of one atmosphere, or 14.7 lb per sq in, weight $\frac{1}{815}$ the weight of water	0.00123	0.0767
Alabaster, carbonate . . . 2.61 to 2.76	2.68	167.1
Alcohol, absolute, at 32° F	0.794	49.5
Alcohol, 50 per cent	0.934	58.24
Alcohol, 95 per cent	0.815	50.82
Alcohol, commercial	0.833	51.95
Alder, dry† . . . 0.42 to 1.01	0.55	34.3
Alum	0.53	33.0
Aluminum, hammered	2.75	171.7
Aluminum, drawn	2.68	167.1
Aluminum, sheet	2.67	166.5
Aluminum, pure	2.67	166.5
Aluminum, cast	2.56	160.0
Amalgam . . . 13.7 to 14.1	13.92	868.0
Amber	1.08	67.4
Ambergris	0.87	54.3
Ammonia, 60° F	0.894	55.81
Antimony, cast	6.70	418.0
Antimony, native	6.67	416.0
Apple-wood, dry† . . . 0.66 to 1.25	0.75	46.8
Arsenic . . . 5.7 to 5.8	5.73	357.3
Asbestos . . . 2.1 to 3.0	2.81	175.0
Asbestos sheathing-paper	1.20	75.0
Ash, American white, dry†	0.61	38.0
Ashes of soft coal, solidly packed	0.70	40 to 45
Asphalt, for street-paving	1.60	100.0
Asphaltum . . . 1.11 to 1.23	1.15	69 to 75
Ballast, brick, gravel	1.79	111.6
Bamboo, dry†	0.36	22.5
Barium	3.88	242.0
Barytes	4.45	277.5
Basalt or trap-rock, average	2.96	184.6
Jersey City, N. J	3.00	187.1
Duluth, Minn	2.95	184.0
Staten Island, N. Y	2.86	178.3
Beech, dry† . . . 0.65 to 1.12	0.74	46.0
Beeswax	0.95	59.0
Benzine	0.69	43.0
Beer	1.04	64.9
Birch, dry† . . . 0.52 to 1.08	0.65	40.6
Bismuth, cast . . . 9.76 to 9.90	9.82	612.3
Blood, at 32° F	1.06	66.2
Bone . . . 1.8 to 2.0	1.90	118.6
Borax . . . 1.7 to 1.8	1.75	109.2
Boxwood, French, dry†	1.33	83.0

* The values given in this table are AVERAGE values.

† The word "dry" in this connection indicates that the wood contains not more than 15% of moisture. Green timbers usually weigh from one-fifth to nearly one-half more than dry; ordinary building-timbers, tolerably seasoned, one-sixth more.

Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb.	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Boxwood, Dutch, dry†	1.035	64 5
Boxwood, Brazilian, dry†		
Brass (copper and zinc), cast .7.8 to 9	8 45	527 0
Brass, rolled	8 56	533 8
Brass, sheet	8 24	513 6
Brass, wire	8 69	542 0
Bricks, common	1 922	120 0
Bricks, light, inferior	1.442	90 0
Bricks, lime-sand	2 163	135 0
Bricks, magnesia	2 643	165 0
Bricks, pressed	2.163	135 0
Bricks, pressed, hard	2 403	150 0
Bricks, soft	1 602	100 0
Bricks, fire	2 403	150.0
Bricks, paving		
Brickwork, pressed bricks, fine joints	2 24	140.0
Brickwork, medium quality	2 00	125.0
Brickwork, coarse, inferior, soft	1 60	100.0
Brickwork, at 125 lb per cu ft, 1 cu yd equals 1 507 tons and 17.92 cu ft equal 1 ton		...
Bromine	3 19	199 0
Bronze, coin	8 66	540 0
Bronze, gun-metal	8 60	536 3
Bronze, ordinary	8 40	524 0
Bronze, aluminum	7 70	480 0
Butter	0 86	53 to 54
Butternut-tree, dry†	0 38	23 7
Cadmium 8 6 to 8 7	8 65	539.4
Calcite .2.6 to 2 8	2 70	168.5
Calcium	1 58	98.6
Camphor, dry	0 99	61.7
Caoutchouc (India Rubber)	0 93	58.0
Carbon disulphide	1 29	80.5
Castor-oil	0.96	59.9
Caustic soda	.	88 0
Cedar, red and white, dry†	0 45	28.1
Cement, Natural (Rosendale), loose	1.04	65.0
Cement, Portland, loose	1 35	84.2
Cement, Natural, solid	2 95	183.9
Cement, Portland, solid	3 15	196.6
Chalk	2 35	146.5
Champagne	0.99	61.7
Charcoal of pines and oaks	.	15 to 30
Cherry, dry†	0 66	41.2
Chestnut, dry†	0.63	39.3
Chromium	5 00	312.0
Cider	1 02	63.5

* The values given in this table for specific gravities and for weights per cubic foot are AVERAGE values.

† The word "dry" in this connection indicates that the wood contains not more than 15% of moisture. Green timbers usually weigh from one-fifth to nearly one-half more than dry; ordinary building-timbers, tolerably seasoned, one-sixth more.

Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu. ft of water, 62.355 lb		Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Cinnabar.		8.12	507.0
Clay, potters', dry..	1.8 to 2.1	1.90	118.5
Clay, dry, in lump, loose........		1.01	63.0
Coal, anthracite, 1 3 to 1 84; of Penn , 1 3 to 1 7.		1.50	93.5
Coal, anthracite, broken, of any size, loose, average		.....	52 to 56
Coal, anthracite, broken, moderately shaken		....	56 to 60
Coal, anthracite, broken, heaped bushel, loose, 77 to 83 lb.		....	....
Coal, anthracite, broken, a ton loose occupies 40 to 43 cu ft.		..	.....
Coal, bituminous, solid, 1.2 to 1 5 .		1 35	84 0
Coal, bituminous, solid, Cambria Co , Pa , 1 27 to 1 34			79 to 84
Coal, bituminous, broken, of any size, loose		.	47 to 52
Coal, bituminous, moderately shaken		.	51 to 56
Coal, bituminous, a heaped bushel, loose, 70 to 78 lb			.....
Coal, bituminous, 1 ton occupies 43 to 48 cu. ft			..
Coke, loose, good quality		...	23 to 32
Coke, loose, a heaped bushel, 35 to 42 lb		.	....
Coke, loose, 1 ton occupies 80 to 97 cu ft		.	....
Concrete, stone.	130 to 150	2 33	145 0
Concrete, cinder	100 to 110	1.68	105 0
Copper, hammered	8.8 to 9 0	8 95	558.0
Copper, rolled.	8 9 to 9 0	8 95	558.0
Copper, drawn wire	8.8 to 9.0	8.89	554.5
Copper, sheet		8 72	543.6
Copper, cast	8.6 to 8.9	8 82	550 0
Copper, melted.		8 23	513 0
Cork, dry		0 24	15.0
Corundum, pure ...	3 92 to 4.01	3.96	247.5
Creosote oil	1 04 to 1 10	1 07	66.8
Cypress, American, dry†		0 55	34 3
Dogwood, dry†		0 75	46.8
Douglas fir, dry†		0 51	31 8
Earth, common loam, perfectly dry, loose			72 to 80
Earth, common loam, perfectly dry, shaken			82 to 92
Earth, common loam, perfectly dry, rammed			90 to 100
Earth, common loam, slightly moist, loose			70 to 76
Earth, common loam, more moist, loose		...	66 to 68
Earth, common loam, more moist, shaken		....	75 to 90
Earth, common loam, more moist, packed.			90 to 100
Earth, common loam, as soft, flowing mud		.	104 to 112
Earth, common loam, as soft, flowing mud, well-pressed			110 to 120
Ebony		1.22	76 0
Eggs		1 09	68.0
Elder-pith		0 076	4.7
Elm, dry†		0 56	35.0
Elm, rock......		0 80	50.0
Emerald		2 70	168 5

* The values given in this table for specific gravities and for weights per cubic foot are AVERAGE values

† The word "dry" in this connection indicates that the wood contains not more than 15% of moisture. Green timbers usually weigh from one-fifth to nearly one-half more than dry; ordinary building-timbers, tolerably seasoned, one-sixth more.

Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb.	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Emery	4.00	249 5
Fats	0.93	58 0
Feldspar	2.57	160 2
Filbert-tree, dry†	0.60	37 5
Fir, Douglas (see Douglas Fir).		
Flint	2 63	164 0
Gamboge	1 20	74 8
Garnet. 3.4 to 4.3	3.85	240 1
Glass, optical	3 45	215 0
Glass, flint	3 00	187 0
Glass, white	2 89	180 2
Glass, plate	2 80	174 6
Glass, green	2 67	166 5
Glass, floor, heavy	2 53	158 0
Glass, window	2 50	156 0
Gneiss (see Granites).		
Gold, pure	19 50	1 215 9
Gold, hammered, native	19 40	1 209 7
Gold, cast	19 258	1 200 8
Granites and gneiss, Connecticut, Greenwich	2 84	177 3
California, Penryn (hornblende)	2.77	172 9
New York	2 74	171 0
Maryland, Port Deposit	2 72	169 6
Massachusetts, Quincy (hornblende)	2 70	168 5
Wisconsin, Athelstane	2 70	168 5
Georgia, Lithornia and Stone Mountain	2 69	167 9
Minnesota	2 68	167 3
California, Rocklin (muscovite)	2 68	167 3
Rhode Island, Westerley	2 67	166 7
Connecticut, New London	2 66	166 0
New Hampshire, Keene	2 66	166 0
Maine, Hallowell	2 65	165 2
New Hampshire, Concord	2 65	165.2
Vermont, Barre	2 65	165 2
Wisconsin, Montello	2.64	164 6
Colorado, Georgetown (biotite)	2 63	164 0
Maine, Fox Island	2 63	164 0
Massachusetts, Rockport	2 61	162 7
Graphite	2 26	140 0
Gravel, dry	1.79	112 0
Gravel, wet	2 00	125 0
Greenstone, trap 2.8 to 3 2	3 00	187 0
Grindstone	2 14	133 5
Gum arabic	1 32	82.5
Gun-metal (see Bronze).		
Gunpowder (granular)	1 00	62 4
Gutta-percha	0 98	61 0

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Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb		Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Gypsum, natural rock, free from surface-water		2.30	143 0
Gypsum, crushed rock, not calcined, all passing 1-in ring		1.52	95.0
Gypsum, ground rock, 90% passing 100 mesh, dried not calcined.. .		1.25	78.0
Gypsum, Plaster-of-Paris, stucco, stiff mortar, set and dried out		1 22	77 0
Gypsum, Plaster-of-Paris, stucco, ground rock, 90% passing 100 mesh, calcined, loose		0 96	60 0
well shaken down or in bins		1 11	70 0
Hackmatack (see Larch)			
Hay, loose in stacks, about 512 cu ft per ton			.
Hemlock, dry†		0 42	26 2
Hickory, pignut, dry†		0 89	55.6
Hickory, mocker-nut, dry†		0 85	53 1
Hickory, shagbark, dry†		0 81	50 6
Hickory, nutmeg, dry†		0 78	48 7
Hickory, bitternut, dry†		0 77	48 1
Hickory, water, dry†		0 73	45 6
Holly		0 76	47 4
Honey		1 45	90 5
Horn		1 69	105.5
Hornblende	3 0 to 3 5	3 25	202.7
Ice	.0 88 to 0 914	0 89	56.0
Indiana Limestone		2 31	144 0
Iodine		4 94	308.0
Iridium, pure		22 12	1 379.0
Iron, cast	6 9 to 7.4	7 2	448.9
Iron, gray, foundry, cold		7 21	450.0
Iron, gray, foundry, molten		6 94	433 0
Iron, wrought		7 70	480.0
Ivory		1 88	117.0
Juniper-wood		0 57	35 6
Kaolin		2 20	137.2
Lava		2 65	165.2
Larch, or hackmatack, dry†		0 55	34.3
Lard		0 94	58.7
Lead, commercial, cast		11 36	708.0
Lead, commercial, sheet		11 40	710.8
Lead, pure		11 42	713.0
Lead, molten		10 40	648.8
Lignum-vitæ, dry†	0 65 to 1 33	0 99	41 to 84
Lime, quick, ground	1.04 to 1 20	1 12	65 to 75
Limestone, Illinois		2.57	160.4
Indiana		2 31	144.0
Kentucky		2 685	167.4
Michigan		2.44	152.1

* The values given in this table for specific gravities and for weights per cubic foot are AVERAGE values.

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Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu. ft of water, 62.355 lb	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Limestone (continued)		
Minnesota	2.655	165.6
Missouri	2.32	144.8
New York	2.71	169.0
Average of limestones	2.57	160.4
Linseed-oil	0.935	58.3
Locust, dry†	0.71	44.3
Magnesite	3.0	187.1
Magnesium, pure	1.72	107.0
Mahogany 0.56 to 1.06	0.81	50.5
Manganese, pure	8.00	499.0
Manganese, ore, red	4.01	250.0
Manganese, ore, black	3.45	215.1
Marble, average 2.6 to 164.4	2.6	162.1
domestic,		
New York	2.83	176.5
California	2.75	171.5
Georgia	2.73	170.2
Vermont, Dorset	2.66	166.0
foreign,		
Parian	2.84	177.1
African	2.80	174.6
Carrara	2.72	169.6
Biscayan	2.71	169.0
British	2.71	169.0
French	2.65	165.2
Marl	2.10	131.0
Masonry, brickwork (see Brickwork).		
Masonry, concrete, stone	2.33	145.3
Masonry, concrete, cinder	1.68	105.0
Masonry, granite, dressed	2.64	165.0
Masonry, granite, rubble in cement	2.48	155.0
Masonry, limestone, dressed	2.60	162.0
Masonry, marble, dressed for buildings	2.72	170.0
Masonry, sandstone	2.41	151.0
Mastic, gum resin	0.85	53.0
Mercury, at 32° F.	13.62	849.0
Mica 2.75 to 3.1	2.93	183.0
Milk, at 32° F.	1.032	64.3
Molybdenum, pure	8.63	538.1
Mortar, lime	1.65	103.0
Mortar, cement	1.68	105.0
Mud, dry, close		80 to 110
Mud, wet, moderately pressed		110 to 130
Mud, wet, fluid		104 to 120
Mulberry-tree, dry†	0.75	46.8
Naphtha-oil, wood, at 32° F	0.85	52.9

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Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb.	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Nickel	8 56	517 to 550
Oak, live, dry† 0 88 to 1 02	0 95	59.3
Oak, white, dry† . 0 66 to 0 88	0 77	48.0
Oak, red and black, dry†		32 to 45
Ochre.	3 50	218 0
Olive-oil, 32° F. .	0 916	57.12
Oolitic stones.......	2 25	140 3
Opal .	2 15	134 0
Opium	1 34	83.6
Orange-tree.. .	0 71	44 3
Palladium... . .	11 80	735 8
Paper.. .	0 95	59.3
Paraffin .. .	0 88	54.9
Pear-tree wood, dry†	0 67	41.8
Peat, pressed .	0 72	45 0
Petroleum, oil. .	0 878	54 8
Pine, Cuban, dry† .	0 63	39 3
Pine, yellow, long-leaf, dry†.	0 61	38 1
Pine, loblolly, dry†	0 53	33 1
Pine, yellow, short-leaf, dry†	0 51	31.8
Pine, red, Norway, dry† .	0 50	31 2
Pine, spruce, dry†. .	0 44	27 5
Pine, white, dry†.. . . .	0 38	23.7
Pitch...... . . .	1 08	67.0
Plaster of Paris (see Gypsum) .	2 25	140 3
Platinum..	21 50	1 340.6
Plumbago........... . . .	2 10	131.0
Poplar, dry†	0 47	29.3
Porcelain, china	2 30	143.4
Porphyry... . ..	2 76	172 3
Potash	2 26	141.0
Potassium .	0 865	54 0
Pumice-stone......	0 92	57.4
Quartz .	2 65	165.3
Quince-tree wood, dry†. . . .	0 71	44.3
Red lead.......	8 94	557.5
Resin..	1 09	68 0
Rock-crystal...	2 60	162.0
Rosewood....... . .	0 73	45 6
Rosin	1 10	68.6
Rubber, India	0 93	58.0
Ruby...	3.90	243.0
Salt, coarse, per struck bushel, Syracuse, N. Y , 56 lb.		45.0
Saltpetre.	2 02	122 to 130
Sand, of pure quartz, perfectly dry and loose .		90 to 106
Sand, or pure quartz, voids full of water		118 to 129
Sand, of pure quartz, very large and small grains, dry		117 0

* The values given in this table for specific gravities and for weights per cubic foot are AVERAGE values.

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Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu ft of water, 62.355 lb	Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Sandstone, average	2 44	152.1
Massachusetts, Longmeadow	2 49	155.4
Connecticut, Portland	2 50	156.0
New York . 2.40 to 2.70	2 60	162.1
New Jersey, Belleville	2 40	149.7
Pennsylvania	2 63	164.2
Virginia, Bristow	2 60	162.0
Ohio	2 22	138.6
Michigan	2.35	146.5
Wisconsin	2 22	138.6
Minnesota	2 25	140.5
Colorado	2 33	145.3
California, Angel Island	2 73	170.0
Shales, red or black	2 60	162.0
Silica	2 66	166.0
Silver	10 50	654.5
Slate	2 76	175.0
Snow, freshly fallen		5 to 12
Snow, moistened, compacted by rain		15 to 50
Soapstone	2 73	170.0
Sodium	0.978	61.0
Spelter	7.10.	443.0
Spirit, rectified	0 824	51.4
Spruce	0.40	25.0
Steel, cast	7.9	492.6
Steel, wrought	7 85	489.6
Sugar	1 60	100.0
Sycamore, dry†	0 58	36.5
Talc	2 81	175.2
Tallow	0 94	58.6
Tamarack	0 38	23.6
Tar	1 00	62.4
Teak	0 70	43.7
Terra-cotta, solid blocks		120 to 122
hollow blocks, 1½-in thick, smaller pieces heaviest		65 to 85
Tiles, solid	2 20	136.5
Tin, rolled	7 40	461.5
Tin, cast	7.30	455.0
Tin, molten	7.02	437.7
Trap (see Basalt).		
Tungsten	19 129	1 192.8
Turpentine	0 87	54.3
Type-metal, cast	10.45	651.8
Uranium	18 49	1 153.0
Vinegar	1 08	67.5
Walnut, black, dry†	0.60	37.5
Water, pure rain, distilled, at 32° F, barometer 30 in.		62.4

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Specific Gravities and Weights per Cubic Foot of Various Substances * (Continued)

The basis for specific gravities is pure water at 62° F., barometer 30 in. Weight of 1 cu. ft of water, 62.355 lb		Average specific gravity. Water = 1	Average weight of 1 cu ft lb
Water, pure rain, distilled, at 62° F, barometer 30 in		1 00	62.355
Water, pure rain, distilled, at 212° F, barometer 30 in			59.7
Water, sea	1 026 to 1.030	1 028	64.1
Wax (see Beeswax).			
Willow.		0 49	30 5
Wine		1 01	63 0
Zinc or spelter	6 8 to 7 2	7 00	436 5

* The values given in this table for specific gravities and for weights per cubic foot are AVERAGE values.

WIRE-GAUGES* AND METAL-DATA

A Wire-Gauge is a method of designating the diameter of wires or the thickness of sheets of metal by the numbers of a table arranged on a certain fixed basis. There are at the present time several gauges, resulting in great confusion, Table XIII, Chapter XI, gives the diameters of the gauges in common use. The only legal gauge in this country is the United States standard gauge. It is used by most of the manufacturers of sheet iron and steel and tinplate. The Brown & Sharpe gauge is commonly used for designating size of copper wires, and for sheet copper and brass. Nearly all copper wire, bare and insulated, is ordered, manufactured, and carried in stock in accordance with this gauge. This might be called the Copper Wire Gauge. The American Steel & Wire Company uses the old Washburn & Moen and Roebling gauges for all their steel and iron wire and also for wire nails. The sectional areas for these gauges are given in Table XIV, Chapter XI, and the table given in this section, taken from the Roebling and American Steel & Wire Company's lists. When placing orders for sheets and wire, it is always best to specify the weight per square or linear foot or the thickness or diameter in thousandths of an inch or in circular mils. The gauge for steel wire, used by the J. A. Roebling's Sons Company, is given in Table XIV, Chapter XI, and the circular-mil-gauge in Table III, Chapter XXXIV. The gauge used by this company is the same as the Washburn & Moen gauge, or the American Steel & Wire gauge, except that the diameters in most cases are given to the nearest mil. This gauge is so generally used for steel wire that it is sometimes called the Steel Wire Guage or the Market Wire Gauge. The Birmingham Wire gauge is the same as Stubs' Iron-Wire gauge, but entirely different from Stubs' Steel-Wire gauge. Galvanized telegraph and telephone-wire, both bare and insulated, and galvanized armor-wire are usually designated by this gauge. Its use is not very extensive and is becoming less. The new British Standard gauge is the legal standard for Great Britain and is used there for all kinds of wire. Its use in this country is very limited. It is known, also, as the English Legal standard gauge and the Imperial Wire gauge.

* See Chapter XI, Article 7.

Weights in Pounds per Square Foot of Sheets of Wrought Iron, Steel, Copper, and Brass

Thickness by American (Brown & Sharpe) gauge

No. of gauge	Thickness in inch	Iron	Steel	Copper	Brass
0000	0 46	18 40	18.77	20 84	19 69
000	0 4096	16 39	16 71	18 56	17 53
00	0 3648	14 59	14 88	16 53	15 61
0	0 3249	12 99	13 25	14 72	13 90
1	0 2893	11 57	11 80	13 11	12 38
2	0 2576	10 31	10 51	11 67	11 03
3	0 2294	9 18	9 36	10 39	9 82
4	0 2043	8 17	8 34	9 26	8 74
5	0 1819	7.28	7 42	8 24	7 79
6	0 1620	6 48	6 61	7 34	6 93
7	0 1443	5 77	5 89	6 54	6 18
8	0 1285	5 14	5 24	5 82	5 50
9	0 1144	4 58	4 67	5 18	4 90
10	0 1019	4 08	4 16	4 62	4.36
11	0 0907	3 63	3 70	4 11	3 88
12	0 0808	3 23	3 30	3 66	3.46
13	0 0720	2 88	2 94	3 26	3 08
14	0 0641	2.56	2 61	2 90	2 74
15	0 0571	2 28	2 33	2 59	2.44
16	0 0508	2 03	2 07	2.30	2.18
17	0 0453	1 81	1.85	2 05	1 94
18	0 0403	1 61	1 64	1 83	1 73
19	0 0359	1 44	1 46	1.63	1 54
20	0 0320	1 28	1.30	1.45	1 37
21	0 0285	1 14	1 16	1.29	1 22
22	0 0253	1 01	1 03	1 15	1 08
23	0 0226	0 903	0 921	1.02	0 966
24	0 0201	0 804	0 820	0 911	0 860
25	0 0179	0 716	0 730	0 811	0 766
26	0 0159	0 638	0 650	0 722	0 682
27	0 0142	0 568	0 579	0 643	0 608
28	0.0126	0 506	0 516	0 573	0 541
29	0 0113	0 450	0 459	0 510	0 482
30	0 0100	0 401	0 409	0 454	0 429
31	0 0089	0 357	0 364	0 404	0 382
32	0 0080	0 318	0 324	0 360	0 340
33	0 0071	0 283	0 289	0 321	0 303
34	0 0063	0.252	0 257	0.286	0.270
35	0 0056	0.224	0 229	0 254	0 240
Specific gravity .		7 704	7 85	8 72	8 24
Weight per cubic foot ..		480.00	489 60	543 6	513.6
Weight per cubic inch..		0 2778	0 2833	0.3146	0.2972

Weights of Sheets and Bars of Lead, Copper and Brass

Thickness or diameter, in	Lead			Copper			Brass			Thickness or diameter, in
	Sheets, per sq ft, lb	Square bars 1 ft long, lb	Round bars 1 ft long, lb	Sheets, per sq ft, lb	Square bars 1 ft long, lb	Round bars 1 ft long, lb	Sheets, per sq ft, lb	Square bars 1 ft long, lb	Round bars 1 ft long, lb	
1/32	1.86	0.005	0.004	1.44	0.004	0.003	1.36	0 004	0 003	1/32
1/16	3.72	0.019	0.015	2.89	0.015	0 012	2.71	0 014	0 011	1/16
3/32	5.58	0.044	0.034	4.33	0.034	0 027	4.06	0 032	0.025	3/32
1/8	7.44	0.078	0.061	5.77	0.060	0 047	5.42	0 056	0.044	1/8
5/32	9.30	0.121	0 095	7.20	0.094	0 074	6.75	0 088	0 069	5/32
3/16	11.20	0.174	0.137	8.66	0.135	0.106	8.13	0 127	0 100	3/16
7/32	13.00	0.237	0.187	10 10	0 184	0.144	9 50	0.173	0 136	7/32
1/4	14.90	0.310	0.244	11.50	0.240	0.189	10 80	0.226	0 177	1/4
5/16	18.60	0 485	0.381	14.40	0 376	0 295	13 50	0.353	0 277	5/16
3/8	22.30	0.698	0.548	17.30	0 541	0 425	16.30	0 508	0 399	3/8
7/16	26.00	0.950	0.746	20.30	0 736	0 578	19.00	0 691	0 543	7/16
1/2	29 80	1.240	0.974	23.10	0 962	0 755	21.70	0 903	0 709	1/2
9/16	33.50	1.570	1.230	26 00	1 220	0 955	24.30	1.140	0 900	9/16
5/8	37.20	1.940	1 520	28.90	1 500	1 180	27.10	1 410	1.110	5/8
11/16	40.90	2.340	1.840	31 70	1 820	1 430	29.80	1.700	1 340	11/16
3/4	44.60	2.790	2.190	34.60	2.160	1 700	32 50	2 030	1 600	3/4
13/16	48.30	3.270	2.570	37 50	2.550	1 990	35 20	2 380	1 870	13/16
7/8	52.10	3.800	2.980	40 40	2.940	2 310	37.90	2 760	2.170	7/8
15/16	56.00	4.370	3.420	43 30	3.380	2 650	40 60	3 180	2 490	15/16
1	59 50	4.960	3.900	46.20	3 850	3 020	43 30	3 610	2 840	1
1 1/8	66.90	6.270	4 920	52.00	4 870	3 820	48.70	4 570	3 600	1 1/8
1 1/4	74.40	7.750	6.090	57.70	6.010	4.720	54 20	5.640	4.430	1 1/4
1 3/8	81.80	9 370	7.370	63.50	7.280	5.720	59 60	6 820	5.370	1 3/8
1 1/2	89.30	11.200	8.770	69.30	8.650	6 800	65 00	8.120	6.380	1 1/2
1 5/8	96.70	13.100	10.30	75.10	10 200	7.980	70.40	9 530	7.490	1 5/8
1 3/4	104.00	15.200	11.90	80.80	11.800	9.250	75.90	11.100	8.680	1 3/4
1 7/8	112.00	17.500	13.70	86.60	13 500	10.600	81.30	12.700	9.970	1 7/8
2	119 00	19 800	15 60	92 30	15 400	12 100	86 70	14 400	11 300	2

Sizes and Weights of Smooth Steel Wire

As made by the American Steel & Wire Company

No. of gauge	Diameters			Sectional area, sq in	Weight *		No. of feet per pound
	Fractions of inch	Decimal of inch	Millimeters		Pounds per 100 feet	Pounds per mile	
000000	. .	0.4615	11.72	0.16728	56 81	2 999.0	1.76
. ..	7/16	0.4375	11.11	0.15033	51.05	2 696.0	1.959
00000	. .	0 4305	10.93	0 14556	49 43	2 610.0	2 023
. .	13/32	0.40625	10 32	0 12962	44 02	2 324.0	2 272
0000	.. . ..	0.3938	10.00	0 12180	41.36	2 184.0	2 418
... .	3/8	0 3750	9.525	0 11045	37 51	1 980.0	2.666
000	. .	0.3625	9.2075	0 10321	35.05	1 851 0	2 853
......	11/32	0.34375	8 731	0 092806	31 52	1 664 0	3 173
00	. .	0 3310	8 407	0 086049	29 22	1 543.0	3 422
... .	5/16	0 3125	7.938	0 076699	26 05	1 375 0	3 839
0	.. .	0.3065	7.785	0 073782	25 06	1 323.0	3.991
1	. . .	0.2830	7.188	0 062902	21.36	1 128.0	4.681
.. ...	9/32	0 28125	7.144	0 062126	21 10	1 114.0	4 740
2	. ..	0 2625	6.668	0.054119	18.38	970.4	5 441
. . .	1/4	0 2500	6.350	0 049087	16 67	880 2	5 999
3	. ...	0.2437	6.190	0.046645	15 84	836.4	6 313
4	. . .	0.2253	5.723	0.039867	13 54	714.8	7.386
.....	7/32	0.21875	5.556	0.037583	12.76	673 9	7.835
5	... .	0.2070	5.258	0 033654	11.43	603 4	8 750
6	.	0 1920	4.877	0 028953	9.832	519 2	10 17
...	3/16	0 1875	4.763	0 027612	9.377	495 1	10 66
7	.....	0 1770	4 496	0 024606	8.356	441.2	11 97
8	. . .	0 1620	4.115	0.020612	7.000	369.6	14.29
......	5/32	0.15625	3.969	0 019175	6.512	343.8	15 36
9	.....	0.1483	3.767	0.017273	5 866	309.7	17 05
10	..	0.1350	3.429	0.014514	4.861	256 7	20 57
.....	1/8	0 125	3.175	0.012272	4.168	220.0	24 00
11	. . .	0.1205	3.061	0.011404	3.873	204.5	25.82
12	. . .	0.1055	2.680	0.0087417	2.969	156.7	33 69
... ..	3/32	0.09375	2.381	0.0069029	2.344	123 8	42.66
13	. . ..	0 0915	2.324	0.0065755	2 233	117.9	44 78
14	... .	0.0800	2.032	0.0050266	1.707	90.13	58 58
15	. . ..	0 0720	1.829	0.0040715	1.383	73 01	72.32
16	1/16	0 0625	1.588	0.0030680	1.042	55.01	95.98
17	......	0.0540	1.372	0.0022902	0.7778	41.07	128.60
18	....	0.0475	1.207	0.0017721	0.6018	31.77	166.20
19	......	0.0410	1.041	0 0013203	0.4484	23.67	223.00
20	......	0.0348	0.8839	0.00095115	0.3230	17.05	309.60
21	. ....	0 0317	0.8052	0.00078924	0.2680	14.15	373.10
......	1/32	0.03125	0.7938	0.00076699	0.2605	13.75	383.90
22	....	0.0286	0.7264	0.00064242	0.2182	11.52	458.40
23	......	0 0258	0.6553	0.00052279	0.1775	9.37	563.30
24	. ...	0.0230	0.5842	0.00041548	0.1411	7.45	708.70

* For iron wire, the values in columns 6 and 7 should be multiplied by 0.98 and for copper wire, by 1.12.

Kinds of Wire Manufactured by the American Steel and Wire Company

Market-wire, Nos. 0000 to 18.

Annealed stone-wire or weaving-wire, Nos. 16 to 47.

Tinned market-wire, Nos. 0 to 18.

Tinned stone-wire, Nos. 16 to 40.

Gun-screw wire, finished with great care as regards roundness and exactness to gauge, Nos. 18 to 50.

Machinery-wire, Nos. 00000 to 18.

Cast-steel wire, ½-in diameter, down to No. 26.

Drill and needle-steel wire, Nos. 12 to 25.

Welding wire:

for Electric Welding,	Nos 3 to 10
for Electric or Acetylene Welding,	Nos. 2 to 7
for Acetylene Welding,	Nos. 2 to 16

The term MARKET-WIRE applies to the ordinary and most used forms of Bessemer ANNEALED, BRIGHT, GALVANIZED, TINNED and COPPERED wires.

Galvanized-Iron-Wire Strand. See Chapter XI.

Weights and Areas of Square and Round Bars and Circumferences of Round Steel Bars *

Weights are for steel, at 489 6 lb per cu ft

Thickness or diameter, in	Weight of □ bar 1 ft long, lb	Weight of ○ bar 1 ft long, lb	Area of □ bar, sq in	Area of ○ bar, sq in	Circumference of ○ bar, in
1/16	0.013	0.010	0.0039	0 0031	0.1963
5/64	0.021	0.016	0.0061	0.0048	0.2454
3/32	0.030	0.023	0.0088	0.0069	0 2945
7/64	0 041	0.032	0.0120	0.0094	0.3436
1/8	0 053	0.042	0 0156	0.0123	0 3927
9/64	0.067	0.053	0 0198	0.0155	0 4418
5/32	0.083	0.065	0.0244	0 0192	0.4909
11/64	0.100	0.079	0.0295	0.0232	0.5400
3/16	0 120	0.094	0.0352	0.0276	0.5890
13/64	0.140	0.110	0 0413	0 0324	0 6381
7/32	0 163	0.128	0.0479	0 0376	0 6872
15/64	0 187	0.147	0.0549	0 0431	0.7363
1/4	0 213	0.167	0 0625	0 0491	0 7854
17/64	0 240	0.188	0 0706	0 0554	0 8345
9/32	0 269	0.211	0 0791	0 0621	0 8836
19/64	0.300	0.235	0.0881	0 0692	0.9327
5/16	0 332	0.261	0.0977	0 0767	0 9817
11/32	0 402	0.316	0.1182	0.0928	1 0799
3/8	0 478	0.376	0.1406	0 1104	1.1781
13/32	0 561	0.441	0.1650	0.1296	1.2763
7/16	0 651	0.511	0.1914	0.1503	1.3744
15/32	0 747	0.587	0 2197	0.1726	1 4726
1/2	0.850	0.668	0 2500	0 1963	1 5708
17/32	0.960	0.754	0.2822	0.2217	1.6690
9/16	1 076	0.845	0 3164	0 2485	1.7671
19/32	1.199	0.941	0.3525	0 2769	1.8653
5/8	1 328	1.043	0.3906	0 3068	1 9635
11/16	1.607	1.262	0.4727	0 3712	2.1598
3/4	1 913	1.502	0.5625	0 4418	2 3562
13/16	2.245	1.763	0.6602	0 5185	2.5525
7/8	2 603	2.044	0.7656	0 6013	2 7489
15/16	2.989	2.347	0.8789	0 6903	2.9452

* Adapted from the Handbook of the Cambria Steel Company, Johnstown, Pa.

Weights and Areas of Square and Round Steel Bars*

Weights are for steel, at 489 6 lb per cu ft

Thickness, in	□ Area, sq in	□ Weight per foot, lb	○ Area, sq in	○ Weight per foot, lb	Thickness, in	□ Area, sq in	□ Weight per foot, lb	○ Area, sq in	○ Weight per foot, lb
1	1 000	3 400	0 785	2 670	3	9.000	30 60	7.069	24.03
1/16	1 129	3.838	0 887	3 014	1/16	9 379	31 89	7 366	25 04
1/8	1 266	4.303	0 994	3 379	1/8	9 766	33 20	7.670	26 08
3/16	1 410	4.795	1 108	3 766	3/16	10 16	34.55	7.980	27.13
1/4	1 563	5.312	1 227	4 173	1/4	10.56	35.92	8.296	28 20
5/16	1 723	5.857	1.353	4 600	5/16	10.97	37.31	8 618	29 30
3/8	1 891	6.428	1.485	5.049	3/8	11 39	38.73	8 946	30.42
7/16	2.066	7.026	1.623	5.518	7/16	11 82	40.18	9.281	31.56
1/2	2 250	7 650	1 767	6.008	1/2	12 25	41.65	9 621	32.71
9/16	2 441	8.301	1.918	6.520	9/16	12.69	43.14	9.968	33 90
5/8	2 641	8.978	2.074	7 051	5/8	13 14	44.68	10 32	35 09
11/16	2 848	9.682	2.237	7.604	11/16	13.60	46.24	10.68	36.31
3/4	3.063	10.41	2.405	8.178	3/4	14.06	47.82	11.05	37.56
13/16	3 285	11.17	2.580	8.773	13/16	14 54	49.42	11.42	38.81
7/8	3 516	11.95	2.761	9 388	7/8	15 02	51.05	11.79	40.10
15/16	3.754	12.76	2.948	10.02	15/16	15.50	52.71	12.18	41.40
2	4 000	13.60	3.142	10.68	4	16.00	54.40	12.57	42.73
1/16	4 254	14.46	3.341	11.36	1/16	16.50	56.11	12.96	44.07
1/8	4 516	15.35	3.547	12 06	1/8	17.02	57.85	13.36	45.44
3/16	4 785	16.27	3.758	12.78	3/16	17.54	59.62	13.77	46.83
1/4	5 063	17.22	3.976	13.52	1/4	18 06	61.41	14.19	48.24
5/16	5 348	18.19	4.200	14.28	5/16	18.60	63.23	14.61	49 66
3/8	5 641	19.18	4 430	15.07	3/8	19.14	65 08	15 03	51.11
7/16	5.941	20.20	4.666	15.86	7/16	19 69	66 95	15.47	52.58
1/2	6.250	21.25	4.909	16.69	1/2	20.25	68.85	15.90	54.07
9/16	6 566	22.33	5.157	17 53	9/16	20.82	70 78	16.35	55.59
5/8	6 891	23.43	5.412	18 40	5/8	21.39	72.73	16.80	57.12
11/16	7.223	24.56	5.673	19.29	11/16	21.97	74.70	17.26	58.67
3/4	7 563	25.71	5.940	20.20	3/4	22 56	76.71	17.72	60.25
13/16	7.910	26.90	6.213	21 12	13/16	23 16	78 74	18.19	61.84
7/8	8.266	28.10	6.492	22 07	7/8	23.77	80.81	18 67	63.46
15/16	8.629	29.34	6 777	23.04	15/16	24 38	82.89	19.15	65.10

* Adapted from the Handbook of the Cambria Steel Company, Johnstown, Pa.

Weights and Areas of Square and Round Steel Bars * (Continued)

Weights are for steel, at 489.6 lb per cu ft

Thickness, in	□ Area, sq in	□ Weight per foot, lb	○ Area, sq in	○ Weight per foot, lb	Thickness, in	□ Area, sq in	□ Weight per foot, lb	○ Area, sq in	○ Weight per foot, lb
5	25 00	85.00	19.64	66.76	7	49.00	166.6	38.49	130 9
1/16	25 63	87.14	20 13	68.44	1/4	52.56	178.7	41 28	140.4
1/8	26.27	89.30	20 63	70.14	1/2	56 25	191.3	44 18	150.2
3/16	26.91	91.49	21.14	71.86	3/4	60.06	204.2	47.17	160 3
1/4	27.56	93.72	21.65	73.60	8	64.00	217.6	50.27	171 0
5/16	28.22	95.96	22.17	75.37	1/4	68.06	231.4	53.46	181 8
3/8	28.89	98.23	22.69	77.15	1/2	72 25	245.6	56 75	193.0
7/16	29 57	100.5	23.22	78.95	3/4	76 56	260.3	60.13	204.4
1/2	30.25	102.8	23.76	80.77	9	81 00	275.4	63 62	216.3
9/16	30.94	105.2	24.30	82.62	1/4	85 56	290.9	67.20	228.5
5/8	31 64	107.6	24.85	84.49	1/2	90 25	306.8	70.88	241.0
11/16	32.35	110.0	25.41	86.38	3/4	95 06	323.2	74 66	253.9
3/4	33.06	112.4	25.97	88.29	10	100 0	340.0	78.54	267.0
13/16	33 79	114.9	26.54	90.22	1/4	105.1	357.2	82.52	280 6
7/8	34.52	117.4	27.11	92 17	1/2	110 3	374.9	86.59	294 4
15/16	35.25	119.9	27.69	94.14	3/4	115.6	392.9	90 76	308.6
6	36.00	122.4	28.27	96 14	11	121.0	411.4	95.03	323 1
1/8	37.52	127.6	29.47	100 2	1/4	126 6	430.3	99.40	337 9
1/4	39 06	132.8	30.68	104.3	1/2	132 3	449.6	103.9	353.1
3/8	40.64	138.2	31.92	108.5	3/4	138.1	469.4	108.4	368.6
1/2	42.25	143.6	33.18	112.8	12	144 0	489.6	113.1	384.5
5/8	43.89	149.2	34.47	117.2	.		...		.
3/4	45 56	154.9	35 79	121.7		.	. . .		.
7/8	47.27	160.8	37.12	126.2	.				.

* Adapted from the Handbook of the Cambria Steel Company, Johnstown, Pa.

Weights in Pounds of Flat Rolled Steel Bars

PER LINEAR FOOT

One cubic foot of steel weighs 489.6 lb

For thicknesses from 1/16 in to 9/16 in and widths from 1/4 in to 3/4 in

Thickness, inches	Width of bar, inches								
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
1/16	0.053	0.066	0.080	0.093	0.106	0.120	0.133	0.146	0.159
5/64	0.066	0.083	0.100	0.116	0.133	0.149	0.166	0.183	0.199
3/32	0.080	0.100	0.120	0.139	0.159	0.179	0.199	0.219	0.239
7/64	0.093	0.116	0.139	0.163	0.186	0.209	0.232	0.256	0.279
1/8	0.106	0.133	0.159	0.186	0.212	0.239	0.266	0.292	0.319
9/64	0.120	0.149	0.179	0.209	0.239	0.269	0.299	0.329	0.359
5/32	0.133	0.166	0.199	0.232	0.266	0.299	0.332	0.365	0.398
11/64	0.146	0.183	0.219	0.256	0.292	0.329	0.365	0.402	0.438
3/16	0.159	0.199	0.239	0.279	0.319	0.359	0.398	0.438	0.478
13/64	0.173	0.216	0.259	0.302	0.345	0.388	0.432	0.475	0.518
7/32	0.186	0.232	0.279	0.325	0.372	0.418	0.465	0.511	0.558
15/64	0.199	0.249	0.299	0.349	0.398	0.448	0.498	0.548	0.598
1/4	0.213	0.266	0.319	0.372	0.425	0.478	0.531	0.584	0.638
17/64	0.226	0.282	0.339	0.395	0.452	0.508	0.564	0.621	0.677
9/32	0.239	0.299	0.359	0.418	0.478	0.538	0.598	0.657	0.717
19/64	0.252	0.315	0.379	0.442	0.505	0.568	0.631	0.694	0.757
5/16	0.266	0.332	0.398	0.465	0.531	0.598	0.664	0.730	0.797
21/64	0.279	0.349	0.418	0.488	0.558	0.628	0.697	0.767	0.827
11/32	0.292	0.365	0.438	0.511	0.584	0.657	0.730	0.804	0.877
23/64	0.305	0.382	0.458	0.535	0.611	0.687	0.764	0.840	0.916
3/8	0.319	0.398	0.478	0.558	0.638	0.717	0.797	0.877	0.956
25/64	0.332	0.415	0.498	0.581	0.664	0.747	0.830	0.913	0.996
13/32	0.345	0.432	0.518	0.604	0.691	0.777	0.863	0.950	1.04
27/64	0.359	0.448	0.538	0.628	0.717	0.807	0.896	0.986	1.08
7/16	0.372	0.465	0.558	0.651	0.744	0.837	0.930	1.02	1.12
29/64	0.385	0.481	0.578	0.674	0.770	0.867	0.963	1.06	1.16
15/32	0.398	0.498	0.598	0.697	0.797	0.896	0.996	1.10	1.20
31/64	0.412	0.515	0.618	0.721	0.823	0.926	1.03	1.13	1.24
1/2	0.425	0.531	0.638	0.744	0.850	0.956	1.06	1.17	1.28
33/64	0.438	0.548	0.657	0.767	0.877	0.986	1.10	1.21	1.31
17/32	0.452	0.564	0.677	0.790	0.903	1.02	1.13	1.24	1.35
35/64	0.465	0.581	0.697	0.813	0.930	1.05	1.16	1.28	1.39
9/16	0.478	0.598	0.717	0.837	0.956	1.08	1.20	1.31	1.43

Weights in Pounds of Flat Rolled Steel Bars (Continued)

PER LINEAR FOOT

For thicknesses from 1/16 to 2 in and widths from 1 to 3 in

Thickness, inches	Width of bar, inches								
	1	1¼	1½	1¾	2	2¼	2½	2¾	3
1/16	0 21	0 26	0 32	0.37	0 43	0 48	0.53	0 58	0 63
⅛	0 42	0 53	0 64	0 75	0 85	0.96	1.06	1 17	1 28
3/16	0 63	0 79	0.96	1 11	1 28	1.44	1.59	1 75	1 91
¼	0 85	1 06	1.28	1 49	1 70	1.91	2 12	2 34	2 55
5/16	1 06	1.33	1.59	1 86	2 12	2.39	2.65	2 92	3 19
⅜	1.28	1 59	1.92	2 23	2.55	2 87	3 19	3 51	3 83
7/16	1 49	1.86	2.23	2 60	2 98	3.35	3.72	4.09	4 46
½	1.70	2.12	2 55	2.98	3 40	3 83	4.25	4.67	5 10
9/16	1 92	2 39	2.87	3.35	3 83	4.30	4.78	5 26	5 74
⅝	2.12	2.65	3 19	3.72	4 25	4.78	5.31	5 84	6 38
11/16	2.34	2.92	3 51	4 09	4 67	5.26	5 84	6 43	7 02
¾	2 55	3 19	3.83	4.47	5 10	5.75	6 38	7.02	7 65
13/16	2.76	3.45	4.14	4.84	5.53	6.21	6.90	7 60	8 29
⅞	2.98	3 72	4.47	5 20	5 95	6.69	7.44	8 18	8 93
15/16	3.19	3 99	4.78	5 58	6 38	7.18	7.97	8 77	9 57
1	3.40	4.25	5.10	5 95	6 80	7.65	8.50	9.35	10.20
1 1/16	3 61	4 52	5.42	6 32	7.22	8.13	9 03	9.93	10 84
1⅛	3 83	4 78	5.74	6.70	7 65	8.61	9 57	10.52	11 48
1 3/16	4 04	5.05	6.06	7.07	8 08	9.09	10.10	11 11	12 12
1¼	4 25	5.31	6.38	7.44	8.50	9.57	10 63	11 69	12 75
1 5/16	4 46	5.58	6 69	7.81	8 93	10.04	11 16	12 27	13 39
1⅜	4 67	5 84	7.02	8.18	9.35	10.52	11.69	12.85	14 03
1 7/16	4.89	6.11	7.34	8.56	9 78	11.00	12.22	13.44	14 66
1½	5.10	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15 30
1 9/16	5 32	6 64	7.97	9.30	10.63	11.95	13.28	14.61	15 94
1⅝	5 52	6.90	8.29	9 67	11 05	12 43	13.81	15.19	16.58
1 11/16	5.74	7.17	8.61	10.04	11 47	12 91	14 34	15.78	17 22
1¾	5.95	7.44	8.93	10.42	11.90	13.40	14.88	16 37	17 85
1 13/16	6.16	7.70	9.24	10.79	12.33	13.86	15.40	16.95	18 49
1⅞	6.38	7.97	9.57	11.15	12.75	14.34	15 94	17.53	19.13
1 15/16	6.59	8.24	9.88	11.53	13.18	14.83	16.47	18.12	19 77
2	6.80	8 50	10.20	11.90	13.60	15.30	17.00	18.70	20.40

Weights in Pounds of Flat Rolled Steel Bars (Continued)

PER LINEAR FOOT

For thicknesses from 1/16 to 2 in and widths from 3½ to 7½ in

Thickness, inches	Width of bar, inches								
	3½	4	4½	5	5½	6	6½	7	7½
1/16	0.75	0.85	0.96	1.06	1.17	1.28	1.39	1.49	1.60
⅛	1.49	1.70	1.92	2.13	2.34	2.55	2.77	2.98	3.19
3/16	2.23	2.55	2.87	3.19	3.51	3.83	4.14	4.46	4.78
¼	2.98	3.40	3.83	4.25	4.67	5.10	5.53	5.95	6.36
5/16	3.72	4.25	4.78	5.31	5.84	6.38	6.90	7.44	7.97
⅜	4.47	5.10	5.74	6.38	7.02	7.65	8.29	8.93	9.57
7/16	5.20	5.95	6.70	7.44	8.18	8.93	9.67	10.41	11.16
½	5.95	6.80	7.65	8.50	9.35	10.20	11.05	11.90	12.75
9/16	6.70	7.65	8.61	9.57	10.52	11.48	12.43	13.39	14.34
⅝	7.44	8.50	9.57	10.63	11.69	12.75	13.81	14.87	15.94
11/16	8.18	9.35	10.52	11.69	12.85	14.03	15.20	16.36	17.53
¾	8.93	10.20	11.48	12.75	14.03	15.30	16.58	17.85	19.13
13/16	9.67	11.05	12.43	13.81	15.19	16.58	17.95	19.34	20.72
⅞	10.41	11.90	13.39	14.87	16.36	17.85	19.34	20.83	22.32
15/16	11.16	12.75	14.34	15.94	17.53	19.13	20.72	22.32	23.91
1	11.90	13.60	15.30	17.00	18.70	20.40	22.10	23.80	25.50
1 1/16	12.65	14.45	16.26	18.06	19.87	21.68	23.48	25.29	27.10
1⅛	13.39	15.30	17.22	19.13	21.04	22.95	24.87	26.78	28.68
1 3/16	14.13	16.15	18.17	20.19	22.21	24.23	26.24	28.26	30.28
1¼	14.87	17.00	19.13	21.25	23.38	25.50	27.62	29.75	31.88
1 5/16	15.62	17.85	20.08	22.32	24.54	26.78	29.01	31.23	33.48
1⅜	16.36	18.70	21.04	23.38	25.71	28.05	30.39	32.72	35.06
1 7/16	17.10	19.85	21.99	24.44	26.88	29.33	31.77	34.21	36.66
1½	17.85	20.40	22.95	25.50	28.05	30.60	33.15	35.70	38.26
1 9/16	18.60	21.25	23.91	26.57	29.22	31.88	34.53	37.19	39.84
1⅝	19.34	22.10	24.87	27.63	30.39	33.15	35.91	38.67	41.44
1 11/16	20.08	22.95	25.82	28.69	31.55	34.43	37.30	40.16	43.03
1¾	20.83	23.80	26.78	29.75	32.73	35.70	38.68	41.65	44.63
1 13/16	21.57	24.65	27.73	30.81	33.89	36.98	40.05	43.14	46.22
1⅞	22.31	25.50	28.69	31.87	35.06	38.25	41.44	44.63	47.82
1 15/16	23.06	26.35	29.64	32.94	36.23	39.53	42.82	46.12	49.41
2	23.80	27.20	30.60	34.00	37.40	40.80	44.20	47.60	51.00

Weights in Pounds of Flat Rolled Steel Bars (Continued)

PER LINEAR FOOT

For thicknesses from 1/16 to 2 in and widths from 8 to 12 in

Thickness, inches	Width of bar, inches								
	8	8½	9	9½	10	10½	11	11½	12
1/16	1.70	1.81	1.91	2.02	2.13	2.23	2.34	2.45	2.55
1/8	3.40	3.61	3.82	4.04	4.25	4.46	4.68	4.89	5.10
3/16	5.10	5.42	5.74	6.06	6.38	6.70	7.02	7.32	7.65
1/4	6.80	7.22	7.65	8.08	8.50	8.92	9.34	9.78	10.20
5/16	8.50	9.03	9.56	10.10	10.62	11.16	11.68	12.22	12.75
3/8	10.20	10.84	11.48	12.12	12.75	13.39	14.03	14.68	15.30
7/16	11.90	12.64	13.40	14.14	14.88	15.62	16.36	17.12	17.85
1/2	13.60	14.44	15.30	16.16	17.00	17.85	18.70	19.55	20.40
9/16	15.30	16.26	17.22	18.18	19.14	20.08	21.02	22.00	22.95
5/8	17.00	18.06	19.13	20.19	21.25	22.32	23.38	24.44	25.50
11/16	18.70	19.86	21.04	22.21	23.38	24.54	25.70	26.88	28.05
3/4	20.40	21.68	22.96	24.23	25.50	26.78	28.05	29.33	30.60
13/16	22.10	23.48	24.86	26.24	27.62	29.00	30.40	31.76	33.15
7/8	23.80	25.30	26.78	28.26	29.75	31.24	32.72	34.21	35.70
15/16	25.50	27.10	28.69	30.28	31.88	33.48	35.06	36.66	38.25
1	27.20	28.90	30.60	32.30	34.00	35.70	37.40	39.10	40.80
1 1/16	28.90	30.70	32.52	34.32	36.12	37.92	39.74	41.54	43.35
1 1/8	30.60	32.52	34.43	36.34	38.25	40.17	42.08	44.00	45.90
1 3/16	32.30	34.32	36.34	38.36	40.38	42.40	44.42	46.44	48.45
1 1/4	34.00	36.12	38.26	40.37	42.50	44.63	46.76	48.88	51.00
1 5/16	35.70	37.93	40.16	42.40	44.64	46.86	49.08	51.32	53.55
1 3/8	37.40	39.74	42.08	44.41	46.75	49.08	51.42	53.76	56.10
1 7/16	39.10	41.54	44.00	46.44	48.88	51.32	53.76	56.21	58.65
1 1/2	40.80	43.35	45.90	48.45	51.00	53.55	56.10	58.65	61.20
1 9/16	42.50	45.16	47.82	50.48	53.14	55.78	58.42	61.10	63.75
1 5/8	44.20	46.96	49.73	52.49	55.25	58.02	60.78	63.54	66.30
1 11/16	45.90	48.76	51.64	54.51	57.38	60.24	63.10	65.98	68.85
1 3/4	47.60	50.58	53.56	56.53	59.50	62.48	65.45	68.43	71.40
1 13/16	49.30	52.38	55.46	58.54	61.62	64.70	67.80	70.86	73.95
1 7/8	51.00	54.20	57.38	60.56	63.75	66.94	70.12	73.31	76.50
1 15/16	52.70	56.00	59.29	62.58	65.88	69.18	72.46	75.79	79.05
2	54.40	57.80	61.20	64.60	68.00	71.40	74.80	78.20	81.60

Rules for Estimating the Weight of any Piece of Wrought Iron, Steel or Cast Iron

Wrought Iron.

One cubic foot of wrought iron weighs	480 lb
One square foot, one inch thick, weighs	40 lb
One square inch, one foot long, weighs	$3\frac{1}{3}$ lb

To find the weight per square foot of sheet iron, multiply the thickness in inches by 40.

To find the weight per linear foot of bars of any section, multiply the cross-sectional area in square inches by $3\frac{1}{3}$.

Steel.

One cubic foot of steel weighs	489.6 lb
(Or just 2% more than wrought iron.)	
One square foot, one inch thick, weighs	40.8 lb
One square inch, one foot long, weighs	3.4 lb

To find the weight per linear foot, of bars of any section, multiply the cross-sectional area in square inches by 3.4; or, if the weight is known, the exact sectional area may be obtained by dividing by 3.4.

Cast Iron.

One cubic foot of cast iron weighs	448.9 lb
One square foot, one inch thick, weighs	$37\frac{1}{2}$ lb
One square inch, one foot long, weighs	$3\frac{1}{8}$ lb
One cubic inch weighs	0.26 lb

The weight of irregular castings must be estimated by the cubic inch.

Rules for Weights of Castings

Multiply the weight of the pattern by 18 for cast iron, 13 for brass, 19 for lead, 12.2 for tin, 11.4 for zinc, the product is the weight of the casting.

Reduction for Round Cores and Core-Prints

Rule. Multiply the square of the diameter by the length of the core in inches, and the product multiplied by 0.017 is the weight of the pine core to be deducted from the weight of the pattern.

Shrinkage in Castings

Pattern-makers' Rule (of an inch longer per linear foot):

Cast iron	$\frac{1}{8}$
Brass	$\frac{3}{16}$
Lead	$\frac{1}{8}$
Tin	$\frac{1}{12}$
Zinc	$\frac{3}{16}$

Weights of Square Cast-Iron Columns in Pounds per Linear Foot *

$a\square b$ $2a+2b$ †	Thickness of metal, inches								
	5/8 in, lb	3/4 in, lb	7/8 in, lb	1 in, lb	1 1/8 in, lb	1 1/4 in, lb	1 1/2 in, lb	1 3/4 in, lb	2 in, lb
12	18.6	21.1	23.3	25.0	26.4	27.3	28 1	. .	.....
14	22.5	25.8	28.7	31.3	33.4	35.1	37.5	.....	....
16	26 4	30.5	34.2	37.5	40.4	43.0	46.9	49.2	50.0
18	30 3	35.2	39.7	43.8	47.4	50 8	56.3	60.2	62.5
20	34 2	39.8	45.1	50.0	54.5	58.6	65.6	71.1	75.0
22	38 1	44.5	50.6	56.3	61.5	66.4	75.0	82.0	87.5
24	42 0	49 2	56.1	62.5	68.5	74.2	84.4	93.0	100 0
26	45 9	53 9	61 5	68.8	75.6	82.0	93 8	103 9	112.5
28	49.8	58 6	67.0	75.0	82 6	89.8	103.1	114.8	125.0
30	53.7	63.3	72.5	81.3	89.6	97.7	112.5	125.8	137.5
32	57 6	68.0	77.9	87.5	96.7	105.5	121.9	136 7	150.0
34	61.5	72.7	83.4	93.8	103.7	113 3	131.3	147.7	162.5
36	65 4	77 3	88.9	100 0	110.7	121.1	140 6	158 6	175.0
38	69 3	82.0	94.3	106.3	117.8	128.9	150.0	169 5	187.5
40	73.2	86.7	99.8	112.5	124.8	136.7	159.4	180.5	200.0
42	77.1	91.4	105.3	118.8	131.8	144 5	168.8	191 4	212.5
44	81.0	96.1	110.8	125.0	138.8	152 3	178.1	202 3	225.0
46	84 9	100 8	116.2	131 3	145.9	160 2	187 5	213.3	237.5
48	88.8	105.5	121.7	137.5	152.9	168.0	196.9	224 2	250.0
50	92.8	110.2	127.2	143.8	159.9	175.8	206.3	235 2	262.5
52	96.7	114.8	132.6	150 0	167.0	183.6	215.6	246.1	275.0
54	100.6	119.5	138.1	156 3	174.0	191.4	225.0	257 0	287 5
56	104 5	124.2	143.6	162 5	181.0	199.2	234 4	268 0	300 0
58	108 4	128.9	149.0	168.8	188.1	207.0	243.8	278.9	312 5
60	112.3	133.6	154.5	175.0	195.1	214.9	253.2	289.8	325.0
62	116.2	138.3	160 0	181 3	202.1	222.7	262.5	300 8	337.5
64	120 1	143.0	165.4	187.5	209 2	230.5	271.9	311.7	350 0
66	124 0	147 7	170 9	193 8	216 2	238 3	281 3	322.7	362 5
68	127.9	152 3	176.4	200.0	223.2	246 1	290.6	333.6	375 0
70	131.8	157.0	181.8	206 3	230.3	253.9	300.0	344.5	387 5
72	135.7	161.7	187.3	212.5	237.3	261.7	309.4	355.5	400 0
74	139 6	166.4	192.8	218 8	244.3	269.5	318.8	366.4	412.5
76	143 5	171.1	198.3	225.0	251.3	277.3	328.1	377.3	425.0
78	147.4	175 8	203.7	231.3	258.4	285.2	337.5	388.3	437.5
80	151 3	180.5	207.2	237.5	265.4	293 0	346.9	399 2	450.0

* Birkmire.

† a and b = either side, outside measurement. $2a + 2b$ = number. Allowance has been made in this table for corners counted twice.

Example. What is the weight per linear foot of a 12 by 16 by 1 in thick column?

Solution. $2a + 2b = 24 + 32 = 56$. Opposite this number, under 1-in-thick metal, we find 162.5, or weight per linear foot of a column 12 by 16 by 1-in-thick.

Note. For flanges, brackets, etc., calculate the cubical contents of same and multiply by 0.26; cast-iron averages 448.9 lb per cu ft.

Weights per Linear Foot of Circular Cast-Iron Columns * †

Outside diameter, inches	Thickness of metal, inches							
	½ in, lb	⅝ in, lb	¾ in, lb	⅞ in, lb	1 in, lb	1⅛ in, lb	1¼ in, lb	1⅜ in, lb
3	12 3	14 6	16 60	18.30	19.6	.	..	
4	17 2	21 0	24.00	27.00	29.5	32.1	33 8	35 4
5	22.1	27.0	31.30	35.50	39.3	43.0	46 0	49.0
6	27 0	33.0	39.00	44.00	49.1	54.1	58.3	62 4
7	32.0	39.1	46 00	53.00	59.0	65.1	70.6	76 1
8	36.8	45.3	53.40	61.20	69.1	76.1	83.1	89 5
9	41 7	51.4	61 10	70 00	78.6	87.1	95 1	103 1
10	46 6	57.5	68 13	78 41	88 4	98.0	107.4	116 4
11	51 6	64 0	75 50	87.10	98.2	109.1	120 1	130 1
12	56 5	70 0	82 87	96 10	108 0	120 0	132 1	143 5
13	61 4	76 0	90 23	104.20	118 1	131 2	144 2	157 1
14	66 3	82.1	97 60	113.20	128.1	142 0	156 5	170 4
15	71.2	88 2	104 96	121.40	137 5	153.3	169 4	184.1
16	76 1	94 4	112.33	130.10	147.3	164.3	181 0	197 4
17	81 0	100.5	120 10	139 10	157.1	175.4	193 3	211 0
18	86.0	107 0	127.00	147.00	167.0	186.4	206.0	224 4
19	91.0	113 0	134.40	156.00	177.1	197.5	218 1	238 0
20	96 0	119 0	142.10	164.30	186.6	208.8	230 1	251 5
21	100 6	125 0	149.10	173.10	196.6	219.6	242 4	265 0
22	105 6	131.2	156 50	181 50	206.2	230.6	255 0	278.0
23	110.5	137.3	164 10	190 10	216 1	242 0	267 0	292.0
24	115 4	143 5	171 20	199 00	226 0	253 0	279 2	305 4

Outside diameter, inches	Thickness of metal, inches							
	1½ in, lb	1⅝ in, lb	1¾ in, lb	1⅞ in, lb	2 in, lb	2⅛ in, lb	2¼ in, lb	2⅜ in, lb
3	. ..	.	.	.	. . .			
4	.	. . .						
5	51.54	54.1	55 84	57.5	.		.	
6	66.30	69.9	73 02	76 0	78.6	80.84	82.83	
7	81.00	85.6	90.20	94 3	98.2	101.70	105.00	107.84
8	95.80	101 8	107 40	112.8	117.8	122.60	127.00	131.20
9	110 50	117.7	124.60	131.2	137.5	143 40	149.10	154.50
10	125 20	133 7	142.00	149.6	157.1	164.30	171.20	177.80
11	140.00	149.6	159.00	168.0	176.8	185.20	193.30	201.10
12	154.70	165 6	176.00	186.4	196.4	206.00	215.40	224.40
13	169.40	181.5	193.30	204 8	216.0	226.90	237.50	247.70
14	184.10	197.4	210.50	223.2	235.7	247.70	259.60	271.10
15	198.90	213.4	227.70	241.6	255.3	268.20	281.70	294.40
16	213.50	229.4	244.90	260.0	274.9	289.50	303 70	317.70
17	228.30	245.3	262.00	278.4	294.5	310.30	325.80	341 00
18	243.00	261.3	279.20	296.8	314.2	331.20	348.00	364.30
19	257.70	277.2	296.40	315 2	338.8	352.10	370.00	387 70
20	272.50	293.2	313 60	333.6	353.4	372 90	392.10	411.00
21	287.20	309.0	330.80	352.1	373.1	393.80	414.20	434.30
22	302.00	325.1	348.00	370.5	393.0	414 60	436.30	457.60
23	316.70	341 0	365.10	388.9	412.3	435.50	458.40	481.00
24	331 40	357 0	382 30	407 3	432 0	456 40	480 50	504.20

* Birkmire.

† The table is arranged for the weight of plain shaft. For brackets, flanges, etc., calculate the cubical contents and multiply by 0.26.

Weight of Cast-Iron Plates

Weights, in Pounds, of Cast-Iron Plates One Inch Thick

Calculated at 450 lb per cu ft

Length, inches	Width, inches									
	6 in, lb	8 in, lb	10 in, lb	12 in, lb	14 in, lb	16 in, lb	18 in, lb	20 in, lb	24 in, lb	30 in, lb
4	6 25	8.3	10 4	12.5	14 6	16.6	18.7	20.8	25	31
6	9 37	12.5	15 6	18 7	21 8	25 0	28 1	31 2	38	47
8	12 50	16.6	20.8	25 0	29 1	33.3	37 4	41 6	50	62
10	15.60	20 8	26.0	31.2	36 4	41.6	46.8	52 0	63	78
12	18 70	25.0	31.2	37.5	43.7	49.9	56.2	62.4	75	94
14	21.80	29.2	36.4	43.7	51.0	58.2	65 5	72 8	88	109
16	24.90	33.3	41 6	50 0	58.2	66.6	74 9	83 2	100	125
18	28.10	37.5	46.8	56 2	65.5	74.9	84.2	93 6	113	140
20	31.20	41.6	52.0	62 3	72.8	83.2	93 6	104 0	125	156
22	34.30	45.8	57 2	68 6	80.1	91.5	103.0	114 4	138	172
24	37.50	50.0	62 4	75.0	87 4	99.8	112.3	124 8	150	187
26	40.60	54.0	67.6	81.2	94.6	108.2	121 7	135 2	163	203
28	43.60	58.2	72.8	87.5	101.9	116.5	131.0	145 6	175	218
30	46.80	62.4	78.0	93.7	109.2	124.8	140 4	156 0	188	234
32	49.80	66 6	83.2	100.0	116.5	133.1	150 3	166 4	200	250
36	56.10	75 0	93 6	112.5	131.0	150.0	168 4	187.2	225	281

For larger plates take size of plate ONE-HALF smaller and multiply by 2. Thus a plate 28 by 32 in will weigh twice as much as one 14 by 32 in. For plates more or less than one inch in thickness multiply weight of plate by thickness in inches.

Approximate Weights of Square-Ribbed Cast-Iron Column-Bases

The following table, giving the weight of cast-iron column-bases, will be useful when estimating the steel and iron in tall buildings.*

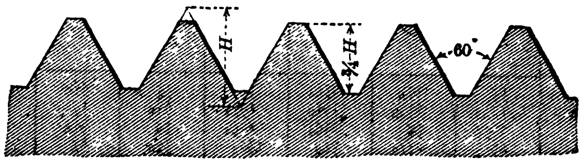
Size of square base, in	Weight, lb	Size of square base, in	Weight, lb
22×22	600	32×32	1 340
24×24	750	34×34	1 450
26×26	880	36×36	1 600
28×28	1 020	38×38	1 720
30×30	1 180	40×40	1 850

* H. G. Tyrrell, in Architects and Builders Magazine.

Screw-Threads, Nuts, and Bolt-Heads

Standard Screw-Threads

Recommended by Franklin Institute, December 15, 1864, and adopted by Navy Department of the United States; by the R. R Master Mechanics' and Master Car-Builders' Associations; by Jones & Laughlin Steel Company; and by many other of the prominent engineering and mechanical establishments of the country.



Angle of thread 60°. Flat at top and bottom 1/8 of pitch.

Diam of screw, in	Threads per inch	Diam at root of thread, in	Area at root of thread, sq in	Diam of screw, in	Threads per inch	Diam at root of thread, in	Area at root of thread, sq in
1/4	20	0 185	0 027	2	4 1/2	1.712	2.302
5/16	18	0 240	0 045	2 1/4	4 1/2	1.962	3.023
3/8	16	0.294	0 068	2 1/2	4	2.176	3 719
7/16	14	0 344	0 093	2 3/4	4	2 426	4.620
1/2	13	0 400	0 126	3	3 1/2	2.629	5 428
9/16	12	0 454	0 162	3 1/4	3 1/2	2 879	6.510
5/8	11	0 507	0 202	3 1/2	3 1/4	3.100	7 548
3/4	10	0 620	0 302	3 3/4	3	3.317	8.641
7/8	9	0 731	0 420	4	3	3 567	9 963
1	8	0 837	0 550	4 1/4	2 7/8	3.798	11.329
1 1/8	7	0 940	0 694	4 1/2	2 3/4	4 028	12.753
1 1/4	7	1.065	0 893	4 3/4	2 5/8	4.256	14.226
1 3/8	6	1 160	1.057	5	2 1/2	4.480	15.763
1 1/2	6	1 284	1.295	5 1/4	2 1/2	4.730	17 572
1 5/8	5 1/2	1.389	1.515	5 1/2	2 3/8	4.953	19.267
1 3/4	5	1.491	1.746	5 3/4	2 3/8	5.203	21.262
1 7/8	5	1.616	2.051	6	2 1/4	5.423	23.098

Nuts and Bolt-Heads are determined by the following rules, which apply to both square and hexagon nuts:

Short diameter of rough nut = 1 1/2 × diam of bolt + 1/8 in.
Short diameter of finished nut = 1 1/2 × diam of bolt + 1/16 in.
Thickness of rough nut = diam of bolt.
Thickness of finished nut = diam of bolt − 1/16 in.
Short diameter of rough head = 1 1/2 × diam of bolt + 1/8 in.
Short diameter of finished head = 1 1/2 × diam of bolt + 1/16 in.
Thickness of rough head = 1/2 short diam of head.
Thickness of finished head = diam of bolt − 1/16 in.

The long diameter of a hexagon nut may be determined by multiplying the short diameter by 1.155, and the long diameter of a square nut by multiplying the short diameter by 1.414.

Standard Dimensions of Nuts and Bolt-Heads

Diam of bolt	Short diam, rough	Short diam, finished	Long diam, rough	Long diam, rough	Thickness, rough. Nut	Thickness, finished. Both	Thickness, rough. Head
1/4	1/2	7/16	37/64	7/10	1/4	3/16	1/4
5/16	19/32	17/32	11/16	10/12	5/16	1/4	19/64
3/8	11/16	5/8	51/64	63/64	3/8	5/16	11/32
7/16	25/32	23/32	9/10	1 7/64	7/16	3/8	25/64
1/2	7/8	13/16	1	1 15/64	1/2	7/16	7/16
9/16	31/32	29/32	1 1/8	1 23/64	9/16	1/2	31/64
5/8	1 1/16	1	1 7/32	1 1/2	5/8	9/16	17/32
3/4	1 1/4	1 3/16	1 7/16	1 49/64	3/4	11/16	5/8
7/8	1 7/16	1 3/8	1 21/32	2 1/32	7/8	13/16	23/32
1	1 5/8	1 9/16	1 7/8	2 19/64	1	15/16	13/16
1 1/8	1 13/16	1 3/4	2 3/32	2 9/16	1 1/8	1 1/16	29/32
1 1/4	2	1 15/16	2 5/16	2 53/64	1 1/4	1 3/16	1
1 3/8	2 3/16	2 1/8	2 17/32	3 3/32	1 3/8	1 5/16	1 3/32
1 1/2	2 3/8	2 5/16	2 3/4	3 23/64	1 1/2	1 7/16	1 3/16
1 5/8	2 9/16	2 1/2	2 31/32	3 5/8	1 5/8	1 9/16	1 9/32
1 3/4	2 3/4	2 11/16	3 3/16	3 57/64	1 3/4	1 11/16	1 3/8
1 7/8	2 15/16	2 7/8	3 13/32	4 5/32	1 7/8	1 13/16	1 15/32
2	3 1/8	3 1/16	3 5/8	4 27/64	2	1 15/16	1 9/16
2 1/4	3 1/2	3 7/16	4 1/16	4 61/64	2 1/4	2 3/16	1 3/4
2 1/2	3 7/8	3 13/16	4 1/2	5 31/64	2 1/2	2 7/16	1 15/16
2 3/4	4 1/4	4 3/16	4 29/32	6	2 3/4	2 11/16	2 1/8
3	4 5/8	4 9/16	5 3/8	6 17/32	3	2 15/16	2 5/16
3 1/4	5	4 15/16	5 13/16	7 1/16	3 1/4	3 3/16	2 1/2
3 1/2	5 3/8	5 5/16	6 7/64	7 39/64	3 1/2	3 7/16	2 11/16
3 3/4	5 3/4	5 11/16	6 21/32	8 1/8	3 3/4	3 11/16	2 7/8
4	6 1/8	6 1/16	7 3/32	8 41/64	4	3 15/16	3 1/16
4 1/4	6 1/2	7 7/16	7 9/16	9 3/16	4 1/4	4 3/16	3 1/4
4 1/2	6 7/8	6 13/16	7 31/32	9 3/4	4 1/2	4 7/16	3 7/16
4 3/4	7 1/4	7 3/16	8 13/32	10 1/4	4 3/4	4 11/16	3 5/8
5	7 5/8	7 9/16	8 27/32	10 19/64	5	4 15/16	3 13/16
5 1/4	8	7 15/16	9 9/32	11 23/64	5 1/4	5 3/16	4
5 1/2	8 3/8	8 5/16	9 23/32	11 7/8	5 1/2	5 7/16	4 3/16
5 3/4	8 3/4	8 11/16	10 5/32	12 3/8	5 3/4	5 11/16	4 3/8
6	9 1/8	9 1/16	10 19/32	12 15/16	6	5 15/16	4 9/16

Weights of One Hundred Bolts with Square Heads and Nuts

INCLUDES WEIGHT OF NUT

Hoopes & Townsend's List

Length under head to point, in	Diameter of bolts								
	¼ in, lb	5/16 in, lb	⅜ in, lb	7/16 in, lb	½ in, lb	⅝ in, lb	¾ in, lb	⅞ in, lb	1 in, lb
1½	4.00	7.00	10.50	15.20	22.50	39.50	63.00		
1¾	4.40	7.50	11.25	16.30	23.82	41.62	66.00		
2	4.75	8.00	12.00	17.40	25.15	43.75	69.00	109.00	163
2¼	5.15	8.50	12.75	18.50	26.47	45.88	72.00	113.25	169
2½	5.50	9.00	13.50	19.60	27.80	48.00	75.00	117.50	174
2¾	5.75	9.50	14.25	20.70	29.12	50.12	78.00	121.75	180
3	6.25	10.00	15.00	21.80	30.45	52.25	81.00	126.00	185
3½	7.00	11.00	16.50	24.00	33.10	56.50	87.00	134.25	196
4	7.75	12.00	18.00	26.20	35.75	60.75	93.10	142.50	207
4½	8.50	13.00	19.50	28.40	38.40	65.00	99.05	151.00	218
5	9.25	14.00	21.00	30.60	41.05	69.25	105.20	159.55	229
5½	10.00	15.00	22.50	32.80	43.70	73.50	111.25	168.00	240
6	10.75	16.00	24.00	35.00	46.35	77.75	117.30	176.60	251
6½			25.50	37.20	49.00	82.00	123.35	185.00	262
7			27.00	39.40	51.65	86.25	129.40	193.65	273
7½			28.50	41.60	54.30	90.50	135.00	202.00	284
8			30.00	43.80	59.60	94.75	141.50	210.70	295
9				46.00	64.90	103.25	153.60	227.75	317
10				48.20	70.20	111.75	165.70	224.80	339
11				50.40	75.50	120.25	177.80	261.85	360
12				52.60	80.80	128.75	189.90	278.90	382
13					86.10	137.25	202.00	295.95	404
14					91.40	145.75	214.10	313.00	426
15					96.70	154.25	226.20	330.05	448
16					102.00	162.75	238.30	347.10	470
17					107.30	171.00	250.40	364.15	492
18					112.60	179.50	262.60	381.20	514
19					117.90	188.00	274.70	398.25	536
20					123.20	206.50	286.80	415.30	558
Per inch additional	1.37	2.13	3.07	4.18	5.45	8.57	12.27	16.70	21.82

Weights of Nuts and Bolt-Heads, in Pounds

For calculating the weight of longer bolts

Diameter of bolt, in inches ...		¼	⅜	½	⅝	¾	⅞
Weight of hexagon nut and head		0.017	0.057	0.128	0.267	0.43	0.73
Weight of square nut and head		0.021	0.069	0.164	0.320	0.55	0.88
Diameter of bolt, in inches.	**1**	**1¼**	**1½**	**1¾**	**2**	**2½**	**3**
Weight of hexagon nut and head	1.10	2.14	3.78	5.6	8.75	17	28.8
Weight of square nut and head	1.31	2.56	4.42	7.0	10.50	21	36.4

Weights of Rivets and Round-Headed Bolts without Nuts. Steel

POUNDS PER HUNDRED

Length, in	⅜ in diam	½ in diam	⅝ in diam	¾ in diam	⅞ in diam	1 in diam	1⅛ in diam	1¼ in diam
1¼	5.5	12.8	22.0	29 3	43 9	66 6	93.3	127
1½	6.3	14.2	24.1	32.4	48.2	72.1	100	136
1¾	7.0	15.5	26.3	35.5	52.5	77.7	107	145
2	7.9	16.9	28.5	38.7	56.7	83.3	114	153
2¼	8.7	18.3	30.7	41.8	61.0	88 8	121	162
2½	9.4	19.7	32.8	44.9	65.2	94.4	128	171
2¾	10.2	21.1	35.0	48.0	69.5	100	136	179
3	11.0	22.5	37.2	51.1	73.7	105	143	188
3¼	11.7	23.9	39.3	54.3	78.0	111	150	197
3½	12.6	25.3	41.5	57.4	82.3	116	157	205
3¾	13.4	26.7	43 7	60.5	86.5	122	164	214
4	14.1	28.1	45.9	63.6	90 8	128	170	223
4¼	14.9	29.4	48.0	66.7	95.0	134	177	231
4½	15.7	30.8	50.2	69.9	99.3	139	185	240
4¾	16.5	32 2	52.4	73.0	104	145	192	249
5	17.2	33.6	54.5	76.1	108	150	199	258
5¼	18.1	35.0	56.7	79 2	112	156	206	266
5½	18.8	36.4	58.9	82 3	116	161	213	275
5¾	19.6	37.8	61.1	85.5	120	166	220	284
6	20.4	39.2	63.2	88.6	124	172	227	292
6½	21.9	42 0	67.6	95.1	133	184	241	310
7	23.5	44.7	71.9	101	142	195	255	327
7½	25.1	47.5	76.1	108	150	206	269	345
8	26.6	50.3	80.6	114	159	217	284	362
8½	28.2	53.1	85.0	120	167	227	298	379
9	29 8	55 9	89 3	126	176	239	312	397
9½	31.3	58.7	93.7	133	185	250	325	414
10	32.8	61.4	98.0	139	193	261	340	431
10½	34.5	64.2	103	145	202	272	354	449
11	36 0	67 0	107	151	210	284	368	466
11½	37.6	69.8	111	158	218	295	382	484
12	39 2	72 5	115	164	227	306	396	501
Heads..	1.8	5 8	11 1	13 6	22 6	39.0	58 0	83.5

For length of shaft required to form rivet-head, see Chapter XII.

NAILS AND SCREWS *

Nails. Based upon the process of manufacture there are three kinds of nails in common use, namely, plate or cut nails, wire nails, and clinch-nails. These are briefly described in the following subdivisions of this article and other data bearing on the subject are included.

(1) **Cut Nails.** Cut nails are made from a strip of rolled iron or steel of the same thickness as the finished nail and a little wider than its length, the fiber of the iron being parallel with the length of the nail. Special machinery cuts the nails out in alternate wedge-shaped slices, the heads are then stamped on them and the finished nails dropped into the casks. Cut nails made from iron are generally preferred for use in exposed positions. Cut nails are made in a variety of shapes to suit special uses. For ordinary use in building, nails of three different shapes are made, and the nails are called COMMON NAILS, FINISH-NAILS and CASING-NAILS. The common nails are used for rough work, finish-nails for finished work, and casing-nails for flooring, matched ceiling and sometimes for pine casings, although the heads are rather too large for finish-work. Cut nails are beginning to return to favor as they have holding power and lasting qualities superior to wire nails.

(2) **Brads.** Brads are thin nails with a small head, used for small finish, panel-moldings, etc. They vary from $\frac{1}{4}$ to 2 in in length.

(3) **Clout-Nails.** Clout-nails are made with broad, flat heads, and are sold in sizes varying from $\frac{3}{8}$ to $2\frac{1}{2}$ in in length. They are used chiefly for fastening gutters and metal-work. Special nails are also made for lathing, slating, shingling, etc.

(4) **Wire Nails.** These have of late years become as common as the cut nails, and are sold at about the same price. They are said to be stronger for driving than the cut nails, not so liable to bend or break, especially when driven into hard woods, and less liable to split the wood; for these reasons they are generally preferred by carpenters. Wire nails are made from wire, of the same section-diameter as the shank of the nail, by a machine which cuts the wire in even lengths, heads and points them, and, when desired, also barbs them. In general, the same classification is used for cut nails. It should be noticed that the gauge of the wire and the shape of the head vary in the different kinds, and that some are barbed, others plain. The various types of wire nails are drawn ROUND, SMOOTH or BARBED, for the domestic trade; for export they are drawn OVAL, SQUARE, or DIAMOND-SHAPED, according to the country to which they are to be shipped and its requirements.

(5) **Clinch-Nails.** These are made from open-hearth or Bessemer-steel wire. Any ordinary wire nail will clinch, especially when made with DUCK-BILL or flattened points for clinching purposes, or even otherwise, if annealed. These nails are used only in places where it is desired to turn over the ends of the nails to form a clinch, as in the case of battens or cleats.

(6) **Length and Weight of Nails.** The length of nails is designated by PENNIES (*d's*). Two explanations are given for the origin of this classification; for example, that tenpenny nails originally sold for tenpence a hundred, or that 1 000 tenpenny nails originally weighed 10 lb. The designation is retained by manufacturers, both for cut and wire nails. The weights expressed in pennies run from two pennies to sixty pennies, the larger sizes being designated by fractions of an inch. The sizes and lengths of various kinds of nails and tacks are given in the following tables.

* Prepared by the late Thomas Nolan.

(7) Sizes of Nails for Different Classes of Work. It is imperative for first-class work that nails of proper size should be used and to insure the best results it is well in certain classes of work to specify the sizes which are to be used. For framing, twentypenny, fortypenny and sixtypenny nails, or spikes, are used, according to the size of the timber. For sheathing and roof-boarding, underfloors and cross-bridging, tenpenny common nails should be used. For over-floors tenpenny floor-nails or casing-nails should be used for jointed boards, and ninepenny or tenpenny for matched flooring, although eightpenny nails are sometimes used. Ceiling when $\frac{3}{4}$ in thick is generally put up with eightpenny casing-nails, and when thinner stuff is used, with sixpenny nails. For inside finish any size of finish-nails or brads from eightpenny down to twopenny is used, according to the thickness and size of the moldings. For pieces exceeding 1 in in thickness, tenpenny nails should be used. Clapboarding is generally put on with sixpenny finish-nails or casing-nails. Threepenny to fourpenny are used for shingling and slating, and threepenny for lathing. For slating, galvanized nails should be used, and they are also better for shingling. Whether wire or cut nails should be used may generally be left to the builder; but in places where there is any danger of the nails being drawn out either by the warping of the boards or from the pull of the nail, cut nails should be used, as they have greater holding power than the wire nails under certain conditions. In regard to the comparative holding power of cut nails and wire nails, and barbed nails and smooth nails, and tests made to determine this property, see tables under (9).

(8) Copper and Brass Nails. Nails are also made of copper and cast brass, and these are sometimes used in connection with boat-building, refrigerator-work, etc. One wing of the Physical Laboratory Building of Harvard College is put together entirely with brass and copper. As the rooms were intended for use in delicate electrical work, no iron was used in their construction.

(9) Cement-coated Wire Nails. The coating consists of various resinous gums mixed by a secret formula, and put on the nails by a baking-process which involves the use of quite complicated machinery. Although the chief market for coated nails is among the users of packages to be shipped, there is a limited market for them among builders, for construction-purposes. The chief merit of the coating is that it gives the nail an adhesive resistance approximately twice that of ordinary wire nails. This quality appeals especially to the manufacturers and users of packages to be shipped, for which strength is particularly wanted. It is desirable for construction-purposes also, but the lack of holding power in plain wire nails is not so apparent in building. About 90% of the output goes to box-factories and large shippers. Cement-coated nails are quite widely used, also, in laying both ordinary and parquetry-flooring. The use of these nails, with a special head which leaves a small hole, gives a firm floor and prevents springing. Though the makers do not claim that the nails are absolutely rust-proof, they do claim that nails thus treated will resist the effects of moisture from 20 to 50% better than the uncoated wire nails. But it is when in use that the non-rusting quality is most evident. There is more coating on the nails than is actually necessary for holding power. The heat caused by the friction of driving the nail softens the coating and the surplus is forced toward the head, completely closing the opening; this prevents the admission of moisture between the wood and the nail. Under similar conditions, the life of a cement-coated nail will be about twice as long as that of an uncoated one. Less force is needed to drive a coated nail as the softened coating forms a lubricant. These nails are made in two types, differing only in the heads, and are either COOLERS or SINKERS. The former

have large flat heads; the latter, heads slightly reinforced by counter-sinking. They are made to replace common nails, in sizes from $\frac{1}{4}$ in to 1 in, and are used for framing, boarding, shingling and staging, and for boxes and crates. Results of tests made with cement-coated nails to determine their adhesive resistance in comparison with the common smooth-wire nails are given below.

The following table shows the result of tests made at the United States Arsenal, Watertown, Mass., in 1902, the wood being pine:

Comparative Adhesive Resistance of Common Smooth-Wire Nails and Cement-Coated Nails

All nails were driven into the same piece and were perpendicular to the grain

Size and name	Diameter, in	Length driven,* in	Adhesive resistance,† lb
Tenpenny, common, smooth.	0.145	$2\frac{1}{2}$	167
Tenpenny, coated	0.117	$2\frac{1}{2}$	418
Ninepenny, common, smooth .	0.132	$2\frac{1}{4}$	182
Ninepenny, coated	0.114	$2\frac{1}{4}$	327
Eightpenny, common, smooth	0.132	2	189
Eightpenny, coated	0.112	2	316
Sixpenny, common, smooth .	0.097	$1\frac{5}{8}$	106
Sixpenny, coated . .	0.092	$1\frac{5}{8}$	226

* All of the nails were left with their heads projecting from $\frac{1}{4}$ to $\frac{1}{2}$ in.

† Average of three trials.

Holding Power of Nails. A committee appointed by the Wheeling nail-manufacturers, a number of years ago, to test the comparative holding power of cut and wire nails. published the following data. The kind of wood is not named. The effect of barbs is slight, and definite conclusions await complete tests.

Pounds Required to Pull Nails Out

	Cut	Wire		Cut	Wire
Twentypenny	1 593	703	Sixpenny .	383	200
Tenpenny ..	908	315	Fourpenny . .	286	123
Eightpenny	597	227			

The holding power of nails varies with the kind of wood into which they are driven. Austin T. Byrne gives the relative holding power of woods as ABOUT as follows: White pine, 1; yellow pine, 1.5; white oak, 3; chestnut, 1.6; beech, 3.2; sycamore, 2; elm, 2; basswood, 1.2.

Comparative Holding Power of Cut and Wire Nails

Very thorough tests of the comparative holding power of wire nails and cut nails OF EQUAL LENGTHS AND WEIGHTS were made at the U. S. Arsenal in 1892 and 1893. From forty series, comprising forty sizes of nails driven in spruce wood, it was found that the cut nails showed an average superiority of 60.50%, the common nails showing an average superiority of 47.51% and the finishing-nails an average of 72.22%. In eighteen series, comprising six sizes of BOX-NAILS driven into pine wood, in three ways the cut nails showed an average superiority of 99.93%. In no series of tests did the wire nails hold as much as the cut nails.

Quantity of Nails Required for Different Kinds of Work

For 1 000 shingles* allow 3½ to 6½ lb fourpenny nails or 3½ to 4½ lb threepenny
- 1 000 laths, 7 lb threepenny fine, or for 100 sq yd of lathing, 10 lb threepenny fine
- 1 000 sq ft of beveled siding, 18 lb sixpenny
- 1 000 sq ft of sheathing, 20 lb eightpenny or 25 lb tenpenny
- 1 000 sq ft of flooring, 30 lb eightpenny or 40 lb tenpenny
- 1 000 sq ft of studding, 15 lb tenpenny and 5 lb twentypenny
- 1 000 sq ft of 1 by 2½-in furring, 12-in centers, 9 lb eightpenny or 14 lb tenpenny
- 1 000 sq ft of 1 by 2½-in furring, 16-in centers, 7 lb eightpenny or 10 lb tenpenny

* Depends upon width and length of shingles and kind of nails.

Cut Steel Nails and Spikes

Sizes, lengths, and approximate number per pound

Taken from the Handbook of the Cambria Steel Company

Sizes	Length, inches	Common	Clinch	Finishing	Casing and box	Fencing	Spikes
2 *d*	1	740	400	1 100			
3 *d*	1¼	460	260	880			
4 *d*	1½	280	180	530	420		
5 *d*	1¾	210	125	350	300	100	
6 *d*	2	160	100	300	210	80	
7 *d*	2¼	120	80	210	180	60	
8 *d*	2½	88	68	168	130	52	
9 *d*	2¾	73	52	130	107	38	
10 *d*	3	60	48	104	88	26	
12 *d*	3¼	46	40	96	70	20	
16 *d*	3½	33	34	86	52	18	17
20 *d*	4	23	24	76	38	16	14
25 *d*	4¼	20					
30 *d*	4½	16½			30		11
40 *d*	5	12			26		9
50 *d*	5½	10			20		7½
60 *d*	6	8			16		6
. . .	6½						5½
	7						5

Sizes	Length, inches	Barrel	Light barrel	Slating	Sizes	Length, inches	Flat-grip, fine	Edge-grip, fine
. . .	⅝	750				¾	1 462	
. . . .	¾	600				⅞	1 300	
	⅞	500			2 *d*	1	1 100	960
2 *d*	1	450		340	3 *d*	1⅛	800	750
	1⅛	310	400		4 *d*	1⅜	650	600
3 *d*	1¼	280	304	280				
. . . . .	1⅜	210			Tobacco		Brads	Shingle
4 *d*	1½	190	224	220				
5 *d*	1¾			180	130			
6 *d*	2				97		120	
7 *d*	2¼				85		94	
8 *d*	2½				68		74	90
9 *d*	2¾				58		62	72
10 *d*	3				48		50	60
12 *d*	3¼						40	
16 *d*	3½						27	

Steel-Wire Nails, Spikes, and Tacks

SIZE, LENGTH, GAUGE AND APPROXIMATE NUMBER TO THE POUND

Compiled from Catalogue of American Steel and Wire Company
American Steel and Wire Company's gauge

Common nails and brads*				Casing-nails†		Finishing-nails†	
Sizes	Length, in	Gauge	Number to pound	Gauge	Number to pound	Gauge	Number to pound
2 *d*	1	15	876	$15\frac{1}{2}$	1 010	$16\frac{1}{2}$	1 351
3 *d*	$1\frac{1}{4}$	14	568	$14\frac{1}{2}$	635	$15\frac{1}{2}$	807
4 *d*	$1\frac{1}{2}$	$12\frac{1}{2}$	316	14	473	15	584
5 *d*	$1\frac{3}{4}$	$12\frac{1}{2}$	271	14	406	15	500
6 *d*	2	$11\frac{1}{2}$	181	$12\frac{1}{2}$	236	13	309
7 *d*	$2\frac{1}{4}$	$11\frac{1}{2}$	161	$12\frac{1}{2}$	210	13	238
8 *d*	$2\frac{1}{2}$	$10\frac{1}{4}$	106	$11\frac{1}{2}$	145	$12\frac{1}{2}$	189
9 *d*	$2\frac{3}{4}$	$10\frac{1}{4}$	96	$11\frac{1}{2}$	132	$12\frac{1}{2}$	172
10 *d*	3	9	69	$10\frac{1}{2}$	94	$11\frac{1}{2}$	121
12 *d*	$3\frac{1}{4}$	9	63	$10\frac{1}{2}$	87	$11\frac{1}{2}$	113
16 *d*	$3\frac{1}{2}$	8	49	10	71	11	90
20 *d*	4	6	31	9	52	10	62
30 *d*	$4\frac{1}{2}$	5	24	9	46		
40 *d*	5	4	18	8	35		
50 *d*	$5\frac{1}{2}$	3	14				
60 *d*	6	2	11				

Spikes‡

Sizes	Length, in	Gauge	Number to pound
10 *d*	3	6	41
12 *d*	$3\frac{1}{4}$	6	38
16 *d*	$3\frac{1}{2}$	5	30
20 *d*	4	4	23
30 *d*	$4\frac{1}{2}$	3	17
40 *d*	5	2	13
50 *d*	$5\frac{1}{2}$	1	10
60 *d*	6	1	9
7″	7	$\frac{5}{16}$	7
8″	8	$\frac{3}{8}$″	4
9″	9	$\frac{3}{8}$″	$3\frac{1}{2}$
10″	10	$\frac{3}{8}$″	3
12″	12	$\frac{3}{8}$″	$2\frac{1}{2}$

Shingle-nails

Size	Length, in	Gauge	Number to pound
3 *d*	$1\frac{1}{4}$	13	429
$3\frac{1}{2}$ *d*	$1\frac{3}{8}$	$12\frac{1}{2}$	345
4 *d*	$1\frac{1}{2}$	12	274
5 *d*	$1\frac{3}{4}$	12	235
6 *d*	2	12	204
7 *d*	$2\frac{1}{4}$	11	139
8 *d*	$2\frac{1}{2}$	11	125
9 *d*	$2\frac{3}{4}$	11	114
10 *d*	3	10	83

Fine nails

Size	Length, in	Gauge	Number to pound
2 *d*	1	$16\frac{1}{2}$	1 351
3 *d*	$1\frac{1}{8}$	15	778
4 *d*	$1\frac{1}{2}$	14	473
2 *d* extra fine	1	17	1 560
3 *d* extra fine	$1\frac{1}{8}$	16	1 015

* Common brads differ from common nails only in the head and point.
† Lengths are the same as common nails for corresponding size.
‡ Spikes are made with chisel-points and diamond points; also with convex heads and flat heads.

Steel-Wire Nails (Continued)

Clinch-nails				Fence-nails*		Slating-nails*	
Size	Length, in	Gauge	Number to pound	Gauge	Number to pound	Gauge	Number to pound
2 *d*	1	14	710	No 5 smallest size		12	411
3 *d*	1¼	13	429			10½	225
4 *d*	1½	12	274			10½	187
5 *d*	1¾	12	235	10	142	10	142
6 *d*	2	11	157	10	124	9	103
7 *d*	2¼	11	139	9	92	Barbed roofing-nails†	
8 *d*	2½	10	99	9	82		
9 *d*	2¾	10	90	8	62	¾"×No 13	714
10 *d*	3	9	69	7	50	⅞"×No 12	469
12 *d*	3¼	9	62	6	40	1"×No 12	411
16 *d*	3½	8	49	5	30	1⅛"×No 12	365
20 *d*	4	7	37	4	23	1¼"×No 11	251

* Length same as clinch-nails of corresponding size.

† Roofing-nails are designated by the length, not by PENNY. These nails are made in lengths up to 2 in.

Wire-Tacks

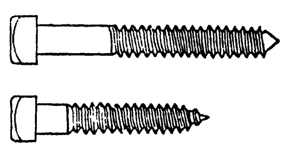
Title, ounce	Length, in	Number per pound	Title, ounce	Length, in	Number per pound	Title, ounce	Length, in	Number per pound
1	⅛	16 000	4	7/16	4 000	14	13/16	1 143
1½	3/16	10 666	6	9/16	2 666	16	⅞	1 000
2	¼	8 000	8	⅝	2 000	18	15/16	888
2½	5/16	6 400	10	11/16	1 600	20	1	800
3	⅜	5 333	12	¾	1 333	22	1 1/16	727
..	.			.	.	24	1⅛	666

Wire carpet-tacks are made polished, blued, tinned, or coppered, there are also upholsterers' and bill-posters' or railroad tacks.

Expansion-bolt

Expansion-Bolts. These are commonly used for bolting wood or iron to masonry that is already built. A hole is drilled in the masonry of such size that the expansion-nut will fit closely, and when the bolt is screwed up the nut expands and binds firmly in the masonry. The illustration shows the Evans expansion-bolt, which is also furnished with screw-head bolts. There are other forms of expansion-bolts on the market. From experiments on expansion-bolts it was found that the holding capacity was 264 lb per sq in when embedded in 1:2 Portland-cement mortar, 843 per sq in when embedded in sulphur and 485 lb per sq in when embedded in lead. For average working unit-stresses it is safe to use about one-fifth of the values given. When the work is exposed to rain or moisture sulphur should not be used as the acid which results will rust the metal and will also tend to disintegrate the masonwork at the point of entrance of the bolt.

Screws. The substitution of screws for nails in building operations is a marked feature of modern work. Trimming hardware of all descriptions is put on with screws, and a great deal of panel-work, inside finish, etc , is put together with them. Stop-beads, the casings of plumbing-fixtures, etc., should be fastened with screws, as well as all kinds of store and office-fixtures, and cabinet-work in general, except where the joints are glued. Screws are also largely used in making furniture. They present a neater appearance than nails, have greater holding power and are less apt to injure the material if it should be removed and replaced. By making holes for the screws with a bit, all danger of splitting the finish is averted. The ordinary type of screw has a gimlet-point by which it can be turned into the wood without the aid of a bit. The heads are made in various forms to suit different uses. Screws are made ordinarily of steel, but sometimes of brass and bronze. The latter sort are used for screwing in place finished hardware of the same material, and have heads finished to correspond with the trimmings. Steel screws, also, are finished with blue, bronze, lacquered, galvanized, or tinned surface, to match the cheaper class of trimmings. The galvanized finish is used in building operations at the seashore. Screws with blue surface, called BLUED SCREWS, are generally used with japanned hardware and for stop-beads, and wherever a cheap round-headed screw is desired. Silver, nickel, and gold-plated screws are also manufactured for use in connection with similar hardware. Steel screws for wood are made in twenty different lengths, varying from ¼ to 6 in, and each length of screw has from six to eighteen varieties in thickness, there being in all thirty-one different gauges; so that altogether there are in the market about two hundred and fifty different sizes of ordinary screws used for woodwork. The most common shapes are the ordinary flat head, round head and oval head. The oval-head screw is tapered for countersinking but is slightly rounded on top



Lag and Coach-screws

Patent diamond-point steel screws are made especially for driving with a hammer. These can be driven with a hammer their entire length into any hard wood, and then held by one or two turns as securely as the ordinary screw. In ordering screws both the length and number of the gauge or diameter of the shank, the material and finish, and the use to which they are to be put, should be given

Screws for Metal have the same diameter throughout and the threads are V-shaped.

Sizes of Screws. The sizes of screws are given in length in inches and the number of the gauge, the gauge denoting the diameter. Thus, a 1-in No. 12 screw is 1 in long and 0.2158 in in diameter. The gauge-numbers range from 0 to 30 and the lengths from ¼ to 6 in. The lengths vary by eighths of an inch up to 1 in, by quarters of an inch up to 3 in and by halves of an inch up to 5 in. Screws from ⅝ to 4½ in long are made in about sixteen different gauge-numbers. Table XIII, Chapter XI, gives the diameter to four places in decimals of an inch of the American screw-gauge. It should be noticed that, unlike the ordinary wire-gauges, the 0 of the screw-gauge indicates the diameter of the smallest screw while the diameter of the screw increases with the number of the gauge.

Lag-Screws and Coach-Screws are large, heavy screws used where great strength is required, as in heavy framing, and for fixing ironwork to timber. Lag-screws with conical point are made with diameters of 5/16, ⅜, 7/16, ½, 9/16,

⅝, ¾, and 1 in, and in lengths from 1½ to 12 in; coach-screws in diameters from 5/16 to ¾ in and in lengths from 1½ to 12 in. For putting in lag-screws a hole should be bored which has a diameter a little greater than the unthreaded shank of the screw and it should be bored to a depth corresponding to the length of the unthreaded shank. A second hole should then be bored at the bottom of the first hole of a diameter somewhat less than that of the threaded shank and to a depth of about half its length.

Holding Power of Lag-Screws

Kind of wood	Size of screw, in	Size of hole bored, in	Length in wood, in	Maximum resistance, lb	Number of tests
Seasoned white oak.	⅝	½	4½	8 037	3
Seasoned white oak.	9/16	7/16	3	6 480	1
Seasoned white oak.	½	⅜	4½	8 780	2
Yellow-pine stick	⅝	½	4	3 800	2
White cedar, unseasoned	⅝	½	4	3 405	2

Hoopes & Townsend give the force required to draw screws out of yellow pine as follows:

Screw...	½ in	⅝ in	¾ in	⅞ in	1 in
Wood, depth...	3½ in	4 in	4 in	5 in	6 in
Force, pounds...	4 960	6 000	7 685	11 500	12 620

Wooden-screws are sold by the gross, lag-screws and coach-screws by the pound.

DATA ON EXCAVATING

Excavating is almost invariably measured by the cubic yard of 27 cu ft. For measuring excavations of irregular depth see Part I. For computing the contents of wells and cesspools, the circular area in square feet may be obtained from the table of Areas and Circumferences, Part I, and this circular area multiplied by the depth in feet will give the contents in cubic feet. The cost of excavating and removing earth is ordinarily made up of the following items:

(1) Loosening the earth for the shovelers;
(2) Loading by shovels into carts or barrows:
(3) Hauling or wheeling it away, including emptying and returning;
(4) Spreading it out on the dump;

For every large job, such as railroad-work, it is also necessary to make an allowance for keeping the hauling-road in repair, for sharpening and repair of tools, and for carts, harness, superintendence and water-carriers. Where the dirt excavated can be spread over the ground immediately surrounding the excavation the loosened dirt may be removed by scrapers without shoveling. Today, practically all excavations are made by mechanically operated shovels, steam, gasoline or electric; hauling is performed by trucks or tractors.

Data for Estimating Cost of Loosening Earth. Two men with a plough and team of horses will loosen from 20 to 30 cu yd of strong, heavy soil per hour or

from 40 to 60 cu yd of ordinary loam. One man with a pick will loosen 1⅛ yd per hour of stiff clay or cemented gravel, 4 yd of common loam, or 6 yd of light sand.

The average quantity of LOOSENED EARTH that a man can shovel into a cart per hour is:

Loam or sand	2.0 cu yd
Clay and heavy soils	1.7 cu yd
Rock	1.0 cu yd

Average earth when loosened swells to from 1⅕ to 1⅓ times its original bulk in place.

The capacity of vehicles used for moving excavated materials is about as follows:

Wheelbarrows	3 to 4 cu ft
One-horse dump-carts	18 to 22 cu ft
Two-horse dump-wagons	27 to 45 cu ft*
Drag-scrapers	3 to 7 cu ft
Wheel-scrapers	10 to 17 cu ft
Dump-cars on rails	27 to 80 cu ft
Steam shovels	¾ cu yd
Motor trucks	1 to 5 tons

The Economical Length of Haul with drag-scrapers is about 150 ft; with wheeled scrapers, 500 ft; with wheelbarrows, 250 ft; with one-horse dump-carts, 600 ft.† The average speed of horses is given as about 200 ft per minute.

Weight of Earth, Sand and Gravel. For general calculations the following average values may be taken:

14 cu ft of chalk weigh 1 ton	19 cu ft of gravel weigh 1 ton
18 cu ft of clay weigh 1 ton	22 cu ft of sand weigh 1 ton
21 cu ft of earth weigh 1 ton	

Rock-Excavation. A cubic yard of rock, in place, when broken up by blasting for removal by wheelbarrows or carts, will occupy a space of about 1⅗ cu yd; consequently, the cost of hauling or removal is about 50% more than for dirt.

DATA ON STONEWORK

Kinds of Stonework. The commonest kind of stonework, that is, for walls, is called RUBBLEWORK. No work whatever is done on the stones except to break them up with a hammer. If the wall is built in courses it is designated COURSED RUBBLE. When the stones showing on the outside face of the wall are squared, the work is designated ASHLAR. Ashlar is of two kinds: COURSED ASHLAR, in which the stones are laid to form courses around the building, all of the stones in any course being of the same height, and BROKEN ASHLAR, in which stones of different heights are used. HAMMER-DRESSED ASHLAR designates work where the stones are roughly squared with a hammer. This is a very cheap class of work. Good ashlar work should be squared on the bench with chisels, and with beds and end-joints cut square to the face. Stonework

* The ordinary load for two-horse wagons such as are commonly used for hauling dirt sand and gravel is from 1¼ to 1½ cu yd

† Inspectors' Pocket-Book, by A. T. Byrne.

which requires a chisel or any other tool except a hammer for dressing is called CUT WORK. Cut work costs considerably more than hammer-dressed work.

Measurement of Stonework. Rough stone from the quarry is usually sold under two classifications: rubble-stone and dimension-stone. Rubble includes the pieces of irregular size most easily obtained from the quarry, and suitable for cutting into ashlar 12 in or less in height and about 2 ft long. Stone ordered to be of a certain size, to SQUARE over 24 in each way and to be of a particular thickness, is called DIMENSION-STONE. The price of the latter varies from two to four times the price of RUBBLE. Rubble is generally sold by the ton or carload. Footings and flagging are usually sold by the square foot; dimension-stone by the cubic foot. In Boston, granite blocks for foundations are usually sold by the ton.

In Estimating on the Cost of Stonework put into a building, the custom varies with different localities, and even among contractors in the same city. Dimension-stone footings, that is, squared stones 2 ft or more in width, are usually measured by the square foot. If built of large rubble or irregular stones the footings are measured in with the wall, allowance being made for the projections of the footings. Rubblework is almost universally measured by the PERCH of $16\frac{1}{2}$ cu ft. In Philadelphia, St. Louis and some sections of Illinois, 22 cu ft are called a perch. Railroad-work is usually measured by the cubic yard. When stonework is let by the perch, the number of cubic feet to the perch should be stated in the contract, and it should be stated, also, whether or not openings are to be deducted. As a rule no deductions are made for openings of less than 70 superficial feet.

Data for Estimating Cost. A ton of most of the different kinds of stones will make from 1 perch to $1\frac{1}{4}$ perches.

The cost of laying one perch of stone may be estimated by the following items:

Labor: mason $2\frac{3}{8}$ hrs, helper $1\frac{2}{3}$ hrs, based on two helpers to three masons; sand $\frac{1}{5}$ load; lime $\frac{3}{4}$ bu, or if laid in all-cement mortar, one perch will require from $\frac{1}{3}$ to $\frac{1}{2}$ bbl of cement.

DATA ON BRICKS AND BRICKWORK *

Definition of Brick. The American Ceramic Society † defines brick as "A structural unit in the form of a rectangular prism (usually solid and $8 \times 3\frac{3}{4} \times 2\frac{1}{4}$ in in size). In the present state of the art the term 'brick,' when used without a qualifying adjective, is generally understood to mean a structural unit of clay or shale formed while plastic and subsequently fired. When substances other than clay or shale are employed, such as lime and sand, cement and sand, fire-clay, adobe, etc., the term 'brick' should be suitably qualified."

Manufacturing Methods. Manufacturing processes, while varied in character, consist essentially of screening, grinding and working the clay to the desired consistency for molding, whether by hand or machine. After molding the brick are dried and then burned in kilns for many hours at high temperatures, approximately 2 000° F. These processes tend to purify the raw prod-

* Much of the material in this section is compiled from data furnished by the Common Brick Manufacturers' Association of America.

† Journal American Ceramic Society, Vol. II, No. 6, June, 1928.

uct, make it uniform and homogeneous, burn out all combustible matter and result in a product which is both chemically stable and physically permanent.

Trade Names. Trade names are derived from the methods of manufacture. Placing soft mud in molds is known as the SOFT-MUD process. Forcing the clay from the orifice of an augur or extrusion machine in a continuous column and then cutting brick of the column is called the stiff-mud process. Molding relatively dry clay under high pressure is the DRY-PRESS method. Brick are SAND-MOLDED if the molds are coated with sand to facilitate the removal of the brick. If the molds are wetted for the same purpose the brick are WATER-STRUCK. Brick made in the extrusion machines are END-CUT or SIDE-CUT according to the face that is cut as the molded column leaves the machine.

Appearance of Brick. The character of clay, or mixtures of clays and shales, the manner of molding or forming, the manner and degree of burning, all influence the appearance of the brick. The whole sweep of color, in smooth to rough textures, is available, from the severe tones of pearl grays or creams, through buff, golden and bronze tints of the descending scale of reds, down to purples, maroons, and even gun-metal blacks. All of the foregoing applies to what are known as COMMON BRICK, which are used for all purposes, including facing. Some demands and usage have led to the controlled production of specific surface treatments, and bricks so specially processed and used in exposed masonry surfaces go by the trade name of FACE-BRICK.

Size and Weight of Brick. In the United States the American Society for Testing Materials has standardized the size of building brick as follows: "The standard sizes shall conform to the following dimensions, with a permissible variation, plus or minus, of $\frac{1}{16}$ in in depth, $\frac{1}{8}$ in in width and $\frac{1}{4}$ in in length." While there are still some departures from these standards, and while, in the opinion of some, changes therefrom are desirable, the majority of manufacturers now conform to them. Specific gravity of building brick ranges from approximately 1.57 for medium-burned, dry-press surface clay brick to 2.32 for very hard-burned stiff-mud shale brick.

Type	Depth, in	Width, in	Length, in
Common brick	$2\frac{1}{4}$	$3\frac{8}{4}$	8
Rough-face brick	$2\frac{1}{4}$	$3\frac{3}{4}$	8
Smooth-face brick	$2\frac{1}{4}$	$3\frac{7}{8}$	8

Classification. There is a very considerable range in the compressive strength of brick, owing to variations in raw materials, methods of manufacture and degree of burning. The examination of tests of several thousand brick indicates, for instance, an average minimum compressive strength of 1 659 lb per sq in and an average maximum of 22 500 lb per sq in. It is probably a safe assertion that more than 75 per cent of all brick now manufactured in the United States have an average compressive strength of more than 3 000 lb per sq in. In its standard specifications for building brick (A.S.T.M. Designation: C 62–29), the American Society for Testing Materials has established the following classification: *

* The classifications are based on strength and do not necessarily measure weather-resistance.

Name of grade	Compressive strength (bricks flatwise), lb per sq in		Modulus of rupture (bricks flatwise), lb per sq in	
	Mean of five tests	Individual minimum	Mean of five tests	Individual minimum
Grade A... .	4 500 or over	3 500	600 or over	400
Grade B	2 500–4 500	2 000	450 or over	300
Grade C....	1 250–2 500	1 000	300 or over	200

The U. S. Government Master Specifications make use of another classification based upon transverse breaking load and absorption characteristics, as follows:

Physical Requirements

Class	Absorption, per cent		Transverse breaking load, lb, 7-in span	
	Average of five	Individual maximum	Average of five	Individual minimum
V.......	5 or less	6	2 170 or more	1 450
H....	5 to 12	15	1 080 or more	725
M	12 to 24	28	810 or more	540
S................	24 or more	No limit	540 or more	360

It is now seriously questioned by investigators,* however, that the water-absorption of building brick is a measure of its durability or tendency to transmit water.

Fire-Bricks are ordinarily made from a mixture of flint clay and plastic clay. They are usually white, or white mixed with brown, in color and are used for the lining of furnaces, fireplaces and tall chimneys.

Paving Brick. Paving brick are very hard brick, usually annealed. They are seldom used in building. The American Society for Testing Materials has included the following in its Tentative Specifications for Paving Brick (A.S.T.M. Designation: C 7–29T):

NOTE. The sizes and varieties recognized by the Permanent Committee on simplification of Varieties and Standards for Vitrified Paving Brick of the U. S. Department of Commerce given below are recommended for use whereever practicable:

	Depth, in	Width, in	Length, in
Plain wire-cut (vertical fiber lugless as usually laid). ...	2½	4	8½
	3	4	8½
	3½	4	8½
Wire-cut lug (Dunn).........	3	3½	8½
	4	3½	8½
Repressed lug................	4	3½	8½

* Report of Committee C-3 on Brick for 1928, Proc. Am. Soc. Test. Mat. 28 (2): 306–10 (1928).
H. Kreuger, Investigations of Climatic Actions on the Exterior of Buildings.
J. W. McBurney, Water Absorption and Penetrability of Brick, Proc. Am. Soc. Test Mat. 29 (2): 711–30 (1929).

The above sizes and varieties are those recognized by the Permanent Committee for the year 1929, and are subject to change from year to year.

Lime-Mortar Bricks. General Description. SAND-LIME BRICKS were originally made of lime mortar, molded in brick form and hardened by exposure to the air. Such bricks are said to have been largely used in ancient times, and it is claimed that remains of such materials are now in evidence and in a good state of preservation. It is known that they were formerly used in Europe in localities where other materials were not readily available, and that they have been used in some localities in this country during the past thirty-five years. The writer knows of several houses in Haddonfield, N. J., built of such bricks, generally with the exterior surfaces plastered. One of them, however, said to be about twenty-five years old, has not been plastered, and an inspection (1915) shows the bricks to be in an excellent state of preservation. Lime-mortar bricks harden by the absorption of carbonic-acid gas from the air. This gas enters into combination with the lime, forming carbonate of lime. The hardening process requires several weeks' exposure under cover and the product has not virtues sufficient to commend it where other materials are available.

Sand-Lime Bricks. It was discovered in Germany about 1875 that lime-mortar bricks could be hardened in a few hours under heat and pressure, and it was found later that the chemical reaction under the new process differs essentially from that just described, and that the percentage of lime can be greatly reduced. Sand-lime bricks were first made in Germany about 1880, and the more extended commercial development of the industry dates back in Europe to about 1888, and in this country, to about 1900.

Manufacture of Sand-Lime Bricks. Pure silica sand, mixed with from 5 to 10% of high-calcium lime and a certain proportion of water, is molded under very high pressure into the form of bricks. These are piled loosely on cars holding about 1 000 bricks each and placed in a steel cylinder large enough to hold from 10 to 20 cars The cylinder is then closed and steam is turned in and maintained at a pressure of from 120 to 135 lb to the square inch for from 8 to 10 hours, when the cylinder is opened and the bricks removed, ready for use. The tremendous pressure, which is said to be 100 tons on each brick, under which the bricks are formed, causes great density and a bringing of the component elements into close contact. The heat in the cylinder dries the bricks and causes a chemical reaction between the lime and a portion of the silica, forming a hydrosilicate of lime, an insoluble and durable element, which bonds the remaining particles of the sand together and forms a comparatively strong cementing material. The small residue of uncombined lime combines, in the course of time, either with silica or with carbonic-acid gas from the air, until no free lime remains. The bricks thus become harder and stronger with age. In regard to the constitution of sand-lime bricks, Edwin C. Eckel says. "It may be safely assumed that a sand-lime brick as marketed consists of (1) sand-grains held together by a network of (2) hydrous lime silicate, with probably (if a magnesian lime is used) some allied magnesium silicate, and (3) lime hydrate or a mixture of lime and magnesia hydrates. These three elements will always be present, and the structural value of the brick will depend in large part on the relative percentage in which the sand and the hydrates occur."

Quality of Sand-Lime Bricks. The quality of the product depends mainly upon the selection and treatment of the sand and the lime. Pure silica sands,

containing a large percentage of fine grains passing through screens of from 80 to 150 mesh, are preferable. Clay or kaolin are dangerous elements and should not be present in quantities of more than 5%. The lime should be, preferably, high-calcium lime, the magnesium silicates formed by impure limes not being as strong as calcium silicates. Some manufacturers use ready-hydrated lime, others hydrate the lime themselves, before mixing it with the sand, and others grind the quicklime, mix it with the sand and slake it in the sand. The other most important element affecting quality is the press. After pressing and before steaming, the bricks are very fragile and the press should be such that they are subjected to no shaking or friction after the pressure is removed from the mold. Vertical clay-brick presses have been commonly used, but do not appear to be well adapted to the purpose. The rotary table-presses seem to be most successful.

Tests of Sand-Lime Bricks. If the sand is reasonably clean and pure, and the lime finely divided, and if the bricks are sound and have a good metallic ring, they will stand weather-exposure well. If a brick stands in still water for an hour and the moisture rises more than ½ in, it is not a first-class brick; if the moisture rises 2 in, its use for facings is questionable; and if the moisture rises 3 in, it should not be used on outside work of any importance. Authentic tests have been made for crushing, fire-resistance, frost-resistance, acid-resistance and absorption, from which it may be concluded that under proper conditions of manufacture sand-lime bricks are produced having the following physical characteristics: Crushing strength, average, between 2 500 and 3 000 lb per sq in, although some specimens have shown over 5 000 lb per sq in; modulus of rupture, average, about 450 lb per sq in; fire-resistance, but little inferior to that of fire-brick; frost-resistance, generally good, acid-resistance, superior; absorption, from 7 to 10% in 48 hours; rate of absorption, slower than for clay bricks; average absorption for complete saturation, 14%; reduction of compressive strength by saturation for absorption-test, average 33%.

Special Properties of Sand-Lime Bricks. The bricks are square, straight, uniform in size and homogeneous in composition and density. They cleave accurately under the stroke of the trowel and present a weather-surface with the good qualities of stone. They can be cut, carved or sand-blasted, are easily washed clean and show no efflorescence. These claims are well established for properly manufactured sand-lime bricks. It should be further stated that common bricks and facings are made in the same press, the only difference being in the selection of the materials and in the handling of the raw bricks. It is therefore claimed that a rational and homogeneous exterior wall-structure is possible, since backings and facings may be built and bonded in even courses, with Flemish or other ornamental bonds. Some factories, however, manufactured, at first, inferior bricks and care should still be taken in selections from their outputs. Frequently, the ordinary runs of sand-lime bricks are not as strong as the average clay building bricks and some of them are too low in their resistance to frost.

Colors of Sand-Lime Bricks. The natural color is pearl-gray, varying in warmth with the composition of the sand. Permanent colors are produced by introducing mineral oxides with the raw materials in quantities varying according to the intensity of color desired; but as the oxides are foreign materials in the bricks, they affect the quality of the latter in proportion to the quantity used.

Glazed and Enameled Bricks. The terms GLAZED BRICKS AND ENAMELED BRICKS as commonly used, refer practically to the same product, and neither includes what is known as SALT-GLAZED BRICK. The enameled or glazed bricks are generally dipped or sprayed and then burned, whereas the salt-glaze is obtained by the introduction of salt into the fire-boxes of kilns while the bricks are being burned. Glazed or enameled bricks are generally divided into two classes: (1) true enameled bricks, which have a glaze containing the coloring matter applied to it without any intermediate SLIP; (2) bricks which have a transparent glaze placed over a white or colored slip, the slip coming between the glaze and the material to be glazed. The latter is the process most used in this country. Manufacturers differ as to which process produces the best bricks although it would seem as though the true enamel would not chip or peel as readily. These bricks can be made in a variety of colors, from white to dark green or chocolate, and either in a HIGHLY GLAZED FINISH or in a DULL, SATIN-FINISH, the latter finish being quite desirable in many instances on account of its doing away with the glare of the more highly glazed bricks or tiles. An enameled surface may be distinguished from a glazed surface by chipping off a piece of the brick. The glazed brick will show the layer of slip between the glaze and the body of the brick; while the enameled brick will show no line of demarcation between the body of the brick and the enamel. American enameled and glazed bricks are now extensively used for the exterior surfaces of buildings, particularly for street-fronts and light-courts, and for interior side walls and partitions of rooms or buildings used for a great variety of purposes.

Sizes of Enameled Bricks. Enameled bricks are made in two regular sizes: (1) English size, 9 by 3-in enameled surface, $4\frac{1}{2}$-in bed, and (2) American size, $8\frac{3}{8}$ by $2\frac{1}{4}$-in enameled surface, $4\frac{1}{8}$-in bed. The English-size bricks cost more than the American, but on account of the saving in the number of bricks, labor of laying and mortar in joints, the former effect a saving. Enameled bricks are made, also, with a 12 by $4\frac{1}{8}$-in enameled surface, $2\frac{1}{4}$-in bed.

Colors of Enameled Bricks. The standard colors carried in stock are white, cream, buff, black, blue, green, etc.

Estimating Quantities of Brickwork

Methods of Calculation. With the adoption of standard sizes for brick the former practice of "rule of thumb" calculations is rapidly giving way to accurate methods of determining quantities of materials and labor, which with unit costs known for a given locality, result in dependable estimates.

Measurements. Surface areas of walls are determined, making deductions for all openings. Dimensions of foundations, piers and chimneys are also essential. With this information in hand, the following tables may be used to advantage. These tables are conservative in so far as labor is concerned and are based upon that required for carefully faced exterior work. For rougher work, the amount of labor required may be reduced, while for intricate work of any character, an increase will be necessary.

Brick Masonry. Materials and Labor Required for 1 000 sq ft Wall—½-in Joints

Character of construction	Thickness of wall, in	Number of bricks	Cu ft of mortar	Approximate* time, laborer, hours	Approximate time, bricklayer, hours: Common bond: Lime or cement-lime mortar	Common bond: Cement mortar	Other bonds†: Lime or cement-lime mortar	Other bonds†: Cement mortar
Basement walls, solid: Outer 4-in thickness laid with all joints filled Remaining brick laid on full bed of mortar, but brick touching end to end. Vertical space between 4-in thicknesses filled with mortar Every fifth course headers	8	12 706	195	97	73	93		
	12	19 252	314	149	110	140		
	16	25 797	433	200	129	159		
Walls above grade, Solid: Same except vertical space between 4-in thickness left open	8	12 706	135	93	84	93		
	12	19 252	195	140	128	140		
	16	25 797	255	187	140	172		
Walls above grade, solid: Outer 8-in thickness laid with as many as possible vertical joints parallel with face of wall left open Remaining brick in thicker walls laid on full mortar bed but with brick touching end to end and vertical space between 4-in thickness left open	8	12 321	195	95			104	110
	12	18 867	255	142			159	168
	16	25 412	314	189			179	197
Walls above grade, solid: All joints filled with mortar.	4	6 161	76	46	62	68		
	8	12 321	195	95	90	99	104	110
	12	18 482	314	144	135	148	156	164
	16	24 642	433	192	152	179	179	197
Rolok-Bak walls, hollow: 4-in facing laid flat Back laid on edge	8	10 500	109	61	92			
	12	15 000	140	73	110			
Rolok-Bak, heavy duty, hollow	12	15 800	161	78	117			
All-Rolok, hollow: Facing and back laid on edge	8	9 000	74	48	72			
	12	13 500	110	67	100			
All-Rolok, Flemish bond, hollow	8	9 000	74	56			111	
	12	13 750	110	69			138	

* Laborer's time includes that required for making mortar.

† Includes Flemish, English and English-Cross bonds.

Footings. Piers and Chimneys

Footings: quantities for 100 lin ft

Construction	Number of brick	Cu ft of mortar	Approximate time laborer, hours	Approximate time brick-layer, hours
8-in wall	2 272	39	18	15
12-in wall	2 812	48	22	16
16-in wall	4 592	78	36	24
Piers: quantities for 10-ft height				
8×12-in solid	124	2 25	1 00	1 75
12×12-in solid	185	3 25	1 50	2 50
12×16-in solid	247	4 50	2 00	3.25
10¾×10¾-in hollow brick laid on edge	113	1 00	1.25	2.00
Chimneys: quantities for 10-ft height				
8×8-in flue	259	4 50	2 00	3 50
12×12-in flue	345	6 00	2 75	4.50
12×12 and 8×12-in flues	539	8 50	4.00	7.25
8×8-in flue	173	3.00	1.50	2.25
12×12-in flue	238	4 00	1.75	3.25
12×12 and 8×12-in flues	367	6.50	2 75	5.00

Materials Required for Mortar. The following quantities of materials are required for 1 000 cu ft of mortar, lump lime in 180-lb barrels, hydrated lime in 50-lb sacks, cement in 94-lb sacks: Lime mortar, 1 : 2½—57 bbl, lump lime or 350 sacks of hydrated lime and 37 cu yd of sand; lime mortar, 1 : 3—47 bbl of lump lime or 292 sacks of hydrated lime and 37 cu yd of sand; cement-lime mortar, 1 : 1 : 6—130 sacks cement, 24 bbl lump lime or 146 sacks hydrated lime and 37 cu yd sand; cement mortar, 1 : 2—442 sacks cement, 16 bbl lump lime or 92 sacks hydrated lime and 34 cu yd sand; cement mortar, 1 : 3—331 sacks cement, 12 bbl lump lime or 69 sacks hydrated lime and 39 cu yd sand; cement mortar, 1 : 4—264 sacks cement, 10 bbl lump lime or 55 sacks hydrated lime and 41 cu yd sand. NOTE: Cement mortars have 10% of the cement content, by weight, replaced with lime.

LIME *

Nature and Properties of Lime. Chemically, lime is calcium oxide. Used in a broader sense, it is the class-name of a great variety of products manufactured by the calcination of LIMESTONE. Limestone consists of the carbonates of calcium and magnesium which vary widely in their ratio to each other. The limestones used in the manufacture of lime products may be

* **Valuable practical data relating to lime and plaster have been furnished by the Charles Warner Company, of Wilmington, Del.**

divided into two classes, CALCIUM LIMESTONES and DOLOMITIC LIMESTONES. High-calcium limestones contain only a relatively low percentage of magnesium carbonate, while dolomitic limestones contain a considerable amount of it. Dolomitic limestone usually corresponds roughly to the theoretical formula of dolomite ($CaCO_3$) ($MgCO_3$). The CALCINATION of limestone consists of heating to expel the carbon dioxide. The product resulting from calcination of limestone is known as QUICKLIME and possesses great affinity for water. SLAKING is the process of adding water to quicklime. During the process of slaking, heat is energetically evolved and much of the water driven off in the form of steam. During this slaking process, also, high-calcium quicklimes must be agitated and stirred continually or a portion will fail to receive the proper quantity of water and will contain unslaked particles which are likely to slake after being used in the work, causing POPPING, PITTING and disintegration. Dolomitic limes do not slake so energetically, and while they should be stirred while slaking, this is not so necessary as with high-calcium limes. Either class of quicklime, through faulty manufacture, is likely to contain over-burned portions which slake with difficulty and may cause popping, etc., if the lime-paste is not carefully screened before use. The SETTING and HARDENING of common lime mortar is due, first, to the drying out and secondly, to the absorption of carbon dioxide from the atmosphere and the formation of crystals of calcium carbonate to which the strength of the mortar is ascribed. In the manufacture and use of common lime mortar, therefore, the raw material, limestone, is first calcined, and the carbon dioxide expelled; it is then slaked with water and forms calcium hydroxide, in which the water is gradually replaced by carbon dioxide. The lime thus eventually returns to its original carbonate form. As far as the ultimate result is concerned, there is generally little difference between high-calcium and dolomitic quicklimes. Owing to greater familiarity with one or the other of the classes of lime, architects and builders in certain sections of the country prefer one to the other.

Specifications for Quicklime. The lime industry has been made the subject of careful study and the following clauses give the various requirements of Standard Specifications for Quicklime adopted by the American Society for Testing Materials.

1. DEFINITION. Quicklime is a material the major part of which is calcium oxide or calcium and magnesium oxides, which will slake on the addition of water.

2. GRADES. Quicklime is divided into two grades:

(*a*) Selected. Shall be well-burned, picked free from ashes, core, clinker or other foreign material.

(*b*) Run-of-Kiln. Shall be well-burned, without selection.

3. FORMS. Quicklime is shipped in two forms:

(*a*) Lump. Shall be kiln-size.

(*b*) Pulverized Lime. Lump lime reduced in size to pass a ¼-in screen.

4. CLASSES. Quicklime is divided into four classes: (*a*) High-Calcium; (*b*) Calcium; (*c*) Magnesian; (*d*) High-Magnesian.

5. BASIS OF PURCHASE. The particular grade, form and class of quicklime desired shall be specified in advance by the purchaser.

I. Chemical Properties and Tests

(A) Sampling

6. LIME IN BULK. When quicklime is shipped in bulk, the sample shall be so taken that it will represent an average of all parts of the shipment from top to bottom, and shall not contain a disproportionate share of the top and bot-

tom layers, which are most subject to changes. The samples shall comprise at least 10 shovelfuls taken from different parts of the shipment. The total sample taken shall weigh at least 100 lb and shall be crushed to pass a 1-in ring and quartered to provide a 15-lb sample for the laboratory.

7. LIME IN BARRELS. When quicklime is shipped in barrels, at least 3% of the number of barrels shall be sampled. They shall be taken from various parts of the shipment, dumped, mixed and sampled as specified in Section 6.

8. LABORATORY SAMPLES. All samples to be sent to the laboratory shall be immediately transferred to an air-tight container in which the unused portion shall be stored till the quicklime is finally accepted or rejected by purchaser.

(B) Chemical Tests

9. CHEMICAL PROPERTIES. (*a*) The classes and chemical properties of quicklime shall be determined by standard methods of chemical analysis. (*b*) Samples shall be taken as specified in Sections 6, 7 and 8. (*c*) Quicklime shall conform to the following requirements as to chemical composition:

CHEMICAL COMPOSITION

Properties considered	High-calcium		Calcium		Magnesium		High-magnesium	
	Selected	Run of kiln	Selected	Run of kiln	Selected	Run of kiln	Selected	Run of kiln
Calcium oxide, per cent..	90 (min)	90 (min)	85–90	85–90			.	. . .
Magnesium oxide, per cent...	.		.		10–25	10–25	25 (min)	25 (min)
Calcium oxide plus magnesium oxide, min, per cent.	90	85	90	85	90	85	90	85
Carbon dioxide, max, per cent	3	5	3	5	3	5	3	5
Silica plus alumina plus oxide of iron, max, per cent	5	7.5	5	7.5	5	7.5	5	7.5

II. Physical Properties and Tests

10. PERCENTAGE OF WASTE. An average 5-lb sample shall be put into a box and slaked by an experienced operator with sufficient water to produce the maximum quantity of lime putty, care being taken to avoid burning or drowning the lime. It shall be allowed to stand for 24 hours and then washed through a 20-mesh sieve by a stream of water having a moderate pressure. No material shall be rubbed through the screens. Not over 3% of the weight of the selected quicklime nor over 5% of the weight of the run-of-kiln quicklime shall be retained on the sieve. The sample of lump lime taken for this test shall be broken so that all of it will pass a 1-in screen and be retained on a ¼-in screen. Pulverized lime shall be tested as received.

III. Inspection and Rejection

11. INSPECTION. (*a*) All quicklime shall be subject to inspection.

(*b*) The quicklime may be inspected either at the place of manufacture or the point of delivery, as arranged at time of purchase.

(*c*) The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the quicklime ordered. The manufacturer shall afford the inspector all reasonable facilities for inspection and sampling, which shall be so conducted as not to interfere unnecessarily with the operation of the works.

(*d*) The purchaser may make the tests to govern the acceptance or rejection of the quicklime in his own laboratory or elsewhere. Such tests, however, shall be made at the expense of the purchaser.

12. REJECTION Unless otherwise specified, any rejection based on failure to pass tests prescribed in accordance with these specifications shall be reported within five days from the taking of samples.

13. REHEARING. Samples which represent rejected quicklime, shall be preserved in air-tight containers for five days from the date of the test-report. In case of dissatisfaction with the results of the tests, the manufacturer may make claim for a rehearing within that time.

Hydrated Lime. The slaking of quicklime is an operation which is almost invariably carried on by laborers who have little or no conception of the importance of their task. As a result, many failures have been charged to lime in the past which actually were due to improper preparation during the slaking operation. The new product known as HYDRATED LIME has been offered widely to the trade in recent years and has met with much success. Hydrated lime is a dry flocculent powder resulting from the slaking of quicklime by mechanical means, with an amount of water which is sufficient to satisfy the calcium oxide, but insufficient to make a paste or putty. Hydrated lime is manufactured in mechanical hydrators in which the batches of quicklime and water used are carefully proportioned by weight. After passing from the hydrator, hydrated lime is subjected to a mechanical system of separation which eliminates the coarse or impure particles which may cause popping, etc. Hydrated lime is sold in bags of definite weight and requires only to be mixed with sand and water to make the mortar. The bags have usually been made of heavy burlap or duck cloth, containing 100 lb, or of paper, containing 40 lb. Several of the more prominent manufacturers of hydrated lime in the United States employ chemists who regularly superintend the manufacture of hydrated lime, just as the chemists in Portland-cement factories superintend the proportioning of the raw mix going to the kilns to be burned for Portland cement. The hydrated lime manufactured under such chemical supervision is a reliable product free from tendencies which might give rise to popping, pitting or disintegration. Hydrated lime of good quality may be used for almost any purpose for which lime mortar is used, and is by some considered a more reliable product than quicklime. Among the newer uses for hydrated lime may be mentioned its employment in cement mortars and concrete. An addition of about 15% of hydrated lime to cement mortar or concrete decreases its permeability to water, reduces the cracking due to shrinkage, etc., and increases the plasticity of the mortar or concrete, thus preventing separation of the sand, stone and cement and causing the mixture to flow and fill the forms more readily.

Specifications for Hydrated Lime. The following clauses give the various

requirements of Standard Specifications for Hydrated Lime adopted by the American Society for Testing Materials.

1. DEFINITION. Hydrated lime is a dry flocculent powder resulting from the hydration of quicklime.

2. CLASSES. Hydrated lime is commercially divided into four classes: (*a*) High-Calcium; (*b*) Calcium; (*c*) Magnesian; (*d*) High-Magnesian.

3. BASIS OF PURCHASE. The particular type of hydrated lime desired shall be specified in advance of purchase.

I. Chemical Properties and Tests

4. SAMPLING. The sample shall be a fair average of the shipment. Three per cent of the packages shall be sampled. The sample shall be taken from the surface to the center of the package. A 2-lb sample to be sent to the laboratory shall immediately be transferred to an air-tight container, in which the unused portion shall be stored until the hydrated lime has been finally accepted or rejected by the purchaser.

5. CHEMICAL PROPERTIES (*a*) The classes and chemical properties of hydrated lime shall be determined by standard methods of chemical analysis. (*b*) The non-volatile portion of hydrated lime shall conform to the following requirements as to chemical composition:

CHEMICAL COMPOSITION

Properties considered	High-calcium	Calcium	Magnesian	High-magnesian
Calcium oxide, per cent . . .	90 (min)	85–90	. . . .	. . . .
Magnesium oxide, per cent	. . . .	. . . .	10–25	25 (min)
Silica plus alumina plus oxide of iron, max, per cent	5	5	5	5
Carbon dioxide, max, per cent .	5	5	5	5
Water.	Sufficient to hydrate the calcium-oxide content	Sufficient to hydrate the calcium-oxide content	Sufficient to hydrate the calcium-oxide content	Sufficient to hydrate the calcium-oxide content

II. Physical Properties and Tests

6. FINENESS. A 100-g. sample shall leave by weight a residue of not over 5% on a standard 100-mesh sieve and not over 0.5% on a standard 30-mesh sieve.

7. CONSTANCY OF VOLUME. Hydrated lime shall be tested to determine its constancy of volume in the following manner: Equal parts of hydrated lime under test and volume-constant Portland cement shall be thoroughly mixed together and gauged with water to a paste. Only sufficient water shall be used to make the mixture workable. From this paste a pat about 3 in in diameter and ½ in thick at the center, tapering to a thin edge, shall be made on a clean glass plate about 4 in square. This pat shall be allowed to harden 24 hours in moist air and shall be without popping, checking, cracking, warping

or disintegration after 5 hours' exposure to steam above boiling water in a loosely closed vessel.

III. Packing and Marking

8. PACKING. Hydrated lime shall be packed either in cloth or paper bags and the weight shall be plainly marked on each package.

9. MARKING. The name of the manufacturer shall be legibly marked or tagged on each package.

IV. Inspection and Rejection

10. INSPECTION. (*a*) All hydrated lime shall be subject to inspection.

(*b*) The hydrated lime may be inspected either at the place of manufacture or the point of delivery, as arranged at the time of purchase.

(*c*) The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the hydrated lime ordered. The manufacturer shall afford the inspector all reasonable facilities for inspection and sampling, which shall be so conducted as not to interfere unnecessarily with the operation of the works.

(*d*) The purchaser may make the tests to govern the acceptance or rejection of the hydrated lime in his own laboratory or elsewhere. Such tests, however, shall be made at the expense of the purchaser.

11. REJECTION. Unless otherwise specified, any rejection based on failure to pass tests prescribed in these specifications shall be reported within five working days from the taking of samples.

12. REHEARING. Samples which represent rejected hydrated lime shall be preserved in air-tight containers for five days from the date of the test-report. In case of dissatisfaction with the results of the tests, the manufacturer may make claim for a rehearing within that time.

Neat Lime. A number of lime manufacturers are at present making a product called NEAT LIME under various trade names. It is composed of various percentages of lime and cement together with hair or fiber, and is used for interior plastering as well as exterior stucco. It is sold in 50-lb paper sacks, and when mixed with sand and water is ready for use. This material, being more sound-absorbing than the gypsum plaster and not being injured by dampness or exposure to water as in the case of gypsum, has certain advantages. In some cases on exterior work an additional amount of cement is added to this neat lime to give it greater strength.

Useful Data on Quicklime. Quicklime is shipped either in barrels or in bulk. In dry climates it will keep for a long time in bulk, but in damp climates and along the coast it soon slakes unless enclosed in barrels By Act of Congress, August 23, 1916, it is required that lime in barrels shall be packed only in barrels containing 280 lb or 180 lb, net weight. When shipped in bulk it is generally sold by the bushel of 80 lb, 3½ bushels or 280 lb, net, of lime being considered as equivalent to a large barrel. Other weights are 180 lb, net, per small barrel, and 64 lb per cu ft. The average yield of LIME-PASTE from the best Eastern limes has been found to be 2.62 times the bulk of unslaked lime. A barrel of good quality well-burned lime should make 8 cu ft, or 20 pails, of lime-paste or putty. Careful experiments conducted by United States engineers have demonstrated that the best mortar is obtained by mixing one part of lime-paste to two parts of sand.

SAND AND GRAVEL

Sand is obtained from banks or pits, from river-beds and from the seashore. Pit-sand or bank-sand, free from clay or earthy materials, is generally considered the best for mortar, although excellent sand is often obtained from river-beds. Sea-sand contains alkaline salts which attract and retain moisture and which, unless thoroughly washed, cause efflorescence when used in brickwork. Both sea-sand and river-sand have more or less rounded grains, to which lime or cement will not adhere as well as to sharp, angular grains. Both are extensively used, however, for lack of better materials. The use of sand in mortar is to prevent excessive shrinkage and to save the cost of lime or cement. Sand, when used in the proportion of 1:2, strengthens lime mortar, but any addition of sand to cement weakens it.

Screening Sand. Sand for mortar must ordinarily be screened. Sand for brown mortar for plastering or common brickwork is ordinarily run through a No. 4 screen having 4 by 4 meshes to the inch. For sand finish and mortar for pressed brickwork, either a No. 10 or a No. 12 screen with 10 by 10 or 12 by 12 meshes to the inch is commonly used. For rubble stonework the sand is not ordinarily screened, unless it contains much gravel, in which case it should be screened through a 3/8-in mesh.

Weight of Sand. Dry sand weighs from 80 to 115 lb per cu ft. The average weight of damp (not wet) sand is about 96 lb per cu ft., or about 2 600 lb per cu yd. The voids for ordinary sand range from 0.3 to 0.5 of the volume, the average for screened sand suitable for mortar being 0.35 of the volume. The more uneven the grains in size the smaller the percentage of the voids. A one-horse load of sand contains about 22 cu ft. Two-horse loads vary from 1¼ to 2 yd. 1¼ yd is a fair load, 1½ yd a good load and 2 yd a large load.

PLASTER LATH, LATHING AND PLASTERING

Wooden Laths should be well seasoned, free from sap, bark and dead knots. Bark on laths is quite sure to stain the plaster. White pine is generally considered the best wood for laths, although spruce and hemlock laths are much used. Hard pine is not a good material, as it contains too much pitch. The regular size of laths is ¼ in by 1½ in by 4 ft. The width and thickness vary somewhat in different mills. Laths are sold by the thousand, in bundles containing 100 laths.

Metal Lathing. (See Chapter XXII.)

Insulation Boards made of cane and wood fiber, cork, etc., are now being used extensively as a base for plaster work. Care must be exercised in their erection, however, so that warping may be reduced to a minimum. It is advisable that the instructions given by the manufacturer be carefully followed.

Plastering on laths is generally done in three coats. The first coat is called the SCRATCH-COAT; the second, the BROWN COAT, and the third, the WHITE COAT, SKIM-COAT, or FINISH. On brickwork or stonework the scratch-coat is generally omitted. For first-class work each coat should be permitted to dry thoroughly before the next coat is applied, and under no circumstances should the finish-coat be applied before the brown coat is thoroughly dry. When plastering is applied to insulation boards two-coat work is often found to be adequate.

Drawn Work is a brown coat applied to a scratch-coat from the same staging, immediately after the scratch-coat is applied. It is a little cheaper than DRY SCRATCH, and much of it is done in the Western States.

The Scratch-Coat should always be made rich in lime, and should contain 1¼ bu of hair, or an equivalent quantity of fiber to each cask of lime, or 1 bu of hair to 2 of lime. A proportion of one part lime-paste to two parts of sand will require 1 cask (2½ bu) of lime to 5½ bbl of screened sand.

The Brown Coat should contain 1 cask (2½ bu) of lime to 7 bbl of screened sand, and 1 bu of hair to 5 of lime. Very little plaster is mixed by measure, however, the usual custom being to mix as much sand with the slaked lime as the mortar-mixer thinks it will stand and give satisfaction, the tendency being always to make the lime go as far as possible.

The Third or Finishing Coat is designated by various terms, such as SKIM-COAT, WHITE COAT, PUTTY-COAT, SAND-FINISH, etc. The skim-coat as used in the Eastern States is generally composed of lime-putty and washed beach-sand in equal proportions.

Sand Finish, which has a rough surface resembling coarse sandpaper, is mixed in the same way, only that coarser sand and more of it is used, and it is finished with a wooden or cork-faced float.

White Coating or Hard Finish generally means a composition of lime-putty and plaster of Paris, to which marble-dust is sometimes added. Plaster of Paris and marble-dust when used should not be mixed with the lime-putty until a few moments before using, and no more should be prepared at one time than can be used up at once, as it soon SETS, after which it should not be used. The skim-coat or hard finish should be finished with a steel trowel and wet brush. The more the work is troweled the harder it becomes. A superior hard finish is obtained by mixing 4 parts of Best's Keene's cement to 1 part lime-putty.

Mortar by Plastering. To make sure that the lime is well slaked, it is customary to require that the mortar for plastering shall be mixed at least seven days before it is used.

Hair such as is used by plasterers is obtained from the hides of cattle, and after being washed and dried is put up in paper bags, each bag being supposed to contain 1 bushel of hair when beaten up. Each package is supposed to weigh from 7 to 8 lb but the weight often falls short. ASBESTOS and MANILLA FIBER are both used in place of hair; they are cleaner than hair and are said to be less injured by the lime. It is much better to add the hair to the lime-paste AFTER IT IS COLD and before mixing in the sand, as hot lime, and the steam caused by the slaking, burn or rot the hair so as to greatly weaken it. The common practice is to put the hair in the mortar-box, run off the hot lime as soon as it is slaked, throw in the sand and mix the whole together. It is then thrown out of the box into a pile and a new batch mixed up.

Machine-Made Mortar. In many cities plants have been equipped for the mixing of mortar by machinery. Machine-mixed mortar is better than the ordinary hand-mixed mortar, for the reason that time can be given for the lime to slake, the lime and sand can be accurately measured, and the hair and lime are not mixed with the lime until just before delivery. The mixing may also be more thoroughly and evenly done by machinery than is possible by hand.

Improved Wall-Plasters. Owing to the difficulty of obtaining sufficient space in building operations in central sections of large cities to properly slake sufficient lime mortar to carry on the plastering with the necessary speed, other

kinds of plastering materials have come into existence in recent years. These are known as GYPSUM PLASTERS or HARD-WALL PLASTERS. The base of these products is calcium sulphate or gypsum which has been calcined to partially expel the water. The setting and hardening of these products is dependent upon their combining chemically with the gauging water and crystallizing in the same chemical form as the material possessed before calcination. All hard-wall plasters contain material added for the purpose of controlling the SET. The straight calcined gypsum sets in a very few minutes, which time would be entirely too short to permit the workmen to apply the plaster to the wall and straighten it up before it had set. These plasters are characterized, also, by their inability to carry as much sand as lime mortar. Many of them contain other substances, such as clay or hydrated lime, added to improve their PLASTICITY. Hard-wall plasters manufactured in the eastern part of of the United States from rock-gypsum invariably contain 15%, more or less, of clay or hydrate, added for this purpose. Plasters made in Kansas, Oklahoma, Texas and other Western and Southwestern States are made from earth-gypsum. In the case of these materials, clay and hydrated lime are not added, for the reason that the earth-gypsum contains considerable clay matter, which renders further additions unnecessary.

Use of Hard-Wall Plasters. Hard-wall plasters are found to be very convenient in cases where space and time are the most important elements in the building operation. They set more rapidly than lime plasters, thus permitting the white coating and finishing of the job to be completed earlier. While hard-wall plasters become extremely hard, this property is sometimes considered objectionable, as it may give rise to what is called the SOUNDING-BOARD effect.

Keene's Cement Plasters. As distinguished from the ordinary hard-wall plasters, there exists another class of gypsum-products which, however, are somewhat different in the method of preparation and behavior. In the manufacture of these materials, the gypsum is calcined, immersed in a bath of alum or similar chemical and recalcined. The name KEENE'S CEMENT is usllly applied to these materials, which are made by several manufacturers in this country. These are slow-setting and ultimately attain great strength and hardness. Keene's cement is generally used with considerable lime-putty or hydrated lime. The use of equal parts of hydrated lime and Keene's cement in making a plastering material is often recommended and found in specifications. (See Neat Lime.)

Advantages of Improved Wall-Plasters. Among the advantages gained by the use of these plasters are uniformity in strength and quality, extra hardness and toughness, freedom from pitting, saving in time required in making and drying, minimum danger from frost while being applied and before set, less weight and moisture in the building, and, in some cases, greater resistance to the action of fire.

Measuring Plasterers' Work. Lathing is always figured by the square yard and is generally included with the plastering, although in small country towns the carpenter often puts on the laths. Plastering on plane surfaces, such as walls and ceilings, is always measured by the square yard, whether it is one-coat, two-coat, or three-coat work, or lime or hard plaster. In regard to deductions for openings, custom varies somewhat in different parts of the country and also with different contractors. Some plasterers allow one-half the area of openings for ordinary doors and windows, while others make no allowance for openings of less than 7 sq yd.

Miscellaneous Details. Returns of chimney-breasts, pilasters and all strips less than 12 in in width should be measured as 12 in wide. Closets, soffits of stairs, etc., are generally figured at a higher rate than plain walls or ceilings, as it is not as easy to get at them. For circular or elliptical work, domes or groined ceilings, an additional price is made. If the plastering cannot be done from trestles an additional charge must be made for staging.

Cornices and Moldings. Stucco cornices and molded work are generally measured by the superficial foot, measuring on the profile of the molding. When less than 12 in in girth they are usually rated as 1 ft. For each internal angle 1 lin ft should be added, and for external angles, 2 lin ft. For cornices on circular or elliptical work an additional price should be charged. Enriched moldings are generally figured by the linear foot, the price depending upon the design and size of the mold.

Quantities of Materials for Lathing and Plastering

Miscellaneous Data. To cover 100 sq yd requires from 1 400 to 1 500 laths, or say 1 450 for an average job, and 10 lb of threepenny fine nails.

Three-coat plastering on wooden laths, plaster-of-Paris finish, will require from 10 to 12 bu of lime, 1½ cu yd of sand, 2 bu of hair and 100 lb of plaster of Paris per 100 sq yd.

If the finish-coat is omitted, deduct 2 bu of lime and all of the plaster of Paris.

If sand-finished, omit the plaster of Paris and add ½ cu yd of sand.

To cover 100 sq yd with two coats on brick or stone walls, the brown coat and finishing coats, will require from 8 to 10 bu of lime, 1½ cu yd of sand, and 100 lb of plaster of Paris, to 100 sq yd.

Using Best's Keene's cement for brown mortar and Keene's finish on expanded-metal lath will require, for brown mortar, 550 lb of cement, 5½ bu of lime, 2 cu yd of sand and 2 bu of hair; for the finish, 300 lb of cement and 1 bu of lime per 100 yd.

Hard plasters on expanded metal lath, plaster-of-Paris finish, require, for brown mortar, 2 000 lb of plaster and 2 cu yd of sand; for the finish, 1 bu of lime and 100 lb of plaster of Paris per 100 yd.

DATA ON LUMBER AND CARPENTERS' WORK

Relative Hardness of Woods. Taking shell-bark hickory as the highest standard of our forest-trees, and calling that 100, other trees will compare with it for hardness as follows:

Shell-bark hickory	100	Yellow oak	60
Pignut hickory	96	Hard maple	56
White oak	84	White elm	58
White ash	77	Red cedar	56
Dogwood	75	Wild cherry	55
Scrub-oak	73	Yellow pine	54
White hazel	72	Chestnut	52
Apple-tree	70	Yellow poplar	51
Red oak	69	Butternut	43
White beech	65	White birch	43
Black walnut	65	White pine	30
Black birch	62		

Weight of Rough Lumber per 1 000 Feet

BOARD-MEASURE, APPROXIMATE

Kind of wood	Green from saw, lb	Shipping-dry, lb	Well-seasoned, lb	Kiln-dried, lb
Ash	...	...	...	...
Chestnut	4 600	...	3 500	3 200
Hemlock	4 200	3 000	...	...
Maple, hard	5 400	4 150	3 900	3 400
Maple, soft	5 000	3 650	3 300	3 000
Oak, red	5 500	4 250	4 000	3 400
Oak, white	5 700	4 500	4 100	3 600
Pine, long-leaf	4 500	3 500	...	...
Pine, white	3 500	2 500	2 400	2 200
Poplar	4 000	3 000	2 900	2 400
Spruce	3 150	2 700	2 300	2 200
Sycamore	4 750	3 200	3 000	...
Walnut, black	4 900	4 000	3 800	...

Framing-Lumber may commonly be purchased in any of the following nominal sizes, except that common pine, spruce, and hemlock cannot usually be obtained in larger sizes than 12 by 12 in.

Nominal Sizes of Framing-Lumber

in	in	in	in
2 × 4	3 × 6	4 × 12	8 × 12
2 × 6	3 × 8	4 × 14	8 × 14
2 × 8	3 × 10	6 × 6	10 × 10
2 × 10	3 × 12	6 × 8	10 × 12
2 × 12	3 × 14	6 × 10	10 × 14
2 × 14	3 × 16	6 × 12	10 × 16
2 × 16	4 × 4	6 × 14	12 × 12
2½ × 12	4 × 6	6 × 16	12 × 14
2½ × 14	4 × 8	8 × 8	12 × 16
2½ × 16	4 × 10	8 × 10	14 × 14
...	...	...	14 × 16

In some of the New England mills, the following sizes, also, are sawed: 2 by 3, 2 by 5, 2 by 7, 2 by 9, 3 by 4 and 3 by 5 in. These sizes are not commonly carried in stock, and in most localities would have to be obtained by ripping larger sizes. Most of the long-leaf yellow pine and Douglas fir is SHIPPED SURFACED ONE SIDE AND EDGE, the actual dimensions being from ¼ in to ⅜ in, and sometimes ½ in, scant of the nominal dimensions. When framing-lumber is required to be full to dimensions it should be ordered IN THE ROUGH, and a special contract made on that understanding.

Lengths of Framing-Timbers. All timber is cut and sold in even lengths, as 10, 12, 14, 16, 18, 20 ft, etc. Odd and fractional lengths are counted as the next higher even length; consequently it is, in certain cases, possible and economical to plan buildings so that timbers of even lengths may be used without waste.

Measurement of Rough Lumber. All rough lumber is sold by the foot, BOARD-MEASURE, one foot being the equivalent of a board 1 ft wide, 1 ft long, and 1 in thick. To compute the board-measure in any board, plank, or timber, divide the nominal sectional area, in inches, by 12, and multiply by the length in feet. Thus the number of FEET in a 2 by 4-in scantling, 8 ft long = $(2 \times 4/12) \times 8 = 5\frac{1}{3}$ ft, board-measure. A 10-in board, 12 ft long, contains $(1 \times 10/12) \times 12 = 10$ ft, board-measure. Extensive tables are published showing the feet, in board-measure, for almost any commercial size of timber. The following table, however, although compact, will enable one to readily estimate the number of FEET in any of the standard sizes of boards, planks, or timbers. To use the table, find the product of the lateral dimensions of the cross-section; then in the column having a heading equal to this product, and in the horizontal line opposite the given length will be found the number of feet in board-measure. Thus, for a 3 by 4, 2 by 6, or 1 by 12-in timber look in the column headed 12; for a 2 by 12, 4 by 6, or 3 by 8-in piece, look in the column headed 24. For lengths not given in the table, take either twice the length and divide by 2, or one-half the length and multiply by 2. Where timbers of the same size abut end to end, it economizes labor in reducing to board-measure to take the full length; for this reason the lengths in the table are carried beyond those for single sticks.

Measurement of Finishing-Lumber, Flooring, Ceiling, Etc. Most, if not all, lumber for finishing is sawed for use in thicknesses of 1 in, $1\frac{1}{4}$ in, $1\frac{1}{2}$ in, and 2 in, and some woods, such as white pine and poplar, are sawed into thicknesses of $2\frac{1}{2}$ in and 3 in.

When surfaced both sides, the thickness is reduced to $\frac{13}{16}$, $1\frac{1}{16}$, $1\frac{5}{16}$, $1\frac{3}{4}$, $2\frac{1}{4}$, and $2\frac{11}{16}$ in.

All dressed stock is measured and sold STRIP-COUNT, that is, full size of rough material necessarily used in its manufacture. Thus $1\frac{1}{16}$-in boards are measured as though $1\frac{1}{4}$ in thick. The number of feet, board-measure, for $1\frac{1}{4}$-in stock ($1\frac{1}{16}$ finished) is $1\frac{1}{4}$ times that in a 1-in board, and in the same way for $1\frac{1}{2}$-in and $2\frac{1}{2}$-in stock. $1\frac{3}{4}$-in planks are always measured 2 in thick, and $2\frac{1}{4}$-in stock, $2\frac{1}{2}$ in thick. Boards less than 1 in thick are measured the same as 1-in boards, but for $\frac{3}{8}$-in and $\frac{5}{8}$-in stock a reduced price is generally made.

Matched Ordinary Flooring. The standard sizes for flooring (other than hardwood, parqueting or parquet-flooring) are 1 by 3, 1 by 4 and 1 by 6; or $1\frac{1}{4}$ by 3, $1\frac{1}{4}$ by 4 and $1\frac{1}{4}$ by 6. The thickness of 1-in flooring should be $\frac{13}{16}$ in, and of $1\frac{1}{4}$-in flooring, $1\frac{3}{32}$ in. 3-in flooring should show $2\frac{1}{4}$ in on the face, after it is laid; 4-in, $3\frac{1}{4}$ in; and 6-in, $5\frac{1}{4}$ in.

Matched Maple Flooring is usually made in 2-in, $2\frac{1}{4}$-in and $3\frac{1}{4}$-in face, and in thicknesses of $\frac{13}{16}$, $1\frac{1}{16}$ and $1\frac{5}{16}$ in.

Ceiling, matched and beaded boards, are regularly made in the same widths as flooring. The standard (nominal) thicknesses of yellow-pine ceiling are $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{4}$ in, the actual thickness of each being $\frac{1}{16}$ in less. The $\frac{3}{8}$-in ceiling is dressed one side only, the other thicknesses both sides.

Yellow Pine Drop-Siding. Dressed and matched yellow pine drop-siding is $\frac{3}{4}$ by $3\frac{1}{2}$ and $\frac{3}{4}$ by $5\frac{1}{2}$ in, showing $3\frac{1}{4}$ and $5\frac{1}{4}$-in face; and worked shiplap is $\frac{3}{4}$ by $3\frac{1}{2}$ and $\frac{3}{4}$ by $5\frac{1}{2}$ in, showing 3 and 5-in face.

Beveled Siding is resawed on a bevel from stock $\frac{13}{16}$ by $3\frac{1}{2}$ and $\frac{13}{16}$ by $5\frac{1}{2}$ in, after surfacing.

Table of Board-Measure

Length in feet	Sectional area in square inches: 4 ft	4 in	6 ft*	8 ft	8 in	10 ft	10 in	12 ft*	14 ft	14 in	16 ft	16 in	18 ft*	20 ft	20 in
6	2	0	3	4	0	5	0	6	7	0	8	0	9	10	0
8	2	8	4	5	4	6	8	8	9	4	10	8	12	13	4
10	3	4	5	6	8	8	4	10	11	8	13	4	15	16	8
12	4	0	6	8	0	10	0	12	14	0	16	0	18	20	0
14	4	8	7	9	4	11	8	14	16	4	18	8	21	23	4
16	5	4	8	10	8	13	4	16	18	8	21	4	24	26	8
18	6	0	9	12	0	15	0	18	21	0	24	0	27	30	0
20	6	8	10	13	4	16	8	20	23	4	26	8	30	33	4
22	7	4	11	14	8	18	4	22	25	8	29	4	33	36	8
24	8	0	12	16	0	20	0	24	28	0	32	0	36	40	0
26	8	8	13	17	4	21	8	26	30	4	34	8	39	43	4
28	9	4	14	18	8	23	4	28	32	8	37	4	42	46	8
30	10	0	15	20	0	25	0	30	35	0	40	0	45	50	0
32	10	8	16	21	4	26	8	32	37	4	42	8	48	53	4
34	11	4	17	22	8	28	4	34	39	8	45	4	51	56	8
36	12	0	18	24	0	30	0	36	42	0	48	0	54	60	0
38	12	8	19	25	4	31	8	38	44	4	50	8	57	63	4
40	13	4	20	26	8	33	4	40	46	8	53	4	60	66	8
42	14	0	21	28	0	35	0	42	49	0	56	0	63	70	0

Length in feet	Sectional area in square inches: 24 ft*	28 ft	28 in	30 ft*	32 ft	32 in	35 ft	35 in	36 ft*	40 ft	40 in	42 ft*	48 ft*
6	12	14	0	15	16	0	17	6	18	20	0	21	24
8	16	18	8	20	21	4	23	4	24	26	8	28	32
10	20	23	4	25	26	8	29	2	30	33	4	35	40
12	24	28	0	30	32	0	35	0	36	40	0	42	48
14	28	32	8	35	37	4	40	10	42	46	8	49	56
16	32	37	4	40	42	8	46	8	48	53	4	56	64
18	36	42	0	45	48	0	52	6	54	60	0	63	72
20	40	46	8	50	53	4	58	4	60	66	8	70	80
22	44	51	4	55	58	8	64	2	66	73	4	77	88
24	48	56	0	60	64	0	70	0	72	80	0	84	96
26	52	60	8	65	69	4	75	10	78	86	8	91	104
28	56	65	4	70	74	8	81	8	84	93	4	98	112
30	60	70	0	75	80	0	87	6	90	100	0	105	120
32	64	74	8	80	85	4	93	4	96	106	8	112	128
34	68	79	4	85	90	8	99	2	102	113	4	119	136
36	72	84	0	90	96	0	105	0	108	120	0	126	144
38	76	88	8	95	101	4	110	10	114	126	8	133	152
40	80	93	4	100	106	8	116	8	120	133	4	140	160
42	84	98	0	105	112	0	122	6	126	140	0	147	168

* The measurements in these columns come out in even feet.

Table of Board-Measure (Continued)

Length in feet	Sectional area in square inches													
	56		60	64		72	80		84	96	100		112	
	ft	in	ft*	ft	in	ft*	ft	in	ft*	ft*	ft	in	ft	in
4	18	8	20	21	4	24	26	8	28	32	33	4	37	4
6	28	0	30	32	0	36	40	0	42	48	50	0	56	0
8	37	4	40	42	8	48	53	4	56	64	66	8	74	8
10	46	8	50	53	4	60	66	8	70	80	83	4	93	4
12	56	0	60	64	0	72	80	0	84	96	100	0	112	0
14	65	4	70	74	8	84	93	4	98	112	116	8	130	8
16	74	8	80	85	4	96	106	8	112	128	133	4	149	4
18	84	0	90	96	0	108	120	0	126	144	150	0	168	0
20	93	4	100	106	8	120	133	4	140	160	166	8	186	8
22	102	8	110	117	4	132	146	8	154	176	183	4	205	4
24	112	0	120	128	0	144	160	0	168	192	200	0	224	0
26	121	4	130	138	8	156	173	4	182	208	216	8	242	8
28	130	8	140	149	4	168	186	8	196	224	233	4	261	4
30	140	0	150	160	0	180	200	0	210	240	250	0	280	0
32	149	4	160	170	8	192	213	4	224	256	266	8	298	8
34	158	8	170	181	4	204	226	8	238	272	283	4	317	4
36	168	0	180	192	0	216	240	0	252	288	300	0	336	0
38	177	4	190	202	8	228	253	4	266	304	316	8	354	8
40	186	8	200	213	4	240	266	8	280	320	333	4	373	4
42	196	0	210	224	0	252	280	0	294	336	350	0	392	0
44	205	4	220	234	8	264	293	4	308	352	366	8	410	8
46	214	8	230	245	4	276	306	8	322	368	383	4	429	4
48	224	0	240	256	0	288	320	0	336	384	400	0	448	0
50	233	4	250	266	8	300	333	4	350	400	416	8	466	8
52	242	8	260	277	4	312	346	8	364	416	433	4	485	4
54	252	0	270	288	0	324	360	0	378	432	450	0	504	0
56	261	4	280	298	8	336	373	4	392	448	466	8	522	8
58	270	8	290	309	4	348	386	8	406	464	483	4	541	4
60	280	0	300	320	0	360	400	0	420	480	500	0	560	0
62	289	4	310	330	8	372	413	4	434	496	516	8	578	8
64	298	8	320	341	4	384	426	8	448	512	533	4	597	4
66	308	0	330	352	0	396	440	0	462	528	550	0	616	0
68	317	4	340	362	8	408	453	4	476	544	566	8	634	8
70	326	8	350	373	4	420	466	8	490	560	583	4	653	4
72	336	0	360	384	0	432	480	0	504	576	600	0	672	0
74	345	4	370	394	8	444	493	4	518	592	616	8	690	8
76	354	8	380	405	4	456	506	8	532	608	633	4	709	4
78	364	0	390	416	0	468	520	0	546	624	650	0	728	0
80	373	4	400	426	8	480	533	4	560	640	666	8	746	8
82	382	8	410	437	4	492	546	8	574	656	683	4	765	4
84	392	0	420	448	0	504	560	0	588	672	700	0	784	0

* The measurements in these columns come out in even feet.

Table of Board-Measure (Continued)

Length in feet	Size and sectional area in inches											
	120 10×12 ft*	140 10×14 ft	in	144 12×12 ft*	160 10×16 ft	in	168 12×14 ft*	192 12×16 ft*	196 14×14 ft	in	224 14×16 ft	in
4	40	46	8	48	53	4	56	64	65	4	74	8
6	60	70	0	72	80	0	84	96	98	0	112	0
8	80	93	4	96	106	8	112	128	130	8	149	4
10	100	116	8	120	133	4	140	160	163	4	186	8
12	120	140	0	144	160	0	168	192	196	0	224	0
14	140	163	4	168	186	8	196	224	228	8	261	4
16	160	186	8	192	213	4	224	256	261	4	298	8
18	180	210	0	216	240	0	252	288	294	0	336	0
20	200	233	4	240	266	8	280	320	326	8	373	4
22	220	256	8	264	293	4	308	352	359	4	410	8
24	240	280	0	288	320	0	336	384	392	0	448	0
26	260	303	4	312	346	8	364	416	424	8	485	4
28	280	326	8	336	373	4	392	448	457	4	522	8
30	300	350	0	360	400	0	420	480	490	0	560	0
32	320	373	4	384	426	8	448	512	522	8	597	4
34	340	396	8	408	453	4	476	544	555	4	634	8
36	360	420	0	432	480	0	504	576	588	0	672	0
38	380	443	4	456	506	8	532	608	620	8	709	4
40	400	466	8	480	533	4	560	640	653	4	746	8
42	420	490	0	504	560	0	588	672	686	0	784	0
44	440	513	4	528	586	8	616	704	718	8	821	4
46	460	536	8	552	613	4	644	736	751	4	858	8
48	480	560	0	576	640	0	672	768	784	0	896	0
50	500	583	4	600	666	8	700	800	816	8	933	4
52	520	606	8	624	693	4	728	832	849	4	970	8
54	540	630	0	648	720	0	756	864	882	0	1 008	0
56	560	653	4	672	746	8	784	896	914	8	1 045	4
58	580	676	8	696	773	4	812	928	947	4	1 082	8
60	600	700	0	720	800	0	840	960	980	0	1 120	0
62	620	723	4	744	826	8	868	992	1 012	8	1 157	4
64	640	746	8	768	853	4	896	1 024	1 045	4	1 194	8
66	660	770	0	792	880	0	924	1 056	1 078	0	1 232	0
68	680	793	4	816	906	8	952	1 088	1 110	8	1 269	4
70	700	816	8	840	933	4	980	1 120	1 143	4	1 306	8
72	720	840	0	864	960	0	1 008	1 152	1 176	0	1 344	0
74	740	863	4	888	986	8	1 036	1 184	1 208	8	1 381	4
76	760	886	8	912	1 013	4	1 064	1 216	1 241	4	1 418	8
78	780	910	0	936	1 040	0	1 092	1 248	1 274	0	1 456	0
80	800	933	4	960	1 066	8	1 120	1 280	1 306	8	1 493	4
82	820	956	8	984	1 093	4	1 148	1 312	1 339	4	1 530	8
84	840	980	0	1 008	1 120	0	1 176	1 344	1 372	0	1 568	0

* The measurements in these columns come out in even feet.

New England Clapboards are 4 ft long, 6 in wide, ½ in thick at the butt, and about ⅛ in thick at the other edge. They are put up in bunches and sold by the thousand.

Rules for Estimating Quantities of Sheathing, Flooring, Etc. For common sheathing laid horizontally on a wall or roof without openings, add one-tenth to the actual superficial area to allow for waste. On the walls of dwellings, figure the walls as though without openings and allow nothing for waste. If sheathing is laid diagonally, add one-sixth to the actual superficial area.

For tight sheathing laid horizontally, add one-fifth for 6-in boards, one-seventh for 8-in boards, and one-ninth for 10-in boards. If laid diagonally add one-fourth for 6-in boards, one-sixth for 8-in boards, and one-eighth for 10-in boards.

For 3-in matched flooring add one-half to the actual superficial area to be covered.

For 4-in flooring add one-third and for 6-in flooring add one-fifth. Ceiling is measured the same as flooring.

For drop-siding, add one-fifth to the superficial area.

For lap-siding, laid 4 in to the weather, add one-half to the actual superficial area; if 4½ in to the weather, add one-third.

BUILDING PAPERS, BUILDING FELTS, QUILTS AND INSULATORS

Sheathing-Papers,* Felts, Quilts, Etc. It is well known that frame buildings when merely sheathed and clapboarded or shingled on the outside and simply lathed and plastered on the inside, are almost sure to be hot in summer and cold in winter; and as the wood almost always shrinks, cracks result through which the wind finds its way. For these reasons some extra provision should be made for keeping out the wind and the heat and cold; and it is generally admitted that there is no material that will do this so well and at so small an expense as good sheathing-papers or sheathing-felts. The papers made for this purpose are commonly known as SHEATHING-PAPERS or BUILDING PAPERS. There is a great variety of sheathing-papers manufactured, many of them of great excellence, and even the best are comparatively inexpensive; so that only the better qualities of any kind of felt or paper should be specified. Where the cost of the sheathing-paper on an ordinary house is only a few dollars, it is poor economy to use a cheap paper, as the labor of applying it is an important item and the poorer the paper the more difficult the work of putting it on. The qualities which good sheathing-paper should possess are: permanence, impenetrability to air and water and sufficient strength to permit of applying without tearing. Protection or proof against vermin and insects is another important requirement. It should not be brittle nor have a lasting strong odor and, for the convenience of the builder, should be clean for handling. There are so many papers possessing all or most of these qualities that it is deemed inexpedient to mention particular brands. The architect should decide for himself, from the samples with which he has probably been furnished, what papers are best adapted to the particular conditions; and he should then specify those brands, giving, also, the manufacturers'

* The terms BUILDING PAPER and SHEATHING-PAPER are by the public indiscriminately applied to all kinds of paper used in connection with building-construction. In the trade, however, the term BUILDING PAPER is confined to the rosin-sized and cheaper grades of paper, while the heavier and better grades are classed as SHEATHING-PAPERS.

names, instead of leaving the choice to the builder, who will be quite sure to be guided by price rather than by quality. Many object to tarred or saturated sheathing-papers and felts because of their tendency to become brittle and because they emit a strong odor and are somewhat disagreeable to handle. On the other hand, the advocates of tarred felts emphasize their cheapness, warmth and even their odor, which makes them vermin-proof. The odor gradually disappears after the clapboards, siding or shingles are put on and the inside walls finished. Sheathing-paper is usually applied just previous to putting on the clapboards, siding, or shingles. It is generally placed horizontally and should lap about 2 in over each sheet and over the paper previously placed around the window and door-frames. If sheathing-quilt or similar material is to be placed under the clapboards or siding, laths should be nailed vertically over it, opposite each stud, and the siding or clapboards nailed to the laths; otherwise it will be difficult to put them on evenly, owing to the thickness and elastic quality of the QUILT. Shingles, however, may be applied directly over it. Sheathing-quilt possesses marked fire-resisting properties. The sheathing-paper and the labor of putting it on should be included in the carpenter's specifications.

Insulating Boards. Of recent years, insulating boards have been used extensively to insulate walls, floors and ceilings. This material has been found especially useful for attics and roofs in preventing heat losses. These boards are made of cork and wood fibers; they possess considerable strength, and as well as serving as insulators, they are used extensively as bases for plastering.

Loose Fill. Another material now used is a loose fill consisting of granules of fibrous or flaky materials, possessing no strength, but useful as an insulating fill between studs, joists and roof-rafters.

Rosin-Sized Building-Papers. These are the common grades of building paper; they are not water-proof, and should not be used on roofs or on walls in damp climates. In dry places they protect from dust, draughts, and to some extent from heat and cold. They are generally either a dull red or gray in color, have a hard, smooth surface, and are clean to handle. They are always put up in rolls 36 in wide and usually contain 500 sq ft. The weight varies from 18 to 40 lb to the roll of 500 sq ft.

Insulating and Deadening-Quilts. Among the insulating and deadening-quilts much in use are those mentioned below. There are also other good materials in this line which are manufactured and used for insulating and deadening purposes

Sheathing-Quilt.* This consists of a felted matting of eel-grass held in place between two layers of strong paper by quilting. "The long, flat fibers of eel-grass cross each other at every angle and form within each layer of quilt innumerable minute dead-air spaces, that make a soft, elastic cushion. This gives the most perfect conditions for non-conduction" Eel-grass is chosen for the filling because of its long, flat fibers, which especially adapt it for felting; because of its great durability, and its resistance to fire; and because, owing to the large percentage of iodine which it contains, it is repellent to rats and vermin. This quilt is made in single, double and triple-ply thickness, and is put up in bales of 125 and 250 sq ft. It is also now made with a covering of asbestos, which tenders it thoroughly fire-proof, and also with a water-proof covering to resist moisture. The material is also efficient for heat-insulation.

* Made by Samuel Cabot (Inc.), Boston, Mass.

When used for this purpose there is no objection to nails passing through it. Quilts are made with bevel edges to permit rapid lap installation.

Keystone Hair Insulator. Another material used for similar purposes is the Keystone Hair Insulator. This consists of thoroughly cleansed cattles' hair, between two layers of strong, non-porous building paper, securely stitched together. The hair is chemically treated, so that it is coated with lime, which makes the finished material vermin-proof and odorless.

Mineral-Wool Deadeners, which are fire-proof sound-deadening quilts of rock-fiber wool stitched between two sheets of building paper or of asbestos paper according to the grade desired, are made by the Union Fibre Company of Winona, Minn., and other firms. This company makes, also, what is called Lith and Feltlino, which are sound-deadening materials in board form. They manufacture, also, Linofelt, a building-quilt of flax-fibers (unbleached linen threads), stitched between water-proof paper or asbestos paper according to need. It is ¼ in thick. Linofelt for sheathing in place of ordinary building paper adds from 1 to 1½% to the cost of a house.

Felt-Papers. There are a great many felt-papers for lining floors and a few are made fire-proof by means of chemicals. As a rule these felts are cheaper than Cabot's QUILT, although the saving in an ordinary residence would be but little, and even among the felts themselves there is quite a difference in cost. In choosing a felt-paper for lining, the architect should select one that is soft and elastic enough to form a cushion, and the thicker the felt, provided it has the above qualities, the greater will be its non-conduction. Some felts are made water-proof by an asphalt center, which is an advantage in case of fire or leaks, but some authorities think that it is doubtful if such felts obstruct the passage of sound as well as felts without the asphalt center. The experience of some acoustical experts seems to show that one of the best methods of deadening is by a combination of heavy hair-felt or felt-paper with sheets of galvanized iron. Two layers of felt, each from ½ to 1 in thick, are placed on either side of a single layer of galvanized iron, the latter resting freely between the felt layers. This form of construction is to be preferred where the deadening-material is not attached to the enclosing woodwork. An additional layer of iron and of felt increases the effectiveness of the combination.

Saturated Felts. Common roofing-felts are made by saturating common dry felt with coal-tar pitch. Roofing-felts are commonly made in weights of 12, 15, and 20 lb to the 100 sq ft. Nothing lighter than 12 lb should be used for roofing. They are usually sold by weight. Asphalt-felts are commonly made in the same weights.

Dry Saturated Tarred Felts are specially run through a tier of calenders to give a hard, uniform surface and contain a minimum amount of coal-tar. They are especially adapted for slaters' use, as they will carry a chalk line and are easy to handle. The rolls are 36 in wide, contain 500 sq ft and weigh about 30 lb.

Asbestos Building Felts are usually made about 6, 10, 14 and 16 lb to the 100 sq ft, although different manufacturers make different weights. They come in rolls 36 in wide and are sold by weight.

Sound-Deadening Felts. These deadening-felts are made by various manufacturers. In one of these felts * the material itself is rather hard and thin, but it is pressed in such a way as to form small indentations or air-cells. This makes it elastic and breaks up the sound-waves.

* Neponset Florian Sound-Deadening Felt.

Asbestos Sheathing. Sheathing-papers or building felts, made of asbestos, are used to a considerable extent for floor-linings and for covering the outside walls of wooden buildings, principally on account of their fire-proof and vermin-proof qualities. These papers are well known in the trade and can be procured without difficulty. They are supplied by the manufacturers in 50 or 100-lb rolls, 36 in wide, on a basis of the following scale of weights:

4 lb to the 100 sq ft	18 lb to the 100 sq ft
6 lb to the 100 sq ft	20 lb to the 100 sq ft
8 lb to the 100 sq ft	24 lb to the 100 sq ft
10 lb to the 100 sq ft	32 lb to the 100 sq ft
12 lb to the 100 sq ft	1/16 in thick
14 lb to the 100 sq ft	3/32 in thick
16 lb to the 100 sq ft	1/8 in thick

The sheathing in the 1/16, 3/32 and 1/8-in thicknesses is used only for special purposes where an unusually thick lining is desired for possible fire-protection around exposed flues, for chimney-breasts, etc. When the weight of paper exceeds 32 lb to the square foot it is known as ROLL-BOARD and is no longer classed by weight per 100 sq ft, but by thickness. For floor-linings, 16-lb paper is generally employed, this weight being sufficiently thick and strong to resist ordinary damage in application and in handling. Asbestos felts and building papers appear to have approximately the same effect in retarding the passage of sound-waves as other felt-papers of a relatively similar thickness and quality, while their fire-proof and vermin-proof qualities are a distinct advantage. The cost of asbestos paper and building-felt, while somewhat greater than that of the ordinary papers used for similar purposes, is not excessive. The market price varies and depends upon the fluctuations of the market.

Water-Proof Papers. Neponset Black Sheathing is water-proof and air-proof, odorless and clean to handle, and is an excellent paper under siding, shingles, slate, or tin. The rolls are 36 in wide, containing 250 and 500 sq ft.

Neponset Red Rope Sheathing and Roofing. This is made of rope-stock, has great strength and flexibility, and is absolutely water-proof and air-tight. It is one of the best sheathing-papers and makes a good cheap roofing for sheds, poultry-houses, etc. The rolls are 36 in wide, containing 100, 250 and 500 sq ft.

Parchment Water-Proof Sheathing. There are various parchment-sheathings on the market which are semitransparent, have smooth surfaces, and are odorless, water-proof, air-proof and vermin-proof. They are adapted for general sheathing purposes In general 1-ply weighs 25 lb to 900 sq ft; 2-ply, 25 lb to 500 sq ft; 3-ply, 25 lb to 275 sq ft. They are 36 in wide.

PAINT AND VARNISH *

Pigments and Vehicles. The solid ingredient of a paint is called the PIGMENT, and is a fine powder, nearly all of which will pass through a brass-wire sieve of 100 meshes to the linear inch; in fact, most pigments are much finer than that, and those formed as precipitates by chemical processes are so fine that there is no way to measure them. The liquid part is called the VEHICLE. This is usually linseed-oil, sometimes with the addition of a little

* The editor is indebted to Professor Alvah H. Sabin and the National Lead Co. of New York for valuable assistance in the revision of the data relating to this subject.

turpentine or other volatile solvent. In the enamel paints it is varnish and in kalsomine and other cold-water paints it is a solution of glue, casein, albumen, or some similar cementing material. The cementing material is sometimes called the BINDER.

Colloidal Paints.* "**Collopakes,**" a new kind of paint, is now made with pure pigments and without fillers. The pigments are not ground but are reduced to a great fineness, and colloidally dissolved in the vehicle. Because of an electrical charge incurred during the process of manufacture they settle very slowly over long periods of time. Advantages claimed by the manufacturer are that this paint does not become hard in the bottom of the container, penetrates the surface of the material to which it is applied, thus insuring against cracking and peeling. This paint is used for exterior and interior surfaces and may be used also as a waterproofing for stucco and brickwork.

Ingredients of Oil-Paint. White lead and white zinc are the common white pigments. Titanium oxide is now a common ingredient of white paint, most frequently as the product called "Titanox," which also contains barium sulphate; this is used in mixtures with white lead and white zinc, is chemically permanent and is white and opaque. There are white pigments of variable composition called leaded zinc and zinc lead, furnace-products, composed of zinc oxide and lead sulphate. There is also a basic lead sulphate, commercially called sublimed white lead, which is a similar furnace-product consisting chiefly of sulphate of lead. These composite white pigments are largely used in mixed paints. LITHOPONE is a mixture of sulphide of zinc and sulphate of barium. It is very white, fine and opaque and largely used as the basis of flat wall-finishes for interior work, but is not durable for exterior work. It is discolored (gray) by strong light, but this is not a very serious practical objection. White lead is used everywhere, but tends to yellow somewhat in the dark. White zinc is chiefly used on interior work, being the whitest paint known. Both are often mixed and both are used in mixed paints. Yellow paint is commonly chromate of lead, or chrome yellow; green is chrome green, which is a mixture of chrome yellow and Prussian blue; blue is ultramarine, or sometimes Prussian blue. The brilliant reds are coal-tar colors as a rule; the dull reds and browns are oxides of iron. Ochres are dull yellow. Carbon forms the base of all black paints, either as lampblack, dropblack (boneblack), or graphite. Linseed-oil is either raw or boiled. Raw oil is the oil in its natural state as it is extracted from the seed; it should be settled and filtered perfectly clear; it is yellow or greenish yellow in color. Boiled oil is raw oil which has been heated to 400° or 500° F. with compounds (usually oxides) of lead and manganese; it is darker in color than raw oil, and dries quicker. Raw oil exposed in a thin film to the air is converted in about five days into a tough leathery substance; boiled oil undergoes this change in from 10 to 24 hours.

Driers. These are compounds of lead and manganese, dissolved in oil, and this solution thinned with turpentine or benzine. They act as carriers of oxygen between the air and the oil, and their addition to a paint makes it dry more rapidly. Some driers are also called JAPANS. Not more than 10% by volume of any of these liquid driers should be added to oil. Excess of drier causes the paint to lack durability. Cheap driers often contain rosin. It is well to specify that driers and japans should be free from ROSIN (not resin, as varnish-resins are present in some of the best driers).

* **Manufactured by Samuel Cabot, Inc., of Boston, Mass.**

Priming Coat. This is the first coat applied to the clean surface. A priming coat for wood is chiefly oil, and is usually equivalent to a gallon of ordinary paint thinned with a gallon of raw linseed-oil. Paint, however, is not thinned to make a priming coat for structural metal. In all wood-work, nail-holes and other defects are filled with putty after the priming coat has been applied; but if the wood is resinous, knots and resinous places must be covered with shellac varnish before the priming coat is put on. Pitchy woods, such as southern yellow pine and cypress, do not readily absorb oil, and turpentine should be substituted for part of the oil. Red lead is successfully used as a primer (2 parts to 1 of white lead) on such woods; this is the standard practice in England, and is better than the use of all white lead.

Outside Painting. The priming coat having largely been absorbed by the wood, a second and third coat of paint are to be applied. The most common paint used on houses is white lead. This is commonly sold as paste white lead, containing 8% of oil; 100 lb of this is equal to 2.8 gal in volume, and is commonly mixed with 3½ gal of raw linseed-oil, 1 qt of turpentine and 1 pt of drier to make 6⅔ gal of paint for the second coat; or with 4 gal of oil, 1 pt of turpentine and 1 pt of drier for the finishing coat. White lead is also put up as a soft paste containing 13% oil; 100 lb bulks 3 25 gal and calls for ¾ gal less oil than the heavy paste formulas. About half of the white-lead paste now made is soft paste because of the ease with which it can be converted into paint. If white zinc is used, 9½ lb of dry zinc oxide and 5.7 lb of oil make 1 gal of paint; to this, turpentine and drier should also be added. White lead, after about a year, begins to CHALK, that is, its surface becomes dry and chalky; this does not indicate failure, however, and it makes a good surface for repainting. Finely reticulated checking, not extending through the film, occurs later, and when sufficiently marked indicates need of repainting. In any paint, when cracks begin to extend through to the wood, repainting is called for; these cracks occur sooner on pitchy woods. White zinc, if used alone on outside (not inside) work, is very hard and tends to peel off. MIXED PAINTS (prepared proprietary paints) generally contain zinc mixed with either white lead or some of the pigments based on basic lead sulphate, and some auxiliary pigments, such as barytes, China clay, etc , ground in oil and turpentine and containing the necessary drier. The best of these are excellent, but some are very poor; the safest way to use them is to specify them by name, and use them according to the maker's directions. RIPOLIN and VITRALITE are types of a paint made with a vehicle consisting of pure linseed-oil heated a long time until it is a viscid liquid when cold; it is then thinned considerably with turpentine, and ground with a suitable pigment, such as white zinc, white lead or mixtures with titanox. Some drier is added. This kind of paint is not very opaque, gives a fairly bright enamel finish and is durable indoors. COLORED PAINTS are commonly made by adding colored pigments to lead or zinc; but some dark paints contain only iron oxides, ochers, etc., as pigments; these weigh from 12 to 14 lb per gal. Painting should always be done in dry weather and no painting should be done until the inside plastering is dry. Paint should not be applied to lumber that is not dry. A week or more should be allowed between successive coats. In painting the outside of a house, the trim should be painted first; then the body-color can be laid neatly against it. The final brushing should be in the direction of the grain of the wood. It is good practice to have the successive coats (except for white paint) vary a little in color, to facilitate inspection. White, light blue and light green are less durable colors than yellow, gray, or dark colors in general, owing to the fact that the chemical rays of light penetrate the former more easily. A

gallon of paint will cover from 400 to 600 sq ft of surface, depending upon the character of the surface. ROOF-PAINTS should contain a larger proportion of oil, and a smaller amount of drier or none at all. Three coats are desirable. Tin roofs and galvanized-iron work should be thoroughly scrubbed and then dried before painting. The shingles on the walls and roofs of a house are sometimes stained with creosote stain, which consists of a pigment suspended in creosote or some similar liquid. The creosote has some preservative effect.

Inside Painting. Door-frames and window-frames should receive a priming coat of paint in the shop; if they are to be finished in varnish this paint will be applied to the back only. As has already been said, before any painting is done any resinous knots should be varnished with shellac. All interior surfaces which are to be painted should be puttied after the priming coat and the putty should be applied with a wooden spatula, not a steel one, to avoid marring the surface. The paint for the second coat should contain as much turpentine as oil, that is, its vehicle should be half oil and half turpentine. The effect of this is to make the paint dry with a dull instead of a glossy surface, FLAT SURFACE being the painter's term. To this the next coat will adhere well. If the next is the final coat, it may be an ordinary oil-paint. When thoroughly dry the gloss may be removed by lightly rubbing it with pumice and water. ENAMEL PAINT consists of pigment with varnish as a vehicle. It is harder and makes a finer finish than oil-paint. It is also more expensive. It is usual to apply it over oil-paint, in which case the last coat of oil-paint should be lightly sandpapered when quite hard and dry. A coat of enamel paint is then put on, and when it is dry it should be sandpapered or rubbed with curled hair. The final coat of enamel is then laid on and it may be rubbed in a like manner if a flat surface is desired, or it may be left with the gloss. It is also common practice for painters to make a final enamel finish by adding varnish to white lead or white zinc, very little oil being used in this case. The best varnish for this purpose is a spar-varnish from a thoroughly reliable maker. The quicker-drying varnishes will crack and ALLIGATOR.

Varnish. There are two principal kinds of varnish, (1) spirit varnishes, of which shellac varnish is the most important, and which consists essentially of a resin dissolved in a volatile solvent, and (2) oleoresinous varnishes, in which the resinous ingredient is combined with linseed-oil, and this compound is dissolved in turpentine or benzine. The oleoresinous varnishes are commercially the more important, and are largely used in interior finishing. A gallon of varnish covers 500 sq ft, one coat. Surfaces to be varnished are treated in the following manner. If the wood is open-grained, as oak, chestnut, or ash, it first receives a coat of paste-filler. Liquid fillers are not desirable, as they form a poor base for subsequent work. A paste-filler is really a sort of paint, the pigment being silex, or ground quartz, and the vehicle is a quick-drying varnish made thin with turpentine or benzine. This is rubbed strongly in on the grain of the wood with a short stiff brush, and as soon as it has set, usually within half an hour, it is rubbed off with a harsh cloth or a handful of excelsior, the rubbing being hard across the grain of the wood. If it is desired to stain the wood, the oil-stain may be mixed with the filler; but if a close-grained wood is used, which needs no filler, the oil-stain may be thinned to the desired color with turpentine or benzine and applied as a wash. In cleaning the filler out of moldings, corners, etc., a suitably shaped stick, but not a steel implement, may be used. If any puttying is necessary it is done next. After two days the first coat of varnish is applied; after five days it should be rubbed with curled hair or fine sandpaper to remove the gloss, so that the next coat will adhere well; then one, two or three more coats of varnish are added, five

days or more apart, each coat being rubbed. The last coat may be rubbed or left with the natural gloss. Outside doors, window-sills, jambs, inside blinds, and all surfaces exposed to the direct rays of the sun, should be varnished with spar-varnish and left glossy. If shellac varnish is used as the interior finish it is applied in the same way, but at least six coats should be applied. Floors which are to be varnished should be treated as has been described; but if they are to be waxed they should receive one or two coats of shellac varnish, then five or six coats of wax, at intervals of a week, each coat being well polished with a weighted floor-brush made for the purpose. Floor-wax is not beeswax, but is a compound wax made for the purpose. Shellac is a good floor-varnish; it discolors the wood less than any other varnish, and dries rapidly.

Painting Plastered Walls. Plastered walls which must be painted are usually washed with a solution of soap and then with a solution of alum. When this is dry it is sponged off, then allowed to dry, then oiled, then painted. If the paint is applied to the fresh plaster the lime in the plaster will attack the paint.

Repainting. The exterior woodwork of a house needs repainting once in five to ten years, according to climate and other conditions, although if not done with proper material or sufficient care it will not last as long as this; the interior should, with good care, stand from fifteen to twenty years, and then may not require complete renewal. Exterior paint sometimes loses its luster, while the body of the paint is still good, and in cases of this kind it is sufficient to wash the surface and then give it a coat of oil. This replaces the oil which has superficially perished, imparts a gloss and brings out the color. If the paint is worn off so as to show the wood in places, or is peeling, it must be very carefully examined. In extreme cases it is necessary to BURN OFF the old paint; this is done with a painter's torch, a lamp which burns alcohol, naphtha, or kerosene, and which furnishes a flaring blast of flame, which is directed against the painted surface just long enough to soften the paint, which is at once removed with a scraper while still hot. The paint is not actually burned, but only softened by the flame; it may, however, be removed as well as softened by this method. Houses covered with pitchy wood, like southern pine, sometimes require this treatment, and the next painting is found to be more lasting. In many cases it is sufficient to thoroughly scrub the surface with a stiff steel-wire brush. Interior surfaces may be cleaned (if the removal of the old paint and varnish is necessary) with varnish-remover; this is a mixture of solvent liquids, which penetrate the old paint or varnish and soften it, when it may be removed with scrapers or brushes. There is less danger of fire with this method than with the burning-off method, but it is slower and costs more. It must not be forgotten that varnish-remover is volatile and highly inflammable and must not be used in a room where there is a fire. It is especially suitable for cleaning out moldings and all irregular surfaces from which the varnish may then be removed with stiff brushes, or with hardwood scrapers. It is especially desirable to have floors occasionally cleaned in this way; but if a house has been varnished originally with a first-class varnish it may be necessary only to wash it thoroughly and then apply another coat of varnish. Smoke and dirt may often be thoroughly removed from ceilings with the crumbs of fresh bread, where washing would not be desirable. A 10% solution of carbonate of soda (sal soda) in hot water may be used to remove old floor-wax.

The Painting of Structural Steel. Steel being usually more perishable than wood, as well as more expensive, and used for service where its strength is essential to the stability of the structure, its protection from corrosion by

painting is of much importance. It must first of all be recognized that the precaution always taken in painting wood, to secure a clean surface for the paint, must not be omitted with steel. Mud and dirt must first be removed from the steel; then it must be examined for rust, and any rust-spots must be thoroughly cleaned. Loose scale may be removed with wire brushes, but thick and closely adherent rust must be removed with steel scrapers, or with hammer and chisel if necessary. No doubt the best way to clean steel is to use the sand-blast, but it is not available for much architectural work. In any case much care must be taken to obtain a clean surface. On wood the priming coat sinks into the wood and forms a perfect bond between it and the succeeding coats; but on metal no such thing is possible and it is a case of simple adhesion, which demands a clean surface for efficient results. The paint for structural metal should be tough and elastic, and to as great a degree as possible it should be water-proof. Less than two coats should never be applied, and three are better. Paint is always thin on edges and angles, and also on bolt and rivet-heads; it is therefore good practice, after the first full coat, to apply a partial or striping coat, covering the angles and edges and the surface for at least 1 in back from the edges, and covering all bolt-heads and rivet-heads. After this striping coat has become dry, the second full coat is applied, and it may then be assumed that the whole surface has received two full coats. At least a week should elapse between coats. In designing the steelwork, all cavities which may be filled with rain during erection should be properly drained; and during erection all small cavities should be filled with cement, and all contact-surfaces thickly painted.

Kinds of Paint for Structural Steel. RED LEAD is more generally used than anything else as a paint for structural steel. It is a "true red lead" (Pb_3O_4), usually made from litharge (PbO), and frequently containing from 10 to 20% of the latter. If it contains much litharge, it rapidly thickens when mixed with oil and finally hardens; this makes it a paint difficult to apply. If, however, the material from which it is made is reduced to a sufficiently fine powder before it is oxidized, an almost completely oxidized red lead is produced, which is as easily worked as white lead, and better in every respect. The requirements of the government of the United States have for years called for red lead of not less than 94% of "true red lead" (Pb_3O_4), and the Navy Department, as well as several large railway companies, is now using large amounts of red lead which has not less than 98% of "true red lead." It may now be obtained in paste-form, similar to white lead and containing about 6½% of raw linseed-oil. Thirty-three pounds of red lead (dry pigment) to 1 gal of oil is the maximum; this is especially suitable for hydraulic work, and is the standard bridge paint on some of the largest railroads in this country, while in England 35 lb is standard on some large railway work; 28 lb to 1 gal of oil (containing 20 lb of pigment in a gallon of paint) is more common, while 25 lb to a gallon of oil is a common requirement for railroad-specifications. Finely ground GRAPHITE in linseed-oil is a favorite paint for metal; it flows well, is easily applied, less expensive than red lead, and if well made gives excellent results. Graphite is sometimes mixed with lampblack, probably with advantage. Boneblack is also an important ingredient of CARBON PAINTS. Formerly oxide of iron in linseed-oil was used more than all other paints for this purpose; but while many engineers still like it, its use has very greatly diminished. ASPHALTUM has been used and is still used, as a varnish either alone or in combination, and some of these asphaltic preparations are fairly satisfactory. The fact is, that a really competent paint-manufacturer can make a reasonably good paint out of any of these, and if the paint is carefully applied

the results will be satisfactory. There are great differences in painters. In regard to the surface of structural steel covered by a gallon of paint, there is a great difference of opinion among experts. Some say from 300 to 400 sq ft, others 1 000 or 1 200 sq ft. The truth is that any paint may be brushed out into an exceedingly thin film by a skilled workman, while ordinary usage results in a film at least twice as thick. The general opinion is that it is not wise to estimate more than 400 sq ft to the gallon for one coat. Varnish-paints cover less than oil-paints, but if well made they are very durable.

Painting on Cement and Concrete. Cement and concrete-work are difficult to paint, because they are strongly alkaline and even caustic when new. Work in these materials should be allowed to stand a year or two if possible before it is painted; then it may be painted with any ordinary paint. A practice which has been highly recommended is to wash the surface, repeatedly if possible, with a strong solution of zinc sulphate, the sulphuric acid uniting with the free lime and the zinc being left in the pores as an oxide or hydrate. Some preparations for this purpose are on the market; and while some are probably good, others are to be distrusted. The best way is to allow the surface to age, if this is at all possible.

WINDOW-GLASS AND GLAZING *†

Glazing. The glazing of windows originally belonged to the painter's trade, and when glass is broken, it is still customary to go to a painter to have it replaced; but custom has so changed in some parts of the country, that when new windows are to be glazed, the work is sometimes done at the mill or factory where the sashes are made, sometimes by the local glass-jobber in the town where the building is being erected, and again, in other localities, the glazing of new buildings is still done by the painter. COMMON WINDOW-GLASS is usually set with putty and secured with triangular pieces of zinc called GLAZIERS' POINTS or SPRIGS, driven into the wood over the glass and covered with putty. In the best work, a thin layer of putty is first put in the rebate of the sash and the glass is then placed on it and pushed down to a solid bearing. The glass is then sprigged and face-puttied after which the sash is turned over and the surplus putty removed with the putty-knife. This is called BEDDING IN PUTTY. Another method is as follows: The glass is placed in the sash, sprigged and face-puttied. The sash is then turned around and the glazier fills in any spaces that may occur between the glass and wood by forcing putty into such voids. This is called BACK-PUTTYING.

Leaded Glass. It was formerly a common practice for architects to name in the specifications a certain sum of money to be allowed by the carpenter for the leaded glass and to be expended under the direction of the architect. Where clear glass was used, the pattern was sometimes shown on the drawings and the glass was specified in the same manner as any other work. When colored glass was to be used, it was customary to make a definite allowance and then to entrust the work to a good art-glass manufacturer. But leaded glass should be designed, furnished and put in place by those who are entirely familiar with its manufacture and its limitations; the purchase of the same should be left entirely in the hands of the owner; and no specification as to its

* Condensed from an article by the late Professor Thomas Nolan.
Much valuable information on this subject has been obtained from the Hires-Turner Glass Co., Philadelphia, Pa.

† See also Glass and Wire-Glass, and Prism and Structural Glass, Article 5, Chapter XXII.

price or make should be used by the architect. The colored glass windows should show as much individual artistic taste as any other picture or decoration used in the building. The cheap and inartistic leaded glass is fast becoming a thing of the past and owners are confining themselves to purely works of art placed in some appropriate location in the building.

Sheet Glass. General Description. Common window-glass is technically known as SHEET GLASS. It is mostly made either by the Libby-Owens or the Fourcault process. The former draws the glass flat and upward from the end of the tank to a height of about 36 in, and then horizontally over a roller into a lehr through which the glass passes for annealing. It is cut off into sheets as it reaches the end of the lehr, a distance of about 200 ft. The Fourcault process also draws the glass in a flat sheet from a tank but it is drawn vertically to a height of about 16 ft, being quickly annealed during the process. The sheets are cut into standard lengths at the top of the draw. Practically no glass is now made by the old-style cylinder process.

Grades and Qualities of Sheet Glass. Sheet glass is graded as DOUBLE-THICK, or SINGLE-THICK, and each thickness is further divided into three qualities, FIRST, SECOND or THIRD, according to its relative freedom from defects. The price varies according to the strength and quality. It should be remembered that sheet glass is always more or less wavy, irrespective of the process by which it is made. Many suppose that by designating sheet glass, CRYSTAL SHEET GLASS, SELECTED-SHEET GLASS, or SHEET GLASS FREE FROM WAVES AND IMPERFECTIONS, a sheet glass free from waves and blemishes can be obtained. The terms and names do not change the nature of this glass, which still remains sheet glass, characterized by the defects inherent in the method by which it is manufactured. To obtain a thin glass, free from waviness, plate glass, 1/8 in thick, formerly known as CRYSTAL PLATE, or plate glass 3/16 in thick, must be specified. Since the improvement in the manufacture of window-glass in this country, scarcely any sheet glass is now imported for glazing purposes. A small amount of Belgian sheet glass is brought to this country and used along the Atlantic seaboard for picture-framing. The low prices of the American sheet glass, and its excellent quality, have practically forced imported sheet glass out of the market. All common sheet glass, without regard to quality, is graded according to thickness, as SINGLE-STRENGTH or DOUBLE-STRENGTH. The thickness of the double-strength glass is approximately 1/8 in while that of the single-strength averages about 1/10 in. It is customary to use the double-strength for sheet glass over 24 in in width. The best quality of sheet glass is specified as AA, the second as A and the third as B. Very little AA or first quality glass is obtainable. The standard for A quality is very high. All AA and A quality glass carries the manufacturer's label on each individual light and should be so specified.

Sizes of Sheet Glass. The regular stock sizes vary by inches from 6 to 16 in in width. Above that they vary by even inches up to 60 in in width and 70 in in length for double strength and up to 30 by 54 in for single strength. However, many sizes in these categories are non-standard sizes and have a higher base list price than standard sizes.

Crystal-Sheet Glass, 26-Ounce. This glass is made by the same process as common window-glass, and most double-strength glass now is 26-ounce. Owing to the method of its manufacture, it is necessarily characterized by a wavy appearance. If good glass is required for first-class residences, hotels, office-buildings, etc., polished plate glass should be used in either 1/8, 3/16 or 1/4 in thickness.

Defects of Sheet Glass. All sheet glass, when looked upon from the outside, has a wavy, watery appearance, like the surface of a lake slightly agitated by the wind; and when the sunshine falls upon it the irregularity of the surface is greatly emphasized. This characteristic of sheet glass is due to its method of manufacture and cannot be wholly avoided. Besides this universal defect, the cheaper grades are often STRINGY, BLISTERY, SULPHURED, SMOKED, or STAINED; so that, in looking through the glass, objects seen at a distance are deformed and distorted.

Plate Glass. General Description. Plate glass is commonly known as POLISHED PLATE GLASS because its surface is finely polished and thus made clear and transparent. It is more largely used every year for windows of fine residences, hotels and office-buildings, where transparency is desired from the inside and an elegant appearance required on the outside. The process of manufacture of plate glass is entirely different from that of sheet glass. In making plate glass the metal, which is prepared with great care, is melted in large pots and then either cast on a perfectly flat cast-iron table and flattened out by a heavy roller, or the Bichereoux process is used wherein the metal is poured onto an angular plane and flows of its own accord between two rollers, which determine its thickness, onto a large steel table which is moving slightly slower than the molten glass is flowing. The rough sheet of glass has thus been formed of the desired thickness and is, as in the former case, taken through a series of ovens and then a long lehr for annealing. The sheet then forms what is known as ROUGH PLATE, which is used for vault-lights, sky-lights, floor-lights and the like. For polished plate the rough plate is carefully examined for flaws, which are cut out, leaving the largest-sized sheet practicable. The old method of grinding and polishing by circular or revolving tables is fast becoming obsolete and is being supplanted by the straight-line continuous grinding and polishing method. In this method, the glass is fastened to large rectangular tables, by means of plaster of Paris, and then passes in a straight line under a number of grinding wheels, shod with cast iron. Sand and water are fed onto the surface. These grinding wheels. operated by electric motors, revolve, going over all parts of the plate, and grind it down to a true plane. Sand in successive degrees of fineness is fed on until the plate is made absolutely smooth and all grit removed. After this, the tables go on under a series of polishing machines shod with very fine felt, also operated by electric motors. Liquid rouge is added for the polishing. When one side is completed, the other side is similarly treated, the plate losing about 30% in weight by the operation.

Qualities of Polished Plate Glass. For glazing purposes there is but one quality of plate glass on the market. The best of this is selected for manufacturing mirrors. At one time, plate glass was extensively imported, but the gradually improving methods of the American manufacturers, as well as the great cheapening of the process, have practically eliminated imported plate glass from the market. The American plate glass is equal in every respect to that which was imported. The usual thickness of polished plate glass is from $\frac{1}{4}$ to $\frac{5}{16}$ in, but it can be made thinner than this; and when required for residence-windows or car-windows, may be obtained in $\frac{3}{16}$ or $\frac{1}{8}$-in thicknesses. The production of $\frac{1}{8}$-in plate glass has increased enormously in the past few years for two reasons: First, the demand for a thin ground and polished glass for use in residences and other buildings where a thick glass is not required; second, the demand for laminated or non-shatterable glass for automobiles. Its cost to-day is approximately the same as the heavier $\frac{1}{4}$-in plate.

Sizes of Polished Plate Glass. Plate glass is cut into stock sizes varying by even numbers from 6 by 6 in up to 144 by 200 in, or 138 by 208 in. Some larger sizes can be had by special manufacture.

Wire-Glass.* The introduction of this material has made it possible to secure fire-protection in many cases, without the necessity of disfigurement due to fire-shutters. Wire-glass is either RIBBED, ROUGH, FIGURED, or POLISHED PLATE, with wire embedded in its center during the process of manufacture.

The temperature at which the wire is embedded in the glass insures adhesion between the metallic netting and the glass, and the two materials become one and inseparable, so that if the glass is broken by shock, by intense heat up to about 1 700° F., or from other cause, it remains intact. It is this property of remaining intact that gives it its fire-retarding qualities. Although fire and water may cause cracks to spread throughout the glass, the wire holds the pieces so firmly that flames cannot pass through it. Many severe tests during actual fires have positively demonstrated the truth of the above claim. For warehouses and factories obscured surfaces are generally preferable; but for offices, or wherever clear transparent glass is desired, the POLISHED PLATE is nearly if not quite as acceptable as the same glass without the wire, the effect being the same as that obtained by looking through a window with a screen on the outside. Where FIRE-RESISTANCE is the desired feature, the following requirements should be satisfied. The thickness of the plate at the thinnest part should be not less than ¼ in, and the plane of the wire mesh should be midway between the two surfaces of the glass. No wire should be smaller than No. 24 Brown & Sharpe gauge. The unsupported surface of the glass should not exceed 720 sq in in any case and should be contained in a metal frame not larger than 5 by 9 ft between supports. As now manufactured by the continuous process, it is rolled in lengths up to about 12 ft and in thicknesses up to ½ in.

Figured Rolled Glass. This is a translucent or OBSCURED glass with a pattern stamped on one surface. As the molten metal is rolled out on the table, the design, cut into the table, imprints itself into the soft glass. This kind of glass has almost entirely supplanted the ordinary ground glass because of its greater cleanliness. There are several popular designs on the market, made by various manufacturers. Some of the designs in common use are known as MOSS, MAZE, SILVERITE and FLORENTINE. This glass is usually made ⅛ in thick and in large sheets from 24 to 42 in wide and from 8 to 10 ft long. MAZE, FLORENTINE and SILVERITE designs can be had either with or without the wire mesh in them. One important property of figured rolled glass is that of diffusing the light which passes through it.

Pressed Prism-Plate Glass. This is manufactured in different patterns and for different purposes and includes (1) Imperial Prism-Plate Ornamental Glass in five different patterns, (2) Imperial Prism-Plate Glass and (3) Imperial Skylight Prism Glass. The general description is as follows:

(1) Imperial Prism-Plate Ornamental Glass is plate glass ground and polished on one side. It is manufactured in plates, 54 by 72 or 72 by 54 in, can be cut into smaller sizes, and is made in five different stock patterns. It is used in modern mercantile, office and public buildings for partitions, transoms, doorlights, vestibule doors, ornamental ceiling-lights, bank-windows and other street-windows, and in all places where semiobscurity and ornamental

* See Glass and Wire-Glass, Article 5, Chapter XXII.

effect are desired. On account of its prismatic qualities it gives a strong diffusion of light for office-use where privacy is desired.

(2) Imperial Prism-Plate Glass. This is manufactured in large sheets, 54 by 72 or 72 by 54 in, and can be cut into smaller sizes. It is made in several different angles in order to obtain the proper diffusion of light for varying conditions. It is a plate glass, ground and polished on one side. There are no wires or bars to collect dirt and retard the light and it is very easily cleaned. It is used in the upper sashes of windows and in transoms, store-fronts, etc.

(3) Imperial Skylight Prism Glass. This is made in unit plates, 18 by 60 in, with a ½-in back, and conforms to the requirements of the Board of Fire Insurance Underwriters. It is used for skylights, roofs over areaways and in light-wells, etc. The possibility of leakage is lessened on account of the large-sized plates in which it may be obtained. These plates, however, can be cut into smaller sizes if required. It is particularly adapted for lighting the rear parts of stores and for railway-stations, sheds, etc.

Prism Glass, for glazing windows, skylights and sidewalk-lights, is now manufactured in a large number of forms in both prisms and sheets, and by several companies. This glass is made with sharp prisms which are glazed horizontally in the windows and by refracting the light throw it back horizontally into the rooms, adding very materially to the interior lighting. It is manufactured by several companies and can be procured from glass-jobbers in practically all the cities of the United States. Glass prisms for lighting are made of pieces of glass of standard dimensions, about 4 in square, with a smooth outer surface and an inner surface divided into a series of prisms. They are, in many cases, formed into plates by the process of electroglazing, the edges of the prism-lenses being welded together, so to speak, by a narrow line of copper which gives the desired stiffness and strength for use in large frames, and also an attractive appearance considered by some to be superior to ordinary leaded work. These prism-plates can be made in any desired size, but for very large surfaces two or more plates, divided by means of metal sash-bars, are generally used.

The commercial value of these prisms depends on that property of glass which causes what is known as REFRACTION. Prism-plates receive the light from the sky, not necessarily from the sun, and refract or turn it back into the room which is to be lighted. With an ordinary window the light from the sky, passing through the glass, strikes the floor at a point not very far distant from the window. As the color of the floor is usually dark, reflecting perhaps only one-tenth part of the light falling on it, the rear parts of the room receive only a small portion of the light which enters the window. For this reason it has been necessary to make very high stories for deep rooms, in order to light, even moderately, those parts which are at a distance from the window. When prisms are substituted for the common window-glass or plate glass, the rays of light as they enter the glass are refracted, and by employing prisms of the proper angle, the rays may be given almost any direction. Moreover, by utilizing different prisms in the same plate, some of the rays may be directed to the rear of the room while others are thrown so as to strike near the front. The prism-plates do not increase the quantity of light entering the window, but simply redistribute it, directing it into those portions of the room in which it is most needed. By thus changing the direction of light-rays a room with a low ceiling can be better lighted than when sheet or plate glass is used. To insure success in the lighting of interiors by means of prisms requires, however, a superior quality of glass, and careful scientific calculations and experiments, besides practical and attractive means of glazing and

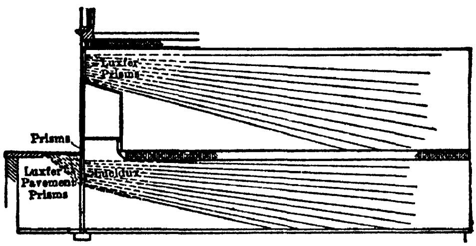
methods of installations. These requirements have been met by the several companies making these prisms and their products may be considered among the relatively new building materials. They have been very successfully applied to the lighting of dark rooms by daylight. The application of prisms to any particular building depends upon the surrounding conditions and requirements, each case requiring some special treatment; but in a general way the various appliances used in the installations may be divided into four classes as follows:

(1) Vertical Plates, which are set directly in the sashes in place of the ordinary window-glass. They are commonly used for the transom-lights of store-windows and the upper sashes of double-hung windows. They may also fill the entire window.

(2) Foriluxes, which are vertical prism-plates set in independent frames and placed in window-openings substantially flush with the face of the wall.

(3) Canopies, which are external prism-plates in independent frames, placed over window-openings and set at an angle with the vertical, a position similar to that of an ordinary awning.

(4) Pavement-Prisms, which are set in iron frames in the pavements or sidewalks, in place of the ordinary bull's-eye lights. In connection with the pavement-prisms, when a well-lighted basement is desired, vertical plates of prisms, hung below and opposite the pavement-lights, are often used. These hanging, vertical plates receive the light from the pavement-prisms, and again changing its direction, project it horizontally into the basement. This feature is illustrated in the figure here given, reproduced through the courtesy of the Luxfer Prism Company.



Refraction and Transmission of Light by Prisms

The canopies may be made either stationary or adjustable and may be employed in a variety of ways, combining the useful with the ornamental. The hanging, vertical plates lend themselves to a highly decorative treatment. In both the fixed and hanging vertical plates the prisms may be arranged to produce ornamental effects, and designs may be inwrought on the face of the prism-plates to correspond with the designs worked into the surfaces of the building and with the style of the entire façade. The prism-plates weigh no more, and often less, than plate glass of the same size, while they are much stronger in resisting wind-pressure, the action of hail and the impact of flying fragments. Although transmitting a very large amount of light, these prism-plates are not transparent in the ordinary sense, and may thus be used as screens to hide unattractive views or to prevent persons looking either in or out of a window. At the same time a maximum quantity of light is admitted. The prism-plates, owing to the stiff, durable manner in which they are united by the electro-glazing process, serve also as a fire-retardant or as a partial substitute for the ordinary iron fire-shutters. The copper glazing forms, as it were, a continuous rivet, which holds the individual prism-lights together, even after they have become badly cracked by the action of fire and water. The details of the various makes of prisms are too complicated to be set forth

in a few pages, but they are well described in the various handbooks and catalogues published by the different manufacturers. From a commercial point of view the special advantages of these systems of interior lighting are manifold. They transform rooms, particularly basements, otherwise too dark for occupancy, into income-producing spaces; in many buildings they do away with the use of light-shafts, thus saving a large amount of valuable floor-space; and in all large or deep rooms they effect a great saving in artificial lighting. Once installed, there is no cost for maintenance. The extent to which these prisms have been used by architects, in both new and old buildings, shows that they have had a decided influence upon commercial architecture.

Glass for Skylights. The glass ordinarily used for skylights is either ROUGH or RIBBED, PLAIN or WIRED. Since ¼-in wire glass is very close in price to the plain glass, the use of it has materially increased. A corrugated glass similar to corrugated iron or asbestos, ¼ in thick, is now being used very extensively for skylights, because it can be laid directly on the purlins without the necessity of a built-up frame, and, by this method of installation expansion and contraction can be taken care of so as practically to reduce breakage in skylight glass. The sizes of flat glass used depend largely upon the pitch of the skylight, small sizes being more desirable when the pitch is slight. The weight of rough or ribbed glass, with or without wire mesh, is approximately as follows:

Weight of Rough or Ribbed Glass

Thickness in inches	⅛	¼	⅜
Weight in pounds	2	3½	5

Glass for Mirrors. Mirrors are made by silvering one side of a sheet of polished plate glass. This is the only kind of glass suitable for making mirrors, because, unless the surface of glass is polished, the reflection is distorted. Originally, mirrors were made by the old-style process of pressing the glass by means of heavy weights onto mercury, backed by tinfoil, the affinity of mercury for tin forming an amalgam which protected the back of the mirrors and gave the reflection. This was a very slow and expensive process. Practically all of the mirrors made to-day are manufactured by what is known as the PATENT-BACK process, in which nitrate of silver is precipitated in a film over the surface of the glass, thus giving it the property of reflecting. This film is afterward covered and protected by shellac, varnish and paint. This modern method of manufacture has made it possible to supply mirrors in considerably less time, and at a very much lower cost, than when manufactured by the old-fashioned MERCURY-BACK process.

Ultra-violet Glass. Within the last few years glasses have been produced which will transmit ultra-violet rays of short wave length which are shut out by ordinary glass. It is these vital ultra-violet rays which tan the skin, improve blood and bone and develop increased resistance to disease. They are of inestimable value to children in affording protection against rickets. Certain makes of this type of glass may be had in sheet, polished plate, obscure and skylight glass. Owing to the variety of brands on the market, some of which are comparatively new, it would be well to see that manufacturers' claims are supported by controlled biological tests and actual experience in use as a means of establishing the efficiency of these glasses.

Anti-glare Glass. The increased use of glass in factory buildings, while it has resulted in greater daylight, has also brought about the problem of glare. In order to offset this condition, a glass has been devised known as ACTINIC GLASS, which intercepts a very large percentage of the ultra-violet (chemically active) and infra-red (heat) rays of the sun. This glass, when glazed, usually affords greater visibility because of its anti-glare properties. At present, it is made in only the ribbed and rough surfaces, 1/8 and 1/4 in thick, either with or without wire.

MEMORANDA ON ROOFING

Shingles. The best shingles are those made from cypress, cedar, redwood, white and yellow pine and spruce, in the order mentioned. Redwood, while perhaps not quite as durable as cypress, is less inflammable; sawed pine shingles are inferior to cedar, and spruce shingles are not suitable for good work.

Number and Weight of Cedar and Pine Shingles per Square of One Hundred Square Feet

Length, in	Assumed width, in	Weather or gauge, in	Number of shingles per square*	Weight per square		Number of nails per square	Weight of nails per square, lb
				Cedar, lb	Pine, lb		
14	4	4	900	210	233	1 800	4.50
15	4	4 1/2	800	200	222	1 600	4 00
16	4	5	720	192	213	1 440	3 60
18	4	5 1/2	655	197	218	1 310	3 28
20	4	6	600	200	222	1 200	3 00
22	4	6 1/2	554	203	226	1 108	2 77
24	4	7	515	206	229	1 030	2 58

* To allow for waste, add from 6 to 10%, the greater allowance being for the shorter shingles.

Sizes of Shingles. Cedar and redwood shingles as commonly sawed are 20 in in length, and cypress shingles usually from 20 to 24 in long, the longer ones allowing a greater exposure to the weather. Redwood shingles and the cedar shingles from the States of Washington and Oregon, which States furnish most of the shingles used west of the Mississippi, are 8/16 and 9/16 in thick at the butt; cypress shingles are usually sawed thicker. Those used in Boston are 9/16 in thick. Ordinary roofing-shingles are of random widths, varying from 2 1/2 to 14 and sometimes 16 in. They are put up in bundles, usually four bundles to the thousand. A THOUSAND common shingles means the equivalent of 1 000 shingles 4 in wide.

Dimension-Shingles are sawed to uniform width, either 4, 5, or 6 in. Dimension-shingles with the butt sawed to various patterns are also carried in stock.

On hip-roofs, or for four valleys, add 5% for cutting. On irregular roofs with dormer-windows, add 10%. It is claimed that redwood shingles will go farther than cedar shingles. With a rise to the roof of from 8 to 10 in to the foot, cedar shingles, or any shingles 16 or 18 in in length, should be laid from 4 to 4 1/4 in to the weather; with a rise from 10 to 12 in, from 4 1/4 to 4 3/8 in to the weather; and on steeper roofs they may be laid from 4 1/2 to 5 in. Red-

wood shingles may be laid ½ in more to the weather. Some authorities allow slightly greater exposures for these lengths. Where the longer shingles are used the exposure to the weather may be increased up to 7 in for the 24-in lengths. On walls cedar shingles are commonly laid 5 in to the weather, and redwood shingles 6 in.

Labor. An average shingler should lay 1 500 shingles in 9 hours on plain work; on irregular roofs with dormers, 1 000 per 9 hours.

Nails. It requires from 3½ to 4½ lb of threepenny or from 3½ to 6½ lb of fourpenny nails to 1 000 shingles, depending upon width and length of shingles.

Slate Roofs

Characteristics of Good Slate. A good slate should be both hard and tough. If the slate is too soft, however, the nail-holes will become enlarged and the slate will become loose. If it is too brittle the slate will fly to pieces in the process of squaring and holing and will be easily broken on the roof. "A good slate should give out a sharp metallic ring when struck with the knuckles; should not splinter under the slater's axe; should be easily HOLED without danger of fracture, and should not be tender or friable at the edges." The surface when freshly split should have a bright metallic luster and be free from all loose flakes or dull surfaces. Very few of the Vermont slates, however, have the metallic luster or ribbons. Some slates contain ribbons or seams which traverse the slate in approximately parallel directions. Slates containing soft ribbons are inferior and should not be used in good work where exposed to weather.

Color. The color of slates varies from dark blue, bluish black, and purple to gray and green. There are also a few quarries of red slate. The color of the slate does not appear to indicate the quality. All slate quarried in Maine is black as is also that quarried in Virginia, while that quarried in Pennsylvania and Maryland is also black but borders on dark blue and is advertised by some firms as dark blue or blue-gray. Slate quarried in New York State is red, of various tints, while that quarried in Vermont is of various colors, such as green, purple, variegated, etc. The red and dark colors were formerly considered the most effective but at the present time the greens are going on some of the largest and finest of the new residences. Some slates are marked with bands or patches of a different color, and the dark-purple slates often have large spots of light green on them.

Color Nomenclature. For the purpose of utilizing the basic natural colors of roofing slate available in large quantities for general usage, it is recommended* that the following color nomenclature be used by architects, contractors, engineers, and others in their specifications.

Black	Gray	Purple	Green	Red
Blue-black	Blue-gray	Mottled purple and green	Purple variegated	

These color designations shall be preceded by the word "unfading" or "weathering," according to the ultimate color effect that may be desired.

For roofs of special treatment certain quarries supply colors and combinations of colors other than those mentioned in the above list, and these should be regarded as "specials."

* Simplified Practice Recommendations, R14–28. Bureau of Standards.

Grading of Slates. The Monson, Me., slates and Brownville, Me., slates are graded as follows: No. 1. Every sheet to be full $\frac{3}{16}$ in thick, both sides smooth and all corners full and square. No pieces to be winding or warped.

No. 2. Thickness may vary from $\frac{1}{8}$ to $\frac{1}{4}$ in, all corners square, one side generally smooth, one side generally rough, no badly warped slates.

The Bangor, Pa., slates are graded.

No. 1 Clear. A pure slate without any faults or blemishes.

No. 1 Ribbon. As well made as No. 1 Clear, except that it contains one or more RIBBONS (a black band or streak across the slate), which, however, are high enough on the slate to be covered when laid, thus presenting a No. 1 roof.

No. 2 Ribbon. This contains several RIBBONS, some of which cannot be covered when laid.

No. 2 Clear. A slate without RIBBONS, made from rough beds.

Hard Beds. A clear Bangor slate, not quite as smooth as No. 1 Clear, but much better than No. 2 Clear.

Ordinary Bent Slate. A smooth slate similar to No. 1 Clear, but bent at a radius of about 12 ft.

Sizes of Roofing Slate *

Dimensions of Slate Shingles for Sloping Roofs; Minimum to a Square

(Each size split * to thickness of $\frac{3}{16}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 inches †)

Face dimensions,‡ in inches	Minimum number to square (3-in lap)	Face dimensions,‡ in inches	Minimum number to square (3-in lap)	Face dimensions,‡ in inches	Minimum number to square (3-in lap)
10 by 6	686	14 by 9	290	18 by 12	160
10 by 7	588	14 by 10	261	20 by 10	169
10 by 8	515	14 by 12	218	20 by 11	154
12 by 6	533	16 by 8	277	20 by 12	141
12 by 7	457	16 by 9	246	20 by 14	121
12 by 8	400	16 by 10	221	22 by 11	138
12 by 9	355	16 by 12	185	22 by 12	126
12 by 10	320	18 by 9	213	22 by 14	109
14 by 7	374	18 by 10	192	24 by 12	115
14 by 8	327	18 by 11	175	24 by 14	98

* The art of splitting slate blocks consists in progressively reducing resultant halves until the desired roofing-slate thickness has been reached or approximated. This hand-wrought characteristic appeals to architects and owners. It is not a simple matter to control precisely the splitting of this natural rock, nor can a uniformity of thickness throughout be assured. The recommended range of thicknesses to be aimed at by operative splitters will meet all normal requirements and will insure the maximum of economy in the utilization of the many sizes of quarried blocks.

† It is customary to regard a thickness falling between two standard thicknesses as a "special," and it is the practice to base the price of the "special" upon the greater of the two standard thicknesses.

‡ For thicknesses $\frac{1}{2}$ in and more, it is not generally considered practicable to use lengths that are less than 16 in, although for roofs of special treatment it may be done in small quantities. In carrying out a desired design on special roofs, it is sometimes necessary to make shingles longer than 24 in, in which case the thicker slates are used.

NOTE.—Where large quantities of thick slates are required of the shortest practical length, one or more of the next longer slates should be permitted for most economical utilization of material from the quarry.

Dimensions of Slate Shingles for Flat Roofs

[Each size split to following thicknesses: For ordinary service, $\frac{3}{16}$ in; for promenade and extraordinary service, $\frac{1}{4}$ in and $\frac{3}{8}$ in]

FACE DIMENSIONS, IN INCHES

6 by 6	10 by 6	12 by 6
6 by 8	10 by 7	12 by 7
6 by 9	10 by 8	12 by 8

Nail-Holes. The standard practice is to punch two nail-holes in all slates.

Dimension Nomenclature.* COMMERCIAL STANDARD THICKNESS (that is, average or basic). The terms "$\frac{3}{16}$-in slate," "full $\frac{3}{16}$-in slate," or "not less than $\frac{3}{16}$-in slate" indicate a desire for a hand-picked selection, regardless of the added labor and cost. "Commercial standard" is the quarry run of production and shows tolerable variations above or below $\frac{3}{16}$-inch. For the thicker slates, however, reasonable plus tolerances only are permissible; thus, a $\frac{1}{4}$-in slate must be a full $\frac{1}{4}$ in or thicker.

Laying. Slates are laid either on a board sheathing (rough, or tongued and grooved) covered with tarred or water-proof paper or felt, or on roofing-laths from 2 to 3 in wide and from 1 to $1\frac{1}{4}$ in thick, nailed to the rafters at distances apart to suit the gauge of the slates. Each slate should lap the slate in the second course below, 3 in. The slates are fastened with two threepenny or fourpenny nails, one near each upper corner. For slates 20 by 10 in or larger, fourpenny nails should be used. Copper, composition, tinned, or galvanized nails should be used. Plain-iron nails are speedily weakened by rust, and they break and allow the slates to be blown off. On iron roofs slates are often placed directly on small iron purlins spaced at suitable distances apart to receive them, and fastened with wire or special forms of fasteners. THE GAUGE of a slate is the portion exposed to the weather, which should be one-half the remainder obtained by subtracting 3 in from the length of the slate. Roofs to be covered with slate should have a rise of not less than 6 in to the foot for 20-in or 24-in slates, or 8 in for smaller sizes An experienced roofer will lay approximately 3 squares of slate in 8 hours.

Elastic Cement. In first-class work, the top course of slate on the ridge, and slate for from 2 to 4 ft from all gutters and 1 ft each way from all valleys and hips, should be bedded in elastic cement.

Flashings. By FLASHINGS are meant pieces of tin, zinc, or copper laid over slate and up against walls, chimneys, copings, etc.

Counterflashings are of lead or zinc, and are laid between the courses in brick, and turned down over the flashings. In flashing against stonework, grooves or reglets often have to be cut to receive the counterflashings.

Closed and Open Valleys. A closed valley is one in which the slates are worked tight to valley line, mitered and flashed in each course and laid in cement. In such valleys no metal can be seen. Close valleys should only be used for pitches above 45°. An open valley is one formed of sheets of copper or zinc 15 or 16 in wide, over which the slates are laid.

Old English Method of Laying Slates. This method of laying slate involves the use of different shades of colored slates in graduated courses and in random widths, beginning at the eaves, for example, with slates 28 in long and $1\frac{1}{4}$ in thick, and using the different thicknesses from $1\frac{1}{4}$ to $\frac{3}{8}$ in, in shorter lengths, in working upward on the roof. The use of this kind of work for

* Simplified Practice Recommendations, R14–28, Bureau of Standards.

roofs has increased in recent years and the method possesses vast possibilities for carrying out architects' ideas for varied artistic effects. The slates are made with rough-cut edges in all thicknesses from $\frac{3}{16}$ to $1\frac{1}{2}$ in, in a combination of the same or various shades carefully selected in such proportion as to produce the best possible harmony, when laid. As all of these colors and shades are unfading, the WEATHERED effect is obtained at once and is permanent. These slates are made not only in usual sizes, but in the OLD ENGLISH STYLE, to be laid in graduated courses of different lengths and in random widths. Architectural or graduated roofing-slates should be specified to secure the light-and-shadow effect, and it is of the utmost importance to specify the thickness desired, as the price is the same for all sizes, while the cost varies according to thickness. When graduated courses are desired, specifications should call for the number of courses to be laid in each length and thickness beginning at the eaves courses, where the thickest slates are used in the largest sizes, sometimes 30 or even 36 in in length, and working upward on the roof with the shorter lengths and thinner slates to the ridges where the smallest sizes and thinnest slates are used. To secure a rough effect at minimum cost, specifications should call for textural or graduated roof, slates to be fully $\frac{1}{4}$ in thick with rough cut edges and graduated courses in sizes ranging from 24 by 16 to 14 by 7 in, with punched nail-holes. To secure the best rough effect, specifications should call for eaves-courses not less than $\frac{3}{4}$ in thick, stating the thickness desired for the eaves, and the number of courses desired in each length and thickness. Among the good specimens of the Old English style of roofing may be mentioned the buildings of Princeton University for the Graduate College, where different shades of unfading-green slates are used in thicknesses running from $1\frac{1}{4}$ in at the eaves to $\frac{3}{8}$ in at the ridge.

Measurement. Slates are sold by the SQUARE, by which is meant a sufficient number of slates of any size to cover 100 sq ft of surface on a roof, with 3 in of lap, over the head of those in the second course below. The square is also the basis on which the cost of laying is measured. "Eaves, hips, valleys and cuttings against walls or dormers are measured extra; 1 ft wide by their whole length, the extra charge being made for waste material and the increased labor required in cutting and fitting. Openings less than 3 sq ft are not deducted, and all cuttings around them are measured extra. Extra charges are also made for borders, figures, and any change of color of the work and for steeples, towers, and perpendicular surfaces."

Weight. Slate roofing $\frac{3}{16}$ in thick will weigh on the roof about $6\frac{1}{2}$ lb per sq ft, and if $\frac{1}{4}$ in thick, $8\frac{3}{4}$ lb, the smaller sizes weighing the most on account of the lap. The actual weight of a square foot of slate $\frac{1}{4}$ in thick is 3.63 lb. A cubic foot of slate weighs approximately 175 lb. The average shipping weight for No. 1, $\frac{3}{16}$-in slates, is approximately 725 lb; for $\frac{1}{4}$-in slates, 1 000 lb; for $\frac{1}{2}$-in slates, 2 000 lb, etc.

Roofing-Tiles

General Notes on Roofing-Tiles. The term ROOFING-TILE is commonly understood to refer to exterior roof-covering made from clay in units of various shapes and laid with overlapping edges. Clay or terra-cotta roof-tiles have long been very largely used in Europe, where their cost is much less than in America. Since the year 1893 the advance here in the character and extent of roofing-tile has been marked and rapid. This material can now be had at much lower prices than formerly prevailed, and the result has been that thou-

sands of squares of terra-cotta tiles have been placed on shops and factories which would under former conditions have been covered with slate or metal. There are so many patterns of roofing-tiles that it is impossible here to enter into a description of them. Of the various patterns, those which interlock are considered, from a practical standpoint, to make the most satisfactory roof.

Laying Roofing-Tiles. Roofing-tiles have been laid directly on a porous book tile or concrete base or on a sheathed surface over such base, or they have been fastened to stripping over the sheathing or wooden or steel purlins by means of copper wires. When thus fastened by wires, the joints were usually pointed on the under side after they were laid, to prevent the entrance of dust or dry snow. Tiles of the older patterns were nailed to the sheathing, but later on this method was superseded by the practice of fastening with copper wires from pierced lugs near the lower ends of the tiles. The best modern method, however, seems to be the one involving a solid continuous base for the roofing-tiles, whether or not purlins are used. "Such purlins should be filled in between either with book tiles or a concrete base and felt should be laid thereon. The book tiles, if used, should be of a porous quality. Instead of regarding the nailing of tiles as a defective method, we have returned to it as the only proper method of fastening tiles and have eliminated the stripping of sheathed roofs and the use of copper wires. Such methods would do in some portions of central Europe where the winds and other climatic conditions are not severe, but through a twenty-five years' experience in the varied climatic conditions of the United States, we have found that the nailing of tiles with copper nails is the only satisfactory method of application. We have also found that a roof should be sheathed and covered with a good asphaltum-felt to prevent wind-suction." * Roofing-tiles weigh from 750 to 1 200 lb per square of 100 sq ft.

Specifications for Tile Roofing

The following specification† contains valuable suggestions for the proper laying of tile roofs:

All pitched roofs shall be covered with (——) tiles with fittings suitable for each pattern unless otherwise selected by the architect. The tiles as specified above are to be hard-burned, of red color, and in accordance with samples deposited in the office of the architect.

(1) Preparation of Roof. Before the roofer is sent for, the owner or general contractor is to construct the roofs in strict accordance with the plans, sheath the roofs TIGHT, have all chimneys and walls above the roof-line completed, have all vent-pipes put through the roofs, furnish all strips of required width used under hip-rolls, furnish all 1½ by ⅞-in cant-strips used under the tiles at the eaves and have all the scaffolding ready for the roofers' use. The metal-contractor is to have all gutters in place on the roof (gutters, whether box, hanging or secret gutters, are to extend over the roof-sheathing and cant-strips, and run under the felt and tiles at least 8 in) and is to have in place, also, all valley-metal, the width of which is to be not less than 24 in, with both edges turned up ¼ in through the entire length of the valley. The valley-metal is to be fastened with clips and never nailed or punctured in any manner. The valley-metal is to be laid over one layer of felt running lengthwise the entire distance of the valley. The metal contractor is to have in readiness all

* Quoted by permission from data on roof-tiling, by the Ludowici-Celadon Company, Chicago, Ill.

† Prepared from data furnished by the Ludowici-Celadon Company, Chicago, Ill.

flashing-metal used alongside and in front of dormers, gables, skylights, towers and perpendicular walls, and around vent-pipes and chimneys, and is to place the same after the arrival of the tile-roofer and under his direction.

(2) Laying the Felt. After the roofs have thus been prepared to receive the felt and tiles, the tile-roofer is to cover the sheathing of the roofs with one thickness of asphalt roofing-felt weighing not less than 30 lb to the square, laying the same with a 2½-in lap and securing it in place by capped nails. The felt is to be laid parallel with the eaves, lapped over all valley-metal about 4 in and laid under all flashing-metal about 6 in.

(3) Laying the Tiles. The roof having thus been prepared, the tile-layer is to fasten the tiles with copper nails. The roofer is to see that the tiles are well locked together and that they lie smoothly, and no attempt is to be made to stretch the courses. The tiles are to be laid so that the vertical lines are parallel with each other and at right-angles to the eaves. The tiles that verge along the hips are to be cut close against the hip-boards, and a water-tight joint made by cementing cut hip-tiles to the hip-boards with elastic cement. Each piece of hip-roll is then to be nailed to the hip-board, and the hip-rolls are to be cemented where they lap each other. The interior spaces of hip-rolls and ridge-rolls are not to be filled with the pointing-material.

Sheet-Metal Tiles. Roofing-tiles stamped from sheet steel, plain or galvanized, and also from sheet copper, in imitation of clay tiles, are made by several manufacturers and have been extensively used for factories and buildings of secondary importance. The first cost of these tiles, except those made of copper, is much less than that of clay tiles and they do not require as heavy roof-framing. Tin or galvanized-iron tiles, however, must be painted every few years, so that for a long period of years they probably cost as much as clay tiles and more than slate.

Tin Roofs

The Sheets. Roofing-plates are made of soft steel of various special analyses, or wrought iron (more commonly of the former), covered with a mixture of lead and tin, and are designated TERNE-PLATES, in distinction from plates coated only with tin and therefore called BRIGHT TIN. Roofing-plates are coated by two methods. (1) The original method of coating the plates consisted in dipping the black plates by hand into the mixture of tin and lead, and allowing the sheets to absorb all the coating that was possible; and at least one brand of roofing-tin is still made by this process. (2) The other process, by which the majority of roofing-plates are now made, is known as the PATENT-ROLLER-PROCESS, by which the plates are put into a bath of tin and lead, and are passed through rolls. The pressure of these rolls leaves on the iron or steel a thickness of coating which, to a great extent, determines the value of the plates. These rolls can be adjusted to leave a relatively large amount of coating on the plate, an ordinary coating, or a very scant coating. The heavier the coating the more valuable the plate. Some makers employ a variation of this patent process, by which the plates are given an extra dip, by hand, in an open pot, to give a HAND-DIPPED FINISH.

Brands. The best roofing-plates always have the BRAND stamped on them, and as the manufacturers have a pecuniary interest in keeping up the reputation of these brands, the only way of being sure of a good tin roof is to specify a brand of tin that has a reputation for quality and durability. Machine-made plates are usually stamped with the weight of coating per box of 112 sheets, 28 by 20-in size.

Sizes of Sheets. The common sizes of tin plates are 10 by 14 in and multiples of that measure. The sizes generally used are 14 by 20 in and 28 by 20 in. The larger size is the more economical to lay, and hence roofers prefer to use it; but for flat roofs the 14 by 20-in size makes the better roof.

Thicknesses of Sheets. Terne-plates are made in two thicknesses, IC, in which the iron body weighs about 50 lb per 100 sq ft, and IX, in which it weighs 62½ lb per 100 sq ft. For roofing, the IC, or lighter weight, is to be preferred, because the seams do not contract and expand as much as they do when the thicker plates are used. For spouts, valleys and gutters, however, IX plates should always be specified, and should preferably be used for flashings, as they are stiffer and less liable to be dented or punched. The thickness of the iron does not add to the durability of the plates, as this depends entirely upon the tin coating.

Weights of Sheets. The standard weight of 14 by 20-in IC terne-plates is 107 lb for 112 sheets, the number usually packed in one box, and of 14 by 20-in IX sheets, 135 lb. The 28 by 20-in sheets should weigh just twice as much. The black sheets, before coating, should weigh, per 112 sheets, from 95 to 100 lb for IC, 14 by 20-in sheets, and from 125 to 130 lb for IX, 14 by 20-in sheets. The difference between the weights of the black sheets and finished sheets is the weight of the tin. A heavily coated tin should weigh from 115 to 120 lb per 112 sheets for IC, 14 by 20-in sheets, and from 145 to 150-lb for IX, 14 by 20-in sheets. The 28 by 20-in sheets should, of course, weigh twice as much.

The Roof. Roofs of less than one-third pitch are made with FLAT SEAMS and should preferably be covered with 14 by 20-in sheets rather than with 28 by 20-in sheets, because the larger number of seams stiffens the surface and helps to prevent buckles and rattling in stormy weather. For a flat-seam roof, the edges of the sheets are turned ½ in, locked together and well soaked with solder. The sheets are fastened to the sheathing-boards by cleats spaced 8 in apart and locked in the seams. Two 1-in barbed and tinned-wire nails are used in each cleat. No nails should be driven through the sheets. The seams must be made with great care and sufficient time taken to properly SWEAT the solder into the seams. Steep tin roofs should be made with STANDING SEAMS and with 28 by 20-in sheets. The sheets are first single-seamed or double-seamed and usually soldered together, preferably end to end, into long strips that reach from eaves to ridge. The sloping seams are composed of two UPSTANDS, interlocked at the upper edge, and held to the sheathing-boards by cleats. The standing seams are usually not soldered but simply locked together with the cleats folded in about 1 ft apart. Nails should be driven into the cleats only. The use of acid in soldering the seams of a tin roof should be carefully avoided as acid coming in contact with the bare iron on the cut edges and corners, where the sheets are folded and seamed together, causes rusting. No other soldering-flux but good rosin should ever be used.

Durability of Tin Roofs. A tin roof of good material, properly put on, and kept properly painted, will last from forty to fifty years, or longer. All traces of rosin left on the roof should be removed as soon as the tin is laid and soldered, and one coat of paint should be applied promptly; a second coat should follow two weeks after the first. One or more layers of felt or water-proof paper should be placed under the tin, to serve as a cushion, and also to deaden the noise produced by rain striking the tin. The durability of tin roofing, and especially of tin gutters, valleys and flashings, is generally increased by painting the tin on the back before laying. An excellent paint for tin roofs is composed of 10 lb of Venetian red, 1 lb of red lead and 1 gal of pure linseed-oil.

Maintenance of Tin Roofs. The tin roof should be given one coat of paint after it is laid and an additional coat of paint at four-year or five-year intervals should be amply sufficient to keep its upper surface in first-class condition as long as the building stands. With each painting the roof is fully restored to its original condition. Graphite and tar paints should be avoided on tin roofs. Metallic brown, Venetian red, red oxide or red lead, only, should be used as pigments, with pure linseed-oil. Tinned gutters should be swept clear of accumulations of leaves, dirt, etc., and if water has a tendency to lie in the gutters they should be painted yearly.

Number of Sheets Required to a Square. For FLAT-SEAM ROOFING a sheet of tin 14 by 20 in, with ½-in edges, measures, when edged or folded, 13 by 19 in, or 247 sq in; but its covering capacity when jointed to other sheets on the roof is only 12½ by 18½ in, or 231.25 sq in. The number of sheets to a square, therefore, equals 14 400 divided by 231.25 or 63, and an area of 1 000 sq ft requires 625 sheets. A box of 112 14 by 20-in sheets will cover, approximately, 180 sq ft. Sheets 28 by 20 in, when edged or folded, have a covering capacity of 490.25 sq in each. To cover 1 000 sq ft (10 squares) requires 294 sheets. For STANDING-SEAM ROOFING the locks require 2¾ in off the width and 1⅛ in off the length of the sheet. A 28 by 20-in sheet, with the seams on the long edges, will cover 463 sq in. To cover 1 000 sq ft requires 312 sheets.

How a Tin Roof Should be Laid *

The Slope of the Roof. If the tin is laid with a flat seam or flat lock, the roof should have an incline of ½ in or more to 1 ft. If laid with a standing seam, there should be an incline of not less than 2 in to 1 ft. Although tin is used on roofs of less pitch than this and on some which are almost flat, a good pitch is desirable to prevent the accumulation of water and dirt in shallow puddles. Gutters, valleys, etc, should have sufficient incline to prevent water from standing in them or backing up far enough to reach standing seams. Tongued and grooved sheathing-boards of well-seasoned dry lumber are recommended. Narrow widths are preferable, and the boards should be free from holes, and of even thickness. A new tin roof should never be laid over old tin, rotten shingles, or tar roofs. Sheathing-paper is not necessary where the boards are laid as specified above. If steam, fumes, or gases are likely to reach the under side of the tin, some good water-proof sheathing-paper should be used. Tarred paper should never be used. No nails should be driven through the sheets.

Flat-Seam Tin Roofing. When the sheets are laid singly, they should be fastened to the sheathing-boards by cleats, using three to each sheet, two on the long side and one on the short side. Two 1-in barbed-wire nails should be used to each cleat. If the tin is put on in rolls the sheets should be made up into long lengths in the shop, and the cross-seams locked together and well soaked with solder. They should be edged ½ in, and fastened to the roof with cleats spaced 8 in apart, and the cleats locked into the seam and fastened to the roof with two 1-in barbed-wire nails to each cleat.

Standing-Seam Tin Roofing. The sheet should be put together in long lengths in the shop, and the cross-seams locked together and well soaked with solder. They should be applied to the roof the narrow way, and fastened with cleats spaced 1 ft apart. One edge of the course is turned up 1¼ in at a right angle, and the cleats are installed. The adjoining edge of the next course is

* These suggestions are in accordance with the standard working specifications adopted by the National Association of Sheet Metal Contractors.

turned up $1\frac{1}{2}$ in, and these edges are locked, turned over and the seam flattened to a rounded edge.

Valleys and Gutters. These should be lined with IX tin, and formed with flat seams, the sheets being applied the narrow way. It is important to see that good solder, bearing the manufacturer's name, is used, that it is guaranteed one-half tin and one-half lead, new metals, and that nothing but rosin is used as a flux. The solder should be well sweated into all seams and joints.

Painting. All painting should be done by the roofer. The tin should be painted one coat on the under side before it is applied to the roof. The upper surface of the tin roof should be carefully cleaned of all rosin-spots, dirt, etc., and immediately painted. The approved paints are metallic brown, Venetian red, red oxide, and red lead, mixed with pure linseed-oil. No patent drier or turpentine should be used. All coats of paint should be applied with a hand-brush, and well rubbed on. A second coat should be applied two weeks after the first and a third coat one year later.

Caution. No unnecessary walking over the tin roof, or use of it for storage of materials, should be allowed at any time. Workmen should wear rubber-soled shoes or overshoes when on the roof. Wherever the slope is steep enough the tin should be laid with standing seams, which allow for expansion and contraction.

Sizes, Weights, Etc., of Roofing-Tin *

Roofing-tin is usually furnished in two sizes, sheets 14 by 20 in and 28 by 20 in, packed 112 sheets to the box. Target-and-Arrow tin is furnished in three thicknesses: IC thickness, approximately No. 30 gauge, U. S. Standard; IX thickness, approximately No. 28 gauge, U. S. Standard; 2X thickness, approximately No. 27 gauge, U. S. Standard, etc. Weight per 100 sq ft laid on the roof, about 65 lb for IC thickness.

Covering Capacity of Roofing-Tin

Flat-Seam Tin Roofing. The following table shows the quantity of 14 by 20-in tin required to cover a given number of square feet with flat-seam tin roofing A sheet 14 by 20 in with $\frac{1}{2}$ in edges measures, when edged or folded, 13 by 19, or 247 sq in, but its covering capacity when joined to other sheets on the roof is only $12\frac{1}{2}$ by $18\frac{1}{2}$ in, or 231.25 sq in. In the following table each fractional part of a sheet is counted a full sheet.

* The following tables of sizes, weights, covering capacities and costs are adapted from useful data compiled for the use of sheet-metal workers by the N. & G. Taylor Company, Philadelphia, Pa.

No. of square feet	100	110	120	130	140	150	160	170	180	190	200
Sheets required....	63	69	75	81	88	94	100	106	112	119	125
No. of square feet	210	220	230	240	250	260	270	280	290	300	310
Sheets required....	131	137	144	150	156	162	169	175	181	187	193
No. of square feet	320	330	340	350	360	370	380	390	400	410	420
Sheets required...	200	206	212	218	224	231	237	243	249	256	262
No. of square feet	430	440	450	460	470	480	490	500	510	520	530
Sheets required....	268	274	281	287	293	299	305	312	318	324	330
No. of square feet	540	550	560	570	580	590	600	610	620	630	640
Sheets required.....	337	343	349	355	362	368	374	380	386	393	396
No. of square feet	650	660	670	680	690	700	710	720	730	740	750
Sheets required....	405	411	418	424	430	436	442	448	455	461	467
No. of square feet	760	770	780	790	800	810	820	830	840	850	860
Sheets required.....	474	480	486	492	499	505	511	517	523	530	536
No of square feet	870	880	890	900	910	920	930	940	950	960	970
Sheets required.....	542	548	554	561	567	573	579	586	592	598	604
No. of square feet	980	990	1000								
Sheets required. .	610	617	625								

A box of 112 sheets 14 by 20 in laid in this way will cover 180 sq ft.

Flat-Seam Tin Roofing. The following table shows the number of 28 by 20-in sheets required to cover a given number of square feet with flat-seam tin roofing. The flat seams edged ½ in take 1½ in off the length and width of the sheet. The covering capacity of each sheet is, therefore, 26½ by 18½ in, or 490.25 sq in. In the following table each fractional part of a sheet is counted a full sheet.

No. of square feet	100	110	120	130	140	150	160	170	180	190	200
Sheets required...	30	33	36	39	42	45	47	50	53	56	59
No. of square feet	210	220	230	240	250	260	270	280	290	300	310
Sheets required...	62	65	68	71	74	77	80	83	86	89	92
No. of square feet.	320	330	340	350	360	370	380	390	400	410	420
Sheets required...	94	97	100	103	106	109	112	115	118	121	124
No. of square feet	430	440	450	460	470	480	490	500	510	520	530
Sheets required...	127	130	133	136	139	141	144	147	150	153	156
No. of square feet.	540	550	560	570	580	590	600	610	620	630	640
Sheets required.....	159	162	165	168	171	174	177	180	183	186	188
No. of square feet..	650	660	670	680	690	700	710	720	730	740	750
Sheets required. .	191	194	197	200	203	206	209	212	215	218	221
No. of square feet.	760	770	780	790	800	810	820	830	840	850	860
Sheets required.....	224	227	230	233	235	238	241	244	247	250	253
No. of square feet..	870	880	890	900	910	920	930	940	950	960	970
Sheets required...	256	259	262	265	268	271	274	277	280	282	285
No. of square feet	980	990	1000								
Sheets required....	288	291	294								

A box of 112 sheets 28 by 20 in laid in this way will cover 381 sq ft.

Standing-Seam Tin Roofing. The following table shows the number of 14 by 20-in sheets required to cover a given number of square feet with standing-seam roofing. The standing seams, edged 1¼ and 1½ in, take 2¾ in

off the width; and the flat cross-seams, edged $\frac{3}{8}$ in, take $1\frac{1}{8}$ in off the length of the sheet. The covering capacity of each sheet is, therefore, $11\frac{1}{4}$ by $18\frac{7}{8}$ in, or 212.34 sq in. In the following table each fractional part of a sheet is counted a full sheet

No of square feet	100	110	120	130	140	150	160	170	180	190	200
Sheets required... .	68	75	82	89	95	102	109	116	123	129	136
No. of square feet	210	220	230	240	250	260	270	280	290	300	310
Sheets required....	143	150	156	163	170	177	184	190	197	204	211
No. of square feet	320	330	340	350	360	370	380	390	400	410	420
Sheets required....	218	224	231	238	245	251	258	265	271	279	285
No. of square feet	430	440	450	460	470	480	490	500	510	520	530
Sheets required.....	292	299	306	312	319	326	333	340	346	353	360
No of square feet .	540	550	560	570	580	590	600	610	620	630	640
Sheets required.....	367	374	379	387	393	401	407	414	421	428	435
No of square feet	650	660	670	680	690	700	710	720	730	740	750
Sheets required. .	441	447	455	462	468	475	482	489	495	501	509
No of square feet	760	770	780	790	800	810	820	830	840	850	860
Sheets required ..	515	523	529	536	543	550	557	563	570	577	584
No of square feet	870	880	890	900	910	920	930	940	950	960	970
Sheets required.	590	597	604	611	618	623	630	637	644	651	658
No of square feet	980	990	1000								
Sheets required	665	672	679								

A box of 112 sheets 14 by 20 in laid in this way will cover 165 sq ft.

Standing-Seam Tin Roofing. The following table shows the number of 28 by 20-in sheets required to cover a given number of square feet with standing-seam roofing. The standing seams take $2\frac{3}{4}$ in off the width, and the flat cross-seams, edged $\frac{3}{8}$ in, take $1\frac{1}{8}$ in off the length of the sheet. The covering capacity of each sheet is, therefore, $26\frac{7}{8}$ by $17\frac{1}{4}$ in, or 463 59 sq in. In the following table each fractional part of a sheet is counted a full sheet.

No of square feet	100	110	120	130	140	150	160	170	180	190	200
Sheets required.....	32	35	38	41	44	47	50	53	56	59	62
No of square feet .	210	220	230	240	250	260	270	280	290	300	310
Sheets required.....	65	68	71	74	77	80	84	87	90	94	97
No. of square feet	320	330	340	350	360	370	380	390	400	410	420
Sheets required ..	100	103	106	109	112	115	118	121	125	128	131
No of square feet	430	440	450	460	470	480	490	500	510	520	530
Sheets required .. .	134	137	141	144	147	150	153	156	159	162	165
No. of square feet .	540	550	560	570	580	590	600	610	620	630	640
Sheets required.....	168	171	174	177	180	184	187	190	193	196	199
No. of square feet .	650	660	670	680	690	700	710	720	730	740	750
Sheets required.....	202	205	208	211	214	218	221	224	227	230	233
No of square feet.	760	770	780	790	800	810	820	830	840	850	860
Sheets required.....	236	239	242	245	249	252	255	258	261	265	268
No. of square feet	870	880	890	900	910	920	930	940	950	960	970
Sheets required.....	271	274	277	280	283	286	289	292	296	299	302
No. of square feet. .	980	990									
Sheets required... .	305	308									

A box of 112 sheets 28 by 20 in laid in this way will cover 360 sq ft.

Laying the Long or Short Way. Sheets 14 by 20 in can be laid either the long or short way. The best roof is made by laying the sheets the 14-in way; similarly, in using the 28 by 20-in sheets, they should always be laid the 20-in way, that is, with the short dimension crosswise

Tin in Rolls, or Gutter-Strips

Number of sheets required per linear foot for 20 and 28-in widths

Feet	Widths		Feet	Widths		Feet	Widths		Hundred feet	Widths	
	20	28		20	28		20	28		20	28
1	1	1	35	16	23	69	31	44	2	89	128
2	1	2	36	16	23	70	32	45	3	134	192
3	2	2	37	17	24	71	32	45	4	178	256
4	2	3	38	17	24	72	32	46	5	223	320
5	3	4	39	18	25	73	33	47	6	267	384
6	3	4	40	18	26	74	33	47	7	312	444
7	4	5	41	19	27	75	34	48	8	356	512
8	4	5	42	19	27	76	34	48	9	401	576
9	4	6	43	20	28	77	35	49	10	445	640
10	5	7	44	20	28	78	35	50	11	495	704
11	5	7	45	20	29	79	36	50	12	540	768
12	6	8	46	21	29	80	36	51	13	585	832
13	6	9	47	21	30	81	36	52	14	630	896
14	7	9	48	22	31	82	37	52	15	675	960
15	7	10	49	22	31	83	37	53	16	720	1 024
16	8	11	50	23	32	84	38	54	17	765	1 088
17	8	11	51	23	33	85	38	54	18	810	1 152
18	8	12	52	24	33	86	39	55	19	855	1 216
19	9	12	53	24	34	87	39	55	20	900	1 280
20	9	13	54	24	34	88	40		21	945	1 344
21	10	14	55	25	35	89	40	57	22	990	1 408
22	10	14	56	25	36	90	40	57	23	1 035	1 472
23	11	15	57	26	36	91	41	58	24	1 080	1 536
24	11	16	58	26	37	92	41	59	25	1 135	1 600
25	12	16	59	27	38	93	42	59	26	1 170	1 664
26	12	17	60	27	38	94	42	60	27	1 215	1,738
27	12	18	61	28	39	95	43	61	28	1 260	1 792
28	13	18	62	28	40	96	43	62	29	1 305	1 856
29	13	19	63	28	40	97	44	62	30	1 350	1 920
30	14	19	64	29	41	98	44	63	31	1 395	1 984
31	14	20	65	29	41	99	44	64	32	1 440	2 048
32	15	21	66	30	42	100	45	64	33	1 485	2 112
33	15	21	67	30	43				34	1 530	2 176
34	16	22	68	31	43				35	1 575	2 240

Tin in Rolls. For the convenience of roofers and for rush-orders, certain brands are put up in rolls 14, 20 and 28 in wide. Each roll contains 108 sq ft (about 63 lin ft, 28 by 20-in sheets laid 20 in wide). The tin is painted on one or both sides, as wanted, with an approved metallic brown paint. The seams are carefully soldered by hand, good 100 to 100 solder and rosin being used as a flux.

Slag or Gravel Roofing

The Ordinary Gravel Roofing over boards is formed by first covering the surface of the roof with dry felt (paper) and over this laying three, four, or five layers of tarred or asphaltic felt lapping each other like shingles, so that only from 6 to 10 in of each layer are exposed. In laying roofs over concrete the dry felts are omitted, a mopping of pitch (or a primary coat and mopping of asphalt) being placed directly over the concrete and the felts embedded therein.

Flashing. Walls, chimneys, curbs of skylights, etc., when of metal, are reinforced behind the metal base by flashing with three plies of felt set in separately, in hot pitch or asphalt, extending out over the roofing felts at least 4 in and up the vertical wall at least 6 in. When felt and plastic elastigum are used for base flashing, the same are set in separately, four trowel coats of plastic and three plies of felt, extending out over the roofing at least 6 in and up the vertical wall at least 10 in. The several layers of felt can be fastened to the walls at the top with metal or creosoted-wood nailing-strips well pointed up with plastic elastigum. Metal counter-flashings to protect the felt are better than wooden strips and should be used when possible. At the eaves and on all exposed edges, metal gravel-stops should be used. After nailing the metal base flashing or metal gravel-stops on the roof, they should be stripped over with two plies of felt, set in hot pitch, before slagging the roof.

A Better Method of Slag or Gravel Roofing is to lay two plies of tarred felt, lapping each other 17 in, and then spreading a coat of pitch over the entire roof. On this three more layers of felt are laid full mopped between and then coated with pitch poured from a dipper, into which the crushed slag or screened gravel is embedded.

Specifications for Pitch-Slag or Gravel Roofing. The following specification-notes * describe the latter method more in detail and also the materials that should be used to secure a first-class job. These roofs are most efficient and durable on comparatively flat inclines. The usual built-up roof consists of successive layers of saturated felt cemented together and surfaced with coal-tar pitch or asphalt, into which is embedded the gravel or slag. Tile is also used as a surfacing material. The saturants used in the felt are generally coal-tar or asphalt-compounds.

(1) Specification for Pitch-Slag or Pitch-Gravel Roofing Over Wooden Sheathing

This specification should not be used when the roof-incline exceeds 2 in to 1 ft.

Lay one thickness of sheathing-paper or unsaturated felt weighing not less than 5 lb per 100 sq ft, lapping the sheets at least 1 in.

Over the entire surface lay two plies † of tarred felt, lapping each sheet 17 in over the preceding one, and nail as often as is necessary to hold them in place until the remaining felt is laid.

* Condensed and adapted from the roofing specifications published by the Barrett Company and known, in their full form, as "The Barrett Specifications"

† In the Western States the number of "plies" is construed to mean the total number of layers, including dry as well as saturated felt, and the terms 3-ply, 5-ply, etc., are hereinafter used on that basis. In the Eastern States, 3-ply, 5-ply, etc., usually refers to the number of layers of saturated felt. The total number of layers should always be specified if there is any doubt as to the exact meaning of the term as used in the specifications.

Coat the entire surface uniformly with pitch.

Over the entire surface lay three plies of tarred felt, lapping each sheet 22 in over the preceding one and mopping with pitch the full 22 in on each sheet, so that in no place felt touches felt. Do such nailing as is necessary so so that all nails are covered by not less than two plies of felt.

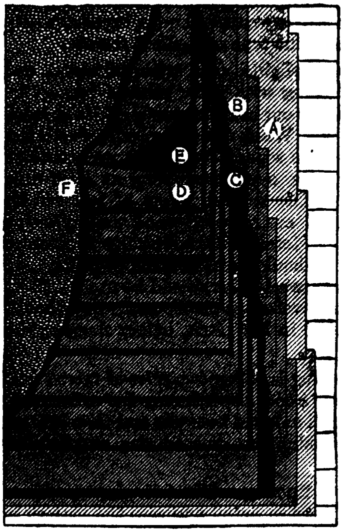


Diagram of Gravel or Slag Roofing on Wooden Sheathing

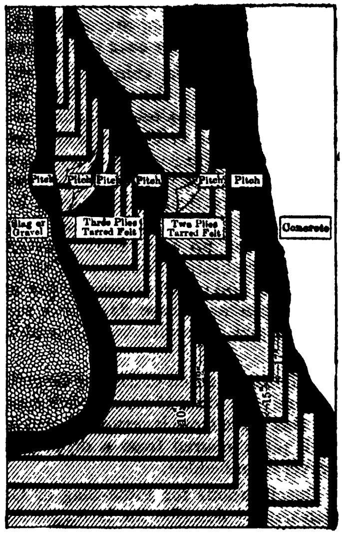


Diagram of Gravel or Slag Roofing on Concrete Base

Spread over the entire surface a uniform coating of pitch, poured from a dipper, into which, while hot, embed not less than 400 lb of gravel or 300 lb of slag to each 100 sq ft. The grains of the gravel or slag are to be from $\frac{1}{4}$ to $\frac{5}{8}$ in in size, and dry and free from dirt.

The roof may be inspected before the gravel or slag is applied, by cutting a slit not less than 3 ft long at right-angles to the direction in which the felt is laid. All felt and pitch is to bear the manufacturer's label.

(2) Specifications for Pitch-Slag or Pitch-Gravel for Roofing over Concrete

This specification should not be used when the roof-incline exceeds 1 in to the foot. When the incline exceeds 1 in to 1 ft the concrete must permit of nailing or nailing-strips must be provided.

Coat the concrete uniformly with hot pitch.

Over the entire surface lay two plies of tarred felt, lapping each sheet 17 in over the preceding one, mopping with pitch the full 17 in on each sheet, so that in no place felt touches felt.

Coat the entire surface uniformly with pitch.

Over the entire surface lay three plies of tarred felt, lapping each sheet 22 in over the preceding one and mopping with pitch the full 22 in on each sheet, so that in no place felt touches felt.

Spread over the entire surface a uniform coating of pitch, poured from a dipper, into which, while hot, embed not less than 400 lb of gravel or 300 lb of slag to each 100 sq ft. The grains of the gravel or slag are to be from ¼ to ⅝ in in size, and dry and free from dirt.

The roof may be inspected before the gravel or slag is applied, by cutting a sample at right-angles to the direction in which the felt is laid, not less than 4 in wide nor less than 3 ft long. After examination, the sample should be replaced and covered over with the same number of plies that constitute the roofing, each set in successively in hot pitch and each 4 in wider and longer than the preceding ply. All felt and pitch are to bear the manufacturer's label, and it is advantageous to have them also carry the Underwriters' label for materials as required for a Class A roof.

Notes on Slag and Gravel Roofing. The difference between slag and gravel roofing is that for the former crushed slag is used instead of gravel. The greater the number of plies of tarred felt, the greater the amount of pitch that it is practical to use, and it is the pitch that gives life to the roof. As there are several different weights and qualities of tarred felt, a specification should state either the minimum weight per 100 sq ft, single thickness (the most practical weight is from 14 to 16 lb), or some known quality, such as Barrett's "Specification Tarred Felt." Felt weighing less than 12 lb per 100 sq ft is not economical even on the cheaper work. To comply with the Barrett specification the materials necessary for each 100 sq ft of completed roof are approximately as follows:

Over boards	Material	Over concrete
108 sq ft	Sheathing-paper	None
80 to 85 lb	Specification tarred felt	80 to 85 lb
120 to 160 lb	Specification-pitch	180 to 225 lb
400 lb	Gravel	400 lb
300 lb	Slag	300 lb

In estimating felt, the average weight is practically 15 lb per 100 sq ft, single thickness, and about 10% additional is required for laps. In estimating pitch the weather-conditions and expertness of the workmen will affect the amount necessary for the moppings and for a proper embedding of the gravel or slag. As there are several qualities of pitch, a specification should either specify it by name, such as "Specification-Pitch" or "Straight-Run Coal-Tar Pitch," or in specifying asphalt-pitch, the brand or origin should be plainly defined. The use of an under-layer of sheathing-paper next to board-sheathing is mainly for the purpose of preventing any pitch which might penetrate the felt from cementing the roofing to the sheathing. It is also of value in preventing the drying out of the roof from below through open joints. Where a less expensive roof is desired, four plies or three plies of saturated felt may be used. With the four plies there should be used from 90 to 100 lb of pitch per 100 sq ft of completed roof; and with the three plies from 70 to 80 lb of pitch.

Durability of Slag or Gravel Roofs. These roofs, mentioned in the preceding paragraph, will last from five to ten years, or even longer, depending upon the quality of the materials used and the care with which they have been applied. Roofing put on strictly as provided for in the standard specifications will last twenty years or more, and if a tile surface is used, instead of gravel or slag, the roofing will last as long as the structure itself.

Resistance to Fire, Acid-Fumes, Etc. The fire-resisting properties of the slag or gravel roof are due principally to the incombustible material on the surface. It is claimed that the gravel or slag tends to prevent the successive layers of felt and pitch from burning and the whole mass has a blanketing influence on fires originating within the building. Some carefully conducted tests seem to indicate that gravel roofing protects a wooden roof better than tin. The general effect of a fire upon gravel roofing is to soften the pitch or asphalt in the roofing, to burn out the inflammable oil in them and to cause the residue to swell and form a porous, incombustible coke. This type of roofing is not attacked by corrosive gases or acid-fumes, and is used extensively on railroad-roundhouses and other structures where the conditions are particularly severe. Coal-tar or tar-oil should not be added to the pitch to soften it.

Bonded Guarantees. Bonded roofing offered by manufacturers of built-up roofing-material has no specific value unless the furnishings of the bond necessitate: (1) Delivery of high quality materials, (2) in such quantity as required by specifications covering proved long-lived roofs, (3) and applied exactly in accordance with such specifications, (4) by roofers of proved experience and integrity, (5) under the inspection supervision of the manufacturer's representative.

Asphalt-Gravel Roofing

Asphalt-Gravel or Asphalt-Slag Roofing differs from coal-tar roofing principally in the substitution of asphalt or asphaltic cement for the coal-tar pitch, for saturating the felt as well as for mopping and surface-coating. It is claimed that the oils of asphalt do not evaporate as quickly as do those of coal-tar pitch under ordinary temperatures and that, therefore, the flexibility and life of asphaltic felts and coatings are not as quickly destroyed. As a matter of fact, asphalt roofs, particularly on comparatively flat decks, have not proved as long-lived as some coal-tar roofs. In sections of the country where moisture is prevalent and water remains on the roof, erosion of the asphalt top coating has released the slag or gravel, thus permitting the weather-protection or fire-resistance value to be removed. Where the pitch of the roof is comparatively steep, however, the water runs off freely and the asphalt is less liable to slippage.

Specifications for Asphalt Roofing. The following specifications are for Warren's heavy standard Anchor-brand roofing. The manner of laying the felting differs from that ordinarily employed for coal-tar roofing.

(1) Specification for Asphalt-Gravel Roofing Over Wooden Sheathing

Cover the roof with two thicknesses of Warren's Composite roofing-felt, manila-paper side down, lapping each sheet 17 in over the preceding one, and securing with nails through tin discs about 2½ ft apart.

Over the entire surface of the Composite felt thus laid, mop an even coating of Warren's Anchor Brand roofing-cement, into which, while hot, lay two thicknesses of Anchor Brand felt, lapping each sheet 17 in over the sheet preceding, sticking these laps the full width with hot Anchor cement and securing with nails through tin discs not more than 20 in apart.

Over the entire surface of the felt thus prepared, spread an even coating of the cement, covering it immediately with a sufficient body of well-screened, dry gravel or crushed slag.

If the roofing is applied in cold weather the gravel or slag must be heated

Slag only should be used if the incline of the roof exceeds 3 in to the foot. All layers of felt must be turned up at least 4 in over battlement-walls, skylight-curbs, or any projections raised above the roof.

(2) Specification for Asphalt-Gravel Roofing Over Concrete

The concrete foundation is to be smooth and perfectly graded to carry the water to the outlets or gutters.

Over the entire surface of the concrete first mop a smooth, even coating of Eclipse Asphalt cement, into which, while hot, lay two thicknesses of Warren's Anchor Brand roofing-felt, lapping each sheet 17 in over the sheet preceding.

Mop back for the full width between the laps of the felt thus laid, with Warren's Anchor Brand roofing-cement.

Over the entire exposed surface of the felt mop an even coating of said Anchor cement, into which, while hot, lay two thicknesses of Anchor Brand felt, lapping each sheet 17 in over the sheet preceding, and sticking these laps thoroughly the full width with hot cement.

Over the entire surface of the felt thus prepared, spread an even coating of the cement, covering it immediately with a sufficient body of well-screened, dry gravel or crushed slag.

If the roofing is applied in cold weather, the gravel or slag must be heated.

Slag only should be used if incline of roof exceeds 3 in to the foot. On steep surfaces nailing-strips should be provided in the concrete, unless the latter is sufficiently soft to admit of nailing. All layers of felt must be turned up at least 4 in over battlement-walls and skylight-curbs, or any projections raised above the roof.

Roof-Incline. Asphalt-gravel or asphalt-slag roofing should not be applied to roofs which are steep enough to make the material run in hot weather. The manufacturers of various roofings will guarantee the permanency of their roofings for certain maximum slopes.

Prepared Roofing. There is a large number of PREPARED ROOFINGS or READY ROOFINGS, which are made by cementing together two, three, or more layers of saturated felt or felt and burlap and then coating the combination either with a hard solution of the same cementing material, or with hot pitch or asphalt into which is embedded sand or fine gravel. These roofings are commonly put up in rolls 36 in wide and are applied by lapping the strips 2 in with a coat of cementing material between, and nailing every 2 or 3 in with tin-capped roofing-nails. A sufficient quantity of cement, nails and tin caps is packed in the middle of the rolls. The particular advantage of these roofings is that no previous experience is required for laying them and no kettles are required; for this reason they are extensively used in the country, and on railroad-shops, factories, and mill-buildings. In cities there is no particular advantage in using them except for roofs that are too steep for coal-tar pitch. Many of these prepared roofings are as durable under ordinary conditions as the light-weight gravel roofs.

Corrugated Iron and Steel Sheets

Corrugated Sheets of iron and steel are very extensively used for the roofing and siding of mills, sheds, grain-elevators and warehouses. The best grades of corrugated sheets are now made of double-refined box-annealed iron or steel. The corrugations are usually made lengthwise of the sheet, either by passing them through rolls or by pressing the plain sheets in a press made to give the desired corrugations. It is claimed that the latter method gives the more per-

fect and uniform corrugations. The weight and thickness of the metal is represented by the gauge-number of the black sheets from which the corrugated sheets are made. The standard gauge for sheet iron and steel in this country is that established by act of Congress, March 3, 1893.

Gauges. The following table gives the weights and thicknesses of the different gauges, from No. 7 to No. 30, for flat BLACK SHEETS. The gauge extends from No. 7–0, ½ in thick, up to No. 40, 0.005469 in thick, but sheet steel is not commonly made thinner than No. 30, and above 3/16 in, the thickness is generally designated by fractions of an inch. Section 3 of the act of Congress provides that in the practical use and application of this gauge, a variation of 2½% either way may be allowed.

United States Standard Gauge for Sheet Iron and Steel

Number of gauge	Thickness		Weights	
	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Weight per square foot in ounces, avoirdupois	Weight per square foot in pounds, avoirdupois
7	3/16	0.1875	120	7.5
8	11/64	0 171875	110	6 875
9	5/32	0 15625	100	6 25
10	9/64	0 140625	90	5 625
11	1/8	0 125	80	5 0
12	7/64	0 109375	70	4 375
13	3/32	0 09375	60	3.75
14	5/64	0.078125	50	3 125
15	9/128	0 0703125	45	2 8125
16	1/16	0 0625	40	2 5
17	9/160	0 05625	36	2 25
18	1/20	0 05	32	2 0
19	7/160	0 04375	28	1.75
20	3/80	0 0375	24	1.50
21	11/320	0 034375	22	1 375
22	1/32	0 03125	20	1 25
23	9/320	0 028125	18	1 125
24	1/40	0 025	16	1 0
25	7/320	0.021875	14	0 875
26	3/160	0 01875	12	0 75
27	11/640	0.0171875	11	0.6875
28	1/64	0.015625	10	0 625
29	9/640	0 0140625	9	0 5625
30	1/80	0.0125	8	0 5

Galvanizing the Sheets adds approximately 2½ oz per sq ft to the above weights. The regular sizes of the corrugations are 2½, 1¼, 5/8 and 3/16 in, measured from center to center. Besides these sizes, 5-in, 3-in and 2-in corrugations are made by one or two corrugating companies. Corrugated sheets are carried in stock in 4-ft, 5-ft, 6-ft, 7-ft, 8-ft, 9-ft and 10-ft lengths. Sheets can be obtained as long as 12 ft at a cost of 5% extra. The 8-ft length, however, is most commonly used. The width of the sheets, as a rule, is 24 in between centers of the outer corrugations, so that the covering width is 24 in when one corrugation is used for the side lap. This applies to all sizes of corrugations, although some mills make wider sheets. The 2-in, 2½-in and 3-in corrugated sheets are made in all gauges from No. 16 to No. 28, the 1¼-in

corrugated sheets from No. 22 to No. 28, the 5/8-in corrugated sheets from No. 24 to No. 28 and the 3/16-in corrugated sheets in Nos. 26, 27 and 28 only. No. 28 gauge is the one commonly used for all purposes. The sheets are generally painted with a red mineral paint before shipping and galvanized sheets, also, can be obtained if desired. All corrugated sheets are sold by the square (100 sq ft), measuring the actual widths and lengths of the corrugated sheets.

Corrugated-Steel Roofing

Useful Data. For covering roofs, either 3-in, 2½-in, or 2-in corrugations should be used, the 2½-in being the most common size. The thickness or gauge depends upon the distance between the supports on which the sheets are laid.

Nos. 26 to 28 gauges should be laid on close sheathing, or strips not more than from 1 to 2 ft on centers. The maximum distances between supports for other gauges should be as follows.

For No. 24 gauge, from 2 to 2½ ft, center to center.

For Nos. 22 and 20 gauge, from 2 to 3 ft, center to center.

For No. 18 gauge, from 4 to 5 ft, center to center.

For No. 16 gauge, 5 to 6 ft, center to center

The least pitch which should be given to roofs that are to be covered with corrugated sheets is 3 in to the foot, and for trussed roofs it is not desirable to

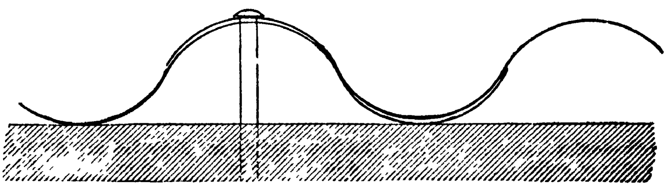


Fig. 1. Approved Method of Laying for Side Lap

have less than one-fourth pitch (6 in to the foot). When laid on a roof, corrugated sheets should have a lap at the lower end of from 3 to 6 in, according to the pitch of the roof. For a 1/3 pitch, a 3-in lap is used; for a ¼ pitch, a 4-in lap; and for a 1/8 pitch, a 5-in lap. For the side lap it is recommended that each alternate sheet be laid upside down and lapped as shown in Fig. 1. By this method, when water is blown through the first lap, it will stop and not pass the half lap, but run down and out at the end of the sheet. A great deal of roofing, however, is laid as in Fig. 2. In applying to sheathing or wooden strips, the sheets are secured by nailing through the tops of the corrugations, the nails being driven through every alternate corrugation at the ends, and about 8 in apart at the sides. When applied to iron or steel purlins, the side laps should extend over at least 1½ corrugations, and the sheets should be riveted together every 8 in on the sides and at every alternate corrugation at the ends. The Cincinnati Corrugating Company makes a patent edge-corrugation which makes a tight joint with a lap of only one corrugation. To fasten the sheets to the purlins, which are usually steel angles, cleats of band-iron, ¾ or 7/8 in wide, may be passed around or under the purlins and riveted at both ends to the sheets, as shown in Fig. 3. By contracting or pressing these cleats toward the web, a tight, secure fastening results, which allows for contraction and expansion of the sheets. Cleats, however, are generally used only with channel or Z-bar purlins. For angle-iron purlins, clinch-nails,

made of soft-iron wire, are commonly used, as shown in Fig. 4; they make very satisfactory fastenings.

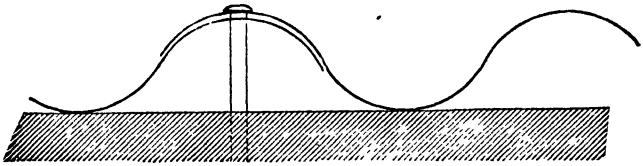


Fig. 2. Common Method of Laying for Side Lap

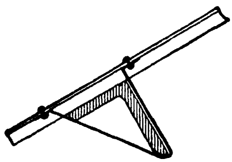


Fig. 3. Sheets Fastened to Angle-purlin by Band-iron Cleats

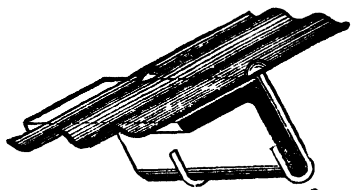


Fig. 4. Sheets Fastened to Angle-purlin by Clinch-nails

The following table shows the sizes of clinch-nails to be used with different sizes of angle-purlins and also the number of nails to the pound in each instance:

Purlin-angles	2×2 in	$2\frac{1}{2}\times3$ in	$3\frac{1}{2}\times3\frac{1}{2}$ in	$4\times4\frac{1}{2}$ in
Lengths of nails	4 in	5 in	6 in	7 in
Number of nails per pound	48	38	33	27

The nails should be placed through the TOP of every second or third corrugation. At the eaves of the building and along the edges of the ventilators special pains should be taken in fastening the roofing, as these are the places where the force of the wind is the greatest and where it tends to strip the roofing from the purlins. For these parts of the roof the best method of fastening is that shown in Fig. 5. These fastenings consist of strips of sheet iron about 2 in wider than the purlins, made of No. 12 iron and riveted to the purlins with $\frac{1}{4}$-in rivets spaced 10 in apart. To these strips the corrugated sheets are riveted, every

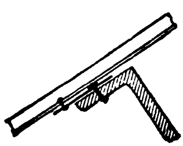


Fig. 5. Approved Fastening for Sheets at Eaves

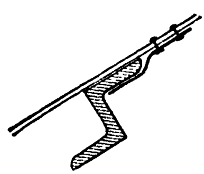


Fig. 6. Alternate Method of Fastening at Eaves

5 in or every two corrugates, with 6-lb rivets. The method of fastening shown in Fig. 6, also, answers very well and is less expensive.

In ordering corrugated sheets an allowance must be made for the laps. The following table gives the number of square feet necessary to cover one square of actual surface, using sheets 8 ft long. If shorter sheets are used, the allowance must be slightly increased.

Number of Square Feet of Corrugated Sheets to Cover 100 Square Feet of Roof

End-laps..	1 in	2 in	3 in	4 in	5 in	6 in
	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft
Side lap, 1 corrugation. .	110	111	112	113	114	115
Side lap, $1\frac{1}{2}$ corrugations. . .	116	117	118	119	120	121
Side lap, 2 corrugations. .	123	124	125	126	127	128

Approximate Weights in Pounds of 100 Square Feet of $2\frac{1}{2}$-in Corrugated Sheets

Gauge..	No. 28	No. 27	No. 26	No. 24	No. 22	No. 20	No. 18	No. 16
Painted .	69	77	84	111	138	165	220	275
Galvanized. .	86	93	99	127	154	182	236	291

Anti-Condensation Lining. Wherever corrugated steel is laid on purlins with no sheathing or paper underneath, if the building is heated, moisture will invariably collect on the under side, and if the air in the building is warm and humid, considerable dripping will result. To prevent this dripping, it is necessary to protect the under side of the corrugated steel with paper or felt. This may be done by first stretching poultry-netting over the purlins, from eaves to ridge, and wiring the strips together at the edges. Over this should be laid one thickness of asbestos paper and one or two layers of saturated felt. The corrugated steel may then be fastened to the purlins in the usual way. The side laps may be secured by stove-bolts, with 1 by $\frac{1}{8}$ by 4-in plate washers on the under side, to support the lining.

Corrugated Steel Siding

For Siding, either the $2\frac{1}{2}$, 2, or $1\frac{1}{4}$-in corrugations are used. The $1\frac{1}{4}$-in size, however, makes the best appearance. For the laps, 1 in at the bottom and one corrugation at the sides are sufficient.

For Sheds, etc., the sheets may be nailed to cross-pieces cut in between the studs horizontally and spaced from 2 to 3 ft apart, the studs being from 3 to 4 ft on centers. For elevators, either cross-corrugated sheets or sheets not more than 32 in long should be used. The nails should be driven in the trough of each alternate corrugation, 2 in above the lower end of the sheet, which will be 1 in ABOVE the top end of the under sheet. This allows the sheet to slide 1 in in 32 in as the building settles, before the nail will strike the upper end of the lower sheet. The side lap should not be nailed.

Ceilings. For the ceilings of stores, stables, etc., $\frac{3}{16}$ or $\frac{5}{8}$-in corrugated sheets are much used; and the construction is an excellent one for this purpose.

Galvanized Iron. This term is commonly applied to all galvanized sheet metal. Formerly most of the galvanized sheets had a steel base, but since

about 1906 a nearly pure iron, called Toncan Metal, has been largely used for sheets of very fine quality. Galvanized sheets come in lengths of 6, 7 and 8 ft in United States Gauge-Nos. 14, 16, 18, 20, 22, 24, 26, 27, 28 and 30, and in widths of 24, 26, 28, 30 and 36 in for all gauges except No. 30, which is made only in widths of 24, 26 and 28 in Sheets of No 28 gauge are also made in widths of 32 and 34 in. The widths commonly carried in stock are 24, 28 and 30 in. Most of the galvanized iron used for cornices and ornamental work is No. 27 gauge. No. 28 is sometimes used for gutters and conductors.

Copper Roofing *

Copper possesses certain characteristics and physical properties which distinguish it from other metals used for roofing purposes. The greatest difference is in expansion and contraction. Copper has a higher coefficient of expansion than have iron and steel, and a lower one than have zinc and lead. This means that for a given temperature-variation there will be more movement in copper than in iron and steel and less than in zinc and lead. The standard length of copper sheets is 96 in. Sheets are stocked in widths in multiples of 2 in. The three usual methods of applying copper roof are: (*a*) RIBBED-SEAM METHOD, (*b*) STANDING-SEAM METHOD, and (*c*) FLAT-SEAM METHOD. The slope of roofs to be covered with flat-seam copper roofing should not be less than ½ in nor more than 3 in to the foot.

Fundamentals in Sheet-Copper Roofing and Flashing Construction.

(1) Use 16-ounce soft (roofing-temper) copper only.

(*a*) Do not use hard (cornice-temper) copper except for cornice work.

(*b*) Do not use lighter than 16-ounce copper.

"Soft" versus "Hard" Copper. Soft copper will give the most satisfactory results. Hard (cornice-temper) copper, though sometimes used for flashings and roofings, is not recommended. The soft sheet is, as can readily be understood, more easily workable, especially where bends, etc , are necessary.

Thickness of Copper Sheets for Roofs. It is not fair to a good material to use too thin a sheet. As copper does not corrode, there is no question of durability in the thinner gauges. Copper sheet weighing 1 lb per sq ft, commonly known as 16-ounce copper, is considered the minimum standard sheet strong enough to withstand extraneous injury. Thinner sheets do not give the best results.

(2) Prepare the laying surfaces carefully and see that it is smooth and even.

(*a*) All copper sheets should be laid on rosin-sized paper or asbestos felt.

(*b*) Sheathing boards should be ship-lap, tongued-and-grooved, or splined.

(*c*) All nail-heads should be set.

Roofing-Boards or Sheathing. It is recommended that ship-lap or tongued-and-grooved roofing-boards be used. All roof-sheathing should be well laid with even joints and secured at all bearings with heavy nails well set. Immediately after laying, the sheathing should be protected by covering it with paper as mentioned below. If possible the sheathing should be exposed to the weather at least 4 weeks before covering it with copper. The wood must be thoroughly dry and seasoned.

Paper over Sheathing. Good practice requires either the ordinary building paper or a rosin-sized or asbestos paper weighing about 6 lb per 100 sq ft.

* Taken from data furnished by the Copper and Brass Research Association, New York City.

On concrete roof-slabs paper is not essential, provided the surface be made smooth and even as outlined below.

Concrete Roof-Slabs. When copper is used over concrete the surface should be made smooth by a wash of neat cement. Elastic cement is sometimes used for this purpose. Cinder concrete should not be used in contact with copper. Where copper is used in this type of construction the concrete should be painted with a heavy coating of asphalt paint before the copper is applied. Wood battens or expansion inserts must be set in the concrete for fastening the cleats.

(3) Avoid sharp bends in copper sheets.

(*a*) Do not crease the sheets or bend them more than 90 degrees.

(*b*) Bend the sheets as little as possible before laying.

(4) Allow for movement at intersections of roof-planes by large loose-locked joints.

(*a*) Never carry a copper sheet more than 3 or 4 in over an angle.

(*b*) Break the sheet and lock it to the adjoining one by means of a loose- or double-locked joint. This allows room for expansion and contraction.

Expansion and Contraction. The temperature at the time the work is done must be taken into consideration by the contractor in allowing for expansion and contraction. A roof laid in July needs little rooom for expansion. It does, however, require ample provision for the contraction which comes with cold weather. The reverse is, of course, true when a roof is laid in cold weather, and under these circumstances the contractor must be careful to provide ample room for movement.

(5) Never nail copper sheets. Use cleats.

(*a*) By "sheet" is meant any piece over 12 in wide.

(*b*) Use two-nail cleats 1½ in wide and place them not more than 12 in apart.

(6) Use copper nails only—never iron or steel—for fastening strips and cleats.

(*a*) Flat-head, wire "slating," or "shingle," nails are preferable.

(*b*) Strip copper should be nailed along one edge only.

(*c*) Nails should be spaced 4 in maximum.

Cleats. Always secure the copper sheet by copper cleats, the cleats only being nailed to the roofing-boards, the battens or wood ribs. Never use nails of iron or steel to fasten copper at any place or under any circumstances. Galvanic action will quickly destroy the ferrous metal.

Copper and Other Metals. If possible, never use copper in contact with another metal, but if the plan of construction requires the use of iron or steel, by all means see that the iron or steel device is heavily tinned or that sheet lead is inserted between the copper and the other metal. The use of brass devices is recommended, especially for gutter and eaves-trough hangers.

(7) Make full-size joints and seams.

(*a*) Standing seams at least 1 in finished.

(*b*) Flat seams (locked) at least ½ in finished.

(*c*) Lapped seams at least 1 in finished.

(*d*) Double or copper-locked seams at least ½ in finished.

(8) Tin carefully and thoroughly.

(*a*) Use heavy tinning-coppers.

(*b*) Use enough tin to cover all the surface.

(9) Use rosin as a flux rather than acid.

(a) If acid is used, see that it is properly and thoroughly killed.

(10) Plenty of solder, well-flowed over, makes strong seams.

(a) Use the best half-and-half solder and lots of it.

(b) Heat the seam thoroughly.

(c) Heavy, hot coppers are best for this.

Cleaning. As soon as a portion of the roof is finished it should be carefully cleaned of all flux, scraps and dirt. Prominent signs should be displayed, where necessary, to prevent walking on the copper, and every reasonable precaution should be exercised to keep the roof tree from all foreign substances, such as mortar, scraps of lumber, paper, etc. As the development of the characteristic green patina of copper is very much retarded by dirt, flux, etc, too much emphasis cannot be placed on thorough cleaning. Accumulation of dirt means uneven, splotchy coloring.

Finish. Copper will develop a beautiful patina in a few months, owing to natural phenomena. When it is desired to obtain this finish immediately, it can be done by the use of the following methods: (1) Clean the copper by washing it thoroughly with a strong solution of soda and hot water to remove the grease acquired in the process of rolling. (2) Apply the following solutions:

(a) One pound of powdered sal ammoniac to 5 gal of water; dissolve thoroughly and let stand 24 hours. Apply to copper with a brush, covering every part. Let stand one day, and then sprinkle surface with clean water.

(b) Use a solution of ½ lb of salt to 2 gal of water. Apply in same manner.

(3) A dark copper finish can be obtained by the following method: Rub off the copper with cotton waste soaked in boiled linseed-oil. Touch up soldered seams with copper bronze.

Painting Copper. It is difficult to obtain a good bond between paint and copper. This is due to the grease and oil of the manufacturing process which is rolled into the fine pores in the surface of the sheets. Paint applied directly to untreated copper will not stand for any length of time, particularly when exposed to the weather. The surface must be thoroughly cleaned and roughened before the paint will adhere. This may be accomplished by washing the copper with a solution of 4 oz of copper sulphate in ½ gal of lukewarm water in a glass or earthen vessel, to which is added ⅛ oz of nitric acid. If the surface is still very smooth, additional roughening must be done by abrasives. Before painting, the surface must be carefully washed with clean water to remove the last trace of the solution and the paint must not be applied until the surface is thoroughly dry. Three coats of paint will give the best results. The first coat should be composed of 15 lb of red lead to 1 gal of raw linseed-oil with not more than ½ pt of oil drier The last two coats should be composed of 15 lb of white lead to 1 gal of raw linseed-oil with not more than 5% of oil drier and the necessary color.

White Lead. White lead in oil is a good substance for filling lock seams in copper work. It is simple to apply, is water-tight, and remains so a long time. White lead has been used on copper roofs, laid many years ago, both in this country and abroad. Notable among roofs of this type is that on the State House in Boston, Mass. This roof was laid in 1887–90 with leaded seams, and is apparently as tight to-day as it was forty years ago. The method of applying consists of smearing the edges of the sheets plentifully with white lead in oil and folding and locking them to form lock seams in the usual way. The viscous lead and oil completely fills the lock, making a water-stop. White lead used in this way has much to recommend it. It is cheaper

than soldering, and it is durable. On flat roofs where water backs up it is better to use solder, but on free-draining surfaces white lead can be used with every assurance of satisfaction. The proper lead to use is that composed of basic lead carbonate and boiled linseed-oil in paste form. It must be smooth and of putty-like consistency. Lumps will make uneven seams and prevent the locks from being completely filled. After the seams have been locked and malleted all excess lead should be carefully removed from the sheets.

Proportioning Gutters and Leaders. The chief consideration in designing gutters and leaders is to conduct the water running off a roof quickly and easily. To do this it is essential that (1) the gutter be large enough to conduct all the water to the outlet; (2) the outlet be large enough to accelerate the velocity of the water in the gutter when it enters the outlet.

A safe system to follow in mill building design is that of the American Bridge Company. Their specifications provide as follows:

Span of roof	Gutters	Leaders
Up to 50 ft	6 in	4 in every 40 ft
50 to 70 ft	7 in	5 in every 40 ft
70 to 100 ft	8 in	5 in every 40 ft

Hanging gutters shall slope 1 in in every 16 ft.

Progressive Steps in Designing Gutters and Leaders.

(1) Locate position of leaders.
(2) Compute area of roof drained by each leader.
(3) Compute size of leader.
(4) Compute size of gutters to supply leaders.

NOTES.

(1) Round leaders should not be less than 3 in in diameter.
(2) Rectangular leaders should not be smaller than $1\frac{3}{4}$ by $2\frac{1}{4}$ in.
(3) Gutters should not be less than 4 in wide.
(4) Gutters should have a fall of not less than 1 in in 16 ft.
(5) Scuppers should be provided for all roofs with a parapet wall built around them. This precaution prevents an overloading of the roof owing to stoppage of the outlet.
(6) All outlets should be provided with screens or strainers.

Built-In or Box Gutter-Linings of Copper. For best results in gutter linings the following practice is recommended.

(1) The design of the gutter should avoid sharp angles. The sides should slope as much as possible to approximate an arc. The inside edge of gutter should be at least 3 in higher than the outside edge. Gutters should be as shallow as possible.

(2) The gutter should be wood-lined to receive the copper.

(3) Sixteen-ounce soft (R.T.) copper sheets which have been crimped are excellent. Their length is optional up to 8 ft maximum; their width should not exceed 36 in.

(4) Longitudinal seams should be double-locked. This provides strength at the seam and with (3) allows plenty of opportunity for expansion.

(5) Seams should be well soldered.

(6) The junction with the roof flashing should be by a large loose-locked joint so placed as to need no solder to make it water-tight.

(7) If long sheets are used a double-cross seam should be used.

Scuppers. One of the most important points of roof-drainage design is often forgotten. This is proper provision for overflow by means of scuppers. A great many buildings have flat roofs enclosed by parapet-walls and inside drainage-systems. When the outlet becomes clogged, water collects on the roof and not only causes an overload, but may also work its way over the flashing and down into the building. On all roofs of this character—that is, where there is not ample provision for escape of the water when the leaders do not work, it is absolutely necessary to provide scuppers through the wall. These should be large enough to preclude any possibility of clogging (at least 4 by 12 in), and should be unobstructed in any way by screens, etc. When copper flashings are used, the scuppers are completely lined with copper and this lining is soldered to the base flashing, the counter-flashing being worked around the hole, or omitted at this point.

Galvanic Action in Roofing-Materials. Dissimilar metals, when in contact in the presence of an electrolyte, set up galvanic action which results in the deterioration of the most electropositive metal Starting with the most electropositive, the commercial metals are listed as follows

1. Aluminum (most electropositive)
2. Zinc
3. Steel
4. Iron
5. Nickel
6. Tin
7. Lead
8. Copper (most electronegative)

This means that when iron (or steel) and copper are in contact with an electrolyte present (which may be water), the iron is corroded. When copper and lead or tin are in contact, a tendency for similar action exists but it is very slight and produces practically no injurious results. This is because the difference in potential is much less and the corrosive action is negligible, especially where water is the electrolyte. Any possibility of galvanic action between copper and iron or steel should be carefully avoided by proper insulation. This insulation is effected in various ways, three of which are: (1) covering the steel member with asbestos, as is frequently done in skylight construction; (2) placing strips of sheet lead between the two metals, as when new copper gutters are placed in old iron hangers; and (3) heavily tinning the iron, as is often done with iron or steel gutter and leader-supports.

MEMORANDA ON TILING

Floor-Tiling and Wall-Tiling

Tile Floors are extensively used in the better class of buildings, and particularly in those portions which are used by the public, on account of their great durability, sanitary qualities and decorative effects. As a matter of fact, a good tile floor is also cheaper in the long run than a wooden floor if it is subject to much wear. The materials used for tiling floors are tiles made from different grades of clay, marble, slate, glass and rubber. Of these probably the most durable and sanitary are the vitreous clay tiles. For walls and wainscotings, glazed tiles, marbles and glass are extensively used.

Floor-Tiles. The following include some of the principal kinds of clay tiles:

(1) **Common Encaustic Tiles.** These belong to the cheapest grades, and are made of naturally colored clays, red, buff, gray, chocolate and black. These tiles are of a porous, absorbent nature and are used for common floors where sanitary requirements are not exacting.

(2) **Semivitreous Tiles.** These belong to a somewhat better grade than the first mentioned and are less porous and absorbent.

(3) **Vitreous Tiles.** These are the hardest tiles known, cannot be scratched by steel or sand, and are non-absorbent and thoroughly aseptic. They are used principally for floors requiring a perfect sanitary condition and are manufactured in white, blue, gray, green and pink colors of great delicacy.

(4) **Ceramic Tiles or Ceramic Roman Mosaic.** This material is made of VITREOUS clay in tesseral pieces representing the tesseræ of the Roman mosaics. It is made into regular tiles ranging from ½ to ¾-in squares and also in hexagonal shapes from ¾ in to 1 in in size. A rounded LOZENGE TILE is also manufactured to be laid in tesseral paving. (See, also, Flooring of Mosaic, Terrazzo, etc)
The material itself is of great hardness and well suited for work of a monumental or public character. The even and regular texture of the tesseræ admits the adoption of DAMASK DESIGNS which have become identified and associated with this material. The minuteness of the tesseræ admits of a great range in designing and the following of the architectural lines. The ceramic Roman mosaic is much preferred to mosaic consisting of natural marbles, because of the great variety in colors and its greater durability. The vitreous-clay tiles are impervious to attacks of any acids contained in the atmosphere, while marbles, especially, are subject to rapid disintegration caused by the sulphuric acid contained in the smoke-laden atmosphere of our cities.

(5) **Florentine Mosaics and Flint Tiles.** These are the largest and heaviest tiles manufactured in this country. They are either plain or inlaid and are in use especially in ecclesiastic work on account of their relation to mediæval application. The material is vitreous, annealed and tougher than it is brittle. It is also in use for exterior polychrome work.

(6) **Aseptic Tiles.** These are large, heavy and thoroughly vitreous tiles used for institute work. They are the only vitreous tiles of large size made in this country. As the tiles are large and generally of hexagonal shape, the joint-spaces are reduced to a minimum, and they are, therefore, especially adapted for hospitals, operating-rooms and wards for contagious diseases.

Enameled Tiles, Wall-Tiles and Mantel-Tiles. The following include some of the enameled tiles:

(1) **White Wall-Tiles.** These are glazed tiles for wainscots. They have a white, soft body and a surface covered with a clear glaze. The brilliancy of this glaze and its reflecting properties make the white wall-tiles especially desirable for dark passages.

(2) **Colored, Glazed or Enameled Tiles.** These tiles are about the same as the former in quality; the GLAZE or ENAMEL, however, is stained with metallic oxides, which produces a brilliant decorative effect.

(3) **Dull-Satin, etc., Finished, Enameled Tiles.** These are glazed tiles with a DULL or BLIND enamel-finish. The dull finish is produced either by sandblasting or by devitrifying enamels. It is principally used for quaint decorative effects in mantel-work.

(4) **Glazed Roman Mosaics.** This is a type of enameled tiling which has great decorative possibilities. It has the same tesseral texture as the ceramic floor-tiles and is readily applied to wainscots and mantel-work.

Setting of Tiles. Clay tiles are set in Portland-cement mortar as a rule, and flooring of this character should always be provided with a substantial

concrete base. Ceramic mosaics are sometimes laid on a flexible base. With this construction wooden floors can be provided with tile covering, and owing to the elasticity and lightness of the material, floors in elevators, boats and other ambulant structures can be safely tiled.

Marble Tiles, from 9 to 12 in square, have been extensively used for flooring, principally on account of their decorative effect. None of the marbles, however, is as hard and consequently as durable as the vitreous and ceramic tiles. When used, they should be 1¼ in thick and not over 12 in square, and should be bedded in cement on a concrete base. Marbles should not be used for flooring in hospitals, as they yield rapidly to the usual antiseptic floor-washes.

Slate, non-absorbent and not affected even by dilute mineral acids. It is frequently used as flooring and as a cover for wiring and pipe-trenches in the floors. As these often follow a wall, it may serve in the capacity of a border and as such be extended around the floor-space. Slate slabs for floors should be about 1¼ in thick.

Marbleithic Tiles or Slabs are made of small pieces or chips of marbles of irregular shapes, set in a backing of sand and Portland cement. After the cement has set, the top surface is rubbed until it becomes flat and smooth. Marbleithic resembles mosaic or TERRAZZO, except that it is laid in the form of tiles instead of being put down on the floor in a plastic condition. Much objection has been made to TERRAZZO because of the cracks which commonly occur in it, due to the slight settlements which are unavoidable in a new building. With tile floors of any material the joints allow for any slight movement of the floor-construction without causing visible cracks. By the process of manufacture, marbleithic is made much harder than it is possible to make mosaic floors that are laid in a plastic condition, so that they have a much better wearing surface. Marbleithic tiles are made of various colored marbles and in different sizes, shapes and patterns, so that a great variety of effects may be produced. Sanitary coved bases, stair-treads, and wainscotings, also, are made of this material.

Cast-Glass Tiles, while quite resistant to a blow when the polish is unbroken, will break very easily when the surface is scratched. All glass tiles should, therefore, be very thick and small or protected by metal framing.

Novus Sanitary Glass is a sanitary structural glass manufactured in all thicknesses from ½ in up to 2 in and in slabs of all widths and lengths up to 100 in in width and 180 in in length. It is made in various colors and designs and in the following finishes: natural-fire finish, hone, semipolished and polished. It can be worked and handled the same as marble, it is readily drilled and shaped to accommodate fixtures, etc., and is very handsome in appearance. It is impervious to discoloration and is non-crazing. These qualities make it especially desirable for floors, wainscoting, tables, shelves, etc., in all places where an absolutely sanitary condition combined with a pleasing appearance is required.

Interlocking Rubber Tiling

General Description. Interlocking rubber tiling, which, because of its being noiseless, non-slippery, and more comfortable to the feet than inelastic substances, has met with great favor for floors in banking-rooms, counting-rooms, vestibules, elevators, stairs, cafés, libraries, churches, etc. For elevators it is one of the most durable and practical floors that can be laid; it is also especially and peculiarly adapted for floors of yachts and steamships. The interlocking feature unites the tiles into a smooth, unbroken sheet of

rubber, unlimited in area. The tiles do not pull apart or come up, and each being distinct, almost any color-scheme can be employed, the tiles being made in a carefully selected variety of colors. The tiles are laid directly over the original floor, like a carpet, except that they are not fastened. Experience has shown that they are very durable. Each tile is 2⅜ in square and ⅜ in thick; 25.5 tiles are required to the square foot. Rubber nosing for stairs is made to interlock with the tiles.

Flooring of Mosaic, Terrazzo, etc.

Flooring of Mosaic Work is largely used. It is composed of small pieces of stone, marble, pottery or glass, usually laid in some ornamental design or pattern. A bed of concrete is first laid and the small pieces of the material used set in a floating of cement and made from ½ to 1 in thick. When cubes of varicolored marble are used, pressed into the cement mortar, it is called ROMAN MOSAIC. A somewhat cheaper flooring is made by spreading marble chips of irregular shape over the surface of the cement, pressing them into it with plasterers' floats and rolling them with iron rollers. This is called TERRAZZO MOSAIC. The following is from the specifications for the new Field Museum, Chicago, Ill., D. H. Burnham, architects: "Filling under terrazzo shall be composed 1 part cement, 2 parts sand and 4 parts brick. Before concrete filling commences to set spread a ¾-in wearing surface composed of marble chips with only enough neat Portland cement to firmly unite the pieces. Trowel and roll, and after the mortar has set, rub the terrazzo to a smooth, even surface and wash clean."

ASPHALTUM

Bitumen, Asphaltum, Asphalt. " Bitumen is the name used to denote a group of mineral substances, composed of different hydrocarbons, found widely diffused throughout the world in a variety of forms which grade from thin volatile liquids to thick semifluids and solids, sometimes in a free or pure state, but more frequently intermixed with or saturating different kinds of inorganic or organic matter. To designate the condition under which bitumen is found, different names are employed; thus the liquid varieties are known as NAPHTHA and PETROLEUM, the semifluid or viscous as MALTHA or MINERAL TAR, and the solid or compact as ASPHALTUM or ASPHALT."

Asphaltum is found in extensive beds or lake-like deposits on both continents; the most notable of these are the PITCH lakes on the island of Trinidad, and at Bermundez, Venezuela. It is also found saturating the limestone and sandstone formations in certain localities. Deposits of very nearly pure asphaltum are found in Utah, Mexico, Cuba, and various parts of the United States. ELATERITE, GILSONITE and WURTZILITE are varieties of very nearly pure asphaltum.

Asphaltic Roofing-Materials are manufactured principally from Trinidad asphalt. These deposits have also been the main source of supply for the asphaltum used in street-paving in the United States.

Rock-Asphalt. The term ROCK-ASPHALT is commonly used to designate the material obtained from the bituminous limestone deposits at Seyssel and Pyrimont, in the valley of the Rhône, France, in the Val-de-Travers, canton of Neuchâtel, Switzerland, and at Ragusa, on the island of Sicily. It is extensively employed for paving purposes throughout Europe, and is considered to make a much more durable pavement than can be made with asphaltum.

Rock-asphalt is prepared for shipment in two forms: (1) COMPRESSED ASPHALT BLOCKS, which are used for paving in much the same way as stone blocks, and (2) MASTIC-ASPHALT, which is put up in cakes of varying shape, generally bearing the manufacturer's trade-mark.

Mastic-Asphalt. In the Eastern States MASTIC-ASPHALT is used for floors of cellars, stores, breweries, malt-houses, hotel-kitchens, stables, laundries, conservatories, public buildings, carriage-factories, sugar-refineries, mills, rinks, etc., and for any place where a hard, smooth, clean, dry, fire-proof and water-proof, odorless and durable covering of a light color is required, either in the basement or upper stories. It can be laid over cement concrete, brick, or wood, in one sheet without seams; also over cement concrete for roofs for fire-proof buildings. For dwelling-house cellars, especially on moist or filled land, this material is especially adapted, being water-tight, non-absorbent, free from mold or dust, impervious to sewer-gases, and for sanitary purposes, invaluable. Mastic-asphalt is also valuable for DAMP-COURSES over foundations, and for covering vaults and arches under ground.

Asphalt Floors and Pavements. For floors of cellars, courtyards, etc., laid on the ground, a base of cement concrete 3 in thick should first be laid; and over this a layer of asphalt from $\frac{3}{4}$ to $1\frac{1}{2}$ in thick, according to the use to which it is to be put. For ordinary cellar-floors, the asphalt need not be more than $\frac{3}{4}$ in thick; for yards on which heavy teams are to drive, it should be $1\frac{1}{2}$ in thick. In specifying asphalt pavement, both the thickness of the concrete and of the asphalt should be given; it should also be remembered that ASPHALT PAVEMENT does not include the CONCRETE FOUNDATION unless so specified. In laying asphalt over planks or boards, a layer of stout, dry, but not tarred, sheathing-paper should first be put down and the asphalt laid on this. Asphalt floors for stables should be at least 1 in thick. Architects and owners desiring to employ ROCK-ASPHALT for any of the above purposes should be careful to secure the genuine VAL-DE-TRAVERS, SEYSSEL, or SICILIAN ROCK-ASPHALT, as there are imitations which are of but little value.

The Bituminous Sandstones of California have been extensively used for paving streets in Western cities. They are prepared for use as paving-materials by crushing to powder. With this powder a considerable proportion of sand or gravel is generally mixed and the mixture heated until it becomes plastic; it is then spread over the roadways and compressed by rolling.

MINERAL WOOL

Sources of Mineral Wool. There are at least two kinds of mineral wool made in this country. The more common quality is made by mixing certain kinds of stone with the MOLTEN SLAG from blast-furnaces and converting the whole mass into a fibrous state. The best slag for the purpose is that which is free from iron. The appearance of the finished product is much that like of wool, being soft and fibrous, but in no other respect are the materials alike. Mineral wool made from slag appears in a variety of colors, principally white, but often yellow or gray, and occasionally quite dark. The color, however, is said to be no indication of the quality, as all of the peculiar properties of the material are present in equal proportions in any of the shades. The other kind of mineral wool is known as ROCK-WOOL, and is made from granite rock raised to 3 000° F. It is claimed that as it is absolutely free from sulphur; it is the only odorless wool manufactured. It has been approved by the United States War Department. It has the same general appearance as that made from slag, and is white in color.

Nature of Mineral Wool. Both of these materials consist of a mass of very fine, pliant, but inelastic, vitreous fibers interlacing in every direction and forming an innumerable number of minute air-cells. Its great value in the insulation and protection of buildings lies in the number of air-cells which it contains, its consequent non-conduction of heat, and its fire-resisting qualities. In wool made from common slag, 92% of the volume consists of air held in minute cells, while in the best grade the proportion of air reaches as high as 96%. This confined air makes it one of the best, if not the best, of the non-conductors of heat. Aside from these qualities it is very durable and contains nothing that can decay or become musty. Being itself incombustible it greatly retards the burning of wooden floors or partitions if their inner spaces are filled with it.

Uses of Mineral Wool. The greatest value of this material is as an insulator of heat, but it is also a valuable non-conductor of sound. It is the general opinion, however, that it can be considered only as a MUFFLER of the sound-waves, for there seems to be no practical way in which it can be used so as to separate entirely the floor and ceiling. It would be crushed by laying floor-cleats upon it. As a muffler or filling between the beams, however, there is probably nothing that is superior.

Manner of Applying Mineral Wool. Mineral wool, when used alone as floor-deadening, may be laid on boards cut in between the joists, or on top of sheathing-lath when that material is used. The wool should be at least 2 in thick. Again, mineral wool is particularly desirable for filling the spaces between the studs of outside walls and partitions and between the rafters of roofs. It may be used to great advantage, also, in partitions around bath-rooms or water-closets, and around water-pipes when placed in partitions. In outside walls and attic roofs, as a protection from the heat of summer or the cold of winter, it is of the greatest value. By lathing the under side of the rafters with sheathing-lath, and spreading on top a layer of 2 or 3 in of mineral or rock-wool, the comfort of the room is greatly increased. Flat roofs over inhabited rooms may be covered with rough boards and 1¾-in cleats nailed on top, the spaces filled with wool, and the roof-sheathing then nailed to the cleats. This not only greatly increases the comfort of the rooms, but greatly retards the progress of fire from the outside. When insulating against heat, nails driven through the insulating material do no harm. When using mineral wool in floors it should be packed in very closely, but not jammed so as to break the fibers, which are naturally very brittle. In partitions it is packed between the studs and laths, so as to completely fill the spaces, the wool being put in after the lathing has reached a height of 2 or 3 ft. More laths are then put on, the spaces filled, and so on to the top. The wool should not be dropped from any considerable height, as the breaking up of the fibers destroys the insulating qualities of the material. In fact the tendency of mineral wool to settle and consolidate, if improperly or too loosely packed, is the only draw-back, except cost, to its use for insulation. The wool behind the lathing will not prevent the plaster from keying.

ESTIMATING THE COST OF BUILDINGS *

Cost of Buildings per Cubic Foot. The method of CUBIC-FOOT VALUES has been used more than any other in estimating the cost of any proposed building, before the plans and specifications are sufficiently complete for taking off the actual quantities. "Comparison of UNIT COSTS is the only scientific criterion by which to judge the economic merit of a structure, a machine or a method of doing work." † Two buildings in the same city, or district, built in the same style and for the same purpose, of the same materials, and on the same scale of wages and prices of materials, should cost the same, or very nearly the same, per cubic foot, although one building may be somewhat larger than the other and of different shape. It therefore follows that if we know the COST PER CUBIC FOOT of different classes of buildings, in different localities, we can approximate quite closely the cost of any proposed building by multiplying its cubic contents in feet by the known cost per cubic foot of a similar building already built in that locality.

Size of Building Proportioned to Cost per Cubic Foot. If the cost of a proposed building must be kept absolutely within a certain sum, the size of the building should be proportioned so that the CUBIC CONTENTS shall not exceed the quotient obtained by dividing the amount appropriated by the AVERAGE COST PER CUBIC FOOT of similar buildings. Even then it may be found, when the bids are opened, that they exceed the appropriation; but the excess is often a relatively small percentage of the total cost and the necessary reductions can be made without altering the main features of the building.

Methods of Computation. In estimating the cost by the METHOD OF CUBIC CONTENTS, it is of course necessary that the contents be computed on the same basis, in both the proposed building and the one already built. The cubic contents are generally computed from the basement or cellar-floor, to the average height of a flat roof, or, if there is a pitched roof, the finished portion of the attic is included, or that part which might be finished, mere air-spaces and open porches not being included. Vaults and areas under sidewalks, etc., are generally included as part of the basement. All measurements are to the outside of the walls and foundations. The estimated cost may or may not include the fees of the architect and other experts.

Other Methods of Estimating the Cost of Buildings. The cost of buildings, such as hospitals, theaters, schools, churches, barracks, dormitories, etc., is sometimes estimated by the COST PER BED, SITTING, INMATE, etc. Estimates are also based upon the COST PER SQUARE FOOT of GROUND OCCUPIED or of all the FLOOR-SPACE, in certain types of buildings.

* The editor is indebted to E. S. Hand and others for valuable data relating to this subject. Readers are referred to the Handbook of Cost Data for Contractors and Engineers, by H. P. Gillette, The New Building Estimator, by William Arthur, and The Building Estimator's Reference Book, by F. R. Walker. Many values given are pre-war values and may be used for relative costs.

† H. P. Gillette, in the preface to his Handbook of Cost Data for Contractors and Engineers.

Data * on Cubic-Foot Values as a Basis for Preliminary Estimates of Building Costs

Notes on Modifying Conditions. Buildings of a given TYPE, such as office-buildings and school-buildings, when similar in construction and finish and built under similar market-conditions as to cost of labor and materials, are found to be nearly identical in CUBIC-FOOT COSTS. The buildings of any such type do not differ widely in bulk, and this is always very considerable when compared with such structures as dwellings and small business buildings. This seems only another way of saying that similar causes produce similar effects, but it goes a step farther by indicating that the results here are virtually identical; so nearly so, that the AVERAGE CUBIC-FOOT COST of a certain kind of building can be relied on to produce an estimate within from 3 to 5% of the actual cost of new work of the same kind and under the same conditions. Other types of large structures, such as public buildings, hotels, churches and theaters, are less subject to standardization because more variable in equipment and finish. This is true also of dwellings, shops and other small structures whose lesser bulk, moreover, renders even less possible a close prediction as to their cost. These uncertainties do not, however, warrant the rejection of the CUBIC-FOOT-COST METHOD for preliminary estimating. They do indicate that it is less closely approximate for some types than for others. But the degree of uncertainty on even the most variable types may be minimized and should be reduced to perhaps 10% under a careful system of cost-computation. Such system should cover a considerable number of examples, taking account of all factors of material influence upon cost in each type, and must follow a consistently uniform method of determining cubic-foot values.

The Factors Which Influence Cost include the following:

(1) Prevailing market prices of labor and materials.

(2) Type of construction employed, depth and kind of foundations and existence of special features such as towers or domes.

(3) Finish: external facing and ornamentation; internal surfacing and decoration.

(4) Equipment: (*a*) number and complexity of heating, lighting, ventilating, sanitary, elevator and other systems; (*b*) extent to which apparatus or equipment, such as laboratory-devices, opera-chairs, bank-counters, etc., is provided for direct use of occupants of building.

(5) Fees of architect and other experts.

(6) Locality: Costs of structures of a given type will vary with the locality because of differing standards of practice and building laws, availability of building materials, labor, etc.

(7) Other items, developing in the experience of the architect.

The Method of Determining Cubage may either simply recognize the GEOMETRICAL VOLUME of the building or, better, may employ a COEFFICIENT OF VALUE for any part whose cost varies materially from the average. The latter method may be preferred as allowing a closer calculation of variations

* The data given in paragraphs: Notes on Modifying Conditions, The Factors Which Influence Cost, The Method of Determining Cubage, and Cubic-Foot Costs are quoted, by permission, from notes relating to this subject, compiled by Dr. Warren P. Laird from the study of a large number of public and private buildings erected in widely separated districts of the United States. For these buildings Dr. Laird acted as the professional adviser for the selection of the architect, and in all cases the estimate of the cost of the buildings was based strictly upon a total number of cubic feet and a fixed unit cost per cubic foot.

from known examples. For instance, an unfinished cellar or other story or a small light-court would cost less per cubic foot than the remainder of the building, while a tower or dome of finished basement containing, also, an expensive mechanical plant, would cost more. Foundations sometimes cost so much that they require figuring to their full depths as though the finished building were carried down to that level.

Cubic-Foot Costs. Subject to the foregoing considerations, the following data on fire-proof buildings may be taken as approximately safe, depending upon the location of the project.

Construction: steel and terra-cotta, stone and brick facings, complete equipment and superior grade of interior finish:

Type of Building	Cents per cubic foot
Office-buildings	65 to 85
Public buildings	80 to 100
School-buildings	45 to 60

Cost-Data of Buildings

Cost of Some Notable Buildings in New York City. Some of the more prominent buildings in the Borough of Manhattan, City of New York, are included in the following table. For all these structures the costs per cubic foot are given. By reason of its height the Woolworth Building may be considered the most notable of the list. The cubic contents total nearly 12 000 000 cu ft. Its foundations are carried to rock, which is about 120 feet below the street-surface. The approximate weight of its steel frame is 23 000 tons.

The Grand Central Station, as a complete terminal, is a very complex structure, but there is a distinct part which contains the passenger-concourse and the waiting-rooms, restaurant and other parts that are considered necessary to care for the traffic. The cubic contents of this part total about 14 000 000 cu ft. Other parts of the building are not considered in the present reference. Some interesting facts as to the main station, only, are:

Cost, about	$8 000 000
Ground-area above street-level, square feet	266 000
Additonal station-facilities under street, square feet,	80 000
Floor-area devoted to station-purposes, square feet	1 188 000
Cubic contents, about, cubic feet	32 857 800
Steel used in construction, tons	35 767
Weight of largest girder used, tons	30

Cost of Various College Buildings. The following table giving the cost per cubic foot of various college buildings is taken from College Architecture in America by Charles Z. Klauder and Herbert C. Wise. In presenting these figures the authors call attention to the fact that data were received from many sources and that, owing to various methods of computing cubage as well as variations in the items considered in computing the actual cost of the buildings, the figures are approximate.

Administration and academic buildings	58 ct
Libraries and museums	65 ct
Chapels and auditoriums	47 ct
Science (laboratory) buildings	75 ct
Engineering and shop buildings	45 ct
Gymnasiums	32 ct
Indoor stadia	23 ct

Cost-Data of Some Notable Pre-War Buildings in New York City *

Name of building	Description	Heights, ft	Ground-areas, sq ft	Total floor-areas, sq ft	Cost	Cubic contents, cu ft	Costs per cubic foot cents
Altman Building [1]	8-story department-store.	152	54 850	495 000	.	.	33
Banker's Trust Co.'s Building [1]	39-story office-building	566	9 721	345 000			70
Heckscher Building [2]	50 E 42nd St	317	10 750	147 172	$700 000	1 793 351	39
Masonic Building [3]	19-story lofts and meeting-rooms	292	23 300	432 000	2 250 000	5 701 000	39
Walker-Lispenard Building [4]	24-story telephone-exchange	365	17 580†			.	40
Woolworth Building [5]	55-story office-building	792	15 600	790 000		7 250 000	62½
United States Rubber Co.'s Building [6]	20-story office-building	273	10 800	226 000	1 628 707	3 090 205	52 7/10
The Madison Avenue Building [7]	20-story office-building	288	14 700	315 000	1 300 000	4 708 000	27½
Auerbach Candy-Factory [8]	11-story factory	103	35 100	343 680	915 000	5 200 000	17½
Æolian Building [9]	17-story lofts	248	15 700	244 724	2 000 000	4 225 000	47 3

* The editor is greatly indebted to Mr. E S Hand and to the architects mentioned for the data for this table. † Typical floor-area.

These buildings were designed by the following architects: [1] Trowbridge & Livingston, [2] Jardine, Hill & Murdock; [3] H P. Knowles; [4] McKenzie, Voorhees & Gmelin; [5] Cass Gilbert; [6] Carrére & Hastings; [7] Charles A. Valentine; [8] Robert D. Kohn; [9] Warren & Wetmore.

"Simplest" Type of Fire-Proof Dormitory for Men*

Georgian style, brick walls, slate roofs No food service, common rooms or faculty suites

Dormitory	Date	Number of men housed	Cost including professional fees: Per cu ft, cents	Cost including professional fees: Per man, dollars	Average net floor-area of room types excluding closets: Single study-bedrooms	Double study-bedrooms	Studies	Single sleeping rooms	Double sleeping rooms	Efficiency of plan with respect to first floor only: Net area student rooms and closets: Men	Net area	Per cent	Tare†: Area	Per cent	Gross plan area	Stairways: Number of stairs	Sq ft of plan area per stairway
Example A	1916	59	43 71	1 520 92	164 22		180 50	94 89		17	3 078	64 1	1 722	35 9	4 800	2	1 200
Example B	1922	98	64 05	2 026 35	181 13	234 21				31	3 911	56 6	2 995	43 4	6 906	3	2 302
Example C	1924	42	70 32	2 303 09	166 53	217 31	168 27	92 48		15	2 196	58 6	1 554	41 4	3 750	2	1 875
Example D	1924	42	70 32	2 303 09	166 53	217 31	168 27	92 48		15	2 196	58 6	1 554	41 4	3 750	2	1 875
Example E	1925	76	67 70	1 668 40		229 11				18	2 276	63 7	1 296	36 3	3 572	2	1 786
Example F	1925	84	66 08	2 971 00	143 69	242 43	231 24	88 40	166 65	29	5 883	64 5	3 239	35 5	9 122	5	1 822
Averages			63 69	2 132 14	164 42	228 07	187 07	92 08	166 65			61 02		38 98		3 2	1 810

* This table has been taken, by permission of the authors, from College Architecture in America, by Charles Z. Klauder and Herbert C. Wise, published by Charles Scribner's Sons, New York.

† "Tare" represents plan area occupied by corridors, stairways, lavatories (toilet-rooms), porters', housemaids' and other hall closets, partitions and exterior walls.

Dwight James Baum was the architect for the following buildings.

Building	Location	Date	Stories	Remarks	Cost per cu ft, ct
West Side Y. M. C. A Building	New York City	1929	14	Cost per cubic foot includes certain complicated equipment common to buildings of this type	68
Ridgewood Bank.	Ridgewood, N J	1930	2 and base-	Cost per cubic foot includes special equipment such as is usually required in a suburban bank, including steel vaults and a complicated system of intercommunication	90
Residence...	Bay Shore, L. I.	1929	$2\frac{1}{2}$	Mr. Baum states that 73 ct per cu ft is the average cost of residences constructed in the neighborhood of New York City for his office.	73

Horace W. Castor was the architect for the following buildings:

Building	Location	Date	Stories	Cu ft	Remarks	Cost per cu ft, ct
Hospital . .	Philadelphia, Pa	1927–28	6 and basement	934 900	Steel frame, concrete arches, brick spandrel walls, tile and terrazzo floors, marble wainscot Building contains private rooms and baths, X-ray department, operating-rooms, laboratories, offices and kitchens	$69\frac{1}{2}$ (With furniture, draperies and lighting fixtures, $71\frac{1}{2}$)
Home for Old Folks	Philadelphia, Pa	1926	6 and basement	894 180	Reinforced-concrete, brick spandrel walls, terrazzo floors Building contains private rooms, infirmary, chapel, dining-hall, kitchens, heating plant, social hall, sun-rooms, recreation-rooms, laundry	$65\frac{1}{2}$
Masonic Building (Consistory)	Philadelphia, Pa	1926–27		2 736 210	Steel frame, concrete arches, limestone facing with terra-cotta trim, marble and terrazzo floors, marble wainscoting. Building contains auditorium (seating 2 100) library, banquet rooms, kitchens, stage and equipment, offices, assembly-rooms, lounges, heating and ventilating equipment.	$46\frac{1}{2}$ (With stage equipment, chairs in auditorium, organ, lighting fixtures, kitchen equipment, furniture and furnishings, $52\frac{1}{2}$)

Rankin and Kellogg were the architects for the following buildings:

Building	Location	Date	Stories	Cu ft	Remarks	Cost per cu ft
Elverson Building (Home of "Philadelphia Inquirer")	Philadelphia, Pa..	1925	13 tower additional	7 427 000	Fire-proof, terra-cotta exterior Building built over railroad.	46 ct, including tower, clock, and chimes.
Provident Trust Co. Building	Philadelphia, Pa ..	1928	12 and basement	1 850 000	Fire-proof, brick and marble exterior; interior, first-class office-building finish.	$ 876, including vaults
Philadelphia Wholesale Drug Co.	Philadelphia, Pa..	1928	.	2 250 000	Fire-proof, brick and artificial stone exterior Considerable portion of interior unplastered and without partitions	$ 261
Montgomery County Court House Annex.	Norriston, Pa...	1930	.	1 140 000	Fire-proof, Vermont light blue marble exterior Interior, first-class office-building finish	$.715
Administration Building, U. S Department of Agriculture	Washington, D C	1930	. .	2 655 000	Fire-proof. White Georgia marble exterior Interior, first-class office-building finish.	$.626
Camden Safe Deposit and Trust Co. Building	Camden, N. J......	1930	3	854 000	Fire-proof. Deer Island granite exterior Entire building used for trust company purposes and finished appropriately.	$1 17, including vaults

Thomas, Martin and Kirkpatrick were the architects for the following buildings:

Building	Location	Date	Stories	Remarks	Cost per cu ft, ct
Y. M. C. A. Building	Easton, Pa	1920	4	Concrete	53
Y. M. C. A. Building	Williamsport, Pa	1923	4	Concrete	40
Y. M. C. A. Building	York, Pa	1925	4	Concrete	53
Y. M. C. A. Building	Norristown, Pa	1923	4	Concrete	57
Office-Building	Philadelphia, Pa	1922	8	Steel	60
Bank and Office-Building	Hazleton, Pa	1923	9	Concrete	44
Hotel	Hazleton, Pa	1923	9	Concrete	66
Hotel	Easton, Pa	1925	10	Steel	61.6
Hotel	Brigantine, N. J.	1926	11	Steel	74
Church	Ardmore, Pa	1922		Stone	51.7
Church	Roxborough, Pa	1927		Stone	55
Church	Columbia, S. C.	1929		Brick	52
Christian Association Building, University of Pennsylvania	Philadelphia, Pa	1928	3	Concrete	70

Paul P. Cret was the architect for the following buildings:

Building	Location	Date	Stories	Cu ft	Remarks	Cost per cu ft
Barnes Foundation	Merion, Pa	1923	2 and basement	398 850	Art gallery, fire-proof, French limestone exterior	$ 876
John Herron Art Institute Art School	Indianapolis, Ind	1928	2 and basement	246 800	Brick, limestone trim, concrete floors	.50
Memorial to Quentin Roosevelt	Chamery, France	1920	.	1 700	Fountain, French limestone	1 63
Residence	Haverford, Pa	1926	2 and basement	103 009	Local stone, fine cabinet work, wood framing.	873
Gatehouse	Villa Nova, Pa.	1928	2 and basement	21 886	Local stone, wood framing	.634

John Russell Pope was the architect for the following buildings: *

Building	Location	Date	Stories	Remarks	Cost per cu ft
Marcus Ward Home.	Maplewood, N. J.	1927–29	2	A group of three buildings Neshaminy Pennsylvania Quarry stone, slate roofs and metal windows. 72 bed-rooms, each with fire-place. Fireproof.	$.98
The D. A. R. Constitution Hall.	Washington, D C.	1928–29		Auditorium seats 4000. The building includes a Library group. Exterior is of limestone, interior is of plaster Span of auditorium, 180 ft	$.58
First Presbyterian Church of New Rochelle.	New Rochelle, N.Y.	1928–29		Local rubble stone and wood trim. Group consists of church, seating 500, tower and Sunday School.	$ 60
Hendricks Chapel, Syracuse University.	Syracuse, N Y	1929–30		Brick and limestone, lead-coated copper dome Seats 1450. Interior, ornamental plaster and wood.	$ 67
Syracuse Memorial Hospital.	Syracuse, N Y.	1927–29		Group consists of Administration Building, Patients' Building and Nurses' Home Brick and limestone, 300 patients.	$ 82
Club House for the Junior League of the City of New York.	New York City, Borough of Manhattan.	1928–29	7	Brick and limestone First, second and third floors for entertaining; fourth floor, bedrooms and offices; fifth floor, nursery for Social Service; sixth and seventh floors, swimming-pool and two hand-ball courts.	$ 96

* The data presented in this table were prepared by the Office of John Russell Pope. The cubic-foot cost has been computed including all equipment of an attached nature necessary to the complete operation of the building, as well as kitchen tables, bread- and meat-cutting machines, clothes-assorting tables, etc The figures given are exclusive of Architect's and Engineer's fees and removable furniture, rugs and draperies.

Detroit Institute of Arts, Detroit, Mich. Architects· Paul P. Cret and Zautzinger, Borie and Medary. This building, erected in 1923–1927, contains an art gallery and large concert hall. It is of fire-proof construction, 2 stories and basement, with a marble exterior, contains 5 661 688 cu ft; the approximate cost per cu ft was 80 ct.

Rodin Museum, Philadelphia, Pa. Architects: Paul P. Cret and Jacques Greber. Erected in 1929, this building is of fire-proof construction having an Indiana limestone exterior, and contains an art gallery. It contains one story

and basement, has a cubage of 248 000 cu ft and cost approximately 80 ct per cu ft.

Hartford County Building, Hartford, Conn. Architects: Paul P. Cret and Smith and Bassette. This building, erected in 1926–1928, contains 3 stories and basement. It is of fire-proof construction, having an Indiana limestone exterior. The cubage is 1 461 051 cu ft and cost approximately $1.04 per cu ft.

Apartments, Bryn Mawr, Pa. Architect: John F. Harbeson. This apartment-building was built in 1923; it contains 3 stories and basement. It is of brick and frame construction, has a cubage of 573 400 cu ft and cost approximately 48 ct per cu ft.

Residence at Chestnut Hill, Philadelphia, Pa. Architect: Robert R. McGoodwin. This building, erected in 1929, was built of local stone for exterior walls with cast-stone trim, tile roof, hardwood floors, tile bathrooms and vapor heat. All framing was of wood. The height was 2½ stories. The approximate cost was 55 ct per cu ft.

Public-School Buildings. Davis and Dunlap were the architects for the following three school buildings:

Junior High School Building

Location: Wyncote, Penna
Date of construction: 1928.

Contents:	Auditorium;	Shops;
	Gymnasium;	Library;
	Cafeteria;	Rest-rooms;
		Class-rooms.

Construction: Fire-proof; stone and concrete.
Pupil capacity: 745.
Total usable floor-space: 24 745 sq ft
Total cost of construction (including architects). $283 699
Cost of equipment, approximately: 20 000

Cost per pupil: $380	Cost per sq ft usable space. $11 42	Number of sq ft per pupil: 33
Cost per pupil with equipment: $407	Cost per sq ft usable space, with equipment $12 27	

Note. Square foot usable floor-space includes: Class-rooms, auditorium, gymnasium, showers and lockers, offices, teachers', library, toilets and special rooms. (No corridors, walls, stairs, boiler-room, janitor, storage, cafeteria)

Cost per cu ft: 28½ ct (without architects' commission or equipment).

Heating:	14.95%
Plumbing:	5.59%
Electric:	3.2 %
Total:	23.74%

Junior High School Building

Location: Camden, N. J.
Date of construction: 1929.

Contents: Auditorium; Shops;
Gymnasium; Library;
Cafeteria; Class-rooms.

Construction: Fire-proof; brick and concrete.
Pupil capacity: 1 400.
Total usable floor-space: 53 692 sq ft.
Total cost of construction (including architects): $656 816
Cost of equipment, approximately: $40 000

Cost per pupil:	Cost per sq ft usable space:	Number of sq ft per pupil:
$462	$10	39.80
Cost per pupil with equipment:	**Cost per sq ft usable space, with equipment:**	
$497	$12	

NOTE. Square foot usable floor-space includes Class-rooms, auditorium, gymnasium, showers and lockers, offices, teachers', library, toilets and special rooms. (No corridors, walls, stairs, boiler-room, janitor, storage, cafeteria)

Cost per cu ft: 30 ct.

Heating·	11.1%
Plumbing	5 9%
Electric.	4 0%
Total	21 0%

CONSOLIDATED SCHOOL BUILDING

Location: Schuylkill Township, Chester Co., Pa
Date of construction: 1930.

Contents: Auditorium and gymnasium combined; class-rooms, etc.

Construction: Slow-burning; stone
Pupil capacity: 300.
Total usuable floor-space: 7 354 sq ft
Total cost of construction (including architects)· $81 418
Cost of equipment, approximately· $5 000

Cost per pupil:	Cost per sq ft usable space:	Number of sq ft per pupil:
$271	$11	24 5
Cost per pupil, with equipment:	**Cost per sq ft usable space, with equipment:**	
$288	$11.71	

NOTE: Square feet usable floor-space includes: Class-rooms, auditorium, gymnasium, showers and lockers, teachers' rooms, toilets. (No corridors, walls, stairs, boiler-room, janitor or storage.)

Cost per cu ft: 28ct.

Heating:	19 41%
Plumbing:	5.63%
Electric:	4.7%
Total:	29.74%

Cost of Certain Public-School Buildings in Various Cities*

Location	Type	Date	Cu ft	Remarks	Approximate cost per pupil	Approximate cost per cu ft, ct
Baltimore, Md.	Elementary	1925	437 971	Wall bearing with flat-slab floor-construction	$441 42	35.2
Baltimore, Md.	Elementary	1925	595 943	Wall bearing with pan-system floor-construction	491 80	41.2
Binghamton, N. Y.	High	1923–24	1 555 504	Wall bearing, auditorium and gymnasium		32
Boston, Mass.	Intermediate	1926	1 123 606	First class, 2 stories, 30 rooms	531 14	57
Boston, Mass.	High	1926	2 505 440	First class, 3 stories	758 65	45
Boston, Mass.	Primary	1924	353 368	12 rooms, first class. 2 stories	373.54	56
Brooklyn, N. Y.	High	1923	4 850 401	Skeleton with flat cinder arch floor-construction	892 52	58.8
Carthage, N. Y.	Elementary	1923–24	713 960	Wall bearing with auditorium and gymnasium		35
Chicago, Ill.	Elementary	1930	1 120 000	Steel and concrete construction, fire-proof. Gymnasium and auditorium; 2 and 3 stories	640.00	63
Chicago, Ill.	Junior High	1930	3 358 000	Steel and concrete construction, fire-proof; 3 gymnasiums, auditorium and natatorium; 3 stories	863 00	57
Chicago, Ill.	Senior High	1930	7 530 000	Steel and concrete construction, fire-proof· 4 gymnasiums, auditorium and natatorium, 4 stories	1 037 00	49
Cincinnati, Ohio	Elementary	1921	1 893 000	Skeleton, concrete-floor construction	491 71	46.7
Cleveland, Ohio	Elementary	1924	955 000	Wall bearing with pan-system floor-construction	492 82	49.5
Denver, Col.	Junior High	1928	1 635 000			30.1
Denver, Col.	Senior High	1926	4 229 000		720 00	35.2

* The data compiled in this table have been taken from various sources.
The buildings noted have been selected at random with no thought of comparison of school-building costs of various cities.

Cost of Certain Public-School Buildings in Various Cities* (continued)

Location	Type	Date	Cu ft	Remarks	Approximate cost per pupil	Approximate cost per cu ft, ct
Detroit, Mich.	Elementary	1922	1 983 308	Wall bearing, concrete columns, reinforced joist	$322.36	38.3
Los Angeles, Calif.	Elementary	1929	..	Two stories, fire-proof corridors and stairs	....	4 26.6
Los Angeles, Calif.	Senior High	1927	.	One and 2 stories, fire-proof corridors and stairs	.	23.1
Milwaukee, Wis.	Elementary	1923	1 284 800	Skeleton, tile and joist	333.33	31
Milwaukee, Wis.	Junior High	1924	1 877 400	Skeleton, tile and joist	421.42	31 5
New York, N. Y.	Elementary	1923	2 016 698	Skeleton, auditorium and gymnasium	. . .	52.3
New York, N. Y.	Private Girls' School	1929	1 312 000	Architect, Benjamin W Morris Auditorium for 590, cafeteria for 130, 2 gymnasiums; steel skeleton with cinder-concrete floor-arches Exterior, cast stone on brick	....	69
New York, N. Y.	Elementary	1929	1 326 500	Steel construction, 4 and 5 stories with auditorium and gymnasium	365.55	48 1
New York, N. Y.	High	1929	4 591 793	Steel construction, 3 and 4 stories with auditorium and 3 gymnasiums	623.71	50.2
Philadelphia, Pa.	Junior High	1930	3 401 262	Auditorium	590 00	34 7
Philadelphia, Pa.	Elementary	1930	1 306 270	Auditorium	349 00	38.4
Philadelphia, Pa.	Senior High	1929	6 036 000	Auditorium	577.00	38
Pittsburgh, Pa.	Elementary	1923	491 865	Wall bearing with tile and concrete floors	420.59	44.3
Syracuse, N. Y.	Junior High	1923–24	1 725 000	Wall bearing, skeleton, auditorium and gymnasium	.	32 5

* The data compiled in this table have been taken from various sources.
The buildings noted have been selected at random with no thought of comparison of school-building costs of various cities.

Federal Buildings.* In presenting data for the following seven Federal Buildings it should be noted that the cubic-foot rates on the projects cited are based upon the lowest bid submitted. Competition on public-building construction results in a large number of bidders and the range in the proposals submitted results in a difference varying from 50% to over 100% over the lowest bidder. It is obvious that any unit rate per cubic foot of volume determined on the basis of the low bid is not a true index to the cost of a particular building.

Worcester, Mass. Contract awarded June, 1930.

A five-story and basement building of fire-proof construction; the exterior walls faced with granite, flat roof, steel sash, main entrance lobby marble wainscot, ornamented plaster ceiling; the first and second floors devoted to postal service and three upper floors of typical office finish.

Ground area	17 700 sq ft
Cubical contents	1 225 000 cu ft
Rate per cu ft	60 ct

White Plains, New York. Contract awarded August, 1930.

Basement, one and part two-story fire-proof building, brick faced with stone trimming, flat roof; interior public lobby having marble wainscot and ornamented plaster ceiling; second floor typical office finish.

Ground area	12 600 sq ft
Cubical contents	466 000 cu ft
Rate per cu ft	41 ct

Lima, Ohio. Contract awarded January, 1930.

One-story and basement fire-proof building; exterior facing of limestone, copper-covered pitched roof, steel sash; main entrance lobby marble wainscot, ornamented plaster ceiling.

Ground area	20 000 sq ft
Cubical contents	712 000 cu ft
Rate per cu ft	41 ct

Flint, Mich. Contract awarded July. 1930.

Basement, one and part two-story building of fire-proof construction, limestone face, slate-covered pitched roof over two-story portion, flat roof elsewhere, steel sash; main lobby marble wainscot, ornamented plaster ceiling.

Ground area	24 000 sq ft
Cubical contents	1 027 000 cu ft
Rate per cu ft	35 ct

San Bernardino, Calif. Contract awarded June, 1930.

Two-story and basement building of fire-proof construction; exterior of brick with stucco facing and architectural terra-cotta trimming, tile roof; main lobby marble wainscot, ornamental plaster ceiling.

Ground area	15 000 sq ft
Cubical contents	598 000 cu ft
Rate per cu ft	39 ct

Asheville, N. C. Contract award July, 1929.

* Data for the seven Federal Buildings here listed have been presented by the Office of Supervising Architect, James A. Wetmore, Acting Supervising Architect.

Three-story and basement, fire-proof construction, granite-faced to first-floor line, limestone-faced elsewhere except rear and courts which are of brick, copper-covered pitched roof, steel sash; main lobby wainscot, ornamented plaster ceiling.

Ground area	24 200 sq ft
Cubical contents	1 380 253 cu ft
Rate per cu ft	45 ct

Springfield, Ill. Contract awarded December, 1929.

Three-story and basement building, fire-proof construction, limestone facing, except rear; copper-covered pitched roof, main lobby limestone wainscot 4 ft high, plaster walls, ornamented ceiling.

Ground area	29 700 sq ft
Cubical contents	1 638 000 cu ft
Rate per cu ft	43 ct

Of new federal buildings in the District of Columbia, those of the Internal Revenue and the Department of Commerce are the only two upon which the office has any definite data. The building of Internal Revenue has a ground area of approximately 150 000 sq ft and 16 100 000 cu ft volume. It is a seven-story building, limestone faced, with Tennessee marble columns, and tile roof. When completed it will have cost approximately 50 ct per cu ft.

The Commerce Building has a ground area of approximately 176 000 sq ft and a content of approximately 27 160 000 cu ft, entirely limestone-faced, tile roof, and will cost, when completed, approximately 60 ct per cu ft.

Percentages of Cost of Items of Construction in Fire-Proof Buildings

The tables * on the following pages show the DIVISION OF THE COSTS of fire-proof buildings among the different materials and parts of the construction, the data having been furnished the compiler by architects and builders in the cities mentioned in the tables. Each column of values in the tables gives the data for an individual building, except the values for New York City, in the second, third and fifth columns, which show the averages for a large number of buildings. The tables on the first four pages include only buildings approximating closely the standard specifications of the National Board of Fire Underwriters. The tables show that the foundations and steel frames, the only parts little damaged in conflagrations, represent, approximately, only 25% of the entire sound value of a building. For example, in the tables on the first four pages, the average cost of all the foundations is 8%, while the average cost of the steel frames is 17.88%.

* The six following tables were compiled by F. J. T. Stewart, Continental Insurance Company. All are reproduced, by permission, from J. K. Freitag's Fire Prevention and Fire Protection.

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings

The figures opposite each item represent percentages of total cost of building

Classified construction	New York					Chicago						Baltimore							
	Offices	Offices	Hotel	Offices	Ware-house	Offices	Offices	Offices	Offices	Mercan-tile	Mercan-tile	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices
Height, in stories	10			12								11	12	7	14	9	8	10	7
Cost, cents per cubic foot				40		30 3	34 8	32 3		14	23 6								
Foundations						*4 4*	*2 34*	*5 67*		*6 00*	*10 74*	*5 62*	*4 37*	*7 25*					*4 3*
Excavations of back-filling												1 22	1 25	2 07					4 3
Shoring banks, etc.													1 06						
Foundation-footings and concrete												4 4	0 74	5					
Rubble-stone and granite pier-caps													0 65						
Sidewalks and curbs													0 13	0 18					
Steel frame	*24 0*	*18 4*	*16 0*	*22 50*	*36 5*	*19 70*	*14 0*	*11 86*	*10 6*	*27 6*	*38 66*	*13 6*	*14 54*	*19 52*	*9 9*	*9*	*10 1*	*11 4*	*17 5*
Material													11 79						
Erection													1 7						
Shop-drawings													0 49						
Painting													0 24						
Teaming													0 32						
Masonwork	*26 56*	*26.2*	*31 5*	*20 10*	*32 5*	*26 31*	*27 96*	*22 3*	*38 5*	*27 6*	*44 79*	*23 76*	*28 5*	*34 15*	*30 15*	*31 31*	*37 2*	*29 9*	*35 5*
Brick, common												10 1	6 59	11 21				8 98	13 5
Brick, faced or pressed													1 82	1 26					
Brick, enameled													0 86						
Brick, cleaning and pointing													0 21						
Terra-cotta	1 41			2 06		10 04		5 76				4 34	7 61	5 07	3.9	2 5		3 5	
Stone	2.75			1.50			10 30					2 55	4 22	9 34	4.05	7 4	27 3	8 7	9 4

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings (continued)

The figures opposite each item represent percentages of total cost of building

Classified construction	New York					Chicago						Baltimore							
	Offices	Offices	Hotel	Offices	Ware-house	Offices	Offices	Offices	Offices	Mercan-tile	Mercan-tile	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices
Marble				10 90												0 83			
Wall-lining or furring												0 77				1 8			
Floor-arches, roof, etc.						11 06	9 16	5 8			22 08	3 87	4.73	3 68	6.8	6 6	3.4	3.6	6.6
Cinder-concrete filling over arches						1 06	1 30	1 24			0.47	0 57	0 58	0 49					6.
Partitions												1 56	1 88	3 1					
Partitions, cleaning and wrecking																			
Safety-deposit vaults															7 6			5 16	
Miscellaneous scaffolding and wrecking																0 48			
Miscellaneous masonry	22.40					4 15	7.2	9 5			22 24				7 8	11 70	6 5		
Equipment	*20 65*	*21 2*	*12 0*	*20 89*	*11 25*	*24 96*	*24 54*	*25 04*	*23 2*	*20 8*	*2 26*	*19 18*	*24 54*	*15 96*	*18 69*	*26 71*	*15 23*	*21 23*	*12 69*
Elevator-plant	5 70			5 28		4 65	7 81	5 48				7 6	7 68	8 15	5	5 68	3 8	7 7	4 02
Plumbing	3 49			4 16		6 58	5 90	6 19			0 85	3 26	3 55	2.61	4 13	3 5	3 7	3 4	3 36
Heating-system	5 85			5 85			0 67	5 88				4 6	7 06	3 54	4 2	3 6	2 2	6 7	4 15
Boiler-plant							1 92	1 86			0.85					1 9			
Lighting-system, wiring and fixtures	5.37			5 6		3 82	2 76					2 54	5 05	1 38	2 03	1 9	1 6	3	1 10
Dynamos, switchboards, etc								3 34								2 18			
Fixtures							0 32	1 24				0 7	0 49		1 05	0 65			
Mail-chute	0 24					0 24	0 06	0 13				0 48	0 27	0 28	0 18		0 33	0 43	
Filter-plant													0 44						
Refrigerating-plant															0 93				

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings (continued)

The figures opposite each item represent percentages of total cost of building

Classified construction	New York					Chicago						Baltimore							
	Offices	Offices	Hotel	Offices	Ware-house	Offices	Offices	Offices	Offices	Mercan-tile	Mercan-tile	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices
Safes																1 9			
Vault-doors and safe-doors						0 24	0.29	0 32			0 56				0 63		3 6		
Flash-signals and indicators															0 54				
Furniture																5 4			
Ventilation						9 43	4 80	0 60											
Trim and finish	*28 40*	*29 8*	*33.25*	*22 26*	*14 75*	*24 44*	*29 45*	*35 28*	*27 7*	*18 3*	*4 39*	*36 06*	*27.56*	*22 83*	*36 20*	*28 78*	*36 63*	*26 89*	*25 04*
Carpentry, rough						8 90	6 53	8 54			2 86	2 03	2 59	0 22	10 9	9 8	5 4	7 9	1.6
Carpentry, finish	20 50			10			1 88					5 27	4 31	0 51					5 7
Hardware, rough						0 96	1 70	1.5				0 32	0 29	1 7		0 64	0 9	1 16	1.95
Hardware, finish												0 74	0 88	5 1	0 83				
Marble	2.83			9 85		5 02	4 17	7.61			0 25	6 74	7 11	1 42	10 5	5 4	10 1	6 2	2 8
Mosaic	1.25											0 14		1 99			1 2		
Glass						1 02	1 32	3 37				1 44	9	1 13	1 4	7 5	1 4		0 89
Slate																			
Plastering	3 82			2 41		2 97	3 68	2 35				2 94	3 05	2 96	2 9	4 21	3 7	2 7	1.4
Fresco						1 84	3 49	1 78				0 88							
Paint and varnish						1 64	2.05	1 39			0 30	2 00	1 34	1.45	2 9	1 6	1 4	2 9	1.5
Office-partitions (wood and glass)												2 65							
Ornamental iron						1 09	2 58	6 19				9 70	5 87	4 62	6 4	5 0	10 5	5 5	6.8
Skylights and sheet metal						1 00	2 05	2 55			0 98	0 71	1 22	1 54	0 37	0 63	0 83	0 53	2 4
Office-grill												0 5		0.19			1 2		
General expenses		*4 4*	*7.70*		*5 00*							*1 83*	*1 92*		*5 17*	*4 31*	*0 36*	*11 2*	*6 94*

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings (continued)

The figures opposite each item represent percentages of total cost of building

Classified construction	Boston																	St. Louis	
	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Mercantile	Mercantile	Mercantile	Mercantile	Mercantile	School	Storage	Mercantile	Mercantile	Offices	Offices
Height, in stories	9	11	10	10	11	11	11	11	7	4	6	3	5	3	7	5	6	12	12
Cost, cents per cubic foot	24	....	35	49	67	41 8	45	34	..	....	21 5	22	29	29 6	..	.	...	....	....
Foundations	*10.6*	*5.13*	*4.32*	*7.5*	*4.42*	*5 8*	....	....	*14.2*	*14*	. .	*11.5*	*13.5*	*7.4*	...	*24 5*	....	*5*	*7*
Excavations and back-filling	....	....	...	...	...	...	...	....	...	.	...	...	...	...	. .		...	..	....
Shoring banks, etc.	....	....	...	...	..	..		....	.	....	....	....	...	...	. .	..	.	..	....
Foundation-footings and concrete	....	....	....	..	....	.	. .	....	.	..	...	...	...	....	.		.	.	..
Rubble-stone and granite pier-caps	. .	...	....	.	...	...	...	...	..	....	. .	....	....	...	. .	.	..	. .	....
Sidewalks and curbs	....	....	..	.	. .	..	...	.	. .				.	.	.			...	....
Steel frame	*14.4*	*27 4*	*22.5*	*15*	*25 7*	*21 8*	*21.4*	*24 6*	*8.25*	*2 2*	*22.1*	*19 32*	*14 65*	*21 2*	*21.6*	*14 3*	*7 95*	*15 0*	*15.0*
Material	....	....	....	....	....	..	...	...	...	...	....	....	....	....	...	...	....	...	....
Erection	...	...	....	....	...	....	...	...	....	....	....	.	....	...	....	....	....	. .	....
Shop-drawings	....	..	....	....	...	...	....	...	...	...	...	....	...	...	. ..	..	....	. .	....
Painting	. .	....	....	....	..	...	...	...	...	...	...	. ..	.	..	..	. .	....	...	....
Teaming	. .	..	..	.		..	....	...	...	...	...	....		.		. .	.	..	....
Masonwork	*41.4*	*15.0*	*32 23*	*41.6*	*33 60*	*27 57*	*34 85*	*26*	*33.65*	*31 45*	*22 88*	*30 6*	*17.42*	*30 46*	*68.62*	*26*	*38.6*	*20.0*	*25.0*
Brick, common	27.5	8 3	18.9	15	19 7	17.75	17.7	16 9	29 2	30	16.8	24	9 75	20.1	64	23.8	....	....	....
Brick, face or pressed	....	. ..	.	..	...	..	...	...	....	....	...	....	....	....	...	...	....	....	....
Brick, enameled	....	....	...	..	...	. .	. .	..	.....	...	..	...	..	....	...	....	....	...	....
Brick, cleaning and pointing	....	..	.	....	...	...	..	...	....	....	. .	...	.	...	....	....	....	...	. .
Terra-cotta	...	...	.	.		.		.	...		..	.		...	.	.	....	....	....
Stone	7.1	1.75	8.17	16 6	5.25	3.55	10 4	4.54	4.45	1 45	3 3	6 6	6 95	6.6	4.62	2.14	....	....	....

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings (continued)

The figures opposite each item represent percentages of total cost of building

Classified construction	Boston																	St. Louis	
	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Mercantile	Mercantile	Mercantile	Mercantile	Mercantile	School	Storage	Mercantile	Mercantile	Offices	Offices
Marble	5.8	4.95	5.16	10	8.65	6.27	6.75	4.56			2.78		0.72	3.76		0.08			
Wall-lining or furring																			
Floor-arches, roof, etc																			
Cinder-concrete filling over arches																			
Partitions																			
Partitions, cleaning and wrecking																			
Safety-deposit vaults																			
Miscellaneous scaffolding and wrecking																			
Miscellaneous masonry																			
Equipment	*9.48*	*30.3*	*18.05*	*15.99*	*24.45*	*20.12*	*23.02*	*23.53*	*8.98*	*23.9*	*18.71*	*12.3*	*11.84*	*16.25*	*2.46*	*10.65*	*19.09*	*17*	*16*
Elevator-plant	3.29	12.4	3.5	4.66	3.92	5.56	4.3	7.1	7.65	1.54		5.07	2.44	2.39		4.25	6.45		
Plumbing	1.74	4.75	3.4	4	4.8	5.45	2.92	5.47	0.51	1.85	3.68	2.9	5.07	4.32		1.25	1.5		
Heating-system	2.71	8.4	4.83	5.33	2.88	2.48	8.46	7.8		9.85	5.5	3.34	2.34	7.64	1.54	1.36	2.52		
Boiler-plant			1.4		5.68	2.37				10.4	7.15					1.07			
Lighting-system, wiring and fixtures	1.74	4.75	3.79	2.00	1.62	4.26	7.34	3.16		0.26	2.38	1.03	1.99	1.9	0.92	1.5	7.2		
Dynamos, switchboards, etc																			
Fixtures			1.13		5.55				0.825							1.22	1.42		
Mail-chute																			
Filter-plant																			
Refrigerating-plant																			

Table Showing Proportion of Value in the Various Items of Construction of Fire-Proof Buildings (concluded)

The figures opposite each item represent percentages of total cost of building

Classified construction	Boston																	St. Louis	
	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Offices	Mercantile	Mercantile	Mercantile	Mercantile	Mercantile	School	Storage	Mercantile	Mercantile	Offices	Offices
Safes																			
Vault-doors and safe-doors																			
Flash-signals and indicators																			
Furniture																			
Ventilation																			
Trim and finish	*20 87*	*22 11*	*22 4*	*19 34*	*12 16*	*24 73*	*21 30*	*26.11*	*34 18*	*28 27*	*36 32*	*26 30*	*41 35*	*24 92*	*7.24*	*24.70*	*34 57*	*33*	*27*
Carpentry, rough																			
Carpentry, finish	16 6	14 3	13.4	13 2	6 54	15 75	13 6	18 9	31 2	24 4	16 8	17 5	29	15 1		19 9	30 8		
Hardware, rough																			
Hardware, finish																			
Marble																			
Mosaic																			
Glass																			
Slate																			
Plastering	3 32	5 5	3 3	2 66	1 93	5 2	2 74	3.55	0 572		12 6	2 0	6 7	4 3	1 54	0 51			
Fresco																			
Paint and varnish	2 9	1.65	4	2 33	2 63	2 6	3 74	2 68	1 14	1 58	5 4	3 26	3 6	2 8	0 30	2 86	2 34		
Office-partitions (wood and glass)																			
Ornamental iron																			
Skylights and sheet metal	2 05	0 66	1 7	1 15	1 06	1 18	1 22	0 986	1 27	2 29	1 52	3 6	2 05	2 72	5 4	1 43	1 43		
Office-grill																			
General expenses																		*10*	*10*

Cost of Elements of Five Buildings

The buildings numbered 1 to 5 in the following table were constructed in New York in 1926. Decimals are proportions of the total cost (labor and materials) of the building:

Building No. 1. Office and Exchange, steel frame (fire-proof), limestone front and trim, 101 by 116; 23 stories and basement and two penthouses, 261 000 sq ft, 3 738 000 cu ft. (Geo. A. Fuller, New York)

Building No. 2. Office and Bank, steel frame (fire-proof), limestone front and trim, 65.5 by 145.25; 12 stories and basement, 125 887 sq ft, 1 569 512 cu ft. (Geo. A. Fuller, New York)

Building No. 3. Hotel, reinforced concrete (fire-proof), face-brick front, terra-cotta trim, 172 by 200; 10 stories and penthouse, 234 631 sq ft, 3 017 015 cu ft. (Geo. A. Fuller, New York)

Building No. 4. Factory, reinforced concrete, $461 711. Brooklyn. (Barney-Ahlers Construction Corporation.)

Building No. 5. Factory, reinforced concrete; $213 666; Long Island City, N. Y. (Barney-Ahlers Construction Corporation.)

Cost of Elements on Five Buildings*

Main operations	Included in main operations	Number of building 1	2	3	4	5
General conditions	Permits, bond, insurance, association fee, damage, heat, plant	0523	.03	.04	.094	.0724
Masonry	Wrecking, excavation, blasting, shoring, waterproofing, pumping, brick stack	.0924	.1650	.1330	.07	.14
Caissons and piling	Concrete piles, creosoting wood piles, test borings	.0973	.0014	...	....	.....
Reinforced concrete	Drawings, royalty, reinforced steel, metal forms, inserts, test, reinforced concrete stack	0233	0710	.1350	.2990	.0431
Cement work and paving	Asphalt, mastic, brick, wood block, stone floors, pavements	.0100	0160	0340	.0298	.0070
Fire-proofing	Tile arch, raised floor, book-tile, partition-tile, furring, column covering, gypsum	.0412	027	0402	0245	.0685
Plastering and lathing	Furring, corner-beads, cement base, stucco, solid and metal stud partitions, suspended ceiling	.0432	0458	0840	0175	.0128
Terra-cotta	Setting, anchors, scaffold, mortar, hoist, models		0057	0170	.	.0112
Cut stone	Setting, anchors, lewising, limestone, sandstone, marble models	1400	.079	0041	0427	0053
Granite	Setting, anchors, lewising, monoliths, polishing, carving, models	.007	0039	0020	.	..
Mill and cabinet work	Frames and sash, trim, priming, cabinet finishing, revolving door, screens	0170	0218	0560	.	.
Carpentry	Rough hardware, window-cleaning devices, weather-strips, calking frames, flag-poles	0020	0510	0494	0097	0096
Roofing	Composition, slate, tile, guarantee	.	0047		0064	0070
Metal windows	Underwriter hardware, glass and glazing, setting frames, store fronts	0125	0053		0064	.
Sheet metal	Cornice, skylights, ventilators, flashing, tin fire-doors	0070		0109	0267	0106
Art-metal doors	Metal jambs and trim, baked enamel finish, kalameined doors	0250	0196	0050	0422	0053
Glass and glazing	Plate, prismatic sheet, art, wire, mirrors, corner-bars	0110	0245	0070		
Painting and decorating	Paint, paper-hanging, floor-finish	0160	0059	0185	0267	0154
Ornamental iron	Bronze, door-hangers, door-opening device, directory board	0620	0328	0262	.	.0047
Structural steel and iron	Details, inspection, sorting, setting, painting, stack, stack lining, C.I bases	0750	1008			

Hardware.	Kick-plates, coat-hooks, bead-screws	.0060	.0071	.0167	0104	.0059
Plumbing and sewerage...	Permits, gas-fitting, meters, filters, tanks and pumps, fixtures and accessories, fire equipment	.0410	.0620	.1947	0491	.0448
Heating and ventilation....	Boilers, temperature regulation, pipe-covering, air-washer, fans, ducts, piping...	.0350	.0700	.	.0468	.0663
Power plant...	Boilers, stokers, generators, switchboard, leads, breeching, coal and ash machinery.	.	.	.0027	..	.
Electric wiring	Telephone, telautograph, carriage call, clock system, burglar, fire and watchman-alarm			.0186	.0299	0163
Lighting fixtures..	Gas, electric, combination, signs, lamps, glassware	.0160	0270	.0159	.	.
Elevators	Sidewalk lift, dumb-waiter, signals, cabs, escalators, automatic gates...	.0720	0647	.0159	.0674	0078
Pneumatic service...	Pneumatic tubes, compressed-air system	0004		.0070	.	.
Vacuum cleaning system	Piping, power, receiver, sweepers	0005	.	0008		.
Sprinkler system	Tank, supply-pipe, pumps, sprinkler heads, automatic alarm	0023	.	0050	0462	.
Refrigeration..	Machines, pumps, tanks, insulation, refrigerators and doors	0020	.	.0144	..	.
Miscellaneous iron.	Lintels, bucks, wheel-guards, curb-angles, flag-poles, safety-treads	.0004	..	0030	0257	0225
Iron doors and shutters....	Strap hinges, automatic hardware, meeker, Peelle doors, rolling shutters	.	.	.	.	0065
Steel sash..	Steel sash doors, setting, glass and glazing hardware, sash operators	0140	..	.0001	0099	0261
Prismatic lights.	C I. frames, reinforced-concrete frame, floor-lights		.	.	.	.0035
Fire-escape	Permit, anchors, platforms	0150	..	0016	.	.
Package-chute	Garbage-chute, dust-chute	0024	.	.0030	.	...
Vaults and vault-doors.....	Steel vault-lining, vault fixtures, safes.	0003	.		.	.
Marble	Slate, carrara glass, toilet hardware.	.0310	.	.0190		
Tile and mosaic.	Ceramic, rookwood, grueby, rubber, cork tile and mats	.0100	0510	0079	.0124	.0016
Art marble and terrazzo...	Scagliola, asbestolith, gustavino	.0040	.	.0020	....	.....
Furniture and fixtures..	Window-shades, carpets, bar, kitchen and laundry fixtures, incinerators	.0050	...	.0085	.	..
Mail-chute	Floor thimbles, backboard, box, angles...	.0010	.0013	.0009	...	....

* This table has been taken by permission from Appraisers and Assessors Manual, by Prouty, Collins and Prouty, published by McGraw-Hill Book Company, Inc., New York.

Cost of Buildings per Square Foot

One-Story Buildings of Large Area, such as exposition-buildings, etc., may be estimated almost as accurately by the square foot of ground covered as by the cubic foot of building, as there are few or no interior partitions, and usually no plastering or interior finish. The cost of buildings for the Chicago World's Fair (1893), the St. Louis Exposition (1904), the San Francisco Exposition (1915) and the Philadelphia Sesqui-Centennial Exposition (1926) are given here for purposes of comparison.

Exposition-Buildings. The cost * of the **World's Fair buildings (Chicago, 1893)** per square foot of ground covered, including sculpture and decoration, was as follows:

Manufactures and Liberal Arts Building	$1 39
Transportation Building	1 08
Electricity Building	1 69
Machinery Hall	2 12
Agricultural Building	1 44
Administration Building	9 18
Horticultural Building	1 41
Mines and Mining Building	1 04
Fisheries Building	2.35
Forestry Building	0.75

Cost of Structures for the St. Louis Exposition (1904). The following figures were issued by Isaac S. Taylor, at that time Director of Works, of the World's Fair, showing the area and cost of the principal exhibition-buildings. The total area of twenty-two buildings was 123.51 acres, and the total cost $6 939 992.26. The cost was for the bare buildings, and did not include sculptural or other decorations, or the architects' compensation.

Building	Dimensions, ft	Area, acres	Total cost	Cost, sq ft
Art	161×346	1 42	$967 833 90	$5 45
Two Art Pavilions, each	144×423	3 14		
Art Building Annex	106×150	0 41	39 388 99	2.48
Government Building	200×736	3 86	328 980 00	2.23
Government Fisheries	136×136	0 42	45 000 00	2.43
Mines and Metallurgy	525×750	9.08	488 848.50	1.24
Liberal Arts	525×750	8 80	471 820 95	1.20
Education and Social Economy	525×758	7.70	323 950.75	0.81
Manufactures	525×1 200	13 47	711 510.00	1.13
Electricity	525×758	6 67	408 531.57	1.03
Varied Industries	525×1 200	10 28	704 067.96	1.12
Machinery	525×1 000	9.48	509 110.50	0 97
Steam, Gas and Fuel	301×326⅔	2 25	135 480.00	1 38
Transportation	525×1 300	15 70	674 853.42	0.99
Horticulture	374×782	5 42	225 342 27	0.77
Agriculture	500×1 600	18 62	520 491.07	0.58
Forestry, Fish and Game	300×600	4.07	168 883.38	0.94
Festival Hall	195 in diameter, exclusive of annex	1.09	215 899.00	. .

* Given by E. C. Chankland, chief engineer.

The Panama-Pacific International Exposition at San Francisco (1915). The following table was taken from the Official History of the International Celebration held at San Francisco, by Frank Morton Todd. The eleven buildings, exclusive of the California Building and the Motor Transportation Building, which were exceptional, covered over 64 acres, including grading, sewers, conduit systems, foundations and everything complete.

Ground Areas and Construction Costs

Building	Area, sq ft	Cost	Cost per sq ft
Fine Arts and Annex*	148 558	$631,929.92	$4.25
Education	205 127	300 183.04	1.46
Food Products	236 752	326 594.69	1 38
Liberal Arts	251 269	325 447.90	1 29
Agriculture	328 419	386 351.48	1 17
Manufactures	234 481	317 436 35	1.35
Transportation	314 345	451 560 70	1 43
Varied Industries	219 453	296 554 07	1 35
Mines and Metallurgy	252 613	338 549.25	1.34
Machinery	369 562	655 336.35	1 77
Horticulture	234 486	352 615.90	1.50
	2 795 065	$4 382 559.65	$1.56 (average)

* Annex had two floors.

Sesqui-Centennial International Exposition at Philadelphia (1926). The table is taken from data prepared by the late John Molitor, City Architect and Supervising Architect of the Exposition, and also from his successor as City Architect, Wm. S. Covell.

Ground Areas and Construction Costs* of Principal Buildings

Building	Ground area, sq ft	Extreme dimensions	Cost	Cost per sq ft	Cost per cu ft
Liberal Arts	338 596	970×392	$976 649 88	$2.88	$.084
Agriculture and Food Products	367 592	970×460	957 427 76	2 60	.076
Auditorium	111 273	445×274	471 314 00	4.23	.10
Transportation	316 124	880×400	938 196 00	2 96	.06
Educational	104 936	524×208	441 389 00	4 20	.14
Fine Arts	68 000	496×258	325 881.86	4 79	.22
Administration (two stories)	9 198	200× 73	70 541 66	7.66	.26

* No heating was installed in any building except the Administration Building.

Depreciation of Buildings

The Average Annual Physical Depreciation of Buildings is given in the following table:*

Building	Annual physical depreciation, per cent
Academies with dormitories	3–4
without dormitories	3½
Agricultural, brick	1–3
frame	2–4
Amusement buildings	1
Apartment-houses, fire-proof	1½
non-fire-proof	3 (5 yr) 2½ (5 yr) 2 for remainder of life (Zangerle)
Asylum, brick or stone	2½–3
frame	3½–4
Auditorium	1
Bakery	3
Bank	2–2½
Barns, brick	3
frame	4
Bath-houses	1
Brick residences, occupied by owner	1–1¼
occupied by tenant	1¼–1½
Frame residences, occupied by owner	2–2½
occupied by tenant	2½–3
Churches, brick	1–2
frame	3–4
Clubhouses	1
Cotton mills	2–3½
Court-houses	2–5
Exhibition halls	1
Factories	
all frame	5
box and door	5
candle, brick	3
frame	5
canning, brick	2
frame	4
cheese, brick	2½
frame	3½
hat	1½–2½
engineering	5–10
starch, brick	3
frame	5
Filling stations, oil, ordinary wood or corrugated steel	20
brick and concrete	10
Fire-proof, modern	2–4
Flour-mill, brick or stone	2–2½
frame	3–3½
Garages	4
Gas-plant	1⅓–3
Glass-works, brick or frame	4
concrete	2½
Grain-elevators	2½–4½
Hardware-works, brick	3½
frame	4
Horticultural	2
Hotel, brick and stone	2½–3½
fire-proof	1½
frame	3½–4½
non-fire-resisting	3 (5 yr) 2½ (5 yr) 2 for remainder of life
Ice-house, brick	3
frame	4–10
Industrial, fire-resisting	1½ (10 yr) 1 for remainder of life
Institutions	2–2¼
Kilns, brick, tile, or concrete	10
frame	25
Lofts and factories	2⅔–3
Machine-tool industry, brick	3
frame	5
Mills, paint	7½
paper	2½–3½
planing	5
rolling, brick	2½
frame	4
saw	7–10
woolen	2–3½
Museums	1
Nail-mill, brick	2½
frame	4

* This table has been compiled from many sources by Prentice-Hall, Inc., of New York City and is published here by permission.

Depreciation of Buildings (Continued)

Building	Annual physical depreciation, per cent	Building	Annual physical depreciation, per cent
Nut or bolt mill, brick	3–3½	Plants, sulfuric acid	15
frame	4–4½	welding and cutting	12½
Office, face-brick or stone	1–1⅓	X-ray	7½
common brick	3⅓–5	Porkhouses	2½–4
fire-proof	1½ (6 yr) 1 for remainder of life	Post fence, ash	11
frame	6	fur oak	6⅔
Plants, absorption, gasoline	8–13⅓	catalpa	6⅔
acetylene gas	10	cherry	10
ash hauling	10	chestnut	6⅔
asphalt paving	10	elm	11
blast furnace	6	hemlock	11
bleaching, dyeing, finishing	7½	locust	4
brass manufacturing	3	mulberry	6
brazing	15	osage orange	3½
cement, Portland, manufacturing	5–10	pine	9
chemical	3–10	red cedar	5
chlorination	5	red oak	15
coal-handling	10	sassafras	11
color, paint and varnish	5–7½	steel	3⅓
cooling water	7½	stone	3
cutting and welding	12½	tamarack	10
disposal sewage	2½	walnut	9
filtration masonry	2	white cedar	7
flat spinning	7½	white oak	7
flour-milling	5–7½	willow	16
galvanizing	12½	concrete	2
gas, natural	20	Post-Office	4
gasoline, natural gas	8–13⅓	Pottery, brick	3½
hotel	7½	frame	5
ice	5	Power plant	2-5
jam factory	5	brick and reinforced concrete	⅔
jewelry manufacturing	5	brick	1
lace, embroidery and muslin manufacturing	7½	frame, heavy	1½
laundry	5–10	light	2
linen-weaving	7½	electric light and railway	2–4
linoleum and floor cloth manufacturing	5–7½	Prison	1
ore production	7½	Quarry	2½
photo-engraving	5	Railroad, frame	5
printing	5–10	Refineries, oil, located at point, assuming long supply of crude oil	5
pumping, mine	5–12½	oil, located at point, assuming supply for several years	10
refrigeration	7	oil-skimming plants and small refineries of poor construction or located at points where supply of crude oil is not assumed for a long period of time	17
silk-manufacturing	7½		
steel-manufacturing	15		

Depreciation of Buildings (Continued)

Building	Annual physical depreciation, per cent	Building	Annual physical depreciation, per cent
Refineries, sugar	7½	Steel construction	2–7
brick	2½	Store, brick	1–5
frame	4	frame	1–6
Retort houses, gas	3⅓–10	fire-proof	1–1½
Rolling mills, brick	2½	Storehouses, frame	6½
frame	4	Substation, public utility	2–7
Roundhouses, railroad	3	Tanneries, brick	2½–3
Rubber factory	12½	frame	3½–4
Saltworks, frame	4	Telephone and telegraph	2–3
Schools	4	Tenement, brick	3⅓–4
Sheds, coal	4–5	frame	6⅔–10
dry	5	Theater	6
poultry, portable	10	Warehouse, brick	2–3
railroad, frame	5	brick and steel	4–6
ship	3⅓	mill-type construction	4
snow	4	steel	3
Shops, blacksmiths, stone or brick	5	wood	3–8
frame	8	Windmills	7½
machine	10–15	Woolen mills, brick	2–2½
brick	3–5	frame	3–3½
frame	4–8	Works, brick-making, brick	3½
railroad	2⅔–3	glass, brick	3–4
1st and 2d class	1⅓–2	frame	4–4½
Stables, brick	3–5	hardware, brick	3½
frame	4–10	frame	4
Starch plant, brick	3	nail, brick	2½
frame	5	frame	4
Stations and waiting-rooms, railroads	2–3	salt, frame	4

THE QUANTITY SYSTEM

Explanation of the System. The QUANTITY SYSTEM is not, as some persons have supposed, merely the taking off of a list of items by one person, probably with uncertain accuracy, for some other person's use. It means the careful measurement by a disinterested expert specially trained in this kind of work, that is, a QUANTITY SURVEYOR. This specialist proceeds in a manner quite different from that of the average contractor. He follows a certain recognized order and system in taking off quantities, abstracting and billing, with a view to eliminating errors. He uses certain uniform standards of measurements and expressions well understood by bidders. His checking and rechecking methods to ensure accuracy must be studied to be appreciated by those to whom the quantity system is unknown. A record is kept of every item, however small, having a money-value. These items are classified and arranged, each under its proper trade or department, in methodical order. Guess-work methods are unknown to the quantity surveyor, while his accuracy and attention to even small details is worthy of comment. Every bidder figures from a copy of the surveyor's quantities furnished to each one, with

(if desired) the plans and specifications. The surveyor who does this work is a professional man similar to the engineer or the architect. He should, in fact, have, and he usually has had, experience in these professions, and in addition, at practical experience acquired in the field in actual contact with and superintendence of construction-work.

Method of Procedure. Such a surveyor, in taking off quantities from an architect's or engineer's drawings, readily detects any discrepancies due to hasty preparation or other cause. The attention of the architect or engineer is called to such matters by the quantity surveyor, as he goes on with his work. Detected in this way, all uncertainties are at once corrected and adjusted, so that by the time the drawings and specifications reach contractors, everything has been made plain and accurate and the possibility of error in quantities can therefore be disregarded. The resulting document, the BILL OF QUANTITIES, is then either printed or otherwise reproduced, and a facsimile copy supplied free of cost to each bidder who inserts his unit price opposite each item and in an hour or two foots up the money-cost in dollars and cents. This is really all that a contractor should be expected to do (for nothing). The BILL OF QUANTITIES contains everything the contractor is called upon to perform or furnish, in order to complete his contract. In short, the bid becomes a proposal to do a certain FIXED QUANTITY of work, no more and no less. This then, briefly, is the main underlying principle of the QUANTITY SYSTEM: a definite quantity of work for a definite price, and the elimination of every condition which now compels bidders to take chances.

Advantages Claimed for the Quantity System. The following are the advantages claimed for the system:

(1) An immense saving of time and money now wasted by bidders; all doing the same thing, going over the same ground, and each arriving at a different result.

(2) Safer bids, as the work to be performed is clearly written out in the bill of quantities, which can be the essence of the contract.

(3) No expense to the bidder; the owner pays for the quantities knowingly. The owners pay now, but this fact is not brought to their attention, and it does not occur to them. The percentage added to a bidder's net cost is not all profit, a certain portion being absorbed in overhead charges, including cost of estimating, which, of course, is ultimately borne by owners.

(4) Saving of disputes arising from ambiguities, oversights, and even errors, all causing extra claims more or less just, but usually vexatious, and sometimes embarrassing.

(5) Better opportunities for the competent bidder, as the bidders all work up and price from the same basis.

(6) Better work and greater harmony If no part of the work is omitted there is less reason to skin the work, a proceeding which produces friction, or worse.

(7) Misunderstandings are reduced. The bill of quantities states clearly what is intended, and is a sort of clearing-house for the drawings and specifications.

(8) Neither party can obtain an advantage over the other on quantity or description of work.

(9) No disputes with subbidders, it being clearly stated what each trade is to furnish.

(10) Contractors have no figuring of quantities to do and can therefore devote more time to buildings in hand and save profits now lost for want of their personal supervision.

(11) Fewer inferior contractors as lowest bidders.

(12) Fewer extras, which are usually a trouble to all concerned.

(13) The architect or engineer has the assistance by collaboration of the professional quantity surveyor, who is available, also, for preliminary figures. This advance-information, now so often furnished by a prospective bidder, creates undesirable obligations.

(14) No change or reorganizing of architects' offices is entailed. Much detail-work now involved in receiving bids could be taken care of in the quantity surveyor's office.

(15) The drawings and specifications having been previously made as complete as possible, subsequent inconvenience to contractors and foremen on the job, and inquiries at the architects' offices for explanations become unnecessary. The BILL OF QUANTITIES gives detailed information which cannot be well given by drawings.

DIMENSIONS AND DATA USEFUL IN THE PREPARATION OF ARCHITECTS' DRAWINGS AND SPECIFICATIONS

Dimensions for Furniture. For the convenience of draughtsmen when designing furniture or providing space for a special article the following approximate dimensions are given:

Chairs and Seats. The average figures taken from a variety of good chairs are: Height of the seat above the floor, 18 in; depth of the seat, 19 in; the top of the back above the floor, 38 in. Usually the seat increases in depth as it decreases in height, while the back is higher and slopes more. Twenty inches inside is a comfortable depth for a seat of moderate size. Chair-arms are about 9 in above the seat. The slope of the back should not be more than one-fifth the depth of the seat. A LOUNGE is 6 ft long and about 30 in wide.

Tables vary in shape and size almost as much as chairs. Writing-tables and dining-tables are made 2 ft 5 in high, and the type of sideboard called a CARVING-TABLE is made 3 ft high to the principal shelf; but tables for general use are 2 ft 6 in high. DINING-TABLES are made from 3 ft 6 in to 4 ft wide and to extend from 12 ft to 16 ft by means of slides within the frame. This frame should not be so deep as to interfere with the knees of any one sitting at the table; that is, there must be about 2 ft clear space between it and the floor. The smallest size practicable for the KNEE-HOLES of desks and library-tables is 2 ft high by 1 ft 8 in wide, the width to be increased as much as possible.

Bedsteads are classed as SINGLE, THREE-QUARTERS, and DOUBLE. A single bed is from 3 to 4 ft wide inside; a three-quarter bed, from 4 ft to 4 ft 6 in; a double bed, 5 ft. Bedsteads are from 6 ft 6 in to 6 ft 8 in long inside. Footboards are from 2 ft 6 in to 3 ft 6 in and headboards from 5 ft to 6 ft 6 in high. Single beds for dormitories are often made only 2 ft 8 in wide.

Bureaus vary in shape and size to such an extent that it is almost impossible to say that any dimension is fixed. Convenient sizes are: body, 3 ft 5 in wide, 1 ft 6 in deep, and 2 ft 6 in high; or 4 ft wide, 1 ft 8 in deep and 3 ft high.

Chiffoniers are about 3 ft wide, 1 ft 8 in deep and 4 ft 4 in high.

Cheval-Glasses are made, if large, 6 ft 4 in high and 3 ft 2 in wide. If small, 5 ft high and 1 ft 8 in wide. If medium, 5 ft 6 in high and 2 ft wide.

Sideboards may be from 4 to 6 ft long and from 20 in to 2 ft 2 in deep.

Upright Pianos vary from 4 ft 10 in to 5 ft 6 in in length, from 4 to 4 ft. 9 in in height and are about 2 ft 4 in deep over all.

Miniature and Baby-Grand Pianos vary from 5 ft 10 in to 6 ft in length, and are about 4 ft 10 in in width.

Grand Pianos vary from 5½ ft to 6 ft 10 in in length, and are about 4 ft 10 in in width.

Concert-Grand Pianos are about 8 ft 10 in in length and 5 ft in width.

Billiard-Tables (Collender), 4 by 8 ft, 4 ft 2 in by 9 ft and 5 by 10 ft. Size of room required 13 by 17 ft, 14 by 18 ft and 15 by 20 ft, respectively.

Dimensions of Plumbing-Fixtures. Enameled-Iron Bath-Tubs. Standard sizes for roll-rim baths are: nominal lengths, 4 ft, 4½ ft, 5 ft, 5½ ft and 6 ft; width over all, from 30 to 34 in. Specially narrow tubs are made from 25 to 29 in wide. The actual length over rim is usually 1 or 2 in more than the nominal length, and 3 in will include an ordinary overflow-pipe.

Lavatories are made in various sizes, the average sizes varying from 20 in by 18 in to 30 in by 24 in.

Water-Closets have the following average dimensions. height to top of tank, 34 in to 39 in; length of tank, 23 in, wall to front, 24 in to 30 in. The smallest space permissible for water-closet compartments, where doors open out, is 2 ft 4 in by 3 ft 3 in. If the doors open in, the compartment should be 3 by 5 ft.

Closet-Ranges, used in schools and factories, are made 24, 27 and 30 in, center to center of partitions. For graded schools, 24 in is ample, and for factories, 27 in. The range usually occupies a space 28 in in depth, if set against a wall.

Urinal-Partitions should be from 24 to 27 in, center to center of partitions; depth of partitions, 20 or 22 in; of ends, 2 ft; of bottom slab, 2 ft; height of partitions, from 4 ft 6 in to 5 ft 6 in. Porcelain stall urinals have nominal widths of 18 and 24 in, and may be set with or without spaces between. Heights are usually about 40 in above the floor-level.

Kitchen-Sinks are made in a great variety of sizes, those most commonly used being 16 by 24 in, 18 by 30 in, 18 by 36 in, 20 by 30 in and 20 by 36 in. The depth inside, for the sizes given, is 6 in.

Cast-Iron Slop-Sinks, common sizes, are 16 by 16 in, 16 by 20 in, 18 by 22 in and 20 by 24 in; 12 in deep.

Copper Pantry-Sinks. Common sizes are 12 by 18 in, 14 by 20 in and 16 by 24 in; 6 in deep.

Laundry-Trays of slate or soapstone are commonly made 2 ft wide over all and 16 in deep. Lengths over all, two-part trays, 4 ft and 4 ft 6 in; three-part trays, 6 ft, 6 ft 6 in and 7 ft. Earthen and porcelain trays come separately, and are connected as required. The dimensions of each tray are 2 ft or 2 ft 7½ in in length, 2 ft 1½ in in width and 15 in in depth, inside. The length required for two 2-ft trays is 4 ft 1 in; for three trays, 6 ft 2 in; and for four trays, 8 ft 3 in.

Range-Boilers are 12 in diameter for 30-gal, 14 in for 40-gal, 16 in for 52-gal and 63-gal, 22 in for 100-gal and 120-gal boilers.

Passenger Automobile. Length from 11 to 19 ft; width, 6 ft; height, 5 ft 6 in to 6 ft 4 in. Turning diameters vary from 34 to 53 ft.

Dimensions of Locomotives and Cars. The dimensions of locomotives and freight-cars vary considerably, but the following will cover those in common use:

Locomotives. From 15 ft 4 in to 16 ft 3 in to top of stack from top of rail; extreme width of cab, 11 ft 6 in. Doors to admit locomotives should be from 12 to 13 ft wide and 18 ft high. Extreme width over cylinders, 10 ft 6 in. to 11 ft 9 in.

Automobile and Furniture-Cars are from 15 ft to 16 ft 2 in, from top of track to top of rake-staff; floor, 3 ft 6 in to 4 ft 2 in from track; extreme width, 10 ft to 11 ft.

Stock-Cars, from 13 ft 5 in to 14 ft 6 in, from top of track to top of brake-staff; floor, 3 ft 6 in to 4 ft 2 in from track; extreme width, 10 ft to 11 ft 2 in.

Refrigerator-Cars, 14 ft 6 in, from top of track to top of brake-staff; floor, 3 ft 6 in to 4 ft 2 in from track; extreme width, 10 ft.

Hopper-Cars, from 10 ft to 12 ft 6 in, from top of rail to top of brake-staff; extreme width, from 10 ft to 10 ft 6 in.

Gondola and Open-top Cars, from 7 ft to 13 ft from top of rail to top of brake-staff. Floor, 3 ft 2 in to 4 ft 6 in; extreme width, 10 ft to 11 ft 3 in.

Doors to admit box-cars should be 22 ft high, to permit a man to stand on top. Minimum width of doors should be 13 ft 11 in.

Passenger-Coaches vary from 14 to 16 ft in height and from 10 to 11 ft in width. Doors to admit cars should give at least 12 in clearance on each side, and 2 ft overhead.

The Gauge of a railroad track is the distance between the inner sides of the heads of the two rails. The STANDARD GAUGE is 4 ft $8\frac{1}{2}$ in.

Capacity of Freight-Cars. Car-Loads. The capacity of freight-cars, and the minimum car-loads, vary so greatly that no accurate general information can be given. For heavy freight, 25 tons is an average load; for light freight, from 12 to 15 tons; for household goods, from 6 to 12 tons; for lime, 15 tons is about a minimum load; for cement, 20 tons. The minimum car-load, to obtain car-load rates, varies with different roads, and also with the rate made; a low rate is usually made on the basis of a big load. Thirty tons is a good load for heavy freight, and 40 tons is about the maximum, except for special cars. Coal-cars hold from 50 to 70 tons

Street Trolley-Cars are about 8 ft 6 in wide for the car proper, and the steps project about 8 in. Height from track to top of coach, 11 ft 6 in; the trolley-stand is 18 in higher. The length varies, up to 42 ft. Trucks for a 41 ft 6 in car are about 24 ft apart. Wheel-base, 4 ft center to center. Radius of shortest curve in Denver, Colo., 35 ft to midway between rails.

Miscellaneous Dimensions. Horse-Stalls. Width, from 3 ft 10 in to 4 ft or else 5 ft or over; length, 9 ft. The width should never be between 4 ft and 5 ft.

Coal-Bins. Volumes of coal bins may be computed on the assumption that a ton of anthracite coal (2 240 lb) occupies 40 to 43 cu ft, while a ton of bituminous coal occupies 43 to 48 cu ft.

Dimensions of Standard Bowling-Alleys.* For ONE PAIR OF ALLEYS· Room necessary, 83 ft over all; 11 ft 6 in wide, 60 ft from foul-line to head pin, 3 ft for pins to back of alley, 4 ft for pin-pit, 8 in deep in front, 6 in in back; alleys

* Dimensions furnished by The Brunswick-Balke-Collender Company, New York City.

of maple flooring, should extend on and beyond the foul-line 12 ft, and then 4 ft more, making a 16-ft approach to the foul-line for the player to run to deliver the ball. For ONE ALLEY: Same length, 83 ft, width, 6 ft 3¼ in; closer dimensions; beds 42 in, gutters 9 in, division-pieces 2¾ in ball-return 9¾ in.

ONE ALLEY:	In	ONE PAIR OF ALLEYS:	In
Ball-return	9¾	Ball-return	9¾
First-division piece	2¾	First-division piece	2¾
Gutter	9	Gutter	9
Bed	42	Bed	42
Gutter	9	Gutter	9
Second-division piece	2¾	Second-division piece	2¾
6 ft 3¼ in =	75¼	6 ft 3¼ in =	75¼

To the 75¼ in of the PAIR OF ALLEYS, should be added

Gutter	9
Bed	42
Gutter	9
Third-division piece	2¾
	138

Additional room should be provided for the bowlers and spectators as these dimensions are for the alleys only.

Dimensions of a Barrel. Diameter of head, 17 in; diameter at bung, 19 in; length, 28 in; volume 7 680 cu in

Weight of Men and Women. The average weight per person of twenty thousand men and women weighed at Boston, Mass, in 1864, was: men, 141½ lb; women, 124½ lb.

Wooden Flagpoles. For a flagpole, extending from 30 to 60 ft above the roof, the following proportions give satisfactory results The diameter at the roof should be $1/50$ the height above the roof, and the top diameter one-half the lower. To profile the pole, divide the height into quarters; make the diameter at the first quarter above the roof, fifteen-sixteenths of the lower diameter; at the second quarter, seven-eighths, and at the third quarter, three-quarters the lower diameter.

Steel Flagpoles.* Many Departments of Education have abandoned the use of wooden flagpoles and are using steel flagpoles. For an ordinary building, 60 ft in height above the curb, a pole 43½ in height is used, which is sufficient for the tackle of a large or post-flag, for the reason that roof-parapets are very low. Each pole is required to be fitted complete with a cast-iron, galvanized, revolving truck, mounted on crucible-steel pins, the cap beneath it, also, being of galvanized iron. The truck is fitted with two 4¾-in bronze sheaves on Tobin-bronze pins, surmounted with an 8-in 20-oz copper ball, acid-cleaned and painted with four coats of the best English weather-proof sizing, and covered with XXXX leaf-gold. One or more field-joints are permitted in the length of the pole, which are determined according to standard details, the bands being secured to the male tube, and both edges of the inner band and the shoe being machine-beveled to insure a perfect fit. The female tube is drilled and secured to the male shoe with tap-screws of sufficient

* From data compiled by E. S. Hand from notes furnished by C. B. J. Snyder.

strength to carry the upper section of the pole, and the ends of the screws are upset. The exposed ends of the female tube are chamfered and calked tight. A steel collar or band, to receive the copper flashing, is secured to the pole and braced just above the roof-lines.

Dimensions of Schoolrooms. The dimensions of schoolrooms as required by the State Board of Education of the State of Pennsylvania are as follows: "Elementary and grade classrooms should measure 23 ft by 30 ft and be at least 12 ft high. A room 24 ft by 32 ft with a minimum ceiling-height of 12 ft is approved for 40 pupils . . . . Every classroom shall have not less than 15 sq ft of floor-space, and not less than 200 cu ft of air-space per pupil."

Heights of Blackboards in Schoolrooms. The heights from floor to base of blackboard should be as follows:

Primary rooms	1 ft 9 in
Elementary rooms	2 ft 1 in
High-school rooms	2 ft 6 in

Slate blackboards are made 3 ft 6 in, 4 ft and 4 ft 6 in high, 4 ft being a very common and satisfactory height.

School Doors and Stairways. Wardrobes when placed at the end of classrooms should be entered from the classrooms only. Classroom-doors should open into the corridors, so as to afford the teacher control in case of panic. All exit-doors should open out. All stairways should be shut off from corridors by means of self-closing doors, which, together with the stairways and the enclosures, should be of fire-proof materials. Stairways should be of sufficient number to permit of the building being vacated within three minutes from the time a signal is given. This can be effected by allowing a linear width of 4 ft for the first 50 persons and 12 in additional for each 100 persons in excess thereof. No stairway is to be less than 4 ft nor more than 5 ft in width. Corridors should be 9 ft in width. Exits should be planned so as to provide 15 lin ft for the first 500 persons and 6 in additional for each 100 persons in excess thereof. No stairway should have more than 15 steps in any one flight, changes in direction being effected by a square platform and no winders being used. No stair-door or exit-door should open out over a step. Platforms are to be provided for such doors and are to extend at least 1 ft beyond the edge of the door when standing open.

Stairs. The RISE of a stair is the height from the top of one step to the top of the next. The TOTAL RISE is the height from floor to floor. The RUN is the horizontal distance from the face of one riser to the face of the next. RISERS are the upright boards or other materials forming the faces of the steps, and the TREADS are the horizontal pieces or surfaces on which the feet tread. Treads are usually from 1¼ to 1¾ in wider than the run, on account of the NOSING. The height of an individual riser or the RISE of any stairs is found by dividing the TOTAL RISE by the number of risers. The RUN of the stairs may be fixed at will unless the space is cramped, but to secure a comfortable stair the run must bear a certain relation to the rise.

Rules for Dimensions of Treads and Risers. For ordinary use a rise of from 7 to 7½ in makes a very comfortable flight of stairs. For schools and for stairs used by children the rise should not exceed 6 in. Stairs having a rise greater than 7¾ in are steep. The width of the run should be determined by the height of the rise; the less the rise the greater should be the run, and *vice versa*. Several rules have been given for proportioning the run to the rise:

(1) THE SUM OF THE RISE AND RUN should be equal to from 17 to 17½ in.

(2) THE SUM OF TWO RISERS AND A TREAD should not be less than 24 nor more than 25 in.

(3) THE PRODUCT OF THE RISE AND RUN should not be less than 70 nor more than 75.

These rules apply only to stairs with nosings. Stone stairs without nosings should have at least 12-in treads for adults.

Height of Hand-Rail. In dwellings, hotels, apartments, étc., the height of the rail should be about 2 ft 6 in above the tread, on a line with the face of the riser. For grand staircases the height may be reduced to 2 ft 4 in. On steep stairs the height should be from 2 ft 7 in to 2 ft 9 in. The rail should also be raised over winders. On landings, the height of the rail should be equal to the height of the stair-rail, measured at the center of the tread, the usual height in residences being from 2 ft 8 in to 2 ft 10 in.

Sash-Cords.* Until a few years ago, linen or cotton cord only was used for connecting weights with the sashes of double-hung windows, and cord is still more extensively used than either ribbons or chains. For windows of ordinary size a good brand of cord will wear for a long time, and this material will probably never be entirely displaced by metal. "Tests made at the Massachusetts Institute of Technology show that cords wear much longer than chains, though they have less tensile strength. Cords should be smooth and round, so that each strand bears its part of the stress, and well glazed, so that they have a smooth surface and consequently less wear from friction with the wheel of the pulley." It has been found that cord can be braided too hard for durability, yet if it is braided so as to be very flexible it may be so soft that it will stretch and cause great annoyance by permitting the weight to hit the bottom of the weight-box. The architect, however, should always specify the particular BRAND and SIZE of cord to be used, and also the diameter of the pulley. Among the leading brands of sash-cord at present are the Samson Spot,† and the Silver Lake A.‡ These brands are superior to the ordinary braided cords, which are made from inferior yarns to meet the jobbers' requirements for price. In addition to other most excellent qualities, the Samson cord offers an additional advantage that architects will appreciate; it has a colored strand woven through it, which shows in spots on the surface and thus enables one to tell at a glance that no other cord has been substituted. The Silver Lake A sash-cord has the name Silver Lake A branded on every foot of cord; but unless the letter A accompanies the name a second grade of cord is denoted. The marking of the cord by color, or any other device, does not alter the quality of the cord. Special marks may be applied to inferior cords as well as to the best. The following numbers should be specified for the different weights of sash-weights:

Relative Sizes of Sash-Cords, Weights and Pulleys

Size-number.	6	7	8	9	10	12
Diameter in inches	3/16	7/32	1/4	9/32	5/16	3/8
Feet per pound . .	66	55	44	36	27	20
Suitable for weights in pounds up to. . . .	5	12	20	30	40	50
Minimum diameter in inches of pulley allowable	1½	1¾	2	2¼	2½	3

* The following notes, relating to Sash-Cords, Sash-Chains, Sash-Ribbons, Sash-Weights and Sash-Balances, are condensed from articles by the late Professor Thomas Nolan.

† Manufactured by the Samson Cordage Works, Boston, Mass.

‡ Manufactured by the Silver Lake Company, Newtonville, Mass.

Table of Treads and Risers*

The figures in the first column can be used for either treads or risers

Height of riser or width of tread, inches	2 ft	2 in	3 ft	3 in	4 ft	4 in	5 ft	5 in	6 ft	6 in	7 ft	7 in	8 ft	8 in	9 ft	9 in	10 ft	10 in	11 ft	11 in	12 ft	12 in	13 ft	13 in	14 ft	14 in
	Number of treads or risers; Story-height or horizontal length of run																									
6	1	0	1	6	2	0	2	6	3	0	3	6	4	0	4	6	5	0	5	6	6	0	6	6	7	0
6¼	1	0½	1	6¾	2	1	2	7¼	3	1½	3	7¾	4	2	4	8¼	5	2½	5	8¾	6	3	6	9¼	7	3½
6½	1	1	1	7½	2	2	2	8½	3	3	3	9½	4	4	4	10½	5	5	5	11½	6	6	7	0½	7	7
6¾	1	1½	1	8¼	2	3	2	9¾	3	4½	3	11¼	4	6	5	0¾	5	7½	6	2¼	6	9	7	3¾	7	10½
7	1	2	1	9	2	4	2	11	3	6	4	1	4	8	5	3	5	10	6	5	7	0	7	7	8	2
7⅛	1	2¼	1	9⅜	2	4½	2	11⅝	3	6¾	4	1⅞	4	9	5	4⅛	5	11¼	6	6⅜	7	1½	7	8⅝	8	3¾
7¼	1	2½	1	9¾	2	5	3	0¼	3	7½	4	2¾	4	10	5	5¼	6	0½	6	7¾	7	3	7	10¼	8	5½
7⅜	1	2¾	1	10⅛	2	5½	3	0⅞	3	8¼	4	3⅝	4	11	5	6⅜	6	1¾	6	9⅛	7	4½	7	11⅞	8	7¼
7½	1	3	1	10½	2	6	3	1½	3	9	4	4½	5	0	5	7½	6	3	6	10½	7	6	8	1½	8	9
7⅝	1	3¼	1	10⅞	2	6½	3	2⅛	3	9¾	4	5⅜	5	1	5	8⅝	6	4¼	6	11⅞	7	7½	8	3⅛	8	10¾
7¾	1	3½	1	11¼	2	7	3	2¾	3	10½	4	6¼	5	2	5	9¾	6	5½	7	1¼	7	9	8	4¾	9	0½
7⅞	1	3¾	1	11⅝	2	7½	3	3⅜	3	11¼	4	7⅛	5	3	5	10⅞	6	6¾	7	2⅝	7	10½	8	6⅜	9	2¼
8	1	4	2	0	2	8	3	4	4	0	4	8	5	4	6	0	6	8	7	4	8	0	8	8	9	4
8¼	1	4½	2	0¾	2	9	3	5¼	4	1½	4	9¾	5	6	6	2¼	6	10½	7	6¾	8	3	8	11¼	9	7½
8½	1	5	2	1½	2	10	3	6½	4	3	4	11½	5	8	6	4½	7	1	7	9½	8	6	9	2½	9	11
9	1	6	2	3	3	0	3	9	4	6	5	3	6	0	6	9	7	6	8	3	9	0	9	9	10	6
9½	1	7	2	4½	3	2	3	11½	4	9	5	6½	6	4	7	1½	7	11	8	8½	9	6	10	3½	11	1
10	1	8	2	6	3	4	4	2	5	0	5	10	6	8	7	6	8	4	9	2	10	0	10	10	11	8
10½	1	9	2	7½	3	6	4	4½	5	3	6	1½	7	0	7	10½	8	9	9	7½	10	6	11	4½	12	3
11	1	10	2	9	3	8	4	7	5	6	6	5	7	4	8	3	9	2	10	1	11	0	11	11	12	10
13	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	13	0	14	1	15	2
14	2	4	3	6	4	8	5	10	7	0	8	2	9	4	10	6	11	8	12	10	14	0	15	2	16	4

* The editor is indebted to T. Z. Talley for the calculations and arrangement of this table.

Table of Treads and Risers* (continued)

The figures in the first column can be used for either treads or risers

Height of riser or width of tread, inches	Number of treads or risers																									
	15		16		17		18		19		20		21		22		23		24		25		26		27	
	Story-height or horizontal length of run																									
	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
6	7	6	8	0	8	6	9	0	9	6	10	0	10	6	11	0	11	6	12	0	12	6	13	0	13	6
6¼	7	9¾	8	4	8	10¼	9	4½	9	10¾	10	5	10	11¼	11	5½	11	11¾	12	6	13	0¼	13	6½	14	0¾
6½	8	1½	8	8	9	2½	9	9	10	3½	10	10	11	4½	11	11	12	5½	13	0	13	6½	14	1	14	7½
6¾	8	5¼	9	0	9	6¾	10	1½	10	8¼	11	3	11	9¾	12	4½	12	11¼	13	6	14	0¾	14	7½	15	2¼
7	8	9	9	4	9	11	10	6	11	1	11	8	12	3	12	10	13	5	14	0	14	7	15	2	15	9
7⅛	8	10⅞	9	6	10	1⅛	10	8¼	11	3⅜	11	10½	12	5⅝	13	0¾	13	7⅞	14	3	14	10⅛	15	5¼	16	0⅜
7¼	9	0¾	9	8	10	3¼	10	10½	11	5¾	12	1	12	8¼	13	3½	13	10¾	14	6	15	1¼	15	8½	16	3¾
7⅜	9	2⅝	9	10	10	5⅜	11	0¾	11	8⅛	12	3½	12	10⅞	13	6¼	14	1⅝	14	9	15	4⅜	15	11¾	16	7⅛
7½	9	4½	10	0	10	7½	11	3	11	10½	12	6	13	1½	13	9	14	4½	15	0	15	7½	16	3	16	10½
7⅝	9	6⅜	10	2	10	9⅝	11	5¼	12	0⅞	12	8½	13	4⅛	13	11¾	14	7⅜	15	3	15	10⅝	16	6¼	17	1⅞
7¾	9	8¼	10	4	10	11¾	11	7½	12	3¼	12	11	13	6¾	14	2½	14	10¼	15	6	16	1¾	16	9½	17	5¼
7⅞	9	10⅛	10	6	11	1⅞	11	9¾	12	5⅝	13	1½	13	9⅜	14	5¼	15	1⅛	15	9	16	4⅞	17	0¾	17	8⅝
8	10	0	10	8	11	4	12	0	12	8	13	4	14	0	14	8	15	4	16	0	16	8	17	4	18	0
8¼	10	3¾	11	0	11	8¼	12	4½	13	0¾	13	9	14	5¼	15	1½	15	9¾	16	6	17	2¼	17	10½	18	6¾
8½	10	7½	11	4	12	0½	12	9	13	5½	14	2	14	10½	15	7	16	3½	17	0	17	8½	18	5	19	1½
9	11	3	12	0	12	9	13	6	14	3	15	0	15	9	16	6	17	3	18	0	18	9	19	6	20	3
9½	11	10½	12	8	13	5½	14	3	15	0½	15	10	16	7½	17	5	18	2½	19	0	19	9½	20	7	21	4½
10	12	6	13	4	14	2	15	0	15	10	16	8	17	6	18	4	19	2	20	0	20	10	21	8	22	6
10½	13	1½	14	0	14	10½	15	9	16	7½	17	6	18	4½	19	3	20	1½	21	0	21	10½	22	9	23	7½
11	13	9	14	8	15	7	16	6	17	5	18	4	19	3	20	2	21	1	22	0	22	11	23	10	24	9
13	16	3	17	4	18	5	19	6	20	7	21	8	22	9	23	10	24	11	26	0	27	1	28	2	29	3
14	17	6	18	8	19	10	21	0	22	2	23	4	24	6	25	8	26	10	28	0	29	2	30	4	31	6

* The editor is indebted to T Z. Talley for the calculations and arrangement of this table.

For hanging sashes weighing over 40 lb, only the largest size of Samson or Silver Lake A cord, or some form of sash-chain or sash-ribbon, should be used, and the pulleys should be selected to fit the çord or chain. A guarantee that the cord will last at least twenty years may be had from either of the manufacturers mentioned above. The Samson wire-center sash-cord has recently been put on the market. This is really a metal sash-cord protected by a braided-cotton surface which acts as a noiseless cushion. It is claimed that it harmonizes with the window-finish and that it has greater durability than other sash-cords or metal devices. The standard color is that of dark mahogany, but this cord is made to order for large buildings in other colors to match the finish.

Sash-Chains. Of several styles of sash-chains on the market, the style most largely used is the flat-link chain. This chain is made either of steel, or of bronze composed of 95% copper and 5% of tin. For suspending very heavy sashes, doors and gates, a cable-chain has been extensively used. Star sash-chain is made of bronze-metal. The manufacturers of the Norris sash-pulley claim that a riveted chain that has joints only one way is almost sure to break when even slightly twisted, and that it is better to use two chains of the link-pattern running side by side over the same pulley. The strongest sash-chains are of steel, made rust-proof by the hot-galvanizing process, and electro-copperplated to give a bronze finish; and of a bronze-mixture which looks like copper, but is tougher and harder. One firm claims that its galvanized-steel sash-chain is from 11 to 45% stronger than any bronze or copper sash-chain and that it will resist fire for a much longer period. The tensile strength of their chain varies from 475 to 850 lb, according to the weight used.

Sash-Ribbons. These are sometimes used in hanging the sashes of buildings The ribbons are made of steel and aluminum-bronze or of some mixture of aluminum, and in 3/8, 1/2, 5/8, 3/4 and 7/8-in widths. They are claimed to be practically indestructible, but according to one series of tests it would appear that in some cases they do not wear as long as sash-cords or sash-chains. Some people object that the ribbons snap against the pulley-stiles, when the sash is raised or lowered, and thus make considerable noise. The 3/8-in ribbon may be used for a sash weighing up to 100 lb and requiring 50-lb weights. Sash-ribbons are now manufactured by a number of firms which also make the necessary attachments for weight and sash. For the best working of windows hung with ribbons, pulleys of the following sizes should be used:

For sashes weighing not over 40 lb,	2 in
For sashes weighing not over 60 lb,	2 1/4 in
For sashes weighing not over 100 lb,	2 1/2 in
For sashes weighing not over 150 lb,	3 in
For sashes weighing not over 250 lb,	3 1/2 in
For sashes weighing not over 300 lb,	4 in
For sashes weighing not over 350 lb,	4 1/2 in

Comparative Strength of Sash-Cords and Chains. The comparative strength and durability of sash-cords and chains have been determined by careful tests, but there is a great variation in both cases, due partly to variation in material, but principally to the relative sizes of the chain and pulley or cord and pulley. The cords or chains may be too light for the weights used, or the pulleys too small in diameter to carry the cord without undue bending. The pulleys may also have too narrow a groove or an uneven groove with sharp edges which cut the cords. The larger the diameter of the pulley, the longer the wear.

Tests* on Wire-Center Sash-Cord and Bronze Sash-Chains. The cord tested was size No. 8, ¼-in diam, Samson solid braided cotton cord with steel-wire cable center, ⅟₁₆ in in diam. The chains tested were of two different makes of bronze, size No. 2, purchased in the open market as typical bronze sash-chains, each recommended by a reputable dealer as the proper chain for use with a 25-lb window-weight. The tests for the better of the two chains are those given. Durability-tests were made by raising and lowering a 25-lb weight over a 2-in pulley, each movement corresponding to once opening and shutting a window. The cord was tested over the regular round grooved pulley ordinarily used for cords, and the chains were tested over the combination grooved pulley usually furnished for sash-chains. For the fire-tests the cords or chains were hung through an asbestos box in which a Bunsen flame under pressure was applied to all alike, the temperature being about 2 200° F. A 25-lb weight was attached in each case to keep the cord or chain under the same tension. The wire-center cord took about twice as long to burn through and wore about seventeen times as long as the bronze chain.

Tests on Wire-Center Sash-Cord and Bronze Sash-Chain

Durability-tests		Fire-tests	
Number of lifts before breaking		Length of time before parting	
Bronze chain	Samson wire-center cord	Bronze chain, sec	Samson wire-center cord, sec
34 944	659 892	42 5	78 5
37 486	592 559	40	75.5
37 381	632 230	39	77
32 948	594 114	32	75
40 356	631 286	.	....
31 234	577 154	. .	.. ..
40 790	504 032		...
27 874	637 796		
Average 35 377	Average 603 633	Average 38 4	Average 76.5

Weights of Sashes and Glass. In figuring the weights of windows, the weight of the glass may be taken at 3½ lb per ft for plate glass, 1⅛ lb for double-strength glass and 1 lb for single-strength glass. For the weight of the wooden sash, add together the height and width, in feet, of each sash, and multiply by 2.1 for 2¼-in sash, by 1⅔ for 1¾-in sash and by 1⅓ for 1⅜-in sash. These values are sufficiently accurate for determining the size of sash-cords and pulleys, but the weights should be determined by weighing each sash after it is glazed, as the weight of the glass varies considerably.

Iron Sash-Weights. The weights ordinarily used for balancing windows are made of cast iron, in the form of solid cylinders from 1⅜ to 2⅛ in in diameter, and from 7½ to 31 in long, with an eye cast in the upper end of each. The lengths vary with the weights, which are from 2 to 25 lb. Flat weights, which usually are called for in the Philadelphia and some other markets, are from 6

* Made at the Massachusetts Institute of Technology, May, 1914, by Professor E. F. Miller.

to $34\frac{1}{2}$ in long, from 2 to 30 lb in weight, and from $1\frac{1}{4}$ by $1\frac{5}{8}$ to $1\frac{5}{8}$ by $2\frac{1}{4}$ in in cross-section. In ordering sash-weights the number of pounds of each weight, and the sections and lengths of the boxes in which the weights will work, should be given. Ordinary weights have very rough eyes for the sash-cords. There are a few manufacturers in the East who make weights with a patent eye that will not cut the cord. A sectional sash-weight made with a well-designed hooking-device which has given satisfaction, is said to be one of the best on the market. Usually from three to six sections are required on each side to balance a sash properly. If the hooking-device fails near the top the upper sash cannot be closed and if at the bottom the window cannot be opened. It is then necessary to open the weight-box and rehang the sections before the window can be operated. In theory, sectional weights are ideal; in practice, however, they are not considered as satisfactory as solid weights. The Brown sectional weights are made $2\frac{1}{4}$ by $2\frac{5}{8}$ in and in weights of 6, 7, 8, 9 and 10 lb. They are made of both cast-iron and lead. It frequently occurs after a contract is let, that the glass is changed from double-thick to plate or prism glass. This means increased weight; but the length of the sash-weight cannot be increased and it, therefore, becomes necessary to increase the area of its cross-section. If the weight-box is detailed to take the regular round sash-weight, its general construction will be such that it will take a 2-in round sash-weight, but not a 2-in square sash-weight. This difficulty can be avoided by a little thought at the start. An added depth of $\frac{1}{4}$ in in the weight-box permits the use of a rectangular cast-iron sash-weight. The Sanborn sectional sash-weight is intended for use in large buildings of heavy construction. Because of the lack of uniformity in the weight of plate glass the required weights of sash-weights cannot be accurately determined previous to the hanging of the sashes. By the use of a sectional sash-weight, combinations of units can be made up to suit the requirements. The units are made square or rectangular in section in order to secure a maximum weight with a minimum length. An opening of 12 in in the side of the pocket is sufficient for hanging the largest unit. These units are manufactured in standard sizes to meet the general conditions found in the building trades.

Lead Sash-Weights. It often happens that for wide and low windows the weights if of iron would be so long that they would touch the bottom of the pocket before the bottom sash was fully raised. In such cases lead weights are usually resorted to, lead being 80% heavier than cast iron. By casting the weights square in section, whether of iron or lead, a considerable saving can be made in the lengths. One sash-weight manufacturer makes a specialty of compressed-lead sash-weights, with wrought and malleable-iron fastenings, centered so that the weights hang perfectly plumb; and when lead weights are necessary the architect will do well to specify the weights made by this company. These weights are made under hydraulic pressure, by which greater smoothness, solidity and density of metal is secured than is possible by the casting-process. A wrought-iron rod is run through the center, to which are securely attached the malleable-iron fittings. In hanging the sashes the weights for the upper sash should be about $\frac{1}{2}$ lb heavier than the sash, and for the lower sash, $\frac{1}{2}$ lb lighter.

Sash-Balances. Several devices have been patented for balancing sashes by means of springs instead of weights. The sash-balance consists of a drum on which the ribbon is wound, and which contains a coiled-steel clock-spring. immersed in oil; the spring sustains the weight of the sash. The common type very much resembles in outward appearance the ordinary sash-pulley, and is

applied practically the same way; the ribbons, which are made usually of aluminum-bronze, are attached to the sashes in the same manner as cords when weights are used. While the sash-balance in its best form works very satisfactorily, it will probably never entirely supplant the weight and axle-pulley for ordinary windows. There are many windows, however, for which sufficient pocket-room for weights cannot be obtained without spoiling the effect desired or narrowing the glass, as in some bay windows, or where it is undesirable to break the frame into the brick jamb. In such cases the sash-balance is almost invaluable. For hanging the glass doors of show-cases, sash-balances are usually preferable to weights Sash-balances are made in both side and top-patterns, but the former are recommended wherever there is room at the side of the frame for the depth of mortise required. For windows of the sizes usually found in residences, the depth of the sash-balance measured from the face of the pulley-stile will vary from 3 to 4 in; this can be provided for usually by cutting a hole, if necessary, in the masonry or studding back of the frame. As sash-balances require only a plank frame, the consequent reduction in the cost of the frame offsets the extra cost of the balance. In remodeling old buildings which have plank frames without weights, sash-balances are found to be a great convenience, since they can easily be inserted in the old frames. An advantage which all spring-balances possess is that they act most strongly when the sash is down, and so enable one to raise a binding window more readily than if it were hung with weights; while when the sash is up the springs barely suffice to hold it in position, and do not offer resistance to drawing it down. Of the various sash-balances on the market, the Pullman and the Caldwell are extensively used, and are undoubtedly reliable. The Pullman Unit sash-balance has been on the market many years and has proved satisfactory. These balances are now made with uniform-size face-plates for the various weights of sash with which they are to be used, and thus make it possible to have all mortises for the balances cut at the mill, as is now done for the regular cord-pulleys. The Caldwell sash-balance, both top and side-types, is much used by the United States Post Office and Navy Departments It is used also by the leading car-builders. The springs are made of high-grade cold-rolled tempered-steel wire, a material similar to that used for clock-springs The manufacturers guarantee these sash-balances for from ten to fifteen years.

Seating-Space in Churches and Theaters. The minimum spacing for pews, back to back, is 30 in. This spacing is fairly comfortable for occupants, but is a little cramped for persons passing by others into or out of the pews. A spacing of 32 in is to be preferred, and if there is abundance of room, the spacing may be made 33 in. Anything over 33 in is a waste of room. A space of 18 in in the length of the pew is considered a SITTING.

Opera or Theater-Chairs are made 19, 20, 21 and 22 in wide, center to center of arms, and in arranging them in rows where the aisles converge, the ends are brought to a line on the aisles by using a few chairs that are either narrower or wider than the standard width. For churches, a standard width of 20 in is the least that is desirable. For theaters, 21 or 22-in chairs are commonly used in the parquet, 20 or 21-in in the dress-circle, and 20 and 19-in in balcony and gallery, although there is no accepted rule in this respect. On account of the seat-lifting, opera or theater-chairs may be comfortably spaced 31 in, back to back, and this is the usual spacing in halls and churches In theaters the chairs are usually set on steps. In the upper gallery these steps should not be more than 30 in wide; in the balcony they are usually made either 30 or 31 in

wide, and in the parquet, 31 or 32 in wide. As a rule the higher-priced seats are more commodious than the lower-priced.

Estimating Seating Capacity. The actual seating capacity of theaters and audience-rooms can be determined only by drawing the seats to an accurate scale, on the floor plan, and then counting the number of chairs, or measuring the linear feet of pews.

Approximate Seating Capacity. For approximate purposes the seating capacity or required size of room may be determined by allowing from 7 to 8 sq ft to each seat, or sitting, when on a curve, and from 6 to 7 sq ft to each sitting when in straight rows, the smaller number being used only for large rooms. This allows for aisles and pulpit-platform. For small concert-halls and narrow rectangular rooms, 6 sq ft per sitting will usually be sufficient allowance, provided only that the actual floor-space utilized for seats and aisles is considered. A reasonable minimum area per seat, omitting aisles, is about 18 in by 30 in, 540 sq in or 3.8 sq ft.

Capacity of Several of the Older Cathedrals, Churches, Theaters and Opera-Houses *

European Cathedrals and Churches Estimating that one person occupies an area of 19 7 inches square			
St. Peter's, Rome.	54 000	Notre Dame, Paris	21 000
Milan Cathedral.......	37 000	Pisa Cathedral.....	13 000
St. Paul's, Rome......	32 000	St Stephen's, Vienna....	12 400
St. Paul's, London	25 600	St Dominic's, Bologna....	12 000
St. Petronio's, Bologna	24 400	St Peter's, Bologna.	11 400
Florence Cathedral.....	24 300	Cathedral of Sienna.	11 000
Antwerp Cathedral....	24 000	St. Mark's, Venice	7 000
St. Sophia's, Constantinople	23 000	Spurgeon's Tabernacle, London...........	7 000
St. John Lateran, Rome ..	22 900		
European Theaters and Opera-Houses			
Carlo Felice, Genoa	2 560	Drury Lane, London.	1 948
Opera-House, Munich	2 370	Covent Garden, London ..	3 000
Alexander, Leningrad...	2 332	Opera House, Berlin. ...	1 636
San Carlo, Naples	2 240	Adelphi, London.......	2 300
Imperial, Leningrad	2 160	Lancaster, London.	1 850
La Scala, Milan	2 113	Globe, London...	1 100
Academy of Paris ..	2 092		

* The above table of seating capacities of some of the earlier churches and theaters is retained for purpose of comparison. So many important structures of these types have been erected in recent years in the larger cities of the world, or are now in process of erection, that it has been found impossible to make any list that would be and would remain, for any length of time, complete.

Seating Capacities of Theaters in Various Cities in the United States *

Theater	Seating capacity	Theater	Seating capacity
Boston, Mass.			
Metropolitan	4 330	Boston.	2 848
Shubert Boston Opera	3 944	Loew's State	2 700
Keith's Memorial	3 500	Grand Opera	2 600
New Palace	3 500	Scollay Square	2 589
Loew's Orpheum	3 100	Keith-Albee	2 500
National	3 000	Columbia	2 200
Brooklyn, N Y			
Paramount	4 126	King (Loew's)	3 690
Fox.	4 089	Metropolitan	3 618
Buffalo, N Y			
Auditorium	12 000	Lafayette	3 400
Shea's Buffalo	4 000	Buffalo	3 397
Great Lakes	3 400	Shea's Century.	3 200
Chicago, Ill			
Coliseum	15 000	Granada	4 200
Chicago	5 000	Paradise	4 000
Marbro.	5 000	Sheridan	4 000
Uptown	5 000	Auditorium.	3 623
Tivoli	4 500	Capitol	3 500
Cleveland, Ohio			
Convention Hall.	12 800	Keith's Palace	3 500
Hippodrome	3 600	Uptown	3 200
State.	3 596	Allen	2 998
Denver, Col			
Auditorium.	12 500	Isis	1 800
Denver	3 000	Orpheum	1 800
Rivoli	2 200	America	1 583
Empress	2 000	Tabor Grand	1 575
Detroit, Mich			
Olympia	12 000	Michigan	4 100
Fox	5 500	Paramount	3 448
Masonic Auditorium	5 000	Hollywood	3 436
Kansas City, Mo.			
Auditorium	12 000	Pantages	2 800
Midland	4 000	Mainstreet	2 500

* The data given in this table have been compiled from various sources. Due to their constant removal and the erection of new buildings, some theaters here listed have now been removed. The capacities of the convention halls at Atlantic City, N. J., and Houston, Texas, are 41 000 and 16 000 respectively.

Seating Capacities of Theaters in Various Cities in the United States (Continued)

Theater	Seating capacity	Theater	Seating capacity
Los Angeles, Cal.			
Outdoor Coliseum	78 000	Grauman's Chinese	2 500
Shrine Civic Auditorium	6 050	Hillstreet....	2 500
Paramount. .	3 000	Boulevard .. .	2 300
Louisville, Ky			
Memorial Auditorium.	3 200	Macauley........	1 900
Keith's National... .	2 400	Strand..	1 800
New Orleans, La			
Saenger. .. .	3 800	Palace. ...	1 800
Loew's State . . .	2 600	Turlane	1 600
Orpheum.. . .	2 400	Louisiana	1 500
New York, N Y			
Madison Square Garden	17 000	Manhattan Opera.. ..	3 246
Roxy..	5 920	Proctor's 86th St.	3 160
Loew's Paradise	3 840	Strand.......	2 989
Academy of Music	3 600	Bronx Art	2 840
State	3 600	Commodore	2 830
Paramount	3 528	Bronx Opera	2 571
Metropolitan	3 305	Lexington	2 559
Philadelphia, Pa			
Convention Hall..	14 500	Stanley...	3 100
Mastbaum...	4 500	Academy of Music..	3 000
Metropolitan..	3 482	State	3 000
Earle. . ..	3 300	Uptown.	3 000
Grand Opera.	3 200	Fox...	2 700
San Francisco, Cal			
Auditorium . . .	10 000	Granada..	3 000
New Fox..... . ..	5 400	Casino ...	2 800
Fox . .	4 647	Keith's Golden Gate	2 800
El Capitan..... . . .	3 000	Pantages........... ...	2 800
St. Louis, Mo			
Arena.	18 000	St. Louis.	3 881
Coliseum. . . .	12 000	Missouri. . ..	3 558
Fox	5 000	Loew's State.........	3 073
Washington, D. C.			
D. A. R. Auditorium. ..	4 000	Tivioli...	2 500
Fox's Washington.......	3 433	Palace.	2 485
Loew's Palace. . .	2 700	Earle	2 240

Dimensions of Certain Theaters and Opera-Houses

The following are the dimensions, in feet, of some of the earlier theaters in this country and in Europe.

Name and location	Auditorium			Proscenium opening		Stage		
	Width	Depth	Height	Width	Height	Width	Depth*	Height‡
Alexander, Leningrad	58	76	58	56		75	84	
—, Berlin	51	78	47	41		92	76	
La Scala, Milan	71	95	64	49		86	78	
San Carlo, Naples	74	73	83	52		66	74	
Grand, Bordeaux	47	56	57	37		80	69	
Salle Lepeletier, Paris	66	76	66	43		78	82	
Covent Garden, London	51	66		32		86	55	
Drury Lane, London	56	64		32		48	80	
Boston, Boston	...	71	58	46		87	68	
Academy of Music, New York	62	87		48		83	71	
Academy of Music, Philadelphia	66	78	74	48		90	72	
Globe, Boston	60	65		30		62	38	
Museum, Boston	68	61		31		68	46	
Metropolitan Opera-House, New York §	.	.		54	50	100	73	88
The Auditorium, Chicago						110	70	95
Empire, New York	69	66		34	34	67	30	65
Knickerbocker, New York	70¾	79	47⅔†	35	34	40	65½	
Fifth Avenue, New York						80	35	
American, New York	74½	74½		39	39	77¾	43⅓	73½
Hudson, New York	67⅓	67		32	30	67⅓	30⅔	
Grand Opera-House, Cincinnati	67	69		36	34	67	41	
Castle Square, Boston¶	79½	85½	70†	40	34	68	45½	
Gaiety, Boston	77	80⅔		.		60	42	70

* From the curtain or back line of proscenium opening.
† Measured from stage to center of ceiling
‡ To the "gridiron" or rigging-loft
§ As remodeled in 1893.
¶ The plan of this theater is in the shape of a horseshoe.

Notes on Theater-Dimensions.* "The utmost distance from the front of the stage to the rear ought not to exceed 75 ft, or the limit the voice is capable of expanding in a lateral direction."

"Measured from the curtain-line, the San Carlo Theater in Naples is 73 ft; the theater at Bologna, 74 ft. Of the London theaters, the Adelphi is 74 ft, Covent Garden 80 ft, the Gaiety 53 ft 6 in, Lancaster 58 ft 4 in, Marylebone 74 ft and the Globe 47 ft 6 in."

* From The Planning and Construction of American Theatres, by Wm. H. Birkmire.

The width of the ideal theater, between inside walls, should be from 70 to 75 ft, and "the ceiling should be from 55 to 65, or even 70 ft above the stage-level."

"The depth of the parquet-floor at the orchestra-rail is governed by the stage level, and is generally from 3 ft 6 in to 4 ft 3 in below the stage. A depth of 3 ft 9 in is a good height, as it fixes the eye of the spectator 5 in above the stage-level."

"The height of the stage, that is, from the floor to the bottom of the 'gridiron' or rigging-loft, should be 2 or 3 ft over twice the height of proscenium opening, in order that the fire-curtain may be raised the full height of the opening." There should be a height of 7 ft above the gridiron to enable the fly-men to adjust their ropes with facility.

ATHLETIC FIELDS AND COURTS

General. The information concerning the dimensions of athletic fields and courts is presented as an aid to architects in designing gymnasiums and playing-fields. All data in this section have been compiled, by permission, from various volumes of Spalding's Athletic Library.

Baseball. Fig. 1 is a diagram showing the official measurements for laying out a baseball field, taken from Official Baseball Rules, adopted by the National and American Leagues and the National Association of Professional Baseball Leagues. The following excerpts are taken from the Official Baseball Rules.

The Ball Ground. The ball ground must be enclosed.* To obviate the necessity for ground rules, the shortest distance from a fence or stand on fair territory to the home base should be 235 ft and from home base to the grandstand 60 ft.

To Lay off the Field.† To lay off the lines defining the location of the several bases, the catcher's and the pitcher's position and to establish the boundaries required in playing the game of baseball, proceed as follows:

DIAMOND OR INFIELD.‡ From a point, A, within the grounds, project a straight line out into the field, and at a point, B, 124 ft from point A, lay off lines BC and BD at right-angles to the line AB; then, with B as a center and 63.63945 ft as a radius, describe arcs cutting the lines BA at F and BC at G, BD at H and BE at I. Draw lines FG, GI, IH, and HF, each 90 ft in length, which said lines shall be the containing lines of the diamond or infield. See Fig. 1.

Football. Fig. 2 is a diagram of the field of play as recommended by the Football Rules Committee. This committee is appointed by the National Collegiate Athletic Association.

Soccer Football. The following excerpts are taken from Laws of the Game, as promulgated by the National Collegiate Athletic Association, and as adopted by the Intercollegiate Soccer Football Association of America.

Dimensions of the Field. See Fig. 3. The dimensions of the field of play shall be: maximum length, 120 yd; maximum breadth, 75 yd; minimum

* Enclosure applies to professional leagues.

† For further detailed information concerning the laying out of a baseball field see Official Baseball Rules.

‡ If it is possible to do so, have the home plate due north and the pitcher's plate due south.

length, 100 yd; minimum breadth, 55 yd. The field of play shall be marked by boundary-lines. The lines at each end are the goal-lines and the lines at each side are the touch-lines. The touch-lines shall be drawn at right-angles with the goal-lines. A flag with a staff not less than 5 ft high shall be placed at each corner. A half-way line shall be marked out across the field of play.

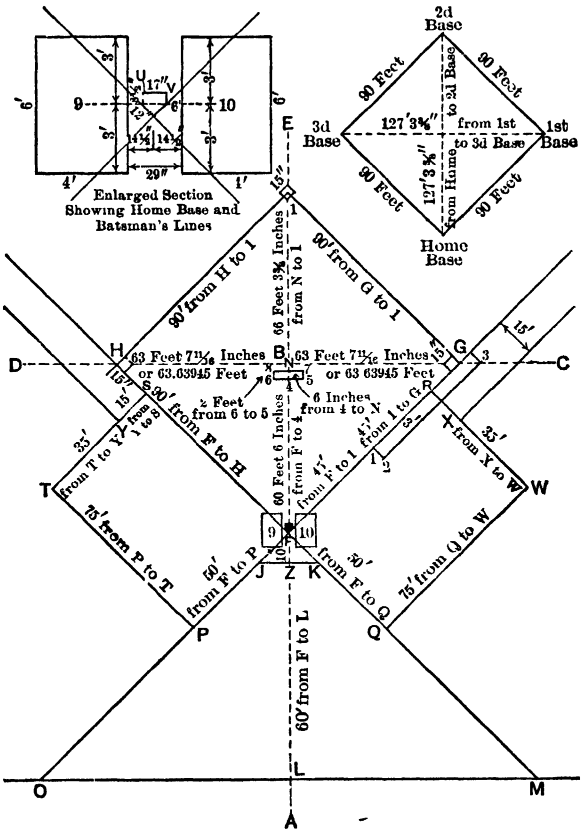


Fig. 1. Measurements for Laying Out a Baseball Field

The center of the field of play shall be indicated by a suitable mark, and a circle with a 10-yd radius shall be made around it. The goals shall be upright posts, fixed on the goal lines, equidistant from the corner flag-staffs, 8 yd apart, with a bar across them 8 ft from the ground. The maximum width of the goal-posts and the maximum depth of the cross-bar shall be 5 in. Each goal shall be provided with regulation goal-nets. Lines shall be marked 6 yd from each goal-post at right-angles to the goal-lines for a distance of 6 yd, and

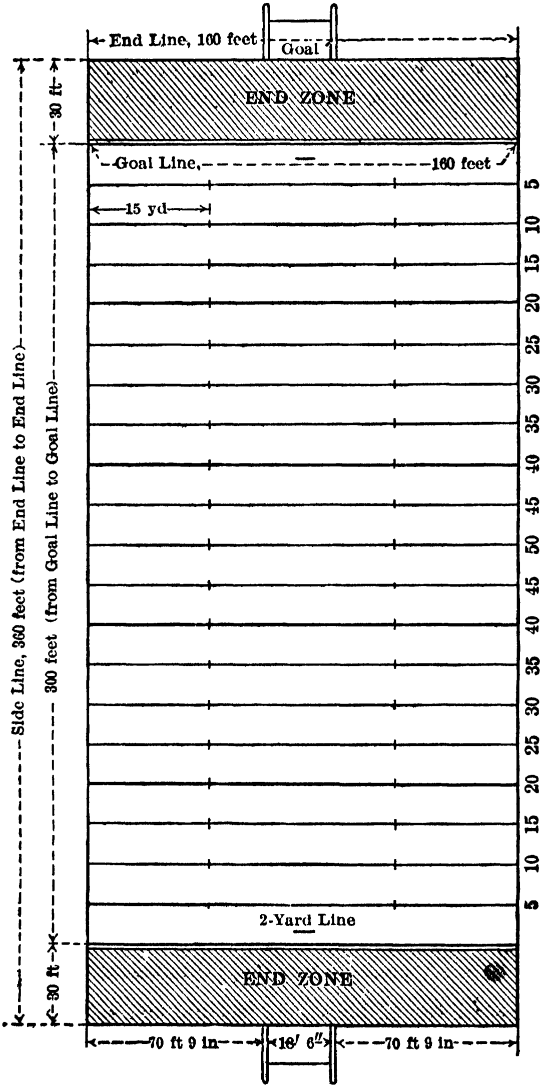


Fig. 2. Diagram of a Football Field

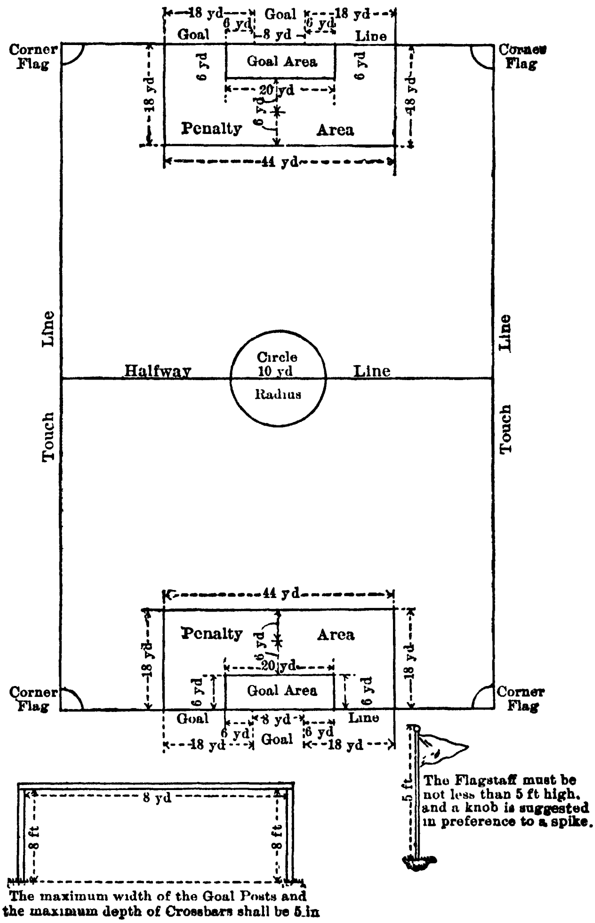


Fig. 3. Diagram of a Soccer Football Field

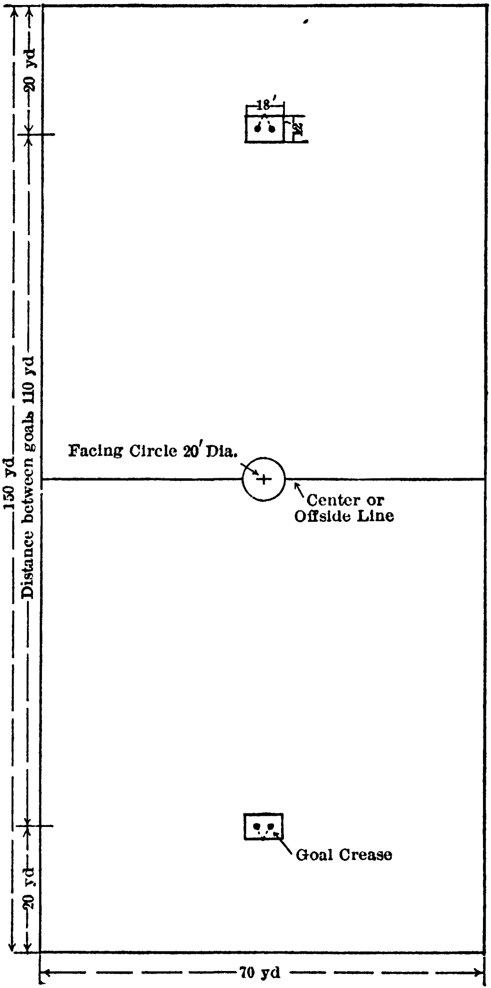


Fig 4 Diagram of a Lacrosse Field

Outer lines should be marked in white. A barrier fence to retain the ball should, if possible, be located a short distance beyond the boundary lines. Dimensions shown are minimum; for maximum, see text.

these shall be connected with each other by a line parallel to the goal-lines: the space within these lines shall be the goal area. Lines shall be marked 18 yd from each goal-post at right-angles to the goal-lines for a distance of 18 yd, and these shall be connected with each other by a line parallel to the goal-lines; the space within these lines shall be the penalty area. A suitable mark shall be made opposite the center of each goal, 12 yd from the goal-line; this shall be the penalty-kick mark.

Lacrosse. See Fig. 4. The following excerpt is taken from Lacrosse Rules, approved by the Executive Committee of the United States Intercollegiate Lacrosse Association. Each goal shall consist of two poles 6 ft apart, and 6 ft high out of the ground, joined by a rigid top cross-bar. The poles must be fitted with a pyramid-shaped cord netting (as shown in sketch) of not more than 1½-in mesh, which pyramid shall extend and be fastened to a stake in the ground at a point 7 ft back of the center of the goal, and the said netting shall be so made as to prevent the passage of the ball put through the goal from the front, and the bottom of the netting must be held close to the ground with tent-pegs or staples. The goals shall be placed 110 yd from each other, with from 20 to 35 yd of clear playing space behind each goal. In matches, they must be furnished by the home club. The width of the field shall be at least 70 yd and not more than 85 yd. The boundaries of the field shall be marked with white lines, and a white line shall be drawn through the center of the field perpendicular to the side-lines.

Field Hockey. See Fig. 5. The instructions given here for laying out a field for field hockey are taken from Learning to Play Field Hockey, Spalding's Athletic Library. The ground should be 100 yd long, and not more than 60 yd, and not less than 55 yd wide. Where possible, always have the ground 60 yd wide. In marking out, note that flag-posts are to be placed at *each corner*, on the junction of the goal and the side-lines. In other cases they are to be one yard outside the touch-line. The 25-yd line must not be fully drawn, but only its extremities (7 yd only to be marked at each end). It is advisable to mark with short lines 3 yd either side of each corner flag, and also 5 yd and 10 yd from each goal-post. The 3-yd marks are where corners are hit from on either side of the flag-post, which may be moved if it impedes the player. The 10-yd marks are the nearest point to the goal-posts from which a penalty corner may be hit. The 5-yd marks are to show the defenders their distance from the strikers of a penalty corner, for no player may stand within 5 yd of it. The 5-yd line, extending the full length of the ground parallel to the touch-line, can be marked either all the way or at intervals of a yard or so. At the roll-in from touch no player may be within this line and the touch-line.

Ice Hockey. See Fig. 6. Instructions given here are taken from Official Ice Hockey Rules, as recommended by the Rules Committee of the National Collegiate Athletic Association. The rink, or playing surface, should be a clear field of ice at least 160 by 60 ft and not greater than 250 by 110 ft. A size of 190 by 85 ft, with rounded corners of 15-ft radius, is recommended.

Lawn Tennis. See Fig. 7.

Basket-Ball. See Fig 8. The dimensions given here are taken from Official Basket-Ball Rules, as adopted by the Joint Basket-Ball Rules Committee. The playing court shall be a rectangular surface free from obstructions and shall have maximum dimensions of 94 ft in length by 50 ft in width and minimum dimensions of 60 ft in length by 35 ft in width. The following dimen-

sions are considered ideal for teams whose players are of the age indicated for each group:

Elementary-school age	40 by 60 ft
High-school age	48 by 75 ft
College age	48 by 84 ft

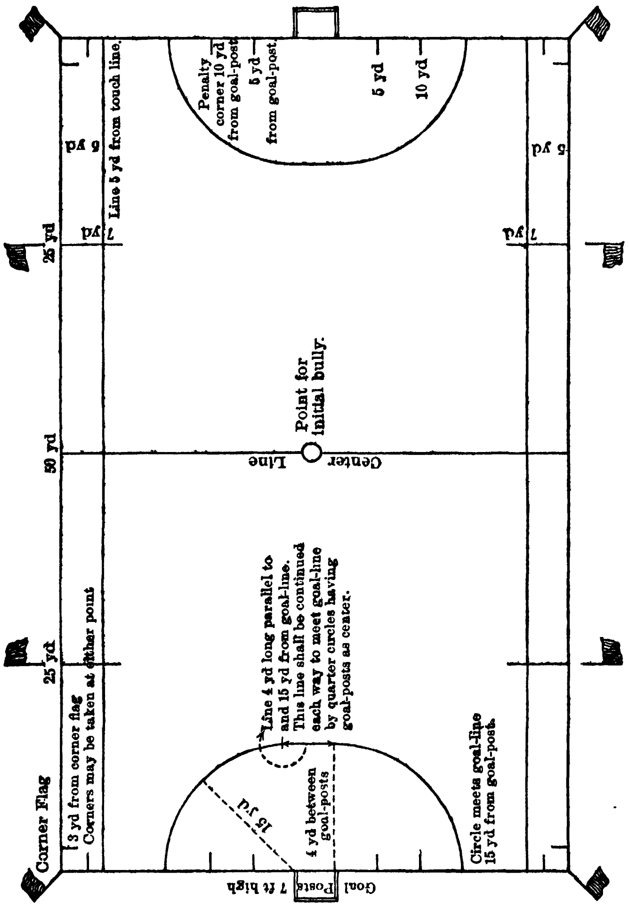


Fig. 5. Diagram of a Field for Field Hockey

These are the dimensions for the playing court only. In planning gymnasiums at least 10 additional feet should be provided on each side and each end.

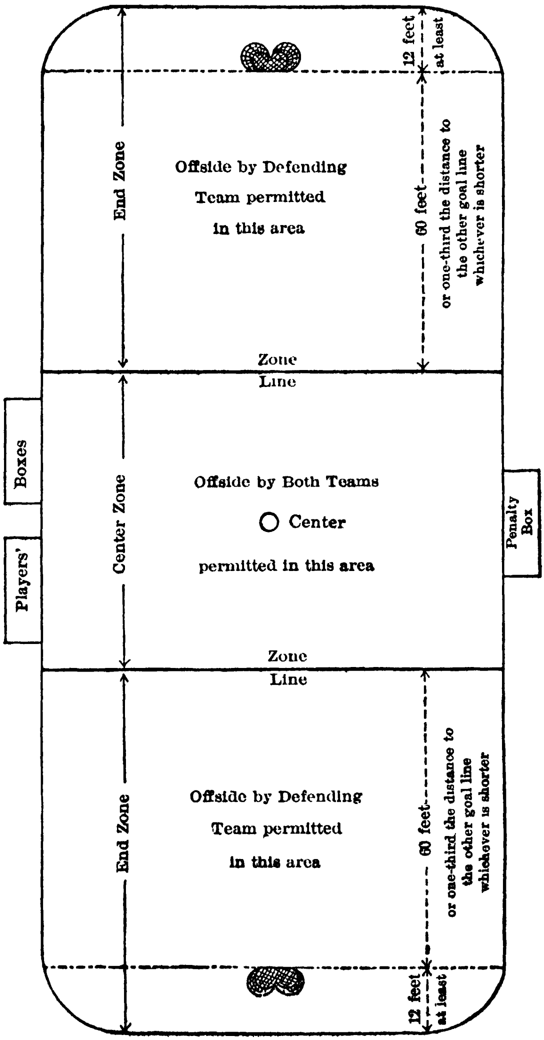
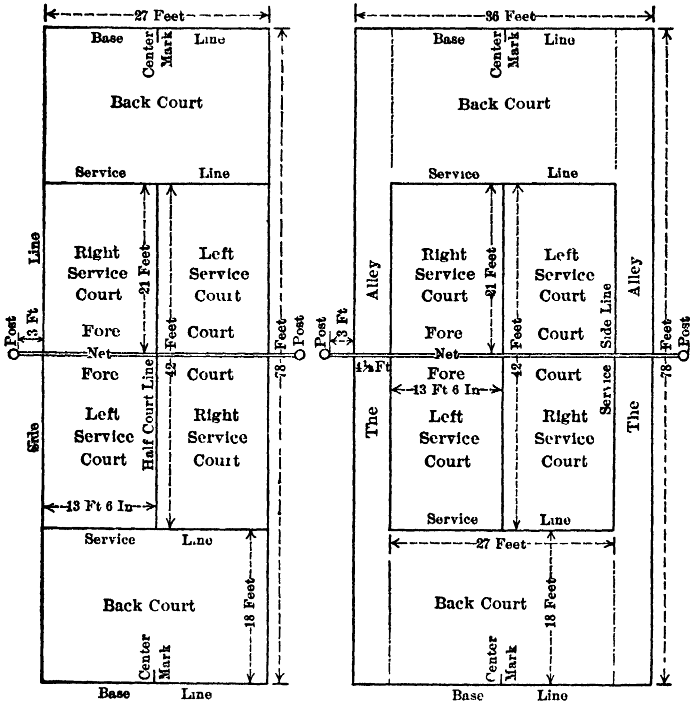


Fig. 6. Diagram of a Rink for Ice Hockey

Backboards must be provided, the dimensions of which shall be 6 ft horizontally and 4 ft vertically. These backboards shall be made of plate glass or wood, or of any other material that is flat and rigid. The faces of the backboards shall be painted white. The backboards shall be located in a position at each end at right-angles to the floor, parallel to the end lines, and with their



The Singles Court

The above diagram shows a court which is marked for the singles game only. The adjoining diagram shows how a court may be marked for both singles and doubles.

The Doubles Court

The service side lines should extend to the service lines only. The dot-and-dash lines show how the service side lines would be extended to the base lines in a court marked for both the singles and doubles game.

Fig. 7. Diagram of Lawn Tennis Courts

lower edges 9 ft above the floor. Their centers shall lie in the perpendiculars erected at the points in the court 2 ft from the midpoints of the end-lines. The faces of the backboards shall be 15 ft from the far edges of the free throw lines.

The ring, 18 in inside diameter, shall be rigidly attached to the backboard; it shall lie in a horizontal plane 10 ft above the floor and shall be equidistant from the two vertical edges of the backboard. The nearest point of the inside edge of the ring shall be 6 in from the face of the backboard.

Women's Basket-Ball. See Fig. 9. The following dimensions are taken from The Official Basket-Ball Guide for Women, Spalding's Athletic Library. Maximum court, 100 ft by 50 ft; use three divisions. Minimum court, 50 ft

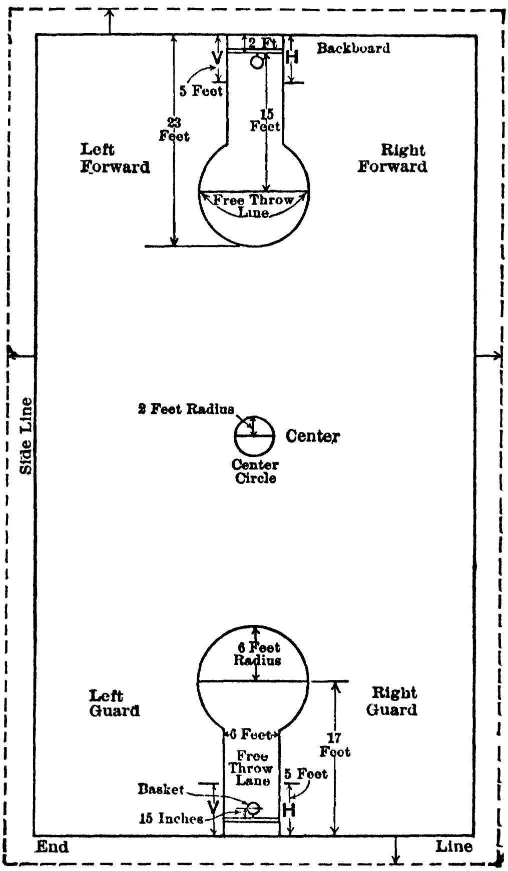


Fig. 8. Diagram of a Basket-Ball Court

For dimensions of ideal-sized courts, see text. A minimum distance of 3 ft., free of all obstructions, should be allowed outside of boundary lines, as indicated by dotted lines.

by 25 ft; use two divisions. Regulation-size court, 90 ft by 45 ft for college players; 70 ft by 35 ft for high-school players; use three divisions. The face of the backboard should be 2 ft from the end-wall; but on short courts, when the backboard is placed against the wall, there shall be an end-line, the inner

edge of which is 2 in out from the wall. On narrow courts when the playing court is the full width of the floor there shall be a side-line, the inner edge of which is 2 in out from the wall. The positions of the players are determined

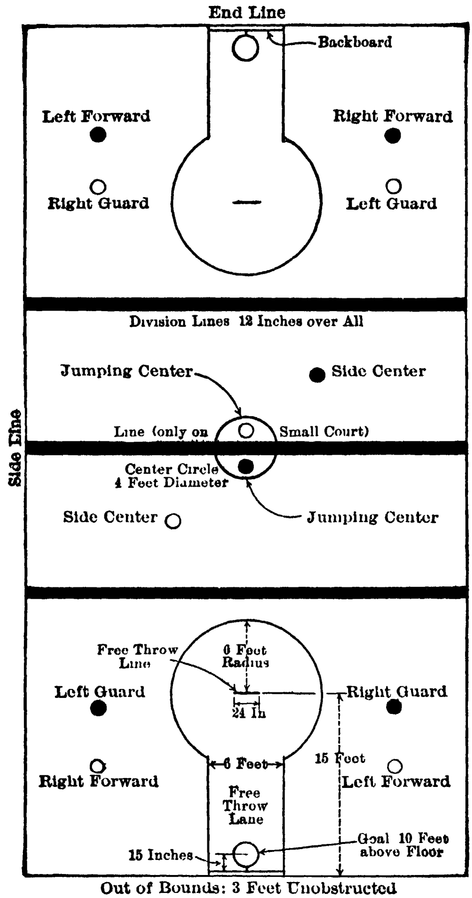


Fig. 9. Diagram of a Women's Basket-Ball Court

by standing with backs toward the goal that is being defended and facing the goal that is being attacked.

Volley-Ball. See Fig. 10. The diagram and dimensions here given are taken from Official Volley-Ball Rules adopted by the National Amateur Athletic Federation. The playing surface shall be a rectangular court 60 ft

long and 30 ft wide, including outer edge of lines, free from obstructions and having a height of 15 ft or more which is free from apparatus or other obstruc-

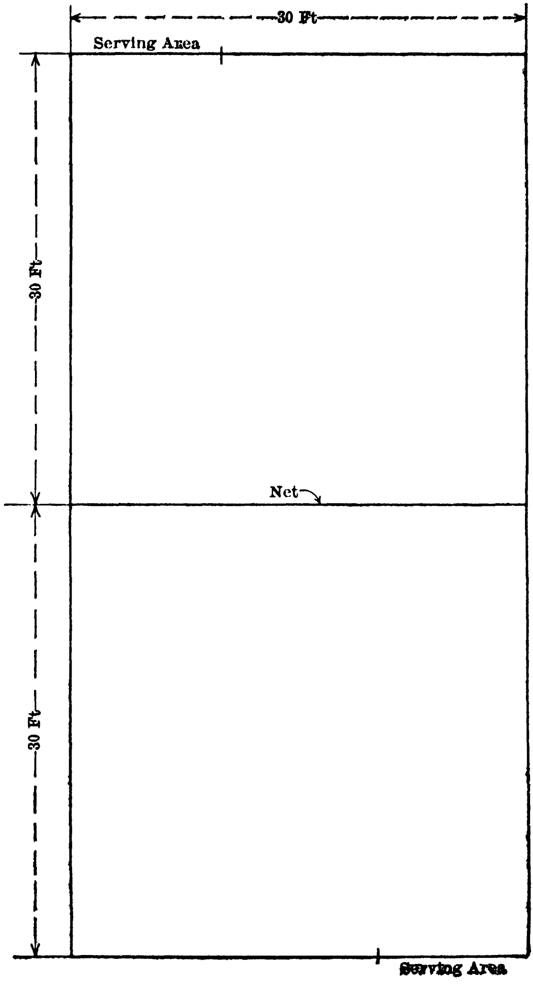


Fig. 10. Diagram of a Volley-Ball Court

tions or projections. The size of the court may be modified for either indoor or outdoor informal games, to accommodate larger or smaller groups to suit local requirements. The court shall be bounded by well-defined lines 2 in

in width, which shall be at every point at least 3 ft from walls or any obstructions. The lines on the short sides of the court shall be termed the end-lines; those on the long sides the side-lines. A center-line, 2 in in width, shall be drawn on the court immediately beneath and parallel to the net.

American Handball. See Fig. 11.

Four-Wall Handball. See Fig. 12. These dimensions are taken from Official Four-Wall Handball Rules of the Amateur Athletic Union of the United States. Standard courts shall have four walls and a ceiling, shall be 22 ft in width, 46 ft in length and 22 ft in height.

The building and use of standard courts is advised, but championships may be held on courts of other sizes as approved by the Handball Committee.

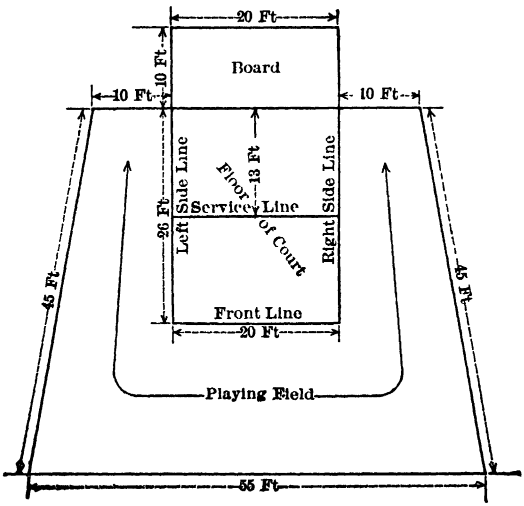


Fig. 11. Diagram of an American Handball Court

Squash Rackets. The following data have been obtained from A. G. Spalding & Bros. The size of the court shall approximate as closely as possible 18 ft 6 in by 32 ft. The service-line on the front wall shall be 6 ft from the floor; the telltale shall be 17 in from the floor and may be made of sheet metal protruding $1\frac{1}{2}$ in from the wall. The line on the back playing wall shall be 6 ft 6 in from the floor, and a strip of wire netting shall be stretched above this line several inches from the floor. The service-court line shall be 9 ft 10 in from the back wall, and the service box shall be 4 ft 6 in in radius, in the shape of an arc of a circle. Halfway from the two side-walls a line shall be drawn from the service-court line to the back-wall, dividing the two service-courts; the general color of the court shall be white. The over-all height of the court shall be from 14 to 22 ft to allow for light. There is no regulation side-wall height, therefore no side-wall line.

Wrestling. See Fig. 13. The following excerpt is taken from National Collegiate Athletic Association Wrestling Rules. The area of the mat shall not be

less than 20 ft by 20 ft, and this dimension shall be considered the standard size, when ropes are used. When ropes are not used a 24 by 24-ft mat shall be considered standard. Whenever possible it is recommended that a "roped-in" area be used in accordance with the following specifications. Three 1-in ropes shall be tightly stretched 2 ft, 3 ft and 4 ft, respectively, above the mat. These ropes shall extend in from four supporting posts, which shall be placed at least 18 in back from the corners of the ring. Cotton ropes are recom-

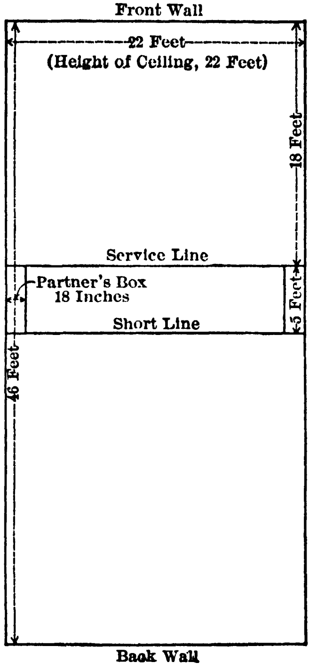


Fig. 12. Diagram of an Official Four-Wall Championship Handball Court

mended, but if manila or sisal ropes are used they must be wrapped with bunting or other soft material to avoid " rope burns." To prevent the spreading of ropes during bouts, they shall be securely fastened together by twelve vertical $\frac{3}{8}$-in ropes, three of which shall be placed equidistant on each side of the ring.

Boxing. The following requirements for equipment are taken from the National Boxing Association Rules.

Ring to be not less than 18 nor more than 20 ft. square * within the ropes, the ring floor to extend beyond the ropes a distance of not less than 18 in.

* The Amateur Athletic Union Boxing Rules call for a ring "not less than 16 ft nor more than 20 ft square . . . The floor of the ring shall extend beyond the lower rope for a distance of not less than 2 ft."

The ring posts shall be not nearer to the ring ropes than 18 in. The ring floor shall be padded with felt, matting or other soft material to a thickness of not less than 1½ in extending over edge of ring platform with a top covering of canvas, duck or similar material tightly stretched and laced to ring platform. Material that tends to gather in lumps must not be used.

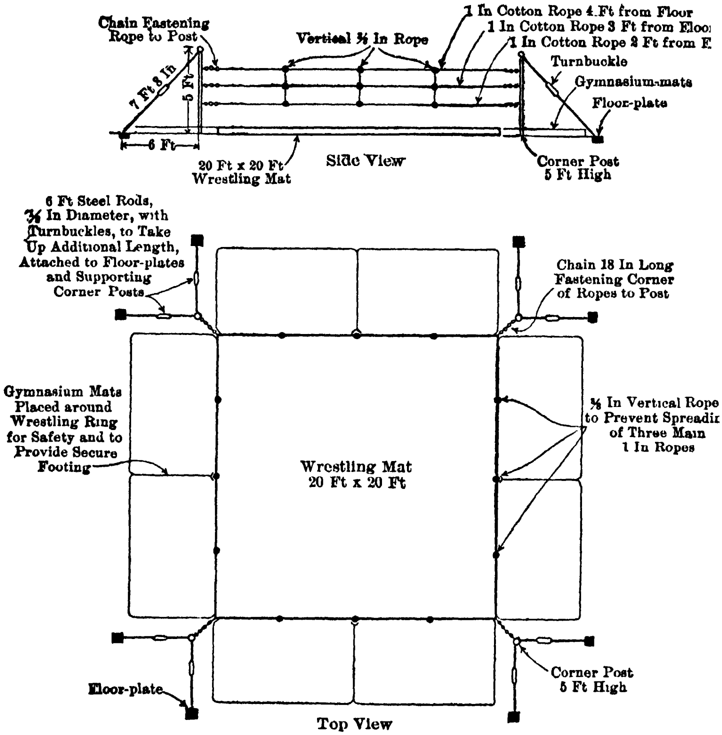


Fig. 13. Diagram of a Wrestling Mat

Height of Ring shall be not more than 4 ft above the floor of the building, and shall be provided with suitable steps for the use of the contestants. Ring posts shall be made of metal not more than 3 in in diameter, extending from floor of building to height of 58 in above ring floor.

Ring Ropes shall be three in number, not less than 1 in in diameter; the lower rope 18 in, the second rope 35 in, and the third rope 52 in above the floor. Ropes shall be wrapped in soft material.

Roque. See Fig. 14.

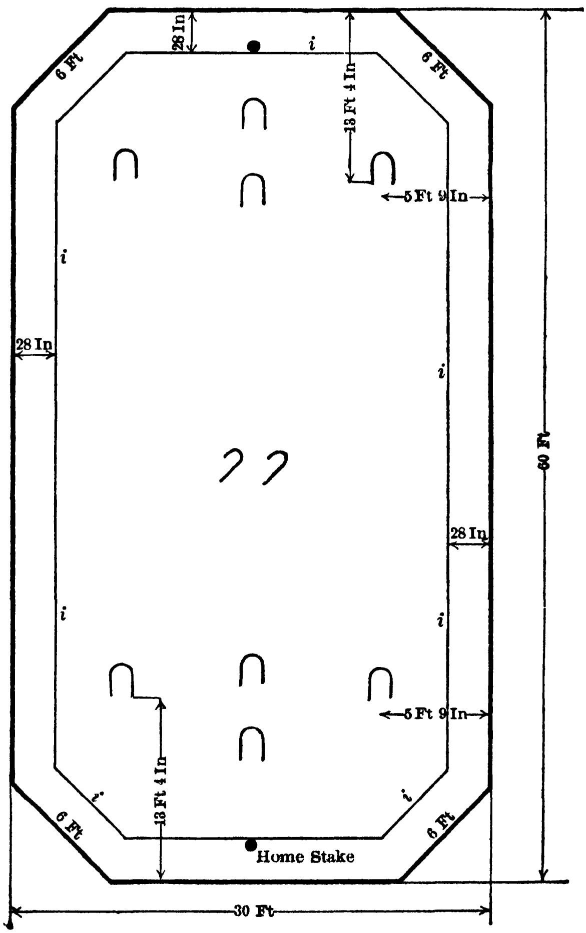


Fig. 14. Diagram of a Roque Court

Badminton. See Fig. 15.

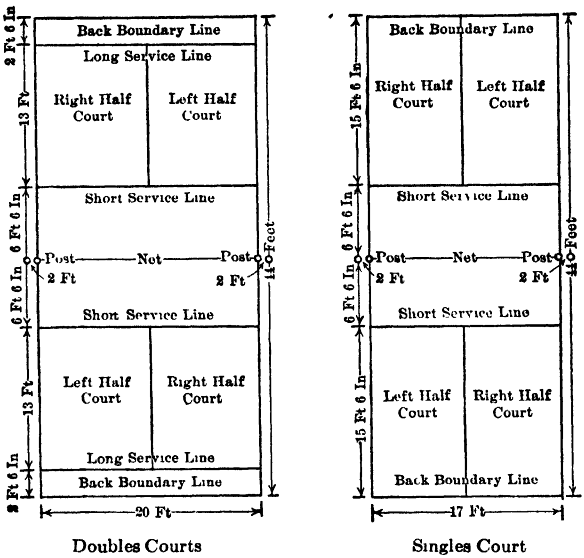


Fig. 15. Diagram of Badminton Courts

The posts shall, if practicable, be placed on the Side Boundary Lines, otherwise at any distance not exceeding 2 ft outside the said lines. On the Singles Court the Back Boundary Lines become the Long Service Lines.

MAIL-CHUTES

General Description. This system of mailing letters by means of a specially constructed chute connected with the receiving-box at the bottom, has come into such general use in public buildings, office-buildings, apartment-houses and hotels, that the restrictions affecting the same and what is required in the way of preparation should be known to architects. The system is installed by the patentees, under regulations of the Post-Office Department governing its construction and location, and for this reason it is well to consult the makers * before permanently locating the apparatus on the plans. It may be placed in any building of more than one story, used by the public, where there is a free delivery and collection-service, in the discretion of the local postmaster; subject to whose approval the contracts are made.

The Chute and Receiving-Box. The chute is required to be made with a removable front and a continuous, rigid, vertical support is absolutely necessary. It must be of metal, its front must be of plate glass, and it must bear the insignia prescribed by the department; and the whole apparatus, when erected and the Government lock put on the box, passes under the exclusive

* The Cutler Mail Chute Company, Rochester, N. Y.

care and control of the Post-Office Department, and the chutes become a part of the receiving-boxes. These boxes may be of various patterns and highly ornamental and are furnished by the makers in connection with the chutes. The work of preparing a rigid support for the chute and cutting and finishing the openings in the floors is of the utmost importance, and details showing the usual arrangements are always given.

Preparatory Work. The requirements for what the manufacturers call PREPARATORY WORK include a flat, vertical, continuous surface not less than 10½ in wide, extending from the floor of the ground-story to a point 4 ft 6 in above the finished floor in the top story, and an opening in each floor directly in front of and centered upon this surface. These openings are neatly finished,

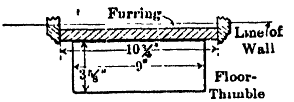


Fig. 1. Wooden Support for Mail-chute

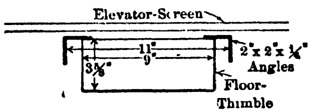



Fig. 2. Steel Support for Mail-chute

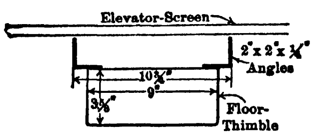


Fig. 3. Alternate Steel Support for Mail-chute

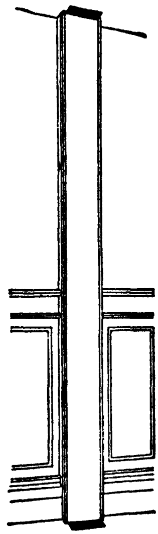


Fig. 4. Preparatory Work Complete for Mail-chute

and their size and shape determined by setting in them thimbles of iron which are furnished and delivered by the patentees as part of their contract. In ordinary installations a casing of wood, suitably molded and finished to match the trim of the building, answers every purpose. Such a casing is shown in plan, Fig. 1, with the opening finished by the iron thimble. In buildings, or sometimes in a few stories, where a more elaborate finish is desired, marble is substituted for wood, the form and construction of the casing being adapted to the material, but of course without disturbing the size and form of the front surface. Steel angles are used where the use of wood is objected to, or where it is necessary to run the chute in front of an elevator-screen, or in other locations where a solid wall is not available to support the casing. Steel square-root angles, 2 by 2 by ¼ in in section, are generally used, and set as in Fig. 2,

but sometimes, where it is desirable to fill up the space between them and the elevator-screen, they are reversed, as in Fig. 3. The angles are usually bolted to the beams, and in any case must be straightened so that they are without twists or kinks, and the surface which receives the mail-chute plumb and flush in all stories. Fig. 4 gives a general view of the mail-chute casings and floor-openings ready to receive the chutes themselves. This work of preparing the building, except the cutting or leaving ready the necessary openings in the floors, is now usually included in the mail-chute contract, as it has been found for many reasons undesirable to separate it. The necessary openings in floors, and all patching around such openings, should be included in the mason's or other proper specifications.

Essential Points to be remembered are (1) that no bends or offsets can be made, a vertical fall being absolutely essential, and (2) that the entire apparatus must be exposed to view and must be accessible, that is, it is not permitted to extend the work behind an elevator-screen or partition or through any part of the building except a public corridor.

REFRIGERATORS *

General Requirements. The following information is given as a guide to architects in providing for refrigerators in large residences, hotels, clubs, hospitals and other institutions. Consultation with a reliable refrigerator-builder, however, is always desirable before deciding upon spaces to be occupied by refrigerators, refrigerating-rooms, etc., as a satisfactory refrigerator cannot be adapted to a badly proportioned space.

Residence-Refrigerators. Care should be taken to select a refrigerator which is simple in operation and easily cleansed, as modern sanitary science has traced much illness to faulty refrigeration. Thorough insulation is an important feature in a refrigerator, as upon this depends economy in the use of ice and the securing and maintaining of the low temperature necessary to the proper preservation of food. Fig. 1 shows a kitchen-refrigerator for use of families of ordinary size. The ice-compartment is located in the middle division. The depth should not be more than 3 ft nor less than 2 ft, and the height may vary from 4 ft 6 in to 7 ft. The length of the front largely determines the capacity and should range from about 4 to 7 ft Fig. 1 shows, also, a most satisfactory method of accomplishing the outside-icing feature which consists of a double outside icing-door complete, with frame and jamb. This is provided by the refrigerator-builder to fit the rough opening furnished by the owner in the outside wall of the building. With this method a minimum outside opening is required to furnish a maximum inside opening for ice. The DRAIN-PIPES should be as short and straight as possible and should be readily detachable for cleansing purposes. The drain should be properly trapped in the floor of the refrigerator and carried through the floor of the building, discharging over the plumber's open connection as shown in the elevation of Fig. 1.

Fig. 2 shows a refrigerator for use in a butler's pantry where economy of space is important. The ice-compartment is of galvanized steel throughout

* Valuable data and the drawings relating to this subject were furnished the author and editor by The Jewett Refrigerator Company, Buffalo, N. Y. Practical data were furnished, also, by The Brunswick-Balke-Collender Company, New York City. There are numerous other reliable firms whose refrigerator-work has the highest reputation.

and is removable for convenience in filling as it slides on roller-bearing runways. When the ice-compartment is replaced in position the outside door closes over it. The adjoining storage-compartment is generally fitted with one removable shelf, below which is a bottle-rack for horizontally placed bottles and a space for standing bottles. The depth should be about 2 ft and the height 2 ft 8 in, under counter-top. The length of the front determines the capacity, but it should never be less than 3 ft. For a double refrigerator with a central ice-compartment and storage-compartments at either side, 5 ft is a convenient length. The exterior finish and hardware should correspond

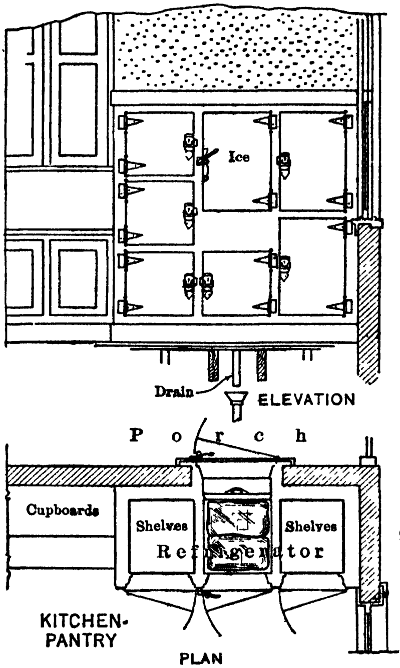


Fig. 1.* Kitchen-refrigerator for Small Family

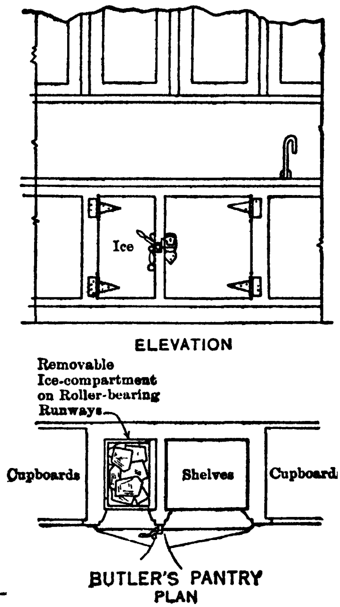


Fig. 2.* Refrigerator for Butler's Pantry

with the adjacent trim. The most sanitary and attractive interior finish for storage-compartments consists of white plate glass for the walls and ceilings and tile for the flooring. The usual complement of refrigerators for use in ordinary families consists of one adjacent to the kitchen and one in the butler's pantry. For large families the number could be the same with the capacity greater.

Refrigerators for Hotels, Clubs, Etc. MECHANICAL REFRIGERATION has largely superseded ICE as a cooling-agent where the refrigerator-equipment

* The Jewett Refrigerator Company

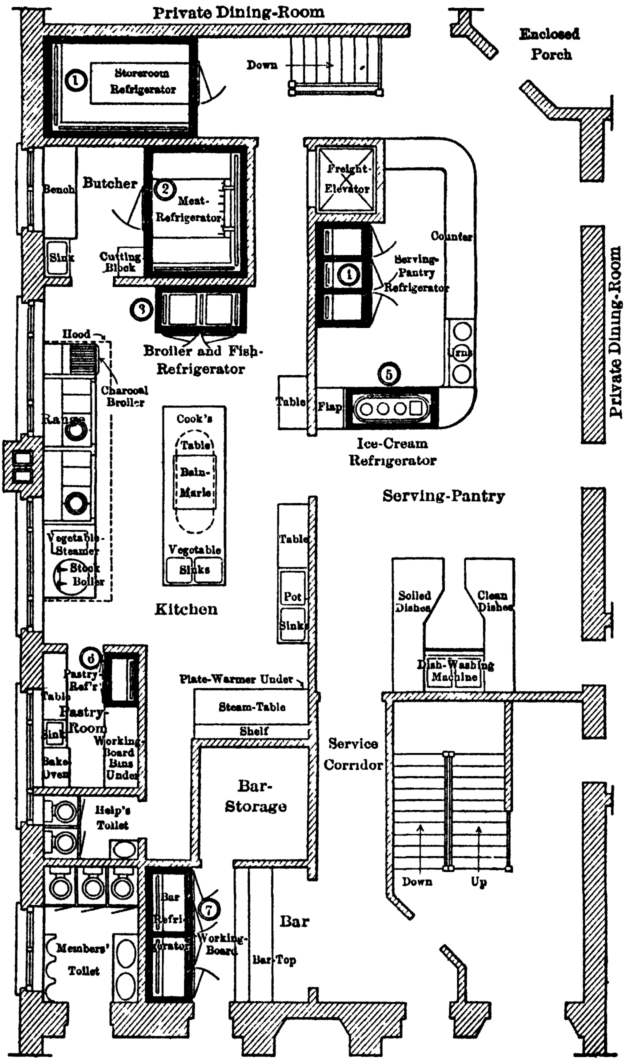


Fig. 3.* Plan of Refrigerators for Large Club-house

* The Jewett Refrigerator Company.

consists of several units, as in hotels, clubs and institutions. (See, also, Mechanical Refrigeration.) The arrangement of refrigerators is similar to that employed where ice is used, as the refrigerating-coils are often contained in compartments corresponding to ice-compartments; the alternative method is to place the coils against walls of storage-compartments. Refrigerating-coils are generally of $1\frac{1}{4}$-in pipe, the length of coil depending upon the temperature required. Fig. 3 shows a practical layout for the working-department of a good-sized club, and illustrates the proper complement of mechanically cooled refrigerators, together with adjacent operating-equipment. No. 1, a storeroom refrigerator, has the front arranged in one full-height door and is fitted with three tiers of shelves throughout. No. 2, a meat-refrigerator, is also accessible through a full-height door and is fitted with shelves and meat-racks. No. 3, a broiler and fish-refrigerator, has the front arranged in two doors, each door opening onto a series of six galvanized sheet-steel pans sliding on self-sustaining roller-bearing runways. No. 4, a serving-pantry refrigerator, is subdivided by an insulated partition into three separate and distinct compartments, those at the left and right being each accessible through two doors, while the middle compartment is accessible through one door, below which is a series of four drawers sliding on self-sustaining roller-bearing runways. The doors open onto removable shelves throughout. No. 5, an ice-cream refrigerator, occupies a position in the serving-pantry counter and has the top arranged in one lift-off cover. Its interior fittings consist of three 20-quart porcelain-lined ice-cream jars and one glacé-frame for fancy forms of ice-cream. No. 6, a pastry-refrigerator, has the front arranged in four doors, two upper doors opening onto removable shelving, and two lower doors onto pastry-pans sliding on angle-iron runways. No. 7, a bar-refrigerator, is subdivided by an insulated partition into two separate and

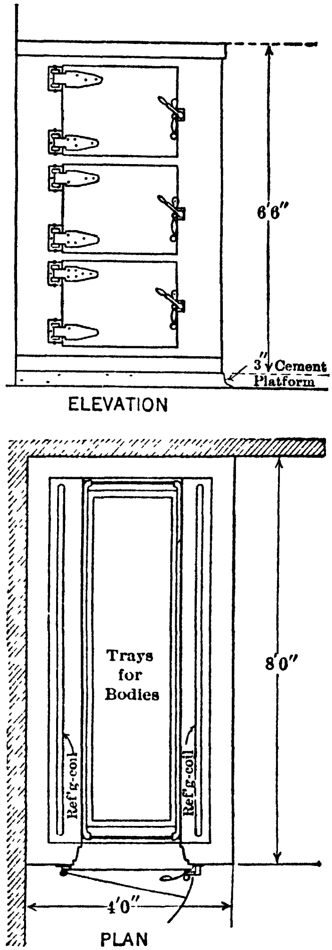


Fig. 4.* Mortuary-refrigerator

* The Jewett Refrigerator Company.

distinct compartments, each accessible through four doors. The upper doors open onto three tiers of removable shelves for standing bottles, while the lower doors open onto five tiers of racks arranged specially for horizontal bottles. The equipment described above will also satisfactorily cover the requirements of a moderate-sized hotel.

Refrigerators for Hospitals. The usual complement of refrigerators for small hospitals consists of one large storage-refrigerator, one refrigerator for the chef's use in or near the kitchen, one for milk and butter and one iron-lined chest for broken ice. For large hospitals the same number with increased capacity and with the addition of small diet-kitchen refrigerators, and possibly a mortuary-refrigerator for two or three bodies, will meet the requirements.

The Height of Large Refrigerators for hotels, clubs and institutions, to be entered through full-height doors, should be from 10 to 12 ft, if equipped with overhead ice or coil-compartments; with side ice-compartments or coils placed against walls, the height should be 7 ft 6 in or 8 ft. The smaller refrigerators, accessible through half-height doors, hinged covers, drawers, etc., should be placed on a 3-in sanitary cement platform finished with cove to floor of building. These refrigerators should not be higher than 6 ft 6 in unless provided with overhead ice or coil-compartments, in which case the height should be from 8 to 9 ft.

Insulation. (See, also, The Value of Good Insulation.) Refrigerators in modern hotels, clubs, institutions, etc., are insulated with Government-standard corkboard, the large refrigerators being constructed of 4-in cork throughout, in two courses of 2 in thickness, and with all joints broken. Cork is applied to adjacent walls of a building with Portland cement, ½ in thick, and this cement is used, also, in applying the inner course of cork to the outer course in walls, partitions and ceilings. Sometimes asphaltum, ⅛ in thick, is used instead of cement. All cork in the flooring is asphalted water-tight. Interior finish may be of Portland cement throughout or of galvanized sheets on walls and ceilings and of Portland cement on floors. Or the walls and ceilings may be of fused-on porcelain or white plate glass, and the floors of tile, all depending upon the grade and character of the building to be equipped. The insulation of smaller refrigerators consists of (1) an exterior course of ⅞-in tongued and grooved lumber, (2) two courses of water-proof insulating-paper and (3) a 3-in thickness of sheet cork in two 1½-in courses, all joints being broken. To this insulation is applied the interior lining.

Mortuary-Refrigerators. Mortuary-refrigerators should be cooled by mechanical refrigeration, the coils being placed longitudinally on both sides of the mortuary-trays. Fig. 4 illustrates a mortuary-refrigerator for three bodies. This may be used as a unit in designing mortuary-refrigerators of larger capacity, or the height may be reduced to 5 ft and the bodies placed in two instead of three horizontal tiers. Mortuary-refrigerators sometimes have both fronts finished and equipped with doors so that bodies are accessible for identification or examination from both fronts.

RECOMMENDATIONS FOR CONSTRUCTION OF COOLING ROOMS AND LARGE REFRIGERATORS *

Definitions. Refrigerator. The term REFRIGERATOR shall apply to any portable refrigerator of the reach-in or service type, built either set-up or sectional.

Cooling-Room. A walk-in refrigerator of portable, sectional construction shall be known as a COOLING-ROOM.

Cold-Storage Room. The term COLD-STORAGE ROOM shall apply to any refrigerator of the built-in type, i.e., built of cement and cork or other insulation material direct to one or more building walls.

Bunker. The space arranged to receive the cooling coils or mechanical equipment shall be known as the BUNKER.

Coil Deck. The term COIL DECK shall apply to the arrangement directly below the cooling coils or other refrigerating equipment to catch the drip from the same.

Drain-Trough. The arrangement along the lower edge of the coil deck to prevent the water from dripping into the storage compartment shall be known as the DRAIN-TROUGH.

Baffle. The term BAFFLE shall apply to the vertical partition separating the warm air duct from the coil space.

Cold-Air Duct. The space through which the cold air descends from the bunker to the storage compartment shall be known as the COLD-AIR DUCT.

Warm-Air Duct. The space through which the warmer air ascends from the storage space to the bunker shall be known as the WARM-AIR DUCT.

Bunker Door. The door provided to allow placing of, or access to, the coils in the bunker shall be known as the BUNKER DOOR.

The following are considered standard sizes for cooling-rooms, the first dimension being the length and the second the depth. The standard height of the sizes is approximately 9 ft 10 in.

7′ 0″ × 5′ 0″	8′ 0″ × 8′ 0″	10′ 0″ × 8′ 0″
8′ 0″ × 6′ 0″	9′ 0″ × 7′ 0″	12′ 0″ × 10′ 0″

When the height of a cooling-room is referred to as 10 ft it may be actually only 9 ft 10 in high over all measured on the body of the room in order that it may be used in a room with a 10-ft ceiling.

Specifications. 1. A grade "A" refrigerator or cooling room is one wherein the structural details and insulation shall be such that over years of service the loss of refrigeration through the walls shall not exceed 3 Btu per sq ft of outside area, per degree difference per 24 hours.

2. The entire upper section of the refrigerator or cooling-room should be devoted to bunker space.

3. The coil decks should be constructed with supports of sufficient structural strength to carry the weight of the cooling coils of refrigerating equipment placed above them, or instead proper supports should be placed above the deck so that coils may be suspended from same by hangers provided as part

* Prepared by the Joint Commercial Refrigeration Committee of the Refrigerating Machinery Association and Commercial Refrigerator Manufacturers.

of the coil equipment. The overhead supports for the coils should be secured to the wall framing and should in no way be fastened to the top or ceiling of the refrigerator.

4. The coil deck should pitch toward the cold-air duct sufficiently to create drainage and provide proper circulation.

5. With cooling-rooms up to and including 8 ft 0 in wide outside and across the decks, a single deck shall be deemed sufficient, with the warm-air duct along one side and the cold-air duct along the opposite side.

6. Cooling rooms over 8 ft 0 in wide outside and across the decks shall have a deck on each side pitched toward the cold-air duct with the warm-air duct opposite the cold-air duct.

7. The baffle forming the warm-air duct should be stopped at a distance from the ceiling equal to the width of the warm-air duct.

8. Cold and warm-air ducts should extend the full length of the cooling-room, and both baffle and deck should touch the end walls.

9. Minimum height between the deck and ceiling of the cooling-room should be 27 in in rooms 10 ft 0 in high and other height rooms in proportion.

10. The coil decks shall be constructed and provided with approved insulation so as to have a heat transmission not exceeding 5 Btu per sq ft, per degree difference each 24 hours.

11. The baffles shall be constructed and insulated so as to have a heat-transmission not to exceed 7.4 Btu per sq ft, per degree difference.

12. The top of the coil deck and inside of the baffle shall be covered with some form of rust-resisting metal.

13. The bunker door should be placed as near as practical in the center of one end of the bunker.

14. Bunker doors shall be at least 2 in higher than the height of the baffle and shall have a minimum width of 27 in. The minimum height of the bunker or coil door in rooms 9 ft up to and including 11 ft in height should be 23 in in the clear. In rooms over 11 ft high the doors should have a minimum height in the clear of 36 in. The heights specified give the measurement of the necessary unobstructed opening.

15. If wedge strips are used on top of the deck for supporting the coils or cooling equipment, they shall be level on top and not less than 2 in high at the low point of the wedge or high point of the deck and higher if possible; such wedge strips should also be notched at the lower end so that they will properly clear the drain-trough and offer no obstruction to the flow of drainage therein.

16. There should be at least 6 in clear space between the baffle and the coils, congealing tanks or tubes, and there should be at least 4 in clear space horizontally between the nearest edge of the cold air duct and these parts of the equipment. Supports for such parts should be arranged so that there will be at least 6 in clearance between their lowest edge and the highest point of the deck.

17. Wherever it is necessary to carry cold pipes through the walls of cooling rooms or refrigerators, the pipe insulation should be carried 1 in inside the cooling-rooms or refrigerator. These openings should be sealed air-tight with a water-proof, odorless sealer inside and outside.

18. Cold pipes should not run through or across warm-air ducts unless it is absolutely necessary. In the event that it is necessary to do so the pipe should be thoroughly insulated or provided with drip troughs to carry the drippage from them to the bunker deck. Cold pipes should not be run through the food compartments unless absolutely necessary. When necessary to do so the pipes should be thoroughly insulated unless their location

is such that proper drip-troughs can be provided to prevent the drippage to the floor or on the produce stored.

19. Wherever cold pipes run through or across cold-air ducts, they should either be well insulated or drip-pans provided to catch the drippings and carry same to the bunker deck.

20. In no case should any pipe be so located as to interfere with the free circulation of air through both the warm and cold-air ducts. In no event should pipes of any kind be run horizontally in either warm-air or cold-air ducts.

21. Pipes that will frost or sweat should not pass through the floor of cooling rooms. If this cannot be avoided, the pipes should then be well insulated not only where they pass through the floor but to the point where they come out to the open below the floor so that frost or moisture may not be permitted to collect between the room-floor and building-floors or foundations. Where pipes pass through floors, there should be flashing extending at least 2 in above the floor sealed water-tight to the covering and the cooling-room floor.

MECHANICAL REFRIGERATION *

A Brief Description of Methods in Common Use for Producing and Applying Refrigeration, with Special Reference to Small Plants

A British Thermal Unit (Btu) is the quantity of heat required to raise the temperature of 1 lb of water 1° F. Heat used in this way, that is, to raise the temperature of water or other substance, is said to be present in that substance as SENSIBLE HEAT, or, in other words, heat, the presence of which we can feel, or sense.

The Heat of Liquefaction, or so-called LATENT HEAT OF LIQUEFACTION of a mass of ice, is the amount of heat it will absorb in melting. One pound of ice at 32° F. will absorb 144 Btu in melting to water at 32° F. Heat coming into a cake of ice is thus absorbed in melting the ice and becomes what is known as LATENT HEAT, or heat absorbed without any rise in temperature. If the ice is at a lower temperature than 32° F., or if the water resulting from the melting rises above 32° F., additional heat will be absorbed as SENSIBLE HEAT.

The Specific Heat of a substance is the ratio of the quantity of heat required to raise the temperature of a certain weight of the substance one degree to that required to raise the same weight of water from 62° to 63° F.

The Heat of Vaporization of water or of any other liquid is the amount of heat it will absorb in vaporizing, in evaporating from a liquid to a gas, or will give out in returning from the gaseous to the liquid state.

Transfer of Heat occurs in three ways: (1) by convection, (2) by radiation and (3) by conduction. For instance, if particles of air in a refrigerator adjacent to a source of heat become warmed they circulate and distribute the heat by CONVECTION through the refrigerator-box. Heat will pass from a warm substance, as from the filament of an incandescent lamp, out into the box by RADIATION. Heat will enter the box through the walls by CONDUCTION.

Heat-Transmission. When the temperatures on opposite sides of any surface, as for instance, a wall, are unequal, heat will pass by conduction through the material from the warmer to the cooler side. The rate of this movement is the RATE OF HEAT-TRANSMISSION and is stated in terms of the quantity of

* Compiled and adapted, by permission, from data included in a paper by R. F. Massa See, also, Refrigerators.

heat (called Btu) which will pass through 1 sq ft of surface in 24 hours, per degree temperature-difference between the two sides of the wall.

Some Advantages Claimed for Mechanical Refrigeration.

(1) Lower temperatures can be obtained with refrigerating-machines than with ice.
(2) The inconvenience of handling ice is avoided.
(3) There is no accumulation of slime in the refrigerators as from the melting of even the best ice.
(4) Refrigerators cooled mechanically are dryer than ice-cooled boxes because the moisture is frozen out of the air and deposited on the cooling surfaces.
(5) There is generally a better air-circulation, resulting in a more uniform temperature and dryer atmosphere throughout the compartment.
(6) With proper design of refrigerator and refrigerating-machine any desired temperature can be obtained.
(7) Refrigeration produced mechanically is often cheaper than refrigeration produced by melting ice.

Operation of Refrigerating-Machines. In almost all methods of producing cold, advantage is taken of the fact that when a liquid evaporates it usually cools both itself and its surroundings, and changes into a gas or vapor. There are several liquids which are easily made to evaporate and produce this cooling effect, and, were it not for their cost, refrigeration could be very simply produced by supplying a steady stream of the liquid and allowing the vapor or gas evaporated to escape into the atmosphere. A refrigerating-machine is practically an apparatus for saving this gas which has evaporated and returning it to its liquid form to be used over again. In this process of recovery and condensation the gas gives out the heat which it has previously absorbed in evaporating This heat is carried away by flowing water, which, in absorbing the heat, rises in temperature.

Types of Refrigerating-Machines. In the (1) COMPRESSION-TYPE of refrigerating-machines the recovery of the gas is effected by drawing it away from the point where it has been evaporated and pumping it under increased pressure into a chamber where it gives out its heat to the water-cooled walls of the chamber and returns to the liquid state ready to be used over again. In the (2) ABSORPTION-TYPE of refrigerating-machines ammonia is generally used and the recovery of the gas is effected by bringing it into contact with water with which it unites chemically. The solution thus formed is pumped into another chamber, and heat is applied to drive off the ammonia-gas which is then condensed under high pressure. It is now ready to be reevaporated and reproduce its cooling effect. In all cases of large units, and in all cases of either large or small units where exhaust-steam is available in sufficient quantities, absorption refrigerating-machines are very economical.

Liquids Used in Refrigerating-Machines. A number of liquids have been used in refrigerating-machines, the ones commonly employed being (1) AMMONIA, (2) CARBON DIOXIDE and (3) SULPHUR DIOXIDE. Various practical considerations determine which is to be used in any particular design of machine. With (1) AMMONIA the advantage is the lower working pressures, from 15 to 200 lb per sq in, which are easy to deal with. An advantage over carbon dioxide is that leaks are very easily located Ammonia-fumes, however, are offensive and sometimes dangerous in case of a break. With (2) CARBON DIOXIDE the advantage is in its inoffensive odor Its disadvantages

are the high pressure at which it works, from 300 to 1 200 lb per sq in, the relative difficulty of holding these pressures and of finding small leaks, owing to its slight odor and chemical inactivity. With (3) SULPHUR DIOXIDE the advantage is its comparatively low working pressure, which is not above 75 lb per sq in. Its great disadvantage is that with moisture it forms an acid which rapidly corrodes the apparatus. At one time this disadvantage was fatal, since with the old-type machines, air and moisture were constantly being drawn into the system more or less rapidly and mixed with the sulphur dioxide. This difficulty has been overcome in modern types of machines in which the refrigerant is hermetically sealed in the machine and chemical action, therefore, prevented.

Rating of Refrigerating-Machines. A 1-TON REFRIGERATING-MACHINE is a machine which, if operated for 24 hours, will absorb the amount of heat which 1 ton of ice would absorb in melting. If the machine is operated a shorter time per day, a less amount of heat will of course be absorbed, and in order to maintain the temperature during the period when the machine is not running, some means must be adopted for storing cold. (See paragraph below.) Refrigerating-machines are sometimes rated in terms of ICE-MAKING CAPACITY, that is, in terms of the amount of ice the machine will make in 24 hours. This is always less than the refrigerating capacity because some refrigerating effect is required to cool the water down to 32° F. before the freezing can begin, and the ice is usually cooled several degrees below 32° F., which requires a still greater capacity. There is also some flow of heat into the apparatus. These elements vary considerably so that from some points of view ICE-MAKING CAPACITY might be considered an unsatisfactory method of rating some refrigerating-machines.

Applying the Cold. According to one classification there are three common systems of applying the cold. These are, (1) the DIRECT-EXPANSION SYSTEM (2) the BRINE-SYSTEM and (3) the COLD-AIR SYSTEM.

(1) In the DIRECT-EXPANSION SYSTEM the refrigerant is evaporated in coils of pipe placed directly in the room to be cooled

(2) In the BRINE-SYSTEM the refrigerant is used to cool brine, which is then circulated through coils of pipe in the room to be cooled.

(3) In the COLD-AIR SYSTEM a current of air is chilled by passing it over coils of pipe cooled directly by the evaporating refrigerant, or by brine, or by passing it through a spray of cold brine; and this chilled air is then passed into the room and circulated back to the cooling-coils, the whole operation being repeated indefinitely.

All of these systems have their advantages and disadvantages. While the brine-system is a little more expensive to operate in large plants, the temperature is more easily controlled than with the direct-expansion system, and in practice in small plants it is found as economical in operation in spite of its theoretical disadvantage. Furthermore, in case of any breakdown in the machine, the temperature can be held for a time by circulating the brine until it becomes too warm to be of use, whereas with direct expansion the temperature will begin to rise immediately upon the stopping of the machine. The cold-air system is not as applicable where any drying of the goods stored would be harmful and there is some risk of carrying fire in the air-passages. It is much used, nevertheless, for such service as chocolate-dipping rooms, ice-cream hardening, fur-storages, etc.

Storage of Cold. When temperatures are to be maintained while the refrigerating-machine is shut down, COLD must be STORED. In the brine-system

this is effected by cooling a comparatively large body of brine which warms slowly as it is circulated. Where the brine-circulating pump as well as the machine must be stopped, so-called PRESSURE-TANKS may be placed in the piping-system in the room being cooled; the mass of brine in these tanks absorbs the heat and helps to maintain an approximately even temperature. Where the direct-expansion system is used, a part of the cooling-coils may be immersed in a tank of brine placed in the room and the remainder of the coils arranged for the direct cooling of the room. In some places the spaces available will not permit the use of brine-storage tanks. In cases of this kind smaller tanks may be used and filled with water, or a weak brine which will freeze at a temperature a little below 32° F. Since 1 lb of ice in melting will absorb 144 Btu and 1 lb of brine rising in temperature, say 20°, will absorb only from 14 to 16 Btu, the saving of space is apparent. It must be absolutely certain that the refrigerant reaches the tank first at the bottom and that the air to be cooled reaches it first at the top so that the ice in forming shall not bulge or burst the tank. If the congealing mass were to freeze from the top down the tank would be strained and finally leak, because of the expansion of the ice in freezing. Another fact to be considered is that where water, only, is frozen, a resulting high temperature may be obtained in the refrigerator, since the brine must be warmer than the ice in order to melt it, and the refrigerator just that much warmer, or warmer than an ice-cooled box. In calculating the proper sizes of tanks for storing brine, it should be remembered that, usually, the period during which the machine is shut down coincides with the period during which the demand for refrigeration in the box is the least. The amount of heat to be absorbed is usually only that entering through the insulation, as the doors are shut and no food is put in or removed.

Description of Refrigerating-Machines. As explained in the preceding paragraphs refrigerating-machines may be divided generally into two classes, (1) the COMPRESSION-TYPE and (2) the ABSORPTION-TYPE.

(1) The Compression-Type of Refrigerating-Machines may be subdivided as follows:

(*a*) The open type of machine, which is made both vertical and horizontal, and both single and double-acting, that is, compressing the gas at one end or at both ends of the cylinder. (*b*) The partially enclosed type of machine, in which all the moving parts of the compressor proper are enclosed within the frame of the compressor, except the fly-wheel and the main shaft which enters the frame of the machine through a stuffing-box. Such valves, also, as are required in the system are exposed. (*c*) The wholly enclosed type of machine, in which all of the working parts are enclosed in a hermetically sealed container.

(*a*) One advantage of the open type of machine is that any lack of adjustment due to wear can be readily corrected; so that, with proper attention, it gives excellent results. For large installations this is considered by many to be a most efficient type of machine.

(*b*) The enclosed type of machine resulted from the effort to reduce the amount of attention required by the open machine, to cheapen its construction and to reduce the possibility of trouble from inexpert tampering. An objection to machines of this type is that when adjustments have to be made the working parts are relatively inaccessible.

(*c*) With the wholly enclosed type of machine it is claimed that the loss of the refrigerant is prevented by the hermetical sealing of the apparatus, and that the working parts, being completely enclosed, are protected from deterioration due to outside causes or tampering.

(2) **The Absorption-Type of Refrigerating-Machines** are of two kinds, differing principally in the proportioning of the parts. In the one machine high-pressure steam is used; in the other the proportions are such that low-pressure or exhaust-steam may be used. Where exhaust-steam is available machines of this type are found to be very economical, and this is true, also, for all large units whether or not exhaust-steam is used. Full descriptions of these machines with detailed plans and layouts may be obtained from the various manufacturers.

Calculations for the Capacity of a Refrigerating-Machine. Heat enters the refrigerated compartments, (1) through the walls, (2) with warm goods, (3) by the interchange of the outside air when doors are opened and by air-leaks, since the cooled air is the heavier and immediately flows out when a door is opened, (4) from lights or from the heat of the bodies of workers, and (5) from any change of state occurring in the goods, such as freezing, fermenting, etc. In large rooms these various sources of heat should be analyzed separately. In small refrigerators, as in hotels, kitchens, dwellings, etc., a rough rule, quite as accurate as a more elaborate analysis, allows a certain number of Btu per cubic feet of refrigerated space per 24 hours. This amount varies with the character and location of the box, the nature of its insulation, the temperatures desired and so on. It will be seen that the insulation, while of great importance, is not by any means the only important factor in this class of boxes. For domestic refrigerators in which a temperature of from 35 to 50° F. is maintained, 300 Btu per cu ft of refrigerator per 24 hours should be allowed. For boxes in hotel or restaurant-kitchens, 600 Btu, or even 900 Btu in extreme cases and where low temperatures are required, should be allowed. For butchers' coolers or large storage-boxes in hotels, etc., from 200 to 250 Btu per cu ft per 24 hours should be allowed. A check on the above figures for the large type of box is the following: * "When the exact conditions under which cold-storage rooms are to be operated are known, namely, the size and shape of the rooms, the quality of the insulation, the kind and quantity of goods to be handled per day and the temperatures at which they are received and at which they are to be held, the amount of refrigeration required can be estimated very closely by the following rule: (1) Calculate the exact area of exposed surface in the walls, floor and ceiling of the room in square feet, multiply the total number of square feet by the number given in the table for the required temperature and divide the product by 288 000. (2) Multiply the amount of goods, in pounds, to be stored per day by the number of degrees of heat to be extracted by the specific heat of the goods, and divide by 288 000. This will give the amount of refrigeration, in tons per day, necessary to maintain the temperature required for the goods. (3) Add these two amounts together. The total will be the amount of refrigeration, in tons per day, required to maintain the temperature required for the goods and for the room. (4) If the goods are to be frozen, the latent heat of freezing should be added to the number of Btu to be extracted."

With small machines it is necessary to allow a greater capacity of machine for a given size of box than with large machines, since, with the latter, one can always throw a large part of the machine-capacity to any given box where special need may exist; whereas to do this with the small machine would almost certainly rob some other box, if indeed there happened to be another box. It is never possible to determine with mathematical certainty exactly how much refrigeration is required for a given case. It is best to allow for this fact and to be sure the machine is amply large. Where an existing ice-cooled box is to

* **Taken from Levey's Refrigeration Memoranda.**

be cooled mechanically one check upon the size of the machine required is the amount of ice used. This check is more apt than any other, however, to lead to erroneous conclusions unless the figures are properly analyzed.

For rooms containing less than 1 000 cu ft			
If maintained at	0° F.	multiply the exposed surface by	1 775
If maintained at	5° F.	multiply the exposed surface by	710
If maintained at	10° F.	multiply the exposed surface by	535
If maintained at	20° F.	multiply the exposed surface by	355
If maintained at	32° F.	multiply the exposed surface by	265
If maintained at	36° F	multiply the exposed surface by	180
For rooms containing from 1 000 to 10 000 cu ft			
If maintained at	0° F	multiply the exposed surface by	1 250
If maintained at	5° F.	multiply the exposed surface by	600
If maintained at	10° F	multiply the exposed surface by	300
If maintained at	20° F	multiply the exposed surface by	190
If maintained at	32° F	multiply the exposed surface by	160
If maintained at	36° F.	multiply the exposed surface by	125
For rooms containing more than 10 000 cu ft			
If maintained at	0° F	multiply the exposed surface by	1 100
If maintained at	5° F.	multiply the exposed surface by	550
If maintained at	10° F	multiply the exposed surface by	275
If maintained at	20° F.	multiply the exposed surface by	180
If maintained at	32° F.	multiply the exposed surface by	140
If maintained at	36° F	multiply the exposed surface by	110

Another Method of Determining the Capacity of a Refrigerating-Machine. The following is a method that gives good results, except that allowance may be made in the larger boxes and where brine-storage tanks are provided in the box for the steadying effect of the mass of cold brine:

(1) The ice-consumption for the hottest month of the year should be determined. This will give the average ice-consumption for that month.

(2) The average temperature that is maintained in the box with ice should then be accurately determined. This will usually be from 55 to 65° F. It will commonly be stated to be anywhere from 40 to 45° F., but these temperatures are seldom obtained. Even if they are, with a full ice-chamber and the box closed for long periods the average will be above these figures. Unless, therefore, there is positive assurance to the contrary, from 55 to 60° F. should be considered the average temperatures.

(3) A calculation should then be made of the heat-inflow through the insulation, with a temperature of 55° F. in the box and with the average summer temperature outside. The difference between the heat-inflow through the insulation and the total heat actually absorbed by the melting of the ice is the amount entering the box from other sources than through the insulation. This access of heat ordinarily occurs during the hours of daytime only, that is, when the box is being opened, since at night the box will remain closed. A machine of sufficient capacity to produce the temperature actually obtained with ice must, therefore, be of larger rated capacity than that indicated by the actual ice-consumption; and how much larger it should be can be determined by this method.

(4) A further fact which it is claimed should be taken into account in deter-

mining the proper size of a machine is that temperatures obtainable with ice are often unsatisfactory. If they were always satisfactory, one reason for putting in cooling-machinery would be done away with. Where 55° F. is obtained with ice, from 35 to 45° F. will be required with mechanical cooling and the machine-size must be further increased in the ratio of the temperature-differences between average summer temperatures and 35° F., and average summer temperatures and 55° F.

(5) The cooling-machine if installed in accordance with these figures would handle average-weather conditions but would not be adequate for extreme hot-weather conditions, the most important conditions to be met by cooling-machinery. It is necessary, therefore, to further increase the size of the machine in the ratio of the difference in temperature between maximum summer temperature and 35° F., and average summer temperature and 35° F.

(6) A further allowance should be considered, namely, the fact that in many cases, for one reason or another, it is not possible, or else not desirable, to operate the machine except during certain periods of the day, and the machine-size must be increased as much as may be required to take care of these conditions.

(7) If the machine is not placed directly at the box to be cooled, allowance must be made for the heat-inflow into the insulated brine-mains. The amount of heat entering from this source is often of considerable importance, particularly with small machines. The table below gives heat transmissions for cork pipe-covering and some other materials.

Water and Milk-Cooling. Mechanical refrigeration as applied to cooling water and milk differs in one respect from other classes of refrigerating-work. A relatively intense quantity of cooling effect is called for in a brief interval of time. For instance, in a drinking-water system the heaviest requirements may come at the noon-hour. In a bakery, also, the demand for chilled water will be intermittent, a large quantity of water being required for the dough-mixing. In dairy-work the milk must be cooled very rapidly to check the development of bacteria which grow with incredible rapidity within the temperature-range of from 110 to 50° F. To install a large enough refrigerating-machine to produce the required cooling effect as it is needed would in most cases call for a very large machine. This is overcome by using a smaller machine and allowing it to operate for a longer time, say throughout the day, storing the refrigerating effect produced by cooling a large body of brine, or melting the ice as rapidly as may be required. For instance, if 50 cans of milk, of 40 qts each, are to be cooled from a temperature of from, say, 75 to 35° F., in 1 hour, the refrigeration required will be 50 cans times 40 qts times 2 lb per qt times (75° F. — 35° F.), which equals 320 000 Btu. Milk is treated in the calculation as having the same specific heat as water, since water forms so large a percentage of its total weight. This amount of refrigeration produced by a machine running 12 hours per day would require the machine to absorb 320 000 Btu divided by 12, or 26 000 Btu per hour. The quantity of brine necessary to store the cooling effect may be calculated closely enough for practical purposes by using the following approximate figures. The specific heat of brine is 0.75. The weight of the brine is 9 lb per gallon. The permissible temperature-range of the brine depends upon the conditions and may be from, say, 30 to 15° F., or lower. In other words, the temperature to which the brine can be permitted to rise is limited to the temperature it must produce in the room or in the subtsance being cooled, and the temperature to which the brine can be cooled in storing cold is limited by the decrease in economy of the refrigerating-machine at the low temperatures.

The Value of Good Insulation. The importance of good insulation cannot be too strongly emphasized. A cold-storage room or refrigerator and its contents may be cooled by ice or mechanical means, but unless the walls are adequately insulated, the demand caused by the inflow of heat through the poor insulation may be more than the ice-supply or refrigerating-machine can meet to maintain the required temperature. The almost universal standard of insulation for cold-storage rooms is a 4-in thickness of pure-cork sheet. The following table shows the heat transmitted through 1 in in thickness of each of the substances, per square foot of exposed surface per degree difference in temperature per 24 hours.

Pure-cork sheets	6.4 Btu
Hair-felt	7 3 Btu
Impregnated cork boards	8 5 Btu
Rock-wool blocks	8.0 Btu
Waterproofing lith-blocks	8 5 Btu
Spruce, clear and dry	16 0 Btu
White oak	26 0 Btu

Incidental Notes on Refrigerators. **Drawers.** In restaurant-kitchens and elsewhere it is sometimes convenient to have a box fitted with a number of refrigerated drawers. The heat-leakage through the many joints, through slides which are invariably only partially closed, and through the poor insulation of the drawers, is very great. Where it is at all possible to do so, it is best to arrange an insulated door covering the entire drawer-space.

Table of Temperatures Recommended for Use in the Equipment Specified Under Normal or Average Operating Conditions *

Description of article	Location of thermometer	From degree named below	To degree named below
Small market cooling-room	Center of rear wall	38	45
Large storage cooling-room	Center of rear wall	36	42
Grocer's refrigerator	Small lower compartment	42	48
Restaurant service-refrigerator	Small lower compartment	42	48
Restaurant storage cooling-room	Center of rear wall	38	45
Florist's refrigerator		48	54
Top display case	Center of bottom	42	48
Floor display counter	Center of bottom	42	48
Floor display counter, heavy construction	Center of bottom	36	40
	Center of top shelf	44	48

* Issued by The Joint Commercial Refrigeration Committee, 422 Murray Bldg., Grand Rapids, Mich.

Anterooms. In storage-rooms of medium to large size the air-interchange due to opening doors is reduced to a minimum by arranging an anteroom or entry which, after it is entered, has its outer door closed before the door to the storage-room proper is opened. Where two rooms are side by side, it is often possible to reduce the interchange of air by treating the one room as an anteroom of the other, having but one door to the outside air.

Doors. Special note should be made as to the design of doors for refrigerated rooms or boxes. There is a common idea that a refrigerator-door should be beveled. As a matter of fact no more certain means of ensuring air-leakage could be devised. A perfectly fitted beveled door, hung accurately in place, could perhaps be made tight in the beginning. This door in service at once begins to sag, since a refrigerator-door is always heavy. It immediately becomes impossible to force it to a tight seat and continuous leakage of air begins. A refrigerator-compartment door is most readily made tight by having a flat surface on the door come up against a corresponding surface on the frame, with a soft gasket of some kind between them. There are several well-made refrigerator-doors on the market at prices low enough to make it doubtful economy to attempt the home-made article.

Approved Cold-Storage Temperatures

Articles stored	Degrees Fahrenheit
Beef	36 to 40
Lamb and mutton	32 to 36
Hogs	29 to 32
Veal	34 to 36
Meats, in pickle or brine	35 to 40
Butter, must be kept separate from other goods	0 to 38
Eggs	29 to 32
Cheese	32 to 34
Lard	38 to 40
Poultry, to freeze	5 to 10
Poultry, when frozen	25 to 28
Game, to freeze	5 to 10
Game, when frozen	25 to 28
Fish, retail fish-counters should be cooled with ice rather than mechanically	25 to 28
Oysters	33 to 45
Beer	33 to 42
Wines	40 to 45
Cider	30 to 40
Fruits	33 to 36
Vegetables	34 to 40
Canned goods	38 to 40
Flour and meal	40
Furs	25 to 32
Brine for ice-cream freezing	5 to 10
Ice-cream, air hardening	5
Ice-cream, serving-temperature	14 to 16

Arrangement of Brine-Mains. In laying out mains to carry brine from the refrigerating-machine to the refrigerator, there are a few simple points to be cared for. For the convenience of the pipe-covering man, the flow and return lines should be placed far enough apart so that he can get his covering onto each pipe without cutting it to pieces, or else they should come close together so as to be covered together. A common difficulty experienced in brine-systems of refrigeration, where the cooling-coils in several compartments are fed from the same main, is that when the adjustment of the valve controlling the flow of brine through one coil is changed, it upsets the adjustment of the whole system. This is due to too small mains or too small a pump, or both. A similar action is observed when the opening of a faucet on a water-pipe checks

the flow from other open faucets on the line. The ideal cross-section area of the brine-mains is as nearly as possible equal to the combined cross-section area of the coils which they serve at any one time. Even with this proportion, however, it is not possible to absolutely ensure that the lower coils will not rob the upper ones, or even drain them completely in some systems of piping. A most effective, even if somewhat expensive method of overcoming this difficulty, is by the addition of a third main. In this arrangement it is not possible for one coil to rob another to the point of draining it.

Calculations for the Necessary Amount of Cooling-Surfaces. No hard-and-fast rule can be given regarding the proper amount of cooling-surface for compartments of various sizes, since the design and arrangement of the cooling-surface and the freedom with which the air circulates over it greatly affect the amount required. As a general guide, however, and where the conditions are such as to permit a good circulation of the air, the following formula will give good results. It will be understoood, of course, that the refrigeration required in the given room has been determined as previously indicated. The cooling-surface required, in square feet, per ton of refrigeration equals $4\,700/(T - t)$ in which T is the temperature desired in the compartment, and t the average temperature of the brine.

TOWER-CLOCKS *

Rule for Diameter of Dials. "To look well and show plainly, dials should be 1 ft in diameter for every 10 ft of elevation and should set out flush with or close to the line of the building or tower." †

Dimensions of Some Large Clock-Faces. Colgate's Factory, Jersey City, N. J. The diameter of the dial is 40 ft. The minute-hand is 20 ft long and 2 ft 11 in in extreme width, and the hour-hand is 15 ft long and 3 ft 10 in in extreme width. The minute-hand weighs 640 lb and the hour-hand 500 lb. This is the largest clock in the world.

Bromo-Seltzer Building, Baltimore, Md. The dials are 24 ft in diameter. The minute-hand is 12 ft 7 in and the hour-hand 9 ft 8 in from tip to tip. The minute-hand weighs 175 lb, the hour-hand 145 lb.

Daniels-Fisher Building, Denver, Colo. The dials are 15 ft 6 in in diameter. The minute-hand is 7 ft 10 in and the hour-hand 5 ft 7 in long.

Maryland Casualty Building, Baltimore, Md. The dials are 17 ft in in diameter. The minute-hand is 8 ft 4 in and the hour-hand 5 ft 11 in long.

Elgin Watch Company's Factory, Elgin, Ill. The dials are 14 ft 6 in in diameter. The minute-hand is 7 ft 4 in and the hour-hand 5 ft 4 in long.

Tower-clock, Station of the Central Railroad of New Jersey, at Communipaw, N. J. The diameter of the single dial is 14 ft 3 in; the minute-hand is 7 ft long and weighs 40 lb; the hour-hand is 5 ft long and weighs 28 lb. The motive power is furnished by a weight of 700 lb, hung from a 3/8-in steel cable.

Four-dial clock, Produce Exchange Building, New York. The diameter of each dial is 12 ft 6 in.

Four-dial clock, Chronicle Tower, ‡ San Francisco, Cal. The diameter of each dial is 16 ft 6 in; length of minute-hands, 8 ft; length of hour-hands, 5 ft 6 in. The mechanism of the clock is 6 ft 1 in high and weighs 3 000 lb.

* For a description of the requirements of installation of tower-clocks, see page 154 of "Churches and Chapels," by F. E. Kidder.

† Seth Thomas Clock Company, Thomaston, Conn.

‡ Destroyed in the earthquake and fire.

Pneumatic clock, City Hall and Court-House, Minneapolis, Minn. The dials are 23 ft 4 in in diameter.

Four-dial clock, City Hall, Philadelphia, Pa. The diameter of each dial is 26 ft.

LIBRARY BOOK-STACKS

The Stack-Work in General. The stack-room of a library is usually cut off by fire-proof doors from the rest of the building. The customary practice among architects is to make the stack-work a separate contract and have the general contractor turn the stack-room over to the stack-contractor with finished floors, walls and ceilings. The stacks, made entirely of incombustible materials, are then built as an independent structure.

Book-Ranges. The book-ranges are usually double-faced and are placed in parallel rows with aisles between. The minimum aisle-width is about 2 ft 4 in. Radial ranges waste space and are costly. Single-faced ranges are relatively more expensive than double-faced ranges.

Tiers. All stacks are divided in their height into tiers by deck-floors in order that all shelves may be easily reached. The regular tier-height is 7 ft or 7 ft 6 in.

Deck-Floors. Deck-floors are composed of slabs of ¾-in rough plate glass or 1¼-in white marble, supported on steel framework A long, narrow opening or deck-slit is left between the edge of each deck-floor and the face of each range to allow proper ventilation of the stack-tiers. The net thickness from top of deck to bottom of steel framework is from 3¼ to 3¾ in for ordinary spans. The deck-floors are carried by the shelf-supports.

Vertical Communication. Continuous flights of stairs of simple design and construction are placed at central points. Books are moved up and down by means of dumb-waiters operated by hand, for short runs, or by electric power controlled by push-buttons.

Shelf-Supports. The shelf-supports are made in various ways, differing with each manufacturer. In the best construction they extend the full width of the shelves so as to hold up the shelves and books without the use of any projecting brackets. They are made of sufficient strength to carry the combined loads of books, deck-floors and superimposed stack-piers. They should provide for a uniform shelf-adjustment at intervals of about 1 in. Compactness is important. Open-work shelf-supports promote proper lighting and ventilation.

Shelves. In each tier of regular height there are usually six rows of adjustable shelves and one row of fixed shelves. Shelves are generally 8 or 10 in wide and 3 ft long. Other sizes are supplied if necessary. The adjustable shelves are made of solid plates of sheet steel or of parallel bars with spaces between. The fixed shelves are placed about 2 in above each floor-level. They are made of solid plates of steel to form dust-stops, fire-stops and water-stops between the tiers.

Finish. The adjustable shelves are always completely finished with baked enamel before delivery. The fixed parts, also, of the stack-construction may be finished at the shop with baked enamel, or preferably with air-drying enamel, after erection at the building, so as to permit repair.

Lighting. Electric-light wires are carried in metal conduits supported by the steel framework of the deck-floors. Lights of 16 candle-power are spaced about 6 ft apart in range-aisles and 12 ft apart in main aisles.

Heating. Indirect radiation is best for books. The lower tiers, only, of a stack should be heated, to prevent the upper tiers from becoming too warm.

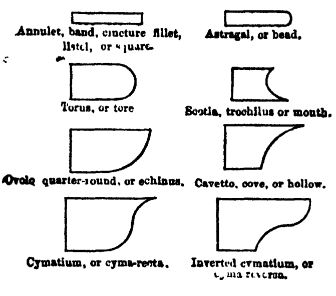
Ventilation. Large stacks are usually ventilated artificially to prevent the entry of dust and outside air through open windows. In the Library of Congress, in Washington, D. C., fresh, filtered and tempered air is forced in at the bottom tier, finds its way up through the stack by means of the deck-slits and is drawn out at the top tier.

Weights. The shelves and shelf-supports weigh from 7 to 10 lb per cu ft of book-range. Books weigh about 20 or 25 lb per cu ft of book-range. The steel deck-floor framing weighs from 4 to 6 lb per sq ft of gross area of deck-floor. Marble floor-slabs, 1¼ in thick, weigh about 20 lb per sq ft, and ¾-in rough, plate-glass slabs, about 10 lb per sq ft of net area.

Book-Capacities. Book-capacities per linear foot of shelf may be figured on the following basis: law-books, 5 volumes; reference books, 6 volumes; scientific books, 7 volumes; general literature, from 8 to 10 volumes. The average in the Library of Congress is 8½ volumes per linear foot. An ordinary stack-tier, 7 shelves high with double-faced ranges 16 in deep (or 8-in shelves) and aisles 32 in wide, with a reasonable allowance made for cross-aisles, stairways, etc., will contain about 22 volumes per sq ft of gross area.

CLASSICAL MOLDINGS

Moldings are so called because they are of the same shape throughout their length as though the whole had been cast in the same mold or form. The regular moldings, as found in remains of classic architecture, are eight in number, as shown in the accompanying illustration, and are known by the following names: The last two are commonly called, also, OGEE MOLDINGS. Some of these terms are derived thus: FILLET, from the French word FIL, a thread; ASTRAGAL, from ASTRAGALOS, a bone of the heel, or the curvature of the heel; BEAD, because this molding, when properly carved, resembles a string of beads; TORUS, or TORE, the Greek for rope, which it resembles when on the base of a column; SCOTIA, from SKOTIA, darkness, because of the strong shadow cast in its hollow, and which is increased by the projection of the torus above it; OVOLO, from OVUM, an egg, which this member resembles when carved, as in the Ionic capital; CAVETTO, from CAVUS, hollow; CYMATIUM, from KUMATON, a wave.



Characteristics of Moldings. None of these moldings is peculiar to any one of the orders of architecture; and although each has its appropriate use, it is by no means confined to any certain position in an assemblage of moldings. The use of the fillet and also of the astragal and torus, which resemble ropes, is to bind the parts. The ovolo and cyma-reversa are strong at their upper extremities, and are therefore used to support projecting parts above them. The cyma-recta and cavetto, being weak at their upper extremities, are not used as supporters, but are placed uppermost to cover and shelter the upper

parts. The scotia is introduced in the base of a column to separate the upper and lower torus, and to produce a pleasing variety and relief. The form of the bead and that of the torus are the same; the reason for giving distinct names to them is that the torus, in every order, is always considerably larger than the bead and is placed among the base-moldings, whereas the bead is never placed there, but on the capital or entablature. The torus, also, is seldom carved, whereas the bead is; and while the torus, among the Greeks, was frequently elliptical in its form, the bead retains its circular shape. While the scotia is the reverse of the torus, the cavetto is the reverse of the ovolo, and the cyma-recta and cyma-reversa are combinations of the ovolo and cavetto.

THE CLASSICAL ORDERS *

Origin of the Orders. "In the classical styles several varieties of column and entablature are in use. These are called the ORDERS. Each order comprises a COLUMN with a BASE, SHAFT and CAPITAL, with or without a PEDESTAL, with its BASE, DIE and CAP, and is crowned by an ENTABLATURE, consisting of ARCHITRAVE, FRIEZE and CORNICE. The entablature is generally about one-fourth as high as the column, and the pedestal one-third, more or less. Among the Greeks the forms used by the Doric race, which inhabited Greece itself and had colonies in Sicily and Italy, were much unlike those of the Ionic race, which inhabited the western coast of Asia Minor, and whose art was greatly influenced by that of Assyria and Persia. Besides the IONIC and DORIC styles, the Romans devised a third, which employed brackets, called MODILLIONS, in the cornice, and was much more elaborate than either of them; this they called the CORINTHIAN. They used also a simple Doric called the TUSCAN, and a cross between the Corinthian and Ionic called the COMPOSITE. These are the FIVE ORDERS. The ancient examples vary much among themselves and differ in different places, and in modern times still further varieties are found in Italy, Spain, France, Germany and England. The best known and most admired forms for the orders are those worked out by Giacomo Barozzi da Vignola in the sixteenth century from the study of ancient examples."

The Tuscan Order. "The distinguishing characteristic of the TUSCAN ORDER (Fig. 1) is simplicity. Any forms of pedestal, column and entablature that show but few moldings, and those plain, are considered to be TUSCAN."

The Doric Order. "The distinguishing characteristics of the DORIC ORDER are features in the FRIEZE and in the BED-MOLD above it called TRIGLYPHS and MUTULES, which are supposed to be derived from the ends of beams and rafters in a primitive wooden construction with large beams. Under each triglyph, and beneath the TÆNIA which crowns the architrave, is a little filet called the REGULA. Under the regula are six long drops, called GUTTÆ, which are sometimes conical, sometimes pyramidal. There are also either eighteen or thirty-six short cylindrical guttæ under the soffit of each mutule. The guttæ are supposed to represent the heads of wooden pins, or treenails. Two different Doric cornices are in use, the MUTULARY with bracket and the DENTICULATED with dentils, the principal difference being in the BED-MOLD." The order shown in Fig. 2 has the denticulated cornice.

* The paragraphs in quotation-marks are taken from The American Vignola by Professor W. R. Ware, by permission of the owners of the copyright, the International Textbook Company, Scranton, Pa., proprietors of the International Correspondence Schools. The engravings were made especially for this book, and correspond with the original drawings prepared by Giacomo Barozzi da Vignola.

The Ionic Order. "The prototypes of the IONIC ORDER (Fig. 3) are to be found in Persia, Assyria, and Asia Minor. It is characterized by BANDS in the architrave and DENTILS in the bed-mold, both of which are held to represent small sticks laid together to form a beam or a roof. But the most conspicuous and distinctive feature is the SCROLLS which decorate the CAPITAL of the

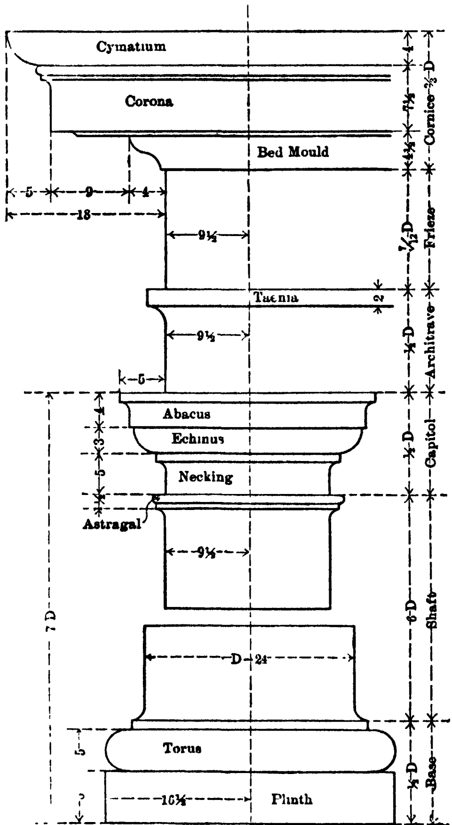


Fig. 1. The Tuscan Order

column. These have no structural significance, and are purely decorative forms derived from Assyria and Egypt. Originally the Ionic order had no FRIEZE and no ECHINUS in the capital. These were borrowed from the Doric order, and, in like manner, the dentils and bands in the Doric were borrowed from the Ionic. The Ionic frieze was introduced in order to afford a place for sculpture, and was called by the Greeks the ZOOPHORUS, or figure-bearer. The typical IONIC BASE is considered to consist mainly of a SCOTIA, as in some

Greek examples. It is common, however, to use instead what is called the ATTIC BASE, consisting of a SCOTIA and two FILLETS between two large TORUSES, mounted on a PLINTH, the whole half a diameter high. The plinth occupies the lower third, or one-sixth of a diameter. Vignola adopted for his

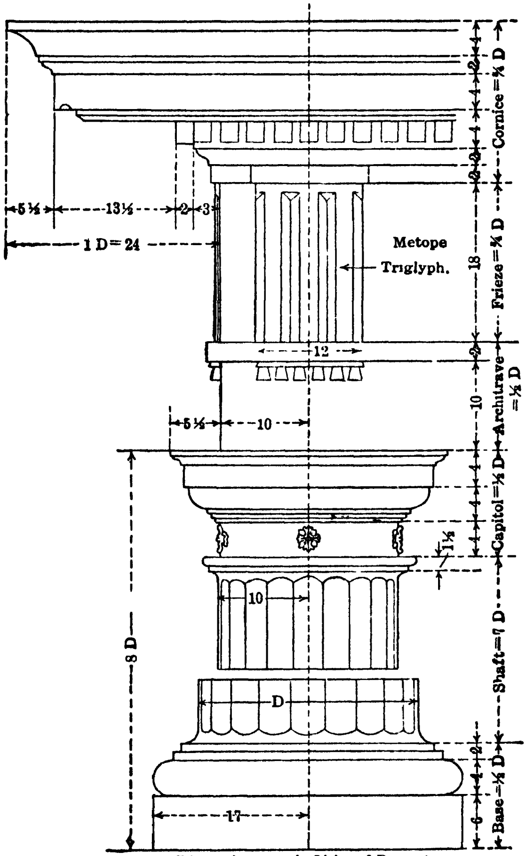
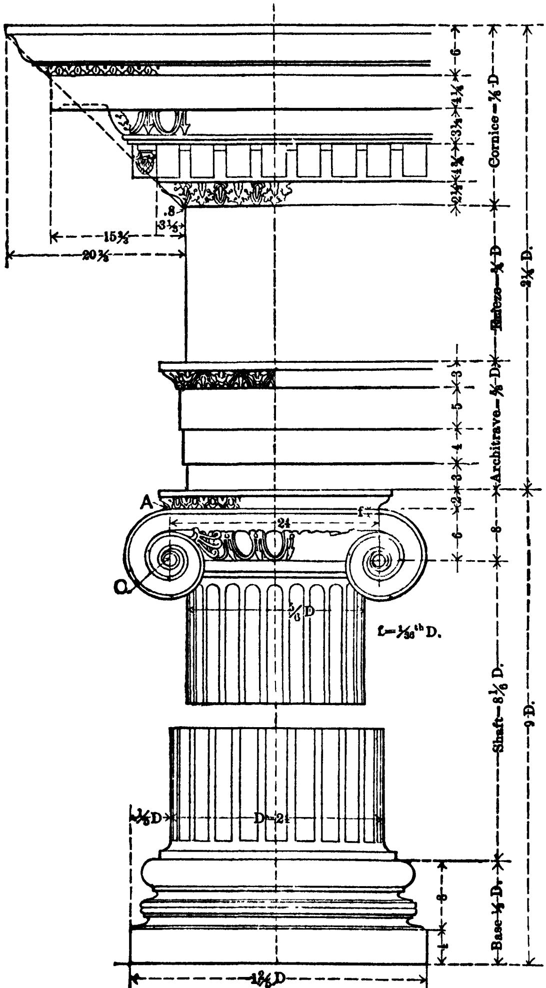


Fig. 2. The Doric Order

Ionic order a modification of the Attic base, substituting for the single large scotia two small ones, separated by one or two beads and fillets, and omitting the lower torus." This is the base shown in Fig. 3. "The Ionic frieze is plain, except for the sculpture upon it. It sometimes has a curved outline, as if ready to be carved, and is then said to be PULVINATED, from pulvinar, a bolster, which it much resembles. The SHAFT of the column is ornamented

with twenty-four FLUTINGS, semicircular in section, which are separated not by an ARRIS, but by a FILLET of about one-fourth their width. This makes



Dimensions are in 24ths of Diameter.

Fig. 3. The Ionic Order

the flutings only about two-thirds as wide as the Doric channels, or about one-ninth of a diameter, instead of one-sixth."

To Describe the Ionic Volute. There are several methods of doing this, the simplest being by means of centers found as shown by the diagram in Fig. 4. First locate the center of the EYE $\frac{1}{4}$ *D* vertically below the point *A*, Fig. 3. Then describe a circle with a diameter equal to $\frac{1}{18}$ *D*, to form the eye. Inside of this circle inscribe a square at 45 degrees to a horizontal; then draw the axes 1–3 and 2–4, and divide each of these into six equal parts. Then with the point 1 as a center, and a radius extending to *A*, Fig. 3, draw a quarter-circle to line 1–2 produced, with 2 as a center, continue the curve until it intersects 2–3 produced; and so on. The centers for the outer curve of the volute are at the points 1, 2, 3, 4, 5, 6, etc. For the centers for the inner curve, start with a point one-third the way from 1 to 5, then a point one-third the way from 2 to 6, and so on.

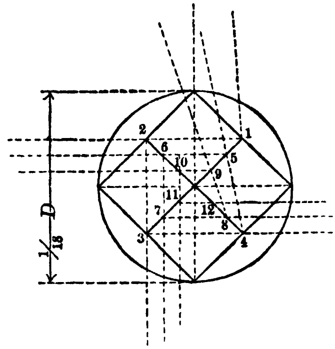
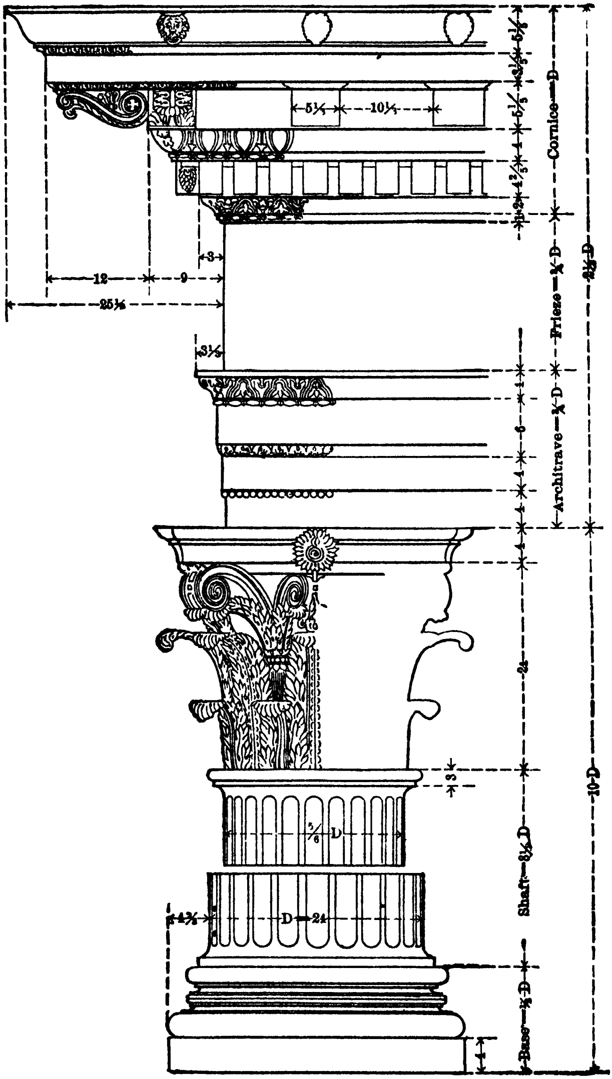


Fig. 4. The Ionic Volute

The Corinthian Order. "The three distinguishing characteristics of the CORINTHIAN ORDER (Fig. 5) are a tall, bell-shaped capital, a series of small brackets called MODILLIONS, which support the cornice instead of MUTULES, in addition to the DENTILS, and a general richness of detail which is enhanced by the use of the ACANTHUS LEAF in both capitals and modillions. Here, again, the ATTIC BASE is commonly used, but sometimes, especially in large columns, a base is used which resembles Vignola's IONIC BASE, except that it has two BEADS between the SCOTIAS instead of one, and also a lower TORUS. The SHAFT is fluted like the Ionic shaft, with twenty-four semicircular FLUTINGS, but these are sometimes filled with a convex molding or CABLE to a third of their height. Almost all the buildings erected by the Romans employ the Corinthian order."

The Composite Order. "The COMPOSITE ORDER is a heavier Corinthian, just as the Tuscan is a simplified Doric. The chief proportions are the same as in the Corinthian order, but the details are fewer and larger. It owes its name to the CAPITAL, in which the two lower rows of leaves and the CAULICOLI are the same as in the Corinthian. But the caulicoli carry only a stunted LEAF-BUD, and the upper row of leaves and the sixteen VOLUTES are replaced by the large ECHINUS, SCROLLS and ASTRAGAL of a complete Ionic capital. Vignola's composite entablature differs from his Ionic chiefly in the shape and size of the DENTILS. They are larger, and are more nearly square in elevation, being one-fifth of a diameter high and one-sixth wide, the INTERDENTIL being one-twelfth, and they are set one-fourth of a diameter apart, on centers. The composite capital is employed in the Arch of Titus in Rome, and elsewhere, with a Corinthian entablature, and the BLOCK CORNICE occurs in the so-called FRONTISPIECE of Nero, as well as in the temple at Athens, in connection with a Corinthian capital."

Egyptian Style. The architecture of the ancient Egyptians is characterized by boldness of outline, solidity, and grandeur. The principal features of the EGYPTIAN STYLE of architecture are: uniformity of plan, never deviating from right lines and angles; thick walls, having the outer surface slightly



Dimensions are in 24ths of Diameter

THE CORINTHIAN ORDER

Fig. 5. The Corinthian Order

deviating inwardly from the perpendicular; the whole building low; roof flat, composed of stones reaching in one piece from pier to pier, these being supported by enormous columns, very stout in proportion to their height; the shaft sometimes polygonal, having no base, but with a great variety of handsome CAPITALS, the foliage of these being of the palm, lotus and other leaves;

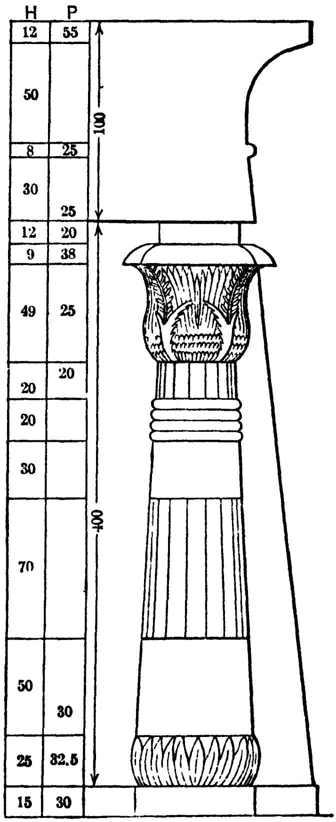
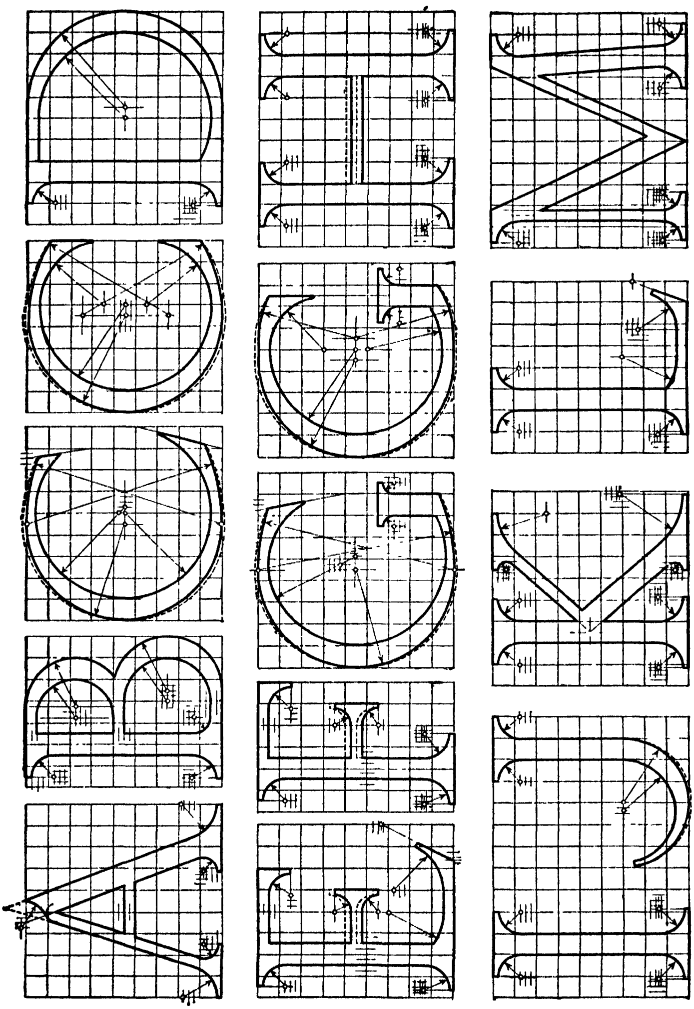
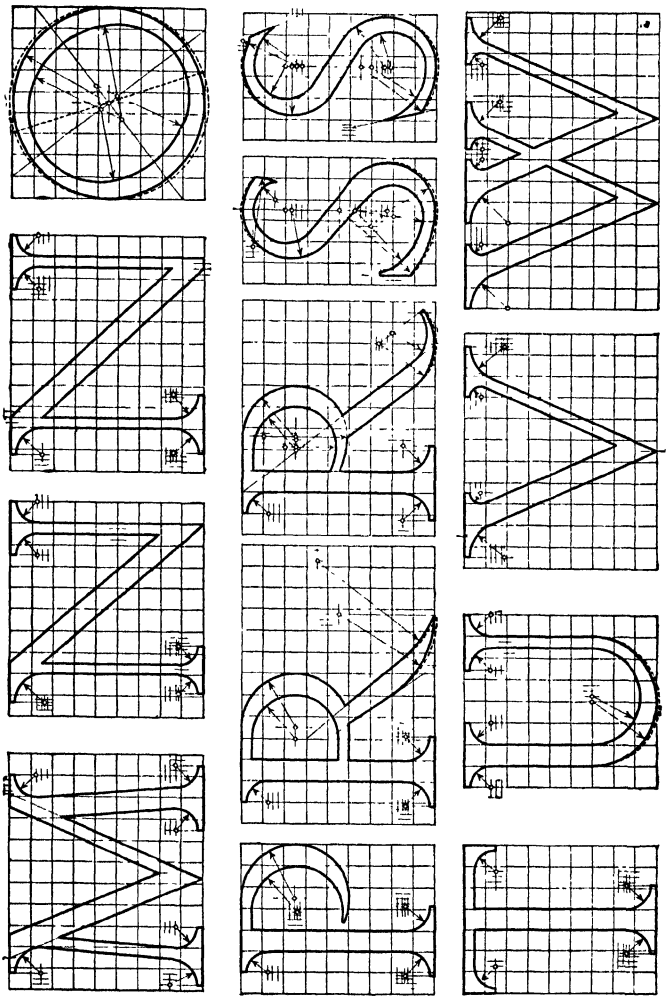


Fig. 6. An Egyptian Order. Diameter Divided into Sixty Parts

ENTABLATURES having simply an ARCHITRAVE, crowned with a huge CAVETO ornamented with sculpture; and the INTERCOLUMNIATION very narrow, usually 1½ diameters and seldom exceeding 2½. A great dissimilarity exists in the proportions, forms and general features of Egyptian columns. For practical use the column shown in Fig. 6 may be taken as a standard of the Egyptian style.

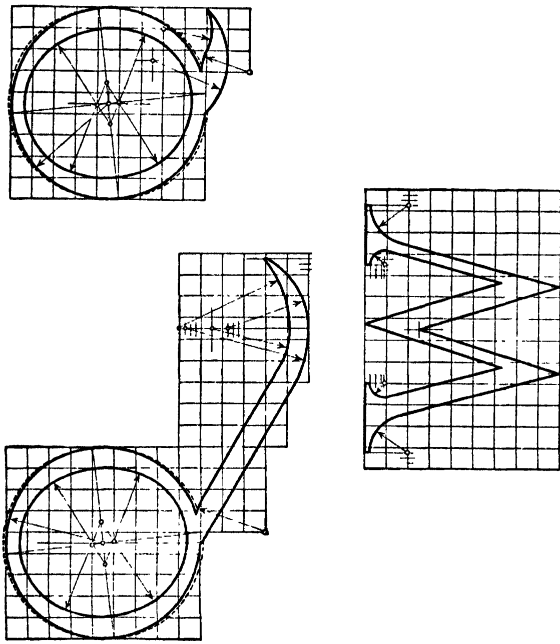




Roman Alphabet *

* From "The Design of Lettering," by Egon Weiss

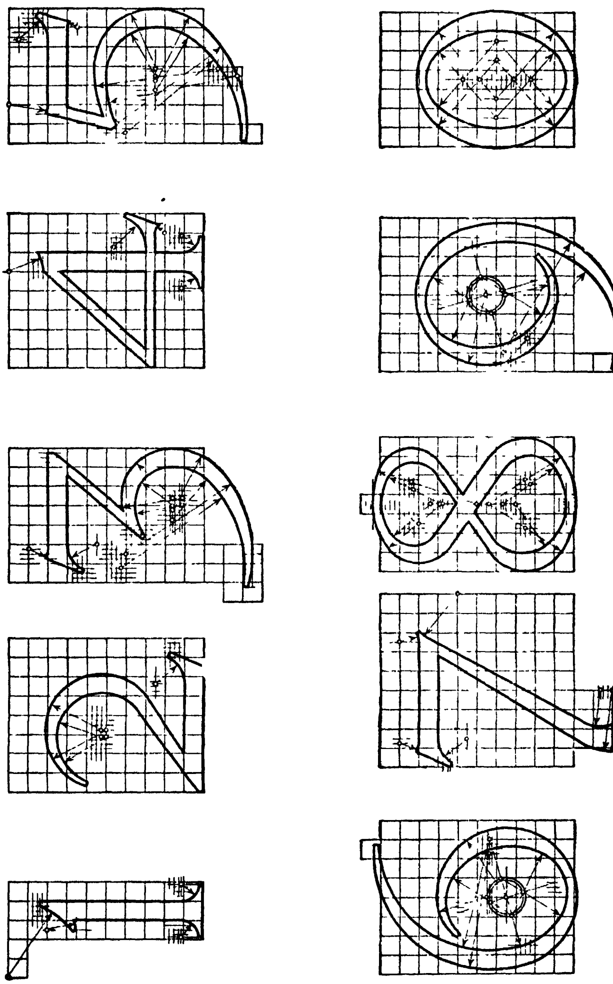
Courtesy of the Pencil Points Press, Publishers.



Letters Q and W of the Roman Alphabet *

Courtesy of the Pencil Points Press, Publishers.

* From "The Design of Lettering," by Egon Weiss.



Arabic Figures to go with Roman Alphabet *

LIGHTNING CONDUCTORS *

Terms and Definitions. Air Terminal. The combination of elevation-rod and brace, or footing placed on upper portions of structures, together with tip or point if used.

Conductor. The portion of a protective system designed to carry the current of a lightning discharge from air terminal to ground.

Copper-clad Steel. Steel with a coating of copper welded to it as distinguished from copper-plated or copper-sheathed material.

Elevation-Rod. The vertical portion of conductor in an air terminal by means of which it is elevated above the object to be protected.

Ground Connection. A buried body of metal with its surrounding soil and a connecting conductor which together serve to bring an object into electrical continuity with the earth.

Conductors. (a) Materials. The materials of which protective systems are made shall be relatively resistant to corrosion or shall be acceptably protected against corrosion. No combination of materials shall be used that forms an electrolytic couple of such nature that in the presence of moisture corrosion is accelerated.

(1) COPPER. Where copper is used it shall be of the grade ordinarily required for commercial electrical work, generally designated as being of 98% conductivity when annealed.

(2) ALLOYS. Where alloys of metals are used they shall be substantially as resistant to corrosion as copper under similar conditions.

The importance of resistance to corrosion of lightning-conductor materials should be emphasized, because corrosion, either soil or atmospheric, leads to deterioration and consequent impairment of the initial degree of reliability of a system and should be forestalled wherever possible. It may also be pointed out that atmospheric conditions in certain seacoast sections of the United States, notably South Atlantic and Gulf coasts, are known to be destructive to galvanized steel, and in such regions galvanized steel should be used with caution, a preference being given to copper. Copper is also to be preferred where corrosive gases are encountered, but it needs to be reinforced with a lead coating under exceptional conditions, such as are found near the tops of smokestacks.

(b) Form and Size. Conductors may be in the form of CABLE, TUBE, STRIP, or ROD, having ROUND, SQUARE, STAR or other solid cross-section. The following give minimum sizes and weights:

(1) COPPER-CABLE. Copper-cable conductors shall weigh not less than 187.5 lb per 1 000 ft (0.279 kg per m). The size of any wire of a cable shall be not less than No. 17 A.W.G. (0.045 in = 0.114 cm diameter).

(2) COPPER TUBE, COPPER SOLID SECTION. Tube or solid-section conductors of copper shall weigh not less than 187.5 lb per 1000 ft (0.279 kg per m). The thickness of any tube wall shall be not less than No. 20 A.W.G. (0.032 in = 0.081 cm). The thickness of any copper ribbon or strip shall be not less than No. 20 A.W.G. (0.051 in = 0.129 cm).

* The information here given is taken from the U. S. Department of Commerce publication, "Code for Protection against Lightning," approved April 4, 1929, by the American Standards Association. Additional information on the subject may be obtained from the Department of Commerce or from the Copper Brass Research Association, 25 Broadway, New York City.

(c) **Joints.** Joints in conductors shall be as few in number as practicable, and where they are necessary they shall be mechanically strong, well made, and provide ample electrical contact. The latter requirement is to be regarded as met by a contact area not less than double the conducting cross-sectional area of the conductor.

(1) Mechanical Strength. On structures exceeding 60 ft in height, joints shall be so constructed that their mechanical strength in tension as shown by laboratory tests is not less than 50% of that of the smallest of the several sections of conductor which are joined.

(2) Electrical Resistance. Joints shall be so made that they have an electrical resistance not in excess of that of 2 ft of conductor.

(d) **Fastenings.** Conductors shall be securely attached to the building or other object upon which they are placed. Fastenings shall be substantial in construction, and of the same material as the conductor, or of such a nature that there will be no serious tendency toward electrolytic corrosion in the presence of moisture. Fasteners shall be spaced, in general, not over 4 ft apart.

Elevation-Rods. (a) **Size.** Elevation-rods shall be at least the equivalent in weight and stiffness of a copper tube having an outside diameter of 5/8 in (1.6 cm) and a wall thickness of No. 20 A.W.G. (0.032 in = 0.081 cm).

(b) **Form.** Elevation-rods may be of any form of solid or tubular cross-section.

(c) **Height.** The height of an elevation-rod shall be such as to bring the tip not less than 10 in (25.4 cm) above the object to be protected.

(d) **Braces.** Elevation-rods shall be amply secured against overturning either by attachment to the object to be protected or by means of substantial tripod or other braces which shall be permanently and rigidly attached to the building.

(1) Materials. The material of which braces are constructed shall be at least the equivalent in strength and stiffness of 1/4 in (0.625 cm) round iron, and the nails or screws used in erecting must comply with the requirements of conductor materials as outlined above in resistance to corrosion or protection against corrosion.

(2) Form and Construction. Braces shall be assembled by means of riveted joints or joints of equivalent strength. Preference should be given to tripod or four-legged braces, and when in place the feet should be spread until the distance between them approximates one-third the height of the brace.

(3) Guides. Where elevation-rods are more than 24 in high, braces shall have guides for holding the elevation-rod at two points located approximately as follows: the lower at a distance above the foot of the rod equal to one-third of its height, the upper at a distance above the lower, equal to one-fourth the height of the rod.

Where elevation-rods are 24 in high or less, braces with a single guide may be used, holding the rod approximately midway of its height. Ten-inch (25.4 cm) elevation-rods may be braced by means of substantial footings.

Location of Air Terminals. (a) **General.** Air terminals shall be provided for all structural parts that are likely to receive, and be damaged by a stroke of lightning.

(b) **Projections.** In the case of projections such as gables, chimneys and ventilators, the air terminal shall be placed on, or attached to, the object to be protected where practicable, otherwise within 2 ft (61 cm) of it.

(c) Ridges, Parapets and Edges of Flat Roofs. Along ridges, parapets and edges of flat roofs, air terminals shall be spaced at intervals not exceeding 25 ft (7.62 m), and intermediate terminals need not be closer than 12.5 ft (3.81 m).

(d) Metal Projections and Parts of Buildings. Ventilators, smokestacks and other objects, that are likely to receive, but not be damaged by, a stroke of lightning, need not be provided with air terminals but shall be securely bonded to the lightning-conductor with metal of the same weight per unit length as the main conductor.

(e) Coursing of Conductors. Conductors shall, in general, be coursed over roofs and down the corners and sides of buildings in such a way as to constitute, as nearly as local conditions will permit, an inclosing network.

(f) Roof-Conductors. Roof-conductors shall be coursed along contours such as ridges, parapets, and edges of flat roofs, and where necessary over flat surfaces, in such a way as to join each air terminal to all the rest.

(g) Down-Conductors. Down-conductors shall preferably be coursed over the extreme outer portions of buildings, such as corners, due consideration being given to the best places for making ground connections and to the location of air terminals.

There shall be at least two down-conductors on any type of building, and these shall be run so as to be as widely separated as practicable. Additional down-conductors shall be installed where necessary to avoid DEAD ENDS, branch conductors ending at air terminals, which exceed 16 ft (4.88 m) in length. Single down-conductors descending flag-poles, spires and similar structures which are adjuncts of buildings shall not be regarded as dead ends, but shall be treated as air terminals.

(h) Bends. No bends in a conductor which embraces a portion of a building, such as an eave, shall have a radius of less than 8 in (20 3 cm). The angle of any turn shall not exceed 90°, and conductors shall everywhere preserve a downward or approximately horizontal course.

VACUUM-CLEANING *

General Description. Vacuum-cleaning of all types has become the recognized standard for cleaning of every description, from the ordinary household to hotels, schools, office-buildings, theaters and industrial plants. The difficulties encountered in the early installations have been largely overcome by equipment of modern design.

Types of Vacuum-Cleaners. There are three recognized types of vacuum-cleaners. First, the HOUSEHOLD type which is largely used as a substitute for the broom in household use. Second, the HEAVY-DUTY PORTABLE type, designed particularly for industrial applications where the machine is subjected to severe use. Third, the CENTRAL CLEANING SYSTEM sometimes designated as the INSTALLED SYSTEM.

The CENTRAL SYSTEM is of the most interest to the architect, as it is this type that is generally used in the modern hotel, school, office building, theater, etc. The fact that this equipment is installed during the construction of a building, and should be included in the general contract, makes it important for the architect to obtain accurate information. The equipment consists of a vacuum-pump and dirt separator located in the basement, from which a

* From data given by the Spencer Turbine Co.

piping system extends throughout the building. Manufacturers of equipment have reliable data concerning the size of equipment necessary for the various types of buildings and should be consulted on the size of the machine, pipe sizes, and the design of the tool.

Irrespective of other specifications, there should be included what is known as the standard orifice performance test, namely: that 2 in Hg. with a $\frac{7}{8}$-in opening, or 3 in Hg. with a $\frac{5}{8}$-in opening, be required for each sweeper the machine is designed to operate. These measurements are to be taken at the end of 50 ft of $1\frac{1}{2}$-in hose distributed at various points throughout the building. The manufacturers of equipment are always willing to supply engineering data and specification sheets and should be consulted in order to insure a correct installation.

WATERPROOFING FOR FOUNDATIONS

The Waterproofing of Substructure Work, is, comparatively speaking, a modern branch of engineering. It is only within recent years that it has become necessary to construct deep basements for buildings. In the past, the more important structures, such as cathedrals, capitols, state-buildings and the like, were usually built upon high ground, and water was prevented from entering the basements of such buildings by means of drainage. Waterproofing, as we now know it, was generally unnecessary. With the advent of the so-called skyscrapers, however, requiring large mechanical plants, deep basements became an actual necessity, and as these basements are usually carried below ground-water level, and in many instances below tide-level, the question became one of utmost importance. Like almost every detail of a modern building, waterproofing is a specialty. Each building presents its own problems, and the safest plan is to leave the solution of these problems to some one expert in the knowledge of waterproofing who has made it a special study and knows how best to overcome the existing difficulties. It may be laid down as an invariable rule that, where conditions are at all serious, the owner or the general contractor will save money in the long run if he employs the services of an expert waterproofer to place his waterproofing-seal, regardless of the method he wishes to use.

Pressure-Resistance Versus Waterproofing. In waterproofing large basements where actual pressure exists, it is a question for the engineer to decide whether it is more economical to attempt to secure an absolute PRESSURE-JOB or a WATER-PROOF JOB in connection with a drainage system. As a general rule, it may be stated that where a building is generating its own power, it is more economical to use a drainage system with an open sump than to construct a pressure-cellar, the cost of pumping being much less than the interest charges on the cost of a floor-slab sufficiently strong to withstand the pressure.

Waterproofing Concrete Foundations. The three following subdivisions of this subject, discussing the causes of permeability of concrete, the addition of substances to render it more water-proof, and the treatment of its surfaces to make it less permeable, embody the conclusions of Committee D-8 of the American Society for Testing Materials: * This committee, since its organization in 1905 has, through laboratory-tests and experiments, together with examinations of work during construction and after completion, as well as the

* This article, as far as the section entitled "Burlap Saturated with Bituminous Substances," is the substance of a Report submitted to the American Society for Testing Materials.

study of literature on the subject, sought to secure sufficient information to enable it to formulate definite methods for securing water-proof concrete structures. The work of the committee was complicated by reason of the facts that there seemed to be so little concordance between results of tests obtained under laboratory-conditions and in the field and that it was necessary to extend its investigations over a period of years in order to determine the permanency of the action noted. The committee reported that while it had not been able to arrive at sufficiently definite conclusions to enable it to formulate specifications for the making of concrete structures water-proof or for materials to be used in such work, it had reached certain general conclusions which might be of assistance to the constructor in securing the desired result of impermeable concrete. Early in the investigation, the work was found to subdivide naturally into three branches, and the conclusions reached will be grouped in order under these subdivisions, which are:

(1) The determination of causes of the permeability of concrete as usually made from mixtures of Portland cement, sand and stone, or other coarse aggregate, in proportions of from 1 of cement, 2 of sand and 4 of stone, to 1 of cement, 3 of sand and 6 of stone, and the best methods of avoiding these causes.

(2) The rendering of concrete more water-proof by adding to ordinary mixtures of cement, sand and stone, other substances which, either by their void-filling or repellent action, would tend to make the concrete less permeable.

(3) The treatment of exposed surfaces after the concrete or mortar has been put in place and hardened more or less, either by penetrative, void-filling or repellent liquids, making the concrete itself less permeable; or by extraneous protective coatings, preventing water from having access to the concrete.

Considering these several subdivisions separately and in the order named, the committee arrives at the following conclusions:

(1) **Causes of Permeability of Concrete.** In the laboratory and under test-conditions where properly graded and sized coarse and fine aggregates are used, in mixtures ranging from 1 of cement, 2 of sand and 4 of stone, to 1 of cement, 3 of sand and 6 of stone, impermeable concrete can invariably be produced. Even with sand of poor granulometric composition, with mixtures as rich as 1 of cement, 2 of sand and 4 of stone, permeable concrete is seldom, if ever, found and is a rare occurrence with mixtures of 1 of cement, 3 of sand and 6 of stone. But the fact remains, nevertheless, that the reverse often obtains in actual construction, permeable concretes being encountered even with mixtures of 1 of cement, 2 of sand and 4 of stone and are of frequent occurrence where the quantity of the aggregate is increased. This the committee attributes to:

(*a*) Defective workmanship, resulting from improper proportioning, lack of thorough mixing, separation of the coarse aggregate from the fine aggregate and cement in transporting and placing the mixed concrete, lack of density through insufficient tamping or spading, improper bonding of work-joints, etc.

(*b*) The use of imperfectly sized and graded aggregates.

(*c*) The use of excessive water, causing shrinkage-cracks and formation of laitance-seams.

(*d*) The lack of proper provision to take care of expansion and contraction, causing subsequent cracking.

Theoretically, none of these conditions should prevail in properly designed and supervised work, and they are avoided in the laboratory and in the field, under test-conditions, where speed of construction and cost are negligible items, instead of being governing features as they must be in actual construction. Properly graded sands and coarse aggregates are rarely, if ever, found

in nature in sufficient quantities to be available for large construction, and the effect of poorly graded aggregates in producing permeable concrete is aggravated by poor and inefficient field-work. Even if the added expense of screening and remixing the aggregates could be afforded, so as to secure proper granulometric composition to give the density required to make untreated concretes impermeable, it is seemingly often a commercial impossibility on large construction to obtain workmanship even approximating that found in laboratory-work.

(2) **Addition of Foreign Substances to Cement Before or During Mixture.** The committee finds that in consequence of the conditions outlined above, substances calculated to make the concrete more impermeable, either incorporated in the cement or added to the concrete during mixing, are often used. This has resulted in the development and placing on the market of numerous patented or proprietary waterproofing-compounds, the composition of which is more or less of a trade-secret. While it has been impossible for the committee to test all of the special waterproofing-compounds being placed on the market, it has investigated a sufficient number of these, as well as the use of certain very finely divided, naturally occurring or readily obtainable commercial mineral products, such as finely ground sand, colloidal clays, hydrated lime, etc., to form a general idea of the value of the different types. The committee finds:

(*a*) That the majority of patented and proprietary integral compounds tested have little or no immediate or permanent effect on the permeability of concrete and that some of these even have an injurious effect on the strength of mortar and concrete in which they are incorporated

(*b*) That the permanent effect of such integral waterproofing-additions, if dependent on the action of organic compounds, is very doubtful.

(*c*) That in view of their possible effect, not only upon the early strength, but also upon the durability of concrete after considerable periods, no integral waterproofing-material should be used unless it has been subjected to long-time practical tests under proper observation to demonstrate its value, and unless its ingredients and the proportion in which they are present are known.

(*d*) That in general, more desirable results are obtainable from inert compounds acting mechanically, than from active chemical compounds whose efficiency depends on change of form through chemical action after addition to the concrete.

(*e*) That void-filling substances are more to be relied upon than those whose value depends on repellent action.

(*f*) That, assuming average quality in sizing of the aggregates and reasonably good workmanship in the mixing and placing of the concretes, the addition of from 10 to 20% of very finely divided void-filling mineral substances may be expected to result in the production of concrete which, under ordinary conditions of exposure, will be found impermeable, provided the work-joints are properly bonded, and cracks do not develop on drying, or through change in volume due to atmospheric changes, or by settlement.

(3) **External Treatment.** While external treatment of concrete would not be necessary if the concrete itself, either naturally or by the addition of waterproofing-material, was impermeable to water, it has been found in practice that in large construction, no matter how carefully the concrete itself has been made, cracks are apt to develop, due to shrinkage in drying out, expansion and contraction under change of temperature and moisture-content, and through settlement. It is, therefore, often advisable in important construction to anticipate and provide for the possible occurrence of such cracks by external

treatment with a protective coating. Such coating must be sufficiently elastic and cohesive to prevent the cracks extending through the coating itself. The application of merely penetrative void-filling liquid washes will not prevent the passage of water due to cracking of the concrete. The committee has, therefore, considered surface-treatment under two heads:

(*a*) Penetrative void-filling liquid washes.

(*b*) Protective coatings, including all surface-applications intended to prevent water coming in contact with the concrete.

Penetrative Washes. While some penetrative washes may be efficient in rendering concrete water-proof for limited periods, their efficiency may decrease with time and it may be necessary to repeat such treatment. Some of these washes may be objectionable, due to discoloring the surface to which they are applied. The committee, therefore, believes that the first effort should be made to secure a concrete that is impermeable in itself and that penetrative void-filling washes should only be resorted to as a corrective measure.

Protective Coatings. While protective extraneous bituminous or asphaltic coatings are unnecessary, so far as the major portion of the surface of the concrete is concerned, provided the concrete, either in itself or through the addition of integral compounds, is made impermeable, they are valuable as a protection where cracks develop in a structure. It is therefore recommended that a combination of inert void-filling substances and extraneous waterproofing be adopted in especially difficult or important work.

Bituminous or Asphaltic Coatings. Considering the use of bituminous or asphaltic coatings, the committee finds:

(*a*) That such protective coatings are often subject to more or less deterioration with time, and may be attacked by injurious vapors or deleterious substances in solution in the water coming in contact with them.

(*b*) That the most effective method for applying such protection is either the setting of a course of impervious brick dipped in bituminous material into a solid bed of bituminous material, or the application of a sufficient number of layers of satisfactory membranous material cemented together with hot bitumen.

(*c*) That their durability and efficiency are very largely dependent on the care with which they are applied. Such care refers particularly to proper cleaning and preparation of the concrete to insure as dry a surface as possible before application of the protective covering, the lapping of all joints of the membranous layers, and their thorough coating with the protective material. The use of this method of protection is further desirable because proper bituminous coverings offer resistance to stray electrical currents, the possible attack from which is referred to in succeeding paragraphs.

Rich Mixtures. So far, the committee has considered only concretes of the usual proportions, namely, those ranging from 1 of cement, 2 of sand and 4 of stone, to 1 of cement, 3 of sand and 6 of stone. It has been suggested that impermeable concretes could be assured by using mixtures considerably richer in cement. While such practice would probably result in an immediate impermeable concrete, it is believed by many that the advantage is only temporary, as richer concretes are more subject to check-cracking and are less constant in volume under changes of conditions of temperature, moisture, etc. Therefore, the use of more cement in mass-concrete would cause increased cracking, unless some means of controlling the expansion and contraction is discovered. With reinforced concretes the objection is not so great, as the tendency to cracking is more or less counteracted by the reinforcement.

Fine Flour Mixtures. It has also been suggested that the presence in the cement of a larger percentage of very fine flour might result in the production of a denser and more impermeable concrete, through the formation of a larger amount of colloidal gels. Neither of these suggestions has been especially investigated by the committee. Both appeal to the committee, however, for the reason that they substitute active cementitious substances for the largely inactive void-filling materials previously recommended, thus increasing the strength of the concrete.

Character of Workmanship. In conclusion, the committee would point out that no addition of waterproofing-compounds or substances can be relied upon to completely counteract the effect of bad workmanship, and that the production of impermeable concrete can only be hoped for where there is determined insistence on good workmanship.

Saline Waters. Electrical Action. The production of impermeable concrete has assumed greater importance since the appointment of this committee, owing to the well-known injurious action of saline or alkaline waters and to the suggested possible effect of the moisture in concrete occasioning or aggravating electrical action from stray currents. Originally, the question of waterproofing involved mainly the physical troubles resulting from water passing through concrete without any special consideration of its effect on its durability, other than a gradual leaching out of the cement. Recent developments suggest the possibility that, owing to the increased conductivity of damp concrete to electrical currents, such currents, if present, may so affect damp concrete as to seriously lessen its integrity; and this possibility further emphasizes the importance of the recommendation that no waterproofing-compound of unknown chemical composition be added to concrete, as recent tests seem to show that the action of electrical currents is aggravated by the presence of certain solutions.

Burlap * Saturated with Bituminous Substances for Use in Waterproofing. The following excerpts are taken from American Society for Testing Materials, Serial Designation: D 174–25, Standard Specifications. "These specifications cover bituminized jute fabric, composed of burlap waterproofed with either asphalt or coal-tar pitch, as specified by the purchaser, for use in the membrane system of waterproofing. In the process of manufacture, the dry burlap shall be thoroughly and uniformly waterproofed with an asphaltic or coal-tar pitch saturant at a temperature and speed which will not injure the fabric. This shall be accomplished by passing the fabric through the saturant and then calendering it in the presence of heat, whereupon it shall be cooled and wound into rolls. The finished material shall be free from visible external defects such as ragged or untrue edges, breaks, rents or cracks. The meshes of the fabric shall not be completely closed or sealed by the process of saturation, but there shall be sufficient porosity maintained to allow successive moppings of the plying cement to seep through. The selvage shall not measure over $3/16$ in. The roll shall be capable of being unrolled easily at atmospheric temperatures above 50° F. (10° C.) without sticking together in such a manner as to injure the fabric. The surface of the fabric shall not be coated or covered with talc or other substances which would tend to interfere with the adhesion between the fabric and the plying cement. The use of silica or wood flour will be permitted. The surface shall be uniformly smooth and free from irregularities, folds or knots."

*** For Standard Specifications for Woven Cotton Fabrics Saturated with Bituminous Substances for Use in Waterproofing, see A. S. T. M. Designation: D 173–27.**

Bituminous Grout for use in Waterproofing below Ground-level. The following excerpts are taken from Standard Specifications of the American Society for Testing Materials, Serial Designation: D 171–25. "These specifications cover the materials for bituminous grout suitable for use in waterproofing below ground-level, either as a protective covering of membrane systems of waterproofing or for bedding brick or filling the joints or flooding the surface of a brick protective covering. The grade of bituminous grout covered by these specifications is suitable for waterproofing tunnels, subways, etc. This bituminous grout is a mixture of substantially 45 parts by weight of bituminous binder and 55 parts by weight of mineral aggregate as coarse as sand, which becomes sufficiently fluid, when heated to approximately 149° C. (300° F.), to flow without mechanical manipulation, and which on cooling congeals to a compact mass.

"The bituminous binder shall consist of either asphalt binder or coal-tar pitch as follows:

"(*a*) The asphalt binder shall conform to the requirements of the Standard Specifications for Asphalt for Use in Dampproofing and Waterproofing Below Ground-Level (Serial Designation: D 40) of the American Society for Testing Materials.

"(*b*) The coal-tar pitch shall conform to either of the following tentative specifications of the American Society for Testing Materials:

"(1) Standard Specifications for High-Carbon Coal-Tar Pitch for Use in Dampproofing and Waterproofing below Ground-Level (Serial Designation: D 42).

"(2) Standard Specifications for High-Bitumen Coal-Tar Pitch for Use in Dampproofing and Waterproofing below Ground-Level (Serial Designation: D 200).

"The mineral aggregate shall consist of silicious sand, all of which will pass a 20-mesh sieve, and not more than 5% of which will pass a 200-mesh sieve."

Waterproofing by External Linings of Brick, Tar, or Asphalt, and Felt. The oldest method of waterproofing is the one involving the use of a tar-and-felt or asphalt-and-felt seal (Fig. 1). This consists of building first a supporting wall and a supporting concrete slab to hold the seal. On the floors, this slab is usually composed of concrete, 4 in thick. The walls are generally of brick from 4 to 8 in thick, but occasionally 4-in terra-cotta tiles are used. Upon this base a swabbing of tar or asphalt is placed and before this has become cold or set, one thickness of paper, saturated with coal-tar, is laid. This paper receives a swabbing of coal-tar and asphalt and another layer of paper is placed, the operation being continued until there are three or more layers of paper with four or more swabbings of the tar or asphalt. For dampproof work, three layers of paper with four swabbings of tar are usually sufficient. For waterproofing-work not less than five and usually six layers of paper with from six to seven swabbings of tar are used. The main walls of the structure are then built against the wall-waterproofing, and after these are in place, the main concrete basement-floor is laid immediately on top of the floor-seal, the idea being to form a continuous waterproof seal enveloping the entire basement below grade. The difficulties of this system consist chiefly in securing perfect laps at all points in the work, and unless extreme care is used and unless there is perfect coöperation between the waterproofer and the mason-contractor, there is apt to be a break somewhere in the seal, usually where the wall-waterproofing is supposed to be joined to the floor-work. The disadvantages of this system are due to the fact that the seal is not permanent

in all soils as the subsurface water frequently contains acids which destroy the seal. Then again, the seal may be easily punctured by the mason-contractor in building his wall against it or in laying the concrete floor upon the flat work. The chief disadvantage, however, is that the waterproofing-seal is invariably buried behind a mass of masonry, either brick or concrete, which means that should there be a leak, due to either carelessness or accident, through the waterproofing-seal, it is frequently impossible to stop it. It not infrequently happens that when a leak has developed in tar-and-felt work,

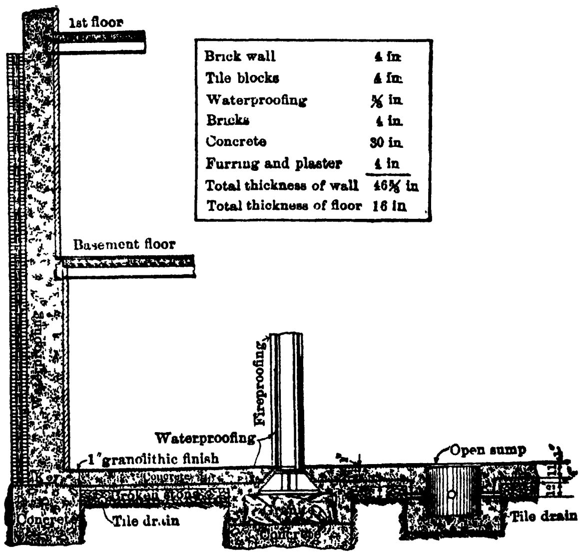


Fig. 1.* Felt-Waterproofing for Foundations

the actual presence of the water does not show opposite the leak, but following some line of least resistance, appears from 50 to 100 ft, or more, away from where the actual damage causing the leak occurs. In actual waterproofing work it is seldom attempted to secure a bottle-tight job with tar and felt. Instead, some system of drainage is installed beneath the water-proof seal which is on the floors of the building, and the water is conducted through tile or other pipes to some central sump from which it is mechanically pumped to a sewer. The purpose of the waterproofing in this case, therefore, is to concentrate or drive the water to this sump. For shallow cellars and especially dampproofing-work, this tar-and-felt method is the most economical and most frequently employed.

* Reproduced, by permission, from a pamphlet published by The Waterproofing Company, New York, and showing the greater thickness of walls and floor required for the outside-surface brick-and-felt method of waterproofing as compared with the inside-surface waterproof-cement coating. Taken from design for waterproofing in a prominent New York building. See, also, Fig 2

Waterproofing by Coating with Water-Proof Cement. For deep and difficult work a comparatively new method of waterproofing is often used (Fig. 2). This consists of placing a coating of water-proof cement upon the interior surface of the exterior walls of the building and over the upper surface of the concrete floor-slab in the basement or sub-basement. Fig. 3 shows a foundation for an engine, the concrete being waterproofed as shown. The pit is made somewhat larger than the foundation, the extra space being filled in with cinders, dry bricks or terra-cotta blocks, which may be readily removed to allow access to the bed-plate bolts for which hand-holes have been cast in

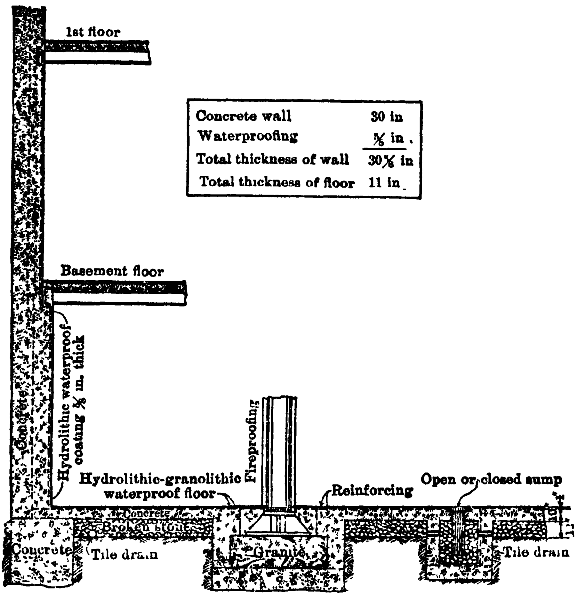


Fig. 2.* Cement Waterproofing for Foundations

the concrete, thus permitting the complete removal of the engine. The figure shows a 2-in sand cushion and a 2-in layer of planks under the engine-foundation. This is not a part of the waterproofing but is put in to prevent the communication of vibration. Fig. 4 shows reinforced-concrete floors for an engine-room and boiler-room, the concrete slab being 12 in thick under the former and 24 in thick under the latter. Both floors are covered with a 1-in course of water-proof cement. The reinforcement is put in as shown and in sizes and spacing as follows:

* From a pamphlet published by The Waterproofing Company, New York, and showing reduced total thickness of walls and floor required for the inside-surface water-proof cement method of waterproofing. Taken from design for waterproofing of the same building shown in Fig. 1. The walls and floors were put in place in the monolithic form.

12-in slab Rods in two courses	24-in slab Rods in three courses
Lower rods, 4 in on centers, 6 in from surface Upper rods, 6 in on centers, 2 in from surface For five rods, total area of cross-section is 0 703 sq in; per square foot of surface, 2.39 lb	Lowest rods, 3 in on centers, 12 in from surface Intermediate rods, 3 in on centers, 7 in from surface Upper rods, 6 in on centers, 2 in from surface For ten rods, total area of cross-section 1 4 sq in; per square foot of surface, 4 78 lb

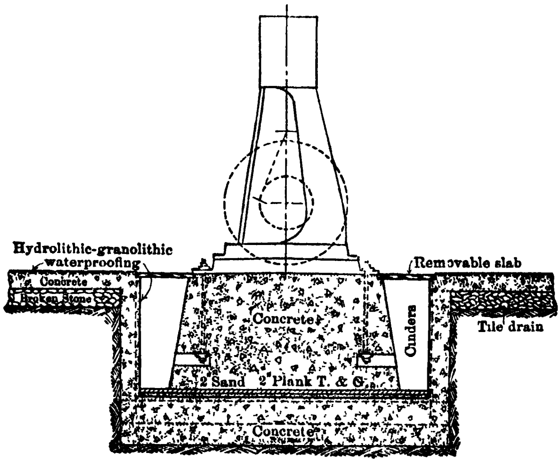


*Fig. 3.** Engine-foundation with Water-proof Cement

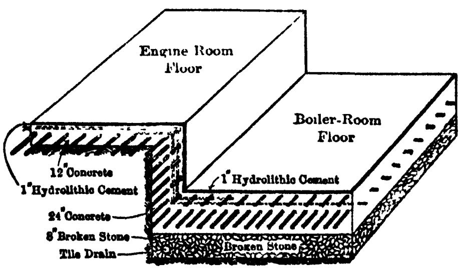


Fig. 4.* Reinforced-concrete Floor with Water-proof Cement

* Reproduced by permission of The Waterproofing Company, New York.

There are many compounds advertised to make cement or concrete water-proof. Besides these, there are water-proof cements manufactured by secret processes and applied by companies that make a specialty of waterproofing. Some of the many waterproofing-compounds have merit; but the main factors of a successful job of waterproofing are the skill and experience of the water-proofers who do the work. It is claimed that to apply cement waterproofing so as to obtain efficient results requires more skill than to apply a tar-and-felt seal; but a cement waterproofing, once properly applied, seems to possess some advantages over the older method of tar and felt. One advantage is that the waterproofing is accessible, and that if any leaks develop, they are apparent and can be readily and economically repaired by cutting out the old waterproofing and placing a new coating where the damage exists. Another advantage claimed is that cement waterproofing is generally permanent and not damaged by the ordinary acids found in solution with water in soil. By the cement method the cost of the brick supporting walls and the concrete supporting slab is eliminated as is also the corresponding cost of the necessary excavation for them; and finally, the waterproofing on the floor serves the double purpose of waterproofing and wearing-surface, thus saving the cost of the cement finish usually found in basements and sub-basements. One of the disadvantages of cement waterproofing is that the material is rigid and is fractured by any settlement of the building or contraction in the concrete upon which it is placed. Experience has shown, however, that settlement-cracks usually take place before the waterproofing contractor has left the building and that there is little or no trouble from these causes after his work is completed. Contraction-cracks in concrete, however, seem to develop at any time within twenty-four months after concrete has been placed. In order to prevent these cracks, users of the cement waterproofing have adopted a system of reinforcement in the concrete, and it is claimed that this reinforcement is, in the long run, an economy, as it permits of less concrete and gives a better and stronger floor or wall. On brick and stone walls no trouble is experienced from contraction and expansion. It should be remembered that this work is all below grade where contraction and expansion are reduced to a minimum, regardless of the materials used.

Waterproofing by Adding Substances to Cement. This is another method of waterproofing now being advocated by some. If this method could always be made efficient, it would be highly advantageous. It is claimed by the manufacturers of these compounds that in order to secure a waterproof-basement, for example, a certain percentage of the compound is to be mixed with the cement before it is incorporated in the concrete. The opponents of this method claim, however, that it is impossible to construct a basement in this way without incurring the danger of serious leaks at the joinings of one day's work with that of another; that leakage at these points of cleavage may be increased by the use of waterproofing-compounds; and that their principal merit is that they produce a very dense mass of concrete. It is always difficult to bond old concrete to new, and if concrete is made water-proof, or, in other words, nonabsorbent, the difficulty of joining new concrete to a nonabsorbent mass of old concrete is increased. This method is effective, however, and is to be recommended in work which can be carried on without interruption, such, for instance, as small elevator-pits or small swimming-pools, where the concrete can be started in the morning and completed by night or before any part of the work has had time to attain its initial set.

FORCE OF THE WIND

Relation Between the Pressure and Velocity of Wind. Accordng to experiments made in 1890 or thereabouts, by C. F. Marvin, United States Signal Service, the relation between wind-pressure and velocity is given very accurately by the formula $p = 0\ 004\ V^2$, where p is the pressure in pounds per square foot on a flat surface normal to the direction of the wind, and V the velocity of the wind in miles per hour. Smeaton considered the pressure as equal to $0.005\ V^2$. The following table, based on Marvin's formula,* is quoted by Turneaure and Ketchum.†

Table Showing the Force of the Wind

Miles per hour	Feet per minute	Feet per second	Force, in pounds, per square foot	Description
1	88	1.47	0 004	Hardly perceptible
2	176	2.93	0 014	Just perceptible
3	264	4 40	0 036	
4	352	5.87	0 064	Gentle breeze
5	440	7.33	0.1	
10	880	14 67	0.4	Pleasant breeze
15	1 320	22 0	0 9	
20	1 760	29 3	1 6	Brisk gale
25	2 200	26.6	2 5	
30	2 640	44.0	3 6	High wind
35	3 080	51.3	4 9	
40	3 520	58.6	6 4	Very high wind
45	3 960	66.0	8 1	
50	4 400	73.3	10.0	Storm
60	5 280	88.0	14 4	Great storm
70	6 160	102 7	19 6	
80	7 040	117.3	25 6	Hurricane
100	8 800	146.6	40.0	

WIND-STRESSES IN TALL BUILDINGS ‡

General Considerations. All buildings should have adequate provision for resisting wind-pressure. Walls, floors and partitions afford a certain amount of resistance, but, in high buildings, the thin walls and light partitions used in modern construction are insufficient for the purpose, and special provision to resist the stresses caused by wind should be made in the steel framing.

Steel columns for tier buildings should be used in lengths of two or more stories and spliced with sufficient plates and rivets to make them continuous with respect to transverse bending. All column splices and the connections of girders and beams to the columns should be riveted. With a properly con-

* If Marvin's formula is written $p = 0.0032\ V^2$ the values in this table will be slightly changed. The formula used by the United States Signal Service is $p = 0.004\ V^2$. The true pressure is probably somewhere between $0.005\ V^2$ and $0.004\ V^2$, near the former for very low velocities and near the latter for high velocities.

† See, also, Trautwine's Pocket-Book.

‡ This article, with slight modifications, is taken by permission from Bethlehem Structural Shapes, Bethlehem Steel Co., Bethlehem, Pa.

structed steel frame of this kind, (See Fig. 1) special wind-bracing will seldom be needed, unless the height of the building is more than twice its least base.

Higher buildings will usually require wind-bracing of some form, and as it is seldom possible to use diagonal rods between the columns, other forms of bracing may be used. Bethlehem H columns afford every facility for the construction of an ideal steel frame for buildings.

Unless specified otherwise by building codes, it is now considered good practice to provide for a horizontal wind-pressure of not less than 20 lb per sq ft on the vertical projection of exposed surfaces during erection, and 15 lb per sq ft on the vertical projection of the finished structure.

The total live, dead and wind-loads should not produce stresses exceeding those of the combined dead and live loads by more than $33\frac{1}{3}\%$.

Wind increases the compression in the leeward columns, and also produces bending in the columns and girders, all of which effects must be considered.

Exact methods, on account of their complexity, are rarely used for the calculation of wind-stresses in building frames.

Several approximate methods are in use, and are based on the general assumption that a building bent acts as a cantilever beam fixed in the ground. The wind-loads are applied horizontally, and the transverse shear in any story is distributed among the columns of the bent. The direct and bending stresses in the columns and girders due to the wind-pressure are then found by the principles of statics. The variation in methods is due primarily to the different assumptions of shear distribution among the columns and the location of the points of contraflexure.*

Cantilever Method. The cantilever method is representative of the approximate methods for calculating wind-stresses and, as modified below, is based on the following assumptions: (1) All columns in any given story have equal sections, and the direct stresses in the columns due to wind are proportional to the distances of the columns from the neutral axis of the bent; (2) the point of contraflexure of each column is at mid-height of the story and the point of contraflexure of each girder is at its mid-length; (3) the resultant of the wind-pressure acting between two successive points of contraflexure is applied at the intersection of column and girder; (4) the joints are perfectly rigid. (See Fig. 2)

Notation. $A, B, C, D, E, \ldots L,$ etc $=$ spans, in feet.

$a, b, c, d, d,'$ etc. = Distances of columns Nos. 1, 2, 3, 4, 5, etc., from the neutral axis of bent, feet.

H = distance from mid-height of the story under consideration to the top of the building, feet.

h_1 = height of the story under consideration, feet.

h_2 = height of story above, feet.

$$h = \frac{h_1}{2} + \frac{h_2}{2}.$$

W = wind-load acting on height H, pounds.

P = wind-load acting on story height h, pounds.

l = distance from mid-height of the story under consideration to the center of gravity of the wind-load W.

M_A, M_B, M_C, M_D, etc. = bending moments at ends of girders in the corresponding spans A, B, C, D, etc., foot-pounds.

M_1, M_2, M_3, M_4, etc. = bending moments of corresponding columns Nos. 1, 2, 3, 4, etc., at top and bottom of story under consideration, foot-pounds.

* Detailed descriptions of these methods may be found in current text-books.

D_1, D_2, D_3, D_4, etc. = direct stresses in the corresponding columns Nos. 1, 2, 3, 4, etc., in story under consideration, pounds.

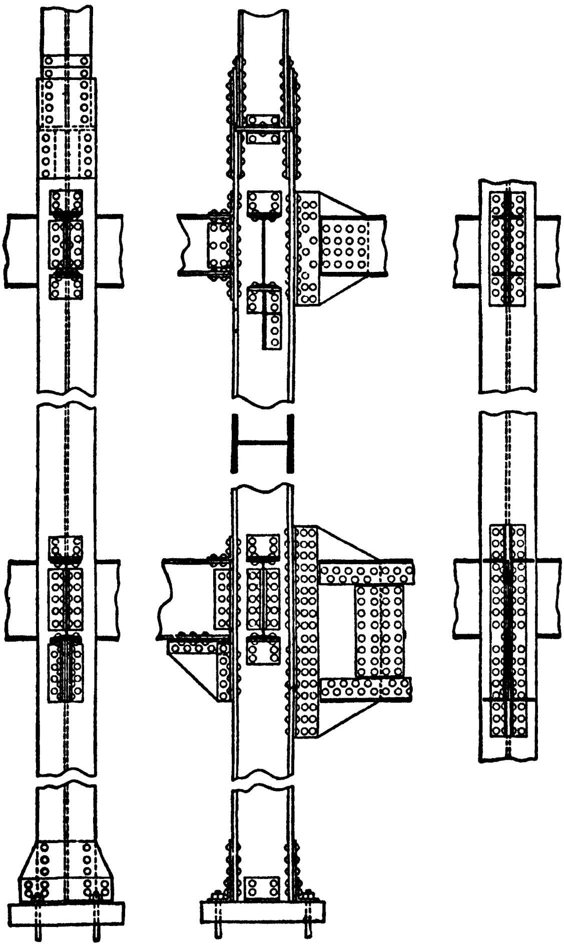


Fig. 1. Typical Bethlehem H Column Wind-Bracing Connections

D_A, D_B, D_C, D_D, etc. = direct stresses in the girders in the corresponding spans A, B, C, D, etc., pounds.

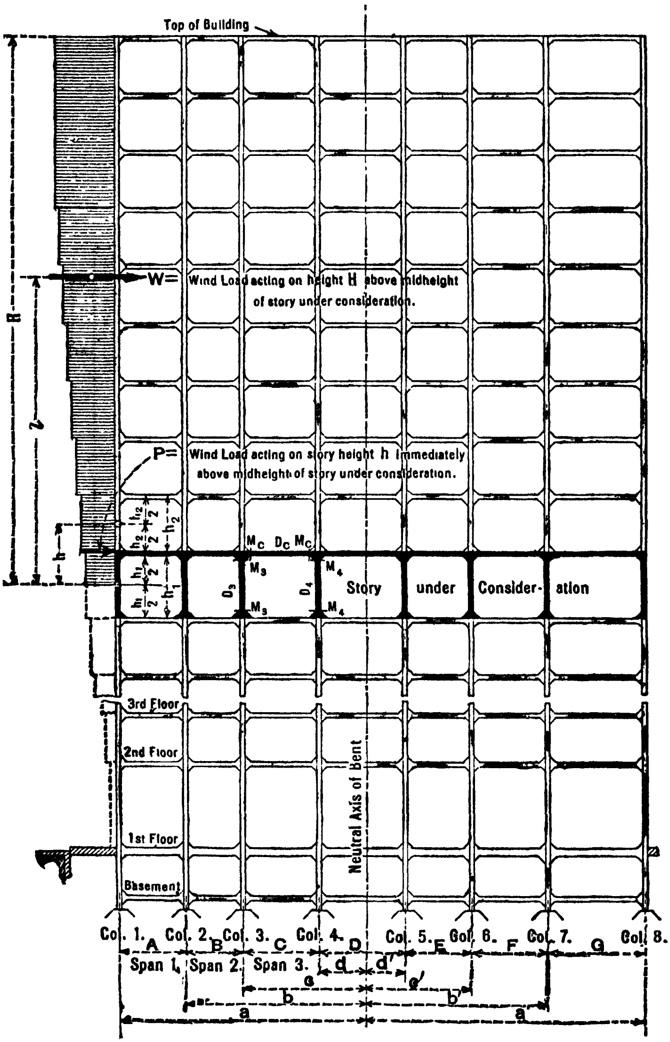


Fig. 2. Wind-Stresses in Buildings

The direct stresses and bending moments due to wind are given in the formulas and tables on the following pages.*

1. Unsymmetrical Building Bent. Varying or Uniform Wind-pressure. Most building codes specify a uniform wind-pressure for the full height of a building, but those of some cities, like Philadelphia and Boston, specify wind-pressures the magnitudes of which increase upwardly from a certain elevation.

The general formulas given below will apply in any case, but their use will be greatly simplified if the wind-pressure is uniform, as generally specified. For uniform wind-pressure the lever arm, l, of the wind-load W becomes equal to half the height H.

In order to find the bending moments and direct stresses due to the wind-load, proceed as follows:

(1) Locate the position of the neutral axis of the bent, so that

$$a + b + c + d + \ldots\ldots\ldots\ldots = a' + b' + c' + d' + \ldots\ldots\ldots\ldots$$

then calculate the quantity Q:

$$Q = (a)^2 + (a')^2 + (b)^2 + (b')^2 + (c)^2 + (c')^2 + (d)^2 + (d')^2 + \ldots\ldots$$

(2) Calculate, for the floor under consideration, the constant factors p, q, r and t.

$$p = \frac{2\,Wh - Ph_2}{4\,Q}, \qquad q = \frac{Wh_1}{2\,Wh - Ph_2}, \qquad r = \frac{Wl}{Q}, \qquad t = \frac{2\,P}{Wh_1}.$$

$$q' = \frac{W}{2\,W - P} \quad \left\{ \begin{array}{l} \text{Where the difference between } h_1 \\ \text{and } h_2 \text{ is negligible.} \end{array} \right.$$

(3) Find the moments at the ends of the girders, as follows:

$$\begin{aligned}
M_A &= pAa \\
M_B &= pB\,(a + b) \\
M_C &= pC\,(a + b + c) \\
M_D &= pD\,(a + b + c + d) = pD\,(a' + b' + c' + d') \\
M_E &= pE\,(a + b + c + d - d') = pE\,(a' + b' + c') \\
&\ldots\ldots\ldots
\end{aligned}$$

(4) Find the moments at the ends of the columns, as follows:

$$M_1 = qM_A \qquad M_2 = q\,(M_A + M_B) \qquad M_3 = q\,(M_B + M_C)$$
$$M_4 = q\,(M_C + M_D) \qquad M_5 = q\,(M_D + M_E)$$
$$\ldots\ldots\ldots$$

(5) Find the direct stresses in the columns, as follows:

$$D_1 = ra \qquad D_2 = rb \qquad D_3 = rc \qquad D_4 = rd \qquad D_5 = rd'$$
$$\ldots\ldots\ldots$$

(6) Find the direct stresses in the girders, as follows

$$D_A = P - tM_1 \qquad D_B = D_A - tM_2 \qquad D_C = D_B = tM_3$$
$$D_D = D_C - tM_4 \qquad D_E = D_D - tM_5$$
$$\ldots\ldots\ldots$$

The direct stresses in the columns are tensile (+) on the windward side and compressive (−) on the leeward side of the neutral axis of the building bent.

The direct stresses in the girders are compressive stresses. In stories for which P is the same, the direct stresses in corresponding girders are generally the same.

* "Tables and Formulas for Wind Stresses in Office Buildings," Engineering News-Record, Sept. 28, 1922.

2. Building Bent with Equal Spans, L. Uniform Wind-Pressure.
Note: The wind load below the story under consideration does not affect the calculations and need not be uniform. If the story under consideration is below ground-level, where all or part of $P = O$, use the formulas for varying wind-pressure.

In order to find the bending moments and direct stresses due to the wind-load, proceed as follows:

(1) Find the moments at the ends of the girders, taking the coefficients K_1 from the table below.

$$M = K_1 P\,(2\,H - h_2)$$

Number of spans in bent	Values of coefficients K_1									
	Span 1	Span 2	Span 3	Span 4	Span 5	Span 6	Span 7	Span 8	Span 9	Span 10
1	1/4									
2	1/8	1/8								
3	3/40	4/40	3/40							
4	2/40	3/40	3/40	2/40						
5	5/140	8/140	9/140	8/140	5/140					
6	3/112	5/112	[illegible]/112	6/112	5/112	3/112				
7	7/336	12/336	15/336	1[illegible]/336	15/336	12/336	7/336			
8	4/240	7/240	9/240	10/240	10/240	9/240	7/240	4/240		
9	9/660	16/660	21/660	24/660	25/660	24/660	21/660	16/660	9/660	
10	5/140	9/140	12/140	14/140	15/140	15/140	11/140	12/140	9/140	5/140

(2) Find the moments at the ends of the columns, taking the coefficients K_2 from the table below.

$$M = K_2 W h_1$$

Number of spans in bent	Values of coefficients K_2										
	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11
1	1/4	1/4									
2	1/8	1/4	1/8								
3	3/40	7/40	7/40	3/40							
4	2/40	5/40	6/40	5/40	2/40						
5	5/140	1[illegible]/140	17/140	17/140	13/140	5/140					
6	3/112	8/112	11/112	12/112	11/112	8/112	3/112				
7	7/336	19/336	27/336	31/336	31/336	27/336	19/336	7/336			
8	4/240	11/240	16/240	19/240	20/240	19/240	16/240	11/240	4/240		
9	9/660	25/660	37/660	45/660	49/660	49/660	45/660	37/660	25/660	9/660	
10	5/140	[illegible]/140	21/140	26/140	29/140	30/140	29/140	26/140	21/140	14/140	5/140

(3) Find the direct stresses in the columns, taking the coefficients K_3 from the table below.

$$D = K_3 W \frac{H}{L}$$

Number of spans in bent	Values of coefficients K_3										
	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11
1	1/2	−1/2									
2	1/4	0	−1/4								
3	3/20	1/20	−1/20	−3/20							
4	2/20	1/20	0	−1/20	−2/20						
5	5/70	3/70	1/70	−1/70	−3/70	−5/70					
6	3/56	2/56	1/56	0	−1/56	−2/56	−3/56				
7	7/168	5/168	3/168	1/168	−1/168	−3/168	−5/168	−7/168			
8	4/120	3/120	2/120	1/120	0	−1/120	−2/120	−3/120	−4/120		
9	9/330	7/330	5/330	3/330	1/330	−1/330	−3/330	−5/330	−7/330	−9/330	
10	5/220	4/220	3/220	2/220	1/220	0	−1/220	−2/220	−3/220	−4/220	−5/220

The direct stresses in the columns are tensile (+) on the windward side and compressive (−) on the leeward side of the neutral axis of the building bent.

(4) Find the direct stresses in the girders, taking the coefficients K_4 from the table below.

$$D = K_4 P$$

Number of spans in bent	Values of coefficients K_4									
	Span 1	Span 2	Span 3	Span 4	Span 5	Span 6	Span 7	Span 8	Span 9	Span 10
1	1/2									
2	3/4	1/4								
3	17/20	10/20	3/20							
4	13/20	11/20	7/20	3/20						
5	65/70	53/70	35/70	19/70	5/70					
6	53/56	45/56	34/56	22/56	11/56	3/56				
7	161/168	142/168	115/168	84/168	53/168	26/168	7/168			
8	116/120	105/120	89/120	70/120	50/120	31/120	15/120	4/120		
9	321/330	296/330	259/330	214/330	165/330	116/330	71/330	34/330	9/330	
10	215/220	201/220	180/220	154/220	125/220	95/220	66/220	40/220	19/220	5/220

The direct stresses in the girders are compressive stresses. In stories for which P is the same, the direct stresses in corresponding girders are the same.

Building Bent with Equal Spans, L. Uniform Wind-Pressure. Example. The direct wind-stresses in the columns and girders and the bending moments at their ends are required for a building bent. The building consists of 20 stories and a pent-house above ground, and a basement and sub-basement below, and is subjected to a wind-load of 15 lb per sq ft of exposed surface. The building bents, each consisting of three equal spans, L, between columns, are spaced 18 ft apart. The story heights are shown in the table below.

Computation of Wind-Stresses

Story considered	Heights, in feet						Wind-loads, pounds	
	Story considered h_1	Story above h_2	Average of h	To top of building H	$2H$	$2H-h_2$	P	W
Pent house	12	0	6	6	12	12	1 620	1 620
20th	10	12	11	17	34	22	2 970	4 590
19th	10 5	10	10 25	27 25	54 5	44 5	2 770	7 360
18th	10.5	10 5	10 5	37 75	75.5	65	2 840	10 200
17th	11	10 5	10 75	48 5	97	86.5	2 900	13 100
16th	11	11	11	59 5	119	108	2 970	16 070
15th	11	11	11	70 5	141	130	2 970	19 040
14th	11	11	11	81 5	163	152	2 970	22 010
13th	11	11	11	92 5	185	174	2 970	24 980
12th	11	11	11	103 5	207	196	2 970	27 950
11th	11	11	11	114 5	229	218	2 970	30 920
10th	11	11	11	125 5	251	240	2 970	33 890
9th	11	11	11	136 5	273	262	2 970	36 860
8th	11	11	11	147 5	295	284	2 970	39 830
7th	11	11	11	158 5	317	306	2 970	42 800
6th	11	11	11	169 5	339	328	2 970	45 770
5th	11	11	11	180 5	361	350	2 970	48 740
4th	11	11	11	191 5	383	372	2 970	51 710
3rd	11	11	11	202 5	405	394	2 970	54 680
2nd	9	11	10	212 5	425	414	2 700	57 380
1st	16	9	12 5	225	450	441	3 380	60 760
Basement	9	16	12 5	237.5			2 160	62 920
Sub-basement	8	9	8 5	246		.	0	62 920

$$M_A = M_C = \frac{3}{40}P\,(2\,H - h_2) \qquad M_1 = M_4 = \frac{3}{40}Wh_1 \qquad D_1 = -\,D_4 = \frac{3}{20}W\frac{H}{L}$$

$$M_B = \frac{4}{40}P\,(2\,H - h_2) \qquad M_2 = M_3 = \frac{7}{40}Wh_1 \qquad D_2 = -\,D_3 = \frac{1}{20}W\frac{H}{L}$$

$a = a' = 24.75$ ft.............. $a^2 = 612\ 5625$

$b = b' = \ \ 8.25$ ft........... .. $b^2 = \ \underline{68\ 0625}$

$$a^2 + b^2 = 680\ 625 = \frac{Q}{2}$$

$$Q = 1361\ 25$$

$$4\,Q = 5445$$

$$D_A = \frac{17}{20}P \qquad D_B = \frac{10}{20}P \qquad D_C = \frac{3}{20}P$$

Story	$2Wh$	Ph_2	$2Wh\text{-}Ph_2$	l	Wh_1	p*	q*	r*	t*
Basement	1 573 000	34 560	1 538 000	121	566 300	282 5	3681	5593	.00763
Sub-basement	1 070 000	0	1 070 000	129 5	503 400	196.4	.4706	5986	0

* Formulas for p, q, r, and t are given under "Unsymmetrical Building Bent. Varying or Uniform Wind-Pressure."

Computation of Wind-Stresses

Bending moments thousands of foot-pounds, at ends of				Direct stresses, thousands of pounds					Story considered
Girders		Columns		Columns		Girders			
$M_A = M_C$	M_B	$M_1 = M_4$	$M_2 = M_3$	$D_1 = -D_4$	$D_2 = -D_3$	D_A	D_B	D_C	
1 5	1 9	1 5	3 4	.1	.0	1.4	8	.2	Pent-house
4 9	6 5	3.4	8 0	.7	.2	2 5	1 5	.4	20th
9.2	12.3	5 8	13.5	1.8	.6	2.4	1.4	.4	19th
13 8	18.4	8 0	18.7	3.5	1.2	2 4	1.4	.4	18th
18 8	25.1	10 8	25 2	5 8	1.9	2 5	1.5	.4	17th
24 1	32 1	13 3	30 9	8 7	2.9	2 5	1.5	.4	16th
29 0	38 6	15 7	36 6	12 2	4 1	2 5	1 5	.4	15th
33.9	45.1	18 2	42.4	16 3	5.4	2 5	1 5	4	14th
38.8	51 7	20.6	48.1	21 0	7.0	2 5	1 5	4	13th
43.7	58 2	23 1	53.8	26 3	8 8	2.5	1.5	4	12th
48.6	64.7	25.5	59.5	32.2	10.7	2.5	1 5	.4	11th
53.5	71.3	28 0	65 2	38.7	12.9	2 5	1.5	.4	10th
58.4	77.8	30 4	70 9	45.7	15.2	2.5	1.5	.4	9th
63 3	84 3	32 9	76.7	53 4	17.8	2 5	1 5	4	8th
68 2	90 9	35.3	82 4	61 7	20.6	2 5	1 5	.4	7th
73 1	97 4	37 8	88.1	70 5	23 5	2.5	1.5	.4	6th
78.0	104.0	40.2	93.8	80.0	26 7	2 5	1.5	.4	5th
82.9	110 5	42.7	99.5	90 0	30 0	2.5	1 5	.4	4th
87.8	117.0	45 1	105.3	100.7	33.6	2 5	1 5	.4	3rd
83 8	111 8	38 7	90 4	110.8	36 9	2.3	1 4	.4	2nd
111.8	149 1	72 9	170 1	124 3	41 4	2 9	1 7	.5	1st
115.4	153.8	42 5	99.1	138 4	46.1	1 8	1 1	3	Basement
80 2	107 0	37.7	88 1	148 1	49.4	0	0	0	Sub-basement

For all floors above ground the formulas and coefficients given in the tables under "2. Building Bent with Equal Spans, L. Uniform Wind-Pressure" are applicable. For the basement and sub-basement the basic formulas under "1. Unsymmetrical Building Bent. Varying or Uniform Wind-Pressure" have been used.

The exposed area for any story equals the product of the average story height, h, by the distance center to center of bays. The wind load, P, at each floor is obtained by multiplying the exposed area by the unit wind-pressure; hence $P = 18 \times h \times 15 = 270\,h$. The total wind-load, W, is obtained by adding the values of P, down to and including the story under consideration.

HORSE-POWER, PULLEYS, GEARS, BELTING AND SHAFTING

Horse-Power. A horse can travel 400 yd at a walk in 4½ min, at a trot in 2 min, and at a gallop in 1 min; he occupies at a picket 3 ft by 9 ft; and his average weight is 1 000 lb. An AVERAGE HORSE carrying 225 lb can travel 25 miles in a day of 8 hr. A DRAUGHT-HORSE can draw 1 600 lb 23 miles a day, weight of carriage included. In a HORSE-MILL a horse moves at the rate of 3 ft in a second. The diameter of the track should not be less than 25 ft.

A Horse-Power, in Machinery, is estimated at 33 000 lb, raised 1 ft in a minute; but as a horse can exert that force but 6 hr a day, one MACHINERY HORSE-POWER is equivalent to that of four hourses.

Rules to Determine the Size and Speed of Pulleys or Gears. The driving-pulley is called the DRIVER, and the driven pulley the DRIVEN. If the number of teeth in the gears are used instead of the diameter, in these calculations, number of teeth must be substituted wherever diameter occurs.

(1) **To Find the Diameter of the Driver,** the diameter of the driven and its revolutions, and also revolutions of driver, being given. Multiply the diameter of the driven by its revolutions, and divide the product by the revolutions of the driver; the quotient will give the diameter of the driver.

(2) **To Find the Diameter of the Driven,** the revolutions of the driven, also the diameter and revolutions of the driver, being given. Multiply the diameter of the driver by its revolutions, and divide the product by the revolutions of the driven; the quotient will give the diameter of the driven.

(3) **To Find the Revolutions of the Driver,** the diameter and revolutions of the driven, also the diameter of the driver, being given. Multiply the diameeter of the driven by its revolutions, and divide the product by the diameter of the driver; the quotient will give the revolutions of the driver.

(4) **To Find the Revolutions of the Driven,** the diameter and revolutions of the driver, also the diameter of the driven, being given Multiply the diameter of the driver by its revolutions, and divide the product by the diameter of the driven; the quotient will give the revolutions of the driven.

Horse-Power Transmitted by Belting. The efficiency of belting to transmit power, or to turn a wheel or PULLEY, depends upon the width and thickness of the belt, the arc-contact with the pulley, the position of the belt, whether horizontal, vertical, or at an angle, and the velocity. The greater the velocity and the thicker or wider the belt, the more power it will transmit. A belt running vertically or inclined will transmit less power than one running horizontally, but in figuring the horse-power capacity of belting only the velocity, width and thickness of belt are usually considered, it being assumed that the pulleys are of proper size and located so that the belt will be nearly horizontal. Belts are commonly assumed to be of LEATHER, unless otherwise designated. The term SINGLE BELT is used to designate a belt made of a single thickness of leather. A DOUBLE BELT or a TRIPLE BELT is made by cementing together two or three thicknesses of leather. The standard average thickness of single belts are ⅛, 5/32 and 3/16 in, and the standard average thicknesses of double belts are ¼, 5/16 and ⅜ in.

Notes on Belting. For continuous use a double belt is the most economical in the long run, except on very small pulleys or for very light duty. Triplex and quadruple belts are sometimes used for very heavy duty, but such belts are not commonly carried in stock. Single belts should always be used with the

hair-side next the pulley. The belt-speed for maximum economy should be from 4 000 to 4 500 ft per minute. IDLER-PULLEYS work most satisfactorily when located on the slack side of the belt about one-quarter way from the driving-pulley.

The best method of installing a leather belt is to splice and cement the ends together, and this method should always be used for electric motor or generator installations as well as for all wide belts or heavy drives. For all other cases the ends may be fastened with hooks or lacing. Belts should be inspected at regular intervals and given any necessary attention or repairs.

Distance from Center to Center of Shafts.* In locating shafts that are to be connected with each other by belts, care should be taken to separate them by a proper distance. This distance should be such as to allow a gentle sag to the belt when in motion.

Rule. A general rule may be stated thus: Where narrow belts are to be run over small pulleys, 15 ft is a good average, the belt having a sag of from 1½ to 2 in. The minimum distance between shafts is about 10 ft. For larger belts, working on larger pulleys, a distance of from 20 to 25 ft does well, with a sag of from 2½ to 4 in. For main belts, working on very large pulleys, the distance should be from 25 to 30 ft, the belts working well with a sag of from 4 to 5 in. If too great a distance is attempted, the belt will have an unsteady flapping motion, which will destroy both the belt and the machinery.

The Horse-Power that Shafting will Transmit

Diameter of shaft		Revolutions per minute						
		100	150	200	250	300	350	400
in	16th	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
0	15	1 2	1 7	2 4	3 1	3 6	4 3	5.0
1	3	2.4	3 7	4 9	6 1	7.3	8.5	9.7
1	7	4.3	6 4	8 5	10.5	12.7	14 8	16.9
1	11	6 7	10 1	13 4	16.7	20 1	23.4	26.8
1	15	10 0	15 0	20.0	25.0	30 0	35 0	40.0
2	3	14 3	21 4	23 5	35 6	42 7	49 8	57.0
2	7	19 5	29 3	39 0	48.7	58.5	68 2	78.0
2	11	26 0	39 0	52.0	65.0	78.0	87.0	104.0
2	15	33.8	50 6	67.5	84.4	101.3	118 2	135.0
3	3	43.0	64 4	85.8	107.3	128.7	150.3	171.6
3	7	53 6	79.4	107.2	134 0	158.8	187 6	214.4
3	11	65 9	97 9	121 8	164.8	195 7	230.7	243.6
3	15	80 0	120 0	160 0	200 0	240 0	280.0	320.0
4	7	113 9	170.8	227 8	284.7	341 7	398 6	455.6
4	15	156 3	234.4	312.5	390.6	468 7	546 8	625 0

Arrangement of Belts and Pulleys.† If possible to avoid it, connected shafts should never be placed one directly over the other, as in such case the belt must be kept very tight to do the work. For this purpose belts should be

* For a discussion of belting, belt-dressing, care of belting, shafting, etc., see Kent's Mechanical Engineers' Pocket-Book.

† See Kent's Mechanical Engineers' Pocket-Book.

carefully selected of well-stretched leather. It is desirable that the angle of the belt with the floor should not exceed 45°. It is also desirable to locate the shafting and machinery so that belts will run off from each shaft in opposite directions, as this arrangement will relieve the bearings from the friction that would result if all pulled one way on the shaft. If possible, machinery should be so placed that the direction of the belt-motion will be from the top of the driving to the top of the driven pulley, so that the sag will increase the arc of contact. The pulley should be a little wider than the belt required for the work, and should have a crowning face, except where the belt is to be shifted. The motion of driving should run with and not against the laps of the belts.

Rubber Belts are cheaper than leather belts but they are not as durable. They should always be kept free from grease or animal oils. If they slip, their inside surfaces should be moistened with boiled linseed-oil. Some fine chalk, sprinkled on over the oil, will help the belt.

Rule for Finding the Lengths of Belts. Add the diameter of the two pulleys together, multiply by $3\frac{1}{7}$, divide the product by 2, add to the quotient twice the distance between the center of the shafts, and the sum will be the required length when the pulleys are of equal diameter.

CHAIN-BLOCKS, HOISTS, HOOKS, ETC.

General Description. These are portable hoisting-devices which enable one man to raise a very heavy load and which sustain the load at any point. In general, they resemble pulleys operated by chains. Since the invention of the differential pulley-block by T. A. Weston, about the year 1863, chain-blocks have come into very general use for economical hoisting, particularly where it is desired to hold the load at any point. Chain-blocks are of three general classes:

(1) The Differential Block. This is the original and the simplest and cheapest form of self-sustaining pulley;

(2) The Screw-Block or Worm-Geared Block. Of these, the Yale & Towne duplex block is an efficient type;

(3) The Triplex Block. This is spur-geared.

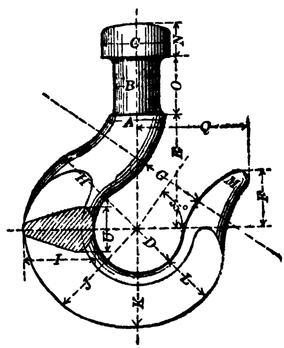
Differential and worm-geared blocks of all kinds depend upon friction to prevent the load from running down. In the triplex block a separate device is introduced which automatically holds the load safely, and yet enables it to be lowered with slight effort and at high velocity but without acceleration or danger. This is the most efficient of all chain-blocks, and the most economical wherever quick work is wanted and economy in time and labor sought. For information as to the kind of block best adapted to any particular service, the manufacturers should be consulted. The following data on the power and efficiency of chain-blocks were supplied by the Yale & Towne Manufacturing Company.

Power and Efficiency of Chain-Hoists. The table below gives the work to be done by the operator at the hand-pulling chain with each size of the various kinds of chain-blocks in lifting the stated capacity, that is, the amount of work or pulling required to lift this load ONE FOOT by stating the force exerted in pounds and the distance in feet of operating-chains to be pulled. The product of these two factors determines the efficiency of the block and the ease and speed of hoisting. The 12, 16, and 20-ton-capacity chain-blocks have each two hand-chains.

Work Done by Operator with Chain-Blocks

Capacity, tons	Triplex spur-geared, lb ft	Duplex worm-geared, lb ft	Differential, lb ft
½	62 × 21	68 × 40	122 × 24
1	82 × 31	87 × 59	216 × 30
1½	110 × 35	94 × 80	246 × 36
2	120 × 42	115 × 93	308 × 42
3	114 × 69	132 × 126	557 × 38
4	124 × 84	142 × 155	. .
5	110 × 126	145 × 195	. . .
6	130 × 126	145 × 252	.
8	135 × 168	160 × 310	.
10	140 × 210	160 × 390	.
12	130 × 126	.	. .
16	135 × 168		.
20	140 × 210		.

The capacities are given in tons. The figures give the number of feet to be operated on each hand-chain. A man cannot pull more than his own weight on the operating chains, and can pull faster in proportion as the pull required is lighter. The maximum pull usually required of one man is 82 lb, and he will do more work with less fatigue if the hand-chain pull is not over 40 lb, because he can then pull the chain hand over hand a little more than twice as fast as he could when pulling twice as hard. When the hand-chain pull is less than 20 lb the speed of hoisting an equal load is diminished, because the man is tired by moving his arms too rapidly, and cannot do as much work as with a heavier pull. The best result is obtained by using a chain-block which has a capacity of double the usual load. The operator then works to the best advantage with average loads, and occasional heavy loads are easily handled without overstraining either the operator or the chain-block, which should never be used beyond its capacity for fear of stretching the chain so that it will not work smoothly.



Proportions of Hooks.* For economy of manufacture hooks of different sizes are made from some regular commercial sizes of round iron. The basis, or initial point, in each case is, therefore, the size of the iron of which the hook is to be made, and it is indicated by the dimension A in the diagram. The dimension D is arbitrarily assumed. The other dimensions, as given by the formulas, are those which, while preserving a proper bearing face on the interior of the hook for the ropes or chains which may be passed through it, give the greatest resistance to spreading and to ultimate rupture which the amount of material in the original bar admits of.

* By Henry R. Towne, in his Treatise on Cranes, which includes the results of an extensive experimental and mathematical investigation.

The symbol Δ is used in the formulas to indicate the NOMINAL CAPACITY of the hook in tons of 2 000 lb. The formulas which determine the lines of the other parts of the hooks of the several sizes are as follows, all the measurements being expressed in inches:

$D = 0.5\ \Delta + 1.25$	$G = 0.75\ D$
$E = 0.64\ \Delta + 1.60$	$O = 0.363\ \Delta + 0.66$
$F = 0.33\ \Delta + 0.85$	$Q = 0.64\ \Delta + 1.60$
$H = 1.08\ A$	$L = 1.05\ A$
$I = 1.33\ A$	$M = 0.50\ A$
$J = 1.20\ A$	$N = 0.85\ B - 0.16$
$K = 1.13\ A$	$U = 0.866\ A$

Example. To find the dimension D, for a 2-ton hook. The formula is:

$$D = 0.5\ \Delta + 1.25$$

and as $\Delta = 2$, the dimension D by the formula is found to be $2\frac{1}{4}$ in. The dimensions A are necessarily based upon the ordinary merchant sizes of round iron. The sizes which it has been found best to select are the following:

Capacities of hooks	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4	5	6	8	10 tons
Dimension A....	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$3\frac{1}{4}$ in

The formulas which give the sections of the hook at the several points are all expressed in terms of A, and can therefore be readily ascertained by reference to the foregoing scale.

Example. To find the dimension I, in a 2-ton hook. The formula is:

$$I = 1.33\ A$$

and for a 2-ton hook, $A = 1\frac{3}{8}$ in. Therefore I, in a 2-ton hook, is found to be $1\frac{13}{16}$ in.

Manner of Failure of Hooks. Experiment has shown that hooks made according to the above formulas will give way first by the opening of the jaw, which, however, will not occur except with a load much in excess of the nominal capacity of the hook. This yielding of the hook when overloaded becomes a source of safety, as it constitutes a signal of danger which cannot easily be overlooked, and which must proceed to a considerable length before rupture occurs and the load is dropped. A comparison of these hooks with most of those in ordinary use shows that the latter are, as a rule, badly proportioned, and frequently dangerously weak.

BELLS

Dimensions and Weights of Church-Bells

Manufactured by Meneely Bell Company, Troy, N. Y.

Bells		Mountings			
Weights, lb	Medium tones	Diameters, in	Sizes of frames, outside, in	Diameters of wheels, ft	Diameters of wheels, in
400	D	27	42×42	4	4
450	C♯	28	42×42	4	4
500	C	29	45×47	4	4
550	C	30	45×47	4	4
600	B	31	45×47	4	9
700	B	33	48×48	5	6
800	B♭	34	48×54	5	6
900	A	36	54×54	5	9
1 000	A	37	54×54	5	9
1 100	A	38	54×59	5	9
1 200	A♭	39	56×59	6	3
1 300	A♭	40	56×59	6	3
1 400	G	41	60×60	6	6
1 500	G	42	60×60	6	6
1 600	G	43	60×60	6	6
1 800	F♯	45	65×68	7	
2 000	F	46	65×68	7	
2 100	F	47	65×68	7	
2 300	E	49	70×72	7	6
2 500	E	50	70×72	7	6
2 800	E♭	51	74×78	8	
3 000	E♭	53	74×78	8	
3 500	D	56	74×78	8	6
4 000	C♯	58	78×81	9	
4 500	C	61	78×81	9	
5 000	C	63	84×84	9	
5 500	B	65	84×84	9	
6 000	B♭	67	84×84	9	6
6 500	B♭	68	90×90	9	6
7 000	B♭	69	101×90	9	6

Meneely School-Bells

Bells		Mountings
Weights, lb	Diameters, in	Sizes of frames, outside, ft in ft in
100	17	2 6 × 2 8
125	18½	2 6 × 2 8
150	19½	2 6 × 2 8
200	21½	2 8 × 3 0
250	23	3 0 × 3 2
300	24½	3 0 × 3 4
350	26	3 0 × 3 4

The Largest Bells in the World

Names and locations of bells	Date cast or erected	Actual vibration	Keynote	Diameter, in	Sound-bow		Weight, lb
					Inches	Stroke	
Moscow, Tzar Kolokol	1733	74	D	272	23	0 84	443 772
Burmah, Mingoon..		94	F#	203?	16?	0.80	201 600
Moscow, St. Ivan's.	1819	105	G#	185	14.75	0.80	127 350
Peiping, Great Bell.	.	..	..	156	.		120 000
Burmah, Maha Ganda..	..	125	B	155	12.5	0.80	95 000
Nishni Novgorod		125	B	151	12	0.80	69 664
Moscow, Ch. of Redeemer	1879	141	C#	136 3?	10.6	0.80	60 736
Nankin, China.	. .	....	...	112	...	.	45 000
New York, Riverside Baptist Church....	1930	....	C				40 926*
Olmutz, Bohemia.....		157	E♭	121	9 125	0 75	40 320
Vienna, Austria . .	1711	157	E♭	118	9.5	0 80	40 200
Philadelphia, Wanamaker Memorial.	1926	...	D	114	. .	. .	38 640
Chicago University Chapel. . .	1930	. .	C#				38 080*
London, St. Paul's....	1881	157	E♭	114.25	8.75	0 76	37 483
Westminster, London	1856	166	E	113 5	9.375	0.83	35 620
Erfurt, Saxony. .	1487	176	F	103.6	9.75	0 75	30 800
Notre Dame, Paris...	1680	166	E	103	7.5	0 73	28 670
Montreal, Canada ..	1847	176	F	103	7.8	0 76	28 560
York, England. .	1845	187	F#	100	8	0.80	24 080
Mountain Lake, Florida.	1929	.	E♭	.	...		23 246*
Nottingham City Exchange		.	...	...	..		23 211
Ottawa, Canada Peace Tower, Parliament Houses.. . .	1927	.	E	..	..	.	22 400*
Bristol University..	.		.		.		21 448
St. Peter's, Rome......	1786	187	F#	97.25	7.5	0 77	18 000
Great Tom, Oxford.	1680	210	G#	84	6.125	0.73	17 024
Cologne, Germany....	1477	198	G	95	7.2	0.76	16 016
Brussels, Belgium....	.	210	G#	95.81	7.75	0.71	15 848
Beverly Minster....		.	.		. .	. .	15 765
Shanghai, Custom House	.		.		.	..	14 000
New Haven Yale Univ			.		.		13 496
State House, Philadelphia	1875	198	G	88	6.375	0.73	13 000
Princeton, Cleveland Tower.... .	1927		G	.	. .	.	12 888*
Netherlands, 's Hertogenbosch.	1925			..	. .		12 100*
Lincoln, England.. .	1834	210	G#	82 85	6	0.73	12 096
Cohasset, Mass., St. Stephen's Church.....	1928	..	G	..	.		11 760*
St. Paul's, London.. .	1716	222	A	81	6.08	0 75	11 500
Wellington, New Zealand	.	....	G#	.	..		11 200*
Albany, N Y., City Hall.	1927	....	G	..	.	. .	11 200*
Indianapolis, Ind , Scottish Rite Cathedral.	1929	...	G			. ..	11 200*
Durham, N. C , University Chapel.........	1931	. .	...	.	...	. .	11 200*
Exeter, England....	1675	210	G#	76	5	0.66	10 080
St Helen's, Leicestershire	1929		G#				10 000*
Old Lincoln, England.	1610	249	B	75 5	5.94	0.78	9 856
Sydney University, Australia . .	1928	..	G#	. .	..	.	9 455*
Loughborough War Memorial, England	1923	. .	.	. ..		...	9 284*
Rotterdam Town Hall ..	1921	. .	.	. .	...	...	9 226*

* Largest bell of a carillon.

SYMBOLS FOR THE APOSTLES AND SAINTS

From the constant occurrence of symbols in the edifices of the Middle Ages and many of the cathedrals of the present day, the following list of symbols, as commonly attached to the apostles and saints, may be found useful:

Holy Apostles

St. Peter. Bears a key, or two keys with different wards.
St. Andrew. Leans on a cross so called from him; called by heralds the saltire.
St. John the Evangelist. With a chalice, in which is a winged serpent. When this symbol is used, the eagle, another symbol of him, is never given.
St. Bartholomew. With a flaying-knife.
St. James the Less. A fuller's staff bearing a small square banner.
St. James the Greater. A pilgrim's staff, hat and escalop-shell.
St. Thomas. An arrow, or with a long staff.
St. Simon. A long saw.
St. Jude. A club.
St. Matthias. A hatchet.
St. Philip. Leans on a spear or has a long cross in the shape of a **T**.
St. Matthew. A knife or dagger.
St. Mark. A winged lion.
St. Luke. A bull.
St. John. An eagle.
St. Paul. An elevated sword, or two swords in saltire.
St. John the Baptist. An Agnus Dei.
St. Stephen. With stones in his lap.

Saints

St. Agnes. A lamb at her feet.
St. Cecilia. With an organ.
St. Clement. With an anchor.
St. David. Preaching on a hill.
St. Denis. With his head in his hands.
St. George. With the dragon
St. Nicholas With three naked children in a tub, in the end whereof rests his pastoral staff.
St. Vincent. On the rack.

THE AMERICAN INSTITUTE OF ARCHITECTS

1. DOCUMENTS OF A PERMANENT NATURE—TITLES AND PRICES

The following documents are published by the American Institute of Architects, The Octagon, 1741 New York Ave , Washington, D. C., and may be obtained by addressing the Executive Secretary.

The Octagon, A Journal of The American Institute of Architects, Yearly	$ 1.00
The Monograph on The Octagon (Thirty Drawings, 12 by 18, Photographs and Text)	15 00
The Handbook of Architectural Practice	5.00
The Standard Contract Documents—	
Agreement and General Conditions in Cover	.25
General Conditions without Agreement	.18
Agreement without General Conditions	.07
Bond of Suretyship	.05
Form of Subcontract	.05
Letter of Acceptance of Subcontractor's Proposal	.05
Cover (heavy paper, with valuable notes)	.01
Complete set in Cover	40
Review of the Standard Documents	1 00
Agenda for Architects	.40
The Standard Form of Agreement between Owner and Architect (Percentage Basis)	.05
A Form of Agreement between Owner and Architect (Fee Plus Cost System)	.05
A Circular of Information on the Fee Plus Cost System of Charges (Explanatory of Owner-Architect Agreement on fee plus cost basis)	.03
A Form of Agreement between Owner and Contractor (Cost Plus Fee Basis)	.10
Circular of Information Relative to Cost Plus Fee System of Contracting (Explanatory of Contractor-Owner Agreement on cost plus fee basis)	.06
Standard Form of Competition Program	.10
A Circular of Advice and Information Relative to Architectural Competions	Free
Architectural Competitions—The Duties of the Professional Adviser and of the Jury	Free
Proceedings of the Convention	Free
Annuary to Institute Members	Free
To Commercial Firms	$5 00
Circular on Functions of the Institute	Free
Manual of the Institute	Free
Circular on the Functions of the Architect	$0 03
Classes of Memberships and Affiliations—Institute and Chapters	Free
Circular of Information Concerning Requirements for Institute Membership	Free
Circular of Information Concerning Requirements for Chapter Associateship	Free
Circular of Information Concerning Requirements for Juniorship	Free
Constitution and By-laws	Free
Standard Form of Chapter Constitution and By-laws	Free
A Circular of Advice Relative to Principles of Professional Practice, The Canon of Ethics	Free
Schedule of Proper Minimum Charges	$0 02
Circular Relative to the Size and Character of Advertising Matter	Free
Standard Symbols for Wiring Plans	Free
The Plan of the Federal City	Free
Model Registration Law	Free
Cubic Contents of Buildings	Free
Standard Filing System (For Information on Building Materials and Appliances)	$0 50
Alphabetical Index to Standard Filing System	.50
Combined Filing System and Index	1.00

2. PROFESSIONAL PRACTICE OF ARCHITECTS *

Details of Service to be Rendered and Schedule of Proper Minimum Charges. Art. VI of the Constitution of The American Institute of Architects is as follows. The Institute shall from time to time adopt a Code or Codes, which shall be standards of professional practice, and it may from time to time recommend a Schedule of Professional Charges, complying with good practice and custom, but such a Schedule shall not be made mandatory.

1. The Architect's professional services consist of the necessary conferences, the preparation of preliminary studies, working drawings, specifications, large scale and full size detail drawings; the drafting of forms of proposals and contracts; the issuance of certificates of payment; the keeping of accounts, the general administration of the business and supervision of the work, for which, except as hereinafter mentioned, a proper minimum charge, based upon the total cost of the work complete, is 6%.

2. On residential work, alterations to existing buildings, monuments, furniture, decorative and cabinet work, and landscape architecture, it is proper to make a higher charge than above indicated.

3. The Architect is entitled to compensation for articles purchased under his direction, even though not designed by him.

4. Where the Architect is not otherwise retained, consultation fees for professional advice are to be paid in proportion to the importance of the question involved and services rendered.

5. The Architect is to be reimbursed the costs of transportation and living incurred by him and his assistants while traveling in discharge of duties connected with the work, and the costs of the services of heating, ventilating, mechanical, and electrical engineers.

6. The rate of percentage arising from Articles 1 and 2 hereof, i.e., the basic rate, applies when all of the work is let under one contract. Should the Owner determine to have certain portions of the work executed under separate contracts, as the Architect's burden of service, expense and responsibility is thereby increased, the rate in connection with such portions of the work is greater (usually by 4%) than the basic rate. Should the Owner determine to have substantially the entire work executed under separate contracts, then such higher rate applies to the entire work. In any event, however, the basic rate, without increase, applies to contracts for any portions of the work on which the Owner reimburses the engineer's fees to the Architect.

7. If, after a definite scheme has been approved, the Owner makes a decision which, for its proper execution, involves extra services and expense for changes in or additions to the drawings, specifications or other documents; or if a contract be let by cost of labor and materials plus a percentage or fixed sum; or if the Architect be put to labor or expense by delays caused by the Owner or a contractor, or by the delinquency or insolvency of either, or as a result of damage by fire or other casualty, he is to be equitably paid for such extra service and expense.

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The words "the cost of the work," as used in Articles 1 and 9 hereof, are ordinarily to be interpreted as meaning the total of the contract sums incurred for the execution of the work not including Architect's and Engineer's fees or the salary of the clerk of the works, but in certain rare cases, e g., when labor or material is furnished by the Owner below its market cost or when old materials are re-used, the cost of the work is to be interpreted as the cost of all materials and labor necessary to complete the work, as such cost would have been if all materials had been new and if all labor had been fully paid at market prices current when the work was ordered, plus contractor's profits and expenses

8. Should the execution of any work designed or specified by the Architect or any part of such work be abandoned or suspended, the Architect is to be paid in accordance with or in proportion to the terms of Article 9 of this Schedule for the service rendered, up to the time of such abandonment or such suspension.

9. Whether the work be executed or whether its execution be suspended or abandoned in part or whole, payments to the Architect on his fee are subject to the provisions of Articles 7 and 8, made as follows:

Upon completion of the preliminary studies, a sum equal to 20% of the basic rate computed upon a reasonable estimated cost.

Upon completion of specifications and general working drawings (exclusive of details) a sum sufficient to increase payments on the fee to 60% of the rate or rates of commission agreed upon, as influenced by Article 6, computed upon a reasonable cost estimated on such completed specifications and drawings, or if bids have been received, then computed upon the lowest bona fide bid or bids.

During the preparation of the preliminary studies and of the specifications and general working drawings, it is proper that payments on account be made at monthly or other intervals, in proportion to the progress of the architect's service, and so as to aggregate in each period not more than the sums described above.

From time to time during the execution of work and in proportion to the amount of service rendered by the Architect, payments are made until the aggregate of all payments made on account of the fee under this Article reaches a sum equal to the rate or rates of commission agreed upon as influenced by Article 6, computed upon the final cost of the work.

Payments to the Architect, other than those on his fee, fall due from time to time as his work is done or as costs are incurred.

No deduction is made from the Architect's fee on account of the use of old materials, penalty, liquidated damages or other sums withheld from payments to contractors.

10. The Owner is to furnish the Architect with a complete and accurate survey of the building site, giving the grades and lines of streets, pavements and adjoining properties; the rights, restrictions, easements, boundaries and contours of the building site, and full information as to sewer, water, gas and electrical service. The Owner is to pay for borings or test pits and for chemical, mechanical or other tests, when required.

11. The Architect endeavors to guard the Owner against defects and deficiencies in the work of contractors, but does not guarantee the performance of their contracts. The supervision of an architect is to be distinguished from the continuous personal superintendence to be obtained by the employment of a clerk of the works.

When authorized by the Owner, a clerk of the works, acceptable to both Owner and Architect, is to be engaged by the Architect at a salary satisfactory to the Owner and paid by the Owner, upon presentation of the Architect's monthly certificates.

12. When requested to do so, the Architect makes or procures preliminary estimates on the cost of the work and he endeavors to keep the actual cost of the work as low as may be consistent with the purpose of the building and with proper workmanship and material, but no such estimate can be regarded as other than an approximation.

13. Drawings and specifications, as instruments of service, are the property of the Architect, whether the work for which they are made be executed or not.

THE GENERAL CONDITIONS OF THE CONTRACT FOR THE CONSTRUCTION OF BUILDINGS *

STANDARD FORM OF THE AMERICAN INSTITUTE OF ARCHITECTS

These General Conditions have received the approval of the National Association of Builders' Exchanges, the Associated General Contractors of America, the Joint Conference on Construction Contracts, the National Association of Master Plumbers, the National Association of Sheet Metal Contractors of the United States, the National Electrical Contractors' Association of the United States, the National Association of Marble Dealers, the Building Granite Quarries Association, the Producers' Council and the Building Trades Employers' Association of the City of New York.

Index to the Articles of the General Conditions

Art. 1. Definitions.

(a) The Contract Documents consist of the Agreement, the General Conditions of the Contract, the Drawings and Specifications, including all modifications thereof incorporated in the documents before their execution. These form the Contract.

(b) The Owner, the Contractor and the Architect are those mentioned as such in the Agreement. They are treated throughout the Contract Documents as if each were of the singular number and masculine gender.

(c) The term Subcontractor, as employed herein, includes only those having a direct contract with the Contractor and it includes one who furnishes material worked to a special design according to the plans or specifications of

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this work, but does not include one who merely furnishes material not so worked.

(d) Written notice shall be deemed to have been duly served if delivered in person to the individual or to a member of the firm or to an officer of the corporation for whom it is intended, or if delivered at or sent by registered mail to the last business address known to him who gives the notice.

(e) The term "work" of the Contractor or Subcontractor includes labor or materials or both.

(f) All time limits stated in the Contract Documents are of the essence of the Contract.

(g) The law of the place of building shall govern the construction of this Contract.

Art. 2. Execution, Correlation and Intent of Documents. The Contract Documents shall be signed in duplicate by the Owner and the Contractor. In case the Owner and the Contractor fail to sign the General Conditions, Drawings or Specifications, the Architect shall identify them.

The Contract Documents are complementary, and what is called for by any one shall be as binding as if called for by all. The intention of the documents is to include all labor and materials, equipment and transportation necessary for the proper execution of the work. It is not intended, however, that materials or work not covered by or properly inferable from any heading, branch, class or trade of the specifications shall be supplied unless distinctly so noted on the drawings. Materials or work described in words which so applied have a well-known technical or trade meaning shall be held to refer to such recognized standards.

Art. 3. Detail Drawings and Instructions. The Architect shall furnish, with reasonable promptness, additional instructions, by means of drawings or otherwise, necessary for the proper execution of the work. All such drawings and instructions shall be consistent with the Contract Documents, true developments thereof, and reasonably inferable therefrom.

The work shall be executed in conformity therewith and the Contractor shall do no work without proper drawings and instructions.

The Contractor and the Architect, if either so requests, shall jointly prepare a schedule, subject to change from time to time in accordance with the progress of the work, fixing the dates at which the various detail drawings will be required, and the Architect shall furnish them in accordance with that schedule. Under like conditions, a schedule shall be prepared, fixing the dates for the submission of shop drawings, for the beginning of manufacture and installation of materials and for the completion of the various parts of the work.

Art. 4. Copies Furnished. Unless otherwise provided in the Contract Documents, the Architect will furnish to the Contractor, free of charge, all copies of drawings and specifications reasonably necessary for the execution of the work.

Art. 5. Shop Drawings. The Contractor shall submit with such promptness as to cause no delay in his own work or in that of any other Contractor, two copies of all shop or setting drawings and schedules required for the work of the various trades, and the Architect shall pass upon them with reasonable promptness. The Contractor shall make any corrections required by the Architect, file with him two corrected copies and furnish such other copies as may be needed. The Architect's approval of such drawings or schedules shall not relieve the Contractor from responsibility for deviations from draw-

ings or specifications, unless he has in writing called the Architect's attention to such deviations at the time of submission, nor shall it relieve him from responsibility for errors of any sort in shop drawings or schedules.

Art. 6. Drawings and Specifications on the Work. The Contractor shall keep one copy of all drawings and specifications on the work, in good order, available to the Architect and to his representatives.

Art. 7. Ownership of Drawings and Models. All drawings, specifications and copies thereof furnished by the Architect are his property. They are not to be used on other work and, with the exception of the signed Contract set, are to be returned to him on request, at the completion of the work. All models are the property of the Owner.

Art. 8. Samples. The Contractor shall furnish for approval all samples as directed. The work shall be in accordance with approved samples.

Art. 9. Materials, Appliances, Employes. Unless otherwise stipulated, the Contractor shall provide and pay for all materials, labor, water, tools, equipment, light, power, transportation and other facilities necessary for the execution and completion of the work.

Unless otherwise specified, all materials shall be new and both workmanship and materials shall be of good quality. The Contractor shall, if required, furnish satisfactory evidence as to the kind and quality of materials.

The Contractor shall at all times enforce strict discipline and good order among his employes, and shall not employ on the work any unfit person or any one not skilled in the work assigned to him.

Art. 10. Royalties and Patents. The Contractor shall pay all royalties and license fees. He shall defend all suits or claims for infringement of any patent rights and shall save the Owner harmless from loss on account thereof, except that the Owner shall be responsible for all such loss when a particular process or the product of a particular manufacturer or manufacturers is specified, but if the Contractor has information that the process or article specified is an infringement of a patent he shall be responsible for such loss unless he promptly gives such information to the Architect or Owner.

Art. 11. Surveys, Permits and Regulations. The Owner shall furnish all surveys unless otherwise specified. Permits and licenses of a temporary nature necessary for the prosecution of the work shall be secured and paid for by the Contractor. Permits, licenses and easements for permanent structures or permanent changes in existing facilities shall be secured and paid for by the Owner, unless otherwise specified.

The Contractor shall give all notices and comply with all laws, ordinances, rules and regulations bearing on the conduct of the work as drawn and specified. If the Contractor observes that the drawings and specifications are at variance therewith, he shall promptly notify the Architect in writing, and any necessary changes shall be adjusted as provided in the Contract for changes in the work. If the Contractor performs any work knowing it to be contrary to such laws, ordinances, rules and regulations, and without such notice to the Architect, he shall bear all costs arising therefrom.

Art. 12. Protection of Work and Property. The Contractor shall continuously maintain adequate protection of all his work from damage and shall protect the Owner's property from injury or loss arising in connection with this Contract. He shall make good any such damage, injury or loss, except such as may be directly due to errors in the Contract Documents or caused by agents or employes of the Owner. He shall adequately protect adjacent property as provided by law and the Contract Documents. He shall provide

and maintain all passageways, guard fences, lights and other facilities for protection required by public authority or local conditions.

In an emergency affecting the safety of life or of the work or of adjoining property, the Contractor, without special instruction or authorization from the Architect or Owner, is hereby permitted to act, at his discretion, to prevent such threatened loss or injury, and he shall so act, without appeal, if so instructed or authorized. Any compensation, claimed by the Contractor on account of emergency work, shall be determined by agreement or Arbitration.

Art. 13. Inspection of Work. The Architect and his representative shall at all times have access to the work wherever it is in preparation or progress and the Contractor shall provide proper facilities for such access and for inspection.

If the specifications, the Architect's instructions, laws, ordinances or any public authority require any work to be specially tested or approved, the Contractor shall give the Architect timely notice of its readiness for inspection, and if the inspection is by another authority than the Architect, of the date fixed for such inspection. Inspections by the Architect shall be promptly made, and where practicable at the source of supply. If any work should be covered up without approval or consent of the Architect, it must, if required by the Architect, be uncovered for examination at the Contractor's expense.

Re-examination of questioned work may be ordered by the Architect and if so ordered the work must be uncovered by the Contractor. If such work be found in accordance with the Contract Documents the Owner shall pay the cost of re-examination and replacement. If such work be found not in accordance with the Contract Documents the Contractor shall pay such cost, unless he shall show that the defect in the work was caused by another Contractor, and in that event the Owner shall pay such cost.

Art. 14. Superintendence: Supervision. The Contractor shall keep on his work, during its progress, a competent superintendent and any necessary assistants, all satisfactory to the Architect. The superintendent shall not be changed except with the consent of the Architect, unless the superintendent proves to be unsatisfactory to the Contractor and ceases to be in his employ. The superintendent shall represent the Contractor in his absence and all directions given to him shall be as binding as if given to the Contractor. Important directions shall be confirmed in writing to the Contractor. Other directions shall be so confirmed on written request in each case.

The Contractor shall give efficient supervision to the work, using his best skill and attention. He shall carefully study and compare all drawings, specifications and other instructions and shall at once report to the Architect any error, inconsistency or omission which he may discover, but he shall not be held responsible for their existence or discovery.

Art. 15. Changes in the Work. The Owner, without invalidating the Contract, may order extra work or make changes by altering, adding to or deducting from the work, the Contract Sum being adjusted accordingly. All such work shall be executed under the conditions of the original contract except that any claim for extension of time caused thereby shall be adjusted at the time of ordering such change.

In giving instructions, the Architect shall have authority to make minor changes in the work, not involving extra cost, and not inconsistent with the purposes of the building, but otherwise, except in an emergency endangering life or property, no extra work or change shall be made unless in pursuance of a written order from the Owner signed or countersigned by the Architect, or a written order from the Architect stating that the Owner has authorized

the extra work or change, and no claim for an addition to the contract sum shall be valid unless so ordered.

The value of any such extra work or change shall be determined in one or more of the following ways:

(a) By estimate and acceptance in a lump sum
(b) By unit prices named in the contract or subsequently agreed upon.
(c) By cost and percentage or by cost and a fixed fee

If none of the above methods is agreed upon, the Contractor, provided he receives an order as above, shall proceed with the work In such case and also under case (c), he shall keep and present in such form as the Architect may direct, a correct account of the net cost of labor and materials, together with vouchers. In any case, the Architect shall certify to the amount, including reasonable allowance for overhead and profit, due to the Contractor. Pending final determination of value, payments on account of changes shall be made on the Architect's certificate.

Art. 16. Claims for Extra Cost. If the Contractor claims that any instructions by drawings or otherwise involve extra cost under this contract, he shall give the Architect written notice thereof within a reasonable time after the receipt of such instructions, and in any event before proceeding to execute the work, except in emergency endangering life or property, and the procedure shall then be as provided for changes in the work. No such claim shall be valid unless so made.

Art. 17. Deductions for Uncorrected Work. If the Architect and Owner deem it inexpedient to correct work injured or done not in accordance with the Contract, an equitable deduction from the contract price shall be made therefor.

Art. 18. Delays and Extension of Time. If the Contractor be delayed at any time in the progress of the work by any act or neglect of the Owner or the Architect, or of any employe of either, or by any other Contractor employed by the Owner, or by changes ordered in the work, or by strikes, lockouts, fire, unusual delay in transportation, unavoidable casualties or any causes beyond the Contractor's control, or by delay authorized by the Architect pending arbitration, or by any cause which the Architect shall decide to justify the delay, then the time of completion shall be extended for such reasonable time as the Architect may decide

No such extention shall be made for delay occurring more than seven days before claim therefor is made in writing to the Architect. In the case of a continuing cause of delay, only one claim is necessary.

If no schedule or agreement stating the dates upon which drawings shall be furnished is made, then no claim for delay shall be allowed on account of failure to furnish drawings until two weeks after demand for such drawings and not then unless such claim be reasonable.

This article does not exclude the recovery of damages for delay by either party under other provisions in the contract documents

Art. 19. Correction of Work before Final Payment. The Contractor shall promptly remove from the premises all materials condemned by the Architect as failing to conform to the Contract, whether incorporated in the work or not, and the Contractor shall promptly replace and re-execute his own work in accordance with the Contract and without expense to the Owner and shall bear the expense of making good all work of other contractors destroyed or damaged by such removal or replacement.

If the Contractor does not remove such condemned work and materials

within a reasonable time, fixed by written notice, the Owner may remove them and may store the material at the expense of the Contractor. If the Contractor does not pay the expenses of such removal within ten days' time thereafter, the Owner may, upon ten days' written notice, sell such materials at auction or at private sale and shall account for the net proceeds thereof, after deducting all the costs and expenses that should have been borne by the Contractor.

Art. 20. Correction of Work after Final Payment. Neither the final certificate nor payment nor any provision in the Contract Documents shall relieve the Contractor of responsibility for faulty materials or workmanship and, unless otherwise specified, he shall remedy any defects due thereto and pay for any damage to other work, resulting therefrom, which shall appear within a period of one year from the date of substantial completion. The Owner shall give notice of observed defects with reasonable promptness. All questions arising under this article shall be decided by the Architect subject to arbitration.

Art. 21. The Owner's Right to Do Work. If the Contractor should neglect to prosecute the work properly or fail to perform any provision of this contract, the Owner, after three days' written notice to the Contractor may, without prejudice to any other remedy he may have, make good such deficiencies and may deduct the cost thereof from the payment then or thereafter due the Contractor provided, however, that the Architect shall approve both such action and the amount charged to the Contractor.

Art. 22. Owner's Right to Terminate Contract. If the Contractor should be adjudged a bankrupt, or if he should make a general assignment for the benefit of his creditors, or if a receiver should be appointed on account of his insolvency, or if he should persistently or repeatedly refuse or should fail, except in cases for which extension of time is provided, to supply enough properly skilled workmen or proper materials, or if he should fail to make prompt payment to subcontractors or for material or labor, or persistently disregard laws, ordinances or the instructions of the Architect, or otherwise be guilty of a substantial violation of any provision of the contract, then the Owner, upon the certificate of the Architect that sufficient cause exists to justify such action, may, without prejudice to any other right or remedy and after giving the Contractor seven days' written notice, terminate the employment of the Contractor and take possession of the premises and of all materials, tools and appliances thereon and finish the work by whatever method he may deem expedient: In such case the Contractor shall not be entitled to receive any further payment until the work is finished. If the unpaid balance of the contract price shall exceed the expense of finishing the work, including compensation for additional managerial and administrative services, such excess shall be paid to the Contractor. If such expense shall exceed such unpaid balance, the Contractor shall pay the difference to the Owner. The expense incurred by the Owner as herein provided, and the damage incurred through the Contractor's default, shall be certified by the Architect.

Art. 23. Contractor's Right to Stop Work or Terminate Contract. If the work should be stopped under an order of any court, or other public authority, for a period of three months, through no act or fault of the Contractor or of anyone employed by him, or if the Architect should fail to issue any certificate for payment within seven days after it is due, or if the Owner should fail to pay to the Contractor within seven days of its maturity and presentation, any sum certified by the Architect or awarded by arbitrators, then the Contractor may, upon seven days' written notice to the Owner and the Architect,

stop work or terminate this contract and recover from the Owner payment for all work executed and any loss sustained upon any plant or materials and reasonable profit and damages.

Art. 24. Applications for Payments. The Contractor shall submit to the Architect an application for each payment, and, if required, receipts or other vouchers, showing his payments for materials and labor, including payments to subcontractors as required by Art. 37.

If payments are made on valuation of work done, such application shall be submitted at least ten days before each payment falls due, and, if required, the Contractor shall, before the first application, submit to the Architect a schedule of values of the various parts of the work, including quantities, aggregating the total sum of the contract, divided so as to facilitate payments to subcontractors in accordance with Art. 37 (e), made out in such form as the Architect and the Contractor may agree upon, and, if required, supported by such evidence as to its correctness as the Architect may direct. This schedule, when approved by the Architect, shall be used as a basis for certificates of payment, unless it be found to be in error. In applying for payments, the Contractor shall submit a statement based upon this schedule, and, if required, itemized in such form and supported by such evidence as the Architect may direct, showing his right to the payment claimed.

If payments are made on account of materials delivered and suitably stored at the site but not incorporated in the work, they shall, if required by the Architect, be conditional upon submission by the Contractor of bills of sale or such other procedure as will establish the Owner's title to such material or otherwise adequately protect the Owner's interest.

Art. 25. Certificates of Payments. If the Contractor has made application as above, the Architect shall, not later than the date when each payment falls due, issue to the Contractor a certificate for such amount as he decides to be properly due.

No certificate issued nor payment made to the Contractor, nor partial or entire use or occupancy of the work by the Owner, shall be an acceptance of any work or materials not in accordance with this contract The making and acceptance of the final payment shall constitute a waiver of all claims by the Owner, other than those arising from unsettled liens, from faulty work appearing after final payment or from requirement of the specifications, and of all claims by the Contractor, except those previously made and still unsettled.

Should the Owner fail to pay the sum named in any certificate of the Architect or in any award by arbitration, upon demand when due, the Contractor shall receive, in addition to the sum named in the certificate, interest thereon at the legal rate in force at the place of building.

Art. 26. Payments Withheld. The Architect may withhold or, on account of subsequently discovered evidence, nullify the whole or a part of any certificate to such extent as may be necessary to protect the Owner from loss on account of:

(a) Defective work not remedied.

(b) Claims filed or reasonable evidence indicating probable filing of claims.

(c) Failure of the Contractor to make payments properly to subcontractors or for material or labor.

(d) A reasonable doubt that the contract can be completed for the balance then unpaid.

(e) Damage to another Contractor.

When the above grounds are removed payment shall be made for amounts withheld because of them.

Art. 27. Contractor's Liability Insurance. The Contractor shall maintain such insurance as will protect him from claims under workmen's compensation acts and from any other claims for damages for personal injury, including death, which may arise from operations under this Contract, whether such operations be by himself or by any subcontractor or anyone directly or indirectly employed by either of them. Certificates of such insurance shall be filed with the Owner, if he so require, and shall be subject to his approval for adequacy of protection.

Art. 28. Owner's Liability Insurance. The Owner shall be responsible for and at his option may maintain such insurance as will protect him from his contingent liability for damages for personal injury, including death, which may arise from operations under this contract.

Art. 29. Fire Insurance. The Owner shall effect and maintain fire insurance upon the entire structure on which the work of this contract is to be done and upon all materials, in or adjacent thereto and intended for use thereon, to at least 80% of the insurable value thereof The loss, if any, is to be made adjustable with and payable to the Owner as Trustee for whom it may concern.

All policies shall be open to inspection by the Contractor If the Owner fails to show them on request, or if he fails to effect or maintain insurance as above, the Contractor may insure his own interest and charge the cost thereof to the Owner. If the Contractor is damaged by failure of the Owner to maintain such insurance, he may recover as stipulated in the contract for recovery of damages.

If required in writing by any party in interest, the Owner as Trustee shall, upon the occurrence of loss, give bond for the proper performance of his duties. He shall deposit any money received from insurance in an account separate from all his other funds and he shall distribute it in accordance with such agreement as the parties in interest may reach, or under an award of arbitrators appointed, one by the Owner, another by joint action of the other parties in interest, all other procedure being as provided elsewhere in the contract for Arbitration. If after loss no special agreement is made, replacement of injured work shall be ordered and executed as provided for changes in the work.

The Trustee shall have power to adjust and settle any loss with the insurers unless one of the Contractors interested shall object in writing within three working days of the occurrence of loss, and thereupon arbitrators shall be chosen as above. The Trustees shall in that case make settlement with the insurers in accordance with the directions of such arbitrators, who shall also, if distribution by arbitration is required, direct such distribution.

Art. 30. Guaranty Bonds. The Owner shall have the right, prior to the signing of the Contract, to require the Contractor to furnish bond covering the faithful performance of the Contract and the payment of all obligations arising thereunder, in such form as the Owner may prescribe and with such sureties as he may approve. If such bond is required by instructions given previous to the submission of bids, the premium shall be paid by the Contractor; if subsequent thereto, it shall be paid by the Owner.

Art. 31. Damages. If either party to this Contract should suffer damage in any manner because of any wrongful act or neglect of the other party or of anyone employed by him, then he shall be reimbursed by the other party for such damage.

Claims under this clause shall be made in writing to the party liable within a reasonable time at the first observance of such damage and not later than the time of final payment except as expressly stipulated otherwise in the case of faulty work or materials, and shall be adjusted by agreement or arbitration.

Art. 32. Liens. Neither the final payment nor any part of the retained percentage shall become due until the Contractor, if required, shall deliver to the Owner a complete release of all liens arising out of this Contract, or receipts in full in lieu thereof and, if required in either case, an affidavit that so far as he has knowledge or information the releases and receipts include all the labor and material for which a lien could be filed, but the Contractor may, if any subcontractor refuses to furnish a release or receipt in full. furnish a bond satisfactory to the Owner, to indemnify him against any lien. If any lien remain unsatisfied after all payments are made, the Contractor shall refund to the Owner all moneys that the latter may be compelled to pay in discharging such a lien, including all costs and a reasonable attorney's fee

Art. 33. Assignment. Neither party to the Contract shall assign the Contract or sublet it as a whole without the written consent of the other, nor shall the Contractor assign any moneys due or to become due to him hereunder, without the previous written consent of the Owner.

Art. 34. Mutual Responsibility of Contractors. Should the Contractor cause damage to any other contractor on the work the Contractor agrees, upon due notice, to settle with such contractor by agreement or arbitration, if he will so settle. If such other contractor sues the Owner on account of any damage alleged to have been so sustained, the Owner shall notify the Contractor, who shall defend such proceedings at the Owner's expense and, if any judgment against the Owner arise therefrom, the Contractor shall pay or satisfy it and pay all costs incurred by the Owner.

Art. 35. Separate Contracts. The Owner reserves the right to let other contracts in connection with this work. The Contractor shall afford other contractors reasonable opportunity for the introduction and storage of their materials and the execution of their work, and shall properly connect and coordinate his work with theirs

If any part of the Contractor's work depends for proper execution or results upon the work of any other contractor, the Contractor shall inspect and promptly report to the Architect any defects in such work that render it unsuitable for such proper execution and results. His failure so to inspect and report shall constitute an acceptance of the other contractor's work as fit and proper for the reception of his work, except as to defects which may develop in the other contractor's work after the execution of his work.

To insure the proper execution of his subsequent work the Contractor shall measure work already in place and shall at once report to the Architect any discrepancy between the executed work and the drawings

Art. 36. Subcontracts. The Contractor shall, as soon as practicable after the signature of the contract, notify the Architect in writing of the names of subcontractors proposed for the principal parts of the work and for such others as the Architect may direct and shall not employ any that the Architect may within a reasonable time object to as incompetent or unfit.

If the Contractor has submitted before signing the contract a list of subcontractors and the change of any name on such list is required in writing by the Owner after signature of agreement, the contract price shall be increased or diminished by the difference in cost occasioned by such change.

The Architect shall, on request, furnish to any subcontractor, wherever practicable, evidence of the amounts certified on his account.

The Contractor agrees that he is as fully responsible to the Owner for the acts and omissions of his subcontractors and of persons either directly or indirectly employed by them, as he is for the acts and omissions of persons directly employed by him.

Nothing contained in the contract documents shall create any contractual relation between any subcontractor and the Owner.

Art. 37. Relations of Contractor and Subcontractor. The Contractor agrees to bind every Subcontractor and every Subcontractor agrees to be bound by the terms of the Agreement, the General Conditions, the Drawings and Specifications as far as applicable to his work, including the following provisions of this article, unless specifically noted to the contrary in a subcontract approved in writing as adequate by the Owner or Architect.

This does not apply to minor subcontracts

The Subcontractor agrees:

(a) To be bound to the Contractor by the terms of the Agreement, General Conditions, Drawings and Specifications, and to assume toward him all the obligations and responsibilities that he, by those documents, assumes toward the Owner.

(b) To submit to the Contractor applications for payment in such reasonable time as to enable the Contractor to apply for payment under Art. 24 of the General Conditions

(c) To make all claims for extras, for extensions of time and for damages for delays or otherwise, to the Contractor in the manner provided in the General Conditions for like claims by the Contractor upon the Owner, except that the time for making claims for extra cost is one week.

The Contractor agrees:

(d) To be bound to the Subcontractor by all the obligations that the Owner assumes to the Contractor under the Agreement, General Conditions, Drawings and Specifications, and by all the provisions thereof affording remedies and redress to the Contractor from the Owner

(e) To pay the Subcontractor, upon the issuance of certificates, if issued under the schedule of values described in Art. 24 of the General Conditions, the amount allowed to the Contractor on account of the Subcontractor's work to the extent of the Subcontractor's interest therein.

(f) To pay the Subcontractor, upon the issuance of certificates, if issued otherwise than as in (e), so that at all times his total payments shall be as large in proportion to the value of the work done by him as the total amount certified to the Contractor is to the value of the work done by him.

(g) To pay the Subcontractor to such extent as may be provided by the Contract Documents or the subcontract, if either of these provides for earlier or larger payments than the above.

(h) To pay the Subcontractor on demand for his work or materials as far as executed and fixed in place, less the retained percentage, at the time the certificate should issue, even though the Architect fails to issue it for any cause not the fault of the Subcontractor.

(j) To pay the Subcontractor a just share of any fire insurance money received by him, the Contractor, under Art. 29 of the General Conditions.

(k) To make no demand for liquidated damages or penalty for delay in any sum in excess of such amount as may be specifically named in the subcontract.

(l) That no claim for services rendered or materials furnished by the Contractor to the Subcontractor shall be valid unless written notice thereof is

given by the Contractor to the Subcontractor during the first ten days of the calendar month following that in which the claim originated.

(m) To give the Subcontractor an opportunity to be present and to submit evidence in any arbitration involving his rights.

(n) To name as arbitrator under arbitration proceedings as provided in the General Conditions the person nominated by the Subcontractor, if the sole cause of dispute is the work, materials, rights or responsibilities of the Subcontractor; or, if of the Subcontractor and any other subcontractor jointly, to name as such arbitrator the person upon whom they agree.

The Contractor and the Subcontractor agree that:

(o) In the matter of arbitration, their rights and obligations and all procedure shall be analogous to those set forth in this contract.

Nothing in this article shall create any obligation on the part of the Owner to pay to or to see to the payment of any sums to any Subcontractor

Art. 38. Architect's Status. The Architect shall have general supervision and direction of the work. He is the agent of the Owner only to the extent provided in the Contract Documents and when in special instances he is authorized by the Owner so to act, and in such instances he shall, upon request, show the Contractor written authority. He has authority to stop the work whenever such stoppage may be necessary to insure the proper execution of the Contract.

As the Architect is, in the first instance, the interpreter of the conditions of the Contract and the judge of its performance, he shall side neither with the Owner nor with the Contractor, but shall use his powers under the contract to enforce its faithful performance by both.

In case of the termination of the employment of the Architect, the Owner shall appoint a capable and reputable Architect, whose status under the contract shall be that of the former Architect.

Art. 39. Architect's Decisions. The Architect shall, within a reasonable time, make decisions on all claims of the Owner or Contractor and on all other matters relating to the execution and progress of the work or the interpretation of the Contract Documents.

The Architect's decisions, in matters relating to artistic effect, shall be final, if within the terms of the Contract Documents.

Except as above or as otherwise expressly provided in the Contract Documents, all the Architect's decisions are subject to arbitration.

Art. 40. Arbitration. All questions subject to arbitration under this Contract shall be submitted to arbitration at the choice of either party to the dispute.

The Contractor shall not cause a delay of the work during any arbitration proceedings, except by agreement with the Owner.

The demand for arbitration shall be filed in writing with the Architect, in the case of an appeal from his decision, within ten days of its receipt and in any other case within a reasonable time after cause thereof and in no case later than the time of final payment, except as otherwise expressly stipulated in the Contract. If the Architect fails to make a decision within a reasonable time, an appeal to arbitration may be taken as if his decision had been rendered against the party appealing.

No one shall be nominated or act as an arbitrator who is in any way financially interested in this Contract or in the business affairs of either the Owner, Contractor or Architect.

Unless otherwise provided by controlling statutes, the parties may agree

upon one arbitrator; otherwise there shall be three, one named in writing, by each party to this Contract, to the other party and to the Architect and the third chosen by these two arbitrators, or if they fail to select a third within fifteen days, then he shall be chosen by the presiding officer of the Bar Association nearest to the location of the work. Should the party demanding arbitration fail to name an arbitrator within ten days of his demand, his right to arbitration shall lapse. Should the other party fail to choose an arbitrator within said ten days, then such presiding officer shall appoint such arbitrator. Should either party refuse or neglect to supply the arbitrators with any papers or information demanded in writing, the arbitrators are empowered by both parties to proceed ex parte.

If there be one arbitrator his decision shall be binding; if three the decision of any two shall be binding. Such decision shall be a condition precedent to any right of legal action, and wherever permitted by law it may be filed in Court to carry it into effect.

The arbitrators, if they deem that the case demands it, are authorized to award to the party whose contention is sustained such sums as they shall deem proper for the time, expense and trouble incident to the appeal and, if the appeal was taken without reasonable cause, damages for delay. The arbitrators shall fix their own compensation, unless otherwise provided by agreement, and shall assess the costs and charges of the arbitration upon either or both parties.

The award of the arbitrators shall be in writing and it shall not be open to objection on account of the form of the proceeding or the award, unless otherwise provided by the controlling statutes

In the event of such statutes providing on any matter covered by this article otherwise than as hereinbefore specified, the method of procedure throughout and the legal effect of the award shall be wholly in accordance with the said statutes, it being intended hereby to lay down a principle of action to be followed, leaving its local application to be adapted to the legal requirements of the jurisdiction having authority over the arbitration.

Art. 41. Cash Allowances. The Contractor shall include in the contract sum of all allowances named in the Contract Documents and shall cause the work so covered to be done by such contractors and for such sums as the Architect may direct, the contract sum being adjusted in conformity therewith. The Contractor declares that the contract sum includes such sums for expenses and profit on account of cash allowances as he deems proper. No demand for expenses or profit other than those included in the contract sum shall be allowed. The Contractor shall not be required to employ for any such work persons against whom he has a reasonable objection.

Art. 42. Uses of Premises. The Contractor shall confine his apparatus, the storage of materials and the operations of his workmen to limits indicated by law, ordinances, permits or directions of the Architect and shall not unreasonably encumber the premises with his materials.

The Contractor shall not load or permit any part of the structure to be loaded with a weight that will endanger its safety.

The Contractor shall enforce the Architect's instructions regarding signs, advertisements, fires and smoking.

Art. 43. Cutting, Patching and Digging. The Contractor shall do all cutting, fitting or patching of his work that may be required to make its several parts come together properly and fit it to receive or be received by work of other contractors shown upon, or reasonably implied by, the Drawings and

Specifications for the completed structure, and he shall make good after them as the Architect may direct.

Any cost caused by defective or ill-timed work shall be borne by the party responsible therefor.

The Contractor shall not endanger any work by cutting, digging or otherwise, and shall not cut or alter the work of any other contractor save with the consent of the Architect.

Art. 44. Cleaning Up. The Contractor shall at all times keep the premises free from accumulations of waste material or rubbish caused by his employes or work, and at the completion of the work he shall remove all his rubbish from and about the building and all his tools, scaffolding and surplus materials and shall leave his work "broom clean" or its equivalent, unless more exactly specified. In case of dispute the Owner may remove the rubbish and charge the cost to the several contractors as the Architect shall determine to be just.

4. THE STANDARD FORM OF AGREEMENT BETWEEN CONTRACTOR AND OWNER FOR CONSTRUCTION OF BUILDINGS*

ISSUED BY THE AMERICAN INSTITUTE OF ARCHITECTS FOR USE WHEN A STIPULATED SUM FORMS THE BASIS OF PAYMENT

This form of agreement has received the approval of the National Association of Builders' Exchanges, the Associated General Contractors of America, the Joint Conference on Construction Contracts, the National Association of Master Plumbers, the National Association of Sheet Metal Contractors of the United States, the National Electrical Contractors' Association of the United States, the National Association of Marble Dealers, the Building Granite Quarries Association, the Producers' Council and the Building Trades Employers' Association of the City of New York.

 THIS FORM IS TO BE USED ONLY WITH THE STANDARD GENERAL CONDITIONS OF THE CONTRACT FOR CONSTRUCTION OF BUILDINGS

THIS AGREEMENT, made the.......................... ....day of
. . . . in the year Nineteen Hundred and ....
by and between . . ..
hereinafter called the Contractor, and ....
. . . .hereinafter called the Owner, ...

WITNESSETH, that the Contractor and the Owner for the considerations hereinafter named agree as follows:

Article 1. Scope of the Work. The Contractor shall furnish all of the materials and perform all of the work shown on the Drawings and described in the Specifications entitled........................ . . . .

(Here insert the caption descriptive of the work as used on the Drawings and in the other Contract Documents)

prepared by. ....
acting as and in these Contract Documents entitled the Architect; and shall do everything required by this Agreement, the General Conditions of the Contract, the Specifications and the Drawings.

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Article 2. Time of Completion. The work to be performed under this Contract shall be commenced ....
and shall be substantially completed.........·.. . ...
(Here insert stipulation as to liquidated damages, if any.)
....(Blank lines)............ . ...

Article 3. The Contract Sum. The Owner shall pay the Contractor for the performance of the Contract, subject to additions and deductions provided therein, in current funds as follows:...... ... .
(State here the lump sum amount, unit prices, or both, as desired in individual cases.)
.... (Blank lines) ....

Where the quantities originally contemplated are so changed that application of the agreed unit price to the quantity of work performed is shown to create a hardship to the Owner or the Contractor, there shall be an equitable adjustment of the Contract to prevent such hardship.

Article 4. Progress Payments. The Owner shall make payments on account of the Contract as provided therein, as follows:
On or about the....................day of each month.. ....
per cent of the value, based on the Contract prices, of labor and materials incorporated in the work and of materials suitably stored at the site thereof up to the . ...day of that month, as estimated by the Architect, less the aggregate of previous payments; and upon substantial completion of the entire work, a sum sufficient to increase the total payments to .. .
per cent of the contract price . . . .
(Insert here any provision made for limiting or reducing the amount retained after the work reaches a certain stage of completion)
. . . .(Blank lines).............. .. ..

Article 5. Acceptance and Final Payment. Final payment shall be due..... days after substantial completion of the work provided the work be then fully completed and the Contract fully performed.

Upon receipt of written notice that the work is ready for final inspection and acceptance, the Architect shall promptly make such inspection, and when he finds the work acceptable under the Contract and the Contract fully performed he shall promptly issue a final certificate, over his own signature, stating that the work provided for in this Contract has been completed and is accepted by him under the terms and conditions thereof, and that the entire balance found to be due the Contractor, and noted in said final certificate, is due and payable.

Before issuance of final certificate the Contractor shall submit evidence satisfactory to the Architect that all payrolls, material bills, and other indebtedness connected with the work have been paid.

If after the work has been substantially completed, full completion thereof is materially delayed through no fault of the Contractor, and the Architect so certifies, the Owner shall, upon certificate of the Architect, and without terminating the Contract, make payment of the balance due for that portion of the work fully completed and accepted. Such payment shall be made under the terms and conditions governing final payment, except that it shall not constitute a waiver of claims.

Article 6. The Contract Documents. The General Conditions of the Contract, the Specifications and the Drawings, together with this Agreement, form the Contract, and they are as fully a part of the Contract as if hereto attached or herein repeated. The following is an enumeration of the Specifications and Drawings:
............................(Blank page)...............................

IN WITNESS WHEREOF the parties hereto have executed this Agreement, the day and year first above written.

5. ARCHITECTURAL COMPETITIONS

Object. The object of a competition is to enable the Owner to secure a choice of designs by different authors, and the hope of those holding a competition is that by appealing to the competitive spirit of the architects they will be inspired to produce designs of outstanding merit.

In order that the competitors may be encouraged to produce the best of which they are capable, there should be a fair agreement between the Owner and the competitors.

All competitions should be conducted in general conformity with the principles set forth in "Competition Code" of the American Institute of Architects. The Institute does not give its approval to a program unless it meets the following essential conditions:

(a) That there be a Professional Adviser.

(b) That there be a Jury of at least three members, one of whom is a practicing Architect.

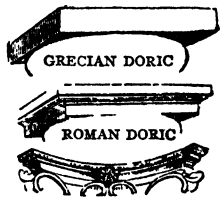
(c) That the program contain an Agreement and Conditions of Contract between Owner and Architect in accord with good practice.

GLOSSARY *

Technical Terms, Ancient and Modern, Used by Architects, Builders, and Draughtsmen

Aaron's-Rod. An ornamental figure representing a rod with a serpent twined about it. It is sometimes confounded with the caduceus of Mercury. The distinction between the caduceus and the Aaron's-rod is that the former has two serpents twined in opposite directions, whereas the latter has but one.

Abacus. The upper member of the capital of a column. It is sometimes square and sometimes curved, forming on the plan segments of a circle called the arch of the abacus, and is commonly decorated with a rose or other ornament in the center, having the angles, called horns of the abacus, cut off in the direction of the radius or curve. In the Tuscan or Doric, it is a square tablet; in the Ionic, the edges are molded; in the Corinthian, its sides are concave and frequently enriched with carving. In Gothic pillars it has a great variety of forms.



CORINTHIAN ABACUS

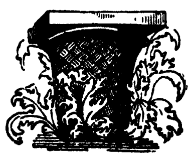
Abbey. A term for the church and other buildings used by conventual bodies presided over by an abbot or abbess, in contradistinction to cathedral, which is presided over by a bishop; and priory, the head of which was a prior or prioress.

Abutment. That part of a pier from which the arch springs.

Abuttals. The boundings of a piece of land on other land, street, river, etc.

Abyssinia, Architecture of. That of the ancient kingdom in northeast Africa. Evidences of Greek and, perhaps, of Egyptian culture have been found, but there seem to be only the remains of monuments erected by conquering chiefs. Portuguese influence in the fifteenth and sixteenth centuries has left some trace. It is evident that curious systems and devices, both in building and in ornamentation, are discoverable in this large, mountainous, and diversified country, containing a very ancient, though newer, high civilization.

Acanthus. A plant found in the south of Europe, representations of whose leaves are employed for decorating the Corinthian and Composite capitals. The leaves of the acanthus are used on the bell of the capital, and distinguish the two rich orders from the three others.


ACANTHUS

Acroteria. The small pedestals placed on the extremities and apex of a pediment. They are usually without bases or plinths, and were originally intended to receive statues.

Adobe. (1) An aluminous earth; (2) unfired brick made from such earth without fire; (3) with the article, a single brick of this kind; (4) a house built of these materials.

* This Glossary was compiled by Mr. Kidder from various sources. Excerpts, principally from "A Dictionary of Architecture and Building," by Russell C. Sturgis, have been added.

Aile, Aisle. The wings; inward side porticos of a church; the inward lateral corridors which enclose the choir, the presbytery, and the body of the church along its sides. Any one of the passages in a church or hall into which the pews or seats open.

Alcove. The original and strict meaning of this word, which is derived from the Spanish *alcoba*, is confined to that part of a bed-chamber in which the bed stands, separated from the other parts of the room by columns or pilasters. It is now commonly used to express any large recess in a room, generally separated by an arch.

Alipterion. In ancient Roman architecture, a room used by bathers for anointing themselves.

Alley. A narrow passageway: (1) between two houses, like a very narrow street; (2) in or under a house, as affording passage directly to the inner court or yard, without entering the rooms of the house; (3) a walk in a garden; (4) a long and narrow building.

Almonry. The place or chamber where alms were distributed to the poor in churches, or other ecclesiastical buildings. At Bishopstone Church, Wiltshire, England, it is a sort of covered porch attached to the south transept, but not communicating with the interior of the church. At Worcester Cathedral, England, the alms are said to have been distributed on stone tables, on each side, within the great porch. In large monastic establishments, as at Westminster, it seems to have been a separate building of some importance, either joining the gate-house or near it, that the establishment might be disturbed as little as possible.

Altar. In ancient Roman architecture, a place on which offerings or sacrifices were made to the gods. In Protestant churches, the communion table is often designated as the Altar, and in Roman Catholic churches it is a square table placed at the east end of the church for the celebration of mass.

Altar of Incense. A small table covered with plates of gold on which was placed the smoking censer in the temple at Jerusalem.

Altar-piece. The entire decorations of an altar; a painting placed behind an altar.

Altar-screen. The back of the altar from which the canopy was suspended, and separating the choir from the lady-chapel and presbytery. The altar-screen was generally of stone, and composed of the richest tabernacle work of niches, finials, and pedestals, supporting statues of the tutelary saints.

Alto-rilievo. High relief. A sculpture, the figures of which project from the surface on which they are carved.

Ambo. A raised platform, a pulpit, a reading-desk, a marble pulpit—an oblong enclosure in ancient churches, resembling in its uses and positions the modern choir.

Ambry. A cupboard or closet, frequently found near the altar in ancient churches to hold sacred utensils.

Ambulatory. An alley, a gallery, a cloister, a place for walking, as in a church or monastery.

Amerind Architecture; Amerindian Architecture. The architecture of the red races of America, the terms Amerind and Amerindian having been adopted by archæologists to replace the obviously inaccurate phrase "American Indian."

Amphiprostylos. A Grecian temple which has a columned portico on both ends.

Amphitheater. A double theater, of an elliptical form on the plan, for the exhibition of the ancient gladiatorial fights and other shows. Its arena or pit, in which those exhibitions took place, was encompassed with seats rising above each other, and the exterior had the accommodation of porticos or arcades for the public.

Amphora. A Grecian vase with two handles, often seen on medals.

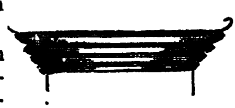
Ancones. The consoles or ornaments cut on the key-stones of arches or on the sides of door-cases. They are sometimes made use of to support busts or other figures.

Angle-capital. In Greek architecture, those Ionic capitals placed on the flank columns of a portico, which have one of their volutes placed horizontally at an angle of a hundred and thirty-five degrees with the plane of the frieze.

Annulated Columns. Columns clustered together by rings or bands; much used in English architecture.

Annular Vault. A vault rising from two parallel walls—the vault of a corridor. Same as *Barrel Vault.*

Annulet. A small square molding used to separate others. The fillet which separates the flutings of columns is sometimes known by this term.



ANNULET

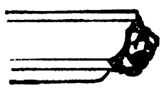
Anta, Antæ. A name given to a pilaster when attached to a wall. Vitruvius calls pilasters *parastatæ* when insulated. They are not usually diminished, and in all Greek examples their capitals are different from those of the columns they accompany.

Antechamber. An apartment preceded by a vestibule and from which another room is approached.

Antechapel. A small chapel forming the entrance to another. There are examples at Merton College, Oxford, and at King's College, Cambridge, England, besides several others. The antechapel to the lady-chapel in cathedrals is generally called the Presbytery.

Antechoir. The part under the rood loft, between the doors of the choir and the outer entrance of the screen, forming a sort of lobby. It is also called the Fore-choir.

Antefixa. In classical architecture (gargoyles, in Gothic architecture), the ornaments of lions' and other heads below the eaves of a temple, through channels in which, usually by the mouth, the water is carried from the eaves. By some this term is applied to the upright ornaments above the eaves in ancient architecture, which hid the ends of the Harmi or joint tiles.



ANTEFIXA

Apophyge. The lowest part of the shaft of an Ionic or Corinthian column, or the highest member of its base if the column be considered as a whole. The Apophyge is the inverted cavetto or concave sweep, on the upper edge of which the diminishing shaft rests.

Apron. A plain or molded piece of finish below the stool of a window, put on to cover the rough edge of the plastering.

Apse. The semicircular or polygonal termination to the chancel of a church. The term also applies to similar forms in other buildings.

Apteral. A temple without columns on the flanks or sides.

Aqueduct. An artificial canal for the conveyance of water, either above or under ground. The Roman aqueducts are mostly of the former construction.

Arabesque. A building after the manner of the Arabs. Ornaments used by the same people, in which no human or animal figures appear. Arabesque is sometimes improperly used to denote a species of ornaments composed of capricious fantastics and imaginary representations of animals and foliage so much employed by the Romans in the decorations of walls and ceilings.

ARABESQUE

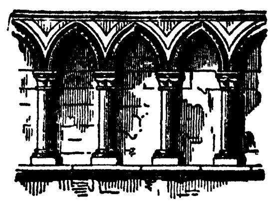
Arabian Architecture. A style of architecture the rudiments of which appear to have been taken from surrounding nations, the Egyptians, Syrians, Chaldeans, and Persians. The best preserved specimens partake chiefly of the Græco-Roman, Byzantine, and Egyptian. It is supposed that they constructed many of their finest buildings from the ruins of ancient cities

Aræostyle. That style of building in which the columns are distant from one another from four to five diameters. Strictly speaking, the term should be limited to intercolumniation of four diameters, which is only suited to the Tuscan order.

Aræosystylos. That style of building in which four columns are used in the space of eight diameters and a half; the central intercolumniation being three diameters and a half, and the others on each side being only half a diameter, by which arrangement coupled columns are introduced.

Arbores. Large bronze candelabra, in the shape of a tree, placed on the floor of ancient churches, so as to appear as if growing out of it.

Arcade. A range of arches, supported either on columns or on piers, and detached or attached to the wall.



ARCADE

Arch. In building, a mechanical arrangement of building materials arranged in the form of a curve, which preserves a given form when resisting pressure, and enables them, supported by piers or abutments, to carry weights and resist pressure.

Arch-buttress. Sometimes called a flying buttress; an arch springing from a buttress or pier.

Architect. A man charged with the preparation for a building, its plan and design, and with the supervision of the work of the builders who actually put it up and finish it. Literally, the term signifies nothing more than the head workman or director of the workmen, and the exact duties of the architect vary with the epoch, the country, the kind of building, and the wishes of the owner.

Architecture. The art of building in such a way as to accord with principles determined, not merely by the ends the edifice is intended to serve, but by high consideration of beauty and harmony. The aim of architecture as an art, is so to arrange the plan, masses and enrichments of a structure as to impart to it interest, beauty, grandeur, unity and power.

Architrave. That part of an entablature which rests upon the capital of a column, and is beneath the frieze.

Architrave Cornice. An entablature consisting of an architrave and cornice, without the intervention of the frieze, sometimes introduced when inconvenient to give the entablature the usual height.

Architrave of a Door. The finished work surrounding the aperture; the upper part of the lintel is called the traverse; and the sides, the jambs.

Archives. A repository or closet for the preservation of writings or records,

Archivolt. A collection of members forming the inner contour of an arch, or a band or frame adorned with moldings running over the faces or the arch-stones, and bearing upon the imposts.

Area. The superficial contents of any figure; an open space or court within a building; also, an uncovered space surrounding the foundation walls to give light to the basement.

Arena. The plain space in the middle of the amphitheater or other place of public resort.

Armory. (1) A room or building serving as a depository for arms and armor. (2) A building or establishment for the manufacture of arms, especially one maintained by the government. (3) A building for the use of a body of militia, with storage for their arms and equipments.

Arris. The meeting of two surfaces producing an angle.

Arsenal. (1) A public storehouse for arms and ammunition. (2) A place supported by government for manufacture of weapons. (3) Anciently, a government dock yard and navy yard.

Artificer, or **Artisan.** A person who works with his hands, and manufactures any commodity in iron, brass, wood, etc

Asarotum. In ancient architecture, a species of painted pavement used by the Romans before the invention of mosaic work.

Ashlar, or **Ashler.** A facing made of squared stones, or a facing made of thin slabs, used to cover walls of brick or rubble. *Coursed ashlar* is where the stones run in level courses all around the building; *random ashlar,* where the stones are of different heights, but level beds. Common freestones of small size, as they come from the quarry, are also called ashlar.

Asia Minor, Architecture of. Although Asia Minor formed a connecting passageway between the East and the West through which ebbed and flowed the art civilization from one to another, but little remains are found there from earlier than the fifth century B C. With few exceptions architectural developments are located on the seaboard. Here are found examples of Greek temples, Roman theaters, colonnaded streets, and tombs. The remains of the Byzantine Empire are comparatively few. The last architectural phase developed toward the end of the twelfth century under the Seljuk Sultans whose chief interest lies in the fact that it forms the foundation of the Turkish Saracenic style. The conical roofs with which all the Seljukian tombs are covered, in contradistinction to the domes which surmount all Persian and Cairene tombs, are probably derived from Armenian and Georgian prototypes.

Astragal. A small semicircular molding, sometimes plain and sometimes ornamented.

Asymptote. A straight line which continually approaches to a curve without touching it.

Atlases, or Atlantes. Figures or half-figures of men, used instead of columns or pilasters to support an entablature; called also Telamones.

Atrium. A court in the interior division of Roman houses.

Attached Columns. Those which project three-fourths of their diameter from the wall.

Attic. A low story above an entablature, or above a cornice which limits the height of the main part of an elevation. Although the term is evidently derived from the Greek, we find nothing exactly answering to it in Greek architecture; but it is very common in both Roman and Italian practice. What are otherwise called tholobates, in St. Peter's and St. Paul's Cathedrals are frequently termed Attics.

ATLANTES

Attic Order. A term used to denote the low pilasters employed in the decoration of an attic story.

Attributes. In painting and sculpture, symbols given to figures and statues to indicate their office and character.

Auditory. In ancient churches, that part of the church where the people usually stood to be instructed in the Gospel, now called the nave.

Aula. A court or hall in ancient Roman houses.

Australia, Architecture of. The natives of Australia were few in number when the land was first explored by Europeans, and were of a low type of savagery. The settlers of European stock have built cities generally laid out on generous lines for future development. The buildings are mostly close copies of modern English architecture of the more conventional and less interesting type. The more important buildings are of the neoclassic style, and cathedrals and university groups are artistic structures.

Austrian States, Architecture of. This presents fewer examples of mediæval architecture, especially churches, than any other German lands. Nature seems to have conspired against Austrian architecture; scarcely a building has escaped earthquake, tempest, or fire. The Cathedral at Vienna represents a valuable example of the beautiful Danubian architecture of the thirteenth century, also the remaining portions of the abbeys.

Aviary. A large apartment for breeding birds.

Axis. The spindle or center of any rotative motion. In a sphere, an imaginary line through the center.

Aztec Architecture. The architecture of the Nahuatl or Aztec tribes of Mexico. Popularly applied to all pre-Columbian Mexican architecture.

Back-choir. A place behind the altar in the principal choir, in which there is, or was, a small altar standing back to back with the former.

Backing of a Rafter or Rib. The forming of an upper or outer surface that it may range with the edges of the ribs or rafters on either side.

Backing of a Wall. The rough inner face of a wall; earth deposited behind a retaining wall, etc.

Back of a Window. That piece of wainscoting which is between the bottom of the sash frame and the floor.

Balcony. A projection from the face of a wall, supported by columns or consoles, and usually surrounded by a balustrade.

Baldachin. A building in the form of a canopy, supported with columns, and serving as a crown or covering to an altar.

Balloon Framing. In the United States a system of framing wooden buildings in which the corner posts and studs are continuous in one piece from sill to roof plate, the intermediate joists being carried by girts spiked to, or let into, the studs, the pieces being secured only by nailing, without the use of mortises and tenons, or the like.

BALDACHIN

Baluster. A small pillar or column, supporting a rail, of various forms, used in balustrades.

Baluster Shaft. The shaft dividing a window in Saxon architecture. At St. Albans are some of these shafts, evidently out of the old Saxon church, which have been fixed up with Norman capitals.

Balustrade. A series of balusters connected by a rail.

Band. A sort of flat frieze or fascia running horizontally round a tower or other parts of a building, particularly the base tables in perpendicular work, commonly used with the long shafts characteristic of the thirteenth century. It generally has a bold, projecting molding above and below, and is carved sometimes with foliages, but in general with cusped circles, or quatrefoils, in which frequently are shields of arms.

Band of a Column. A series of annulets and hollows going round the middle of the shafts of columns, and sometimes of the entire pier. They are often beautifully carved with foliages, etc., as at Amiens. In several cathedrals there are rings of bronze apparently covering the junction of the frusta of the columns. At Worcester and Westminster they appear to have been gilt; they are there more properly called Shaft-rings.

Baptistery. A separate building to contain the font, for the rite of baptism. Baptisteries are frequent on the Continent; that at Rome, near St. John Lateran, and those at Florence, Pisa, Pavia, etc., are all well-known examples. The only examples in England are at Cranbrook and Canterbury; the latter, however, is supposed to have been originally part of the treasury.

Barbican. An outwork for the defence of a gate or drawbridge; also, a sort of pent-house or construction of timber to shelter warders or sentries from arrows or other missiles.

Barge Board. See *Verge Board*.

Bartizan. A small turret, corbeled out at the angle of a wall or tower, to protect a warder and enable him to see around him. It generally is furnished with oylets or arrow-slits.

BARTIZAN

Basement. The lower part of a building, usually in part below the grade of the lot or street.

Base Moldings. The moldings immediately above the plinth of a wall, pillar, or pedestal.

Base of a Column. That part which is between the shaft and the pedestal or, if there be no pedestal, between the shaft and the plinth. The Grecian Doric had no base, and the Tuscan has only a single torus, or a plinth.

Basilica. A term given by the Greeks and Romans to the public buildings devoted to judicial purposes.

Bas-relief. See *Basso-rilievo.*

Basse-cour. A court separated from the principal one, and destined for stables, etc.

Basso-rilievo, or Bas-relief. The representations of figures projected from a background without being detached from it. It is divided into three parts: Alto-rilievo, when the figure projects more than one-half; Mezzo-rilievo, that in which the figure projects one-half; and Basso-rilievo, when the projection of the figure is less than one-half, as in coins.

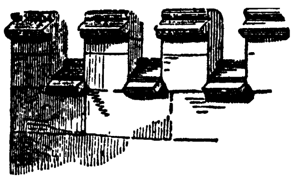
Bat. A part of a brick.

Batten. Small scantlings, or small strips of boards, used for various purposes. Small strips put over the joints of sheathing to keep out the weather.

Batten Door. A door made of sheathing, secured by strips of board, put crossways, and nailed with clinched nails.

Batter. A term used by bricklayers, carpenters, etc., to signify a wall, piece of timber, or other material, which does not stand upright, but inclines from you when you stand before it; but when, on the contrary, it leans toward you, it is said to overhang.

Battlement. A parapet with a series of notches in it, from which arrows may be shot, or other instruments of defence hurled on besiegers. The raised portions are called merlons; and the notches, embrasures or crenelles. The former were intended to cover the soldier while discharging his weapon through the latter. Their use is of great antiquity; they are found in the sculptures of Nineveh, in the tombs of Egypt, and on the famous François vase, where there is a delineation of the siege of Troy. In ecclesiastical architecture the early battlements have small shallow embrasures at some distance apart. In the Decorated period they are closer together, and deeper, and the moldings on the top of the merlon and bottom of the embrasure are richer. During this period, and the early part of the Perpendicular, the sides or cheeks of the embrasures are perfectly square and plain. In later times the moldings were continued round the sides, as well as at top and bottom, mitring at the angles, as over the doorway of Magdalen College, Oxford, England. The battlements of the Decorated and later periods are often richly ornamented by paneling, as in the last example. In castellated work the merlons are often pierced by narrow arrow-slits. (See *Oylet.*) In South Italy some battlements are found strongly resembling those of old Rome and Pompeii; in the Continental ecclesiastical architecture, the parapets are very rarely embattled.



BATTLEMENT

Bay. Any division or compartment of an arcade, roof, etc. Thus each space, from pillar to pillar, in a cathedral, is called a bay, or severy.

Bay-window. Any window projecting outward from the wall of a building, either square or polygonal on plan, and commencing from the ground. If they are carried on projecting corbels, they are called Oriel windows. Their use seems to have been confined to the later periods. In the Tudor and Elizabethan styles they are often semicircular in plan, in which case some think it more correct to call them Bow Windows.

Bazaar. A kind of Eastern mart, of Arabic origin.

Bead. A circular molding. When several are joined, it is called Reeding; when flush with the surface, it is called Quirk-bead; and when raised, Cock-bead.

Beam. A piece of timber, iron, stone, or other material, placed horizontally, or nearly so, to support a load over an opening, or from post to post.

Bearing. The portion of a beam, truss, etc., that rests on the supports.

Bearing Wall, or **Partition.** A wall which supports the floors and roofs in a building.

Beaufet, or **Buffet.** A small cupboard, or cabinet, to contain china. It may either be built into a wall, or be a separate piece of furniture.

Bed. In bricklaying and masonry, the horizontal surfaces on which the stones or bricks of walls lie in courses.

Bed of a Slate. The lower side.

Bed Moldings. Those moldings in all the orders between the corona and frieze.

Belfry. Properly speaking, a detached tower or campanile containing bells, as at Evesham, England, but more generally applied to the ringing-room or loft of the tower of a church. See *Tower*.

Belgium, Architecture of. The Architecture of Belgium is essentially Germanic in spirit, the style of detail, the disposition and character of buildings of all the periods showing the Teutonic origin of the people, though much of the mediæval work is inspired directly from France, the Renaissance reflects the influence of the Spanish domination and the modern buildings are almost wholly French by derivation. The most characteristic phase of the national architecture is that afforded by its civic buildings, which are in many respects unrivaled anywhere in the world. The Renaissance organ fronts also deserve study, notably in Louvain, Liège, and Brussels. In dignity of composition and masterly handling of mass the Brussels Palais de Justice is perhaps the most striking modern building of Europe.

Bell-cot, Bell-gable, or **Bell-turret.** The place where one or more bells are hung in chapels, or small churches which have no towers. Bell-cots are sometimes double, as at Northborough and Coxwell, England; a very common form in France and Switzerland admits of three bells. In these countries, also, they are frequently of wood, and attached to the ridge. Those which stand on the gable, dividing the nave from the chancel, are generally called Sanctus Bells. A very curious and, it is believed, unique example at Cleves Abbey, England, juts out from the wall. In later times bell-turrets were much ornamented; these are often called Flèches.

Bell of a Capital. In Gothic work, immediately above the necking is a deep, hollow curve; this is called the bell of a capital. It is often enriched with foliages. It is also applied to the body of the Corinthian and Composite capitals.

Belt. A course of stones or brick projecting from a brick or stone wall, generally placed in a line with the sills of the windows; it is either molded, fluted, plane, or enriched with patras at regular intervals. Sometimes called Stone String.

Belvedere, or **Look-out.** A turret or lantern raised above the roof of an observatory for the purpose of enjoying a fine prospect.

Bema. The semicircular recess, or exedra, in the basilica, where the judges sat, and where in aftertimes the altar was placed. It generally is roofed with a half-dome or concha. The seats of the priests were against the wall, looking into the body of the church, that of the bishop being in the center. The bema is generally ascended by steps, and railed off by cancelli.

Bench Table. The stone seat which runs round the walls of large churches, and sometimes round the piers; it very generally is placed in the porches.

Bevel. An instrument for taking angles. One side of a solid body is said to be beveled with respect to another, when the angle contained between those two sides is greater or less than a right angle.

Bezantee. A name given to an ornamental molding much used in the Norman period, resembling bezants, coins struck in Byzantium.

Billet. A species of ornamented molding much used in Norman, and sometimes in Early English work, like short pieces of stick cut off and arranged alternately.

Blocking, or **Blocking-course.** In masonry, a course of stones placed on the top of a cornice crowning the walls.

Bond. In bricklaying and masonry, that connection between bricks or stones formed by lapping them upon one another in carrying up the work, so as to form an inseparable mass of building, by preventing the vertical joints falling over each other. In brickwork there are several kinds of bond. In common brick walls in every sixth or seventh course the bricks are laid crossways of the wall, called Headers. In face work, the back of the face brick is clipped so as to get in a diagonal course of headers behind. In Old English bond, every alternate course is a header course. In Flemish bond, a header and stretcher alternate in each course.

Bond-stones. Stones running through the thickness of the wall at right angles to its face, in order to bind it together.

Bond-timbers. Timbers placed in a horizontal direction in the walls of a brick building in tiers, and to which the battens, laths, etc., are secured. In rubble work, walls are better plugged for this purpose.

Border. Useful ornamental pieces around the edge of anything.

Boss. An ornament, generally carved, forming the key-stone at the intersection of the ribs of a groined vault. Early Norman vaults have no bosses. The carving is generally foliage, and resembles that of the period in capitals, etc. Sometimes they have human heads, as at Notre Dame at Paris, and sometimes grotesque figures. In Later Gothic vaulting there are bosses at every intersection.

Boutell. The mediæval term for a round molding, or torus. When it follows a curve, as round a bench end, it is called a Roving Boutell.

Bow. Any projecting part of a building in the form of an arc of a circle. A bow, however, is sometimes polygonal.

Bow Window. A window placed in the bow of a building.

Brace. In carpentry, an inclined piece of timber, used in trussed partitions, or in framed roofs, in order to form a triangle, and thereby stiffen the framing. When a brace is used by way of support to a rafter, it is called a strut. Braces in partitions and span-roofs are, or always should be, disposed in pairs, and introduced in opposite directions.

Brace Mold. [{] Two ressaunts or ogees united together like a brace in printing, sometimes with a small bead between them.

Bracket. A projecting ornament carrying a cornice. Those which support vaulting shafts or cross-springers of a roof are more generally called Corbels.

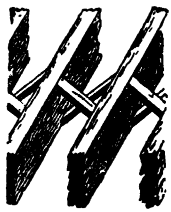
Break. Any projection from the general surface of a building.

Breaking Joint. The arrangement of stones or bricks so as not to allow two joints to come immediately over each other. See *Bond.*

Breast of a Window. The masonry forming the back of the recess and the parapet under the window-sill.

Bressummer. A lintel, beam, or iron tie, intended to carry an external wall and itself supported by piers or posts; used principally over shop windows. This term is now seldom used, the word *beam,* or *girder,* taking its place.

Bridging. A method of stiffening floor joist and partition studs, by cutting pieces in between. Cross-bridging of floor joist is illustrated in cut.


CROSS-BRIDGING

Bulwark. In ancient fortification, nearly the same as Bastion in modern.

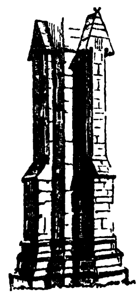
Burse, or **Bourse.** A public edifice for the assembly of merchant traders; an exchange.

Bust. In sculpture, that portion of the human figure which comprises the head, neck, and shoulders.

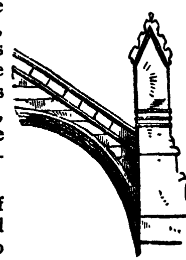
Buttery. A store-room for provisions.

Butt-joint. Where the ends of two pieces of timber of molding butt together.

Buttress. Masonry projecting from a wall, and intended to strengthen the same against the thrust of a roof or vault. Buttresses are no doubt derived from the classic pilasters which serve to strengthen walls where there is a pressure of a girder or roof-timber. In very early work they have little projection, and, in fact, are " strippilasters." In Norman work they are wider, with very little projection, and generally stop under a cornice or corbel table. Early English buttresses project considerably, sometimes with deep sloping weatherings in several stages, and sometimes with gabled heads. Sometimes they are chamfered, and sometimes the angles have jamb shafts. At Wells and Salisbury, England, they are richly ornamented with canopies and statues. In the Decorated period they became richly paneled in stages, and often finish with niches and statues and elegantly carved and crocketed gablets, as at York, England. In the Perpendicular period the weatherings became waved, and they frequently terminate with niches and pinnacles.


BUTTRESS

Buttress, Flying. A detached buttress or pier of masonry at some distance from a wall, and connected therewith by an arch or portion of an arch, so as to discharge the thrust of a roof or vault on some strong point.


FLYING BUTTRESS

Buttress Shafts. Slender columns at the angle of buttresses, chiefly used in the Early English period.

Byzantine Architecture. A style developed in the Byzantine Empire. The capitals of the pillars are of endless variety and full of invention; some are founded on the Greek Corinthian, some resemble the Norman and the Lombard style, and are so varied that no two sides of the same capital are alike. They are comprised under the style Romanesque, which comprehends the round-arch style. Byzantine architecture reached its height in the Church of St. Sophia at Constantinople.

Cabinet. A highly ornamented kind of buffet or chest of drawers set apart for the preservation of things of value.

Cabling. The flutes of columns are said to be cabled when they are partly occupied by solid convex masses, or appear to be refilled with cylinders after they had been formed.

Caduceus. Mercury's rod, a wand entwined by two serpents and surmounted by two wings. The rod represents power; the serpents, wisdom; and the wings, diligence and activity.

CADUCEUS

Caisson. (1) A panel sunk below the surface in flat or vaulted ceilings. See *Cassoon.* (2) In bridge building, a chest or vessel in which the piers of a bridge are built, gradually sinking as the work advances till its bottom comes in contact with the bed of the river, and then the sides are disengaged, being so constructed as to allow of their being thus detached without injury to its floor or bottom. They are also used for foundations of buildings.

Caliber, or Caliper. The diameter of any round body; the width of the mouth of a piece of ordnance.

Camber. In carpentry, the convexity of a beam upon the surface, in order to prevent its becoming concave by its own weight, or by the burden it may have to sustain.

Campanile. A name given in Italy to the bell-tower of a town-hall or church. In that country this is almost always detached from the latter.

Canada, Architecture of. That of the British possessions to the north and northeast of the United States. The work of the aboriginal tribes included architectural features of great interest. Only the eastern section has developed architecturally.

Candelabrum. Stand or support on which the ancients placed their lamps. Candelabra were made in a variety of shapes and with much taste and elegance. The term is also used to denote a tall ornamental candlestick with several arms, or a bracket with arms for candles.

Canopy. The upper part or cover of a niche, or the projection or ornament over an altar, seat, or tomb. The word is supposed to be derived from conopæum, the gauze covering over a bed to keep off the gnats; a mosquito curtain. Early English canopies are generally simple, with trefoiled or cinque-foiled heads; but in the later styles they are very rich, and divided into compartments with pendants, knots, pinnacles, etc. The triangular arrangement over an Early English and Decorated doorway is often called a canopy. The triangular canopies in the North of Italy are peculiar. Those in England are generally part of the arrangement of the arch moldings of the door, and form, as it were, the hood-molds to them, as at York. The former are above and independent of the door moldings, and frequently support an arch with a tympanum, above which is a triangular canopy, as in the Duomo at Florence. Sometimes the canopy and arch project from the wall, and are carried on small jamb shafts, as at San Pietro Martiro at Verona. Canopies are often used

over windows, as at York Minster over the great west window, and lower ties in the towers. These are triangular, while the upper windows in the towers have ogee canopies.

Capital. The upper part of a column, pilaster, pier, etc. Capitals have been used in every style down to the present time. That mostly used by the Egyptians was bell-shaped, with or without ornaments. The Persians used the double-headed bell, forming a kind of bracket capital. The Assyrians apparently made use of the Ionic and Corinthian, which were developed by the Greeks, Romans, and Italians into their present well-known forms. The Doric was apparently an invention or adaptation by the Greeks, and was altered by the Romans and Italians. But in all these examples, both ancient and modern, the capitals of an order are all of the same form throughout the same building, so that if one be seen the form of all the others is known. The Romanesque architects altered all this, and in the carving of their capitals often introduced such figures and emblems as helped to tell the story of their building. Another form was introduced by them in the curtain capital, rude at first, but afterward highly decorated. It evidently took its origin from the cutting off of the lower angles of a square block, and then rounding them off. The process may be distinctly seen, in its several stages, in Mayence Cathedral. But this form of capital was more fully developed by the Normans, with whom it became a marked feature. In the early English capitals a peculiar flower of three or more lobes was used spreading from the necking upward in most graceful forms. In Decorated and Perpendicular styles this was abandoned in favor of more realistic forms of crumpled leaves, enclosing the bell like a wreath. In each style bold abacus moldings were always used, whether with or without foliage.

Caravansary. A huge, square building, or inn, in the East, for the reception of travelers and lodging of caravans

Carriage. The timber or iron joist which supports the steps of a wooden stair.

Carton, or **Cartoon.** A design made on strong paper, to be transferred on the fresh plaster wall to be afterward painted in fresco; also, a colored design for working in mosaic tapestry.

Cartouche. An ornament which like an escutcheon, a shield or an oval or oblong panel has the central part plain, and usually slightly convex, to receive an inscription, armorial bearings, or an ornamental or significant piece of painting or sculpture. Frequently used in French Renaissance and Modern Architecture.


CARYATID

Caryatides. Human female figures used as piers, columns or supports. *Caryatic* is applied to the human figure generally, when used in the manner of caryatides.

Cased. Covered with other materials, generally of a better quality.

Casement. A glass frame which is made to open by turning on hinges affixed to its vertical edges.

Cassoon, or **Caisson.** A deep panel or coffer in a soffit or ceiling. This term is sometimes written in the French form, *caisson;* sometimes derived directly from the Italian *cassone,* the augmentative of cassa, a chest or coffer.

Cast. A term used in sculpture for the impression of any figure taken in plaster of Paris, wax, or other substances.

Catacombs. Subterranean places for burying the dead. Those of Egypt, and near Rome, are believed to be the most important.

Catafalco. An ornamental scaffold used in funeral solemnities.

Cathedral. The principal church, where the bishop has his seat as diocesan.

Cauliculus. The inner scroll of the Corinthian capital. It is not uncommon, however, to apply this term to the larger scrolls or volutes also.

Causeway. A raised or paved way.

Cavetto. A concave ornamental molding, opposed in effect to the ovolo—the quadrant of a circle.

Ceiling. That covering of a room which hides the joists of the floor above, or the rafters of the roof. Most European churches either have open roofs, or are groined in stone. At Peterborough and St. Albans, England, there are very old flat ceilings of boards curiously painted. In later times the boarded ceilings, and, in fact, some of those of plaster, have molded ribs, locked with bosses at the intersection, and are sometimes elaborately carved. In many English churches there are ceilings formed of oak ribs, filled in at the spandrels with narrow, thin pieces of board, in exact imitation of stone groining. In the Elizabethan and subsequent periods the ceilings are enriched with most elaborate ornaments in stucco. Matched and beaded boards, planed and smoothed, used for wainscoting. In the New England States it is called sheathing.

Cenotaph. An honorary tomb or monument, distinguished from monuments in being empty, the individual it is to memorialize having received interment elsewhere.

Centaur. A poetical imaginary being of heathen mythology, half-man and half-horse.

Central America, Architecture of. That of the five states: Guatemala, Honduras, Nicaragua, San Salvador, and Costa Rica, and the colony of British Honduras. This region is rich in ruined structures that belong to the pre-Columbian class of American Architecture. The modern houses are usually small huts made of cane and palm leaves, or of sticks plastered with adobe mud, and whitewashed. In the towns adobe bricks are largely used, sometimes with a cut-stone foundation, as adobe disintegrates most rapidly near the ground. Aside from churches, which are numerous, there are few structures of importance.

Centring. In building, the frames on which an arch is turned.

Châlet. A wooden dwelling house of the type common in Switzerland The châlets are of two different types as for the structure: (1) The type derived from the log house, of heavy beams placed one upon the other and crossed at the angles; Oberland, Uri, Schwitz, etc.; (2) the framework building, a structure of posts and beams, with the wall merely filled in with thick boards, or even bricks, etc.; Zurich, Saint Gallen, Appenzell. A few châlets combine the two types.

Chamfer, Champfer, or **Chaumfer.** When the edge or arris of any work is cut off at an angle of 45° in a small degree, it is said to be chamfered; if to a large scale, it is said to be a canted corner. The chamfer is much used in mediæval work, and is sometimes plain, sometimes hollowed out, and sometimes molded.

Chamfer Stop. Chamfers sometimes simply run into the arris by a plane face; more commonly they are first stopped by some ornament, as by a bead·

they are sometimes terminated by trefoils, or cinquefoils, double or single, and in general form very pleasing features in mediæval architecture.

Chancel. A place separated from the rest of a church by a screen. The word is now generally used to signify the portion of a liturgical church containing the altar.

Chantry. A small chapel, generally built out from a church. They generally contain a founder's tomb, and are often endowed places where masses might be said for his soul. The officiator, or mass priest, being often unconnected with the parochial clergy. The chantry has generally an entrance from the outside.

Chapel. A small, detached building used as a substitute for a church in a large parish; an apartment in any large building, a palace, a nobleman's house, a hospital or prison, used for public worship; or an attached building running out of and forming part of a large church, generally dedicated to different saints, each having its own altar, piscina, etc., and screened off from the body of the building.

Chapter House. The chamber in which the chapter or heads of the monastic bodies assembled to transact business. They are of various forms; some are oblong apartments, some octagonal, and some circular.

Chaptrel. In Gothic architecture, the capital of a pier or column which receives an arch.


CHAPTREL

Charnel House. A place for depositing the bones which might be thrown up in digging graves. Sometimes it was a portion of the crypt; sometimes it was a separate building in the church-yard; sometimes chantry chapels were attached to these buildings. M. Viollet-le-Duc has given two very curious examples of *ossuaires*—one from Fleurance, the other from Faouet.

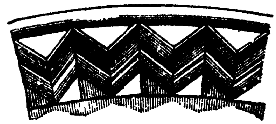
Charrette. (Fr.) Final period of work on design problem, a period of stress because of a fixed date of delivery.

China, Architecture of. Most ancient of nations, center of Asiatic civilization at different epochs, original source of Japanese culture, architecturally China yet remains, for European scholars, an unworked field. By three thousand years of civil wars, dynastic fighting, and Tartar and Mongol invasions all trace of palace or temple have been swept away. It is from the Japan architecture of fifty years since, that we must infer what we can of China's. In the Horiuji temples near Nara (Japan) are sixth-century structures built by Korean Architects; at this period Korea's art was that of China. Like the modern palaces, the ancient ones undoubtedly consisted of many small buildings crowded together, and very probably masonry entered largely into their construction, for we know that a thousand years before the Christian era brick was the common building material. The Hia dynasty was founded 2205 B.C. The fatal barbarism of Mongol and Manchu influences are everywhere visible in what passes for modern architecture, all that remains of architectural excellence being the dignity of great masses, and the contrast, often very agreeable, between light, generally wood-built superstructures, with the walls of the basements, as in the "Winter Palace" at Peiping, and its great gateway structures, and in some of the pagoda towers. The architecture of that imperial city and of the seaport towns is nearly all of a very recent period, and it is safe to conclude, in general, that the Chinese architecture of the last three hundred years is immeasurably inferior to that which must have gone before. That at one time Chinese architecture was a thing of the utmost beauty one cannot doubt. We know that the lofty civilization created by the

wise emperors of the Sung dynasty could have expressed itself only in noble art. The larger and more richly decorated of the now existing dwellings are interesting, because of the completely different ordering of the architectural members from that which is familiar to Europeans.

Cherub—Gothic. A representation of an infant's head joined to two wings, used in the churches on key-stones of arches and corbels.

Chevron—Gothic. An ornament turning this and that way, like a zigzag, or letter Z.



CHEVRON

Chiaroscuro. The effects of light and shade in a picture.

Choir. That part of a church or monastery where the breviary service, or "horæ," is chanted.

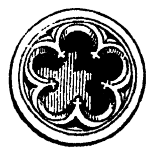
Christian Architecture. (1) Architecture of any style assumed to be especially under the influence of Christian religious belief. In this sense the English Gothic styles were called Christian by the Gothic revivalists of 1850 and thereafter. The styles which came of direct imitation of Greco-Roman architecture were considered pagan in contradistinction to that of the Middle Ages. (2) The architecture of the European world since the general triumph of Christianity. In this sense the earliest Christian Architecture is that of the basilicas and baptisteries of the reign of Constantine; and all buildings erected since that time in lands occupied or settled by Europeans are Christian, except where obviously the work of persons of other religions.

Church. A building for the performance of public worship. The first churches were built on the plan of the ancient basilicæ, and afterward on the plan of a cross: a church is said to be in Greek cross when the length of the transverse is equal to that of the nave; in Latin cross, when the nave is longer than the transverse part; in rotundo, when it is a perfect circle; simple, when it has only a nave and choir; with aisles, when it has a row of porticos in form of vaulted galleries, with chapels in its circumference.

Ciborium. A tabernacle or vaulted canopy supported on shafts standing over the high altar.

Cincture. A ring, list, or fillet at the top and bottom of a column, serving to divide the shaft of the column from its capital and base.

Cinquefoil. A sinking or perforation, like a flower, of five points or leaves, as a quatrefoil is of four. The points are sometimes in a circle. and sometimes form the cusping of a head.



CINQUEFOIL

Civic Crown. A garland of oak-leaves and acorns, given as honorary distinction among the Romans to such as had preserved the life of a fellow-citizen.

Clere-story, Clear-story. When the middle of the nave of a church rises above the aisles and is pierced with windows, the upper story is thus called. Sometimes these windows are very small, being mere quatrefoils, or spherical triangles. In large buildings, however, they are important objects both for beauty and utility. The window of the clere-stories of Norman work, even in large churches, are of less importance than in the later styles. In Early English they became larger; and in the Decorated they are more important still, being lengthened as the triforium diminishes. In Perpendicular work the latter often disappears altogether, and in many later churches the clere-

stories are close ranges of windows. The word *clere-story* is also used to denote a similar method of lighting other buildings besides churches, especially factories, depots, sheds, etc.

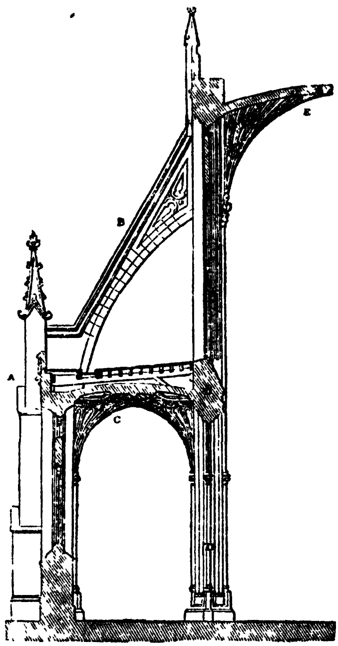
Cliff-Dwelling. An Amerindian house built in a cliff, either in a cave-like opening, a gallery, or on a ledge. Especially applied to structures in the cliffs of the Southwestern United States and Northern Mexico, built by Indians similar to the present Pueblos.

Cloister. An enclosed square, like the atrium of a Roman house, with a walk or ambulatory around, sheltered by a roof, generally groined, and by tracery windows, which were more or less glazed.

Close. The precinct of a cathedral or abbey. Sometimes the walls are traceable, but now generally the boundary is only known by tradition.

Close String, or **Box String.** A method of finishing the outer edge of stairs, by building up a sort of curb string on which the balusters set, and the treads and risers stop against it.

Clustered. In architecture, the coalition of several members which penetrate each other.



Bath Abbey

FLYING BUTTRESS AND CLERE-STORY

A, buttress with pinnacle; *B*, flying buttress supporting clere-story; *C*, vaulted roof of aisle; *D D*, pier dividing nave from aisle; *E*, vaulted roof of nave

Clustered Column. Several slender pillars attached to each other so as to form one. The term is used in Roman architecture to denote two or four columns which appear to intersect each other at the angle of a building to answer at each return.

Coat. A thickness or covering of paint, plaster, or other work, done at one time. The first coat of plastering is called the scratch coat, the second coat (when there are three coats) is called the brown coat, and the last coat is variously known as the slipped coat, skimcoat, or white coat. It varies in composition in different localities.

Coffer. A deep panel in a ceiling.

Coffer Dam. A frame used in the building of a bridge in deep water, similar to a caisson.

Collar Beam. A beam above the lower ends of the rafters, and spiked to them.

CLUSTERED COLUMN

Colonial Architecture. The architecture of a colony or colo-

nies; especially in American use as late as the beginning of the nineteenth century. (See *Georgian Architecture.*)

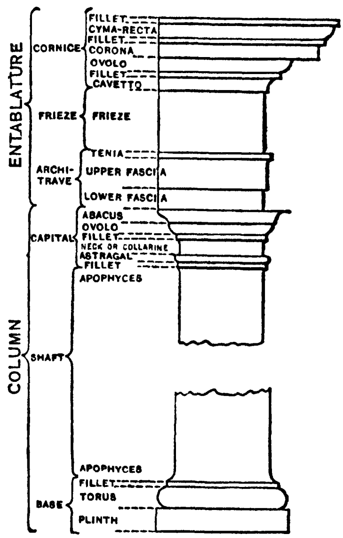
Colonnade. A row of columns. The colonnade is termed, according to the number of columns which support the entablature: Tetrastyle, when there are four; hexastyle, when six; octostyle, when eight, etc. When in front of a building they are termed porticos; when surrounding a building, peristyle; and when double or more, polystyle.

Colosseum, or Coliseum. The immense amphitheater built at Rome by Flavius Vespasian, A.D. 72, after his return from his victories over the Jews. It would contain ninety thousand persons sitting, and twenty thousand more standing. The name is now employed to denote an unusually large audience building, generally of a temporary nature.

Colossus. The name of a brazen statue which was erected at the entrance of the harbor at Rhodes, one hundred and five feet in height. Vessels could sail between its legs.

Column. A round pillar. The parts are the base, on which it rests; its body, called the shaft; and the head, called the capital. The capital finishes with a horizontal table, called the abacus, and the base commonly stands on another, called the plinth. Columns may be either insulated or attached. They are said to be attached or engaged when they form part of a wall, projecting one-half or more, but not the whole, of their substance.

Common. A line, angle, surface, etc., which belongs equally to several objects. Common centring is a centring without trusses, having a tie beam at bottom. Common joists are the beams in naked flooring to which the joists are fixed. Common rafters in a roof are those to which the laths are attached.



SECTION OF COLUMN AND ENTABLATURE

Composite Arch. Is the pointed or lancet arch.

Composite Order. The most elaborate of the orders of classical architecture.

Compound Arch. A usual form of mediæval arch, which may be resolved into a number of concentric archways, successively placed within and behind each other.

Concour. (Fr.) Competition.

Conduit. A long narrow passage between two walls or underground for secret communication between different apartments; also, a canal or pipe for the conveyance of water.

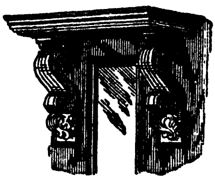
Confessional. The seat where a priest or confessor sits to hear confessions.

Conge. Another name for the echinus or quarter round.

Conservatory. A building for the protection and rearing of tender plants, often attached to a house as an apartment. Also, a public place of instruction, designed to preserve and perfect the knowledge of some branch of learning or the fine arts; as a *conservatory* of music.

Consistory. The judicial hall of the College of Cardinals at Rome.

Consol, or Console. A bracket or truss, generally with scrolls or volutes at the two ends, of unequal size and contrasted, but connected by a flowing line from the back of the upper one to the inner convolving face of the lower.


CONSOLES

Coping. The capping or covering of a wall. This is of stone, weathered to throw off the wet. In Norman times, as far as can be judged from the little there is left, it was generally plain and flat, and projected over the wall with a floating to form a drip. Afterward it assumed a torus or bowtell at the top, and became deeper, and in the Decorated period there were generally several off-sets. The copings in the Perpendicular period assumed something of the wavy section of the buttress caps, and mitred round the sides of the embrasure, as well as the top and bottom.

Corbel. The name, in mediæval architecture, for a piece of stone jutting out of a wall to carry any superincumbent weight. A piece of timber projecting in the same way was called a tassel or a bragger. Thus, the carved ornaments from which the vaulting shafts spring at Lincoln are corbels. Norman corbels are generally plain. In the Early English period they are sometimes elaborately carved. They sometimes end with a point, apparently growing into the wall, or forming a knot, and often are supported by angles and other figures. In the later periods the foliage or ornaments resemble those in the capitals. In modern architecture, a short piece of stone or wood projecting from a wall to form a support, generally ornamented.

Corbel Out. To build out one or more courses of brick or stone from the face of a wall, to form a support for timbers.

Corbel Table. A projecting cornice or parapet, supported by a range of corbels a short distance apart, which carry a molding, above which is a plain piece of projecting wall forming a parapet, and covered by a coping. Sometimes small arches are thrown across from corbel to corbel, to carry the projection.

Cornice. The projection at the top of a wall finished by a blocking-course, common in classic architecture. In Norman times, the wall finished with a corbel table, which carried a portion of plain projecting work, which was finished by a coping, and the whole formed a parapet. In Early English times the parapet was much the same, but the work was executed in a much better way, especially the small arches connecting the corbels. In the Decorated period the corbel table was nearly abandoned, and a large hollow, with one or two subordinate moldings, substituted; this is sometimes filled with the ball-flowers, and sometimes with running foliages. In the Perpendicular style the parapet frequently did not project beyond the wall-line below; the molding then became a string (though often improperly called a cornice), and was ornamented by a quatrefoil, or small rosettes, set at equal intervals immediately under the battlements. In many French examples the molded string is very bold. and enriched with foliage ornaments.

Corona. The brow of the cornice which projects over the bed moldings to throw off the water.

Corridor. A long gallery or passage in a mansion connecting various apartments and running round a quadrangle. Any long passageway in a building.

Countersink. To make a cavity for the reception of a plate of iron, or the head of a screw or bolt, so that it shall not project beyond the face of the work.

Coupled Columns. Columns arranged in pairs.

Course. A continued layer of bricks or stones in buildings; the term is also applicable to slates, shingles, etc.

Court. An open area behind a house, or in the center of a building and the wings. Courts admit of the most elegant ornamentations, such as arcades, etc.

Cove, Coving. The molding called the cavetto, or the scotia inverted, on a large scale, and not as a mere molding in the composition of a cornice, is called a cove or a coving.

Cove-bracketing. The wooden skeleton mold or framing of a cove, applied chiefly to the bracketing of a cove ceiling.

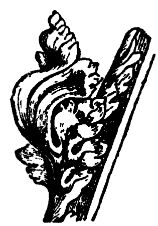
Cove Ceiling. A ceiling springing from the walls with a curve.

Coved and Flat Ceiling. A ceiling in which the section is the quadrant of a circle, rising from the walls and intersecting in a flat surface.

Cradling. Timber work for sustaining the lath and plaster of vaulted ceilings.

Cresting. An ornamental finish in the wall or ridge of a building, which is common on the Continent of Europe. An example occurs at Exeter Cathedral, the ridge of which is ornamented with a range of small fleurs-de-lis in lead.

Crocket. An ornament running up the sides of gablets, hood-molds, pinnacles, spires; generally, a winding stem like a creeping plant, with flowers or leaves projecting at intervals, and terminating in a finial.


CROCKET

Cross. This religious symbol is almost always placed on the ends of gables, the summit of spires, and other conspicuous places of old churches. In early times it was generally very plain, often a simple cross in a circle. Sometimes they take the form of a light cross, crosslet, or a cross in a square. In the Decorated and later styles they became richly floriated, and assumed an endless variety of forms. Of memorial crosses the finest examples are the Eleanor crosses, erected by Edward I. Of these a few yet remain, one of which has recently been reërected at Charing Cross. Preaching crosses were often set up by the wayside as stations for preaching; the most noted is that in front of St. Paul's, England. The finest remaining sepulchral crosses are the old elaborately carved examples found in Ireland.

Cross-aisle. An old name for a transept.

Cross-springer. The transverse ribs of a vault.

Cross-vaulting. A common name given to groins and cylindrical vaults.

Crown. In architecture the uppermost member of the cornice; called also Corona and Larmier.

Crypt. A vaulted apartment of greater or less size, usually under the choir.

Cupola. A small room, either circular or polygonal, standing on the top of a dome. By some it is called a Lantern.

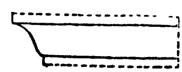
Curb Roof, or Mansard Roof. A roof formed of four contiguous planes, each two having an external inclination.

Curtail Step. The first step in a stair, which is generally finished in the form of a scroll.

Cusp. The point where the foliations of tracery intersect. The earliest example in England of a plain cusp is probably that at Pythagoras School, at Cambridge, of an ornamental cusp, at Ely Cathedral, where a small roll, with a rosette at the end, is formed at the termination of a cusp. In the later styles the terminations of the cusps were more richly decorated; they also sometimes terminate not only in leaves or foliages, but in rosettes, heads, and other fanciful ornaments.

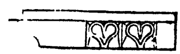
Cyclostyle. A structure composed of a circular range of columns without a core is cyclostylar; with a core, the range would be a peristyle. This is the species of edifice called by Vitruvius monopteral.

Cyma. The name of a molding of very frequent use. It is a simple, waved line, concave at one end and convex at the other, like an italic *f*. When the concave part is uppermost it is called a cyma recta, but if the convexity appear above, and the concavity below, it is then a cyma reversa.



CYMA RECTA

Cymatium. When the crowning molding of an entablature is of the cyma form, it is termed the Cymatium.



CYMA REVERSA

Cyrtostyle. A circular projecting portico. Such are those of the transept entrances of St. Paul's Cathedral, London.

Dado, or Die. The vertical face of an insulated pedestal between the base and cornice, or surbase. It is extended also to the similar part of all stereobates which are arranged like pedestals in Roman and Italian architecture.

Dais. A part of the floor at the end of a mediæval hall, raised a step above the rest of the floor. On this the lord of the mansion dined with his friends at the great table, apart from the retainers and servants. In mediæval halls there was generally a deep recessed bay window at one or at each end of the dais, supposed to be for retirement, or greater privacy than the open hall could afford. In France the word is understood as a canopy or hanging over a seat; probably the name was given from the fact that the seats of great men were then surmounted by such an ornament.

Darby. A flat tool used by plasterers in working, especially on ceilings. It is generally about seven inches wide and forty-two inches long, with two handles on the back.

Decastyle. A portico of ten columns in front.

Decorated Style. The second stage of the Pointed or Gothic style of architecture, considered the most complete and perfect development of Gothic architecture, the best examples of which are found in England.

Demi-metope. The half of a metope, which is found at the retiring or projecting angles of a Doric frieze.

Denmark, Architecture of. The peninsula of Jutland as far south as Ribe, the islands of Zealand, Funen, Laaland, and Borneholm, with some smaller islands, are the territories properly included in Denmark. In considering the architecture of the region it is to be noted that its influence on the rest of Europe has been slight, and, however attractive the buildings are to the traveler, they have never been studied with any thoroughness. The earliest buildings of consequence are the round churches, of which four are on the island of Borneholm, one on the island of Funen, two on the island of Zealand, and one only on the mainland. They are generally alike in having a choir of great length, and the entrance porch at the opposite side sometimes carried up into a tower. Other churches in Denmark are generally Romanesque, simple in character such as corresponds to the plainer churches of the second half of the twelfth century in Germany. A curious cruciform church without aisles has five towers, a square one at the crossing and an octagonal one at the end of each arm of the cross; as no group of towers known has greater possibilities, it would have been interesting had the Gothic builders pushed the scheme to a final result. The Danish builders took a marked interest in the German Neoclassic Architecture of the sixteenth or seventeenth centuries, although there are many signs of independent thought. A most interesting state house, and two royal castles with lofty dormers are mainly of the sixteenth century. Later buildings are of the fantastic eighteenth century. The principal royal palace is of simple and tasteful design.

Dentil. The cogged or toothed member, common in the bed-mold of a Corinthian entablature, is said to be dentiled, and each cog or tooth is called a dentil.

Depressed Arches, or Drop Arches. Those of less pitch than the equilateral.

Design. The plans, elevations, sections, and whatever other drawings may be necessary for an edifice, exhibit the design, the term plan having a restricted application to a technical portion of the design.

Detail. As used by architects, detail means the smaller parts into which a composition may be divided. It is applied generally to moldings and other enrichments, and again to their minutiæ.

Diameter. The line in a circle passing through its center, or thickest part, which gives the measure proportioning the intercolumniation in some of the orders.

Diameters. The diameters of the lower and upper ends of the shaft of a column are called its inferior and superior diameters, respectively; the former is the greatest, the latter the least diameter of the shaft

Diaper. A method of decorating a wall, panel, stained glass, or any plain surface, by covering it with a continuous design of flowers, rosettes, etc., either in squares or lozenges, or some geometrical form resembling the pattern of a diapered table-cloth, from which, in fact, the name is supposed by some to have been derived.

Diastyle. A spacious intercolumniation, to which three diameters are assigned.

Dipteros. A double-winged temple. The Greeks are said to have constructed temples with two ranges of columns all around, which were called dipteroi. A portico projecting two columns and their interspaces is of dipteral or pseudo-dipteral arrangement.

Discharging Arch. An arch over the opening of a door or window, to discharge or relieve the superincumbent weight from pressing on the lintel.

Distemper. Term applied to painting with colors mixed with size or other glutinous substance. All the cartoons of the ancients, previous to the year 1410, are said to be done in distemper.

Distyle. A portico of two columns. This is not generally applied to the mere porch with two columns, but to describe a portico with two columns *in antis*.

Ditriglyph. As intercolumniation in the Doric order, of two triglyphs.

Dodecastyle. A portico of twelve columns in front. The lower one of the west front of St Paul's Cathedral, London, is of twelve columns, but they are coupled, making the arrangement pseudo-dodecastyle. The Chamber of Deputies in Paris has a true dodecastyle.

Dog-tooth. A favorite enrichment used from the latter part of the Norman period to the early part of the Decorated. It is in the form of a four-leaved flower, the center of which projects, and probably was named from its resemblance to the dog-toothed violet.

Dome. A cupola or inverted cup on a building. The application of this term to its generally received purpose is from the Italian custom of calling an archiepiscopal church, by way of eminence, Il Duomo, the temple; for to one of that rank, the Cathedral of Florence, the cupola was first applied in modern practice. The Italians themselves never call a cupola a dome; it is on this side of the Alps the application has arisen, from the circumstance, it would appear, that the Italians use the term with reference to those structures whose most distinguishing feature is the cupola, tholus, or (as we now call it) dome.

Domestic Architecture. That branch which relates to private buildings.

Donjon. The principal tower of a castle, generally containing the prison.

Door Frame. The surrounding case into and out of which the door shuts and opens. It consists of two upright pieces, called jambs, and a head, generally fixed together by mortices and tenons, and wrought, rebated, and beaded.

Doric Order. The oldest of the three orders of Grecian architecture.

Dormer Window. A window belonging to a room in a roof, which consequently projects from it with a valley gutter on each side. They are said not to be earlier than the fourteenth century. In Germany there are often several rows of dormers, one above the other. In Italian Gothic they are very rare; in fact, the former have an unusually steep roof, while in the latter country, where the Italian tile is used, the roofs are rather flat.

Dormitory. A room, suite of rooms, or building used to sleep in. The name was first applied to the place where the monks slept at night. It was sometimes one long room like a barrack, and sometimes divided into a succession of small chambers or cells. The dormitory was generally on the first floor, and connected with the church, so that it was not necessary to go out of doors to attend the nocturnal services. In the large houses of the Perpendicular period, and also in some of the Elizabethan, the entire upper story in the roof formed one large apartment, said to have been a place for exercise in wet weather, and also for a dormitory for the retainers of the household, or those of visitors.

Double Vault. Formed by a duplicate wall; wine cellars are sometimes so formed.

Dovetailing. In carpentry and joinery, the method of fastening boards or other timbers together, by letting one piece into another in the form of the expanded tail of a dove.

Dowel. A pin let into two pieces of wood or stone, where they are joined together. A piece of wood driven into a wall so that other pieces may be nailed to it. This is also called plugging.

Draw-bridge. A bridge made to draw up or let down, much used in fortified places. In navigable rivers, the arch over the deepest channel is made to draw or revolve, in order to let the masts of ships pass through.

Drawing-room. A room appropriated for the reception of company; a room to which company withdraws from the dining-room

Dresser. A cupboard or set of shelves to receive dishes and cooking utensils.

Dressing. Is the operation of squaring and smoothing stones for building; also applied to smoothing lumber.

Dressing-room. An apartment appropriated for dressing the person.

Drip. A name given to the member of a cornice which has a projection beyond the other parts for throwing off water by small portions, drop by drop. It is also called Larmier.

Drip-stone. The label molding which serves on a canopy for an opening, and to throw off the rain. It is also called Weather Molding.

Drop-scene. A curtain suspended by pulleys, which descends or drops in front of the stage in a theater.

Drum. The upright part of a cupola over a dome; also, the solid part or vase of the Corinthian and Composite capitals.

Dry-rot. A rapid decay of timber, by which its substance is converted into a dry powder, which issues from minute cavities resembling the borings of worms.

Dungeon. The prison in a castle keep, so called because the Norman name for the latter is donjon, and the dungeons, or prisons, are generally in its lowest story.

Dwarf Wall. The walls enclosing courts above which are railings of iron; low walls, in general, receive this name.

Eaves. In slating and shingling, the margin or lower part of the slating hanging over the wall, to throw the water off from the masonry or brickwork.

Echinus. A molding of eccentric curve, generally cut (when it is carved) into the forms of eggs and anchors alternating, whence the molding is called by the name of the more conspicuous. It is the same as Ovolo.

ECHINUS

Edifice. Is synonymous with the terms building, fabric, erection, but is more strictly applicable to architecture distinguished for size, dignity, and grandeur.

Efflorescence. In architecture, the formation of a whitish loose powder, or crust, on the surface of stone or brick walls.

Egyptian Architecture. The earliest civilization and cultivation of the arts was in Upper Egypt. The most remarkable and most ancient monuments of the Egyptians, with the exception of the pyramids, are nearly all included in Upper Egypt. The buildings of Egypt are characterized by solidity and mas-

siveness of construction, originality of conception, and boldness of form. The walls, the pillars, and the most sacred places of their religious buildings were ornamented with hieroglyphics and symbolical figures, while the ceilings of the porticos exhibited zodiacs and celestial planispheres. The temples of Egypt were generally without roofs, and, consequently, the interior colonnades had no pediments, supporting merely an entablature, composed of only architrave, frieze, and cornice, formed of immense blocks united without cement and ornamented with hieroglyphics.

Element. The outline of the design of a Decorated window, on which the centers for the tracery are formed. These centers will all be found to fall on points which, in some way or other, will be equimultiples of parts of the openings. To draw tracery well, or understand even the principles of its composition, much attention should be given to the study of the element.

Elevation. The front façade, as the French term it, of a structure; a geometrical drawing of the external upright parts of a building.

Elizabethan Architecture. The architecture of the reign and time of Queen Elizabeth in England. That which grew out of the Tudor style mainly by means of the great country houses built by noblemen and landholders in England.

Embattlement. An indented parapet; battlement.

Emblazon. To adorn with figures of heraldry, or ensigns armorial.

Embossing. Sculpture in rilievo, the figures standing partly out from the plane.

Embrasure. The opening in a battlement between the two raised solid portions or merlons, sometimes called a crenelle.

Encaustic. Pertaining to the art of burning in colors, applied to painting on glass, porcelain, or tiles, where colors are fixed by heat; hence, encaustic tiles, bricks, etc.

Engaged Columns. Are those attached to, or built into walls or piers, a portion being concealed.

England, Architecture of. The history of architecture in England begins with the history of her people; the styles of architecture were modified by the changes in the various political systems: Roman, Saxon and Norman were the influence in early times. From the French came the first inspiration for Gothic work, which went through its various phases into the Tudor. At that time the architectural interest centered on the domestic instead of the outstanding cathedrals and monasteries, and the Elizabethan, Queen Anne, and English classic are developed. The Oxford movement caused a study of all periods in the history of architecture, and resulted in the multiplicity of styles. Among them all there is, however, much that has a sterling ring. There is good sound Gothic, full of vigor and life; there is Gothic touched with the sixteenth-century spirit and Italian detail. There is classic, as of Wren, in its simple robustness, there is Renaissance of Flanders and the Lowlands; occasionally some French Renaissance There are Tudor, Elizabethan, and Jacobean, all these and more in Victoria's days.

Enrichment. The addition of ornament, carving, etc., to plain work; decoration; embellishment.

Ensemble. Means the whole work or composition considered together, and not in parts.

Entablature. The assemblage of parts supported by the column. It consists of three parts: the architrave, frieze, and cornice.

Entail. In Gothic architecture, delicate carving.

Entasis. The swelling of a column, etc. In mediæval architecture, some spires, particularly those called " broach spires," have a slight swelling in the sides, but no more than to make them look straight; for, from a particular " deceptio visus," that which is quite straight, when viewed at a height, looks hollow.

Entry. A hall without stairs or vestibule.

Epistyle. This term may with propriety be applied to the whole entablature, with which it is synonymous; but it is restricted in use to the architrave, or lowest member of the entablature.

Escutcheon. (Her.) The field or ground on which a coat-of-arms is represented. (Arch) The shields used on tombs, in the spandrels of doors, or in string-courses; also, the ornamented plates from the center of which door rings, knockers, etc., are suspended, or which protect the wood of the keyhole from the wear of the key. In mediæval times these were often worked in a very beautiful manner.

Eskimo Architecture. The architecture of the tribes occupying the polar shores and regions of North America, and to a limited extent Siberia, who are called Eskimos or Esquimaux.

Esquisse. (Fr.) Sketch from which project is developed with least possible deviation.

Esquisse-Esquisse. (Fr.) A quickly presented sketch problem.

Etching. A mode of engraving on glass or metal (generally copper) by means of lines, eaten in or corroded by means of some strong acid.

Etruscan Architecture. The remains of Etruscan architecture are so few that they might be passed over, were it not for the very important part they play in the foundation of Roman architecture. There are two classes: (1) Structures erected in stone have in part lasted to the present day; as the roads, walls and arched entrance gates; (2) those built of ephemeral materials have long since passed away, of which only conjecture can be made from writings of historians, collections in museums, and evidence given in designs of their tombs.

Eustyle. A species of intercolumniation to which a proportion of two diameters and a quarter is assigned. This term, together with the others of similar import—pycnostyle, systyle, diastyle, and aræostyle—referring to the distance of columns from one another in composition, is from Vitruvius, who assigns to each the space it is to express. It will be seen, however, by reference to them individually, that the words themselves, though perhaps sufficiently applicable convey no idea of an exactly defined space, and, by reference to the columnar structures of the ancients, that no attention was paid by them to such limitations. It follows, then, that the proportions assigned to each are purely conventional, and may or may not be attended to without vitiating the power of applying the terms. Eustyle means the best or most beautiful arrangement; but, as the effect of a columnar composition depends on many things besides the diameter of the columns, the same proportioned intercolumniation would look well or ill according to those other circumstances, so that the limitation of Eustyle to two diameters and a quarter is absurd.

Exedra. In classical archæology, a place partly enclosed and roofed over, and provided with seats; also, a covered room or hall opening from a colonnade, and arranged as a lecture-room or court-room; an outhouse, or shed; a hall for meetings. In English usage, usually a semicircular, or nearly semi-

circular, raised seat, the back of which forms a low wall, and especially one of considerable size and of some pretensions. This sense is derived from the rounded apses of the Roman basilicas, which were fitted with a permanent seat, and served regularly, or on special occasions, for court-rooms.

Exercise en loge. (Fr.) Drawings prepared without assistance, criticism, or the use of documents.

Extrados. The exterior or convex curve forming the upper line of the arch stones; the term is opposed to the intrados, or concave side.

Eye of a Dome. The apperture at its summit.

Eye of a Volute. The circle in its center.

Façade, or Face. The whole exterior side of a building that can be seen at one view; strictly speaking, the principal front.

Face Mold. The pattern for marking the plank or board out of which ornamental hand-railings for stairs and other works are cut.

Fan Tracery. The very complicated mode of roofing used in the Perpendicular style, in which the vault is covered by ribs and veins of tracery.

Fascia. A flat, broad member in the entablature of columns or other parts of buildings, but of small projection. The architraves in some of the orders are composed of three bands, or fasciæ; the Tuscan and the Doric ought to have only one. Ornamental projections from the walls of brick buildings over any of the windows, except the uppermost, are called Fasciæ.

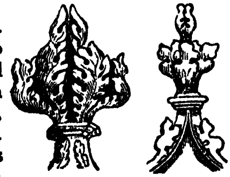
Fenestral. A frame, or "chassis," on which oiled paper or thin cloth was strained to keep out wind and rain when the windows were not glazed.

Festoon. An ornament of carved work, representing a wreath or garland of flowers or leaves, or both, interwoven with each other. It is thickest in the middle, and small at each extremity, where it is tied, a part often hanging down below the knot.


FESTOON

Fillet. A narrow vertical band or listel of frequent use in congeries of moldings, to separate and combine them, and also to give breadth and firmness to the upper edge of a crowning cyma or cavetto, as in an external cornice. The narrow slips of breadth between the flutes of Corinthian and Ionic columns are also called fillets. In mediæval work the fillet is a small, flat, projecting square, chiefly used to separate hollows and rounds, and often found in the outer parts of shafts and boutels. In this situation the center fillet has been termed a keel, and the two side ones, wings; but, apparently, this is not an ancient usage.

Finial. The flower, or bunch of flowers, with which a spire, pinnacle, gablet, canopy, etc., generally terminates. Where there are crockets, the finial generally bears as close a resemblance as possible to them in point of design. They are found in early work where there are no crockets. The simplest form more resembles a bud about to burst than an open flower. They soon became more elaborate, as at Lincoln, and still more, as at Westminster and the Hôtel Cluny at Paris. Many perpendicular finials are like four crockets bound together. Almost every known example of a finial has a sort of necking separating it from the parts below.


FINIALS

Fireplace. That part of a building which is arranged for the making of fires, as for warmth; especially, such a provision when made for open fires of coal or wood, as distinguished from furnaces, stoves, or the elaborate holocaust of the Romans. In this sense the fireplace is either the hearth in the middle of the room, as common in primitive times, the smoke escaping through openings in the roof, or a part of a chimney. The latter sense is the ordinary use of the term fireplace, and implies a recess in the wall or chimney-breast which connects directly with a flue for smoke. It is a recess or a space enclosed by two jambs or cheeks, and terminating above in the flue. The decoration of the fireplace has always been important, because that one part of the room is wholly different in its uses, and probably in its material as well, from the rest.

Fish-joint. A splice where the pieces are joined butt end to end, and are connected by pieces of wood or iron placed on each side and firmly bolted to the timbers, or pieces joined.

Flags. Flat stones, from 1 to 3 inches thick, for floors.

Flamboyant. A name applied to the Third Pointed style in France, which seems to have been developed from the Second, as the English Perpendicular was from the Decorated. The great characteristic is, that the element of the tracery flows upward in long wavy divisions like flames of fire. In most cases, also, every division has only one cusp on each side, however long the division may be. The moldings seem to be as much inferior to those of the preceding period as the Perpendicular moldings were to the Early English, a fact which seems to show that the decadence of Gothic architecture was not confined to one country.

Flange. A projecting edge, rib, or rim. Flanges are often cast on the top or bottom of iron columns, to fasten them to those above or below; the top and bottom of I-beams and channels are called the flange.

Flashings. Pieces of lead, tin, or copper, let into the joints of a wall so as to lap over gutters or other pieces; also, pieces worked in the slates or shingles around dormers, chimneys, and any rising part, to prevent leaking.

Flatting. Painting finished without leaving a gloss on the surface.

Flèche. A general term in French architecture for a spire, but more particularly used for the small, slender erection rising from the intersection of the nave and transepts in cathedrals and large churches, and carrying the sanctus bell.

Fleur-de-lis. The royal insignia of France, much used in decoration.

Flight. A run of steps or stairs from one landing to another.

Floating. The equal spreading of plaster or stucco on the surface of walls, by means of a board called a float; as a rule, only rough plastering is floated.

Floriated. Having florid ornaments, as in Gothic pillars.

Flue. The space or passage in a chimney through which the smoke ascends. Each passage is called a flue, while all together make the chimney.

Flush. The continued surface, in the same plane, of two contiguous masses.

Flute. A concave channel. Columns whose shafts are channeled are said to be fluted, and the flutes are collectively called Flutings.

Flying Buttress. An arched buttress used when extra strength was required for the upper part of the wall of the nave, etc., to resist the outward thrust of a vaulted ceiling. The flying buttress generally rests on the wall and buttress of the aisle.

Foils. The small arcs in the tracery of Gothic windows, panels, etc.

Foliage. An ornamental distribution of leaves on various parts of buildings.

Foliation. The use of small arcs or foils in forming tracery.

Font. The vessel used in the rite of baptism. The earliest extant is supposes to be that in which Constantine is said to have been baptized; this is a porphyry labrum from a Roman bath. Those in the baptisteries in Italy are all large, and were intended for immersion; as time went on, they seem to have become smaller. Fonts are sometimes mere plain hollow cylinders, generally a little smaller below than above; others are massive squares, supported on a thick stem, round which sometimes there are smaller shafts. In the Early English this form is still pursued, and the shafts are detached; sometimes, however, they are hexagonal and octagonal, and in this and the later styles assume the form of a vessel on a stem. Norman fonts frequently have curious carvings on them, approaching the grotesque; in later times the foliages, etc., partook absolutely of the character of those used in other architectural details of their respective periods. The font in European churches is usually placed close to a pillar near the entrance, generally that nearest but one to the tower in the south arcade; or, in large buildings, in the middle of the nave, opposite the entrance porch, and sometimes in a separate building. In Protestant churches in this country, the font is generally placed inside the communion rail, or on the steps of the chancel.

Footings. The spreading courses at the base or foundation of a wall. When a layer of different material from that of the wall (as a bed of concrete) is used, it is called the Footing.

Foundation. That part of a building or wall which is below the surface of the ground.

Foxtail Wedging. Is a peculiar mode of mortising, in which the end of the tenon is notched beyond the mortise, and is split and a wedge inserted, which, being forcibly driven in, enlarges the tenon and renders the joint firm and immovable.

Frame. The name given to the wood-work of windows, doors, etc.: and in carpentry, to the timber works supporting floors, roofs, etc.

Framing. The rough timber work of a house, including the flooring, roofing, partitioning, ceiling, and beams thereof.

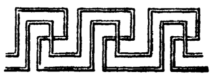
France, Architecture of. The architecture of the modern state of France; it should be divided into its various architectural sections. This region was all included in the Roman Gallia, and Roman remains are found in all parts. Many of the sections were independent states, or nearly so, and as architecture is developed according to the mode of living, each province should be taken separately to understand the architectural vicissitudes of France. The mere mentioning of a few of these divisions, like Parisian France, Flemish France, Normandy, Brittany, tells the diversity of the people. This paragraph, however, will treat France as a whole, and the simple statement will be made that it was not until the beginning of the Renaissance that the country became one in the modern sense. France is the richest country in Europe in buildings of value to the Western student (later than the time of the fall of the Roman Imperial dominion). In the eleventh century France takes the lead in Europe, with noblest and richest as well as most varied Romanesque. In the twelfth century the lead is still more decided, in spite of the magnificent German round-arched cathedrals, for the Gothic art beginning about 1150 is entirely French in the strictest sense, all the other European lands having

taken their primary and most of their subsequent impulses from the French royal domain. Practically the same may be said of the works of the later Gothic style and of the Renaissance in France, which developed the styles of Louis XV and XVI. Present-day architecture is deserting the conventional, and the final development is awaited with interest. On the whole, then, the architecture of France since the beginning of the Romanesque period is the most important for the modern student.

Freestone. Stone which can be used for molding, tracery, and other work required to be executed with the chisel. The oolitic and sandstone are those generally included by this term.

Fresco. The method of painting on a wall while the plastering is wet. The color penetrates through the material, which, therefore, will bear rubbing or cleaning to almost any extent. The transparency, the chiaroscuro, and lucidity, as well as force, which can be obtained by this method, cannot be conceived unless the frescos of Fra Angelico or Raphael are studied. The word, however, is often applied improperly to painting on the surface in distemper or body color, mixed with size or white of egg, which gives an opaque effect.

Fret. An ornament consisting of small fillets intersecting each other at right angles.


FRET

Frieze. That portion of an entablature between the cornice above and and architrave below. It derives its name from being the recipient of the sculptured enrichments either of foliage or figures which may be relevant to the object of the sculpture. The frieze is also called the Zoösphorus.

Frigidarium. An apartment in the Roman bath, supplied with cold water.

Furniture. Formerly a name given to the metal trimmings of doors, windows, and other similar parts of a house. In this country the word "hardware" is more generally used to denote the same thing. Household furnishings.

Furrings. Flat pieces of timber used to bring an irregular framing to an even surface.

Gable. When a roof is not hipped or returned on itself at the ends, its ends are stopped by carrying up the walls under them in the triangular form of the roof itself. This is called the gable, or, in the case of the ornamental and ornamented gable, the pediment. Of necessity, gables follow the angles of the slope of the roof, and differ in the various styles. In Norman work they are generally about half-pitch; in Early English, seldom less than equilateral, and often more. In Decorated work they become lower, and still more so in the Perpendicular style. In all important buildings they are finished with copings or parapets. In the Later Gothic styles gables are often surmounted with battlements, or enriched with crockets; they are also often paneled or perforated, sometimes very richly. The gables in ecclesiastical buildings are mostly terminated with a cross; in others, by a finial or pinnacle. In later times the parapets or copings were broken into a sort of steps, called corbic steps. In buildings of less pretension the tiles or other roof covering passed over the front of the wall, which then, of course, had no coping. In this case, the outer pair of rafters were concealed by molded or carved verge boards.

Gable Window. A term sometimes applied to the large window under a gable, but more properly to the windows in the gable itself.

Gabled Towers. Those which are finished with gables instead of parapets. Many of the German Romanesque towers are gabled.

Gablets. Triangular terminations to buttresses, much in use in the Early English and Decorated periods, after which the buttresses generally terminate in pinnacles. The Early English gablets are generally plain, and very sharp in pitch. In the Decorated period they are often enriched with paneling and crockets. They are sometimes finished with small crosses, but oftener with finials.

Gain. A beveled shoulder on the end of a mortised brace, for the purpose of giving additional resistance to the shoulder.

Gallery. Any long passage looking down into another part of a building, or into the court outside. In like manner, any stage erected to carry a rood or an organ, or to receive spectators, was latterly called a gallery, though originally a loft. In later times the name was given to any very long rooms, particularly those intended for purposes of state, or for the exhibition of pictures.

Gambrel Roof. A roof with two pitches, similar to a mansard or curb roof.

Gargoyle, or Gurgoyle. The carved termination to a spout which conveyed away the water from the gutters, supposed to be called so from the gurgling noise made by the water passing through it. Gargoyles are mostly grotesque figures.


GARGOYLE

Gate-house. A building forming the entrance to a town, the door of an abbey, or the enceinte of a castle or other important edifice. They generally had a large gateway protected by a gate, and also a portcullis, over which were battlemented parapets with holes (machicolations) for throwing down darts, melted lead, or hot sand on the besiegers. Gatehouses always had a lodge, with apartments for the porter, and guard-rooms for the soldiers; and, generally, rooms over for the officers, and often places for prisoners beneath. The name is now commonly applied to the gate-keeper's lodge on large estates.

Gauge. To mix plaster of Paris with common plaster to make it set quickly, called gauged mortar. A tool used by carpenters, to strike a line parallel to the edge of a board.

Georgian Architecture. The architecture of the reigns of the four Georges in England—1714 to 1830. The term is more usually employed for that of the earlier reigns. Architecture of the same epoch in America has been called generally "Colonial," or "Old Colonial," but the term Georgian is applied to this also, as an expletive more accurate or more descriptive.

Germany, Architecture of. That of the states constituting the present Republic. Since it is inhabited by people of various races, languages, manners, and religions, one might reasonably expect great contrasts in architecture; but as a rule, whatever is borrowed from without or originates within is welded into an unmistakable German form. The early inhabitants appear to have commenced their architectural operations by scooping out caves in the rocks or hillsides. Examples remain of rock-cut churches. Fragments of Roman buildings exist all over Germany. Typical German work shows the highest reach of constructional excellence of the pure Romanesque style. An Eastern influence is conspicuous in the Romanesque buildings which chiefly exhibits itself in churches planned and constructed in cubes, polygons, and circles, covered with domical vaults, semidomes, and lunette vaulting. Gothic cathedrals have had an important influence on German architecture.

The present-day architecture of Germany is working along original lines and is dealing more in masses and undecorated surfaces.

Girder. A large timber or iron beam, either single or built up, used to support joists or walls over an opening.

Glyph. A vertical channel in a frieze.

Gothic Style. The name of Gothic was given to the various Mediæval styles at a period in the sixteenth century when a great classic revival was going on, and everything not classic was considered barbarian, or Gothic. The term was thus originally intended as one of stigma, and, although it conveys a false idea of the character of the Mediæval styles, it has long been used to distinguish them from the Grecian and Roman. The true principle of Gothic architecture is the vertical division, relation and subordination of the different parts, distinct and yet at unity with each other, and while this principle was adhered to, Gothic architecture may be said to have retained its vitality.

Grange. A word derived from the French, signifying a large barn or granary. Granges were usually long buildings with high wooden roofs, sometimes divided by posts or columns into a sort of nave and aisles, with walls strongly buttressed. In England the term was applied not only to the barns, but to the whole of the buildings which formed the detached farms belonging to the monasteries; in most cases there was a chapel either included among these or standing apart as a separate edifice.

Greco-Roman. An architecture of the beam and column, the lintel being used pure and unmixed by the Greeks, but forced into union with the arch by the Romans.

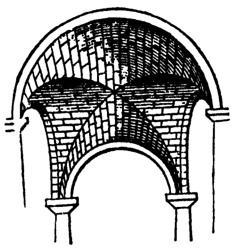
Greece, Architecture of. That of the Republic of Greece, South Balkan peninsula and neighboring islands. Architectural history may be divided into periods coinciding with the country's political history. The beginning of the prehistoric period is lost in the mists of antiquity. The ruins of massive fortification walls indicate a state of society resembling Feudal Europe in the Middle Ages. The surpassing loveliness of the scenery and the facility of intercourse with Oriental countries naturally served to stimulate the early artistic development of the people near the Argive Gulf, and Herodotus tells of the commercial attractions offered by this section of the Phoenician traders. The "classic" period (500 to 400 B.C.) is the culminating era. Artists had developed apace with rapid growth of wealth and refinement, and, as in the times of all good art, the architects, sculptors, and painters interlocked arms or were embodied in the same individuals. The octastyle Parthenon on the Athenian Acropolis represents the perfect Greek, temple. The beautiful Doric and Ionic temples are found in many places. Temples, stoas, gymnasia, stadia, theaters, gateways, propylæa, and acropolises reached perfection. The choragic monument of Lysicrates serves as a type for the Corinthian order. In Greece wood was originally used for column and entablature. Stone was gradually substituted, retaining and petrifying the traditional wooden forms. The stone columns were usually built of several drums, and were often given a thin coat of lime plaster and painted an orange yellow. When marble was used, its smooth surface rendered the plaster unnecessary. The members of the entablature, ceilings and carved ornaments were also painted, chiefly in reds, blues and yellows. The Byzantine period finds Greece still but a province of the vast Roman empire, and only small and unimportant examples remain. During the Turkish period (1453–1831) Greece lay prostrate. Then followed the modern period with Austrians and Germans designing most of her buildings.

Grillage. A framework of beams laid longitudinally and crossed by similar beams laid upon them, used to sustain walls to prevent irregular setting.

Grille. The iron-work forming the enclosure screen to a chapel, or the protecting railing to a tomb or shrine; more commonly found in France than in England. They are of wrought iron, ornamented by the swage and punch, and put together either by rivets or clips. In modern times grilles are used extensively for protecting the lower windows in city houses, also the glass opening in outside doors.

Groin. By some described as the line of intersection of two vaults where they cross each other, which others call the groin point; by others the curved section or spandrel of such vaulting is called a groin, and by others the whole system of vaulting is so named.

Groin Arch. The cross-rib in the later styles of groining, passing at right angles from wall to wall, and dividing the vault into bays or travees.



GROINED VAULTING

Groin Ceiling. A ceiling to a building composed of oak ribs, the spandrels of which are filled in with narrow, thin slips of wood. There are several in England; one at the Early English church at Warmington, and one at Winchester Cathedral, exactly resembling those of stone.

Groin Centring. In groining without ribs, the whole surface is supported by centring during the erection of the vaulting. In ribbed work the stone ribs only are supported by timber ribs during the progress of the work, any light stuff being used while filling in the spandrels.

Groin Point. The name given by workmen to the arris or line of intersection of one vault with another where there are no ribs.

Groin Rib. The rib which conceals the groin point or joints, where the spandrels intersect.

Groined Vaulting. The system of covering a building with stone vaults which cross and intersect each other, as opposed to the barrel vaulting, or series of arches placed side by side. The earliest groins are plain, without any ribs, except occasionally a sort of wide band from wall to wall, to strengthen the construction. In later Norman times ribs were added on the line of intersection of the spandrels, crossing each other, and having a boss as a key common to both; these ribs the French authors call *nerfs en ogive.* Their introduction, however, caused an entire change in the system of vaulting; instead of arches of uniform thickness and great weight, these ribs were first put up as the main construction, and spandrels of the lightest and thinnest possible material placed upon them, the haunches only being loaded sufficiently to counterbalance the pressure from the crown. Shortly after, half-ribs against the walls (formerets) were introduced to carry the spandrels without cutting into the walling, and to add to the appearance. The work was now not treated as continued vaulting, but as divided into bays, and it was formed by keeping up the ogive, or intersecting ribs and their bosses; a sort of construction having some affinity to the dome was formed, which added much to the strength of the groining. Of course, the top of the soffit or ridge of the vault was not horizontal, but rose from the level of the top of the formeret-rib to the boss and fell again; but this could not be perceived from below. As this system of construction got more into use, and as the vaults were required to be of greater span and of higher pitch, the spandrels became larger, and required more support. To

give this, another set of ribs was introduced, passing from the springers of the ogive ribs, and going to about half-way between these and the ogive, and meeting on the ridge of the vault; these intermediate ribs are called by the French *tiercerons*, and began to come into use in the transition from Early English to Decorated. About the same period a system of vaulting came into use called *hexpartite*, from the fact that every bay is divided into six compartments instead of four. It was invented to cover the naves of churches of unusual width. The filling of the spandrels in this style is very peculiar, and, where the different compartments meet at the ridge, some pieces of harder stone have been used, which give rather a pleasing effect. The arches against the wall, being of smaller span than the main arches, cause the center springers to be perpendicular and parallel for some height, and the spandrels themselves are very hollow. As styles progressed, and the desire for greater richness increased, another series of ribs, called *liernes*, was introduced; these passed crossways from the *ogives* to the *tiercerons*, and thence to the *doubleaux*, dividing the spandrels nearly horizontally. These various systems increased in the Perpendicular period, so that the walls were quite a net-work of ribs, and led at last to the Tudor, or, as it is called by many, fan-tracery vaulting. In this system the ribs are no part of the real construction, but are merely carved upon the voussoirs, which form the actual vaulting. Fan-tracery is so called because the ribs radiate from the springers, and spread out like the sticks of a fan. These later methods are not strictly groins, for the pendentives are not square on plan, but circular, and there is, therefore, no arris intersection or groin point.

Groins, Welsh, or Underpitch. When the main longitudinal vault of any groining is higher than the cross or transverse vaults which run from the windows, the system of vaulting is called underpitch groining, or, as termed by the workmen, Welsh groining. A very fine example is at St. George's Chapel, Windsor, England.

Groove. In joinery, a term used to signify a sunk channel whose section is rectangular. It is usually employed on the edge of a molding, stile, or rail, etc., into which a tongue corresponding to its section, and in the substance of the wood to which it is joined, is inserted.

Grotesque. A singular and fantastic style of ornament found in ancient buildings.

Grotto. An artificial cavern.

Ground Floor. The floor of a building on a level, or nearly so, with the ground.

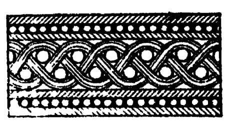
Ground Joist. Joist that is blocked up from the ground.

Grounds. Pieces of wood embedded in the plastering of walls to which skirting and other joiner's work is attached. They are also used to stop the plastering around door and window openings.

Grouped Columns. Three, four, or more columns put together on the same pedestal. When two are placed together, they are said to be coupled.

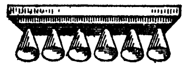
Grout. Mortar made so thin by the addition of water that it will run into all the joints and cavities of the mason-work, and fill it up solid.

Guilloche, or Guillochos. An interlaced ornament like net-work, used most frequently to enrich the torus.



GUILLOCHE

Guttæ. The small cylindrical drops used to enrich the mutules and regulæ of the Doric entablature are so called.



GUTTÆ

Gutter. The channel for carrying off rain-water. The mediæval gutters differed little from others, except that they are often hollows sunk in the top of stone cornices, in which case they are generally called channels in English, and *chcneaux* in French.

Gymnasium. A building classed in the first rank by the Greeks; it was in them they instructed the youth in all the arts of peace and war; a building for athletic exercises.

Hall. The principal apartment in the large dwellings of the Middle Ages. used for the purposes of reception, feasts, etc. In the Norman castle the hall was generally in the keep above the ground floor, where the retainers lived, the basement being devoted to stores and dungeons for confining prisoners. Later halls—indeed, some Norman halls (not in castles)—are generally on the ground floor, as at Westminster, approached by a porch either at the end, as in this last example, or at the side, as at Guildhall, London, having at one end a raised dais or estrade. The roofs are generally open and more or less ornamented. In the middle of these was an opening to let out the smoke, though in later times the halls have large chimney-places with funnels or chimney-shafts for this purpose. At this period there were usually two deeply recessed bay windows at each end of the dais, and doors leading into the withdrawing-rooms, or the ladies' apartments; they are also generally wainscoted with oak, in small panels, to the height of five or six feet, the panels often being enriched. Westminster Hall was originally divided into three parts, like a nave and side aisles, as are some on the Continent of Europe. A room or passageway at the entrance of a house, or suite of chambers. A place of public assembly as a town-hall, a music-hall.

Halving. The junction of two pieces of timber, by letting one into the other.

Hammer Beam. A beam in a Gothic roof, not extending to the opposite side; a beam at the foot of a rafter.

Hanging Buttress. A buttress not rising from the ground, but supported on a corbel, applied chiefly as a decoration and used only in the Decorated and Perpendicular styles.

Hanging Style. Of a door, is that to which the hinges are fixed.

Hangings. Tapestry; originally invented to hide the coarseness of the walls of a chamber. Different materials were employed for this purpose, some of them exceedingly costly and beautifully worked in figures, gold and silk.

Hatching. Drawing parallel lines close together for the purpose of indicating of a section of anything. The lines are generally drawn at an angle of 45° with a horizontal.

Haunches. The sides of an arch, about half-way from the springing to the crown.

Headers. In masonry, are stones or bricks extending over the thickness of a wall. In carpentry, the large beam into which the common joists are framed in framing openings for stairs, chimneys, etc.

Heading Courses. Courses of a wall in which the stone or brick are all headers.

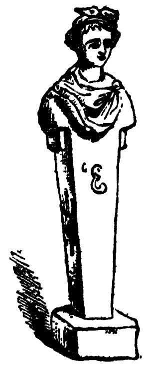
Head-way. Clear space or height under an arch, or over a stairway, and the like.

Heel. Of a rafter, the end or foot that rests upon the wall plate.

Height. Of an arch, a line drawn from the middle of the chord to the intrados.

Helix. A small volute or twist like a stalk, representing the twisted tops of the acanthus, placed under the abacus of the Corinthian capital.

Hermes. A rough quadrangular stone or pillar, having a head, usually of Hermes or Mercury, sculptured on the top, without arms or body, placed by the Greeks in front of buildings.



HERMES

Herring-bone Work. Bricks, tile, or other materials arranged diagonally in building.

Hexastyle. A portico of six columns in front is of this description.

High Altar. The principal altar in a cathedral or church. Where there is a second, it is generally at the end of the choir or chancel, not in the lady chapel.

Hip-knob. The finial on the hip of a roof, or between the barge boards of a gable.

Hip-roof. A roof which rises by equally inclined planes from all four sides of the building.

Hippodrome. A place appropriated by the ancients for equestrian exercises.

Hips. Those pieces of timber placed in an inclined position at the corners or angles of a hip-roof.

Hood-mold. A word used to signify the drip-stone for label over a window or door opening, whether inside or out.

Hors Coucours. (Fr.) Disqualification from competition.

Hôtel de Ville. The town-hall, or guild-hall, in France, Germany, and Northern Italy. The building, in general, serves for the administration of justice, the receipt of town dues, the regulation of markets, the residence of magistrates, barracks for police, prisons, and all other fiscal purposes. As may be imagined, they differ very much in different towns, but they have almost invariably attached to them or closely adjacent, a large clock-tower containing one or more bells, for calling the people together on special occasions.

Hôtel Dieu. The name for a hospital in mediæval times. In England there are but few remains of these buildings, one of which is at Dover; in France there are many. The most celebrated is the one of Angers, described by Parker. They do not seem to differ much in arrangement of plan from those in modern days, the accommodation for the chaplain, medicine, nurses, stores, etc., being much the same in all ages, except that in some of the earlier, instead of the sick being placed in long wards like galleries, as is now done, they occupied large buildings, with naves and side aisles, like churches.

Housing. The space taken out of one solid to admit the insertion of another. The base on a stair is generally housed into the treads and risers; a niche for a statue.

Hypæthros. A temple open to the air, or uncovered. The term may be the more easily understood by supposing the roof removed from over the nave of a church in which columns or piers go up from the floor to the ceiling, leaving the aisles still covered.

Hypogea. Constructions under the surface of the earth, or in the sides of a hill or mountain.

Ichnography. A horizontal section of a building or other object, showing its true dimensions according to a geometric scale, a ground plan.

Impluvium. The central part of an ancient Roman court, which was uncovered.

Impost. A term in classic architecture for the horizontal moldings of piers or pilasters, from the top of which spring the archivolts or moldings which go round the arch.

In Antis. When there are two columns between the antæ of the lateral walls and the cella.

Inca Architecture. The architecture of the pre-Columbian inhabitants of Peru and neighboring countries.

Incise. To cut in; to carve; to engrave.

Indented. Toothed together.

India, Architecture of. That of the great peninsula bounded on the north by the Himalaya mountains, and reaching from the lower Ganges on the east to the frontiers of Afghanistan on the west. The history of architecture in this region presents many difficult problems, especially as to its origin and early periods. The natural divisions of the art, as of the civilization of India follow religious lines. They comprise the Buddhist, the Jaina, the Brahman, the Mohammedan or Indo-Moslem Styles. A predilection for minute and profuse ornament covering every part of the architecture is a characteristic common to all the divisions. The second characteristic is the multiplication of horizontal bands and lines, giving an appearance as of stratification even to towers and minarets. A third characteristic is the indefinite reduplication and multiplication of the same motive, which is often itself a miniature of the tower or other construction of which it is a part. Lastly, it should be observed that, in Hindu design generally, the dominant masses and chief features of a building are in no wise evolved out of structural requirements, but are purely artistic creations, in which fancy and tradition both have had a share. Modern architecture of India is following the old traditions even with the many buildings which have been erected under the British rule.

Inlaying. Inserting pieces of ivory, metal, or choice woods, or the like, into a groundwork of some other material, for ornamentation.

Insulated. Detached from another building. A church is insulated, when not contiguous to any other edifice. A column is said to be insulated, when standing free from the wall; thus, the columns of peripteral temples were insulated.

Intaglio. A sculpture or carving in which the figures are sunk below the general surface, such as a seal the impression of which in wax is in bas-relief; opposed to Cameo.

Intercolumniation. The distance from column to column, the clear space between columns.

Interlaced Arches. Arches where one passes over two openings, and they consequently cut or intersect each other.

Intrados. Of an arch, the inner or concave curve of the arch stones.

Inverted Arches. Those whose key-stone or brick is the lowest in the arch.

Ionic Order. One of the orders of Classical architecture.

Iron Work. In mediæval architecture, as an ornament, is chiefly confined to the hinges, etc., of doors and of church chests, etc. In some instances not only do the hinges become a mass of scroll work, but the surface of the doors is covered by similar ornaments. In almost all styles the smaller and less important doors had merely plain strap hinges, terminating in a few bent scrolls, and latterly in fleur-de-lis. Escutcheon and ring handles, and the other furniture, partook more or less of the character of the time. On the Continent of Europe the knockers are very elaborate. At all periods doors have been ornamented with nails having projecting heads, sometimes square, sometimes polygonal, and sometimes ornamented with roses, etc. The iron work of windows is generally plain, and the ornament confined to simple fleur-de-lis heads to the stanchions. The iron work of screens enclosing tombs and chapels is noticed under *Grille*, *q.v.*

Ireland, Architecture of. Ireland has preserved vestiges of architectural forms which have almost disappeared from Europe. In her own Gaelic she has curious records of builders' methods in osier, wattle, wood and stone. The Irish did not use mortar, for the stones were carried away for other uses upon abandonment. The outer walls of forts and houses, however, were whitewashed. The beehive huts and round towers are the only distinctive architecture handed down by the ancient Irish. There are many modern picturesque churches, city halls, merchants' houses, abbeys, cathedrals, castles, and ruins of forts and monasteries.

Italian Architecture. The architecture of Italy; more especially that of any style assumed to have taken its origin in Italy. The term is used carelessly with the especial significance of a neoclassic style of great severity and of systematic design and fixed proportion, as if of the Italian sixteenth century. The term also refers to the architecture of the continental part of the kingdom of Italy together with San Marino. All this region is rich in ancient architecture, architectural sculpture, mosaic, and in portable works of art of durable material; also in painting of epochs, beginning with the thirteenth century and with a few remains of early painting. Here are Italian pre-Roman structures with those of Etruscan and Oscan remains of unique value. Grecian architecture of Italy is of peculiar importance. At the same time the historical buildings of Rome are the foundations of our knowledge of this most important epoch in building—the root and stem of all modern European architecture. Latin architecture can be studied only in Italy. There is no country in Europe in which so wide a field of art exists as in Italy, nor any country in the world in which so many styles, each representative and peculiar, are to be seen and studied, side by side.

Jack. An instrument for raising heavy loads, either by a crank, siren and pinion, or by hydraulic power, and in all cases worked by hand.

Jack Rafter. A short rafter, used especially in hip-roofs.

Jamb. The side-post or lining of a doorway or other aperture. The jambs of a window outside the frame are called Reveals.

Jamb-shafts. Small shafts to doors and windows with caps and bases; when in the inside arris of the jamb of a window they are sometimes called Esconsons.

Japan, Architecture of. That of the modern state of that name, and chiefly of the southern islands. Previous to the coming of the first Buddhist priests from Korea in A.D. 552, we know nothing of the architecture of Japan. At that time the islands lay in almost complete barbarism and the efforts at shelter were characteristic of a mingling of Tartar and Malay blood with the primitive

stock. Traces of this primitive mode may be found in the modern Shinto temples, where grotesque attempts at ornament and the preservation of tradition still remain. In 593 when Buddhist priests, architects, scholars and teachers were invited into the country from Korea, the architectural style introduced, already perfectly developed in China, was seized upon by the Japanese and made their own. The buildings are architecturally beautiful solely because of the subtlety of their proportions, dignity of composition, amazing refinement of line, and vitality of curves that characterize them. With the transferring of the capital to Kyoto the second period of Japanese civilization and art began. In architecture the purest Chinese models were followed, while architectural decoration became amazingly rich and splendid. A kind of palace pavilion at Uji, built in 1052, in subtle grace, perfection of curves and refinement of composition, touches the highest point in Japanese architecture. Two centuries of civil conflict and martial activity succeeded this epoch of culture. This was followed by the Kamakura period and, though painting was the favorite mode of artistic expression, architecture reached the last phase of greatness before the first trace of decay showed itself. For two hundred and twenty-five years, in the seventeenth century, when Japan was closed to the world, a great period of industrial development and steady progressing civilization unhampered by wars ensued. It is only in their domestic work that the traditions of true architecture remain.

Joggle. A joint between two bodies so constructed by means of jogs or notches as to prevent their sliding past each other.

Joinery. That branch in building confined to the nicer and more ornamental parts of carpentry.

Joist. A small timber to which the boards of a floor or the laths of ceiling are nailed. It rests on the wall or on girders.

Keep. The inmost and strongest part of a mediæval castle, answering to the citadel of modern times. The arrangement is said to have originated with Gundulf, the celebrated Bishop of Rochester. The Norman keep is generally a very massive square tower, the basement or stories partly below ground being used for stores and prisons. The main story is generally a great deal above ground level, with a projecting entrance, approached by a flight of steps and drawbridge. This floor is generally supposed to have been the guard-room or place for the soldiery; above this was the hall, which generally extended over the whole area of the building, and is sometimes separated by columns; above this are other apartments for the residents. There are winding staircases in the angles of the buildings, and passages and small chambers in the thickness of the walls. The keep was intended for the last refuge, in case the outworks were scaled and the other buildings stormed. There is generally a well in a mediæval keep, ingeniously concealed in the thickness of a wall, or in a pillar. The most celebrated of Norman times are the White Tower in London, the castles at Rochester, Arundel, Newcastle, Castle Hedingham, etc. The keep was often circular.

Key-stone. The stone placed in the center of the top of an arch. The character of the key-stone varies in different orders. In the Tuscan and Doric it is only a simple stone projecting beyond the rest; in the Ionic it is adorned with moldings in the manner of a console; in the Corinthian and Composite it is a rich sculptured console.

King-post. The middle post of a trussed piece of framing for supporting the tie-beam at the middle and the lower ends of the struts.

Knee. A piece of timber naturally or artificially bent to receive another to relieve a weight or strain.

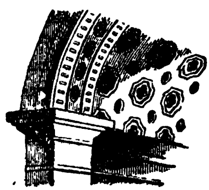
Knob, Knot. The bunch of flowers carved on a corbel, or on a Boss.

Kremlin. The Russian name for the citadel of a town or city.

Label. Gothic: the drip or hood-molding of an arch, when it is returned to the square.

Label Terminations. Carvings on which the labels terminate near the springing of the windows. In Norman times those were frequently grotesque heads of fish, birds, etc., and sometimes stiff foliage. In the Early English and Decorated periods they are often elegant knots of flowers, or heads of kings, queens, bishops, and other persons supposed to be the founders of churches. In the Perpendicular period they are often finished with a short square, mitred return or knee, and the foliages are generally leaves of square or octagonal form.

Lacunar. A paneled or coffered ceiling or soffit. The panels or cassoons of a ceiling are by Vitruvius called lacunaria.


LACUNARS IN CEILING

Lady-chapel. A small chapel dedicated to the Virgin Mary, generally found in ancient cathedrals.

Lancet. A high and narrow window pointed like a lancet, often called a lancet window.

Landing. A platform in a flight of stairs between two stories; the terminating of a stair.

Lantern. A turret raised above a roof or tower and very much pierced, the better to transmit light. In modern practice this term is generally applied to any raised part in a roof or ceiling containing vertical windows, but covered in horizontally. The name was also often applied to the louver or femerall on a roof to carry off the smoke; sometimes, too, to the open constructions at the top of towers, as at Ely Cathedral, probably because lights were placed in them at night to serve as beacons.

Lanterns of the Dead. Curious small slender towers, found chiefly in the center and west of France, having apertures at the top, where a light was exhibited at night to mark the place of a cemetery. Some have supposed that the round towers in Ireland may have served for this purpose.

Lath. A slip of wood used in slating, tiling, and plastering.

Latin Architecture. The early Christian architecture of historic Europe. It is reasonably called Latin, because it was the architecture of the Latin Church, and was developed among the Latin races of Italy. From Italy it passed into Gaul, Germany, and probably into Spain, and even Britain. Beginning with Constantine, it held its place from the fourth century to the eighth, and only in the ninth began to give way to Romanesque, which grew up under the impulse of the Teutonic races, and which, displacing it, was the architecture of the Western Church from the tenth century till the end of the twelfth. The earliest Christian Churches started with the simple outlines of the basilicas. Another family of Latin Churches was round or polygonal, and often surrounded by an aisle. The form that grew out of this by the addition of four arms of a cross naturally led to a square center, over which a dome was set almost from the beginning. The only ornament of the early churches was in their altars and the surrounding furniture, which were lavishly enriched, and in the pictorial decoration of their surfaces. The chief of this was mosaic, which began at Constantinople, spread to Ravenna, and became the one

artistic achievement of Rome, where in Italy it centered. It was applied first to the apses; in the richer churches it was spread over the interior walls, and in a simpler ornamental form over the floors. When mosaic could not be had, or was too costly, painting was habitually substituted; the interiors were brilliant with Bible stories and legends of the Church. In many this decoration overflowed upon the outside, and the fronts, usually smoothly stuccoed over, were covered with mosaic or painting.

Lattice. Any work of wood or metal made by crossing laths, rods, or bars, and forming a net-work. A reticulated window, made of laths or slips of iron, separated by glass windows, and only used where air rather than light is to be admitted, as in cellars and dairies.

Lavabo. The lavatory for washing hands, generally erected in cloisters of monasteries. A very curious one at Fontenay, surrounding a pillar, is given by Viollet-le-Duc. In general, it is a sort of trough, and in some places has an almry for towels, etc.

Lavatory. A place for washing the person.

Lean-to. A small building whose rafters pitch or lean against another building, or against a wall.

Lectern. The reading-desk in the choir of churches.

Ledge, or Ledgement. A projection from a plane, as slips on the side of window and door frames to keep them steady in their places.

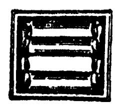
Ledgers. The horizontal pieces fastened to the standard poles or timbers of scaffolding raised around buildings during their erection. Those which rest on the ledgers are called putlogs, and on these the boards are laid.

Lewis. An iron clamp dovetailed into a large stone to lift it by.

Lich-gate. A covered gate at the entrance of a cemetery, under the shelter of which the mourners rested with the corpse, while the procession of the clergy came to meet them. There are several examples in England.

Light. A division or space in a sash for a single pane of glass; also a pane of glass.

Linen Scroll. An ornament formerly used for filling panels, and so called from its resemblance to the convolutions of a folded napkin.



LINEN SCROLL

Lining. Covering for the interior, as casing is covering for the exterior surface of a building; also, such as linings of a door for windows, shutters, and similar work.

Lintel. The horizontal piece which covers the opening of a door or window.

Lip Mold. A molding of the Perpendicular period like a hanging lip.

List, or Listel. A little square molding, to crown a larger; also termed a fillet.

Lithograph. A print from a drawing on stone.

Lobby. An open space surrounding a range of chambers, or seats in a theater; a small hall or waiting room.

Lodge. A small house in a park.

Loft. The highest room in a house, particularly if in the roof; also, a gallery raised up in a church to contain the rood, the organ, or singers.

Loggia. An outside gallery or portico above the ground, and contained within the building.

Lombard Architecture. A name given to the round-arched architecture of Italy, introduced by the conquering Goths and Ostrogoths, and which superseded the Romanesque. It reigned between the eighth and twelfth centuries, during the time that the Saxon and Norman styles were in vogue in England, and corresponded with them in its development into the Continental Gothic.

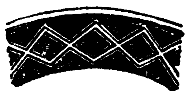
Loop-hole. An opening in the wall of a building, very narrow on the outside and splayed within, from which arrows or darts might be discharged on an enemy. They are often in the form of a cross, and generally have round holes at the ends.

Lotus. A plant of great celebrity among the ancients, the leaves and blossoms of which generally form the capitals of Egyptian columns.

Louver. A kind of vertical window, frequently in the peaks of gables, and in the top of towers, and provided with horizontal slats which permit ventilation and exclude rain.

Lozenge Molding. A kind of molding used in Norman architecture, of many different forms, all of which are characterized by lozenge-shaped ornaments

Lunette. The French term for the circular opening in the groining of the lower stories of towers, through which the bells are drawn up.



LOZENGE MOLDING

LOUVER WINDOW

Machicolation. A parapet or gallery projecting from the upper part of the wall of a house or fortification, supported by brackets or corbels, and perforated in the lower part so that the defenders of the building might throw down darts, stones, and sometimes hot sand, molten lead, etc.; upon their assailants below.

Man-hole. A hole through which a man may creep into a drain, cesspool, steam-boiler, etc.

Manor-house. The residence of the suzerain or lord of the manor; in France the central tower or keep of a castle is often called the *manoir*.

Mansard Roof. Curb roof, invented by François Mansard, a distinguished French architect, who died in 1666.

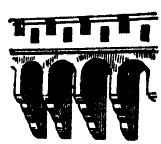
Mansion. A residence of considerable size and pretension.

Mantel. The work over a fireplace in front of a chimney; especially, a shelf, usually ornamented, above the fireplace.

Marquetry. Inlaid work of fine hard pieces of wood of different colors, also of shells, ivory, and the like.

Mausoleum. A magnificent tomb or sumptuous sepulchral monument.

Maya Architecture. The architecture of the pre-Columbian inhabitants of Yucatan and adjacent regions.



MACHICOLATION

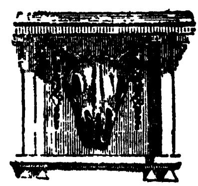
Medallion. Any circular tablet on which are embossed figures or busts.

Mediæval Architecture. The architecture of England, France, Germany, etc., during the Middle Ages, including the Norman and Early Gothic styles. It comprises also the Romanesque, Byzantine and Saracenic, Lombard, and other styles.

Members. The different parts of a building, the different parts of an entablature, the different moldings of a cornice, etc.

Merlon. That part of a parapet which lies between two embrasures.

Mesopotamia, Architecture of. That of the great plain between the rivers Tigris and Euphrates, which was the seat of a civilization which is perhaps as ancient as that of Egypt. The countries in which developed these two most ancient civililzations are in many respects strikingly similar. The southern portion of the Mesopotamia plain is Chaldea; its principal cities in the earliest times were Ur and Larsam, and later the more northerly Babylon. The northern portion of the plain is Assyria, whose capital in earliest times was Assur, and later Nineveh. (1) *Chaldean or Babylonian:* The absence of stone forced builders to the development of brick construction. The temples seem to have been vast stepped pyramids, rising sometimes to a height comparable to that of the great Egyptian pyramids, if not higher, with a shrine at the top. The palaces were erected on great platforms, and consisted of a collection of apartments about courtyards. (2) *Assyrian:* Although building in a more hilly country the Assyrians still continued to raise their palaces upon platforms of earth or crude brick as did the Chaldeans; but they faced these mounds with retaining walls of massive cut stone easily obtained and brought down the Tigris from the neighboring mountains. Their temples seem to have been similar in character to, though smaller in scale than, the Babylonian temples. That the Assyrian builders were thoroughly familiar with the vault is clear, from the city gates and the vaulted drains beneath the palaces. If a method of vault construction was used, as seems probable, it becomes easier to explain the existence of the great vaults of similar construction in Persia; and we get a glimpse of a continuous history of vault construction in the East from the time of the Assyrian to the time of the Byzantine empire; a system of construction radically different from that employed in the West.


METOPE

Metope. The square recess between the triglyphs in a Doric frieze. It is sometimes occupied by sculptures.

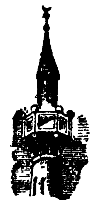
Mexico, Architecture of. That of the Republic as it has existed since 1854. *Pre-Columbian:* Within the borders of Mexico is contained almost the entire range of aboriginal American architecture. At one end of the scale are shelters of the rudest kind, and at the other the really extraordinary stone ruins of Yucatan. In the wide intervals are cliff dwellers, houses of jacál, of pisé, of cajon, and of adobe bricks. In their own way, ethnologically, archæologically, and artistically, they are of surpassing interest, exhibiting, as they do, one of the most extraordinary race developments of the world. It is fairly well established that the strictly Aztec structures are comparatively modern. *Modern:* The architecture of Mexico has been developed in a dry equable climate, necessitating protection from the sun rather than from cold or rain, by a race which mingles the polish and traditions of Europe with the barbaric taste of the Indian. Although stone serves for the best work, a native clay is most commonly used; as sun-dried brick it is known as adobe; it further serves when kiln-dried as terra-cotta ornament, roofing tiles, brick, etc., and colored glazed tile are also made of it. It is often applied to masonry in the form of stucco and plaster, and such walls are frequently covered with colored washes. Wrought iron in the form of balconies and grilles is common, but wood is scarce, and rarely used in building. The work is unique in its boldness of color and outline, as shown in its churches, patios, portals, markets, government buildings, fountains, and aqueducts.

Mezzanine. A low story between two lofty ones. It is called by the French *entresol,* or inter-story.

Mezzo-rilievo. Or mean relief, in comparison with alto-rilievo, or high relief.

Minaret. Turkish: a circular turret rising by different stages or divisions, each of which has a balcony.

Minster. Probably a corruption of monasterium — the large church attached to any ecclesiastical fraternity. If the latter be presided over by a bishop, it is generally called a Cathedral; if by an abbot, an Abbey; if by prior, a Priory.


MINARET

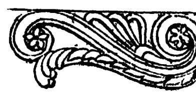
Minute. The sixtieth part of the lower diameter of a column; it is the measure used by architects to determine the proportions of an order.

Miserere. A seat in a stall of a large church made to turn up and afford support to a person in a position between sitting and standing. The under side is generally carved with some ornament, and very often with grotesque figures and caricatures of different persons.

Mission Architecture. Pertaining to a building or group of buildings composing a frontier or outpost settlement of one of the Spanish religious orders in America. At first these missions were rude huts, but they developed into extensive structures in the form of a quadrilateral building surrounding a court. There were churches, hospitals, workshops,. schoolrooms, etc. They were of simple outline with artistic masses. Large stone basins, which serve as fountains, are commonly found in the cloisters or near the entrance doorways of the missions.

Miter. A molding returned upon itself at right angles is said to miter. In joinery, the ends of any two pieces of wood of corresponding form, cut off at 45°, necessarily abut upon one another so as to form a right angle, and are said to miter.

Modillion. So called because of its arrangement in regulated distances; the enriched block or horizontal bracket generally found under the cornice of the Corinthian entablature. Less ornamented, it is sometimes used in the Ionic.

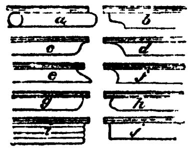

MODILLION

Module. This is a term which has been generally used by architects in determining the relative proportions of the various parts of a columnar ordinance. The semi-diameter of the column at its base is the module, which, being divided into thirty parts called minutes, any part of the composition is said to be of so many modules and minutes, or minutes alone, in height, breadth or projection. The whole diameter is now generally preferred as a module, it being a better rule of proportion than its half.

Mogul Architecture. (1) The architecture of the Mogul dynasty. This was established in northwest India in the sixteenth century. The characteristics are very different from those of the purely Indian styles. (2) Loosely, and because of the little study which the Mohammedan architecture in India has received, any architecture of Mohammedan races or dynasties in India.

Molding. When any work is wrought into long regular channels or projections, forming curves or rounds, hollows, etc., it is said to be molded, and each separate member is called a molding. In mediæval architecture the principal

moldings are those of the arches, doors, windows, piers, etc. In the Early English style, the moldings, for some time, formed groups set back in squares, and frequently very deeply undercut. The scroll molding is also common. Small fillets now become very frequent in the keel moldings, from its resemblance in section to the bottom of a ship; sometimes, also, it has a peculiar hollow on each side, like two wings. Later in the Decorated style the moldings are more varied in design, though hollows and rounds still prevail. The undercutting is not so deep, fillets abound, ogees are more frequent, and the wave mold, double ogee, or double ressaunt, is often seen. In many places the strings and labels are a round, the lower half of which is cut off by a plain chamfer. The moldings in the later styles in some degree resemble those of the Decorated, flattened and extended; they run more into one another, having fewer fillets, and being, as it were, less grouped. One of the principal features of the change is the substitution of one, or perhaps two (seldom more), very large hollows in the set of moldings. These hollows are neither circular nor elliptical, but obovate, like an egg cut across, so that one half is larger than the other. The brace mold also has a small bead, where the two ogees meet. Another sort of molding, which has been called a lip mold, is common in parapets, bases, and weatherings.



MOLDINGS

a, astragal; *b*, ogee; *c*, cymatium; *d*, cavetto; *e*, scotia, or casement; *f*, apophyges; *g*, ovolo, or quarter round; *h*, torus; *i*, reeding; *j*, band.

Moldings, Ornamented. The Saxon and early Norman moldings do not seem to have been much enriched, but the complete and later styles of Norman are remarkable for a profusion of ornamentation, the most usual of which is what is called the zigzag. This seems to be to Norman architecture what the meander or fret was to the Grecian; but it was probably derived from the Saxons, as it is very frequently found in their pottery. Bezants, quatrefoils, lozenges, crescents, billets, heads of nails, are very common ornaments. Besides these, battlements, cables; large ropes round which smaller ropes are turned, or, as our sailors say, " wormed "; scallops, pellets, chains, a sort of conical barrels, quaint stiff foliages, beaks of birds, heads of fishes, ornaments of almost every conceivable kind, are sculptured in Norman moldings; and they are used in such profusion as has been attempted in no other style. The decorations on Early English moldings are chiefly the dog-tooth, which is one of the great characteristics of this style, though it is to be found in the Transition Norman. It is generally placed in a deep hollow between two projecting moldings, the dark shadow in the hollow contrasting in a very beautiful way with the light in these moldings. In this period and in the next the tympanum over doorways, particularly if they are double doors, is highly ornamented. Those of the Decorated period resemble the former, except that the foliage is more natural and the dog-tooth gives way to the ball-flower. Some of the hollows, also, are ornamented with rosettes set at intervals, which are sometimes connected by a running tendril, as the ball-flowers are frequently. Some very pleasing leaf-like ornaments in the labels of windows are often found in Continental architecture. In the Perpendicular period the moldings are ornamented very frequently by square four-leaved flowers set at intervals, but the two characteristic ornaments of the time are running patterns of vine leaves, tendrils, and grapes in the hollows, which by old writers are called

"vignettes in casements," and upright stiff leaves, generally called the Tudor leaf. On the Continent moldings partook much of the same character.

Monastery. A set of buildings adapted for the reception of any of the various orders of monks, the different parts of which are described in the separate article, *Abbey.*

Monastic Architecture. The architecture of religious organizations having permanent rules of conduct and life, and divided into considerable bodies of men or women who devote themselves in common to a life of worship and labor. Many religions have recognized these religious bodies which, when in Christian nations, are called Orders of Monks and of Nuns; terms which are used also in an extended sense as applicable to similar communities in other religions.

Monotriglyph. The intercolumniations of the Doric order are determined by the number of triglyphs which intervene, instead of the number of diameters of the column, as in other cases; and this term designates the ordinary intercolumniation of one triglyph.

Moorish Architecture. The architecture of the Mohammedan races of North Africa and of the Kindgoms which they established in Spain. The work of these last-named states may be designated Hispano-Moresque.

Mosaic. Pictorial representations, or ornaments, formed of small pieces of stone, marble, or enamel of various colors. In Roman houses the floors are often entirely of mosaic, the pieces being cubical. The best examples of mosaic work are found in St. Mark's, at Venice.

Moslem Architecture. The architecture of the Mohammedans, particularly of the Arab conquerors in Syria and Egypt, of the Moors in North Africa and Spain, of the Persians, of the Mohammedans of India, and of the Turks. Except in Persia, Moslem architecture has everywhere begun with Christian models. The Arabs who conquered the Mediterranean littoral in the seventh century were no artists; neither were the Osmauli Turks, nor the Moguls who overran India and Persia. Bringing with them no artistic traditions and no established types, they employed Coptic, Syrian, Greek and Byzantine builders to erect and decorate their mosques and palaces in the prevailing local styles. Civilized in time by their contact with the Western world, they developed, in the course of the centuries, decided predilections and aptitudes in design, by which their art was given a form and spirit wholly unlike those of the arts on which it was based, emphatically Oriental in character. Moslem architecture started under Arab dominion, early in the seventh century.

Mosque. A Mahometan temple, or place of worship.

Mullion, Munion. The perpendicular pieces of stone, sometimes like columns, sometimes like slender piers, which divide the bays or lights of windows or screen-work from each other. In all styles, in less important work, the mullions are often simple plain chamfered, and more commonly have a very flat hollow on each side. In larger buildings there is often a bead or boutell on the edge, and often a single very small column with a capital. As tracery grew richer, the windows were divided by a larger order of mullion, between which came a lesser or subordinate set of mullions, which ran into each other. The term is also applied to a wood or iron division between two windows.

Multifoil. A leaf ornament consisting of more than five divisions, applied to foils in windows.

Muntin. A small, slender mullion in light framing, as a sash bar, a middle stile of a door.

Mutule. The rectangular impending block under the corona of the Doric cornice, from which guttæ or drops depend. Mutule is equivalent to modillion but the latter term is applied more particularly to enriched blocks or brackets, such as those of Ionic and Corinthian entablatures.

Narthex. The long arcaded porch forming the entrance into the Christian basilica. Sometimes there was an inner narthex, or lobby, before entering the church. When this was the case, the former was called exo-narthex, and the latter eso-narthex. In the Byzantine churches this inner narthex forms part of the solid structure of the church, being marked off by a wall or row of columns, whereas in the Latin churches it was usually formed only by a wooden or other temporary screen.

Natural Beds. In stratified rocks, the surface of a stone as it lies in the quarry. If not laid in walls in their natural bed the laminæ separate.

Nave. The central part between the arches of a church, which formerly was separated from a chancel or choir by a screen. It is so called from its fancied resemblance to a ship. In the nave were generally placed the pulpit and font. In continental Europe it often also contains a high altar, but this is of rare occurrence in England.

Necking. The annulet or round, or series of horizontal moldings, which separate the capital of a column from the plain part or shaft.

Neoclassic Architecture. The architecture of modern times, beginning with the Italian Renaissance of the fifteenth century; and especially that which is carefully studied from Greco-Roman examples.

Netherlands, Architecture of. The architecture of the low-lying country at the mouth of the Rhine, where this river and the Meuse unite, and also divide and subdivide into many branches. Its common name among foreigners is Holland, which is properly the name of the largest and wealthiest of the little states of which it was formerly composed. The country inevitably followed closely the immensely powerful impulse given by the architecture of the French lands lying not far to the south. The Protestant Reformation, which in the sixteenth century divided the state as above described from Belgium, came too late to affect the growth of architecture in those styles which were swayed by religious enthusiasm. The Dutch seventeenth century architecture is unique and altogether national in character, owing to mingled influences of domestic feeling and the northern climate. The attraction in all the country is nearly the same as in Amsterdam, namely, the fascinating private houses, town halls, weighing houses, etc. For the study of architecture, domestic and civic of the simpler kind, no country in Europe surpasses the Netherlands.

Newel. In mediæval architecture, the circular ends of a winding staircase which stand over each other, and form a sort of cylindrical column.

Newel Post. The post, plain or ornamented, placed at the first, or lowest step, to receive or start the hand-rail upon.

Niche. A recess sunk in a wall, generally for the reception of a statue. Niches sometimes terminate by a simple label, but more commonly by a canopy, and with a bracket or corbel for the figure, in which case they are often called tabernacles.

Norman Style. Was that species of Romanesque which was practised by the Normans, and which was introduced and fully developed in England after they had established themselves in it. The chief features of this style are plainness and massiveness. The arches, windows, and doorways were semi-

circular, the pillars were very massive, and often built up of small stones laid like brickwork.

Norse American Architecture. The architecture in the western hemisphere which shows Scandinavian influence. A few examples of early Norse buildings are found in ruins in southern Greenland, and there are traces in Labrador.

North African Architecture. The architecture of the vast tract of country bordering on the Mediterranean, comprising Tripoli, Tunisia, Algeria, and Morocco.

Nosings. The rounded and projecting edges of the treads of a stair, or the edge of a landing.

Obelisk. Lofty pillars of stone, of a rectangular form, diminishing toward the top, and generally ornamented with inscriptions and hieroglyphics among the ancient Egyptians.

Observatory. A building erected on an elevated spot of ground for making astronomical observations.

Octostyle. A portico of eight columns in front.

Offsets. When the face of a wall is not one continued surface, but sets in by horizontal jogs, as the wall grows higher and thinner, the jogs are called offsets.

Ogee. The name applied to a molding, partly a hollow and partly a round, and derived no doubt from its resemblance to an O placed over a G. It is found in Norman work, and is not very common in Early English. It is of frequent use in Decorated work, where it becomes sometimes double, and is called a wave molding; and later still, two waves are connected with a small bead, which is then called a brace molding. In ancient MSS. it is called a Ressaunt.

Orchestra. In ancient theaters, where the chorus used to dance; in modern theaters, where the musicians sit.

Order. A column with its entablature and stylobate is so called. The term is the result of the dogmatic laws deduced from the writings of Vitruvius, and has been exclusively applied to those arrangements which they were thought to warrant.

Oriel Window. Gothic: a projecting angular window, commonly of a trigonal or pentagonal form, and divided by mullions and transoms into different bays and compartments.

Orthography. A geometrical elevation of a building or other object in which it is represented as it actually exists or may exist, and not perspectively, or as it would appear.

Orthostyle. A columnar arrangement in which the columns are placed in a straight line.

Ovolo. Same as *Echinus*.

Oylet. A small hole, an eyelet.

Pagoda. A name given to temples in India and China.

Palace. The dwelling of a king, prince, or bishop.

Pale. A fence picket, sharpened at the upper end.

Palladian Architecture. Relating to the art or style of Palladio. This work belonged to the period of decline in Italian neoclassic architecture, when classic formality and the punctilious observance of rules were taking the place of the grace, freedom, and life which characterized the earlier period. The classicism of Palladio was noted for a certain cold and correct purity of

form. A favorite motive of his, known as the Palladian Motive, was the use of a minor and major order of columns in the same composition, the former supporting the arches which occurred between the latter, as in his two-storied arcade about the mediæval basilica of Vicenza. His writings had the good fortune to be considered the most authoritative expositions of the principles of classic architecture in the seventeenth and eighteenth centuries throughout Europe, and his monuments were models for the classic art of that period; the name Palladian, therefore, is descriptive of that variety of neoclassic architecture distinguished for cold, inelastic, and unimaginative, but correct, elegant, and studied classicism. It is generally held as true that the English classical revivalists followed Palladio, whereas the French were rather under the influence of Vignola. *Andrea Palladio, Architect:* born about 1518; died August, 1580.

Pane. Probably a diminutive of panneau, a term applied to the different pieces of glass in a window; same as *Light.*

Panel. Properly a piece of wood framed within four other pieces of wood, as in the styles and rails of a door, filling up the aperture, but often applied both to the whole square frame and the sinking itself; also to the ranges of sunken compartments in wainscoting, cornices, corbel tables, groined vaults, ceilings, etc.

Pantograph, or **Pentagraph.** An instrument for copying on the same, or an enlarged or reduced scale.

Pantry. An apartment or closet in which bread and other provisions are kept.

Papier-maché. A hard substance made of a pulp from rags or paper mixed with size or glue, and molded into any desired shape. Much used for architectural ornaments.

Parapet. A dwarf wall along the edge of a roof, or round a terrace walk, etc., to prevent persons from falling over, and as a protection to the defenders in case of a siege. Parapets are either plain, embattled, perforated, or paneled. The last two are found in all styles except the Norman. Plain parapets are simply portions of the wall generally overhanging a little, with coping at the top and corbel table below. Embattled parapets are sometimes paneled, but oftener pierced for the discharge of arrows, etc. Perforated parapets are pierced in various devices—as circles, trefoils, quatrefoils and other designs—so that the light is seen through. Paneled parapets are those ornamented by a series of panels, either oblong or square, and more or less enriched, but are not perforated. These are common in the Decorated and Perpendicular periods.

Pargeting. A species of plastering decorated by impressing patterns on it when wet. These seem generally to have been made by sticking a number of pins in a board in certain lines or curves, and then pressing on the wet plaster in various directions, so as to form geometrical figures. Sometimes these devices are in relief, and in the time of Elizabeth represent figures, birds, foliages, etc. Rough plastering, commonly adopted for the interior surface of chimneys.

Parging. Thin coat of plastering to smooth off rough brick or stone walls.

Parlor. A room in a house which the family usually occupy for society and conversation, and for receiving visitors. The apartment in a monastery or nunnery where the inmates are permitted to meet and converse with each other, or with visitors and friends from without.

Parochial. Belonging or relating to a parish.

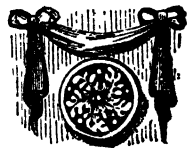
Parquetry, or **Marquetry.** A kind of inlaid floor composed of small pieces of wood either square or triangular, which are capable of forming, by their disposition, various combinations of figures; this description of joinery is very suitable for the floors of libraries, halls, and public apartments.

Parthian Architecture. The architecture of the peoples subjected to the Parthian dynasty, which lasted for nearly five centuries, from 250 B.C. to A.D. 226. The Parthians were not a building race, but details unearthed of various decorative features with the remains of palaces and temples help to reconstruct parts of these buildings, with the vaulted rooms and the carvings of the sun god, moon, bulls' heads and griffins at the entrances.

Parti. (Fr.) Scheme.

Party Walls. Partitions of brick or stone between buildings on two adjoining properties.

Patera. A circular ornament resembling a dish, often worked in relief on friezes, etc.


PATERA

Pavilion. A turret or small insulated building, and comprised beneath a single roof; also, the projecting part in front of a building which marks the center, and which sometimes flanks a corner, when it is termed an angular pavilion.

Pedestal. The square support of a column, statue, etc., and the base or lower part of an order of columns; it consists of a plinth for a base, the die, and a talon crowned for a cornice. When the height and width are equal, it is termed a square pedestal; one which supports two columns, a double pedestal; and if it supports a row of columns without any break, it is a continued pedestal.

Pediment. A low triangular crowning, ornamented, in front of a building, and over doors and windows. Pediments are sometimes made in the form of a segment; the space enclosed within the triangle is called the tympanum. Also, the gable ends of classic buildings, where the horizontal cornice is carried across the front, forming a triangle with the end of the roof.

Pendent. A name given to an elongated boss, either molded or foliated, such as hang down from the intersection of groins, especially in fan tracery, or at the end of hammer beams. Sometimes long corbels, under the wall pieces, have been so called. The name has also been given to the large masses depending from enriched ceilings, in the later works of the Pointed style.

Pendent Posts. A name given to those timbers which hang down the side of a wall from the plate in hammer beam trusses, and which receive the hammer braces.

Pendentive. A name given to an arch which cuts off, as it were, the corners of a square building internally, so that the superstructure may become an octagon or a dome. In mediæval architecture these arches, when under a spire in the interior of a tower, are called Squinches.

Pendentive Bracketing, or **Cove Bracketing.** Springing from the rectangular walls of an apartment upward to the ceiling, and forming the horizontal part of the ceiling into a circle or ellipse.

Pentastyle. Having five columns in front.

Pent-roof. A roof with a slope on one side only.

Perch. A measure used in measuring stone work, being $24\frac{3}{4}$ cu ft and $16\frac{2}{3}$ cu ft, according to locality and custom.

Periptery. An edifice or temple surrounded by a peristyle.

Peristyle. A range of columns encircling an edifice, such as that which surrounds the cylindrical drum under the cupola of St. Paul's. The columns of a Greek peripteral temple form a peristyle also, the former being a circular, and the latter a quadrilateral peristyle.

Perpendicular Style. The third and last of the Pointed or Gothic style; also called the Florid style.

Persian Architecture. The architecture of the lands included in the modern kingdom of Persia, and those immediately adjoining which have received and retained Persian artistic influence. Architecturally, Persia may be considered under the following rules: (1) Darius, Xerxes, and their successors, 500 to 334 B.C. (2) Alexander and his successors, starting 334 B.C. (3) Parthian rule in the second and third centuries A.D. (4) Sassanians, A.D. 226 until the Moslem conquest. (5) The Moslem rule, resulting about the ninth century in an independent Mohammedan Kingdom. The later conquests, as of the Seljuks and by Genghis Khan do not alter the national character of the Persian Mohammedan art. The origin of the architecture of Persia must be sought for in that of the two countries conquered by her, viz., Babylon and Media. From the former she derived the raised platform, or terrace, on which her palaces were built, and the winged bulls. To the Medes she owed her porticoes and halls of columns. The precise position of Persian art in the history of the Mohammedan styles is debatable but some scholars consider it the fountain head. The varied buildings are treated with remarkable uniformity of style. The great mosques, medresseh, and caravanseries are built around large courts, usually square or rectangular, entered by an imposing vaulted portal and surrounded by arcades, usually in two stories, which give access to the various chambers, rooms, or cells behind them. As to details, it should be noted that neither the horseshoe nor the cusped arch prevails in Persia. The equilateral pointed arch occurs but rarely; the characteristic Persian arch resembles somewhat the Tudor arch of the Perpendicular. In the matter of ornament the Persians surpassed the western Moslems in ceramic decoration and in the flowing grace and freedom of their patterns as far as they were inferior to them in variety and splendor of ornament. It was Persian artists who carried the art of wall tiling into Turkey, Egypt, Sicily and Spain. The Persians are adepts in a certain kind of formal gardening, which adds to the effect of their palaces and squares and which was carried to India by Persian artists in the train of the Moguls in the sixteenth century.

Perspective Drawing. The art of making such a representation of an object upon a plane surface as shall present precisely the same appearance that the object itself would to the eye situated at a particular point.

Pews. A word of uncertain origin, signifying fixed seats in churches, composed of wood framing, mostly with ornamented ends. They seem to have come into general use early in the reign of Henry VI, and to have been rented and "well paid for" before the Reformation. Some bench ends are certainly of a decorated character, and some have been considered to be of the Early English period. They are sometimes of plain oak board, two and a half to three inches thick, chamfered, and with a necking and finial, generally called a poppy head; others are plainly paneled with bold cappings; in others the panels are ornamented with tracery or with the linen pattern, and sometimes with running foliages. The divisions are filled in with thin chamfered boarding, sometimes reaching to the floor, and sometimes only from the capping to the seat.

Picket. A narrow board, often pointed, used in making fences; a pale or paling.

Pier-glass. A mirror hanging between windows.

Piers. The solid parts of a wall between windows, and between voids generally. The term is also applied to masses of brickwork or masonry which are insulated to form supports to gates or to carry arches, posts, girders, etc.

Pilasters. Are flat square columns, attached to a wall, behind a column, or along the side of a building, and projecting from the wall about a fourth or a sixth part of their breadth. The Greeks had a slightly different design for the capitals of pilasters, and made them the same width at top as at bottom, but the Romans gave them the same capitals as the columns, and made them of diminished width at the top, similar to the columns.

Pile. A large stake or trunk of a tree, driven into soft ground, as at the bottom of a river, or in made land, for the support of a building.

Pillar, or Pyller. A word generally used to express the round or polygonal piers, or those surrounded with clustered columns, which carry the main arches of a building. Saxon and Early Norman pillars are generally stout cylindrical shafts built up of small stones. Sometimes, however, they are quite square, sometimes with other squares breaking out of them (this is more common in French and German work), sometimes with angular shafts, and sometimes they are plain octagons. In Romanesque Norman work the pillar is sometimes square, with two or more semicircular or half-columns attached. In the Early English period the pillars become loftier and lighter, and in most important buildings are a series of clustered columns, frequently of marble, placed side by side, sometimes set at intervals round a circular center, and sometimes almost touching each other. These shafts are often wholly detached from the central pillar, though grouped round it, in which case they are almost always of Purbeck or Bethersden marbles. In Decorated work the shafts on plan are very often placed round a square set anglewise, or a lozenge, the long way down the nave; the center or core itself is often worked into hollows or other moldings, to show between the shafts, and to form part of the composition. In this and the latter part of the previous style there is generally a fillet on the outer part of the shaft, forming what has been called a keel molding. They are also often, as it were, tied together by bands formed of rings of stone and sometimes of metal. The small pillars at the jambs of doors and windows, and in arcades, and also those slender columns attached to pillars, or standing detached, are generally called shafts.

Pin. A cylindrical piece of wood, iron, or steel, used to hold two or more pieces together, by passing through a hole in each of them, as in a mortise and tenon joint, or a pin joint of a truss.


PINNACLE

Pinnacle. An ornament originally forming the cap or crown of a buttress or small turret, but afterwards used on parapets at the corners of towers and in many other situations. It was a weight to counteract the thrust of the groining of roofs, particularly where there were flying buttresses; it stopped the tendency to slip of the stone copings of the gables, and counterpoised the thrust of spires; it formed the piers to steady the elegant perforated parapets of later periods; and in France, especially, served to counterbalance the weight of overhanging corbel tables, huge gargoyles, etc. In the Early English period the smaller buttresses

frequently finished with gablets, and the more important with pinnacles supported with clustered shafts. At this period the pinnacles were often supported on these shafts alone, and were open below; and in larger work in this and the subsequent periods they frequently form niches and contain statues. In France, pinnacles, like spires, seem to have been in use earlier than in England. There are small pinnacles at the angles of the tower in the Abbey of Saintes. At Roullet there are pinnacles in a similar position, each composed of four small shafts, with caps and bases surmounted with small pyramidal spires. In all these examples the towers have semicircular headed windows.

Pitch of a Roof. The proportion obtained by dividing the height by the span; thus, we speak of its being one-half, one-third, one-fourth. When the length of the rafters is equal to the breadth of the building it is denominated Gothic.

Pitching-piece. A horizontal timber, with one of its ends wedged into the wall at the top of a flight of stairs, to support the upper end of the rough strings.

Place. An open piece of ground surrounded by buildings, generally decorated with a statue, column, or other ornament.

Plan. A horizontal geometrical section of the walls of a building; or indications, on a horizontal plane, of the relative positions of the walls and partitions, with the various openings, such as windows and doors, recesses and projections, chimneys and chimney-breasts, columns, pilasters, etc. This term is often incorrectly used in the sense of Design.

Planceer. Is sometimes used in the same sense as soffit, but is more correctly applied to the soffit of the corona in a cornice.

Plastering. A mixture of lime, hair, and sand, to cover lath-work between timbers or rough walling, used from the earliest times, and very common in Roman work. In the Middle Ages, too, it was used not only in private, but in public constructions. On the inside face of old rubble walls it was not only used for purposes of cleanliness, rough work holding dirt and dust, but as a ground for distemper painting (tempera, or, as it is often improperly called, fresco), a species of ornaments often used in the Middle Ages. At St. Albans Abbey, England, the Norman work is plastered, and covered with lines imitating the joints of stone. The same thing is found in English Perpendicular work. On the outside of rubble walls, and often of wood framing, it was used as roughcast; when ornamented in patterns outside, it is called pargeting.

Plate. The piece of timber in a building which supports the end of the rafters.

Plinth. The square block at the base of a column or pedestal. In a wall, the term plinth is applied to the projecting base or water table, generally at the level of the first floor.

Plumb. Perpendicular; that is, standing according to a plumb line, as, the post of a house or wall is plumb.

Plumbing. The lead and iron pipes and other apparatus employed in conveying water, and for toilet purposes in a building; originally the art of casting and working in lead.

Ply. Used to denote the number of thicknesses of roofing paper, as three ply, four ply, etc.

Podium. A continued pedestal; a projection from a wall, forming a kind of gallery.

Polytriglyph. An intercolumniation in the Doric order of more than two triglyphs.

Poppy Heads. Probably from the French *poupée:* the finials or other ornaments which terminate the tops of bench ends, either to pews or stalls. They are sometimes small human heads, sometimes richly carved images, knots of foliage, or finials, and sometimes fleurs-de-lis simply cut out of the thickness of the bench end and chamfered.

POPPY HEAD

Porch. A covered erection forming a shelter to the entrance door of a large building. The earliest known are the long arcaded porches in front of the early Christian basilicas, called Narthex. In later times they assume two forms—one, the projecting erection covering the entrance at the west front of cathedrals, and divided into three or more doorways, etc ; and the other, a kind of covered chambers open at the ends, and having small windows at the sides as a protection from rain.

Portal. A name given to the deeply recessed and richly decorated entrance doors to the cathedrals in Continental Europe.

Portcullis. A strong framed grating of oak, the lower points shod with iron, and sometimes entirely made of metal, hung so as to slide up and down in grooves with counterbalances, and intended to protect the gateways of castles, etc.

Portico. An open space before the door or other entrance to any building, fronted with columns. A portico is distinguished as prostyle or *in antis* according as it projects from or recedes within the building, and is further designated by the number of columns its front may consist of.

Portugal, Architecture of. The architecture of the westernmost of all the states of Europe, now a republic. Although there are a few buildings here which date from the tenth century, it is almost impossible to trace through them any continuous architectural development, for the numerous earthquakes and hostile invasions have made havoc. There is a decorative feature of Portuguese architecture which deserves mention, namely, the extensive use of wall tiling.

Post. Square timber set on end. The term is especially applied to those which support the corners of a building, and are framed into bressummers or crossbeams under the walls.

Posticum. A portico behind a temple.

Presbytery. A work applied to various parts of large churches in a very ambiguous way. Some consider it to be the choir itself; others, what is now named the sacrarium. Traditionally, however, it seems to be applied to the vacant space between the back of the high altar and the entrance to the lady-chapel, as at Lincoln and Chichester; in other words, the back- or retro-choir.

Priming. The laying on of the first shade of color, in oil paint, and generally consisting mostly of oil, to protect and fill the wood.

Priory. A monastic establishment, generally in connection with an abbey, and presided over by a prior, who was a subordinate to the abbot, and held much the same relation to that dignitary as a dean does to a bishop.

Profile. The outline; the contour of a part, or the parts composing an order, as of a base, cornice, etc.; also, the perpendicular section. It is in the just proportion of their profiles that the chief beauties of the different orders

of architecture depend. The ancients were most careful of the profiles of their moldings.

Projet. (Fr.) Finished presentation of a design problem.

Proscenium. The front part of the stage of ancient theaters, on which the actors performed.

Prostyle. A portico in which the columns project from the building of which it is attached.

Protractor. A mathematical instrument for laying down and measuring angles on paper, used in drawings or plotting.

Pseudo-classic Architecture. That phase of neoclassic architecture which marked the most stilted period of post Renaissance art, when, under the influence of Vitruvius' writings, and those of his modern disciples, the most formal imitation of Roman architecture prevailed, and it was the aim to revive the whole art of Rome.

Pseudo-dipteral. False double-winged. When the inner row of columns of a dipteral arrangement is omitted and the space from the wall of the building to the columns is preserved, it is pseudo-dipteral.

Puddle. To settle loose dirt by turning on water, so as to render it firm and solid.

Pugging. A coarse kind of mortar laid on the boarding, between floor joists, to prevent the passage of sound; also called deafening.

Pulpit. A raised platform with enclosed front, whence sermons, homilies, etc., were delivered. Pulpits were probably derived in their modern form from the ambones in the early Christian church. There are many old pulpits of stone, though the majority are of wood. Those in the churches are generally hexagonal or octagonal; and some stand on stone bases, and others on slender wooden stems, like columns. The designs vary according to the periods in which they were erected, having paneling, tracery, cuspings, crockets, and other ornaments then in use. Some are extremely rich, and ornamented with color and gilding. A few also have fine canopies or sounding boards. Their usual place is in the nave, mostly on the north side, against the second pier from the chancel arch. Pulpits for addressing the people in the open air were common in the Mediæval period, and stood near a road or cross. Thus, there was one at Spitalfields, and one at St. Paul's, London. External pulpits still remain at Magdalen College, Oxford, and at Shrewsbury, England.

Purlins. Those pieces of timbers which support the rafters to prevent them from sinking.

Putlog. Horizontal pieces for supporting the floor of a scaffold, one end being inserted into putlog holes, left for that purpose in the masonry.

Putty in Plastering. Lump lime slacked with water to the consistency of cream, and then left to harden by evaporation till it becomes like soft putty. It is then mixed with plaster of Paris, or sand, for the finishing coat.

Puzzolana. A grayish earth used for building under water.

Pyramid. A solid, having one of its sides, called a base, a plane figure, and the other sides triangles, these points joining in one point at the top, called the vertex. Pyramids are called triangular, square, etc., according to the form of their bases.

Pyx. In Roman Catholic churches, the box in which the host, or consecrated wafer, is kept.

Quadrangle. A square or quadrangular court surrounded by buildings, as was often done formerly in monasteries, colleges, etc.

Quarry. A pane of glass cut in a diamond or lozenge form.

Quarry-face. Ashlar as it comes from the quarry, squared off for the joints only, with split face. In distinction from Rock-face, in that the latter may be weather-worn, while Quarry-face should be fresh split. The terms are often used indiscriminately.

Quatrefoil. Any small panel or perforation in the form of a four-leaved flower. Sometimes used alone, sometimes in circles and over the aisle windows, but more frequently in square panels. They are generally cusped, and the cusps are often feathered.

Queen Truss. A truss framed with two vertical tie-posts, in distinction from the king-post, which has but one. The upright ties are called Queen-posts.

Quirk Moldings. The convex part of Grecian moldings when they recede at the top, forming a reëntrant angle, with the surface which covers the moldings.

Quoins. Large squared stones at the angles of buildings, buttresses, etc., generally used to stop the rubble or rough stone work, and that the angles may be true and stronger. Saxon quoin stones are said to have been composed of one long and one short stone alternately. Early quoins are generally roughly axed; in later times they had a draught tooled by the chisel round the outside edges, and later still were worked fine from the saw.

Rabbet. A continuous small recess, generally understood as having a right-angle included between its sides, especially one whose sides enclose a relatively restricted area; one formed by two planes, very narrow as compared with their length, such as the small recess on a door-frame, into which the edge of a door is made to fit, the recess of a brick jamb to receive a window-frame, and the like.

Rafters. The joist to which the roof-boarding is nailed. *Principal rafters* are the upper timbers in a truss, having the same inclination as the common rafters.

Rail. A piece of timber or metal extending from one post to another, as in fences, balustrades, staircases, etc. In framing and paneling, the horizontal pieces are called rails, and the perpendicular, *stiles*.

Raking. Moldings whose arrises are inclined to the horizon.

Ramp. A concavity on the upper side of hand railings formed over risers, made by a sudden rise of the steps above. Any concave bend or slope in the cap or upper member of any piece of ascending or descending workmanship.

Rampant. A term applied to an arch whose abutments spring from an inclined plane.

Random Work. A term used by stone-masons for stones fitted together at random without any attempt at laying them in courses. *Random Coursed Work* is a like term applied to work coursed in horizontal beds, but the stones are of any height, and fitted to one another.

Range Work. Ashlar laid in horizontal courses; same as coursed ashlar.

Rebate. A groove on the edges of a board.

Recess. A depth of some inches in the thickness of a wall, as a niche, etc.

Refectory. The hall of a monastery, convent, etc., where the religious took their chief meals together. It much resembled the great halls of mansions, castles, etc., except that there frequently was a sort of ambo, approached by steps, from which to read the Legenda Sanctorum, etc., during meals.

Reglet. A flat, narrow molding, used to separate from each other the parts of members of compartments and panels, to form frets, knots, etc.

Renaissance (a new birth). A name given to the revival of Roman architecture which sprang into existence in Italy as early as the beginning of the fifteenth century, and reached its zenith in that country at the close of the century. There are several divisions of this style as developed in different localities; viz.

The Florentine Renaissance, of which the Pitti Place, by Brunelleschi, is one of the best examples.

The Venetian Renaissance, characterized by its elegance and richness.

The Roman Renaissance, which originated in Rome, under the architects known as Bronte, Vignola, and Michael Angelo. Of this style the Farnese Palace, St. Peter's, and the modern Capitol at Rome are the best examples.

The French Renaissance, introduced into France in the latter part of the fifteenth century, by Italian architects, where it flourished until the middle of the seventeenth century. The Renaissance style was introduced into Germany about the middle of the sixteenth century, and into England about the same time by John of Padua, architect to Henry VIII. This style in England is generally known under the name of Elizabethan.

Rendering. In drawing, finishing a perspective drawing in ink or color, to bring out the spirit and effect of the design. The first coat of plaster on brick or stone work.

Rendu. (Fr.) Delivery of the projet.

Reredos, Dorsal, or **Dossel.** The screen or other ornamental work at the back of an altar. In some large English cathedrals, as Winchester, Durham, St. Albans, etc., this is a mass of splendid tabernacle work, reaching nearly to the groining. In smaller churches there are sometimes ranges of arcades or panelings behind the altars; but, in general, the walls at the back and sides of them were of plain masonry, and adorned with hangings or paraments. In the large churches of Continental Europe the high altar usually stands under a sort of canopy or ciborium, and the sacrarium is hung round at the back and sides with curtains on movable rods.

Reticulated Work. That in which the courses are arranged in a form like the meshes of a net. The stones or bricks are square and placed lozenge-wise.

Return. The continuation of a molding, projection, etc., in an opposite direction.

Return Head. One that appears both on the face and edge of a work.

Reveal. The two vertical sides of an aperture, between the front of a wall and the window or door-frame.

Rib. A molding or projecting piece upon the interior of a vault, or used to form tracery and the like. The earliest groining had no ribs. In early Norman times plain flat arches crossed each other, forming ogive ribs. These by degrees became narrower, had greater projection, and were chamfered. In later Norman work the ribs were often formed of a large roll placed upon the flat band, and then of two rolls side by side with a smaller roll or a fillet between them, much like the lower member. Sometimes they are enriched with zigzags and other Norman decorations, and about this time bosses

became of very general use. As styles progressed the moldings were more undercut, richer, and more elaborate, and had the dog-tooth or ball-flower or other characteristic ornament in the hollows. In all instances the moldings are of similar contours to those of arches, etc., of the respective periods. Later, wooden roofs are often formed into cants or polygonal barred vaults, and in these the ribs are generally a cluster of rounds, and form square or stellar panels, with carved bosses or shields at the intersections.

Ridge. The top of a roof which rises to an acute angle.

Ridge-pole. The highest horizontal timber in a roof, extending from top to top of the several pairs of rafters of the trusses, for supporting the heads of the jack rafters.

Relievo, or Relief. The projection of an architectural ornament.

Rise. The distance through which anything rises, as the rise of a stair, or inclined plane.

Riser. The vertical board under the tread in stairs.

Rococo Style. A name given to that variety of the Renaissance which was in vogue during the seventeenth and the latter part of the sixteenth century.

Roman Imperial Architecture. The architecture of the Roman dominion in Europe, Asia, and Africa, ending with the fourth century A.D. With the exception of fragments of walls and of a few simple buildings, Roman architecture is represented by those monuments which were erected during the Empire, and as the style was first developed in Rome and then spread throughout the various countries subjugated, with such modifications as the materials of the country and the labor to be obtained required and as the climate suggested, the term " Roman Imperial Architecture " has been adopted to prevent confusion. Etruscan architecture is the first source of what may be called its elements and in one sense the most important. It was mainly through her paved roads that Rome was able to bring into connection all the chief cities of the Empire, by the solid construction of her walls to render the cities she founded secure against attack, and by employment of arched construction to roof over her buildings with an imperishable material, to build bridges and aqueducts (the latter of vital importance in all her Eastern possessions), and lastly to drain off water from marshy districts. The second source was Greek architecture. The orders, Doric, Ionic and Corinthian, where employed in the temple, the basilica and thermæ, underwent but little change; the great scientific advance, however, made in the quarrying of stone and marble and in the transport of large masses of stone, enabled the Roman architect to substitute the monolith for the Greek column built in several courses, and the greater display of richness in the Corinthian order led to its almost universal adoption. There is one other order introduced by the Romans, the Composite, which is a mixture of two orders. The earliest example of the super-imposition of orders is found in Rome. The notable feature of Imperial Roman architecture, when compared with the history of more ancient styles, is the immense variety of buildings of every type. By far the greatest development in Roman Imperial architecture is that which in the thermæ and palaces, is found in the employment of the barrel vaults, the intersecting barrel vault, and the dome. These features led to erection of buildings entirely homogeneous in the material employed, and of so lasting and durable a nature that, but for earthquakes and the destructive action of mankind, they might all have remained perfect to the present day. The other structures which were developed were the arched entrances to towns, tri-

umphal arches, tombs, viaducts and colonnaded streets. In some instances a knowledge of a type of iron construction is shown.

Romanesque Style. The term Romanesque embraces all those styles of architecture which prevailed between the destruction of the Roman Empire and the beginning of Gothic architecture. In it are included the Early Roman Christian architecture, Byzantine, Mahometan, and the later Romanesque architecture proper, which was developed in Italy, France, England, and Germany. This later Romanesque, which was quite different from the preceding, came into vogue during the tenth century, and reached its height during the twelfth century, and in the thirteenth century gave way to the Pointed or Gothic style. In England, Romanesque architecture is known under the name of the Saxon, Norman, and Lombard styles, according to the different political periods.

Rood. A name applied to a crucifix, particularly to those which were placed in the rood-loft or chancel screens. These generally had not only the image of the crucified Saviour, but also those of St. John and the Virgin Mary standing one on each side. Sometimes other saints and angels are by them, and the top of the screen is set with candlesticks or other decoration.

Rood-loft, Rood-screen, Rood-beam, Jube Gallery, etc. The arrangement to carry the crucifix or rood, and to screen off the chancel from the rest of the church during the breviary services, and as a place whence to read certain parts of those services. Sometimes the crucifix is carried simply on a strong transverse beam, with or without a low screen, with folding-doors below but forming no part of such support. In European churches the general construction of wooden screens is close paneling beneath, about 3 feet to 3 feet 6 inches high, on which stands screen work composed of slender turned balusters or regular wooden mullions, supporting tracery more or less rich, with cornices, cresting, etc., and often painted in brilliant colors and gilded. These not only enclose the chancels, but also chapels, chantries, and sometimes even tombs. In English mansions, and some private houses, the great halls were screened off by a low passage at the end opposite to the dais, over which was a gallery for the use of minstrels or spectators. These screens were sometimes close and sometimes glazed.

Rood-tower. A name given by some writers to the central tower, or that over the intersection of the nave and chancel with the transepts.

Roof. The covering or upper part of any building.

Roofing. The material put on a roof to make it water-tight.

Rose Window. A name given to a circular window with radiating tracery; called also wheel window.

Rostrum. An elevated platform from which a speaker addresses an audience.

Rotunda. A building which is round both within and without. A circular room under a dome in large buildings is also called the rotunda.

Roughcast. A sort of external plastering in which small sharp stones are mixed, and which, when wet, is forcibly thrown or cast from a trowel against the wall, to which it forms a coating of pleasing appearance. Roughcast work has been used in Europe for several centuries, where it was much used in timber houses, and when well executed the work is sound and durable. The mortar for roughcast work should always have cement mixed with it.

Rubble Work. Masonry of rough, undressed stones. When only the roughest irregularities are knocked off, it is called scabbled rubble, and when

the stones in each course are rudely dressed to nearly a uniform height, ranged rubble.

Rudenture. The figure of a rope or staff, which is frequently used to fill up the flutings of columns, the convexity of which contrasts with the concavity of the flutings, and serves to strengthen the edges. Sometimes, instead of a convex shape, the flutings are filled with a flat surface; sometimes they are ornamentally carved, and sometimes on pilasters, etc. Rudentures are used in relief without flutings, as their use is to give greater solidity to the lower part of the shaft, and secure the edges. They are generally only used in columns which rise from the ground and are not to reach above one-third of the height of the shaft.

Russia, Architecture of. The architecture of as much of that country as lies west of the Ural Mountains. In this vast region there are many remains of ancient building showing great capacity for art of a semibarbaric sort, and much of that picturesqueness which comes from massive building of unusual height and expressive outline, as in fortification. The Byzantine influence entered not earlier than the twelfth century and came partly from the monasteries of the Greek church in the south and partly from Constantinople direct. The simple log buildings of the peasantry, including churches as well as large and small residences and village houses, are extremely attractive. Beginning with the reign of Peter the Great, buildings of modern European form have been imitated in Russia, and palaces for the sovereign and for the nobles have followed the neoclassic style of the epoch, but generally without fortunate result. The taste is often barbarous without being effective, the exteriors pompous without being stately.

Rustic or Rock Work. A mode of building in imitation of nature. This term is applied to those courses of stone work the face of which is jagged or picked so as to present a rough surface. That work is also called rustic in which the horizontal and vertical channels are cut in the joinings of stones, so that when placed together an angular channel is formed at each joint. *Frosted rustic work* has the margins of the stones reduced to a plane parallel to the plane of the wall, the intermediate parts having an irregular surface. *Vermiculated rustic work* has these intermediate parts so worked as to have the appearance of having been eaten by worms. *Rustic chamfered work,* in which the face of the stones is smooth, and parallel to the face of the wall, and the angles beveled to an angle of one hundred and thirty-five degrees with the face so that two stones coming together on the wall, the beveling will form an internal right angle.

Sacristy. A small chamber attached to churches, where the chalices, vestments, books, etc., were kept by the officer called the sacristan. In the early Christian basilicas there were two semicircular recesses or apsides, one on each side of the altar. One of these served as a sacristy, and the other as the bibliotheca or library. Some have supposed the sacristy to have been the place where the vestments were kept, and the vestry that where the priests put them on; but we find from Durandus that the sacrarium was used for both these purposes. Sometimes the place where the altar stands enclosed by the rails has been called sacrarium.

Saddle Bars. Narrow horizontal iron bars passing from mullion to mullion, and often through the whole window, from side to side, to steady the stone work, and to form stays, to which the lead work is secured. When the bays of the windows are wide, the lead lights are further strengthened by upright bars passing through eyes forged on the saddle bars, and called stanchions.

When saddle bars pass right through the mullions in one piece, and are secured to the jambs, they have sometimes been called stay bars.

Sagging. The bending of a body in the middle by its own weight, or the load upon it.

Salient. A projection.

Salon. A spacious and elegant apartment for the reception of company, or for state purposes, or for the reception of paintings, and usually extending through two stories of the house. It may be square, oblong, polygonal, or circular.

Sanctuary. That part of a church where the altar is placed; also, the most sacred or retired part of a temple. A place for divine worship; a church.

Sanctus Bell-cot, or Turret. A turret or enclosure to hold the small bell sounded at various parts of the service, particularly where the words "Sanctus," etc., are read. This differs but little from the common bell-cot, except that it is generally on the top of the arch dividing the nave from the chancel. Sometimes, however, the bell seems to have been placed in a cot outside the wall. In England sanctus bells have also been placed over the gables of porches. In Continental Europe they run up into a sort of small slender spire, called *flèche* in France, and *guglio* in Italy.

Saracenic Architecture. That Eastern style employed by the Saracens, and which distributed itself over the world with the religion of Mahomet. It is a modification and combination of the various styles of the countries which they conquered.

Sarcophagus. A tomb or coffin made of stone, and intended to contain the body.

Sash. The framework which holds the glass in a window.

Scabble. To dress off the rougher projections of stones for rubble masonry with a stone axe or scabbling hammer.

Scagliola. An imitation of colored marbles in plaster work, made by a combination of gypsum, glue, isinglass, and coloring matter, and finished with a high polish, invented between 1600 and 1649.

Scandinavia, Architecture of. The architecture of the great peninsula now occupied by the two kingdoms of Sweden and Norway. The Scandinavian lands are sometimes held to include Denmark which is here treated separately. Norway contains some ancient timber churches, construction of which is not unlike those in New England in the seventeenth and eighteenth centuries. Romanesque stone vaulted churches are not unknown. The neoclassic architecture of Scandinavia is largely a matter of the residences of the nobility. Some of these residences are of extraordinary interest, containing a character of bold pseudo-Renaissance design reminding one of good seventeenth century German work, but of still greater independence and daring in the treatment of the semiclassical details. The late seventeenth century and the early eighteenth century have left civic buildings of grave and sedate aspect.

Scantling. The dimensions of a piece of timber in breadth and thickness; also, studding for a partition, when under five inches square.

Scarfing. The joining and bolting of two pieces of timber together transversely, so that the two appear as one.

Sconce. A fixed hanging or projecting candlestick.

Scotia. A concave molding, most commonly used in bases, which projects a deep shadow on itself, and is thereby a most effective molding under the eye,

as in a base. It is like a reversed ovolo, or, rather, what the mold of an avolo would present.

Scratch Coat. The first coat of plaster, which is scratched to afford a bond for the second coat.

Screeds. Long narrow strips of plaster put on horizontally along a wall, and carefully faced out of wind, to serve as guides for plastering the wide intervals between them.

Screen. Any construction subdividing one part of a building from another, as a choir, chantry, chapel, etc. The earliest screens are the low marble podia shutting off the chorus cantantium in the Roman basilicas, and the perforated cancelli enclosing the bema, altar, and seats of the bishops and presbyters. The chief screens in a church are those which enclose the choir or the place where the breviary services are recited. In Continental Europe this is done not only by doors and screen work, but also, when these are of open work, by curtains, the laity having no part in these services. In England screens were of two kinds: one, of open wood-work, generally called rood-screens or jubes, and which the French call *grilles, clôtures du chœur,* the other, massive enclosures of stone work enriched with niches, tabernacles, canopies, pinnacles, statues, crestings, etc., as at Canterbury, York, Gloucester, and many other places.

Scribing. Fitting wood-work to an irregular surface

Scupper. Outlets for overflow of water in outside or court walls, to drain floor of water from automatic sprinklers or fire-hose in case of fire.

Section. A drawing showing the internal heights of the various parts of a building. It supposes the building to be cut through entirely, so as to exhibit the walls, the heights of the internal doors and other apertures, the heights of the stories, thicknesses of the floors, etc. It is one of the species of drawings necessary to the exhibition of a Design.

Sedilia. Seats used by the celebrants during the pauses in the mass. They are generally three in number—for the priest, deacon, and sub-deacon—and are in England almost always a species of niches cut into the south walls of churches, separated by shafts or by a species of mullions, and crowned with canopies, pinnacles, and other enrichments more or less elaborate. The piscina and ambry sometimes are attached to them. In Continental Europe the sedilia are often movable seats; a single stone seat has rarely been found.

Setback. A flat, plain set-off in a wall; also, a setting back of the outside wall of a building for some distance from the street-line, especially, in tall buildings, such a setting back at a particular height in the building, or one of a number of such settings back at different heights, for the purpose of allowing better light and ventilation in the street.

Set-off. The horizontal line shown where a wall is reduced in thickness, and, consequently, the part of the thicker portion appears projecting before the thinner. In plinths this is generally simply chamfered. In other parts of work the set-off is generally concealed by a projecting string. Where, as in parapets, the upper part projects before the lower, the break is generally hid by a corbel table. The portions of buttress caps which recede one behind another are also called set-offs.

Shaft. In Classical architecture that part of a column between the necking and the apophyge at the top of the base. In later times the term is applied to slender columns either standing alone or in connection with pillars, buttresses, jambs, vaulting, etc.

Shed Roof, or Lean-to. A roof with only one set of rafters, falling from a higher to a lower wall, like an aisle roof.

Shore. A piece of timber placed in an oblique direction to support a building or wall temporarily while it is being repaired or altered.

Shrine. A sort of ark or chest to hold relics. It is sometimes merely a small box, generally with a raised top like a roof; sometimes an actual model of churches; sometimes a large construction, like that of Edward the Confessor at Westminster, of St. Genevieve at Paris, etc. Many are covered with jewels in the richest way; that of San Carlo Borromeo, at Milan, is of beaten silver.

Sills. Are the timbers on the ground which support the posts and superstructure of a timber building. The term is most frequently applied to those pieces of timber or stone at the bottom of doors or windows.

Skewback. The inclined stone from which an arch springs.

Skintled Brickwork. Irregularly formed brickwork arranged with variations in projections on the exterior face-wall, usually formed of bricks of irregular shape.

Skirtings. The narrow boards which form a plinth around the margin of a floor, now generally called the base.

Sleeper. A piece of timber laid on the ground to receive floor joists.

Soffit. The lower horizontal face of anything as, for example, of an entablature resting on and lying open between the columns, or the under face of an arch where its thickness is seen.

Sound Board. The covering of a pulpit to deflect the sound into a church.

South America, Architecture of. *Ancient:* The most interesting development occurred previous to the conquest of Peru by the Spaniards in 1532. The extent and time of its beginning are not known. At the time of the Spanish invasion, the entire territory west of the Andes was in possession of the Inca dynasty. Inca architecture is characterized by distinctly stone construction. There is no suggestion of wooden types as in Greece and Egypt. The ruins are of large palaces or public buildings. The palaces and temples are built around courts. *Modern:* The most interesting monuments of modern architecture are situated in Peru. The architecture of Brazil is quite recent, built mainly of brick, the splendid woods of the country being used in the interior. Like the cities of Brazil, those of Venezuela, Colombia, Chile, the Argentine Republic, and other South American countries are quite modern.

Spall. Bad or broken brick; stone chips.

Spain, Architecture of. The Iberians, or pre-Roman inhabitants of Spain, left no surviving monuments beyond a few slightly known structures in the northern Basque provinces. Only fragmentary examples of Roman aqueducts, temples and theaters remain. A few scattered remains of seventh and eighth century mediæval buildings present a curious analogy, in style, plan, and arrangement, to some of the Roman or Early Christian work of Syria. The Moorish invasion amalgamated rival interests into the mediæval kingdom of Spain and introduced the Renaissance. In the south, the Moors found little to preserve when they first entered the kingdom, and their own architectural work strongly impressed the Christian conquerors, so that as the Moslems were driven out and the Christian influence gained the ascendancy, the Moorish work was, on the whole, little disturbed, resulting in our finding it in its present condition today, with but a slight admixture of the Renaissance. The existing architectural remains present, in their geographical distribution, a curious analogy to the conditions of the soil and climate, the Gothic and

Romanesque being chiefly found in the north, the Moorish in the south, and the neoclassic along the central plateau. There does not appear to be in Spain a well-defined development or consecutive growth from the mediæval to the modern styles. Rather, each style appears broadly and frankly borrowed outright, the only essentially Spanish factors being displayed in a few peculiarities of plan and in the spirit of the detail, the noticeable lack being in inventive progression rather than in ability to adapt.

Span. The distance between the supports of a beam, girder, arch, truss, etc.

Spandrel, or Spandril. The space between any arch or curved brace and the level label, beams, etc., over the same. The spandrels over doorways in Perpendicular works are generally richly decorated. Also, an exterior non-bearing wall in skeleton construction built between columns or piers and wholly supported at each story.

Specification. *Architect's.* The designation of the kind, quality, and quantity of work and material to go in a building, in conjunction with the working drawings.

Spire. A sharply pointed pyramid or large pinnacle, generally octagonal in England, and forming a finish to the stops of towers. Timber spires are very common in England. Some are covered with lead in flat sheets, others with the same metal in narrow strips laid diagonally. Very many are covered with shingles. In Continental Europe there are some elegant examples of spires of open timber work covered with lead.

Splayed. The jamb of a door, or anything else of which one side makes an oblique angle with the other.

Springer. The stone from which an arch springs; in some cases this is a capital, or impost; in other cases the moldings continue down the pier. The lowest stone of the gable is sometimes called a springer.

Squinches. Small arches or corbled set-offs running diagonally and, as it were, cutting off the corners of the interior of towers, to bring them from the square to the octagon, etc., to carry the spire.

Squint. An oblique opening in the wall of a church; especially, in mediæval architecture, an opening so placed as to afford a view of the high altar from the transept or aisles.

Staging. A structure of posts and boards for supporting workmen and material in buildings.

Stall. A fixed seat in the choir for the use of the clergy. In early Christian times the thronus cathedra, or seat of the bishop, was in the center of the apsis or bema behind the altar, and against the wall; those of the presbyters also were against the wall, branching off from side to side around the semicircle. In later times the stalls occupied both sides of the choir, return seats being placed at the ends for the prior, dean, precentor, chancellor, or other officers. In general, in cathedrals, each stall is surmounted by tabernacle work, and rich canopies, generally of oak.

Stanchion. A word derived from the French *étançon*, a wooden post, applied to the upright iron bars which pass through the eyes of the saddle bars or horizontal irons to steady the lead lights. The French call the latter *traverses*, the stanchions *montants*, and the whole arrangement *armature*. Stanchions frequently finish with ornamental heads forged out of the iron.

Steeple. A general name for the whole arrangement of tower, belfry, spire, etc.

Stereobate. A basement, distinguished from the nearly equivalent term stylobate by the absence of columns.

Stile. The upright piece in framing or paneling.

Stilted. Anything raised above its usual level. An arch is stilted when its center is raised above the line from which the arch appears to spring.

Stoa. A portico used by the ancient Greeks as a shady promenade or meeting place, where conversation might be held or speakers heard. The stoa took the form of a roof structure, the length of which was great as compared to its depth. Ordinarily, there was at the back an enclosing wall, and in front, facing the street or public square, a colonnade. The floor was generally raised a few steps above the street. Some were two stories in height.

Stoop. A seat before the door; often a porch with a balustrade and seats on the sides.

Stoup. A basin for holy water at the entrance of Roman Catholic churches, into which all who enter dip their fingers and cross themselves.

Straight Arch. A form of arch in which the intrados is straight, but with its joints radiating as in a common arch.

Strap. An iron plate for connecting two or more timbers, to which it is screwed by bolts. It generally passes around one of the timbers.

Stretcher. A brick or block of masonry laid lengthwise of a wall.

String Board. A board placed next to the well-hole in wooden stairs, terminating the ends of the steps. The string piece is the piece of board put under the treads and risers for a support, and forming the support of the stair.

String-course. A narrow, vertically faced and slightly projecting course in an elevation. If window-sills are made continuous, they form a string-course; but if this course is made thicker or deeper than ordinary window-sills, or covers a set-off in the wall, it becomes a blocking-course. Also, horizontal moldings running under windows, separating the walls from the plain part of the parapets, dividing towers into stories or stages, etc. Their section is much the same as the labels of the respective periods; in fact, these last, after passing round the windows, frequently run on horizontally and form strings. Like labels, they are often decorated with foliages, ball-flowers, etc.

Stucco. Any material used as a covering for walls and the like, put on wet and drying hard and durable. Plaster where applied to walls in the usual way is a kind of stucco, and the hard finish is almost exactly like fine Roman stucco except that it is applied in only one thin coat instead of many. Vitruvius speaks of three coats mixed with sand and three coats mixed with marble dust, but does not give the thickness of coats, nor, what would answer the same purpose, how wet the mixture was made. He speaks of well-finished stucco shining so as to reflect the images falling upon it, and states that persons used to get slabs of plaster from ancient walls and use them for tables, the material being so beautiful in itself. The term is generally applied to out-of-door work. Even in modern fireproof buildings the decorative use of fine plastering to replace woodwork, as for dados and the like, is not in the United States called stucco, but takes the name of the material used, generally a proprietary name. The term is used commonly for outer walls. The practical value of stucco is very great as being so nearly impervious to water; thus an excellent wall three stories high, or even higher, may be built with eight inches of brick on the inner side, four inches of brick on the outer side, an air space of two or four inches across which the outer and the inner walls are well tied,

and two coats of well-mixed and well-laid stucco on the exterior, this being finally painted with oil paint.

Studs, or Studding. The small timbers used in partitions and outside wooden walls, to which the laths and boards are nailed.

Style. The term style in architecture has obtained a conventional meaning beyond its simpler one, which applies only to columns and columnar arrangements. It is now used to signify the differences in the moldings, general outlines, ornaments, and other details which exist between the works of various nations, and also those differences which are found to exist between the works of any nation at different times.

Stylobate. A basement to columns. Stylobate is synonymous with pedestal, but is applied to a continued and unbroken substructure or basement to columns, while the latter term is confined to insulated supports. The Greek temples generally had three or more steps all around the temple, the base of the column resting on the top step; this was the stylobate.

Subsellium. A name sometimes given to the seat in the stalls of churches; same as miserere.

Summer. A girder or main-beam of a floor; if supported on two-story posts and open below, it is called a Brace-summer.

Sump. A swamp, bog, or muddy pool (now provincial English); also, a pit, well, or the like, in which water or other liquid is collected; in mining, a space at the bottom of a shaft where water is allowed to collect; in mechanics, a reservoir for oil situated at the lowest point in a circulating system for lubricating-oil in an internal-combustion engine.

Surbase. A cornice or series of moldings on the top of the base of a pedestal-podium, etc.; a molding above the base.

Surface. To make plane and smooth.

Syria, Architecture of. The architecture of the country stretching from the eastern coast of the Mediterranean eastward to the Euphrates scarcely possesses an indigenous architectural style, as the Semitic tribes who peopled this region were not a building race. In all the earlier work of the country, down to the Mohammedan invasion in the seventh century, there are two special characteristics not found elsewhere: (1) Monolithism, employment of immense blocks of masonry, essentially Phœnician. (2) Drafted masonry; in order to obtain a fine and accurate joint, the masons worked a draft round the edge of each stone, thus constituting what is known as rusticated masonry. The architecture may be grouped under the following divisions: Herodian and Roman work, synagogues, Roman work of second century and later, colonnaded streets, Roman temples, Roman tombs, theaters, Sassanian or doubtful buildings, Christian buildings, Syrian round-arched style, Crusaders' work, Moslem work.

Systyle. An intercolumniation to which two diameters are assigned.

Tabernacle. A species of niche or recess in which an image may be placed. They are generally highly ornamented and often surmounted with crocketed gables. The word tabernacle is also often used to denote the receptacle for relics, which was often made in the form of a small house or church.

Tabernacle Work. The rich ornamental tracery forming the canopy, etc., to a tabernacle, is called tabernacle work; it is common in the stalls and screens of cathedrals, and in them is generally open or pierced through.

Tail Trimmer. A trimmer next to the wall, into which the ends of joists are fastened to avoid flues.

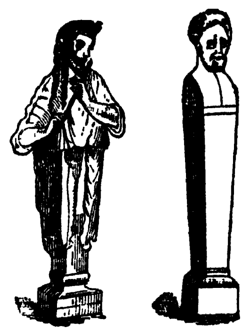
Tamp. To pound the earth down around a wall after it has been thrown in.

Tapestry. A kind of woven hangings of wool or silk, ornamented with figures, and used formerly to cover and adorn the walls of rooms. They were often of the most costly materials and beautifully embroidered.

Temple. An edifice destined, in the earliest times, for the public exercise of religious worship.

Templet, or **Template.** A mold used by masons for cutting or setting work. A short piece of timber sometimes laid under a girder

Terminal. Figures of which the upper parts only, or perhaps the head and shoulders alone, are carved, the rest running into a parallelopiped, and sometimes into a diminishing pedestal, with feet indicated below, or even without them, are called terminal figures.



ANCIENT TERMINI

Terra-cotta. Baked clay of a fine quality. Much used for bas-reliefs for adorning the friezes of temples. In modern times employed for architectural ornaments, statues, vases, etc.

Terrazzo. A concrete pavement used for floors in the province of Venetia, even in houses of some pretensions to elegance. Lime-mortar made unusually dry is the principal material; in this are inlaid small pieces of marble, usually not too large to pass through a ring an inch and a half in diameter. The whole is beaten hard, rubbed down, and polished.

Tessellated Pavements. Those formed of tesseræ, or, as some write it, tessellæ, or small cubes from half an inch to an inch square, like dice, of pottery, stone, marble, enamel, etc.

Tetrastyle. A portico of four columns in front.

Tholobate. That on which a dome or cupola rests. This is a term not in general use, but it is not the less of useful application. What is generally termed the attic above the peristyle and under the cupola of St. Paul's, London, would be correctly designated the tholobate. A tholobate of a different description, and one to which no other name can well be applied, is the circular substructure to the cupola of the University College, London.

Throat. A channel or groove made on the under-side of a string-course, coping, etc., to prevent water from running inward toward the walls.

Tie. A timber, rod, chain, etc., binding two bodies together, which have a tendency to separate or diverge from each other. The *tie-beam* connects the bottom of a pair of principal rafters, and prevents them from bursting out the wall.

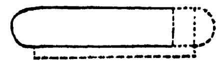
Tiles. Flat pieces of clay burned in kilns, to cover roofs in place of slates or lead. Also, flat pieces of burned clay, either plain or ornamented, glazed or unglazed, used for floors, wainscoting, and about fireplaces, etc. Small square pieces of marble are also called tile.

Tongue. The part of a board left projecting, to be inserted into a groove.

Tooth Ornament. One of the peculiar marks of the Early English period of Gothic architecture, generally inserted in the hollow moldings of doorways, windows, etc.

Torso. A multilated statue of which nothing remains but the trunk. Columns with twisted shafts have also this term. Of this kind there are several varieties.

Torus. A protuberance or swelling, a molding whose form is convex, and generally nearly approaches a semicircle. It is most frequently used in bases, and is generally the lowest molding in a base.



TORUS

Tower. An elevated building originally designed for purposes of defence. Those buildings are of the remotest antiquity, and are, indeed, mentioned in the earliest Scriptures. In mediæval times they were generally attached to churches, to cemeteries, to castles, or used as bell-towers in public places of large cities. In churches, the towers of the Saxon period were generally square. Norman towers were also generally square. Many were entirely without buttresses; others had broad, flat, shallow projections which served for this purpose. The lower windows were very narrow, with extremely wide splays inside, probably intended to be defended by archers. The upper windows, like those of the preceding style, were generally separated into two lights, but by a shaft or short column, and not by a baluster. Early English towers were generally taller, and of more elegant proportions. They almost always had large projecting buttresses, and frequently stone staircases. The lower windows, as in the former style, were frequently mere arrow-slits; the upper were in couplets or triplets, and sometimes the tower top had an arcade all around. The spires were generally broach spires; but sometimes the tower tops finished with corbel courses and plain parapets, and (rarely) with pinnacles. There are a few Early English towers which break into the octagon from the square toward the top, and still fewer which finish with two gables. Both these methods of termination, however, are common in Continental Europe. At Vendôme, Chartres, and Senlis the towers have octagonal upper stages surrounded with pinnacles, from which elegant spires arise. In the North of Italy, and in Rome, they are generally tall square shafts in four to six stages, without buttresses, with couplets or triplets or semicircular windows in each stage, generally crenellated at top, and covered with a low pyramidal roof. The well-known leaning tower at Pisa is cylindrical, in five stories of arcaded colonnades. In Ireland there are in some of the churchyards very curious round towers.

Tracery. The ornamental filling in of the heads of windows, panels, circular windows, etc., which has given such characteristic beauty to the architecture of the fourteenth century. Like almost everything connected with mediæval architecture, this elegant and sometimes fairy-like decoration seems to have sprung from the smallest beginnings. The circular-headed window of the Normans gradually gave way to the narrow-pointed lancets of the Early English period, and, as less light was afforded by the latter system than by the former, it was necessary to have a greater number of windows; and it was found convenient to group them together in couplets, triplets, etc. When these couplets were assembled under one label, a sort of vacant space or spandrel was formed over the lancets and under the label. To relieve this, the first attempts were simply to perforate this flat spandrel, first by a simple lozenge-shaped or circular opening, and atterward by a quatrefoil. By pierc-

ing the whole of the vacant spaces in the window head, carrying moldings around the tracery, and adding cusps to it, the formation of tracery was complete, and its earliest result was the beautiful geometrical work such as is found at Westminster Abbey.

Transept. That portion of a church which passes transversely between the nave and choir at right angles, and so forms a cross on the plan.

Transom. The horizontal construction which divides a window into heights or stages. Transoms are sometimes simple pieces of mullions placed transversely as cross-bars, and in later times are richly decorated with cuspings, etc.

Traverse. To plane in a direction across the grain of the wood, as to traverse a floor by planing across the boards.

Tread. The horizontal part of a step of a stair.

Trefoil. A cusping the outline of which is derived from a three-leaved flower or leaf, as the quatrefoil and cinquefoil are from those with four and five.

Trellis. Lattice-work of metal or wood for vines to run on.

Trestle. A movable frame or support for anything; when made of a cross-piece with four legs it is called by carpenters a horse.

Triforium. The arcaded story between the lower range of piers and arches and the clere-story. The name has been supposed to be derived from *tres* and *fores*—three doors, or openings—that being a frequent number of arches in each bay.

Triglyph. The vertically channeled tablets of the Doric frieze are called triglyphs, because of the three angular channels in them—two perfect and one divided—the two chamfered angles or hemiglyphs being reckoned as one. The square sunk spaces between the trigylphs on a frieze are called metopes.

Trim. Of a door, sometimes used to denote the locks, knobs, and hinges.

Trimmer. The beam or floor joist into which a header is framed.

Trimmer Arch. An arch built in front of a fireplace, in the thickness of the floor, between two trimmers. The bottom of the arch starting from the chimney and the top pressing against the header.

Tuck-pointing. Marking the joints of brickwork with a narrow parallel ridge of fine putty.

Tudor Style. The architecture which prevailed in England during the reign of the Tudors; its period is generally restricted to the end of the reign of Henry VIII.

Turret. A small tower, especially at the angles of larger buildings, sometimes overhanging and built on corbels, and sometimes rising from the ground.

Tuscan Order. The plainest of the five orders of Classic architecture.

Tympanum. The triangular recessed space enclosed by the cornice which bounds a pediment. The Greeks often placed sculptures representing subjects connected with the purposes of the edifice in the tympana of temples, as at the Parthenon and Ægina.

Under-croft. A vaulted chamber under ground.

United States, Architecture of. The architecture of the whole territory of the republic, which might be considered under the following heads: (1) *Pre-Columbian Era;* Many structures were only of one story, but some were of four or five. From the rude bough wickyup of the Arizona Pai Ute, lightly

abandoned on every change of camp, to the massive and elaborately ornamental stone buildings of the Maya of Yucatan. It is possible, therefore, in America to take up the study of house building where the thread is lost in Europe, and with both follow the line of development from the wickyup to the Parthenon. (2) *Colonial Era;* Influenced by Great Britain, Spain, Holland, Sweden, and France. (3) *Modern Era;* Starting with World's Fair at Chicago in 1893 and culminating in the tall building or "skyscraper" office-building of the twentieth century.

Upset. To thicken, and shorten as by hammering a heated bar of iron on the end.

Vagina. The upper part of the shaft of a terminus, from which the bust or figure seems to rise.

Valley. The internal angle formed by two inclined sides of a roof.

Valley Rafters. Those which are disposed in the internal angle of a roof to form the valleys.

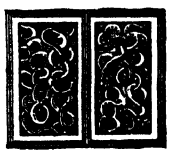
Vane. The weathercock on a steeple. In early times it seems to have been of various forms, as dragons, etc.; but in the Tudor period the favorite design was a beast or bird sitting on a slender pedestal, and carrying an upright rod, on which a thin plate of metal is hung like a flag, ornamented in various ways.

Vault. An arched ceiling or roof. A vault is, indeed, a laterally conjoined series of arches. The arch of a bridge is, strictly speaking, a vault. Intersecting vaults are said to be groined. See *Groined Vaulting* for fuller description of vaults.

Verge. The edge of the tiling, slate or shingles, projecting over the gable of a roof, that on the horizontal portion being called eaves.

Verge Board. Often corrupted into Barge Board; the board under the verge of gables, sometimes molded, and often very richly carved, perforated, and cusped, and frequently having pendants, and sometimes finials, at the apex.

Vermiculated. Stones, etc., worked so as to have the appearance of having been worked by worms.



VERMICULATED

Vestibule. An ante-hall, lobby, or porch.

Vestry. A room adjoining a church, where the vestments of the minister are kept and parish meetings held.

Viaduct. A structure of considerable magnitude, and usually of masonry, for carrying a railway across a valley.

Vignette. A running ornament, representing, as its name imports, a little vine, with branches, leaves, and grapes. It is common in the Tudor period, and runs or roves in a large hollow or casement. It is also called Trayle.

Villa. A somewhat pretentious country house.

Volute. The convolved or spiral ornament which forms the characteristic of the Ionic capital. Volute, scroll, helix, and cauliculus are used indifferently for the angular horns of the Corinthian capital.

Voussoir. One of the wedge-like stones which form an arch; the middle one is called the key-stone.

Wainscot. The wooden lining of walls, generally in panels.

Wall Plates. Pieces of timber which are placed on top of brick or stone walls so as to form the support to the roof of a building.

Wane. The natural curvature of a log or of the edge of a board sawed from an unsquared log.

Warped. Twisted out of shape by seasoning.

Water Table. A slight projection of the lower masonry or brickwork on the outside of a wall a few feet above the ground as a protection against rain.

Weather Boarding. Boards lapped over each other to prevent rain, etc., from passing through.

Weathering. A slight fall on the top of cornices, window-sills, etc., to throw off the rain.

West Indies, Architecture of. The architecture of these islands, their country districts and their towns. Some of the towns are of ancient establishment and contain buildings of the sixteenth century and those which have immediately succeeded them, while others are in architectural character entirely of the nineteenth century, and not of its earliest years.

Wicket. A small door opening in a larger. They are common in mediæval doors, and were intended to admit single persons, and guard against sudden surprises.

Wind. A turn, a bend. A wall is *out of wind* when it is a perfectly flat surface.

Wing. A side building less than the main building.

Withes. The partition between two chimney flues in the same stack.

ARCHITECTURAL TERMS AS DEFINED IN VARIOUS BUILDING CODES

The following definitions have been taken from building codes or laws of various cities. The letters following the definitions indicate the city or code from which they are taken, as follows: (B.), Boston; (C.), Chicago; (D.), Detroit; (L. A.), Los Angeles; (N. Y.), New York; (P. C.), Pacific Coast Uniform Building Code; (P), Philadelphia; (S. F.), San Francisco; (S.), Seattle; (St. L.), St. Louis.

Addition. . . . any change to a structure which increases any of its exterior dimensions. (P.)

Alley. . . . any public thoroughfare in the rear of a lot which fronts on a public street. (St. L.) . . . a public thoroughfare which is less than twenty feet in width, or one which is twenty feet or more in width but unnamed. (S.) . . . any public space, public park or thoroughfare less than sixteen feet but not less than ten feet in width which has been deeded to the public for public use. (P. C.)

Alteration. . . . any change to a structure which does not increase any of its exterior dimensions. (P.) . . . any change, addition or modification in construction or grade of occupancy. (D.) . . . any change or addition. (S. F.) . . . as applied to a building or structure, is any change or rearrangement in the structural parts or in the exit facilities, or any enlargement, whether by extending on any side or by increasing in height, or the moving from one location to another. (N. Y.) . . . any change or addition. (L. A.)

Apartment. (See *Apartment-House.*) . . . room, or suite of two or more rooms, which is or are occupied as a home for one or more persons. (P) . . . shall be taken to mean a room or suite of two or more rooms in a tenement-house occupied or suitable for occupation as a residence for one family doing its own cooking in the apartment or on the premises. One person may be construed to be a family. (St. L.) Apartment or Flat, . . . any number of rooms occupied as the dwelling-place of but one family, except that it shall not include a residence as herein defined. (S) . . . room or suite of rooms in an apartment-house, which is occupied or which is intended or designed to be occupied by one family for living and sleeping purposes. (P. C.) . . . a room or suite of two or more rooms occupied or intended or designed to be occupied as a family domicile. (C.)

Apartment-House. (See *Apartment.*) . . . any building, or portion thereof, containing three or more apartments. For the purpose of this code an apartment-house is a tenement. (P.) Apartment-House or Flat Building, . . . a building containing one or more apartments. (S.) . . . in any building or portion thereof which is designed, built, rented, leased, let or hired out to be occupied, or which is occupied as the home or residence of three or more families living independently of each other and doing their own cooking in the said building, and shall include flats and apartments. (P. C.) Tenement-House and Apartment-House, . . . any dwelling coming within the definition of a tenement-house as defined in the State Tenement House Law. (S. F.) Apartment-House or Tenement-House, . . . a building containing separate apartments for three or more families, and having a street entrance common to all. (L. A.)

Areaway. . . . any sub-surface space adjacent to a building, for lighting or ventilating cellars or basements, or for other purposes. (P.) . . . a space below the surface of the ground adjacent to and outside of a building and used in connection therewith. (S.)

Attic. . . . that part of a building wholly or partly within the roof framing, unfinished, not inhabited nor used for sleeping purposes. (P.) . . . any unfinished space immediately below the roof of a building or an upper room having a height of less than eight feet. (St. L.) Attic or Attic Story, . . . any story situated wholly, or partly in the roof, so designated, arranged or built as to be used for business, storage or habitation. (P. C.)

Basement. . . . a story, or portion thereof, partly but not more than one-half below the average grade of the ground surrounding the building, as shown on the drawings upon which the permit was granted, and counted as a story in determining the height of the building. (P.) . . . shall be taken to mean a story, partly, but not more than one-half below the level of the street grade or ground nearest the building. (St. L.) . . . that portion of a building between floor and ceiling, which is partly below and partly above grade . . . , but so located that the vertical distance from grade to the floor below is less than the vertical distance from grade to ceiling. (P. C.) . . . a story partly, but not more than one-half below the level of the inside sidewalk grade of the street nearest the building. If the floor of such basement is less than two feet below such grade, or if the ceiling of such basement is more than seven feet six inches above said grade, said story shall be classed as the first story of the building in which it occurs; provided, however, that the ceiling height may be raised above the height of seven feet six inches heretofore given, not more than one-third of an inch for every foot of such distance said building is set back from the street-line of the street nearest the building, but in no case shall any rise of ceiling be allowed for any distance beyond thirty feet said building may be set back from the line of the street nearest the building, and in such cases all rises in the basement ceiling shall be computed according to the distance between the street-line and the outside wall of the building nearest to said street-line. Provided, further, that the yard, or ground level, or walks, or other improvements thereon for a distance of twelve feet at every point from all outside walls of said building shall not be lower than eight feet three inches below the floor-level of the first story of said building. (C.) . . . a lower story of which a part, but less than one-half, is below the level of the curb-line of the street or of the general level of the ground. (S. F.) . . . that part of a building not more than 40% nor less than 35% of which is below the mean grade of the curb of the principal street upon which the building abuts, or if it does not abut upon a street, then below the mean grade of the land adjoining the building. When the building abuts on two or more streets the commissioner shall determine which is the principal street. (B.) . . . that story of a building not more than five feet of the height of which at any point is below the grade of the street upon which the principal entrance to the building opens. (L. A.)

Bay. . . . that space within a building between two adjacent rows of columns, or between a row of columns and a bearing-wall. (P.) Bay or Panel, . . . one of the intervals or spaces into which a building front is divided by columns, piers or division-walls. The floor-space included between the intersection of parallel rows of columns or walls of bays at right-angles to each other and the face of a wall between two adjoining pilasters or piers is called a panel. (D.)

Bay-Window. (See *Oriel Window.*) . . . any window extending beyond the wall of a building. (P.) . . . a rectangular, curved or polygonal window supported on a foundation which projects from the balance of the enclosing wall. (D.) Oriel Window, . . . a projecting window similar to a bay-window, but carried on brackets or corbels. The term " Bay-Window " may also be applied to an oriel window, projecting over the street line. (D.) . . . a rectangular, curved or polygonal window, supported on a foundation extending beyond the main wall of the building. (P. C.)

Building. . . . any structure for the support, shelter or enclosure of persons, animals or chattels; and when separated by division walls, from the ground up, and without openings, then each portion of such building shall be deemed a separate building. (St. L.) . . . any structure built for the support, shelter, or enclosure of persons, animals or chattels, and when separated by division walls without openings, each portion so separated shall be deemed a separate building. (S.) . . . any structure built for the support, shelter and enclosure of persons, animals, chattels, or movable property of any kind; and when separated by an "Absolute Fire Separation " each portion of such building so separated shall be deemed a separate building. (P. C.) Building or Structure, . . . any construction the arrangement of which may affect the health, safety or general welfare of man or animals. (S. F.)

Building-Line. . . . a line beyond which property owners or others have no legal or vested right to extend a building or any part thereof, without provision therefor in this code, or without special permission and approval of the proper authorities; ordinarily a line of demarcation between public and private property. (P.)

Cellar. . . . a story, or portion thereof, more than one-half below the average grade of the ground surrounding the building, as shown on the drawings upon which the permit was granted, and not counted as a story in determining the height of the building. (P.) . . . shall be taken to mean a story more than one-half below the level of the street grade or ground nearest the building. A cellar shall not be included in designating the height or number of stories of building referred to in any part of this ordinance. (St. L.) . . . that portion of a building between floor and ceiling which is wholly or partly below grade . . . and so located that the vertical distance from grade to the floor below is equal to or greater than the vertical distance from grade to ceiling. (P. C.) . . . a story more than one-half below the level of the inside sidewalk grade of the street nearest the building. Where the grade of a street adjacent to a tenement-house varies, the average grade of such street opposite the lot containing the tenement-house shall be regarded as the grade of such street within the meaning of this chapter. (C.) . . . a lower story of which one-half or more is below the level of the curb-line of the street, or streets, on which it faces, or of the general level of the ground. (S. F.) . . . when there is a basement, that part or parts of a building below the basement. When there is no basement, that part of a building more than 40% of which is below the mean grade of the curb of the principal street upon which the building abuts, or if it does not abut upon a street not more than 40% below the mean grade of the land adjoining the building. When the building abuts on two or more streets, the commissioner shall determine which is the principal street. (B.)

Club. . . . a building used for the mutual entertainment, recreation and lodging of the members only of an organized club or society and their guests. (S.)

Court. . . . an open unoccupied space other than a yard on the same lot with a building. (P.) . . . an open, unoccupied space on the same lot with a tenement-house other than a yard. (St. L.) Exterior, . . . a court which is bounded on one or more sides by a street or alley. Interior, . . . any court other than an exterior court, as herein defined. (S.) . . . an open, unoccupied space, bounded on two or more sides by the walls of the building. An inner court is a court entirely within the exterior walls of a building. All other courts are outer courts. (P. C.) . . . an open, unoccupied, unobstructed space, other than a yard, on the same lot with a tenement-house; a court entirely surrounded by a tenement-house is an inner court; a court bounded on one side and both ends by a tenement-house, and on the remaining side by a lot line, is a lot-line court; a court extending to a street, alley or yard is an outer court. (C.) . . . an open, unoccupied space other than a yard on the same lot as the building. A court extending to the yard or street is an outer court. A court extending to the lot line is a lot-line court. (S. F.)

Dwelling. . . . any house or building not a lodging-house, tenement, rooming-house or inn; all or any part of which is occupied as a home or residence of a family, or of two families, living independent of each other. (P.) . . . a residence. (S.) . . . a building which shall be intended or designed for or used as the home or residence of not more than two separate and distinct families or households, and in which not more than fifteen rooms shall be used for the accommodation of boarders, and no part of which structure is used as a store or for any business purpose. Two or more such dwellings may be connected on each story and used for boarding purposes, provided the halls and stairs of each house shall be left unaltered and kept open and in use as such. (S. F.) . . . a building intended for the residence of not over two families. (L. A.) . . . Dwelling-Place, . . . a building, or part thereof, which is occupied by one or more families, who live therein and cook their own food upon said premises. (S)

Factory. The term shall include every building in which goods, wares and merchandise are manufactured. (D.) . . . a building used for manufacturing articles by machinery. (S.) . . . a building, the whole or greater portion of which is used for manufacturing purposes. (L. A.)

First Story. (See *Story.*) . . . the story, the floor of which is at or first above the level of the sidewalk, or adjoining the ground. The other stories are to be numbered in regular succession, counting upward. (D.) . . . the first story more than 65% of the height of which is above the mean grade of the curb of the principal street upon which the building abuts or if it does not abut upon a street, the first story of a building, more than 65% of the height of which is above the mean grade of the land adjoining the building. Where there is a basement, that story next above the basement shall be the first story of a building. Where there is a cellar and no basement that story next above the cellar shall be the first story of a building. When the building abuts on two or more streets the commissioner shall determine which is the principal street. (B.)

Flats. (See *Apartment.*) . . . a building of two or more stories containing separate self-contained dwellings, each dwelling having an independent entrance on the level of the street or from an outside vestibule on the level of the first floor. (S. F.) . . . a building of two or more stories containing independent dwellings, each having its own street entrance. (L. A.)

Footing. . . . the spreading course at the base or bottom of a foundation

wall, pier or column. (P.) . . . the projecting course or courses at the bottom of a foundation wall or pier. (D.)

Foundation. (*a*) . . . all that portion of a building or a structure below the top of footings, or basement or cellar floor. (*b*) . . . the earth upon which the structure rests. (D.) . . . shall be taken to mean that portion of a building below ground and in contact with the earth. (St. L.)

Front. . . . that face thereof which contains the principal entrance to said building. (L. A.)

Garage. A public garage shall mean any place where space is rented for the storage of more than two automobiles or motor-driven vehicles. A private garage shall mean any place other than a public garage where motor-driven vehicles are stored. (P.) . . . a building, or portion thereof, in which a motor vehicle is kept for storage, repair or for any other purpose, except that it shall not be considered to include a building, or portion thereof, in which the only motor vehicles kept are in transit or are kept temporarily during the daylight hours only for changing of tires or batteries, or are unused motor vehicles temporarily in storage and containing no inflammable fuel. (S.) . . . a building or portion thereof in which a motor vehicle containing gasoline, distillate or other volatile, inflammable liquid in its tank, is stored, repaired, used or kept. (P. C.)

Grade. . . . the surface of the ground, court, lawn or sidewalks adjoining a building. The "established grade" is the level of the street curb as fixed by the municipality. (P.) The established sidewalk level at the building line of any street, or if the building be not built on the building line of a street, then the exposed surface of the earth adjoining any wall shall be taken to be the grade for that wall. (St. L.) Established Grade, . . . the sidewalk or alley elevation along the property-line as established by public improvements. (S.) Natural Grade, . . . the undisturbed natural surface of the ground. (S.) (1) . . . for buildings adjoining one street only, the elevation of the sidewalk at the center of that wall adjoining the street. (2) . . . for buildings adjoining more than one street, the average of the elevations of the sidewalk at centers of all walls adjoining streets. (3) . . . for buildings having no wall adjoining the street, the average level of the ground (finished surface) adjacent to the exterior walls of the building. All walls approximately parallel to and not more than five feet from a street-line are to be considered as adjoining a street. (P. C.)

Ground Floor. (See *First Story.*) . . . the floor at or near the level of the grade when used for public purposes. (D.)

Height of Building. (*a*) . . . the vertical distance of the highest point of the roof, in the case of flat roofs; and for high-pitched roofs, the average of the height of the gable above the mean grade of the curbs of all the streets, or the mean grade of the natural ground adjoining the building if the said grade or ground is not below the grade of the curb. (*b*) . . . the number of stories in a building, including the basement, but not including the cellar. (P.)

Hospital. . . . any building for housing sick persons, including maternity hospitals, sanitariums, etc. (S.) Hospital or Sanitarium, . . . a building used for the keeping and care of sick, invalid and infirm people, and having accommodation for more than five such people. (S. F.) Hospital, Sanatorium, Sanitarium or Asylum, . . . a building in which sick, demented, injured, infirm, aged or orphaned persons are housed or intended to be housed (L. A.)

Hotel. . . . a building in which is conducted the business of lodging the public and which is occupied by more than six guests or lodgers. (S.) . . . a building or part thereof intended, designed or used for supplying food and shelter to residents or guests and having a general public dining-room or café, or both, and containing more than fifteen guests' rooms. (S. F.)

Lodging-House. . . . a building containing more than fifteen rooms in which persons are or may be accommodated with sleeping apartments for hire, by the day, week or month. (S. F.)

Lot. . . . a subdivision of a block, as shown by a recorded plot of an addition to, or subdivision of, the City, or any portion of land, whether plotted or unplotted, considered as a unit of property and described by metes and bounds; if one or more lots are built upon as a unit of property, they shall, for the purpose of this ordinance, be considered as a single lot. (S.)

Masonry. . . . brick, stone, tile or terra-cotta laid in mortar or concrete. (S.) . . . shall apply to brick, stone, interlocking hollow tiles, concrete or reinforced-concrete construction. (S. F.) . . . includes such parts of a structure as are constructed with stone, bricks of burnt clay, cement, or a sand lime, hollow blocks of burnt clay or concrete, and stone or cinder concrete, both plain and reinforced work. (B.) . . . means brick, stone or concrete. (L. A)

Motion-Picture Theater. (See *Theater.*) . . . any public hall or room in which motion pictures are displayed, in which the seating capacity does not exceed six hundred persons and in which there is no stage or scenery. (N. Y.)

Office-Building. . . . a building divided into rooms, intended and used for office purposes, and no part of which shall be used for living purposes, except by the janitor and his family. (S. F.)

Owner. . . . any person having title to, or control as guardian or trustee of, a building or property. (S.) . . . includes his duly authorized agent or attorney, a purchaser, devisee, and any person entitled to an interest in the property in question. (N. Y.)

Oriel Window. (See *Bay-Window.*) . . . a window that projects from the main line of an enclosing wall of a building and is carried on brackets or corbels. (P. C.)

Panel. (See *Bay.*)

Pent-House. . . . any structure erected on the roof of a building for the purpose of enclosing stairways to the roof, elevator machinery, water tanks, ventilating apparatus, exhaust chambers or other building equipment machinery. (P.) . . . a room constructed above the roof of a building and used exclusively to give head room above a stair, to house elevator machinery, or provide working space above the elevator sheaves. (S.)

Public Hall. . . . a corridor or passageway used in common by all of the occupants of a building. (S.) . . . a hall, corridor or passageway not within an apartment. (C.) . . . a room for public assemblages, not including a theater, having a total seating capacity of one hundred or more persons. (L. A.)

Repairs. . . . the reconstruction or renewal of any existing part of a building, or of its fixtures or appurtenances, by which the strength of the fire risk is not affected or modified. (S. F.) . . . the reconstruction or renewal of any existing part of a building, or of its fixtures or appurtenances. (L A.)

Shaft. . . . includes exterior and interior shafts, whether for air, light, elevator, dumb-waiter or any other purpose; a "vent-shaft" is one used solely to ventilate or light a water-closet compartment, bath-room or pantry. (C.) . . . in a building is any open space other than a court, extending through the building for two or more stories, exterior or interior, whether for light, air, elevator, dumb-waiter or any other purposes. A vent-shaft is one used solely to ventilate or light, or both, a water-closet compartment or bath-room. (S. F.)

Stair-Hall. . . . the stairs, stair landings and those portions of the public halls through which it is necessary to pass in getting from the entrance floor to the top story. (C.) . . . the stairs, stair landings, hallways or passages through which it is customary to pass in going from the entrance to the roof. (S. F.)

Stores. The term "stores" shall include every building used for the sale of goods, wares and merchandise. (D.) A "store building" is a building used wholly or in part for the purposes of exhibiting for sale goods, wares or merchandise. (L. A.)

Story. (*a*) . . . that portion of any building included between the upper surface of any floor and the upper surface of the floor next above. (*b*) A "half-story" is that part of any building wholly or partly within the roof framing; finished and inhabited. (P.) . . . that portion of a building included between the surface of any floor and the surface of the next floor above it, or if there be no floor above it, then the space between such floor and the ceiling next above it. (St. L.) . . . that portion of a building included between the upper surface of the floor next above, except that the topmost story shall be that portion of a building included between the upper surface of the topmost floor and the ceiling or roof above. A basement or cellar shall not be considered a story unless the ceiling thereof is more than five feet above grade. (P. C) . . . that part of a building between the top of any floor level and the top of the floor or roof level next above. (B.) . . . that part of any building comprised between any floor and the floor or roof next above. (N. Y.) . . . that portion of a building between the top of any floor-beams and the top of the floor or ceiling-beams next above. (C.) A "story and a half building" shall be taken to mean a building that is more than one story in height and less than two stories in height, wherein any portion of the space above the first-story ceiling is used or intended to be used or occupied for storage, living or sleeping purposes. (L. A.)

Tenement-House. (See *Apartment-House.*)

Theater. . . . a building which contains seats for the public, and to which an admission fee is charged, and in which movable scenery is used. (S. F.) . . . a room, hall, or auditorium having a stage either with or without scenery, used or designed to be used for the public entertainment of persons, and adapted to the presentation of plays, operas, spectacles, or similar forms of entertainment. (L. A.)

Walls. (See under various kinds of walls.)

Wall, Bearing. . . . a wall which supports any vertical load other than its own weight. (P.) . . . a wall on which joists, beams, girders and trusses of floor or roof-construction rest. (D.) . . . a wall designed to carry, in addition to its own weight, some portion of the roof or floors of a building. (S.) . . . a wall which supports any load other than its own weight. (P. C.) . . . any wall carrying all or part of the interior load of a building. (S. F.) . . . a wall carrying a portion of the interior load of a building. (L. A.)

Wall, Curtain. . . . a non-bearing wall between columns and/or piers and which is not supported by girders or beams. (P.) . . . the non-bearing and enclosing wall of an iron or steel, or reinforced-concrete skeleton frame, or the non-bearing portion of an enclosing wall between piers. (D.) . . . an enclosing wall built and supported between columns or piers, and on girders or other support, and sustaining no weight but its own. (St. L.) . . . a non-bearing wall built between columns and extending through two or more stories without intermediate support. (S.) . . . a non-bearing wall between columns or piers which is not supported by girders or beams. (P. C.) . . . any wall supported at intervals on the frame of a building, or a wall which is self-supporting only on the exterior of a building. (S. F.)

Wall, Division. . . . a bearing wall running between two exterior walls subdividing building into several parts (D.) . . . a masonry wall entirely dividing or separating one building from another. (St. L.) . . . an interior wall dividing a building or extending from the ground to and through the roof. (S.) . . . any wall other than an exterior wall, or a party wall, which extends the full height of a building and through the roof, and such walls shall be constructed in all respects as provided for party walls. Such walls may be bearing walls or self-supporting only. (S. F.) . . . any wall, other than an exterior wall or a party wall which extends the full height of the building and through the roof. (L. A.)

Wall, Exterior. . . . any outside wall which serves as a vertical enclosure of a building, including a party wall when not used in common. (P.) . . . every outer wall or vertical enclosure of a building. (S. F.) . . . every outer wall or vertical enclosure of a building other than a party wall. (L. A.)

Wall, Fire. . . . a wall of solid masonry or reinforced concrete which subdivides a building to restrict the spread of fire and which starts at the foundations and extends continuously through all stories to and above the roof. (P.) . . . a division-wall which extends through and at least eighteen inches above the roof and in which all openings are protected by fire-doors. Also any division or partition-wall dividing spaces into limited areas for fire-protection purposes. (D.) . . . is a wall of masonry or reinforced concrete which subdivides a building to prevent the spread of fire by starting at the foundation and extending continuously through all stories to and above the roof. (P C.) . . . shall apply to all walls built for the purpose of fire-resistance. The term also applies to that portion of walls above roof-surface. (S. F.) . . . is that part of a masonry or reinforced concrete wall extending above the roof, immeately adjoining such wall. (L. A.)

Wall, Fire Division. . . . a wall of solid masonry or reinforced concrete which subdivides a building to restrict the spread of fire, but is not necessarily continuous through all stories nor extended through the roof. (P.)

Wall, Foundation. . . . that portion of an enclosing wall below the first tier of floor-joists or beams nearest to and above the grade-line, and that portion of any interior wall or pier below the basement or cellar-floor. (D.) . . . that part of a wall below the level of the street curb, or, if a wall below the level of the highest ground next to the wall, or, in the discretion of the commissioner, that part or partition-wall below the cellar floor. (B.)

Wall, Non-Bearing. (See *Wall, Curtain.*) . . . a wall designed to carry only its own weight. (S.)

Wall, Partition. . . . an interior non-bearing wall wholly supported at each story. (P.) . . . any wall running at any angle to the bearing-walls

and subdividing the interior of a building into compartments. (D.) . . . any interior wall of masonry. (St. L.) . . . any interior wall other than a division-wall. (S. F.) . . . an interior wall of masonry in a building. (B.) . . . any interior wall in a building other than a division-wall. (L. A.) . . . an interior subdivision-wall. If constructed of other material than masonry, the name or class of the structural material is usually prefixed. (D.)

Wall, Party. . . . a wall which, or portion of which, encroaches upon ground of an adjoining owner and is used jointly or adapted to joint use. (P.) . . . a wall that separates two or more buildings, and used or to be used jointly by separate buildings. (D.) . . . a masonry wall used or built to be used for the common separation or support of adjoining buildings of separate owners. (St. L.) . . . a wall used, or designed to be used, in common by two buildings. (S.) . . . a wall used or adapted for joint service between two buildings. (P. C.) . . . a wall used, or built to be used, in common by two or more buildings. (S. F.) . . . a wall that separates two or more buildings, and is used or adapted for the use of more than one building. (B.) . . . a wall used or erected to be used, in common as a structural wall by two or more adjoining buildings. (L. A.)

Wall, Retaining. . . . a wall used to resist the lateral thrust of any material. (P.) . . . a sub-surface wall built to resist lateral pressure of the adjoining earth and to prevent its caving in. Also any enclosing wall built to resist the lateral pressure of internal loads. (D.) . . . a wall subjected to lateral pressure. (S.) . . . shall apply to all walls constructed for the purpose of holding back or supporting earth. (S. F.)

Wall, Spandrel. . . . an exterior non-bearing wall in skeleton construction built between columns or piers and wholly supported at each story. (P.)

Wall, Thickness. . . . minimum thickness as given in this ordinance and measured on the bed. (D.) . . . the minimum thickness . . . measured between any two floors, or between floor and ceiling or roof. (S. F.) . . . the minimum thickness of such wall. (B.) . . . means the minimum thickness of a wall between the floors and the ceiling or roof of a building. (L. A.)

Warehouse. . . . a building or portion thereof, used principally for the storage of merchandise. (S.) . . . a building used exclusively for the storage of merchandise. (S. F.) . . . a building used for the storage of goods, wares or merchandise. (L. A.) . . . every building used for storage of goods, wares and merchandise. (D.)

Yard. . . . an open, unoccupied space on the same lot with a tenement, rooming-house or dwelling, and which space extends in its full width or depth between opposite lot-lines. (P.) . . . an open, unoccupied space on the same lot with a tenement-house for the full width of the lot. (St. L.) . . . an open, unoccupied space, other than a court, unobstructed from the ground to the sky, except where specifically provided by this code, and on the lot on which a building is situated. (P. C.) . . . an open, unoccupied space on the same lot with a tenement-house, separating every part of every building on the lot from the rear line of the lot. (C.) . . . an open, unoccupied space on the same lot as the house, between the extreme rear line of the house and the rear line of the lot. (S. F.)

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